



**Department of Economics**

# **Trade Finance, Bank Bail-outs and Profit Taxation in an Interconnected World**

**Tim Schmidt-Eisenlohr**

Thesis submitted for assessment with a view to obtaining the degree of  
Doctor of Economics of the European University Institute

Florence  
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# *Introduction*

Countries are increasingly linked internationally. The three models developed in this thesis shed light on how firms and governments respond to the increasing interconnectedness of the world economy, analyzing profit taxation, trade finance and government intervention in the event of a contagious banking crisis. They can help understand in how far integration is beneficial or harmful and what optimal policies might be.

My first paper, which is joint work with Sebastian Krautheim, is based on the finding that larger firms are more likely to use tax haven operations to exploit international tax differences. We study a tax game between a large country and a tax haven modeling heterogeneous monopolistic firms, which can shift profits abroad. We show that a higher degree of firm heterogeneity increases the degree of tax competition, i.e. it decreases the equilibrium tax rate of the large country, leads to higher outflows of its tax base and thus decreases its equilibrium tax revenue. Similar effects hold for a higher substitutability across varieties.

My second paper takes a first step towards building a theory of trade finance. Cross border transactions are conducted using different payment contracts, the usage of which varies across countries and over time. I build a model that can explain this observation and study its implications for international trade. In the model exporters switch between payment contracts optimally, trading off differences in enforcement and efficiency between financial markets in different countries. I find that the ability of firms to switch contracts is key to understand how trade flows respond to variations in financial conditions. Numerical experiments with a two-country version of the model suggest that limiting the choice between payment contracts reduces traded quantities by up to 60 percent.

The third paper, which is joint work with Friederike Niepmann, analyzes ex-post intervention by governments in response to international banking crises under different cooperation regimes. Financial institutions are increasingly linked internationally and engaged in cross-border operations. As a result, financial crises and potential bail-outs by governments have important international implications. Extending Allen and Gale (2000) we provide a model of international contagion allowing for bank bail-outs financed by distortionary taxes. We single out inefficiencies due to spillovers, free-riding and limited burden-sharing. When countries are of equal size, an increase in cross-border deposit

holdings improves, in general, the non-cooperative outcome. For efficient crisis management, ex-ante fiscal burden sharing is essential as ex-post contracts between governments do not achieve the same global welfare.

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# Chapter 1

## Heterogeneous Firms, 'Profit Shifting' FDI and International Tax Competition

with Sebastian Krautheim, Paris School of Economics

### Abstract

Larger firms are more likely to use tax haven operations to exploit international tax differences. We study a tax game between a large country and a tax haven modeling heterogeneous monopolistic firms, which can shift profits abroad. We show that a higher degree of firm heterogeneity (a mean-preserving spread of the cost distribution) increases the degree of tax competition, i.e. it decreases the equilibrium tax rate of the large country, leads to higher outflows of its tax base and thus decreases its equilibrium tax revenue. Similar effects hold for a higher substitutability across varieties.

JEL: *F23, H25, H87*

Keywords: *heterogeneous firms, tax competition, profit shifting, tax havens*

## 1.1 Introduction

With globalization tax havens have become more important. The increased opportunities for multinational firms to shift profits towards these low-tax jurisdictions have changed the strategic tax game for international profits. Recent empirical evidence by Desai, Foley, and Hines (2006) shows that larger firms use tax haven operations more intensively.<sup>1</sup> This suggests that firm heterogeneity is relevant for international tax competition. Theory, however, has mainly focused on models with homogeneous firms.<sup>2</sup>

We introduce a tractable model of tax competition with heterogeneous firms between a large country and a tax haven. Firm heterogeneity is introduced in exactly the way that has been found to be relevant empirically: heterogeneity in productivity and size. The analysis reveals that economies with a higher degree of firm heterogeneity (relatively many productive firms) and higher substitutability across goods (low monopolistic market power) face stronger international tax competition.

In a large country, firms in a monopolistically competitive industry make positive profits, which are taxed by the government.<sup>3</sup> Given the tax rate firms can decide to avoid paying taxes at home by opening an affiliate in a tax haven and shifting profits abroad. The governments of the large country and the tax haven set their tax rates non-cooperatively. Our setup allows us to derive the pure strategy Nash equilibrium of the tax game between a large country and a tax haven.

In equilibrium the tax haven undercuts the large country which gives firms an incentive to do ‘profit shifting’ FDI. While the fixed cost of opening an affiliate in the tax haven is the same for all firms, the gains from profit shifting depend on the level of profits a firm is making. In line with the findings of Desai, Foley, and Hines (2006), in equilibrium the most productive (and thus largest and most profitable) firms shift profits while less productive firms continue to pay taxes at home.<sup>4</sup>

Tax competition is strongest when the distribution of profits across firms is such that the most productive firms account for a large share of aggregate profits. This is the case when firms are very heterogeneous and when monopolistic market power is low. In this case,

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<sup>1</sup>In line with these findings Graham and Tucker (2006) show that larger firms are more likely to avoid taxes through corporate tax shelters.

<sup>2</sup>Some notable exceptions will be discussed below.

<sup>3</sup>While, in practice, pure profits of firms are hard to determine, in the model we assume that firm profits are observable.

<sup>4</sup>Desai, Foley, and Hines (2006) analyze data on American multinational firms from the Bureau of Economic Analysis annual survey of U.S. Direct Investment Abroad for the years 1982 to 1999. Grouping countries with US affiliates into tax havens and non-havens allows them to find correlations between tax haven activities and firm level characteristics.

the large country suffers a substantial outflow of its tax base. The tax haven gains as it can set a relatively high tax rate and still attract a considerable fraction of the tax base. When instead there is low firm heterogeneity and high monopolistic market power, the tax base does not react strongly to tax differences and the large country is ‘protected’ from international tax competition. It can set a relatively high tax rate without losing much of its tax base. The tax haven is forced to strongly undercut the large country in order to attract some of the tax base.<sup>5</sup>

In our model tax competition creates a distortion. Welfare is thus higher when the large country is more ‘protected’ from tax competition which is the case when firms are more homogeneous. If taxes themselves were distortionary, welfare effects could be different.

The effect of a change in the tax rate on tax revenues of governments is given by the own-tax elasticity of tax revenues. We show that this elasticity can be decomposed into the *intensive margin* and the *extensive margin* of taxation. An increase in the tax rate of the large country has two opposing effects. On the one hand it increases tax revenues collected from firms that continue paying taxes at home (intensive margin). On the other hand a higher tax rate makes it profitable for some firms to start profit shifting. This reduces the tax base and thereby tax revenues (extensive margin). Along their best response functions the two governments optimally trade off these two opposing margins.

To complement our findings, we also analyze the role of the fixed costs of profit shifting. When these fixed costs are high it is more costly for firms to shift profits. This allows the large country to set a higher tax rate, which in turn makes it more profitable for firms to shift profits. In equilibrium these two effects exactly offset each other and the fraction of firms shifting profits and the fraction of profits shifted abroad remains constant. With a higher tax rate and a constant tax base, the equilibrium tax revenues in the large country increase.

We also compare our model to a model with homogeneous firms. We confirm that the model with homogeneous firms is the limit case of our model with heterogeneous firms and find that tax competition is lowest when firms are perfectly homogeneous. Accounting for firm heterogeneity increases the degree of tax competition by increasing the mobility of the tax base.<sup>6</sup>

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<sup>5</sup>For analytical tractability we focus the analysis on a proportional profit tax. If the government in the large country could additionally set a deductible, this instrument could be used to reduce pressures from tax competition by putting relatively more tax burden on less mobile firms. As long as the government is unable to perfectly discriminate between firms with different profits, tax haven operations are more profitable for larger firms.

<sup>6</sup>Heterogeneity affects tax competition through the distribution of the tax base (profits) across firms. Thus any policies or other factors that increases the heterogeneity of firm profits increase tax competition

Starting with Zodrow and Mieszkowski (1986) and Wilson (1986) a large and growing theoretical literature has analyzed the increasing competitive pressures on governments to reduce corporate tax rates.<sup>7</sup> While this literature tends to focus on outflows of capital, several theoretical contributions have considered the possibility of multinational firms to shift profits to jurisdictions with lower tax rates.<sup>8</sup> Recent empirical studies have shown that the mobility of profits has a considerable impact on the ability of governments to increase tax income by increasing tax rates.<sup>9</sup> The quantitative importance of tax havens in this context has been documented by Hines and Rice (1994), Hines (2004), Sullivan (2004) and Desai, Foley, and Hines (2006).<sup>10</sup>

Several recent contributions have addressed firm heterogeneity in international tax competition. In these papers different approaches to modeling firm heterogeneity have been chosen. One way to generate some degree of heterogeneity is e.g. to assume like Ogura (2006) (following Mansoorian and Myers (1993)) that otherwise identical firms have different costs of investing outside their home region. This type of heterogeneity is unrelated to the productivity of a firm. Burbidge, Cuff, and Leach (2006) introduce a related type of firm heterogeneity into a model with perfect competition, immobile labor and mobile capital. They model firm heterogeneity as an idiosyncratic, exogenous comparative advantage in one of the locations. So firms are heterogeneous in the sense that they are more productive in one country or the other. Equivalently, countries are heterogeneous in their ability to fruitfully accommodate particular firms.

A different strand of the literature is based on models of tax competition in a ‘New Economic Geography’ (NEG) context.<sup>11</sup> These models typically consider the location decision of (homogeneous) monopolistically competitive firms between two asymmetric countries. Baldwin and Okubo (2008) outline a New Economic Geography (NEG) model with tax

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in a similar way. For example the presence of multi-product firms as in Bernard, Redding, and Schott (forthcoming), Mayer, Melitz, and Ottaviano (2009) and Eckel and Neary (2006) would imply larger variance of the profit distribution for a given productivity distribution and thus increase tax competition.

<sup>7</sup>See Wilson (1999), Wilson and Wildasin (2004) or Fuest, Huber, and Mintz (2005) for surveys.

<sup>8</sup>See e.g. Elitzur and Mintz (1996), Haufler and Schjelderup (2000), Janeba (2000), Mintz and Smart (2004), Peralta, Wauthy, and van Ypersele (2006) and Bucovetsky and Haufler (2008).

<sup>9</sup>See Bartelsman and Beetsma (2003), Clausing (2003), Mintz and Smart (2004), Huizinga and Laeven (2008), Grubert and Mutti (1991), Hines and Rice (1994), Hines (1999) and Egger, Eggert, Winner, and Keuschnigg (2008).

<sup>10</sup>Theoretical discussions on the role of tax havens are provided by Hong and Smart (forthcoming) and Slemrod and Wilson (2006).

<sup>11</sup>See e.g. Kind, Knarvik, and Schjelderup (2000), Ludema and Wooton (2000), Baldwin and Krugman (2004), Borck and Pflueger (2006) and Ottaviano and van Ypersele (2005)

competition and firm heterogeneity in productivity. They do not derive the equilibrium of the tax game. Instead they assume an exogenous tax difference and focus their analysis on the trade-off between base-widening and rate-lowering tax reforms.

Davies and Eckel (2010) also propose an NEG-type model of tax competition with heterogeneous firms and endogenous location choice. They show that in the case of symmetric countries no equilibrium in pure strategies exists. Due to the complexity of their model an equilibrium does only exist under very particular conditions.

In recent work, Haufler and Stähler (2009) build on this and provide a model of tax competition with heterogeneous firms and endogenous firm location that is sufficiently simple to prove the existence of an equilibrium if countries are sufficiently asymmetric. They show that an increase in demand increases tax competition. Since they keep the endogenous location choice, their model remains too complex to derive equilibrium tax rates and revenues in closed form.

The remainder of the paper is structured as follows. Section two presents the case of a large country in financial autarky. Section three introduces profit shifting. The equilibrium is derived in section four. Section five discusses the main results. Section six concludes.

## 1.2 Financial Autarky

We first outline the structure of the large country in financial autarky. Labor is the only input in production. There is a unit mass of workers each of which inelastically supplies one unit of labor. There are two sectors, one producing varieties of a differentiated good and one producing a homogeneous good with constant returns to scale. The homogeneous good is used as the numeraire with its price normalized to one. We only consider equilibria in which the homogeneous good is produced. This implies that wages are unity. There is a fixed and exogenous measure of firms that are owned by consumers in the large country.

**Preferences:** The workers are all identical and share the same quasi-linear preferences over consumption of the two goods and a good provided by the government:

$$U = \alpha \ln Q + \beta G + q_0 \quad \text{with} \quad Q = \left( \int_{\Omega} q(\omega)^{\frac{\sigma-1}{\sigma}} d\omega \right)^{\frac{\sigma}{\sigma-1}}, \quad (1.1)$$

where  $q(\omega)$  is the quantity consumed of variety  $\omega$ . The elasticity of substitution between varieties is given by  $\sigma > 1$  and  $Q$  thus represents consumption of a preference weighted basket of differentiated goods.  $G$  is the quantity of a public good provided by the government. The consumption of the numeraire good is given by  $q_0$ .  $\alpha$  and  $\beta$  are parameters with  $0 < \alpha < 1 < \beta$ .<sup>12</sup> The parameter  $\beta$  represents marginal utility from the public good while marginal utility from the private good is unity. Setting  $\beta > 1$  assures that the government has an incentive to provide the public good also when the distortion from tax competition is introduced.<sup>13</sup> Demand for one particular variety is:

$$q(\omega) = \frac{p(\omega)^{-\sigma}}{P^{-\sigma}} Q. \quad (1.2)$$

$p(\omega)$  is the price of variety  $\omega$ , the aggregate price index of the differentiated goods sector is given by  $P = \left( \int_0^{a_m} p(a)^{1-\sigma} dF(a) \right)^{\frac{1}{1-\sigma}}$  and  $Q = \alpha/P$ .<sup>14</sup>

**The government:** The only tax instrument of the government is a proportional tax on the profits of firms in the home country.<sup>15</sup> Tax income is used to provide government services  $G$  to the consumers. The government can transform one unit of the numeraire good into one unit of the government services. It is assumed to maximize welfare of its own citizens.

**Firms:** In the homogeneous good sector firms produce with a constant returns to scale technology and earn zero profits. There is a fixed and exogenous measure of firms in the differentiated good sector that is without loss of generality normalized to one. Each firm produces a different variety. Firms differ in their levels of marginal cost, which is constant for each firm. We assume that these marginal cost levels are distributed Pareto on  $[0, a_m]$  with the distribution function given by:

$$F(a) = \left( \frac{a}{a_m} \right)^\gamma$$

<sup>12</sup>These preferences are similar to those used in Baldwin and Okubo (2006). To obtain closed form solutions, we assume linear utility from the public good.

<sup>13</sup>Although we do not formally impose an upper bound on it, we think of this marginal utility to be reasonably close to unity in particular to be smaller than two.  $\beta = 2$  would imply that a unit of the numeraire spent by the government yields twice the utility of one unit spent on private consumption.

<sup>14</sup> $F(a)$  is the distribution function of cost levels of firms and  $a_m$  is the maximum cost level.

<sup>15</sup>Ottaviano and van Ypersele (2005) add a lump sum tax on labor, which leads to negative tax rates on capital. This would not be the case in our model. As firms cannot shift production, in our setting a lump sum tax on labor would make the capital tax redundant.

where  $a_m$  is the highest marginal cost level. The degree of firm heterogeneity (i.e. the variance of the cost distribution) is determined by  $a_m$  and the shape parameter  $\gamma$  of the Pareto distribution. When the parameter  $\gamma$  is high a large mass of firms is located at the high cost levels. When  $\gamma$  is low, there are relatively many low cost firms.<sup>16</sup> We assume  $\gamma > \sigma - 1$  in order for aggregate profits to be finite. There is no fixed cost of production for firms, so in equilibrium all firms produce.

Firms in the differentiated good sector charge a constant mark-up over marginal cost:

$$p(a) = \frac{\sigma}{\sigma - 1} a. \quad (1.3)$$

The level of the mark-up depends on the elasticity of substitution between varieties. When  $\sigma$  is high, firms have a low degree of monopolistic market power and can only charge a low mark-up.

A firm's gross profits are given by:

$$\pi(a) = a^{1-\sigma} T_1 \quad \text{with} \quad T_1 = \frac{\alpha}{\sigma} \left( \frac{\gamma - (\sigma - 1)}{\gamma} \right) a_m^{\sigma-1}. \quad (1.4)$$

$T_1$  is a constant that only depends on parameters of the model.<sup>17</sup>

Net profits are given by  $\pi(a)^{net} = (1 - t) \pi(a)$ , where  $t \in [0, 1]$  is a tax rate set by the government taken as given by the firm. Firm choices that maximize gross profits also maximize net profits. The tax is thus not distorting the optimal production decision of firms.

In financial autarky all firms pay taxes at home. The tax base is thus given by aggregate profits of firms:

$$\Pi_H^A = \int_0^{a_m} \pi(a) dF(a) = \frac{\alpha}{\sigma} \quad (1.5)$$

which is constant.

**Optimal tax rate in autarky:** Households have income from labor and receive the net profits of firms in their country. In autarky the aggregate income  $I^A$  of consumers is

<sup>16</sup>The relation between  $\gamma$  and firm heterogeneity (measured as the variance of the cost distribution) is discussed in the proof of Proposition 1.1.

<sup>17</sup> $T_1 = \left( \frac{\sigma-1}{\sigma} \right)^{\sigma-1} \frac{\alpha P^{\sigma-1}}{\sigma}$ . The price index is defined as  $P = \left( \int_0^{a_m} p(a)^{1-\sigma} dF(a) \right)^{\frac{1}{1-\sigma}}$ . Evaluating the integral using (1.3) leads to  $P = \frac{\sigma}{\sigma-1} a_m \left( \frac{\gamma}{\gamma - (\sigma-1)} \right)^{\frac{1}{1-\sigma}}$  which is a constant. This implies  $T_1 = \frac{\alpha}{\sigma} \left( \frac{\gamma - (\sigma-1)}{\gamma} \right) a_m^{\sigma-1}$ .

thus:

$$I^A = L + (1 - t_H^A)\Pi_H^A.$$

Welfare in financial autarky is:

$$U^A = \bar{U} + (1 - t_H^A)\Pi_H^A + \beta t_H^A \Pi_H^A. \quad (1.6)$$

$\bar{U} \equiv \alpha \ln\left(\frac{\alpha}{P}\right) - \alpha + 1$  collects terms that are unaffected by the taxation decision. The first term in  $\bar{U}$  reflects utility of consuming the basket of differentiated products,  $\alpha$  is the cost of this basket and 1 is labor income. The second term in (1.6) are profits retained by consumers. The last term represents utility from the consumption of the public good. In financial autarky the welfare maximizing tax rate of the large country is then given by  $t_H^A = 1$ . Since no outflows of tax base are possible in financial autarky,  $\beta > 1$  implies that it is optimal for the government to collect the highest possible amount of tax revenues.<sup>18</sup>

### 1.3 Profit Shifting

Next we consider the case where firms in the large country have the possibility to open an affiliate in the tax haven, which allows them to shift profits abroad. These profits are then taxed according to the tax rate in the tax haven, but not at home, where the firm declares zero profits. Opening an affiliate in the tax haven requires paying a fixed cost  $f_t$ .

**Individual firm behavior and the tax base:** Whether an individual firm chooses to pay the fixed cost of shifting profits abroad depends on the tax differential and on the level of profits the firm generates. Firms with a lower marginal cost have higher profits and are thus more likely to pay the fixed cost of ‘profit-shifting’ FDI.

We define the ‘profit shifting cutoff cost level’ as the cost level  $a^*$  for which a firm is indifferent between paying taxes at home and paying taxes in the tax haven. Note that profit shifting only takes place for a positive tax difference  $\rho = t_H - t_X > 0$ . In this case

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<sup>18</sup>This is a very stylized result. We have chosen the simplest possible way to create an incentive to collect taxes without adding any trade-off in autarky. This preserves tractability when the trade-off we are interested in is introduced with profit shifting: the trade-off between the intensive margin and the extensive margin. These margins are discussed in detail in subsection 1.5.1. With profit shifting an interior solution exists on which we focus our analysis. For this, only the tax difference is important.



the cutoff cost level is determined by the following condition:

$$(1 - t_H) \pi(a^*) = (1 - t_X) \pi(a^*) - f_t.$$

Where  $\pi(a^*)$  are gross profits of a firm with marginal cost of  $a^*$ ,  $t_H$  is the domestic tax rate and  $t_X$  is the rate set by the tax haven. When the tax difference is zero or negative, no profit shifting takes place. Rewriting the cutoff condition gives:

$$\pi(a^*) = f_t / \rho \tag{1.7}$$

with the corresponding cost cutoff level:

$$a^* = \left( \frac{\rho T_1}{f_t} \right)^{\frac{1}{\sigma-1}}. \tag{1.8}$$

Under financial integration the most productive firms (with a cost level below  $a^*$ ) self-select into profit-shifting FDI. The mass of firms is thus endogenously split into multinationals and domestic firms. The measure of profit shifting firms is:

$$N_x = F(a^*) = (a^*)^\gamma a_m^{-\gamma} \tag{1.9}$$

This productivity sorting is in line with the empirical evidence on the determinants of the use of tax haven operations.

**Tax base:** The tax base in the home country is given by aggregate profits of firms that have not become multinationals and thus pay taxes at home:  $\Pi_H = \int_{a^*}^{a_m} \pi(a) dF(a)$ . The tax base taxed in the tax haven is given by  $\Pi_X = \int_0^{a^*} \pi(a) dF(a)$ . Evaluating the integrals leads to:

$$\Pi_X = \left( \frac{\alpha}{\sigma} \right)^{\epsilon+1} \left( \frac{\rho}{f_t} \frac{\epsilon}{\epsilon+1} \right)^\epsilon \tag{1.10}$$

$$\Pi_H = \Pi_H^A - \Pi_X \tag{1.11}$$

with  $\epsilon \equiv \frac{\gamma}{\sigma-1} - 1$ . Thus the tax base flowing to the tax haven only depends on constant terms and the tax difference.

$\epsilon$  combines two of the crucial parameters of the model: the shape parameter of the cost distribution and the elasticity of substitution between varieties. Recall that above we

have assumed that  $\gamma > (\sigma - 1)$  which implies  $\epsilon > 0$ .

**Household Income:** In addition to their income from labor, households receive the net profits of firms paying taxes at home and of firms paying taxes in the tax haven. Under financial integration, the aggregate income  $I$  of consumers is thus given by:

$$I = L + (1 - t_H)\Pi_H + (1 - t_X)\Pi_X - N_x f_t, \quad (1.12)$$

where the last term accounts for the fact that net profits of firms paying taxes in the tax haven are also net of the fixed cost payed to do profit shifting FDI.<sup>19</sup>

**Governments:** Under financial integration governments have to take into account the tax rate set in the other jurisdiction. Taxes are set in a simultaneous one-shot game.<sup>20</sup> To analyze the tax game, we first derive the best response functions of the two governments.

## 1.4 Equilibrium under Financial Integration

As in financial autarky, the only variable governments can set are the profit tax rates in their jurisdictions. In this section we derive the best response functions of the two governments in the international tax game.

### 1.4.1 Optimization of the Tax Haven

The structure of the tax haven is kept as simple as possible. It does not have a tax base of its own. Its only source of revenues stems from taxing multinational companies that have an affiliate in the tax haven. Taking the tax rate in the large country as given, the tax haven maximizes total revenues  $V = t_X \Pi_X$ .

One can think of the tax haven as the limit case of a very small country. The measure of differentiated goods firms is proportional to the mass of consumers, which are close to zero. In this case the tax haven's own tax base is 'almost zero'. The same holds for demand for the differentiated good (imported from the large country with zero trade cost). Then welfare maximization of the government in the tax haven is equivalent to maximization of tax revenues.

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<sup>19</sup>We assume that the fixed cost of tax avoidance is not tax-deductible, i.e. profits are not taxed net of these fixed costs.

<sup>20</sup>The case where one country has a first mover advantage is discussed in a complementary appendix available upon request.

The attracted tax base  $\Pi_X$  is only positive if the tax haven sets a lower tax rate than the large country. Thus for any given (positive) tax rate of the large country  $t_H$ , it will always be optimal for the tax haven to undercut, so that  $\rho > 0$ . Revenue maximization leads to:

$$t_X = \frac{t_H}{\epsilon + 1} = \frac{\sigma - 1}{\gamma} t_H = \min \left\{ \frac{\rho}{\epsilon}; \frac{1}{\epsilon + 1} \right\}, \quad (1.13)$$

where the *min* reflects the fact that  $t_H$  is bounded from above by unity. Note that  $\epsilon + 1 = \gamma/(\sigma - 1) > 1$ . The tax haven sets a tax rate that is a constant fraction of the rate of the large country. The extent to which the tax haven undercuts the large country is determined by the degree of firm heterogeneity and the elasticity of substitution.<sup>21</sup> We can now state the following proposition:

**Proposition 1.1** *Under financial integration when firms have the possibility to do profit shifting,*

- (i) *the tax haven always undercuts the large country.*
- (ii) *the undercutting is the stronger, the lower the degree of firm heterogeneity (higher  $\gamma$ ) and the stronger the market power of individual firms (lower  $\sigma$ ).*

**Proof:** see Appendix A.1.

## 1.4.2 Optimization of the Large Country

For any given tax rate of the tax haven  $t_X$ , the government of the large country sets its tax rate  $t_H$  to maximize welfare of its citizens  $U(t_H, t_X)$ .

For certain parameter values the best response function of the large country is discontinuous. In these cases we find that there is a threshold level of  $t_X$  depending on parameters. Below this threshold level the large country chooses a tax rate implying a strictly positive tax difference. Above, the large country chooses a tax difference of zero, i.e.  $t_H = t_X$  which, given (1.10) can never be an equilibrium. The discontinuity in the response function as well as the conditions for equilibrium existence are discussed in detail in Appendix A.3. There we show that for empirically relevant parameter values the

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<sup>21</sup>To analyze the impact of firm heterogeneity we consider a mean preserving spread of the cost distribution, which involves a variation in  $\gamma$  and the maximum cost level  $a_m$ . We show in the proof of Proposition A.1 that for variables that are independent of  $a_m$  an increase in the shape parameter  $\gamma$  is identical to a decrease in firm heterogeneity. This is the case in equation (1.13).

equilibrium exists.

Under financial integration the implicit best response of the large country for a given  $t_X$  is:

$$t_H^{\rho > 0} = \min \left\{ \frac{(\beta - 1)(T_2 - \rho^\epsilon)}{\epsilon \beta \rho^{\epsilon-1}}; 1 \right\} \quad (1.14)$$

as long as the implied value of  $t_H$  is large enough to satisfy:

$$\rho^\epsilon \geq \frac{(1 - \epsilon)(\beta - 1)}{\frac{\epsilon}{\epsilon+1} + (\beta - 1)} T_2, \quad (1.15)$$

with  $T_2 \equiv f_t^\epsilon T_1^{-\epsilon} a_m^{\gamma - (\sigma-1)}$ . Otherwise the best response is given by:

$$t_H^{\rho \leq 0} = t_X. \quad (1.16)$$

These results are derived in Appendix A.1. Appendix A.2 states and derives a condition on  $t_X$  which is analog to (15). If  $t_X$  satisfies this condition, the large country optimally sets its tax rate according to (14).

### 1.4.3 Equilibrium of the Tax Game

We now turn to the equilibrium of the tax game.

**Proposition 1.2** (i) *An equilibrium of the tax game exists iff*

$$\beta (\epsilon^3 + 2\epsilon^2 - 1) - \epsilon^2 \geq 0 \quad \text{or} \quad \left( \frac{\epsilon + 1}{\epsilon} \right)^2 \left( \frac{\beta - 1}{\epsilon\beta + 2\beta - 1} \right)^{\frac{1}{\epsilon}} \frac{\sigma}{\alpha} f_t \geq 1. \quad (1.17)$$

(ii) *The equilibrium tax rates are then given by:*

$$t_H^* = \min \left\{ \left( \frac{\epsilon + 1}{\epsilon} \right)^2 \left( \frac{\beta - 1}{\epsilon\beta + 2\beta - 1} \right)^{\frac{1}{\epsilon}} \frac{\sigma}{\alpha} f_t, 1 \right\}. \quad (1.18)$$

$$t_X^* = \min \left\{ \frac{\epsilon + 1}{\epsilon^2} \left( \frac{\beta - 1}{\epsilon\beta + 2\beta - 1} \right)^{\frac{1}{\epsilon}} \frac{\sigma}{\alpha} f_t, \frac{1}{\epsilon + 1} \right\}. \quad (1.19)$$

**Proof:** The equilibrium tax difference  $\rho^*$  can be derived taking the difference of (1.14)

and (1.13) and solving for the tax difference:

$$\rho^* = \min \left\{ \frac{\epsilon + 1}{\epsilon} \left( \frac{\beta - 1}{\epsilon\beta + 2\beta - 1} \right)^{\frac{1}{\epsilon}} \frac{\sigma}{\alpha} f_t, \frac{\epsilon}{\epsilon + 1} \right\}. \quad (1.20)$$

The first condition in (i) follows directly from plugging (1.20) into condition (1.15) and simplifying which is the relevant condition for interior solutions.<sup>22</sup> Before turning to the second condition, note that the equilibrium tax rates in (ii) follow directly from combining (1.20) and (1.13). Now it is obvious from (1.18) that the second condition in (i) is necessary and sufficient for the economy to be in a corner solution of  $t_H^* = 1$ . Thus when at least one of the two conditions in (i) holds, equilibrium existence is assured. **q.e.d.**

Based on Proposition 1.2, we can derive all relevant equilibrium objects for interior solutions. The equilibrium cost cutoff is given by:

$$a^{**} = \left( \frac{\beta - 1}{\epsilon\beta + 2\beta - 1} \right)^{\frac{1}{\epsilon(\sigma-1)}} a_m. \quad (1.21)$$

The equilibrium number of firms choosing ‘profit shifting’ FDI is:

$$N_X^* = \left( \frac{\beta - 1}{\epsilon\beta + 2\beta - 1} \right)^{\frac{\epsilon+1}{\epsilon}}. \quad (1.22)$$

The tax base that flows to the tax haven in equilibrium is:

$$\Pi_X^* = \frac{\beta - 1}{\epsilon\beta + 2\beta - 1} \frac{\alpha}{\sigma}. \quad (1.23)$$

Overall government income from taxation is then given by:

$$G_H^* = \frac{\beta - 1}{\beta} \frac{(\epsilon + 1)^3}{\epsilon^2} \left( \frac{\beta - 1}{\epsilon\beta + 2\beta - 1} \right)^{\frac{\epsilon+1}{\epsilon}} f_t. \quad (1.24)$$

For some parameter values (e.g. very high fixed costs of profit shifting) the government would like to set a tax rate larger one. In this case there is a corner solution with  $t_H = 1$  and  $t_X = \frac{1}{\epsilon+1}$ . We focus our analysis on interior solutions. For completeness we report all relevant variables for the corner solution in appendix A.4.<sup>23</sup>

<sup>22</sup>This result allows to narrow down the possible range for  $\epsilon$  in our model. Given  $\beta > 1$  we find a lower bound for existence of an interior equilibrium at about 0.618. This can be obtained by solving the equation  $-1 + 2\epsilon^2 + \epsilon^3 = 0$ , which represents the limit case for  $\beta \rightarrow \infty$ .

<sup>23</sup>In the corner solution, the trade-off between the intensive- and extensive margin (as defined in subsection 1.5.1) is distorted as the government cannot set  $t_H > 1$ .

## 1.5 The Role of Industry Structure and Market Power

### 1.5.1 Margins of Taxation

Along the best response functions (i.e. for a given foreign tax rate) the overall effect of a change in the own tax rate on tax revenues is determined by the own-tax rate elasticities.

For the tax haven and the large country these elasticities are given by:

$$\frac{d(\Pi_X t_X)}{dt_X} \frac{1}{\Pi_X} = 1 - \epsilon \frac{t_X}{\rho} \quad \text{and} \quad \frac{d(\Pi_H t_H)}{dt_H} \frac{1}{\Pi_H} = 1 - \epsilon \frac{t_H}{\rho} \frac{\Pi_X}{\Pi_H}. \quad (1.25)$$

This implies that for a given tax rate of the other country, a one percent increase in the own tax rate changes tax revenues by  $\left(1 - \epsilon \frac{t_X}{\rho}\right)$  percent for the tax haven and by  $\left(1 - \epsilon \frac{t_H}{\rho} \frac{\Pi_X}{\Pi_H}\right)$  percent for the large country. These elasticities can be decomposed into an *intensive margin* and an *extensive margin* of taxation:

**Proposition 1.3** *Under financial integration when firms have the possibility to do profit shifting,*

(i) *For a given tax rate of the large country, the own-tax rate elasticity of tax revenues of the tax haven can be decomposed into the intensive margin  $IM_X = 1$  and the extensive margin  $EM_X = -\epsilon \frac{t_X}{\rho}$ .*

(ii) *For a given tax rate of the tax haven, the own-tax rate elasticity of tax revenues of the large country can be decomposed into the intensive margin  $IM_H = 1$  and the extensive margin  $EM_H = -\epsilon \frac{t_H}{\rho} \frac{\Pi_X}{\Pi_H}$ .*

**Proof:** see appendix A.1

Consider the problem of the large country. For a given tax rate of the tax haven, increasing the tax rate has two opposing effects on tax revenues. On the one hand firms which continue paying taxes at home increase their tax payments. Holding the set of firms constant, this increase in tax revenues represents the intensive margin of taxation. The intensive margin in our model is very simple: holding the tax base constant a one percent increase in the tax rate increases revenues by one percent.<sup>24</sup> On the other hand

<sup>24</sup>In principle, it is possible to consider richer setups implying a more complex intensive margin. We could for example generalize our model allowing firms to choose the fraction of their profits to be shifted abroad. Consider the case of a convex shifting cost which is increasing in the fraction shifted and

a higher tax rate gives an incentive for some firms to start shifting profits abroad. This reduces the tax base and thereby decreases tax revenues. This is the extensive margin of taxation. Along their best response functions, both governments optimally trade off the impact of the intensive and extensive margins on their tax revenues.<sup>25</sup>

## 1.5.2 Industry Structure and Tax Competition

In this subsection we analyze the impact of firm heterogeneity on the degree of tax competition and the equilibrium tax difference. We consider two measures for the strength of tax competition. The fraction of the tax base leaving the country in equilibrium,  $\Pi_X^*/\Pi_H^A$  and the deviation of the equilibrium tax rate of the large country  $t_H^*$  from its autarky level. The former is a very direct measure of the equilibrium effects of tax competition on the large country. The latter measures to which extent tax competition pushes the large country to deviate from its optimal autarky tax rate of 1.<sup>26</sup> We find that the degree of firm heterogeneity does indeed affect the degree of tax competition the large country is facing as well as the equilibrium tax difference.

**Proposition 1.4** *Under financial integration when firms have the possibility to do profit shifting,*

- (i) *the degree of tax competition measured as  $\Pi_X^*/\Pi_H^A$  is higher when firms are more heterogeneous (low  $\gamma$ ) and when monopolistic market power of firms is low (high  $\sigma$ ).*
- (ii) *for  $\beta < 2$ , the degree of tax competition measured as the deviation of the equilibrium tax rate of the large country from its autarky level,  $1 - t_H^*$  is higher when firms are more heterogeneous (low  $\gamma$ ).*
- (iii) *When firms are more heterogeneous (low  $\gamma$ ) the equilibrium tax difference  $\rho$  decreases.*

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proportional to profits. Then, the intensive margin becomes smaller as firms respond to an increase in the tax rate by adjusting the fraction of shifted profits. In such a setup the impact of firm heterogeneity on the extensive margin does not change qualitatively. Derivations of the intensive and extensive margin of taxation for this extended model variant are available upon request from the authors.

<sup>25</sup>The tradeoff between the intensive and extensive margin determines the level of tax revenues. For the tax haven welfare maximization implies revenue maximization. The government of the large country faces an additional tradeoff between public and private income. There, welfare depends not only on the level of collected taxes but also on the level of private consumption crowded out by taxation.

<sup>26</sup>In the working paper version of this paper (Krautheim and Schmidt-Eisenlohr (2009)) we also analyze tax revenues of the large country, which is a combination of the other two measures.

**Proof:** see appendix A.1

Firm heterogeneity matters for the degree of tax competition as it determines the shape of the distribution of profits across firms, which represent the tax base. The first two graphs in Figure 1 illustrate this point. The first graph shows a case where tax competition is strong. When firms are very heterogeneous there are relatively many very productive firms (dashed line) which accordingly account for a relatively large fraction of the tax base (solid line). Since these firms are the first ones to shift profits, a given tax difference leads *ceteris paribus* to strong outflows of the tax base. This allows the tax haven to attract a sizeable fraction of the tax base by setting only a moderate tax difference.

The case of relatively homogeneous firms is illustrated in the graph on the right: the mass of firms is concentrated at the high cost levels. It is thus difficult for the tax haven to attract tax base. This leads to lower equilibrium outflows, allows the large country to set a higher tax rate and forces the tax haven to set a low tax rate to attract a sufficient tax base.

In the two graphs in Figure 2 the fraction of the tax base shifted as well as the equilibrium tax rates for different degrees of firm heterogeneity are plotted. A higher shape parameter  $\gamma$ , corresponds to higher firms heterogeneity.<sup>27</sup> The solid line represents the fraction of shifted profits, which decreases as firms become more homogeneous. The dashed line represents the number of firms accounting for these profits. The last graph shows the equilibrium tax rates for different values of the shape parameter  $\gamma$ . As firms become more homogeneous (higher  $\gamma$ ), the large country can set a higher tax rate. The tax haven undercuts more strongly, which leads to a larger tax difference.

### 1.5.3 Fixed Cost of Profit Shifting and Tax Competition

The fixed costs of profit shifting play a central role in our model as they affect the cutoff between firms paying taxes at home and abroad. In this subsection we discuss in more detail which equilibrium values get affected by  $f_t$  and which are independent of this parameter.

First note that both tax rates are linearly increasing in the fixed costs. Suppose fixed costs increase. Then it is more costly for firms to avoid taxes and, for given tax rates,

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<sup>27</sup>This corresponds to a mean-preserving spread, see footnote 21.



less firms are shifting their profits abroad. In this situation the large country raises its tax rate in order to optimally trade off the intensive and extensive margins of taxation. As the tax haven undercuts by a constant fraction its tax rate goes up as well while the absolute tax difference  $\rho$  increases.

This increase in the absolute tax difference exactly offsets the effect of the initial increase in the fixed costs such that the cutoff cost level  $a^{**}$  does not change. Thus for interior solutions the cutoff cost level  $a^{**}$ , the share of firms shifting profits  $N_X^*$  and the tax bases  $\Pi_H^*$  and  $\Pi_X^*$  are independent of the fixed costs  $f_t$ . With an increasing tax rate and an unchanged tax base, equilibrium tax income  $G_H^*$  increases.

### 1.5.4 Homogeneous Firms as the Limiting Case

In this subsection we provide a brief discussion of the limit case for  $\gamma \rightarrow \infty$ . In this case our model collapses to a model with homogeneous firms.

Consider first the case with homogeneous firms: either all firms shift profits or all firms pay taxes at home. Assume that indifferent firms do not do profit shifting. For any given  $t_X$ , the large country optimally sets a tax rate such that firms are indifferent i.e. the zero cutoff profit condition holds with equality i.e.  $\rho\pi = f_t \Leftrightarrow \rho = \frac{\sigma}{\alpha}f_t$ . The tax haven always tries to undercut sufficiently in order to attract tax base, but its tax rate is bounded from below by zero. Thus the large country sets a tax rate that ensures that all firms pay taxes at home even for  $t_X = 0$ . This implies an optimal limit tax of  $t_H^{*h} = \frac{\sigma}{\alpha}f_t$ .

In equilibrium the whole tax base stays in the large country. Thus:  $\Pi_H^{*h} = \frac{\alpha}{\sigma}$  and total tax income is  $G^{*h} = f_t$ . As should be expected these results coincide with the limiting result of our heterogenous firms model for  $\gamma \rightarrow \infty$ , i.e.:  $t_H^{*lim} = \lim_{\gamma \rightarrow \infty} t_H^* = \frac{\sigma}{\alpha}f_t$ ;  $\Pi_H^{*lim} = \lim_{\gamma \rightarrow \infty} \Pi_H^* = \frac{\alpha}{\sigma}$  and  $(t_H^* \Pi_H^*)^{*lim} = \lim_{\gamma \rightarrow \infty} t_H^* \Pi_H^* = f_t$ .

Finally note that the tax base and total tax income are always larger with homogenous firms than with heterogenous firms.<sup>28</sup> This complements the results in Proposition 1.4: the more homogeneous firms are, the lower is the degree of tax competition. In the limit case of homogeneous firms, tax competition is lowest.

<sup>28</sup>To see this note that differences between the limit values and interior values for government income are

$t_H^* \Pi_H^* - (t_H^* \Pi_H^*)^{*lim} = f_t \left( 1 - \frac{\beta(1+\epsilon)^3 \left( \frac{\beta-1}{\epsilon\beta+2\beta-1} \right)^{1+\frac{1}{\epsilon}}}{(\beta-1)\epsilon^2} \right)$ . This expression is always positive. Thus government income with homogenous firms is always larger than with heterogenous firms. The same holds for the tax base.

## 1.6 Conclusions

In this paper we provide a benchmark model of tax competition with heterogeneous firms. It captures two features of the data. First, larger firms tend to use tax havens operations more intensively. Second, tax havens play a central role for international tax planning strategies of multinational firms. In line with empirical evidence we consider firms with heterogeneous marginal productivities and thus heterogeneous profits. We achieve analytical tractability by focusing on the case of ‘very’ asymmetric countries: a large country and a tax haven.

We derive expressions for equilibrium tax rates, equilibrium tax base and equilibrium government revenues in closed form. This allows us to analyze the effects of different variables on the equilibrium allocations. We show that stronger firm heterogeneity (a mean-preserving spread of the cost distribution) increases the degree of tax competition: it decreases the equilibrium tax rate of the large country, leads to higher outflows of its tax base and thus decreases its equilibrium tax revenues. Similar effects hold for a higher substitutability across varieties. The rationale behind these results is that firm heterogeneity and monopolistic market power shape the distribution of aggregate profits, the tax base. Since more profitable firms are more prone to shift profits abroad, the two governments face a tradeoff between the intensive and the extensive margins of taxation.

Our benchmark model could be extended in several ways. Two large but asymmetric countries could be considered. Allowing for firm entry and endogenous reallocation would give rise to an interesting tradeoff. While a country could lose part of its tax base due to profit shifting, this could be the only way for it to keep production of very productive firms at home.<sup>29</sup>

Another extension would be to enlarge the set of tax instruments. Firm heterogeneity creates a problem for a government as it cannot discriminate between firms with different productivity levels. Thus any instrument that allows the government to treat firms asymmetrically may alleviate the pressures from tax competition. An analysis of this extension would very likely require numerical solution methods.

Finally our benchmark model provides a starting point to test empirically the effect of firm heterogeneity on tax rates chosen by governments. According to our model countries with more heterogeneous firms should, *ceteris paribus*, set lower tax rates. In addition,

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<sup>29</sup>Due to the complexity of a setup with two asymmetric countries, we conjecture that this case could only be analyzed numerically. For this the closed form solutions derived in this paper could provide a valuable benchmark.

the model implies that the government would like to impose different tax rates in different sectors. If this is not feasible in practice, the government could still impose rules e.g. on deductability of capital investment or depreciation rules that would affect sectors with different capital structure differently.

# Appendix A

## Appendix to Chapter 1

### A.1 Proofs

**Proof of Proposition 1.1** The standard way to analyze the impact of firm heterogeneity is to consider a mean-preserving spread of the cost distribution. The variance of the Pareto distributed cost levels is  $Var(a) = \frac{\gamma}{(\gamma+1)^2(\gamma+2)}a_m^2$ . A mean preserving spread implies that when  $\gamma$  is varied,  $a_m$  is adjusted such that the mean stays constant. The expected value of the Pareto distributed cost levels is given by  $E(a) = \frac{\gamma}{\gamma+1}a_m$ . To keep  $E(a)$  constant at  $E$ ,  $a_m$  needs to be adjusted according to  $a_m = \frac{\gamma+1}{\gamma}E$ . Plugging this into the definition of the variance, we find  $Var(a) = \frac{E^2}{\gamma(\gamma+2)}$ . It can be seen that the Variance decreases in  $\gamma$ .

The impact of firm heterogeneity on any variable in our model can thus be studied by replacing all expressions for  $a_m$  by the equation given above and then taking the derivative with respect to  $\gamma$ . To analyze the impact of firm heterogeneity on any object that is *independent* of  $a_m$ , it is therefore sufficient to simply take the derivative with respect to  $\gamma$ .

We can now prove (i) and (ii). (i) follows from the fact that for  $t_H > 0$ , a tax rate of  $t_X \geq t_H$  implies  $\Pi_X = 0$  and thus  $V = 0$ , while any  $0 < t_X < t_H$  implies  $\Pi_X > 0$  and thus  $V > 0$ .

It follows directly from (1.13) that a higher  $\gamma$  and a lower  $\sigma$  imply stronger undercutting. Since the optimal tax rate in (1.13) does not depend on  $a_m$  (as  $t_H$  is taken as given), a higher  $\gamma$  corresponds to lower firm heterogeneity. This proves (ii). **q.e.d.**

**Proofs for the Discontinuous Response Function** This appendix derives the discontinuous best response function of the large country as well as parameter condition

(1.15) which determines whether the best response function is given by (1.14) or (1.16). We first derive equations (1.16) and (1.14). They represent two different cases which we discuss separately. In case 1 the large country sets a tax rate below or equal the rate of the tax haven. In case 2 it sets a higher rate. Then we show that condition (1.15) determines which of the two cases represents the best response.

**Case 1:** First consider the case in which the government sets a tax rate below or equal to the tax haven's rate. All firms then pay taxes at home. Total welfare in this case is:

$$U^{\rho \leq 0}(t_H, t_X) = \bar{U} + (1 - t_H) \Pi_H^A + \beta t_H \Pi_H^A. \quad (\text{A.1})$$

Since the government values public expenditure more than expenditure on the homogeneous good by its citizens ( $\beta > 1$ ) it sets the highest possible tax rate that satisfies  $t_H \leq t_X$ .<sup>1</sup> This tax rate is given by (1.16).

**Case 2:** Now consider the case where the government sets a tax rate that satisfies  $t_H > t_X$ . Welfare in the large country is then given by

$$U^{\rho > 0}(t_H, t_X) = \bar{U} + (1 - t_H) \Pi_H + (1 - t_X) \Pi_X - N_X f_t + \beta t_H \Pi_H, \quad (\text{A.2})$$

where  $\Pi_H = \Pi_H^A - \Pi_X$  reflects the fact that some of the tax base in the home country flows to the tax haven when the tax differential is positive.

From the first order condition of the maximization problem, we derive (1.14). Where  $t_X$  and  $t_H^{\rho > 0}$  enter in the tax differential  $\rho$ . Equation (1.14) provides an implicit solution for the best response function of the government which only depends on the tax differential and parameters of the model. To complete the proof, we proceed in two steps. We first derive condition (1.15) under the *assumption* that the second order condition for a welfare maximum holds for (1.14). In a second step we show that this is indeed the case whenever (1.14) is the countries' best response function derived in step 1.

**Step 1:** We first determine when, according to the FOC (assuming the SOC to be satisfied), the large country will set its tax rate according to (1.14). This is the case as long as  $U(t_H^{\rho > 0}, t_X) \geq U(t_H^{\rho \leq 0}, t_X)$ . Plugging (A.2) and (A.1) into this condition, rearranging

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<sup>1</sup>Note that in the autarky equilibrium, the large country also sets the highest possible tax rate under the condition that  $t_H^A \leq 1$ .

and using (1.16) and  $(1 - t_X) - (1 - t_H^{\rho > 0}) = \rho$  we get:

$$\rho (\beta - 1) \Pi_H^A \geq (\beta t_H^{\rho > 0} - \rho) \Pi_X + N_X f_t. \quad (\text{A.3})$$

Now first using  $f_t N_x = \Pi_X \rho \frac{\epsilon}{\epsilon+1}$ , then  $\Pi_H^A = T_3 T_1^{-\epsilon} a_m^{1-\sigma}$  and  $\Pi_X = \rho^\epsilon T_3 f_t^{-\epsilon} a_m^{-\gamma}$ , with  $T_3 = \frac{\epsilon+1}{\epsilon} T_1^{\epsilon+1}$  simplifying and solving for  $t_H^{\rho > 0}$  gives:

$$t_H^{\rho > 0} \leq \frac{(\beta - 1)T_2 + \rho^\epsilon \frac{1}{\epsilon+1}}{\beta \rho^{\epsilon-1}}. \quad (\text{A.4})$$

To see for which values of  $\rho$  the tax rate in (1.14) satisfies condition (A.4), we plug (1.14) into (A.4), which gives:

$$\rho^\epsilon \geq \frac{(1 - \epsilon)(\beta - 1)}{\frac{\epsilon}{\epsilon+1} + (\beta - 1)} T_2.$$

As long as for a given  $t_X$  the  $t_H$  implied by (1.14) is high enough to satisfy this condition, we will have  $U(t_H^{\rho > 0}, t_X) \geq U(t_H^{\rho \leq 0}, t_X)$  and thus the best response function of the large country given by (1.14). When the above condition is violated, we have  $U(t_H^{\rho > 0}, t_X) < U(t_H^{\rho \leq 0}, t_X)$ . In this case the best response of the large country is given by (1.16) because it maximizes  $U(t_H^{\rho \leq 0}, t_X)$ .

**Step 2:** It remains to be shown that for any given  $t_X$  equation (1.14) is a welfare maximum (and not a minimum) for all relevant values of  $t_H$ . Relevant values are all values of  $t_H$  that satisfy (1.15) for a given  $t_X$ .<sup>2</sup>

First note that the second derivative of the welfare function with respect to the tax rate of the large country is given by:

$$\frac{\partial^2 U}{\partial t_H^2} = -2(\beta - 1) \frac{\partial \Pi_X}{\partial t_H} - \frac{\partial^2 \Pi_X}{\partial^2 t_H} [(\beta - 1)t_H + t_X] - \frac{\partial^2 N_x}{\partial^2 t_H} f_t.$$

For notational convenience, we define  $T_4 \equiv T_3 \epsilon \rho^{\epsilon-1} f_t^{-\epsilon} a_m^{-\gamma} > 0$ . We then use  $\frac{\partial \Pi_X}{\partial t_H} = T_4$  together with  $\frac{\partial^2 \Pi_X}{\partial t_H^2} = \frac{\epsilon-1}{\rho} T_4$  and  $\frac{\partial^2 N_x}{\partial t_H^2} f_t = \epsilon T_4$  to get:

$$\begin{aligned} \frac{\partial^2 U}{\partial t_H^2} &= -T_4 \left( 2(\beta - 1) + \epsilon + \frac{\epsilon - 1}{\rho} [(\beta - 1)t_H + t_X] \right) \\ &= -T_4 \left( 2(\beta - 1) + \epsilon - (\epsilon - 1) + (\epsilon - 1) \beta \frac{t_H}{\rho} \right). \end{aligned}$$

<sup>2</sup>All values that do not satisfy (1.15) are irrelevant as in these cases the best response of the large country is given by the ‘case 1’ best response (1.16) anyway.

From (1.14) it follows that  $\frac{t_H}{\rho} = \frac{(\beta-1) T_2}{\epsilon \beta \rho^\epsilon} - \frac{(\beta-1)}{\epsilon \beta}$  so that:

$$\frac{\partial^2 U}{\partial t_H^2} = -T_4 \left( 2(\beta - 1) + \epsilon + \frac{(\beta - 1)(\epsilon - 1)}{\epsilon} \left( \frac{T_2}{\rho^\epsilon} - 1 \right) - \epsilon + 1 \right). \quad (\text{A.5})$$

We need to show that equation (1.15) is a sufficient condition for (A.5) to be negative. To do so, we proceed in two steps. We first show that it is negative when the above condition holds with equality. We then show that this is also true for larger values of  $\rho$ .

Define  $\rho^e$  as the value of  $\rho$ , where (1.15) holds with equality. We will first determine the sign of (A.5) for  $\rho^e$  i.e.  $\frac{\partial^2 U}{\partial t_H^2} \Big|_{\rho=\rho^e}$ .

$$\frac{\partial^2 U}{\partial t_H^2} \Big|_{\rho=\rho^e} = -T_4 \left( 2(\beta - 1) + 1 - T_j - \frac{(\beta - 1)(\epsilon - 1)}{\epsilon} \right), \quad (\text{A.6})$$

$$\text{with } T_j \equiv \frac{(\beta-1)(1-\epsilon)}{\epsilon} \frac{\frac{\epsilon}{\epsilon+1} + (\beta-1)}{(\beta-1)(1-\epsilon)} = \frac{1}{\epsilon+1} + \frac{\beta-1}{\epsilon} > 0$$

Simplifying and recalling that  $T_4 > 0$  then gives:

$$\frac{\partial^2 U}{\partial t_H^2} \Big|_{\rho=\rho^e} = -T_4 \left( (\beta - 1) + \frac{\epsilon}{\epsilon + 1} \right) < 0. \quad (\text{A.7})$$

This implies that for  $\rho = \rho^e$  the second order condition holds and (1.14) is indeed the optimal response.

To see that this is true for all  $\rho \geq \rho^e$ , note that any value of  $\rho \geq \rho^e$  can be written as  $\rho = x \rho^e$  with  $x \geq 1$ . In order to obtain  $\frac{\partial^2 U}{\partial t_H^2} \Big|_{\rho=\rho^e}$ , we have plugged in  $\rho^e$  into (A.5). Now considering any value of  $\rho \geq \rho^e$ , we can plug  $\rho = x \rho^e$  into (A.5).

It can be seen in (A.5) that  $\rho$  enters twice in the expression for  $\frac{\partial^2 U}{\partial t_H^2}$ . Entering via  $T_4$  it does not affect the sign. To see the effect of a higher  $\rho$  on the second term, note that when we use  $\rho = x \rho^e$ ,  $T_j$  in equation (A.6), is replaced by  $T_j \frac{1}{x^\epsilon} < T_j$ . The positive effect of  $T_j$  on the sign of  $\frac{\partial^2 U}{\partial t_H^2}$  is thus dampened for a any  $\rho > \rho^e$ . This shows that (1.15) is indeed a sufficient condition for (1.14) to be a utility maximum. **q.e.d.**

**Proof of Proposition 1.3** The proof of Proposition 1.3 follows Chaney (2008) who decomposes the trade cost elasticity of aggregate trade flows into an intensive and an extensive margin. First note that the tax base of the tax haven is given by  $\Pi_X = \int_0^{a^*} \pi(a) dF(a)$ . Applying the Leibniz rule for differentiation under the integral sign, we can decompose

the own-tax rate elasticity into the intensive and extensive margins of taxation:

$$\frac{d(\Pi_X t_X)}{dt_X} \frac{1}{\Pi_X} = \underbrace{\frac{1}{\Pi_X} \int_0^{a^*} \frac{\partial}{\partial t_X} \left[ t_X \pi(a) \frac{dF(a)}{da} \right] da}_{IM_X} + \underbrace{\frac{1}{\Pi_X} t_X \pi(a^*) \frac{dF(a^*)}{da^*} \frac{da^*}{dt_X}}_{EM_X}. \quad (\text{A.8})$$

The first term on the right hand side, the intensive margin, takes the cutoff level  $a^*$  (and thus the set of tax payers) as constant and varies payments by individual firms. The second term on the right represents the extensive margin. Here, the tax payments of a firm at the cutoff are fixed ( $t_X \pi(a^*)$ ), while the effect of the variation in the set of tax paying firms is accounted for. By Leibniz rule, these two margins add up to the own-tax rate elasticity on the left hand side.

For the tax haven we thus have  $IM_X = 1$  and  $EM_X = -\epsilon \frac{t_X}{\rho}$ . Noting that  $\frac{\partial EM_X}{\partial \gamma} > 0$ , this proves (i).

The tax base of the large country is given by  $\Pi_H = \int_{a^*}^{a_m} \pi(a) dF(a)$  so that the own-tax elasticity can be decomposed using Leibniz rule:<sup>3</sup>

$$\frac{d(\Pi_H t_H)}{dt_H} \frac{1}{\Pi_H} = \underbrace{\frac{1}{\Pi_H} \int_{a^*}^{a_m} \frac{\partial}{\partial t_H} \left[ t_H \pi(a) \frac{dF(a)}{da} \right] da}_{IM_H} - \underbrace{\frac{1}{\Pi_H} t_H \pi(a^*) \frac{dF(a^*)}{da^*} \frac{da^*}{dt_H}}_{EM_H}. \quad (\text{A.9})$$

From this it follows that  $IM_H = 1$  and  $EM_H = -\epsilon \frac{t_H}{\rho} \frac{\Pi_X}{\Pi_H}$  which, together with  $\frac{\partial EM_H}{\partial \gamma} > 0$ , proves (ii). **q.e.d.**

**Proof of Proposition 1.4** Recall that in the proof of Proposition 1.1 we have shown that to analyze the impact of firm heterogeneity on any object that is *independent* of  $a_m$ , it is sufficient to simply take the derivative with respect to  $\gamma$ . This is the case for all equilibrium objects considered in Proposition 1.4.

Note that the fraction of equilibrium outflows from the large country are  $\frac{\Pi_X^*}{\Pi_H^A} = \frac{\beta-1}{\epsilon\beta+2\beta-1}$ . We thus have  $\frac{\partial \Pi_X^*/\Pi_H^A}{\partial \gamma} = -\frac{\beta(\beta-1)}{(\epsilon\beta+2\beta-1)^2(\sigma-1)} > 0$ . This proves the statement on  $\gamma$  and thus on firm heterogeneity in (i). In addition,  $\frac{\partial (\Pi_X^*/\Pi_H^A)}{\partial \sigma} = \frac{(\beta-1)}{(\epsilon\beta+2\beta-1)^2} \frac{\beta\gamma}{(\sigma-1)^2} > 0$ , which proves the statement on  $\sigma$ .

To prove (ii), note that the derivative of  $t_H^*$  with respect to  $\gamma$  is given by:

$$\frac{\partial t_H^*}{\partial \gamma} = -\frac{(\epsilon+1) \left( \frac{\beta-1}{\epsilon\beta+2\beta-1} \right)^{1+\frac{1}{\epsilon}} \left( \epsilon(3\epsilon\beta+5\beta-2) + (\epsilon+1)(\epsilon\beta+2\beta-1) \text{Log} \left[ \frac{\beta-1}{\epsilon\beta+2\beta-1} \right] \right)}{\epsilon^4(\beta-1)(\sigma-1)} \frac{\sigma}{\alpha} f_t.$$

<sup>3</sup>Note that now  $a^*$  is the lower bound of the integral, therefore the second term on the right is subtracted.



The sign of which is equal to the sign of:

$$S_{t_H\gamma}(\beta, \epsilon) = -\epsilon(3\epsilon\beta + 5\beta - 2) - (\epsilon + 1)(\epsilon\beta + 2\beta - 1) \text{Log} \left[ \frac{\beta - 1}{\epsilon\beta + 2\beta - 1} \right]$$

Using numerical minimization, we can determine its global minimum under the restriction of  $\beta < 2$ . The function attains its global minimum at  $\beta \rightarrow 2$  and  $\epsilon \approx 2.40$  with a value of  $\approx 0.72$ , which is positive. This proves (ii).

Finally, we prove (iii) using the equilibrium tax difference  $\rho^* = \frac{\sigma}{\alpha} \frac{\epsilon+1}{\epsilon} \left( \frac{\beta-1}{\epsilon\beta+2\beta-1} \right)^{\frac{1}{\epsilon}} f_t$ . We take the first derivative and simplify to get:

$$\frac{\partial \rho^*}{\partial \gamma} = -\frac{\sigma}{\alpha} \left( \frac{\beta - 1}{\epsilon\beta + 2\beta - 1} \right)^{\frac{\epsilon+1}{\epsilon}} \frac{\epsilon(2\epsilon\beta + 3\beta - 1) + (1 + \epsilon)(\epsilon\beta + 2\beta - 1) \log \left( \frac{\beta-1}{\epsilon\beta+2\beta-1} \right)}{(\beta - 1)(\sigma - 1)\epsilon^3} f_t.$$

This expression has the same sign as:

$$S(\beta, \epsilon) = -\epsilon(2\epsilon\beta + 3\beta - 1) - (1 + \epsilon)(\epsilon\beta + 2\beta - 1) \log \left( \frac{\beta - 1}{\epsilon\beta + 2\beta - 1} \right)$$

We use numerical minimization to determine the global minimum of this expression which for  $\epsilon \rightarrow 0$  and  $\beta \approx 1.3$  is  $S(\beta, \epsilon) \approx 2.68$ . This implies that given the parameter constraints from the model ( $\epsilon > 0$  and  $\beta > 1$ ) this expression is always positive. **q.e.d.**

## A.2 Maximum $t_X$ for Equilibrium Existence

The values of  $t_X$  for which the best response is given by (1.14) is

$$t_X \leq \left( (\beta - 1) \left( \frac{(1 - \epsilon)(\beta - 1)}{\frac{\epsilon}{\epsilon+1} + (\beta - 1)} \right)^{\frac{1-\epsilon}{\epsilon}} - (\beta - 1 + \epsilon\beta) \left( \frac{(1 - \epsilon)(\beta - 1)}{\frac{\epsilon}{\epsilon+1} + (\beta - 1)} \right)^{1/\epsilon} \right) \frac{T_2^{1/\epsilon}}{\epsilon\beta}. \quad (\text{A.10})$$

**Proof:** Define  $\rho^{jump}$  as the tax differential at the discontinuity. For  $\rho^{jump}$ , (1.15) holds with equality. Note that  $t_X^{jump} = t_H^{\rho>0}[\rho^{jump}] - \rho^{jump}$  combining this with (1.14) and (1.15), the value of  $t_X$  for which the best response for the large country switches from  $\rho > 0$  to  $\rho = 0$  is given by

$$t_X^{jump} = \frac{(\beta - 1) T_2}{\epsilon\beta \left( \frac{(1-\epsilon)(\beta-1)}{\frac{\epsilon}{\epsilon+1}+(\beta-1)} T_2 \right)^{\frac{\epsilon-1}{\epsilon}}} - \frac{(\beta - 1) + \epsilon\beta}{\epsilon\beta} \left( \frac{(1 - \epsilon)(\beta - 1)}{\frac{\epsilon}{\epsilon+1} + (\beta - 1)} T_2 \right)^{1/\epsilon}.$$

This can be simplified to

$$t_X^{jump} = \left( (\beta - 1) \left( \frac{(1 - \epsilon)(\beta - 1)}{\frac{\epsilon}{\epsilon + 1} + (\beta - 1)} \right)^{\frac{1 - \epsilon}{\epsilon}} - (\beta - 1 + \epsilon\beta) \left( \frac{(1 - \epsilon)(\beta - 1)}{\frac{\epsilon}{\epsilon + 1} + (\beta - 1)} \right)^{1/\epsilon} \right) \frac{T_2^{1/\epsilon}}{\epsilon\beta}.$$

To prove the inequality in (A.10), it remains to be shown that all values of  $t_X$  below  $t_X^{jump}$  imply that the large country sets its tax rate such that  $\rho > 0$ .

We know that when  $t_X = t_X^{jump}$ , the government is indifferent between  $\rho = 0$  and  $\rho^{jump}$ . A sufficient condition for the government to prefer  $\rho > 0$  for any  $t_X < t_X^{jump}$  is that  $\rho$  decreases in  $t_X \forall t_X \leq t_X^{jump}$ . From before we have:

$$t_H = \frac{(\beta - 1)(T_2 - \rho^\epsilon)}{\epsilon\beta\rho^{\epsilon-1}}$$

subtracting  $t_X$  on both sides and multiplying by  $\epsilon\beta\rho^{\epsilon-1}$  we get:

$$\epsilon\beta\rho^\epsilon = (\beta - 1)T_2 - (\beta - 1)\rho^\epsilon - t_X\epsilon\beta\rho^{\epsilon-1}.$$

This can be rewritten as:

$$t_X = Q(\rho) = \frac{1}{\epsilon\beta} [(\beta - 1)T_2\rho^{1-\epsilon} - (\epsilon\beta + \beta - 1)\rho].$$

It remains to show that  $Q'(\rho) < 0 \forall t_X \leq t_X^{jump}$

$$Q'(\rho) = (1 - \epsilon)(\beta - 1)T_2\rho^{-\epsilon} - (\epsilon\beta + \beta - 1) < 0.$$

This condition can be rewritten to:

$$\rho^\epsilon > \frac{(1 - \epsilon)(\beta - 1)}{(\epsilon\beta + \beta - 1)} T_2.$$

Condition (1.15) gives a lower bound for  $\rho$ . Plugging in this bound into the previous condition delivers:

$$\frac{(1 - \epsilon)(\beta - 1)}{\frac{\epsilon}{\epsilon + 1} + \beta - 1} T_2 > \frac{(1 - \epsilon)(\beta - 1)}{(\epsilon\beta + \beta - 1)} T_2$$

which can be simplified to:

$$\beta > \frac{1}{\epsilon + 1}$$

which is always true. **q.e.d.**

### A.3 Discontinuity and Equilibrium Existence

In this appendix we illustrate the existence condition as stated in Proposition 1.2 graphically and provide some intuition for the potential discontinuity of the best response function of the large country.

In the two graphs in Figure 3 the best response functions of the large country and the tax haven are plotted for different values of  $\epsilon$ . In the first graph we choose parameters such that  $\epsilon > 1$ . In this case the best response of the large country (solid line) is continuous and an equilibrium exists at the intersection with the best response of the tax haven (dashed line). The second graph illustrates the case of  $\epsilon < 1$ . In this case the best response function of the large country is discontinuous: for high values of the tax havens' rate  $t_X$  the optimal response of the large country is along the diagonal i.e. choosing  $t_H = t_X$ . For lower values of  $t_X$  it jumps to  $t_H > t_X$ . For the parameter values considered here ( $\epsilon = 0.9$ ) the equilibrium exists. When  $\epsilon$  is too low (i.e. when condition (1.15) is violated) the discontinuity lies so far to the left that no equilibrium exists.

The intuition for the discontinuity is best explained using the intensive and extensive margins of taxation introduced in subsection 1.5.1. The large country faces a trade off between the two margins: a higher tax rate implies higher revenues from firms that stay in the large country (intensive margin) but also an outflow of tax base (extensive margin). When  $\epsilon$  is very low, the most productive firms account for a large fraction of profits. This makes the extensive margin so strong that for high levels of the tax havens' tax rate it becomes optimal for the large country to set a tax rate low enough to keep the whole tax base at home i.e. to set  $t_H = t_X$ .

We have shown in footnote 22 above that a value of  $\epsilon = 0.618$  is sufficient to assure the existence of the equilibrium. Using French firm level data, Eaton, Kortum, and Kramarz (2008) estimate  $\epsilon = 1.46$  using the method of simulated moments. For the empirically relevant parameter values, the discontinuity does thus not seem to be a major concern.<sup>4</sup>

<sup>4</sup>A possible way to address situations of  $\epsilon < 0.618$  could be to consider mixed strategy equilibria.

## A.4 Corner Solution of Maximum Tax Rates

This appendix reports the expression for the main equilibrium objects when parameters are such that the governments choose the maximum tax levels. This case arises when the large country would optimally set a tax rate larger than one. In this corner solution the large country cannot optimally trade off the intensive and extensive margins of taxation anymore as there is an upper bound for the tax rate. Since the focus of this paper is to analyze the effect of these two margins of the tax game, the case of  $t_H^* = 1$  is only briefly outlined for completeness.

The main equilibrium objects are given by  $a^{**c} = \left( \frac{\epsilon^2}{(\epsilon+1)^2} \frac{\alpha}{\sigma f_t} \right)^{\frac{1}{\sigma-1}} a_m$ ,  $N_X^{*c} = \left( \frac{\epsilon^2}{(\epsilon+1)^2} \frac{\alpha}{\sigma f_t} \right)^{\epsilon+1}$  and  $\Pi_X^{*c} = \left( \frac{\epsilon^2}{(\epsilon+1)^2} \frac{\alpha}{\sigma f_t} \right)^\epsilon \frac{\alpha}{\sigma}$ , where the superscript  $c$  stands for corner solution.

One reason why the large country might want to set a tax rate above one is when the fixed costs of profit shifting are very high. As the large country cannot optimally trade off its intensive and extensive margins, changes in fixed costs also move  $a^*$ ,  $N_X$  and the tax bases  $\Pi_H$  and  $\Pi_X$ . As  $\rho$  increases less than would be optimal for the large country, the cutoff cost level decreases in fixed costs. The share of firms shifting profits abroad decreases and the home tax base increases.

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# Chapter 2

## Towards a Theory of Trade Finance

### Abstract

Cross border transactions are conducted using different payment contracts, the usage of which varies across countries and over time. In this paper I build a model that can explain this observation and study implications from this for international trade. In the model exporters optimally choose payment contracts, trading off differences in enforcement and efficiency between financial markets in different countries. I find that the ability of firms to switch contracts is central to the reaction of trade to variations in financial conditions. Numerical experiments with a two-country version of the model suggest that limiting the choice between payment contracts reduces traded quantities by up to 60 percent.

JEL: *F12, F3, G21, G32*

Keywords: *trade finance, payment contracts, trade patterns, financial crisis*

### 2.1 Introduction

Transactions in international trade are conducted using different payment contracts. These can be broadly classified into Cash in Advance, Open Account and bank intermediated contracts. Evidence suggests that usage of payment contracts varies both across countries and over time. Current trade theory does not address these contracts. To un-

derstand why different payment contracts are used by different firms and what effects these have on international trade, a suitable model is needed.

In this paper I develop a theory of trade finance that explains the co-existence of the different financing forms depending on enforcement and cost of financing, and analyze the trade-offs faced by exporters. The model shows that the impact of variations in financial conditions on trade significantly depends on the ability of firms to switch contracts. Furthermore, it predicts that the choice of payment contract shapes variable trade costs and that FOB prices vary systematically with payment contracts and financial market conditions. In a two-country version of the model numerical experiments are conducted. I find that limiting the choice between payment contracts reduces traded quantities by up to 60 percent. When introducing repeated transactions, the model can help to explain the empirical observation that trade credit intensive industries are less affected by a financial crisis.

While several recent papers have analyzed the effect of the exporter's financial market on participation decisions or exported quantities, none have examined the interaction of financial conditions in the sending and receiving countries. A setup featuring the choice of payment contract between an importer and an exporter has not been studied before. Taking into account characteristics of both financial markets gives rise to interesting interactions. Depending on which payment contract is chosen by an exporter, different parameters of the two financial markets are relevant. Choosing a payment contract allows to substitute away from the least favorable conditions. In models that only consider the financial markets of exporters, this substitution is not possible. Then, exporters are fully affected by any changes of financial conditions in the country they are located in.

Three types of payment contracts are modeled: Cash in Advance (CIA), Open Account (OA) and Letter of Credit (LC). With Cash in Advance, the importer pays the trade before receiving the goods. Under Open Account the importer only pays after receiving the goods. The third payment contract considered is a Letter of Credit, in which banks act as intermediaries resolving the enforcement problem.<sup>1</sup>

The main findings are as follows. CIA is used if enforcement at home is strong and if financing costs abroad are low. OA is used if enforcement abroad is strong and financing costs at home are low. A LC is used if enforcement is relatively weak and interest rate

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<sup>1</sup>I represent all bank intermediated transactions by the LC. They usually involve banks both in the importer's and exporter's country and the usage of some form of documentation upon which payment is being made by a bank to the exporter. For an introduction to the different types of trade finance payment contracts see U.S. Department of Commerce (2008).

costs are relatively low in both countries. Costs arising from trade finance take the form of variable costs that are proportional to the value traded, i.e. they correspond to the iceberg formulation often used in international trade. When financing costs or enforcement probabilities change, firms can react by switching payment contracts. In this respect, the time horizon in which firms are able to switch payment contracts is important. In the short run switching contracts might be difficult. In particular it might imply sunk costs that limit the usage of this option. It might, however, still be easier for two trading partners to switch payment contracts than to switch banks for any of them. Therefore, if a bank supplying credit to one of the trading partners is hit by a negative shock, a payment contract switch might be the best response to reduce the impact of the bank's problems on the trade relationship.

When repeated transactions are considered, under certain conditions, trigger strategies can improve on the one shot equilibria when CIA or OA are used. These trigger strategies are more likely to be optimal when the expected number of repeated transactions is large and when enforcement probabilities are high. In this case CIA and OA become more attractive relative to the LC. This prediction is in line with the notion that LCs are used relatively more when trade relationships have a shorter time horizon, whereas especially OA is more common in long term relationships. It can also explain the finding that trade relationships, in which more trade credit, i.e CIA and OA, is used, might be less affected by financial crisis than relationships using trade credit less intensively, as reported by Levchenko, Lewis, and Tesar (2009).

The model predicts asymmetric reactions of trade flows to financial turmoil. If there is country-specific financial turmoil, firms are able to partially mitigate adverse effects by switching payment contracts. If there is global financial turmoil that affects both the financial markets of the exporter and the importer, this possibility does no longer exist and trade flows react more strongly to a crisis. This suggests that in the recent global financial crisis trade finance might have had a stronger effect on aggregate trade flows than in former more locally concentrated ones. The model predicts differences between South-North and South-South trade volumes that are absent in models where only the exporter's financial market conditions are taken into account. Payment contracts affect FOB prices in two ways. They determine a part of variable trade costs and they specify the timing of transactions. The latter fixes who has to bear the financing costs, which is then reflected in the price charged.

There are several theoretical papers that have addressed the issue of financial market

conditions and international trade. Kletzer and Bardhan (1987) show how sovereign default risk and credit market imperfections can result in differences in interest rates and tightness of credit rationing in equilibrium respectively and thus create comparative advantage. In Matsuyama (2005) the share of revenues an entrepreneur can pledge towards wage payments differs between countries leading to comparative advantage.<sup>2</sup>

Chaney (2005) develops a theoretical model analyzing financial constraints in a heterogeneous firms trade model based on Melitz (2003). Firms have to finance their fixed entry cost into foreign markets through own liquidity and domestic operating profits. Liquidity is introduced as a second type of heterogeneity. He derives conditions on productivity and liquidity under which a firm exports. Manova (2008) extends this model and estimates it using the methodology suggested by Helpman, Melitz, and Rubinstein (2008). She introduces a default probability similar to the one used in this paper. While in her model there is a domestic enforcement problem between a bank and a firm, in this paper the enforcement problems arise between firms in different countries.

Her model has been further extended to a dynamic setting taking into account capital accumulation by Suwantaradon (2008). In Chaney (2005), Manova (2008) and Suwantaradon (2008) only domestic financial market conditions are relevant for the exporting decisions of firms. In this paper, in contrast, the effects of conditions in the financial markets of both the exporter and the importer, determined by the optimal choice between payment contracts, are analyzed. Furthermore, here, the focus is on financing needs specific to international trade. In comparison, the contributions discussed above, are concerned with the more general problem of external finance, which is relevant to domestic production as well.

Some articles study empirically the interaction between financial conditions and trade. The role of financial development for trade in manufactures is studied in Beck (2002) and Beck (2003). Using data from 65 and 56 countries respectively he finds that financial development of a country has a strong effect on export volumes of manufactures and that this effect is stronger in external finance intensive industries.<sup>3</sup>

Some recent papers test for effects of financial constraints on the extensive margin of trade using firm level data. Greenaway, Guariglia, and Kneller (2007) use UK manufacturing firm level data and find no evidence for a causal effect of ex-ante financial health

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<sup>2</sup>The broader issue of institutional constraints, trade and outsourcing has been studied extensively. For a survey see Helpman (2006).

<sup>3</sup>Svaleryd and Vlachos (2005) find evidence on patterns of industrial specialization and financial market condition that point in the same direction.

on export participation. French firm level data analyzed by Berman and Héricourt (2008) seems to point in the opposite direction.

There is a policy debate on the effects and importance of trade finance in general and during a financial crisis.<sup>4</sup> The effects of financial crisis on trade have been assessed empirically by several papers. Ronci (2004) analyzes data from 10 financial crises and finds evidence for a negative effect of financial crises both on imports and exports. Iacovone and Zavacka (2009) analyze data from 23 banking crises. They find a differential effect of a banking crisis on firms that rely on bank finance versus firms that rely on inter-bank credit. This supports the prediction of the model that inter-firm credit relations are an important factor for mitigating adverse effects of financial crises. Evidence on firm level trade finance of African exporters is documented by Humphrey (2009). Berman and Martin (2009) analyze whether financial crises affected trade and find that disruption from financial crises was stronger and longer lasting for African than for other countries. Using data on U.S. imports, Levchenko, Lewis, and Tesar (2009) analyze causes of the current international trade decline and argue that trade credit did not play a role. I discuss in Section 5 how this finding can be explained and why trade finance might be more important than suggested by this result. Amiti and Weinstein (2009) use data on Japanese exporters matched with Japanese banks to study the transmission of financial shocks during the financial crisis in the 1990s. They find that one third of the decline in Japanese exports in that period can be explained by a bank firm trade finance channel. This confirms the relevance of financing conditions on the side of exporters. The theory proposed here suggests, that in addition to this, there are effects arising from the financing conditions of importers and the interaction of the two.

The rest of the paper is organized as follows. Section 2 develops a microeconomic model of the choice of payment contracts. In Section 3 this model is put into an intra-industry trade framework. Section 4 derives the general equilibrium and studies a quantitative example. Section 5 extends the analysis to repeated transactions. Section 6 discusses implications of payment contracts during financial crisis, on trade patterns, and on FOB prices. Section 7 concludes.

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<sup>4</sup>See Stephens (1998), Auboin and Meier-Ewert (2003) and Auboin (2007). In the context of the current financial crisis trade finance has received a renewed attention. See Chauffour and Farole (2009) and Dorsey (2009).

## 2.2 Payment Forms

### 2.2.1 Overview

There are three types of payment contracts available to finance trade transactions: Cash-in Advance, Open Account and bank intermediated transactions.

Figure 1 based on data from IMF (2009) illustrates some survey evidence on the usage of these different payment forms.<sup>5</sup> While Open Account seems to be the dominant financing form, the share of the other two groups is also quantitatively important. The survey suggests that during the current financial crisis the usage of open account has declined, whereas the two payment forms that limit the risk of exporters have increased their shares. This is in line with the prediction of the model that changes in financial market conditions cause switches of payment contracts. Comments on the effect of the crisis on the choice of payment forms in the ICC (2009) survey suggest that the current crisis might lead to a renaissance of the Letter of Credit. That is, given rising uncertainty, firms rely more on the safest payment contract available, which is the Letter of Credit. It is also stated there that 50-60 percent of China's foreign trade is financed with LCs. This indicates that in a less financially developed country like China bank intermediated transactions are used relatively more often.

*Here figure 1*

### 2.2.2 Micro Model

The following section introduces a simple microeconomic model for the choice of finance of an exporter. Besides transport costs, I consider two additional cost factors in international trade relative to domestic trade. First, the time delay between production and the realization of sales is longer for international than domestic transactions. Hummels (2001) discusses the effect of physical transport time while Djankov, Freund, and Pham (2006) analyze the effect of time delay from the factory gate to the means of transportation on international trade. Both show that international trade has a relevant time dimension. The latter document that procedures specific to trade across international borders are responsible for additional delays. Second, enforcement at the international level is more difficult than domestically.<sup>6</sup> This can be due to differences in legal traditions, working

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<sup>5</sup>Unfortunately to my knowledge currently only survey data is available. Thus the numbers should be seen as a reference point.

<sup>6</sup>Enforcement might also be a problem at the domestic level. In the model I abstract from this and analyze only international enforcement problems. The enforcement probability is thus best interpreted

languages or the limited willingness of countries to enforce international contracts to the same extent as national ones.<sup>7</sup> Due to the two additional sources of costs in international trade, time delay and enforcement problems, the choice of a payment contract is a determinant of variable trade costs.

**Timing** The three payment contracts differ in the timing of payments relative to the delivery of goods and the amount of risk that is incurred by the exporter and the importer respectively. Suppose there is one exporter and one importer. The exporter can make a take it or leave it offer to the importer.<sup>8</sup> There are two points in time. One before production, transport and sales and one afterwards. If producing, the exporter incurs production cost  $K$  at  $t = 1$ . If importing the importer realizes sales revenues of  $R$  at  $t = 2$ .<sup>9</sup>

First consider only CIA and OA. The difference between these two forms of finance is whether the payment is made before or after realization of the revenues. If the importer pays before the exporter delivers the goods, I call it CIA. If the importer pays after receiving the goods and realizing sales revenues, I call it OA.<sup>10</sup> A third option is a bank-intermediated payment contract. As discussed above, I represent all contracts belonging to this group by a Letter of Credit. This is an instrument offered by financial institutions to improve security in international trade. First the importer pays the amount due in advance to a local bank.<sup>11</sup> This bank then transfers the money to the bank of the exporter,

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as representing the additional enforcement problems arising in the international context.

<sup>7</sup>A complementary factor to the enforcement risk could be individual firm default. As counter-party risks can be hard to observe across borders, whether a trading partner will turn out to be insolvent or illiquid can be seen as a random variable from the point of view of the other party. Different to international enforcement risk though, the probability of failure of a firm would have an effect on the interest rate a firm is charged.

<sup>8</sup>In the analysis the exporter has all negotiation power. This leads to the result that under open account there are positive expected profits for the importer. If the importer would have all negotiation power, there would be positive expected exporter profits under CIA instead. There could also be some type of intermediate surplus sharing rules. The main mechanism of the model, which is driven by financing costs and enforcement, should not be affected by the distribution of negotiation power.

<sup>9</sup>In this part of the paper trade transactions in the model are one shot games. There is a large literature analyzing advantages of trade credit due to repeated transactions and supply-chain relationships. See Petersen and Rajan (1997), Biais and Gollier (1997), Wilner (2000), Burkart and Ellingsen (2004), Cunat (2007) and Fabbri and Menichini (forthcoming). In section 5, I introduce a survival probability of trade relationships and analyze under which conditions a simple trigger strategy can be implemented to improve upon the equilibrium of the one shot game.

<sup>10</sup>One could also think about intermediate types of contracts combining elements of both CIA and OA. Evidence does not suggest that these are used very much though. In the official brochure of the U.S. Department of Commerce (U.S. Department of Commerce (2008)) e.g. no intermediate forms are mentioned. One reason for a limited usage of these might be legal considerations, as ownership claims would be less clear.

<sup>11</sup>Often firms do not actually pay the amount to the bank in cash, but receive a credit for the amount and period of the LC against a fee. As I assume perfect enforcement in the domestic financial market, the two are equivalent as long as firms discount at the lending rate.

which upon receipt of delivery documents pays out the money received. Assuming full enforcement at the bank level, a LC completely solves the enforcement problem.<sup>12</sup> A LC implies additional costs as there are advance finance requirements for both the exporter and the importer plus any additional fees charged by the banks for offering LCs.<sup>13</sup> The choice between these three forms of financing is relevant because of two imperfections in financing costs and enforcement.

**Interest rates** I assume that financial markets to finance international trade are segmented between countries and that the efficiencies of financial intermediaries in these countries differ. As a result interest rates to finance trade faced by firms in different countries can differ. In the Appendix I discuss a simple model rationalizing the different interest rates. There, both countries are small open economies facing a world interest rate. Due to segmented markets regarding credit at the firm level, differences in the efficiency of the financial intermediaries lead to differences between interest rates faced by firms in the two countries. Given this imperfection, *ceteris paribus*, it is optimal for the firm located in the country with the lower interest rate to finance a transaction.

**Enforcement** There is limited enforcement of contracts in two ways. First, there is an exogenous country-specific probability that a contract will be enforced, in the case that a firm does not want to fulfill it voluntarily.<sup>14</sup> In the case of CIA, it is the probability that the exporter is forced to deliver the goods after receiving the payment. In the case of OA, it is the probability that an importer has to pay the agreed upon price for the goods after receiving and selling them.<sup>15</sup> Second, the amount specified in the contract to be paid by the importer for the goods imported cannot exceed their total value at market prices.<sup>16</sup> The imperfection of limited enforcement leads, *ceteris paribus*, to finance being

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<sup>12</sup>This assumption will be relaxed in section 6 where changes in the national default risk will be explicitly allowed for.

<sup>13</sup>In the current version of the model I do not introduce any additional fees on top of the interest rates being charged to firms. Introducing a fee would make LCs less attractive. If during a crisis this fee changed to a different degree than interest rates, this would have an effect on the optimal choice of payment contracts and thus on outcomes of the model.

<sup>14</sup>This captures the reduced form of an enforcement game played between the importer and the exporter, which is affected by the legal institutions of the two countries. This could be extended to a model in which firms choose their legal expenditures to achieve or prevent enforcement. In that case the enforcement probability would change with the value at stake.

<sup>15</sup>For simplicity these two enforcement probabilities are assumed to be equal. It would be an interesting extension to consider an asymmetry here. This could be rationalized by the difference between the in-kind nature of an OA trade credit versus the cash nature of a CIA credit. For a formalization of this argument see Burkart and Ellingsen (2004).

<sup>16</sup>In order to enforce a trade contract in court the value specified in the contract has to be in some proportion to the real value of the goods traded. Technically this assumption is necessary in order for



optimally done by the firm in the country with lower enforcement. Finally, LCs are used when enforcement problems are so large that they outweigh financing costs in the form of interest payments.

Thus, for moderate levels of enforcement problems, trade is, *ceteris paribus*, financed by the country with the lower interest rate and with the lower contract enforcement. As the two factors, interest rate costs and enforcement probabilities, can be found in different combinations across countries, the choice of the financing form is non-trivial. In the following I formally describe the three mentioned financing forms and derive conditions under which firms choose one of them over the others.

**Cash in advance** Under cash in advance the importer first pays an amount  $C^{CIA}$  to the exporter. Then with probability  $\lambda$  the contract is enforced. In this case the exporter produces the goods at cost  $K$  and delivers them to the importer, who sells them for  $R$ . The exporter makes a take it or leave it offer and has to respect the limited value of contract and the importer participation constraint. The expected profit maximization problem is:

$$\begin{aligned} \max_C E [\Pi_E^{CIA}] &= C^{CIA} - \lambda K, \\ \text{s.t. } C^{CIA} &\leq R, && \text{(limited value of contract)} \\ \text{and } E [\Pi_I^{CIA}] &= \lambda R - (1 + r^*)C^{CIA} \geq 0. && \text{(participation constraint importer)} \end{aligned}$$

Under CIA the trade transaction is financed by the importer. The exporter receives the payment before production and delivery of the product. There is the possibility that the exporter keeps the money and does not deliver the good. This happens with the exogenous probability of non-enforcement  $1 - \lambda$ . The participation constraint of the importer assures that taking the default probability into account, the expected profits of the importer are non-negative and thus the importer is willing to agree on the trade and to pre-finance it. As all negotiation power lies with the exporter and the limited value of contract never binds under CIA, the participation constraint binds under the optimal contract. The optimal payment  $C^{CIA}$  and optimal expected profits of the exporter are:

$$C^{CIA} = \frac{\lambda}{1 + r^*} R, \quad E [\Pi_E^{CIA}] = \frac{\lambda}{1 + r^*} R - \lambda K. \quad (2.1)$$

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imperfect enforcement to have an effect on outcomes in the open account case. If this condition does not bind, any changes in enforcement risk would be offset by a proportional increase in the period 2 payment  $C^{OA}$ .

Note that  $\lambda$  appears in the optimal payment  $C^{CIA}$ . Thus the importer's payment is discounted by the probability of non-payment by the exporter. Under CIA production and delivery only takes place with probability  $\lambda$ .

**Open account** Under open account first the exporter produces the goods at cost  $K$  and then delivers them to the importer. Next, the importer sells the goods for  $R$ . With probability  $\lambda^*$  the contract is enforced and the importer pays the amount  $C^{OA}$  to the exporter. The profit maximization problem of the exporter is:

$$\begin{aligned} \max_C E [\Pi_E^{OA}] &= \frac{1}{1+r} (\lambda^* C^{OA} - K(1+r)), \\ \text{s.t. } C^{OA} &\leq R, && \text{(limited value of contract)} \\ \text{and } E [\Pi_I^{OA}] &= \frac{1}{1+r^*} (R - \lambda^* C^{OA}) \geq 0, && \text{(participation constraint importer)} \end{aligned}$$

assuming that the exporter and importer discount profits with their local interest rates.<sup>17</sup>

Now, the exporter pre-finances the trade transaction. The importer receives the goods before payment. With probability  $1 - \lambda^*$  the importer is not forced to pay the exporter. Due to the limited value of contract constraint the maximum payment  $C^{OA}$  that is contractible is the sales value of the product  $R$ . Under this maximum payment the exporter is not able to extract all rents from the importer. Thus under open account the importer has positive ex-ante expected profits. There is no contract that allows the exporter to extract all surplus.<sup>18</sup> The optimal payment amount  $C^{OA}$  and the optimal discounted expected exporter profits can be derived as:

$$C^{OA} = R, \quad E [\Pi_E^{OA}] = \frac{\lambda^*}{1+r} R - K. \quad (2.2)$$

**Letter of credit** Under LC the financial transaction is secured via a bank in the country of the exporter and the importer, respectively. Under the assumption of no default at the bank level, this completely resolves the enforcement problem at the individual contract

<sup>17</sup>In order to be able to compare profits between CIA and OA they have to be discounted to the same time period.

<sup>18</sup>This is true for any finite upper bound on the contractible payment level. Without an upper bound any change in enforcement probability could be offset by a proportional increase in  $C^{OA}$ . It is conceivable though that courts would not enforce amounts too disconnected from the real value of the trade.

level.<sup>19</sup> The maximization problem of the exporter is:

$$\begin{aligned} \max_C E [\Pi_E^{LC}] &= \frac{1}{1+r} (C^{LC} - K(1+r)), \\ \text{s.t. } C^{LC} &\leq R, && \text{(limited value of contract)} \\ \text{and } E [\Pi_I^{LC}] &= \frac{1}{1+r^*} (R - (1+r^*)C^{LC}) \geq 0. && \text{(participation constraint importer)} \end{aligned}$$

With LCs both the exporter and the importer pre-finance the transaction and incur costs due to interest rate payments. The contract enforcement problem is resolved by an indirect transaction with banks as intermediaries. The importer does not directly pay the exporter, but first pays the amount  $C^{LC}$  to a local bank. The bank cooperates with a bank in the country of the exporter. The latter guarantees payment upon proof of delivery. Thus the exporter knows for certain that the payment will be made, but only receives it after production and delivery. In the case of LCs the participation constraint of the importer is binding. The optimal payment  $C^{LC}$  and discounted expected exporter profits are:

$$C^{LC} = \frac{R}{1+r^*}, \quad E [\Pi_E^{LC}] = \frac{1}{(1+r)(1+r^*)} R - K. \quad (2.3)$$

Note that as pre-financing takes place on both sides the interest rates from both markets affect profits. As enforcement risk is completely resolved, profits are independent of the enforcement parameters  $\lambda$  and  $\lambda^*$ .

**Comparison** At this point, before an explicit demand structure is imposed, some statements on the optimality of the different payment contracts can be made. The four financial market parameters  $r, r^*, \lambda, \lambda^*$  together with the production cost  $K$  and sales revenue  $R$  determine a unique ordering of the different payment forms as stated below<sup>20</sup>:

**Proposition 2.1** *The optimal choice of payment contract is uniquely determined by the*

<sup>19</sup>It is conceivable that full enforcement at the banking level is more likely than at the firm level. As banks tend to have more long-term relationships, reputation building and repeated transactions ease enforcement between them.

<sup>20</sup>Here I assume  $K$  and  $R$  to be exogenous and the same for all payment contracts. When introducing an explicit demand in the next section, different payment contracts imply different optimal levels of  $K$  and  $R$ .

following three conditions:

$$\begin{aligned}
 i) \quad OA \prec CIA &\iff \frac{K}{R} < \frac{\lambda^*(1+r^*) - \lambda(1+r)}{(1+r)(1+r^*)(1-\lambda)}, \\
 ii) \quad OA \prec LC &\iff \lambda^*(1+r^*) > 1, \\
 iii) \quad LC \prec CIA &\iff \frac{K}{R} < \frac{1 - \lambda(1+r)}{(1+r)(1+r^*)(1-\lambda)}.
 \end{aligned}$$

**Proof.** Follows directly from comparing optimal discounted expected profits, Equations (1)-(3). ■

Proposition 1 summarizes the different effects of the parameters on expected profits under the different financing forms. The usage of open account is increasing in enforcement abroad  $\lambda^*$  and decreasing in financing costs at home  $r$ . CIA is more likely to be used if enforcement at home  $\lambda$  is strong and if financing costs abroad  $r^*$  are high. LCs are used if enforcement is relatively weak (low  $\lambda, \lambda^*$ ) and if financing costs are relatively low (low  $r, r^*$ ).

## 2.3 The Trade Model

In the following I introduce a CES demand structure that is standard in intra-industry trade models. I analyze the effects of trade finance at the firm level in partial equilibrium. This allows to derive predictions of the model in a trade framework that could be taken to the data.

### 2.3.1 Basic Setup

**Preferences** Assume the following preferences:

$$U = Q^\mu q_0^{1-\mu} \quad \text{with } Q = \left( \int_{\Omega} q(\omega)^{\frac{\sigma-1}{\sigma}} d\omega \right)^{\frac{\sigma}{\sigma-1}}. \quad (2.4)$$

$Q$  is a CES basket of a continuum of differentiated goods and  $q_0$  is a homogenous good. There is a Cobb-Douglas utility function between the homogeneous good and the differentiated goods implying constant shares of income spent on differentiated goods and the homogeneous good, respectively. The demand for a single variety of the differentiated good has the standard form:

$$q(\omega) = p(\omega)^{-\sigma} P^\sigma Q.$$

$\omega$  denotes a variety of the differentiated goods.  $P = \left( \int_{\omega \in \Omega} p(\omega)^{1-\sigma} \right)^{1-\sigma}$  is the price index of the optimal CES basket.  $\sigma > 1$  is the elasticity of substitution between varieties and  $Q$  is the total demand for differentiated goods.

**Technology** Labor is the only input factor. Firms in the homogenous goods sector face perfect competition. They operate a constant returns to scale technology requiring one unit of labor per unit of output. The homogenous good is freely traded. I only consider equilibria in which every country produces the homogenous good. This equalizes wages, which are normalized to one making the homogenous good the Numeraire. In the differentiated goods sector firms face monopolistic competition. Each variety is produced by only one firm. There is a fixed cost of entry into the market  $f$ . The production of one unit of the differentiated product requires  $a$  units of labor input. Thus the total labor requirement of a firm operating and producing quantity  $q$  is  $l(a) = f + aq$ . When selling a differentiated good abroad, a firm incurs iceberg type trade costs. In order for one unit of a differentiated product to arrive in the destination country,  $\tau > 1$  units have to be shipped.

### 2.3.2 Optimal Behavior of Firms

**Differentiated sector** Firms in the differentiated sector maximize their expected profits. Given CES utility and monopolistic competition firms optimally do markup pricing. Domestic prices, quantities and profits can be derived as:

$$p_d = \frac{\sigma}{\sigma - 1} a, \quad q_d = (p_d)^{-\sigma} P^\sigma Q, \quad \Pi_d = q_d \left[ \frac{a}{\sigma - 1} \right]. \quad (2.5)$$

Next, I derive the optimal behavior of firms on the export market. Optimal profits under all financing forms can be represented by the general expression:

$$E[\Pi_x] = \alpha R - \beta K.$$

This problem can be rewritten to  $E[\tilde{\Pi}_x] = E\left[\frac{\Pi_x}{\alpha}\right] = R - \frac{\beta}{\alpha} K$ . Maximizing the original objective function  $E[\Pi]$  implies the same optimal decisions as maximizing the new function  $E[\tilde{\Pi}]$ . This price setting problem is equivalent to the standard case with new per unit production costs of  $\frac{\beta}{\alpha} a$ .

Thus the optimal export decision of a firm implies:

$$p_x = \frac{\beta}{\alpha} \tau p_d^*, \quad E[q_x] = A \tau^{1-\sigma} q_d^*, \quad E[\Pi_x] = A \tau^{1-\sigma} \Pi_d^*, \quad (2.6)$$

with  $A = \alpha^\sigma \beta^{1-\sigma}$ .<sup>21</sup>

The finance profit factor  $A$  fully summarizes the effects of payment contracts on expected profits and expected quantities at the factory gate.  $A = 1$  corresponds to no financing frictions, which would be the case if  $r = r^* = 0$  and  $\lambda = \lambda^* = 1$ . Then, expected profits are the same as in the standard model. Note that the parameters  $\alpha$  and  $\beta$  enter the problem in a multiplicative form with the value of exports. Thus the model endogenizes some of the variable trade costs of trade arising from financing costs and the enforcement problem.<sup>22</sup>

Now that the explicit demand structure is defined, I can derive new conditions for the optimal financing forms. The profits and quantities under financing form 1 are larger than under financing form 2 iff:

$$A^1 > A^2.$$

Plugging in the different values for  $\alpha$  and  $\beta$  delivers:<sup>23</sup>

**Corollary 2.2** *The optimal choice of payment contract is uniquely determined by the following three conditions:*

$$\begin{array}{ll} i) & OA \prec CIA \iff \frac{(\lambda^*)^\sigma}{\lambda} \left( \frac{1+r^*}{1+r} \right)^\sigma > 1, \\ ii) & OA \prec LC \iff \lambda^*(1+r^*) > 1, \\ iii) & LC \prec CIA \iff \lambda(1+r)^\sigma < 1. \end{array}$$

<sup>21</sup> $E[q_x]$  is the expected quantity at the factory gate, i.e. including iceberg trade costs and taking into account that under CIA only a fraction  $\lambda$  of export contracts is enforced. That is:  $E[q_x] = \tau \beta (p_x^*)^{-\sigma} (P^*)^\sigma Q^*$ .

<sup>22</sup>It would be interesting to combine the value-dependant variable costs introduced in this model with some unit-dependent transport costs.

<sup>23</sup>In the case of CIA the parameters are:  $\alpha = \frac{\lambda}{1+r^*}$  and  $\beta = \lambda$ , under OA  $\alpha = \frac{\lambda^*}{1+r}$  and  $\beta = 1$  and under LC  $\alpha = \frac{1}{(1+r)(1+r^*)}$  and  $\beta = 1$

**Proof.** These conditions follow directly from a comparison of expected profits.<sup>24</sup> ■

The expressions simplify, but the same factors as in the general case can be identified: CIA is more likely to be used if enforcement at home  $\lambda$  is strong and financing costs abroad  $r^*$  are low. OA is more likely used if enforcement abroad  $\lambda^*$  is strong and financing costs at home  $r^*$  are low. LCs are used if enforcement is relatively weak (low  $\lambda, \lambda^*$ ) and if financing costs are relatively low (low  $r, r^*$ ).

### 2.3.3 Payment Contract Switching

**Optimal switching** An exporter switches payment contract when this maximizes expected profits. In this subsection I discuss how the ability to switch contracts allows a firm to mitigate partially adverse effects from financial markets. First, note that for any financial variable  $\theta \in \{r, r^*, \lambda, \lambda^*\}$  there is at least one payment contract under which expected profits, revenues and quantities are independent of this variable. Thus if there is a deterioration of one of these variables at some point a firm will switch its payment contract. There are also pairs of financial variables of which some payment contracts are independent.

Thus there are cases when the possibility of a payment contract switch can at some point completely mitigate any adverse effects on trade. However, there can be changes in several financial variables for which there is no financing form that completely mitigates the effects. Nevertheless, a switch of payment contract can reduce the impact of financial market changes.

**Effects of contract switching** In the following I illustrate these different cases with some graphs for trade between firms in different countries.<sup>25</sup>

Figure 2 illustrates the difference between a change in only one financial variable, the effects of which can be fully mitigated, and a simultaneous change in two financial

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<sup>24</sup>Prices and profits for the three financing forms are:

$$\begin{aligned}
 p^{CIA} &= (1 + r^*)\tau p_d^* & \mathbb{E} [\Pi_E^{CIA}] &= \left( \frac{1}{1 + r^*} \right)^\sigma \lambda \tau^{1-\sigma} \Pi_d^* \\
 p^{OA} &= \frac{1 + r}{\lambda^*} \tau p_d^* & \mathbb{E} [\Pi_E^{OA}] &= \left( \frac{\lambda^*}{1 + r} \right)^\sigma \tau^{1-\sigma} \Pi_d^* \\
 p^{LC} &= (1 + r)(1 + r^*)\tau p_d^* & \mathbb{E} [\Pi_E^{LC}] &= ((1 + r)(1 + r^*))^{-\sigma} \tau^{1-\sigma} \Pi_d^*
 \end{aligned}$$

<sup>25</sup>All graphs are calculated in partial equilibrium, i.e. keeping foreign demand fixed. In Section 4 the same experiments are studied under general equilibrium. For all graphs the following baseline calibration is used:  $1 + r^N = 1.02, 1 + r^S = 1.04, \lambda^N = 0.98, \lambda^S = 0.95$ .

variables, the negative effects of which cannot be fully eliminated by switching contracts.

*Here figure 2*

In the left graph,  $r^N = 1.02$  is constant and only the Southern interest rate varies. The initial payment contract is Cash in Advance. When the interest rate in the South passes a threshold, there is a switch of payment contract to Open Account. From that point on changes in the Southern interest rate do not affect trade volumes. In the right graph,  $r^N = r^S - 0.02$  is a constant value below the southern interest rate. Initially both interest rates are low and a Letter of Credit is used. When both interest rates increase beyond a certain point, there is a switch to Cash in Advance. Note that the switch reduces the impact of the financial changes, but cannot fully eliminate them.

A switch of payment contract takes place when the ordering of the finance profit factors  $A$  changes. This is illustrated in Figure 3, which shows the values of profit factors in the experiment discussed above for the different payment forms. Quantities depend only on the factor  $A$  specific to the payment form actually used, which is the maximum of the three lines.

*Here figure 3*

Figure 4 illustrates the case where two financial parameters change and it is still possible to fully eliminate effects on trade from some point onwards. Now enforcement in the South and in the North and South are changed respectively.

*Here figure 4*

In the left graph,  $\lambda^N = 0.98$  is constant and only the Southern enforcement probability changes. For low values of enforcement, Cash in Advance is used. When the enforcement rate in the South gets very close to the Northern one, then the payment contract is switched to Open Account in order to profit from the lower interest rate in the North. In the right graph,  $\lambda^N = \lambda^S + 0.03$  is a constant value above the enforcement value in the South. For low values of enforcement, Letter of Credit is chosen, which fully eliminates any effect of the enforcement probabilities on trade quantities. For higher values of enforcement, there is a switch to Cash in Advance and trade rises with enforcement in the North.

**Effects of no contract switching in the short run** The ability of an exporter to switch to a different payment contract is important to optimally adjust to the changes in financial efficiencies and enforcement probabilities. When an exporter is not able to switch payment contracts in the short run, this has an adverse affect on traded quantities



and expected profits. This is illustrated in Figure 5. In the example, interest rates are relatively low to begin with and a Letter of Credit is used. Now suppose there is a change in the Northern interest rate. Then at some point the exporter would like to switch to Cash in Advance. The solid line represents the standard case when the firm can switch contracts. Then reductions in profits and quantities are limited to about 10 percent. The dashed line represents the case when the firm is not able to substitute away and the effect of the interest rate change cannot be mitigated. In the example the change of 10 percentage points in the interest rates leads to a 30 percent drop in profits and quantities. Thus the ability to switch payment contracts is crucial for an exporter to mitigate the effects of adverse financial conditions. As a consequence, the time horizon, in which a switch is possible, is an important determinant of the reaction of trade flows to changes in financial market conditions.

*Here figure 5*

## 2.4 General Equilibrium

In the following the trade model is analyzed in general equilibrium. First, the equilibrium is derived. Second, a numerical example is studied to evaluate the quantitative importance of payment contracts. Third, some graphs are presented, illustrating additional effects arising in general equilibrium.

### 2.4.1 Equilibrium

Suppose there are two countries H and F.<sup>26</sup> As derived in the previous sections, domestic and foreign profits of firms in the differentiated goods sector are:

$$\Pi_d^i = q_d^i \frac{a}{\sigma - 1} \text{ and } \Pi_x^i = A^i \Pi_d^j,$$

with  $A^i = (\alpha^i)^\sigma (\beta^i)^{1-\sigma} \tau^{1-\sigma}$ .

**Free entry** As in Krugman (1980), a free entry condition pins down both the number and the size of firms in equilibrium. It requires that fixed cost of entry have to equal

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<sup>26</sup>For subsequent equations I always use  $i, j \in \{H, F\}, i \neq j$

expected profits from domestic sales and exports:

$$f = \Pi_d^i + \Pi_x^i.$$

Next, I combine the two equations resulting from free entry and solve for domestic quantities:<sup>27</sup>

$$q_d^i = \frac{\sigma - 1}{a} f \frac{1 - A^i}{1 - A^i A^j}.$$

Export quantities at the factory gate are given by:

$$q_x^i = A^i q_d^j = \frac{\sigma - 1}{a} f \frac{A^i (1 - A^j)}{1 - A^i A^j}.$$

Note that due to the Cobb Douglas structure in preferences the expenditure share on differentiated products is fixed. Thus output in the differentiated sector, measured in labor, is constant.

Taking partial derivatives allows to analyze effects of A on quantities:

$$\frac{\partial q_d^i}{\partial A^j} = -\frac{\partial q_x^i}{\partial A^j} = \frac{\sigma - 1}{a} f \frac{(1 - A^i) A^i}{(1 - A^i A^j)^2} > 0,$$

and

$$\frac{\partial q_d^i}{\partial A^i} = -\frac{\partial q_x^i}{\partial A^i} = -\frac{\sigma - 1}{a} f \frac{1 - A^j}{(1 - A^i A^j)^2} < 0.$$

**Lemma 2.3** *In the two country trade model exported (domestic) quantities are*

- i) increasing (decreasing) in profit factor  $A^i$ ,*
- ii) decreasing (increasing) in profit factor  $A^j$ .*

Thus in the two country model domestic quantities are decreasing in the own export profit parameter  $A$  and increasing in the foreign export parameter  $A^*$ . Due to the choice of payment contracts all financial parameter  $r, r^*, \lambda, \lambda^*$  can affect both  $A$  and  $A^*$ . Thus while the effect of the profit parameters  $A$  and  $A^*$  on quantities is unambiguous, no general result can be established for the effect of the four financial parameters on traded and domestic quantities.

<sup>27</sup>Plugging in the values derived for profits from before delivers:

$$f = \frac{a}{\sigma - 1} \left[ q_d^i + A^i q_d^j \right].$$

Total (expected) quantities at the factory gate are constant:<sup>28</sup>

$$q^i = \frac{\sigma - 1}{a} f.$$

**Labor market clearing** The number of firms in both countries is determined by labor market clearing.<sup>29</sup> Given the CD preferences, a constant fraction of labor is employed by the differentiated sector:

$$L_Q^i = \mu L^i.$$

Labor in the differentiated sector is used for entry and production:

$$L_Q^i = L_E^i + L_P^i = n^i(f + aq).$$

From this the following result for the number of firms can be derived:

$$n = \frac{\mu L^i}{f + aq} = \frac{\mu L^i}{\sigma f}.$$

## 2.4.2 Numerical Example

In the following, properties of the model are illustrated using a numerical example. The quantitative importance of payment contract switches is evaluated.

**Country Types** Suppose there are three types of countries. Type I has a very efficient financial market and strong enforcement. Type II has a relatively efficient financial market, but enforcement is weak. Type III has a less efficient financial market, but relatively strong enforcement. The countries are characterized by the parameters in Table 1:

here table 1

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<sup>28</sup>Quantities are:

$$q^i = q_d^i + q_d^j A^i.$$

Plugging in domestic quantities delivers total quantities per firm:

$$q^i = \frac{\sigma - 1}{a} f \frac{1}{1 - A^S A^N} [(1 - A^N) + (1 - A^S) A^N].$$

The last expressions can be simplified to obtain the standard result as stated above.

<sup>29</sup>For tractability, I assume that the positive expected profits of importers under Open Account do not enter the demand for differentiated good. These profits occur under Open Account as only a share of importers fulfill the contract and payment  $C^{OA}$  is limited to be smaller or equal to  $R$ . It would be interesting to analyze these 'informal' profits explicitly in the general equilibrium. This could be relevant for countries with very low enforcement rates.

**Optimal Contracts** As shown before these country characteristics can be mapped uniquely into the optimal choice of payment contract for each exporter-importer country combination. The optimal payment contracts chosen are:

*here table 2*

The rows correspond to the country type of the exporter, the columns to the country type of the importer. On the diagonal the countries are symmetric. In this case an exporter would never choose OA as this is dominated by CIA. Thus given symmetric countries either CIA or LC is chosen depending on the domestic interest rate and domestic enforcement. For trade between countries of type II where enforcement is low, LC is chosen, whereas country I and III choose CIA. Country type II has low enforcement. Therefore, its exporters choose contracts that circumvent this problem, i.e. OA and LC. The main factor for the choice of an exporter in a country of type III is the relatively high home interest rate. The only payment contract that is independent of this interest rate is CIA, which is the dominant payment contract for trade with all three types of trading partners. Given these payment contracts, traded quantities can be calculated:

*here table 3*

To evaluate quantitatively the effects of the choice of payment contracts on trade, two experiments are studied. First, the differences in trade volumes between the optimal contract and the worst contract are calculated. Second, the differences in trade volumes between the optimal contract and any of the three available contracts are calculated.<sup>30</sup>

**Worst contracts** The contracts that minimize quantities and profits for exporter-importer pairs are:

*here table 4*

The percentage decreases in quantities relative to the optimal payment contracts are:

*here table 5*

If, for example, an exporter in a country of type I trading with an importer in country type II would be forced to use OA instead of CIA, this would reduce the traded quantity by 49.4 percent. The table shows that the choice of payment contract has a quantitatively important effect on traded volumes. Furthermore, this effect is heterogeneous

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<sup>30</sup>For both experiments I assume that exporters in the home country are constrained, while exporters in the country abroad choose the optimal payment contract. I do not report the reverse trade flows (from abroad to home) here. Nevertheless, they have an impact on price levels and thus on traded quantities of home country exporters.

across country pairs. Some trade, like trade between type I countries, does not depend very much on the payment contract in use. Trade between countries of type II on the other hand, which have low enforcement, benefits a lot from the ability of exporters to avoid OA. Note that, whether symmetric or asymmetric countries trade with each other, is not a good predictor of potential losses from non-optimal payment contracts. Which country-pair combinations profit most from a free choice of payment contracts depends on the interaction of all four financial parameters as shown in the previous sections.

The percentage decreases in traded quantities relative to the optimal contract if exporters are forced to use one specific contract are:

*here table 6*

### 2.4.3 Graphical Illustrations

In the following, additional effects arising in the two country general equilibrium model are discussed. The experiments shown in Figures 2 to 5 are repeated in Figures 6 to 8. The dashed lines represent the previously discussed partial equilibrium responses. The solid lines represent the full general equilibrium results. There are two new general equilibrium effects. Changes in financial efficiency and enforcement affect the finance profit factor  $A$  of Northern exporters and thus their prices and quantities. This has an effect on price level in the South and thus affects the export decision of Northern exporters. A second effect arises from the Southern exporters. Their finance profit factor is also affected by changes in the efficiency and enforcement parameters, altering export quantities and prices and thus price levels. Furthermore, when financial parameters change, exporters in the South might optimally switch payment contracts. These switches have an impact on the Northern exporters through price levels, too. Note that the switching decisions of Northern exporters is independent of general equilibrium effects as it only depends on exogenous financial parameters.

Figure 6 illustrates changes in interest rates. It corresponds to Figure 2 in partial equilibrium. In the left graph, Northern exporters switch from CIA to OA at the same point as before. Now there is a new effect from a switch by exporters in the South, though. The vertical solid line marks the threshold at which they switch their payment contract from LC to CIA in order to substitute away from the rising Southern interest rate. Now with a further rise in  $r^*$ , Northern exports become relatively more expensive. Thus exports fall steeper than in partial equilibrium. At the vertical dashed line, the

Northern exporters switch as well and all trade becomes independent of the Southern interest rate.

In the right graph, when both Northern and Southern interest rates rise jointly, there is no switch by Southern exporters who use CIA. The fall of Northern exports is less steep in general equilibrium as also Southern exporters are affected by the changes in interest rates. Thus the relative price of Northern exporters rises less quickly and trade decreases by less than in partial equilibrium.

*Here figure 6*

Figure 7 illustrates changes in enforcement. It corresponds to Figure 4 in partial equilibrium. In the left graph as before Northern exporters switch from CIA to OA. Exporters in the South use CIA and do not switch. In the left part of the figure, the solid line is now decreasing as Southern exporters become more competitive with a rise in  $\lambda^*$ , i.e. the relative price of Northern exporters rises and their quantities decrease. In the right graph, initially exporters in both countries use a LC and then at some point switch to CIA. Southern exporters do so first as the interest rate in the North is lower. From the point of the switch onwards, they become relatively more competitive and quantities of Northern exporters decrease. This is the case until Northern exporters themselves switch to CIA and their sales increase with the further rise of Northern enforcement.

*Here figure 7*

Figure 8 illustrates the case of no contract switching in the short run. It corresponds to Figure 5 in partial equilibrium. As before, if they can do so in the short run, Northern exporters switch from LC to CIA. Exporters in the South first use CIA and then later switch to OA. After the Northern exporters switch contracts and before the Southern exporters change their choice, all exporters use CIA. Then, only exporters in the South are affected by the increase in the Northern interest rate. This makes Northern exporters relatively more competitive and their exports rise. When Southern exporters switch as well, trade becomes independent of the Northern interest rate. If exporters in the North cannot switch contracts, their exports decrease in the whole range. First, when also exporters in the South are affected by the interest rate, the decrease is relatively smaller. Then, when Southern exporters switch contracts and their sales become independent of the Northern interest rate, the slope of the line representing trade gets steeper and the competitiveness of Northern exporters deteriorates more quickly.

*Here figure 8*

## 2.5 Repeated Transactions

In the previous sections the relationship between an exporter and an importer consisted of only one transaction. Often though, trade relationships are longer lasting, i.e. there is the possibility that the two trading partners interact again in subsequent periods. Repeated transactions give rise to a continuation value of a trade relationship, which makes the non-fulfillment of a contract less desirable. In this section I introduce the possibility of repeated transactions between an exporter and an importer and study under which conditions a simple trigger strategy can be implemented to improve upon the equilibrium of the one shot game. I find that the ability to sustain a trigger strategy equilibrium increases with enforcement probabilities and with the survival probability of the trade relationship. Under CIA the ability also increases in the markup  $\frac{\sigma}{\sigma-1}$  and iceberg trade costs  $\tau$ .

Suppose that trade can happen more than once between two trading partners. Let  $\gamma$  denote the probability that a given trade relationship can be continued in the next period. That is  $\gamma$  is the trade relationship survival rate. As before a match between an exporter and an importer is analyzed. No new trading relationships are created, i.e. there are no outside options to trade with another partner.<sup>31</sup>

**Cash in Advance** First, I analyze the case of Cash in Advance. Consider the following trigger strategy: The importer pays the full revenue amount discounted by the interest rate, i.e.  $C = \frac{R}{1+r^*}$ . If the exporter ever fails to deliver, the importer punishes by ending the trade relationship. The exporter always pays the money and never runs away. The equilibrium exists if the exporter has no incentive to deviate, i.e. to take the money and not deliver the good. Yet, even when deviating, the exporter is forced to fulfill the contract with probability  $\lambda$ . That is, the higher the enforcement probability at home, the less likely the exporter does profit from a deviation.

In order for the trigger strategy to be an equilibrium, the value of the relationship for the exporter  $V_E$  has to be larger than the one period deviation payoff. The trigger strategy equilibrium exists iff:

$$V_E = \sum_{n=0}^{\infty} \left( \frac{\gamma}{1+r} \right)^n (C - K) = \frac{C - K}{1 - (\gamma/(1+r))} > (1 - \lambda)C.$$

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<sup>31</sup>It would be an interesting extension to allow for the creation of new relationships via searching and matching. This would increase the value of the outside option and make it more difficult to sustain a trigger strategy equilibrium.

Using  $C^{CIA} = \frac{R}{1+r^*}$  this condition holds iff:<sup>32</sup>

$$\frac{\sigma}{\sigma-1}\tau \left[ 1 - \left( 1 - \frac{\gamma}{1+r} \right) (1-\lambda) \right] > 1$$

There are different factors, which make the condition more likely to hold. It holds more likely for a higher  $\lambda$ . As the expected gain from a deviation decreases in domestic enforcement, implementation of the trigger strategy is the easier, the better enforcement. Furthermore, the trigger strategy equilibrium is easier to sustain when the ratio  $R/K$  increases, i.e. when revenues are relatively large compared to production costs. This is the case when the markup and iceberg trade costs are higher. Finally, the higher  $\gamma$ , the higher the value of the trade relationship, the easier it is to implement the trigger strategy.<sup>33</sup>

**Open Account** Under OA the relevant deviation is by the importer. When OA is used an importer has positive expected profits in the one shot game:

$$E [\Pi_I^{OA}] = \frac{R(1-\lambda^*)}{1+r^*}.$$

In a trigger strategy equilibrium the importer has to receive at least as large expected profits as in the one shot game. The equilibrium considered is as follows: The importer always pays amount  $C$ . If the importer ever fails to pay the amount, the exporter stops the relationship. First note that for the importer to have positive expected profits, the payment  $C$  has to be strictly below the revenues  $R$  for any  $\lambda^* < 1$ . The amount  $C$  that makes the importer indifferent between adhering to the trigger strategy equilibrium and deviating is characterized by:

$$V_I = \sum_{n=0}^{\infty} \left( \frac{\gamma}{1+r^*} \right)^n \left( \frac{R-C}{1+r^*} \right) = \frac{R-C}{(1+r^*)(1-(\gamma/(1+r^*)))} = \frac{R(1-\lambda^*)}{1+r^*}.$$

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32

$$C - K > (1 - (\gamma/(1+r)))(1-\lambda)C \Leftrightarrow C \left[ 1 - \left( 1 - \frac{\gamma}{1+r} \right) (1-\lambda) \right] > K$$

Next use  $C^{CIA} = \frac{R}{1+r^*}$  to get:

$$\frac{R}{K} \left[ 1 - \left( 1 - \frac{\gamma}{1+r} \right) (1-\lambda) \right] > (1+r^*)$$

Plugging in  $\frac{R^{CIA}}{K^{CIA}} = (1+r^*)\frac{\sigma}{\sigma-1}\tau$  delivers the result.

<sup>33</sup>Note that so far only one specific trigger strategy is considered. It would be interesting to look at a wider set of strategies allowing for different forms of punishment.



This condition corresponds to the equilibrium chosen by the exporter, who has all negotiation power. It determines the highest incentive compatible  $C$  and thus maximizes expected profits of the exporter. The maximum payment  $C$  is:

$$C^{TR,OA} = R \left[ 1 - \left( 1 - \frac{\gamma}{1+r^*} \right) (1 - \lambda^*) \right].$$

The amount increases in the survival probability  $\gamma$  and the enforcement probability abroad  $\lambda^*$ . Under OA a trigger strategy always improves on the one shot game whenever  $\gamma > 0$ . To see this note that the expected payment in the one shot game is  $\lambda^*R$ . The expected payment under a trigger strategy with open account is  $C^{TR,OA}$ . The latter is larger iff:

$$R \left[ 1 - \left( 1 - \frac{\gamma}{1+r^*} \right) (1 - \lambda^*) \right] > \lambda^*R.$$

This condition can be simplified to obtain  $\gamma > 0$ .<sup>34</sup>

**Letter of Credit versus other Payment Contracts** As shown above, under the discussed conditions, trigger strategies can improve upon one shot equilibria in the cases of CIA and OA. A LC on the other hand already resolves all enforcement problems. Trigger strategies can thus not improve upon the one shot equilibrium when a LC is used. As a result, the introduction of trigger strategies and repeated transactions makes CIA and OA more attractive while leaving the LC unaffected, implying a worsening of the relative attractiveness of LCs. This is especially the case when transaction horizons are long, characterized by high relationship survival probabilities  $\gamma$ , and when enforcement probabilities  $\lambda$  and  $\lambda^*$  are high. Thus long lasting relationships and trade between countries with high enforcement imply a higher usage of trigger strategies and less reliance on the LC.

Levchenko, Lewis, and Tesar (2009) find that trade credit intensive industries are not more strongly affected by the financial crisis than others. This finding might be explained by the presence of both one shot equilibria and repeated games equilibria in the data. Inter-firm trade credit usually corresponds to CIA and OA. The model predicts that these two payment contracts are used more intensively when trigger strategies can be

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<sup>34</sup>Intermediate steps are:

$$1 - \lambda^* > \left( 1 - \frac{\gamma}{1+r^*} \right) (1 - \lambda^*) \Leftrightarrow 1 > 1 - \frac{\gamma}{1+r^*}$$

implemented more easily. In a successful trigger strategy equilibrium, only one financial variable affects the traded quantities, i.e.  $r^*$  for CIA and  $r$  for OA. Thus trade credit intensive industries, which use relatively more CIA and OA, should be less affected by shocks to financial markets than industries that rely more on the LC.<sup>35</sup>

## 2.6 Implications of Endogenous Payment Contracts

### 2.6.1 Financial and Economic Turmoil

In the following I analyze the effects of different forms of financial and economic turmoil on trade profits and quantities.

**Enforcement and Country level risk** Suppose that the enforcement probability  $\lambda$  is a combination of national risk and individual firm level risk. National risk corresponds to risk of expropriation, risk of sovereign or bank level default while individual risk is the probability that a specific contract is enforced. The important difference between the two is that while the banking system can cope with idiosyncratic risk at the contract level, it is not able to solve the problem of national risk. Before, a Letter of Credit could fully resolve the problem of enforcement for the exporter. Now, with national risk, the only way to fully mitigate changes in  $\lambda$  is to switch to Open Account. Let overall risk be the product of national risk and individual contract risk:

$$\lambda = \lambda^N \lambda^I$$

Given this, trade finance reacts differently depending on the source of the enforcement problems. An decrease of enforcement at the contract level in the exporter's country implies a shift towards Open Account or Letter of Credit. A decrease in the importer's country contract enforcement leads to a shift towards Cash in Advance and Letter of Credit. An increase in national risk, however, cannot be solved by switching to a Letter of Credit. In this case an exporter can only switch to Open Account and an importer can only switch to a Cash in Advance to fully eliminate the effect of domestic enforcement problems. In the following I discuss different types of turmoil that can be observed in reality and elaborate on some predictions of the model.

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<sup>35</sup>Trade using a trigger strategy is less affected by changes in financial parameters as long as the trigger strategy equilibrium can be sustained, i.e. as long as changes in parameters do not lead to a breakdown of the trigger strategy equilibrium.

**Interest rate changes** There are two potential sources of interest rate changes. First, due to events that might take place outside the two trading countries, the world interest rate  $r^w$  can change, moving both interest rates simultaneously. Second, there can be some problems of banks in one or both of the countries so that their efficiency parameters  $\varphi_i$  change, which affects interest rates.<sup>36</sup> The effect of these changes crucially depends on the time horizon in which firms are able to switch their payment contracts. If they are fully flexible in their choice of payment contracts, then the effects of interest rates changes are captured by Figure 2 as discussed above. If firms are not flexible in the short run, the effects of interest rate changes might be stronger as shown in Figure 5. If an exporter can change the payment contract in the time horizon studied, then a unilateral change in the interest rate can, from some point on, be fully mitigated while a change in both interest rates cannot. If there is a short run inability to switch contracts, then we should not observe a differential effect of unilateral and multilateral changes in interest rates.

**Contract level enforcement changes** As introduced above there are enforcement problems at the national and at the contract level. From these two, the national risk seems to be more susceptible to turmoil. The individual contract enforcement is mainly determined by legal institutional factors, which should in general not change in the short run.<sup>37</sup> Yet, perceived national risk like sovereign default or expropriation probabilities can change relatively quickly. As discussed above, national risk cannot be mitigated by a Letter of Credit. Thus the only way two trading partners can react to a deterioration in national risk in one of the two countries, is to switch to the one contract that is not affected by this change: Open Account in case of a deterioration in the exporter country and Cash in Advance in case of a worsening in the importer country. Note, though, that this depends on the effect of changes in national risk on the interest rate in the affected country. An increase in sovereign default could imply a proportional increase in the national interest rate. That is, the risk free interest rate would then be scaled up by the national risk. In this case no contract switching is possible to mitigate the effect. However, any deviations from a proportional change in interest rates to changes in national risk allow improvements through a change of payment contract.

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<sup>36</sup>For details on  $r^w$  and  $\varphi_i$  see the interest rate model in the Appendix.

<sup>37</sup>If one would extend the individual contract enforcement by factors changing expected firm level insolvency and illiquidity this would be different. Then, an economic crisis in a country could change contract fulfillment expectations at the firm level.

## 2.6.2 Trade Patterns

One aspect of trade that has been analyzed are differences in trade flows and trade volumes between developed countries, between developed and developing countries and between developing countries. Taking into account that for trade both the financial conditions in the importer and exporter country are relevant can help explain differences in trade patterns. A model that looks at exporter financial markets only can e.g. not predict differences between South-South and South-North trade as a result of financial conditions. All differences in such a framework would be attributed to the demand side of trade. The new feature of my model, namely, that the importer's financial market and enforcement conditions matter, leads to the following two propositions about trade volumes, export participation and importing country conditions:

### Intensive margin

**Proposition 2.4** *For given domestic financial conditions  $r, \lambda$  and foreign demand conditions  $P^*$  and  $Q^*$ , exports of a firm increase in foreign financial market conditions.*

*i) strictly if both  $r^*$  decreases and  $\lambda^*$  increases*

*ii) weakly if  $r^*$  decreases or  $\lambda^*$  increases*

**Proof.** See Appendix. ■

Exports of a country increase if the financial conditions in the country it is exporting to improve. Thus, in order to predict trade flows, it is not sufficient to look at financial market conditions in the country of the exporter alone, but one has to take into account the conditions in both countries jointly. Analyzing payment contracts allows to understand how these conditions interact and influence trade quantities.

**Extensive margin** Similar effects arise in a model with an extensive margin, where firms decide whether to export or not to another country. To see this suppose for this paragraph that there is a fixed cost  $f_x$  of serving a foreign market, which has to be incurred by any firm which wants to export. Then a firm will only export if the expected profits from entering the foreign market are at least as large as the fixed cost that have to be incurred. Suppose that the fixed cost can be financed without any additional financial problems. Then:

**Proposition 2.5** *For given domestic financial conditions  $r, \lambda$  and foreign demand conditions  $P^*$  and  $Q^*$ , a firm is more likely to export if foreign financial market conditions are better.*

*i) strictly if both  $r^*$  decreases and  $\lambda^*$  increases*

*ii) weakly if  $r^*$  decreases or  $\lambda^*$  increases*

**Proof.** See Appendix. ■

Not only do trade volumes depend on the conditions in financial markets in the countries of the exporter and the importer, but also the decision of firms to export, the extensive margin. That is, the probability of exporting cannot be fully predicted by conditions in the financial market of the exporter. Conditions in the financial market of the importer play a role, too.

### 2.6.3 FOB Prices

Given per unit cost  $a$  and iceberg transportation costs  $\tau$ , different payment contracts imply different FOB prices. To see this, note that from before the agreed on payment amounts  $C$  differ by payment contract, i.e.:

$$C^{CIA} = \frac{\lambda}{1+r^*} R^{CIA}, \quad C^{OA} = R^{OA}, \quad C^{LC} = \frac{R^{LC}}{1+r^*}.$$

$R$  is the amount of sales revenues in the importing country when trade takes place:

$$R = \left( \frac{\beta}{\alpha} \tau \right)^{1-\sigma} r_d^*.$$

The following payment amounts corresponding to FOB prices can be derived:

$$C^{CIA} = \lambda(1+r^*)^{-\sigma} \tilde{r}, \quad C^{OA} = (\lambda^*)^{\sigma-1} (1+r)^{1-\sigma} \tilde{r}, \quad C^{LC} = (1+r)^{1-\sigma} (1+r^*)^{-\sigma} \tilde{r},$$

with  $\tilde{r} = \tau^{1-\sigma} r_d^*$ .

From this it can be seen that the amounts specified to be paid for the traded goods vary with financial market parameters in a systematic way. Depending on the payment form used financial parameters affect FOB prices differentially. In an empirical analysis of FOB price data it might thus be relevant to control for differences in payment contracts. Estimates regarding FOB prices and financial indicators might otherwise be biased.

## 2.7 Conclusions

In this paper I propose a new theory explicitly modeling the choice of different payment contracts in international trade. I analyze the trade-offs taken into account by an exporter choosing between these different forms of payment, which are determined by enforcement probabilities and financing costs. The model shows that the choice of payment contracts is quantitatively important. Financing decisions of trade transactions are driven by factors in the financial markets of the exporter and the importer. The ability to freely choose and switch between payment contracts is central for firms to adapt to different constellations of financial conditions in different country pairs and over time. Limiting this choice can reduce traded quantities significantly, increase prices, and reduce the ability of exporters to react to short term fluctuations in financial conditions.

The model maps enforcement probabilities and financial market efficiencies of an exporter-importer country pair into an optimal payment contract. In a richer model that could be brought to the data one might want to include extensions concerning heterogeneity both in the firm and in the product dimension. Product differences could imply different degrees of enforceability in court or different time horizons of trade relationships (high or low  $\gamma$ ). Firm differences in size could affect relative negotiation power between the exporter and the importer, the ability to enforce contracts in court, the ability to punish deviations from a trigger strategy and the ability to switch contracts in the face of sunk costs. Another extension would be to explicitly introduce currencies and to study the interaction of the payment contract decision with exchange rate risk. This would give a suitable framework to study different aspects: first, which new effects arise from payment contracts for the optimal decision in which currency to price exports, and second, how this affects the transmission mechanism of international shocks.

There is little data available on the usage of the different payment contracts across countries, firms and time. To assess the model empirically, available firm level data, that does not contain direct evidence on payment contracts, could be used to test predictions of the model. Furthermore, a new data set on payment contracts could be built to directly test for determinants of the choice of payment contracts.

# Appendix B

## Appendix to Chapter 2

### B.1 Proofs

**Proposition 1 Proof.** The statement is true for trade measured both in expected quantities and in expected revenues at factory gate. Note that:

$$E[q_x] = A\tau^{1-\sigma}q_d^* = \alpha^\sigma\beta^{1-\sigma}\tau^{1-\sigma}q_d^*$$

and

$$E[r_x] = \frac{\beta\tau}{\alpha}A\tau^{1-\sigma}r_d^* = \alpha^{\sigma-1}\beta^{2-\sigma}\tau^{2-\sigma}r_d^*$$

Taking partial derivatives delivers the following signs for  $A = \alpha^\sigma\beta^{1-\sigma}$ :

	CIA	OA	LC
$A = \alpha^\sigma\beta^{1-\sigma}$	$(1+r^*)^{-\sigma}\lambda$	$(1+r)^{-\sigma}(\lambda^*)^\sigma$	$((1+r)(1+r^*))^{-\sigma}$
$\partial A/\partial r^*$	–	0	–
$\partial A/\partial \lambda^*$	0	+	0

As  $\sigma > 1$  the expression for revenues  $\alpha^{\sigma-1}\beta^{2-\sigma}$  delivers the same signs:

	CIA	OA	LC
$\alpha^{\sigma-1}\beta^{2-\sigma}$	$(1+r^*)^{1-\sigma}\lambda$	$(1+r)^{1-\sigma}(\lambda^*)^{\sigma-1}$	$((1+r)(1+r^*))^{1-\sigma}$
$\partial A/\partial r^*$	–	0	–
$\partial A/\partial \lambda^*$	0	+	0

If  $r^*$  decreases, both quantities and revenues increase under CIA and LC. If  $\lambda^*$  increases, quantities and revenues increase under OA. ■

**Proposition 2 Proof.** The extensive margin is determined solely by expected profits. Thus it is sufficient to consider expected profits. Note that:

$$E[\Pi_x] = A\tau^{1-\sigma}\Pi_d^* = \alpha^\sigma\beta^{1-\sigma}\tau^{1-\sigma}\Pi_d^*$$

Thus profits are monotonously increasing in  $A$ . This combined with the partial derivatives above proves Proposition 2. ■

## B.2 A Simple Model of Interest Rates

Suppose the importer and the exporter are in a small open economy and take the world interest rate  $r^w$  as given. In both countries there is a competitive sector of financial intermediation. Due to technological and legal differences, the efficiency of the intermediation technology of banks in different countries can differ. The intermediation technology displays constant returns to scale. Let  $\varphi_i \leq 1$  be the efficiency of banks in country  $i$  for handling transactions, such that the cost of lending one unit is  $1/\varphi_i$ . Zero profit implies that the interest rate of a country charged to a firm is  $r^i = \frac{r^w}{\varphi_i}$ . Thus both interest rates change with changes in  $r^w$ . Furthermore, the interest rate of a country changes with the efficiency parameter  $\varphi$ . This parameter captures both short- and long-run effects. When there is financial turmoil in a single country and the domestic interest rate rises, this is captured by a change in  $\varphi$  in that country.

When extending the model to the two types of enforcement problems, national and contract level, the interest rate in a country will likely react to the former, i.e. the risk free world interest rate will most likely be scaled up, reflecting national default risk. Let  $\gamma$  be the transmission factor measuring the effect of changes in national risk on the local interest rate. That is,  $r^i = \frac{r^w}{(\lambda^N)^\gamma \varphi_i}$ . If  $\gamma = 0$ , then national risk has no effect on the interest rate. If on the other hand  $\gamma = 1$ , the interest rate perfectly adjusts to any changes in national risk.

## B.3 Tables



Table B.1: Country Types

Country Type	I	II	III
$\lambda$	0.99	0.85	0.97
$1 + r$	1.02	1.03	1.08

Table B.2: Optimal Payment Contracts

from / to	I	II	III
I	CIA	CIA	OA
II	OA	LC	OA
III	CIA	CIA	CIA

Table B.3: Traded Quantities

Country	I	II	III
I	2.94	2.9	2.6
II	2.76	2.64	2.53
III	3.02	2.93	2.45

Table B.4: Worst Payment Contracts

from / to	I	II	III
I	LC	OA	LC
II	CIA	OA	CIA
III	LC	OA	LC

Table B.5: Optimal vs. Worst Payment Contract, percentage changes of quantities

from / to	I	II	III
I	-8.0	-49.4	-19.4
II	-9.3	-44.6	-23.2
III	-27.4	-59.2	-26.3

Table B.6: Optimal vs. Only 1 Payment Contract, percentage changes of quantities

from / to	Only CIA			Only OA			Only LC		
	I	II	III	I	II	III	I	II	III
I	0	0	-12.8	-3.6	-49.4	0	-8.0	-7.8	-19.4
II	-9.3	-4.9	-23.2	0	-44.6	0	-4.5	0	-19.2
III	0	0	0	-24.1	-59.2	-9.7	-27.4	-27.1	-26.3

## B.4 Figures

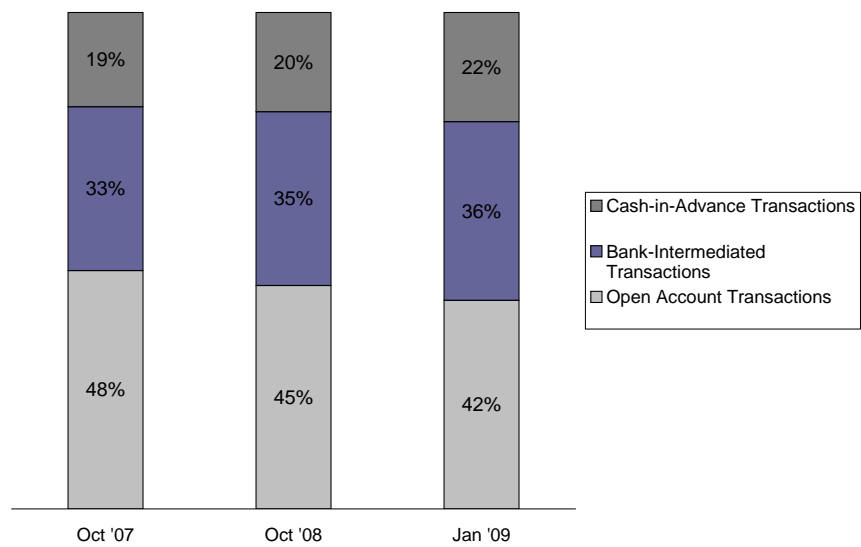


Figure B.1: Source: IMF World Economic Outlook

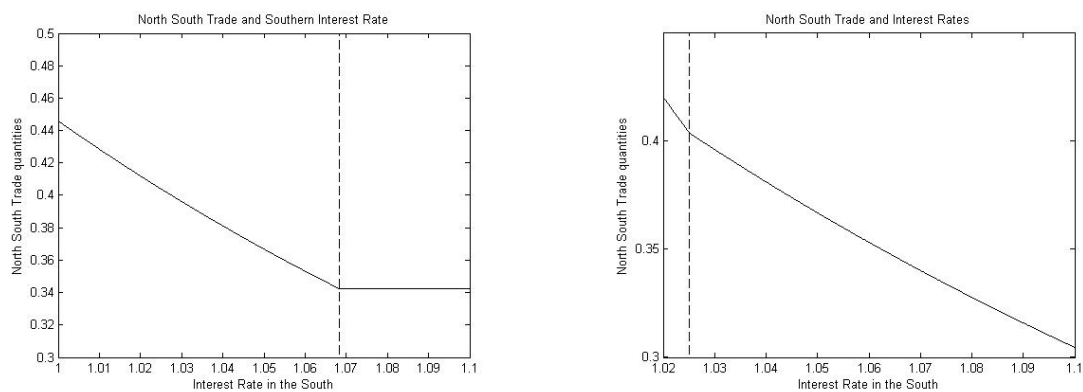


Figure B.2: The effect of a change in interest rates on North-South trade flows. Solid line: traded quantities. Vertical dashed line: change of payment contract.

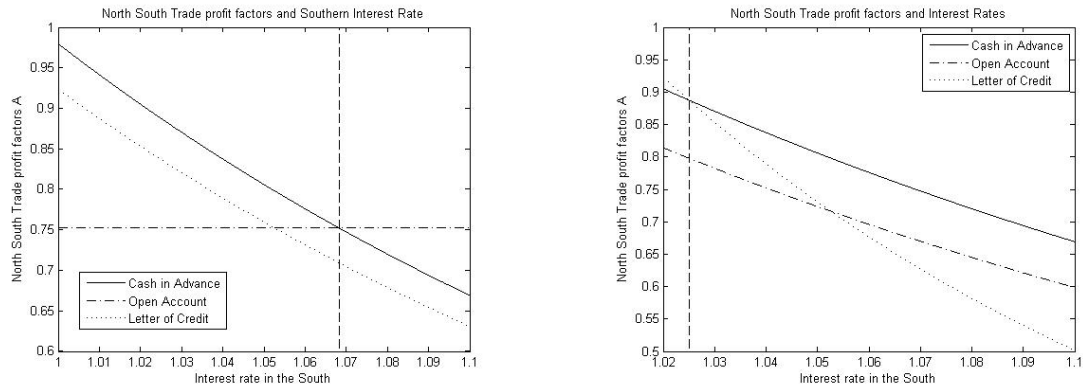


Figure B.3: The effect of a change in interest rates on the finance profit factor A. Vertical dashed line: change of payment contract.

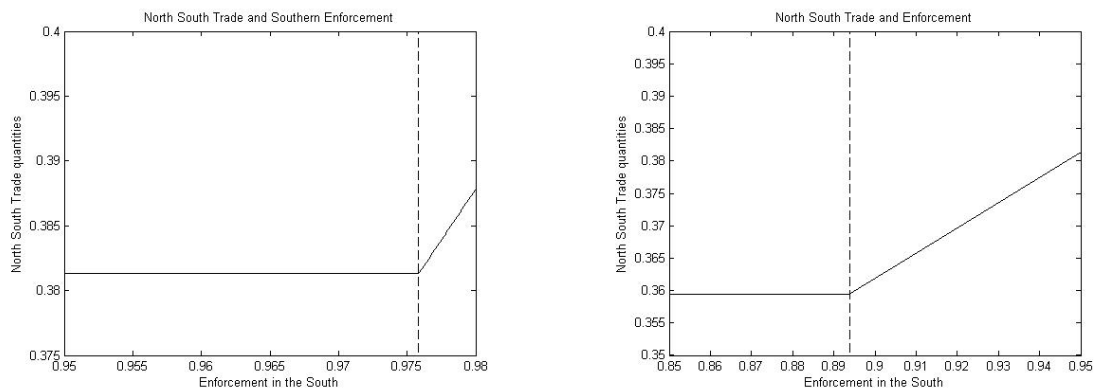


Figure B.4: The effect of a change of enforcement probabilities on North-South Trade flows. Solid line: traded quantities. Vertical dashed line: change of payment contract.

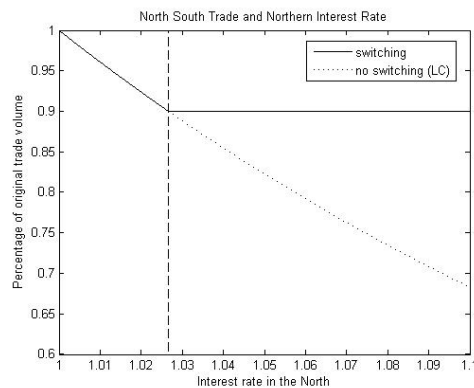


Figure B.5: Percentage effect of a change in the interest rate in the North on North-South trade flows with and without payment contract change. Dashed line: trade change with contract change. Solid line top: trade change without contract change. Vertical dashed line: change of payment contract.

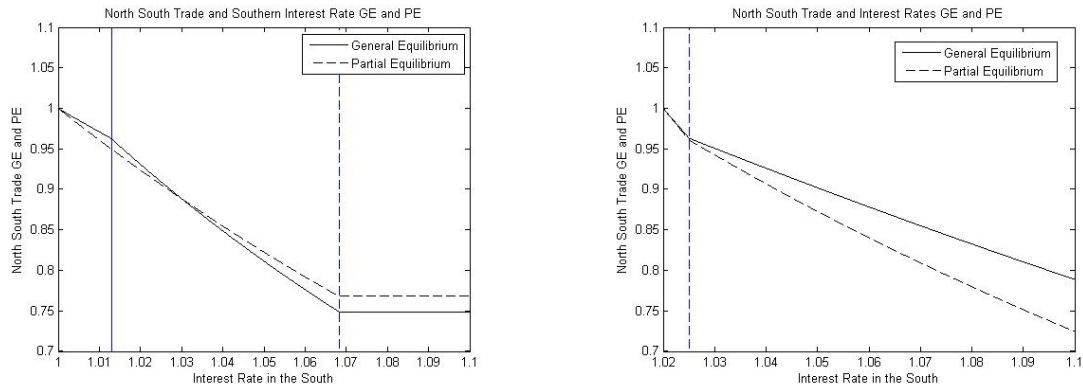


Figure B.6: The effect of a change in interest rates on North-South trade flows, GE versus PE. Vertical dashed line: change of contract foreign exporter. Vertical solid line: change of contract domestic exporter. Solid line: traded quantities general equilibrium. Dashed line: traded quantities partial equilibrium.

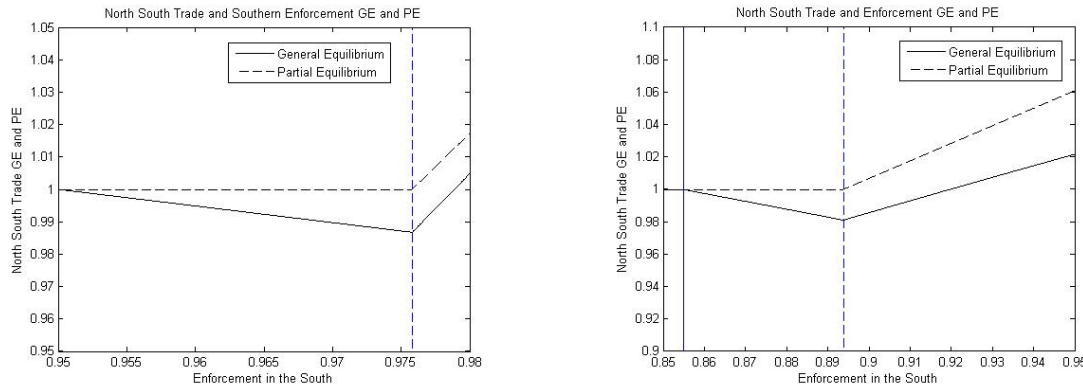


Figure B.7: The effect of a change of enforcement probabilities on North-South trade flows, GE versus PE. Vertical dashed line: change of contract foreign exporter. Vertical solid line: change of contract domestic exporter. Solid line: traded quantities general equilibrium. Dashed line: traded quantities partial equilibrium.

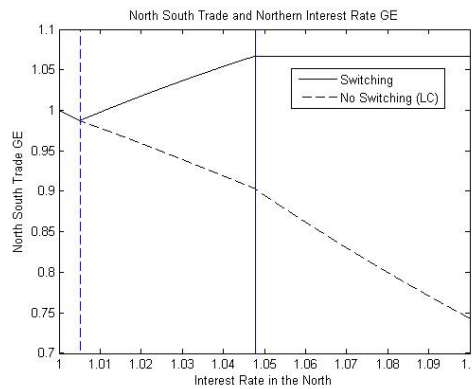


Figure B.8: Percentage effect of a change in the interest rate in the North on North-South trade flows with and without payment contract change in general equilibrium. Vertical dashed line: change of contract foreign exporter. Vertical solid line: change of contract domestic exporter. Dashed line: trade change with contract change. Solid line: trade change without contract change.

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# Chapter 3

## Bank Bail-outs, International Linkages and Cooperation

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### Abstract

Financial institutions are increasingly linked internationally and engaged in cross-border operations. As a result, financial crises and potential bail-outs by governments have important international implications. Extending Allen and Gale (2000) we provide a model of international contagion allowing for bank bail-outs financed by distortionary taxes. In the sequential game between governments, there are inefficiencies due to spillovers, free-riding and limited burden-sharing. When countries are of equal size, an increase in cross-border deposit holdings improves, in general, the non-cooperative outcome. For efficient crisis management, ex-ante fiscal burden sharing is essential as ex-post contracts between governments do not achieve the same global welfare.

JEL: *F36, F42, G01, G28*

Keywords: *bail-out, contagion, financial crisis, international institutional arrangements*



## 3.1 Introduction

With globalization, banks have become more and more linked internationally and engaged in cross-border operations. As a result, banking crises today have international dimensions which make bail-out decisions less of a domestic and more of an international issue, with governments in different countries responding and consumers in different countries being affected. In this paper we present a theoretical model of international contagion, where governments decide domestically upon ex-post intervention and explore inefficiencies from unilateral decision-making. We analyze how different forms of cooperation can improve upon the non-cooperative outcome.

Banking crises are a frequent phenomenon. Caprio and Klingebiel (2003) compiled a data set with information on 117 systemic banking crises that have occurred in 93 countries since the late 1970s. In most countries, banking crises led to intervention by national governments, the fiscal costs of which have been considerable. Honohan and Klingebiel (2000) study a sample of 40 countries and find that, in order to restore the financial system, governments spent on average 12.8 percent of national GDP. Often, this implies rising government debt. Reinhart and Rogoff (2009) investigate the aftermath of financial crisis and find that the real value of government debt tends to increase sharply. While the collapse in tax revenues resulting from contractions of real output is the major reason for this, bail-out and recapitalization costs can be an important factor, too. During the recent financial crisis, governments all over the world have responded with significant financial support. In a report, the IMF provides an overview of taken measures and related costs. Upfront commitments for financing pledged support operations are estimated at 5.7 % of GDP for the advanced G-20 as of August 2009. Pledged capital injections alone amount to 2.2% of G-20 GDP. (see Tables 3 and Annex Tables 3 and 4 of IMF (2009)).

The recent global financial crisis, which emanated from the United States, showed how crisis can spread from one country to another. Degryse and Penas (2009) analyze cross-border contagion risk for the period from 1996 to 2006 using data on cross-border exposure of 17 countries. They find that the speed of propagation of contagion has increased over the past years and that a shock which affects the liabilities of one country may undermine the stability of the global financial system. In the face of contagion risk due to increasing cross-border exposures and high economic costs associated with banking crises, banks in distress become an international issue. Claessens (2009) investigates financial nationalism in the context of the current financial crisis. He gives evidence for international effects of

unilateral government intervention. Guarantees on deposits and other liabilities issued by individual countries had beggar-thy-neighbor effects. Countries also started to ring-fence assets in their jurisdictions when cross-border entities showed signs of failing, like the German government that froze Lehman assets in 2008. Other examples for conflicting international interests in the context of financial institutions in distress are the AIG bail-out by the U.S. Federal Reserve in 2008, which benefited several non-U.S. financial institutions, as well as the bankruptcy case of Icesave in 2009 concerning Dutch and British depositors.

Such conflicting international interests give rise to potential gains from cooperation between governments. The initiative of the European Union toward a new financial architecture, a response to the current financial crisis, is, amongst other things, meant to foster cooperation in crisis management. It has been argued that the resolution of distressed cross-border banks within the EU is not handled efficiently. Crisis management is limited to the national level or ad-hoc cross-border solutions (see De Larosiere Report (2009)). Correspondingly, little cooperation of governments was observed during the current crisis. An exception are Belgium, the Netherlands and Luxembourg, with some involvement of France, who cooperated to a certain extent in order to resolve Dexia and Fortis (see Claessens (2009)). The first plans of the EU Commission to endow a European regulator with the right to enjoin individual governments to bail out a national bank at their own expense was dismissed. However, due to the perceived need for EU-wide coordination, the European Systemic Risk Board, a new EU institution, has been given the task to issue policy recommendations on how to deal with distressed banks.

In the following we study a model of international contagion where banks are linked through interbank deposits. We analyze a sequential game between two governments facing a potentially contagious banking crisis. Governments can prevent bankruptcy by doing a bail-out financed by distortionary taxes, which has spillover effects on the other country. We show that inefficiencies in the bail-out decision of governments arise when there is no international cooperation. We introduce private cross-country deposits to study their effects on efficiency in the non-cooperative outcome. In a next step, we focus on fiscal burden sharing, which can further improve efficiency. Finally, we model Nash bargaining between governments and consider how a central authority with mandating and/or fiscal power or contracts impact global and national welfare.

**Contagion model** Building upon Allen and Gale (2000), we model cross-border financial linkages in form of interbank deposits. Interbank deposits allow for risk sharing of idiosyncratic liquidity needs, but induce systemic risk. Crisis spreads from the one bank to the other bank in the system when the former goes bankrupt due to unexpected liquidity needs and interbank deposits cannot be repaid fully. Facing a bankrupt bank, governments can decide not to take any action. Then, the bankrupt bank is liquidated. Alternatively, the government may bail out its bank financed by distortionary taxes. This action directly affects depositors of the bank that is saved and domestic households that are taxed. Moreover, it impacts the foreign bank and its depositors, either because contagion is avoided (spillover effect on the *affected country*) or because its liquidation value is raised (spillover effect on the *crisis country*).

We study inefficiencies in government intervention when there is no international authority. In the sequential bail-out game the country where the crisis arises moves first, the other one follows. We identify three distinct sources of inefficiencies. First, externalities arise from the fact that governments maximize national welfare, but do not take the spillover effects into account which impact the welfare of the other country. In this context, we find a second source of inefficiency, a free-riding problem that arises through the sequential nature of our model. The crisis country may not bail out its domestic bank because it knows that then the affected country will do a bail-out. This, in turn, benefits the crisis country through the positive spillover in form of increased returns on interbank deposits. In this way, as the first mover, it can free ride on the bail-out carried out by the affected country. The larger the interbank linkages, the larger the spillovers and the bigger the incentives to free-ride. A third inefficiency arises due to no burden-sharing, i.e. the inability, in the non-cooperative game, to share the costs of a bail-out that arise in one country between the two governments. Furthermore, we analyze the effects of cross-country deposits and asymmetric country sizes on government intervention. Due to private cross-country deposits, governments take spillover effects in their bail-out decision into account. Whether cross-country deposits tend to increase or decrease efficiency depends on the extent of cross-country deposits and potential asymmetries in country sizes. We find that, if country sizes are equal, cross-country deposits tend to reduce inefficiencies.

We study three different forms of cooperation: a central authority with mandating power and/or fiscal power, and contracts. Our main findings are the following: A central authority with mandating and fiscal power achieves the best ex-post outcome trading off

income inequality with distortions from taxation. At an optimum, burden sharing is such that the affected country always contributes more than the crisis country. This can, but does not need to imply a Pareto improvement compared to the non-cooperative solution. A central authority with mandating power alone cannot induce a Pareto improvement compared to the non-cooperative solution, whereas contracts imply this by their very nature. The two regimes alleviate different inefficiencies. A clear ranking between them in terms of efficiency is therefore not possible. In general, mandating power alone or ex-post contracts do not achieve as high a welfare level as an authority with mandating and fiscal power.

**Related literature** There are several approaches to model contagion. One is to model contagion as driven by information. Bad news about one bank imply, due to a correlation in the value of assets, bad news for another institution (see Acharya and Yorulmazer (2008) and Chen (1999)). A different mechanism is proposed in Diamond and Rajan (2005) who show that contagion can be an equilibrium phenomenon because bankruptcy of one bank may reduce the available aggregate liquidity, which then causes bankruptcy of potentially all banks in the system. The third approach, which we follow in this paper, explains contagion by the fact that banks are connected through their balance sheets. In Allen and Gale (2000) systemic risk arises because banks are linked through interbank deposits that insure banks against idiosyncratic liquidity risk. Default of one bank reduces the value of assets in other banks, the ultimate consequence of which can be the failure of the entire banking system. Dasgupta (2004) applies the theory of global games, developed by Carlsson and van Damme (1993) and introduced to this setting by Morris and Shin (2003), to a similar setup. Depositors receive private, but correlated signals about the fundamentals of the bank they deposited in. Depending on the signal, a bank run occurs and crisis spreads along the channels of interbank deposits.

There is a large literature on optimal government intervention at the national level and the role for a Lender of Last Resort (LLR). The classic doctrine of Bagehot (1873) that central banks should lend to illiquid, but solvent banks has been studied extensively (see i.e. Rochet and Vives (2004), Freixas, Parigi, and Rochet (2000) and Freixas and Parigi (2008) for a summary of the literature on the LLR). Castiglionesi (2007) investigates the role of a central bank in the framework of Allen and Gale (2000) where it can prevent financial contagion by setting appropriate reserve requirements. Bail-outs have also been studied by Diamond and Rajan (2002). In their paper, bail-outs increase the

aggregate liquidity available, which can lead to ambiguous effects on the stability of the banking system. In Gorton and Huang (2004), private hoarding of liquidity is socially costly. A government can reduce this hoarding by subsidizing distressed banks as it can issue government securities backed by future tax revenues. Goodhart and Huang (2005) consider the trade-off between liquidity support and moral hazard costs which central banks face and rationalize the argument for so-called 'constructive ambiguity'. Cordella and Yeyati (2003) show that bail-outs can instead have risk-reducing effects. The optimal institutional arrangement for the LLR function is the focus of some further studies e.g. Kahn and Santos (2005) and Repullo (2000).

Some recent contributions put banking theory in an international perspective. One focus of the literature on regulation is on analyzing the scope for international cooperation among bank regulators, i.e. on understanding the interactions between internationally operating banks, which are to be regulated, and national regulatory incentives. Holthausen and Roende (2005) study multinational banks and optimal closure policies, where each authority maximizes national welfare. Asymmetries of operating banks and of regulators' preferences across countries lead to suboptimal closure decisions in a 'cheap talk' game. Acharya (2003) considers the international standardization of ex-ante capital adequacy ratios and ex-post closure policies in a framework where domestic banks compete internationally. In his model, competition among banks leads to inefficient closure policies and finally increased risk in the banking system. Dell'Ariccia and Marquez (2006) look at the scope for centralized regulation when competing banks are heterogeneous across countries. Again, asymmetries lead to suboptimally low standards due to a trade-off between flexibility in policy design and increased stability of the banking system.

There is a policy debate on regulation and the LLR function in the context of a globalizing banking sector, especially within the European Union. Vives (2001) argues in favor of a bigger role for the European Central Bank in crisis management and more centralized supervisory arrangements. Teixeira and Schinasi (2006) raise the issue of cross-border externalities and the need for a centralized LLR for the EU Single Market. Pauly (2008) considers crisis management in Europe and makes the case for financial burden sharing rules among national governments. Claessens (2009) reviews government interventions taken during the current financial crisis with respect to competition policy and points at the need for international coordination.

While there is considerable theoretical work on international issues related to regulation and supervision, there are only few papers that study cooperation problems arising

in an international banking crisis. Agur (2009) treats the question of the optimal institutional structure for a LLR in an international framework where depositors have imperfect information about the solvency of banks and government intervention has a signaling effect. While national governments do not internalize the contagion effect, a central authority has limited signaling power. Therefore, the maximum welfare is achieved by central coordination rather than pure centralization of the bail-out decision. Freixas (2003) discusses externalities from bail-outs in a multi-country setting. In a standard public goods model, he analyzes cooperation mechanisms which implement first-best bail-outs. Goodhart and Schoenmaker (2009) extend this model and consider ex-ante fiscal burden sharing rules in order to induce optimal cooperation. Our model goes beyond the pure public goods problem of bank bail-outs that the latter two papers study and the framework of Agur (2009), as we explicitly model international interbank linkages and bail-out incentives in a contagion framework based on Allen and Gale (2000).

The remainder of this paper is structured as follows. Section 2 introduces the model. Section 3 discusses centralized and decentralized decision making and their efficiency properties. Section 4 extends the model by allowing for private cross-country deposits. Section 5 discusses fiscal burden sharing. Section 6 considers contracts. Section 7 compares different forms of cooperation with respect to efficiency and Pareto improvements. Section 8 studies ring-fencing. Section 9 concludes.

## 3.2 The Model

Our model builds on Allen and Gale (2000). We use their basic framework to model interbank linkages and contagion. We extend the analysis to an international setting with two countries and allow for government interventions in case of bankruptcy after uncertainty about liquidity needs has been resolved. Moreover, we introduce a production sector operating at date  $t = 1$ , employing labor whose income can be taxed by the government in order to finance interventions.

### 3.2.1 Setup

There are three time periods indexed by  $t = 0, 1, 2$  and a continuum of ex-ante identical agents of measure one. Each agent is endowed with one unit of a single consumption good at date  $t = 0$ . It serves as numéraire and can be invested in two different assets, a short asset and a long asset. The short asset represents a storage technology. For each

unit invested at date  $t$ , it pays out one unit at date  $t + 1$ . The investment in the long asset can only take place at date  $t = 0$ , but gives a higher pay-off  $R > 1$  at date  $t = 2$ . The long asset can be liquidated in period  $t = 1$ , but early liquidation is costly. For one unit invested at date  $t = 0$ , only  $r < 1$  units can be recovered. At date  $t = 1$ , each agent decides on its supply of labor to the perfectly competitive production sector. Each unit of labor produces 1 unit of the consumption good. Consumers have Diamond-Dybvig preferences. With probability  $\lambda$ , an agent only values consumption at date  $t = 1$  (early type), while with probability  $1 - \lambda$  it is of the late type and values consumption only at date  $t = 2$ . Individual preferences are given by:

$$U(c_1, c_2) = \begin{cases} u(c_1) & \text{with probability } \lambda \\ u(c_2) & \text{with probability } 1 - \lambda, \end{cases} \quad (3.1)$$

where  $u$  is assumed to be twice continuously differentiable, increasing and strictly concave. Consumption of an agent of type  $i$   $c_i$ , is composed of three different elements: the return from the investment  $d_i$  made at date  $t = 0$ , labor income  $n$ , which depends on the labor supplied at date  $t = 1$ , and disutility of work expressed in consumption terms.<sup>1</sup> That is:

$$c_i = d_i + \eta \left( n - \frac{n^2}{\kappa} - \frac{\kappa}{4} \right). \quad (3.2)$$

Disutility of work is quadratic, with shape parameters  $\kappa$  and  $\eta$ . Due to our assumptions on the utility function, the labor supply decision of the agent is independent of its type and we can drop the subscript  $i$ .<sup>2</sup> The first order condition for labor of both types of individuals is:

$$\frac{du(c_i)}{dn} = u'(c_i)\eta \left( 1 - \frac{2n}{\kappa} \right) = 0. \quad (3.3)$$

Therefore, in an optimum each agent supplies  $n = \kappa/2$  units of labor. With the last term of Equation 3.2, we normalize the utility contribution of labor for the optimal labor supply to 0, which is convenient for our subsequent analysis.<sup>3</sup>

By the law of large numbers, the probability  $\lambda$  of being an early consumer, is also the fraction of early consumers in the economy. We assume that the population is divided into two groups of consumers, each of mass one. Within each group, the fraction of early

<sup>1</sup>Our setup is similar to Cooper, Kempf, and Peled (2008).

<sup>2</sup>Late consumers, who only consume at date  $t = 2$ , store their labor income from date 1 to date 2. The disutility of labor, although conceptually arising at date  $t = 1$ , unfolds only at  $t = 2$ .

<sup>3</sup>Due to the normalization, the date-0 investment decision of the bank is not impacted by the level of the expected labor income and the bank's optimization problem that we consider later can be formulated as is standard in the literature.

consumers is stochastic. Across groups it is perfectly negatively correlated. There are two possible states of nature  $S_1$  and  $S_2$ , which are summarized in Table 1, where  $\lambda_H$  denotes a high fraction of early consumers and  $0 < \lambda_L < \lambda_H < 1$ . Both states occur with equal probability. Groups of consumers are thus identical in expectation and aggregate demand for liquidity is the same in both states.

Table 3.1: Liquidity shocks

	Group A	Group B	Probability
State $S_1$	$\lambda_h$	$\lambda_l$	0.5
State $S_2$	$\lambda_l$	$\lambda_h$	0.5

### 3.2.2 Optimal Risk Sharing and the First-Best Allocation

To start with, we analyze the first-best allocation given the two available investment technologies. As there is no aggregate uncertainty, the optimal allocation implies perfect risk-sharing and allocations are independent of the two states. In order to find the first-best solution, we consider a social planner that has perfect information, hence knows the type of each consumer. The planner chooses investment at date  $t = 0$  so as to maximize overall expected utility treating all consumers alike. The maximization problem includes three feasibility constraints and is as follows:

$$\begin{aligned} \max_{\{d_1, d_2\}} \quad & \bar{\lambda}u(c_1(d_1)) + (1 - \bar{\lambda})u(c_2(d_2)) & (3.4) \\ \text{s.t.} \quad & x + y \leq 1, \\ & \bar{\lambda}d_1 \leq y, \\ & (1 - \bar{\lambda})d_2 \leq Rx, \end{aligned}$$

where  $x$  and  $y$  are the per capita amounts invested in the long and the short asset, respectively. The three constraints represent the resource constraints at date  $t = 0$ ,  $t = 1$  and  $t = 2$ . The social planner anticipates the optimal labor supply by the agents. Substituting  $n = \kappa/2$  yields the standard objective function, which is independent of labor income:  $\bar{\lambda}u(d_1) + (1 - \bar{\lambda})u(d_2)$ . The unique solution to the problem is characterized by the condition:

$$u'(\bar{d}_1) = Ru'(\bar{d}_2), \quad (3.5)$$



where  $\bar{d}_1$  and  $\bar{d}_2$  denote the consumption levels that early and late consumers receive.

As aggregate consumption at dates 1 and 2 is constant and liquidation of the long asset is costly, it is optimal to provide date-1 consumption by investing in the short asset and date-2 consumption by investing in the long asset. Thus, at the optimum, all constraints bind, and  $\bar{d}_1 = y/\bar{\lambda}$  as well as  $\bar{d}_2 = R(1 - y)/(1 - \bar{\lambda})$ .

### 3.2.3 Decentralization and Interbank Deposits

In this section, we introduce a banking sector and show that the first-best allocation from above can be decentralized as an equilibrium with competitive banks. First, we describe the decentralized setting and define the equilibrium. Second, we discuss a specific deposit and interbank deposit contract and derive the corresponding equilibrium. Third, we show that the resulting allocation coincides with the first-best and prove equilibrium existence.

Assume that only banks can invest in the long asset. Therefore, they have two advantages: They can invest in both assets and, by pooling endowments, provide insurance against liquidity risk. The banking sector is perfectly competitive. This implies that, in order to attract funds, banks have to offer a contract that maximizes the expected utility of its depositors. We consider the following deposit contract  $D$  that is not contingent on the state of nature. A depositor can choose to withdraw at date  $t = 1$  or at date  $t = 2$ . Per unit deposited, the contract specifies a repayment  $d_1$  to depositors that withdraw at date  $t = 1$ . A late withdrawer receives a pro rata share of the bank assets remaining at date  $t = 2$ , which is denoted by  $d_2$ . If the bank cannot serve all withdrawals at date  $t = 1$ , it is bankrupt and all depositors receive the same pro rata repayment. As there is no sequential service constraint, no expectation driven bank runs occur.<sup>4</sup>

We assume that each bank faces uncertainty about the fraction of early and late depositors as described in Table 1. Thus Group A corresponds to the depositors of Bank A and Group B to depositors of Bank B. This uncertainty introduces an incentive for banks to sign contracts with each other in order to insure against liquidity risk. Interbank deposit contracts have the same specification as the deposit contracts between consumers and banks. Repayments per unit of deposit are contingent on the date of withdrawal  $(d_1^I, d_2^I)$ , but independent of the state. If the counterpart bank goes bankrupt, a bank receives a pro rata share of the counterpart's liquidation value and thus incurs a loss on

<sup>4</sup>Note that our model does not feature multiple equilibria as e.g. in Diamond and Dybvig (1983), Cooper and Ross (1998), Ennis and Keister (2006) and Ennis and Keister (2009). Therefore, government intervention in our model is not driven by an equilibrium selection motive, but by a liquidity shortage.

the interbank deposits. The amount of interbank deposits each bank makes is denoted by  $z$ .

The timing is as follows. At  $t = 0$ , consumers sign deposit contracts with the banks and deposit their endowments. Banks sign interbank deposit contracts and each deposit amount  $z$ . Moreover, banks invest in the long and the short asset. At date  $t = 1$  uncertainty resolves and consumers learn privately about their types. At that stage, consumers can decide to withdraw their claims or wait until next period. Early consumers withdraw always, as they only have utility from consumption at date  $t = 1$ . Late consumers decide whether to withdraw at date  $t = 1$  and store the good or to withdraw at date  $t = 2$ , depending on payoffs  $d_1$  and  $d_2$ . Banks do the same and decide whether to withdraw the claims  $d_1^I z$  that they have at date  $t = 1$ . Define a withdrawal strategy for each bank  $w^I$  and late consumers  $w^L$  contingent on the state:

$$\begin{aligned} w^I(d, \lambda) &: \{d_1^I, d_2^I\} \times \{\lambda\} \rightarrow \{0, 1\}, \\ w^L(d) &: \{d_1, d_2\} \rightarrow \{0, 1\}, \end{aligned}$$

where 1 stands for withdrawal at date  $t = 1$  and 0 for withdrawal only at date  $t = 2$ . Consumers and banks know the state that occurred and the fraction of early depositors in each bank.

An equilibrium is characterized by a deposit and an interbank deposit contract, and withdrawal strategies of banks and late consumers.

**Definition 3.1** *An equilibrium is a deposit contract  $\{D = (d_1, d_2)\}$ , an interbank deposit contract  $\{D^I = (d_1^I, d_2^I)\}$ , an amount of interbank deposits  $\{z\}$ , a withdrawal strategy of banks  $\{w^I(d, \lambda) : \{d_1, d_2\} \times \{\lambda\} \rightarrow \{0, 1\}\}$ , a withdrawal strategy of late consumers  $\{w^L(d) : \{d_1, d_2\} \rightarrow \{0, 1\}\}$  such that*

- (i) *the deposit contract  $D$  maximizes expected depositor utility given  $\{D^I, z\}$ ,*
- (ii) *there does not exist another interbank deposit contract and amount of interbank deposits  $\{D^I, z'\}$  that imply a higher expected utility for depositors,*
- (iii) *given  $\{D^I, z, \lambda\}$ , bank withdrawal strategies are optimal,*
- (iv) *given  $\{D, z\}$ , late consumer strategies are optimal.*

Next, we describe an equilibrium characterized by a set of  $D$ ,  $D^I$ ,  $z$ ,  $w^I(d^I, \lambda)$  and  $w^L(d)$  and show that its allocation coincides with the first-best allocation derived before.

Consider the following equilibrium: The deposit and interbank deposit contracts are

identical and given by  $D = D^I = (\bar{d}_1, \bar{d}_2)$ , as determined by Condition 3.5, and the interbank deposits are given by  $z = \bar{d}_1(\bar{\lambda} - \lambda_L)$ . Banks withdraw early if their fraction of early consumers is low and wait if the fraction is high.<sup>5</sup> Late consumers only withdraw at date  $t = 2$ .

For an illustration of the equilibrium, suppose that State  $S_1$  occurs. The liquidity needs of early consumers of Bank A are  $\bar{d}_1\lambda_H$ . Given the high liquidity needs, Bank A calls in its interbank claims and withdraws  $z\bar{d}_1 = \bar{d}_1(\bar{\lambda} - \lambda_L)$ . The budget constraint Bank A faces at date  $t = 1$  is  $y + \bar{d}_1(\bar{\lambda} - \lambda_L) = \bar{d}_1\lambda_H$ , which corresponds to  $y = \bar{\lambda}\bar{d}_1$ , the date-1 budget constraint of the social planner. Bank B with the low liquidity needs does not withdraw its interbank deposits, but pays out the amount requested by Bank A. Therefore, the budget constraint Bank B faces at date  $t = 1$  is  $\lambda_L\bar{d}_1 = y - \bar{d}_1(\bar{\lambda} - \lambda_L)$ , which again coincides with the budget constraint of the social planner. Similarly one can show that the budget constraints at date  $t = 2$  given the interbank deposits and withdrawal decision of the banks and the consumers specified above reduce to the one of the social planner. Thus, given the interbank contracts, interbank deposits and withdrawal decisions, the bank's investment problem reduces to the one of the social planner. The first-best allocation is implemented.

In order to prove existence of the characterized equilibrium, consider the following argument. Consumers choose the deposit contract that maximizes their expected utility. Due to perfect competition in the banking sector, this deposit contract has to be the constrained maximum that can be attained by a pair of banks given the investment technology and the instruments  $\{D, D^I, z\}$ . As shown, the values for  $\{D, D^I, z\}$  suggested above attain the global first-best given the investment technology. Thus, there is no possible deviation by any pair of banks that would allow them to offer deposit contracts with the same level of expected utility but positive profits.

### 3.2.4 Contagion

Now, we introduce the possibility of bank runs and contagion. The decentralized first-best allocation is fragile in so far as a perturbation can lead to bankruptcy of all banks in the system. Following Allen and Gale (2000), we perturb the banking system by introducing

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<sup>5</sup>We assume that interbank deposits equal the minimum amount necessary to implement the first-best allocation. Interbank deposits could also be larger. However, if there is a small but positive probability on the perturbation state, which we introduce in the next section, then banks optimally hold only the minimum amount of interbank deposits necessary to implement liquidity insurance for states  $S_1$  and  $S_2$  in order to minimize the contagion risk.

a third state that is assigned a zero-probability at date  $t = 0$ . This assumption is a departure from rational expectations. It allows us to focus on bankruptcy, contagion and ex-post intervention as it shuts down potential effects from ex-ante expectations on the real investment allocation.

Contracts and investment decisions at date  $t = 0$  are the first-best allocation, as in the previous section. In the third state  $\bar{S}$ , aggregate liquidity needs are higher than expected. As illustrated in Table 3.2, there is an additional fraction  $\epsilon$  of early consumers in Bank A. If states  $S_1$  or  $S_2$  occur, then the allocations at date  $t = 1$  and  $t = 2$  are first-best. However, if State  $\bar{S}$  occurs, the continuation equilibrium is different. As we are interested in reactions to bankruptcy and potential contagion, we concentrate our attention in the following on this state.

Table 3.2: Liquidity shocks with perturbation

	Bank A	Bank B	Probability
State $S_1$	$\lambda_h$	$\lambda_l$	0.5
State $S_2$	$\lambda_l$	$\lambda_h$	0.5
State $\bar{S}$	$\bar{\lambda} + \epsilon = \frac{\lambda_h + \lambda_l}{2} + \epsilon$	$\bar{\lambda}$	0

Given state  $\bar{S}$ , we can specify conditions under which firstly, bankruptcy of Bank A and secondly, contagion and hence bankruptcy of Bank B occurs. In order to make liquidation of the long asset the least attractive option, we assume that the following condition holds:

$$\frac{R}{r} \geq \frac{\bar{d}_2}{\bar{d}_1}. \quad (3.6)$$

From this a "pecking order" can be derived comparing the costs of obtaining date-1 consumption in terms of future consumption. Costs increase in the following order: short asset, interbank deposits, long assets. In state  $\bar{S}$ , the short assets of Bank A are not enough to satisfy its date-1 liquidity needs  $\bar{d}_1(\bar{\lambda} + \epsilon)$  as the optimal investment decision at date  $t = 0$  implies  $y = \bar{d}_1\bar{\lambda}$ . Therefore, facing the additional fraction of early withdrawers and given the assumption on the pecking order, Bank A calls in its interbank claims before starting to liquidate the long asset.

Bank A is bankrupt if by liquidating all assets, it still cannot meet demands of its depositors. More specifically, it goes bankrupt if it has to liquidate so much of the long asset in order to satisfy liquidity needs of early consumers that late consumers would receive a payoff smaller than  $d_1$ . Anticipating this, late consumers then decide to withdraw

their funds early and a bank run occurs. Due to losses from early liquidation of the long asset, all consumers get less than the pay-offs originally promised to early depositors in the deposit contract and  $d_1 < \bar{d}_1$ . We can derive a condition for bankruptcy of Bank A:

$$\epsilon \bar{d}_1 > r \left( (1 - y) - \frac{(1 - \bar{\lambda} - \epsilon) \bar{d}_1}{R} \right). \quad (3.7)$$

The term on the left hand side of Equation 3.7 represents the additional liquidity needs that cannot be satisfied by the investment in the short asset. As discussed, Bank A calls in its interbank claims. This, however, entails that also Bank B withdraws its interbank claims early as it faces more liquidity needs than it can satisfy with its short asset. Therefore, the interbank claims, which are of the same size, cancel out and do not appear in Equation 3.7. In order to avoid a run of its (late) depositors, Bank A must keep so much of the long asset that the return that it yields at date  $t = 2$  is at least as high as to give every late consumer the pay-off promised to early consumers  $\bar{d}_1$ . Therefore the bank must keep  $\frac{(1 - \bar{\lambda} - \epsilon) \bar{d}_1}{R}$  units of the long asset. Only  $(1 - y) - \frac{(1 - \bar{\lambda} - \epsilon) \bar{d}_1}{R}$  units of the long asset can be liquidated, which yield a return of  $r \left( (1 - y) - \frac{(1 - \bar{\lambda} - \epsilon) \bar{d}_1}{R} \right)$  at date  $t = 1$ . If these resources, the so-called buffer, are not enough to satisfy the unexpected demand of the fraction  $\epsilon$  of additional early depositors, then Bank A is bankrupt.

Bankruptcy of Bank A has an impact on the other bank through the interbank deposits. Facing a higher than expected fraction of early consumers, Bank A calls in its deposits from Bank B. In order to meet the demand by Bank A, Bank B equally withdraws its deposits from Bank A. However, Bank A is bankrupt and is liquidated. Similar to Bank A's private depositors, Bank B receives only a pro rata share of Bank A's liquidation value. This share is necessarily smaller than the value of the actual claims ( $r < 1$ ). Whether these losses are sufficient to cause bankruptcy of Bank B depends on whether the buffer of Bank B is large enough to provide sufficient liquidity at date  $t = 1$ . The following expression gives the bankruptcy condition for Bank B:

$$z(\bar{d}_1 - q^A) > r \left( (1 - y) - \frac{(1 - \bar{\lambda}) \bar{d}_1}{R} \right), \quad (3.8)$$

where  $q^A$  represents the liquidation value of Bank A. This value is affected by bankruptcy of Bank B. If both banks go bankrupt, then each bank receives a pro rata share of the other's liquidation value. With symmetric interbank deposits, the mutual claims cancel out and each bank's liquidation value  $q^j$  is given by the value of the short asset plus the

liquidation value of the long asset, hence  $q^j = \bar{q} = y + (1 - y)r$ ,  $\forall j \in \{A, B\}$ . If the bank hit by contagion (Bank B) does not go bankrupt, but fully repays the interbank deposit claims, then the liquidation value of Bank A is raised. This link is important for our later analysis. For bankruptcy of Bank B, Condition 3.8 has to hold in the case where Bank B fully repays Bank A's claims, hence when  $q^A = \hat{q} = \frac{y+r(1-y)+z\bar{d}_1}{1+z}$ . This is a sufficient condition for bankruptcy as lower liquidation values move this condition further towards bankruptcy.

Consider again Conditions 3.7 and 3.8 and assume that they hold with equality, i.e. that there is no bankruptcy. All other things equal, a lower liquidation value  $r$  or a lower return of the long asset  $R$  causes bankruptcy. The lower the liquidation value of the long asset, the more long assets have to be liquidated early in order to meet date-1 demand. Similarly, the lower the return on the long asset, the fewer long assets can be liquidated without lowering the pay-off of late depositors below  $\bar{d}_1$ , which would cause a bank run. Moreover, a larger amount  $\bar{d}_1$  promised to early depositors leads to bankruptcy.  $\bar{d}_1$  depends on the relative risk aversion of the consumers. If the relative risk aversion coefficient is above 1, then the promised pay-off to early depositors exceeds the return of the storage technology,  $\bar{d}_1 > 1$ , and liquidity insurance is provided to early depositors.

### 3.2.5 Government Intervention within one Jurisdiction

From now on, we assume that the two bankruptcy conditions 3.7 and 3.8 hold. We cast the model in an international setting with two banks and two countries. Bank A is located in Country A, Bank B in Country B. Banks are linked internationally through interbank deposits as described before. Each country has a government that maximizes welfare of its population and that can decide to intervene when faced with potential bankruptcy of its domestic bank. In order to finance an intervention, it can tax the labor income of domestic agents at date  $t=1$ . It has to have a balanced budget.<sup>6</sup> In this section, we discuss possible forms of government intervention and the determinants of optimal policy responses within a country. We formulate the decision problem of the government in a way that is valid for both countries.

At date  $t=1$ , a government that faces potential bankruptcy of its domestic bank chooses between two actions.<sup>7</sup> Firstly, it can decide not to intervene at all. Given our

<sup>6</sup>We assume that the government cannot borrow. If we allowed for this possibility, the government would have to raise taxes in the future to pay back its debt. The possibility to smooth taxes over time can reduce distortions. However, as long as raising funds is costly, the main trade-off remains unaffected.

<sup>7</sup>Notice that we do not consider the option of stopping convertibility. Stopping convertibility would

assumptions, this leads immediately to bankruptcy and liquidation of its bank. Each depositor receives a pro rata share  $q$  of the liquidated bank. Late consumers store the return and consume at date  $t = 2$ . If there is no intervention, which we denote by the subscript  $n$ , the welfare level  $V$  of the country is given by:

$$V_n = u(q). \quad (3.9)$$

Secondly, in order to prevent bankruptcy, the government can bail out its bankrupt bank. The cost of a bail-out can be derived from bankruptcy conditions 3.7 and 3.8, respectively. For a bail-out a government has to supply at least the additional liquidity that the bank needs in order to prevent a bank run. That is each depositor, independently of his type, has to receive at least  $\bar{d}_1$ . If the bail-out sum is larger than this minimal amount, the bank liquidates less long assets and late consumers get a higher pay-off. Let  $b$  denote the pay-off that late depositors receive when the bank is bailed out. Let  $gap$  be the additional unexpected liquidity needs that occur in state  $\bar{S}$  and let  $\lambda$  be the fraction of early depositors that the bank faces. Then the general formula for the costs of a bail-out is given by:

$$G(b) = gap - r \left( (1 - y) - \frac{(1 - \lambda)b}{R} \right). \quad (3.10)$$

We distinguish different degrees of bail-outs by the amount  $b$  that late depositors receive. We define a partial bail-out as the case where the minimum amount of liquidity is provided to avoid a bank run, i.e. all consumers of the bank receive  $\bar{d}_1$ .<sup>8</sup> In contrast, we define a full bail-out as a situation where so much liquidity is provided that late depositors receive the return they expected ex-ante, i.e.  $b = \bar{d}_2$ . Bail-out costs are linearly increasing in  $b$ . The optimal bail-out may be different from the two discussed above. It trades off the losses from liquidation with the costs of providing government funds. In order to finance the bail-out, the government taxes labor income at date  $t = 1$ . We assume that consumers observe the bail-out and know the tax rate  $\tau \geq 0$ , which the government imposes. With this information, agents decide upon how much labor they are going to supply. Thus, taxes distort the agents' labor supply decision. The labor supply function is now given

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avoid a bank run at no direct costs. However, a fraction  $\epsilon$  of early consumers would not be able to withdraw, which would reduce their consumption to zero.

<sup>8</sup>A deposit insurance would guarantee the same pay-offs to consumer. However, there is a difference between a deposit insurance and a partial bail-out. In case of deposit insurance, the bank goes bankrupt and the government pays the difference between the liquidation value of a bank and the deposits. A partial bail-out is less costly as the provided funds avoid the early liquidation of some part of the long asset. For each unit of liquidity that the government provides,  $R/r$  funds are recovered.

by:

$$n(\tau) = (1 - \tau) \frac{\kappa}{2}. \quad (3.11)$$

In order to raise a total amount of taxes  $G$ , the government has to set the tax rate so that:

$$G = \tau n(\tau) = \frac{\kappa}{2} \tau (1 - \tau). \quad (3.12)$$

This equation describes the Laffer curve the government faces. The government always chooses the smaller tax rate to finance any given spending. The tax rate  $\tau^{max} = \frac{1}{2}$ , which yields the maximum tax income, is independent of the parameter  $\kappa$ . In contrast, the maximum funds that the government can raise depend on  $\kappa$  and government intervention can only be financed if  $G \leq G^{max} = \frac{\kappa}{8}$ . In what follows we assume that this condition holds and each country can finance a domestic bail-out. In order to facilitate the notation, we let  $\tau(G)$  denote the tax rate that has to be set if the government wants to collect  $G$ . Furthermore, we define  $Z(G)$  as the total utility loss in terms of consumption due to distortionary taxation. As discussed before, for a tax rate of zero this effect is normalized to 0. Thus, for  $\tau > 0$ , this utility loss is strictly positive and is given by:

$$\begin{aligned} Z(G) &= -\eta \left( n(\tau(G))(1 - \tau(G)) - \frac{n(\tau(G))^2}{\kappa} - \frac{\kappa}{4} \right) \\ &= \frac{\kappa \eta}{4} \left[ 1 - \left( \frac{1}{2} + \sqrt{\frac{1}{4} - \frac{2}{\kappa} G} \right)^2 \right], \end{aligned} \quad (3.13)$$

where we substituted, in the second line of the expression, the optimal labor supply and tax rate, which are functions of  $G$ .<sup>9</sup> If the government decides to intervene and to bail out the bank, then early consumers receive the promised amount  $\bar{d}_1$ , while late consumer receive a pro rata share of the bank asset left at date  $t = 2$ . The latter, amount  $b$ , depends on how much liquidity was provided by the government. At the same time the government raises taxes to finance the bail-out. Due to equal taxation, each consumer incurs the same utility loss from taxation  $Z(G(b)) > 0$ , which increases with  $b$ . Welfare in the economy is then:

$$V_{bo}(b) = \lambda u(\bar{d}_1 - Z(G(b))) + (1 - \lambda) u(b - Z(G(b))). \quad (3.14)$$

When considering to do a bail-out, the government chooses  $b \in \{\bar{d}_1, \bar{d}_2\}$  so as to maximize

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<sup>9</sup>It would be sufficient for  $Z(G)$  to be increasing, convex and twice continuously differentiable. We assume a specific functional form for illustrative purposes.



this equation. The first-order conditions of this problem imply:

$$\frac{\lambda u'(c_1)}{(1-\lambda)u'(c_2)} = \frac{1 - Z'(G(b))G'(b)}{Z'(G(b))G'(b)} \quad (3.15)$$

A necessary condition for a bail-out beyond  $b = \bar{d}_1$  to be optimal is that the utility of late consumers is increasing in  $b$ , thus  $1 - Z'(G(b))G'(b)$  must be positive. In the two corner solutions  $b = \bar{d}_1$  and  $b = \bar{d}_2$ , the FOC does not need to hold. We assume that when governments are indifferent between intervention and no intervention, they do not intervene, hence a government chooses a bail-out if  $V_{bo}(b^*) > V_n$ , where  $b^*$  is the solution to 3.15.

There is a notable difference between a partial bail-out where just enough liquidity is provided as to avoid bankruptcy and a bail-out where liquidity is provided beyond the minimum required amount  $G(\bar{d}_1)$ . From Equations 3.9 and 3.14 for a partial bail-out to be optimal, we must have that  $d_1 - Z(G(\bar{d}_1)) > q$ . A partial bail-out, if chosen by the government, thus implies a Pareto improvement. Any liquidity that is provided beyond  $b = \bar{d}_1$  benefits only late depositors, while early depositors face a higher tax rate and thus a higher disutility from work. This can be optimal, because the utility gain from increasing late depositors' pay-offs may be bigger than the early consumers' utility loss. However, moving from partial bail-out to any other degree of bail-out never implies a Pareto improvement.<sup>10</sup>

**Proposition 3.2** *Any additional liquidity provided beyond the amount required for a partial bail-out redistributes resources among agents, but does not induce a Pareto improvement.*

**Proof.** Omitted. ■

Whether no intervention or a bail-out yields higher welfare depends crucially on the curvature of the utility function and the function  $Z(\cdot)$ . They determine the trade-off a government faces. With changes in parameters, pay-offs are modified. Such changes may also imply that the bankruptcy conditions no longer hold. Assume in the following that the bankruptcy conditions continue to hold. From Equations 3.9 and 3.14 we see that an increase in the return on the long asset decreases the costs of a bail-out and increases pay-offs to late consumers, therein raising the overall welfare level from a bail-out relative to no intervention. Thus the incentives to intervene increase in  $R$ . The impact of the liquidation rate  $r$  on the optimal government intervention is ambiguous. A bigger loss from

<sup>10</sup>This would not be the case if the government could tax early and late consumers differently.

early liquidation decreases welfare levels for all forms of intervention. Relative changes depend on the exact parameter values.

The investment in the short asset  $y$ , decided upon at date  $t = 0$ , implicitly impacts the government decision as well. The welfare level given no intervention depends on the liquidation value of the bank, which is a function of  $y$ . The higher the amount invested in the short asset, the higher the liquidation value of the bank, the higher welfare if there is no intervention by the government. Moreover,  $y$  affects the costs of a bail-out. A higher  $y$  corresponds to a higher pay-off  $\bar{d}_1$  promised to early consumers. Thus, liquidity needs or the *gap* increase with  $y$  as well.

### 3.2.6 Differences across Banks and Countries

The analysis in the previous section is valid for both countries and points out the trade-offs that each government faces within its own country when deciding on intervention. Now we analyze in more detail the differences between the two countries, which can lead to different optimal decisions of the governments. Country A is the source of the crisis. Its domestic bank faces an additional unexpected amount of early depositors  $\epsilon$ . We call Country A therefore the *crisis country*. Bankruptcy of Bank A causes bankruptcy of Bank B. Therefore, we call Country B the *affected country*. Each government can decide to intervene or to bail-out the bank within its jurisdiction. Besides the direct effect of a bail-out on domestic welfare, there is a spillover effect on the welfare of the other country. This is due to the fact that banks are connected through interbank deposits. However the spillover effects of a bail-out are asymmetric and differ between the crisis country and the affected country. A bail-out of Bank A avoids contagion and saves Bank B. In contrast, a bail-out of Bank B increases the liquidation value of Bank A as interbank deposits are fully repaid. Figure 3.1 illustrates the setup and linkages of our model.

As a consequence of the different sources of bankruptcy across banks, bail-out costs differ between countries. Equation 3.10 is valid for both countries, but the *gap* and  $\lambda$  differ. From Equation 3.7, the explicit bail-out costs for Bank A are:

$$G^A(b) = \epsilon \bar{d}_1 - r \left( (1 - y) - \frac{(1 - \bar{\lambda} - \epsilon)b}{R} \right). \quad (3.16)$$

For Bank B, we have from Equation 3.8 that:

$$G^B(b) = z(\bar{d}_1 - \hat{q}) - r \left( (1 - y) - \frac{(1 - \bar{\lambda})b}{R} \right). \quad (3.17)$$

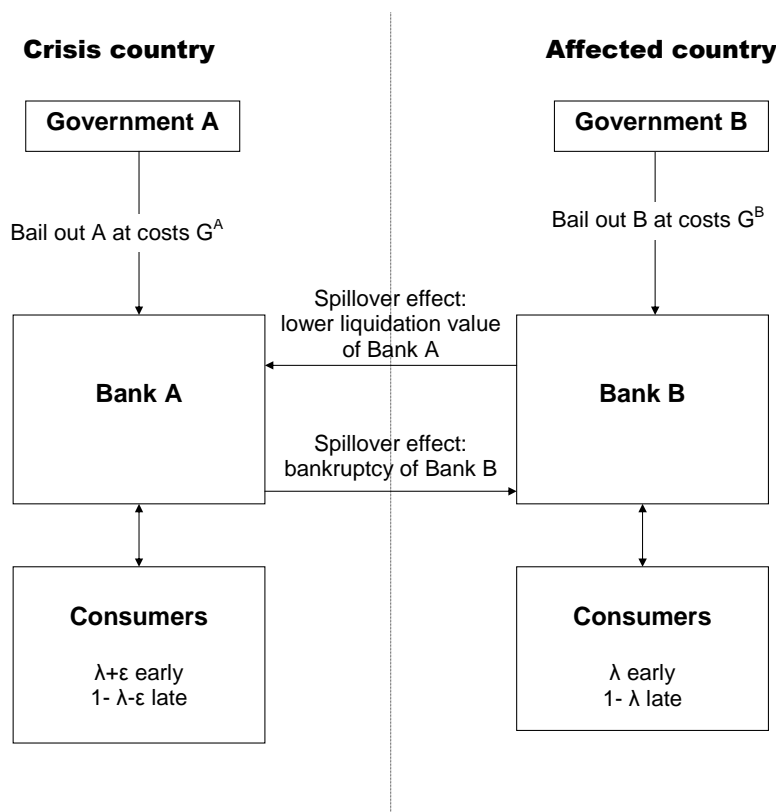


Figure 3.1: Model setup

While bankruptcy of Bank A is caused by unexpected liquidity needs creating a maturity mismatch, the reason for bankruptcy of Bank B lies in a real loss of assets.<sup>11</sup> In Country A, more individuals want to consume early. Funds however have been invested in the long asset at date  $t = 0$ . By providing funds, the government can avoid liquidation costs and due to the smaller fraction of late depositors can implicitly collect some return on the long asset. Given the same *gap*, a bail-out is cheaper in Country A.<sup>12</sup> Another difference between countries lies in the fraction of early and late depositors. Due to the exogenous shock, Bank A faces a larger fraction of early depositors  $\bar{\lambda} + \epsilon$ . Therefore, the government of Country A puts more weight on the welfare of early consumers than the government

<sup>11</sup>Suppose a government can raise non-discriminatory lump-sum taxes. Then, Country A would always prefer a bail-out over no intervention as there is a pure liquidity problem. The free-riding problem, though, remains and therefore no clear preference by Country A of a bail-out of Bank A compared to a bail-out of Bank B can be established. Bank B faces real losses in assets. Therefore, a bail-out of Bank B is desirable if the liquidation loss that can be avoided exceeds resources that have to be provided for the bail-out. It can be shown that this is always the case. We derive these results on lump-sum taxation in the Appendix.

<sup>12</sup>As the share of late consumers is smaller than expected given the shock, the government in Country A could raise the return of late consumers beyond the expected level  $\bar{d}_2$  by providing funds in order for  $b > \bar{d}_2$ . If the government could provide the funds conditional on becoming a residual claimant of the bank, it could collect the residual value of the bank after it has paid  $\bar{d}_2$  to all late consumers, thereby potentially increasing efficiency. By restricting  $b \in \{\bar{d}_1, \bar{d}_2\}$  in the optimization problem of the government, we do not consider this case.

of Country B, where early consumers represent only a fraction of  $\bar{\lambda}$ . Due to asymmetric bail-out costs and different fractions of early and late depositors, the optimal decision between no intervention and bail-out as well as the choice of  $b$  typically differ between governments.

We state the welfare levels of each country separately. They depend on the actions taken by both governments. We denote welfare of country  $j$  by  $U_{s^A, s^B}^j$ , where the first subscript stands for Country A's intervention decision, while the second subscripts captures the action of Country B. We subsume the pair of actions taken by both countries by  $a$ . The general welfare function can then be formulated as:

$$V_a^j = \lambda^j u(c_1^j(a)) + (1 - \lambda^j) u(c_2^j(a, b)) \quad (3.18)$$

If neither country intervenes, all agents receive a pro rata share of the liquidation value of the bank. Each consumer, no matter in which bank it deposited its endowment, obtains  $\bar{q}$  as interbank claims offset each other:

$$V_{n,n}^A = V_{n,n}^B = u(\bar{q}). \quad (3.19)$$

If the government of the crisis country decides to bail-out its domestic bank, then contagion is avoided and the bank in Country B remains unaffected by the crisis in Country A. Welfare of Country A from bailing out its bank is:

$$V_{bo,n}^A(b) = (\bar{\lambda} + \epsilon) u(\bar{d}_1 - Z(G^A(b))) + (1 - \bar{\lambda} - \epsilon) u(b - Z(G^A(b))), \quad (3.20)$$

where  $G^A(b)$  is given by Equation 3.16. As there is no contagion and Bank B remains unaffected, there is no scope for intervention and Country B's welfare attains the maximum:

$$V_{bo,n}^B = \bar{\lambda} u(\bar{d}_1) + (1 - \bar{\lambda}) u(\bar{d}_2). \quad (3.21)$$

However, if Country A does not intervene, there is contagion and the government of Country B has to decide whether or not to intervene. If Country B does a bail-out, Country A's welfare is raised compared to the case where both banks go bankrupt. This is because, as Bank B is saved and does not go bankrupt, it is able to pay the full amount of the interbank claims. Instead of a pro rata share of Bank B's liquidation value, Bank A now obtains  $z\bar{d}_1$ . Thus Bank A's liquidation value increases to  $q^A = \hat{q} = \frac{y+r(1-y)+z\bar{d}_1}{1+z} > \bar{q}$

and welfare of Country A is given by:

$$V_{n,bo}^A = u(\hat{q}). \quad (3.22)$$

The welfare level of the affected country, if it does a bail-out, is:

$$V_{n,bo}^B(b) = \bar{\lambda}u(\bar{d}_1 - Z(G^B(b))) + (1 - \bar{\lambda})u(b - Z(G^B(b))), \quad (3.23)$$

where  $G^B(b)$  is given by Equation 3.17. Note that the bail-out of Bank B has a positive feedback effect on itself. Because Bank B does not go bankrupt, it can fully repay Bank A's claims. This in turn, raises the liquidation value of Bank A, of which Bank B receives a pro rata share.

### 3.3 Centralized versus Decentralized Decision-Making

In this section, we study and compare possible equilibria given decentralized and centralized decision making, respectively. This, in turn, allows us to evaluate the role of government cooperation in times of international banking crisis. As a benchmark, we consider the decentralized solution with strategic interaction between governments, where governments play a sequential bail-out game. Then, we introduce a central authority with the power to mandate actions to be taken by both governments. We analyze whether and to what extent this central authority can improve upon the non-cooperative equilibrium.<sup>13</sup>

#### 3.3.1 Non-Cooperative Bail-out Game

When there is no coordination, each government decides on its own whether and how to intervene. Strategic interaction arises as welfare from each form of government intervention depends on the action of the other government. Note that the welfare level of each country, however, is independent of the liquidity that is provided for a bail-out of the foreign bank. Therefore,  $b$  is not a strategic variable. The bail-out game in extensive form is illustrated in Figure 3.2. The set of strategies of the government in Country A is given by  $S_A = \{n, bo\}$  and in Country B by  $S_B = \{n, bo\}$ . We consider subgame-perfect Nash equilibria in pure strategies of the game with sequential moves. The crisis country moves first and the affected country is the follower.

<sup>13</sup>In the Appendix, we study the game with simultaneous moves by the governments.

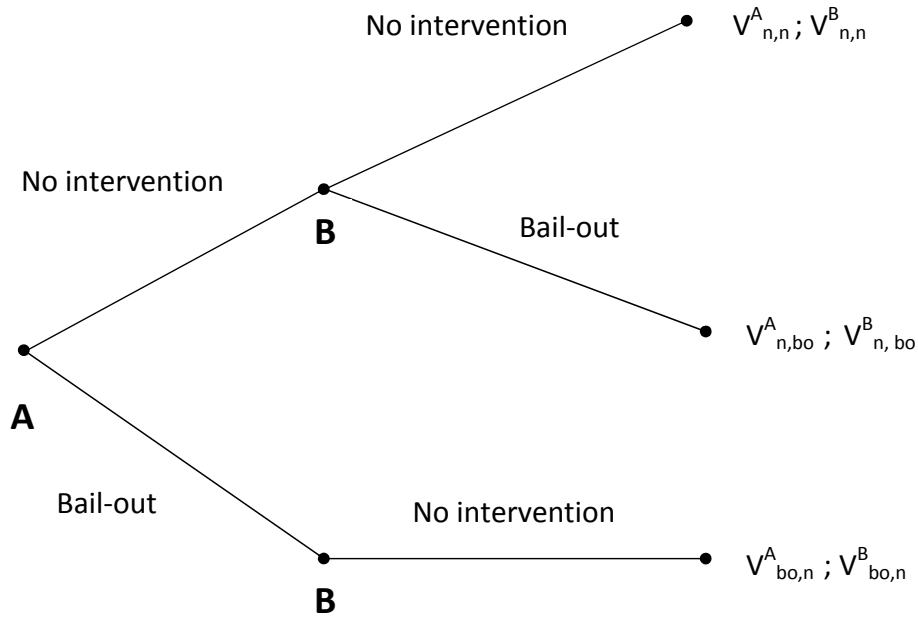


Figure 3.2: Bail-out game in extensive form

**Definition 3.3** *The profile  $a^* \equiv (s^{A^*}, s^{B^*})$  is a subgame-perfect Nash equilibrium (SPNE) of the game described in Figure 3.2 iff*

- (i) *the government in Country B maximizes its domestic welfare, given the strategy of Country A, i.e.  $V^B(a^*) \geq V^B(s^B, s^{A^*}) \forall s^B \in S^B$ ,*
- (ii) *the government in Country A maximizes its domestic welfare, given the strategy of Country B, i.e.  $V^A(a^*) \geq V^A(s^A, s^{B^*}) \forall s^A \in S^A$ ,*
- (iii) *and  $a^*$  is a Nash equilibrium in every proper subgame.*

There are three possible equilibria of the sequential game. In the following we state conditions under which each of these three possible outcomes of the game occurs:

**Proposition 3.4** (i)  $a^* = (n, n)$  is a SPNE iff  $V_{n,bo}^B \leq V_{n,n}^B$  and  $V_{bo,n}^A \leq V_{n,n}^A$ .

(ii)  $a^* = (n, bo)$  is a SPNE iff  $V_{n,bo}^B > V_{n,n}^B$  and  $V_{bo,n}^A \leq V_{n,bo}^A$ .

(iii)  $a^* = (bo, n)$  is a SPNE iff  $V_{bo,n}^A > V_{n,bo}^A$  or  $V_{n,bo}^B \leq V_{n,n}^B$  and  $V_{bo,n}^A > V_{n,n}^A$ .

**Proof.** Note that, from Equations 3.19 to 3.23 it follows that  $V_{n,bo}^A > V_{n,n}^A$  and  $V_{bo,n}^B > V_{n,n}^B$  as well as  $V_{bo,n}^B > V_{n,bo}^B$  and  $V_{bo,n}^B = V_{bo,bo}^B$ . ■

### 3.3.2 Central Authority with Mandating Power

Now we derive the optimal decision of a central authority with a mandate to decide upon intervention. As before, three possible pairs of actions  $a$  are possible: no intervention of both countries  $(n,n)$ , bail-out of Bank A  $(bo,n)$  or bail-out of Bank B  $(n,bo)$ . The objective function of the central authority is the weighed sum of national welfare levels. Attributing welfare weight  $\Theta^j$  to country  $j$ , it solves the following problem:

$$\max_{a \in \{(n,n), (n,bo), (bo,n)\}} V = \sum_{j \in \{A,B\}} \Theta^j V_a^j(d_1^j(a), d_2^j(a,b), Z(G^j)). \quad (3.24)$$

The pair of actions which maximizes this objective is denoted by  $a'$ . It is not necessary for the central authority to have the power to mandate  $b$  as governments automatically choose the optimal degree of the bail-out (see Equation 3.15).

Bail-out costs differ between banks. Therefore, although a bail-out of Bank A prevents contagion and raises Country B's welfare to the maximum, a bail-out of Bank A does not necessarily dominate a bail-out of Bank B. If  $G^A(b)$  is sufficiently large, the optimal solution to the central authority's problem can imply saving Bank B only. Therefore, without any restrictions on parameters, any of the three possible combinations of government actions can be optimal.

### 3.3.3 Inefficiencies in the Bail-out Game

We compare the solution of the central authority with mandating power with the equilibrium of the sequential game. For this we assume that the welfare weights attributed to each country are the same. In a first step we study relations between outcomes of the sequential game and decisions taken by the central authority. Proposition 3.5 states which actions can be the solution to Expression 3.24 for a given equilibrium of the sequential game:

**Proposition 3.5** (i) *If the SPNE is  $a^* = (bo,n)$ , then this equilibrium coincides with the optimal solution of the central authority  $a'$ .*

(ii) *If the SPNE is  $a^* = (n,bo)$ , then  $a' \in \{(bo,n), (n,bo)\}$ .*

(iii) *If the SPNE is  $a^* = (n,n)$ , then  $a' \in \{(bo,n), (n,bo), (n,n)\}$ .*

**Proof.** See the Appendix. ■

If  $(bo, n)$  is the SPNE, a central authority chooses the same outcome. However, when  $(n, bo)$  is the SPNE, a bail-out of Bank A or Bank B can be optimal. Finally, all actions can be optimal to be mandated when  $(n, n)$  is the equilibrium.<sup>14</sup>

Next, we consider the relations in the opposite direction and ask which actions can be the equilibrium of the sequential game, if the central authority finds a certain sequence of actions  $a'$  optimal:

**Corollary 3.6** (i) *If  $a' = (bo, n)$ , then all actions  $a^* \in \{(bo, n), (n, bo), (n, n)\}$  can be the SPNE of the bail-out game.*

(ii) *If  $a' = (n, bo)$ , then the actions  $a^* \in \{(n, bo), (n, n)\}$  can be the SPNE of the bail-out game.*

(iii) *If  $a' = (n, n)$ , then  $a^* = (n, n)$  is the only SPNE of the bail-out game.*

**Proof.** Follows directly from Proposition 3.5. ■

First, if the central authority finds a bail-out of Bank A optimal, then all three outcomes are possible equilibria of the sequential game and  $a^* \in \{(n, n); (n, bo); (bo, n)\}$ . Second, if a central authority does not find a bail-out of Bank A optimal, then it follows that the government in Country A itself does not choose to bail-out its domestic bank either. Finally, if the central authority finds that no country should intervene, then  $a^* = (n, n)$  is also the SPNE of the sequential game. This follows from the fact that for the central authority to choose no intervention, we must have  $u(\bar{q}) > V_{bo, n}^A$  and  $u(\bar{q}) > V_{n, bo}^B$ , a situation in which neither Country A nor Country B choose a bail-out.

Note that all distortions are towards to little intervention. If no intervention is optimal in both countries, there is no bail-out in the non-cooperative equilibrium. It is only possible that the "wrong" bank is the subject of the bail-out in that a bail-out of Bank B is implemented although a bail-out of Bank A maximizes overall welfare. Suboptimal decisions in the form of  $a^* \neq a'$  only occur if a central authority finds a bail-out optimal and if Country A does not choose a bail-out in the bail-out game.

When comparing decisions in the sequential game with those taken by a central authority with mandating power, two sources of inefficiencies can be identified. First, due to the interbank linkages, there are spillover effects (*externalities*), which are not taken into account by the governments. Second, there is a *free-riding* problem due to the sequential

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<sup>14</sup>The situation in which the bankruptcy condition of Country B, Condition 3.8, holds and  $a' = (bo, n)$  can be interpreted as representing the case of "too big to fail". While saving Bank A might not be optimal per se, i.e. only taking into account effects on Bank A depositors, a bail-out of Bank A might be optimal when the potential failure of Bank B is taken into account, too.



nature of the bail-out game.<sup>15</sup> The crisis country may not bail out its domestic bank because it knows that then the affected country will do a bail-out. As the first mover it can free ride on the bail-out carried out by Country B. This is captured in the following proposition:

**Proposition 3.7** *An anticipated bail-out in the affected country lowers the incentives for a bail-out in the crisis country.*

**Proof.** Country A does a bail-out iff  $V_{bo,n}^A > V_{n,.}^A$ .

$$V_{n,.}^A = \begin{cases} V_{n,n}^A & \text{if Country B does not intervene} \\ V_{n,bo}^A & \text{if Country B does a bail-out,} \end{cases}$$

and  $V_{n,bo}^A > V_{n,n}^A$ . ■

Given that the crisis country does not intervene, an anticipated bail-out of the affected country raises the welfare level in the crisis country because Bank B can fully repay Bank A's interbank claims, which raises Bank A's liquidation value. The magnitude of the effect of a bail-out in the affected country on the crisis country is determined by the size of the interbank deposits:

**Proposition 3.8** (i) *The incentives for a bail-out in the affected country decrease in the interbank deposits  $z$ .*

(ii) *If the affected country bails out its domestic bank, then the incentives for a bail-out in the crisis country decrease in the interbank deposits  $z$ .*

**Proof.**

$$(i) \frac{\partial G^B(b)}{\partial z} = \bar{d}_1 - q^A > 0 \Rightarrow \frac{\partial(V_{n,bo}^B - V_{n,n}^B)}{\partial z} > 0.$$

$$(ii) \frac{\partial \hat{q}}{\partial z} = \frac{\bar{d}_1 - (y+r(1-y))}{(1+z)^2} > 0 \Rightarrow \frac{\partial(V_{bo,n}^A - V_{n,bo}^A)}{\partial z} < 0. \quad \blacksquare$$

Interbank deposits are a function of the investment in the short asset  $y$  and the anticipated date-0 uncertainty captured by  $\bar{\lambda} - \lambda_L$ . Note from Expression 3.17 that with increasing interbank deposits  $z$ , a bail-out in Country B becomes more costly as the loss from bankruptcy of Bank A grows. This makes it less attractive for the affected country to bail out its bank.

A central authority that can dictate actions internalizes the externalities and eliminates the free-riding problem. This form of coordination can therefore improve global welfare.

<sup>15</sup>In the simultaneous bail-out game, there are multiple Nash equilibria. Instead of the free-riding problem, a coordination problem may occur.

## 3.4 Cross-Country Deposits and Country Sizes

In this section, we introduce cross-country deposits as an additional form of international linkages, i.e. banks compete for customers in both countries who can decide freely on where to deposit their endowment. Furthermore, we allow for differences in country size.

In the previous section, we identified two sources of inefficiency that can arise in the bail-out game: externalities due to spillovers and free-riding due to the sequential nature of the game. As discussed, they imply that a central authority with mandating power can in some cases improve upon the non-cooperative equilibrium. In the following, we study how the presence of cross-country deposits and differences in country size affect the outcome of the non-cooperative game and the decision taken by a central authority, as well as the two sources of inefficiencies. We find that with equal country sizes the incentives to free ride, in general, decrease in the share of cross-border deposit holdings. The optimal choice of  $b$  decreases in the size of cross-country deposits. Finally, asymmetric country sizes affect optimal decisions as they imply different tax distortions given the same level of government expenditures.

### 3.4.1 Extended Model Setup

The modified setup with cross-country deposits is illustrated in Figure 3.4. Let  $\alpha$  ( $\beta$ ) denote the fraction of depositors of Bank A (Bank B) that live in Country A (Country B) and let  $(1 - \alpha)$  ( $(1 - \beta)$ ) denote the fraction of agents that are depositors of Bank A (Bank B) that live in Country B (Country A).<sup>16</sup> Banks remain of equal size, each hosting a unit mass of deposits. We assume that the liquidity shock  $\epsilon$  hits a bank.<sup>17</sup> Therefore, bail-out costs  $G$  are independent of the distribution of depositors. However, how easily a bail-out can be financed depends on the tax base of a country, hence its size. The smaller the population, the higher the tax rate required to raise the funds for a bail-out, the higher the distortion. Countries differ in size if  $\alpha \neq \beta$ . The population of Country A is  $P^A = \alpha + (1 - \beta)$ . In Country B it is  $P^B = \beta + (1 - \alpha)$ . With asymmetric country sizes, the disutility from work becomes country-specific and depends on the population size  $P^j$ .

<sup>16</sup>Ex-ante agents are indifferent where to deposit their endowments. We assume that each agent deposits its entire endowment either abroad or at home.

<sup>17</sup>An alternative would be a shock hitting a country. This would lead to shocks in form of additional early depositors in both banks, i.e.  $\epsilon\alpha$  in Bank A and  $\epsilon(1 - \alpha)$  in Bank B for the case where the shock hits nationals of Country A.

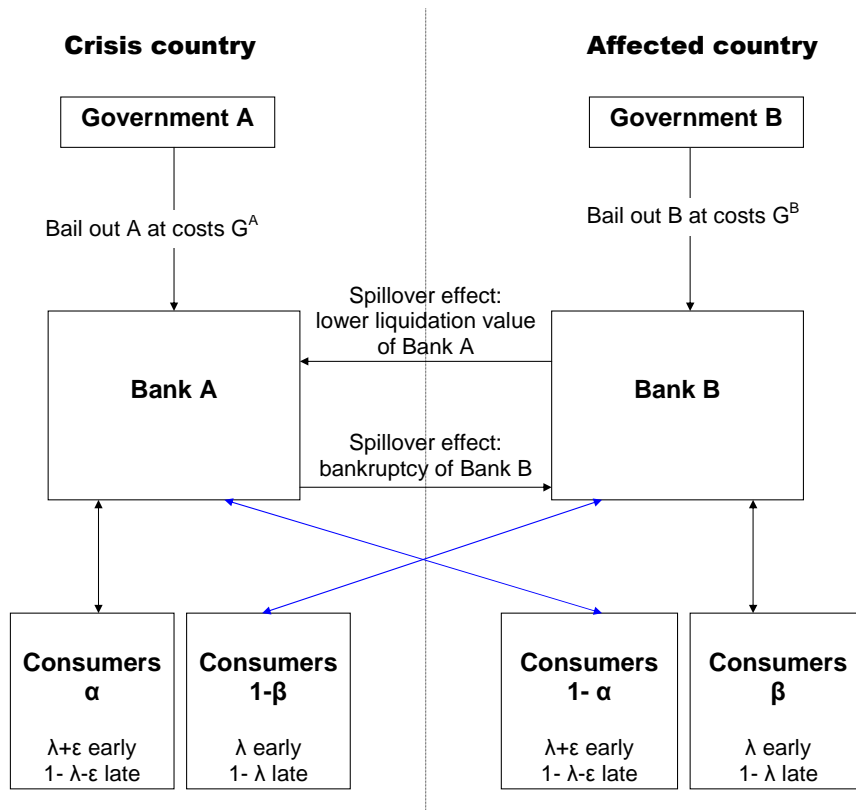


Figure 3.3: Extended model setup

Substituting for the tax rate  $\tau(G, P^j)$  we obtain the following expression:

$$Z^j(G, P^j) = \frac{\kappa\eta}{4} \left[ 1 - \left( \frac{1}{2} + \sqrt{\frac{1}{4} - \frac{2}{P^j\kappa}G} \right)^2 \right]. \quad (3.25)$$

Cross-country deposits change the bail-out game in three ways. Firstly, governments now take the spillover effects into account because they care about domestic consumers that invested abroad. The intervention has, through the spillover, a direct impact on their pay-offs. Secondly, the amount of liquidity  $b$  that is provided by a government enters the welfare function of the other country. Without cross-country deposits, a country's welfare was only impacted by the bail-out decision that the other country took and not by  $b$ . Nevertheless,  $b$  is as before not a strategic variable.<sup>18</sup> When deciding upon an action, each government takes the optimal  $b^*$  of the other country as given. Thirdly, as liquidity shocks are attributed to a bank rather than a country, the fraction of early and late depositors of each country in State  $\bar{S}$  is altered. The fraction of early depositors in

<sup>18</sup>Consider two possible cases. i) If country A does a bail-out, then there is no action by Country B. ii) If Country B does a bail-out, there is no strategic effect as it is the second mover without commitment power.

Country A is reduced and is only  $\alpha(\lambda + \epsilon) + (1 - \beta)\lambda$ , while the equivalent fraction in Country B is increased and is  $(1 - \alpha)(\lambda + \epsilon) + \beta\lambda$ . As a consequence the welfare weights that are attributed to early and late consumers are modified.

The derivation of the pay-offs of the bail-out game with cross-country deposits is straightforward. If both countries do not take any action, total welfare of Country-A agents is

$$V_{n,n}^A = \alpha u(\bar{q}) + (1 - \beta)u(\bar{q}). \quad (3.26)$$

For Country B we have equivalently:

$$V_{n,n}^B = (1 - \alpha)u(\bar{q}) + \beta u(\bar{q}). \quad (3.27)$$

If both countries are of equal size, then both expressions reduce to Equation 3.19. If Country A bails out its bank, it obtains the following welfare level:

$$\begin{aligned} V_{bo,n}^A(b) &= \alpha [(\bar{\lambda} + \epsilon)u(\bar{d}_1 - Z^A(G^A(b))) + (1 - \bar{\lambda} - \epsilon)u(b - Z^A(G^A(b)))] + (3.28) \\ &+ (1 - \beta) [\bar{\lambda}u(\bar{d}_1 - Z^A(G^A(b))) + (1 - \bar{\lambda})u(\bar{d}_2 - Z^A(G^A(b)))] \end{aligned}$$

Country B benefits from the bail-out in Country A because firstly, there is no need for intervention as contagion is avoided, and secondly, because domestic consumers that deposited in Bank A receive higher pay-offs than under bankruptcy, namely the ones expected at date  $t = 0$ . The welfare in Country B is:

$$V_{bo,n}^B(b) = (1 - \alpha) [(\bar{\lambda} + \epsilon)u(\bar{d}_1) + (1 - \bar{\lambda} - \epsilon)u(b)] + \beta [\bar{\lambda}u(\bar{d}_1) + (1 - \bar{\lambda})u(\bar{d}_2)]. \quad (3.29)$$

If Country A does not intervene but Country B bails out its bank, welfare in Country A is:

$$V_{n,bo}^A(b) = \alpha u(\hat{q}) + (1 - \beta) [\bar{\lambda}u(\bar{d}_1) + (1 - \bar{\lambda})u(b)]. \quad (3.30)$$

Utility of Country-A consumers is positively impacted when Country B bails out its bank. In addition to domestic consumers holding deposits abroad, agents having invested domestically benefit through the positive effect of the bail-out on the liquidation value of Bank A as  $\hat{q} > \bar{q}$ . Welfare in Country B is:

$$V_{n,bo}^B(b) = (1 - \alpha)u(\hat{q} - Z^B(G^B(b))) + \beta [\bar{\lambda}u(\bar{d}_1 - Z^B(G^B(b))) + (1 - \bar{\lambda})u(b - Z^B(G^B(b)))] . \quad (3.31)$$

Proposition 3.3 also pins down the equilibrium with cross-country deposits.

### 3.4.2 Cross-Country Deposits, Country Sizes and Inefficiencies

To continue with, we analyze which impact cross-country deposits and differences in country size have on the decision of the central authority, the outcome of the non-cooperative game and the two sources of inefficiencies discussed before. If the central authority gives equal weight to every consumer and country sizes are equal, cross-country deposits do not change its problem. Differences in country size, in contrast, have an effect as they imply different tax bases and thus country-specific tax distortions. Cross-country deposits also affect the decision by each country on how much liquidity to provide in a bail-out.<sup>19</sup> Due to the fact that not all domestic agents deposit in the domestic bank, an additional unit of liquidity has a lower marginal contribution to national welfare and  $b^*$  will be lower than without cross-country deposits:

**Proposition 3.9** *For a given country size, the smaller the fraction of domestically held deposits, the lower the optimal liquidity  $b^*$  provided by the government in case of a bail-out.*

**Proof.** See the Appendix. ■

Furthermore, cross-country deposits have an impact on the non-cooperative outcome. As the incentives for bail-outs change, the extent of the free-riding problem changes as well. For equally sized countries, the effects can be summarized in the following proposition:

**Proposition 3.10** *Suppose countries are of equal size ( $\alpha = \beta$ ). Then, an increase in the fraction of deposits abroad*

*(i) decreases the incentives for a bail-out in the affected country.*

*(ii) increases the incentives for a bail-out in the crisis country if it anticipates no bail-out in the affected country.*

**Proof.** See the Appendix. ■

The incentives for Country B to do a bail-out decrease with the fraction of deposits abroad. This is the case because the direct effect of a bail-out on consumers who have invested domestically is larger than the indirect effect of a bail-out on domestic deposit holdings abroad through the rise in the liquidation value of Bank A. While a partial bail-out raises pay-offs by  $\bar{d}_1 - \bar{q}$  for Bank-B depositors, the increase for Bank-A depositors

<sup>19</sup>If the central authority could also mandate  $b$ , this could improve welfare as it would take the externalities from the liquidity provision into account.

is only a fraction of that,  $\hat{q} - \bar{q} = \frac{z}{1+z}(\bar{d}_1 - \bar{q})$ . If  $b^* > \bar{d}_1$ , late consumers that invested domestically benefit more than early consumers, while the additional liquidity support does not impact the pay-off to consumers that invested abroad.

If Country A anticipates that Country B will not bail out its bank, then the incentives of Country A to do a bail-out increase with the fraction of domestic deposits abroad. A bail-out in Country A can avoid contagion and has therefore a large effect on the pay-offs to Country-A consumers that invested abroad. With a growing fraction of domestic endowments deposited in Bank B, the benefits from a bail-out increase for Country A. This is due to the fact that a partial bail-out in Country A is enough to guarantee that late depositors that invested in Bank B receive the originally promised amount  $\bar{d}_2$ . In order to induce the same welfare of domestic depositors, costs are lower when we have  $\beta > 0$ . Therefore, given Country B is not willing to bail-out its bank, the incentives of the government in Country A for a bail-out of Bank A increase in  $\beta$ .

If Country A anticipates that there will be a bail-out in Country B, then the effect of the fraction of deposits held abroad on its incentives to bail-out Bank A is ambiguous. The welfare of Country A from a domestic bail-out increases with the fraction of agents that have invested abroad. At the same time, however, the benefits for Country-A depositors from a bail-out of Bank B by Country B increase. How the optimal decision changes with  $\beta$  is ambiguous. If there is a bail-out of Bank B, then early depositors of Bank B receive  $\bar{d}_1$ , while late depositors get the pay-off  $b$ . If Country A avoids contagion incurring cost  $G^A(b)$ , then late consumers of Country A with endowments in Bank B receive the full promised return  $\bar{d}_2$ . Therefore, a bail-out by Country A can be optimal as it can result in a higher return for its late depositors that invested in Bank B. The exact effect of an increase in domestic Bank-B depositors on the decision of Country A therefore depends on the bail-out costs and the optimal bail-out level chosen by Country B. Clearly, the lower the liquidity provided to Bank B by Government B, the lower the pay-off to late depositors of Bank B, the higher the incentive of Government A to bail out its domestic bank in order to avoid contagion.

Next, we study the effect of country size. For the special case where 50% of consumers in each country hold deposits abroad, the following proposition can be stated:

**Proposition 3.11** *Suppose countries hold an equal share of deposits at home and abroad ( $\alpha = 1 - \beta$ ). Then, holding  $b$  constant,*

- (i) *the incentives for a bail-out in the crisis country increases with its size ( $\alpha$ ),*
- (ii) *the incentives for a bail-out in the affected country increases with its size ( $\beta$ ).*

**Proof.** See the Appendix. ■

In the model country size corresponds to the size of the tax base. If  $\alpha$  is bigger than  $\beta$ , the crisis country is bigger than the affected country. As a consequence, to cover the same costs of intervention, the necessary tax rate and therefore distortions from taxation are lower in the crisis country than in the affected country. This makes a bail-out in the crisis country relatively more attractive. At the same time, the free-riding problem is smaller, as a bail-out in the affected country is less likely if its tax base is smaller.

So far we have isolated the effects of cross-country deposits and differences in country sizes. For other combinations of parameters, these two effects interact. We find that, whether cross-country deposits tend to reinforce inefficiencies or alleviate them, depends on the nature of the asymmetry between countries.

If  $1 - \beta \geq \alpha > 1 - \alpha \geq \beta$ , then the crisis country is larger than the affected country and the majority of domestic agents holds deposits abroad. The size of the crisis country increases Government A's willingness to finance a bail-out. The fact that most of the domestic endowments are deposited abroad reinforces the positive effect of size because by bailing out Bank A domestic consumers that invested abroad are saved at the same time while the small tax base of Country B makes a bail-out there unlikely.

If  $1 - \alpha \geq \beta > 1 - \beta \geq \alpha$ , then the situation is the other way around. The crisis country is smaller than the affected country, but still the majority of depositors in the domestic bank are foreigners. In this case, it will be difficult for Country A to finance a bail-out of its domestic bank due to the small tax base. At the same time, it is easier for Country B to finance the bail-out of Bank B due to the larger size of the country. By a bail-out of Bank B, in turn, the majority of the deposits of Country A, which are abroad, is saved, and through this indirect effect, the incentives for bail-out in Country A can decrease.

### 3.5 Fiscal Burden Sharing

In the context of the European initiative toward a new financial architecture, it has been argued that fiscal burden sharing is required for efficient crisis management (see for example De Larosiere Report (2009)). In this section, we consider a central authority with mandating *and* fiscal power, i.e. it can now decide on the action to be taken and set a contribution to be financed by each country. We study how the new instrument of fiscal burden sharing can improve efficiency compared to the non-cooperative outcome. As dis-

cussed, a central authority without fiscal power can improve global welfare by mandating the efficient actions as this removes the inefficiencies due to externalities and free-riding. Introducing burden sharing can further improve upon the equilibrium allocation.

Without joint financing, there is an inefficiency due to *no burden sharing*, which comes from the convexity of function  $Z(\cdot)$  due to labor distortions. A bail-out becomes cheaper in utility terms when it is financed by both countries because these distortions are reduced. Now, even if the optimal actions prescribed by a central authority coincide with the outcome of the sequential game, i.e.  $a^* = a'$ , a situation in which the central authority without fiscal power could not improve upon the equilibrium global welfare, the central authority now brings an improvement by implementing optimal burden sharing. The possibility of burden sharing increases the number of cases in which a bail-out is desirable. That is there are cases in which a central authority without fiscal power may decide for no intervention, while a central authority with fiscal power may choose to mandate a bail-out. Compared to the optimal solution chosen by a central authority with fiscal power, the equilibrium of the sequential game is, as before, distorted towards too little intervention and Proposition 3.5 remains valid.

**No cross-country deposits** We start with an analysis of the problem of a central authority with fiscal power when there are no cross-country deposits. The central authority decides upon an intervention and a burden sharing rule. If a bail-out is optimal, the authority chooses the bail-out level  $b$  and the country-specific contributions  $X^A$  and  $X^B$  such that the sum of the contributions equals the funds required for the bail-out  $G(a,b)$ , where  $G(a,b)$  is given by Equation 3.16 if  $a = (b, n)$  or by Equation 3.17 if  $a = (n, b)$ . We restrict contributions to be non-negative. The central authority solves the following maximization problem:

$$\begin{aligned} \max_{a \in \{(n,n), (n,bo), (bo,n)\}, b \leq \bar{d}_2, X^A} \quad & V = \sum_{j \in \{A,B\}} \Theta^j V_a^j(d_1^j(a), d_2^j(a,b), Z(X^j)) & (3.32) \\ \text{s.t. } G(a,b) &= X^A + X^B, \\ X^A &\geq 0, \\ X^B &\geq 0, \\ Z(X^j) &= \frac{\kappa\eta}{4} \left[ 1 - \left( \frac{1}{2} + \sqrt{\frac{1}{4} - \frac{2}{\kappa} X^j} \right)^2 \right], \forall j \in \{A, B\}. \end{aligned}$$



The first-order conditions with respect to the contribution levels imply:

$$\frac{\Theta^A}{\Theta^B} = \frac{\bar{\lambda}u'(d_1^B(a) - Z(X^B)) + (1 - \bar{\lambda})u'(d_2^B(a, b) - Z(X^B))}{(\bar{\lambda} + \epsilon)u'(d_1^A(a) - Z(X^A)) + (1 - \bar{\lambda} - \epsilon)u'(d_2^A(a, b) - Z(X^A))} \frac{Z'(X^B)}{Z'(X^A)}. \quad (3.33)$$

By setting contribution levels, the central authority can induce discriminatory taxation. On the one hand, disutilities from labor taxation, which in our setup are independent of individual income levels, prescribe an equalization of contribution levels between countries, i.e. tax smoothing. On the other hand, differences in income levels between countries, resulting from asymmetric effects of the banking crisis, give rise to a consumption smoothing motive. Thus, a central authority trades off a tax smoothing *and* a consumption smoothing motive. Given a banking crisis, Country A is always poorer than Country B. Due to this fact, we can derive the following result regarding contribution levels:

**Proposition 3.12** *Suppose countries have the same welfare weights. Then, under a central authority with fiscal power, the contribution for a bail-out  $X^B$  of the affected country will be larger than the contribution  $X^A$  of the crisis country .*

**Proof.** See the Appendix. ■

Note that the ability to induce discriminatory taxes is an additional instrument in the international context, which is not available in a national crisis. Imagine a closed economy model where the two countries integrate and form one country with one government. Then the maximization problem of the government that hosts both Banks A and B is the same as the problem of the central authority stated above with the exception that the government cannot set discriminatory taxes, i.e. there is perfect tax-smoothing. The depositors of each bank have to bear the same distortion from taxation and  $X^A = X^B = G(a, b)/2$ . As consumers have different pay-offs in case of a banking crisis, discriminatory taxation is desirable. A central authority has thus an advantage relative to the case of full integration. The additional policy instrument of tax discrimination allows it to attain a higher welfare level.

The central authority also chooses the degree of the bail-out  $b$ . Note that, in general, the authority will mandate a level of liquidity provision different from the choice that a government would make on its own as it can raise taxes more efficiently. The central authority's first-order condition for  $b$  is stated in the Appendix.

**With cross-country deposits** The problem of the central authority changes with the introduction of cross-country deposits. The relative income of the two countries depends

on  $\alpha$  and  $\beta$ . Cross-country deposits alter the fraction of early and late depositors each country has as well as the pay-offs to them. Moreover, the degree of the bail-out  $b$  now impacts the welfare level of both countries. In addition, the ability of each country to raise taxes, which is characterized by the country-specific function  $Z^j(\cdot)$ , is affected when countries differ in size. These factors modify the consumption smoothing versus tax smoothing trade-off that a central authority with fiscal power faces. Consequently, Proposition 3.12 does not hold with cross-country deposits.<sup>20</sup>

**Unilateral financing** There is the possibility that a country may not be able to finance a bail-out on its own, which we have ruled out so far. This is the case when the maximum tax income it can raise is not sufficient to finance the costs of a bail-out, i.e.  $G^{max} < G^j(b)$ . With cross-country deposits, the relative size of banks to population is no longer bounded from above. The size of a domestic bank can exceed the country size. This makes the case of an absolute financing constraint more likely.

Such a financing constraint reduces the strategy set of the governments as well as the set of possible equilibria. Thereby, it can improve welfare if there is a free-riding problem. If Country B is not able to finance a bail-out, the free-riding problem simply disappears (see Appendix E). However, it may also be desirable to circumvent a financing constraint. If a breakdown of the financial system can be avoided, global welfare may be boosted by a bail-out. A central authority with mandating power alone cannot bypass a financing constraint as the financial burden between countries is not shared. Only an authority that can also set contribution levels can make a bail-out feasible in such a situation. This implies that burden sharing becomes more important when country sizes are very asymmetric.

## 3.6 Contracts

So far we have analyzed the non-cooperative solution and contrasted it with the outcomes under a central planner with fiscal power and/or mandating power. Our focus has been on revealing inefficiencies without cooperation. In this section, we investigate an additional form of cooperation: contracts. While a central authority corresponds to an ex-ante agreement on how to coordinate actions during a crisis, contracts reflect ex-post cooperation. We address the question what contracts between governments that are signed after

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<sup>20</sup>In our setting, the ability to tax people according to the bank they deposited in could bring an improvement when we have cross-country deposits.

the crisis has occurred can achieve compared to a central authority.

Contracts specify actions to be taken and a burden sharing rule. We model the negotiation process between governments via Nash bargaining with symmetric negotiation power. As contracts are voluntary, governments only sign a contract if doing so weakly improves their domestic welfare level. The Nash bargaining problem is as follows:

$$\begin{aligned} \max_{a \in \{(n, bo), (bo, n)\}, X^A} & (V^A(a, b, X^A) - V^B(a^*)) (V^B(a, b, X^B) - V^B(a^*)) & (3.34) \\ \text{s.t.} & G(a, b) = X^A + X^B. \end{aligned}$$

If a contract is signed, it implies a Pareto improvement compared to the non-cooperative benchmark. One necessary condition for a Pareto improvement is that the actions  $\tilde{a}$  that are prescribed by the contract differ from the equilibrium actions  $a^*$  of the bail-out game. A contract cannot be an agreement on burden sharing or a different degree of bail-out alone as the participation constraint of one government would be violated. A second necessary condition is that the country where the bail-out does not take place helps finance the bail-out in the other country. This follows from the fact that governments maximize domestic welfare in the bail-out game. A change in actions without compensation must therefore reduce domestic welfare of at least one country. From these two conditions together with Proposition 3.5, it follows that there are two different outcomes of the non-cooperative game that each allow for specific types of contracts to be signed. We summarize our findings in the following proposition:

**Proposition 3.13** (i) *If  $a^* = (n, n)$  and contracts allow for a Pareto improvement, then  $\tilde{a} = (n, bo)$  with  $\tilde{X}^A > 0$  or  $\tilde{a} = (bo, n)$  with  $\tilde{X}^B > 0$ .*  
(ii) *If  $a^* = (n, bo)$  and contracts allow for a Pareto improvement, then  $\tilde{a} = (bo, n)$  with  $\tilde{X}^B > 0$ .*

**Proof.** See the Appendix. ■

Case (i) captures situations where neither country intervenes, but welfare can be improved by a bail-out. In order for Country B to agree on bailing out its domestic bank, Country A has to subsidize the bail-out and vice versa. In Case (ii), Country B would bail-out its bank without any cooperation between countries. However, each country's welfare can be increased if Bank A instead of Bank B is bailed out and Country B subsidizes the bail-out.

The first-order conditions of the Nash bargaining problem imply:

$$\frac{Z'(X^B)}{Z'(X^A)} = \frac{(\bar{\lambda} + \epsilon)u'(d_1(a) - Z(X^A)) + (1 - \bar{\lambda} - \epsilon)u'(d_2(a, b) - Z(X^A)) (V^B(a, b, X^B) - V^B(a^*))}{\bar{\lambda}u'(d_1(a) - Z(X^B)) + (1 - \bar{\lambda})u'(d_2(a, b) - Z(X^B)) (V^A(a, b, X^A) - V^B(a^*))}. \quad (3.35)$$

In a similar way, the FOC for  $b$  differs from the one of a central authority with fiscal power. Every marginal utility is weighted by the other country's Nash factor. Due to differing FOC, the action  $\tilde{a}$ ,  $\tilde{b}$  and the burden sharing rule in form of  $\tilde{X}^A$ ,  $\tilde{X}^B$  will in general not coincide with the solution of the central authority with fiscal power. Whether countries are able to agree upon the efficient actions depends on the amount of redistribution that is required by the Nash bargaining solution. Redistribution is costly. Therefore, when countries move away from the solution of a central authority with fiscal power and choose  $\tilde{X}^j \neq X^j$ , the surplus from changing the actions shrinks with the redistribution.

## 3.7 Winners and Losers from Cooperation

In this section, we consider the different types of cooperation discussed so far with respect to gains and losses for individual countries and potential Pareto improvements compared to the non-cooperative outcome. Analyzing which countries in their role as crisis or affected country benefit from the different types of cooperation is important for understanding their incentives to agree on cooperation.

### 3.7.1 Central Authority without Fiscal Power

The introduction of a central authority with mandating power only has an effect on the equilibrium if it mandates actions different from the ones taken in the non-cooperative game. As it has no fiscal power, which would allow for burden sharing, it cannot induce Pareto improvements:

**Proposition 3.14** *Suppose no country is indifferent between a bail-out and no intervention. Then, a central authority with mandating power can only increase global welfare at the expense of the welfare of one country. It cannot induce a Pareto improvement.*

**Proof.**

Suppose there is no intervention by the governments or a bail-out of Bank B. Then, if the central authority mandates  $a' = (bo, n)$ , Country A is made worse off as  $V_{bo, n}^A <$

$$\max\{V_{n,bo}^A; V_{n,n}^A\}.$$

Suppose there is no intervention by the governments. Then, if the central authority mandates  $a' = (n, bo)$ , Country B is made worse off as  $V_{n,bo}^B < V_{n,n}^A$ . ■

If, in the non-cooperative equilibrium, Country A does not choose to bail-out its bank, then if it is mandated to do so, given that there is no burden sharing, this can only deteriorate its domestic welfare. The other country benefits from this. It can also be optimal to have a bail-out of Bank B. If Country B is mandated to do a bail-out, although in the non-cooperative equilibrium it did not choose this option, then the decision of the central authority raises welfare of Country A at the expense of Country B.

### 3.7.2 Central Authority with Fiscal Power

The decision of a central authority with fiscal power implies that the welfare of one country increases, while the other country can experience a gain or a loss in welfare. To shed some light on this, we discuss under which conditions Pareto improvements are possible.

To start with, we consider cases where the actions taken in the non-cooperative equilibrium coincide with the choice of the central authority. Then, the central authority only modifies the costs that each country has to bear together with  $b$ . In this case, no Pareto improvement is possible, which we have already proven as part of Proposition 3.13. If Country B does a bail-out and this is globally optimal, then burden sharing will lead to a nonnegative contribution of Country A,  $X^A \geq 0$ , which can benefit Country B but harm Country A. If instead the latter does a bail-out, the central authority will use this instrument to implement some consumption smoothing across countries. From Proposition 3.12, we know that the contribution of Country B will be greater than zero,  $X^B > 0$ . This increases welfare of Country A, but decreases welfare of Country B.

When the central planner does not only introduce burden-sharing, but also mandates actions different from the ones taken in the sequential game, this can, but does not need to imply improvements for both countries. There are three cases where the central authority with fiscal power may coordinate on actions different from the ones taken in the sequential game:

- (i)  $a^* = (n, n)$  and  $a' = (n, bo)$ ,
- (ii)  $a^* = (n, n)$  and  $a' = (bo, n)$ ,
- (iii)  $a^* = (n, bo)$  and  $a' = (bo, n)$ .

In all cases, welfare in Country B is strictly higher than in Country A. Furthermore, it can be shown that there is a strictly positive net gain for Country B due to the introduction of a central authority with fiscal power relative to the outcome of the sequential game for Cases (i) and (ii). This is because both countries would have the same utility in the non-cooperative game. For Case (iii), we find that each country can be a net winner or loser. As  $G^A$  might be larger than  $G^B$ , it is possible that  $X^B > G^B$  and that Country B loses welfare. These results are derived in the Appendix.

### 3.7.3 Discussion

Table 3.3: Winners from cooperation

<i>Form of Cooperation</i>	<i>Number of Winners</i>
Central authority without fiscal power	1
Central authority with fiscal power	1-2
Contracts	2

Table 3.3 summarizes the findings of the previous sections regarding winners and losers from the different forms of cooperation. Contracts imply, by their very nature, that both countries benefit. In contrast, the introduction of a central authority with mandating power only always makes one country worse off compared to the non-cooperative outcome. A central authority with additional fiscal power may be able to bring welfare improvements for both countries at the same time, though this is not guaranteed. It can be efficient to make one country worse off.

Among the three cooperation regimes we look at, a central authority with fiscal power achieves the highest overall welfare, which is our benchmark for efficiency. A clear ranking between contracts and a central authority with mandating power only, with respect to efficiency, is not possible. A central authority with mandating power cannot alleviate the inefficiency due to no burden sharing, but can fully internalize spillover effects. While contracts allow for some form of burden sharing, redistribution limits the overall gains that can be realized by contracts. Due to their limitations, neither contracts nor a central authority with mandating power guarantee the implementation of the efficient actions.

As mentioned, contracts correspond to a form of ex-post cooperation. Knowing their roles in the bail-out game, after the realization of the third state, governments negotiate.

A central authority represents a form of ex-ante cooperation. The setup described in Table 3.2 identifies Country A as the crisis country and Country B as the affected country, which is a reduced formulation in order to highlight the mechanisms in our model. We can think of a fourth state that is the mirror image of the third state, where the roles of the countries are interchanged. In this symmetric world, each country can take the role of the crisis country or the affected country when crisis occurs. Cooperation ex-ante takes place under uncertainty about the role each country has in case of a crisis. In this light, the above considerations may shed some light on the desirability and the incentives of countries to agree ex-ante on a form of cooperation.

### 3.8 Ring-Fencing

Another form of government intervention, which can be observed during banking crises, is the ring-fencing of assets, i.e. the freeze of foreign asset holdings in domestic banks. During the current crisis, the German government froze assets of Lehman in order for domestic depositors to be reimbursed. (See Claessens (2009).) Furthermore, in the context of the bankruptcy case of Barings, counterparties and customers faced constraints in accessing their funds during the resolution process. When the Bank for Credit and Commerce International (BCCI) was resolved, California and New York ring-fenced assets in order to secure a higher share of the liquidation value for local depositors. Ring-fencing applied to all assets up to the total value of liabilities towards local depositors. (See Herring (2005).)

In our model, we define ring-fencing as an asset freeze, that is foreign depositors (either a bank or private households) that would like to withdraw early are prevented from doing so. Governments observe the state of the world and decide on the form of intervention before any claims are paid. Furthermore, we assume that a country only does ring-fencing if this implies a strictly higher welfare than any alternative.

**No cross-country deposits** To start with, we consider the case with interbank deposits only. In this case, ring-fencing is equivalent to an interbank deposit freeze at date  $t = 1$ . We extend the bail-out game as illustrated in Figure 3.4 and introduce the additional form of intervention called *ring-fencing*. The figure is a reduced form of the game given the optimality of mutual ring-fencing. We show in the Appendix that the best response to ring-fencing is ring-fencing. Therefore, it does not matter for the pay-offs to depositors

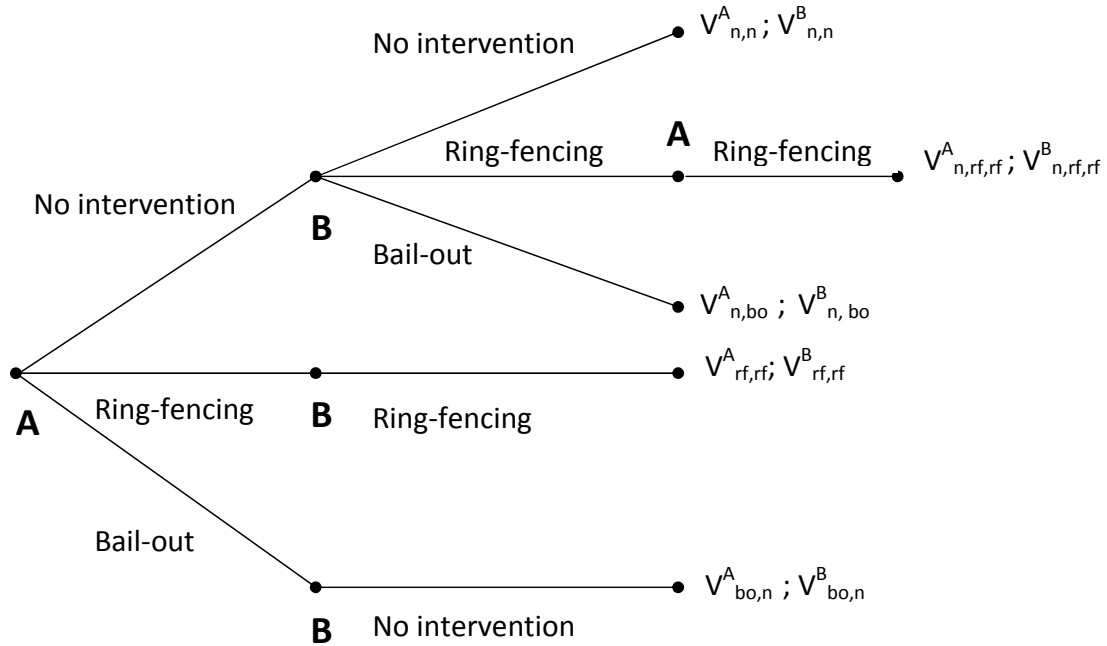


Figure 3.4: Intervention game in extensive form

and thus the welfare levels of the countries which country chooses to ring-fence interbank deposits first. As interbank claims exactly offset each other, ring-fencing cannot prevent bankruptcy in Country A. Bank A has to be liquidated and the welfare level of Country A is the same as in the case where neither country intervenes:

$$V_{rf,rf}^A = V_{n,rf,rf}^A = u(\bar{q}). \tag{3.36}$$

In contrast, ring-fencing has a positive effect on the welfare level of Country B. Because interbank claims net out, contagion is avoided and welfare of Country B attains the first-best:

$$V_{rf,rf}^B = V_{n,rf,rf}^B = \bar{\lambda}u(\bar{d}_1) + (1 - \bar{\lambda})u(\bar{d}_2). \tag{3.37}$$

The following result on optimal strategies can be derived:

**Proposition 3.15** *In the non-cooperative game without cross-country deposits, the crisis country never chooses to ring-fence assets although this could avoid contagion. Without any costs to ring-fencing, the affected country will always choose this option.*

**Proof.** See the Appendix. ■

As Country B always chooses to ring-fence foreign assets in order to avoid contagion, the number of SPNE reduces to the following two:



**Proposition 3.16** (i)  $a^* = (n, rf, rf)$  is a SPNE iff  $V_{bo,n}^A \leq V_{n,rf,rf}^A$ .

(ii)  $a^* = (bo, n)$  is a SPNE iff  $V_{bo,n}^A > V_{n,rf,rf}^A$ .

**Proof.** Omitted. ■

We compare the SPNE of the game with the possible choices of a central authority with mandating power only:

**Proposition 3.17** (i) If the SPNE is  $a^* = (bo, n)$ , then this equilibrium coincides with the optimal solution of the central authority with mandating power  $a'$ .

(ii) If the SPNE is  $a^* = (n, rf, rf)$ , then  $a' \in \{(n, rf, rf), (n, bo)\}$ .

**Proof.** See the Appendix. ■

As burden sharing makes bail-outs less costly, a central authority with fiscal power may find a bail-out of Bank A optimal. Therefore, in Case (ii) the set of possibly efficient actions given fiscal burden sharing changes to  $a' \in \{(n, rf, rf), (n, bo), (bo, n)\}$ . In the modified game, there is only scope for one specific type of contract as defined in Expression 3.34 because Country B always attains the maximum welfare level:  $\tilde{a} = (n, bo)$  with  $X^A = G^B(\bar{d}_2)$  and  $X^B = 0$ , i.e. Country A fully finances the bail-out of Bank B.

So far we have abstracted from any costs that ring-fencing might have. Yet, one can imagine that a country, which ring-fences assets, could be punished for its behavior, for example through the exclusion from the international interbank market in the future. Suppose the country that ring-fences suffers from a utility loss due to some penalty. Then ring-fencing may be no longer the dominant strategy of Country B. The severity of the punishment determines whether ring-fencing initiated by Country B is observed in equilibrium. Punishment could also be endogenous, giving an additional role to cooperation.

**With cross-country deposits** With cross-country deposits, the scope for ring-fencing increases. Countries can freeze interbank assets as well as private assets. The motivation of a government for ring-fencing becomes twofold. As before, freezing deposits prevents the withdrawal of assets, thereby eventually alleviating the liquidity problem at date  $t = 1$  and preventing contagion. Moreover, by ring-fencing assets, a government can change the de-facto seniority of claims. It allows for a compensation of domestic depositors at the expense of foreigners. When a large fraction of assets deposited in the domestic bank is owned by foreigners, the incentives to ring-fence may therefore increase. It is crucial whether governments can discriminate between interbank and private deposits and freeze these assets independently of each other.

### 3.9 Conclusions

The financial system is more and more linked internationally. This has important implications for international crises and corresponding interventions by governments. We provide a model of contagion in an international setting with endogenous bail-out decisions. We study efficiency properties under different forms of cooperation in contrast to the non-cooperative outcome, identify winners and losers and point out factors that make cooperation more important.

Among the three different cooperation regimes we consider, a central authority with mandating and fiscal power achieves the highest overall welfare. A central authority with mandating power only can improve the global welfare as well, but to a lesser extent. Decisions by both types of central authority do not imply gains for both countries, i.e. Pareto improvements. While under contracts an agreement always implies a Pareto improvement, ex-post negotiations between governments can fall short on global efficiency both in terms of actions chosen and in terms of burden-sharing.

The model provides a framework to understand potential gains and losses for different countries from different cooperation regimes. In this regard, it can help guide current policy reforms like the one by the European Commission and provide some intuition for the negotiation processes. An important result of our analysis is that with equal country sizes larger cross-country deposit holdings improve the non-cooperative equilibrium. Therefore, cross-border deposit holdings by consumers can have positive effects by reducing externalities and free-riding problems. Another relevant aspect highlighted by our model is the case of asymmetric country sizes. The potential need for a bail-out at some point in time implies that it might be good to limit the ratio of the financial sector of a country to its population, that is the tax base, when there is no ex-ante cooperation in crisis management.

The model may be a suitable framework to study other aspects of bail-outs. While we consider fiscal bail-outs, banks could also be saved through a bail-out financed by monetary policy where central banks are left to pick-up the bail-out bill. This would entail different trade-offs as e.g. the labor-leisure choice might be distorted by inflation.

# Appendix C

## Appendix to Chapter 3

### C.1 Proofs

**Proof of Proposition 3.5 Proof.**

(i)  $a^* = (bo, n)$  iff  $V_{bo,n}^A > V_{n,bo}^A$  or  $V_{n,bo}^B \leq V_{n,n}^B$  and  $V_{bo,n}^A > V_{n,n}^A$ .

If  $V_{bo,n}^A > V_{n,bo}^A$ , using  $V_{bo,n}^B > V_{n,bo}^B \Rightarrow V_{bo,n}^A + V_{bo,n}^B > V_{n,bo}^A + V_{n,bo}^B$ .

Using  $V_{n,bo}^A > V_{n,n}^A$  and  $V_{bo,n}^B > V_{n,n}^B \Rightarrow V_{bo,n}^A + V_{bo,n}^B > V_{n,n}^A + V_{n,n}^B$ .

If  $V_{bo,n}^A > V_{n,n}^A$  and  $V_{n,n}^B \geq V_{n,bo}^B \Rightarrow V_{bo,n}^A + V_{n,n}^B > V_{n,n}^A + V_{n,bo}^B$ .

Using  $V_{bo,n}^B - V_{n,n}^B > V_{n,bo}^A - V_{n,n}^A \Rightarrow V_{bo,n}^A + V_{bo,n}^B > V_{n,bo}^A + V_{n,bo}^B$ ,

and as  $V_{bo,n}^B > V_{n,n}^B \Rightarrow V_{bo,n}^A + V_{bo,n}^B > V_{n,n}^A + V_{n,n}^B$ .

(ii)  $a^* = (n, bo)$  iff  $V_{n,bo}^B > V_{n,n}^B$  and  $V_{bo,n}^A \leq V_{n,bo}^A$ .

As  $V_{n,bo}^A > V_{n,n}^A \Rightarrow V_{n,bo}^A + V_{n,bo}^B > V_{n,n}^A + V_{n,n}^B$ .

(iii)  $a^* = (n, n)$  iff  $V_{n,bo}^B \leq V_{n,n}^B$  and  $V_{bo,n}^A \leq V_{n,n}^A \Rightarrow V_{bo,n}^A \leq V_{n,n}^A < V_{n,bo}^A$ ,

and  $V_{n,bo}^B \leq V_{n,n}^B < V_{bo,n}^B$ ,

which implies that in case (iii) no general statement can be made on actions maximizing global welfare. ■

**Proof of Proposition 3.9 Proof.** With constant country size implying  $\bar{P} - \alpha = (1 - \beta)$ , the first-order condition of Country A for  $b$  implies:

$$\frac{\alpha(\bar{\lambda} + \epsilon)u'(\bar{d}_1 - Z(G(b))) + (\bar{P} - \alpha)[\bar{\lambda}u'(\bar{d}_1 - Z(G(b))) + (1 - \bar{\lambda})u'(\bar{d}_2 - Z(G(b)))]}{\alpha(1 - \bar{\lambda} - \epsilon)u'(b - Z(G(b)))} = \frac{1 - Z'(G(b))G'(b)}{Z'(G(b))G'(b)}.$$

The derivative of the left hand side (LHS) with respect to  $\alpha$ , holding  $b$  constant, implies:

$$\begin{aligned} & \frac{\partial LHS}{\alpha} \Big|_{b=\bar{b}} [\alpha(1 - \bar{\lambda} - \epsilon)u'(b - Z(G(b)))]^2 \\ &= [(\bar{\lambda} + \epsilon)u'(\bar{d}_1 - Z(G(b))) - \bar{\lambda}u'(\bar{d}_1 - Z(G(b))) - (1 - \bar{\lambda})u'(\bar{d}_2 - Z(G(b)))] \\ & \alpha(1 - \bar{\lambda} - \epsilon)u'(b - Z(G(b))) \\ & - [\alpha(\bar{\lambda} + \epsilon)u'(\bar{d}_1 - Z(G(b))) + (\bar{P} - \alpha)[\bar{\lambda}u'(\bar{d}_1 - Z(G(b))) \\ & + (1 - \bar{\lambda})u'(\bar{d}_2 - Z(G(b)))](1 - \bar{\lambda} - \epsilon)u'(b - Z(G(b))) \\ &= -[\bar{\lambda}u'(\bar{d}_1 - Z(G(b))) + (1 - \bar{\lambda})u'(\bar{d}_2 - Z(G(b)))]\alpha(1 - \bar{\lambda} - \epsilon)u'(b - Z(G(b))) \\ & - (\bar{P} - \alpha)[\bar{\lambda}u'(\bar{d}_1 - Z(G(b))) + (1 - \bar{\lambda})u'(\bar{d}_2 - Z(G(b)))](1 - \bar{\lambda} - \epsilon)u'(b - Z(G(b))) < 0. \end{aligned}$$

The derivative of the LHS with respect to  $b$  implies:

$$\begin{aligned} & \frac{\partial LHS}{b} (\alpha(1 - \bar{\lambda} - \epsilon)u'(b - Z(G(b))))^2 \\ &= -\alpha(\bar{\lambda} + \epsilon)u'(\bar{d}_1 - Z(G(b))) + (\bar{P} - \alpha)[\bar{\lambda}u'(\bar{d}_1 - Z(G(b))) + (1 - \bar{\lambda})u'(\bar{d}_2 - Z(G(b)))] \\ & \alpha(1 - \bar{\lambda} - \epsilon)u''(b - Z(G(b))) > 0. \end{aligned}$$

The derivative of  $Z'(G(b))G'(b)$  with respect to  $b$  is:

$$\frac{\partial Z'(G(b))G'(b)}{b} = Z''(G(b))(G'(b))^2 + Z'(G(b))G''(b) = Z''(G(b))(G'(b))^2 > 0.$$

Therefore, the derivative of the right hand side (RHS) with respect to  $b$  is negative. The statements on the derivatives above imply that an increase in  $\alpha$  leads to a higher bailout level  $b$  being chosen by the government in Country A. The proof for Country B is analogous. ■

**Proof of Proposition 3.10 Proof.**

(i) Country B does a bail-out iff  $V_{n,bo}^B > V_{n,n}^B$ . Now:

$$\begin{aligned} \frac{\partial(V_{n,bo}^B - V_{n,n}^B)}{\partial(1 - \alpha)} \Big|_{b=\bar{b}} &= u(\hat{q} - Z(G(b))) - [\bar{\lambda}u(\bar{d}_1 - Z(G(b))) + (1 - \bar{\lambda})u(b - Z(G(b)))] < 0 \\ \Rightarrow \forall \alpha, \alpha' \in [0, 1] \text{ and } \forall b \in [\bar{d}_1, \bar{d}_2] : \alpha' > \alpha &\Leftrightarrow V_{n,bo}^B(b; \alpha') > V_{n,bo}^B(b; \alpha). \end{aligned}$$

Let  $b = \arg \max V_{n,bo}^B(b; \alpha)$  and  $b' = \arg \max V_{n,bo}^B(b; \alpha')$  with  $\alpha' > \alpha$ . Then, from optimal behavior of Country B and above:  $V_{n,bo}^B(b'; \alpha') \geq V_{n,bo}^B(b; \alpha') > V_{n,bo}^B(b; \alpha)$ .

(ii) Country A does a bail-out iff  $V_{bo,n}^A > V_{n,n}^A$ . Now:

$$\begin{aligned} \frac{\partial(V_{bo,n}^A - V_{n,n}^A)}{\partial(1 - \beta)} \Big|_{b=\bar{b}} &= - [(\bar{\lambda} + \epsilon)u(\bar{d}_1 - Z^A(G^A(b))) + (1 - \bar{\lambda} - \epsilon)u(b - Z^A(G^A(b)))] \\ &+ [\bar{\lambda}u(\bar{d}_1 - Z^A(G^A(b))) + (1 - \bar{\lambda})u(\bar{d}_2 - Z^A(G^A(b)))] > 0 \\ \Rightarrow \forall \beta, \beta' \in [0, 1] \text{ and } \forall b \in [\bar{d}_1, \bar{d}_2] : \beta > \beta' &\Leftrightarrow V_{bo,n}^A(b; \beta') > V_{bo,n}^A(b; \beta). \end{aligned}$$

Let  $b = \arg \max V_{bo,n}^A(b; \beta)$  and  $b' = \arg \max V_{bo,n}^A(b; \beta')$  with  $\beta > \beta'$ . Then, from optimal behavior of Country A and above:  $V_{bo,n}^A(b'; \beta') \geq V_{bo,n}^A(b; \beta') > V_{bo,n}^A(b; \beta)$ . ■

**Proof of Proposition 3.11 Proof.**

(i) For Country A, we have:

$$Z^A(G) = \frac{\kappa\eta}{4} \left[ 1 - \left( \frac{1}{2} + \sqrt{\frac{1}{4} - \frac{G}{\alpha\kappa}} \right)^2 \right].$$

Holding  $b$  constant, we have:

$$\frac{\partial Z^A}{\partial \alpha} \Big|_{b=\bar{b}} = -\frac{G\eta}{4\alpha^2} \left( \frac{1}{2} + \sqrt{\frac{1}{4} - \frac{G}{\alpha\kappa}} \right) \left( \frac{1}{4} - \frac{G}{\alpha\kappa} \right)^{-\frac{1}{2}} < 0.$$

Country A does a bail-out if:

$$\frac{V_{bo,n}^A}{V_{n,n}^A} = \frac{(2\bar{\lambda} + \epsilon)u(\bar{d}_1 - Z(G^A(b))) + (1 - \bar{\lambda} - \epsilon)u(b - Z(G^A(b))) + (1 - \bar{\lambda})u(\bar{d}_2 - Z(G^A(b)))}{2u(\hat{q})} > 1$$

and

$$\frac{V_{bo,n}^A}{V_{n,bo}^A} = \frac{(2\bar{\lambda} + \epsilon)u(\bar{d}_1 - Z(G^A(b))) + (1 - \bar{\lambda} - \epsilon)u(b - Z(G^A(b))) + (1 - \bar{\lambda})u(\bar{d}_2 - Z(G^A(b)))}{u(\hat{q}) + \bar{\lambda}u(\bar{d}_1) + (1 - \bar{\lambda})u(b)} > 1.$$

Now:

$$\begin{aligned} & \frac{\partial(V_{bo,n}^A/V_{n,n}^A)}{\partial Z} \Big|_{b=\bar{b}} \\ &= - \frac{(2\bar{\lambda} + \epsilon)u'(\bar{d}_1 - Z(G^A(b))) + (1 - \bar{\lambda} - \epsilon)u'(b - Z(G^A(b))) + (1 - \bar{\lambda})u'(\bar{d}_2 - Z(G^A(b)))}{2u(\bar{q})} < 0 \\ &\Rightarrow \frac{\partial(V_{bo,n}^A/V_{n,n}^A)}{\partial \alpha} \Big|_{b=\bar{b}} > 0, \end{aligned}$$

and

$$\begin{aligned} & \frac{\partial(V_{bo,n}^A/V_{n,bo}^A)}{\partial Z} \Big|_{b=\bar{b}} \\ &= - \frac{(2\bar{\lambda} + \epsilon)u'(\bar{d}_1 - Z(G^A(b))) + (1 - \bar{\lambda} - \epsilon)u'(b - Z(G^A(b))) + (1 - \bar{\lambda})u'(\bar{d}_2 - Z(G^A(b)))}{u(\hat{q}) + \bar{\lambda}u(\bar{d}_1) + (1 - \bar{\lambda})u(b)} < 0. \\ &\Rightarrow \frac{\partial(V_{bo,n}^A/V_{n,bo}^A)}{\partial \alpha} \Big|_{b=\bar{b}} > 0. \end{aligned}$$

(ii) For Country B, we have:

$$Z^B(G) = \frac{\kappa\eta}{4} \left[ 1 - \left( \frac{1}{2} + \sqrt{\frac{1}{4} - \frac{G}{\beta\kappa}} \right)^2 \right].$$

Holding  $b$  constant, we have:

$$\frac{\partial Z^B}{\partial \beta} \Big|_{b=\bar{b}} = - \frac{G\eta}{4\beta^2} \left( \frac{1}{2} + \sqrt{\frac{1}{4} - \frac{G}{\beta\kappa}} \right) \left( \frac{1}{4} - \frac{G}{\beta\kappa} \right)^{-\frac{1}{2}} < 0.$$

Country B does a bail-out if:

$$\frac{V_{n,bo}^B}{V_{n,n}^B} = \frac{u(\hat{q} - Z(G^B(b))) + \bar{\lambda}u(\bar{d}_1 - Z(G^B(b))) + (1 - \bar{\lambda})u(b - Z(G^B(b)))}{2u(\bar{q})} > 1.$$

Now:

$$\begin{aligned} & \frac{\partial(V_{n,bo}^B/V_{n,n}^B)}{\partial Z} \Big|_{b=\bar{b}} \\ &= - \frac{u'(\hat{q} - Z(G^B(b))) + \bar{\lambda}u'(\bar{d}_1 - Z(G^B(b))) + (1 - \bar{\lambda})u'(b - Z(G^B(b)))}{2u(\bar{q})} < 0 \\ &\Rightarrow \frac{\partial(V_{n,bo}^B/V_{n,n}^B)}{\partial \beta} \Big|_{b=\bar{b}} > 0. \end{aligned}$$

■

**Proof of Proposition 3.12 Proof.**

(1) Interior solution, where Condition 3.33 holds. For the case (n,bo), we have:

$$1 = \frac{\bar{\lambda}u'(\bar{d}_1 - Z(X^B)) + (1 - \bar{\lambda})u'(b - Z(X^B))}{u'(\hat{q} - Z(X^A))} \frac{Z'(X^B)}{Z'(X^A)}.$$

Suppose that  $X^A \geq X^B$ . Then:

$$\begin{aligned} & \bar{\lambda}u'(\bar{d}_1 - Z(X^B)) + (1 - \bar{\lambda})u'(b - Z(X^B)) \geq u'(\hat{q} - Z(X^A)) \\ \Rightarrow & u'(\bar{d}_1 - Z(X^B)) \geq u'(\hat{q} - Z(X^A)) \\ \Rightarrow & Z(X^B) - Z(X^A) \geq \bar{d}_1 - \hat{q} > 0 \\ \Rightarrow & X^B > X^A, \end{aligned}$$

which is a contradiction. Therefore,  $X^B > X^A$ .

For the case (bo,n), we have:

$$1 = \frac{\bar{\lambda}u'(\bar{d}_1 - Z(X^B)) + (1 - \bar{\lambda})u'(\bar{d}_2 - Z(X^B))}{(\bar{\lambda} + \epsilon)u'(\bar{d}_1 - Z(X^A)) + (1 - \bar{\lambda} - \epsilon)u'(b - Z(X^A))} \frac{Z'(X^B)}{Z'(X^A)}.$$

Suppose that  $X^A \geq X^B$ . Then, given  $b \in [\bar{d}_1, \bar{d}_2]$ :

$$\begin{aligned} & \lambda u'(\bar{d}_1 - Z(X^B)) + (1 - \bar{\lambda})u'(\bar{d}_2 - Z(X^B)) \geq (\bar{\lambda} + \epsilon)u'(\bar{d}_1 - Z(X^A)) + (1 - \bar{\lambda} - \epsilon)u'(b - Z(X^A)) \\ \Rightarrow & \lambda u'(\bar{d}_1 - Z(X^B)) + (1 - \bar{\lambda})u'(\bar{d}_2 - Z(X^B)) \geq (\bar{\lambda} + \epsilon)u'(\bar{d}_1 - Z(X^A)) + (1 - \bar{\lambda} - \epsilon)u'(\bar{d}_2 - Z(X^A)) \\ \Rightarrow & \bar{\lambda}[u'(\bar{d}_1 - Z(X^B)) - u'(\bar{d}_1 - Z(X^A))] + (1 - \bar{\lambda})[u'(\bar{d}_2 - Z(X^B)) - u'(\bar{d}_2 - Z(X^A))] \\ \geq & \epsilon[u'(\bar{d}_1 - Z(X^A)) - u'(\bar{d}_2 - Z(X^A))] > 0 \\ \Rightarrow & Z(X^B) > Z(X^A) \Rightarrow X^B > X^A, \end{aligned}$$

which is a contradiction. Therefore,  $X^B > X^A$ .

(2) Corner solutions. Condition 3.33 might not bind as contributions by countries are bounded from below, i.e.  $X^A, X^B \geq 0$ . There are two possible corner solutions in which this could be the case:

$$(i) X^B = 0 \text{ and } X^A = G,$$

$$(ii) X^A = 0 \text{ and } X^B = G.$$

We now show that only case (ii) can be chosen optimally by a central authority, which implies that in any corner solution  $X^B > X^A$ .

Suppose (i), i.e.  $X^B = 0$  and  $X^A = G$ . Then, the FOC of the central authority with respect to  $X^A$  becomes:

$$\begin{aligned} \frac{\partial V}{\partial X^A} = & -[(\bar{\lambda} + \epsilon)u'(d_1^A(a) - Z(G)) + (1 - \bar{\lambda} - \epsilon)u'(d_2^A(a, b) - Z(G))]Z'(G) \\ & + [\bar{\lambda}u'(d_1^B(a)) + (1 - \bar{\lambda})u'(d_2^B(a, b))]Z'(0). \end{aligned}$$

This can be rearranged to:

$$\begin{aligned} \frac{\partial V}{\partial X^A} = & \bar{\lambda}[u'(d_1^B(a))Z'(0) - u'(d_1^A(a) - Z(G))Z'(G)] \\ & + (1 - \bar{\lambda} - \epsilon)[u'(d_2^B(a, b))Z'(0) - u'(d_2^A(a, b) - Z(G))Z'(G)] \\ & + \epsilon[u'(d_2^B(a, b))Z'(0) - u'(d_1^A(a) - Z(G))Z'(G)] < 0, \end{aligned}$$

as  $u()$  is concave,  $Z()$  is convex,  $d_2^B \geq d_2^A \geq d_1^A$  and  $d_1^B \geq d_1^A$ . Welfare could be improved by decreasing  $X_A$  and increasing  $X_B$ . Therefore,  $X_A = G$  cannot be the optimal solution chosen by a central authority. ■

### Proof of Proposition 3.13 Proof.

(1) A contract  $(\tilde{a}, \tilde{X}^A)$  must imply a Pareto improvement as otherwise the participation constraint of one country would be violated.

(2) A change in actions alone (excluding the case of Country A or Country B being indifferent between a bail-out and another action) cannot induce a Pareto improvement. Suppose there is no intervention by the governments or a bail-out of Bank B, i.e.  $a^* \in \{(n, n), (n, bo)\}$ . Then, if the central authority mandates  $(bo, n)$ , Country A is made worse off as  $V_{bo, n}^A < \max\{V_{n, bo}^A; V_{n, n}^A\}$ .

Suppose there is no intervention by the governments, that is  $a^* = (n, n)$ . Then, if the central authority mandates  $(n, bo)$ , Country B is made worse off as  $V_{n, bo}^B < V_{n, n}^A$ .

(3) A change in the contribution levels without a change of actions cannot bring a Pareto improvement. Suppose  $\tilde{a} = a^*$ . Then, any change in the contribution levels makes one country better off and one country worse off as at an optimum  $G(a, b) = X^A + X^B$  binds.

(4) A change in the degree of the bail-out cannot induce a Pareto improvement as the degree of the bail-out  $b$  does not affect the welfare of the country that does not conduct



the bail-out.

(5)  $a^*$  can only be inefficient if  $a^* \in \{(n, bo), (n, n)\}$  (see Proposition 3.5). Furthermore, it follows that if  $a^* = (n, bo)$ , then  $\tilde{a} \in \{(bo, n)\}$  and if  $a^* = (n, n)$ , then  $\tilde{a} \in \{(bo, n), (n, bo)\}$ .

(6) If a country did not do a bail-out before, and agrees in the contract to do so, then it has to receive a positive payment. This is the case as the country weakly preferred not to do a bail-out and as the gains from the change of action are shared. ■

**Proof of Proposition 3.15 Proof.** We consider the two cases where (1) Country A ring-fences first, and (2) Country B ring-fences first. We solve each case by backward induction.

(1) Given ring-fencing by Country A, the welfare levels of Country B for the different responses (no intervention, bail-out, ring-fencing) are as follows:

$$V_{rf,n}^B = u(q_r), \text{ with } q_r = \frac{(1-y)r+y}{1+z} < \bar{q},$$

$$V_{rf,bo}^B = \bar{\lambda}u(\bar{d}_1 - G(b)) + (1 - \bar{\lambda})u(b - G(b)), \quad \text{with } G(b) = z\bar{d}_1 - r \left( (1-y) - \frac{(1-\bar{\lambda})b}{R} \right),$$

$$V_{rf,rf}^B = \bar{\lambda}u(\bar{d}_1) + (1 - \bar{\lambda})u(\bar{d}_2).$$

It follows from this that the best response of Country B to ring-fencing by Country A is ring-fencing. The welfare level for Country A, if it ring-fences first, is:

$$V_{rf,rf}^A = u(\bar{q}).$$

Comparing this with Equations 3.19, 3.20 and 3.22, it can be seen that Country A never chooses to ring-fence first.

(2) Given ring-fencing by Country B, the welfare levels of Country A for the different responses (no intervention, ring-fencing) are:

$$V_{n,rf,n}^A = u(q_r), \text{ with } q_r = \frac{(1-y)r+y}{1+z} < \bar{q},$$

$$V_{n,rf,rf}^A = u(\bar{q}).$$

Note that a bail-out by Country A will not be chosen in the third round. If Country A prefers that option, it does already choose it in the first round. Therefore, given that Country B ring-fences, a bail-out cannot be optimally chosen by Country A. The best

response of Country A to ring-fencing by Country B is ring-fencing. The welfare level of Country B, if it ring-fences first, is:

$$V_{n,rf,rf}^B = \bar{\lambda}u(\bar{d}_1) + (1 - \bar{\lambda})u(\bar{d}_2),$$

which is strictly higher than all other welfare levels except for the case where Country A does a bail-out. Therefore, if Country A does not do a bail-out, Country B always ring-fences. ■

### Proof of Proposition 3.17 Proof.

(i)  $a^* = (bo, n) \Rightarrow V_{bo,n}^A > V_{n,rf,rf}^A$ . Therefore, together with  $V_{bo,n}^B = V_{n,rf,rf}^B$ , we have  $V_{bo,n}^A + V_{bo,n}^B > V_{n,rf,rf}^A + V_{n,rf,rf}^B$ .

(ii)  $a^* = (n, rf, rf) \Rightarrow V_{bo,n}^A \leq V_{n,rf,rf}^A \Rightarrow V_{bo,n}^A + V_{bo,n}^B \leq V_{n,rf,rf}^A + V_{n,rf,rf}^B$ .

A clear ranking in terms of welfare between  $(n, bo)$  and  $(n, rf, rf)$  is not possible because  $V_{n,bo}^A > V_{n,rf,rf}^A$ , but  $V_{n,rf,rf}^B < V_{n,bo}^B$ . ■

## C.2 Lump-Sum Taxation

With lump-sum taxation, Country A always prefers  $(bo, n)$  over  $(n, n)$ , as long as  $\bar{\lambda} < 1$ . Due to the free-riding problem, no clear statement can be made on  $(bo, n)$  vs.  $(n, bo)$ .

**Proof.**

$$\begin{aligned} V_{bo,n}^A &> V_{n,n}^A \\ \Leftrightarrow (\bar{\lambda} + \epsilon)u(\bar{d}_1 - \epsilon\bar{d}_1) + (1 - \bar{\lambda})u(\bar{d}_2 - \epsilon\bar{d}_1) &> u(\bar{q}) \\ \Rightarrow u(\bar{d}_1 - \epsilon\bar{d}_1) &> u(\bar{q}) \\ \Leftrightarrow \bar{d}_1 - \epsilon\bar{d}_1 &> \bar{q} \\ \Leftrightarrow \bar{d}_1 = \frac{y}{\bar{\lambda}} > \frac{y + (1 - y)r}{1 - \epsilon} = \frac{\bar{q}}{1 - \epsilon}. \end{aligned}$$

Using  $\epsilon \leq 1 - \bar{\lambda}$  implies

$$\Rightarrow \frac{y}{\bar{\lambda}} > \frac{y + (1 - y)r}{\bar{\lambda}}.$$

This is true as  $\bar{\lambda} < 1 \Rightarrow y < 1$ .

■

With lump-sum taxation, Country B always does a bail-out. In the following we prove

that a full bail-out implies a higher welfare in Country B than no intervention.

**Proof.**

$$\begin{aligned}
V_{n,bo}^B &> V_{n,n}^B \\
&\Leftrightarrow \bar{\lambda}u(\bar{d}_1 - z(\bar{d}_1 - \hat{q})) + (1 - \bar{\lambda})u(\bar{d}_2 - z(\bar{d}_1 - \hat{q})) > u(\bar{q}) \\
&\Rightarrow u(\bar{d}_1 - z(\bar{d}_1 - \hat{q})) > u(\bar{q}) \\
&\Leftrightarrow \bar{d}_1 - z(\bar{d}_1 - \hat{q}) > \bar{q} \\
&\Leftrightarrow \bar{d}_1(1 - z) + z\frac{\bar{q} + z\bar{d}_1}{1 + z} - \bar{q} > 0 \\
&\Leftrightarrow \bar{d}_1(1 - z) + \frac{z^2\bar{d}_1 - \bar{q}}{1 + z} > 0 \\
&\Leftrightarrow \bar{d}_1 - \bar{q} > 0.
\end{aligned}$$

■

### C.3 First-Order Conditions of the Central Authority with Fiscal Power

Suppose the central authority implements a bail-out of Bank A, then it faces the following FOCs:

$$\begin{aligned}
\frac{\partial V}{\partial b}\Big|_{a=(bo,n)} &= \Theta^A(1 - \bar{\lambda} - \epsilon)u'(b - Z(X^A)) \\
&- \Theta^B [\bar{\lambda}u'(\bar{d}_1 - Z(G^A(b) - X^A)) + (1 - \bar{\lambda})u'(\bar{d}_2 - Z(G^A(b) - X^A))] Z'(G^A(b) - X^A)G^{A'},
\end{aligned}$$

and

$$\begin{aligned}
\frac{\partial V}{\partial X^A}\Big|_{a=(bo,n)} &= -\Theta^A [(\bar{\lambda} + \epsilon)u'(\bar{d}_1 - Z(X^A)) + (1 - \bar{\lambda} - \epsilon)u'(b - Z(X^A))] Z'(X^A) \\
&+ \Theta^B [\bar{\lambda}u'(\bar{d}_1 - Z(X^B)) + (1 - \bar{\lambda})u'(\bar{d}_2 - Z(X^B))] Z'(X^B).
\end{aligned}$$

Suppose the central authority implements a bail-out of Bank B, then it faces the following FOCs:

$$\begin{aligned}
\frac{\partial V}{\partial b}\Big|_{a=(n,bo)} &= -\Theta^B G^{B'} \\
&[\bar{\lambda}u'(\bar{d}_1 - Z(G^B(b) - X^A))Z'(G^B(b) - X^A) + (1 - \bar{\lambda})u'(b - Z(G^B(b) - X^A))(Z'(G^B(b) - X^A) - 1)],
\end{aligned}$$

and

$$\frac{\partial V}{\partial X^A} \Big|_{a=(n,bo)} = -\Theta^A u'(\hat{q} - Z(X^A)) Z'(X^A) + \Theta^B [\bar{\lambda} u'(\bar{d}_1 - Z(X^B)) + (1 - \bar{\lambda}) u'(b - Z(X^B))] Z'(X^B).$$

## C.4 The Bail-out Game with Simultaneous Moves

Note that the following three inequalities hold:

$$V_{n,bo}^A > V_{n,n}^A,$$

$$V_{bo,n}^B > V_{n,n}^B,$$

$$V_{bo,n}^B > V_{n,bo}^B.$$

Therefore, all possible welfare orderings for Country A are:

$$(A-i) \quad V_{bo,n}^A > V_{n,n}^A \quad \text{and} \quad V_{bo,n}^A > V_{n,bo}^A,$$

$$(A-ii) \quad V_{bo,n}^A > V_{n,n}^A \quad \text{and} \quad V_{bo,n}^A \leq V_{n,bo}^A,$$

$$(A-iii) \quad V_{bo,n}^A \leq V_{n,n}^A.$$

For Country B all possible welfare orderings are:

$$(B-i) \quad V_{n,bo}^B > V_{n,n}^B,$$

$$(B-ii) \quad V_{n,bo}^B \leq V_{n,n}^B.$$

Combining the two countries, there are in total 6 different orderings possible. The following proposition reports all equilibria for all cases:

**Proposition C.1** *(i) Suppose (A-i) and (B-i) hold, then the game has one Nash equilibrium  $a^* = (bo, n)$ .*

*(ii) Suppose (A-i) and (B-ii) hold, then the game has one Nash equilibrium  $a^* = (bo, n)$ .*

*(iii) Suppose (A-ii) and (B-i) hold, then the game has two Nash equilibria  $a_1^* = (bo, n)$  and  $a_2^* = (n, bo)$ .*

*(iv) Suppose (A-ii) and (B-ii) hold, then the game has one Nash equilibrium  $a^* = (bo, n)$ .*

*(v) Suppose (A-iii) and (B-i) hold, then the game has one Nash equilibrium  $a^* = (n, bo)$ .*

*(vi) Suppose (A-iii) and (B-ii) hold, then the game has one Nash equilibrium  $a^* = (n, n)$ .*

Note that in case (iii) there are two Nash equilibria. While in this case  $V_{n,n}$  is always smaller than  $V_{n,bo}$ , there is no clear ordering between  $V_{bo,n}$  and  $V_{n,bo}$ . Both  $(bo, n)$  and  $(n, bo)$  are equilibria of the simultaneous game and can, depending on parameters, be efficient. Thus, in the simultaneous game, coordination failure can occur. This is the main difference to the sequential game where instead a free-riding problem arises.

## C.5 Unilateral Commitment

Suppose a country can commit to an action. From the structure of the sequential game it is obvious that only commitment of the second mover is relevant. Commitment of Country B can tackle the potential inefficiency due to free-riding by Country A. In a situation described by (ii) in Proposition 3.3 commitment can change the equilibrium outcome. If Country B commits not to intervene, then the subgame-perfect Nash equilibrium is  $a^* = (bo, n)$ . As  $V_{bo,n}^B > V_{n,bo}^B$ , Country B benefits from the possibility to commit, while Country A loses as  $V_{bo,n}^A < V_{n,bo}^A$ . The same considerations apply to the case with cross-country deposits. With cross-country deposits, it may also be sufficient for removing the free-riding problem that Country B commits to a degree of bail-out  $b < b^*$  in order to induce the equilibrium  $a^* = (bo, n)$ . A lower  $b$  decreases the welfare level of Country A given a bail-out in Country B just as the commitment to no intervention does.

## C.6 Winners and Losers under a Central Authority with Fiscal Power

There are three possible cases in which a central authority with fiscal power chooses a set of actions different to the outcome of the sequential game. These are:

- (i)  $a^* = (n, n)$  and  $a' = (n, bo)$ ,
- (ii)  $a^* = (n, n)$  and  $a' = (bo, n)$ ,
- (iii)  $a^* = (n, bo)$  and  $a' = (bo, n)$ .

In the following we study for each of these cases, which of the two countries wins and loses due to the presence of a central authority with fiscal power. The results are derived assuming symmetric welfare weights, i.e.  $\Theta^A = \Theta^B$ .

- (i)  $a^* = (n, n)$  and  $a' = (n, bo)$ . We show that in this case, under a central authority

with fiscal power, welfare in Country B is strictly higher than in Country A, i.e.  $V_{n,bo}^B > V_{n,bo}^A$ . In Proposition 3.12, we showed that  $X^B > X^A$ .

Therefore:

$$\begin{aligned} \frac{u'(\hat{q} - Z(X^A))}{\bar{\lambda}u'(\bar{d}_1 - Z(G^B(b) - X^A)) + (1 - \bar{\lambda})u'(b - Z(G^B(b) - X^A))} &= \frac{Z'(X^B)}{Z'(X^A)} > 1 \\ \Leftrightarrow \bar{\lambda}u'(\bar{d}_1 - Z(G^B(b) - X^A)) + (1 - \bar{\lambda})u'(b - Z(G^B(b) - X^A)) &< u'(\hat{q} - Z(X^A)). \end{aligned}$$

Due to concavity of  $u'()$ , it follows that:

$$\begin{aligned} \bar{\lambda}u(\bar{d}_1 - Z(G^B(b) - X^A)) + (1 - \bar{\lambda})u(b - Z(G^B(b) - X^A)) &> u(\hat{q} - Z(X^A)) \\ \Leftrightarrow V_{n,bo}^B > V_{n,bo}^A. \end{aligned}$$

As both countries would have the same welfare level in the sequential game, that is  $V_{n,n}^A = V_{n,n}^B = u(\bar{q})$ , this implies that the net gain of Country B from the presence of a central authority with fiscal power is also larger than that of Country A. Note that while the former is strictly positive, it is possible for the latter to be negative.

(ii)  $a^* = (n, n)$  and  $a' = (bo, n)$ . We show that in this case, under a central authority with fiscal power, welfare in Country B is strictly higher than in Country A, i.e.  $V_{bo,n}^B > V_{bo,n}^A$ . In Proposition 3.12, we showed that  $X^B > X^A \forall b \in [\bar{d}_1, \bar{d}_2]$ . Therefore:

$$\begin{aligned} \frac{u'(\bar{d}_1 - Z(X^A))}{u'(\bar{d}_1 - Z(G^B(b) - X^A))} &> 1 \\ \Leftrightarrow u'(\bar{d}_1 - Z(G^B(b) - X^A)) &< u'(\bar{d}_1 - Z(X^A)) \\ \Leftrightarrow u(\bar{d}_1 - Z(G^B(b) - X^A)) &> u(\bar{d}_1 - Z(X^A)) \\ \Leftrightarrow V_{bo,n}^B > V_{bo,n}^A. \end{aligned}$$

Again, as welfare in the two countries in the sequential game is equal, this implies that Country B enjoys a larger and strictly positive net gain. Country A can gain or loose due to the presence of a central authority.

(iii)  $a^* = (n, bo)$  and  $a' = (bo, n)$ . The proof on the welfare ordering under  $a' = (bo, n)$  from (ii) remains valid. Therefore, welfare in Country B is strictly larger than welfare in Country A. In this case though, no clear statement can be made on who enjoys higher net gains relative to the sequential game, as  $G^A$  might be larger than  $G^B$ , and therefore it is possible that  $X^B > G^B$ . Thus, each country might gain or loose due to the presence

of a central authority with fiscal power.

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