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Population Aging and Economic Growth: the effect of health expenditure*

Atsue Mizushima†

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Abstract

Rising longevity has led to population aging in developed countries, causing increasing concerns about its economic impact. Specially, the trend of population aging increases health expenditure in developed countries, and 70% to 80% of health expenditure is funded by public sector. Therefore, this paper focuses on the health demand in an aging economy and examines how the aging of the population and public health funding (PHF) affects agents' behavior. For this purpose, we construct a simple growth model and examine the effect of aging and PHF on saving and the growth rate. We show that an increase in life expectancy increases the growth rate in the economy without PHF, but that it has an inverted U-shaped relation in the economy with PHF. From the welfare standpoint, we show that it increases the intergenerational conflict between current and future generation and that PHF has the result of alleviating the conflict.

Key words: life expectancy, household production, economic growth, social welfare

Classification Numbers: I10; J14; O41

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1 Introduction

Global population aging is one of the most important demographic dynamics affecting individuals and societies throughout the world. Population projections show that in more developed regions, the population aged 65 or over is expected to nearly double (from 245 million in 2005 to 406 million in 2050), whereas the number of persons aged 0-24 is likely to decline (from 372 million in 2005 to 131 million in 2050). People aged 65 or over in more developed countries represent 15.3% (2005) and are estimated at 26.1% (2050) of the entire population; and people aged 0-24 represent 30.7% (2005) and are also estimated at 25.8% (2050) (See, United Nations (2007)). Population aging has many influences on the economy (for example, pay-as-you-go social security and the retirement age), causing increasing concerns about its economic impacts. In the paper, we focus our attention on the health expenditure in an aging economy and examine the effect of population aging on saving, the growth rate, and public policy. Because health status tends to deteriorate with age, though, the trend of population aging increases health expenditure and it counts for about 9% of GDP in the OECD countries; and because approximately 70 to 80 % of health expenditure is funded by public sectors except in Greece, Mexico, and the United States (See, OECD (2007)), the health expenditure represents the important factor in an aging economy.

To examine these issues, the paper extends a two-period overlapping generations model by introducing uncertain longevity (Pecchenino and Pollard (1997)) and household health production (Grossman (1972) and Jacobson (2000)). In the model, we introduce the family as the producer of health; this provides the essential insight that the family's joint resources are determinate for health investments (expenditures) in family members.

Previous research has examined the potential for uncertain longevity to explain capital accumulation or the economic growth. Earlier studies rely on Yaari (1965) and Blanchard (1985), and they show that an increased rate of life longevity increases capital stock. In some more recent work, de la Croix and Licandro (1999) examine the economy where the effect of a reduction in the mortality rate has consequences on affects the duration of education, and they show that the growth rate becomes higher for a high mortality rate, but lower for a low mortality rate. Using the uncertain lifetime horizon and endogenous retirement age model, Boucekkine, de la Croix, and

Licandro (2002) derive the same result. **?** also obtain the same results, not through own education time but through expenditure on children's education time. Indeed, the results we present in the paper show that greater longevity has different effects on the economy depending on whether this is with or without public funded health expenditure; and the paper also gains this result by a different channel.

The increasing health costs associated with an aging population have, in fact, received much attention from other scholars. Using American data, Palumbo (1999) shows that uncertain out-of pocket medical expense in old age is the motivation for much precautionary saving . Correspondingly, using Taiwanese data, **?** show that health insurance, which reduces uncertainty in future medical expenses, can reduce households' precautionary saving. Our results in this paper confirm their conclusions and show that the demand for health in old age provides another important motive for saving decisions and economic growth.

My analysis builds primarily on previous research by Nakanishi and Nakayama (1993) and Tabata (2005). Nakanishi and Nakayama (1993) give a microfoundation to the work in Grossman (1972) by applying a cost minimization analysis to determine the demand for health. In his theoretical paper, Tabata (2005) constructs a growth model which incorporates health care in an aging economy. The former analysis does not consider how the health demand affects individuals' saving decision and economic growth; the latter analysis does not consider cost minimization in the role of the determinants of aggregate health expenditure. When the family is the producer of health, they have an incentive to minimize the cost, thus the cost minimization approach will achieve satisfactory outcomes. My methodology, therefore, naturally allows an alternative growth model by using the channel of the demand for health.

From a policy standpoint, it is important to understand the macroeconomic impact of the demand for health of old agents. If agents save for old-age health expenditure for precautionary reasons, then changes in government policy influence these agents' saving decisions. Therefore, in this paper, we also construct a model of publicly funded health expenditure where the government reimburses a part of the direct health care of agents; we then examine the macroeconomic effect of publicly funded health expenditure on economic growth and welfare.

The remainder of this paper is organized as follows. We set up the model in section 2. Section 3 analyzes the equilibrium of the economy without PHF. Section 4 analyzes the economy with PHF. Section 5 examines the welfare of PHF. Section 6 contains some concluding remarks.

2 The Model

Consider an infinite-horizon economy composed of agents and perfectly competitive firms. A new generation, referred to as generation t, is born in each period $t = 1, 2, 3, \cdots$. Generation $t \geq 1$ is composed of a continuum of $N_t > 0$ units of agents who live maximum for two periods, young and old age. The net rate of population growth is constant $n > 0 : N_t = (1+n)N_{t-1}$.

Firms

Firms are considered as perfectly competitive profit maximizers that produce output using a Romer (1986) type production function $Y_t = A(K_t)^{\alpha}(\bar{k}_t L_t)^{1-\alpha}$, where Y_t is the aggregate output, A is the parameter representing the technology level, K_t is the aggregate productive capital, L_t is the aggregate labor, and $\bar{k}_t$ is the average level of capital per worker in the economy. The production function can be rewritten in an intensive form as $y_t = (k_t)^{\alpha}(l_t\bar{k}_t)^{1-\alpha}$, where $k_t \equiv K_t/N_t$ is a per capita capital stock and $l_t \equiv K_t/N_t$ is a per capita labor supply in period t. We assume that capital depreciates completely in the process of production. Since firms are price takers, they take the wage, w_t and real rental rate, $1 + r_t$ as given and hire labor and capital up to the point where their marginal products equal to their factor prices in period t. Noting $k_t = \bar{k}_t$ and $l_t = 1$ in equilibrium, the wage and the real rental rate are given as follows:

$$w_t = (1-\alpha)Ak_t, \quad 1 + r_t = R_t = \alpha A. \tag{1}$$

Agents

The model of individual behavior is based on that developed by Pecchenino and Pollard (1997). The probability that an agent survives through the period of old age is $p \in (0, 1)$. The probability that an individual dies at the beginning of the period of old age, after having had a child is $1 - p$.

In the model, we introduce the family as the producer of health, thus the each agent's health status in old age determined by the health expenditure of their own health expenditure (generation t) and that of his or her children (generation $t + 1$). Using the Grossman (1972) and Jacobson

(2000) model, we assume the household produced-health function as follows:

$$h_{t+1} = \delta I_{t+1}^t + (O_{t+1}^t)^\gamma (Q_{t+1})^{1-\gamma} \quad \delta > 1, \quad \gamma \in (0,1), \tag{2}$$

where I_{t+1}^t is the indirect health expenditure; that is, exercise, food, and preventive medicine; O_{t+1}^t is the direct health expenditure; that is, hospital, medicine, and nursing care; and $Q_{t+1} = (1+n)q_{t+1}^{t+1}$ is health expenditure of generation $t+1$. The output from the above household-produced health technology equalized with the health status of old generation.

In young age, each agent is endowed with one unit of labor, which supplies inelastically to firm, and obtains wage income. A fraction p of young agents are of type a, whose parents are survive. Type d agents, whose parents die constitutes a fraction $1-p$ of young agents. Type d agents in generation t consume a part of their income, $c_{d,t}^t$ and save the remainder, $s_{d,t}^t$ for consumption in old age. Type a agents differ from type d agents in that they are a producer of household health production and derive utility from contributing household health production, q_t^t. They also consume, $c_{a,t}^t$ and save the remainder, $s_{a,t}^t$. In what follows, we refer the type of young agents as index $i = a, d$. The budget constraint for a young agent in generation t is given as follows:

$$w_t = \phi(c_{a,t}^t + q_t^t + s_{a,t}^t) + (1-\phi)(c_{d,t}^t + s_{d,t}^t), \tag{3}$$

where ϕ is an index indicating a agent's type and take $\phi = 1$ or 0. ϕ, which is realized at the beginning of date t immediately after agents of generation t are born, is distributed independently and identically across agents and time with the probability distribution: $\phi = 1$ with probability $p \in (0,1)$, $\phi = 0$ with probability $1-p$.

In old age, type a agents, whose is survive is also constitute a fraction $p \in (0,1)$ of old agents. If an agent dies, his or her annuitized wealth is transferred to the agents who live throughout old age (see, Yaari (1965) and Blanchard (1985)). As the capital depreciate 100% in one period, agents take R_{t+1}/p units of returns.

When we assume that old agents does not leave bequests to his or her children, old agents whose young period's status is $i = a, d$ have following budget constraint:

$$\frac{R_{t+1}}{p} s_{i,t} = c_{t+1}^t + I_{t+1}^t + O_{t+1}^t, \quad i = a, d. \tag{4}$$

We assume that each agent in generation t has the expected utility function of the form:

$$Eu_{i,t} = \ln c_{i,t}^{\ t} + \phi\beta \ln q_t^t + EV(c_{t+1}^t, h_{t+1}; p), \quad i = a, d, \tag{5}$$

where $\beta \in (0, 1)$ shows the altruism towards their parents; $EV(c_{t+1}^t, h_{t+1}; p)$ is the expected value in old age. We assume that the expected value $EV(c_{t+1}^t, h_{t+1}; p)$ takes the following log-linear form:

$$EV(c_{t+1}^t, h_{t+1}; p) = p[\ln \bar{\sigma} + \sigma \ln c_{t+1}^t + (1 - \sigma) \ln h_{t+1}] + (1 + p)0, \tag{6}$$

where $\sigma \in (0, 1)$ is a weight attached to the utility from his or her consumption and health stasus; and $\bar{\sigma} \equiv 1/\sigma^\sigma(1 - \sigma)^{1-\sigma}\delta^{1-\sigma}$. Each agent of generation t maximizes his or her utility (5) subjects (2), (3), (4), and (6). The timing is decided as follows:

1. Each agent maximizes his or her expected utility (5) subject to budget constraints (3) taking w_t and R_{t+1} as given.

2. If an agent survives in their old age, he or she maximizes his or her old period's value (6) subject to budget constraints (4).

3. An agent decides his or her household's health status by minimizing their cost $I_{t+1}^t + O_{t+1}^t$ subject to home production function (2), taking the health expenditure of his or her children Q_{t+1} as given.

3 Equilibrium

As a benchmark case, we first describe an economy in which there is no government. In Section 4, we introduce the public sector which funds the health expenditure of old generation. In order to derive the equilibrium in benchmark case, we solve each agent's problem by following the three timing which we showed in Section 2 by backward.

At first, let us derive the indirect and direct health expenditure in old age. An agent produces his or her health status by minimizing the cost, that is, minimizing $I_{t+1}^t + O_{t+1}^t$ subject to (2). The demand for each health expenditure is decided as $O_{t+1}^t = (\frac{\gamma}{\delta})^{\frac{1}{1-\gamma}} Q_{t+1}$ and $I_{t+1}^t = \frac{1}{\delta}[h_{t+1} - (\frac{\gamma}{\delta})^{\frac{\gamma}{1-\gamma}}]Q_{t+1}$, thus, we have the aggregate demand for health as follows:

$$I_{t+1}^t + O_{t+1}^t = \frac{1}{\delta} h_{t+1} - Q_{t+1} \Big(\frac{\gamma}{\delta}\Big)^{\frac{1}{1-\gamma}} \frac{1-\gamma}{\gamma}. \tag{7}$$

The second term in the right-hand side of (7) shows the value of aggregate health expenditures of generation $t+1$. An decrease (increase) in the health expenditures of generation $t+1$ increases (decrease) the demand for health.

Next, we examine the utility maximizing problem in old age. An agent who survives in his or her old age, decides his or her old period's consumption and health status by maximizing the expected value (6) subjects to (4) and (7). The first order condition for this problem yields the solution of the consumption and the health expenditure to household health production as follows:

$$
\begin{aligned}
c_{t+1}^{t} &= \sigma\Big[\frac{R_{t+1}}{p}s_{i,t}^{t}+Q_{t+1}\Big(\frac{\gamma}{\delta}\Big)^{\frac{1}{1-\gamma}}\frac{1-\gamma}{\gamma}\Big], \qquad i=a,d, &(8)\\
h_{t+1} &= (1-\sigma)\delta\Big[\frac{R_{t+1}}{p}s_{i,t}^{t}+Q_{t+1}\Big(\frac{\gamma}{\delta}\Big)^{\frac{1}{1-\gamma}}\frac{1-\gamma}{\gamma}\Big], \quad i=a,d. &(9)
\end{aligned}
$$

Since the first term in bracket on the right-hand side of (8) and (9) shows the return from saving and the second term shows the value of aggregate health expenditure of generation $t+1$, the inside of the bracket shows the old periods income of generation t. Therefore, in old age, the expected income is allocated to the consumption and health status according to the weighted parameter.

Substituting (8) and (9) into (6), we have the value in his or her old age whose young period's status is $i=a,d$ as follows:

$$
EV(c_{t+1}^{t},h_{t+1};p)=p\Big[\frac{R_{t+1}}{p}s_{i,t}^{t}+Q_{t+1}\Big(\frac{\gamma}{\delta}\Big)^{\frac{1}{1-\gamma}}\frac{1-\gamma}{\gamma}\Big], \quad i=a,d. \tag{10}
$$

Finally, we derive the saving function, which is, maximizing (5) subjects to (3) and (10). The first order condition for this problem yields the solution of the young period's consumption, health expenditure, and saving as follows:

$$
\begin{aligned}
c_{a,t}^{t} &= \frac{1}{1+p+\beta}\Big[w_t+p\frac{Q_{t+1}}{R_{t+1}}\Big(\frac{\gamma}{\delta}\Big)^{\frac{1}{1-\gamma}}\frac{1-\gamma}{\gamma}\Big],\\
q_t^{t} &= \frac{\beta}{1+p+\beta}\Big[w_t+p\frac{Q_{t+1}}{R_{t+1}}\Big(\frac{\gamma}{\delta}\Big)^{\frac{1}{1-\gamma}}\frac{1-\gamma}{\gamma}\Big],\\
s_{a,t}^{t} &= \frac{p}{1+p+\beta}\Big[w_t-(1+\beta)\frac{Q_{t+1}}{R_{t+1}}\Big(\frac{\gamma}{\delta}\Big)^{\frac{1}{1-\gamma}}\frac{1-\gamma}{\gamma}\Big],\\
c_{d,t}^{t} &= \frac{1}{1+p}\Big[w_t+p\frac{Q_{t+1}}{R_{t+1}}\Big(\frac{\gamma}{\delta}\Big)^{\frac{1}{1-\gamma}}\frac{1-\gamma}{\gamma}\Big],\\
s_{d,t}^{t} &= \frac{p}{1+p}\Big[w_t-\frac{Q_{t+1}}{R_{t+1}}\Big(\frac{\gamma}{\delta}\Big)^{\frac{1}{1-\gamma}}\frac{1-\gamma}{\gamma}\Big].
\end{aligned} \tag{11}
$$

Since aggregate saving is the weighted sum of each agent[1], we have:

$$s_t = \frac{p}{(1+p+\beta)(1+p)}\Big[(1+p+\beta-p\beta)w_t - (1+p+\beta+p^2\beta)\frac{Q_{t+1}}{R_{t+1}}\Big(\frac{\gamma}{\delta}\Big)^{\frac{1}{1-\gamma}}\frac{1-\gamma}{\gamma}\Big]. \tag{12}$$

The market clearing condition of capital is $K_{t+1} = s_t N_t$, which expresses the equality of the total savings by young agents in generation $t, s_t N_t$, to the stock of aggregate physical capital in period $t+1, K_{t+1}$. Dividing both sides by N_t leads the following:

$$(1+n)k_{t+1} = s_t. \tag{13}$$

In period 1, there are young agents in generation 1 and the initial old agents in generation 0. The initial old agents of generation 0 is endowed with k_1 units of capital. Each initial old agents rents his or her capital to the insurance firms and earns an income R_1/pk_1, which is then spent for consumption and health expenditure. The measure of initial old agents is $pN_0 > 0$. The utility of an agent in generation 0 is $p(\ln c_1^0 + \ln h_1)$.

Definition 1 *An economic equilibrium is a sequence of allocations and prices which satisfy the following conditions at each date.*

- *Agents and firms optimize, taking the wage rate and the rate of interest as given; that is, (1) and (12) hold.*
- *Markets for goods, capital, and labor clear; that is, (13) and $l_t = 1$ is hold.*
- *The health expenditure q_{t+1}^{t+1} of generation $t+1$, which is taken as given by each agent of generation $t \geq 1$ in his or her maximization problem is realized.*

In equilibrium, we guess the aggregate health expenditure of generation $t+1$ from (12):

$$\frac{Q_{t+1}}{R_{t+1}}\Big(\frac{\gamma}{\delta}\Big)^{\frac{1}{1-\gamma}}\frac{1-\gamma}{\gamma} = \frac{p(1+p+\beta-p\beta)w_t - (1+p+\beta)(1+p)s_t}{p(1+p+\beta+p^2\beta)}. \tag{14}$$

To express the equilibrium in this economy in a compact manner, we note, first, that the third condition in definition together with (11) and (14) imply:

$$Q_{t+1} = (1+n)\frac{\beta(1+p)}{1+p+\beta+p^2\beta}(w_{t+1} - s_{t+1}). \tag{15}$$

[1] Aggregate saving in period t is derived as $s_t = ps_{a,t}^t + (1-p)s_{d,t}^t$.

Substituting (15) into (12), we have the equilibrium saving function as follows:

$$s_t = \frac{p}{(1+p+\beta)(1+p)}\Big[(1+p+\beta-p\beta)w_t - \frac{\beta(1+p)(1+n)}{R_{t+1}}\Big(\frac{\gamma}{\delta}\Big)^{\frac{1}{1-\gamma}}\frac{1-\gamma}{\gamma}(w_{t+1}-s_{t+1})\Big]. \quad (16)$$

Substituting (1) and (13) into (16) to obtain:

$$\begin{aligned}
&\frac{(1+n)^2}{A}\frac{\beta(1+p)}{\alpha}\Big(\frac{\gamma}{\delta}\Big)^{\frac{1}{1-\gamma}}\frac{1-\gamma}{\gamma}k_{t+2} \\
&-(1+n)\Big[\frac{\beta(1+p)(1-\alpha)}{\alpha}\Big(\frac{\gamma}{\delta}\Big)^{\frac{1}{1-\gamma}}\frac{1-\gamma}{\gamma}+\frac{(1+p+\beta)(1+p)}{p}\Big]k_{t+1} \\
&+(1+p+\beta-p\beta)(1-\alpha)Ak_t = 0. \qquad (17)
\end{aligned}$$

The general solution to the second-order linear difference equation (17) is given by :

$$\begin{aligned}
k_t &= Z_1 g_1^t + Z_2 g_2^t, && (18)\\
b_1 &= A\Big(\frac{\Delta-\sqrt{\Theta}}{(1+n)2\Omega}\Big) > 0, && (19)\\
b_2 &= A\Big(\frac{\Delta+\sqrt{\Theta}}{(1+n)2\Omega}\Big) > 0,
\end{aligned}$$

where Z_1 and Z_2 is arbitrary constants. In addition, $\Delta \equiv \frac{\beta(1+p)(1-\alpha)}{\alpha}(\frac{\gamma}{\delta})^{\frac{1}{1-\gamma}}\frac{1-\gamma}{\gamma}+\frac{(1+p+\beta)(1+p)}{p}$, $\Theta \equiv [\frac{\beta(1+p)(1-\alpha)}{\alpha}(\frac{\gamma}{\delta})^{\frac{1}{1-\gamma}}\frac{1-\gamma}{\gamma}]^2 + 2\frac{\beta(1+p)(1-\alpha)}{\alpha}\frac{(1-p)(1+p+\beta)+2p^2\beta}{p}(\frac{\gamma}{\delta})^{\frac{1}{1-\gamma}}\frac{1-\gamma}{\gamma}+[\frac{(1+p+\beta)(1+p)}{p}]^2$, and $\Omega \equiv \frac{\beta(1+p)}{\alpha}(\frac{\gamma}{\delta})^{\frac{1}{1-\gamma}}\frac{1-\gamma}{\gamma}$.

Lemma 1 *The equilibrium condition must hold*

$$g_{t+1} \equiv \frac{k_{t+1}}{k_t} \le \frac{(1-\alpha)A}{1+n}.$$

The equilibrium path which satisfies this condition is g_1. Then the general solution is rewritten as $k_t = Z_1 g_1^t$.

Proof. See Appendix 1. ■

Proposition 1 *Suppose that*

$$A > \frac{(\Delta+\sqrt{\Theta})(1+n)}{2p(1-\alpha)},$$

then there exists a unique equilibrium such that $k_t = k_1 g_1^{t-1}$, where g_1 is given as (19) and $g_1 > 1$ for each $t \ge 1$.

Proof. See Appendix 2. ■

Proposition 1 makes it clear that when the productivity parameter A is sufficiently large, the economy grows at the positive constant rate, g_1. We can see that the output or GDP of this economy grows at the same constant rate, g_1. In addition, wage, w_t, young-period consumption, c_t, young-period transfer, q_t, and saving, s_t also grow at the same constant rate, g_1. Because our research interest is the aging economy in developed countries, the restriction of proposition 1 is satisfactory.

The growth rate depends on the rate of life expectancy, p. The following proposition formalizes the results of comparative statics result and its proof is given in Appendix 3.

Proposition 2 *Suppose that*

$$\delta > \gamma\Big(\frac{1-\alpha}{\alpha}\frac{1-\gamma}{\gamma}\Big)^{1-\gamma},$$

an increase in the life expectancy increases the growth rate.

Proof. See Appendix 3. ■

An increase in life expectancy can be interpreted as the rate of time preference, as incorporated in models such as those of Yaari (1965) and Blanchard (1985). Therefore, an increase in life expectancy lowers each agent's rate of time preference and increases saving. In addition, the logic leading to the effect of the productivity of health, δ is explained by the precautionary saving motive. When health productivity increases, agents of generation t expect the health expenditure of generation $t+1$ to decrease (see, (14)); they thus have an incentive to prepare for future expenditure, which results in an higher saving and growth rate.

4 Public Policy

In this section, we introduce a government that funds the health expenditure of old generation. We assume that a government funds the health expenditure by reimbursing a part of direct health expenditure. At each time, government levies payroll tax, τ_t on young agents (generation t), then transfers these resources to old generation (generation $t+1$) as a reimbursement of health expenditure. For analytical simplicity, we assume that government strategically decides reimbursement rate, ϵ to the direct health spending for a given rate of life expectancy, p. Thus, we have the budget constraint of government as follows:

$$\tau_t w_t = p\epsilon O_t^{t-1}. \tag{20}$$

The left-hand side of (20) represents the aggregate revenue from income tax and the right-hand side shows the aggregate spending of public funded health. The tax rate in period t is decided to satisfy the budget constraint (20), an increase in life expectancy, p and the reimbursement rate, ϵ increases the tax rate. Taking R_{t+1}, w_t, p, τ_t and ϵ as given, each agent maximizes his or her utility (5) subjects to (2) and following constraints:

$$\begin{aligned} (1-\tau_t)w_t &= \phi({c_a}_t^t + q_t^t + {s_a}_t^t) + (1-\phi)({c_d}_t^t + {s_d}_t^t), && (21) \\ \frac{R_{t+1}}{p}s_{i_t} &= c_{t+1}^t + I_{t+1}^t + (1-\epsilon)O_{t+1}^t, \quad i = a, d. && (22) \end{aligned}$$

Solving this problem using the similar method to that used in Section 3, we respectively have the demand for health expenditure as follows:

$$O_{t+1}^t = \Big(\frac{\gamma}{(1-\epsilon)\delta}\Big)^{\frac{1}{1-\gamma}} Q_{t+1}, \quad I_{t+1}^t = \frac{1}{\delta}\Big[h_{t+1} - \Big(\frac{\gamma}{(1-\epsilon)\delta}\Big)^{\frac{\gamma}{1-\gamma}} Q_{t+1}\Big]. \tag{23}$$

Public health funding (PHF) increases the demand of direct health expenditure, O_{t+1}^t. Since the health production is the joint production function of indirect health expenditure and direct health expenditure, an increase the demand for direct health expenditure leads to decrease the demand for indirect health expenditure. The aggregate demand for health is derived as follows:

$$I_{t+1}^t + (1-\epsilon)O_{t+1}^t = \frac{1}{\delta}h_{t+1} - Q_{t+1}\Big(\frac{1}{1-\epsilon}\Big)^{\frac{\gamma}{1-\gamma}}\Big(\frac{\gamma}{\delta}\Big)^{\frac{1}{1-\gamma}}\Big(\frac{1-\gamma}{\gamma}\Big). \tag{24}$$

By comparing (7) and (24), we found that PHF also decreases the aggregate demand for health of old agents. Solving the utility maximization problem as a same manner in section 3, we have the aggregate saving in period t as follows:[2]

$$s_t = \frac{p}{(1+p+\beta)(1+p)}\Big[(1+p+\beta-p\beta)(1-\tau_t)w_t - (1+p+\beta+p^2\beta)\frac{Q_{t+1}}{R_{t+1}}\Big(\frac{1}{1-\epsilon}\Big)^{\frac{\gamma}{1-\gamma}}\Big(\frac{\gamma}{\delta}\Big)^{\frac{1}{1-\gamma}}\frac{1-\gamma}{\gamma}\Big]. \tag{25}$$

In equilibrium, we guess the aggregate health expenditure of generation $t+1$ as follows:

$$Q_{t+1} = (1+n)\frac{\beta(1+p)}{1+p+\beta+p^2\beta}\Big[(1-\tau_{t+1})w_{t+1} - s_{t+1}\Big]. \tag{26}$$

Comparing (14) and (26), we found that the tax burden of PHF decreases the health expenditure of generation $t+1$.

[2] See, appendix 4 in details.

Using (25) and (26), we have the equilibrium saving function as follows:

$$s_t = \frac{p}{(1+p+\beta)(1+p)}\Big\{(1+p+\beta-p\beta)(1-\tau_t)w_t$$
$$-\frac{(1+n)\beta(1+p)}{R_{t+1}}\Big(\frac{1}{1-\epsilon}\Big)^{\frac{\gamma}{1-\gamma}}\Big(\frac{\gamma}{\delta}\Big)^{\frac{1}{1-\gamma}}\frac{1-\gamma}{\gamma}[(1-\tau_{t+1})w_{t+1}-s_{t+1}]\Big\}. \tag{27}$$

By comparing the saving function with and without PHF (See, (16) and (27)), we found that PHF has three effects on saving. The first effect is shown in the first term in the bracket on the right-hand side of (27). Income tax decreases disposable income; this effect has a negative impact on saving. We call this effect a direct tax effect. The second effect is a health cost effect. Since PHF reduces the demand for health, the incentive to prepare the expense of health in old age decreases. Therefore, a health cost effect also has a negative impact on saving. The last effect is a transfer effect. As mentioned in the first effect, as PHF decreases disposable income, the health expenditure of generation $t+1$ decreases. A decrease in the health expenditure of generation t creates the incentive to prepare to spend on health in his or her old period. Thus, this effect has a positive impact on saving. The second and third effects are shown in the second term in the bracket on the right-hand side of (27).

Since we have not considered the debt, the budget of government must be balanced each time; that is, (20) holds each time. Substituting (23) and (26) into (20), we have the equilibrium tax rate as follows:

$$\tau_t = \frac{\frac{p(1+p)X}{p(1+p\beta)+B}(w_t-s_t)}{[1+(1+n)\frac{p(1+p)X}{p(1+p\beta)+B}]w_t}, \tag{28}$$

where $X \equiv \beta(1+n)\epsilon(\frac{\gamma}{(1-\epsilon)\delta})^{\frac{1}{1-\gamma}}$, and $B \equiv 1+\beta$. Since $\partial\tau_t/\partial p > 0$,[3] an increase in life expectancy increases tax rate.

Substituting (28) and (13) into (27), we can rewrite the saving function as follows:

$$\begin{aligned} s_t &= \frac{p}{(1+p)(1+p+\beta+pX)} \\ &\Big[(1+p+\beta-p\beta)w_t - \frac{\beta(1+p)(1+n)}{R_{t+1}}\Big(\frac{1}{1-\epsilon}\Big)^{\frac{\gamma}{1-\gamma}}\Big(\frac{\gamma}{\delta}\Big)^{\frac{1}{1-\gamma}}\frac{1-\gamma}{\gamma}(w_{t+1}-s_{t+1})\Big]. \end{aligned} \tag{29}$$

[3] sign($\partial\tau_t/\partial p$)= sign $[(1+2p)(1+\beta)+2p^2(1-\beta)] > 0$.

We then substitute equilibrium conditions (1) into (29), to obtain:

$$
\begin{aligned}
&(1+n)^2\frac{(1+p)\beta}{\alpha A}\Big(\frac{1}{1-\epsilon}\Big)^{\frac{\gamma}{1-\gamma}}\frac{1-\gamma}{\gamma}\Big(\frac{\gamma}{\delta}\Big)^{\frac{1}{1-\gamma}}k_{t+2}\\
&-(1+n)\Big[\frac{\beta(1+p)(1-\alpha)}{\alpha}\Big(\frac{1}{1-\epsilon}\Big)^{\frac{\gamma}{1-\gamma}}\frac{1-\gamma}{\gamma}\Big(\frac{\gamma}{\delta}\Big)^{\frac{1}{1-\gamma}}\frac{1-\gamma}{\gamma}+\frac{1+p}{p}(1+p+\beta+pX)\Big]k_{t+1}\\
&+(1+p+\beta-p\beta)(1-\alpha)Ak_t=0 \qquad (30)
\end{aligned}
$$

The general solution of this second-order linear difference equation is given by:

$$
\begin{aligned}
\tilde{k}_t &= \tilde{Z}_1\tilde{g}_1^t+\tilde{Z}_2\tilde{g}_2^t,\\
\tilde{g}_1 &= A\Big(\frac{\tilde{\Delta}-\sqrt{\tilde{\Theta}}}{(1+n)2\tilde{\Omega}}\Big), \qquad (31)\\
\tilde{g}_2 &= A\Big(\frac{\tilde{\Delta}+\sqrt{\tilde{\Theta}}}{(1+n)2\tilde{\Omega}}\Big),
\end{aligned}
$$

where $\tilde{Z}_1$ and $\tilde{Z}_2$ is arbitrary constants. In addition, $\tilde{\Delta}\equiv\frac{\beta(1+p)(1-\alpha)}{\alpha}(\frac{1}{1-\epsilon})^{\frac{\gamma}{1-\gamma}}(\frac{\gamma}{\delta})^{\frac{1}{1-\gamma}}\frac{1-\gamma}{\gamma}+\frac{1+p}{p}[1+p+\beta+pX]>0$, $\tilde{\Theta}\equiv[\frac{\beta(1+p)(1-\alpha)}{\alpha}(\frac{1}{1-\epsilon})^{\frac{\gamma}{1-\gamma}}(\frac{\gamma}{\delta})^{\frac{1}{1-\gamma}}\frac{1-\gamma}{\gamma}]^2+\frac{2\beta(1+p)(1-\alpha)}{\alpha}(\frac{1}{1-\epsilon})^{\frac{\gamma}{1-\gamma}}(\frac{\gamma}{\delta})^{\frac{1}{1-\gamma}}\frac{1-\gamma}{\gamma}[\frac{(1+p+\beta)(1-p)}{p}+(1+p)X+2p\beta]+[\frac{1+p}{p}(1+p+\beta+pX)]^2>0$, and $\tilde{\Omega}\equiv\frac{\beta(1+p)(1-\alpha)}{\alpha}(\frac{1}{1-\varepsilon})^{\frac{\gamma}{1-\gamma}}(\frac{\gamma}{\delta})^{\frac{1}{1-\gamma}}\frac{1-\gamma}{\gamma}>0$. Lager root $\tilde{g}_2$ does not satisfy the resource constraint, the general equation is rewritten as $\tilde{k}_t=\tilde{Z}_1\tilde{g}_1^t$.[4]

Proposition 3 *Suppose that*

$$A>\frac{(\tilde{\Delta}+\sqrt{\tilde{\Theta}})(1+n)}{2p(1-\alpha)},$$

then, there exists a unique equilibrium such that $\tilde{k}_t=\tilde{k}_1\tilde{g}_1^{t-1}$*, where* $\tilde{g}_1$ *is given as (31) and* $\tilde{g}_1>1$ *for each* $t\geq 1$*.*

Proof. See Appendix 5. ■

Suppose that the productive parameter A is sufficiently large, the economy grows at a constant rate of $g_{t+1}\equiv k_{t+1}/k_t=\tilde{g}_1$ at each date $t\geq 1$. It is easy to see that the output or GDP of this economy grow at the same constant rate $\tilde{g}_1$. The following two propositions respectively shows the impact of PHF and life expectancy on growth rate and proof are given in Appendix 6 and Appendix 7.

Proposition 4 *For a given level of life expectancy, PFH decreases the growth rate.*

Proof. See, Appendix 6. ■

Since the rate of reimbursement, ϵ determines the size of PHF, the impact of the rate of reimburse-

[4]Let us define the characteristic equation of (30) as $F(\tilde{g})$. Deriving the condition of resource constraint as the same way in Lemma 1, we have $F[(1-\alpha)A/(1+n)]=-(1-\alpha)A[2+\beta+(2+\beta)(1+n)X]<0$. Thus, we have: $F(\tilde{g}_1)<F[(1-\alpha)A/(1+n)]<F(\tilde{b}_2)$.

ment on the growth rate represents the PHF effect on the growth rate. Although the tax burden reduces the health expenditure of generation $t+1$ and leads to a positive incentive on saving, a negative health cost effect and tax burden effect on saving dominate the positive effect result in reducing the saving and growth rate.

Proposition 5 *An increase in life expectancy increases (decreases) the growth rate when the rate of life expectancy is small (large).*

Proof. See, Appendix 7. ■

When the rate of life expectancy is small, the public sector does not find it necessary to fund a large amount of health funding. Thus, the negative tax burden and health cost effect on saving becomes small; and these effects are dominated by the positive transfer effect. On the other hand, when the life expectancy is large, the public sector has to fund the large amount of health funding. Thus, the negative tax burden and health cost effect on saving becomes large, and these effects dominate the positive transfer effect.

5 The Effect of Public Funded Health Spending

In this section examines whether public funded health expenditure accelerates the welfare level or not. Let superscripts "n" and "p" respectively denote "the economy with no public funded health spending or the economy with $e=0$", "the economy with public funded health spending or the economy with $\epsilon \in (0,1)$". The welfare of each member is measured by his or her expected utility which given by (5). To examine the welfare, let us define the Benthamite social welfare function; that is, the welfare level of period t is measured by the sum of the utility of generation $t-1$ and generation t who live in period t. This sum is formulated as follows:

$$W^t = p(\ln c_{a,t}^t + \beta \ln q_t^t) + (1-p)\ln c_{d,t}^t + EV(c_t^{t-1}, h_t; p). \tag{32}$$

Substituting the equilibrium values, we respectively have the welfare level of period t as follows:[5]

$$
\begin{aligned}
W^{t,n} &= \Lambda + (1+p+\beta)\Big\{ t\ln(g^n) + \ln\Big(\frac{1}{p+p^2\beta+B}\Big) \\
&+ \ln\Big\{\frac{(p+p^2\beta+B)(1-\alpha)A}{g^n} + p(1+p)D[(1-\alpha)A-(1+n)g^n]\Big\}\Big\}, \\
W^{t,p} &= \Lambda + (1+p+\beta)\Big\{(t-1)\ln(g^p) + \ln\Big(\frac{1}{p+p^2\beta+B+p(1+p)X}\Big) \\
&+ \ln\Big\{\frac{(p+p^2\beta+B)(1-\alpha)A}{g^p} + (1+n)p(1+p)X + p(1+p)D\Big(\frac{1}{1-\epsilon}\Big)^{\frac{\gamma}{1-\gamma}}[(1-\alpha)A-(1+n)g^p]\Big\}\Big\},
\end{aligned}
$$

where $\Lambda \equiv p\ln(\frac{1}{1+p+\beta}) + p\beta\ln(\frac{\beta}{1+p+\beta}) + (1-p)\ln(\frac{1}{1+p}) + p^2\ln(\frac{\alpha A}{1+p+\beta}) + p(1-p)\ln(\frac{\alpha A}{1+p}) + (1+p+\beta)\ln(k_1)$, and $D \equiv \frac{\beta(1+n)}{\alpha A}(\frac{\gamma}{\delta})^{\frac{1}{1-\gamma}}\frac{1-\gamma}{\gamma}$.

To determine how PHF accelerates the welfare level or not, we subtract $W^{t,n}$ from $W^{t,p}$:

$$
\begin{aligned}
& W^{t,p} - W^{t,n} = (1+p+\beta) \\
& \Big\{\Big[t[\ln(g^p)-\ln(g^n)]\Big] \\
& +\Big[\ln\Big(\frac{1}{p(1+p\beta)+B+p(1+p)X}\Big) - \ln\Big(\frac{1}{p(1+p\beta)+B}\Big)\Big] \\
& +\Big[\ln\Big\{\frac{(1+p+\beta+p^2\beta)(1-\alpha)A}{g^p} + (1+n)X + p(1+p)D\Big(\frac{1}{1-\epsilon}\Big)^{\frac{\gamma}{1-\gamma}}[(1-\alpha)A-(1+n)g^n]\Big\} \\
& -\ln\Big\{\frac{(p+p^2\beta+B)(1-\alpha)A}{g^n} + p(1+p)D[(1-\alpha)A-(1+n)g^n]\Big\}\Big]\Big\} \qquad (33)
\end{aligned}
$$

The first term of the brackets in the right-hand side of (33) represents the growth effect of PHF. PHF decreases the growth rate, it has negative impact on the welfare. The second brackets in the right-hand side of (33) represents the tax burden effect of PHF. PHF increases the tax burden, it also has the negative impact on the welfare. The third brackets in the right-hand side of (33) represents the transfer effect of PHF. PHF increases the transfers, it has positive impact on the welfare.

Lemma 2 *An increase in life expectancy increases the positive and negative value of each effect*

Proof. See Appendix 9. ∎

(1) Since the economy without PHF has a higher growth rate than that with PHF, and an increase in life expectancy increases the growth rate of the economy without PHF, the result of growth effect is obvious. (2) An increase in life expectancy increases the tax burden; thus the negative tax burden effect on PHF increases with an increase in life expectancy. (3) An increase in the

[5]See Appendix 8 for derivation.

growth rate increases the opportunity cost of the health expenditure of the young generation, thus an increase in the growth rate decreases the health expenditure of the young generation. Because the growth rate on the economy without PHF is higher than that of the economy without PHF, the positive impact of PHF on welfare increases.

Proposition 6

1. *Suppose that*

$$D(X + (1+\beta))g^n[(1-\alpha)A - (1+n)g^n] < (1+\beta)[D(\frac{1}{1-\epsilon})^{\frac{1}{1-\gamma}}g^p - X][(1-\alpha)A - (1+n)g^p],$$

PHF accelerates (deteriorate) the welfare of current generation when the rate of life expectancy is large (small).

2. *PHF deteriorates the welfare level of future generation.*

3. *An increase in life expectancy yields the intergenerational conflict between current and future generation at the economy without PHF. However, PHF alleviates the conflict.*

Proof. See, Appendix 10. ■

Since the growth rate of the economy without PHF is higher than that of the economy with PHF, the speed of a decrease in transfer effect is larger than that of the economy with PHF. Therefore, when the growth rate of the economy with PHF is sufficiently large, an increase in life expectancy accelerates the welfare level of PHF. The assumption of the condition also shows these results.

The growth rate tends to increase monotonically and takes maximum values as $t \to \infty$, thus for the future generation, the impact of the growth effect dominates the other effects. Since PHF has a negative impact on the growth effect, it deteriorates the welfare level of the future generation.

In an economy without PHF, an increase in life expectancy increases the growth rate, and the welfare level of current and future generation tends to large when the life expectancy is large. On the other hand, in an economy with PHF, there is an inverted-U shaped relation between growth rate and life expectancy; thus an increase in life expectancy decreases growth rate and results in the alleviation of intergenerational conflict.

6 Conclusion

In this paper, we focus on the increased amount of health expenditure in an aging economy and examine the effect of life expectancy on saving and the growth rate by using the health demand in old age. In the first part of this paper, we construct the benchmark model of the demand for health and then introduce the government, who is the authority of public funded health spending (PHF).

By analyzing the model, we show that an increase in the life expectancy increases the growth rate in an economy without PHF. On the other hand, in an economy with PHF, an increase in life expectancy increases (decreases) the growth rate when the rate of life expectancy is small (high). We also show that the PFH accelerates the welfare level of current generations when the rate of life expectancy is large, but it deteriorates the welfare level of future generations, and as the generation goes on, it results in a lower level of welfare. Although the intergenerational conflict between current and future generations get bigger with an increase in life expectancy, we show that PHF has a role to play in alleviating the intergenerational conflict.

Appendix

Appendix 1

We have the characteristic equation of (17) as follows:

$$
\begin{aligned}
&(1+n)^2\frac{\beta(1+p)}{\alpha A}\Big(\frac{\gamma}{\delta}\Big)^{\frac{1}{1-\gamma}}\frac{1-\gamma}{\gamma}g^2\\
&-(1+n)\Big[\frac{\beta(1+p)(1-\alpha)}{\alpha}\Big(\frac{\gamma}{\delta}\Big)^{\frac{1}{1-\gamma}}\frac{1-\gamma}{\gamma}+\frac{(1+p+\beta)(1+p)}{p}\Big]g+(1+p+\beta-p\beta)(1-\alpha)A=0.
\end{aligned}
$$

Define the left-hand side of the characteristic equation as follows:

$$
\begin{aligned}
&f(g)=(1+n)^2\frac{\beta(1+p)}{\alpha A}\Big(\frac{\gamma}{\delta}\Big)^{\frac{1}{1-\gamma}}\frac{1-\gamma}{\gamma}g^2\\
&-(1+n)\Big[\frac{\beta(1+p)(1-\alpha)}{\alpha}\Big(\frac{\gamma}{\delta}\Big)^{\frac{1}{1-\gamma}}\frac{1-\gamma}{\gamma}+\frac{(1+p+\beta)(1+p)}{p}\Big]g+(1+p+\beta-p\beta)(1-\alpha)A.
\end{aligned}
$$

The budget constraint (3) is rewritten as $s_t \leq w_t$. Substituting equilibrium values (1) and (13) into the condition, we just derived before, we have $k_{t+1} \leq (1-\alpha)Ak_t/(1+n)$. Arranging this condition, to obtain

$$
g_{t+1} \equiv \frac{k_{t+1}}{k_t} \leq \frac{(1-\alpha)A}{1+n}. \tag{34}
$$

Substituting (34) into (34), we have $f[(1-\alpha)A/(1+n)] = -(1-\alpha)A(1+p+\beta+p^2\beta)/p < 0$ and $g_1 < (1-\alpha)A/(1+n) < g_2$. As any equilibrium path must satisfy the resource constraint (3), the general solution of (18) must be such that Z_2 is zero, that is, the equilibrium path is given by $k_t = Z_1 g_1^t$.

Appendix 2

The necessary and sufficient condition that the growth rate is positive is $b_1 > 1$. From (19), the condition is derived as follows:

$$A > \frac{(\Delta + \sqrt{\Theta})(1+n)}{2p(1-\alpha)}.$$

The equilibrium condition must satisfy its initial condition, $k_1 = Z_1 g_1$. Arranging this, we have $Z_1 = k_1/g_1$, thus the equilibrium path is given by $k_t = k_1(g_1)^{t-1}$ for each $t \geq 1$.

Appendix 3

(i) As the growth rate in this economy is given by $g \equiv k_{t+1}/k_t = b_1$(constant), the total differentiate of (17) with respect to p derives the following:

$$\frac{dg}{dp} \equiv -\frac{\frac{\partial f(g)}{\partial p}}{\frac{\partial f(g)}{\partial b}} = -\frac{\frac{(1+n)^2}{\alpha A}(\frac{\gamma}{\delta})^{\frac{1}{1-\gamma}}\frac{1-\gamma}{\gamma}\beta g^2 - (1+n)[\frac{1-\alpha}{\alpha}(\frac{\gamma}{\delta})^{\frac{1}{1-\gamma}}\frac{1-\gamma}{\gamma}\beta - \frac{(1+p)(1-p)+\beta}{p^2}]g + (1-\alpha)A(1-\beta)}{(2(1+n)\Omega b - \Delta)}.$$

Substituting (19) into the denominator follows $-(1+n)\sqrt{\Theta} < 0$, to obtain sign$\frac{dg}{dp}$ = sign$\frac{\partial f(g)}{\partial p}$. When $p \to 0$, the second term of $\frac{\partial f(g)}{\partial p}$ tends to $+\infty$, then $\frac{\partial f(g)}{\partial p} = 0$ has two negative roots. It leads to $\frac{\partial f(g)}{\partial p} > 0$ for all $g > 0$. When $p \to 1$, the second term of $\frac{\partial f(g)}{\partial p}$ tends to positive suppose that $\delta > \gamma(\frac{1-\alpha}{\alpha}\frac{1-\gamma}{\gamma})^{1-\gamma}$, to obtain $\frac{\partial f(g)}{\partial p} > 0$ for all $g > 0$.

Appendix 4

The old age consumption and health level is determined by maximizing (6) subjects to (22) and (24). The first order condition derive the following consumption and health demand in his or her old age as follows:

$$\begin{aligned} c_{t+1}^t &= \sigma\Big[\frac{R_{t+1}}{p}s_{i_t} + Q_{t+1}\Big(\frac{1}{1-\epsilon}\Big)^{\frac{\gamma}{1-\gamma}}\Big(\frac{\gamma}{\delta}\Big)^{\frac{1}{1-\gamma}}\frac{1-\gamma}{\gamma}\Big], \quad i = a, d \\ h_{t+1} &= (1-\sigma)\delta\Big[\frac{R_{t+1}}{p}s_{i_t} + Q_{t+1}\Big(\frac{1}{1-\epsilon}\Big)^{\frac{\gamma}{1-\gamma}}\Big(\frac{\gamma}{\delta}\Big)^{\frac{1}{1-\gamma}}\frac{1-\gamma}{\gamma}\Big], \quad i = a, d. \end{aligned}$$

The utility maximization problem in his or her young period is solved in the same manner as section 3. The first order condition for this problem yields the solution for the young period's consumption, transfer towards their parents, and saving as follows:

$$\begin{aligned}
c_{a,t}^{t} &= \frac{1}{1+p+\beta}\Big[(1-\tau_t)w_t + p\frac{Q_{t+1}}{R_{t+1}}\Big(\frac{1}{1-\epsilon}\Big)^{\frac{\gamma}{1-\gamma}}\Big(\frac{\gamma}{\delta}\Big)^{\frac{1}{1-\gamma}}\frac{1-\gamma}{\gamma}\Big],\\
q_t^{t} &= \frac{\beta}{1+p+\beta}\Big[(1-\tau_t)w_t + p\frac{Q_{t+1}}{R_{t+1}}\Big(\frac{1}{1-\epsilon}\Big)^{\frac{\gamma}{1-\gamma}}\Big(\frac{\gamma}{\delta}\Big)^{\frac{1}{1-\gamma}}\frac{1-\gamma}{\gamma}\Big],\\
s_{a,t}^{t} &= \frac{p}{1+p+\beta}\Big[(1-\tau_t)w_t - (1+\beta)\frac{Q_{t+1}}{R_{t+1}}\Big(\frac{1}{1-\epsilon}\Big)^{\frac{\gamma}{1-\gamma}}\Big(\frac{\gamma}{\delta}\Big)^{\frac{1}{1-\gamma}}\frac{1-\gamma}{\gamma}\Big],\\
c_{d,t}^{t} &= \frac{1}{1+p}\Big[(1-\tau_t)w_t + p\frac{Q_{t+1}}{R_{t+1}}\Big(\frac{1}{1-\epsilon}\Big)^{\frac{\gamma}{1-\gamma}}\Big(\frac{\gamma}{\delta}\Big)^{\frac{1}{1-\gamma}}\frac{1-\gamma}{\gamma}\Big],\\
s_{d,t}^{t} &= \frac{p}{1+p}\Big[(1-\tau_t)w_t - \frac{Q_{t+1}}{R_{t+1}}\Big(\frac{1}{1-\epsilon}\Big)^{\frac{\gamma}{1-\gamma}}\Big(\frac{\gamma}{\delta}\Big)^{\frac{1}{1-\gamma}}\frac{1-\gamma}{\gamma}\Big].
\end{aligned}$$

Appendix 5

The necessary and sufficient condition that the growth rate is positive is $\tilde{g}_1 > 1$. From (31), the condition is derived as

$$A > \frac{(\tilde{\Delta}+\sqrt{\tilde{\Theta}})(1+n)}{2p(1-\alpha)}.$$

The equilibrium condition must satisfy its initial condition, $\tilde{k}_1 = \tilde{Z}_1\tilde{g}_1$. Arranging this, we have $\tilde{Z}_1 = \tilde{k}_1/\tilde{g}_1$, thus the equilibrium path is given by $\tilde{k}_t = \tilde{k}_1(\tilde{g}_1)^{t-1}$ for each $t \geq 1$.

Appendix 6

Since the economy without PHF is the economy with $\epsilon = 0$ and that of with PHF is the economy with $\epsilon \in (0,1)$, an effect of ϵ on the economic growth shows the effect of PHF on the economic growth. The growth rate in this economy is given by $g \equiv k_{t+1}/k_t = \tilde{g}_1$ (constant), total differentiate of (30) with respect to ϵ yields the following:

$$\begin{aligned}
&\frac{d\tilde{g}_1}{d\epsilon} \equiv -\frac{\frac{\partial F(g)}{\partial \epsilon}}{\frac{\partial F(b)}{\partial b}} = \frac{-1}{(1+n)[2(1+n)\tilde{\Omega}g-\tilde{\Delta}]}\\
&\Big\{\frac{(1+n)^2(1+p)\beta}{\alpha A}(1-\epsilon)^{-\frac{1}{1-\gamma}}(\frac{\gamma}{\delta})^{\frac{1}{1-\gamma}}\frac{1-\gamma}{\gamma}g^2-(1+n)\\
&[\frac{(1+p)\beta(1-\alpha)}{\alpha}(1-\epsilon)^{-\frac{1}{1-\gamma}}(\frac{\gamma}{\delta})^{\frac{1}{1-\gamma}}\frac{1-\gamma}{\gamma}+(1+p)p\beta(1+n)[(\frac{\gamma}{\delta(1-\epsilon)})^{\frac{1}{1-\gamma}}+\epsilon(1-\epsilon)^{-\frac{1}{1-\gamma}}(\frac{\gamma}{\delta})^{\frac{1}{1-\gamma}}]]g\Big\}
\end{aligned}$$

Substituting (31) into the denominator follows $-(1+n)\sqrt{\tilde{\Theta}} < 0$, to obtain sign$\frac{dg}{dp}$ = sign$\frac{\partial F(g)}{\partial \epsilon}$. Noting that $\lim_{g\to 0}\frac{\partial F(g)}{\partial \epsilon} = 0$, $\lim_{g\to\frac{(1-\alpha)A}{1+n}}\frac{\partial F(g)}{\partial \epsilon} = -\frac{(1+p)p\beta(1+n)(1-\alpha)A}{P}[(\frac{\gamma}{\delta(1-\epsilon)})^{\frac{1}{1-\gamma}}+\epsilon(1-\epsilon)^{-\frac{1}{1-\gamma}}(\frac{\gamma}{\delta})^{\frac{1}{1-\gamma}}] < 0$ and $1 < \tilde{g}_1 < \frac{(1-\alpha)A}{1+n}$, we have $\frac{dg}{d\epsilon} < 0$.

Appendix 7

Total differentiate of (30) with respect to p yields the following:

$$\frac{d\tilde{g}_1}{dp} \equiv -\frac{\frac{\partial F(g)}{\partial p}}{\frac{\partial F(g)}{\partial b}} = \frac{-1}{2(1+n)\tilde{\Omega}g - \tilde{\Delta}}$$
$$\Big\{\frac{(1+n)^2}{\alpha A}(\frac{1}{1-\epsilon})^{\frac{1}{1-\gamma}}(\frac{\gamma}{\delta})^{\frac{1}{1-\gamma}}\frac{1-\gamma}{\gamma}\beta g^2$$
$$-(1+n)[\frac{1-\alpha}{\alpha}(\frac{1}{1-\epsilon})^{\frac{1}{1-\gamma}}(\frac{\gamma}{\delta})^{\frac{1}{1-\gamma}}\frac{1-\gamma}{\gamma}\beta + \frac{-(1+p)(1-p)-\beta}{p^2} + (1+N)\beta X]g$$
$$+(1-\alpha)A(1-\beta)\Big\}.$$

When $p \to 0$, the second term of $\frac{\partial F(b)}{\partial p}$ tends to $+\infty$, then $\frac{\partial F(b)}{\partial p}$ has two negative roots. It leads to $\frac{\partial F(b)}{\partial p} > 0$ for all $g > 0$. To examine the case of $p \to 1$, let us define $\lim_{p\to 1}\frac{\partial F(g)}{\partial p} \equiv \Psi(g) = \frac{(1+n)^2}{\alpha A}(\frac{1}{1-\epsilon})^{\frac{1}{1-\gamma}}(\frac{\gamma}{\delta})^{\frac{1}{1-\gamma}}\frac{1-\gamma}{\gamma}\beta g^2 - (1+n)\beta[\frac{1-\alpha}{\alpha}(\frac{1}{1-\epsilon})^{\frac{1}{1-\gamma}}(\frac{\gamma}{\delta})^{\frac{1}{1-\gamma}}\frac{1-\gamma}{\gamma} - 1 + (1+n)X]g + (1-\alpha)A(1-\beta)$. Since $\Psi(\frac{(1-\alpha)A}{1+n}) = (1-\alpha)A[1-\beta(1+n)X]$, thus we have $\Psi(\frac{(1-\alpha)A}{1+n}) < 0$, when $(\frac{\gamma}{\delta(1-\epsilon)})^{\frac{1}{1-\gamma}} > \frac{1}{\epsilon\beta(1+n)}$.

Now let us define the $\bar{g}_1$ and $\bar{g}_2$ as the solution of $\Psi(\frac{(1-\alpha)A}{1+n}) = 0$ and assume the $\hat{g}_1$ which satisfy the following:

$$\bar{g}_1 < \frac{[\beta\frac{1-\alpha}{\alpha}(\frac{1}{1-\epsilon})^{\frac{1}{1-\gamma}}(\frac{\gamma}{\delta})^{\frac{1}{1-\gamma}}\frac{1-\gamma}{\gamma} - \beta + (1+n)\beta X]^2}{2(1+n)\frac{\beta}{\alpha A}(\frac{1}{1-\epsilon})^{\frac{1}{1-\gamma}}(\frac{\gamma}{\delta})^{\frac{1}{1-\gamma}}\frac{1-\gamma}{\gamma}} \equiv \hat{g}_1.$$

We have a results that $\Psi(\tilde{g}_1) < 0$, when $\hat{g}_1 < \tilde{g}_1$. By comparing $\hat{g}_1$ and $\tilde{g}_1$, we have the following:

$$\hat{g}_1 = \frac{(\beta\frac{1-\alpha}{\alpha}(\frac{1}{1-\epsilon})^{\frac{1}{1-\gamma}}(\frac{\gamma}{\delta})^{\frac{1}{1-\gamma}}\frac{1-\gamma}{\gamma} - \beta + (1+n)\beta X)^2}{2(1+n)\frac{\beta}{\alpha A}(\frac{1}{1-\epsilon})^{\frac{1}{1-\gamma}}(\frac{\gamma}{\delta})^{\frac{1}{1-\gamma}}\frac{1-\gamma}{\gamma}} < \frac{(\beta\frac{1-\alpha}{\alpha}(\frac{1}{1-\epsilon})^{\frac{1}{1-\gamma}}(\frac{\gamma}{\delta})^{\frac{1}{1-\gamma}}\frac{1-\gamma}{\gamma} + (2+\beta+(1+n)\beta X)^2}{(1+n)\frac{\beta}{\alpha A}(\frac{1}{1-\epsilon})^{\frac{1}{1-\gamma}}(\frac{\gamma}{\delta})^{\frac{1}{1-\gamma}}\frac{1-\gamma}{\gamma}} = \tilde{g}_1.$$

Therefore there exists $p^\star \in (0,1)$ such that $\frac{\partial F(g)}{\partial p} = 0$, and have $\frac{\partial F(g)}{\partial p} > 0$ for $p \in (0, p^\star]$ and $\frac{\partial F(g)}{\partial p} < 0$ for $p \in [p^\star, 1)$.

Appendix 8

At first, let us derive the the welfare in the economy without PHF. Noting that

$$\ln c_{a,t}^{n,t} = \ln\{\frac{1}{1+p+\beta}[(1-\alpha)Ak_t + \frac{p(1+p)D}{\alpha A(p+p^2\beta+B)}[(1-\alpha)Ak_{t+1} - (1+n)k_{t+2}]]\},$$
$$\ln q_t^{n,t} = \ln\{\frac{\beta}{1+p+\beta}[(1-\alpha)Ak_t + \frac{p(1+p)D}{\alpha A(p+p^2\beta+B)}(\frac{\gamma}{\delta})^{\frac{1}{1-\gamma}}\frac{1-\gamma}{\gamma}[(1-\alpha)Ak_{t+1} - (1+n)k_{t+2}]]\},$$
$$\ln c_{a,t}^{n,t} = \ln\{\frac{1}{1+p}[(1-\alpha)Ak_t + \frac{p(1+p)D}{\alpha A(p+p^2\beta+B)}[(1-\alpha)Ak_{t+1} - (1+n)k_{t+2}]]\},$$
$$EV_{t+1}(c_t^{n,t-1}, h_t^n; p) = P(p\ln\{\frac{R_{t+1}}{p}s_{a,t} + Q_{t+1}(\frac{\gamma}{\delta})^{\frac{1}{1-\gamma}}\frac{1-\gamma}{\gamma}\} + (1-p)\ln\{\frac{R_{t+1}}{p}s_{d,t} + Q_{t+1}(\frac{\gamma}{\delta})^{\frac{1}{1-\gamma}}\frac{1-\gamma}{\gamma}\})$$
$$= p(1-p)\ln\{\frac{\alpha A}{1+p}[(1-\alpha)Ak_t + \frac{p(1+p)D}{\alpha A(p+p^2\beta+B)}[(1-\alpha)Ak_{t+1} - (1+n)k_{t+2}]]\},$$

then substituting these values into (32) we have the welfare of generation t at the economy without PHF.[6]

Next, let us derive the welfare in the economy with PHF. Noting that

$$\ln c_{a,t}^{p,t} = \ln\{\frac{1}{1+p+\beta}[(1-\alpha)Ak_t + \frac{(1+n)Xk_{t+1}}{p+p^2\beta+B} + \frac{p(1+p)D}{\alpha A(p+p^2\beta+B)}(\frac{1}{1-\epsilon})^{\frac{1}{1-\gamma}}[(1-\alpha)Ak_{t+1}-(1+n)k_{t+2}]]\},$$
$$\ln q_t^{n,t} = \ln\{\frac{\beta}{1+p+\beta}[(1-\alpha)Ak_t + \frac{(1+n)Xk_{t+1}}{p+p^2\beta+B} + \frac{p(1+p)D}{\alpha A(p+p^2\beta+B)}(\frac{1}{1-\epsilon})^{\frac{1}{1-\gamma}}[(1-\alpha)Ak_{t+1}-(1+n)k_{t+2}]]\},$$
$$\ln c_{a,t}^{n,t} = \ln\{\frac{1}{1+p}[(1-\alpha)Ak_t + + \frac{(1+n)Xk_{t+1}}{p+p^2\beta+B} + \frac{p(1+p)D}{\alpha A(p+p^2\beta+B)}(\frac{1}{1-\epsilon})^{\frac{1}{1-\gamma}}[(1-\alpha)Ak_{t+1}-(1+n)k_{t+2}]]\},$$
$$EV_{t+1}(c_t^{n,t-1}, h_t^n; p) = P(p\ln\{\frac{R_{t+1}}{p}s_{a,t} + Q_{t+1}(\frac{\gamma}{\delta})^{\frac{1}{1-\gamma}}\frac{1-\gamma}{\gamma}\} + (1-p)\ln\{\frac{R_{t+1}}{p}s_{d,t} + Q_{t+1}(\frac{\gamma}{\delta})^{\frac{1}{1-\gamma}}\frac{1-\gamma}{\gamma}\})$$
$$= p(1-p)\ln\{\frac{\alpha A}{1+p}[(1-\alpha)Ak_t + + \frac{(1+n)Xk_{t+1}}{p+p^2\beta+B} + \frac{p(1+p)D}{\alpha A(p+p^2\beta+B)}(\frac{1}{1-\epsilon})^{\frac{1}{1-\gamma}}[(1-\alpha)Ak_{t+1}-(1+n)k_{t+2}]]\},$$

and substituting these values into (32) we have the welfare of generation t at the economy with PHF.

Appendix 9

(1) Growth effect:

Let us define $\Upsilon(p) \equiv \ln(\frac{g^n}{g^p})$. $\Upsilon'(p) = \frac{(\partial g^n/\partial p)g^p - (\partial g^p/\partial p)g^n}{g^p g^n}$. Nothing that $\frac{\partial g^p}{\partial p} < 0$ for $p \in [p^\star, 1)$, $\Upsilon'(p) > 0$ for $p \in [p^\star, 1)$. Since $\lim_{p\to 0}\Upsilon'(p) = 0$ and $\Upsilon'(p) > 0$ for all $p \in (0,1)$, an increase in life expectancy increases the negative growth effect of PHF.

(2) Tax effect:

Let us define $\Theta(p) \equiv \ln(\frac{p+p^2\beta+B+p(1+p)X}{p+p^2\beta+B})$. Since $\Theta'(p) = \frac{X[p+p^2\beta+B+p(1+p-2p\beta+2p\beta)]}{(p+p^2\beta+B)(p+p^2\beta+B+p(1+p)X)} > 0$, $\Theta''(p) = -\frac{X[p+p^2\beta+B+p(1+p)X][(p+p^2\beta+B)(1+p)+2p(1-\beta)+2\beta+(1+2P)X+p(1+p+2\beta(1-p))(1+2p\beta)]}{[(p+p^2\beta+B)(p+p^2\beta+B+p(1+p)X)]^2} < 0$, and $\lim_{p\to 0}\Theta'(p) = \frac{1}{1+\beta} > 0$. Therefore an increase in the life expectancy increases the negative tax burden effect of PHF.

(3) Transfer effect:

Let us define $N \equiv \frac{(p+p^2\beta+B)(1-\alpha)A}{g^n} + p(1+p)D[(1-\alpha)A - (1+n)g^n]$, $P \equiv \frac{(p+p^2\beta+B)(1-\alpha)A}{g^p} + (1+n)p(1+p)X + p(1+p)D(\frac{1}{1-\epsilon})^{\frac{\gamma}{1-\gamma}}[(1-\alpha)A - (1+n)g^p]$. The difference of the transfer effect with PHF and without PHF is shown as $\ln P - \ln N$. Since $P = N$ when $\epsilon = 0$ and an increase in ϵ increases the value of P, we have $\ln(\frac{P}{N}) > 0$.

To examine the effect of life expectancy on transfer effect, let us compare the effect of life expectancy on P and N as follows: $\frac{\partial P}{\partial p} - \frac{\partial N}{\partial p} = (1+2P\beta)(1-\alpha)A(\frac{g^n - g^p}{g^n g^p}) + (p+p^2\beta+B)(1-$

[6] where $B = 1+\beta$ and $D = \frac{\beta(1+n)}{\alpha A}(\frac{\gamma}{\delta})^{\frac{1}{1-\gamma}}(\frac{1-\gamma}{\gamma})$.

$\alpha)A(\frac{1}{(g^n)^2}\frac{\partial g^n}{\partial p} - \frac{1}{(g^p)^2}\frac{\partial g^p}{\partial p}) + (1+2p)D\{(\frac{1}{1-\epsilon})^{\frac{\gamma}{1-\gamma}}[(1-\alpha)A-(1+n)g^p] - [(1-\alpha)A-(1+n)g^n]\} +$ $(1+n)(1+2p)X + p(1+p)(1+n)D(\frac{\partial g^n}{\partial p} - (\frac{1}{1-\epsilon})^{\frac{\gamma}{1-\gamma}}\frac{\partial g^p}{\partial p})$. Since $\frac{\partial g^p}{\partial p} > 0$ for $p \in [p^*, 1)$, $\frac{\partial P}{\partial p} - \frac{\partial N}{\partial p} > 0$ for the regime. In addition, $\lim_{p\to 0}(\frac{\partial P}{\partial p} - \frac{\partial N}{\partial p}) = 0$, we have the result that life expectancy increases the positive transfer effect of PHF.

Appendix 10

(1) Let us define $\Phi(p) \equiv \ln(\frac{P}{N})$, then $\Phi'(p) = \frac{1}{NP}(\frac{\partial P}{\partial p}N - \frac{\partial N}{\partial p}P) > 0$. Since $\lim_{p\to 0}\Upsilon'(p) = 0$, $\lim_{p\to 0}\Theta'(p) = \frac{1}{1+\beta} > 0$, and $\lim_{p\to 0}\Phi'(p) = 0$ (see, lemma 2), $\Upsilon(0) + \Theta(0) > \Phi(0)$. Therefore, if $\Upsilon(1) + \Theta(1) < \Phi(1)$, there exists unique $p^{**} \in (0,1)$ such that $\Upsilon(p) + \Theta(p) = \Phi(p)$.

Since $\Upsilon(1)+\Theta(1) = \ln\{\frac{g^n}{g^p}(1+\frac{2X}{1+\beta+B})\} > 1$, and $\Phi(1) = \ln\{\frac{\frac{2(1+\beta)(1-\alpha)A}{g^p}+(1+n)2X+2D(\frac{1}{1-\epsilon})^{\frac{\gamma}{1-\gamma}}[(1-\alpha)A-(1+n)g^p]}{\frac{2(1+\beta)(1-\alpha)A}{g^n}+2D[(1-\alpha)A-(1+n)g^n]}\} >$ 1, we compare the follows:

$$1 + \frac{X}{(1+\beta)} < 1 + \frac{(1+n)g^pX + D\{(\frac{1}{1-\epsilon})^{\frac{1}{1-\gamma}}g^p[(1-\alpha)A-(1+n)g^p] - g^n[(1-\alpha)A-(1+n)g^n]\}}{(1+\beta)(1-\alpha)A + Dg^n[(1-\alpha)A-(1+n)g^n]}.$$

Arranging this, we have

$$D(X+(1+\beta))g^n[(1-\alpha)A-(1+n)g^n] < (1+\beta)[D(\frac{1}{1-\epsilon})^{\frac{1}{1-\gamma}}g^p - X][(1-\alpha)A-(1+n)g^p].$$

Therefore, if it holds the relation, we have the result of $\Upsilon(1) + \Theta(1) < \Phi(1)$.

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