Department of Economics

Three Essays on Household Consumption

Judith Ay

Thesis submitted for assessment with a view to obtaining the degree of
Doctor of Economics of the European University Institute

Florence, April 2008

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Prof. Salvador Ortigueira, EUI, supervisor
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Contents

Part I

Introduction

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Introduction

The central topic of this thesis is household consumption. The first two papers address consumption in a life-cycle framework, with particular focus on the effect of time-inconsistent preferences if households consume perishable goods, but also accumulate durable goods. The third paper investigates the sensitivity of total household consumption to family size, which has implications for life-cycle modelling, as well as for the analysis of welfare and inequality.
The common motivation behind the different chapters is to show that the driving forces behind household consumption are very complex, but not contradictory to the life-cycle hypothesis as formulated by Friedman in 1957. The original formulation simply states that households seek to smooth the marginal utility of consumption over time, which initially was interpreted as a prediction that consumption profiles themselves should be flat.
Indeed, one of the particular criticisms towards the life-cycle model has been, that flat life-cycle consumption profiles were never observed empirically. Instead data clearly show that household consumption is low when households are young, subsequently rises and reaches its peak during mid age, and then falls significantly as households approach retirement age.
However, the predictions of the life-cycle model are driven by some decisive features. Firstly, it is important which assumptions are made on the income process and on income uncertainty. While older life-cycle papers, for technical reasons, had to draw on assuming perfect foresight or certainty equivalence, now these assumptions are no longer needed. By now a variety of different income processed can be modelled, from more traditional Monte Carlo simulations (Laibson, 2001) to Bayesian Learning (Guvenen, 2005), all having different implications on the household's ability - and willingness - to smooth consumption.
Secondly, the marginal utility of consumption is not only determined by the level of consumption. Unless one assumes that consumption and leisure are additively separable, or that labor supply is exogenous, the marginal utility of consumption is determined by the choice of labor effort and vice versa.
Thirdly, consumption possibilities are also limited by liquidity, and especially by borrowing constraints. In addition, households consume not only perishable goods but also invest in housing and other durable goods which yield a stream of services. While durable goods are depreciating with time, even in a life-cycle context it is evident that they can be carried across periods. This means that in each period utility does not only depend on current consumption, but also on past investments in the durables stock, which, depending on the relationship between durable goods

and perishables, can significantly affect the predictions of the model, and also the household's sensitivity to income risk. The most prominent example for a durable good is housing, which not only provides shelter but also can serve as collateral for loans and alleviates liquidity and borrowing constraints, which may change the pattern of effective precautionary savings in the face of income uncertainty. For these reasons, as Fernández-Villaverde and Krüger (2005) show, including a durable good into the life-cycle model significantly contributes to replicating the hump-shaped pattern of expenditure observed in real data.

Fourthly, household size changes over the life-cycle. This may affect not only the amount of goods demanded by the household, but also household preferences over time.

Furthermore, time preferences play an important role in determining how the future is weighted against the present, and demographics may have significant impact not only on the level of consumption needed to reach a given level of utility, but also on intertemporal substitution.

For reasons of tractability, and given computational constraints, all these features cannot be included at the same time into any theoretical life-cycle framework. Not only that, they also are difficult to capture by empirical estimation. However, the richness of the household problem has to be kept in mind when judging the ability of the life-cycle model to replicate observed consumption profiles.

Given the nature of the household problem, there also is reason and room for including elements from behavioral economics into life-cycle analysis. Behavioral economics investigates alternatives to standard formulations of individuals' preferences, and can provide explanations for cases where observed behavior is not reconcilable with standard economic theory based on the assumptions of consistency and rationality. Behavioral impetus often comes from real-life observations of individual behavior, like loss aversion shaping risk taking behavior, or like (costly) commitment devices being used to guarantee adherence to plans for the future. Indeed, in the life-cycle context, difficulties to adhere to a retirement savings plan can be a valid assumption. In modelling terms this phenomenon can be captured by taking up Phelbs' and Pollak's formulation of present-biased preferences and introducing this feature into the life-cycle model as first done with surprising results by Laibson: A present-bias leads to time-inconsistency in households' preferences, and manifests itself in a self-control problem, which in turn affects their savings behavior. If households are aware of their self-control problem, they may have the incentive to look for commitment tools, which changes their preferences over different types of assets. For example, in Laibson's specification time-inconsistent households find a solution for their self-control problem in saving for retirement in form of an illiquid asset, which is preferred over standard life-cycle savings in liquid assets.

The second part of this thesis centers around the effect of children on household welfare. While children are doubtlessly a very important factor in a household's life, household size is usually not taken into consideration in macroeconomic research on consumption. This is perfectly logical for most areas of macroeconomic analysis, but can have repercussions on life-cycle modelling and, more importantly, on the effectiveness of welfare and inequality analysis. In the former, family size can be approximated by rescaling consumption by equivalence scales, where the evolution of family size is deterministic, so that children do not contribute to inequality between households, but only induce a scale effect on consumption. In inequality and welfare analysis, household data on consumption are made comparable across different household sizes by using equivalence scales in order to establish the welfare of a 'representative' household member. However, this means that additional important information useful for inter-household comparison of welfare is quasi 'out of sight'. In the context of observed growing income inequality demographic factors may well play a more important role than generally recognized in academic literature.

The three different chapters can be summarized accordingly. **Chapter 1** constructs a very simple version of the life-cycle model, in which the behavior of households with standard and time-inconsistent time-preferences is compared. Given that no income uncertainty is assumed, all saving will be for retirement, and there will not be any precautionary savings motive. Saving is possible only in form of a standard liquid asset. There are no borrowing constraints, with the exception to the No-Ponzi condition at the very end of household life. It is found that in this setting, assuming that long-run discount rates for both types of households are the same, present-bias and self-control problem merely lead to a delay in retirement savings, but not to lower savings at the point of retirement. As a second step a durable good is introduced into the model, where it is assumed that the durable and the perishable good are complements. Deriving the Hyperbolic Euler Equation for the durable good, I find that the perishable and the durable good are affected differently by the self-control problem. As the Euler Equations and the simulation results show, a durable good reduces the overall effect of the self-control problem on household decision making. It not only serves as an asset, but its intertemporal properties smooth utility over time and reduce the impact of short-run impulses. In addition, they make consumption decisions less irreversible, so that excess spending on the durable good can be turned into savings in the liquid asset, so that the self-control problem is less of an discouragement for saving.

Chapter 2 takes up and extends the idea of the first chapter. Again households consume a perishable good, accumulate the durable good and save in form of a liquid financial asset. The goal now is to achieve some power of prediction on the impact of the self-control problem and to learn, whether the model yields trends strong enough to imply that time-inconsistency could be visible in household data. Consequently the model is enriched by income and life-horizon uncertainty, a bequest motive, changes in demographics, and by adjustment costs to the durables stock. Also now there is a borrowing constraints and durables serve as collateral and in part can be financed. The simulation results clearly show that, since the durable good serves as an asset and also reduces the consumption response to changes in income and family size, the self-control problem hardly affects savings and consumption patterns. The only notable difference between the two types is greater volatility in the durables stock and in savings of time-inconsistent households. Conditional on the assumption that long-run patience is greater for hyperbolic households, the simulation results suggest that households with time-inconsistent preferences are not easily identifiable from households with standard preferences. While undoubtedly in this life-cycle framework, time-inconsistent preferences help address plausible aspects of life-cycle behavior, they also highlight more closely the role durable goods play in shaping household consumption patterns. Even more importantly, it is found that the predictions of a realistic life-cycle model are robust to reasonable departures from exponential discounting.

Chapter 3 is an empirical paper with focus on the effect of children on household expenditure from a macroeconomic perspective. While household expenditure is widely used as the chief indicator for economic welfare, there remains a lack of transparency on how expenditure translates into actual consumption, and how consumption is allocated within households. Nevertheless, if expenditure does not react to an increase in household size, then household welfare of all household members can be expected to decline. In order to learn more about the comparability of the sensitivity of expenditure to family size, regressions are run for different income groups on the basis of CEx Data. The estimates on the effect of children show that the effect of children on household expenditure significantly differs for households with different income profiles. A propensity score matching exercise strongly supports the result that for households with a college degree, and, also to some extent for High School graduates, children substantially affect the level of total household expenditure, as was to be expected from economic theory. However, for households without High School degree, children do not seem to affect total expenditure. This implies strong liquidity constraints for low income households, and, more generally, the absence of savings, which not only results in great vulnerability of low income households to adverse income shocks, but also raises significant welfare concerns. Regarding welfare analysis, this result indicates that in studies on welfare inequality the use of equivalence scales contributes to additional measured inequality.

Part II

Chapters

Chapter 1

Self-Control and Durable Goods – A Simple Study on Life-Cycle-Savings

Abstract
The aim of this study is to investigate whether the impact of time-inconsistency of preferences on life-cycle savings changes when durable goods are included in the life-cycle model. Deriving the Hyperbolic Euler Equation for the durable good, I find that the perishable and the durable good are affected differently by the self-control problem. Furthermore, given its properties as an asset, including a durable good into the model quasi eliminates the impact of the self-control problem on life-cycle savings. This result builds on the assumption that the two goods are inseparable in the utility function. Since the focus is on the life-cycle aspect of saving, there is no uncertainty in the model.

JEL classification: D11, D91, E21
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1.1 Introduction

In the life-cycle literature it is a common short cut to model utility in terms of consumption of a perishable 'consumption' good. Although allowing for greater tractability, this assumption neglects that households also derive utility from the services provided by accumulated durable goods. In this paper I show that in a very simple, deterministic life-cycle framework, modelling durables accumulation as well as perishables consumption not only changes the general dynamics of household behavior, as demonstrated in Fernández-Villaverde and Krüger (2005), but also changes the impact of time-inconsistency of preferences and the resulting self-control problem. Given the asset properties of the durable good, the Hyperbolic Euler Equations suggest that on overall the durable stock is affected less by the self-control problem than the perishable good. This result is reflected in the simulated savings and consumption profiles: When only perishable goods are taken into account, the effect of the self-control problem is to delay retirement savings; this effect disappears once durable goods are introduced into the model.

The notion of preferences being in fact time-inconsistent comes from the domain of behavioral economics and economic psychology. Assuming time-inconsistency not only poses an intellectual challenge to the standard fashion of time-discounting, it also - potentially - questions standard economic predictions of human behavior. Given the implications of this divide for the economic modelling of intertemporal decision making, it is important to assess when and under which conditions, theoretical predictions are determined by the stand taken on the issue of time-discounting.

In the life-cycle literature the phenomenon of time-inconsistency has been investigated in a series of papers where households can choose between saving in form of a liquid and an illiquid asset (Laibson *et al.*, 1998, 2001). In this framework, it is found that time-consistent and time-inconsistent households indeed behave differently, for the latter struggle with a self-control problem. Here the illiquid asset helps the time-inconsistent household commit to a savings plan, so that even in the face of income uncertainty and the resulting need for precautionary savings, the time-inconsistent household prefers illiquid to liquid assets.

While in Laibson *et al.* (1998, 2001), the mode of time-discounting has sizable implications for predicted life-cycle savings, this does not necessarily have to be the case in other specifications of the life-cycle model. As will be shown in this paper, the introduction of a liquid durable good quasi eliminates the predicted impact of the self-control problem on household behavior. From deriving the Hyperbolic Euler Equation for the durable good, it becomes clear that the self-control problem affects the durables decision in a more complex way than it affects the consumption of the perishable good. In the case of the perishable good, only the trade-off between current and future consumption is influenced by the self-control problem. For durable goods, in addition to this trade-off, the value of the durable stock as an asset is determined by the degree to which, due to short run impatience, future consumption is financed by sales of the durable good. These two forces counteract each other, so that the overall effect of the self-control problem on the durable decision is reduced. This finding is supported by a simulation exercise, where when durables are taken into account, the effect of the time-inconsistency nearly disappears. However, unlike in Laibson *et al.* (1998, 2001), this finding is not due to the introduction of a commitment device since the durable good considered here is perfectly liquid.

Section 2 offers a brief overview of the concept of time-inconsistent preferences and hyperbolic discounting. Section 3 will present the standard life-cycle model without durables as well as the simulation results, while in Section 4 durables are introduced into the model and the impact of the durable good on the self-control problem is analyzed. Section 5 concludes.

1.2 The concept of time-inconsistent preferences

In behavioral studies individuals often are observed to make plans for the future, but to change their mind when it comes to realizing them, even though external circumstances have remained unchanged. This is mainly due to a conflict between long term goals and the need for immediate gratification. Examples where this conflict becomes evident are the dismissal of plans on self-improvement or investments, that have an immediate cost but yield returns or benefits only with some delay. Often this pattern of behavior is known to individuals themselves, who then attempt to commit their future selves to following a savings or investment plan. On a day-to-day basis the search for commitment devices is reflected in the demand for long-term memberships in health clubs (DellaVigna and Malmendier, 2004), and even in the subscription prices for magazines with investment characteristics but low entertainment value (Oster and Scott, 2005). These observations are explained by time-inconsistency of preferences, as described in Rabin (1998), O'Donoghue and Rabin (1999), Loewenstein and Prelec (1992), and Della Vigna and Malmendier (2004). Indeed it seems that time-discounting, far from being constant over periods, varies strongly with respect to the time-span between the 'now' and the period under consideration. A far greater degree of impatience is observed between today and tomorrow than between 'next month' and 'next month and a day'. This is true for consumption behavior as well as for the allocation of work and leisure, and decisions regarding when to face cumbersome tasks. Hence, over time there is a change in relative preferences concerning the timing of events and determining intertemporal choice, so that, for example, unpleasant activities are often and repeatedly deferred. As O'Donoghue and Rabin (1999a) express it, preferences are "present biased [in the sense that] when considering trade-offs between two future moments, present biased preferences give stronger relative weight to the earlier moment as it gets closer". Furthermore there is a contrast between "long run patience and short run impatience" (Harris and Laibson, 2001).

Experimental data have allowed to measure the discount function, and there is a consensus in the literature that the true discount factor can be approximated by a hyperbolic function of the form $(1+\alpha\tau)^{\frac{\gamma}{\alpha}}$, with $\alpha, \gamma > 0$ and τ as the distance to the period to be discounted (Laibson *et al.*, 2001). The way in which time-inconsistent preferences are expressed in economics goes back to Strotz (1958), Pollak (1968), Phelps and Pollak (1968), who use an approximation to the above function, the quasi-hyperbolic discount factor, taking the form $\{1, \beta\delta, \beta\delta^2, \beta\delta^3, ...\}$. Figure 1.1 in the Appendix plots the two hyperbolic discount functions against the traditional exponential discount function. In these early studies quasi-hyperbolic discounting is introduced mainly in order to abstract from the rather strong assumption of perfect altruism *across* generations. However, in most later studies, including this one, it is used in order to capture time-inconsistency in the *intra*personal decision making process.

In short, the general idea behind time-inconsistent discounting is that the discount rate is determined not only by the time-span between the reference point and the event in question, but also by the distance between the the reference point and the present. This leads to relative short run impatience contrasting with long run patience, so that individuals exhibit greater patience when allocating consumption (or leisure) between two future dates, than when deciding whether to save (or work) today or tomorrow. In other words, from the point of view of today, the future is discounted extra just because it is the future, while 'within the future' this distinction does not matter. As the date for which a decision is to be made draws closer, the discount factor changes and additional weight is given to the present which can change the way the allocation problem is solved.

From this it follows that plans made at present *for* the future will most likely not be followed up *in* the future, and individuals have a self-control problem. The effect of the self-control problem depends on whether it is anticipated by the individual. Here assuming rationality is equivalent to the assumption that the time-inconsistency of her preferences is known to the individual, who takes it into account when making the allocation decision. Given rationality, the solution to the intratemporal allocation problem lies in an intrapersonal game played by the current self against all future selves. This also is the reason why in Laibson *et al.* (1998, 2001) illiquidity of an asset stimulates savings: By investing in an illiquid asset the current self de facto commits the future selves to a savings plan. In the literature rational individuals are commonly referred to as 'sophisticated'[1].

1.3 A simple life-cycle model with time-inconsistent preferences

1.3.1 Model set-up

The model used in this section is a very simple version of the standard life-cycle model, except for the introduction of nonstandard, time-inconsistent preferences of the type described above. Time is finite and discrete so that $t \in \{t_1, ..., T+N-1, T+N\}$, where T is the time spent working and N is the time spent in retirement. Labor supply is inelastic and income is exogenous. Since there is neither income nor life-horizon uncertainty, the setting is purely deterministic. Households can borrow against future income and there is no borrowing limit, except for the households not to die in debt at the end of period T+N. Since in this model households cannot leave any bequests and due to the absence of uncertainty, all saving will be motivated by life-cycle concerns.
Each period households are able to invest in an interest bearing financial asset a by saving. Apart from investing in assets households consume a perishable good c, yielding instantaneous utility

$$u(c_t) = \frac{c_t^{1-\sigma} - 1}{1-\sigma} \tag{1.1}$$

The continuation profit of the hyperbolic household at time t equals

$$CP_t = \beta \sum_{i=1}^{N+T-t} \delta^i u(c_{t+i}) \tag{1.2}$$

where β represents the additional discounting of the future due to short run impatience; δ is the standard discount factor and hence denotes long run impatience. Consequently at time t, the household chooses c_t as to maximize

$$U(c_t) = \frac{c_t^{1-\sigma} - 1}{1-\sigma} + \beta \sum_{i=1}^{N+T-t} \delta^i u(c_{t+i}) \tag{1.3}$$

subject to the budget constraint

$$c_t + a_{t+1} = Ra_t + y_t = x_t \tag{1.4}$$

[1]While rationality is a common assumption in most studies investigating long-term planning, many studies focussing on the behavioral aspects of time-inconsistency allow for individuals to be irrational. Here irrationality is identified with 'naivety', derived from the inability to recognize that short run impatience is an inherent part of time-discounting and will influence future decisions as well as current ones. Hence in each period the naive individual will firmly believe that from tomorrow onwards she will act as today she thinks she should. Each recurrent lapse into impatience is considered the last one.

In the budget constraint x_t summarizes cash-on-hand held by the household at time t; the price of the consumption good has been normalized to one. Households receive income Y_t for $t \in \{t_0 \leq t \leq T\}$, where income is an increasing concave function of age; while for $t \in \{T+1 \leq t \leq T+N\}$, $Y_t = \kappa \times Y_T$. Initial asset holdings are zero.

1.3.2 Equilibrium conditions

Following Harris and Laibson (2001), the value functions are derived and the resulting policy functions will be the equilibrium strategy of an intrapersonal Markov game, as explained in Section 2.
Due to hyperbolic discounting self t conceptually distinguishes between the *current* value of cash-on-hand in period t, $W_{t,t}(x_t)$ and the *continuation* value $V_{t,t+1}(x_{t+1})$ in period t of cash on hand in period $t+1$.

$$W_{t,t}(x_t) = \max_{c_t}\{u(c_t) + \beta\delta V_{t,t+1}(x_{t+1})\} \tag{1.5}$$

$$V_{t,t+1}(x_{t+1}) = \max_{c_{t+1}}\{u(c_{t+1}) + \delta V_{t+1,t+2}(x_{t+2})\} \tag{1.6}$$

subject to the budget constraint (1.4). In the absence of bequests, it holds that for $t = N + T$

$$V_{T+N,T+N+1}(x_{T+N+1}) = 0$$

$$W_{T+N,T+N}(x_{T+N}) = \max_{c_{T+N}}\{u(c_{T+N})\}$$

Hyperbolic discounting implies that self t attaches a different value to next period cash-on-hand than self $t+1$ does. In the absence of a perfect commitment technology this inconsistency will give rise to a self-control problem, since self t cannot control self $t+1's$ savings decisions. Since households are assumed to be rational, or 'sophisticated', they foresee by how far future selves will diverge from any current savings plan. Hence self t makes her own decisions based on her expectation on the influence of the future selves' impatience on future savings. The equilibrium strategies played in this game are a sequence of policy functions $\{c_t^*\}_1^{T+N}$ that solve for $W_{t,t}(x_t)$ and $V_{t,t+1}(x_{t+1})$, subject to the conditions imposed by sophistication.
By construction, the sophisticated self t is thus fully aware of the time-inconsistency of her preferences but cannot control the consumption and savings decisions of self $t+1$. Nevertheless self t can foresee the level of consumption chosen by self $t+1$ given the state variable x_{t+1} in period $t+1$.
Hence self t anticipates

$$W_{t+1,t+1}(x_{t+1}) = u(c_{t+1}^*) + \beta\delta V_{t+1,t+2}(x_{t+2}^*)$$

From this it follows that

$$\delta V_{t+1,t+2}(x_{t+2}^*) = -\frac{1}{\beta}u(c_{t+1}^*) + \frac{1}{\beta}W_{t+1,t+1}(x_{t+1})$$

Substituting this expression into (1.6) yields

$$V_{t,t+1}(x_{t+1}, D_t) = \left(1 - \frac{1}{\beta}\right) u\left(c_{t+1}^*, D_{t+1}^*\right) + \frac{1}{\beta}W_{t+1,t+1}(x_{t+1}, D_t)$$

Clearly, unless $\beta = 1$, in which case preferences are time-consistent and one returns to the standard case,

$$W_{t+1,t+1}(x_{t+1}) \neq V_{t,t+1}(x_{t+1})$$

In short at any time t, a sophisticated household foresees that the present bias will repeat itself in the next period. This will be taken into account when making the savings and consumption decisions at time t.

Proposition 1 *The Hyperbolic Euler Equation (HEE) for the consumption good equals:*

$$\frac{\partial u(c_t^*(x_t))}{\partial c_t} = R\frac{\partial u(c_{t+1}^*(x_{t+1}))}{\partial c_{t+1}}\left[\beta\delta\frac{\partial c_{t+1}^*(x_{t+1})}{\partial x_{t+1}} + \delta\left(1 - \frac{\partial c_{t+1}^*(x_{t+1})}{\partial x_{t+1}}\right)\right] \tag{1.7}$$

The proof for Proposition 1 can be found in Section B in the Appendix.

Equation (1.7) is the Hyperbolic Euler Equation derived by Harris and Laibson (2001), and applied to the risk-free case. The HEE is very similar to the standard Euler Equation, the only difference being that the term in squared brackets has replaced the usual exponential discount factor. Here, the effective discount factor consists of the discount factor at period t, $\beta\delta$, weighted by the marginal propensity in period $t+1$ to consume out of period t savings, plus the long run discount factor δ, weighted by the marginal propensity to save in period $t+1$. The more of period t savings will be consumed by the next self, the smaller is the effective discount factor, and hence the greater is period t consumption. The more heavily self t expects self $t+1$ to consume out of her own savings, the less she is willing to save. As long as self t can only foresee but not control self $t+1$'s consumption decisions, she will react to future impatience by giving greater weight to her own need for immediate gratification. Hence for the same δ, the effective hyperbolic discount factor will always be smaller than the exponential discount factor unless $\frac{\partial c_{t+1}^*(x_{t+1})}{\partial x_{t+1}} \leq 0$.
Obviously, setting $\beta = 1$ one returns to the standard Euler Equation with standard exponential discounting, where preferences are time-consistent and the self-control problem does not arise.

1.3.3 Calibration

The values used for the discount factors and risk aversion come from the calibration results in Laibson *et al.* (2001). For simplification households lend and borrow at the same interest rate; the value for the fraction of income received at retirement, κ, is hypothetical, although intuitively sensible. The other values lie within the range commonly assumed in the literature.

Table 1.1: Calibrated parameters

β	δ	σ	R	κ
0.7	0.95	2	1.04	0.4

Further it should be noted, that in order to isolate the effect of the time-inconsistency, in the following the outcome for households with time-inconsistent preferences will be compared with the counterfactual outcome for households with time-consistent preferences. Although in the literature it is the norm to assume that for time-inconsistent individuals, long run patience is greater than in the standard time-consistent case, in order to allow for better comparison between the two cases, I assume the same long run discount factor for both types of households.

1.3.4 Results

Figure 1.2 in the Appendix compares savings of households with time-inconsistent preferences (hyperbolics) and with time-consistent preferences (exponentials). Both households spend most of their working life in debt, and only accumulate retirement savings during the last decade before retirement.
Although it is a common finding that young households borrow against future income, here the size of their debt is unusually large. This has two reasons. Firstly no borrowing constraint, with the exception of the No Ponzi condition, has been imposed. Secondly, due to the absence of uncertainty in this model, there is no motivation for precautionary savings.
From Figure 1.2 two things are evident. The effect of the time-inconsistency on savings at retirement is very small. However, the self-control problem has a clear impact on the timing of retirement savings. While for very young households, time-inconsistency increases debt only marginally, it does delay debt repayment and saving for retirement. Returning to the Hyperbolic Euler Equation (1.7), this is not surprising and confirms that the savings decision today is determined by the decision on whether to save in the future. Given current impatience, the self-control problem prevents early debt repayment as long as future selves are predicted to diverge from the savings plan. Or, in other words, future impatience drives current debt.
During retirement, both types of households behave very similarly. Surprisingly, hyperbolic households slightly delay dissaving in order to forestall against the consequences of impatience in the very last years of household life. Although this result is counter-intuitive at first, it is explained by the Inada conditions and the very high marginal utility at low levels of consumption. For newly retired households, the fear of very low consumption levels at the very end of life overweighs their own short run impatience, so that the propensity to consume becomes negative. This leads to the surprising result that during most of retirement the time-inconsistent household acts more patiently than the time-consistent household.
To summarize, the self-control problem mainly affects the *timing*, but not the *amount* of saving for retirement. This is equivalent to less consumption smoothing for time-inconsistent households.

1.4 A life-cycle model with time-inconsistent preferences and durable goods

1.4.1 Model set-up

Now a durable good is added to the model presented in the previous section. Since now households derive utility from the services provided by the durable good, the instantaneous utility function is equal to:

$$u\left(c_t, D_t\right) = \frac{g\left(c_t, D_t\right)^{1-\sigma} - 1}{1-\sigma} \tag{1.8}$$

where $g\left(c_t, D_t\right)$ is assumed to be of the CES type, so that

$$g\left(c_t, D_t\right) = \left[\alpha c_t^{\tau} + (1-\alpha) D_t^{\tau}\right]^{\frac{1}{\tau}} \tag{1.9}$$

The elasticity of substitution τ between durable and perishable goods determines whether the two goods are inseparable in the utility function. When $\tau < 1$, the marginal utility of perishables consumption

depends on the durables stock and vice versa.
Durable goods depreciate at rate γ and in each period households can sell what is left of the previous period's durables stock and use the returns to purchase new durable and perishable goods. Further, in order to make durables completely liquid, there are no adjustment costs to changes in the durables stock.
The continuation payoff of the hyperbolic household at time t equals

$$CP_t = \beta \sum_{i=1}^{N+T-t} \delta^i u(c_{t+i}, D_{t+i})$$

Again β signifies the additional discounting of the future due to short run impatience. Consequently, a time t the household chooses c_t and D_t as to maximize

$$U\left(c_t, D_t\right) = \frac{g\left(c_t, D_t\right)^{1-\sigma} - 1}{1-\sigma} + \beta \sum_{i=1}^{N+T-t} \delta^i u(c_{t+i}, D_{t+i}) \tag{1.10}$$

subject to the budget constraint

$$c_t + D_t + a_{t+1} = x_t + (1-\gamma) D_{t-1} \tag{1.11}$$

where

$$x_t = Ra_t + y_t$$

such that

$$x_{t+1} = R\left(x_t + c_t - D_t + (1-\gamma)\, D_{t-1}\right) + y_{t+1} \tag{1.12}$$

In the budget constraint (1.11), x_t summarizes cash-on-hand held by the household at time t; the price of durables and perishables has been normalized to one. This also implies that households buy and sell durables at the same price.

1.4.2 Equilibrium conditions

Again following Harris and Laibson (2001) as a guideline, the two value functions for the hyperbolic household are derived. The resulting policy functions will be the equilibrium strategies of an intertemporal Markov game. Due to hyperbolic discounting, self t conceptually distinguishes between the *current* value in period t, $W_{t,t}(x_t, D_{t-1})$ of cash-on-hand and the preexisting durables stock, and the *continuation* value, $V_{t,t+1}(x_{t+1}, D_t)$ of cash-on-hand and durables carried over into period $t+1$. Hence the equilibrium is defined by two value functions

$$W_{t,t}(x_t, D_{t-1}) = \max_{c_t, D_t} \{u(c_t, D_t) + \beta\delta V_{t,t+1}(x_{t+1}, D_t)\} \tag{1.13}$$

$$V_{t,t+1}(x_{t+1}, D_t) = \max_{c_{t+1}, D_{t+1}} \{u(c_{t+1}, D_{t+1}) + \delta V_{t+1,t+2}(x_{t+2}, D_{t+1})\} \tag{1.14}$$

subject to the budget constraint (1.11).
In the absence of bequests it holds that for $t = T + N$

$$V_{T+N,T+N+1}(x_{T+N+1}, D_{T+N}) = 0$$

and hence

$$W_{T+N,T+N}(x_{T+N}, D_{T+N-1}) = \max_{c_{T+N}, D_{T+N}} \{u(c_{T+N}, D_{T+N})\}$$

The solution to the household problem is a sequence of policy functions $\{c_t^*, D_t^*\}_1^{T+N}$ solving for $W_{t,t}(x_t, D_{t-1})$ and $V_{t,t+1}(x_{t+1}, D_t)$, plus the conditions imposed by sophisticated households. Again sophistication implies that at time t, self t knows that in period $t+1$, self $t+1$ will maximize

$$W_{t+1,t+1}(x_{t+1}, D_t) = \max_{c_{t+1}, D_{t+1}} \{u(c_{t+1}, D_{t+1}) + \beta\delta V_{t+1,t+2}(x_{t+2}, D_{t+1})\}$$

subject to

$$c_{t+1} + a_{t+2} + D_{t+1} = x_{t+1} + (1-\gamma)D_t$$

where x_{t+1} is given by (1.12). Self t is thus fully aware of the time-inconsistency of preferences, but cannot control the consumption and savings decision of self $t+1$. Nevertheless self t can foresee the perishables consumption and durables stock chosen by self $t+1$, given the states in period $t+1$, $x_{t=1}$ and D_t. Hence self t anticipates that

$$W_{t+1,t+1}(x_{t+1}, D_t) = u(c_{t+1}^*, D_{t+1}^*) + \beta\delta V_{t+1,t+2}(x_{t+2}^*, D_{t+1}^*)$$

From this it follows that

$$\delta V_{t+1,t+2}(x_{t+2}^*, D_{t+1}^*) = -\frac{1}{\beta} u(c_{t+1}^*, D_{t+1}^*) + \frac{1}{\beta} W_{t+1,t+1}(x_{t+1}, D_t)$$

Substituting this expression into (2.8) yields

$$V_{t,t+1}(x_{t+1}, D_t) = \left(1 - \frac{1}{\beta}\right) u(c_{t+1}^*, D_{t+1}^*) + \frac{1}{\beta} W_{t+1,t+1}(x_{t+1}, D_t)$$

Clearly, unless $\beta = 1$, in which case one returns to the standard case with time-consistent preferences,

$$W_{t+1,t+1}(x_{t+1}, D_t) \neq V_{t,t+1}(x_{t+1}, D_t)$$

Proposition 2 *In the presence of a durable good the Hyperbolic Euler Equation for the perishable good (HEEP) is as follows, where $c_t^* = c_t^*(x_t, D_{t-1})$ and $D_t^* = D_t^*(x_t, D_{t-1})$ are the policy functions for the perishable and the durable good.*

$$\begin{aligned} \frac{\partial u(c_t^*, D_t^*)}{\partial c_t} &= \delta R \frac{\partial u(c_{t+1}^*, D_{t+1}^*)}{\partial c_{t+1}} \\ &- (1-\beta)\delta R \left[\frac{\partial u(c_{t+1}^*, D_{t+1}^*)}{\partial c_{t+1}} \frac{\partial c_{t+1}^*}{\partial x_{t+1}} + \frac{\partial u(c_{t+1}^*, D_{t+1}^*)}{\partial D_{t+1}} \frac{\partial D_{t+1}^*}{\partial x_{t+1}} \right] \end{aligned} \quad (1.15)$$

The Proof for Proposition 2 is found in the Appendix, Section C.

The first term on the RHS of the Hyperbolic Euler Equation for the perishable good (1.15) is consistent with the standard Euler Equation in the time-consistent case. The term in squared brackets shows, that, when preferences are time-inconsistent, the marginal propensity to spend current savings on perishables and durables in the next period, decreases the marginal cost of current consumption. Consequently the sophisticated, present-biased hyperbolic household will consume more of the perishable good in period t when the future marginal propensity to consume out of her savings is positive. This is a direct consequence of the self-control problem, given that households are sophisticated and anticipate the influence of the present-bias on future consumption and savings decisions, but cannot control their future behavior.

Proposition 3 *The Hyperbolic Euler Equation for the durable good (HEED) is as follows with* $c_t^* = c_t^*(x_t, D_{t-1})$ *and* $D_t^* = D_t^*(x_t, D_{t-1})$ *as the policy functions for the perishable and the durable good.*

$$\begin{aligned}\frac{\partial u(c_t^*, D_t^*)}{\partial D_t} &= \delta(R-1+\gamma)\frac{\partial u(c_{t+1}^*, D_{t+1}^*)}{\partial c_{t+1}} \\ &+ (1-\beta)\delta\frac{\partial u(c_{t+1}^*, D_{t+1}^*)}{\partial c_{t+1}}\left[\frac{\partial c_{t+1}^*}{\partial D_t} - R\frac{\partial c_{t+1}^*}{\partial x_{t+1}}\right] \\ &+ (1-\beta)\delta\frac{\partial u(c_{t+1}^*, D_{t+1}^*)}{\partial D_{t+1}}\left[\frac{\partial D_{t+1}^*}{\partial D_t} - R\frac{\partial D_{t+1}^*}{\partial x_{t+1}}\right] \end{aligned} \quad (1.16)$$

The proof for Proposition 3 can be found in Section C in the Appendix.

The first term on the RHS of the Hyperbolic Euler Equation for the durable good (1.16) is identical to the RHS of the standard Euler Equation for durables when preferences are time-consistent. Note, that the durable good functions as an asset, where $(R-1+\gamma)$ is the return foregone by not having invested in the financial asset.

From the terms in squared brackets it is apparent that the self-control problem affects the durables decision in two offsetting ways. Firstly, as in the previous case, the marginal propensity to consume out of savings in period $t+1$, reduces the marginal cost of durables purchases in terms of foregone savings, and hence increases spending on durables in period t. Secondly the durable good not only yields utility directly, but also serves as an asset. Consequently, the greater is the marginal propensity to consume out of the sales of durables in period $t+1$, the lower is the value of holding the durable good as an asset. This is amplified by the durable good being return dominated by the financial asset, and reduces investment in the durables stock in period t. The two effects of the self-control problem on the durables decision are of opposite sign and hence offset each other.

The overall impact of the time-inconsistency on household behavior is determined by the relationship between durables and perishables in the utility function. When the two goods are inseparable in the utility function, the marginal utility of the perishable good depends on the durables stock. If the overall effect of the self-control problem is lower for the durable good, this can 'feed through' to the consumption decision for the perishable good. Hence, in a model with a durable good, the behavior of households with time-inconsistent preferences should be more similar to the standard time-consistent case than when durables are not taken into account.

1.4.3 Calibration

The calibration values for the old parameters remain unchanged. Regarding the parameters concerning the durable good, I follow Fernández-Villaverde and Krüger (2005) in assuming zero substitutability between perishables and durables, drawing on past estimates on the elasticity of substitution in the utility function (Ogaki and Reinhart (1998), and Rupert *et al.* (1995)), so that perishables and durables are complements and (1.9) takes the Cobb-Douglas form. The rate of durables depreciation, γ, and the weight of the perishable good in the utility function, α, are hypothetical but within the range commonly assumed.

Table 1.2: Calibrated parameters

β	δ	σ	α	R	γ	κ
0.7	0.95	2	0.7	1.04	0.1	0.4

Again, in order to isolate the effect of the time-inconsistency, in the following the outcome for households with time-inconsistent preferences will be compared to the counterfactual outcome for time-consistent households. In order to allow for better comparison between the two types, I assume the same long-run discount factor for both kinds of households.

1.4.4 Results for the model with durables

As Figures (1.3) and (1.4) in the Appendix show, now the savings profiles and the durables stock for hyperbolic and exponential households are very similar. Compared to the simple model in section 3, both types of households start at a higher level of debt and reach their maximum amount of debt earlier. The reason is that at the beginning of household life, households invest heavily in the durables stock, which then remains largely unchanged with only minor readjustments due to depreciation. While this phenomenon is commonly observed in the literature (Fernández-Villaverde and Krüger, 2005), here it is reinforced by the deterministic income process and the resulting lack of a precautionary motive for savings or debt-restraint. Retirement savings are now somewhat lower while households consume their savings faster and run into debt again at the end of their lives. The end-of-life debt is paid off by the sale of the remaining durables in the last period before death, so that the No Ponzi condition remains satisfied.

In this version of the life-cycle model the time-inconsistency no longer delays retirement savings. Now hyperbolics and exponentials only differ in that savings and durables stock of hyperbolic households are rather volatile. While exponential households initially invest heavily in the durables stock and then maintain it at its level, hyperbolic households alternate between buying and selling durables. The reason for this volatility is that the hyperbolic household, due to her short-run impatience, consumes too much of the perishable good, and, since the two goods are complements, overinvests in the durables stock. Subsequently, however, the household will need to save for retirement. Since the two goods are complements, instead of solely reducing consumption of the perishable good, she will also sell a portion of her durables stock and reconvert them into savings. Depending on age, and on the two state variables, these two phases alternate.

Given that they are return dominated by savings, as a result of the short-run impatience durables are used as an asset excessively, which is anticipated by the sophisticated household. From the Hyperbolic Euler Equations (1.16) we see that the overall impact of the self-control problem is smaller for the durable good. At the same time, since the durable good is a tangible asset, with respect to life-cycle savings, over-investing in durables is not the same as overconsumption of perishables, since the durable stock can be reconverted into savings. For these reasons, in the presence of a durable good, future impatience does not discourage savings to the degree observed in the case when there are only perishables. As a result, the life-cycle profiles of hyperbolic and exponential households are very similar.

1.5 Conclusion

From the Hyperbolic Euler Equations as well as from the simulation results it has become clear that, at least in a deterministic setting, the introduction of a durable good essentially eliminates the impact of the self-control problem on life-cycle savings of hyperbolic households.

This is due to the liquid durable good allowing the hyperbolic household to recover excessive spending since durables can be reconverted into savings. Nevertheless, this strategy is inefficient, and, since the sophisticated household anticipates future sales of the durable good, the overall effect of the self-control problem is reduced. Here, the durable good is perfectly liquid, so that it does not function as a commitment device between current and future selves and the self-control problem is not solved.

1.6 Bibliography

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Appendix

1.A Figures

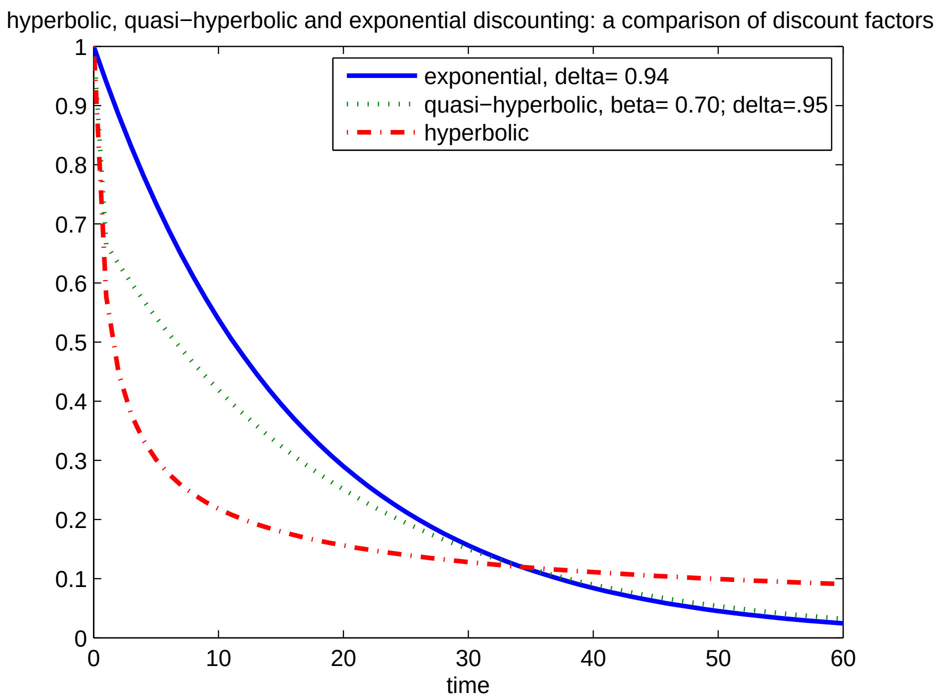

Figure 1.1: Comparison of Discount factors

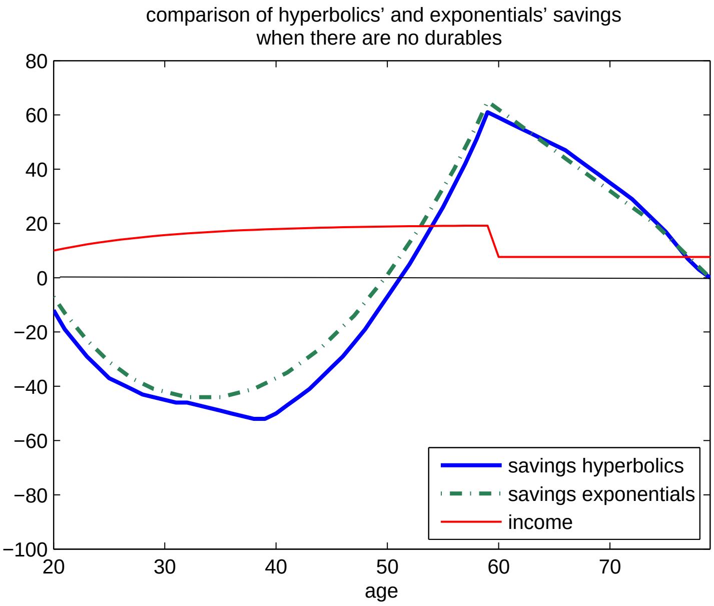

Figure 1.2: Comparison of hyperbolic and exponential life-cycle savings in the simple model

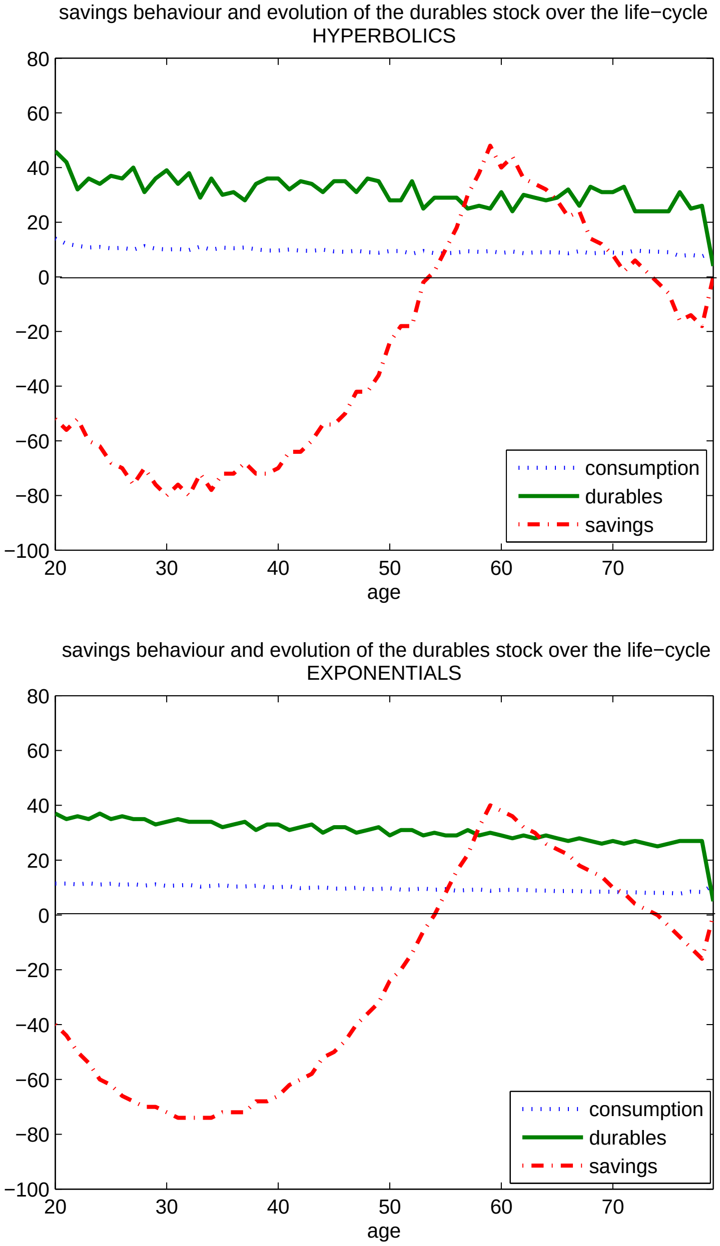

Figure 1.3: Comparison of hyperbolic and exponential life-cycle profiles in the the model with durables

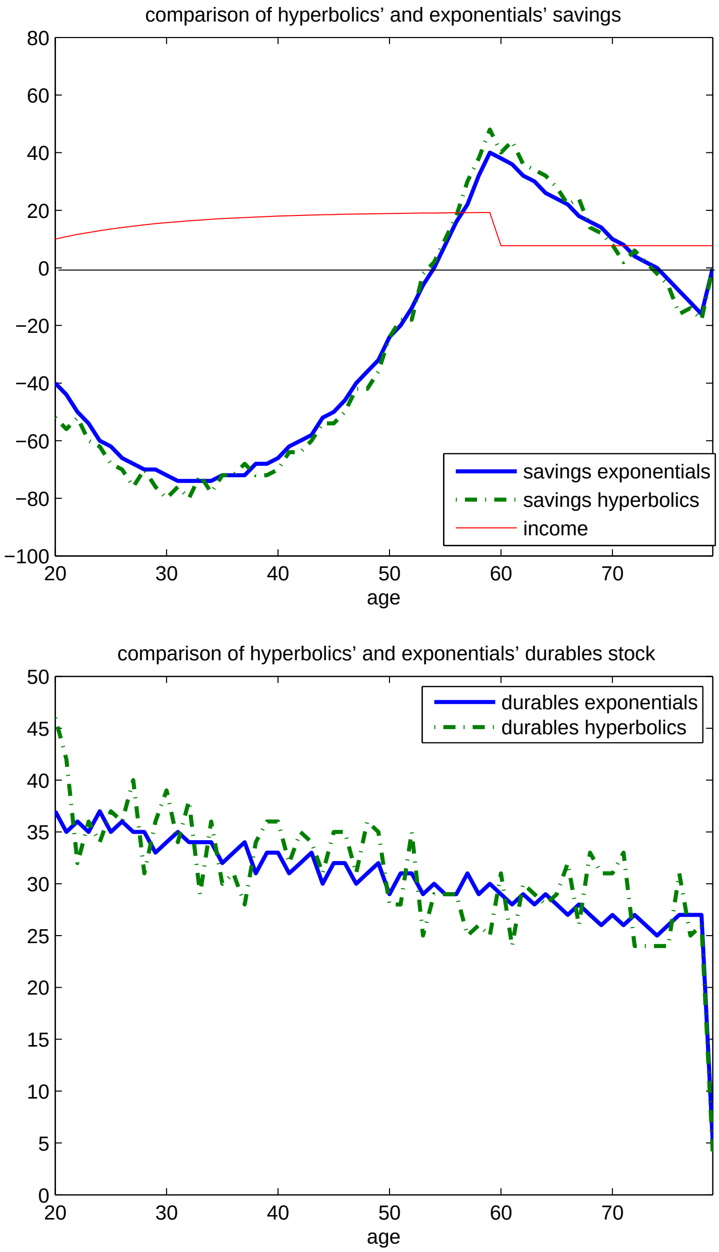

Figure 1.4: Comparison of hyperbolic and exponential life-cycle savings and durables stock

1.B The model without a durable good

Definition of the equilibrium

The equilibrium is defined by two value functions

$$W_{t,t}(x_t) = \max_{c_t} \{u(c_t) + \beta\delta V_{t,t+1}(x_{t+1})\} \tag{1.17}$$

$$V_{t,t+1}(x_{t+1}) = \max_{c_{t+1}} \{u(c_{t+1}) + \delta V_{t+1,t+2}(x_{t+2})\} \tag{1.18}$$

subject to the budget constraint

$$c_t + a_{t+1} = x_t$$

where

$$x_t = R \cdot a_t + y_t$$

and a sequence of policy functions $\{c_t^*\}_0^{T+N}$ that solve for $W_{t,t}(x_t)$ and $V_{t,t+1}(x_{t+1})$, plus the conditions imposed by sophisticated households. Sophistication implies that at time t self t knows that in period t+1, self t+1 will maximize

$$W_{t+1,t+1}(x_{t+1}) = \max_{c_{t+1}} \{u(c_{t+1}) + \beta\delta V_{t+1,t+2}(x_{t+2})\}$$

subject to

$$c_{t+1} + a_{t+2} = x_{t+1}$$

Self t is thus fully aware of the time-inconsistency of preferences but cannot control the consumption and savings decisions of self t+1. Nevertheless self t can foresee the level of consumption chosen by self t+1 given the state variable in period t+1, x_{t+1}. Hence self t anticipates that

$$W_{t+1,t+1}(x_{t+1}) = u\left(c_{t+1}^*\right) + \beta\delta V_{t+1,t+2}\left(x_{t+2}^*\right)$$

From this it follows that

$$\delta V_{t+1,t+2}\left(x_{t+2}^*\right) = -\frac{1}{\beta} u\left(c_{t+1}^*\right) + \frac{1}{\beta} W_{t+1,t+1}(x_{t+1})$$

Substituting this expression in (1.18) yields

$$V_{t,t+1}(x_{t+1}) = \left(1 - \frac{1}{\beta}\right) u\left(c_{t+1}^*\right) + \frac{1}{\beta} W_{t+1,t+1}(x_{t+1}) \tag{1.19}$$

Clearly, unless $\beta = 1$, in which case preferences are time consistent and one returns to the standard case,

$$W_{t+1,t+1}(x_{t+1}) \neq V_{t,t+1}(x_{t+1})$$

In short at any time t a sophisticated household foresees that the present bias will repeat itself in the next period. This will be taken into account when making the saving and consumption decisions at time t.

Proof of Proposition 1

Taking the derivative w.r.t. c_t of the objective function $W_{t,t}(x_t)$ of self t yields

$$\frac{\partial u(c_t^*)}{\partial c_t} + \beta\delta \frac{\partial V_{t,t+1}(x_{t+1}^*)}{\partial c_t} = 0$$

Since

$$\frac{\partial V_{t,t+1}(x_{t+1}^*)}{\partial c_t} = -R\frac{\partial V_{t,t+1}(x_{t+1}^*)}{\partial x_{t+1}}$$

it follows that

$$\frac{\partial u(c_t^*)}{\partial c_t} = \beta\delta R \frac{\partial V_{t,t+1}(x_{t+1}^*)}{\partial x_{t+1}} \tag{1.20}$$

Replacing $V_{t,t+1}(x_{t+1}^*)$ by expression (1.19) on arrives at

$$\frac{\partial u(c_t^*)}{\partial c_t} = \beta\delta R\left[\left(1-\frac{1}{\beta}\right)\frac{\partial u(c_{t+1}^*)}{\partial x_{t+1}} + \frac{1}{\beta}\frac{\partial W_{t+1,t+1}(x_{t+1}^*)}{\partial x_{t+1}}\right] \tag{1.21}$$

In order to derive $\frac{\partial W_{t+1,t+1}(x_{t+1})}{\partial x_{t+1}}$ I use the envelope condition for the state variable x_{t+1} in period t+1. In the following $u(c_{t+1}^*) = u_{t+1}^*$. Hence

$$\frac{\partial W_{t+1,t+1}(x_{t+1})}{\partial x_{t+1}} = \frac{\partial u_{t+1}^*}{\partial c_{t+1}}\frac{\partial c_{t+1}^*}{\partial x_{t+1}} + \beta\delta\frac{\partial V_{t+1,t+2}(x_{t+2}^*)}{\partial x_{t+2}}\left[\frac{\partial x_{t+2}^*}{\partial x_{t+1}} + \frac{\partial x_{t+2}^*}{\partial c_{t+1}}\frac{\partial c_{t+1}^*}{\partial x_{t+1}}\right]$$

Since $\frac{\partial x_{t+2}^*}{\partial c_{t+1}} = -R$, collecting the derivatives of the policy function yields

$$\begin{aligned}\frac{\partial W_{t+1,t+1}(x_{t+1})}{\partial x_{t+1}} &= \frac{\partial c_{t+1}^*}{\partial x_{t+1}}\left[\frac{\partial u_{t+1}^*}{\partial c_{t+1}} - \beta\delta R\frac{\partial V_{t+1,t+2}(x_{t+2}^*)}{\partial x_{t+2}}\right] \\ &\quad + \beta\delta R\frac{\partial V_{t+1,t+2}(x_{t+2}^*)}{\partial x_{t+2}}\end{aligned}$$

Using the FOC for c_t (1.20), one sees that the term in the squared bracket cancels out, so that the expression is reduced to

$$\frac{\partial W_{t+1,t+1}(x_{t+1})}{\partial x_{t+1}} = \beta\delta R\frac{\partial V_{t+1,t+2}(x_{t+2}^*)}{\partial x_{t+2}}$$

Using (1.20) again, one arrives the envelope condition

$$\frac{\partial W_{t+1,t+1}(x_{t+1})}{\partial x_{t+1}} = \left.\frac{\partial u(c_{t+1})}{\partial c_{t+1}}\right|_{c_{t+1}=c_{t+1}^*} \tag{1.22}$$

Substituting the envelope condition into (1.21) gives the Hyperbolic Euler Equation for the perishable good:

$$\frac{\partial u(c_t^*)}{\partial c_t} = R\frac{\partial u(c_{t+1}^*)}{\partial c_{t+1}}\left[\frac{\partial c_{t+1}^*}{\partial x_{t+1}}\beta\delta + \left(1-\frac{\partial c_{t+1}^*}{\partial x_{t+1}}\right)\delta\right] \tag{1.23}$$

Now the marginal propensity to spend current savings on next period consumption decreases the marginal cost of current consumption. This is due to the household being fully aware of the time-inconsistency of her preferences, so that, although the current self cannot control future consumption decisions, she takes them into account when making her own savings decision. Hence the *effective* discount factor is the term in squared brackets.

1.C The model with a durable good

Definition of the equilibrium

The equilibrium is defined by two value functions

$$W_{t,t}\left(x_t, D_{t-1}\right) = \max_{c_t, D_t}\left\{u\left(c_t, D_t\right) + \beta\delta V_{t,t+1}\left(x_{t+1}, D_t\right)\right\} \tag{1.24}$$

$$V_{t,t+1}\left(x_{t+1}, D_t\right) = \max_{c_{t+1}, D_{t+1}}\left\{u\left(c_{t+1}, D_{t+1}\right) + \delta V_{t+1,t+2}\left(x_{t+2}, D_{t+1}\right)\right\} \tag{1.25}$$

subject to the budget constraint

$$c_t + a_{t+1} + D_t = x_t + (1-\gamma)D_{t-1}$$

where the state variable x_t is given by past savings plus period t income so that

$$x_t = R \cdot a_t + y_t$$

and hence

$$x_{t+1} = R\left(x_t - c_t - D_t + (1-\gamma)D_{t-1}\right) + y_{t+1} \tag{1.26}$$

and a sequence of policy functions $\{c_t^*, D_t^*\}_0^{T+N}$ that solve for $W_{t,t}\left(x_t, D_{t-1}\right)$ and $V_{t,t+1}\left(x_{t+1}, D_t\right)$, plus the conditions imposed by sophisticated households. Sophistication implies that at time t self t knows that in period t+1, self t+1 will maximize

$$W_{t+1,t+1}\left(x_{t+1}, D_t\right) = \max_{c_{t+1}, D_{t+1}}\left\{u\left(c_{t+1}, D_{t+1}\right) + \beta\delta V_{t+1,t+2}\left(x_{t+2}, D_{t+1}\right)\right\}$$

subject to

$$c_{t+1} + a_{t+2} + D_{t+1} = x_{t+1} + (1-\gamma)D_t$$

where x_{t+1} is given by (1.26). Self t is thus fully aware of the time-inconsistency of preferences but cannot control the consumption and savings decisions of self t+1. Nevertheless self t can foresee the level of consumption and durables purchases chosen by self t+1 given the states in period t+1, x_{t+1} and D_t. Hence self t anticipates that

$$W_{t+1,t+1}\left(x_{t+1}, D_t\right) = u\left(c_{t+1}^*, D_{t+1}^*\right) + \beta\delta V_{t+1,t+2}\left(x_{t+2}^*, D_{t+1}^*\right)$$

From this it follows that

$$\delta V_{t+1,t+2}\left(x_{t+2}^*, D_{t+1}^*\right) = -\frac{1}{\beta}u\left(c_{t+1}^*, D_{t+1}^*\right) + \frac{1}{\beta}W_{t+1,t+1}\left(x_{t+1}, D_t\right)$$

Substituting this expression in (1.25) yields

$$V_{t,t+1}\left(x_{t+1}, D_t\right) = \left(1 - \frac{1}{\beta}\right)u\left(c_{t+1}^*, D_{t+1}^*\right) + \frac{1}{\beta}W_{t+1,t+1}\left(x_{t+1}, D_t\right) \tag{1.27}$$

Clearly, unless $\beta = 1$, in which case one returns to the standard case with time-consistent preferences,

$$W_{t+1,t+1}\left(x_{t+1}, D_t\right) \neq V_{t,t+1}\left(x_{t+1}, D_t\right)$$

In short at any time t a sophisticated household foresees that the present bias will repeat itself in the next period. This will be taken into account when making the saving and consumption decisions at time t.

First Order Conditions

The First Order Condition for the Perishable Good
Taking the derivative w.r.t. c_t of the objective function $W_{t,t}(x_t, D_{t-1})$ of self t yields

$$\frac{\partial u\left(c_t^*, D_t^*\right)}{\partial c_t} + \beta\delta \frac{\partial V_{t,t+1}\left(x_{t+1}^*, D_t^*\right)}{\partial c_t} = 0$$

Since

$$\frac{\partial V_{t,t+1}\left(x_{t+1}^*, D_t^*\right)}{\partial c_t} = -R\frac{\partial V_{t,t+1}\left(x_{t+1}^*, D_t^*\right)}{\partial x_{t+1}}$$

it follows that

$$\frac{\partial u\left(c_t^*, D_t^*\right)}{\partial c_t} = \beta\delta R\frac{\partial V_{t,t+1}\left(x_{t+1}^*, D_t^*\right)}{\partial x_{t+1}} \tag{1.28}$$

The First Order Condition for the Durable Good
Taking the derivative of $W_{t,t}(x_t, D_{t-1})$ of self t w.r.t D_t yields

$$\frac{\partial u\left(c_t^*, D_t^*\right)}{\partial D_t} + \beta\delta \frac{\partial V_{t,t+1}\left(x_{t+1}^*, D_t^*\right)}{\partial D_t} = 0$$

Since

$$\begin{aligned}\frac{\partial V_{t,t+1}\left(x_{t+1}^*, D_t^*\right)}{\partial D_t} &= \frac{\partial x_{t+1}^*}{\partial D_t}\frac{\partial V_{t,t+1}\left(x_{t+1}^*, D_t^*\right)}{\partial x_{t+1}} + \frac{\partial V_{t,t+1}\left(x_{t+1}^*, D_t^*\right)}{\partial D_t} \\ &= -R\frac{\partial V_{t,t+1}\left(x_{t+1}^*, D_t^*\right)}{\partial x_{t+1}} + \frac{\partial V_{t,t+1}\left(x_{t+1}^*, D_t^*\right)}{\partial D_t}\end{aligned}$$

the first order condition for the durable good equals

$$\frac{\partial u\left(c_t^*, D_t^*\right)}{\partial D_t} = \beta\delta R\frac{\partial V_{t,t+1}\left(x_{t+1}^*, D_t^*\right)}{\partial x_{t+1}} - \beta\delta\frac{\partial V_{t,t+1}\left(x_{t+1}^*, D_t^*\right)}{\partial D_t} \tag{1.29}$$

Proof of proposition 2

In the FOC for the perishable good (1.28), replacing $V_{t,t+1}\left(x_{t+1}^*, D_t^*\right)$ by expression (1.27) on arrives at

$$\begin{aligned}\frac{\partial u\left(c_t^*, D_t^*\right)}{\partial c_t} &= \beta\delta R\left[\left(1-\frac{1}{\beta}\right)\frac{\partial u\left(c_{t+1}^*, D_{t+1}^*\right)}{\partial x_{t+1}}\right. \\ &\quad \left. +\frac{1}{\beta}\frac{\partial W_{t+1,t+1}\left(x_{t+1}^*, D_t^*\right)}{\partial x_{t+1}}\right]\end{aligned} \tag{1.30}$$

In order to derive $\frac{\partial W_{t+1,t+1}(x_{t+1},D_t)}{\partial x_{t+1}}$, I use the envelope condition for the state variable x_{t+1} in period t+1. In the following $u\left(c^*_{t+1}, D^*_{t+1}\right) = u^*_{t+1}$, hence

$$\begin{aligned}\frac{\partial W_{t+1,t+1}\left(x_{t+1}, D_t\right)}{\partial x_{t+1}} &= \frac{\partial u^*_{t+1}}{\partial c_{t+1}}\frac{\partial c^*_{t+1}}{\partial x_{t+1}} + \frac{\partial u^*_{t+1}}{\partial D_{t+1}}\frac{\partial D^*_{t+1}}{\partial x_{t+1}} \\ &+\beta\delta\left[\frac{\partial V_{t+1,t+2}\left(x^*_{t+2}, D^*_{t+1}\right)}{\partial x_{t+2}}\left[\frac{\partial x^*_{t+2}}{\partial x_{t+1}}\right.\right. \\ &\left.+\frac{\partial x^*_{t+2}}{\partial c_{t+1}}\frac{\partial c^*_{t+1}}{\partial x_{t+1}} + \frac{\partial x^*_{t+2}}{\partial D_{t+1}}\frac{\partial D^*_{t+1}}{\partial x_{t+1}} + \frac{\partial x^*_{t+2}}{\partial D_t}\frac{\partial D_t}{\partial x_{t+1}}\right] \\ &\left.+\frac{\partial V_{t+1,t+2}\left(x^*_{t+2}, D^*_{t+1}\right)}{\partial D_{t+1}}\frac{\partial D^*_{t+1}}{\partial x_{t+1}}\right]\end{aligned}$$

Since $\frac{\partial x^*_{t+2}}{\partial c_{t+1}} = \frac{\partial x^*_{t+2}}{\partial D_{t+1}} = -R$, and $\frac{\partial D_t}{\partial x_{t+1}} = 0$, collecting the derivatives of the policy functions yields

$$\begin{aligned}\frac{\partial W_{t+1,t+1}\left(x_{t+1}, D_t\right)}{\partial x_{t+1}} &= \frac{\partial c^*_{t+1}}{\partial x_{t+1}}\left[\frac{\partial u^*_{t+1}}{\partial c_{t+1}} - \beta\delta R\frac{\partial V_{t+1,t+2}\left(x^*_{t+2}, D^*_{t+1}\right)}{\partial x_{t+2}}\right] \\ &+\frac{\partial D^*_{t+1}}{\partial x_{t+1}}\left[\frac{\partial u^*_{t+1}}{\partial D_{t+1}} - \beta\delta R\frac{\partial V_{t+1,t+2}\left(x^*_{t+2}, D^*_{t+1}\right)}{\partial x_{t+2}}\right. \\ &\left.+\beta\delta\frac{\partial V_{t+1,t+2}\left(x^*_{t+2}, D^*_{t+1}\right)}{\partial D_{t+1}}\right] \\ &+\beta\delta R\frac{\partial V_{t+1,t+2}\left(x^*_{t+2}, D^*_{t+1}\right)}{\partial x_{t+2}}\end{aligned}$$

Using the FOCs for c_t (1.28) and D_t (1.29), one sees that the terms in the squared brackets cancel out, so that the expression is reduced to

$$\frac{\partial W_{t+1,t+1}\left(x_{t+1}, D_t\right)}{\partial x_{t+1}} = \beta\delta R\frac{\partial V_{t+1,t+2}\left(x^*_{t+2}, D^*_{t+1}\right)}{\partial x_{t+2}}$$

Using the first order condition for consumption (1.28) one arrives at the envelope condition

$$\frac{\partial W_{t+1,t+1}\left(x_{t+1}, D_t\right)}{\partial x_{t+1}} = \left.\frac{\partial u\left(c_{t+1}, D_{t+1}\right)}{\partial c_{t+1}}\right|_{c_{t+1}=c^*_{t+1}, D_{t+1}=D^*_{t+1}}$$

Substituting the envelope condition in (1.30) gives the Hyperbolic Euler Equation (HEE) for the perishable good (1.31):

$$\begin{aligned}\frac{\partial u\left(c^*_t, D^*_t\right)}{\partial c_t} &= \delta R\frac{\partial u\left(c^*_{t+1}, D^*_{t+1}\right)}{\partial c_{t+1}} - (1-\beta)\,\delta R\left[\frac{\partial u\left(c^*_{t+1}, D^*_{t+1}\right)}{\partial c_{t+1}}\frac{\partial c^*_{t+1}}{\partial x_{t+1}}\right. \\ &\left.+\frac{\partial u\left(c^*_{t+1}, D^*_{t+1}\right)}{\partial D_{t+1}}\frac{\partial D^*_{t+1}}{\partial x_{t+1}}\right] \qquad (1.31)\end{aligned}$$

Now the marginal propensity to spend current savings on next period perishable goods and durables purchases decreases the marginal cost of current consumption. This is due to the household being fully aware of the time-inconsistency of her preferences, so that, although the current self cannot control future consumption decisions, she takes them into account when making her own savings decision.

Proof of proposition 3

In the first order condition for the durable good (1.29), replacing $V_{t,t+1}\left(x^*_{t+1}, D^*_t\right)$ by expression (1.27) on arrives at

$$\begin{aligned}\frac{\partial u\left(c^*_t, D^*_t\right)}{\partial D_t} &= R\left[(\beta-1)\,\delta\frac{\partial u(c^*_{t+1}, D^*_{t+1})}{\partial x_{t+1}} + \delta\frac{\partial W_{t+1,t+1}(x^*_{t+1}, D^*_t)}{\partial x_{t+1}}\right] \\ &\quad - \left[(\beta-1)\,\delta\frac{\partial u(c^*_{t+1}, D^*_{t+1})}{\partial D_t} + \delta\frac{\partial W_{t+1,t+1}(x^*_{t+1}, D^*_t)}{\partial D_t}\right] \end{aligned} \tag{1.32}$$

In order to obtain $\frac{\partial W_{t+1,t+1}(x_{t+1}, D_t)}{\partial D_t}$ we use the envelope condition for the state variable D_t at time t+1. Again, in the following $u\left(c^*_{t+1}, D^*_{t+1}\right) = u^*_{t+1}$, hence

$$\begin{aligned}\frac{\partial W_{t+1,t+1}(x_{t+1}, D_t)}{\partial D_t} &= \frac{\partial u^*_{t+1}}{\partial c_{t+1}}\frac{\partial c^*_{t+1}}{\partial D_t} + \frac{\partial u^*_{t+1}}{\partial D_{t+1}}\frac{\partial D^*_{t+1}}{\partial D_t} \\ &\quad + \beta\delta\frac{\partial V_{t+1,t+2}\left(x^*_{t+2}, D^*_{t+1}\right)}{\partial x_{t+2}}\left[\frac{\partial x^*_{t+2}}{\partial x_{t+1}}\frac{\partial x_{t+1}}{\partial D_t}\right. \\ &\quad \left. + \frac{\partial x^*_{t+2}}{\partial c_{t+1}}\frac{\partial c^*_{t+1}}{\partial D_t} + \frac{\partial x^*_{t+2}}{\partial D_{t+1}}\frac{\partial D^*_{t+1}}{\partial D_t} + \frac{\partial x^*_{t+2}}{\partial D_t}\right] \\ &\quad + \beta\delta\frac{\partial V_{t+1,t+2}\left(x^*_{t+2}, D^*_{t+1}\right)}{\partial D_{t+1}}\frac{\partial D^*_{t+1}}{\partial D_t}\end{aligned}$$

where $\frac{\partial x_{t+1}}{\partial D_t} = 0$ and $\frac{\partial x^*_{t+2}}{\partial D_t} = R(1-\gamma)$. Collecting $\frac{\partial c^*_{t+1}}{\partial D_t}$ and $\frac{\partial D^*_{t+1}}{\partial D_t}$ one arrives at

$$\begin{aligned}\frac{\partial W_{t+1,t+1}(x_{t+1}, D_t)}{\partial D_t} &= \frac{\partial c^*_{t+1}}{\partial D_t}\left[\frac{\partial u^*_{t+1}}{\partial c_{t+1}} - \beta\delta R\frac{\partial V_{t+1,t+2}\left(x^*_{t+2}, D^*_{t+1}\right)}{\partial x_{t+2}}\right] \\ &\quad + \frac{\partial D^*_{t+1}}{\partial D_t}\left[\frac{\partial u^*_{t+1}}{\partial D_{t+1}} - R\beta\delta\frac{\partial V_{t+1,t+2}\left(x^*_{t+2}, D^*_{t+1}\right)}{\partial x_{t+2}}\right. \\ &\quad \left. + \beta\delta\frac{\partial V_{t+1,t+2}\left(x^*_{t+2}, D^*_{t+1}\right)}{\partial D_{t+1}}\right] \\ &\quad + \beta\delta R(1-\gamma)\frac{\partial V_{t+1,t+2}\left(x^*_{t+2}, D^*_{t+1}\right)}{\partial x_{t+2}}\end{aligned}$$

Due to the two FOCs (1.28) and (1.29), the terms in the first two squared brackets cancel and the envelope condition for D_t reduces to

$$\frac{\partial W_{t+1,t+1}\left(x_{t+1}, D_t\right)}{\partial D_t} = \beta\delta R(1-\gamma)\frac{\partial V_{t+1,t+2}\left(x^*_{t+2}, D^*_{t+1}\right)}{\partial x_{t+2}}$$

Hence, again using the FOC for the perishable good (1.28) yields

$$\frac{\partial W_{t+1,t+1}\left(x_{t+1}, D_t\right)}{\partial D_t} = (1-\gamma)\frac{\partial u\left(c_{t+1}, D_{t+1}\right)}{\partial c_{t+1}}\Bigg|_{c_{t+1}=c^*_{t+1}, D_{t+1}=D^*_{t+1}}$$

Returning to the reformed first order condition for the durable good (1.32) we note that the first term in squared brackets is the RHS of the HEE for the perishable good at time t, as derived above in (1.31). Further substituting the envelope condition for the Durable stock one can rewrite the first order condition for the durable good as the following:

$$\begin{aligned}\frac{\partial u\left(c_t^*, D_t^*\right)}{\partial D_t} &= \delta R\frac{\partial u\left(c_{t+1}^*, D_{t+1}^*\right)}{\partial c_{t+1}} + (\beta-1)\,\delta R\left[\frac{\partial u\left(c_{t+1}^*, D_{t+1}^*\right)}{\partial c_{t+1}}\frac{\partial c_{t+1}^*}{\partial x_{t+1}}\right.\\ &\left. + \frac{\partial u\left(c_{t+1}^*, D_{t+1}^*\right)}{\partial D_{t+1}}\frac{\partial D_{t+1}^*}{\partial x_{t+1}}\right] - \delta(1-\gamma)\frac{\partial u\left(c_{t+1}^*, D_{t+1}^*\right)}{\partial c_{t+1}}\\ &- (\beta-1)\,\delta\left[\frac{\partial u\left(c_{t+1}^*, D_{t+1}^*\right)}{\partial c_{t+1}}\frac{\partial c_{t+1}^*}{\partial D_t} + \frac{\partial u\left(c_{t+1}^*, D_{t+1}^*\right)}{\partial D_{t+1}}\frac{\partial D_{t+1}^*}{\partial D_t}\right]\end{aligned}$$

Rearranging the terms yields the HEE for the durable good (1.33):

$$\begin{aligned}\frac{\partial u\left(c_t^*, D_t^*\right)}{\partial D_t} &= \delta\left(R-1+\gamma\right)\frac{\partial u\left(c_{t+1}^*, D_{t+1}^*\right)}{\partial c_{t+1}}\\ &+ (1-\beta)\,\delta\left[\frac{\partial u\left(c_{t+1}^*, D_{t+1}^*\right)}{\partial c_{t+1}}\left[\frac{\partial c_{t+1}^*}{\partial D_t} - R\frac{\partial c_{t+1}^*}{\partial x_{t+1}}\right]\right.\\ &\left. + \frac{\partial u\left(c_{t+1}^*, D_{t+1}^*\right)}{\partial D_{t+1}}\left[\frac{\partial D_{t+1}^*}{\partial D_t} - R\frac{\partial D_{t+1}^*}{\partial x_{t+1}}\right]\right] \qquad (1.33)\end{aligned}$$

Equation (1.33) clearly shows that the marginal benefit from investing in the durable stock at time t has to equal the marginal cost. From the first term on the RHS it is clear that the Durable good has the character of an asset, so that the marginal cost of purchasing the durable good in terms of consumption in the next period is determined by the difference between the interest rate and the survival rate of the durable good. This part of the HEE for the durable good is identical to the EE for durables in the standard setting with time-consistent preferences. The second term shows how the durable decision at time t is affected by the time-inconsistency of preferences. On the one hand, as in the case of the perishable good, the propensity to consume in period t+1 out of period t savings reduces the marginal cost of the durable good in period t and hence increases the amount of durables purchased. On the other hand, self t also knows by how much self t+1 will increase her spending out of the proceeds from the durables sales in period t+1. Hence the propensity to consume out of future durables sales reduces the amount of the durable good purchased today, since it reduces the value of the durable good as an asset. Note further that assuming $\beta = 1$ reduces the problem to the standard case with time-consistent preferences and the exponential discount factor δ. As can be shown easily, setting $\beta = 1$ in the FOCs reduces them to the standard Euler Equation for the perishable good and to the equivalent to (1.33) for the time-consistent case.

Chapter 2

Eat Your Pie And Have It - Does Time-Inconsistency Show In A Life-cycle Model With Durable Goods?

Abstract

This paper investigates the impact of time-inconsistent preferences in a life-cycle model with a durable good. Since the durable good serves as an asset, and also reduces the consumption response to changes in income and family size, the self-control problem affects retirement savings less than in a model where only perishable goods are taken into account. Conditional on the standard assumption that long-run patience is greater for hyperbolic households, the simulation results suggest that households with time-inconsistent preferences are not easily identifiable from households with standard preferences. The predictions of a realistic life-cycle model are robust to reasonable departures from exponential discounting.

JEL classification: D11, D91, E21

Keywords: Life-cycle savings, durables, time-inconsistent preferences, hyperbolic discounting

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2.1 Introduction

Introducing time-inconsistent preferences into a model with durable goods, this paper incorporates two new approaches in the life-cycle literature. The objective is to learn how the impact of time-inconsistency, and the resulting self-control problem, is affected by the presence of a durable good, which provides utility but also preserves its value, and hence serves as an asset. Indeed, the simulated savings and consumption profiles for households with time-inconsistent preferences are nearly indistinguishable from the profiles of households with standard preferences. Conditional on the different discount rates calibrated for the two household types and given the dual role played by the durable good, the predictions of a realistic life-cycle model are robust to reasonable departures from exponential discounting.

The motivation behind this study is threefold. Firstly, the essence of modern life-cycle literature is the idea that households smooth utility, and hence consumption, over the different stages of life, as has been formulated first by Friedman in 1957. However, as long as only perishable goods are taken into account, the empirically observed hump shape of consumer expenditure over the life-cycle suggests, that this hypothesis is problematic. As Fernández-Villaverde and Krueger (2005) show, this changes once durable goods are introduced. Secondly, in the life-cycle context the effect of the self-control problem has only been investigated when the household had to choose between saving in financial assets and consuming a perishable good. Assuming that at least one of the goods from which the household derives utility is durable, reduces the trade-off between current and future gratification. Since the self-control problem is a problem of imperfect intratemporal resource allocation, it is intuitive to expect, that a durable good will narrow the gap between current and future interests, and should hence reduce the effect of time-inconsistency on savings and consumption decisions. Thirdly, the concept of time-inconsistency itself is a challenge to the standard approach taken towards modelling intertemporal choice. Given this potential divide it is important to assess, in how far a stand taken on this issue actually determines economic predictions on human behavior. While the life-cycle model is only one of many potential applications to this question, its very subject makes it one of the most interesting ones.

In the light of the general performance of the life-cycle model, the introduction of a durable good in addition to the standard "perishable" or "consumption" good is very promising. Durable goods not only affect the intertemporal allocation of consumption and saving, but have meaningful implications for the Euler Equation of the consumption good, when the two goods are not perfect substitutes in the utility function. As observed in Padula (1999) complementarity between the two goods also markedly improves the empirical support for the Euler Equation. Fernández-Villaverde and Krueger (2005) find that durable goods, especially when they serve as collateral for loans, help replicate the hump in expenditure observed over the life-cycle. Here the accumulation of durable goods by young households also serves as a precautionary savings device in the face of income uncertainty. Pavan (2005) furthermore investigates the interplay of durables accumulation and bankruptcy legislation in the life-cycle framework, and again finds that durables acquisitions are subject to precautionary savings motives.

The introduction of time-inconsistent preferences into the life-cycle model goes back to Laibson (1997) and Laibson *et al.* (1998, 2001), where the implications of time-inconsistent preferences, or hyperbolic discounting, on savings behavior are investigated. The concept of time-inconsistent preferences has been extensively addressed in the previous chapter. Again, however, it is necessary to stress that the contrast between short run impatience and long run patience can affect individuals' preferences for savings instruments. In Laibson (1997) and Laibson *et al.* (1998, 2001) the key finding is that an illiquid financial asset acts as a commitment device, and hence helps a household overcome her self-control problem and consequently stimulates savings. In the previous chapter it is found that, to a large extent, the intertemporal properties of the durable good can eliminate the effect of the self-control problem on retirement savings. This chapter builds on this result, but does not conjecture that, when durable goods are taken into account, an illiquid asset is no longer needed. Nevertheless, since investment in the durables stock reduces the need for savings in form of a financial asset, finding a commitment device is less important.

Section 2 presents the model and Section 3 explains the calibration of the model parameters. In Section 4 the simulation results are presented and explained, Section 5 concludes.

2.2 A life-cycle model with durable goods

2.2.1 Model set-up

The version of the life-cycle model used here is a partial equilibrium model with inelastic labor supply and exogenous interest rate and income process. The key features of the model are the inclusion of a durable good and labor income uncertainty. Further taken into account are subjective life-horizon uncertainty, bequests and changes in family size, where the fertility decision is not modelled, so that family composition evolves exogenously. Although from the perspective of the household, the time of death is uncertain, all households live for T+N periods. They work for T periods and spend their remaining life in retirement. Since labor income is uncertain for the first T periods, each period t households receive income according to

$$y_t^W = f\left(age_t\right) + u_t$$

where $f\left(age_t\right)$ is a cubic function of age and u_t expresses labor income uncertainty. During retirement households are certain to receive income

$$y_t^R = g\left(age_t\right)$$

Each period t, households decide how much to save, how much of a perishable good to consume and how much to invest in a durable good. Household utility is given by the standard CES function

$$u\left(c_t, D_t, n_t\right) = n_t \frac{\left(\left(\alpha\left(\frac{c_t}{n_t}\right)^{\tau} + (1-\alpha)\left(\frac{D_t}{n_t}\right)^{\tau}\right)^{\frac{1}{\tau}}\right)^{1-\rho} - 1}{1-\rho} \tag{2.1}$$

where c_t is consumption of the perishable good, D_t is the household's durables stock, and n_t is effective family size. Since within the household neither good is a public good, and since the wellbeing of all household members is taken into account, effective family size is included in the utility function in order to capture that bigger households need greater amounts of both goods.

The expected payoff for the household in period t, subject to quasi-hyperbolic discounting, is

$$\begin{aligned} CP_t &= \beta E_t \sum_{i=1}^{T+N+1-t} \delta^i \left(\prod_{j=1}^{i-1} s_{t+j}\right) \left[s_{t+i} u\left(c_{t+i}, D_{t+i}, n_{t+i}\right)\right. \\ &+ \left.\left(1 - s_{t+i}\right) B\left(a_{t+i}, D_{t+i-1}, n_{t+i-1}\right)\right] \end{aligned}$$

Here the product term in round brackets expresses the probability of the household surviving until the end of period $t+i-1$. With probability s_{t+i} the households survives into period $t+i$, while $(1-s_{t+i})$ is the probability of dying between period $t+i-1$ and period $t+i$, for which case $\mathbf{B}\left(a_{t+i}, D_{t+i-1}, n_{t+i-1}\right)$ defines the bequest value of the household's assets and durables stock at the time of death. Note that s_{T+N+1} is zero, since the maximum age a household can reach is $T+N$. The hyperbolic household's short run impatience is captured by β, while δ is equivalent to the standard exponential discount factor. Each period the household maximizes

$$U\left(c_t, D_t, n_t\right) = u\left(c_t, D_t, n_t\right) + CP_t \tag{2.2}$$

subject to the budget constraint

$$c_t + d_t + h(d_t) + a_{t+1} = Ra_t + y_t \tag{2.3}$$

where cash-on-hand in period t, $Ra_t + y_t = x_t$, is determined by savings with interest in period $t-1$ and the realization of income in period t. Hence cash-on-hand in period $t+1$ evolves according to

$$x_{t+1} = R\left(x_t - c_t - d_t - h(d_t)\right) + y_{t+1} \tag{2.4}$$

The amount of durables held in period t is determined by the investment in the durables stock d_t and the amount of durables carried over from period $t-1$, where γ is the rate at which the durables stock depreciates between periods. Hence

$$D_t = d_t + (1-\gamma)\, D_{t-1} \tag{2.5}$$

The price of durables and perishables has been normalized to one and households can sell off their durables stock at the same price at which they purchase new durables. Nevertheless the durable good is illiquid in the sense that whenever durables are traded, a transition cost $h(d_t)$ proportional to the size of the transaction has to be paid.

Furthermore, borrowing is restricted to fraction $(1-\theta)$ of current durables holding D_t, so that durables also serve as collateral for loans. Hence for all periods up to $t = T+N-1$ it holds that

$$a_{t+1} \geq -(1-\theta) D_t \tag{2.6}$$

All debt has to be repaid in period T+N at latest. The interest rate is constant and the same for borrowing and lending.

2.2.2 Equilibrium Conditions

In order to characterize the equilibrium, I follow the approach taken by Harris and Laibson (2001) and derive the two value functions for the hyperbolic household. The resulting policy functions will be the equilibrium strategies of an intertemporal Markov game. Due to hyperbolic discounting, self t conceptually distinguishes between the *current* value in period t, $W_{t,t}(x_t, D_{t-1}, n, s)$ of cash-on-hand and the existing durables stock, and the *continuation* value, $V_{t,t+1}(x_{t+1}, D_t, n, s)$ of cash-on-hand and durables carried over into period $t+1$, both determined by the evolution of family size n, the vector of survival probabilities s, and the expectation on future income. Hence the equilibrium is defined by the two value functions

$$\begin{aligned} W_{t,t}(x_t, D_{t-1}, n, s) &= \max_{c_t, d_t} \{u(c_t, D_t, n_t) + \beta\delta (s_{t+1} E_t V_{t,t+1}(x_{t+1}, D_t, n, s) \\ &+ (1 - s_{t+1}) B(a_{t+1}, D_t, n_t))\} \end{aligned} \tag{2.7}$$

$$\begin{aligned} V_{t,t+1}(x_{t+1}, D_t, n, s) &= \max_{c_{t+1}, d_{t+1}} \{u(c_{t+1}, D_{t+1}, n_{t+1}) + \delta (s_{t+2} E_{t+1} V_{t+1,t+2}(x_{t+2}, D_{t+1}, n, s) \\ &+ (1 - s_{t+2}) B(a_{t+2}, D_{t+1}, n_{t+1}))\} \end{aligned} \tag{2.8}$$

subject to the budget constraint (2.3) and the borrowing constraint (2.6). Due to the bequest motive it holds that for $t = T + N$

$$V_{T+N,T+N+1}(x_{T+N+1}, D_{T+N}, n, s) = B(a_{T+N+1}, D_{T+N}, n_{T+N})$$

and hence

$$\begin{aligned} W_{T+N,T+N}(x_{T+N}, D_{T+N-1}, n, s) &= \max_{c_{T+N}, d_{T+N}} \{u(c_{T+N}, D_{T+N}, n_{T+N}) \\ &+ \beta\delta B(a_{T+N+1}, D_{T+N}, n_{T+N})\} \end{aligned}$$

A sequence of policy functions $\{c_t^*, d_t^*\}_1^{T+N}$ solves for $W_{t,t}(x_t, D_{t-1}, n, s)$ and $V_{t,t+1}(x_{t+1}, D_t, n, s)$, given the conditions imposed by sophisticated households. Sophistication implies that at time t, self t knows, that, given the realization of income in period $t+1$, self $t+1$ will maximize

$$\begin{aligned} W_{t+1,t+1}(x_{t+1}, D_t, n, s) &= \max_{c_{t+1}, d_{t+1}} \{u(c_{t+1}, D_{t+1}, n_{t+1}) + \beta\delta (s_{t+2} E_{t+1} V_{t+1,t+2}(x_{t+2}, D_{t+1}, n, s) \\ &+ (1 - s_{t+2}) B(a_{t+2}, D_{t+1}, n_{t+1}))\} \end{aligned}$$

subject to the budget constraint for period $t+1$, where x_{t+1} is given by (2.4).
Self t is thus fully aware of the time-inconsistency of preferences but cannot control the consumption and savings decisions of self $t+1$. Nevertheless, self t can foresee the level of consumption and durables purchases chosen by self $t+1$, given the states in period $t+1$, x_{t+1} and D_t, as well as the evolution of family size n and survival probabilities s. Hence self t anticipates that

$$W_{t+1,t+1}(x_{t+1}, D_t, n, s) = u\left(c^*_{t+1}, D^*_{t+1}, n_{t+1}\right) + \beta\delta\left(s_{t+2}E_{t+1}V_{t+1,t+2}\left(x^*_{t+2}, D^*_{t+1}, n, s\right) + (1 - s_{t+2})\, B\left(a^*_{t+2}, D^*_{t+1}, n_{t+1}\right)\right)$$

From this it follows that

$$\delta s_{t+2}E_{t+1}V_{t+1,t+2}\left(x^*_{t+2}, D^*_{t+1}, n, s\right) = -\frac{1}{\beta}u\left(c^*_{t+1}, D^*_{t+1}, n_{t+1}\right) + \frac{1}{\beta}W_{t+1,t+1}(x_{t+1}, D_t, n, s) - \delta(1 - s_{t+2})\, B\left(a^*_{t+2}, D^*_{t+1}, n_{t+1}\right)$$

Substituting this expression into the original value function (2.8) yields

$$V_{t,t+1}(x_{t+1}, D_t, n, s) = \left(1 - \frac{1}{\beta}\right) u\left(c^*_{t+1}, D^*_{t+1}, n_{t+1}\right) + \frac{1}{\beta}W_{t+1,t+1}(x_{t+1}, D_t, n, s) \qquad (2.9)$$

From (2.9) it is apparent that self t, when making her own savings and consumption decisions, will take into account how self $t + 1$ will react to a change in period t savings and durables stock. Clearly, unless $\beta = 1$, in which case one returns to the standard case with time-consistent preferences,

$$E_tW_{t+1,t+1}(x_{t+1}, D_t, n, s) \neq E_tV_{t,t+1}(x_{t+1}, D_t, n, s)$$

After the model parameters have been calibrated, the model is solved using numerical solution methods and a backwards induction algorithm as described in Appendix B. In the following, in order to evaluate the effect of the time-inconsistency on household decisions, the profiles of time-inconsistent (hyperbolic) households will be compared to the counterfactual outcome for time-consistent (exponential) households.

2.3 Calibration

The income process, evolution of family size, interest rate and the time-discount factors are taken from Laibson *et al.* (2001). The parameters regarding the durable good are those calibrated by Fernández-Villaverde and Krueger (2005).

2.3.1 Income process

Laibson *et al.* (2001) estimate the income process using PSID data (1983-1990) and distinguish between different levels of education (no High school certificate, High School certificate and college diploma). In order to account for variations in income unrelated to household age and education, they include cohort and cyclical dummies in the regression. The authors also distinguish between a pre-retirement and a post-retirement income process. Hence in the model, for each level of education h, pre-retirement household income evolves according to

$$y_t^{W,h} = f^h\left(age_t\right) + u_t^h$$

where $f^h\left(age_t\right)$ is a cubic function of age and u_t^h is labor income uncertainty in form of a 2-stage Markov process with support $[-\theta^h, \theta^h]$ and constant transition probabilities $[p^h, 1-p^h]$.
Retired households receive income

$$y_t^{R,h} = g^h\left(age_t\right)$$

where $g^h\left(age_t\right)$ is a linear, decreasing function of age. During retirement there is no income uncertainty. Table 3.2 and Table 2.2 in Appendix A show the parameter values of the respective income processes as estimated by Laibson *et al.* (2001), Figure 2.1 shows the evolution of the deterministic part of household income over the life-cycle for the three types of households.

2.3.2 Demographics

Household life begins at age 20, and retirement, as in Laibson *et al.* (2001) depends on the household type. Households with incomplete High School education retire at age 61, households with a High School certificate at age 63, and households with a college degree at age 65. The population weights for the three groups are 0.25, 0.50 and 0.25 respectively.
Table 2.3 reports the estimation results for household decomposition from Laibson *et al.* (2001), also to be seen in Figure 2.2. In order to capture the effect of family size and decomposition on household utility, I construct effective family size using the official OECD scale, which gives weight 1 to the first adult, weight 0.67 to the following adults and weight 0.43 to each child. In the following a household is assumed to consist at least of household head and spouse.
Household mortality is such, that from the perspective of the household the time of death is uncertain, but the maximum age households can reach is 99 years. The subjective survival probabilities are taken from the U.S. Decennial Life-Tables for 1979-81 and reported in Figure 2.3 in Appendix A.
Bequests are set to be the sum of financial assets and remaining durables, where depreciation γ, and selling costs $h(D_t)$ of the entire remaining durables stock are taken into account. Hence the utility derived from leaving a bequest is given by the following

$$B\left(a_{t+1}, D_t, n_t\right) = n_t \frac{\left(\max\left(0, \frac{a_{t+1}}{n_t} + \frac{(1-\gamma)D_t - h(D_t)}{n_t}\right)\right)^{1-\rho} - 1}{1-\rho}$$

2.3.3 The durable good

The price of the durable good and the price of the perishable good are normalized to one. With respect to the depreciation rate of the durable good, I follow Fernández-Villaverde and Krueger (2005), who

calibrate $\gamma = .081$ on the basis of the ratio of expenditure on perishables and durables using NIPA data. Although the household can sell her durables at any time, durables are illiquid in the sense, that, when durables are bought or sold, an adjustment - or transactions cost has to be paid. In order to capture the size of these costs, I draw on the results in Martin and Gruber (2005), who using CES data for housing sales, derive a proportional purchasing cost of 2.5% and a proportional selling cost of 7% for the mean household. Intuitively it is sensible to assume, that transaction cost are nonlinear and concave. They increase with the amount of the durable good purchased or sold, however less so as the amount of the durable good traded becomes large. Using the above estimates of the transaction costs for housing, and assuming that housing is the largest durable good, I model transaction costs as concave functions of the amount of the durable traded, converging to 2.5% and 7% respectively as the amount of durables purchased or sold increases.
Furthermore it is assumed that durables serve as collateral for loans. Equivalently, borrowing cannot exceed fraction $(1-\theta)$ of the durables stock; hence the borrowing constraint is endogenous. I follow Fernández-Villaverde and Krueger (2005) as well as Diaz and Luengo-Prado (2005) who assume a fixed down payment of 20% of the durables purchase, so that $\theta = 0.2$.

2.3.4 Utility function

As in Fernández-Villaverde and Krueger (2005), perishables and durables are modelled as complements. This decision is motivated by the results of Rupert *et al.* (1995) and Ogaki and Reinhart (1998), who empirically estimate the substitutability between perishables and durables, and find that on overall it is not significantly different from zero. Consequently (2.1) takes the Cobb-Douglas form

$$u(c_t, D_t, n_t) = n_t \frac{\left(\left(\frac{c_t}{n_t}\right)^{\alpha}\left(\frac{D_t}{n_t}\right)^{1-\alpha}\right)^{1-\rho} - 1}{1-\rho}$$

where household utility takes into account effective family size n_t, which according to the OECD weights is given by

$$n_t = 1.67 + 0.67 \times dependent.adults_t + 0.43 \times kids_t$$

As calibrated in Fernández-Villaverde and Krueger (2005), α is set to 0.81 and the intertemporal elasticity of substitution ρ is set to 2, which is also the value chosen in Laibson *et al.* (2001).

2.3.5 Interest rate and time discount rates

I follow Laibson *et al.* (2001) in assuming a return rate of $r = 3.75\%$ to the liquid asset. Based on this assumption, I also draw on these authors' calibration results for the time-discount rates for households with and without time-consistent preferences, as shown in Table 2.4. Since the discount factors for hyperbolics and exponentials are calibrated using the same SCF data on savings at retirement age, but assuming $\beta = .7$ for all hyperbolic households, the calibrated long-run discount rates δ are greater in the time-inconsistent case. The observed amount of savings differs over education types, which suggests different rates of long run discounting for households with different education levels.

2.4 Results

Figures 2.4 and 2.5 show the average simulation results over all education types of households, given the share of each education type in the population. Figures 2.6, 2.7 and 2.8 separately show the simulation results for households without completed High School education, with High School certificate and with a college degree. Clearly the pattern of durables stock and savings is robust over the different education types and income profiles.

2.4.1 General results

The simulation of the model predicts hyperbolic and exponential households to behave very similarly. The same general pattern applies to all households. As explained in Fernández-Villaverde and Krueger (2005), durable goods play a dual role. On the one hand they provide utility, on the other hand, accumulating durables is a form of precautionary savings and protects the household from negative income shocks. Hence young households borrow in the financial asset in order to build up a durables stock, which then serves as collateral for future loans, and therefore as a precautionary savings device. Simultaneously, since the two goods are complements in the utility function, a lower durables stock reduces the marginal utility from perishables consumption. Hence overall consumption is low when the durables stock is low; as the durables stock grows, marginal utility derived from consumption in the perishable good increases and both, durables holdings and consumption grow together. Together with changes in family size, this mechanism can account for the hump shaped pattern of life-cycle expenditure observed in consumption data. In addition, in this model the durables stock also functions as bequest, and even sustains debt for very old households.

With respect to the role of time-inconsistency in this model, at first the result that hyperbolic and exponential households are predicted to be observationally equivalent is surprising, especially since for college graduates savings for hyperbolic households slightly exceed exponentials' savings (Figure 2.5). Another point of interest is the greater volatility of the durables stock for time-inconsistent households.

2.4.2 The effect of the time-inconsistency

In order to explain the similarity in life-cycle profiles it is best to separately consider the two key differences between hyperbolic and exponential households. The first difference is the time-inconsistency itself, in the sense that for hyperbolic households $\beta < 1$, due to their great short run impatience. The second difference is the greater long run patience of the hyperbolic households since $\delta_{hyp} > \delta_{exp}$ for all education types.

Beginning with the effect of the inconsistency, the durable good reduces the impact of short-run impatience on household saving through three channels. The durables stock not only yields utility but also serves as a precautionary savings device, so that households invest heavily in durables at the beginning of their lives. Every future saving and consumption decision is made on the basis of the accumulated durables stock, so that current utility depends less on current spending than it would if there was only a perishable good in the model. In fact, given the decreasing marginal utilities of durables and perishables consumption, the durables stock smooths utility over the life-cycle. Hence the scope for short-run impatience to affect the savings and consumption decision is lower and the self-control problem is less pronounced. This becomes especially important when hyperbolic households become older and when income is declining while the number of dependent adults in the household still is large. Since family size is decreasing, the middle aged household will not have to further invest in the durables stock, but can let it depreciate while only spending on the perishable good. Hence she will be less tempted to prematurely consume out of her savings, and retirement savings will be affected less by the self-control problem.

The other channel is that durables serve as an asset, however subject to adjustment costs and return dominated by the financial asset. Thus the consequences of short-run impatience are less severe, since either the household can simply spend less on the durable good in the next periods, or more importantly, part of the expenditure on durable goods can be recovered by selling off durables at a later date. In this sense, hyperbolic households willing to save and consume at the same time, are able to 'eat their cake and have it'. Here also lies the explanation for the observed volatility of the hyperbolics' durables stock.

The third channel is probably the most obvious one. Introducing durables into the model reduces the need for retirement savings in general, since it reduces consumption of the perishable good. If, however, savings are less important, then a self-control problem preventing these savings also has less of an impact. Overall, when durable goods are taken into account, the self-control problem is reduced to a debt-repayment problem for young households with a growing family. Thereafter savings rates are not much affected.

The second key difference between hyperbolic and exponential households is the difference in long run patience. Since the calibrated rate of long-run discounting is smaller for the hyperbolic household, they ideally want to save more than exponential households would. The observation that, in fact, savings are nearly identical for the two types of households independently of their income process, shows that the self-control problem indeed reduces savings. However, given the calibrated parameter values and given the effect the durable good has on the self-control problem, as well as on the role of retirement savings, the two types of households cannot be told apart.

Returning to the modus in which the time-discount rates were calibrated in Laibson *et al.* (2001), it is important to keep in mind that for both types of households the same data on wealth and savings were used. The fact, that, given the calibrated discount factors, this model replicates the same savings and durables profiles for the two types of households lends support to including durables into the life-cycle model.

2.4.3 Robustness

In order to assess whether the result of observational equivalence is driven by the illiquidity of the durable good, the exercise is repeated under the assumption that there are no adjustment costs for changes in the durables stock, so that the durable good is perfectly liquid. Figures 2.9, 2.10 and 2.11 show the results for the different education types. Although the spending and savings patterns remain very similar, two things should be noted. Firstly, the volatility of the hyperbolics' durables stock has increased, since now a revision of the durables stock is cost free and the household will give in to her short-run impatience more easily. Secondly, at least for households with a college degree, the difference in savings of hyperbolic

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and exponential households has increased. Since college graduates have the steepest income profile, their savings are least affected by the self-control problem. Also college graduates have the highest demand for durables and hyperbolic college graduates profit the most, when it is cost-free to convert their durables into savings when family size is decreasing, i.e. "when the kids move out".

The other feature of the model which might be suspected of causing the similarity in the profiles of the two types, is the collateral condition (2.6), since imposing a constraint on household borrowing could reduce the effect of the self-control problem. However, Figures 2.12, 2.13 and 2.14 clearly show, that the effect of time-inconsistency does not change when borrowing is unconstrained. In other words, for both types of households, debt is collateralized by the durables stock, even if the borrowing constraint is not imposed explicitly.

2.5 Conclusion

While the results of this exercise lend great support towards the inclusion of durable goods into the life-cycle model, they do not give any clear indication in how far the assumption of time-inconsistent preferences is helpful. The simple reason is that the effect of the time-inconsistency tends to disappear given the difference in long run discounting, but also the asset value of the durable good and its effect of reducing the impact of the self-control problem by smoothing household expenditure.

Unlike in the case where a commitment device in form of an illiquid asset is introduced, here the self-control problem is not solved. On the contrary, apart from its asset character, the advantage of the durable good lies in its (comparative) liquidity. The hyperbolic household derives immediate gratification from the services provided by the durable good, while simultaneously the durables stock also serves as savings. Indeed, up to a degree, she is able to "eat her pie and have it".

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Appendix

2.A Figures and Tables

The deterministic part of the income profile consists of the effect of age on income (cubic polynomial) and the effect of household size and composition.

Table 2.1: Estimated Age - Non Asset Income Profiles*

	Less than High School	High School Graduates	College Graduates
In the labor force			
Age	0.077	0.118	0.223
$Age^2/100$	-0.172	-0.201	-0.390
$Age^3/10000$	0.092	0.081	0.204
N head&spouse	0.668	0.548	0.462
N kids	0.012	-0.033	-0.023
N dep. adults	0.167	0.170	0.022
Other effects[a]	7.958	7.439	6.029
Retired			
Age	-0.039	-0.002	-0.009
N head&spouse	0.656	0.554	0.327
N kids	0.042	0.199	-0.560
N dep. adults	0.421	0.204	0.162
Other effects[a]	9.927	8.433	10.172

**Source Laibson* et al. *(2001), Table 4*

Includes the effects of a constant, cohort dummies, and the unemployment rate in the household's state of residence

The stochastic component of the income process is expressed as a 2 stage Markov process. Depending on household type h, for households in the work force this process has symmetric support $\left[-\theta^h, \theta^h\right]$, and constant transition probabilities $p^h, \left(1 - p^h\right)$, where

$$\theta^h = \sqrt[2]{\frac{\sigma_h^2}{1-\alpha_h^2}}$$

$$p^h = \frac{\alpha_h + 1}{2}$$

Table 2.2: Estimated Age-Income Processes*

	Less than High School	High School Graduates	College Graduates
α	0.881	0.782	0.967
σ^2	0.024	0.029	0.019

**Source Laibson* et al. *(2001), Table 5*

where α_h and σ_h^2 are the coefficients and the estimated variance from the residuals of the above regression.

For each education group Figure 2.1 shows the deterministic component of the income profile for households consisting at least of household head and spouse, and given the number of dependents as estimated by Laibson *et al.* (2001) and reported in Table 2.3

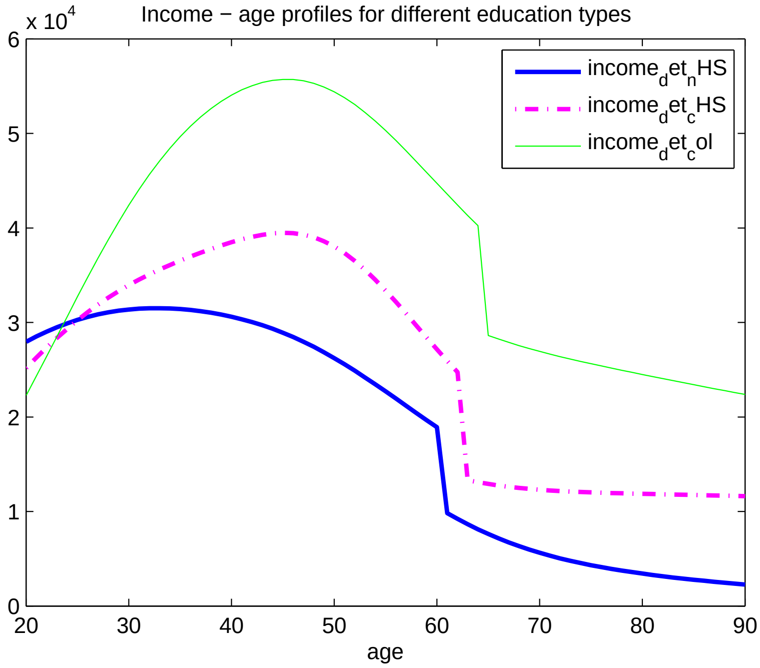

Figure 2.1: Income-Age Profiles For The Different Education Types*
** Own simulation based on parameter values estimated in Laibson* et al. *(2001)*

Table 2.3: Estimated Age - Number of Children and Dependent Adults Profiles*

	Children	Dependent Adults
Less than High School		
constant	0.12143	0.00002
Age	0.16690	0.41411
Age^2	0.00238	0.00396
High School Graduates		
constant	0.00613	8E-09
Age	0.32402	0.72718
Age^2	0.00450	0.00713
College Graduates		
constant	0.00005	4E-12
Age	0.55628	1.00347
Age^2	0.00729	0.00965

**Source Laibson* et al. *(2001), Table 3*

coefficients for regression

$$x_{i,t} = \beta_0 exp\left(\beta_1 age_{i,t} - \beta_2 age_{i,t}^2\right) + \varepsilon_{i,t}$$

Table 2.4: Time Discount Rates*

	δ^{nHS}	δ^{HS}	δ^{Col}
hyperbolics	0.930	0.956	0.956
exponentials	0.912	0.944	0.945

**Source Laibson* et al. *(2001)*

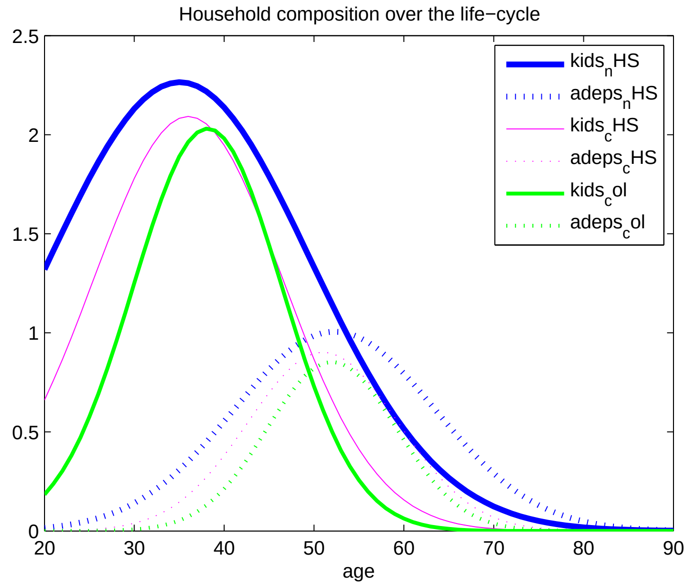

Figure 2.2: Estimated Household Composition*
** Source Laibson* et al. *(2001)*

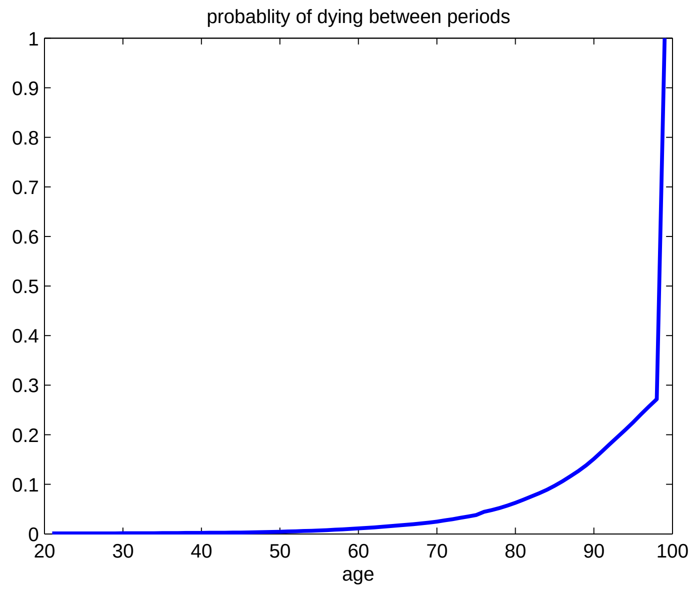

Figure 2.3: Probabilities of dying between periods*
** Source Life-tables, (OECD)*

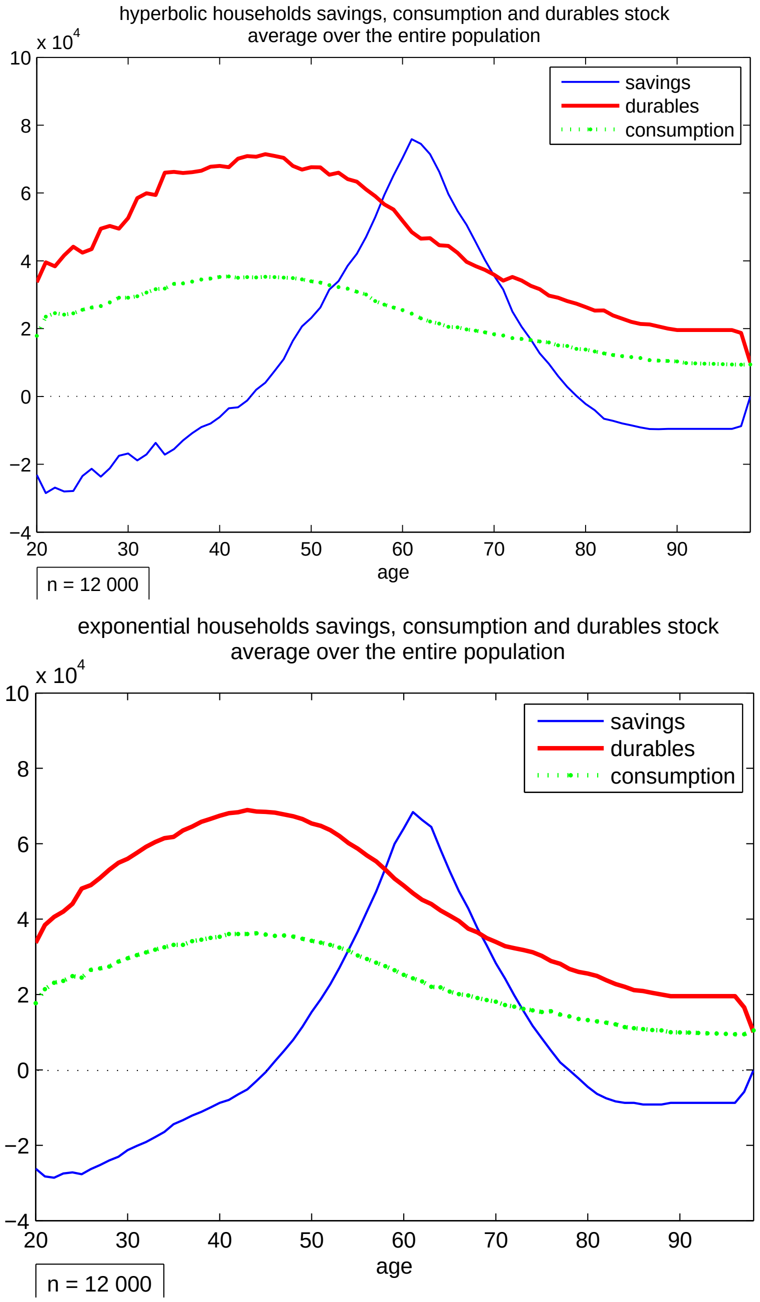

Figure 2.4: Life-cycle profiles for hyperbolic and exponential households

Averages over entire population

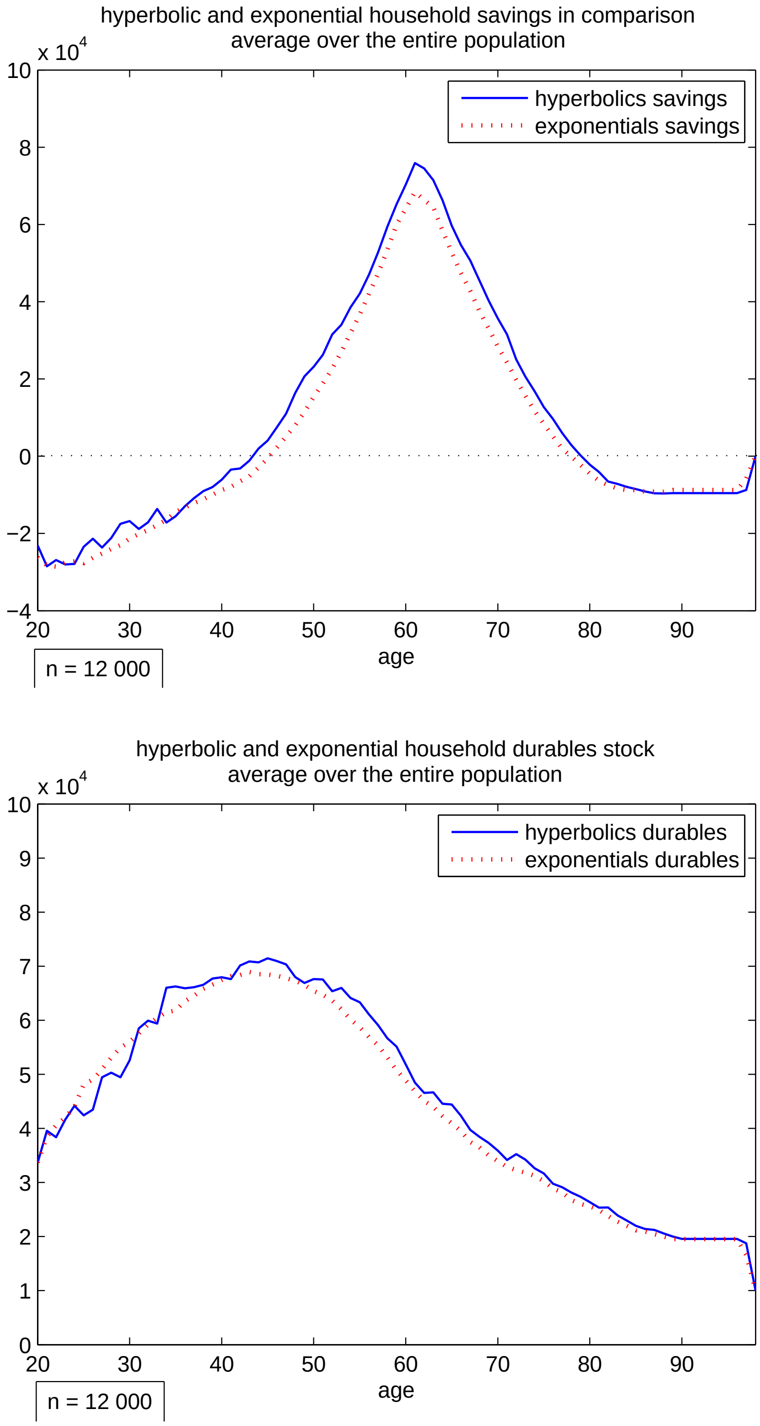

Figure 2.5: Comparison of savings and durables stock of hyperbolic and exponential households

Averages over entire population

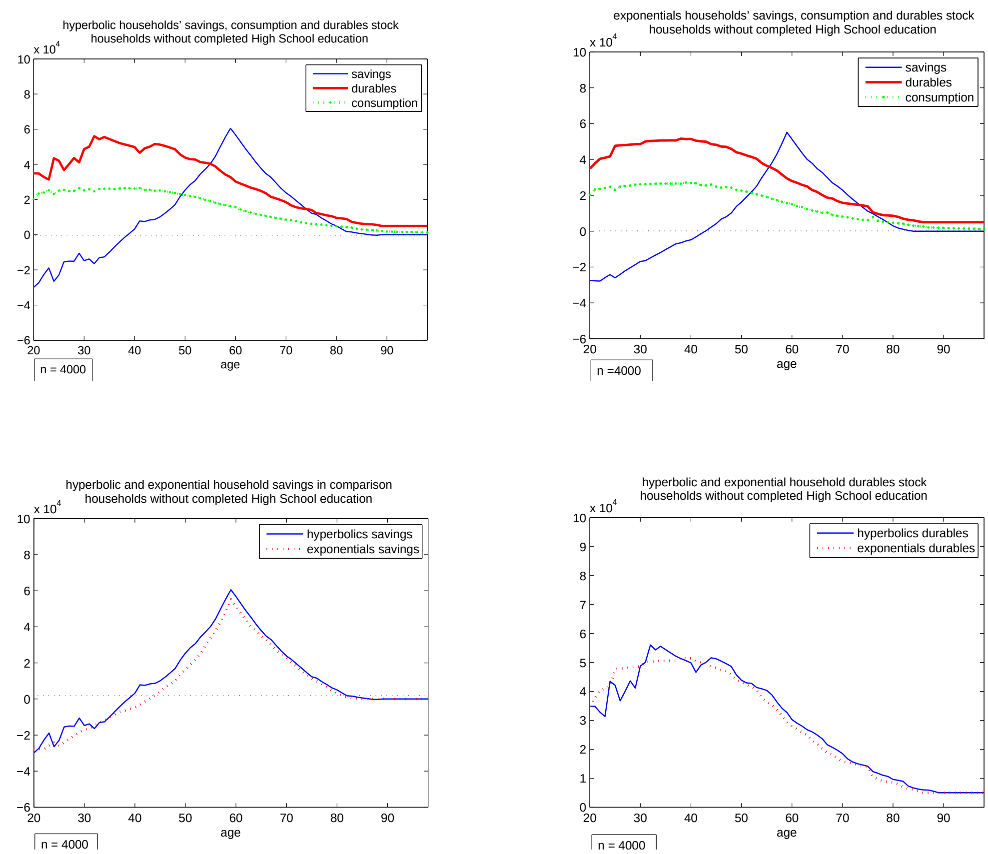

Figure 2.6: Comparison of savings and durables stock of hyperbolic and exponential households

Households without completed High School education

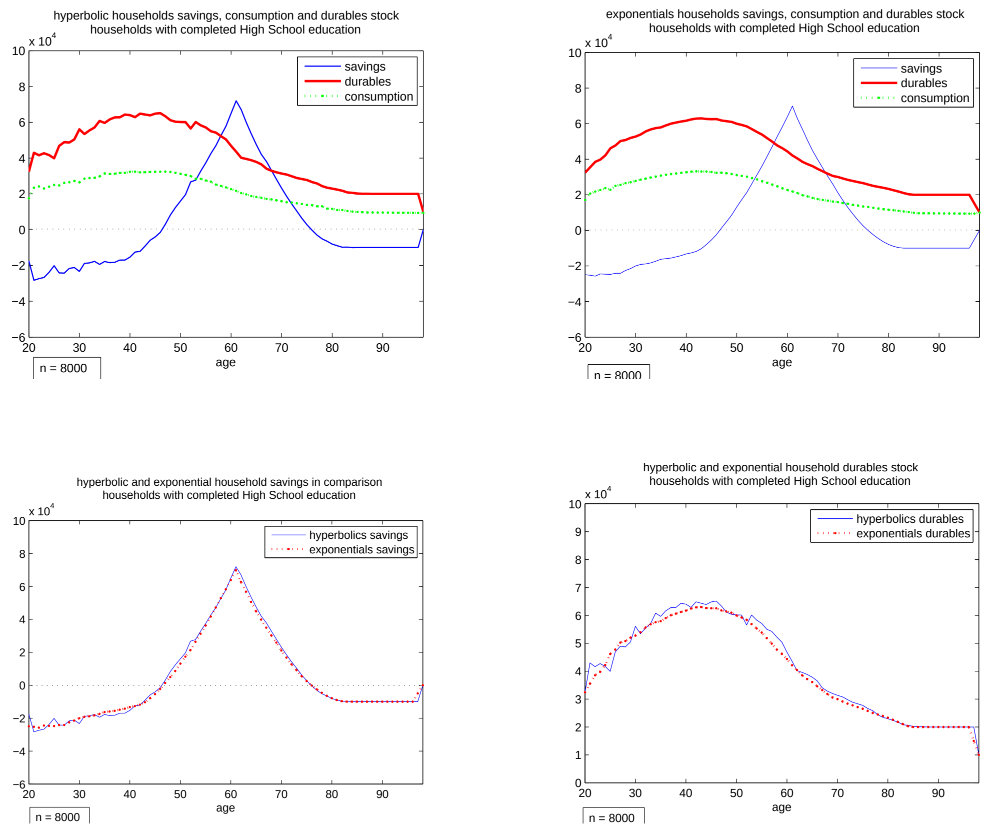

Figure 2.7: Comparison of savings and durables stock of hyperbolic and exponential households

Households with completed High School education

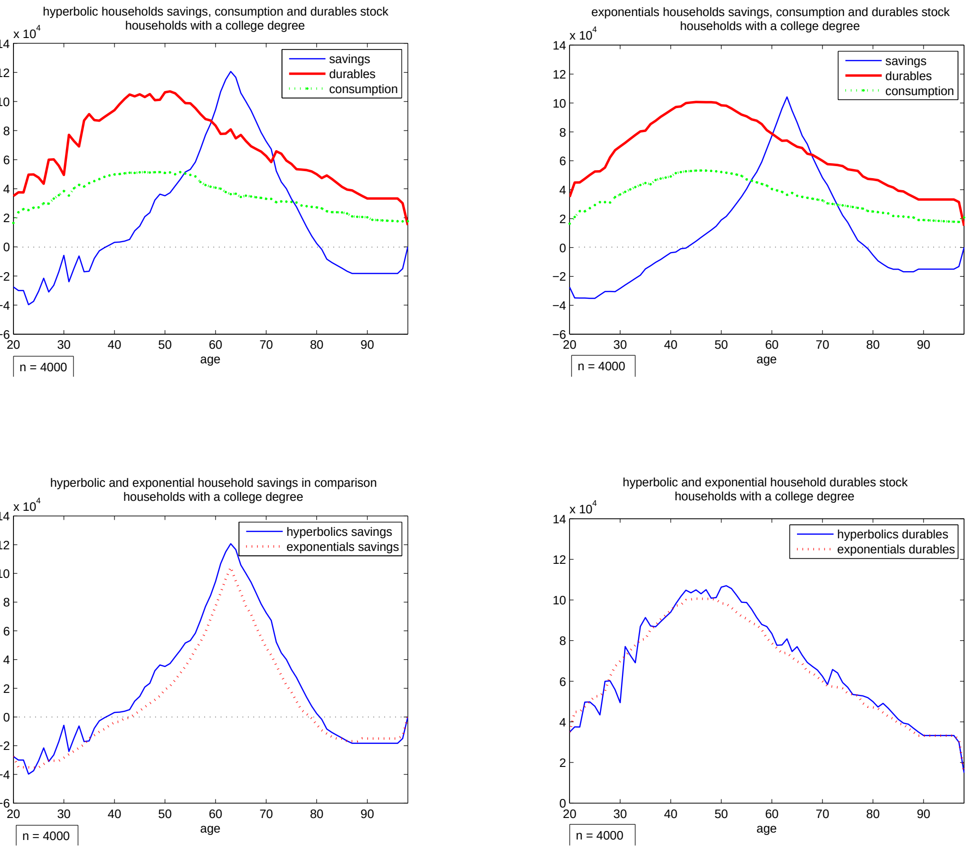

Figure 2.8: Comparison of savings and durables stock of hyperbolic and exponential households

Households with college degree

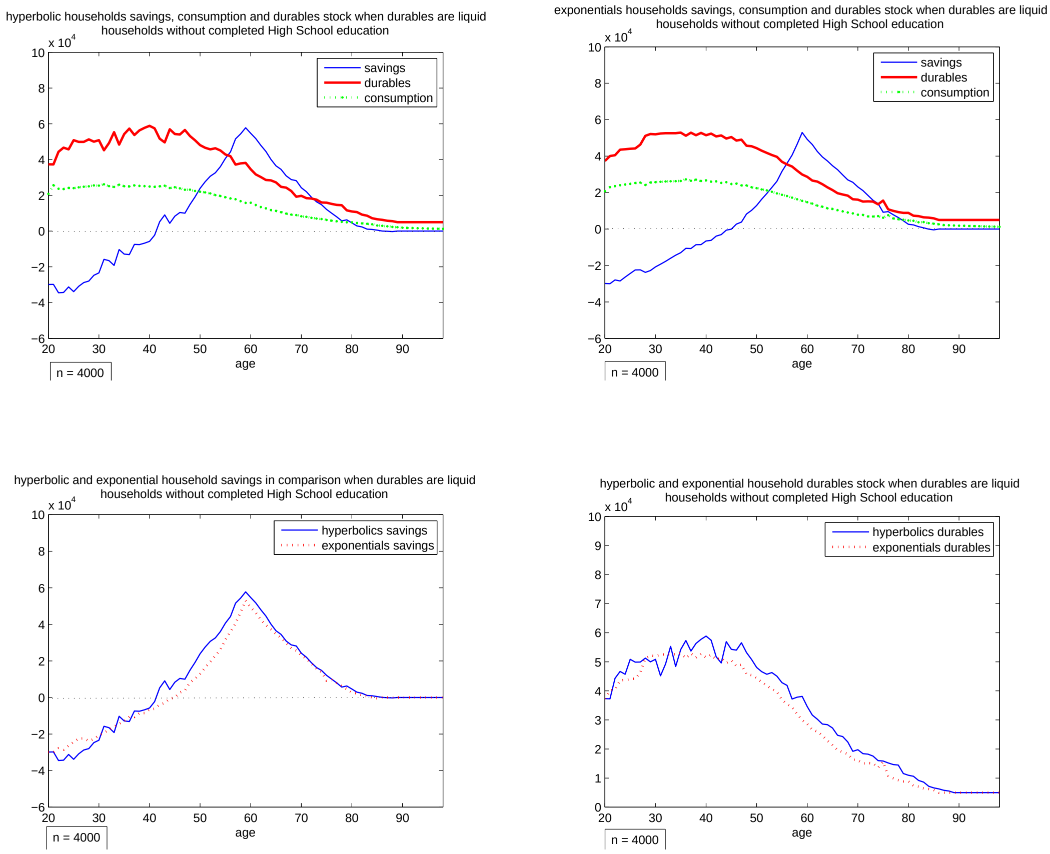

Figure 2.9: Comparison of savings and durables stock of hyperbolic and exponential households when durables are liquid

Households without completed High School education

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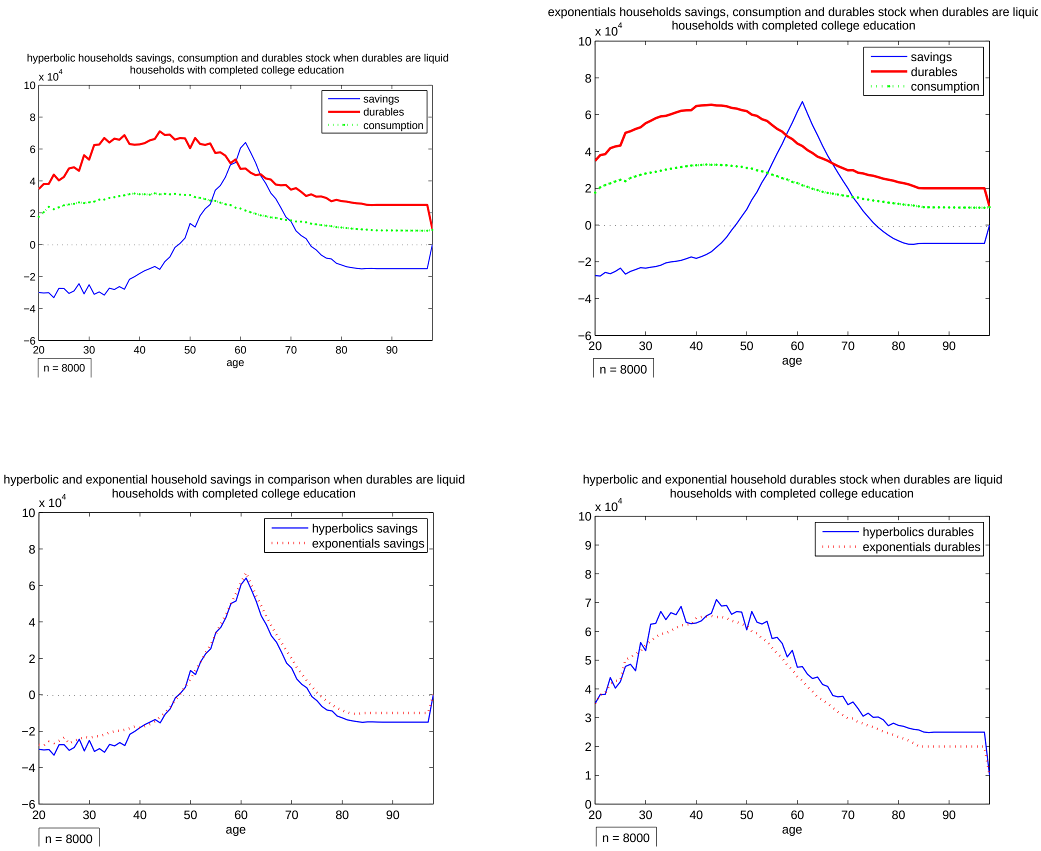

Figure 2.10: Comparison of savings and durables stock of hyperbolic and exponential households when durables are liquid

Households with completed High School education

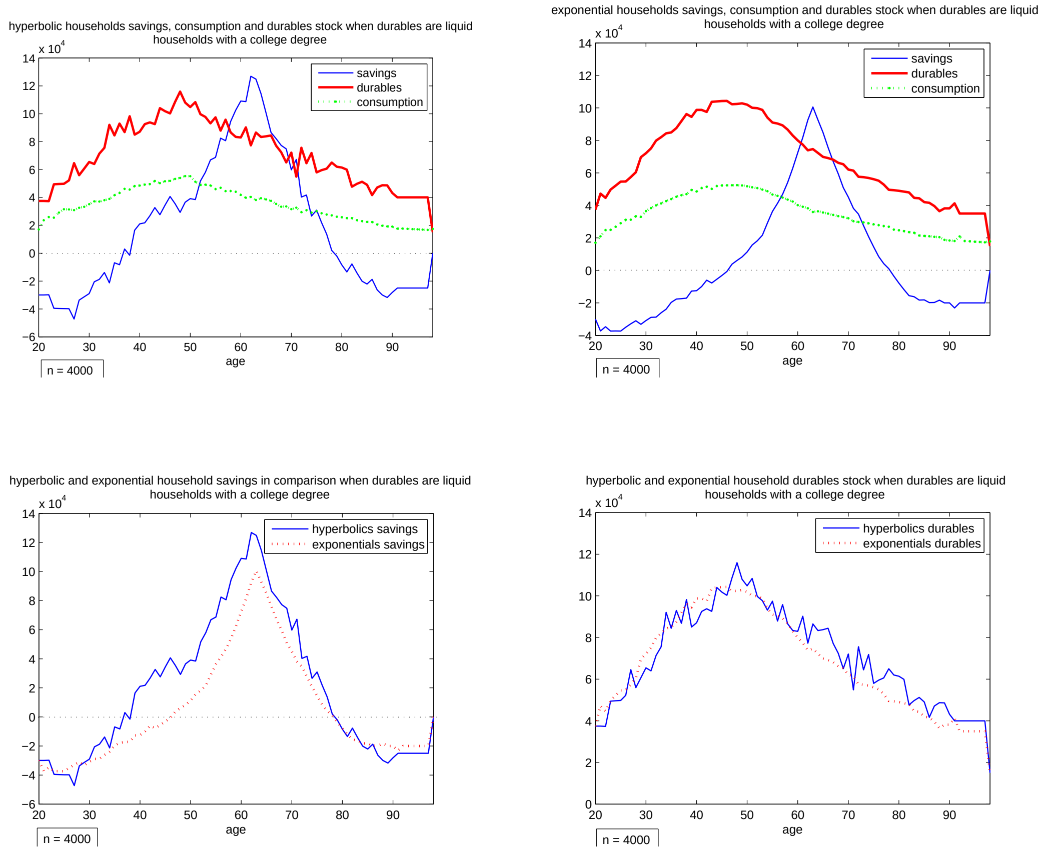

Figure 2.11: Comparison of savings and durables stock of hyperbolic and exponential households when durables are liquid

Households with college degree

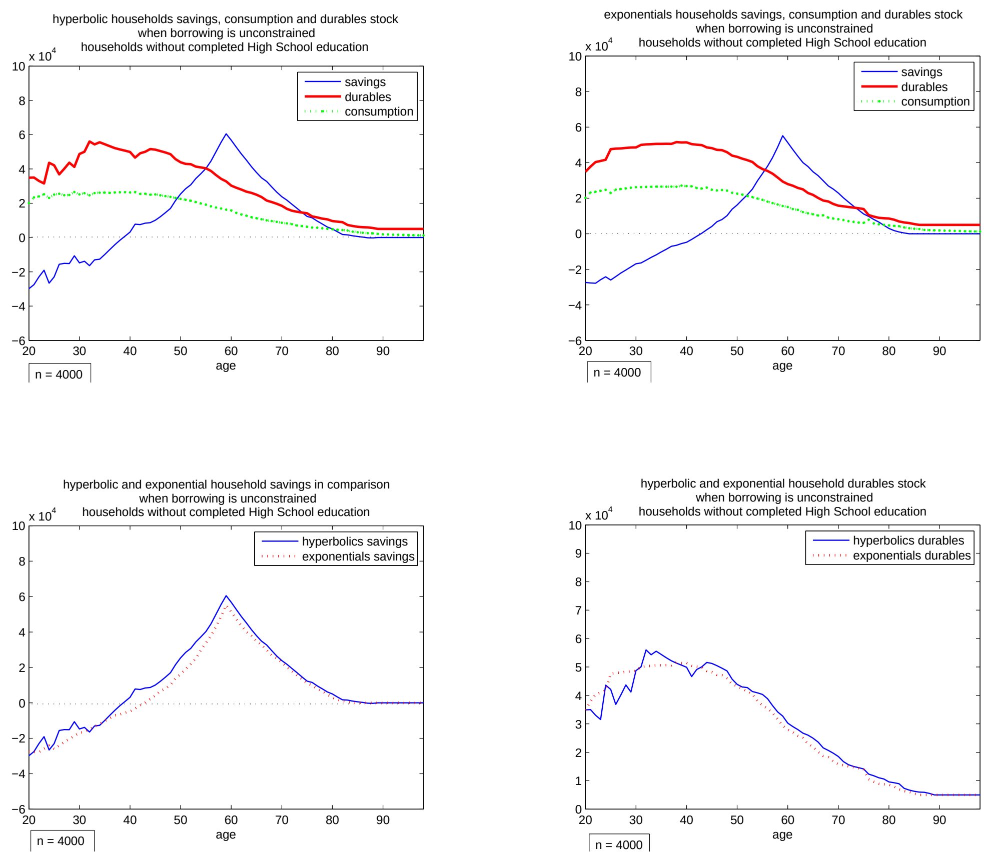

Figure 2.12: Comparison of savings and durables stock of hyperbolic and exponential households when borrowing is unconstrained

Households without completed High School education

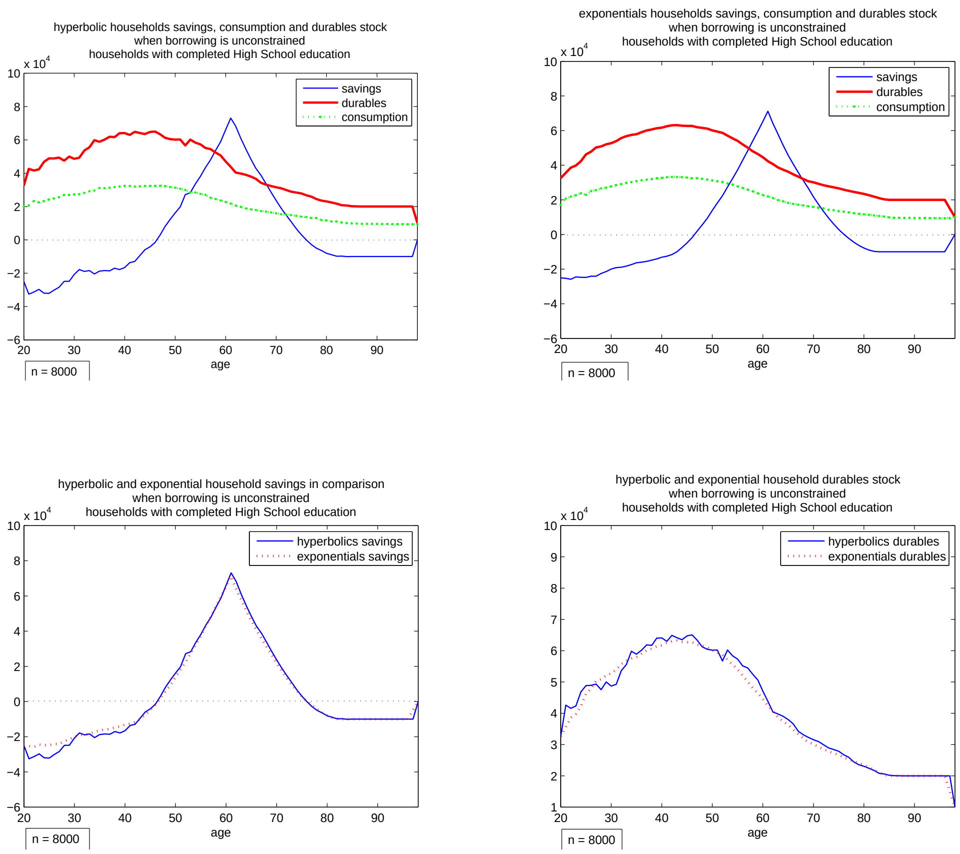

Figure 2.13: Comparison of savings and durables stock of hyperbolic and exponential households when borrowing is unconstrained

Households with completed High School education

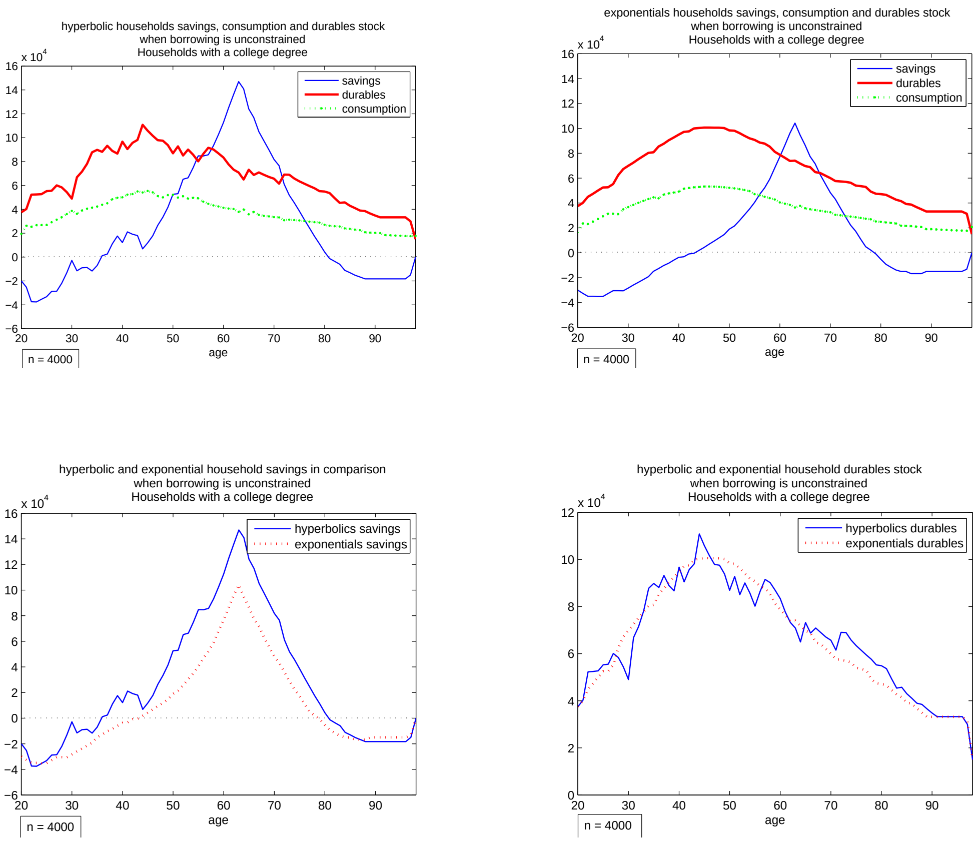

Figure 2.14: Comparison of savings and durables stock of hyperbolic and exponential households when borrowing is unconstrained

Households with college degree

Appendix II

2.B Algorithm

The algorithm used to find the policy functions, is explained using a simpler version of the model, abstracting from the collateral condition, adjustment costs, bequests, life-horizon uncertainty and from changes in family size. From the equilibrium conditions one sees, that each period t the household solves her maximization problem according to the current value (2.10) and the continuation value (2.11)

$$W_{t,t}(x_t, D_{t-1}) = \max_{c_t, D_t} \{u(c_t, D_t) + \beta\delta E_t V_{t,t+1}(x_{t+1}, D_t)\} \tag{2.10}$$

$$V_{t,t+1}(x_{t+1}, D_t) = \max_{c_{t+1}, D_{t+1}} \{u(c_{t+1}, D_{t+1}) + \delta E_{t+1} V_{t+1,t+2}(x_{t+2}, D_{t+1})\} \tag{2.11}$$

subject to the budget constraint, and taking into account sophistication. This is equivalent to solving the problem with durables stock and *savings* as control variables, with the current value function

$$W_{t,t}(x_t, D_{t-1}) = \max_{a_{t+1}, D_t} \{u(x_t - a_{t+1} - d_t, D_t) + \beta\delta E_t V_{t,t+1}(x_{t+1}, D_t)\} \tag{2.12}$$

and continuation value

$$\begin{aligned} V_{t,t+1}(x_{t+1}, D_t) \quad = \quad & \max_{a_{t+2}, D_{t+1}} \{u(x_{t+1} - a_{t+2} - d_{t+1}, D_{t+1}) \\ & + \delta E_{t+1} V_{t+1,t+2}(x_{t+2}, D_{t+1})\} \end{aligned} \tag{2.13}$$

where

$$\begin{aligned} d_t &= D_t - (1-\gamma) D_{t-1} \\ x_{t+1} &= y_{t+1} + R a_{t+1} \end{aligned}$$

Now the household chooses savings and durables holdings according to the policy functions $\left\{a^*_{t+1}, D^*_t\right\}_1^{T+N}$.
Again sophistication implies that the household foresees the recurrence of her present-bias and knows the future savings and durables policies given the state variables and the realization of the income shock.
In order to solve this problem numerically, I closely follow the solution method in Laibson *et al.* (2001), who employ a backwards induction algorithm based on a multidimensional grid search, with a grid for the two states each (durables and asset holdings), as well as a grid for the different realizations of the income shock. Note that the control variables in period t translate into the states in period t+1.

The problem in period T+N

Given the No Ponzi condition that the household cannot die in debt, and given that there is no uncertainty with respect to the life horizon, I specify the asset grid such that minimum feasible savings in period $T+N$ equals zero. For each level of retirement income and for each initial position on asset and durables holdings at the very beginning of period $T+N$, household self $T+N$ chooses a_{T+N+1}, D_{T+N} according to

$$W_{T+N,T+N}(x_{T+N}, D_{T+N-1}) = \max_{a_{T+N+1}, D_{T+N}} \{u\left(x_{T+N} - a_{T+N+1}/R - d_{T+N}, D_{T+N}\right)\}$$

where $x_{T+N} = y_{T+N} + a_{T+N}$ and $a_{T+N+1} \geq 0$. Note that the grid is specified such that in period t, the control variable for savings is a_{t+1}/R and the state variable of asset holdings in period $t+1$ is a_{t+1}. For each realization of the income shock this yields a $(n \times k)$ matrix of savings and durables policies given the combination of state variables at the beginning of $T+N$.

The next step is to update the value function. For each income state the continuation value in period $T+N-1$ is determined by the policy choices in $T+N$.

$$V_{T+N-1,T+N}(x_{T+N}, D_{T+N-1}) = u\left(x_{T+N} - a^*_{T+N+1}/R - D^*_{T+N} + (1-\gamma)D_{T+N-1}, D^*_{T+N}\right) \quad (2.14)$$

The problem in period T+N-1

For each income state and for each initial level of asset and durables holdings, the household in period T+N-1 maximizes her current value according to

$$\begin{aligned} W_{T+N-1,T+N-1}(x_{T+N-1}, D_{T+N-2}) &= \max_{a_{T+N}, D_{T+N-1}} \{u\left(x_{T+N-1} - a_{T+N}/R - d_{T+N-1}, D_{T+N-1}\right) \\ &+ \beta\delta E_{T+N-1} V_{T+N-1,T+N}(x_{T+N}, D_{T+N-1}) \end{aligned}$$

where $V_{T+N-1,T+N}(x_{T+N}, D_{T+N-1})$ is given by (2.14)

Again this yields a $(n \times k)$ matrix for each of the policies for the control variables at each current income state. It also determines the realized continuation value for period $T+N$, $V^*_{T+N-1,T+N}(x_{T+N}, D_{T+N-1})$, which depends on the consumption and savings choices in period $T+N-1$. So far, the solution procedure for the problem of the time inconsistent households is nearly identical to the standard backwards induction algorithm for the time-consistent household. Given the policies a^*_{T+N} and D^*_{T+N-1}, for each income state the continuation value in period $T+N-2$ equals

$$\begin{aligned} &V_{T+N-2,T+N-1}(x_{T+N-1}, D_{T+N-2}) \\ &= u\left(x_{T+N-1} - a^*_{T+N}/R - D^*_{T+N-1} + (1-\gamma)D_{T+N-2}, D^*_{T+N-1}\right) \\ &+\delta E_{T+N-1} V^*_{T+N-1,T+N}(x_{T+N}, D_{T+N-1}) \end{aligned}$$

Here the effect of time inconsistency and sophistication is clearly visible.

While the period $T+N-1$ households discount period $T+N$ utility at rate $\beta\delta$, the household in period $T+N-2$ discounts between period $T+N-1$ and period $T+N$ according to the standard exponential factor δ. Since the problem is solved by backwards induction, each self t of the household makes her savings and consumption decisions taking into account the future policy choices and the present-bias of her preferences (sophistication).

The problem for periods T+N-2 to period 1

The procedure described above is repeated successively for all time periods, up to the maximization problem of the newly born household at period 1. Given income uncertainty, at each income state the expectation of the continuation value $E_t V_{t,t+1}(x_{t+1}, D_t)$ is determined by the transition probabilities from one state into the other, conditional on the current state. As above, the continuation payoff for period $t-1$ is determined by the solution to period t households maximization problem, reflecting the idea of the intrapersonal game played by the different selves. Overall, the algorithm yields $T + N \times m$ matrices of size $(n \times k)$ for each control variable, where m is the number of states in the income process.

General Comments

Since the algorithm is based on a grid search there is a trade-off between accuracy on the one hand and time and storage space saving on the other. Furthermore, due to the time-inconsistency of preferences, the results are sensitive to the fineness of the grid and the minimum value of the asset grid has to be large and negative. Since the grid has to be fine unless it biases the results of the intrapersonal Markov game, computation time can be reduced only by reducing the number of search iterations in each period, which is done by restricting search to those values of the asset grid which lie within a reasonable radius from the preceding policy function. Hence the search range in period t is determined by the policy functions chosen in period $t + 1$. For a large enough search radius this works well since households smooth utility over periods.

Chapter 3

Children and Inequality

Abstract

This study presents some empirical findings on the effect of children on household expenditure in the United States. Estimates from a regression and a propensity score matching exercise using US CEx data show that the effect of children on household expenditure strongly differs for households with different income profiles. For households with a college degree, and, also to some extent for High School graduates, children substantially affect the level of total household expenditure, as was to be expected from economic theory. However, for households without a High School degree, children do not seem to increase total expenditure. The lack of sensitivity of consumption to household size implies that there are strong liquidity constraints for low income households and raises welfare concerns. Also, for welfare analysis the result indicates an additional, permanent source of inequality between households.

JEL classification: D13 D63 E21 I31 I32 J13
Keywords: Household Consumption, Equivalence Scales, Children, Welfare, Inequality

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3.1 Introduction

Even before children enter college and parents have to pay tuition fees, children are expensive. Or, to be more precise, children are expensive, if parents can afford to spend money on them. This may not always be the case and the aim of this paper is to see, whether overall household expenditure is affected differently by children for households of different income categories.
The motivation for addressing this issue in a macroeconomic context is threefold. Firstly, the presence of children affects consumption patterns over the life-cycle. However, life-cycle literature usually abstracts from within-household decision making and the impact of changes in family size usually is treated in a uniform way for all types of households. Secondly research on inequality implicitly assumes that the impact of children on household expenditure is - at least proportionately - the same for all income groups. This means that, so far, household size is not a criterion in the macroeconomic analysis of welfare or consumption inequality. Thirdly, if it is found that liquidity constraints severely restrict some households' ability to react to an increase in family size for increasing expenditure, this strongly points at very low savings or binding borrowing constraints for low income households. Also, to the best of my knowledge, as yet the impact of family size on savings and consumption, and hence on consumption smoothing, so far has not been addressed in macroeconomics.
When assessing inequality or investigating household consumption, macroeconomic theory broadly distinguishes between three types of income profile, determined by the highest level of education reached by a household. Apart from differently calibrated parameter values and income profiles, these groups are assumed to behave according to the same rules. However, macroeconomic studies on inequality commonly do not specifically account for demographic characteristics of households. Nevertheless, it is a very strong, albeit implicit, assumption that the response of household consumption to an increase in the number of household members should follow the same rules, independently of the household's income profile. This, however, does not necessarily hold true.
In order to investigate this issue, a very simple model on household expenditure is estimated on the basis of U. S. Consumer Expenditure Survey data. The results from this estimation indicate that for households with a college degree (and the corresponding income process) household expenditure reacts strongly to the presence of children in the household. To a slightly less extent, this also holds for households with completed High School education. For households without a High School degree, the response of expenditure to children is substantially less pronounced. These findings are reconfirmed in a propensity score matching exercise, which shows that indeed for the households with the lowest income profile, the average treatment effect of children on total expenditure for households with children is not significantly different from zero. The result holds independently of the number of children.
This finding does not only suggest that demographic characteristics should be taken into account when interpreting observations and estimates on consumption inequality, it also hints at strong budget constraints for low income households. In particular regarding rising inequality, the group for which household income has not increased on real terms over the last decades, is also the group whose overall household expenditure does not increase with the presence of children. As a consequence children from low income households will be further disadvantaged by rising inequality. This observation alone raises severe welfare concerns.
Section 2 gives a short summary on equivalence scales and on research on intra-household allocation of resources, and explains how family size commonly is accounted for in macroeconomics. Section 3 describes the data set used in this study, and Section 4 explains the model to be estimated. Section 5 presents the results from a OLS regression and from a propensity score matching exercise. Section 6 concludes.

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3.2 Literature Review

Household size and household composition affect household expenditure and consumption patters. Nevertheless, it is very difficult to establish a causal relationship, since intra-household decisions are barely observable for the researcher.
Microeconomic theory addresses the effect of children on household welfare, or the 'costs' of children. Although there is a richness in frameworks addressing the impact of children on demand, they usually rest on restrictive assumptions. Also, if the effect of children on labor supply is not accounted for, estimates of the effect of children on consumption are likely to be biased (Browning, 1992).
In order to capture the cost of children, the effect of household size on consumption and household welfare is often expressed in equivalence scales. Equivalence scales themselves go back to estimates of physical requirements and the estimation of "what households could consume, rather than on actual household behavior" (Nelson, 1993). They were originally calculated in the 1960ies in order to allow conclusions on the additional amount of income that could equalize consumption of essential goods per scaled household member in poor households of different sizes. Equivalence scales, however, do not take into account that households with children may have different dynamics altogether, i.e. that they have a demand for different goods, and that they may have different preference structures.
Table 3.1 shows examples of different equivalence scales. The scales in the first row go back to the empirical estimates from a demand system analysis in Phibbs (1998). The other two rows are official equivalence scales, which are recommended for the use in poverty analysis. It is worth noting that the equivalence scales from the U.S. Department of Commerce explicitly take into account scale effects in household consumption. The equivalence scales should reflect the 'cost of children' for poor households and denote the additional amount of income a household would have to receive in order to arrive at the same welfare level per household member as a comparable childless household.

Table 3.1: Examples for Equivalence Scales

Family Type: couple with	0 children	1 child	2 children	3 children
Phibbs (1998) - Canada	1	1.16	1.28	1.38
OECD(modified) - unspecified	1.5	1.8	2.1	2.4
U.S Dept. of Commerce - U.S. **	1.68	2.11	2.50	2.88

**Formula: $(adults + 0.7 * kids)^{0.75}$

Another important issue concerns decision making within households. While in macroeconomic models, household utility is assumed to be maximized for the household as a whole, microeconomic theory makes a distinction between the different members' utility and allows for different preferences of household members. Also the decision power within the household can be concentrated in the household head or be distributed among members. In the analysis of intra household decision making, economists usually distinguish between public (within the household) and private goods.
Different empirical studies have tried to distinguish between two extreme scenarios. The first one is the case where only the household head's utility is taken into account, so that in fact there is substitution of resources away from the other household members and towards consumption of the household head. The other possibility is a scenario where all member's utility is maximized equally or according to weights.

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This is either due to perfect altruism of the decision making household head, or due to sharing of decision making power. Consequently intra-household decision making can be a cooperative process which also is determined by the distribution of resources within a household.

Browning and Chiappori (1998) find empirical support that for couples household decisions are made in a collective setting. Utility theory applies to each household member and decisions on the household level are made collectively and are Pareto efficient. Also on the basis of this finding Lise and Seitz (2004) show that intra-household decision is determined by bargaining power within the household. This is important, since the use of equivalence scales for adults would be valid for the determination of adult consumption only if households pool their income on an equal level. Indeed, the authors arrive at the conclusion that between 1968 and 2001 in the United Kingdom within-household consumption inequality has been decreasing partly because female labor income has been increasing steadily. Given that children have zero income, their position in the collective setting is very weak.

In macroeconomics there are two areas of research for which intra-households demographics can have particularly strong implications: life-cycle analysis and empirical work on welfare inequality.

For once, a household's optimum consumption decisions strongly vary over the life cycle. From an empirical perspective household age and household composition are strongly correlated, so that it is difficult to separate the effect of life-cycle factors from the effect of children. Also the expectation of future fertility can be an important driving force behind current consumption and savings patters, but remains entirely unobserved. In addition, household decomposition often is endogenous, since the decision to have children is consciously made.

The key assumption in the life-cycle framework is that households smooth consumption over periods as to keep the marginal utility of wealth constant as specified by Friedman (Friedman, 1957). However, there is no consensus how household utility is to be specified, and a common assumption is that consumption and labor supply are additively separable in the utility function, while there is no standard way to include demographics. As Blundell *et al.* (1994) find, this approach can be quite misleading, since it implies that household consumption is decided on independently of labor supply and demographics characteristics.

Along the same lines, in the past a main criticism of the life-cycle model has been that - in its simple specification - it is not able to replicate the empirically observed hump-shaped consumption profile. More recently, technological progress has allowed to move beyond certainty equivalence and perfect foresight, towards a more realistic treatment of uncertainty, and a replicated life-cycle consumption profiles have come closer to observed patterns. Importantly, however, Attanasio *et al.* (1999) show that demographic factors can improve the performance of the life-cycle model remarkably, even if certainty equivalence and perfect foresight still are assumed. They find that when the discount factor is estimated empirically, it strongly grows with household size. As a result, simulated household consumption is high when household size peaks, even if family size does not directly enter the utility function.

Independently of the specification of the household problem, it is a fixed feature of the life-cycle literature that the decision making body is the household per se, and that no distinction is made between different members. While family size may enter the part of the utility function concerning consumption (since preferences usually are assumed to be additively separable), as in Laibson *et al.* (2001), usually this only is done in order to reflect that larger households consume more. Depending on the weights given to household size, there is room for economies of scale of consumption within the household, even if consumption (at least of perishable goods) is considered partly private. Nevertheless, the question which household members decide on consumption is not addressed, also the allocation of consumption between the different household members is assumed stable and the same for all households. Consequently, while household size can drive total household expenditure on an 'ad hoc basis', equal allocation of consumption within the household is presumed rather than resting on theoretical or empirical foundations.

The second field of macroeconomic research for which household size and composition could matter is welfare and inequality analysis. In order to learn about household welfare, household expenditure is used as a proxy for consumption. Given that households are assumed to smooth consumption, expenditure is considered to better reflect expectations life-time resources that current income does (Blundell and Preston, 1998). Volatility in expenditure also allows for conclusions on insurance towards income shocks. Blundell and Preston (1998) assess the reliability of consumption as a measure of welfare, and find that consumption volatility can serve as a measure for welfare inequality within cohorts, but that across cohorts comparison is problematic, since inequality increases with age. Taking this into account, the authors find a strong increase in short term income uncertainty in Great Britain during the 1980ies. Nevertheless income inequality has increased faster than consumption inequality.

In welfare analysis, a very straight forward strategy of dealing with households of different size and composition is to concentrate on consumption per capita within a household. In order to capture economies of scales, consumption per capita is calculated on the basis of equivalence scales, where economies of scale mainly result from shared consumption of a public good. The use of equivalence scales also is conform to the idea of all household members' utility being accounted for in the consumption decision, so that the objective is to effectively neutralize household composition and household size as household characteristics. After this adjustment, direct comparison of the welfare of members of households of different sizes is possible.

This technique has also been used in Blundell and Preston (1998) in their assessment of the evolution of inequality in Great Britain. Other prominent examples where equivalence scales have been use to facilitate comparison of welfare of households of different sizes are Deaton and Paxson (1994) and Krueger and Perri (2006).

Deaton and Paxson (1994) investigate whether, within age cohorts, the increase in inequality over lifetime, as predicted by the Permanent Income Hypothesis is observed in consumption data for Great Britain the United States and Taiwan. The authors recognize that household composition and structure change over the life-cycle, but find that any alternative to specifying consumption at the household level would be arbitrary. Consequently the authors compare their results for consumption dispersion for the household level and for adult equivalents. In both cases, for the three countries the empirical results support the Permanent Income Hypothesis. However, the effect of dispersion of family size on dispersion of consumption is very strong in Taiwan, but very small in the United States.

Krueger and Perri (2006) compare the evolution of income and consumption inequality in the United States on the basis of CEx data for the period between 1980-2003. The authors distinguish 'between-group' and 'within-group' inequality, where households are divided into groups according to education, sex and race. Again equivalence scales are used in order to eliminate the effect of household size on consumption inequality across households. The key finding is that within those groups, consumption inequality has increased a lot less that income inequality. Between-group inequality in consumption, however, closely tracks income inequality. An explanation is that while idiosyncratic income inequality has increased, financial markets now allow households to insure against income fluctuations. This hypothesis also is supported by the observation that consumer credit and income inequality both show a very similar upwards trend.

This result is contradicted in Gorbachev (2007), where PSID data on food expenditure are used to investigate consumption inequality in the United States between 1970 and 2002. After taking into account changes in interest rates and household composition, there is evidence that volatility in food consumption has increased over the sample period and that inequality has increased especially among disadvantaged

households. Those households are also those most likely to be unable to smooth consumption over transitory income shocks.
Studying inequality in a life-cycle context, Kaplan (2007) evaluates the life-cycle model along various dimensions of inequality using structural estimation techniques on the basis of the PSID and the CEx data sets. Here, welfare is assumed to be determined not only by consumption but also by labor supply decisions, making the the analysis of welfare inequality more complex. While this very rich paper does not investigate the evolution of inequality within income cohorts, it illustrates that for welfare inequality over the entire life-cycle, shocks after the entry into the labor market have a similar impact as differences in initial human wealth. Nevertheless, since equivalence scales are used in order to control for differences in family size, the effect of children on inequality - either directly affecting welfare via consumption, or indirectly via labor supply decisions -, remains unexplored.
Welfare inequality also is closely related to the issue of poverty. In context of the present study, it is child poverty that carries particular weight. In the United States, poverty is measured in absolute levels, and a national poverty standard is used which has been kept fix in real terms since the late 1960ies. Consequently changes in inequality are not necessarily reflected by official poverty rates, as described in Dickens and Ellwood (2003). On the Background of rising income inequality in the United States, an increase in relative poverty may not even be captured in official statistics on child poverty. Since child poverty is measured in terms of parents' income, it is not only essential to learn how income inequality is translates into consumption inequality, but also whether households expenditure can react to the presence of children. If liquidity constraints prevent poor households from increasing their expenditure even if family size increases, then this is a clear indicator for child poverty, irrespectively of any official benchmark defined by parent's income.

3.3 Data

The data used in this paper come from the fmly files of the CEx Data set, covering 20 years from 1986 to 2005. The data source are interviews with US households, conducted by the Bureau of Labor Statistics. Ideally, each quarter, the households report information of family characteristics, income and quarterly expenditures in five consecutive interviews. The advantages of this data set are that the expenditure data are very detailed, and that there is a wide range of information on household composition and other characteristics. A shortcoming is that all household information is self-reported, and hence subject to moral hazard and issues of misspecification. Furthermore the short span of coverage for each household reduces makes intertemporal analysis difficult and often necessitates the construction of pseudo-panels with all their shortcomings. While expenditure and income are originally reported at their nominal level, in this paper the increase in consumer prices has been taken into account and expenditure and income are converted to 1984 USD, on the basis of the CPI. Only data of households reporting to all five consecutive interviews is taken into account.
Also in the following only households under 65 years and with an after tax income above the minimum wage[1] and below USD 200.000 are included.
All household members younger than 18 years are considered children, and the maximum number of children is restricted to three.
The measure of expenditure in this paper does not contain expenditure on any goods and services related

[1]The annual minimum wage is roughly calculated by taking the US. Fed minimum hourly wage of USD 5.15, multiplied by 40 (weekly hours), and then by 50 (working weeks).

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to education. This is mainly due to expenses on education like tuition and school fees being strongly dependent on the age of children. Furthermore tuition fees, and expenditure on education in general, can not necessarily be compared to other expenditure on consumption goods. Another reason for this decision is that expenditure on education is very heterogenous and likely to be highly correlated with household income.

3.4 Model

The model assumes that household expenditure is determined by household income, age of the household head, race, the number of grownups present in the household, the number of children, and the household head being single or part of a couple. This is formulated in a linear regression model, where logs are taken for expenditure and income (3.1) The estimation further controls for cohort and year effects.

$$\begin{aligned} ln(cons_i) &= \alpha_1^j ln(income_i) + \alpha_2^j age_i + \alpha_3^j age_i^2 + \alpha_4^j race_i + \alpha_5^j grownups_i \\ &+ \alpha_6^j kids_i + \alpha_7^j couple_i + \alpha_8^j cohort_i + \alpha_9^j year_i + u_i \end{aligned} \tag{3.1}$$

where $j = \{nHS, HS, Col\}$.
The households are separated into education categories, since education is considered the main determinant for the household's income process. Regarding the dummy variables, if the household head is white, $race = 1$, and if the household head is black $race = 2$. If the household head is single, $couple = 1$, if the household head is part of a couple, $couple = 2$.
This model of course is a very simple formulation of the relationship between household expenditure and demographic characteristics. Therefore the results should not be interpreted beyond their descriptive value. In this sense the main message of the regression results will be, whether overall household expenditure is affected differently by children for households of different income categories.
The main reason for this limitation is that it is reasonable to assume that the fertility decision is correlated with age and income, and also with the household head being part of a couple or being single. Consequently, while endogeneity of the explanatory variable maybe less of a problem, strong causal relationships between the explanatory variables are possible. This calls for caution regarding a causal interpretation of the coefficients. Consequently it is not within the scope of this analysis to establish a clear causal link between children and household expenditure.

3.5 Results

3.5.1 Estimation Results of the Model

Table 3.2 shows the OLS results for the estimated model. Nearly all explanatory variables are highly significant. Only the year of the observation does not have any significant effect on expenditure. The negative coefficient for the household cohort shows that older cohorts have greater expenditure, and might capture additional age-group effects. What is striking is that the effect of children on household expenditure increases substantially with the education of the household head. Indeed, the effect of children on household log expenditure for households with a college educated head is roughly three times as large as for households, where the household head does not hold a high school certificate. Given the semi-log formulation of the model, this is equivalent to saying that household expenditure grows three

time faster with each child for college educated households than for households without High School degree, and two times faster when the household head completed his High School education.
The results clearly show that the household's income process has a strong effect on how household consumption - and hence welfare - adjust to the presence of children. This hints at important liquidity constraints for the group of households with the lowest income.
Of course it is not observable how children affect consumption within the household itself. Also it is very likely that households substitute a larger number of cheaper goods for more expensive ones when liquidity constraints are binding. Often there are also scale effects if goods can be shared among household members. Nevertheless, as the theory of equivalence scales indicates, all household members are worse of if overall household expenditure cannot adequately increase with the number of household members. Alternatively, depending on the intra-household allocation of resources, there can be substantial substitution of consumption from children to parents. From a policy perspective, viewed from any of those two angles, the results described above raise severe welfare concerns.

3.5.2 Matching Exercise

Given the fact that the data analyzed are very heterogeneous, the results from the above estimation are more closely examined in a matching exercise. In order to account for at least some of the possible correlation between the explanatory variables, in particular between the number of children, age and income, I use the propensity score matching technique developed by Andrea Ichino and Sascha Becker for the estimation of Average Treatment Effects on the Treated (ATT). This technique has been originally designed in order to reduce the selection bias, which is problematic in the estimation of treatment effects. The matching method compares the outcomes for treated subjects with the outcomes of a control group. Importantly, based on their observable characteristics, subjects of the treatment and control group are allocated in clusters. Within each cluster subjects of both group have approximately the same probability of undergoing the treatment.
In the current framework, this translates into estimating the effect of children on household expenditure by comparing expenditure of households with children to households which are very similar in all characteristics but are childless. Given that in this case there are 3 different treatments, namely one child, two children or three children in the household, all three treatment situations have to be separately compared to the control group of childless households. The propensity score, as defined by Rosenbaum and Rubin (1983) is the conditional probability of receiving the treatment given the pre-treatment characteristics, where $D = \{0, 1, 2, 3\}$ is the indicator for exposure to the different treatments, and X the multidimensional vector of pretreatment characteristics.

$$p_g(X) \equiv Pr\{D = g \mid X\} = E\{X \mid X\}$$

where $g = \{1, 2, 3\}$.
For each education group, and as explained in Becker and Ichino (2001), the Average Treatment Effect on the Treated (ATT) of the "treatment" children on expenditure Y, is estimated as follows

$$\begin{aligned} ATT_g &\equiv E\{Y_{gi} - Y_{0i} \mid D_i = g\} \\ &= E\{E\{Y_{gi} - Y_{0i} \mid D_i = g, p(X_i)\}\} \\ &= E\{E\{Y_{gi} \mid D_i = g, p(X_i)\} - E\{Y_{0i} \mid D_i = 0, p(X_i)\} \mid D_i = g\} \end{aligned}$$

In order to allow for efficient matching, the sample is narrowed down to a subsample. Only white households with household head and spouse, and without any additional grownups are compared. Also excluded are households who derive means from other sources than income. In order to allow for a better comparison, and also due to demographic reasons, the household age is restricted to above 23 and below 55 (65 for households with college education and 3 children). Also, there is an upper bound to household income, to further ensure comparability [2]. It should be noted that in the following the levels effects on expenditure are compared, so that the coefficients show the absolute effects of the explanatory variables.

3.5.3 Results from the Matching Exercise

The results from this matching exercise are are shown in Tables 3.3, 3.4 and 3.5 for households with respectively, no High School certificate, completed High School education, and a college degree. Four different standard estimation techniques are used and mostly the results for the different estimator are rather similar. For households with unfinished High School education, the ATTs in Table 3.3 are not significantly different from zero, independently of the estimation technique and the number of children assigned as treatment. Compared with comparable childless households, neither one child, nor two, nor three children increase household expenditure. This is a very strong result and in line with the welfare concerns mentioned above. The result further implies that High School drop outs are not able to save or borrow due to liquidity constraints imposed by their low income, since otherwise expenditure could react to changes in household size. Consequently this group of households is likely to be particularly vulnerable to negative income shocks. This observation also is in line with the result in Gorbachev (2007) which points to greater inequality in consumption between poorly educated households.

As Table 3.4 shows, for households with a High School degree, expenditure increases when there are children, although the increase is greater when the household has two children, than when there are three children. Applying the idea of equivalence scales to this group of households, household members in households with three children should be worse off, than in households with only two children. This may in part be explained by endogenous decisions on labor supply. While only households with household head and spouse are analyzed, in households with three children there might be a strong motive for one parent to stay at home, which overall reduces household earnings. This could somewhat reduce expenditure, as compared to households with only two children.

When the household head holds a college certificate, the reaction of expenditure is most striking, as seen from Table 3.5. Household expenditure steadily increases with the number of children. College educated households seem the only ones where the welfare (in consumption terms) of the individual household members is not affected by an increase in the number of children. It appears that this group of households is the only group not to be substantially constrained by illiquidity.

The results are summarized in Figure 3.1, where the Kernel estimates for the ATTs are compared with each other for households with High School certificate and college degree. Since the ATTs for households without completed High School education are not significantly different from zero, they are not included in the graph.

Of course, this exercise focuses rather on differences in sensitivity of expenditure to children, than on establishing actual level effects. For the latter, even after the sorting into propensity scores, the sample

[2] For households without High School degree, maximum annual income after tax is 65 000 USD, for households with a High School degree maximum income is 75 000 USD and when the household head holds a college degree maximum income is restricted to USD 85 000.

still is far too heterogeneous, which also is reflected in the relatively high standard errors.
The real message of these results is the following. Abstracting from the direct effect of children on parents's happiness, in terms of consumption per household member, households with children are clearly worse off than households without children, if consumption cannot increase adequately with family size. In macroeconomic data analysis family size is controlled for by applying equivalence scales to households of different sizes, which have different per capita levels of welfare even at the same income and at similar levels of expenditure (the very concept of equivalence scales). Provided that the chosen equivalence scale is suitable, this correctly reflects welfare inequality between representative members of households of different sizes, but does not attribute it to its actual source. In welfare analysis, the consequence is that part of the observed inequality in scaled per capita consumption is not due to income inequality, or income volatility, depending on the framework of analysis. This, however, should not be (positively) interpreted as greater consumption smoothing across transitive income shocks, or as overall lower inequality. To the contrary, it means that a considerable amount of inequality between households with and without children is inherent in the permanent income component. And, given the results above, the negative effect of children on household welfare is much greater for households with lower permanent income than for households in the high income brackets. In other words, a great part of welfare inequality among the poor, but also within the lower middle class, is attributable to differences in the number of mouths to feed and in strong liquidity constraints on how this can be done. Consequently it is unrelated to transitory consumption smoothing and also cannot be alleviated by improving the performance of financial markets for consumer credit.

3.6 Conclusion

From the above exercise it is clear that the response of household consumption to an increase in the number of household members strongly depends on on the household' income profile. For life-cycle analysis this has the implication, that taking into account family size demands a very careful approach. Regarding welfare inequality the following issues are of importance.
Firstly, assessing welfare across households while controlling for differences in household size by using equivalence scales, still yields very valuable results. Nevertheless, when, for example, welfare inequality is investigated, part of the differences in variety of consumption across different income groups will be due to differences in the effect of household size on overall household consumption. While this correctly reflects differences in welfare, the link between consumption or liquidity constraints and household size is not longer recognizable as an additional source of inequality.
Secondly the results raise concrete welfare concerns. The observation that for low-income households expenditure hardly reacts to the presence of children strongly hints at binding liquidity constraints. Consequently it is very likely that for those households precautionary and life-cycle savings are restricted by low income. This issue demands closer investigation, since it would mean that irrespectively of the presence of children, low income households are extremely vulnerable to income shocks. In addition, if low income is reflected in minimum expenditure on children, then this is a strong argument for increased child support of low income families with children. This holds true especially in the light of growing income inequality in the US, and given the observation that real wages for low income workers have hardly increased in the last twenty years.

3.7 Bibliography

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Appendix I

3.A Figures and Tables

Table 3.2: OLS estimation results

	no High School certificate	High School certificate	College degree
constant	4.3222$^{*\cdot**}$	4.6433$^{*\cdot**}$	4.8985$^{*\cdot**}$
	(0.1436)	*(0.0662)*	*(0.0847)*
α_1	0.5057$^{*\cdot**}$	0.4840***	0.4265$^{*\cdot**}$
	(0.0078)	*(0.0037)*	*(0.0049)*
α_2	.0122$^{*\cdot**}$	0.0138***	0.0266$^{*\cdot**}$
	(0.0034)	*(0.0017)*	*(0.0027)*
α_3	$-$.0002$^{*\cdot**}$	$-$0.0002***	$-$0.0003$^{*\cdot**}$
	(0.00004)	*(0.00002)*	*0.00003*
α_4	$-$0.017*	$-$0.0528$^{*\cdot**}$	$-$0.0445$^{*\cdot**}$
	(0.0069)	*(0.0035)*	*(0.004)*
α_5	0.0613$^{*\cdot**}$	0.0679***	0.0686$^{*\cdot**}$
	(0.0054)	*(0.0033)*	*(0.0054)*
α_6	0.0139$^{*\cdot**}$	0.0259***	0.0395$^{*\cdot**}$
	(0.0038)	*(0.0021)*	*(0.0031)*
α_7	0.2233$^{*\cdot**}$	0.2058***	0.2352$^{*\cdot**}$
	(0.0114)	*0.0054*	*(0.0076)*
α_8	$-$0.0336$^{*\cdot**}$	$-$0.0260$^{*\cdot**}$	$-$0.0356$^{*\cdot**}$
	(0.0080)	*(0.0038)*	*(0.0051)*
α_9	0.00002	-0.00003	-0.000004
	(0.00005)	*(0.00002)*	*(0.00002)*
R^2	0.5653	0.5518	0.5187

$*$ significant at 10 % level, $**$ significant at 5% level

$$\begin{aligned} ln(cons_i) &= \alpha_1^j ln(income_i) + \alpha_2^j age_i + \alpha_3^j age_i^2 + \alpha_4^j race_i + \alpha_5^j grownups_i \\ &+ \alpha_6^j kids_i + \alpha_7^j couple_i + \alpha_8^j cohort_i + \alpha_9^j year_i + u_i \end{aligned}$$

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Table 3.3: No High School Certificate - Average effect of Treatment on Treated

ATT	1 child	2 children	3 children
nearest neighbor	-0.722	1.307	-0.547
	(1.235)	*(1.182)*	*(1.174)*
radius matching	-0.142	-0.125	-0.793
	(-0.837)	*(0.733)*	*(0.721)*
kernel matching	-0.034	0.076	-0.490
	(0.710)	*(0.715)*	*(0.776)*
stratification	-0.066	0.114	-0.365
	(0.864)	*(0.894)*	*(0.916)*

in thousand USD

$*$ significant at 10 % level, $**$ significant at 5% level

Table 3.4: High School Certificate - Average effect of Treatment on Treated

ATT	1 child	2 children	3 children
nearest neighbor	$1.345^{*,**}$	$2.198^{*,**}$	0.747
	(0.520)	*(0.475)*	*(0.683)*
radius matching	0.687	$1.442^{*,**}$	0.875^{*}
	(0.333)	*(0.334)*	*(0.363)*
kernel matching	0.625	$1.514^{*,**}$	$1.021^{*,**}$
	(0.302)	*(0.327)*	*(0.361)*
stratification	0.689	$1.445^{*,**}$	$1.378^{*,**}$
	(0.361)	*(0.343)*	*(0.471)*

in thousand USD

$*$ significant at 10 % level, $**$ significant at 5 % level

Table 3.5: College degree - Average effect of Treatment on Treated

ATT	1 child	2 children	3 children
nearest neighbor	1.584	2.199*,**	1.845
	(0.883)	*(0.729)*	*(1.141)*
radius matching	1.359*	2.487*,**	3.312*,**
	(0.554)	*(0.552)*	*(0.659)*
kernel matching	1.438*	2.479*,**	3.362*,**
	(0.561)	*(0.552)*	*(0.742)*
stratification	1.592*	2.572*,**	3.726*,**
	(0.630)	*(0.596)*	*(0.741)*

in thousand USD

∗ significant at 10 % level, ∗∗ significant at 5 % level

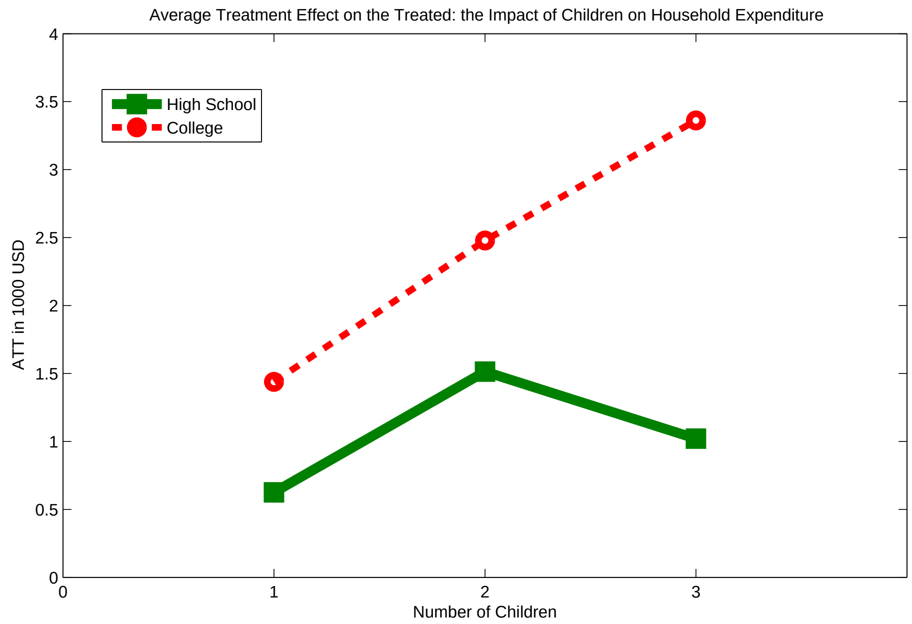

Figure 3.1: Comparison of ATTs, for households with college and High School education: Kernel estimators