European University Institute

Department of Economics

Essays in Applied Macroeconomics

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Thesis submitted for assessment with a view to obtaining the degree of Doctor of Economics of the European University Institute

Florence
December 2007

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Dedicated to Julie and Lia

Acknowledgements

The first person to thank is my supervisor Giancarlo Corsetti, who has taught me a lot. Giancarlo never failed to motivate me in my work and was of great help from the start. I am also very grateful for his unconditional support on the academic job market. I would also like to thank my second supervisor, Rick van der Ploeg, and Morten Ravn for their advice, teachings and sense of humor. Pascal Courty and Pierpaolo Benigno have also generously helped me in many ways.

This thesis, and other work that has not been included, has benefited greatly from conversations with many other people, among which I list: Jeffrey Campbell, Fabio Canova, Wouter Den Haan, John Duca, Aitor Erce, René Garcia, Mark Gertler, Francesco Lippi, Helmut Lütkepohl, John Muellbauer, Elvira Prades, Pontus Rendahl, Thomas Sargent, Fabio Schiantarelli, Harald Uhlig, and Sanne Zwart.

I am indebted to Nobuhiro Kiyotaki, Rick van der Ploeg, Danny Quah, Rachel Ngai and Andrei Sarychev for introducing me to modern macroeconomics at the London School of Economics. I am also grateful to Marc de Clercq, Geert Dhaene, Jozef Plasmans, Rudi Vander Vennet, Dirk van de gaer and Freddy Heylen for introducing me to economics and for encouraging me to continue my studies abroad after my time at Ghent University.

This thesis would not exist without the financial support of the Belgian government, the European University Institute, and most importantly, my parents.

Most of all, I am indebted to my wife, Julie Serreyn, who followed me to Florence and who has never failed to support me.

Contents

Preface

This thesis contains several lines of research conducted during my four years at the European University Institute. It consists of three chapters that are all in the area of applied macroeconomics, but are built on distinct ideas.

The first chapter, "*How the Removal of Deposit Rate Ceilings Has Changed Monetary Transmission in the US: Theory and Evidence*" is concerned with US monetary history and the impact of institutional changes in the late 1970s and early 1980s on the monetary transmission mechanism. The chapter presents evidence on a phenomenon of disintermediation occurring during the major recessions in the 1960s and 1970s, but absent ever since. Using a novel data set, I show that disintermediation is closely linked to the existence of deposit rate ceilings that were imposed under regulation Q of the Federal Reserve System. Disintermediation potentially has real effects if the resulting shortage of loanable funds forces banks to cut back on lending to borrowers that rely on intermediated finance. The main hypothesis of Chapter 1 is that regulation-induced disintermediation affected the monetary transmission mechanism during the 1960s and 1970s and provided the Federal Reserve with greater leverage over real activity than afterwards. In a monetary DSGE model that incorporates deposit rate ceilings as occasionally binding constraints, I show how the regulation alters the behavior of money aggregates and exacerbates the drop in economic activity following a monetary tightening. The results of a threshold VAR lend support to the main theoretical predictions of the model. This chapter contributes to establishing the existence and nature of changes in the conduct and transmission of monetary policy since the mid 1980s, which is key in understanding the remarkable macroeconomic performance of the US since then.

Chapter 2, titled "*The Role of Expectations in Sudden Stops*", studies the abrupt declines in capital inflows, usually accompanied by depression-sized, but short-lived, contractions in economic activity, that have plagued so many countries in the last 25 years. It proposes a new framework for the analysis of these "Sudden Stops" and applies it to the case of the Korean Crisis in 1998. The chapter presents a flexible-price small open economy model that faces a "peso problem" in productivity states. Agents rationally adjust their beliefs about future productivity growth after the arrival of news. A downward revision of expectations triggers a Sudden Stop, together with large declines in GDP, employment, consumption and investment. One of the distinctive features of the model is that there need not be any actual change in productivity growth to generate large fluctuations. Quantitatively, the model goes a long way in matching the 1998 Korean Crisis and subsequent swift recovery by considering a sudden deterioration of expectations about the future.

The last chapter, "*Business Cycle Analysis and VARMA Models*", is joint work with Christian Kascha and is more methodological in nature. We address the question whether long-run identified Structural Vector Autoregressions (SVARs), which are a popular tool in applied macroeconomics, can in practice discriminate between competing models. Some authors have recently suggested that SVARs may fail to do so partly because they are finite-order approximations to infinite-order stochastic processes. We estimate VARMA and state space models, which are not misspecified, using simulated data and compare true with estimated impulse responses of hours worked to a technology shock. We find little gain from using VARMA models. However, state space models do outperform SVARs. In particular, subspace methods consistently yield lower mean squared errors, although even these estimates remain too imprecise for reliable inference. Our findings indicate that long-run identified SVARs perform weakly because of small sample problems, not because of the finite-order approximation.

Chapter 1

How the Removal of Deposit Rate Ceilings Has Changed Monetary Transmission in the US: Theory and Evidence

1.1 Introduction

Output and inflation volatility in the US have dropped considerably since the early 1980s, which suggests a fundamental change in the dynamics of the economy.[1] So far, no consensus has emerged on the fundamental causes of this Great Moderation. Many, such as Clarida, Gali and Gertler (2000) and Cogley and Sargent (2001, 2005), focus on shifts in monetary policymaking, arguing that the Federal Reserve has become more successful in fighting inflation and stabilizing economic activity. Others, such as Bernanke and Mihov (1998) and Sims and Zha (2006), find little evidence for a break in the conduct of monetary policy. The focus of this paper is on one aspect of monetary policymaking for which structural change is a historical fact: Regulatory deposit rate ceilings and their removal in the early 1980s.

After the banking collapse of the 1930s, US legislators imposed a regulatory structure on the US banking sector aimed at restoring financial stability. The Banking Act of 1933 introduced regulation Q of the Federal Reserve, prohibiting interest payments on demand deposits and imposing interest rate ceilings on time and savings deposits at commercial banks. The purpose of the regulation was to shelter banks from excessive competition, discourage risky investment policies and prevent future bank failures. Most of the ceilings were phased out between 1980 and 1986.

This paper provides evidence, based on data constructed from historical Federal Reserve releases, that binding deposit rate ceilings gave rise to a phenomenon of "disintermediation": Unable to raise deposit rates above the legal ceilings, banks could not compete effectively with market instruments and failed to manage their liabilities in the same way as without binding regulations. Disintermediation potentially has real effects if the resulting shortage of loanable funds forces banks to cut back on lending to borrowers that rely on

[1] The evidence on the Great Moderation is discussed by Kim and Nelson (1999), as well as McConnell and Perez-Quiros (2000) and Blanchard and Simon (2001).

intermediated finance. In that case, regulation Q affects the monetary transmission mechanism and provides the Federal Reserve with greater leverage over real activity. The Fed's tighter control over bank liabilities with binding ceilings may even have contributed to business cycle volatility during the 1960s and 1970s. This hypothesis deserves attention, since in contrast with the post-1980 years, every recession during this period is associated with outflows in all deposit categories at US banks.

To formalize the argument, I construct a theoretical DSGE model that incorporates occasionally binding interest rate ceiling constraints and solve it numerically. Attention is restricted to monetary policy innovations and I compute impulse responses to an unanticipated decrease in money supply growth with and without a binding constraint. With binding regulation, the same decrease in money growth produces a larger drop in output. The model has several implications regarding the relevance of a regulation-induced disintermediation channel of monetary transmission in US data: In the presence of a binding ceiling, the spread between market interest rates and the ceiling widens; interest-bearing bank liabilities are constrained; the liquidity effect of a monetary tightening is larger; non-interest bearing money holdings respond little to interest rate increases; the contraction in lending to firms is more severe and inventories-to-sales ratios are lower.

I confront the theoretical predictions with the data by estimating a structural VAR that allows for regime shifts in the autoregressive coefficients. The empirical model captures the nonlinearities of binding deposit rate ceilings by exploiting information contained in the spread between market interest rates and the regulatory ceilings. In response to an identified positive innovation in the Federal Funds rate, binding ceilings exacerbate the contraction in output. Moreover, the response of the spread, the monetary aggregates and real lending are consistent with the theoretical predictions. These findings lend support for a structural change in the monetary transmission mechanism since the early 1980s that is due to the disappearance of regulation-induced disintermediation effects. The results also imply a role for regulation Q in shaping macroeconomic outcomes during the 1960s and 1970s in the US. A counterfactual experiment, in which the effects of regulation Q are removed from the 1960s-1970s data, accounts for over half of the reduction in output volatility since the early 1980s.

The rest of the paper is organized as follows: Section 1.2 presents evidence of the disintermediation phenomenon during the 1960s and 1970s; Section 1.3 describes the monetary DSGE model and its quantitative properties; Section 1.4 lays out the empirical strategy, and discusses the estimated impulse responses and the counterfactual experiment; Section 1.5 concludes.

1.2 Disintermediation and Deposit Rate Ceilings

Regulation-induced disintermediation occurs when depository institutions experience drops in deposit inflows because legal ceilings prevent the payment of the higher interest rates offered on market instruments. This phenomenon was described by, for instance, Friedman (1970) as well as Ruebling (1970), Cook (1978) and Gilbert and Lovati (1979). In this paper, I distinguish two major components of the liability side of depository institutions' balance sheets: *Core deposits*, consisting of all checkable and savings deposits; and *managed liabilities*, which are defined as the sum of small and large denomination time deposits, dollar-denominated deposits issued by foreign banks (also known as eurodollar deposits), and security repurchase agreements. Core deposits have traditionally been, and still are, quite interest-sensitive, with rising interest rates leading to outflows as agents substitute towards higher yielding alternatives. By varying the rate of interest offered on other types of deposits, banks can control the total deposit inflow and "manage" their liabilities. To the extent that investors substitute towards these alternatives, banks are able to maintain the pool of loanable funds. The larger the spread between rates on alternative saving instruments and the rates offered by depository institutions, the bigger the opportunity cost of holding deposits. If regulations constrain interest rates paid on managed liabilities, banks, not being able to offer competitive yields, may fail to offset losses in core deposits. The disintermediation effect occurs when, for these reasons, binding regulatory ceilings cause slowing or negative growth in all deposit categories.

Figure 1.1 plots real growth of core deposits and managed liabilities at US depository institutions from 1960 to 2005. From the late 1970s onwards, there is a negative correlation between the two series, which suggests that banks have successfully counteracted losses in core deposits. Before, however, the picture is quite different: Every NBER-dated recession is associated with slowing or negative growth in *both* core deposits and managed liabilities. If this fact is to be explained by a regulation-induced disintermediation effect, it needs to be the case that the ceilings constituted binding constraints at those instances. Note for example that there is one early episode (1966 and early 1967) of negative growth in core deposits during which banks were able to expand their use of managed liabilities. To identify the periods in which regulation Q was binding, I constructed a monthly data set from historical issues of the Federal Reserve's Annual Statistical Digest.[2] I refer the interested reader to the Appendix for more detail on the regulatory adjustments and financial market innovations that have influenced deposit rate regulation over time.

Core Deposits. Figure 1.2 plots real growth of core deposits against the difference between the 3-month T-bill rate and the ceiling on savings deposits from 1960 until the removal in 1986. The Figure shows a stable negative relationship between the spread and deposit growth. It also reveals how on each occasion up to 1983, a positive spread is associated with drops in core deposit growth. In fact, every NBER-dated recession is associated

[2] The Annual Statistical Digest is published by the Board of Governors of the Federal Reserve System and documents ceiling adjustments on the various types of deposits. A monthly data set is available from the author.

with troughs in core deposit growth and peaks in the ceiling spread. Finally, note that the large spike in core deposit growth in 1983 is due to the nationwide authorization of a string of new transaction and savings deposit instruments (including ATS, NOW, Super-NOW and MMDA accounts). The ceilings on core deposits were lifted in March 1986.

Managed Liabilities. To assess the relevance of deposit rate ceilings on managed liabilities, it is important to first have a look at their composition. Figure 1.3 depicts the components of managed liabilities as a share of the total. Although steadily declining over time, small denomination time deposits (STDs) have traditionally constituted the largest share of managed liabilities. Because of the creation of large negotiable certificates of deposit (CDs), by the mid 1960s, the share of large time deposits (LTDs) had risen from 10% to almost 40% of managed liabilities. After 5 years of subsequent decline, 1970 marked a turning point, after which the share of LTDs has remained relatively stable at about 25%, until recently climbing back up to 40%. The volumes of Eurodollar deposits and Repurchase Agreements have historically remained relatively small compared to the volume of time deposits.

Although usually higher than those for savings deposits, maximum rates also applied to time deposits. Figure 1.4 plots the real growth rate of small and large time deposits, together with the spreads between the relevant market rates and the ceilings for various maturities. Real growth in small term deposits, depicted in the left panel of Figure 1.4, was negative during 1960, when the spread on all maturities was positive. A series of upward adjustments of the ceilings in 1962, 1963, 1964 and 1965 brought STD rates back in line with market rates. The revisions initiated an expansion in STDs during the course of 1966 and early 1967. This counteracted the contemporaneous drop in core deposits on which the ceilings were still binding. In contrast, at the onset of the recessions in 1969-1970 and 1973-1975, market rates exceeding the ceilings had brought about significant reductions in

STD growth. The authorization of Money Market Certificates and Small Saver Certificates in 1978 and 1979 explains why the growth in STDs remained relatively stable while interest rates soared in the early 1980s (see Appendix). In September 1983, the ceilings on small time deposits were eliminated.

Real growth of LTDs, shown in the right panel of Figure 1.4, also displays evidence of disintermediation effects. Major ceiling rates adjustments in the first half of the 1960s ensured that banks could offer competitive rates as the market for LTDs expanded. However, positive spreads towards the end of the 1960s caused a dramatic fall in the amount of LTDs outstanding, also explaining the decline in their relative use, evident in Figure 1.3. The contraction in LTDs outstanding even led to the removal of ceilings for certain smaller maturities in 1970. In 1973, regulation Q ceilings were lifted on all LTDs.

The analysis of bank liabilities and deposit rate ceilings leads to the following conclusions: *First*, on multiple occasions, there have been contemporaneous contractions in core deposits and managed liabilities growth during the 1960s and 1970s. In contrast, after the late 1970s, the correlation between the growth rates of both deposit categories is consistently negative. *Second*, the contractions in deposits coincide exactly with periods in which market rates exceeded the deposit rate ceilings imposed under regulation Q, thus providing evidence for a regulation-induced disintermediation effect. *Third*, episodes in which disintermediation appears to have taken place include the periods surrounding the 1960 and 1970 recessions. Also the 1973-1975 recession seems to have been associated with disintermediation, despite the earlier removal of ceilings on large time deposits. Regulatory changes and innovations in banking explain why the impact of the deposit rate ceilings was reduced towards the end of the 1970s, even though regulation Q was not formally removed until the early 1980s.

1.3 Effects of Disintermediation: A Monetary DSGE Model

If the regulation-induced disintermediation observed in the data forces banks to cut back on lending to borrowers that depend on intermediated finance, it also potentially exacerbates the real effects of a monetary tightening. This section develops a Dynamic Stochastic General Equilibrium model in which interest rate regulations, incorporated as occasionally binding constraints, alter the transmission of stochastic monetary shocks primarily by affecting the availability of bank credit: Binding ceilings reduce the demand for deposits and constrain the pool of loanable funds. The model also incorporates inventory investment decisions, because the availability of short-term credit directly affects firms' optimal inventory response to a monetary-induced drop in sales. Since most short-term business credit consists of bank loans, the disintermediation effect could alter the sensitivity of inventory investment to cash flow fluctuations and the ability of firms to smooth production.

1.3.1 A Monetary DSGE Model of Deposit Rate Ceilings

The model builds on a money-in-the-utility-function framework. Apart from the regulation Q price controls and the possibility of inventory investment, the model features a finance constraint: As in Christiano and Eichenbaum (1992) or Fuerst (1992), firms must finance working capital expenses by taking out a bank loan. Furthermore, I assume that prices are set before the realization of the current shock. This price-setting friction allows for more realistic effects of monetary shocks on interest rates and real variables. The economy contains a household sector, a firm sector, a banking sector and a government and the only source of uncertainty is a shock to the money growth rate.

The Household. The economy is inhabited by an infinitely-lived representative household that starts every period $t = 0, ..., \infty$ with the economy's entire last period money stock

M_t. The household decides to deposit an amount $M_t - Q_t$ in a bank and keep Q_t as nominal cash balances. Deposits earn a gross nominal deposit rate R_t.

The specification of household preferences is identical to Christiano and Eichenbaum (2005). Lifetime utility is

$$\mathrm{E}_0 \sum_{t=0}^{\infty} \beta^t \left[\log(c_t) - \frac{\Psi}{2} h_t^2 + \frac{\Omega}{1 - 1/\phi} \left(\frac{Q_t}{P_t} \right)^{1-\frac{1}{\phi}} \right] , \tag{1.1}$$

where E_0 is the time 0 conditional expectation operator; c_t is the period t value of a consumption index to be defined below; h_t is hours worked in period t; and P_t is the period t consumption-based money price index. The parameters of the utility function are the discount factor, $1 > \beta > 0$; the time allocation parameter, $\Psi > 0$; the utility weight of real cash balances, $\Omega > 0$; and the interest rate elasticity of the demand for cash, $\phi > 0$.

There is a continuum of differentiated final consumption goods, indexed by $j \in (0,1)$, that enter the household utility function through the index c_t, given by

$$c_t = \left[\int_0^1 \left(\frac{n_{jt}}{n_t} \right)^{\frac{\xi}{\varepsilon}} s_{jt}^{\frac{\varepsilon-1}{\varepsilon}} dj \right]^{\frac{\varepsilon}{\varepsilon-1}} .$$

Here, s_{jt} is the amount of good j purchased by the household for consumption in period t, whereas n_{jt} denotes the *total stock* of good j that is available for sale in period t. This specification of the consumption index implies that, at a given price, finished goods inventories facilitate sales. This modelling approach is followed by, for instance, Bils and Kahn (2000) and is also related to models that incorporate inventories as a factor of production, such as Kydland and Prescott (1982) and Christiano (1988). Note that here, following Jung and Yun (2006), the stock of good j available in stores generates higher utility for the household only to the extent it is higher than the economy-wide average level $n_t = \int_0^1 n_{jt} dj$. The parameters of the consumption goods index c_t are the elasticity of demand with respect to stock available for sales, $\xi > 0$; and the price elasticity of demand for good j, $\varepsilon > 1$. Letting P_{jt} denote the price of good j, intratemporal cost minimization implies the following demand function for the j-th good:

$$s_{jt} = \left(\frac{n_{jt}}{n_t}\right)^{\xi} \left(\frac{P_{jt}}{P_t}\right)^{-\varepsilon} c_t \ , \tag{1.2}$$

where the utility-based price index P_t is given by

$$P_t = \left[\int_0^1 \left(\frac{n_{jt}}{n_t}\right)^{\xi} {P_{jt}}^{1-\varepsilon} dj\right]^{\frac{1}{1-\varepsilon}} \ .$$

Household assets evolve according to

$$M_{t+1} = R_t(M_t - Q_t) + D_t + W_t h_t + Q_t - P_t c_t \ . \tag{1.3}$$

The first term on the right hand side, $R_t(M_t - Q_t)$, denotes total interest earnings on deposits held with the banks. Without loss of generality, asset markets are not modeled explicitly and D_t simply represents combined period t dividend payments from the household and banking sector. The third term, $W_t h_t$ is total labor earnings, where W_t is the period t nominal wage. Finally, Q_t are cash balances and $P_t c_t$ is total consumption expenditure. The intertemporal decision problem of the household consists of choosing sequences of cash holdings, consumption levels and labor effort, contingent on the history of realizations of the shock, in order to maximize lifetime utility as defined in (1.1), subject to the constraint defined in (1.3).

The Firms. Each final consumption good $j \in (0,1)$ is produced by a monopolistic firm using labor as the only input. The technology for period t production for each firm j is given by

$$y_{jt} = \begin{cases} h_{jt}^{\alpha} - \theta & \text{if } h_{jt}^{\alpha} \geq \theta \\ 0 & \text{otherwise} \end{cases} , \tag{1.4}$$

where $\theta > 0$ is a fixed cost of production and $1 > \alpha > 0$ and h_{jt} is labor input by firm j. There is no entry or exit in the market for good j. Each firm's stock of goods available for sale in period t is

$$n_{jt} = n_{jt-1} - s_{jt-1} + y_{jt} , \tag{1.5}$$

where s_{jt-1} are period $t-1$ sales of good j.

It is assumed that workers must be paid in advance and that firms must borrow the wage bill by lending L_{jt} from a bank. As a result, each firm j faces a financing constraint $W_t h_{jt} \leq L_{jt}$. Loans are repaid at the end of the period at the gross interest rate r_t. Firms set prices in period t before the current realization of the shock and treat V_t, the marginal value of one unit of cash to the household in period t, as exogenous. Noting that, as long as $r_t > 1$, the financing constraint holds with equality, the objective of each firm j is to choose sequences of prices and production levels contingent on the realization of uncertainty in order to maximize

$$\mathrm{E}_0 \sum_{t=0}^{\infty} \beta^t V_t \left[P_{jt} s_{jt} - r_t W_t h_{jt} \right] ,$$

taking into account the demand function for its differentiated final consumption good, given by (1.2), and subject to (1.4) and (1.5).

The Banks. The representative bank collects deposits $M_t - Q_t$ and receives X_t as a cash injection from the government. Hence, bank lending is restricted by the availability of funds as follows

$$L_t \leq M_t - Q_t + X_t .$$

The market for business loans is competitive and as long as $r_t \geq R_t > 1$, the intermediary lends all the available funds to the firms. Government regulation stipulates the following restriction on interest rates paid on deposits:

$$R_t \leq R^q ,$$

where $R^q > 1$ is an exogenous deposit rate ceiling. This inequality captures regulation Q in a simple and straightforward manner. The banks' net cash position at the end of each period t is distributed as dividends to the household, the ultimate owner of the banks. Entry and exit in the banking sector is ruled out.

The Government. The money stock evolves according to $M_{t+1} = M_t + X_t$. Define $\mu_t \equiv M_{t+1}/M_t$ as the growth rate of money in period t, which evolves according to the univariate stochastic process

$$\ln(\mu_t) = (1-\rho)\ln(\mu) + \rho\ln(\mu_{t-1}) + \chi_t , \tag{1.6}$$

where $\mu > 1$ is the average gross growth rate of the money stock, $1 > \rho > 0$ measures the persistence and χ_t is a normal i.i.d. random variable with mean zero and variance $\sigma^2 > 0$. Christiano, Eichenbaum and Evans (1998) provide arguments in favor of this simple representation of monetary policy for evaluating the effect of monetary shocks in theoretical models.

Market Clearing and Equilibrium. An equilibrium is a set of sequences of prices and quantities such that all agents maximize their objective functions and goods, labor and asset markets clear. The Appendix provides a more detailed equilibrium definition.

Note that loan market clearing implies that

$$W_t h_t = M_{t+1} - Q_t \ . \tag{1.7}$$

Competition in the market for loans ensures that $R_t = r_t$ as long as the regulation Q-constraint is not binding. Else, in the case where the interest rate constraint binds, it must be the case that $r_t > R_t = R^q$.

1.3.2 Parametrization and Numerical Solution Methodology

Parametrization. Following standard practice, I choose parameter values to obtain certain properties of the steady state of a *non-stochastic* version of the model. All the values used for generating the results are summarized in Table 1.1. The time period in the model corresponds to one quarter. The parameters can be partitioned into three groups. The *first* group contains the household's preference parameters, namely β, Ψ, Ω, ϕ, ε and ξ. The discount factor is $\beta = 1.03^{-0.25}$, implying an annualized real interest rate of 3% in the non-stochastic steady state. The value for the time allocation parameter Ψ normalizes the steady-state value of hours worked h to unity. The value for the utility weight Ω yields a ratio of cash balances to the total money stock Q/M equal to 0.3. The interest rate elasticity of money demand ϕ is 0.24. Obtaining reliable estimates for this parameter is difficult because of structural changes in financial markets. I take the value estimated by Teles and Zhou (2005), who use a relatively stable monetary aggregate, MZM, and a more accurate measure of the opportunity cost. The value of 0.24 is higher than the one estimated by Christiano and Eichenbaum (2005) but lower than those obtained by Lucas (1988) and Chari, Kehoe and McGrattan (2000). Next, the value of the price elasticity of demand ε yields a steady state non-competitive markup of 20% (Rotemberg and Woodford, 1995). Finally, the value for ξ matches the average inventories-to-sales ratio in manufacturing and trade in US postwar data. The *second* group contains the technology parameters, θ and α. The choice of $\alpha = 0.64$ corresponds to the labor income share in the data and the fixed cost θ is set to ensure that firms make zero excess profits in the non-stochastic steady state. The *third* group are the parameters governing the money growth process. I borrow these values from Christiano and Eichenbaum (1992): The average quarterly gross growth rate is $\mu = 1.012$, the autoregressive parameter ρ is 0.3 and the standard deviation of the monetary shock is $\sigma = 0.012$.

Numerical Solution Methodology. I compute the model solution by solving the system of Euler equations describing the equilibrium behavior of the various macroeconomic variables, which is given in the Appendix. The numerical method is based on time iteration, as described by Coleman (1990). The main advantages of the procedure are its straightforward application to non-Pareto optimal economies and the fact that convergence is usually achieved. Time iteration also does not rely on discretization of the state space, but instead requires interpolation techniques that preserve the continuous nature of the state space. In addition, as showed by Rendahl (2006), time iteration is relatively easily to implement in the presence of inequality constraints.[3] However, time iteration is generally slow. Therefore the iterative scheme is augmented by the application of the method of endogenous gridpoints, developed by Carrol (2006). This method reduces the number of nonlinear equations that need to be solved numerically in every iteration.

The model features two endogenous state variables, n_t and p_t, and one exogenous state χ_t. All functions are approximated by a linear interpolation scheme based on a grid of the state space. The grid contains 20 nodes for both endogenous state variables, and 9 nodes for the shock. Taking expectations requires integration over the continuous state space. The integrals with respect to the normal density are approximated using Gauss-Hermite quadrature

[3]In fact, Rendahl (2006) proves numerical convergence for the case of Pareto optimal economies.

with 10 quadrature nodes (Judd, 1998).

1.3.3 Quantitative Properties of the Model

Figure 1.5 plots the response of some of the main variables to a one-standard-deviation (1.2%) unexpected decrease in the money growth rate in period $t = 1$. The solid line represents the model without deposit rate ceiling (R^q is set arbitrarily high), the striped line represents the model with a ceiling R^q of 1.02. The response of the net interest rate $r_t - 1$, inflation P_t/P_{t-1} and the interest rate-ceiling spread $r_t - R^q$ are in levels, whereas those for production y_t, the inventories-to-sales ratio $(n_t - c_t)/c_t$, real lending L_t/P_t, real cash balances Q_t/P_t and real deposits $(M_t - Q_t)/P_t$ are all in percentage deviations from the state prior to the shock. This initial state, at $t = 0$, is the one to which the economy converges when the value of the exogenous shock is set to $\chi_t = 0$ for an arbitrarily long period. Note that this state does not correspond to the steady state of the non-stochastic version of the model. For instance, in the absence of a ceiling on the deposit rate, $r_0 = R_0 = 1.0192$ is slightly below its non-stochastic steady-state level of $\mu/\beta = 1.0195$ because of Jensen's inequality. When a ceiling of $R^q = 1.02$ is imposed, the loan rate is 1.0202, which is higher than μ/β. This is because, in those states of the world where the ceiling is binding, a higher loan rate r_t is required to clear the market for loanable funds, as the ceiling decreases the opportunity cost of holding cash and lowers the demand for deposits. For the consumption Euler equation to hold, interest rates also need to be higher in states of the world where the ceiling is not binding. The fact that interest rates increase on average with regulation Q is interesting in itself, since one of the motivations during the 1960s and 1970s to keep the ceilings below the market interest rates was to *lower* loan rates charged by banks. In the current setting, this reasoning breaks down. Because of the finance constraint, the higher interest rates raise the marginal cost of hiring labor and therefore lower output and consumption relative to the regulation-free model. Note that for $R^q = 1.02$, the ceiling is binding in the initial state. Hence, in simulations, deposit rates are at the ceiling more often than not, which is the case in 1960s and 1970s data.

In response to a negative money growth shock, both models display a "liquidity effect", i.e. interest rates rise after a money tightening. To see why, write the real supply of loanable funds in the economy as

$$\frac{L_t^s}{P_t} = \frac{X_t}{P_t} + \left(\frac{M_t}{P_t} - \left(\frac{R_t - 1}{\Omega c_t} \right)^{-\phi} \right) . \tag{1.8}$$

Loanable funds come from two sources: The first term in equation (1.8) represents the exogenous injections by the government, the second term reflects the endogenous supply of deposits by the households derived from the household's first order condition with respect to cash balances. The demand for deposits depends positively on the deposit rate R_t and negatively on consumption through the equalization of the marginal utilities of holding cash and consuming. Using the firms' first order condition for labor, loan demand can be written as

$$\frac{L^d}{P_t} = \alpha \frac{MC_t}{P_t} \frac{y_t + \theta}{r_t} . \tag{1.9}$$

Loan demand depends negatively on the cost of borrowing r_t and the markup P_t/MC_t and positively on production y_t, all of which determine the size of the real wage bill in equilibrium. The monetary shock leads to a leftward shift of the loan supply curve through a decrease in X_t/P_t. Clearing of the loan market is achieved by a reduction in demand through an increase in r_t and decreases in y_t and MC_t/P_t. In the regulation-free model, an endogenous reaction of deposit demand through an increase in R_t and a decrease in c_t counteracts the decrease in the supply of funds. The two lower right panels of Figure 1.5 show how households substitute out of cash balances and into interest-bearing deposits.

Despite this shift into deposits, the net effect for the loan market is a decrease in L_t/P_t and an increase in the borrowing cost r_t. In the model with a binding regulation Q constraint, there is no first order effect on deposit demand through an increase in R_t. The cash position of the household changes only through the decrease in c_t. This is the model equivalent of the disintermediation effect of deposit rate ceilings. As a result, a larger part of the adjustment must occur through the net effect of the increase in r_t and the decreases in y_t and MC_t/P_t. The immediate consequence of binding regulation is to alter the relation between money and interest rates: Disintermediation amplifies the liquidity effect of a monetary contraction.

In both models, a monetary tightening leads to a fall in hours worked and output: On the one hand, higher borrowing costs increase the marginal cost of hiring labor, which leads to a leftward shift of the labor demand curve; on the other hand, the labor supply curve shifts rightward because of a drop in consumption. Both effects contribute to a decline of the real wage W_t/P_t. Nevertheless, the net result is a decline in h_t, which implies that the marginal cost of hiring labor, R_tW_t/MC_t has risen. Because borrowing costs rise more, the decline in output with a binding regulation Q ceiling is more pronounced than for the regulation-free model.

To understand the response of inventories, it useful to consider the firms' first order condition for inventory investment, which in a symmetric equilibrium leads to the following relation:

$$P_t\xi\frac{c_t}{n_t}+\beta\mathrm{E}_t\left[\frac{V_{t+1}}{V_t}MC_{t+1}\right]\left(1-\xi\frac{c_t}{n_t}\right)=MC_t\ . \qquad (1.10)$$

Adding an extra unit to the stock of goods available for sale is associated with a marginal cost MC_t, and yields an increase in sales of $\xi c_t/n_t$, valued at P_t. To the extent the extra unit adds to inventories and is not sold, production is shifted from $t+1$ to t, saving the firm the present discounted value of tomorrow's marginal production cost MC_{t+1}. For notational ease, let $z_t = MC_t/P_t$ denote the inverse of the markup charged over marginal cost and $z_t^e = \beta E_t\left[z_{t+1}P_{t+1}V_{t+1}/P_t/V_t\right]$ the expected discounted value of next period's inverse markup. Using (1.10), the inventories-to-sales ratio can be written as

$$\frac{n_t - c_t}{c_t} = \xi\left(\frac{1 - z_t^e}{z_t - z_t^e}\right) - 1\ . \tag{1.11}$$

Note that the inventories-to-sales ratio is increasing in today's markup, and decreasing in the expected markup tomorrow. The reason is that, since the return to inventory investment is largely the ability to boost current sales, firms wish to build up the stock of goods available for sale at times when profit margins are high. The real marginal cost of producing one additional good is

$$z_t = r_t \frac{W_t}{P_t}\frac{1}{\alpha}(y_t + \theta)^{\frac{1-\alpha}{\alpha}}\ . \tag{1.12}$$

The drop in y_t and W_t/P_t contributes to a decrease in z_t, whereas the increase in r_t tends to increase z_t. The net effect in both models, however, is a decrease in z_t and therefore an increase in the markup P_t/MC_t. In the next period, prices adjust and there is a drop in the markup. The countercyclical reaction of today's markup and the lower markup tomorrow are the reason why firms wish to increase the inventories-to-sales ratio, which is achieved both through a drop in sales and an increase in the stock of inventories. Because borrowing costs rise more in the model with regulation Q, the increase in the profit margin is smaller and the positive response of inventories is reduced relative to the regulation-free model.

Since the shock to monetary growth rate is persistent, in period 2 inflation falls. Whereas the supply of loanable funds on behalf of the government is still low, equilibrium in the market for funds is now attained mainly through the drop in prices, even to the extent that, in real terms, lending increases in period 2. Through the household's consumption Euler equation, consumption and therefore sales rise in period 2. Afterwards, as the effect of the money shock dies out, consumption reverts to the initial state. Because of the price adjustment, markups today are small relative to those tomorrow, which leads firms to decrease the stock of inventories relative to sales. Note how, because of the ceiling, the liquidity effect in the regulation Q model is quite persistent. This additional gradual decline in borrowing costs further raises next period's markup relative to today's, and therefore the inventories-to-sales ratio is reduced more in the regulation Q model. As markups and sales stabilize, inventory levels return to the initial state.

The model response to a contractionary monetary shock is qualitatively consistent with the consensus view: Interest rates rise; aggregate output, employment, the price level and monetary aggregates fall. Admittedly, the model is too simple to replicate the precise timing and persistence encountered in the data. Nevertheless, it provides several clear predictions about the impact of deposit rate ceilings after a contractionary monetary shock. First and foremost, regulation Q affects the relation between monetary aggregates and real activity. With binding regulation, the same decrease in the growth rate of the money stock, measured as M_t, is associated with a larger drop in output. The regulation-induced disintermediation effect provides the monetary authority with greater leverage over loanable funds and real activity. In order to find evidence for this hypothesis, the following additional facts should be observed in the data: After an (unexpected) tightening of the monetary stance that results in rising interest rates and with binding deposit rate ceilings,

1. the spread between market rates and the regulatory ceilings widens, i.e. the monetary authority does not offset the monetary shock by raising the ceiling;

2. the response of (interest-bearing) bank deposits displays a disintermediation effect, i.e. the response is constrained relative to the nonbinding case;

3. non-interest bearing money holdings respond little to the interest rate increase;

4. the contraction in real lending is more severe than otherwise;

5. firms' inventories-to-sales ratios are lower than otherwise.

If the data support these facts, then there is evidence for a role of ceiling regulations in the monetary transmission mechanism and in shaping the macroeconomic outcomes of the 1960s and 1970s in the US. It is important to note that these predictions are likely to hold in more elaborate model settings. It is true that the biggest banks gradually gained access to unregulated financial markets, mostly abroad and therefore beyond the jurisdiction of the Fed. It is also true that some of the big firms gained access to nonintermediated (short-term) funds, such as commercial paper. But as long as these do not account for a sufficiently large fraction of economic activity, which is all the more realistic for the 1960s and 1970s, regulation-induced disintermediation is likely to be relevant. Another realistic extension of the model is to expand the menu of assets available to the households beyond cash and bank deposits. But as long as the ceiling regulation creates an opportunity cost of holding bank deposits relative to unregulated alternatives, there will be a disintermediation effect. Also, to the extent that bank deposits are the closest substitutes for non-interest bearing assets, their interest rate sensitivity is reduced when the ceilings bind. Finally, the rationing of bank credit in the model occurs exclusively through the price mechanism, i.e. via increases in the loan interest rate. It is probable that, instead, some of the adjustment in bank lending occurs through quantity rationing. However, this would not decrease the relevance of the disintermediation channel of monetary transmission.

1.4 Confronting the Theory with the Data

To evaluate the theoretical predictions empirically, I estimate impulse responses to an identified monetary policy innovation in a structural Vector Autoregressive Model (VAR). The objective is to compare the impulse responses for two separate regimes: One in which deposit rate ceilings are binding, the other in which they are not. Implicitly, this means that the autoregressive coefficients in the VAR must vary across these two regimes. Researchers focussing on regime dependence in monetary VARs have traditionally relied either on subsample analysis (Boivin and Giannoni, 2002), threshold VARs (Balke, 2000) or on Bayesian estimation of regime-switching models (Sims and Zha, 2006). My empirical strategy is to directly exploit information about the presumed source of regime switching. I assume that observations on the interest-rate ceiling spreads contain sufficient information to identify the asymmetric effects of deposit rate regulation. A VAR in which the interest-rate ceiling spread is a threshold variable and 0 the threshold value is a natural framework to assess the role of binding ceilings as a nonlinear propagator of monetary policy shocks.

1.4.1 Empirical Framework and Stability Tests

Model Specification. Let $\mathbf{y}_t$ be a $(n \times 1)$ vector of time series including a price index PI_t; a commodity price index $Pcom_t$; a measure of economic activity Y_t; the Federal Funds rate FF_t; and a monetary aggregate MZM_t, all of which are variables that are commonly included in monetary VARs. In this paper, $\mathbf{y}_t$ also contains some additional variables of interest, namely the real volume of managed liabilities ML_t; the real volume of loans LNS_t; and the inventories-to-sales ratio In_t, such that $n = 8$ and $\mathbf{y}_t = [Pcom_t \;\; PI_t \;\; Y_t \;\; In_t \;\; FF_t \;\; MZM_t \;\; LNS_t \;\; ML_t]'$. Let $\mathbf{x}_t = [1 \; \mathbf{y}_{t-1}' \; ... \; \mathbf{y}_{t-p}']'$ be a vector containing a constant and lagged observations, where p denotes the number of lags. I use $p = 6$ in all the estimations.[4] A traditional structural VAR is a system of equations:

$$A\mathbf{y}_t = B\mathbf{x}_t + \Sigma\varepsilon_t \; . \tag{1.13}$$

[4] In a previous version of this paper, I used 12 lags which yielded very similar results.

The structural coefficients are contained in the matrices $A_{n\times n}$, $B_{n\times(np+1)}$ and the positive definite, diagonal matrix $\Sigma_{n\times n}$. The structural shocks ε_t are i.i.d. $\sim N(0, I_n)$. The system (1.13) can be written in reduced form as

$$\mathbf{y}_t = G\mathbf{x}_t + \mathbf{u}_t \ , \tag{1.14}$$

where $G = A^{-1}B$ and the relationship between between the fundamental shocks and the VAR-disturbances is given by $A\mathbf{u}_t = \Sigma\varepsilon_t$.

This paper evaluates a non-linear threshold model, as in for instance Balke (2000), as an alternative to the traditional linear specification in (1.13). Let S_t be the period t observation of the spread between the market interest rate and the legal deposit rate ceiling and define $\tilde{\mathbf{y}}_t = [\mathbf{y}_t' \ \ I(S_t > 0)S_t]'$ and $\tilde{\mathbf{x}}_t = [1 \ \tilde{\mathbf{y}}_{t-1}' \ ... \ \tilde{\mathbf{y}}_{t-p}']'$. $I(S_t > 0)$ is an indicator variable that equals one when $S_t > 0$ and zero otherwise. Consider the following specification:

$$\begin{aligned} A\mathbf{y}_t &= B\tilde{\mathbf{x}}_t + CS_t + \Sigma\varepsilon_t \ , \quad && \text{if } S_t > 0 \ , \\ A\mathbf{y}_t &= B\tilde{\mathbf{x}}_t + \Sigma\varepsilon_t \ , && \text{if } S_t < 0 \ , \end{aligned} \tag{1.15}$$

where the matrices $A_{n\times n}$, $B_{n+1\times((n+1)p+1)}$ and $C_{n\times 1}$ contain the structural coefficients. S_t is the threshold variable that determines the regime of the dynamic system: When $S_t > 0$, the market rate exceeds the ceiling and the system is in the binding regulation regime; when $S_t < 0$, the system is in the nonbinding regime. The latter includes periods where the regulation was in place but did not constrain the banks, as well as periods in which the regulation was no longer active. Specification (1.15) differs from (1.13) by the inclusion of S_t in the binding regulation regime. Following Duca (1998), (1.15) also adds lagged observations of S_t, truncated at zero, to the dynamic system. The implicit assumption is therefore that the only source of misspecification in the linear model is the omission of an important variable: the market rate-ceiling spread S_t. Positive values of S_t contain useful information about the impact of binding regulation on the economy.[5] I posit the following law of motion for S_t:

$$S_t \quad = \quad A^s \mathbf{y}_t + B^s \left[\mathbf{x}_t' \; \mathbf{s_t}'\right]' + \sigma^s \varepsilon_t^s \,, \tag{1.16}$$

where $A^s_{1\times n}$, ${B^s}_{1\times(np+1)}$ are coefficient matrices, $\mathbf{s_t}$ is a row vector containing p lags of S_t, $\sigma^s > 0$ is a scalar, and ε_t^s is i.i.d. $\sim N(0,1)$.

Substituting (1.16) into (1.15) and defining $\hat{S}_t = B^s \mathbf{x}_t + \sigma^s \varepsilon_t^s$, the system can be written in reduced VAR form as

$$\mathbf{y}_t = I(S_t > 0)(A - CA^s)^{-1}\left[B\tilde{\mathbf{x}}_t + C\hat{S}_t\right] + I(S_t < 0)A^{-1}B\tilde{\mathbf{x}}_t + \mathbf{u}_t \,, \tag{1.17}$$

or

$$\mathbf{y}_t = I(S_t > 0)G^1\left[\tilde{\mathbf{x}}_t' \; \hat{S}_t\right]' + I(S_t < 0)G^2\tilde{\mathbf{x}}_t + \mathbf{u}_t \,. \tag{1.18}$$

The relationship between the fundamental shocks and the VAR-disturbances is given by $\left(I(S_t > 0)(A - CA^s) + I(S_t < 0)A\right)\mathbf{u}_t = \Sigma\varepsilon_t$. When $C = 0$, the threshold model in (1.18) reduces to the standard linear specification of (1.14).

Data Series. The monthly series included in $\mathbf{y_t}$ are PCEPI less food and energy for PI_t; the index of sensitive materials prices for $Pcom_t$; interpolated real GDP for Y_t; the effective Federal Funds rate for FF_t; money at zero maturity for MZM_t; the ratio of real inventories to real sales in trade and manufacturing for In_t; commercial and industrial loans, deflated by the price index, for LNS_t; and the volume of managed liabilities, deflated by the price

[5]In principle, it is possible to also allow for regime dependence in the matrices A and B. I abstain from doing so because observations of S_t are only available for the subsample in which the regulation was active, the number of parameters to be estimated per equation is large and the estimation procedure is plagued by numerical difficulties that are due to local maxima.

index, for ML_t.[6] All variables are expressed in natural logs, except for the Federal Funds rate. The series used for S_t is the spread between the average of the 3 and 6 month T-bill rates and the regulatory ceiling on time deposits with maturity between 3 and 6 months.

The choice of variables is motivated by a number of considerations. $Pcom_t$ is usually included in VARs to mitigate the so-called "price puzzle", the finding that prices rise persistently after a contractionary monetary shock. Real GDP constitutes the broadest measure of economic activity. I repeated the estimations with the log index of industrial production, which is available at monthly frequency, but obtained very similar results. Money zero maturity is the preferred money stock measure because of its stability, because it does not include any of the components of ML_t, and because it is a good data equivalent of Q_t/P_t in the theoretical model.[7] I use C&I loans rather than total bank loans, since business loans are the more appropriate measure in testing for a credit channel affecting production. Moreover, recent research by Den Haan, Sumner and Yamashiro (Forthcoming) stresses the importance of distinguishing between the components of banks' loan portfolios. Finally, ML_t is a good data equivalent of real deposit holdings $(M_t - Q_t)/P_t$ in the theoretical model. An issue in VAR estimation is whether or not to remove time trends. The inventories-to-sales ratio displays a clear low frequency shift, most likely due to changes in management techniques during the early 1980s. Also, innovations in financial markets have caused much higher trend growth in the series for managed liabilities before 1980s than afterwards. Since these low frequency movements are outside the scope of this paper, they are removed from the ML_t and In_t series with the HP filter. The smoothing parameter is 86,400, as opposed to the conventional choice of 14,400, such as not to remove too much business cycle variation.

[6] Detailed descriptions and the sources of all series can be found in the Appendix.

[7] Money zero maturity (MZM) roughly equals $M2$ but excludes small time deposits. See Teles and Zhou (2005) for evidence on the stability of the MZM measure.

Sample. The monthly data set covers the period 1959:1-2004:4, spanning a period of about 45 years and containing a total of 544 observations for each series in $\mathbf{y_t}$. However, the estimation of the VAR is conducted using only a subset of the available data points, including the periods 1959:7-1978:6 and 1983:10 to 2004:4. This means that 63 observations (1978:7 to 1983:9) are dropped during the estimations in addition to the loss of observations due to the inclusion of lagged terms. The omission of a range of intermediate observations is motivated by two considerations. *First*, the break date of June 1978 marks the permission of retail money market certificates (MMCs). As explained in the Appendix describing the history of the regulation, these and other subsequent new instruments offered market-determined interest rates and greatly facilitated the ability of banks to raise funds well before the formal removal of regulation Q. The second break date, October 1983, marks the elimination of the remaining ceilings on small time deposits. The omitted data points comprise the relevant transition period during which the effects of the regulation gradually vanished, although the spread variable S_t takes on very large positive values. A *second* reason for the omission is the short-lived experiment of nonborrowed reserve targeting adopted by the Federal Open Market Committee from 1979 to 1982, which resulted in excessive volatility of the Federal Funds rate during this period. As shown by Bernanke and Mihov (1998) and Sims and Zha (2006), this period constitutes a significant regime shift that invalidates the use of a single policy indicator over the entire sample. However, Bernanke and Mihov (1998) conclude that using the funds rate prior to 1979 and after 1982 produces reasonable results. Finally, note that in the estimation of the S_t equation in (1.16), necessarily only the first subsample is used. There are 127 data points for which the spread is positive, roughly 25% of the entire sample.

Reduced-Form Stability Tests. Before proceeding to the estimation of the structural model, I perform a number of stability tests based on the reduced-form VAR in (1.14). Table 1.2 reports the bootstrapped p-values of the Wald-statistic testing for the hypothesis of stable coefficients. Under the null, the VAR-coefficients and the error variance estimated for the nonbinding periods ($S_t < 0$) are equal to those estimated for the binding periods ($S_t > 0$). The rows report the p-values resulting from the application of the test to each equation in the VAR-system separately. The results indicate that instability across regulation regimes is important: In three of the eight equations (PI_t, FF_t, MZM_t), the null of stability is rejected at the 5% level. In one other equation (LNS_t), the null is rejected at the 10% level. Therefore, a nonlinear model that takes into account the effect of binding deposit rate ceilings is more appropriate than the traditional linear specification.

1.4.2 Identification and Estimation Procedure

Identification. Recovering the structural coefficients of the threshold model in (1.17) is impossible without identification assumptions. In correspondence with a large part of the literature on monetary VARs, I base the strategy for estimating the effects of a monetary shock on the recursiveness assumption, together with the assumption that the Federal Funds rate is a good measure of monetary policy. Bernanke and Mihov (1998) and Christiano, Eichenbaum and Evans (1999) provide arguments why this strategy is reasonable. Monetary policy is characterized by the rule

$$FF_t = f(\Omega_t) + \varepsilon_t^m \,, \tag{1.19}$$

where $f(\cdot)$ is a linear function and Ω_t is the information set of the monetary authority. A monetary shock, denoted by ε_t^m, is a shock orthogonal to the elements of Ω_t. As in the benchmark model of Christiano et al. (1999), the information set Ω_t contains the current value of $Pcom_t$, Y_t, PI_t, together with all of the lagged values in $\tilde{\mathbf{x}}_t$. In this paper, Ω_t also includes the current value of the spread S_t, as well as the inventories-to-sales ratio In_t.

Given the use of monthly data, this definition of a monetary shock therefore assumes that the Federal Reserve has within-month data on prices, economic activity, inventories, sales and market interest rates.

The theoretical model points to a potential caveat in using the Federal Funds rate as a measure of the monetary stance. A given change in the loan rate reflects a different monetary intervention depending on the regulation regime, because the liquidity effect is larger when the ceiling binds. The same could be true for the interest rate in the Federal Funds market, which implies that, for a given funds rate shock, the actual intervention of the Fed in the binding regime is smaller than in the nonbinding regime. But with a finding of a larger effect on output and lending in the binding regime, this objection only corroborates the evidence for the hypothesis that regulation Q gave the Fed greater leverage over output.

The recursiveness assumption is not sufficient for identification and, following common practice, I further assume that A is a lower triangular matrix with diagonal elements equal to one. If $C = 0$, the structural VAR can be estimated by standard OLS and subsequent Choleski decomposition of the estimated variance-covariance matrices. If $C \neq 0$, estimation of (1.17) requires some more restrictions, since A^s and $\hat{S}_t$ are unknown. *First*, I assume that Y_t, PI_t, $Pcom_t$ and In_t are predetermined relative to ε_t^s, which implies that the corresponding elements of C are zero. In other words, a financial market shock to S_t does not affect output, prices and inventories and sales within the same month. *Second*, I assume that there is no contemporaneous relation between S_t on the one hand and Y_t, PI_t, $Pcom_t$ and In_t on the other hand, which implies that the corresponding elements of A^s are zero. Note however that shocks to Y_t, PI_t, $Pcom_t$ and In_t still affect S_t contemporaneously through their effect on the other variables.

Estimation Procedure. With the above identification scheme, I estimate (1.17) in the following two steps:

1. Obtain estimates of A^s and $\hat{S}_t$ by instrumental variables estimation of equation (1.16), using Y_t, PI_t, $Pcom_t$ and In_t as instruments;

2. Use these to estimate the coefficients in A, B, C and Σ with conventional maximum likelihood techniques.

Because relative to the linear structural model, there are only four additional coefficients to be estimated in the maximum likelihood step, it did not prove to be problematic in practice.

1.4.3 Regime-Dependent Impulse Responses and Counterfactual Experiment

Figure 1.6 plots the estimated regime-dependent impulse response to a one-standard-deviation (0.25%) shock to the Federal Funds rate in the threshold model (1.17), occurring in period $t = 1$. The solid line represents the case where the deposit rate ceiling is nonbinding in period 1 and all periods following the shock. The dashed line represents the case where the regulation Q constraint is binding in $t = 1$ and all subsequent periods. For comparison, the dotted line represents the results for the traditional linear specification (1.13). The grey areas are the (centered) 68% bootstrapped confidence regions for the traditional linear structural VAR.

According to Figure 1.6, the responses of the main economic aggregates to an unanticipated monetary policy tightening are qualitatively in line with the established facts: the rise in FF_t is persistent; there is a sustained drop in Y_t and the price level PI_t; the money stock MZM_t goes down. Also, in correspondence with the theoretical model of this paper, the inventories-to-sales ratio rises after the shock and, as prices adjust, reverts back to trend and eventually declines in the long run. At the same time, Figure 1.6 reveals some important differences between the responses with and without a binding regulation Q constraint. A first observation is that a binding ceiling exacerbates the output decline from 6 months after the shock onwards. Moreover, the difference in the output response is quantitatively quite large and lies outside of the 68% confidence region of the stable VAR impulse response. The extent to which this finding can be explained by presence of the regulation Q depends on whether the other variables behave according to the theoretical predictions of the model:

First, an unanticipated funds rate hike of 25 basis points causes the spread between the market rate and the regulatory ceiling to widen up to 19 basis points in the third month after the shock. Thus, the pass-through from the funds rate to the spread is considerable.

Second, without constraint, the real volume of managed liabilities ML_t expands relative to trend, peaking 9 months after the shock. With binding deposit rate ceilings, ML_t *declines* until 6 months after the shock, after which there is a slow reversion back to trend. In the longer run, the real volume of managed liabilities drops in both cases. The striking difference during the first 18 months after the shock constitutes evidence that a tighter monetary stance induces a disintermediation effect and is consistent with the second theoretical prediction. The responses of ML_t in both regimes lie well outside of the confidence region of the stable VAR impulse response.

Third, the negative response of money holdings MZM_t is much less pronounced with binding regulation than without and is outside of the confidence region of the stable VAR. This result is in line with the conjecture that the binding regulatory ceilings significantly disrupt the relation between money demand and interest rates. It is therefore consistent with the third theoretical prediction, that non-interest bearing money holdings respond little to the interest rate increase if regulation Q binds.

Fourth, although real lending to firms increases immediately after the shock, before declining in the longer run, the initial increase is much more short-lived with a binding ceiling. The finding of an increase in business lending is consistent with previous results in the literature, see for instance Den Haan, Sumner and Yamashiro (Forthcoming). There is a contraction in real lending in the longer run for the regime where the ceiling binds. In general, it is very hard to draw firm conclusions about the direction of casuality between lending and economic activity from this type of evidence. Some researchers, such as Romer and Romer (1990), see the lack of a drop in lending that precedes the decline in output as evidence against a causal role for bank credit. However, the fact that business lending, at least initially, expands after a monetary shock can be explained by the desire of firms to finance an inventory build-up and smooth production. What is more important to the argument of this paper is the *difference* between the responses with and without binding regulation, for which the theory provides a rationale. The evidence is therefore seen as consistent with the fourth theoretical prediction, that the reduction in firm lending after a monetary tightening is more severe when the regulatory constraint is binding. Also, the negative response of firm lending in the binding regime moves outside of the stable VAR confidence region.

Fifth, the response of the inventories-to-sales ratio in the first few months after the shock is initially slightly negative with a binding ceiling, remains below the regulation-free response for the first six months and lies marginally outside of the confidence region for the stable VAR impulse response. This is in line with the theory, according to which firms postpone production to a greater extent because larger increases in borrowing cost negatively affect current profit margins. The inventory build-up lasts longer with the regulation Q constraint, which is more at odds with the theory.[8] One possible explanation is that technological improvements in inventory management since the early 1980s influence the results for the nonbinding case. A second contributing factor potentially lies in the reaction of the price level. Without the ceiling, the price level starts to adjust almost immediately after the shock. In contrast, the drop in the price level occurs much later with the binding

[8] This is in line with McCarthy and Zakrajsek (2003) who find, using subsample analysis, a smoother and shorter-lived response of inventories-to-sales ratios after a contractionary monetary shock in recent samples.

constraint, after about 16 months. To the extent that the price response implies a faster adjustment of markups in the nonbinding case, this could be reflected in a swifter adjustment of inventories.

To gain some further insight in the role of the regulation-induced disintermediation effect, Figure 1.7 examines the response to an identified one-standard-deviation positive shock to the interest rate-ceiling spread in equation (1.16). An exogenous increase in S_t raises the opportunity cost of holding bank deposits. The figure shows that the rise in the spread causes an immediate decline in managed liabilities ML_t, reflecting substitution out of time deposits. Interestingly, this decline is associated with a relatively quick drop in real lending to firms, which constitutes additional evidence that the disintermediation effect negatively affected the ability of banks to lend during the 1960s and 1970s. The response of output displays a swift and fairly persistent decline shortly after the shock. There is also a short-run decrease in the inventories-to-sales ratio. Perhaps more so than for the monetary shock, the inventories response to a spread shock is in line with the hypothesis that firms postpone production because increases in borrowing cost negatively affect current profit margins. This evidence points to effects of a spread shock that are in accordance with a channel of transmission operating through bank lending. Note how the money stock increases immediately after the shock. This fact potentially reflects a reaction of the Federal Reserve to counteract the negative spread shock by a money supply expansion. There is a slight increase in the price level, and no clear pattern in the response of the Federal Funds rate, which could be consistent with this conjecture.

Given the evidence of regime dependence in the VAR coefficients, it is informative to make a comparison with the results from the standard linear specification in (1.13) with stable coefficients. An interesting question is how the failure to explicitly take into account the regulation-induced disintermediation effects of the 1960s and 1970s biases the estimated responses in the standard structural VAR. Perhaps the most obvious observation concerns the behavior of monetary aggregates. The linear specification tends to underestimate the interest elasticity of money demand. The fundamentally different response of ML_t also makes evident why the MZM money definition, which roughly corresponds to M2 *excluding* small time deposits, is a more useful measure of the money stock as a macroeconomic indicator over the sample period. In addition, not taking into account the disintermediation effect during periods with binding ceilings leads to overestimation of the contractionary effect on output of a monetary tightening in other periods.

Business Cycle Volatility: A Counterfactual Experiment. This final section addresses the question to what extent disintermediation effects have contributed to business cycle volatility during the 1960s and 1970s. The answer involves estimating how the economy would have evolved, had the regulation Q not been in place. I perform a counterfactual experiment using the estimated threshold model (1.17). The structural shocks obtained from the data are fed into the VAR system, but with the indicator variable I_t equal to zero in every period. This implies that the spread no longer plays any role for the dynamics of $\mathbf{y_t}$. Figure 1.8 reports the true and simulated HP-filtered paths of real GDP for the 1959:7-1978:6 sample period.[9] The figure shows how all three of the NBER-dated recessions would have been considerably milder. By and large, most of the differences between the true and simulated series occur during the 1970s. This is primarily because, in contrast to the 1960s, the continuous upward revisions of the ceilings were much less sufficient to keep pace with rising market interest rates in that period. Table 1.3 reports the standard deviation of HP-filtered real GDP. In accordance with the evidence on the Great Moderation, output volatility in the data is about twice as large in the 1959:7-1978:6 than in the 1983:10-2004:4 sample. Removing the effect of the regulation Q ceilings, the output volatility in the 1959:7-1978:6 sample drops from 1.16% in the data to 0.82% in the counterfactual

[9]The HP-filter employs a smoothing parameter of 14,400.

experiment. In other words, the experiment suggests that more than half of the reduction in output volatility can be explained by removing the effects of deposit rate ceilings in the data.

1.5 Conclusion

Assessing the role of changes in the monetary transmission mechanism is essential for understanding the performance of the US economy since the mid 1980s. This paper provides evidence for a disintermediation phenomenon in the 1960s and 1970s that is linked to the existence of deposit rate ceilings under regulation Q. In a monetary DSGE model that incorporates interest rate ceilings as occasionally binding constraints, I show how binding ceilings change the behavior of monetary aggregates and their relation with real activity. In an environment where deposit rate ceilings are binding, the Federal Reserve has greater leverage over real activity during a monetary tightening. The main predictions of the model are supported in the data after estimating a threshold VAR aimed at capturing the nonlinearities implied by binding regulation.

In the debate on the stability of monetary VARs, my empirical results suggest a significant amount of regime dependence in the autoregressive coefficients that is due to changes in the financial landscape, and to regulation-induced disintermediation effects in particular. Regarding changes in the volatility of the business cycle, my counterfactual experiment indicates that output volatility is significantly reduced by removing the effects of deposit rate ceilings during the 1960s and 1970s. This result suggests that financial markets deregulation and innovation deserve more attention in future research as an explanation for the structural changes in the US economy over the last couple of decades.

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Appendix: A Brief History of Regulation Q

Until the second half of the 1950s, regulation Q was of limited significance in US banking, as the legal ceilings remained well above market rates and the average rates paid on savings and term deposits. However, the gradual rise of interest rates in the 1950s made the ceilings bind for the first time since their inception. In order to provide banks with greater flexibility in competing for funds, in 1957 legislators decided to raise the ceilings for the first time in over 20 years. Nevertheless, the continuing updrift in market rates throughout the 1960s and 1970s meant banks were frequently unable to offer competitive yields. The recurrent problems in the banking sector stirred reactions by both regulators and banks: On the one hand the maximum rates payable on various types of deposits were frequently revised; on the other hand, banks actively sought to decrease their reliance on heavily regulated sources of funds through innovation.

The rising interest rates necessitated frequent adjustments of the ceilings. As an illustration, Figure 1.9 depicts the ceiling on savings deposits at commercial banks, together with the interest rate on 3-month Treasury bills. The Figure shows how the maximum rate payable was revised upwards on multiple occasions: To 3.5% in 1962, 4% in 1964, 4.5% in 1970, 5% in 1973, 5.25% in 1979 and finally to 5.5% in 1984. Similar revisions were made for the other deposit categories. But despite all these adjustments, market rates repeatedly rose above the ceilings. Quarterly surveys conducted by the Federal Home Lone Bank Board during the 1970s confirm that the vast majority of banks were indeed paying the maximum rates.[10] One major regulatory change occurred during the recession of 1970, when ceilings on certain types of large time deposits were suspended. The suspension took place after the volume outstanding of these instruments had shrunk dramatically because ceilings had hampered banks to compete effectively. Later, in May 1973, all large denomi-

[10]See Cook (1978) for a discussion of the survey results.

nation time deposits were freed of interest rate restrictions. Towards the end of the 1970s, the regulatory attitude started to move more in pace with developments in the financial markets. Finally, the Monetary Control Act (MCA) of 1980 initiated the phaseout of regulation Q. In practice, the remaining ceilings on small term deposits were eliminated in October 1983, and those on savings deposits in March 1986.

After the experience of the late 1950s, depository institutions started to reduce their reliance on heavily regulated sources of funds. The binding ceilings unleashed a cat-and-mouse game between banks and regulators: Banks would develop a new instrument, after which the Federal Reserve would declare it a deposit and subject it to regulation Q. Before 1960, almost all deposits at US banks consisted of demand and savings deposits. Figure 1.10, which plots the ratio of managed liabilities to core deposits, shows the shift in deposit categories throughout the 1960s and 1970s. One of the key innovations in this respect was the creation in 1961 of large denomination negotiable CDs by a New York bank, together with the creation of a secondary market. The advantage of the negotiable CD was that it was a liquid, interest-bearing asset that was marketable when nearing maturity. This contrasted with standard time deposits which could not bear interest at maturities below 30 days. Within a couple of years, total domestic negotiable CDs outstanding had risen dramatically. Another innovation were the Eurodollar deposits, which, falling outside of the jurisdiction of the Federal Reserve, were not subject to regulatory ceilings. After interest rates had risen well above the maximum rates during the 1973-1975 recession, banks put strong pressure on legislators for deregulation. In June 1978, the use of retail money market certificates (MMCs), a new category of 6-month time deposits was permitted. The ceiling rate on newly issued MMCs was adjusted weekly to the current discount yield on 6-month T-bills and then remained constant until maturity. Similarly, 1979 saw the introduction of small saver certificates (SSCs), which had large maturities but also paid market-determined interest rates. The subsequent growth in the use of MMCs and SSCs, which were effectively ridden of interest rate ceilings, greatly improved the ability of banks to compete with money market instruments from 1978 onwards. In fact, Gilbert et al. (1979) observe how the authorization of MMCs by Federal regulators arose precisely in an attempt to reduce the extent of disintermediation.

Theoretical Appendix

Attention is confined to symmetric equilibria in which $P_{jt} = P_t$, $N_{jt} = N_t$, $y_{jt} = y_t$, $h_{jt} = h_t$ and $s_{jt} = s_t$ for all $j \in (0,1)$. In addition, the state space is confined to yield equilibria where all net nominal interest rates are positive, and it is verified that the inequality in (1.4) never binds. Although the method used to solve the model allows to take into account the zero bound on nominal interest rates as well as the inequality in (1.4), both are irrelevant for the choice of parameters discussed in Section 1.3.2 and are therefore ignored in the equilibrium definition below.

It is useful to scale the model's nominal quantities by the beginning-of-period money stock and define $[p_t \ q_t \ w_t] \equiv [P_t \ Q_t \ W_t]/M_t$. In addition, let z_t be the inverse markup due to imperfect competition in the goods markets. An equilibrium is conveniently defined as a set of sequences of quantities $\{y_t, h_t, c_t, n_t, q_t\}_{t=0}^{\infty}$ and prices $\{p_t, r_t, R_t, z_t, w_t\}_{t=0}^{\infty}$ such that in every period t and given the initial conditions $p_0, n_0 - c_0$ and the period t history of realizations of χ_t, the system of equations, given by (B.1)-(B.11) is satisfied.

Household

$$\Psi h_t = \frac{1}{c_t}\frac{w_t}{p_t} \quad \text{(B.1)}$$

$$1 = \beta \mathrm{E}_t \left[\frac{c_t}{c_{t+1}} R_{t+1} \frac{p_t}{p_{t+1}\mu_t}\right] \quad \text{(B.2)}$$

$$R_t = 1 + \Omega \left(\frac{q_t}{p_t}\right)^{-\frac{1}{\phi}} c_t \quad \text{(B.3)}$$

Policy

$$R_t = \min(r_t, R^q) \quad \text{(B.4)}$$

$$\ln(\mu_t) = (1-\rho)\ln(\mu) + \rho \ln(\mu_{t-1}) + \chi_t \quad \text{(B.5)}$$

Firms

$$y_t = h_t^{\alpha} - \theta \quad \text{(B.6)}$$

$$n_t = y_t + n_{t-1} - c_{t-1} \quad \text{(B.7)}$$

$$\frac{w_t}{p_t} = \alpha \frac{z_t}{r_t}\frac{y_t+\theta}{h_t} \quad \text{(B.8)}$$

$$1 = \frac{\varepsilon}{\varepsilon - 1}\mathrm{E}_t \beta \left[\frac{c_{t+1}}{c_{t+2}} z_{t+2}\right] \quad \text{(B.9)}$$

$$\left(\xi \frac{c_t}{n_t} - z_t\right) = \beta \mathrm{E}_t \left[\frac{c_t}{c_{t+1}} z_{t+1}\right]\left(\xi \frac{c_t}{n_t} - 1\right) \quad \text{(B.10)}$$

Loan Market

$$q_t = \mu_t - w_t h_t \quad \text{(B.11)}$$

The first set of equilibrium conditions summarizes the household's optimal behavior. Equation (B.1) determines labor supply by equating the marginal disutility of working to the real wage, weighted by the marginal utility of consumption $1/c_t$. The household's saving and consumption behavior is governed by the standard Euler condition, given by equation (B.2). Equation (B.3) specifies the demand for cash. Equation (B.4) stipulates the regulation Q restriction on the deposit rate and (B.5) specifies monetary policy. Equation (B.6) specifies the production technology and (B.7) states that the current aggregate stock of final consumption goods available for sale equals the sum of newly produced goods and inventories carried over from the previous period. Labor demand is determined by equating the real wage with the marginal product of labor, as in (B.8). Equation (B.9) determines the typical firm's optimal price-setting behavior, with prices set as markups over expected discounted future marginal costs. Similar to Jung et al. (2006), the possibility of inventory investment leads firms to take into account marginal cost two periods ahead rather than one. Equation (B.10) dictates the firms' optimal inventory accumulation and is identical to the first-order condition of inventory investment in the partial equilibrium setting of Bils et al. (2000). Finally, clearing in the market for loans requires that the nominal wage bill equals the total volume of loanable funds, as in equation (B.11).

Data Appendix

Data used in Figures 1.1-1.4 and 1.9-1.10

The data on the ceiling rates on the various types of deposits was constructed from the Annual Statistical Digest published by the Board of Governors of the Federal Reserve System. The monthly time series are available from the author. Market interest rates are from Federal Reserve release H.15: Selected Interest Rates. The seasonally adjusted data on deposits is from Federal Reserve release H.6: Money Stock Measures. The following series were used to construct Figures 1.1-1.4.

Figure 1.1: Annualized growth in managed liabilities is the year-on-year growth in the sum of small and large time deposits, repurchase agreements and eurodollar deposits at all depository institutions, deflated by the Personal Consumption Expenditures price index; annualized growth in core deposits is year-on-year growth in savings and checkable deposits at all depository institutions in the US, deflated by the Personal Consumption Expenditures price index.

Figure 1.2: Spread is the difference between the 3-month Treasury Bill rate and the ceiling on savings deposits at commercial banks with maturity less than 12 months; annualized

growth in core deposits is year-on-year growth in savings and checkable deposits at all depository institutions in the US, deflated by the Personal Consumption Expenditures price index. Core deposits include Money Market Deposit Accounts.

Figure 1.3: Components of managed liabilities are small time deposits, large time deposits, repurchase agreements and eurodollar deposits at all depository institutions, each divided by the sum of small and large time deposits, repurchase agreements and eurodollar deposits at all depository institutions.

Figure 1.4: Annualized growth in small term deposits is year-on-year growth in small term deposits (STDs) at all depository institutions, deflated by the Personal Consumption Expenditures price index; Spreads are the differences between 1) 3-month T-bill and ceiling on STDs with maturity from 30 to 89 days, 2) average of 3-month and 6 month T-bill rate and the ceiling on STDs from 90 to 179 days, 3) average of 6-month T-bill rate and 1 year T-security rate and the ceiling on STDs from 180 days to 1 year, 4) average of 1-year and 2-year T-security rate and the ceiling on STDs from 1 year to 2 years. Annualized growth in large term deposits is year-on-year growth in large term deposits (LTDs) at all depository institutions, deflated by the Personal Consumption Expenditures price index. Spreads are the differences between 1) 3-month T-bill rate and ceiling on LTDs with maturity from 30 to 59 days, 2) 3-month T-bill rate and the ceiling on LTDs from 60 to 89 days, 3) average of 3-month and 6-month T-bill rate and the ceiling on LTDs from 90 to 179 days, 4) average of 6-month T-bill rate and 1-year T-security rate and the ceiling on LTDs from 180 days to 1 year.

Figure 1.9: Ceiling on savings deposits at commercial banks with maturity less than 12 months; 3-Month Treasury Bill rate (secondary market).

Figure 1.10: The ratio of managed liabilities is the sum of small and large time deposits, repurchase agreements and eurodollar deposits at all depository institutions divided by the sum of savings and checkable deposits at all depository institutions.

Additional Data used in VARs

Price Index: *Personal Consumption Expenditures Chain-Type Price Index Less Food and Energy*, Federal Reserve Bank of St.Louis, http://research.stlouisfed.org/

Commodity Price Index: *Index of Sensitive Materials Prices*, Business Cycle Indicators, The Conference Board Series, http://www.conference-board.org

Real GDP: *Real Gross Domestic Product, Seasonally Adjusted Annual Rate*, U.S. Department of Commerce: Bureau of Economic Analysis (NIPA), http://www.bea.gov/

Inventories-to-Sales Ratio: *Ratio of Manufacturing and Trade Inventories (Chained 2000 $) to Manufacturing and Trade Sales (Chained 2000 $)*, Business Cycle Indicators, The Conference Board Series, (mnemonics A0M070 and A0M057) http://www.conference-board.org

Federal Funds Rate: *Effective Federal Funds Rate*, The Federal Reserve System's H.15 release, http://www.federalreserve.gov/releases/h15.

Commercial and Industrial Loans: *Commercial and Industrial Loans at All Commercial Banks*, H.8 Statistical Release of the Federal Reserve System,
http://www.federalreserve.gov/releases/h8.

Tables and Figures

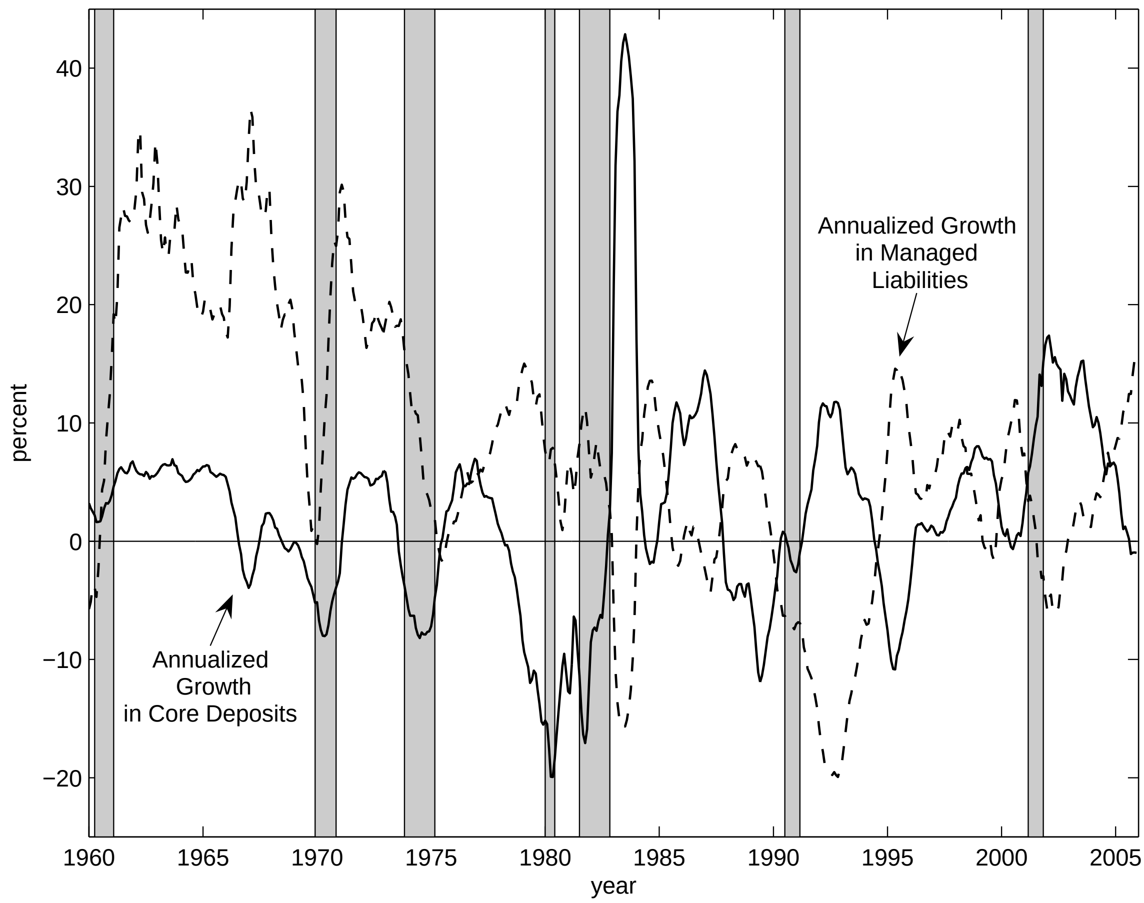

Figure 1.1: Annualized Real Growth in Core Deposits and Managed Liabilities. Monthly data (Jan 1960-Dec 2005). Grey areas indicate NBER-dated recessions (peak-to-through). Data Sources: See Appendix.

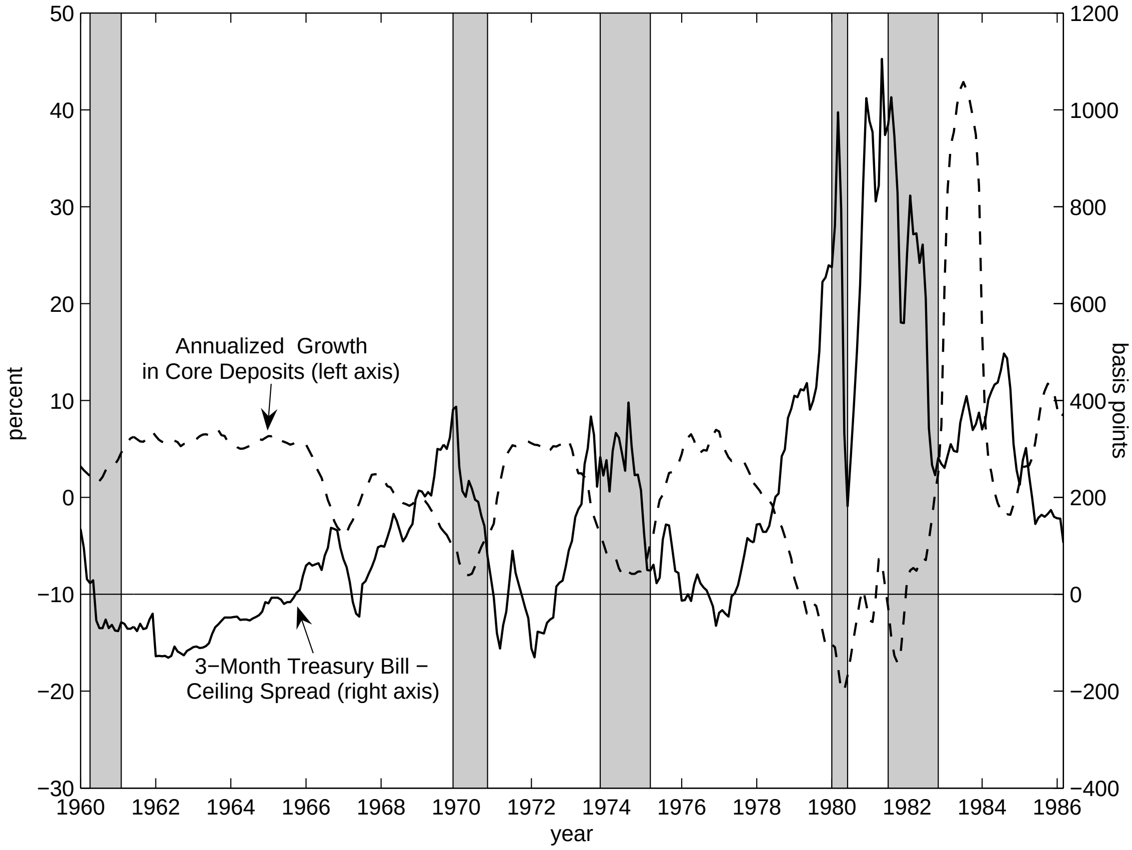

Figure 1.2: Annualized Real Growth in Core Deposits and 3-Month Treasury Bill - Ceiling Spread.
Monthly data (Jan 1960- Mar 1986). Grey areas indicate NBER-dated recessions (peak-to-through). Data Sources: See Appendix.

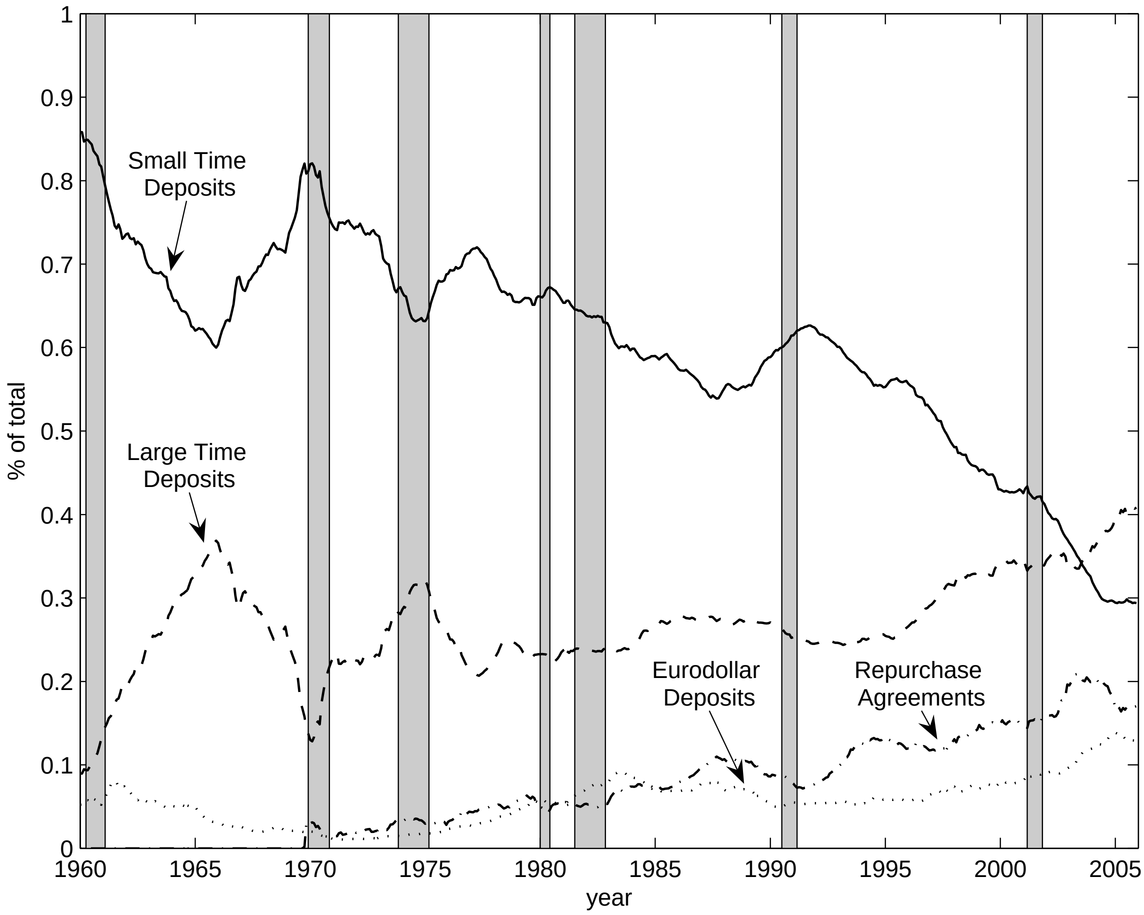

Figure 1.3: Components of Managed Liabilities.
Monthly data (Jan 1960-Dec 2005). Grey areas indicate NBER-dated recessions (peak-to-through). Data Sources: See Appendix.

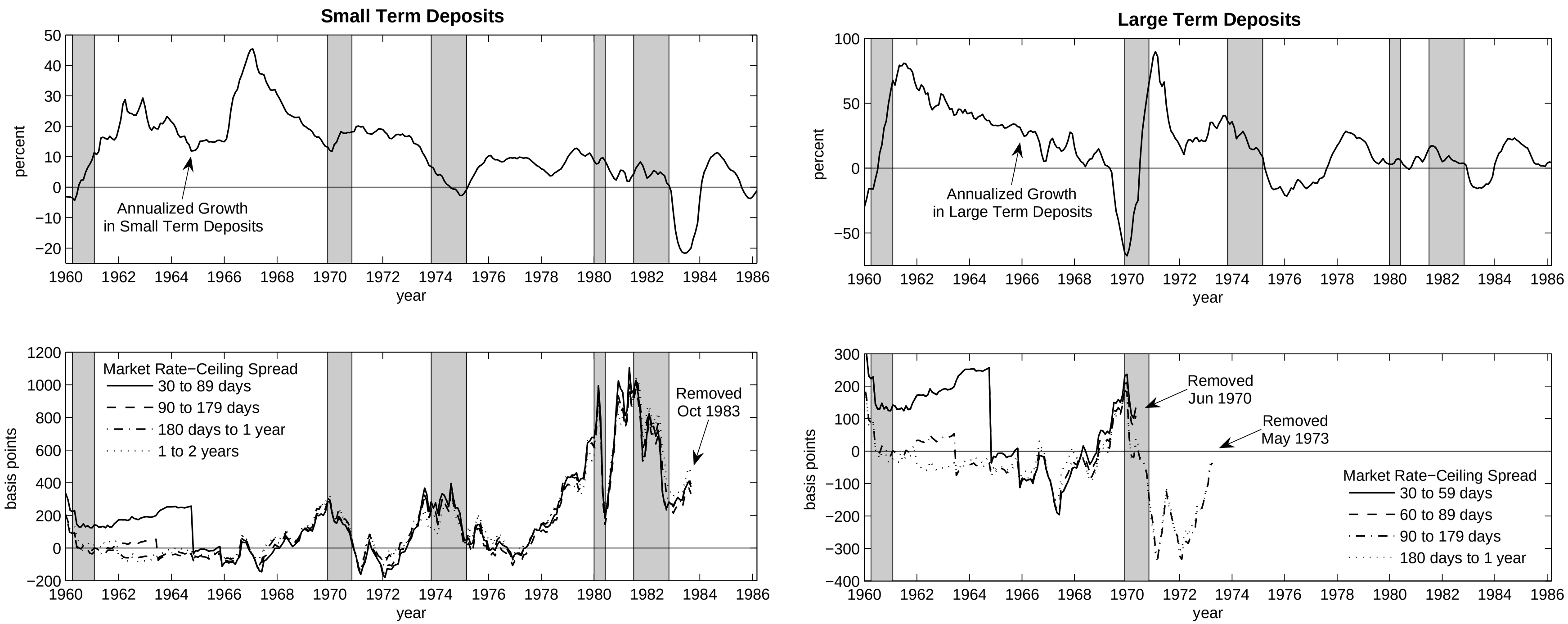

Figure 1.4: Annualized Real Growth in Small and Large Time Deposits and Spreads between the Market Rate on Treasury Securities with Similar Maturity and the Deposit Rate Ceiling.
Monthly data (Jan 1960-Mar 1986). Grey areas indicate NBER-dated recessions (peak-to-through). Data Sources: See Appendix.

Table 1.1: Parameter Values of the Theoretical Model

Household Preferences		
β	$1.03^{-0.25}$	Household discount factor
Ψ	0.64	Time allocation parameter
Ω	$4.38e^{-5}$	Utility weight of real cash balances
ϕ	0.24	Interest rate elasticity of money demand
ε	5.79	Price elasticity of demand for goods
ξ	0.08	Sales stock elasticity of demand for goods
Technology		
α	0.64	Labor input elasticity of production
θ	0.18	Fixed cost of production
Monetary Policy		
μ	1.012	Average money growth rate
ρ	0.3	Persistence of the shock
σ	0.012	Standard deviation of the shock

Table 1.2: Reduced-Form Stability Tests

Equation:	$PCOM_t$	PI_t	Y_t	In_t	FF_t	MZM_t	LNS_t	ML_t
p-value	0.105	0.001	0.4628	0.1478	0.041	0.002	0.063	0.237

The numbers are p-values for the Wald statistic testing for the null hypothesis that the reduced-form coefficients in the nonbinding regulation Q periods ($S_t < 0$) equal those of the binding periods ($S_t > 0$). The test is applied jointly to the autoregressive coefficients and the error variance. The p-values are based on 5000 bootstrap simulations.

Table 1.3: Standard Deviation of HP-filtered Real GDP

Series	Sample	St.Dev.
Data	1960:1-1978:6	1.16%
Experiment no Reg Q	1960:1-1978:6	0.82%
Data	1983:10-2004:4	0.58%

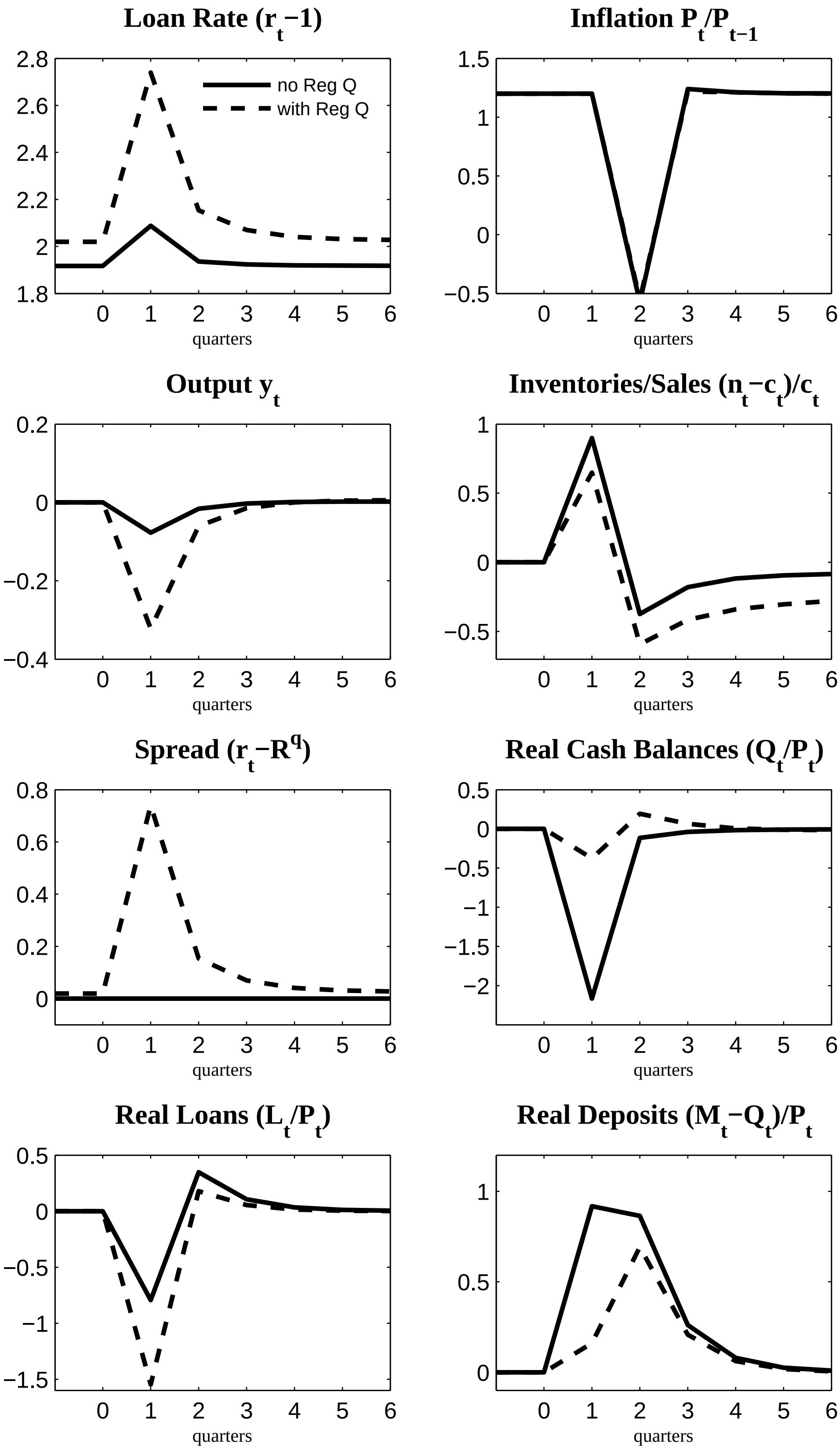

Figure 1.5: Theoretical Response to a One-St.-Dev. Negative Money Growth Shock. The shock occurs in period 1. The solid line depicts the response without regulation Q and the dashed line depicts the response with binding regulation Q.

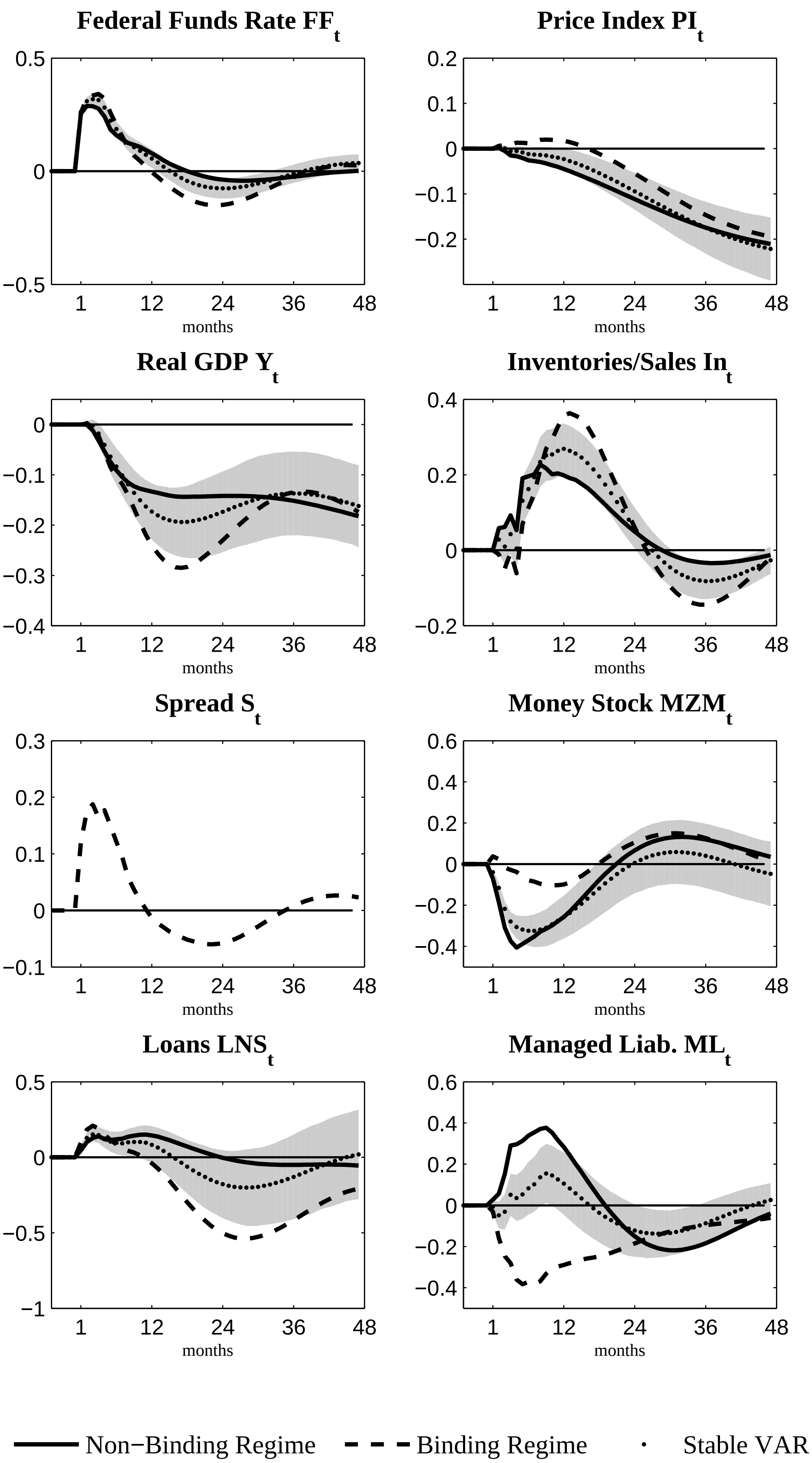

Figure 1.6: Response to a One-St.-Dev. Positive Funds Rate Shock in period 1.

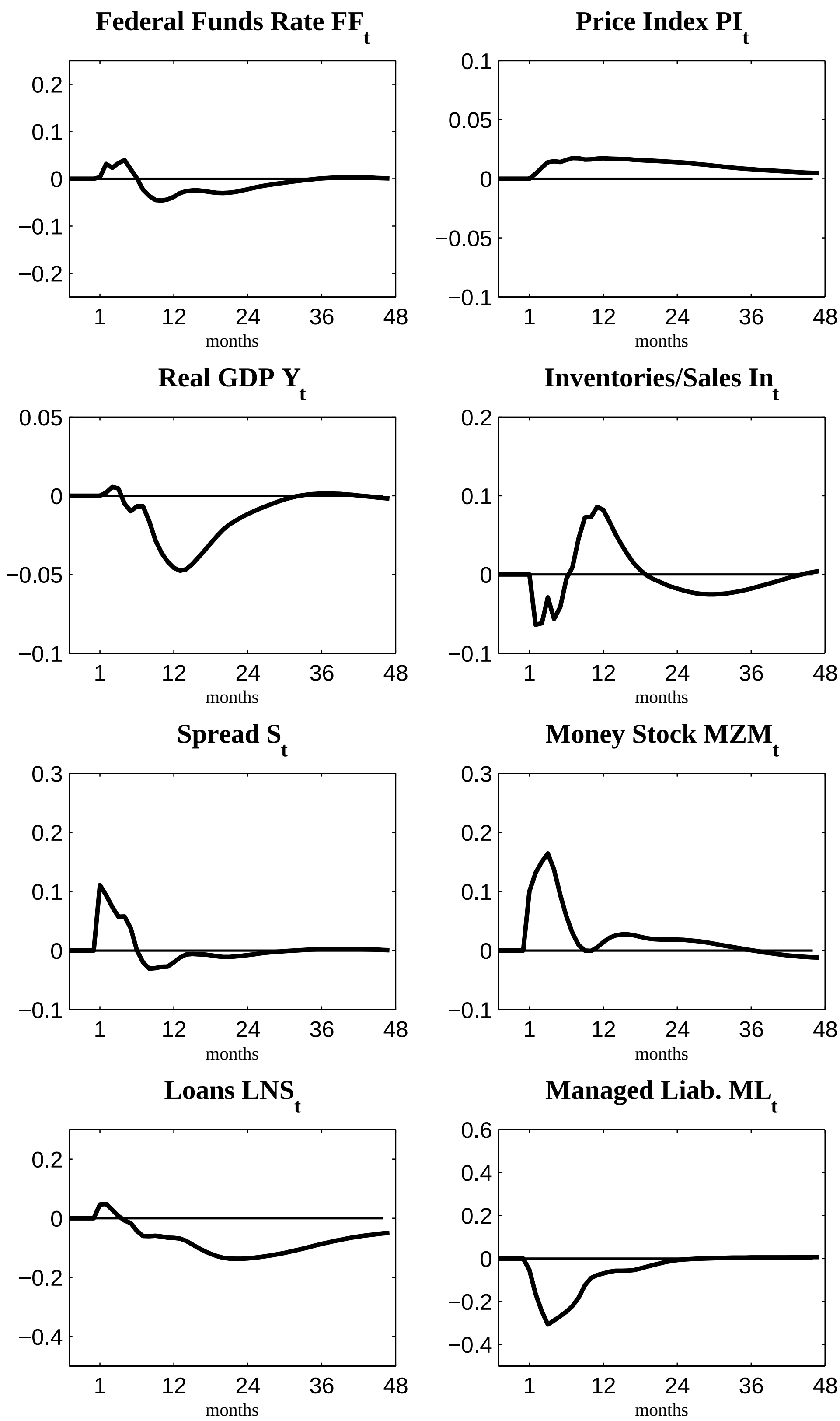

Figure 1.7: Response to a One-St.-Dev. Positive Spread Shock in period 1.

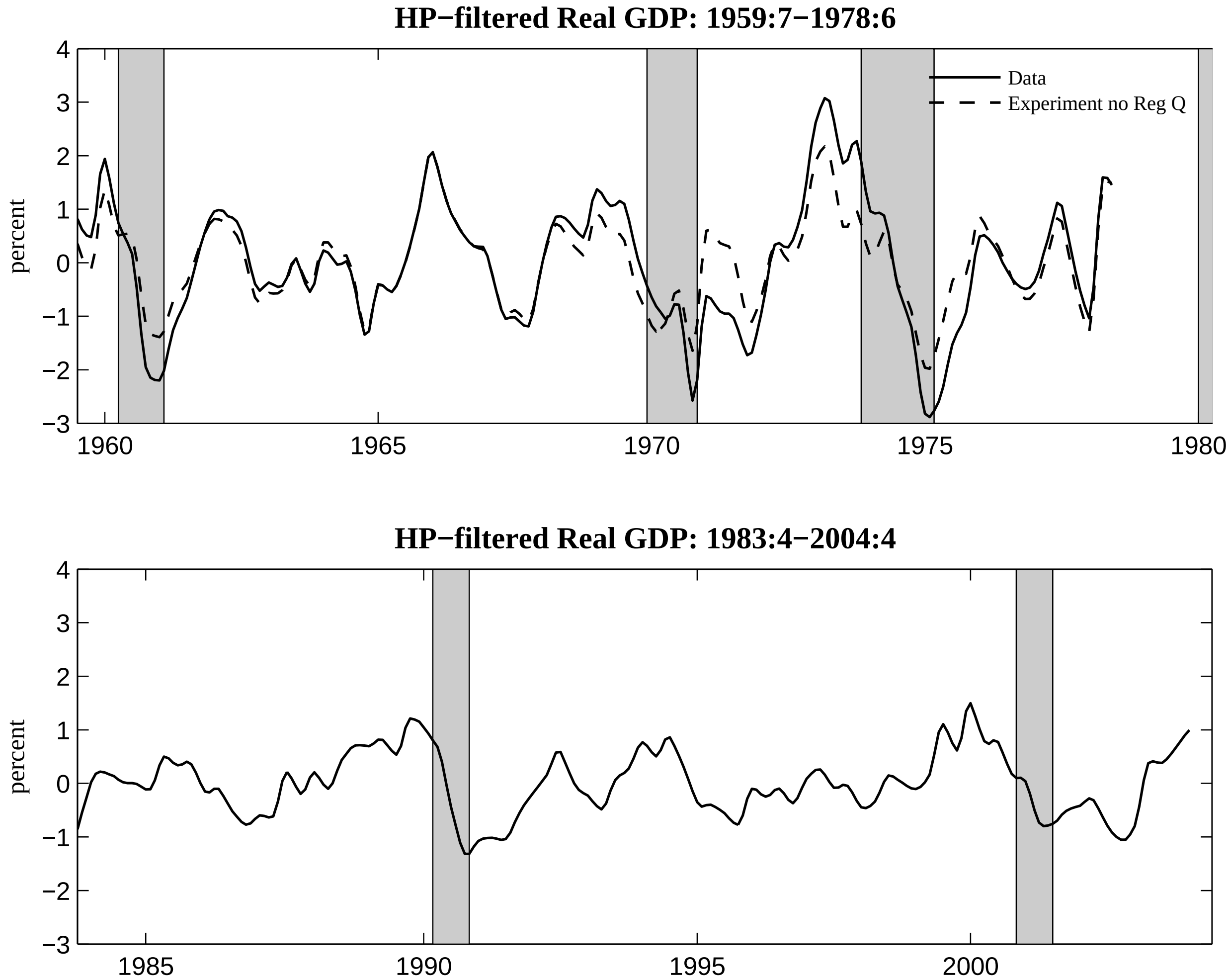

Figure 1.8: Real GDP Counterfactual Experiment.
Grey areas indicate NBER-dated recessions (peak-to-through). The solid line depicts the true data and the dashed line depicts the simulated path of real GDP for the experiment without regulation Q.

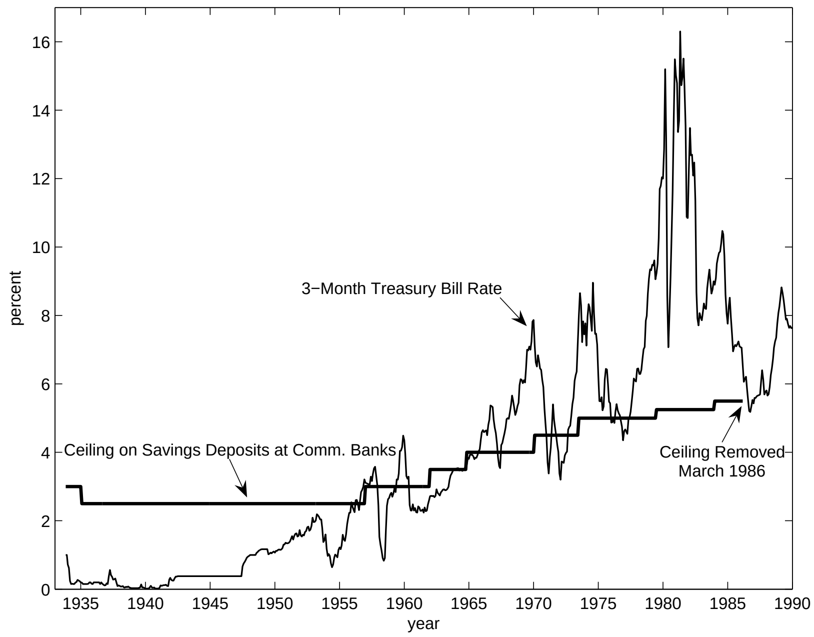

Figure 1.9: 3-Month T-Bill Rate and Ceiling on Savings Deposits at Commercial Banks. Monthly data (Nov 1933-Dec 1989). Data Sources: See Appendix.

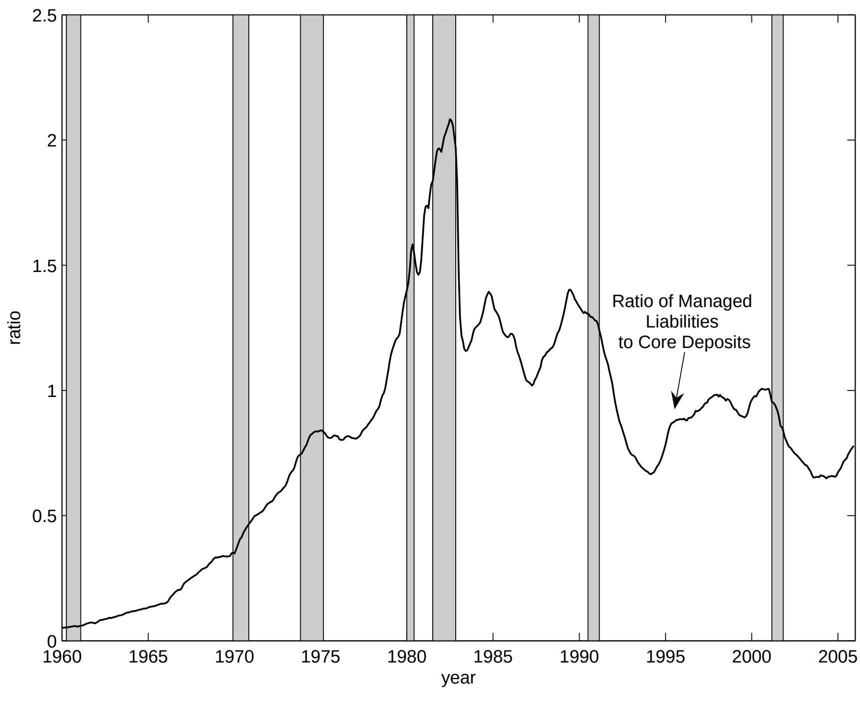

Figure 1.10: Ratio of Managed Liabilities to Core Deposits.
Monthly data (Jan 1960-Dec 2005). Grey areas indicate NBER-dated recessions (peak-to-through). Data Sources: See Appendix.

Chapter 2

The Role of Expectations in Sudden Stops

2.1 Introduction

In October 1997, Standard & Poor's downgraded South Korea's sovereign risk status. During the first quarter of 1998, Korea's net exports-to-GDP ratio rose by more than 18%, GDP contracted by 8.7%, consumption by 13%, and investment by 30%. One year later, GDP, consumption and investment were growing at 10% or more. What causes these abrupt declines in capital inflows, known as "Sudden Stops" (Calvo, 1998), and why are they accompanied by depression-sized, but short-lived, contractions in economic activity?

This paper argues that a shift in expectations about future productivity growth can trigger a Sudden Stop such as the one experienced by South Korea. I propose a small open economy model that suffers from a "peso problem": There is a non-zero probability that productivity growth switches to a bad regime. In response to news signals about future productivity growth, agents revise the probability of the bad regime occurring. The model displays equilibrium paths which, after agents receive bad news, are characterized by an increase in net exports, and a decrease in aggregate output, employment, investment and consumption. Agents are fully rational and during the Sudden Stop, there is never any actual change in productivity growth. When the news signal turns out to be false, the economy quickly reverts to its previous growth path, which requires a period of above-trend growth. To quantitatively account for the Korean experience without unrealistic adjustments in expectations, the model relies on a number of amplification mechanisms, such as variable capacity utilization, predetermined labor, a working capital constraint and an expectation-elastic country risk premium. Because Sudden Stops are phenomena that lead the economy far away from steady states and given the focus on expectations, I solve the model using a nonlinear global approximation method.

The idea that shifts in expectations can drive macroeconomic fluctuations goes back as far as Pigou (1927), but has received renewed attention in the context of modern business cycle models.[1] This is also not the first paper to explore the role of adverse expectations in emerging markets crises. They are an inherent feature of all models in which crises are self-fulfilling events, such as for instance Obstfelt (1986) or Chang and Velasco (2001). However, the model in this paper does not display equilibrium indeterminacy. With Corsetti, Pesenti and Roubini (1999) and Burnside, Eichenbaum and Rebelo (2001), it shares the emphasis that future, rather than current, events cause crises.[2] But the model in this paper is not one of perfect foresight, such that the occurrence of a crisis does not hinge on the future event materializing. The implication is that it is not only difficult to predict crises, they may also be hard to rationalize ex post.

In the literature, there are several approaches to modeling Sudden Stops in a Dynamic Stochastic General Equilibrium (DSGE) framework. In a sense, the most closely related is by Aguiar and Gopinath (2007). They show in a standard small open economy model that a large and persistent decrease in productivity growth generates a Sudden Stop, together with contractions in GDP, consumption and investment. With a *persistent* shock, agents also revise expectations about future productivity growth. The persistence is crucial for obtaining responses that are typical of Sudden Stops. In contrast with Aguiar and Gopinath (2007), in this paper a Sudden Stop occurs without any change in productivity growth. Aguiar and Gopinath (2007) base their evidence for a negative TFP growth shock on the estimation of Solow residuals. However, as the authors themselves point out, the large decreases in measured TFP are to a large extent only indicative of endogenous links between measured

[1] Examples include Danthine, Donaldson and Johnsen (1999), Beaudry and Portier (2004), Jaimovich and Rebelo (2006), Den Haan and Kaltenbrunner (2007) and Prades (2007).

[2] In the case of Corsetti et al. (1999) and Burnside et al. (2001), it are the large prospective fiscal deficits associated with implicit bailout guarantees to failing banks that trigger currency crises.

TFP and Sudden Stops. I allow for variable capacity utilization such that a Sudden Stop is accompanied by a change in the *measured* Solow residual.

Many models in the literature focus on the role of credit frictions. One common approach is to model a Sudden Stop as an exogenous tightening of a collateral constraint. Chari, Kehoe and McGrattan (2005) show that this type of shock tends to stimulate output unless further frictions are included. Christiano, Gust and Roldos (2004) impose advance payment constraints on intermediate inputs to produce output drops. Gertler, Gilchrist and Natalucci (2003) model the Korean crisis as caused by an exogenous increase in the country risk premium and rely on a financial accelerator framework to explain the depth of the crisis. I also incorporate a financial propagation mechanism in the form of a working capital constraint and an expectation-elastic risk premium. The approach in this paper is different, however, as both the Sudden Stop and the associated recession arise endogenously after an adjustment in expectations. The financial friction is not necessary to *qualitatively* match the Korean crisis experience, but is nevertheless important to obtain fluctuations that are *quantitatively* similar.

In the business cycle models of Mendoza (2006) and Mendoza and Smith (2006), Sudden Stops are also endogenous. In their setup, when the economy moves towards a high debt state, shocks of standard magnitudes can force a collateral constraint to bind, triggering highly nonlinear dynamics that resemble Sudden Stops. Their model does not hinge on large unexpected shocks; it captures precautionary motives; and Sudden Stops are rare events nested within regular business cycle movements. The model in this paper can also explain Sudden Stops without large exogenous shocks to TFP, interest rates or the terms of trade. However, to trigger a Sudden Stop, a fairly large (but not unrealistic) shock to expectations is necessary. As in Mendoza (2006), agents engage in precautionary behavior, and since I model shocks to expectations necessary to generate a Sudden Stop as rare events, they arise infrequently within regular business cycles. In contrast with Mendoza (2006), the economy does not need to be in a high debt state to experience a crisis. Indeed, this property is important for the Korean case, as its foreign debt-to-GDP ratio was far lower at the onset of the crisis than for instance in the 1980s.

The rest of the paper is organized as follows: Section 2.2 describes the theoretical model; Section 2.3 discusses the calibration to South Korean data and the numerical solution technique; Section 2.4 presents the model response to a news shock, compares it with the Korean experience and performs a sensitivity analysis of the key parameters; Section 2.5 concludes.

2.2 The Model

The model is that of a single good neoclassical small open economy that faces stochastic shocks to the growth rate of productivity. Agents receive stochastic news about future productivity growth and are fully rational in judging the reliability of the news. Both households and domestic firms trade a non-contingent real bond. As in Neumeyer and Perri (2005), Mendoza (2006) and Uribe and Yue (2006), the latter trade in the asset because of the presence of a working capital constraint that requires firms to advance the wage bill before final output is available. Following Neumeyer and Perri (2005), the model also allows for a country risk premium that is decreasing in expected future productivity. Together, these two features amplify news shocks through changes in interest rates. Two other model ingredients are variable capacity utilization and the requirement that firms choose labor input before the realization of present period uncertainty. Variation in the utilization rate of the capital stock leads to propagation mainly through the effect on the marginal product of labor. Predetermined labor input means that firms hire workers based on expected rather than actual productivity levels. The rest of the model closely resembles the canonical small open economy real business cycle models in for instance Mendoza (1991), Correia, Neves and Rebelo (1995) or Schmitt-Grohé and Uribe (2003).

Time is discrete and in each period t, there are two subperiods: t^- in the beginning of t and t^+ at the end of t. Time t^+ and $(t+1)^-$ are arbitrarily close. All uncertainty is revealed to the agents in period t^-. Table 2.1 summarizes the timing of events.

Firms and Technology. At time t^- a representative firm rents capital services k_t^s and, in combination with labor input h_t, produces y_t of an international tradable good, which becomes available in t^+. The firm's labor input decision must be made in $(t-1)^+$, i.e. *before* the realization of period t uncertainty. The firm is entirely owned by domestic households. Factor markets are perfectly competitive, and production occurs through the constant returns to scale technology

$$\begin{aligned} y_t &= (k_t^s)^\alpha (\Gamma_t h_t)^{1-\alpha} \ , 0 < \alpha < 1 \ , \\ \Gamma_t &= g_t \Gamma_{t-1} \ . \end{aligned} \tag{2.1}$$

Γ_t measures the level of labor-augmenting technology, which grows at a stochastic rate $\ln(g_t)$. The firm needs to borrow working capital in advance: In order to transfer $w_t h_t$ to workers in t^+, where w_t is the real wage, the firm needs to issue bonds worth $w_t h_t$ in t^- at a rate R_{t-1}. Given prices, the firm chooses k^s to maximize time t^+ profits $y_t - r_t k_t^s - R_{t-1} w_t h_t$, subject to the technological constraint in (2.1). The firm chooses h_t to maximize the appropriately weighted expectation of time t^+ profits, given by

$$\mathrm{E}_{t-1} \left[\lambda_t \left(y_t - r_t k_t^s - R_{t-1} w_t h_t \right) \right] \ . \tag{2.2}$$

The firm takes λ_t, the marginal utility of consumption of its owner, as exogenous.

Households and Preferences. The economy is populated by identical, infinitely-lived households with preferences described by

$$\mathrm{E}_0 \sum_{t=0}^{\infty} \exp\left(-\sum_{\tau=0}^{t-1} \beta\left(c_\tau, l_\tau\right)\right)\left[\frac{\left(c_t - \Gamma_{t-1}\zeta\frac{l_t^{1+\psi}}{1+\psi}\right)^{1-\gamma}\Gamma_{t-1}^{\gamma}}{1-\gamma}\right], \tag{2.3}$$

$$\begin{aligned}\beta(c_\tau, l_\tau) &= \xi \ln\left(1 + \Gamma_{\tau-1}^{-1} c_\tau - \zeta\frac{l_\tau^{1+\psi}}{1+\psi}\right), \\ &\psi > 0\,, \gamma > 1\,, 0 < \xi \leq \gamma\,, \zeta > 0\,,\end{aligned}$$

where c_t denotes consumption and l_t is time spent in the workplace. As in Mendoza (1991), these preferences feature an endogenous rate of time preference that increases with the past level of consumption. The inclusion of an endogenous discount factor is one way to avoid a unit root in bond holdings and otherwise has little implications for the model dynamics (Schmitt-Grohé et al., 2003). The momentary utility function is of the form proposed by Greenwood, Hercowitz and Huffman (1988). With this specification, optimal labor effort depends only on the contemporaneous real wage. These preferences are popular in small open economy models because they generate more realistic business cycles moments (Correia et al., 1995). They also facilitate the numerical solution procedure by eliminating a root-finding operation. The term Γ_{t-1} enters the utility function to ensure the existence of a balanced growth equilibrium. Given these preferences, the marginal utility of consumption λ_t is given by

$$\lambda_t = \left(c_t - \Gamma_{t-1}\zeta\frac{l_t^{1+\psi}}{1+\psi}\right)^{-\gamma}\Gamma_{t-1}^{\gamma} - \xi\left(1 + \Gamma_{t-1}^{-1}c_t - \zeta\frac{l_t^{1+\psi}}{1+\psi}\right)^{-1} e^{-\beta(c_t,l_t)}\frac{V_t}{\Gamma_{t-1}}, \tag{2.4}$$

where

$$V_t = \mathrm{E}_t\left[\frac{\left(c_{t+1}-\Gamma_t\zeta\frac{l_{t+1}^{1+\psi}}{1+\psi}\right)^{1-\gamma}}{1-\gamma}\Gamma_t^{\gamma}+e^{-\beta(c_{t+1},l_{t+1})}V_{t+1}\right] .$$

At time t^-, households supply labor and capital services. At time t^+ they receive factor payments and make consumption and investment decisions. Households own a stock of capital k_t, and capital services k_t^s are equal to the product of the capital stock and the rate of capacity utilization u_t. The households' budget constraint in period t is

$$c_t + x_t + R_{t-1}d_t \leq d_{t+1} + w_t l_t + r_t u_t k_t \ , \tag{2.5}$$

where x_t are resources for investment and d_{t+1} is the households' foreign debt position. Long-run solvency is enforced by imposing an upper bound on foreign debt, $d_{t+1} < \Gamma_t \bar{d}$. This condition precludes households from running Ponzi-type schemes.[3]

The law of motion for capital is

$$k_{t+1} = x_t + \left(1-\delta-\eta\frac{u_t^{1+\omega}}{1+\omega}\right)k_t - \frac{\phi}{2}\left(\frac{k_{t+1}}{k_t}-\mu\right)^2 k_t \ , \tag{2.6}$$
$$\phi > 0 \ , \mu > 1 \ , \eta > 0 \ , \omega > 0 \ .$$

As in Baxter and Farr (2001), the rate of capital depreciation depends positively on capital utilization. There is a quadratic capital adjustment cost, and μ is the economy's average productivity growth factor.

The households' problem is to choose state-contingent sequences of c_t, l_t, x_t, u_t, k_{t+1} and d_{t+1} to maximize expected utility (2.3), subject to the budget constraints (2.5), the borrow-

[3] Choosing a large value for $\bar{d}$, the probability of reaching the debt limit in the stochastic steady state can be made arbitrarily small.

ing constraints and the law of motion for capital (2.6), for given prices w_t, r_t and R_t and initial values k_0 and d_0.

The Interest Rate. A large mass of international investors is willing to purchase the economy's bonds at a rate R_t. The bonds are risky assets because default on payments to foreigners is possible. The interest rate faced by the small open economy is given by

$$R_t = \rho D_t \,, \tag{2.7}$$

where ρ is the international rate for riskless assets and D_t is the country risk premium. As in Neumeyer and Perri (2005), private domestic lenders always receive the full loan plus interest, but there is a probability that the local government will confiscate all interest payments to foreign lenders. Foreign bond holders determine the interest rate and, given the small open economy assumption, domestic agents take R_t as given. To be consistent with this interpretation, I verify that in the numerical analysis foreign lenders always lend positive amounts in equilibrium. Default decisions are not modelled explicitly. As in Neumeyer and Perri (2005), the risk premium depends negatively on expected future productivity. For practical purposes, the dependence on expected productivity is captured by the following functional form:

$$D_t = \chi_1 \left(1 + \mathrm{E}_t\left[g_{t+1}\right] - \mu\right)^{-\chi_2} \,, \chi_1 > 1 \,, \chi_2 \geq 0 \,. \tag{2.8}$$

Arrelano (2006) provides a model in which a negative relation between default incentives and expected productivity arises endogenously.

News and States of Technology. I discipline the modelling of the news and technology processes by maintaining the assumption of rational expectations and by pursuing extreme parsimony in the number of parameters. This is to counter any suggestion that a purely arbitrary formation of expectations explains the fit of the model.

Productivity growth g_t is a discrete Markov chain with support $\mu = \{\mu_B, \mu_G\}$, i.e. there is a "bad" state and a "good" state. The transition matrix is

$$P = \begin{bmatrix} p_{BB} & 1 - p_{GG} \\ 1 - p_{BB} & p_{GG} \end{bmatrix}, \tag{2.9}$$

where the ij-th entry is $\Pr(g_{t+1} = \mu_i | g_t = \mu_j)$. Agents receive news n_t about the growth rate two periods in advance. A two period lead is the minimum to ensure that firms alter labor input in response to news, while larger leads come at significant computational cost. The agents' perception of the news accuracy is captured by a matrix Q, given by

$$Q = \begin{bmatrix} q & 1 - q \\ 1 - q & q \end{bmatrix}, \tag{2.10}$$

where the ij-th element is $\Pr(n_t = \mu_i | g_{t+2} = \mu_j)$. The parameter $0.5 \leq q \leq 1$ is a measure for the news precision. To avoid over-parametrization, the news signal contains no information about uncertainty in $t+1$ and its accuracy is independent of the history of shocks up to time t. Suppose in period t the economy is in the good state μ_G and news arrives of a switch to the bad state in $t+2$. When $q = 0.5$ the news signal does not contain any information and the time t expectation of productivity growth in $t+2$ equals the unconditional expectation. When $q = 1$, the signal is perfect and expected productivity in $t+2$ equals μ_B. When $0.5 < q < 1$, agents expect productivity growth to be in between these two values.

Given the rationality assumption, the agents' subjective assessment of the news accuracy corresponds to the objective accuracy. Let n_t^- denote the previous period value of n_t and let x_t^i be shorthand notation for $x_t = \mu_i$. Then all the above assumptions imply the following transition probabilities for the technology/news processes:

$$\Pr(g^i_{t+1}, n^n_{t+1}, n^{-v}_{t+1} \mid g^j_t, n^l_t, n^{-s}_t) = d_{vl} \Pr(n^n_{t+1} \mid g^i_{t+1}, g^j_t, n^l_t, n^{-s}_t) \Pr(g^i_{t+1} \mid g^j_t, n^l_t, n^{-s}_t) \ . \tag{2.11}$$

The first term in (2.11), d_{vl}, equals 1 if $v = l$ and zero otherwise. The second term is

$$\Pr(n^n_{t+1} \mid g^i_{t+1}, g^j_t, n^l_t, n^{-s}_t) = \sum_k \Pr(n^n_{t+1} \mid g^k_{t+2}, g^i_{t+1}, g^j_t, n^l_t, n^{-s}_t) \Pr(g^k_{t+2} \mid g^i_{t+1}, g^j_t, n^l_t, n^{-s}_t) \ , \tag{2.12}$$

where $\Pr(n^n_{t+1} \mid g^k_{t+2}, g^i_{t+1}, g^j_t, n^l_t, n^{-s}_t) = \sum_w Q_{nw} P_{wk}$ and $\Pr(g^k_{t+2} \mid g^i_{t+1}, g^j_t, n^l_t, n^{-s}_t) = Q_{lk} P_{ki} / \sum_k Q_{lk} P_{ki}$. Finally, the third term is

$$\Pr(g^i_{t+1} \mid g^j_t, n^l_t, n^{-s}_t) = \frac{\Pr(n^l_t \mid g^i_{t+1}, g^j_t, n^{-s}_t) \Pr(g^i_{t+1} \mid g^j_t, n^{-s}_t)}{\sum_i \Pr(n^l_t \mid g^i_{t+1}, g^j_t, n^{-s}_t) \Pr(g^i_{t+1} \mid g^j_t, n^{-s}_t)} \ , \tag{2.13}$$

where $\Pr(g^i_{t+1} \mid g^j_t, n^{-s}_t) = Q_{si} P_{ij} / \sum_i Q_{si} P_{ij}$ and $\Pr(n^l_t \mid g^i_{t+1}, g^j_t, n^{-s}_t) = \sum_k Q_{lk} P_{ki}$. The state transition matrix is fully determined by only three parameters: The productivity transition probabilities p_{GG} and p_{BB} and the news accuracy parameter q.

Equilibrium and Balanced Growth. Given initial conditions k_0 and d_0 and a sequence for productivity growth g_t and news n_t, an *equilibrium* is a sequence of allocations $\{k_{t+1}, h_t, l_t, d_{t+1}, c_t, x_t, u_t\}_{t=0}^{\infty}$ and prices $\{w_t, r_t, R_t\}_{t=0}^{\infty}$ such that the allocations solve the firms' and households' problems at the equilibrium prices and all markets clear. A balanced growth equilibrium is an equilibrium where $[k_t, d_{t+1}, c_t, x_t]/\Gamma_{t-1}$ are stationary variables. The balanced growth equilibrium is summarized by a system of Euler equations, which is given in the Appendix.

2.3 Calibration and Solution Methodology

The time period in the model corresponds to six months. This choice follows from a trade-off between the computational burden of a larger number of news leads and the ability to match the Korean Crisis data.

News and States of Technology. The parametrization of the state space and transitions probabilities requires numerical values for five parameters: μ_B, μ_G, p_{GG}, p_{BB} and q. All of these determine the size of the change in expectations following a news shock. If a news shock is to explain the large macroeconomic volatility during Sudden Stop episodes, the shock can be thought to be fairly large. At the same time, large news shocks should be restricted to occur infrequently, as Sudden Stops within the same country are rare. Therefore, a natural approach for setting the parameter values is to construct a "peso problem", as in for instance Danthine and Donaldson (1999). I think of μ_B as a depression state with a very low probability of occurring and of μ_G as the actual productivity growth rate in the sample of observations. In practice, the probability that the economy moves from the good to the bad state μ_B is 1%, i.e. $p_{GG} = 0.99$, and $\mu_G = \mu = 1.019$ equals the value calibrated below for 1980-2002 Korean data. Since one period in the model corresponds to 6 months, the expected duration of the high growth regime is 50 years, the same as in Danthine and Donaldson (1999).

It is possible to compute $\mathrm{E}_t\left[g_{t+2} \mid g_t = \mu_G, n_t = \mu_B\right]$, the expected productivity growth in $t+2$ conditional on being in the good state in period t with a bad signal. Figure 2.1 plots this expectation for various values of μ_B and q. Evidently, it is non-increasing in productivity growth in the bad state μ_B, and decreasing in news precision q. When $q = 0.5$, the news signal does not contain any information and the time t expectation of productivity in $t+2$ equals the unconditional expectation, which is very close to μ_G. When $q = 1$, the signal is perfect and expected productivity growth in $t+2$ is μ_B. When $0.5 < q < 1$, the expectation lies in between μ_G and μ_B. Figure 2.1 makes clear that generating sizeable changes in expectations requires high values for q. The benchmark calibration will therefore have $q = 0.99$. I choose the value of μ_B such that $\mathrm{E}_t\left[g_{t+2} \mid g_t = \mu_G, n_t = \mu_B\right] = 1$. Hence, when bad news arrives in period t, agents revise their forecast for semiannual productivity growth in $t+2$ downwards from 1.9% to 0%. This choice strikes a balance between having an adjustment that is sufficiently large and infrequent, and one that reasonably lies within the agents' belief set. In practice, $\mu_B = 0.985$ and the conditional probability that the bad state realizes given bad news is about 0.55. Given the choice for p_{GG}, the probability p_{BB} that the bad state persists the next period has only a small effect on $\mathrm{E}_t\left[g_{t+2} \mid g_t = \mu_G, n_t = \mu_B\right]$. Nevertheless, p_{BB} is an important parameter as the expected duration of the bad state has consequences for the agents' savings decision. In the benchmark calibration, I set p_{BB} to 0.25.[4]

Given the peso problem setup, three key parameters thus determine the dynamics of expectations following bad news: The bad state value μ_B, the news accuracy q, and the expected duration of the bad state, captured by p_{BB}. In the robustness section, I explore alternative values for these parameters, and therefore smaller and larger shocks to expectations. Although a switch to the bad technology state is extremely rare, a bad news shock is more frequent. Under the benchmark calibration, bad news arrives in the good state once every 23 years. Hence, roughly one out of two occurrences of the bad news shock will not be followed by an actual change in productivity growth. This property is consistent with the presumption that in the sample of observations for Korea the depression state has not occurred, whereas a Sudden Stop has.

[4]For comparison, Danthine and Donaldson (1999) set the persistence of their depression state to 0.20.

Model Parameters. I set the gross annual interest rate to 1.05, the average 3 month T-bill rate that prevailed in the years surrounding the crisis. The country risk premium in tranquil times is one percent, the approximate average value of the EMBI global spread for Korea in non-crisis years, and also the value at the onset of the crisis. Together, this choice yields an annual interest rate of 6%.

The labor elasticity of output α takes on the conventional value of 0.36, which implies a labor share in total factor earnings for Korea that is in between the value of 0.5 used by Gertler et al. (2003) and the value 0.7 calculated by Young (1995). The average productivity growth factor is $\mu = 1.019$ in order to match the average gross capital formation-to-GDP ratio of approximately 0.31 in 1980-2002 in a non-stochastic version of the model which excludes the bad state. The annual productivity growth rate is therefore 3.8%. The parameters δ and η normalize the rate of capacity utilization to one and generate an annual depreciation rate of 0.1 in the non-stochastic model. A difficult parameter to calibrate is ω, the elasticity of marginal depreciation with respect to capital utilization. Basu and Kimball (1997) obtain a point estimate for the US of unity, but their 95% confidence bounds are wide: $[-0.2, 2]$. Baxter et al. (2001) find in the context of an international two-country RBC model that lower values, $\omega = 0.05$ or 0.10, fit the data well. The benchmark calibration in this paper will take a value of 0.05, but the numerical analysis also considers higher values.

The value for ζ normalizes hours worked to one in the non-stochastic model. The wage elasticity of labor supply is 2.2 (or $\psi = 0.45$), as in Mendoza (1991). The elasticity of the discount factor $\xi = 0.061$ matches the average net foreign debt position-to-GDP ratio of 0.21 in Korea, which I obtain by averaging annual data from of Lane and Milesi-Ferretti (2006) for the period 1980 to 1997. The implied consumption-to-GDP ratio is 0.68, which is roughly in line with the value of 0.71 in the data. The parameters γ and ϕ are 2 and 2.5 respectively, well within the range of conventional values. Finally, I choose $\chi_2 = 0.76$ to match the 5% increase in the risk premium during the Korean Crisis episode. Table 2.2 summarizes all the values used for generating the results under the benchmark calibration.

Numerical Solution Technique. I obtain the approximate model solution to the system of Euler equations describing the equilibrium behavior of the various macroeconomic variables, given in the Appendix. The numerical method used is time iteration, as described by Coleman (1990). Time iteration is generally slow and therefore the iterative scheme is augmented by the application of the method of endogenous gridpoints, developed by Carrol (2006). This method reduces the number of nonlinear equations that need to be solved numerically in every iteration.

The model features two endogenous state variables, k_t and d_t (detrended), and five exogenous state variables: The current and previous period value of g_t, the news shock n_t and the two lags n_{t-1}, n_{t-2}. The additional lags of n_t and g_t are necessary to evaluate lagged expectations. All functions are approximated by a linear interpolation scheme based on a (11×11) grid of the (k_t, d_t) space, which means their continuous nature is preserved. As a result, each function is approximated over a total number of 3872 nodes. Further increasing the number of nodes does not lead to any noticeable changes in the results.

2.4 Quantitative Properties of the Model

2.4.1 Matching the Korean Crisis Episode

Figure 2.2 plots equilibrium paths of the key macroeconomic variables, together with their data equivalents for the years surrounding the Korean Crisis. The first row depicts the annualized growth rate of real GDP, consumption and investment. The second row plots hours worked (in percentage deviation from the 1996 value), the country risk premium and the net exports-to-GDP ratio. The equilibrium paths are for the following sequence of shocks: Prior to the second half of 1997 (1997:2), the model economy has been in high growth state μ_G for an arbitrarily long period with the news signal correctly predicting future states. In 1997:2, bad news arrives about productivity growth in 1998:2. The bad news persists in 1998:1 and when 1998:2 arrives, the signal returns to predicting μ_G. During the whole experiment, there is never any change in productivity growth. The transition probabilities in (2.11), which imply that the news shock displays persistence of 0.14, lead to the following expectation dynamics: In 1997:2, bad news shifts expectations, which are rationally adjusted downwards from 1.9% to 0% productivity growth for 1998:2. When 1998:1 brings bad news about 1999:1, agents incorporate this additional information into their forecasts and expected productivity growth in 1998:2 drops further to -1.3%. In 1998:2, the information that the initial news signal was false and the good news signal about 1999:2 both lead to an upward revision to 0.4% of the forecast for 1999:1. In 1999:1, agents return to anticipating 1.9% growth for the subsequent periods. The first column in Table 2.3 summarizes the dynamics of expected productivity growth in response to the news sequence.[5]

The model succeeds in capturing the key aspects of the Korean crisis experience. Consistent with the data, the news shock causes a rise in net exports and contractions in GDP, consumption, investment and hours. Quantitatively, the responses are of magnitudes associated with Sudden Stop episodes: Annual GDP growth plummets from 3.8% to -7.7%; hours worked decline by 12%; consumption falls by 6.2% and investment contracts with 31% on an annual basis. The net exports-to-GDP ratio shoots up from 0.3% to 11%. The model also closely matches the subsequent swift recovery after the crisis. As in the data, GDP, consumption and investment grow above trend in the year following the crisis.

In some respects, the model performs less well. Consumption falls more than GDP dur-

[5] An alternative experiment has the news signal switch back to μ_G in 1998:1. The results are qualitatively very similar to the experiment in which the bad news persists. Without persistent news, the recession is quantitatively smaller and lasts for one period only. The results are available on request.

ing the crisis, whereas the model yields the reverse. Because the model underpredicts the consumption drop, the increase in net exports is smaller than in the data. Also, the net exports-to-GDP ratio remains high after the crisis, but reverts in the model. Another issue is timing. According to the theory, investment leads GDP and consumption. In reality, the contraction and recovery of these variables is more simultaneous. Nevertheless, the experiment shows that the shock to expectations goes a long way in explaining the Sudden Stop, the associated economic crisis and the quick recovery experienced by South-Korea.

To understand the mechanics of the response to a news shock in period $t = 1997{:}2$, it is useful to see what drives the eventual reduction in real activity in $t+2 = 1998{:}1$. The output drop is primarily caused by a decrease in hours worked. The equilibrium in the labor market can be loosely summarized by the following equations:

$$0 = \mathrm{E}_{t+1}\left[\lambda_{t+2}\left(R_{t+1}w_{t+2} - (1-\alpha)\Gamma_{t+2}^{1-\alpha}\left(\frac{u_{t+2}k_{t+2}}{h_{t+2}}\right)^{\alpha}\right)\right], \qquad \text{(Labor Demand)}$$

$$w_{t+2} = \zeta l_{t+2}^{\psi}\Gamma_{t+1}, \qquad \text{(Labor Supply)}$$

$$l_{t+2} = h_{t+2}. \qquad \text{(Labor Market Clearing)}$$

The labor demand and supply equations follow from the firm's and households' optimality conditions. Three ingredients of the model are key for inducing a large decline in hours: Predetermined labor, the working capital constraint and the expectation-elastic country risk premium. Firms choose labor input for $t+2$ production before the realization of time $t+2$ uncertainty, and lower expected labor productivity induces them to demand less labor. Since the country's risk premium in $t+1$ increases with lower expected productivity, the interest rate R_{t+1} rises. Because firms need to finance the wage bill in advance by issuing bonds, the rise in R_{t+1} increases the cost of hiring labor in $t+2$, causing a further reduction in labor demand. In general equilibrium, both these effects dominate and hours drop through a decrease in the real wage. Because expectations for $t+3$ productivity growth are still relatively low in $t+2$, the cut in hours persists in $t+3$.

To assess the relative importance of predetermined labor and the financial propagation mechanism, Figure 2.3 plots the equilibrium paths for the same shock sequences in three models: The benchmark model, a version in which labor responds contemporaneously to productivity shocks ("Variable Labor"), and a version without financial propagation mechanism in which the elasticity of the risk premium χ_2 is zero ("R Fixed"). In the benchmark model, hours worked decline by 12%. Without financial propagation mechanism, hours fall by 6.4%, and with variable labor by 5.3%. Hence, both predetermined labor and the financial propagation stemming from the working capital constraint and the expectation-elastic risk premium are important for generating a large response of hours worked.

The decline in economic activity after the news shock is also due to reductions in the capital stock and in the rate of capacity utilization. In order to obtain a drop in investment that is comparable to the data, the expectation-elastic risk premium is the key model ingredient, at least under the benchmark calibration. Figure 2.3 shows how investment falls in all variants of the model, but only when the risk premium increases is the reaction similar in size to the data. To see why, note that the households' asset choice is determined by an arbitrage condition stating that the expected return in utils of buying one additional bond or investing one more unit should be equal,

$$\mathrm{E}_t\left[\lambda_{t+2}\right] R_{t+1} = \mathrm{E}_t\left[\lambda_{t+2}(1+r^k_{t+2})\right] , \qquad \text{(Arbitrage)}$$

where the return on capital investment r^k_{t+2} is given by

$$r^k_{t+2} = \frac{r_{t+2} + 1 - \delta - \eta \frac{u_{t+2}^{1+\omega}}{1+\omega} + \Phi\left(\frac{k_{t+3}}{k_{t+2}}\right)}{1+\phi\left(\frac{k_{t+2}}{k_{t+1}} - \mu\right)} - 1 , \qquad \text{(Return to Capital)}$$

where $\Phi\left(\frac{k_{t+3}}{k_{t+2}}\right) = \phi\left(\frac{k_{t+3}}{k_{t+2}} - \mu\right)\frac{k_{t+3}}{k_{t+2}} - \frac{\phi}{2}\left(\frac{k_{t+3}}{k_{t+2}} - \mu\right)^2$. In equilibrium, the rental rate of capital equals the marginal product of capital services,

$$r_{t+2} = \alpha u_{t+2}\left(\frac{u_{t+2}k_{t+2}}{\Gamma_{t+2}h_{t+2}}\right)^{\alpha-1}. \qquad \text{(Rental Rate of Capital)}$$

The households' optimality condition for capacity utilization is

$$\eta u_{t+2}^{\omega} = \alpha\left(\frac{u_{t+2}k_{t+2}}{\Gamma_{t+2}h_{t+2}}\right)^{\alpha-1}, \qquad \text{(Capacity Utilization)}$$

which states that the marginal benefit of higher utilization equals the marginal cost in terms of higher capital depreciation. Households expect the marginal product of capital to be lower in $t+2$ because of the decline in hours and because of lower expected productivity in period $t+2$. The decline in hours also causes a fall in capacity utilization in $t+2$, which has an additional negative effect on the marginal product of capital. On the other hand, lower utilization reduces capital depreciation, which raises the $t+2$ return to capital. Overall, the equalization of capital and bond returns requires a contraction of period $t+1$ investment. Evidently, if the bond rate rises after the shock to expectations, the required drop in investment is much larger. Because adjusting the capital is stock is costly, the households, who anticipate the course of events, start cutting investment in period t. Since hours do not react significantly until $t+2$, the resulting lower capital stock in $t+1$ yields a slight increase in capacity utilization before a significant decrease in $t+2$. In $t+2$, investment growth remains negative because of lower expectations for productivity growth in $t+3$. From $t+3$ onwards, investment growth picks up in order to catch up with the trend. Because of adjustment costs, the capital stock remains below trend for a longer period, resulting in high levels of capacity utilization during the recovery.

To understand the response of consumption, consider the households' Euler equation for bond holdings

$$\lambda_t = e^{-\beta(t)} R_t \mathrm{E}_t \left[\lambda_{t+1}\right] \qquad \text{(Euler eq. for Bonds)}$$

Given the choice of preferences, in equilibrium, $\lambda_t \approx (c_t/\Gamma_{t-1} - w_t h_t/(1+\psi))^{-\gamma}$, implying that consumption growth and changes in hours are positively related. Therefore all the elements of the model causing the decline in hours are also responsible for the drop in consumption in $t+2$. Consumption falls in period in t and $t+1$ because of lower anticipated future income. Of course, in the case of an expectation-elastic risk premium, there is the additional direct negative effect on consumption of a higher interest rate. Figure 2.3 shows how in all variants of the model consumption falls in response to the news shock. The reaction is the largest when all factors magnifying the decline in hours are present.

The response of net exports is positive as savings increase and investment falls. Figure 2.3 shows that this is true with and without expectation-elastic risk premium and with and without predetermined labor. However, only in the version of the model with the expectation-elastic risk premium and the associated large negative effect on investment is the magnitude of the reaction roughly of the same order as in the data.

Figure 2.4 allows to assess the role of variable capacity utilization as an amplification mechanism by plotting the model response for different values of the elasticity of depreciation with respect to capital utilization ω. In order to maintain the calibrated depreciation and utilization rate, I adjust the parameters δ and η correspondingly. The main effect of higher values for ω is to dampen movements in the marginal product of labor and therefore in hours worked, GDP and consumption. At the same time, higher values for ω make investment react more. The reason is that the effect on depreciation dominates the one on the marginal product of capital. Overall, the return of capital tends to decrease more with higher ω and a larger adjustment of investment is necessary to equalize returns on capital and bonds.

To summarize, in a small open economy model it is relatively easy to generate a response to bad news about future productivity growth that is characterized by reductions in real activity, employment, consumption and investment, together with an increase in net exports. However, to obtain fluctuations of similar magnitude as during the Korean crisis, all model features are important. Predetermined labor, the working capital constraint and the expectation-elastic risk premium all contribute to obtaining large declines in hours, consumption and real activity after the shock to expectations. For investment and net exports, the expectation-elastic risk premium is the most important element of the model under the present calibration. What value of the elasticity of marginal depreciation ω is better suited to match the Korean experience is ambiguous. On the one hand, a lower ω contributes to explaining the large decline in real activity, as well as the measured Solow residual during Sudden Stops (Aguiar and Gopinath, 2007). On the other hand, a lower ω increases the reliance on other factors to rationalize the observed fluctuations in investment and net exports.

2.4.2 Changing the Shock to Expectations

This section explores how the model response to bad news is affected when the expectations dynamics are different from the benchmark calibration. Recall that three parameters are key in determining the expectation dynamics: Productivity in the bad state μ_B, the probability that the bad state persists p_{BB}, and the news precision q.

Figure 2.5 plots the response for the benchmark value of $\mu_B = 0.985$, together with those for $\mu_B = \{0.97, 1\}$. The second and third column in Table 2.3 give the dynamics of expected productivity growth for the new values. Lower values of μ_B imply larger drops in expected productivity growth. The probabilities of the bad state are unchanged. For each value of

μ_B, I change the elasticity of the risk premium χ_2 to keep the response of the risk premium in $t+1$ identical. The resulting values are $\chi_2 = \{0.53,\ 1.4\}$ respectively. The main effect of lowering μ_B is to enlarge the negative response of hours. The reason is that, since labor is predetermined, lower expected productivity causes larger reductions in labor demand. As a consequence, the reactions of output and consumption are also larger for lower values for μ_B. The response of investment and net exports does not change dramatically, because the reaction of the interest rate is unchanged.

Figure 2.6 plots the equilibrium paths when $p_{BB} = \{0.5,\ 0.75\}$, together with the benchmark case of $p_{BB} = 0.25$. The fourth and fifth column in Table 2.3 give the dynamics of expected productivity growth for these values. The parameter p_{BB} has only a negligible effect on expected productivity in $t+2$, both in period t and period $t+1$. However, period $t+2$ expectations about growth in $t+3$ are higher for larger p_{BB}. The higher the persistence of the bad technology state, the more the incorrect first news signal and the arrival of a good signal about $t+4$ reduce the probability of the bad regime in $t+3$. Again, I adjust the values for χ_2 to obtain an identical response of the risk premium in $t+1$ (to 0.73 and 0.72 respectively). The main change induced by an increase in the expected duration of the bad state is in the reaction of investment and net exports. The response of hours and GDP is only significantly affected in $t+2$ because of differences in the risk premium. The reason is that, although labor is predetermined, adjusting the labor stock is not costly. Therefore the firm's optimal choice of labor input depends on the expectation of next period productivity. Adjustments of the capital stock, however, are costly, and the change in investment is affected by expected productivity beyond the next period, and therefore by the expected duration of the bad state. As a result, the response of investment in t and $t+1$ is magnified by higher values of p_{BB}. There is also an effect on consumption. When the bad state persistence is higher, expected future income is lower. Therefore consumption falls more in period t and $t+1$ for higher p_{BB}. Larger reductions in investment and consumption also cause bigger increases in net exports. This result establishes a second element of the model that can explain the large reaction of investment and net exports during the Korean Crisis besides the rise in the risk premium: If the bad state is more persistent, the fluctuations in investment and net exports are considerably larger.

Finally, Figure 2.7 plots the response for two alternative values of the precision parameter, $q = \{0.95, 1-\varepsilon\}$, where ε is an arbitrarily small number. The sixth and seventh column in Table 2.3 provide the dynamics of expected productivity growth for these values. The main effect of altering the value of q is a change in the probability of the bad technology state. Higher values of q therefore lower expected productivity growth. Once more, I adjust the values for χ_2 to obtain the same response of the risk premium in $t+1$ (to 1.51 and 0.70 respectively). Higher precision q magnifies the decline in hours, GDP and consumption in $t+2$. When the news signal is more precise, expectations in $t+2$ about growth in $t+3$ rely more on the $t+1$ news signal and less on the state of technology or news in $t+2$. That is why the decline in expected productivity growth and the drop in economic activity are more persistent in $t+3$ for higher q. There is also a more subtle effect on the period t response of consumption and investment. When the bad news shock arrives in period t, agents put higher probability weight on the bad state occurring beyond $t+2$ if q is higher. If the news signal is more precise and the bad state is persistent, the arrival of bad news about $t+2$ also increases the probability of bad news in $t+1$ and of the bad technology state in $t+3$. For instance, the period t probability of additional bad news in $t+1$ is 0.10 when $q = 0.95$, 0.14 when $q = 0.99$ and arbitrarily close to p_{BB} when $q = 1-\varepsilon$. Because of capital adjustment costs, the period t contraction in investment is larger when the precision is higher.

2.5 Conclusion

In their analysis of equilibrium models of Sudden Stops, Chari et al. (2005) challenge future research to explore an alternative approach, in which: *[...] private agents see events that lead them to predict future drops in a country's output, and as a result, these agents pull their capital from the country. [...] anticipated output drops drive the Sudden Stops, rather than the reverse.* But *[...], whether quantitative evidence can be found to support it is an open issue.* This paper provides quantitative evidence that an adverse shift in expectations about future productivity growth can trigger a Sudden Stop and output drop. In a small open economy that faces a peso problem in productivity growth states, a news shock announcing a switch to a bad regime generates an increase in net exports and decreases in economic activity, consumption and investment. To quantitatively match the 1998 Korean Crisis with reasonable shifts in expectations, the model relies on several amplification mechanisms. Predetermined labor input, variable capacity utilization and financial frictions contribute most to explaining the large declines in hours, GDP and consumption for a given adjustment in expectations. The financial friction and a larger expected duration of the bad regime are the most important elements for generating large fluctuations in investment and net exports.

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Appendix: The Equilibrium Conditions

Define $\hat{x}_t = \frac{x_t}{\Gamma_{t-1}}$. The solution of the model is obtained by finding approximations for $\hat{c}_t$, h_t, $\hat{y}_t$, $\hat{V}_t$, λ_t, u_t, $\hat{k}_{t+1}$, $\hat{d}_{t+1}$ and $\hat{x}_t$ as functions of the model's state variables that solve the following system of equations:

$$\lambda_t = U_c(t) - \beta_c(t) e^{-\beta(t)} \hat{V}_t \tag{2.14}$$

$$\hat{V}_t = g_t E_t \left[\left(\hat{U}(t+1) + e^{-\beta(t+1)} \hat{V}_{t+1} \right) \right] \tag{2.15}$$

$$0 = \mathrm{E}_{t-1} \left[\lambda_t \left(\zeta h_t^{\psi} R_{t-1} - g_t (1-\alpha) \left(\frac{u_t \hat{k}_t}{g_t h_t} \right)^{\alpha} \right) \right] \tag{2.16}$$

$$\eta u_t^{\omega} = \alpha \left(\frac{u_t \hat{k}_t}{g_t h_t} \right)^{\alpha - 1} \tag{B.4}$$

$$\lambda_t = e^{-\beta(t)} R_t \mathrm{E}_t \left[\lambda_{t+1} \right] \tag{2.17}$$

$$\lambda_t \left(1 + \phi \left(g_t \frac{\hat{k}_{t+1}}{\hat{k}_t} - \mu \right) \right) = e^{-\beta(t)} \mathrm{E}_t \left[\lambda_{t+1} \left(\alpha u_{t+1} \left(\frac{u_{t+1} \hat{k}_{t+1}}{g_{t+1} h_{t+1}} \right)^{\alpha-1} + 1 - \delta - \eta \frac{u_{t+1}^{1+\omega}}{1+\omega} + \Phi \left(\frac{\hat{k}_{t+2}}{\hat{k}_{t+1}} \right) \right) \right] \tag{2.18}$$

$$g_t \hat{k}_{t+1} = \hat{x}_t + \left(1 - \delta - \eta \frac{u_t^{1+\omega}}{1+\omega} \right) \hat{k}_t - \frac{\phi}{2} \left(g_t \frac{\hat{k}_{t+1}}{\hat{k}_t} - \mu \right)^2 \hat{k}_t \tag{2.19}$$

$$R_{t-1} \hat{d}_t + \hat{c}_t + \hat{x}_t = \alpha \hat{y}_t + \frac{(1-\alpha)}{R_{t-1}} \hat{y}_t + g_t \hat{d}_{t+1} \tag{2.20}$$

$$\hat{y}_t = (u_t \hat{k}_t)^{\alpha} (g_t h_t)^{1-\alpha} \tag{2.21}$$

where

$$\hat{U}(t) = \frac{\left(\hat{c}_t - \zeta \frac{h_t^{1+\psi}}{1+\psi} \right)^{1-\gamma}}{1-\gamma}, \quad U_c(t) = \left(\hat{c}_t - \zeta \frac{h_t^{1+\psi}}{1+\psi} \right)^{-\gamma}, \quad R_t = \rho \chi_1 \left(1 + \mathrm{E}_t \left[g_{t+1} \right] - \mu \right)^{-\chi_2}$$

$$\beta_c(t) = \xi \left(1 + \hat{c}_t - \zeta \frac{h_t^{1+\psi}}{1+\psi} \right)^{-1}, \quad \Phi \left(\frac{\hat{k}_{t+2}}{\hat{k}_{t+1}} \right) = \phi \left(g_{t+1} \frac{\hat{k}_{t+2}}{\hat{k}_{t+1}} - \mu \right) g_{t+1} \frac{\hat{k}_{t+2}}{\hat{k}_{t+1}} - \frac{\phi}{2} \left(g_{t+1} \frac{\hat{k}_{t+2}}{\hat{k}_{t+1}} - \mu \right)^2 .$$

Tables and Figures

Table 2.1: Timing of Events in the Model

$(t-1)^+$	
t^-	shocks are revealed;
	firms rent capital and issue bonds at rate R_{t-1}
t^+	firms produce and decide on period $t+1$ labor input;
	bonds issued in $(t-1)^+$ and t^- mature;
	households consume, invest and trade bonds at rate R_t
$(t+1)^-$	

Table 2.2: Benchmark Parameter Values

Technology		
α	0.36	Labor input elasticity of output
δ	−0.026	Capital depreciation parameter
η	0.078	Capital depreciation parameter
ω	0.05	Utilization elasticity of marginal depreciation
ϕ	2.5	Capital adjustment cost parameter
μ	1.019	Semiannual productivity growth factor
Household Preferences		
ψ	0.45	Inverse wage elasticity of labor supply
ξ	0.061	Elasticity of discount factor
γ	2	Coefficient of relative risk aversion
Interest Rate		
ρ	$1.05^{0.5}$	World riskless interest rate
χ_1	$1.01^{0.5}$	Country risk premium parameter
χ_2	0.76	Elasticity of country risk premium

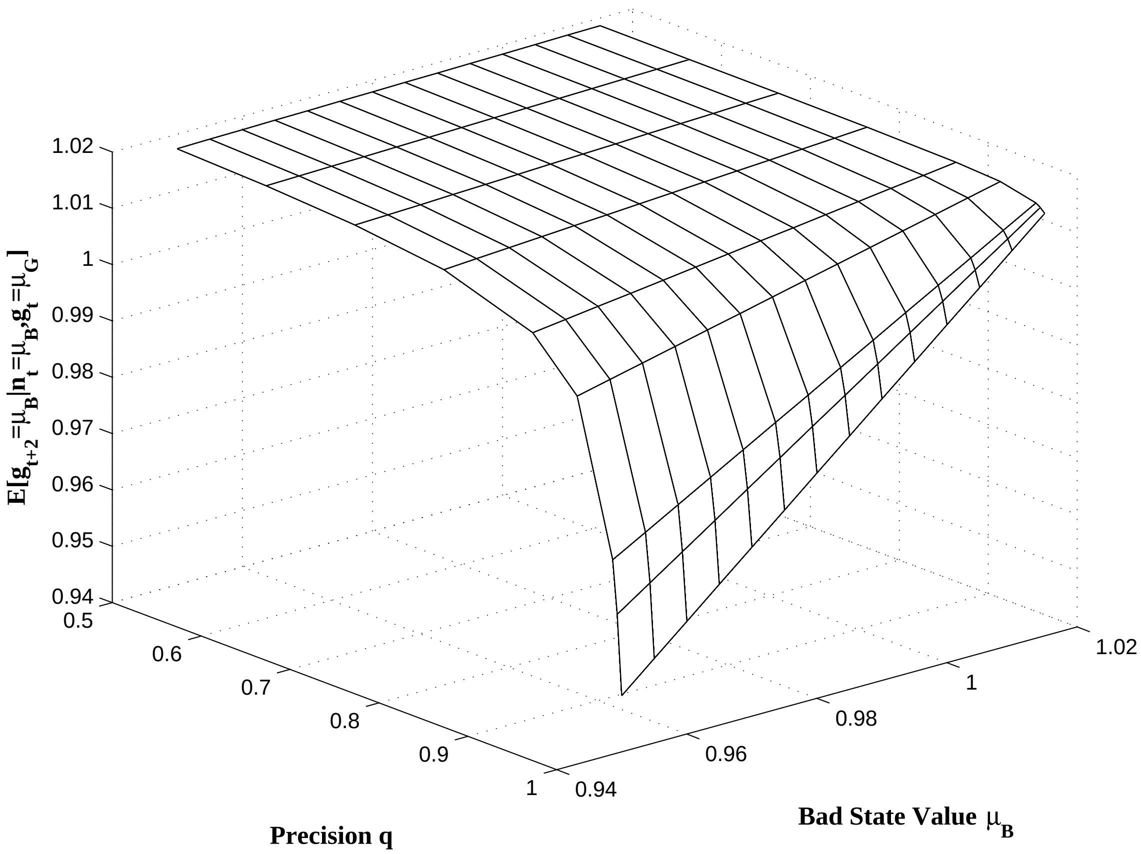

Figure 2.1: Expected Productivity in $t+2$ Conditional on time t Bad News and Good Technology State

Table 2.3: Expectation Dynamics

Period	Expected	Benchmark	$\mu_B = 0.97$	$\mu_B = 1$	$p_{BB} = 0.5$	$p_{BB} = 0.75$	$q = 0.95$	$q = 1 - \varepsilon$
$t = 1997{:}2$	$\ln g_{t+1}$	1.9%	1.9%	1.9%	1.9%	1.9%	1.9%	1.9%
	$\ln g_{t+2}$	0.0%	−0.9%	0.8%	0.0%	0.0%	1.2%	−1.5%
$t+1 = 1998{:}1$	$\ln g_{t+2}$	−1.3%	−2.6%	0.1%	−1.4%	−1.4%	0.3%	−1.5%
	$\ln g_{t+3}$	0.0%	−0.9%	0.8%	0.0%	0.0%	1.2%	−1.5%
$t+2 = 1998{:}2$	$\ln g_{t+3}$	0.4%	−0.2%	1.1%	0.7%	1.2%	1.4%	−1.5%
	$\ln g_{t+4}$	1.9%	1.9%	1.9%	1.9%	1.9%	1.9%	1.9%

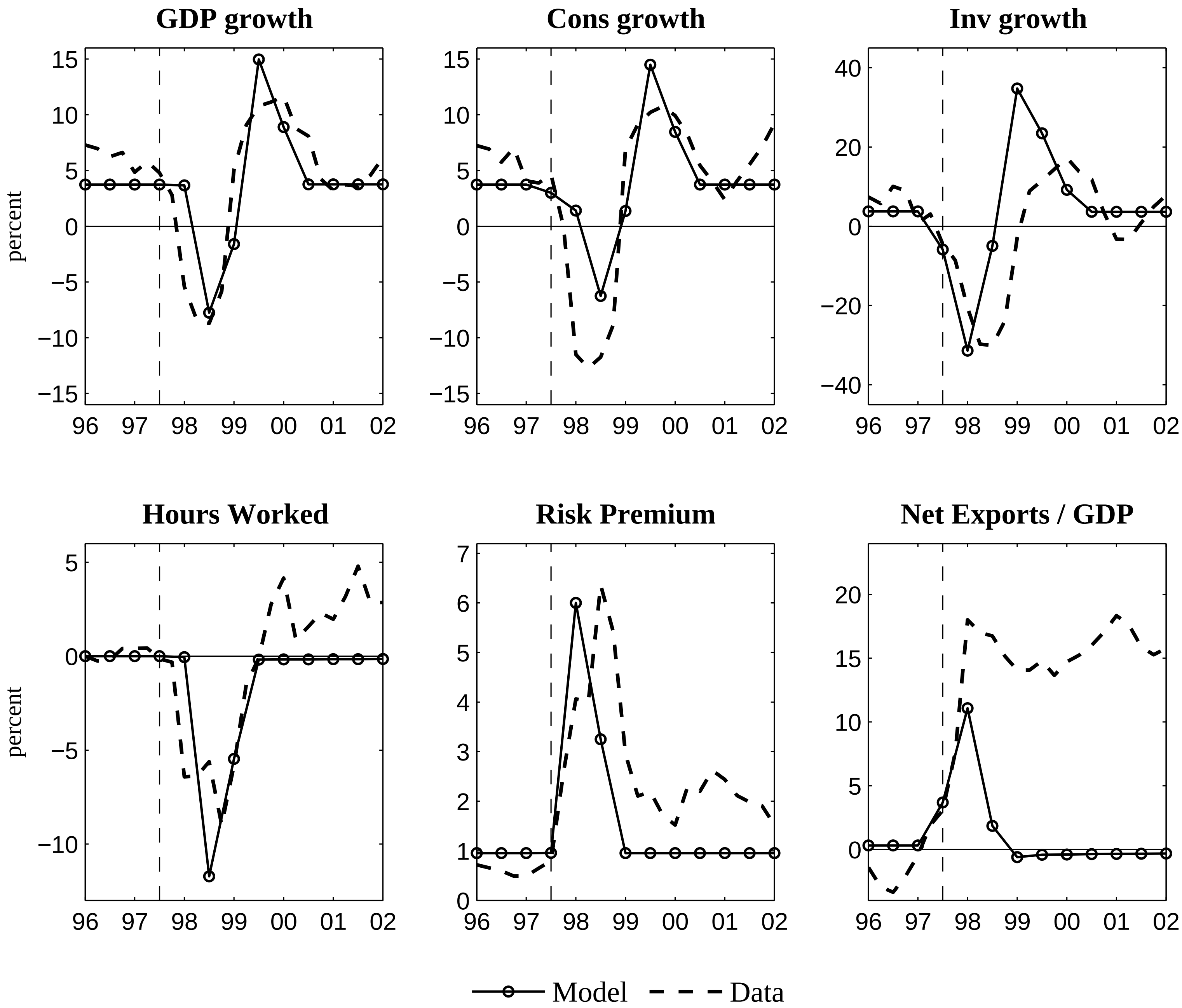

Figure 2.2: The Korean Crisis and the Model Response to News Shock in 1997:2. The Vertical Line Marks the Period of the Shock.

GDP, consumption and investment are year on year growth rates. Hours worked is in percentage deviations from the 1996:1 value. Data Sources: **GDP**: Gross domestic product at constant prices, quarterly levels, OECD ; **Investment**: Gross Fixed Capital Formation, quarterly levels, OECD ; **Consumption**: Private plus Government Final Consumption Expenditure at constant prices , quarterly levels, OECD ; **Hours Worked**: Total Employment Multiplied by Weekly Hours per Employee in Non-Agricultural Activities, ILO; **Risk Premium**: EMBI Global Spread Korea, JP Morgan, obtained from Neumeyer et al. (2005); **Net Exports/GDP**: obtained from Neumeyer et al. (2005). All variables are seasonally adjusted by the publishing agency, except for weekly hours and employment, which I seasonally adjusted using the Census Bureau's X12 method.

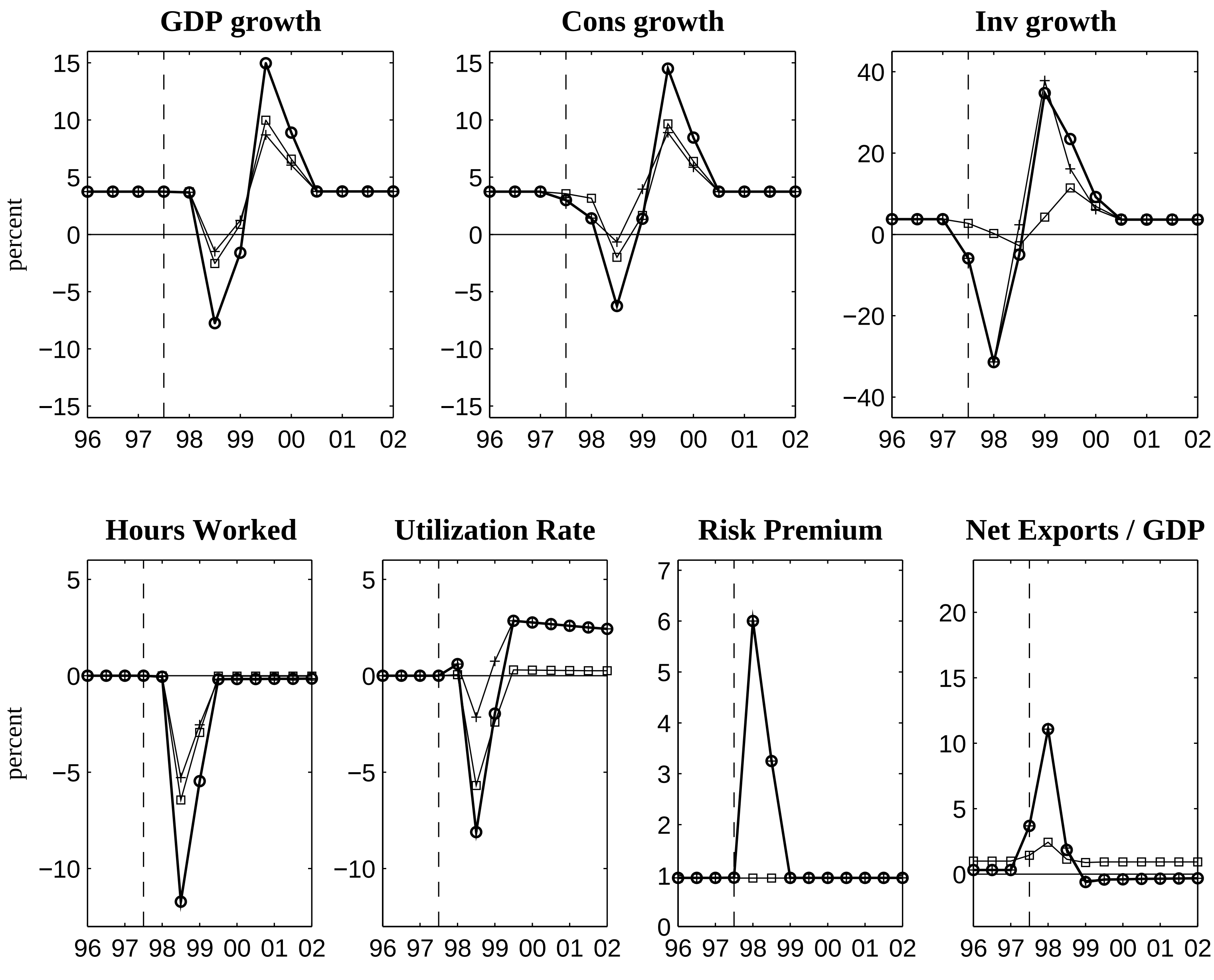

Figure 2.3: Response to News Shock in Different Models. The Vertical Line Marks the Period of the Shock.

GDP, consumption and investment are year on year growth rates. Hours worked and the rate of capacity utilization are in percentage deviations from the 1996:1 value.

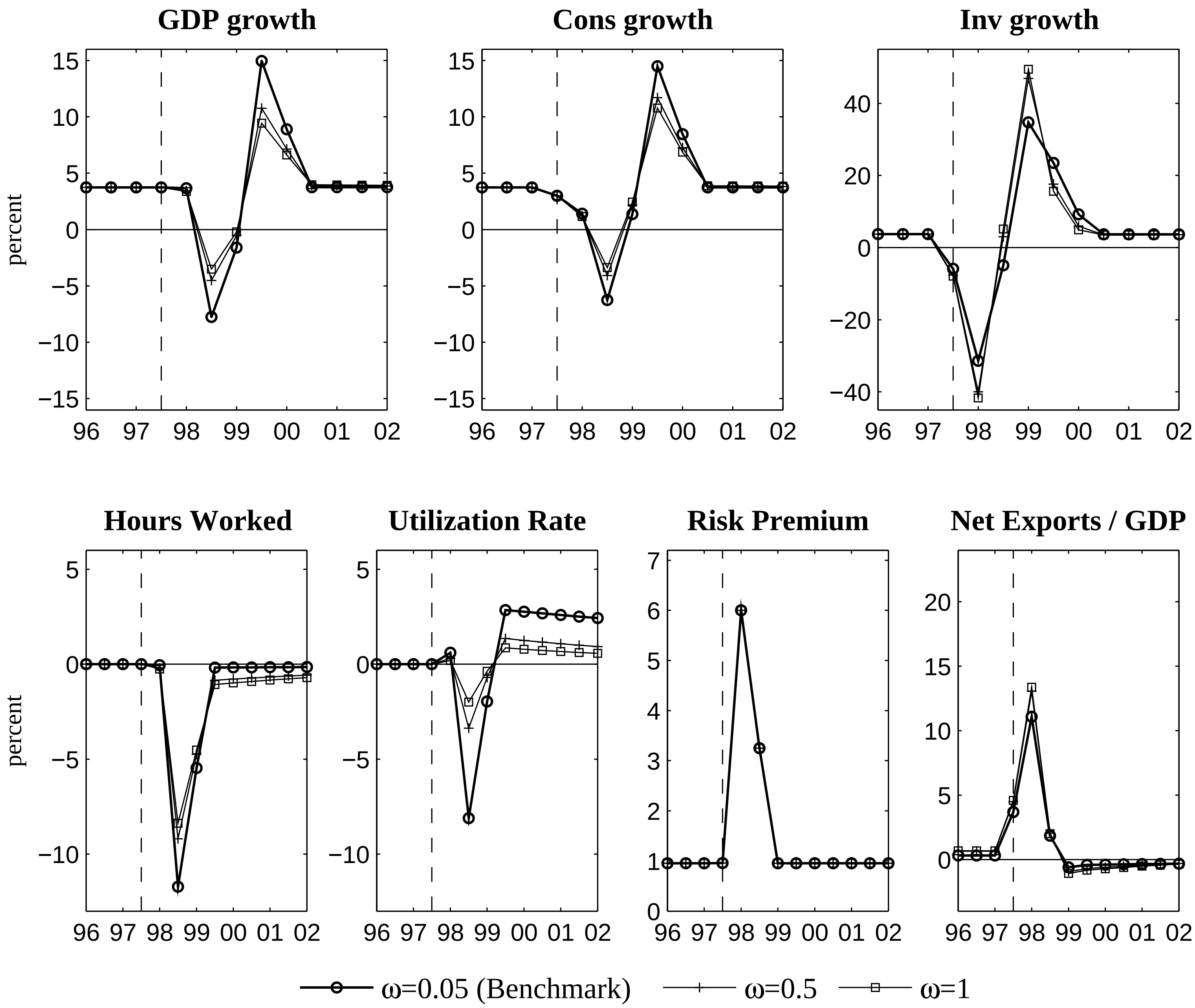

Figure 2.4: Model Response to News Shock: Different Values for ω. The Vertical Line Marks the Period of the Shock.

GDP, consumption and investment are year on year growth rates. Hours worked and the rate of capacity utilization are in percentage deviations from the 1996:1 value.

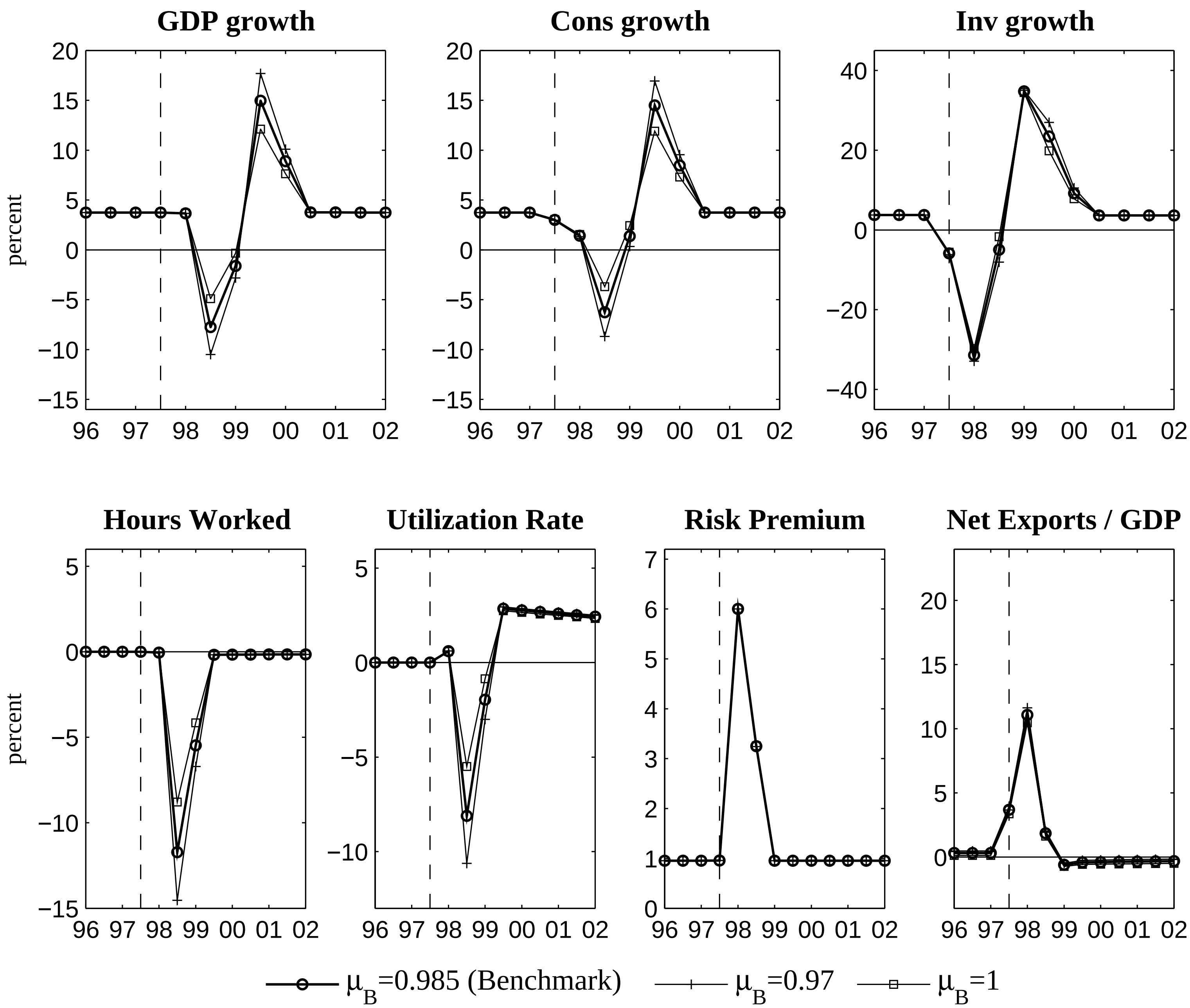

Figure 2.5: Model Response to News Shock: Different Values for μ_B. The Vertical Line Marks the Period of the Shock.

GDP, consumption and investment are year on year growth rates. Hours worked and the rate of capacity utilization are in percentage deviations from the 1996:1 value.

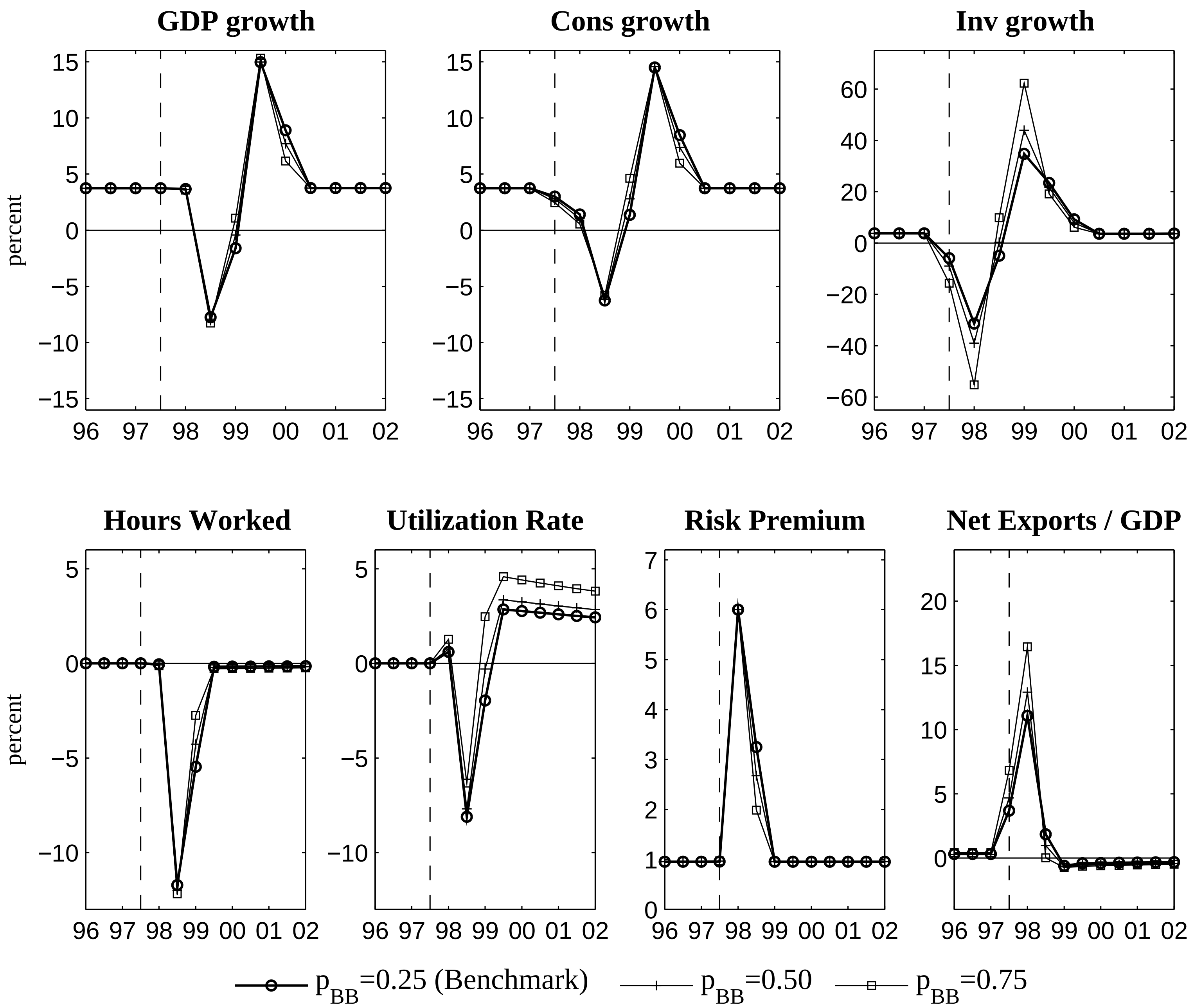

Figure 2.6: Model Response to News Shock: Different Values for p_{BB}. The Vertical Line Marks the Period of the Shock.

GDP, consumption and investment are year on year growth rates. Hours worked and the rate of capacity utilization are in percentage deviations from the 1996:1 value.

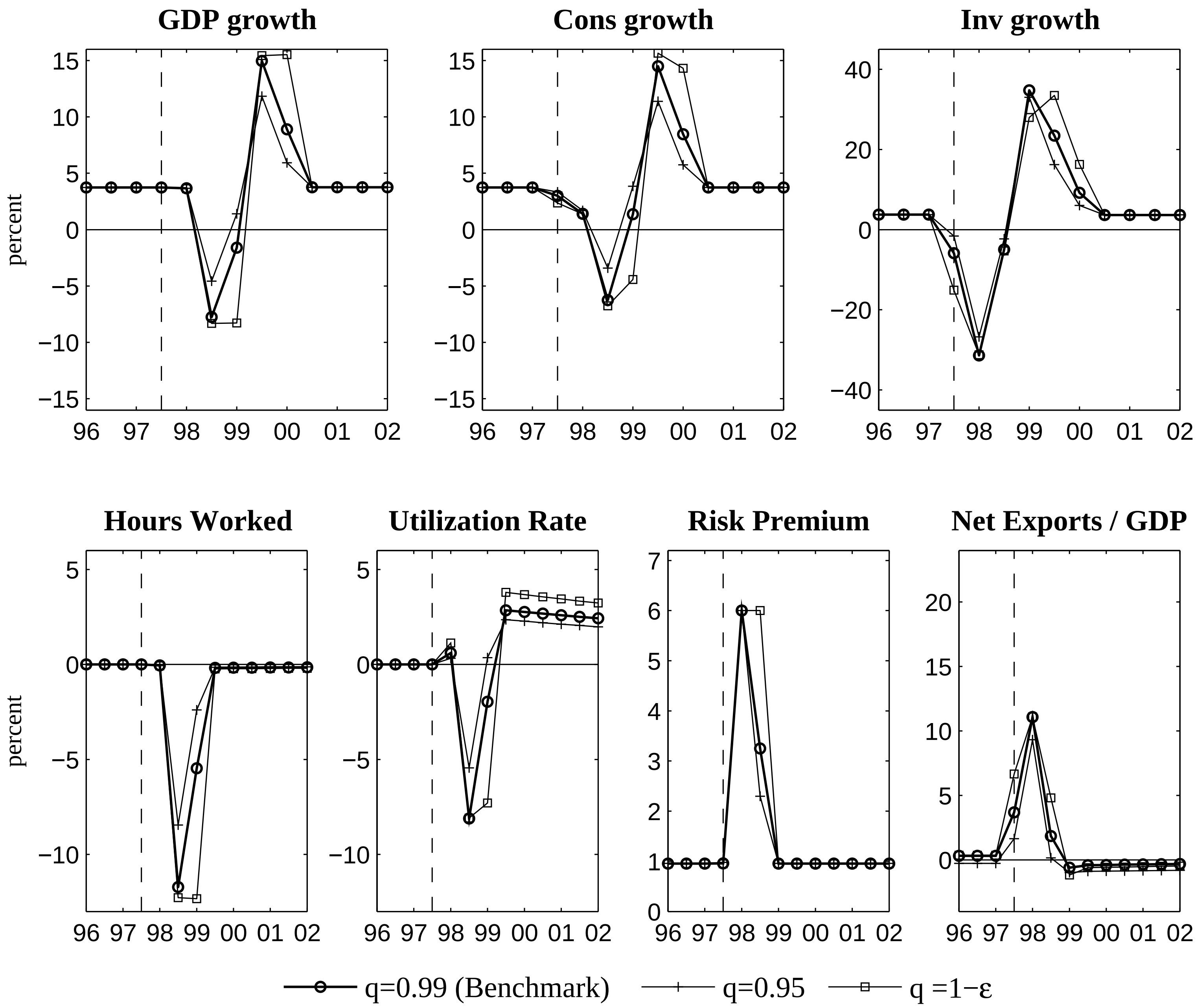

Figure 2.7: Model Response to News Shock: Different Values for q. The Vertical Line Marks the Period of the Shock.

GDP, consumption and investment are year on year growth rates. Hours worked and the rate of capacity utilization are in percentage deviations from the 1996:1 value.

Chapter 3

Business Cycle Analysis and VARMA Models

Joint with Christian Kascha

3.1 Introduction

Structural vector autoregressions (SVARs) are a widely used tool in empirical macroeconomics, particularly for the evaluation of dynamic stochastic general equilibrium (DSGE) models.[1] The results from SVARs are often viewed as stylized facts that economic models should replicate. However, there is some debate whether SVARs can in practice discriminate between competing DSGE models and whether their sampling properties are good enough to justify their popularity in applied macroeconomics. In response to a seminal paper by Gali (1999), the discussion has focused on the impact of technology shocks on hours worked, identified using restrictions on the long-run impact matrix of the structural errors. Chari, Kehoe and McGrattan (2005) and Christiano, Eichenbaum and Vigfusson (2006) investigate the properties of the estimators based on SVARs by simulating an artificial data generating process (DGP) derived from a prototype real business cycle (RBC) model and by comparing true with estimated impulse responses.

According to Chari et al. (2005), long-run identified SVARs fail dramatically for both a level and difference specification of hours worked. Even with a correct specification of the integration properties of the series, the SVAR overestimates in most cases the impact of technology on labor and the estimates display high variability. However, Christiano et al. (2006) argue that the parametrization chosen by Chari et al. (2005) is not very realistic. With their preferred parametrization, Christiano et al. (2006) find that both long-run and short-run identification schemes display only small biases and argue that, on average, the confidence intervals produced by SVARs correctly reflect the degree of sampling uncertainty.[2] Nevertheless, they also find that the estimates obtained via a long-run identification

[1]Examples in the literature are, among many others, Blanchard and Quah (1989), as well as King, Plosser, Stock and Watson (1991), Christiano and Eichenbaum (1992) and Gali (1999).

[2]In addition, Christiano et al. (2006) find that short-run identification schemes work much better compared to identification via long-run restrictions.

scheme are very imprecise. These results have been further confirmed by Erceg, Guerrieri and Gust (2005). In the end, with long-run restrictions, it is often very difficult to even make a correct inference about the *sign* of the structural impulse responses. The question is therefore if one should use this type of identification scheme at all. However, long-run identification is attractive from a theoretical point of view, since it requires much weaker assumptions than short-run identification and is in any case a useful additional tool for model evaluation.

The failure of finite-order SVARs is sometimes attributed to the fact that they are only approximations to infinite-order VAR processes or to the possibility that there does not exist a VAR representation at all. For example, Cooley and Dwyer (1998) give an example of an economic model that implies a vector autoregressive moving-average (VARMA) representation of the data series and state: "*While VARMA models involve additional estimation and identification issues, these complications do not justify* systematically *ignoring these moving average components, as in the SVAR approach*". As further shown by Fernández-Villaverde et al. (Forthcoming), DSGE models generally imply a state space system that has a VARMA and eventually an infinite VAR representation. Fernández-Villaverde et al. (Forthcoming) propose the inclusion of moving-average terms if the DSGE model at hand does not permit an infinite VAR representation. Christiano et al. (2006) state that "*Given our data generating processes, the true VAR of the data has infinite lags. However, the econometrician can only use a finite number of lags in the estimation procedure. The resulting specification error is the reason why in some of our examples the sum of VAR coefficients is difficult to estimate accurately*".

This paper explores the possible advantages of structural VARMA and state space models that capture the full structure of the time series representation implied by DSGE models, while imposing minimal theoretical assumptions. We investigate whether estimators based on these alternative models can outperform SVARs in finite samples.[3] This question is important for several reasons. First, it is useful to find out to what extent the poor performance of SVARs in these simulation studies is due to the omission of moving-average components. Second, whether estimators based on alternative representations of the same DGP have good sampling properties is interesting in itself. Employing these alternatives enables researchers to quantify the robustness of their results by comparing different estimates.

In order to assess whether the inclusion of a moving-average component leads to important improvements, we stick to the research design of Chari et al. (2005) and Christiano et al. (2006): We simulate DSGE models and fit different reduced form models to recover the structural shocks using the same long-run identification strategy. We then compare the performance of the models by focusing on the estimated contemporaneous impact to a technology shock. We employ a variety of estimation algorithms for the VARMA models, and both a prediction error method and a subspace algorithm for the state space models. One of the findings is that one can indeed perform better by taking the full structure of the DGP into account: While the algorithms for VARMA models and the prediction error method do not perform significantly better (and sometimes worse), the subspace algorithm for state space models consistently outperforms SVARs in terms of mean squared error. Unfortunately, we also find that even these alternative estimators are highly variable and are therefore not necessarily much more informative for discriminating between different DSGE models. One of the implications is that SVARs do not perform poorly in these simulation studies because they are only finite-order approximations. Given the properties of the data generating process, the disappointing performance of SVARs is most likely due to

[3]McGrattan (2006) is closely related to our paper. In a similar setting, McGrattan (2006) also investigates whether state space or VARMA models with minimal structural assumptions can uncover statistics of interest. Her work focusses on different business cycle statistics, while we are exclusively concerned with classical structural estimation.

the fact that the long-run identification approach is inappropriate with small samples.

The rest of the paper is organized as follows. In section 2 we present the RBC model used by Chari et al. (2005) and Christiano et al. (2006) that serves as the basis for our Monte Carlo simulations. In section 3 we discuss the different statistical representations of the observed data series. In section 4 we present the specification and estimation procedures and the results from the Monte Carlo simulations. Section 5 concludes.

3.2 The Data Generating Process

The DGP for the simulations is based on a simple RBC model taken from Chari et al. (2005). In the model, a technology shock is the only shock that affects labor productivity in the long-run, which is the crucial identifying assumption made by Gali (1999) to assess the role of technology shocks in the business cycle.

Households choose infinite sequences, $\{C_t,\, L_t,\, K_{t+1}\}_{t=0}^{\infty}$, of per capita consumption, labor and capital to maximize expected lifetime utility

$$E_0 \sum_{t=0}^{\infty} [\beta(1+\gamma)]^t \left[\log C_t + \psi \frac{(1-L_t)^{1-\sigma}-1}{1-\sigma}\right] , \tag{3.1}$$

given an initial capital stock K_0, and subject to a set of budget constraints given by

$$C_t + (1+\tau_x)\left((1+\gamma)K_{t+1} - (1-\delta)K_t\right) \leq (1-\tau_{lt})w_t L_t + r_t K_t + T_t , \tag{3.2}$$

for $t = 0, 1, 2, ...$, where w_t is the wage, r_t is the rental rate of capital, T_t are lump-sum government transfers and τ_{lt} is an exogenous labor tax. The parameters include the discount factor $\beta \in (0,1)$, the labor supply parameters, $\psi > 0$ and $\sigma > 0$, the deprecation rate $\delta \in (0,1)$, the population growth rate $\gamma > 0$ and a constant investment tax τ_x. The production technology is

$$Y_t = K_t^{\alpha}(X_t L_t)^{1-\alpha} , \tag{3.3}$$

where X_t reflects labor-augmenting technological progress and $\alpha \in (0,1)$ is the capital income share. Competitive firms maximize $Y_t - w_t L_t - r_t K_t$. Finally, the resource constraint is

$$Y_t \geq C_t + (1+\gamma)K_{t+1} - (1-\delta)K_t . \tag{3.4}$$

The model contains two exogenous shocks, a technology shock and a tax shock, which follow the stochastic processes

$$\log X_{t+1} = \mu + \log X_t + \sigma_x \varepsilon_{x,t+1} , \tag{3.5a}$$

$$\tau_{lt+1} = (1-\rho)\bar{\tau}_l + \rho \tau_{lt} + \sigma_l \varepsilon_{l,t+1} , \tag{3.5b}$$

where $\varepsilon_{x,t}$ and $\varepsilon_{l,t}$ are independent random variables with mean zero and unit standard deviation and $\sigma_x > 0$ and $\sigma_l > 0$ are scalars. $\mu > 0$ is the mean growth rate of technology, $\bar{\tau}_l > 0$ is the mean labor tax and $\rho \in (0,1)$ measures the persistence of the tax process. Hence, the model has two independent shocks: a unit root process in technology and a stationary AR(1) process in the labor tax.

3.3 Statistical Representations

Fernández-Villaverde et al. (Forthcoming) show how the solution of a detrended, log-linearized DSGE model leads to different statistical representations of the model-generated data. This section presents several alternative ways to write down a statistical model for the bivariate, stationary time series

$$\mathbf{y}_t = \begin{bmatrix} \Delta \log(Y_t/L_t) \\ \log(L_t) \end{bmatrix} . \tag{3.6}$$

Labor productivity growth $\Delta \log(Y_t/L_t)$ and hours worked $\log(L_t)$ are also the series analyzed by Gali (1999), as well as Chari et al. (2005) and Christiano et al. (2006). The Appendix provides more detail on the derivations. Given the log-linearized solution of the RBC model of the previous section, we can write down the law of motion of the logs

$$\log k_{t+1} = \phi_1 + \phi_{11} \log k_t - \phi_{11} \log x_t + \phi_{12} \tau_t , \tag{3.7a}$$

$$\log y_t - \log L_t = \phi_2 + \phi_{21} \log k_t - \phi_{21} \log x_t + \phi_{22} \tau_t , \tag{3.7b}$$

$$\log L_t = \phi_3 + \phi_{31} \log k_t - \phi_{31} \log x_t + \phi_{32} \tau_t , \tag{3.7c}$$

where $k_t = K_t/X_{t+1}$ and $y_t = Y_t/X_t$ are capital and output detrended with the unit-root shock and the ϕ's are the coefficients of the calculated policy rules. Following Fernández-Villaverde et al. (Forthcoming) the system can be written in state space form. The state transition equation is

$$\begin{aligned} \begin{bmatrix} \log k_{t+1} \\ \tau_t \end{bmatrix} &= K_1 + A \begin{bmatrix} \log k_t \\ \tau_{t-1} \end{bmatrix} + B \begin{bmatrix} \varepsilon_{x,t} \\ \varepsilon_{lt} \end{bmatrix} , \\ \mathbf{x}_{t+1} &= K_1 + A\mathbf{x}_t + B\varepsilon_t , \end{aligned} \tag{3.8}$$

and the observation equation is

$$\begin{aligned}\begin{bmatrix}\Delta\log(Y_t/L_t)\\ \log L_t\end{bmatrix} &= K_2+C\begin{bmatrix}\log k_t\\ \tau_{t-1}\end{bmatrix}+D\begin{bmatrix}\varepsilon_{x,t}\\ \varepsilon_{lt}\end{bmatrix}, \\ \mathbf{y}_t &= K_2+C\mathbf{x}_t+D\varepsilon_t\ ,\end{aligned} \tag{3.9}$$

where K_1, A, B, K_2, C and D are constant matrices that depend on the coefficients of the policy rules and therefore on the "deep" parameters of the model. The state vector is given by $\mathbf{x}_t = [\log k_t,\ \tau_{t-1}]'$ and the noise vector is $\varepsilon_t = [\varepsilon_{xt},\ \varepsilon_{lt}]'$. Note that the system has a state vector of dimension two with the logarithm of detrended capital and the tax rate shock as state components.

The above state space system is still a structural model, since the formulation contains the non-observable state vector and the structural errors. We now show different representations of the system for $\mathbf{y}_t$, which can be estimated in practice. Given certain invertibility conditions on the system matrices, A, B, C, D, there is an **infinite VAR representation**:

$$\mathbf{y}_t = K_3 + C\left(I-(A-BD^{-1}C)L\right)^{-1}BD^{-1}\mathbf{y}_{t-1} + D\varepsilon_t\ , \tag{3.10}$$

or

$$\mathbf{y}_t = K_3 + \sum_{i=1}^{\infty}\Pi_i\mathbf{y}_{t-i} + u_t\ , \tag{3.11}$$

where K_3 and $\Pi_i, i = 1, 2, \ldots$ are constant coefficient matrices, L denotes the lag operator, I denotes an identity matrix of suitable dimensions, $u_t = D\varepsilon_t$ and $u_t \sim iid\, N(0, \Sigma_u)$, $\Sigma_u = DD'$, where Σ_u is the covariance matrix of u_t. Note that a condition for the existence of an infinite VAR representation is that the eigenvalues of $(A - BD^{-1}C)$ are strictly less than one in modulus. In practice, it is only possible to approximate this structure by a finite-order

VAR.

Alternatively, the system can be written as a **state space model** in "innovations form":

$$
\begin{aligned}
\mathbf{x}_{t+1} &= K_1 + A\mathbf{x}_t + Ku_t \ , \\
\mathbf{y}_t &= K_2 + C\mathbf{x}_t + u_t \ ,
\end{aligned}
\tag{3.12}
$$

where the innovation, u_t, is defined as above and $K = BD^{-1}$. In contrast to the VAR representation in (3.10), it is possible to estimate (3.12) exactly.

Finally, the underlying DGP can be represented by a **VARMA(1,1) representation**:

$$
\begin{aligned}
\mathbf{y}_t &= K_4 + CAC^{-1}\mathbf{y}_{t-1} + \left(D + (CB - CAC^{-1}D)L\right)\varepsilon_t \ , \\
\mathbf{y}_t &= K_4 + A_1\mathbf{y}_{t-1} + u_t + M_1u_{t-1} \ ,
\end{aligned}
\tag{3.13}
$$

where the last equation defines A_1, M_1 and u_t is defined as above. As with the state space representation, the VARMA(1,1) representation can also be estimated exactly.

Given the conditions stated in Fernández-Villaverde et al. (Forthcoming), all three representations are algebraically equivalent. That is, given the same input sequence $\{\varepsilon_t\}$, they produce the same output sequence $\{\mathbf{y}_t\}$. The representations are however not statistically equivalent: the properties of estimators and tests depend on the chosen statistical representation. It should be emphasized that we are always interested in the same process and ultimately in the estimation of the same coefficients, i.e. those associated with the first-period response of $\mathbf{y}_t$ to a unit shock in $\varepsilon_{x,t}$ to the technology process. However, the different representations give rise to different estimation algorithms and therefore our study can be regarded as a comparison of different algorithms to estimate the same linear system.

3.4 The Monte Carlo Experiment

3.4.1 Monte Carlo Design and Econometric Techniques

To investigate the properties of the various estimators, we simulate 1000 samples of the vector series $\mathbf{y}_t$ in linearized form and transform log-deviations to values in log-levels. As in the previous Monte Carlo studies, the sample size is 180 quarters. We use two different sets of parameter values: The first is due to Chari et al. (2005) and is referred to as the CKM-specification, while the second is the one used by Christiano et al. (2006) and is labeled the KP-specification, referring to estimates obtained by Prescott (1986).[4] The specific parameter values are given in Table 3.1 for the CKM and KP benchmark specifications. Christiano et al. (2006) show that the key difference between the specifications is the implied fraction of the variability in hours worked that is due to technology shocks. Table 3.1 also provides the eigenvalues of the autoregressive and moving-average matrices of the corresponding VARMA representations, together with the eigenvalues of the Kalman gain K. In terms of these values, the time series properties are very similar and indicate why the estimation of both systems could be difficult. Note that the moving-average part is not of full rank and the associated eigenvalue is close to unity in modulus. Also, the eigenvalues of the autoregressive part are close to one and close to the eigenvalue of the moving-average part in modulus. The fact that one eigenvalue of the moving-average part is close to one eigenvalue of the autoregressive part could imply that the VARMA(1,1) representation is close to being not identified (Klein et al., 2005).

To check the robustness of our results, we also consider variations of the benchmark models. As in Christiano et al. (2006), we consider different values for the preference

[4]Both parameterizations are obtained by maximum likelihood estimation of the theoretical model, using time series on productivity and hours worked in the US. However, because of differences in approach, both papers obtain different estimates.

parameter σ and the standard deviation of the labor tax, σ_l. These variations change the fraction of the business cycle variability that is due to technology shocks. The different values for σ are reported in Table 3.2. For the CKM specification, we also consider cases where σ_l assumes a fraction of the original benchmark value.

Turning to the issue of identification, consider the following infinite moving-average representation of $\mathbf{y}_t$ in terms of u_t:

$$\mathbf{y}_t = \sum_{i=0}^{\infty} \Phi_{u,i} u_{t-i} = \Phi_u(L) u_t \ , \tag{3.14}$$

where we abstract from the intercept term and $\Phi_u(L)$ is a lag polynomial, $\Phi_u(L) = \sum_{i=0}^{\infty} \Phi_{u,i} L^i$. Analogously, we can represent $\mathbf{y}_t$ in terms of the structural errors using the relation $u_t = D\varepsilon_t$:

$$\mathbf{y}_t = \sum_{i=0}^{\infty} \Phi_{u,i} D\varepsilon_{t-i} = \Phi_\varepsilon(L)\varepsilon_t \ , \tag{3.15}$$

where $\Phi_\varepsilon(L) = \sum_{i=0}^{\infty} \Phi_{u,i} D L^i$. The former lag polynomial, evaluated at one,

$$\Phi_u(1) = I + \Phi_{u,1} + \Phi_{u,2} + \dots \tag{3.16}$$

is the long-run impact matrix of the reduced form error u_t. Note that the existence of this infinite sum depends on the stationarity of the series. If the stationarity requirement is violated or "nearly" violated, then the long-run identification scheme is not valid or may face difficulties. Also note that the matrix D defined in section 3.3 gives the first-period impact of a unit shock in ε_t. Using the above relations, we know that $\Phi_\varepsilon(1) = \Phi_u(1)D$ and further $\Sigma_u = DD'$, where $\Phi_\varepsilon(1)$ is the long-run impact matrix of the underlying structural errors. The identifying restriction on $\Phi_\varepsilon(1)$ is that only the technology shock has a permanent effect on labor productivity. This restriction implies that in our bivariate system the long-run impact matrix is triangular,

$$\Phi_{\varepsilon}(1) = \begin{bmatrix} \Phi_{11} & 0 \\ \Phi_{21} & \Phi_{22} \end{bmatrix} , \tag{3.17}$$

and it is assumed that $\Phi_{11} > 0$. Using $\Phi_{\varepsilon}(1)\Phi'_{\varepsilon}(1) = \Phi_u(1)\Sigma_u\Phi'_u(1)$ we can obtain $\Phi_{\varepsilon}(1)$ from the Cholesky decomposition of $\Phi_u(1)\Sigma_u\Phi'_u(1)$. The contemporaneous impact matrix can be recovered from $D = [\Phi_u(1)]^{-1}\Phi_{\varepsilon}(1)$. Correspondingly, the estimated versions are

$$\hat{\Phi}_{\varepsilon}(1) = \text{chol}[\hat{\Phi}_u(1)\widehat{\Sigma}_u\hat{\Phi}'_u(1)] , \tag{3.18a}$$

$$\hat{D} = [\hat{\Phi}_u(1)]^{-1}\hat{\Phi}_{\varepsilon}(1) . \tag{3.18b}$$

Only the first column of $\hat{D}$ is identified and is our estimate of the first-period impact of the technology shock.

Next, we comment on the estimation techniques. First, note that for each representation there are several possible estimation methods. We chose algorithms that are both popular in the literature and known to work well in general. Of course, it is possible that there are algorithms that work slightly better for one of the representations in the current setting. However, the aim of this study is primarily to quantify whether the inclusion of the moving-average term alone leads to important gains in terms of more precise estimates of the structural parameters.

Vector Autoregressive Models. VARs are well known, so we comment only on a few issues. Fernández-Villaverde et al. (Forthcoming) show that for the CKM-specification there exists an infinite VAR representation. We verified that the same is true for the benchmark KP-specification. As in the previous Monte Carlo studies, the VAR lag length is set at four. However, for different sets of parameter values a VAR with different lags may yield slightly better results. We have chosen to stick to the VAR(4) because we want to facilitate comparison with the results of Christiano et al. (2006) and because there was no lag order that performed uniformly better for all DGPs.

State Space Models. There are many ways to estimate a state space model, e.g., the Kalman-based maximum likelihood methods and subspace identification methods such as N4SID of Van Overschee and De Moor (1994) or the CCA method of Larimore (1983). An obvious candidate is maximum likelihood. Therefore, we included a prediction error method that is implemented with the PEM routine in the MATLAB system identification toolbox. However, it is well-known that maximum likelihood methods can face numerical problems that are due to the dependence on starting values, nonlinear optimization or local maxima. Indeed, these problems also apply to our setting. Therefore, we also use the CCA subspace algorithm that is asymptotically equivalent to maximum likelihood and was previously found to be remarkably accurate in small samples. As argued in Bauer (2005), CCA might be the best algorithm for econometric applications. The idea of subspace methods is that the state, $\mathbf{x}_t$, summarizes all information of the past that can be used for mean square prediction. Thus, the center of attention is the state that is estimated in a first step. In a second step the coefficient matrices are estimated by OLS. The different subspace algorithms use the structure of the state space representation in various ways. See Bauer (2005) for a more general introduction to subspace methods and the Appendix for a detailed description of the algorithm that is employed in this paper.

While implementing the algorithm, we chose the correct dimension of the state vector, $n = 2$.[5] To calculate the long-run effect of the prediction errors, it is necessary to solve the

[5] There are two auxiliary parameters in the subspace algorithm, f, $\mathfrak{p}$, which determine the row and column dimension of a Hankel matrix which is estimated in an intermediate step (see Bauer (2005) and the Appendix). They have been set to $f = \mathfrak{p} = 8$. These parameters are of no importance asymptotically as long as they increase at certain rates with the sample size. In the literature it has been suggested to set $f = \mathfrak{p} = 2\hat{p}$ where

state space equations $\mathbf{x}_{t+1} = A\mathbf{x}_t + Ku_t$, $\mathbf{y}_t = C\mathbf{x}_t + u_t$, where the deterministic component is omitted. The lag polynomial of the infinite moving-average representation is given by

$$\Phi_u(L) = I + \sum_{j=0}^{\infty} CA^j L^{j+1} K = I + LC(I - LA)^{-1} K \ . \tag{3.19}$$

An estimate of the long-run impact matrix $\Phi_u(1)$ can be obtained from the estimated system matrices, $\hat{\Phi}_u(1) = I + \hat{C}(I - \hat{A})^{-1}\hat{K}$. Henceforth, the estimation of the contemporaneous impact matrix is entirely analogous to long-run identification in a standard VAR setting. That is, we recover $\Phi_\varepsilon(1)$ by a Cholesky decomposition and then obtain an estimate of D.

Vector Autoregressive Moving-Average Models. The VARMA representation given in (3.13) implies that we can represent $\mathbf{y}_t$ in terms of the innovations as

$$\mathbf{y}_t = (I - A_1 L)^{-1}(I + M_1 L)u_t = A(L)^{-1} M(L) u_t \ , \tag{3.20}$$

where $A(L)$ and $M(L)$ are the autoregressive polynomial and the moving-average polynomial, respectively, and the intercept term has been omitted. The long-run impact of u_t is given by $\Phi_u(1) = A(1)^{-1}M(1)$ and D can be recovered as before. The representation in (3.13) is however not the most useful representation in practice. It is more useful to choose a specific representation which guarantees that all parameters are identified and the number of estimated parameters is minimal. For an introduction to the identification problem in VARMA models see Lütkepohl (2005). Here we employ a final moving-average (FMA) representation that can be derived analogously to the final equation form (Dufour et al., 2002). In our case, this results in a VARMA $(2,1)$ representation in final moving-average

$\hat{p}$ is the order of the chosen autoregressive approximation Bauer (2005).

form (see Appendix).[6]

As in the case of state space models there are many different estimation methods for VARMA models. Examples are the methods developed by Hannan and Rissanen (1982), Koreisha and Pukkila (1990), Mauricio (1997) or Kapetanios (2003). We report results for a simple two-stage least squares method as in Hannan and Rissanen (1982), an iterative least squares estimation algorithm proposed by Kapetanios (2003) and the three-stage procedure developed by Hannan and Kavalieris (1984). The two-stage least squares method starts with an initial "long" autoregression in order to estimate the unobserved residuals. The estimated residuals are then plugged into equation (3.13) and a (generalized) least squares regression is performed. The iterative least squares procedure takes these estimates as initial parameters and uses them to computes new residuals which are again used in a second least squares regression in order to update the parameter estimates. The updated parameter estimates are then used to update the residual estimates and so on, until convergence. The last algorithm is regression-based and is the first step of a Gauss-Newton procedure for the maximization of the likelihood, conditional on initial values. First, a high-order VAR is fitted to get initial estimates of the innovations. In the second stage these estimates are used to estimate the autoregressive and moving-average parameters by least squares. In the third stage the estimated coefficients are used to form new residuals and the coefficient estimates from the second stage are refined (see, e.g. Hannan and Kavalieris (1984) or Hannan and Deistler (1988)). In the Appendix we provide further details on the estimation algorithms. We use a VAR with lag length $n_T = 0.5\sqrt{T}$ for the initial long autoregression.[7]

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[6] We experimented with other identified representations such as the final equation representation or the Echelon representation. However, the final moving-average representation always yielded the best results.

[7] We also tried full information maximum likelihood maximization as, for example, in Mauricio (1997). However, this procedure proved to be highly unstable and was therefore not considered to be a practical alternative. One likely reason is that the roots of the AR and the MA polynomials are all close to the unit circle.

3.4.2 Results of the Monte Carlo Study

Table 3.2 summarizes the results of the Monte Carlo simulation study. We tabulate Monte Carlo means and standard deviations of the estimates of the contemporaneous impact of a technology shock on productivity and hours worked for the various estimators. We also tabulate the MSE of the different estimators relative to the MSE of the estimator resulting from the benchmark SVAR. For the VARMA algorithms the estimation method is indicated in parenthesis, where 2*SLS* refers to the two-stage least squares method and *ILS* and 3*SLS* refer to the iterative least squares algorithm and the Hannan-Kavalieris method, respectively. Figures 3.1 and 3.2 depict the average estimated impulse responses of hours worked, together with the true impulse responses for the SVAR and the CCA subspace algorithm. In the figures, the bands around the mean lines correspond to the 0.025% and 0.975% quantiles of the estimated impulse responses at each point of time.

Our SVAR results confirm the findings of both Christiano et al. (2006) and Chari et al. (2005). While the SVAR is unbiased for the KP-specification (first row in Table 3.2), the same is not true for the CKM-specification (fourth row in Table 3.2). The associated pictures for both parameterizations show that the 95% bands around the mean impulse responses comprise a large region ranging from negative values to very high positive values. Also, for the different variations of the benchmark model we find that the SVAR is often biased and/or displays high variability. As can be seen from row 2, 3, 5 and 6 in Table 3.2, both the biases and standard deviations are larger for the models with higher Frisch elasticities of labor supply (lower σ), as in the model this decreases the proportion of the variation in hours worked that is due to the technology shock. From row 7 and 8 it is clear that reducing the relative importance of the tax shock by lowering σ_l by 1/2 and 1/3 reduces the bias and the standard deviations.

The algorithms based on the state space representation perform quite differently. The

PEM routine performs uniformly worse for all sets of parameter values. Although, in contrast to the SVAR, the state space model nests the DSGE model, the small sample performance of this estimation algorithm is much poorer. The low accuracy of the PEM routine can be attributed to the near non-stationarity and non-invertibility of the series that cause difficulties for the optimization procedure. The results of the PEM routine illustrate that using a formally exact representation of the DGP does not automatically lead to more precise estimates because the associated estimation algorithm may be numerically unreliable or not robust to the near violation of the underlying assumptions. For the CCA subspace algorithm, however, we find that the associated MSE of the estimated first-period impulse response is almost uniformly lower for both series and across different specifications. Only in two cases does the MSE of the CCA-based estimates exceed the MSE of the SVAR, and only by a very small amount. In particular, the first-period impact on hours worked is estimated more precisely up to a relative reduction to 85% in terms of MSE for the KP-specification. Figure 3.1 shows that the 95% interval is narrower for the estimated state space model, but still rather wide. In almost all cases the bias is at least slightly reduced. Although the response of hours worked is usually estimated more precisely, the performances of the subspace algorithm and the SVAR seem to be related: in cases where the SVAR does poorly, the state space model does so too. The advantage of the CCA algorithm over the SVAR-based least squares algorithm should be due to the use of the more general state space representation.[8]

The results for the different algorithms based on the VARMA representation are either similar to or worse than those for the VAR approximation. Generally, the less sophisticated methods give better results than the more complex estimation algorithms. Improvements in terms of bias are compromised by increases in variance and a higher MSE. The reason for

[8]It is also worth mentioning that since the CCA algorithm is based on OLS regressions, it is computationally not more intensive than a VAR. The same is not true for the PEM routine and most VARMA estimation algorithms.

this pattern is that we face an ill-conditioned problem that cannot be remedied by rescaling the data because of the presence of eigenvalues close to the unit circle of both the autoregressive and moving-average parts. Furthermore, the roots of the moving-average part and the autoregressive part imply that the model is close to being not identified. The iterative least squares method and the Hannan-Kavalieris method face consequently more problems than the simple two-stage least squares method. In comparison to the SVAR model, the structural estimates obtained from the VARMA algorithms perform relatively well in estimating the impact on hours worked, but worse in estimating the response of productivity to a technology shock. While the VARMA model fully nests the underlying DGP, this representation is not very efficient in our context.

A problem common to all algorithms is that the stationarity requirement is nearly violated for the DGPs at hand. As we have seen in section 3.4.1, the stationarity assumption lies at the heart of the long-run identification scheme. However, as the eigenvalues in Table 3.1 indicate, this assumption is nearly violated for the benchmark models. This problem is independent of the chosen representation and, therefore, does not vanish even when we control for omitted moving-average terms. Apart from problems specific to the algorithms, this common problem may explain the relatively weak performance of all algorithms - a problem that could be overcome in larger samples.[9]

3.5 Conclusions

There has been some debate whether long-run identified SVARs can in practice discriminate between competing DSGE models and whether their sampling properties are good enough to justify their widespread use. Several Monte Carlo studies indicate that SVARs

[9]Our estimation results are in line with the findings of McGrattan (2006). However, McGrattan (2006) stresses the need to impose more theoretical restrictions to obtain informative statistics from empirical models.

based on long-run restrictions are often biased and usually imprecise. Some authors have suggested that SVARs do poorly because they are only approximate representations of the underlying DGPs. Therefore, we replicate the simulation experiments of Chari et al. (2005) and Christiano et al. (2006) and apply more general models to their simulated data. In particular, we use algorithms based on VARMA and state space representations of the data and compare the resulting estimates of the underlying structural model. For our simulations, we find that one can do better by taking the full structure of the DGP into account. While our VARMA-based estimation algorithms and the prediction error algorithm for state space models are not found to do significantly better and often even worse, the CCA subspace algorithm seems to consistently outperform the SVAR. However, the estimates display high variability and are often biased, regardless of the reduced form model used. Furthermore, the performances of the different estimators are strongly correlated. This finding suggests that long-run identified SVARs do not fail because they are simple finite-order approximations. Instead, we find that the simulated processes are nearly violating the most basic assumptions on which long-run identification schemes are based. Given these properties of the data series, the poor performance seems almost entirely a small sample problem in this type of simulation studies.

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Appendix: Final MA Equation Form

Consider a standard representation for a stationary and invertible VARMA process

$$A(L)\mathbf{y}_t \quad = \quad M(L)u_t \ . \tag{3.21}$$

Recall that $M^{-1}(L) = M^*(L)/|M(L)|$, where $M^*(L)$ denotes the adjoint of $M(L)$ and $|M(L)|$ its determinant. We can multiply the above equation with $M^*(L)$ to get

$$M^*(L)A(L)\mathbf{y}_t \quad = \quad |M(L)|u_t \ . \tag{3.22}$$

This representation therefore places restrictions on the moving-average polynomial which is required to be a scalar operator, $|M(L)|$. Dufour and Pelletier (2002) show that this restriction leads to an identified representation. More specifically, consider the VARMA(1,1) representation in (3.13). Since the moving-average part is not of full rank we can write the system as

$$\begin{bmatrix} 1-a_{11}L & -a_{12}L \\ -a_{21}L & 1-a_{22}L \end{bmatrix} \mathbf{y}_t = \begin{bmatrix} 1+m_{11}L & \alpha m_{11}L \\ m_{21}L & 1+\alpha m_{21}L \end{bmatrix} u_t \ , \tag{3.23}$$

where α is some constant not equal to zero.

Clearly, $\det(M(L)) = 1 + (m_{11} + \alpha m_{21})L$ and we can write

$$\begin{bmatrix} 1+\alpha m_{21}L & -\alpha m_{11}L \\ -m_{21}L & 1+\alpha m_{11}L \end{bmatrix} \begin{bmatrix} 1-a_{11}L & -a_{12}L \\ -a_{21}L & 1-a_{22}L \end{bmatrix} \mathbf{y}_t = [1 + (m_{11} + \alpha m_{21})L]u_t \ . \tag{3.24}$$

Because of the reduced rank we end up with a VARMA $(2,1)$. Note that the moving-average part is indeed restricted to be a scalar operator.

Appendix: Statistical Representations

This section elaborates on the derivation of the infinite VAR, VARMA and state space representations that result from our DSGE model in order to get an insight into the relationship between the economic model and the implied time series properties.

Consider again the law of motion of the logs

$$\log k_{t+1} = \phi_1 + \phi_{11} \log k_t - \phi_{11} \log x_t + \phi_{12} \tau_t \ , \tag{3.25a}$$

$$\log y_t - \log L_t = \phi_2 + \phi_{21} \log k_t - \phi_{21} \log x_t + \phi_{22} \tau_t \ , \tag{3.25b}$$

$$\log L_t = \phi_3 + \phi_{31} \log k_t - \phi_{31} \log x_t + \phi_{32} \tau_t \ , \tag{3.25c}$$

and the exogenous states

$$\log x_{t+1} = \mu + \sigma_x \varepsilon_{x,t+1} \ , \tag{3.26a}$$

$$\tau_{t+1} = (1-\rho)\bar{\tau}_l + \rho \tau_t + \sigma_l \varepsilon_{l,t+1} \ . \tag{3.26b}$$

From these equations the state space representation can be derived as follows. First write down the law of motion of labor productivity in differences:

$$\Delta \log(Y_t/L_t) = \log x_t + \phi_{21} \Delta \log k_t - \phi_{21} \Delta \log x_t + \phi_{22} \Delta \tau_t \ . \tag{3.27}$$

Thus the observed series can be expressed as

$$\begin{aligned} \Delta \log(Y_t/L_t) &= \phi_{21} \log k_t - \phi_{21} \log k_{t-1} + (1-\phi_{21}) \log x_t \\ &\quad + \phi_{21} \log x_{t-1} + \phi_{22} \tau_t - \phi_{22} \tau_{t-1} \ , \end{aligned} \tag{3.28a}$$

$$\log L_t = \phi_3 + \phi_{31} \log k_t - \phi_{31} \log x_t + \phi_{32} \tau_t \ . \tag{3.28b}$$

Next, rewrite the law of motion for capital as

$$\log k_{t-1} = -\phi_{11}^{-1}\phi_1 + \phi_{11}^{-1}\log k_t + \log x_{t-1} - \phi_{11}^{-1}\phi_{12}\tau_{t-1} \ , \tag{3.29}$$

in order to substitute for capital at time $t-1$:

$$\begin{aligned} \Delta\log(Y_t/L_t) &= \phi_{21}\phi_{11}^{-1}\phi_1 + \phi_{21}(1-\phi_{11}^{-1})\log k_t \\ &\quad +(1-\phi_{21})\log x_t + \phi_{22}\tau_t + (\phi_{21}\phi_{11}^{-1}\phi_{12} - \phi_{22})\tau_{t-1} \ . \end{aligned} \tag{3.30}$$

Using the laws of motion for the stochastic shock processes, substitute the current exogenous shocks to get

$$\begin{aligned} \Delta\log(Y_t/L_t) &= \left[\phi_{21}\phi_{11}^{-1}\phi_1 + (1-\phi_{21})\mu + \phi_{22}(1-\rho)\bar{\tau}_l\right] + \phi_{21}(1-\phi_{11}^{-1})\log k_t \qquad \text{(3.31a)} \\ &\quad +(\phi_{21}\phi_{11}^{-1}\phi_{12} - (1-\rho)\phi_{22})\tau_{t-1} + (1-\phi_{21})\sigma_x\varepsilon_{x,t} + \phi_{22}\sigma_l\varepsilon_{l,t} \ , \\ \log L_t &= [\phi_3 - \phi_{31}\mu + \phi_{32}(1-\rho)\bar{\tau}_l] + \phi_{31}\log k_t + \phi_{32}\rho\tau_{t-1} \qquad \text{(3.31b)} \\ &\quad -\phi_{31}\sigma_x\varepsilon_{x,t} + \phi_{32}\sigma_l\varepsilon_{l,t} \ . \end{aligned}$$

Next, consider the law of motion for capital and express future capital in terms of the current states as

$$\begin{aligned} \log k_{t+1} &= [\phi_1 - \phi_{11}\mu + \phi_{12}(1-\rho)\bar{\tau}_l] + \phi_{11}\log k_t + \phi_{12}\rho\tau_{t-1} \\ &\quad -\phi_{11}\sigma_x\varepsilon_{x,t} + \phi_{12}\sigma_l\varepsilon_{l,t} \ . \end{aligned} \tag{3.32}$$

Collecting the above equations, the system can be written in state space form according to Fernández-Villaverde, Rubio-Ramírez, Watson and Sargent (Forthcoming). The state transition equation is

$$\begin{bmatrix} \log k_{t+1} \\ \tau_t \end{bmatrix} = K_1 + A \begin{bmatrix} \log k_t \\ \tau_{t-1} \end{bmatrix} + B \begin{bmatrix} \varepsilon_{x,t} \\ \varepsilon_{lt} \end{bmatrix}, \tag{3.33}$$

where the system matrices are given by

$$K_1 = \begin{bmatrix} \phi_1 - \phi_{11}\mu + \phi_{12}(1-\rho)\bar{\tau}_l \\ (1-\rho)\bar{\tau} \end{bmatrix},$$

$$A = \begin{bmatrix} \phi_{11} & \phi_{12}\rho \\ 0 & \rho \end{bmatrix},$$

and

$$B = \begin{bmatrix} -\phi_{11}\sigma_x & \phi_{12}\sigma_l \\ 0 & \sigma_l \end{bmatrix}.$$

The observation equation is

$$\begin{bmatrix} \Delta\log(Y_t/L_t) \\ \log L_t \end{bmatrix} = K_2 + C \begin{bmatrix} \log k_t \\ \tau_{t-1} \end{bmatrix} + D \begin{bmatrix} \varepsilon_{x,t} \\ \varepsilon_{lt} \end{bmatrix}, \tag{3.34}$$

with system matrices

$$K_2 = \begin{bmatrix} \phi_{21}\phi_{11}^{-1}\phi_1 + (1-\phi_{21})\mu + \phi_{22}(1-\rho)\bar{\tau}_l \\ \phi_3 - \phi_{31}\mu + \phi_{32}(1-\rho)\bar{\tau}_l \end{bmatrix},$$

$$C = \begin{bmatrix} \phi_{21}(1-\phi_{11}^{-1}) & \phi_{21}\phi_{11}^{-1}\phi_{12} - (1-\rho)\phi_{22} \\ \phi_{31} & \phi_{32}\rho \end{bmatrix},$$

and

$$D = \begin{bmatrix} (1-\phi_{21})\sigma_x & \phi_{22}\sigma_l \\ -\phi_{31}\sigma_x & \phi_{32}\sigma_l \end{bmatrix}.$$

This representation permits us to derive the infinite VAR and VARMA representation in compact form.

Let $\mathbf{y}_t$ denote the vector of observables, $\mathbf{x}_t$ the vector of states, and ε the white noise shocks. Then we have as above

$$\mathbf{x}_{t+1} = K_1 + A\mathbf{x}_t + B\varepsilon_t \ , \tag{3.35a}$$

$$\mathbf{y}_t = K_2 + C\mathbf{x}_t + D\varepsilon_t \ . \tag{3.35b}$$

If D is invertible, it is possible to use $\varepsilon_t = D^{-1}(\mathbf{y}_t - K_2 - C\mathbf{x}_t)$ in the transition equation to obtain

$$\mathbf{x}_{t+1} = K_1 + A\mathbf{x}_t + BD^{-1}(\mathbf{y}_t - K_2 - C\mathbf{x}_t) \ , \tag{3.36a}$$

$$(I-(A-BD^{-1}C)L)\mathbf{x}_{t+1} = [K_1 - BD^{-1}K_2] + BD^{-1}\mathbf{y}_t \ . \tag{3.36b}$$

If the eigenvalues of $(A-BD^{-1}C)$ are strictly less than one in modulus we can solve for $\mathbf{x}_{t+1}$:

$$\mathbf{x}_{t+1} = \left(I-(A-BD^{-1}C)L\right)^{-1}\left([K_1 - BD^{-1}K_2] + BD^{-1}\mathbf{y}_t\right) \ . \tag{3.37}$$

Using this relation in the observation equation yields the infinite VAR representation for $\mathbf{y}_t$:

$$\mathbf{y}_t = K_2 + C\left(I-(A-BD^{-1}C)L\right)^{-1}\left([K_1 - BD^{-1}K_2] + BD^{-1}\mathbf{y}_{t-1}\right) + D\varepsilon_t \ , \tag{3.38}$$

$$\mathbf{y}_t = K_3 + C\left(I-(A-BD^{-1}C)L\right)^{-1}BD^{-1}\mathbf{y}_{t-1} + D\varepsilon_t \ .$$

Note that the condition for the existence of an infinite VAR-representation is that $I-(A-BD^{-1}C)$ is invertible. If this condition does not hold, impulse responses from a VAR are unlikely to match up those from the model.

If C is invertible, it is possible to rewrite the state as $\mathbf{x}_t = C^{-1}(\mathbf{y}_t - K_2 - D\varepsilon_t)$ and use it in the transition equation:

$$\begin{aligned} C^{-1}(\mathbf{y}_{t+1} - K_2 - D\varepsilon_{t+1}) &= K_1 + AC^{-1}(\mathbf{y}_t - K_2 - D\varepsilon_t) + B\varepsilon_t \,, \\ \mathbf{y}_{t+1} - CAC^{-1}\mathbf{y}_t &= CK_1 + K_2 - CAC^{-1}K_2 + (CB - CAC^{-1}D)\varepsilon_t + D\varepsilon_{t+1} \,. \end{aligned} \tag{3.39}$$

Therefore, we obtain a VARMA(1,1) representation of $\mathbf{y}_t$:

$$\mathbf{y}_t = K_4 + CAC^{-1}\mathbf{y}_{t-1} + \left(I + (CBD^{-1} - CAC^{-1})L\right) u_t \,. \tag{3.40}$$

with $u_t \sim N(0, DD')$.

Appendix: Estimation Algorithms

Two-Stage Least Squares and Iterative Least Squares

These two methods are computationally very easy to implement. The iterative least squares method has been introduced by Kapetanios (2003).

We discuss the methods in the framework of a standard VARMA (p,q) representation

$$y_t = A_1 y_{t-1} + \ldots + A_p y_{t-p} + u_t + M_1 u_{t-1} + \ldots + M_q u_{t-q} \,. \tag{3.41}$$

Usually additional restrictions need to be imposed on the coefficient matrices to ensure identification of the parameters.

Given that the moving-average polynomial is invertible, there exists an infinite VAR representation of the process, $y_t = \sum_{i=1}^{\infty} \Pi_i y_{t-i} + u_t$. In the first step of both algorithms, this representation is approximated by a "long" VAR to get an estimate of the residuals. More precisely, the following regression equation is used

$$y_t = \sum_{i=1}^{n_T} \Pi_i y_{t-i} + u_t \ , \tag{3.42}$$

where n_T is large and goes to infinity as the sample size grows. The estimated residuals are denoted by $\hat{u}_t^{(0)}$. Given these estimates, we might obtain estimates of the parameter matrices by performing a (restricted) regression in

$$y_t = A_1 y_{t-1} + \ldots + A_p y_{t-p} + u_t + M_1 \hat{u}_{t-1}^{(0)} + \ldots + M_q \hat{u}_{t-q}^{(0)} \ . \tag{3.43}$$

Denote the estimated coefficient matrices by $A_1^{(1)}, A_2^{(1)}, \ldots, A_p^{(1)}$ and $M_1^{(1)}, M_2^{(1)}, \ldots, M_q^{(1)}$. These estimates are the final estimates of the two-stage least squares method. These initial parameter estimates can be used to obtain a new estimate of the residuals. Denote by $\hat{U}^{(1)}$ the vector collecting the estimated new residuals. We can then use $\hat{U}^{(1)}$ again in the above equation to obtain new estimates of the coefficient matrices. Denote the vector of estimated residuals at the i^{th} iteration by $\hat{U}^{(i)}$. Kapetanios (2003) proposes to iterate least squares regressions until $||U^{(i-1)} - \hat{U}^{(i)}|| < c$, according to some pre-specified number c.

Hannan-Kavalieris Method

This method goes originally back to Durbin (1960) and has been introduced by Hannan and Kavalieris (1984) for multivariate processes.[10] It is a Gauss-Newton procedure to maximize the likelihood function conditional on $y_t = 0$, $u_t = 0$ for $t \leq 0$, but its first iter-

[10] See also Hannan and Deistler (1988), sections 6.5, 6.7, for an extensive discussion.

ation has been sometimes interpreted as a three-stage least squares procedure (Dufour and Pelletier (2002)). We discuss also this method in the framework of a standard VARMA (p,q) representation

$$y_t = A_1 y_{t-1} + \ldots + A_p y_{t-p} + u_t + M_1 u_{t-1} + \ldots + M_q u_{t-q} \; . \tag{3.44}$$

To consider zero restrictions, we use the following notation. The vector of all parameters is denoted by $\beta = \text{vec}[A_1, \ldots, A_p, M_1, \ldots M_q]$. The vector of free parameters, γ, can be defined by introducing a restriction matrix R such that the vectors are related by $\beta = R\gamma$.

Given that the moving-average polynomial is invertible, there exists an infinite VAR representation of the process, $y_t = \sum_{i=1}^{\infty} \Pi_i y_{t-i} + u_t$. In the first step of the algorithm, this representation is approximated by a "long" VAR to get an estimate of the residuals. More precisely, the following regression equation is used

$$y_t = \sum_{i=1}^{n_T} \Pi_i y_{t-i} + u_t \; , \tag{3.45}$$

where n_T is large and goes to infinity as the sample size grows. Given an estimate of the residuals, $\hat{u}_t$, we might obtain starting values for future iterations by performing a (restricted) regression in

$$y_t = A_1 y_{t-1} + \ldots + A_p y_{t-p} + u_t + M_1 \hat{u}_{t-1} + \ldots + M_q \hat{u}_{t-q} \; . \tag{3.46}$$

Denote the estimated coefficient matrices by $\tilde{A}_1, \tilde{A}_2, \ldots, \tilde{A}_p$ and $\tilde{M}_1, \tilde{M}_2, \ldots, \tilde{M}_q$. The first iteration of the conditional maximum likelihood algorithm can be expressed in a simple regression framework. One forms new residuals, ε_t, and new matrices, ξ_t, η_t and $\hat{X}_t$, according to

$$\varepsilon_t = y_t - \sum_{j=1}^{p} \tilde{A}_j y_{t-j} - \sum_{j=1}^{q} \tilde{M}_j \varepsilon_{t-j} , \quad (3.47a)$$

$$\xi_t = -\sum_{j=1}^{q} \tilde{M}_j \xi_{t-j} + \varepsilon_t , \quad (3.47b)$$

$$\eta_t = -\sum_{j=1}^{q} \tilde{M}_j \eta_{t-j} + y_t , \quad (3.47c)$$

$$\hat{X}_t = -\sum_{j=1}^{q} \tilde{M}_j \hat{X}_{t-j} + (Y_t' \otimes I_K) R , \quad (3.47d)$$

for $t = 1, 2, \ldots, T$, $Y_t = [y_t', \ldots, y_{t-p+1}', \hat{u}_t', \ldots, \hat{u}_{t-q+1}']'$ and $y_t = \varepsilon_t = \xi_t = \eta_t = 0$ and $\hat{X}_t = 0$ for $t \leq 0$. The final estimate is

$$\hat{\gamma} = \left(\sum_{m+1}^{T} \hat{X}_{t-1}' \widehat{\Sigma}_t^{-1} \hat{X}_{t-1} \right)^{-1} \left(\sum_{m+1}^{T} \hat{X}_{t-1} \widehat{\Sigma}^{-1} (\varepsilon_t + \eta_t - \xi_t) \right) , \quad (3.48)$$

where $\widehat{\Sigma} = T^{-1} \sum \varepsilon_t \varepsilon_t'$, $m = \max\{p, q\}$. This procedure is asymptotically efficient under certain conditions (Lütkepohl (2005)).

Subspace Algorithms

Subspace algorithms rely on the state space representation of a linear system. The CCA algorithm is originally due to Larimore (1983). The basic idea behind subspace algorithms lies in the fact that if we knew the unobserved state, x_t, we could estimate the *system matrices*, A, K, C, by linear regressions as can be seen from the basic equations

$$x_{t+1} = A x_t + K u_t , \quad (3.49a)$$

$$y_t = C x_t + u_t . \quad (3.49b)$$

Given the state and the observations, $\hat{C}$ and $\hat{u}_t$ could be obtained by a regression of y_t on x_t and $\hat{A}$ and $\hat{K}$ could be obtained by a regression of x_{t+1} on x_t and $\hat{u}_t$. Therefore, the problem is to obtain in a first step an estimate of the n-dimensional state, $\hat{x}_t$. This is analogous to the idea of a long autoregression in VARMA models that estimates the unobserved residuals in a first step which is followed by a least squares regression.

Solving the state space equations, one can express the state as a function of past observations of y_t and an initial state for some integer $\mathfrak{p} > 0$ as

$$\begin{aligned} x_t &= (A-KC)^{\mathfrak{p}} x_{t-\mathfrak{p}} + \sum_{i=0}^{\mathfrak{p}-1} (A-KC)^i K y_{t-i-1}, \\ &= (A-KC)^{\mathfrak{p}} x_{t-\mathfrak{p}} + \mathcal{K}_{\mathfrak{p}} Y^-_{t,\mathfrak{p}} , \end{aligned} \tag{3.50}$$

where $\mathcal{K}_{\mathfrak{p}} = [K, (A-KC)K, \ldots, (A-KC)^{\mathfrak{p}-1}K]$ and $Y^-_{t,\mathfrak{p}} = [y'_{t-1}, \ldots, y'_{t-p}]'$. On the other hand, one can express future observations as a function of the current state and future noise as

$$y_{t+j} = CA^j x_t + \sum_{i=0}^{j-1} CA^i K u_{t+j-i-1} + u_{t+j} , \tag{3.51}$$

for $j = 1, 2, \ldots$. Therefore, at each t, the best predictor of y_{t+j} is a function of the current state only, $CA^j x_t$, and thus the state summarizes in this sense all relevant information in the past up to time t.

Define $Y^+_{t,f} = [y'_t, \ldots, y'_{t+f-1}]'$ for some integer $f > 0$ and formulate equation (3.51) for all observations contained in $Y^+_{t,f}$ simultaneously. Combine these equations with (3.50) in order to obtain

$$Y^+_{t,f} = O_f \mathcal{K}_{\mathfrak{p}} Y^-_{t,\mathfrak{p}} + O_f (A-BC)^{\mathfrak{p}} x_{t-\mathfrak{p}} + \mathcal{E}_f E^+_{t,f} , \tag{3.52}$$

where $O_f = [C', A'C', \ldots, (A^{f-1})'C']'$, $E^+_{t,f} = [u'_t, \ldots, u'_{t+f-1}]'$ and $\mathcal{E}_f$ is a function of the system matrices. The above equation is central for most subspace algorithms. Note that if the maximum eigenvalue of $(A - KC)$ is less than one in absolute value, we have $(A - KC)^{\mathfrak{p}} \approx 0$ for large $\mathfrak{p}$. This condition is satisfied for stationary and invertible processes. This reasoning motivates an approximation of the above equation by

$$Y_{t,f}^{+} = \beta Y_{t,\mathfrak{p}}^{-} + N_{t,f}^{+} , \tag{3.53}$$

where $\beta = O_f \mathcal{K}_{\mathfrak{p}}$ and $N_{t,f}^{+}$ is defined by the equation. Most popular subspace algorithms use this equation to obtain an estimate of β that is decomposed into O_f and $\mathcal{K}_{\mathfrak{p}}$. The identification problem is solved implicitly during this step.

For given integers, n, $\mathfrak{p}$, f, the employed algorithm consists of the following steps:

1. Set up $Y_{t,f}^{+}$ and $Y_{t,\mathfrak{p}}^{-}$ and perform OLS in (3.53) using the available data to get an estimate $\hat{\beta}_{f,\mathfrak{p}}$.

2. Compute the sample covariances

$$\hat{\Gamma}_f^{+} = \frac{1}{T_{f,\mathfrak{p}}} \sum_{t=\mathfrak{p}+1}^{T-f+1} Y_{t,f}^{+}(Y_{t,f}^{+})' \, , \, \hat{\Gamma}_{\mathfrak{p}}^{-} = \frac{1}{T_{f,\mathfrak{p}}} \sum_{t=\mathfrak{p}+1}^{T-f+1} Y_{t,\mathfrak{p}}^{-}(Y_{t,\mathfrak{p}}^{-})',$$

   where $T_{f,\mathfrak{p}} = T - f - \mathfrak{p} + 1$.

3. Given the dimension of the state, n, compute the singular value decomposition

$$(\hat{\Gamma}_f^{+})^{-1/2} \hat{\beta}_{f,\mathfrak{p}} (\hat{\Gamma}_{\mathfrak{p}}^{-})^{1/2} = \hat{U}_n \hat{\Sigma}_n \hat{V}_n' + \hat{R}_n,$$

   where $\hat{\Sigma}_n$ is a diagonal matrix that contains the n largest singular values and $\hat{U}_n$ and $\hat{V}_n$ are the corresponding singular vectors. The remaining singular values are neglected and the approximation error is $\hat{R}_n$. The reduced rank matrices are obtained as

$$\hat{O}_f \hat{\mathcal{K}}_{\mathfrak{p}} = [(\hat{\Gamma}_f^+)^{1/2} \hat{U}_n \hat{\Sigma}_n^{1/2}][\hat{\Sigma}_n^{1/2} \hat{V}_n' (\hat{\Gamma}_{\mathfrak{p}}^-)^{-1/2}].$$

4. Estimate the state as $\hat{x}_t = \hat{\mathcal{K}}_{\mathfrak{p}} Y_{t,\mathfrak{p}}^-$ and estimate the system matrices using linear regressions as described above.

Although the algorithm looks quite complicated at first sight, it is actually very simple and is regarded to lead to numerically stable and accurate estimates. There are certain parameters which have to be determined prior to estimation, namely the dimension of the state and the integers f and $\mathfrak{p}$. See the text for the employed values. For the asymptotic consequences of various choices see Bauer (2005).

Tables and Figures

Table 3.1: Benchmark Calibrations and Time Series Properties

Parameters	Common	CKM-specification	KP-specification
α	0.33		
β	$0.98^{1/4}$		
σ	1		
δ	$1-(1-0.6)^{1/4}$		
ψ	2.5		
γ	$1.01^{1/4}-1$		
μ	0.00516		
$\bar{L}$	1		
$\bar{\tau}_l$	0.243		
τ_x	0.3		
ρ		0.94	0.993
σ_τ		0.008	0.0066
σ_x		0.00568	0.011738
Selected time series properties			
$\text{eig}(A_1)$		0.9573, 0.9400	0.9573, 0.9930
$\text{eig}(M_1)$		-0.9557, 0	-0.9505, 0
$\text{eig}(K)$		$-1.7779 \pm 0.51i$	$-2.0298 \pm 0.35i$

Parameter values of the CKM and KP benchmark calibrations. The last three rows display some properties of the implied VARMA and state space representation. $\text{eig}(A_1)$ and $\text{eig}(M_1)$ denote the eigenvalues of the autoregressive and the moving-average matrix, respectively. $\text{eig}(K)$ denotes the eigenvalues of the matrix K in the state space model in innovations form.

Table 3.2: Simulation Results

		VAR (4)			PEM (2)			SS (2,8,8)			VARMA (2SLS)			VARMA (ILK)			VARMA (3SLS)		
Variable	True Value	Mean	Std.	MSE	Mean	Std.	MSE	Mean	Std.	MSE	Mean	Std.	MSE	Mean	Std.	MSE	Mean	Std.	MSE
KP Benchmark																			
Prod.	0.69	0.55	0.19	1	0.53	0.23	1.44	0.57	0.18	0.86	0.54	0.18	1.02	0.54	0.19	1.06	0.53	0.21	1.23
Hours	0.28	0.31	0.43	1	0.32	0.50	1.33	0.31	0.40	0.85	0.33	0.42	0.92	0.32	0.42	0.94	0.36	0.47	1.22
KP, $\sigma = 0$ (Indivisible Labor)																			
Prod.	0.65	0.48	0.23	1	0.45	0.28	1.49	0.50	0.22	0.92	0.47	0.22	1.02	0.47	0.22	1.02	0.47	0.23	1.09
Hours	0.43	0.56	0.56	1	0.58	0.67	1.44	0.52	0.54	0.92	0.57	0.54	0.93	0.56	0.55	0.98	0.62	0.63	1.30
KP, $\sigma = 6$ (Frisch elasticity=0.63)																			
Prod.	0.75	0.61	0.16	1	0.57	0.24	2.03	0.63	0.14	0.80	0.59	0.16	1.10	0.59	0.16	1.16	0.59	0.17	1.26
Hours	0.11	0.10	0.19	1	0.10	0.24	1.52	0.10	0.18	0.89	0.10	0.19	0.98	0.10	0.19	1.00	0.11	0.23	1.37
CKM Benchmark																			
Prod.	0.34	0.10	0.17	1	0.13	0.20	1.03	0.11	0.18	1.00	0.09	0.16	1.07	0.09	0.17	1.08	0.10	0.17	1.04
Hours	0.14	0.65	0.39	1	0.61	0.53	1.21	0.62	0.40	0.95	0.67	0.37	1.02	0.66	0.40	1.03	0.70	0.41	1.18
CKM, $\sigma = 0$ (Indivisible Labor)																			
Prod.	0.31	-0.12	0.21	1	-0.07	0.30	1.02	-0.12	0.23	1.05	-0.15	0.19	1.09	-0.13	0.21	1.06	-0.12	0.22	1.04
Hours	0.21	1.26	0.49	1	1.16	0.84	1.20	1.24	0.54	1.01	1.31	0.45	1.06	1.28	0.50	1.03	1.33	0.55	1.17
CKM, $\sigma = 6$ (Frisch elasticity=0.63)																			
Prod.	0.36	0.30	0.08	1	0.29	0.11	1.67	0.31	0.08	0.95	0.29	0.08	1.04	0.29	0.09	1.21	0.30	0.09	1.24
Hours	0.05	0.12	0.17	1	0.12	0.20	1.35	0.11	0.17	0.92	0.13	0.17	0.97	0.12	0.17	1.02	0.14	0.18	1.11
CKM, $\sigma_l/2$																			
Prod.	0.34	0.25	0.10	1	0.25	0.12	1.45	0.26	0.10	0.93	0.25	0.09	1.06	0.25	0.10	1.07	0.25	0.10	1.17
Hours	0.14	0.26	0.22	1	0.26	0.26	1.35	0.24	0.21	0.88	0.27	0.21	0.97	0.26	0.22	1.01	0.28	0.23	1.19
CKM, $\sigma_l/3$																			
Prod.	0.34	0.28	0.07	1	0.28	0.09	1.63	0.29	0.07	0.89	0.28	0.07	1.08	0.28	0.07	1.13	0.28	0.07	1.11
Hours	0.14	0.18	0.15	1	0.18	0.18	1.44	0.17	0.14	0.87	0.19	0.14	0.96	0.18	0.15	1.01	0.20	0.16	1.14

Percent contemporaneous impact on productivity and hours of one standard deviation shock to technology.
The entries are Monte Carlo means and standard deviations. MSEs are relative to the MSE of the SVAR.

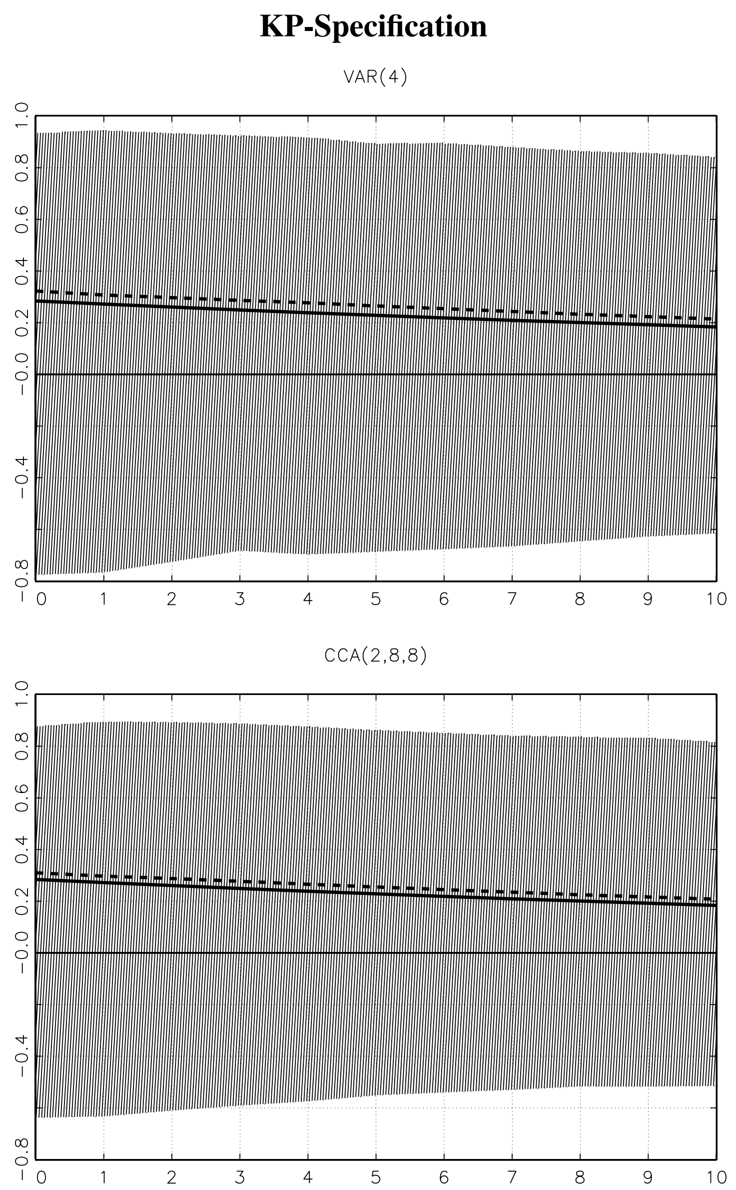

Figure 3.1: Mean impulse response (- -), true impulse response (–) and 95% intervals of hours worked to one standard deviation shock to technology for the VAR and the CCA subspace algorithm.

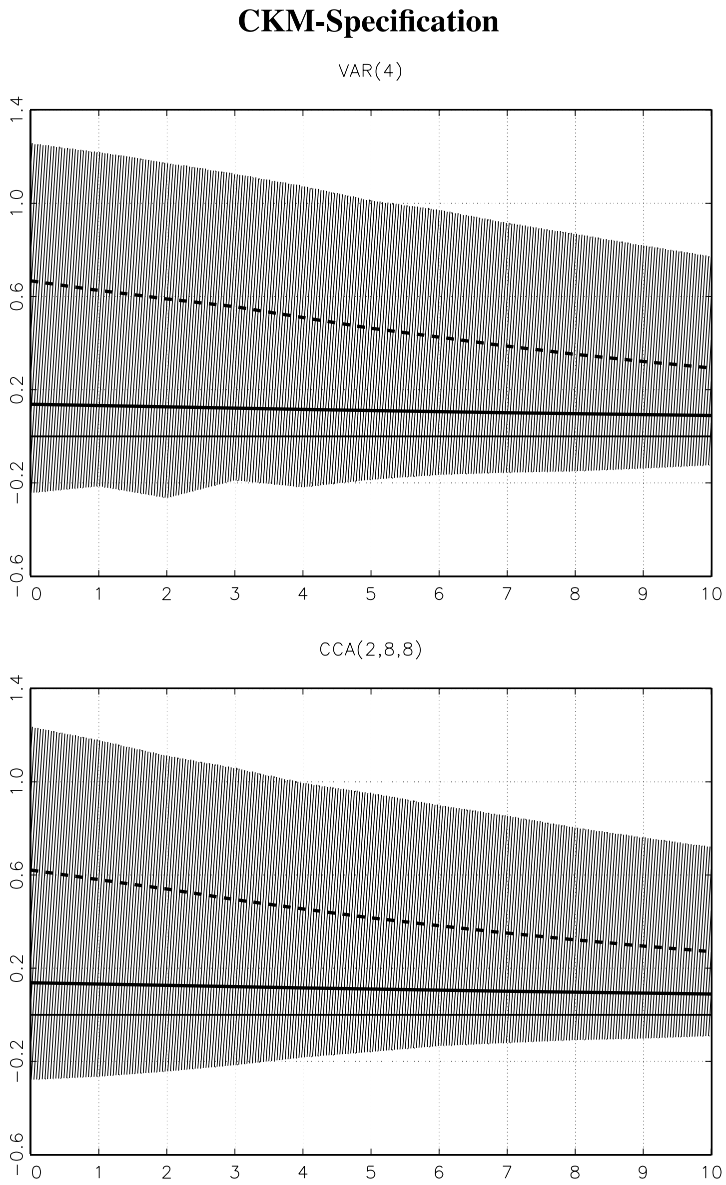

Figure 3.2: Mean impulse response (- -), true impulse response (–) and 95% intervals of hours worked to one standard deviation shock to technology for the VAR and the CCA subspace algorithm.