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Of Inefficient Firewalls and Efficient Monopolies

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Abstract

In this paper we study, within a formal model, market environments where information is costly to acquire and is of use also to potential competitors. Agents may then sell, or buy, reports over the information acquired and choose the trades in the market on the basis of what they learnt. Reports are unverifiable - cheap talk messages - hence the quality of the information transmitted depends on the conflicts of interest faced by the senders. We find that, in equilibrium, information is acquired when its costs are not too high and in that case it is also sold, though reports are typically noisy. Also, the market for information tends to be a monopoly, and there is inefficiency given by underinvestment in information acquisition. Regulatory interventions in the form of firewalls, limiting the access to the sale of information to agents uninterested in trading the underlying object, only make the inefficiency worse. Efficiency can be attained with a monopolist selling differentiated information, provided entry is blocked. The above findings hold when information has a prevalent horizontal differentiation component. When the vertical differentiation element is more important firewalls can in fact be beneficial.

JEL Classification: D83, C72, G14.

Keywords: Information sale, Cheap talk, Conflicts of interest, Information Acquisition, Firewalls, Market efficiency.

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1 Introduction

Motivation and Objectives It is common to observe potential competitors in a market exchanging information about issues pertaining to that market. To take an example from the labor market, managers often discuss the characteristics of employees in their sector. Analogous situations arise in the housing market, or in financial markets. This is somewhat surprising since the information supplied often has a rival nature. A striking example of this rivalry, this time taken from the private equity market, appears in the following quote from The Economist: "Buy-out firms complained that banks which were supposedly advising or lending to them sometimes snatched deals from under their noses. A notorious example was the battle for Warner Chilcott, a British drugmaker, in late 2004: while working with buy-out firms bidding for the company, Credit Suisse teamed up with JPMorgan Chase to launch a bid of its own."[1] This reveals a fundamental conflict of interest arising in information markets due to the rival nature of the information. The firm manager or the private equity banker mentioned above may prefer to be the first to use the discovery of an important event. As a consequence, as providers of information they may not be trusted to make truthful reports over the information they acquired: the banker can say that his research indicates the company on sale is in trouble, when in fact it is doing really well. There is, in fact, a serious concern by regulators about the objectivity and the conflicts of interest present in the financial sector.[2]

To further understand this problem notice that, in many situations, information may be quite costly to acquire. Getting to know whether a particular company is doing well (and thus worth bidding over) may require nontrivial effort for the private equity banker in charge of the research. These costs, together with the fact that information is of common interest, generate a clear incentive for setting up a market for information, where the agents who acquired information can provide reports over it, possibly in exchange for the payment of a price, to other agents.

At the same time, the mere existence of information transmission may seem surprising,

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[1] The Economist, October 12, 2006: "Banks and buy-outs: Follow the money".

[2] Not only regulators, of course. The Economist (July 27, 2007) entitles an article "Analyst for sale. Price: membership of a good club", reporting on an unpublished (even as a working paper) study by James Westphal and Michael Clement in which they survey thousands of analysts and executives. "Almost two-thirds of the analysts admitted to receiving favors from the firms they cover. And those favors appeared to sway their recommendations to their clients... If a company suffers subpar profits, doing a good turn for an analyst cuts the likelihood of a downgrade by half. In the wake of a big acquisition, which analysts tend to frown on, the likelihood falls by 65%."

given the rivalry in its use that we posit. Why would the banker part with the information about a company rather than make the maximal profit through purchasing it? As we will see, this may happen more easily if different individuals have different values for the same bit of information, or if some specific skills or features are needed to profit from a given news. The analyst could find out, through his research, valuable information about a technology firm about which his bank has no expertise, whereas the customer with whom he shares the information may be interested in precisely such kind of firms. In the language of industrial organization, we will see that information about a horizontal dimension, instead of a vertical one, is particularly amenable to profitable exchanges. The possibility of exchanging, or selling information to other traders may in turn affect the agents' incentive to acquire information.

In this paper we present a model which allows us to analyze information acquisition and transmission in an environment where the veracity of this information is neither verifiable nor contractible. Hence the quality of the information transmitted depends on the conflicts of interest faced by the information providers. We examine when information is acquired; if so, whether a market for information forms, and if it does how it is organized. That is, who and how many traders sell information, who and how many traders purchase it, hence how competitive is the market? And how truthful is the information transmitted?

We are also interested in studying the effects of the acquisition and transmission of information for the performance of the underlying market. We will then investigate the efficiency properties of equilibrium allocations and hence the possible benefits of alternative regulatory interventions. In particular, it is immediate to realize that the conflict of interest mentioned above is sensitive to the extent by which the information providers are also interested in trading in the underlying market. Thus, in the wake of financial scandals after the dot-com bust, one typical recommendation of regulators in various countries was to demand a separation between who provides information on a market from who trades in it ("firewalls").[3] It is then particularly of interest to analyse the welfare implications of such restrictions.

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[3] Section 501 of title V in the Sarbanes-Oxley act (significantly entitled "Analysts conflicts of interest") requires financial firms to establish specific safeguards to ensure the independence and separation of analysts from traders. The compensation of analysts should be independent from the performance in trading and information flows between the different arms are severely restricted. It also prescribes these firms to disclose conflicts of interest. Similar provisions appear in The Global Settlement reached on April 28, 2003 between the SEC, the NASD, the NY State Attorney General and NYSE and in the report of the European Commission Forum Group (2003).

The model and results We consider a model which, although admittedly stylized in some dimensions, allows us to capture what we believe are some key factors at play in the issues described above (information acquisition, its transmission via non verifiable reports and underlying market performance). We examine a market where a single, indivisible unit of an object is up for sale. The market is organized as a (second price) auction, where several potential buyers can participate. The good comes in different possible varieties, and each buyer only likes one randomly chosen variety. In addition to buyers, there is the seller, who initially owns the object and has no utility for it, and some other agents who are not interested in trading the object. The true variety is not known ex-ante by anybody, but can be ascertained, incurring a given cost, by any market participant. Besides the market for the commodity there is another market where information is traded: any agent who acquired information can set a price at which he sells a report over his information to other potential buyers. The information transmitted, as we said, is non verifiable, thus reports are pure "cheap talk" messages. As can be seen from the brief description above, the model displays a number of important simplifications. We show however in one of the final sections on robustness checks, that the main conclusions survive most natural extensions (concerning, e.g., the auction format, the specification of the agents' possible valuations, the timing of the game and so on).

We characterize an important class of equilibria of such game (where the degree of informativeness of the chap talk reports sent by agents is maximal). We find that, when information costs are not too high, information is acquired in equilibrium and in that case it is also sold. That is, the market for information is active. However, the information sold in that market can be noisy: when the provider of information is the seller of the commodity, he tends to hype the value of the good he declares for the agents who purchase information, while when he is a potential buyer, he tends to depress it.

Typically, only one trader acquires information in equilibrium, hence the market for information is a monopoly. Information is either sold at a positive but sufficiently low price such that all the uninformed buyers except one purchase it or, when the cost of acquisition of information is low, at a zero price so that all uninformed buyers purchase it. To understand this, notice that the seller of information may benefit even by transmitting information for free as this allows him to manipulate the behavior of uninformed traders in the auction and hence to increase the amount of surplus he can appropriate in the auction.

We also show that, if information is acquired at all, the commodity ends up in the hands of the agent who values it the most, that is the allocation is ex post efficient. But the

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level of investment in information acquisition is not efficient, in particular there is typically underinvestment. Interestingly, this inefficiency is present no matter what is the identity of the agent who acquires and sells information, i.e. not only when he is a potential buyer or the seller, but also when he is an agent not interested in trading the object. Actually, in the last case the inefficiency is even more severe. Hence restricting the possibility of selling information in the market only to agents not interested in trading upon it (as with the introduction of "firewalls"), while improving the truthfulness of the information transmitted, makes the overall market outcome worse. The reason is that when the seller of information is also interested in trading the object, he gets an additional benefit from the information, due to the possibility of trading directly on it. Hence the investment in information is higher. In this respect, it is interesting to note that Boni (2005) and Kolasinsky (2006) document significant decreases in the number of analysts following stocks after the Global Settlement[4] and the passage of Sarbanes-Oxley.[5]

An efficient outcome can be attained if the informed agent can sell different types of reports, of different quality, (or equivalently if we permit the resale of information). We show that in this way the information provider, when he is a potential buyer, can appropriate all the increase in social surplus generated by his information acquisition and dissemination. At the same time, in this case entry in the market for information is often profitable. Thus some regulatory intervention may still be needed to get efficiency, protecting monopoly situations in the market for information with barriers to entry, though regulators usually frown upon such practices.[6]

In most of the paper we consider the case we described where the uncertainty concerns the true "variety" of the object up for sale, over which buyers have different preferences. We can thus say, as mentioned above, that information only concerns a horizontal differentiation element. At the end of the paper we examine the case where an element of vertical differentiation is introduced: the commodity can now also be of high or low quality, and all buyers prefer, at least weakly, high to low quality. In that case, the degree of truthfulness of the reports transmitted deteriorates, both when the provider is a potential buyer or when he is the seller of the commodity. Hence, if quality is sufficiently important for buyers, firewalls

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[4]See footnote 3.

[5]There is not a clear consensus as to the reasons for this drop (see Parker 2005, Kolasinsky 2006, Zingales 2006). Since that discussion is mostly atheoretical, we believe that our model can shed some light on the mechanisms which underlie this empirical observation.

[6]It even landed a jail sentence to Henry Blodget, the noted Merril Lynch analyst who was issuing an "accumulate" recommendation for Excite at Home while in an internal e-mail he was writing "ATHM is such a piece of crap!"

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could be welfare improving.

The conclusion that emerges and is in fact robust to various elements of the model is that firewalls have both a negative effect on the incentives to invest in information acquisition and a positive one on the quality of the information transmitted. The second effect is significant and prevails over the first one only when the vertical differentiation component of the information is sufficiently important.

The paper is organized as follows. The environment is described in Section 2 while a characterization of the equilibria and their welfare properties when information providers are potential buyers is given in Section 3. The following section investigates how the properties of equilibria, in particular their efficiency, vary with other types of providers of information, establishing the adverse effect on welfare of firewalls. Section 5 presents some alternative ways in which the inefficiency problem can be solved. The robustness of our results and the extension to the case where information also regards a vertical differentiation element are then discussed in Section 6.

Literature This paper is related to different strands in the literature. More obviously, it is related to the seminal work of Crawford and Sobel (1982) on strategic information transmission. The primary focus of such work and the ensuing literature is the message game and the relationship between information transmission and alignment of the preferences of sender and receiver (or the 'conflict of interest' among them). To that literature, we add a richer game structure. The amount of information available and who 'owns' it are endogenously determined, as a result of the information acquisition decisions of every agent. We also allow messages to be transmitted for the payment of a price, thus formalizing a market for information. And we examine the consequences of the acquisition and transmission of information for the properties of the equilibria in the underlying market for the commodity. Finally, with regard to the message (sub)game, in our set-up the degree of coincidence of the objectives of sender and receivers is not common knowledge, as it depends on the realization of the true variety of the object and of the preferred variety of the seller of information, which is only privately known to him.

There is also a rather large empirical literature which studies the behavior of financial analysts, and in particular the presence of biases in their reports, and its effects for the performance of asset markets; e.g., Womack (1996), Michaely and Womack (1999), Barber et al. (2001), Agrawal and Chen (2006), Bradshaw, Richardson and Sloan (2003), Jegadeesh et al. (2004), and the recent survey by Mehran and Stulz (2007).

On the other hand there is much less theoretical work on markets for information. A good

part of the attention has received the case where the quality of the information transmitted is perfectly verifiable, thus abstracting from the problem posed by the possibility of untruthful reports. Admati and Pfleiderer (1986, 1990) look at a situation where market participants act as price takers, where the "paradox" arises that when information is too precise, asset prices are perfectly revealing, so that information is worthless. Therefore, providers need to add some noise in order to profit from information sales.[7] When traders are strategic, information transmission may also provide a strategic advantage, as pointed out by Vives (1990) in a general oligopoly framework, and Fishman and Hagerty (1995) in the case of financial markets.

The case where the information transmitted is non verifiable, as in our set-up, has been considered by Morgan and Stocken (2003), who study the information transmitted by an analyst when his incentives may not be aligned with those of investors, as he may be either a type that enjoys higher utility when the price of the underlying asset is high, or a type that enjoys telling the truth. Unlike in our set-up, such preferences are taken as primitives, there is no choice concerning the acquisition of information acquisition nor the price at which it is sold and the equilibrium in the market for the underlying asset is not considered. They find that the analyst always "hypes" the stock; see also Kartik, Ottaviani and Squintani (2007). This is in line with our results for the case in which the information provider is the seller of the object. Bolton, Freixas and Shapiro (2007) study how a cost for lying and competition can mitigate the tendency of financial intermediaries to sell to their customers products that are not appropriate for their tastes. They share with our work the feature that the information transmitted has a horizontal dimension, but they focus on the incentives for truth-telling by sellers who may sometimes carry all existing varieties and some other times only one.

Our analysis, being cast in a static framework, abstracts from reputational concerns. These may arise in a dynamic framework, where providers of information and traders repeatedly interact, and may mitigate the tendency of providers to send untruthful reports which may damage their future reputation, as shown by Benabou and Laroque (1992), and Ottaviani and Sorensen (2006).

Allen (1990) focuses on a different problem affecting information transmission in markets: traders do not know whether advisors are actually informed or not. He considers the case where advisors have no reason to lie if informed[8], thus the only reason not to fully trust

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[7]A similar point is made in Milgrom (1981) who explores how to inform others, if providing too good information can hurt the provider.

[8]This is because advisors provide information only to few people, whose small volume of trading does not

their reports is their possible lack of information. It is shown that advisors can give credibility to the fact that they are informed by investing in more risky portfolios when really informed. Allen's approach is complementary to ours. Unlike his, our advisors are known to be informed, but (again unlike his) they might be biased in their reports because they compete with advisees when they use their information in choosing their trades.

2   Model

There is one object for sale, initially owned by an agent, indicated as the seller of the object, who has no utility for it. The type of the object is uncertain: there are $K \geq 2$ possible varieties, all with the same ex-ante probability. Let the true type of the object be $v \in \mathcal{K} = \{1, 2, ..., K\}$. There are then $N \geq 3$ potential buyers, agents who may be interested in purchasing the object. We denote buyer $i \in \mathcal{N} = \{1, ..., N\}$ by B_i; such buyer only cares (has positive utility) for one particular variety in $\mathcal{K}$ indicated as θ_i. The variables $\{\theta_i\}_{i \in \mathcal{N}}$ and v are all i.i.d. over $\mathcal{K}$, thus for all $i, j \in \mathcal{N}$, θ_i is uncorrelated with θ_j and v; all elements of $\mathcal{K}$ have then the same probability, $1/K$. The object is allocated to buyers via a second price auction.

Information structure. The realization of θ_i is private information of individual i. On the other hand, the type of the object for sale is not known to any trader. Before the auction takes place, anybody can acquire, by paying a cost c, a signal over the type of the object, which we assume is perfectly informative. If a trader acquires such signal he can in turn 'sell information' to other traders.

The utility of buyer B_i can then be written as

$$\pi_{B_i} = I_v - cI_e - t_{B_i},$$

where t_{B_i} is the sum of the net monetary payments made by B_i to the seller to gain possession of the object and/or to the other traders to purchase or sell information to them. I_v is an indicator variable that takes the value 1 if B_i gains the object and $v = \theta_i$ (i.e., the object is of the type B_i likes) and 0 otherwise. Finally I_e is another indicator that takes the value 1 if B_i decides to acquire the signal over the type of the object, and 0 otherwise.

In this paper we are interested in situations where the information sold is not verifiable, i.e. the seller of information sends a report, which is pure 'cheap talk', over the signal he received. Since we abstract from reputational concerns (there is a single period), information

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affect prices, so that information does not have a rival nature.

is transmitted by the seller only when he cannot profit by distorting his report. This in turn requires us to examine the consequences of transmitting false or truthful information for the behavior of buyers in the market for the good. In the environment described, different buyers might, but also might not, be interested in the same type of object, the uncertainty only concerns a horizontal differentiation component of the object. Hence the information transmitted, while being rival, has also an element of non-rivalry. One could interpret the specification of the model as capturing situations where the agent who sells information is not always able to profit directly from the information acquired. For example, leveraging the information may require the possession of complementary assets or skills, which he may lack. In Section 6 we extend the model to allow also for the presence of a vertical differentiation component (e.g. quality, on which most if not all traders agree).

We examine first the case where information can be acquired and transmitted by potential buyers of the object. The possibility that information is sold by other traders, as the seller of the object and/or agents not interested in trading the object, is considered later, in section 4. Furthermore, we assume that each seller of information sells a single, identical report to all buyers, at the same price; we refer to this situation as no differentiation of the quality of the information sold. In section 5.1 we discuss the case where different kinds of reports may be sold by the same trader, at different prices.

Timing of the game.

1. First, each potential buyer decides whether or not to acquire the signal over the type of the object. The cost of the signal is c. The decision to acquire information, but obviously not the information itself, is commonly observable by all agents.[9]

2. Any potential buyer who has chosen to acquire information, before learning the realization of the (perfectly informative) signal over the type of the object, can post a price p at which he is willing to sell a report over the signal, which will be sent after receiving it.[10]

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[9]See the work by Allen (1990) mentioned in the introduction for an analysis of the case where the informational status of the providers of information is not known by the other traders, though in a set-up where they have no other reason to lie.

[10]Thus the price posted has no signaling content. In section 6 we discuss the effects of allowing the price to be set only after learning the realization of the signal, as well as alternative auction formats, and other robustness checks.

3. Each of the buyers who did not choose to acquire information in stage 1. decides whether or not to purchase information from any of the traders selling information (possibly from more than one seller). Each of these buyers has then a final chance, after the market for information closes, to acquire the signal at a cost of c.[11]

4. All agents who paid the cost c of information acquisition learn the realization of the signal. They then issue a report to the buyers who purchased information from them.

5. A second price auction takes place among all the buyers for allocating the object.

Message subgame. We consider the case where the set of messages available to a seller of information is the set of direct messages plus one other message, denoted message zero, where no type is announced. We will also refer to this last message as the *empty* message. Thus, the set of messages is:

$$\mathcal{M} = \{0, 1, 2, ..., K\},$$

i.e. it coincides with the set $\mathcal{K}$ of possible types of the object, plus the *empty* message 0. The report sent by the seller is then given by an element of $\mathcal{M}$.

The structure of the game, as well as the preferences, have been simplified to make the analysis as transparent as possible, given the inherent complication of the phenomena we study. The main conclusions, however, are robust to natural extensions. For example, we have assumed that the price for information is posted before learning the type of the object. In this way, the price posted has no signaling content. In section 6 we show that the equilibrium in which we focus still exists under the alternative timing where prices are posted after learning the type of the object. The main novelty would be to create alternative, less efficient (and arguably less natural) equilibria. The lower efficiency of those equilibria would in fact strengthen our main point. In section 6 we also discuss the consequences of allowing the buyers of information, rather than the seller, to post a price; the impact of alternative auction formats; of having more than one unit for sale or of allowing more than one nonzero valuation for the object. The main qualitative conclusions obtained in the benchmark model are still valid under all those variations.

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[11]This last opportunity of direct information acquisition, with no opportunity to resell it, limits the ability of the sellers of information to corner the market and extract surplus from buyers. Evidently, without it, the efficiency properties would be worsened. The timing considered reflects the fact that acquiring information entails a simpler technology and takes less time than organizing a market for selling reports over it.

3 Equilibrium and welfare

3.1 Equilibrium

Since information transmission needs not be truthful, the game (and in particular the message subgame, starting from stage 4 of the game) has many equilibria, as is common in other kinds of "cheap talk" games (see Crawford and Sobel 1982). We restrict our attention to equilibria where any agent, whenever he is indifferent between lying and telling the truth, tells the truth. We think this is a natural way to select equilibria in the message subgame, which can be formalized by assuming that players experience a small cost of lying (as in Kartik 2008), either from an intrinsic small disutility, or because they may be caught and penalized with a small probability.

We also focus on pure-strategy equilibria in the information acquisition decision. It is easy to see that there are equilibria where all potential buyers acquire information with positive probability. Those would lead to inefficient information acquisition with positive probability (sometimes too little and sometimes too much), and our inefficiency results would then be strengthened.[12] Besides their inefficiency, the mixed-strategy equilibria look less plausible, and both the experimental and field evidence for entry games, which share a similar strategic structure, tend to favor pure strategy equilibria.[13]

We will show that an equilibrium always exists where the seller of information adopts the following reporting strategy (both in and out of equilibrium):

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m_i = \begin{cases} v, & \text{if } v \neq \theta_i \\ 0, & \text{if } v = \theta_i \end{cases} \tag{1}
$$

where B_i denotes the buyer selling information, m_i is the report issued by him. Therefore, trader B_i tells the truth about the type of the object when the true variety of the object does not coincide with his own type (i.e. with the variety he likes). On the other hand, when the two coincide B_i faces a conflict of interest as he wishes to get the good at the lowest possible price, and this price generally depends on the report sent by him. Thus, he will send the *empty* message, 0. One could interpret this message as issuing no report to buyers of information.

It should be clear, also from the following analysis, that the reporting strategy described

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[12]Most analyses of efficiency in entry tend to focus on pure strategy equilibria for this reason. See e.g. Mankiw and Whinston (1986) or Vives (2001, p.107).

[13]See, e.g. Erev and Rapoport (1998) and references therein for the experimental evidence, and Berry (1992) or Ericson and Pakes (1995) for the field evidence.

in (1) entails the maximal degree of information transmission at an equilibrium. Since there is no cost for not announcing the true type of the object, the seller is only willing to tell the truth when he cannot gain a strictly higher payoff in the market (i.e. in the auction) by avoiding to give that information. This happens when he is not interested in the object. If the seller's reports are informative and hence affect buyers' beliefs and bids, we should expect a buyer, upon receipt of a report saying that he likes the object, to raise the belief that he indeed likes the object and hence his bid, and decrease them otherwise. Hence when the seller is interested in getting the object he wants to deceive buyers and send the message which induce buyers to make the lowest bid, which he can achieve by sending the *empty* message, that is a completely uninformative report.

Another, more limited, source of multiplicity of equilibria comes from traders' behavior in the auction, in the final stage of the game. In the set-up under consideration in this section the auction subgame is a standard second price auction with private values, where the only weakly undominated strategy for each trader consists in making a bid equal to the trader's expected value of the object.

We will characterize the perfect bayesian equilibria of the game described in the previous section and evaluate their welfare properties for different parameter configurations (in particular, for different levels of the cost of information acquisition, c). Given the selection of equilibria in the message and auction subgames specified above (quite natural, we would like to argue, given our purposes), we will show that the overall equilibrium is, for almost all parameter values, unique:

THEOREM **1** *For all $c \geq 0$ there exists a perfect bayesian equilibrium of the game with no differentiation of the quality of information sold where sellers of information adopt the reporting strategy in (1) and participants in the auction adopt a weakly undominated bidding strategy. Furthermore, when $K \geq N - 2$:*

1. *If $c \geq c^I \equiv \frac{1}{K}\left(\frac{K-1}{K}\right) + (N-2)\frac{1}{K}\left(\frac{K-1}{K}\right)^{N-1}$, no buyer chooses to acquire information; the object is then gained by a randomly chosen buyer, at a price $1/K$.*

2. *If $c < c^I$, one buyer acquires information and sells a report over it at a price $p = \min\left\{\frac{1}{K}\left(\frac{K-1}{K}\right)^{N-1}, c\right\}$, at which all the other buyers except one purchase information; the object is then always gained by a buyer who likes it, if such a buyer exists, at a price equal to $1/K$ (when either the seller of information, or only one buyer of information likes the object), 0 (when neither the seller nor any buyer of information likes the object) and 1 otherwise.*
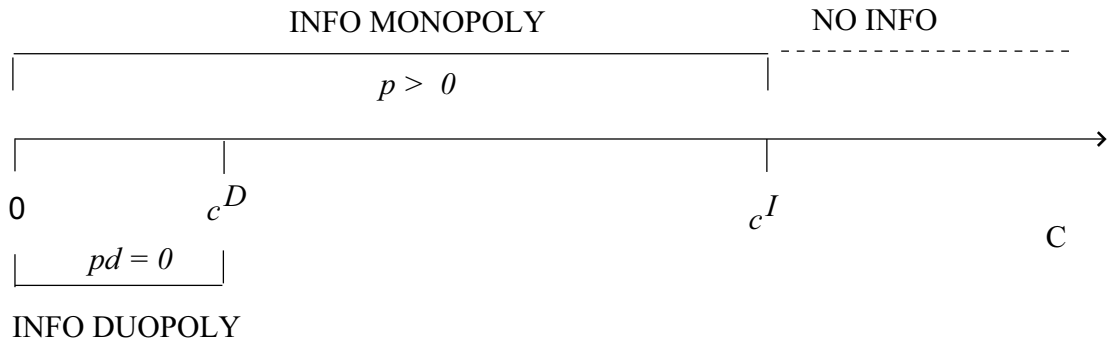
Figure 1: Equilibrium with homogeneous messages

When $K < N - 2$ the equilibrium has the same properties, with two main exceptions: the threshold for information to be acquired is different, $c^{I'} \equiv \frac{1}{K}\left(\frac{K-1}{K}\right) + \left(\frac{K-1}{K}\right)^{K-1}$, and the monopolist seller of information chooses to set a price at a higher level so that more than one buyer chooses to remain uninformed. Hence when neither the seller nor any buyer of information like the object, this is gained by one, randomly chosen, uninformed buyer, at a price $1/K$.

This is the unique equilibrium outcome with reporting strategies exhibiting maximal degree of truthfulness and weakly undominated bidding strategies for all possible values of c, with the only exception of a subset of region 2., given by $c^D \equiv \frac{1}{K}\left(\frac{K-1}{K}\right)^{N-1} \geq c$, where another equilibrium exists, with two buyers acquiring information and each of them selling a report over it to all other buyers, at a price $p = 0$; the object is then always gained by a buyer who likes it, if such a buyer exists, at a price of 0 if nobody else likes it, and 1 otherwise.

Thus when information costs are low enough, information is acquired in equilibrium. Whenever it is acquired, information is transmitted via a report that in some events is informative while in others is not. Information is sold for a low enough price so that all buyers except one purchase it (when $K \geq N - 2$). The market for information is typically a monopoly. Furthermore, the seller of information always gets the object when he likes it; when he does not like it, the object goes to one of the buyers of information who likes it, if such buyer exists and otherwise, in case 2. goes to the buyer not purchasing information. Figure 1 summarizes the result.

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3.2 Proof of Theorem 1

We present here the main steps of the proof of the Theorem while leaving some details in the Appendix. First, the consistent beliefs of buyers associated with the sellers' reporting strategy in equation (1) are determined. Then we study the traders' strategies in each stage of the game, and establish their optimality given the beliefs.

Beliefs With the message structure in (1) there are no out-of-equilibrium messages. Thus, we can find the beliefs for an uninformed buyer, say buyer B_j, who receives a report from an informed buyer, say buyer B_i, using Bayes' rule in all cases:

- When buyer B_j receives from B_i a message $m_i = \theta_j$ he knows for sure that he likes the object (the message is truthful). That is, $\Pr(v = \theta_j | m_i = \theta_j) = 1$.

- When buyer B_j receives a message announcing a type different from his type, B_j again can say for sure that the message is truthful, and hence that he does not like the object, so that $\Pr(v = \theta_j | m_i \neq \theta_j, m_i \neq 0) = 0$.

- When buyer B_j receives the *empty* message, 0, he knows the seller of information likes the object, but nothing more. Since types are uncorrelated, beliefs are equal to prior beliefs and $\Pr(v = \theta_j | m_i = 0) = 1/K$.

Finally, the buyers who neither acquired information directly, nor indirectly by purchasing it in the market, have beliefs equal to their prior beliefs. That is, $\Pr(v = \theta_j) = 1/K$. The beliefs of a buyer who is purchasing two (or more) distinct reports from two (or more) informed buyers are similar.

Stage 5: Behavior in the auction

Given the beliefs of buyers who purchased one report described above, in the auction a subset of bidders is fully informed about the true value of the object while the rest has no information. Since traders' valuation are independent and privately known, by a standard argument it can be shown that any participant has a unique weakly undominated strategy, consisting in making a bid equals his expected value of the object (thus 1 or 0 if he is fully informed and $1/K$ if he is uninformed).

Stage 4: Behavior in the message subgame

We show first that the reporting strategy we postulated for a seller of information is indeed optimal for such trader. A key element in the argument is that, by changing the message strategy, the seller of information cannot affect the outcome of the auction in his favor.

Seller: There are two possible deviations which need to be considered for the seller of information. When he likes the object, he may deviate and announce a type from 1 to K. If he does that, with positive probability the price he must pay to gain the object will increase, since with positive probability the announced type will coincide with the type of some buyer of information who then will bid 1. Moreover, if there is at least one buyer not purchasing information, the price he pays cannot decrease, as the uninformed buyers always bid $1/K$. If all other buyers purchase information the price paid may decrease to zero (when the announced type is different from the type of all buyers of information), but it is easy to verify that the seller still ends up paying more, in expectation, for the object than if he had followed the equilibrium message strategy[14], so he never wants to make such deviation.

Second, when the seller does not like the object, he may deviate by announcing a type different from the true one. But that only changes the outcome in the auction, which has no effect on the seller's utility in this case since he is not interested in the object. So the seller does not gain with such a deviation either.

Stage 3: Purchase of information.

In this stage each uninformed buyer has to choose whether to purchase information from one - or more - of the informed buyers who are selling information, at the price posted by them, or alternatively to acquire the information directly (at the cost c), or do nothing of the two.

Stage 2: Sale of information.

This is the key stage of the game, where the market for information opens and each trader who at stage 1 has chosen to acquire information posts a price at which he is willing to sell a report over it to any other buyer. The price is set at the level which maximizes the

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[14]The probability of the price paid being 0 is in fact $[(K-1)/K]^{N-1}$, hence the expected value of the price paid is $1 \cdot \left\{ 1 - [(K-1)/K]^{N-1} \right\}$ clearly higher than $1/K$.

utility of the seller of information, i.e. his expected payoff in the auction plus the revenue from the sale of information, taking as given the strategies of the other sellers, if any, and the response strategies of buyers in the next stage to the prices posted. The main elements of the analysis are summarized below.

The **maximal willingness to pay for information** of an uninformed buyer is given by the amount by which the buyer's payoff in the auction increases if he purchases information and hence becomes indirectly informed, relative to the best of his two alternatives: acquire information directly at the cost c, or remain uninformed. The expected payoff in the auction of the buyer of information is in turn determined by the probability that he gains the object with a positive surplus, which occurs when he likes it and no other trader who is directly or indirectly informed likes it, and the price at which the object is gained. Letting J be the total number of buyers who did not acquire information, either directly or indirectly, the probability of this event is $1/K \left[(K-1)/K \right]^{N-J-1}$, and is clearly higher the lower is the number of agents who are purchasing information. The price paid to win the object in the auction in this event, given the bidding strategies described in the previous steps, is 0 if $J = 0$, i.e. if no buyer remains uninformed, and $1/K$ otherwise.

If an uninformed buyer were not to purchase information but to acquire it directly in the next stage of the game, his payoff in the auction would be exactly the same, minus the cost c. This implies that, for the sale of information to take place, its price p cannot be higher than c.[15] On the other hand, if the buyer remains uninformed his payoff is zero if there are other uninformed buyers, while if he is the only uninformed trader it is positive and equal to $1/K \left[(K-1)/K \right]^{N-1}$. By comparing this expression with the ones obtained above and in the previous paragraph, it is immediate to see that a buyer is only willing to pay a positive price for information when not all the other buyers are either directly or indirectly informed ($J \geqslant 1$). In this case (i.e. when $J \geqslant 1$), the maximal price he is willing to pay is:

$$
\min \left\{ c, \frac{1}{K} \left(\frac{K-1}{K} \right)^{N-J} \right\},
$$

which is strictly decreasing in $N - J$. Thus the demand for information is strictly decreasing in its price p, for $p < c$.

When there is a **monopolist seller of information**, he always sets the price so as to extract all the surplus from the number of buyers he wishes to attract, i.e. equal to their maximal willingness to pay. The issue is then to determine the number of buyers of information which maximizes the seller's payoff. In the Appendix, we show the following:

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[15]Also, if $p < c$ no buyer chooses to acquire information directly in stage 3 of the game.

CLAIM 1 *When $K \geq N - 2$, the **revenue from the sale of information** for a monopolist seller of information is maximized by setting the price p low enough that all uninformed buyers, except one, purchase information, i.e. such that $J = 1$. When $K < N - 2$, the **revenue from the sale of information** for a monopolist seller of information is maximized by setting the price p so that the number of agents who do not buy information is larger than 1 ($J = N - K - 1$ or lower, when c is low).*

To better understand this result, note that the price elasticity of the market demand for information depends on the magnitude of K relative to N. When K is large relative to N, the competition among buyers is not very intense so that market demand is rather inelastic to the price and the seller's revenue is then maximized by selling to the maximum number of buyers willing to pay a positive price (that is, $J = 1$). On the other hand, when $K < N - 2$, competition among buyers is more intense, market demand is more elastic and revenue is maximized by selling only to part of the buyers.

The second component in the payoff of the seller of information is given by his **payoff in the auction.** Given the reporting and bidding strategies described above, the sale of information has no influence on the fact that a monopolist seller always gains the object whenever he likes it. Hence its only possible consequence is on the price at which the seller gains the object in the auction, via its effect on buyers' bids. But the price at which he purchases the object when he likes it is $1/K$ whatever is J. This is so since the buyers who purchase information from him are given the *empty* message and bid $1/K$, like the uninformed buyers. Hence the optimal choice of the monopolist seller of information is to set the price for information at the level which maximizes the revenue from the sale of information found above.

With an **information oligopoly,** the equilibrium price of information is always zero. Note first that, when there are two or more sellers of information, the additional benefit for an uninformed buyer of purchasing a second report is always zero. This follows from the fact that purchasing information from one seller allows to gain a positive surplus in the auction only when the buyer likes the object and nobody else who is informed, either directly or indirectly, likes the object. Purchasing information also from a second seller allows the buyer to have more precise information in the event in which one of the two sellers of information likes the object (since the other tells the truth); however in such event no positive surplus can be gained since the seller who likes the object bids one.

Furthermore, the benefit for a buyer of purchasing one report is essentially the same as when there is a monopolist seller; in particular, it is positive only if not all the other buyers

purchase information, i.e. buy at least one report. Given that each buyer is willing to pay a positive price only for one signal, and only if not all other buyers purchase information, the only possible equilibrium with positive prices would entail a split of the buyers between the different providers of information, with at least one buyer not purchasing information. But then each of the sellers would have an incentive to undercut. By lowering his price the seller would retain all those already buying from him and manage to steal the buyers from the other sellers of information. This produces a discrete jump not only in his revenue from the sale of information but also in his payoff in the auction; the latter is in fact positive (and equal to $1 - 1/K$ if at least one trader is not purchasing information) when the seller likes the object and neither the other sellers of information, nor any other buyer that is purchasing information from the *other* sellers, likes the object. Hence the probability that a seller has a positive surplus increases with the number of buyers who purchase information only from him. Since such incentive to undercut persists as long as the posted prices for information are positive, the only possible equilibrium obtains when all sellers post a zero price for information and all uninformed buyers purchase information from every seller. In this situation, the sellers of information have the same payoff as the buyers of information, less the cost of acquiring information, thus their overall payoff is lower.

Stage 1: Information acquisition

Having determined the benefits for a buyer of acquiring information, we immediately find when this is profitable:

CLAIM **2** *Let $K \geq N - 2$. When the cost of acquiring information is so high ($c \geq c^I$) that it exceeds the maximal gains that a monopolist seller of information can get from the sale of information ($[(N-2)/K][(K-1)/K]^{N-1}$) plus the gains from obtaining the object in the auction ($(1/K)[(K-1)/K]$), no buyer chooses to acquire information. On the other hand, when $c \leq c^I$ one buyer always acquires information. If $K < N - 2$ the same result holds, with the threshold given by $c^{I'}$ instead of c^I.*

As argued above in the discussion of Step 2, when a buyer acquires information he always chooses to sell a report over it. Furthermore, the entry in the market for the sale of information by a second buyer is never strictly profitable, as the payoff of an informed duopolist is less or equal than the payoff that a trader would get if he did not acquire information directly but rather purchase it from the other informed buyer. We show in the Appendix:

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CLAIM 3 *For a range of intermediate values of c, $c^D \geq c$,[16] the payoff is exactly equal for an informed duopolist and a buyer of information with a monopolist seller of information. In this case there are two equilibria, one with a monopolist seller of information and the other with two sellers of information. Outside this range there is a unique equilibrium with a monopolist seller.*

3.3 Welfare

We now discuss the welfare properties of the equilibria described in the previous section. In particular we are interested in comparing the equilibria to the Pareto efficient allocations, i.e. to the allocations which could be attained by a planner who (knows the buyers' types but) is also uninformed about the type of the object and may acquire information, at the same cost c, over it. Given the assumed transferable property of traders' utilities, welfare can be simply evaluated by considering the total surplus, or the sum of the payoffs of all buyers and the seller of the object.

Notice first that, if information is acquired by some buyer, when $K \geq N - 2$ the resulting equilibrium allocation is always ex post efficient, in the sense that the object always goes to a buyer who likes it the most. This is no longer true when the competition among buyers is more intense ($K < N - 2$) as the object with positive probability ends up in the hands of one randomly picked uninformed buyer. Since our aim in the rest of this section and in the following, Section 4, is to identify the conditions under which efficiency holds we will thus focus on the first case, $K \geq N - 2$. There, the only possible source of inefficiency may lie in the information acquisition decision: is that also efficient at equilibrium, or rather is there overinvestment, or underinvestment in information? Evidently, the equilibrium with two buyers both acquiring information is always inefficient as the duplication of the investment in information acquisition is always wasteful. On the other hand, at an equilibrium where only one buyer acquires information there is no wasteful duplication. Such equilibrium was shown to exist for all $c \leq c^I$, while for $c > c^I$ no information is acquired.

To assess the efficiency of such equilibrium we need then to find the threshold for information to be acquired at an efficient allocation and compare it to c^I. If information is acquired, the object can always be allocated to a buyer who likes it, when such buyer exists. In that event the total surplus of traders from the object equals one, while it is zero otherwise. Total

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[16]For c in this interval, a monopolist seller sets the price of information at $p = c$.

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welfare is then obtained by subtracting the cost of information:

$$W_1 = P(\exists i | v = \theta_i) - c = 1 - \left(\frac{K-1}{K}\right)^N - c.$$

On the other hand, if information is not acquired the total surplus is one only if the agent who receives the object (and, with no information, this agent can only be randomly chosen) happens to like it. Thus total welfare is in that case:

$$W_0 = \frac{1}{K}.$$

By comparing W_0 and W_1 we find that it is socially efficient for information acquisition to take place if, and only if, $1 - ((K-1)/K)^N - c \geq 1/K$, or:

$$\left(\frac{K-1}{K}\right)\left(1 - \left(\frac{K-1}{K}\right)^{N-1}\right) \geq c. \tag{2}$$

We show in what follows that this threshold is lower than c^I:

PROPOSITION 1 *In equilibrium there is a less than efficient level of investment in informa-tion.*[17] *In particular, for values of c lying in the following, non empty interval:*

$$c^I < c \leq \left(\frac{K-1}{K}\right)\left(1 - \left(\frac{K-1}{K}\right)^{N-1}\right) \tag{3}$$

no information is acquired in equilibrium, though it would be socially efficient to acquire it.[18]

Thus there is a range of values of c for which acquiring information is efficient but in equilibrium the gains from information acquisition are too low so that nobody chooses to become informed. To understand the reasons for this result, it is useful to examine first the distribution of the welfare gains and losses across agents when we compare the situation where no information is acquired to the equilibrium with a monopolist seller of information, in particular when c is below but close to its threshold value c^I.

Who gains and who loses from information acquisition   When $c \leq c^I$, in equilibrium there is one buyer, say B_1, who acquires information directly and then sells it, as a monopo-list, and another buyer, say B_N, who remains uninformed. B_1 clearly gains, with respect to

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[17]It is immediate to see that a similar underinvestment result also holds when $K < N - 2$. Unlike the ex post (allocational) efficiency, the underinvestment is robust to various extensions of the model.

[18]The proof of this and the following propositions are in the Appendix.

the situation where no information is acquired, as his payoff goes from 0 to a strictly positive level (except when $c = c^I$); so does B_N, whose payoff[19]

$$\pi_{B_N} = \frac{1}{K} \left[\frac{K-1}{K} \right]^{N-1} \tag{4}$$

is strictly positive. On the other hand, the payoff of the remaining buyers, who acquire information indirectly by purchasing a report in the market, is unchanged at zero when c is smaller than c^I.

What about the seller of the object? His payoff, in the region under consideration is given by

$$\left[\left(\frac{K-1}{K} \right) \left(1 - \left(\frac{K-1}{K} \right)^{N-2} - (N-2) \frac{1}{K} \left(\frac{K-1}{K} \right)^{N-3} \right) \right] + \frac{1}{K} \left[\frac{1}{K} + (N-2) \frac{1}{K} \left(\frac{K-1}{K} \right)^{N-2} \right]$$

As claimed in 2. of Theorem 1, the price at which the object is gained in the auction can be 1, $1/K$ or 0. The terms in square brackets above are then the probabilities of the auction price being, respectively, 1 and $1/K$. The difference in the revenue of the seller of the object between this case and the one where nobody is informed (where the price in the auction is always $1/K$) is then:

$$\Delta\pi_S = \left(1 - \frac{1}{K} \right) \left[\left(\frac{K-1}{K} \right) \left(1 - \left(\frac{K-1}{K} \right)^{N-2} - (N-2) \frac{1}{K} \left(\frac{K-1}{K} \right)^{N-3} \right) \right] - \frac{1}{K} \left(\frac{K-1}{K} \right)^{N-1}, \tag{5}$$

which is positive if, and only if:

$$1 > \left(\frac{K-1}{K} \right)^{N-3} \left(\frac{K+N-2}{K} \right) \tag{6}$$

As we show in section 4.2 below, this inequality is satisfied for some, but not all admissible values of K and N.

The source of the inefficiency From the ex post efficiency of the equilibrium allocations with a monopolist seller of information it follows that the sum of the changes in the payoff of all traders between the equilibrium with and without information acquisition equals the difference between the levels of maximal total welfare in these two situations, $W_1 - W_0$. The analysis of the distribution of the welfare changes across agents in the previous section allows us so to gain some further understanding of the source of the inefficiency result we obtained. Since, as we said, the payoff of the indirectly informed buyers is zero in both

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[19]See equation (16) in the Appendix.

situations (when c is smaller but close to c^I), the change in total welfare $W_1 - W_0$ equals the change in the payoff of the seller of the object $\Delta \pi_S$ plus the payoff of the buyer who acquires and sells information and the payoff of the buyer who remains uninformed:

$$W_1 - W_0 = \Delta \pi_S + \pi_{B_1} + \pi_{B_N}. \tag{7}$$

Underinvestment in information obtains if $\pi_{B_N} + \Delta \pi_S > 0$, i.e. if the trader acquiring information is unable to recoup all the gains in social surplus generated by his decision. From (4) and (5) we get:

$$\pi_{B_N} + \Delta \pi_S = \left(1 - \frac{1}{K}\right) \left[\left(\frac{K-1}{K}\right) \left(1 - \left(\frac{K-1}{K}\right)^{N-2} - (N-2)\frac{1}{K}\left(\frac{K-1}{K}\right)^{N-3}\right) \right] > 0, \tag{8}$$

which is strictly positive if and only if the interval of values of c defined by (3) is non empty, always true by Proposition 1. It is also useful to notice that the expression in (8) is equal to the first term of (5), describing the gains which accrue to the seller when at least two indirectly informed buyers happen to like the object, so their bids raise to 1 the price at which the object is won in the auction. We refer so to such term as *rent dissipation* by indirectly informed buyers, since these are rents generated by the information acquisition that the buyer who makes the investment in information will not appropriate, and go instead to the seller of the good.

As argued in the previous section, the term π_{B_N} is strictly positive. Thus the uninformed buyer appropriates some informational rents, by successfully free riding on the information acquisition of all the other buyers, which allows him to get the object at a zero price when nobody else likes it. We indicate then this term as *free riding*. Note that it is exactly equal to the second, negative, term in expression (5) for $\Delta \pi_S$, which reveals that the free riding happens entirely at the expense of the seller of the object and entails so a pure transfer of surplus from the seller to B_N, and hence does not undermine the incentives for efficient information acquisition. What does undermine such incentives, and shows in equation (8), is thus only the rent dissipation.

4 Who should sell information?

Does the inefficiency we found depend on the fact that information is sold by a trader who is also interested in purchasing the object? We examine here the efficiency properties of equilibria when other types of traders can be the providers of information.

4.1 Disinterested traders

As we said in the introduction, a common proposal for solving inefficiencies in information transmission in markets is the separation between information providers and traders. We model this by introducing a new type of agents, who do not own the object nor have any utility for it and hence have no interest in participating in the market where the object is traded. To keep the comparison clean, we assume that also this type of disinterested traders has to pay a cost c to acquire the information.

The reporting strategy of a disinterested trader is clearly different from that of an interested trader since the first one never has an interest in lying over the type of the object. The optimal reporting strategy with maximal degree of truthfulness for the disinterested trader is then to always tell the truth. Hence the quality of the information transmitted is clearly higher and so information can be sold at a higher price. Does this imply the equilibrium with a disinterested trader as provider of information has better efficiency properties? We will show that the answer to such question is negative, the efficiency properties are actually worse in this case, as the incentives for information acquisition are weaker.

PROPOSITION **2** *When information can be sold only by disinterested traders, in equilibrium there is again a less than efficient level of investment in information. Furthermore, the interval of values of c for which information is not acquired in equilibrium though it is socially efficient to acquire it is*[20]

$$(N-1)\frac{1}{K}\left(\frac{K-1}{K}\right)^{N-1} < c < \left(\frac{K-1}{K}\right)\left(1 - \left(\frac{K-1}{K}\right)^{N-1}\right),$$

which is larger than the one found in Proposition 1 when information is sold by potential buyers.

The intuition for this result is not too hard to get. Notice first that the payoff of a disinterested trader is only given by his revenue from the sale of information, as he never gets any payoff in the auction. Hence information is acquired in equilibrium if the revenue from the sale of information alone exceeds the cost c. As a consequence, there can be an equilibrium where information is sold by a disinterested trader only if he is the monopolist provider of information and information is sold at a strictly positive price (with two or more sellers of information, by the argument given in the previous section, the price of information is always zero).

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[20]For the reasons explained in Section 3.3, in stating the results in this section we still focus on the case $K \geq N - 2$.

For the incentives to acquire information to be stronger in the present situation, the higher revenue from the sale of better quality information should more than compensate the lack of any payoff in the auction. The disinterested trader has one additional customer than a potential buyer as he can sell information at a positive price to $N-1$ rather than $N-2$ buyers. Since, as shown in the proof, the price at which information is sold is the same, this means an extra gain from the sale of information equal to $1/K \left[(K-1)/K\right]^{N-1}$. On the other hand, a disinterested trader does not gain any surplus in the auction, so he loses, with respect to a potential buyer, the surplus this one gets in the auction, which is $(K-1)/(K)^2$. Clearly, the loss is larger than the gain, which explains the greater region of inefficiency for the disinterested trader.

4.2   The seller of the good

We examine next whether the owner of the good as a provider of information could allow to overcome the inefficiency problem we found. If the seller has no information over the buyers' types, he is willing to report truthfully the type of the object, like the uninterested trader; hence the information sold is of the highest quality.

PROPOSITION **3** *When only the owner of the good can be seller of information, in equilibrium there is a less than efficient level of investment in information. In particular, for values of c lying in the following, non empty interval:*

$$\left(\frac{K-1}{K}\right)\left(1-\left(\frac{K-1}{K}\right)^{N-2}\right) \leq c \leq \left(\frac{K-1}{K}\right)\left(1-\left(\frac{K-1}{K}\right)^{N-1}\right) \qquad (9)$$

information acquisition is socially efficient, but does not take place in equilibrium.

This result can also be easily understood in the light of the discussion in section 3.3. Like in the case where information is sold by a potential buyer of the good, whenever information is acquired equilibrium allocations are always ex-post efficient. Hence, the sum of the changes in the payoff of all traders between the situation with and without information acquisition equals the change in total welfare, $W_1 - W_0$. Since the payoff of buyers who purchase information is again zero in both cases (for c below but close to the threshold for information to be acquired), the change in total welfare equals the change in the payoff of the seller, $\Delta \pi_S^S$,

plus the payoff of the buyer (B_N) who remains uninformed.[21] That is[22]:

$$W_1 - W_0 = \pi^S_{B_N} + \Delta\pi^S_S. \tag{10}$$

Thus, underinvestment obtains whenever $\pi^S_{B_N} > 0$. Since the uninformed buyer gets the same payoff as when information is sold by a potential buyer, i.e. $\pi^S_{B_N} = \frac{1}{K}\left(\frac{K-1}{K}\right)^{N-1}$, the result follows. Notice that the source of the inefficiency is here only the informational *free riding* of the uninformed buyer, not the *rent dissipation* by the indirectly informed buyers, in contrast to what we found in section 3.3 when the information provider is a buyer.

Another natural question is whether information acquisition is more efficient if carried out by the owner of the good rather than by a potential buyer. Comparing the threshold for a potential buyer to acquire information derived in theorem 1, c^I, with the one obtained in proposition 3 for the owner of the good, we find that inefficiency is more severe with the owner of the good as information provider when:

$$\frac{1}{K}\left(\frac{K-1}{K}\right) + (N-2)\frac{1}{K}\left(\frac{K-1}{K}\right)^{N-1} > \left(\frac{K-1}{K}\right)\left(1 - \left(\frac{K-1}{K}\right)^{N-2}\right),$$

or equivalently,[23]

$$\left(\frac{K-1}{K}\right)^{N-3}\left(\frac{K+N-2}{K}\right) > 1 \tag{11}$$

As shown in Claim 4 in the Appendix, this condition is satisfied if $K - N$ is sufficiently large, or if $K < 6$. When N is much smaller than K, the probability that two agents like the same type of object is quite small. Hence, the *rent dissipation*, the source of inefficiency for the buyer as information provider, is small, while the payoff from *free riding* is large with N small.

Should the seller of the good always be trusted as a more reliable source of information than a potential buyer of the good? As we have argued above, when the seller of information does not know the preferences of the buyers, he always announces the true type of the object, hence his report is more informative than the one sent when the seller of information

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[21] As shown in the proof of Proposition 3 in the Appendix, when $K \geq N - 2$ the seller's optimal choice is always to sell information to all buyers but one.

[22] We use a superscript S to denote variables at equilibria where the owner of the good is the seller of information.

[23] Note that (11) is the reverse inequality of (6), which describes the condition under which $\Delta\pi_S > 0$. By comparing (7) and (10) we see in fact that a buyer is more efficient than the seller of the good as provider of information, or (11) holds, when $\pi^S_{B_N} > \Delta\pi_S + \pi_{B_N}$; that is, since $\pi^S_{B_N} = \pi_{B_N}$, when $\Delta\pi_S < 0$.

is another potential buyer of the good. This is no longer true, however, if he knows something about the types of the buyers; in that case the seller of information would also face a conflict of interest in his reporting strategy as he could enhance his revenues in the auction by giving false information. In particular, in the following proposition we show that when he knows perfectly the type of buyers[24], he could (and would choose to in equilibrium) announce a "popular" type of the object, one that more than one buyer liked, even if such type was not in fact the one of the object for sale:

PROPOSITION 4 *Let $c > \frac{1}{K}\left(\frac{K-1}{K}\right)$. If the owner of the good has acquired information and is fully informed about the type of all buyers, in a maximal truth-telling equilibrium he sometimes reports a lie $m \neq v$. In addition, his expected equilibrium payoff is lower than when he does not know the buyers' type.*

To give an intuition for the proof in the Appendix, suppose the owner of the good finds that no more than one of the buyers likes the object but at least two other buyers like a different type of the object, i.e. say $v = \theta_i$ only for $i = 1$ but $\theta_2 = \theta_3 \neq \theta_1$. Then he prefers to report θ_2 rather than the truth. The reason is that this lie allows to increase buyers' bids for the object and thus the price at which the object is sold in the auction. On the other hand, when he finds that at least two of the buyers like the object, the owner of the good is willing to report the true type of the object. Thus his report still contains some, though now imperfect, information over the true type of the object.

Given the reporting strategy of the seller described in the previous paragraph, the equilibrium allocation will not always be ex post efficient, as sometimes the object will end up in the hands of a buyer who does not like it even when there is another buyer who likes it. This inefficiency together with the lower informational content of the reports sold adversely affect the buyers' willingness to pay for information and hence the seller's revenue from the sale of information, as well as his revenue from the auction when buyers happen to like the object. We show in the proof that altogether such negative effects prevail over the positive effect on the auction revenue which obtains when buyers of information do not like the object, so that the expected payoff of the owner of the good is higher if he does not know the buyers' types.

The conflict of interest we just identified is an important one. Lying about the type of the good in order to increase the demand of the object in the market is akin in fact to the "hyping" of securities by analysts which inspired the counter-measures in title V of the Sarbanes-Oxley act (as well as the authors of the report of the European Commission Forum

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[24]As we will see in Section 5.1, such information can be gathered by asking buyers to report their types.

Group 2003). Such lies happen in spite of the fact that information here is not about the quality, but the variety of the good for sale. It will occur 'a fortiori' when information concerns quality as well, i.e. when elements of vertical differentiation of the information are introduced (see our discussion in section 6).

4.3   Who will sell information?

So far, we considered situations where only one type of trader has the 'license' to sell information. We discuss here briefly what happens when all three types of traders (potential buyers, disinterested traders and owner of the good) can compete among them for the acquisition and sale of information.

The first thing to note is that the entry of a second agent as a seller of information will drive the price of information down to zero, for reasons analogous to those explained in Section 3.2. Given this, the only incentive for an agent to enter the market and disrupt the equilibrium with a monopolist provider of information may come from the effects that entering has on the agent's payoff in the auction. This immediately implies that an uninterested trader, whose only payoff is given by the revenue from the sale of information, can never gain from entering and hence never wants to disrupt any monopolist equilibrium. Also, as argued in that same section, when there is already one agent selling information the payoff in the auction of a buyer is always the same whether he acquires information directly or indirectly. It is true that by purchasing information directly he can then sell the information to to others, but remember that with two providers of information, its price drops to zero. Since the price at which a monopolist sells information is always less or equal to c, a buyer never strictly gains by entering. This leaves the owner of the good as the only potential disrupter of monopolist equilibria.

When a disinterested trader is a monopolist seller of information, as shown in Proposition 2, he sells information to all buyers except one and always reveals the true value of the object. Hence in that case entry only affects the information available to the single buyer who is not purchasing information and we can verify (see proof of the next Proposition) that the change in the auction revenue of the owner of the object if he enters is zero so that entry is never profitable. But when the monopolist seller of information is a potential buyer, he does not always tell the truth and then entry also affects the information revealed to the agents purchasing information. With a potential buyer as an information monopolist the auction price is either $0, 1/K$, or 1 as stated in Theorem 1. If the owner of the good enters the market, the auction price will increase from $1/K$ to 1 in the event where the directly

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informed buyer likes the good and at least one other buyer also likes it, but it will decrease from $1/K$ to 0 when only one directly or indirectly informed buyer likes the object, and the uninformed buyer does not like it.

More formally, we have:

PROPOSITION **5** *Suppose all types of traders are allowed to acquire information and compete among them for its sale. In this case:*

1. *All the equilibria we found when only the owner of the object or only disinterested traders are allowed to sell information remain (monopolist) equilibria in this case. The same is true also for the (duopoly) equilibria with two potential buyers selling information.*

2. *An equilibrium with a potential buyer as an information monopolist exists only when*

$$\frac{1}{K}\left(1 - \left(\frac{K-1}{K}\right)^{N-1}\right)\left(1 - \frac{1}{K}\right) < (N-1)\frac{1}{K^2}\left(\frac{K-1}{K}\right)^{N-1} + c.$$

Thus we can say that the owner of the object is the most aggressive trader in the pursuit of the sale of information.

5 How can Efficiency be Attained?

We discuss in this section some possible ways to overcome the inefficiencies described in the previous sections. They vary according to the type of the agent selling information.

5.1 Differentiation of the information sold

We allow here informed traders to also seek information over the preferences of uninformed buyers and to sell then, on such basis, different kinds of reports over their information, at different prices (as argued below its effects are essentially the same as those of allowing the resale of information). Thus there can be differentiation of the information transmitted, and the extent of such differentiation will be optimally chosen by the seller. We consider first, in the next subsection, the case where the seller of information is a potential buyer and is always a monopolist, i.e. entry to the market for the sale of information is restricted to a single trader. We then discuss in the following subsection the consequences of eliminating this restriction and allowing, as in the previous sections, for free entry in the market for information.

A (potential buyer as a) monopolist seller

To see why the differentiation of information may allow to increase the profits of the seller of information, recall that, as we saw, when a single type of report is sold the price a buyer is willing to pay for information depends on the number J of other buyers who choose to remain uninformed. Purchasing information is indeed valuable for the buyer because it gives him an informational advantage over the uninformed buyers, which manifests itself in a priority over uninformed buyers in obtaining the good when the buyer likes it. When a single type of report is sold, there are up to three information, and hence priority, levels. First, the directly informed buyer, then all the indirectly informed buyers (who share the same priority level), finally the uninformed buyers, when they exist. The larger is the number J of buyers in the last level, the more valuable is the information of indirectly informed buyers. We now show that, by differentiating the reports sold, the seller of information can arrange the indirectly informed buyers into several distinct priority levels, and by so doing can increase his revenue. Hence, the incentives for information acquisition improve, as the *rent dissipation,* which was shown in Section 3.3 to be at the root of inefficiency in information acquisition, is reduced (if not eliminated).

Notice that, to implement an effective differentiation of the reports sold, the seller must have some information over buyers' preferences, so we will allow him to ask buyers about their preferences. Formally, the seller of information follows now a two-step procedure to disseminate information:

a. Each uninformed buyer who has agreed to purchase information sends first a report over his type to the seller of information (the set of available messages for an uninformed buyer is the set of possible types of the buyer, given by $\mathcal{K}$). Such report is observed only by the seller of information, not by the other buyers.

b. Subsequently, the seller of information sends some reports over what he learned to the buyers who purchased information from him. He chooses now the number of types of reports and the prices at which he is willing to sell them. We will consider in particular the case where the different types of reports offered for sale can always be arranged in a hierarchy of reports of decreasing quality, or informativeness. Let L denote the number of different types of reports sold. The hierarchy of the qualities of the different reports is modeled by assuming that the seller issues L messages and buyers purchasing a report of type l, $l \in \{1, .., L\}$ observe all the messages m_j, $j = l, .., L$. The information provided by the reports has then a nested structure, in the sense that receiving any report $i > l$ conveys no additional information when compared to report l, while the

reverse is not true. Hence report 1 has the highest quality and report L the lowest. We consider the case where the set of possible messages available to the seller of information for any l is the set of direct messages, $m_l \in \mathcal{K} = \{1, 2, ..., K\}$.[25]

Summarizing, there are two phases in the reporting of messages. First each buyer of information sends a report over his type to the seller of information. Subsequently the seller sends the messages m_l, $l = 1, .., L$ and, for any $l \in \{1, .., L\}$, the buyers of report of type l receive the messages $(m_l, m_{l+1}, .., m_L)$.

We characterize first the equilibria of the subgame starting from the node where a single buyer, say B_1, has acquired information and is selling differentiated information in the market. For any given level of L we determine the optimal choice of B_1 concerning the prices posted for the different reports p_l, $l = 1, .., L$ and the equilibrium strategies in the rest of the subgame (which reports are purchased by each uninformed buyer B_i, $i = 2, .., N$, the reporting strategies and bids in the auction). On this basis we can then find the level of L which maximizes the revenue of the seller B_1. Finally, we compare this value of the revenue to the cost c; when it is higher we conclude that information acquisition is worthwhile for the seller and will take place in equilibrium.

We still focus our attention on the equilibria where agents' reporting is characterized by the maximal degree of truthfulness and is now also consistent with the differentiation of information in L levels. To describe the seller's reporting strategies, it is convenient to adopt some notational conventions. Given the hierarchical structure of the information, we will sometimes refer to the buyers purchasing from B_1 a report of quality l as the buyers in layer l of the hierarchy. For any $l \geq 2$, let $\mathcal{N}_l(B_1)$ denote the set of buyers purchasing a report of type $i \geq l$ (i.e. who are in layer l or below) and $N_l(B_1)$ the number of different realizations of θ_i across all buyers $B_i \in \mathcal{N}_l(B_1)$; hence $\mathcal{N}_l(B_1)/\mathcal{N}_{l+1}(B_1)$ indicates the set of buyers in layer l. $\mathcal{N}_1(B_1)$ is similarly defined and indicates the set of buyers who purchased any type of report from B_1.

We will show that there is an equilibrium where the uninformed buyers always report their type (as lying does not allow then to affect the outcome of the auction in their favor) and the reporting strategy of the seller B_1 for the messages $m_1, .., m_L$ is defined recursively

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[25]It will be clear from the analysis which follows that the *empty* message is not needed anymore.

as follows:[26]

$$m_1 = \begin{cases} v, \text{ if } v \neq \theta_1 \text{ or } v \neq \theta_j \ \forall j \in \mathcal{N}_1(B_1) \\ y, \begin{cases} \text{with probability } \frac{1}{[K-N_1(B_1)]}, \\ \text{for all } y \neq \theta_j, \ B_j \in \mathcal{N}_1(B_1) \end{cases}, \text{ if } v = \theta_1 \end{cases} \qquad (12)$$

and, for $l = 2, .., L$

$$m_l = \begin{cases} m_{l-1}, \text{ if } m_{l-1} \neq \theta_i \text{ for all i} \in \mathcal{N}_{l-1}(B_1)/\mathcal{N}_l(B_1) \\ y, \begin{cases} \text{with probability } 1/[K - N_1(B_1)], \\ \text{for all } y \neq \theta_j, \ B_j \in \mathcal{N}_1(B_1) \end{cases}, \text{ if } m_{l-1} = \theta_i \text{ for some } B_i \in \mathcal{N}_{l-1}(B_1)/\mathcal{N}_l(B_1) \end{cases}$$

$$(13)$$

Thus at each layer l the informed trader tells the truth as long as the true variety of the object does not coincide with his own type or with the type of any buyer who has purchased information of higher quality.[27] Otherwise, the informed trader randomizes over any variety different from the type of any of the agents who purchased information; in this event the seller wants in fact to induce buyers to make the lowest bid and we can interpret such message as telling buyers the object is not appropriate for any of them. Thus when the sender of information does not tells the truth he does not simply keep receivers in the dark, he actively misleads them.

The fact that the seller of information is now informed about buyers' preferences and adopts a reporting strategy as the one above has also some important implications for the traders' behavior in the auction. A buyer can now receive either a message equal to his type, in which case again he infers that for sure he likes the object, or a message different from his type. The latter can happen in two alternative events. The first one is when the buyer likes the object, but so does the sender of the message or one of the buyers purchasing a higher quality report. In that case the buyer cannot win the object in the auction with a positive surplus, i.e. at a price lower than his valuation because he faces a trader who, unlike him, is perfectly informed of the fact that he likes the object. The other possibility is that the buyer does not like the object, in which case any positive bid, if successful, would yield him a negative surplus (with positive probability). Thus any positive bid yields a negative expected payoff. The optimal bid of a buyer of information, after receiving a message different from

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[26]The expression below is always well-defined as long as $K > N$, that is the number of possible types of the object is strictly greater than the number of potential buyers. The results obtained in this section extend however to the case where $K \leq N$, provided the set of possible messages is suitably expanded to allow for the possibility that a separate message is sent to each buyer of information.

[27]Strictly speaking, he also tells always the truth when the true variety of the object is different from the type of any buyer of information as in this case a conflict of interest never arises.
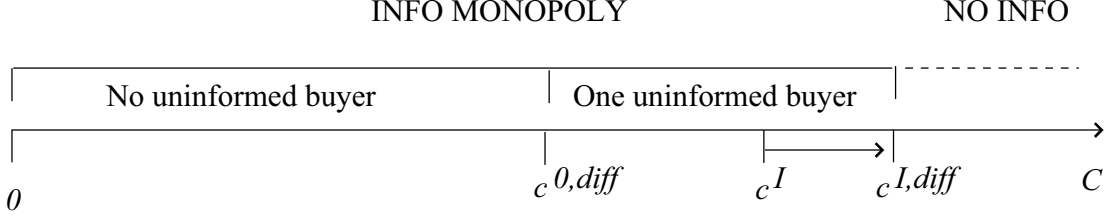
Figure 2: Equilibrium with heterogeneous messages

his type, is then equal to zero, which is his expected value of the object conditional on winning the auction. Hence, even though we are in a second price auction, the buyer's bid is not always equal to his posterior belief over the value of the object. This is due to the correlation of the information of traders induced by the sender's reporting strategy. The information conveyed by winning the auction should then also be taken into account.

Thus in equilibrium each buyer of information bids again 1 when the message received equals his type, but bids 0 otherwise and each uninformed buyer still bids 0.

With this message structure the equilibrium exhibits some new interesting features:

PROPOSITION 6 *When we allow for the differentiation of the information sold, with a monopolist seller of information there is an equilibrium where information acquisition takes place if and only if*[28]

$$c \leq \left(\frac{K-1}{K}\right)\left(1 - \left(\frac{K-1}{K}\right)^{N-1}\right) = c^{I,diff}, \tag{14}$$

the seller adopts the truthful reporting strategy (12), (13) and we have maximal differentiation (a different report is sold to each buyer). Also, information is sold to all other buyers (i.e. $L = N - 1$) when

$$c \leq \left(1 - \left(\frac{K-1}{K}\right)^{N-1}\right)\frac{1}{N-2} = c^{0,diff} \tag{15}$$

and otherwise to all other buyers except one ($L = N - 2$).

Figure 2 summarizes the result.

Hence the optimal choice for a monopolist seller is to design the hierarchy of reports sold and the prices posted so that each buyer chooses to purchase one report, and each type of report is purchased by only one agent. We have thus only one buyer of information in each

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[28]Note that this threshold holds now for all values of N, K.

layer, in each priority level, i.e. the maximal degree of differentiation as possible. The key insight in the argument of the proof for this result is that the payoff of a buyer purchasing a given report only depends on the total number of other buyers who are **equally or better informed** than him. Hence the price such buyer is willing to pay for information does not change if anybody who is equally informed gets instead a higher quality report. The one who does that, however, improves his priority level and hence his payoff, and therefore is willing to pay more. Thus when the same type of report is sold to two buyers, the seller of information can always increase his revenue from the sale of information by improving the quality of the report which is sold to one of them. Furthermore, this will have no effect on the seller's payoff in the auction (since, given the bidding strategies described above, the price at which he gets the object only depends on whether or not there are uninformed traders). By induction, the best is to completely differentiate the information sold to each buyer who is purchasing information.

Notice that the threshold for information acquisition to take place with a monopolist seller of differentiated information, $c^{I,diff}$ in (14), is the same as the one found in (2) for the efficiency of the investment in information. Hence the equilibrium is now efficient, not only with regard to the properties of the allocation but also of the information acquisition. This is due to the fact that, by selling a different report to each buyer, the seller of information, B_1, can ensure that all buyers are completely ranked, so that there will never be ties in the auction and hence no *rent dissipation*. Hence B_1 can appropriate now all the social surplus generated by the acquisition of information.[29]

Condition (15) reflects the fact that the monopolist seller of information faces now a tradeoff in his choice of the prices at which the various reports are sold. His payoff in the auction is now maximal when information is purchased by all buyers, since in that case the seller of information gains the object at a zero price (while he does so at a price $1/K$ when there is some uninformed buyer). The same is true for the payoff in the auction of the buyers of information. However their maximal willingness to pay for information also depends on their outside option, that is on the payoff a buyer would get if he were to stay uninformed and this is clearly higher when there is no uninformed buyer. We show in the proof that the revenue from the sale of information is still the highest when there is one buyer who does not purchase information ($J = 1$). The seller of information has then to choose between the set of prices which maximize his revenue from the sale of information and those which maximize

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[29]Notice that the *free-riding* by the uninformed buyer still occurs in this case. However, as argued in section 3.3, this only negatively affects the owner of the good, not the seller of information.

his revenue in the auction. When c is low, $c \leq c^{0,diff}$, the prices at which information is sold are low so that the second effect prevails and $J = 0$ is optimal, while the reverse happens for $c > c^{0,diff}$.

REMARK 1 *Notice that there is a close relationship between differentiation and* **resale** *of information. Our results can in fact be reinterpreted as describing the outcome when the agents who purchase information are able to resell it. In that case in equilibrium a single buyer purchases information from the informed trader, and then sells, at a lower price, a noisier report which is purchased by only one buyer, who in turn sells an even noisier report to a single buyer, and so on.*

Free Entry

In the previous section we have seen that the differentiation of the information sold allows a monopolist to achieve an efficient outcome. However, we show here that it also makes the monopolist's position much more vulnerable to entry. In the case of homogeneous information, we know from Theorem 1 that with free entry in the market for information in equilibrium there is at most a single seller and monopoly is the only outcome for most parameter values. Now this is no longer true.

PROPOSITION 7 *With differentiation of the information sold and free entry in the market for information, when the cost of information c is not too high there is always an equilibrium with at least two sellers of information (clearly inefficient). Each of them chooses to sell the same number of different reports as in the monopoly equilibrium of Proposition 6 and to adopt the same reporting strategies, (12) and (13).*

To understand this result we have to remember what prevented multiple entry when homogeneous information is sold. In that situation a buyer of information was never willing to pay a positive price to purchase an additional report from a second seller. His position was in fact in the intermediate priority level, between the informed sellers of information and the uninformed buyers, and such position was unaffected (i.e. his surplus in the auction unchanged) by the purchase of a second report. Given this, the only possible outcome was intense competition among sellers of information driving its price down to zero.

On the contrary, when the information providers sell differentiated reports every buyer who is considering to buy a report not of the lowest quality needs to buy the same type of report from each seller to be able to retain the position in the hierarchy of information - and hence the priority level - such report is meant to deliver. If he fails to buy this report from

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one of the providers, the traders who buy reports of lower quality from such provider would be able to compete with him for the object on the same terms, hence the buyer would not get the object with a positive payoff when both he and any of these other traders like the object. This incenive to buy the same type of report from each seller creates the possibility of a collusive outcome where each seller of information, instead of lowering prices so as to attract all customers only to him, prefers to set prices in such a way that every buyer will purchase a report from all the sellers. In this way the seller can gain positive profits by sharing the monopoly rents with the other sellers.[30]

5.2 Entry fee for the auction

We propose here an alternative way to restore first best efficiency, when the seller of information is the owner of the good. Suppose he could charge any potential buyer an entry fee to participate in the auction. Then if the owner of the good sets such fee equal to the rent obtained by the uninformed individual in the equilibrium without entry fee (which is, remember, $(K-1)^{N-1}/K^N$) and rebates it to all those traders who purchase information from him, he can extract from all the buyers the full surplus generated by the information. Hence the social incentives for information acquisition are aligned with the individual incentives for the owner of the good and efficiency obtains. At the same time, we should point out that this is only true if the owner of the object has no information over the buyers' types and can credibly commit not to seek such information. When these conditions are not met, as we saw in Section 4.2, the outcome is always inefficient when the seller of information is the owner of the good.

6 Discussion

The model considered is quite versatile and has allowed us to study information acquisition, transmission and trade upon this information in a number of setups. The information transmitted may be homogeneous or heterogeneous, the seller of information may be a potential

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[30]The outcome with a monopolist seller of heterogeneous information can also be sustained at an equilibrium with free entry. It can in fact be easily verified that the strategies where, whenever another trader enters the market for the sale of information, each seller chooses not to differentiate the information, i.e. to sell a homogeneous report, constitute another equilibrium of the subgame. In such situation, as shown in Section 3, information is sold at a zero price and hence entry is not profitable. However this requires the sellers of information to coordinate on what is, from their point of view, a Pareto inferior subgame equilibrium.

buyer of the good, the seller or a "neutral" third party. We show next that the analysis and conclusions drawn are fairly robust with respect to changes in various features of the specification. Finally we discuss the consequences of introducing some elements of vertical differentiation in the uncertainty concerning the object traded for the efficiency of the market and for the conclusions drawn concerning the desirability of different forms of regulatory interventions.

6.1 Some robustness checks

6.1.1 Organization of the market for information

How would the equilibrium properties change with alternative assumptions concerning who and when the price for information is posted? Suppose the sellers of information were to post the price after - rather than before - having learnt the signal realization. Then the price posted would have a signaling value, as a seller may want to post a different price after having learnt that he likes or does not like the object. In this case we should expect a large set of equilibria to exist. Clearly an equilibrium with properties analogous to the ones found in Theorem 1 (where the price posted conveys no information) still exists. There are other pooling equilibria, but with lower prices (and thus lower efficiency),[31] and under maximal truth-telling there is no separating equilibrium.[32]

Consider next the case where the price is posted by buyers, rather than sellers of information, again before the latter have learnt the realization of the signal. Each buyer can then post a price contingent on the number of other buyers who also purchase information[33]. We claim that in such case the equilibria would have similar, though not exactly identical, features to the ones we found. In particular, each buyer of information sets a price equal to his maximal willingness to pay for the information, except when the number of the other buyers of information equals its maximum minus one (e.g. $N-3$ with a monopolist seller given by a potential buyer), in which case he sets a price equal to zero, which is lower than his

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[31]Those are sustained by the off-equilibrium beliefs that a deviator posting a higher price is a buyer who likes the object, from whom nobody wants to purchase information.

[32]At a separating equilibrium there are two distinct prices which signal that a seller of information likes, respectively does not like, the object. At the first one the seller is expected to bid 1 and so no buyer is willing to pay anything for information. At the second one the seller is expected to tell the truth (under maximal truth-telling), thus information is valuable and its sale also allows to decrease the price at which the object is gained in the auction. Thus, the seller of information would strictly gain by deviating when he likes the good and posting the second price.

[33]It is easy to verify such property is needed for the equilibrium not to be trivial.

true maximal willingness to pay for information.[34] Given these strategies of the buyers, the optimal response of the seller of information is to sell at a positive price to all the potential buyers less two (rather than one, as in Theorem 1). The payoff of all potential buyers of information, both those who actually purchase it and those who don't, is then zero.

6.1.2 Auction Format

We have assumed throughout that the object is sold via a second price auction. This is without loss of generality in the case where sellers of information send the same report to all buyers of information, as bidders' valuations remain independent after the transmission of information, and we can apply the revenue equivalence theorem. With the sale of differentiated information, however, things are different. In that case information transmission generates correlation in the values of the agents, so we can no longer invoke the revenue equivalence result. However, we can argue at least that a first price auction is likely to generate less rents for the informed buyer and possibly ex-post misallocations (hence worse incentives for information acquisition and lower welfare). The reason is that with a first price auction a pure strategy equilibrium does not exist. To fix ideas, think of a message strategy as in (12, 13). Any (pure strategy) bid by the seller of information (when the good is of his preferred type) *above* the expected value of the other buyers (after receiving the report sent by the seller, that the object is not of a type they like) would induce them to bid zero, as that would be their valuation conditional on winning the auction. This would make the bid of the informed buyer suboptimal. But any (pure strategy) bid by the informed buyer *below* this expected value would induce the other buyers to bid above the informed buyer, which would not be optimal for him either. Pure strategy bids by indirectly informed or uninformed buyers could not be equilibrium strategies for analogous reasons. A mixed strategy equilibrium clearly leads to ex-post suboptimal allocations with positive probability, higher prices in the auction for the informed buyer and lower revenues from selling information.

6.1.3 Buyers' preferences and number of objects for sale

We considered so far the case where a single unit of the object is available for sale. Suppose instead multiple - say $Q > 1$ - units of the good were up for sale and each buyer has a positive

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[34]This is because, if the seller were indeed to sell to a total of $N - 2$ buyers, i.e. to all potential buyers but one, each buyer of information would have an incentive to deviate and post a lower price. By so doing the buyer would make sure he is the one not purchasing information, i.e. remaining uninformed, which gives him a strictly positive payoff.

utility only for one unit, thus a limited capacity for "enjoying" the good in the market. Let the good be sold via a $(Q + 1)$-th price auction. When the seller of information happens to like the object, if there is no differentiation of the quality of the information sold, he will still not tell the truth and send a zero message to all buyers so as to lower competition in the auction, as he does when a single unit is up for sale. But now this will lead to some units of the object being sold to buyers who do not like them and hence to a possible *ex-post* inefficiency of the equilibrium allocation. Otherwise, the equilibrium strategies are similar to those of Theorem 1, underinvestment in information is still present and even more severe. Notice that this problem does not arise if the seller differentiates the quality of the information sold, as in Section 5.1; when such differentiation is allowed, the efficiency of the equilibrium could be preserved.

In a similar fashion, suppose that the utility of an arbitrary buyer B_i when he likes the object, that is when $v = \theta_i$, is given by U_i, a positive number but no longer equal to 1 for all buyers. In this context, there exists an equilibrium, whose structure is again similar to that of Theorem 1, where the seller of information is the buyer of type i^* with the maximal potential valuation for the object $U_{i^*} > U_i$ for all i. There are, however, other equilibria in which the seller of information is a different buyer $j \neq i^*$ with a lower potential valuation. An important difference, though, is that the equilibrium may now be *ex-post* inefficient. In the event where $v = \theta_j = \theta_{i^*}$ buyer B_j will get the object, even though B_{i^*} also likes it and values it strictly more.

6.2   Introducing Vertical Differentiation of the Information

In the environment considered so far the uncertainty only concerns a horizontal differentiation element, the type or variety of the object over which buyers have idiosyncratic tastes. It is thus important to examine the consequences of allowing also for the presence of a vertical differentiation element, quality, over which buyers' preferences tend to agree.

To this end, consider the following extension of the model. Suppose the good not only comes in one of the K types we described, but also in one of 2 quality levels, H (High) and L (Low). Formally, the true type of the good is now $v = (k, q) \in \mathcal{S} \equiv \mathcal{K} \times \{H, L\}$. Suppose, in addition, that buyers are also of two types: while all buyers only care for one, randomly drawn variety, some of them are sensitive to quality (Se) and others are insensitive to quality (In). An In consumer has a constant valuation of 1 for a good of the variety he likes. A Se consumer values a good of the type he likes V if the good is of H quality, and 0 if it is of L quality. Let us assume for simplicity that H and L have identical probabilities for each

variety of good, and that consumers have identical probabilities to be of type Se and In.

Also, consider again the case where the set of available messages to the seller of information is the set of direct messages: $\mathcal{M}' = \mathcal{S}$, so that a generic message m is now a pair (k, q), where $k \in \mathcal{K}$, $q \in \{H, L\}$. We show next that, when the seller of information is the owner of the object, whenever he perfectly reveals in equilibrium the true variety k of the object, he never reveals any information over the quality of the object:

PROPOSITION 8 *Suppose the set of possible types of the good is given by $\mathcal{S}$ and the owner of the object is the seller of information. Then at any equilibrium where the seller truthfully reveals the variety of the object, that is where for all $m \in \mathcal{M}'$, $\Pr(k = i|m) = 1$ for some $i \in \mathcal{K}$, we have $\Pr(q = H|m) = \Pr(q = H) = \frac{1}{2}$. Thus, if $V > 2$, the Se type buyers always bid more for the good than the In type buyers, and vice versa if $V < 2$.*

Hence we can never have at the same time perfect revelation of the true variety and the true quality of the object.[35] It follows that the equilibrium allocation of the good is ex post inefficient, which in turn implies that not all social surplus can be appropriated by the seller, and so that there will be underinvestment in information acquisition.

The source of the inefficiency is that when the seller of information is the owner of the object he faces a conflict of interest, analogous to the one we found in the second part of Section 4.2, which induces him to want to lie to exaggerate how much buyers like the good so as to increase his revenue from the sale of the good. Notice that analogous features - ex post inefficiency due to false reports - obtain when the seller of information is a potential buyer (the circumstances and form of the lie are however different in that case: the seller wishes to lie only when he likes the object and tells buyers the variety and quality of the good is not "right for them"). The only case where the report sent is always truthful and the equilibrium allocation remains ex post efficient is the one where the seller of information is a disinterested trader.

7 Conclusion

Good quality information is key to a properly functioning market. This has long been understood by academics as well as by practitioners and regulators. But market participants are *not endowed* always with all necessary information, and typically *obtain* it, often from

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[35]Some revelation over the true quality of the object could be obtained at equilibrium, but only at the cost of partial revelation over the true variety.

other actors in the market. For this reason, authorities have established numerous rules on the amount and kinds of communication between market participants and information providers. Surprisingly, there is little research into the interplay between acquisition of information, its transfer and actions in the market, which would be necessary to provide foundations for such policy. We partly fill this gap by building a formal model of a market environment with costly acquisition and unverifiable transmission of information. In this set-up we can investigate the conflicts of interest faced by the information providers, see how they vary according to the type of the provider, in which directions they limit the extent of truthful transmission of information and examine the consequences for the performance of the market.

We find that when information concerns a prevalent horizontal differentiation component, there are typically inefficiencies because of underinvestment in information acquisition. Usual regulatory interventions, such as firewalls, or limiting the sale of information to parties which have no interest in trading the underlying object, worsen the inefficiencies. In contrast, efficiency can be attained with a monopolist selling differentiated information, if additional entry is blocked. When, on the other hand, the vertical differentiation element is more relevant, firewalls can be beneficial. As we argued in the introduction, both the horizontal and the vertical elements are likely to be part of the information problem in real markets. We thus provide a tool to assess the potential benefits of establishing various kinds of regulations.

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Appendix

Proof of Theorem 1 (further details)

We provide here the missing details of the proof of the Theorem.

Optimal pricing rules and payoffs with a monopolist seller of information

As argued in Section 3.2, to find the optimal pricing strategy of a monopolist seller of information in the subgame starting in stage 2 of the game, we derive first the maximal willingness to pay for information of an uninformed buyer for each given number J of buyers who choose not to acquire information. Let us denote such situation as configuration J, where $J \in \{0, 1, .., N - 2\}$.

Let, w.l.o.g., B_1 be the seller of information, $B_2, .., B_{N-J}$ indicate the traders buying information from the single seller and $B_{N-J+1}, ..., B_N$ be the J buyers not purchasing information in configuration J, i.e. when there are $J \geq 0$ buyers not purchasing information. The payoff of buyer B_i in such configuration is then denoted by $\pi_{B_i}^J$. The value of the outside option for the buyers purchasing information is given by $\max\left\{\pi_{IC}^J, \pi_U^J\right\}$, where π_{IC}^J (resp. π_U^J) indicate the expected utility of buyer $B_2, .., B_{N-J}$ if, rather than purchasing information, he were to acquire information directly (resp. to stay uninformed). For the buyers not purchasing information it is given by $\max\left\{\pi_{UC}^J, \pi_I^J\right\}$, where π_{UC}^J (resp. π_I^J) is now the expected utility of a buyer $B_{N-J+1}, ..., B_N$ if, rather than staying uninformed, he were to acquire information directly (resp. to purchase information). Let then $p(J)$ be the price posted by the seller of information, that is the price which maximizes his revenue among all the prices that support such configuration.

J > 0. From the discussion in Section 3.2, it is immediate to see that when $J > 0$ the payoffs of the N potential buyers are:

$$\pi_{B_1}^J = \frac{1}{K}\left(1 - \frac{1}{K}\right) + (N - (J+1))p(J) - c \tag{16}$$

$$\pi_{B_i}^J = \frac{1}{K}\left(\frac{K-1}{K}\right)^{N-(J+1)}\left(1 - \frac{1}{K}\right) - p(J), \text{ for } i = 2, ..., N - J$$

$$\pi_{B_N}^J = \begin{cases} ... = \pi_{B_{N-J+1}}^J = 0 \text{ if } J \geq 2 \\ \left(\frac{K-1}{K}\right)^{N-1}\frac{1}{K} \text{ if } J = 1 \end{cases}$$

The value of the outside options for those who are also buyers of information is then

$$\pi_{IC}^J = \frac{1}{K}\left(\frac{K-1}{K}\right)^{N-(J+1)}\left(1 - \frac{1}{K}\right) - c$$

$$\pi_U^J = 0$$

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while for the uninformed buyers it is

$$\pi_{UC}^J = \frac{1}{K}\left(\frac{K-1}{K}\right)^{N-J}\left(1-\frac{1}{K}\right) - c$$

$$\pi_I^J = \frac{1}{K}\left(\frac{K-1}{K}\right)^{N-J}\left(1-\frac{1}{K}\right) - p(J)$$

if $J \geq 2$ and

$$\pi_{UC}^1 = \frac{1}{K}\left(\frac{K-1}{K}\right)^{N-1} - c$$

$$\pi_I^1 = \frac{1}{K}\left(\frac{K-1}{K}\right)^{N-1} - p(1)$$

if $J = 1$.

It is then immediate to see that the optimal pricing rule of the monopolist seller of information in any configuration $J \geq 1$ is to set the price equal to the maximal willingness to pay for information of traders $B_2, .., B_{N-J}$, i.e.:

$$p(J) = \min\left\{c, \frac{1}{K}\left(\frac{K-1}{K}\right)^{N-J}\right\}. \tag{17}$$

Furthermore, configuration $J = 1$ is always attainable since from the above expressions we see that the condition $\pi_{B_N}^1 = \left(\frac{K-1}{K}\right)^{N-1}\frac{1}{K} \geq \max\left\{\pi_{UC}^1, \pi_I^1\right\}$ holds with the pricing rule in (17), while any other configuration $J > 1$ is only attainable as long as

$$c \geq \frac{1}{K}\left(\frac{K-1}{K}\right)^{N-J+1}. \tag{18}$$

When (18) holds, $\pi_{B_{N-J+1}}^J = ... = \pi_{B_N}^J = 0 \geq \max\left\{\pi_{UC}^J, \pi_I^J\right\}$ is also satisfied, when it is violated no configuration $J > 1$ is attainable since the uninformed buyers always prefer, no matter what is the level of p, to become directly informed.

J = 0. In the configuration $J = 0$ (no one stays uninformed) the payoff expressions are the same as when $J > 1$ with two exceptions: the payoff of a buyer of information becomes:

$$\pi_{B_i}^0 = \frac{1}{K}\left(\frac{K-1}{K}\right)^{N-1} - p(0) \text{ for } i = 2, .., N$$

(since he gets the object at a zero price when he is the only one to like it) while the value of his outside option of staying uninformed is

$$\pi_U^0 = \frac{1}{K}\left(\frac{K-1}{K}\right)^{N-1}$$

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Thus $\pi_U^0 > \pi_C^0$ and the optimal pricing rule for such configuration is

$$p(0) = 0,$$

again equal to the maximal willingness to pay of buyers.

On this basis we can provide now the:

Proof of Claim 1.   Suppose first that

$$c \geq \frac{1}{K}\left(\frac{K-1}{K}\right)^2,$$

so that all configurations are sustainable, since (18) holds for all $J > 1$, and $p(J) = \frac{1}{K}\left(\frac{K-1}{K}\right)^{N-J}$ for all $J = 1, .., N-2$. We then show that the revenue from the sale of information is always higher in configuration J than in $J+1$:

$$
\begin{aligned}
(N-(J+1))p(J) \quad &\geq \quad (N-(J+2))p(J+1) \qquad\qquad\qquad (19)\\
&\Longleftrightarrow \quad (N-(J+1))\frac{1}{K}\left(\frac{K-1}{K}\right)^{N-J} \geq (N-(J+2))\frac{1}{K}\left(\frac{K-1}{K}\right)^{N-(J+1)}\\
&\Longleftrightarrow \quad \frac{(N-(J+1))}{(N-(J+2))} \geq \frac{K}{K-1}\\
&\Longleftrightarrow \quad K-1 \geq N-(J+2).
\end{aligned}
$$

This condition is satisfied for all $J \geq 1$ when $K \geq N-2$, so in this case the maximum obtains at $J = 1$. When $K < N-2$ it is only satisfied for $J \geq N-K-1$, while for $J < N-K-1$ the revenue is always higher at $J+1$ than at J; hence if $K < N-2$ the maximum obtains at $J = N-K-1$, larger than 1.

Consider next the case where

$$\frac{1}{K}\left(\frac{K-1}{K}\right)^{N-\bar{J}+1} \leq c < \frac{1}{K}\left(\frac{K-1}{K}\right)^{N-\bar{J}} \qquad\qquad (20)$$

for some $\bar{J} \in \{2, .., N-2\}$, so that only configurations $J = 1, .., \bar{J}$ are sustainable, $p(J) = \frac{1}{K}\left(\frac{K-1}{K}\right)^{N-J}$ for $J = 1, .., \bar{J}-1$ and $p(\bar{J}) = c$. When $K \geq N-2$, by the same argument as above, the revenue is higher at $J = 1$ than at any other $J = 2, .., \bar{J}-1$. Thus we only need to compare the revenue in configuration $J = 1$ with the one at $J = \bar{J}$ (where $p(\bar{J}) = c$) and show that, for all $\bar{J} \in \{2, .., N-2\}$:

$$
\begin{aligned}
(N-2)p(1) \quad &\geq \quad \left(N-(\bar{J}+1)\right)p(\bar{J})\\
&\Longleftrightarrow \quad (N-2)\frac{1}{K}\left(\frac{K-1}{K}\right)^{N-1} \geq (N-(\bar{J}+1))c
\end{aligned}
$$

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Clearly it suffices to show this property for the minimum value of $\bar{J}$, $\bar{J} = 2$:

$$(N-2)\frac{1}{K}\left(\frac{K-1}{K}\right)^{N-1} \geq (N-3)c$$

Using the same argument as in the previous paragraph we can show that the following inequality, which is stronger by (20), always holds when $K \geq N-2$:

$$(N-2)\frac{1}{K}\left(\frac{K-1}{K}\right)^{N-1} \geq (N-3)\frac{1}{K}\left(\frac{K-1}{K}\right)^{N-2},$$

thus establishing the result.

When $K < N-2$, again by the same argument as in the first paragraph, the revenue is higher at $J^* = \min\{N-K-1, \bar{J}-1\}$ than at any other $J = 2, .., \bar{J}-1$. Thus it still suffices to compare the revenue in configuration $J = J^*$ with the one at $J = \bar{J}$ (where $p(\bar{J}) = c$), for all $\bar{J} \in \{2, .., N-2\}$. If $J^* = N-K-1$, the maximal revenue obtains at $J = J^*$ since:

$$\begin{aligned}(N-J^*-1)p(J^*) &= \frac{1}{K}\left(\frac{K-1}{K}\right)^{K+1} \geq \left(N-(\bar{J}+1)\right)\frac{1}{K}\left(\frac{K-1}{K}\right)^{N-\bar{J}} \\ &\geq \left(N-(\bar{J}+1)\right)p(\bar{J}) = \left(N-(\bar{J}+1)\right)c,\end{aligned}$$

where the first inequality holds by (19) and the second one by (20). If then $J^* = \bar{J}-1$ the maximum revenue is attained either at $J = \bar{J}-1$ or at $\bar{J}$.[36]

The value of J found above gives the maximal revenue among all values of $J \geq 1$. To conclude the proof it suffices to observe that this also gives always a higher value than $J = 0$, since the value of $p(J)$ given by (17) is strictly positive for all $J \geq 1$ while $p(0) = 0$.

∎

Equilibrium pricing rules and payoffs with an information oligopoly   An alternative situation which may arise in the subgame starting in stage 2 has $M \geq 2$ sellers of information (w.l.o.g. let them be $B_1, ..., B_M$) and $N-M$ buyers of information ($B_{J+1}, ..., B_N$). Let us denote it as configuration $OL(M)$. In this case, as we argued in the main text, each seller posts a zero price for information and each uninformed buyer purchases information from all sellers. The payoffs in this case are:

$$\pi_{B_1}^{OL(M)} = ... = \pi_{B_M}^{OL} = \frac{1}{K}\left(\frac{K-1}{K}\right)^{N-1} - c$$
$$\pi_{B_i}^{OL(M)} = \frac{1}{K}\left(\frac{K-1}{K}\right)^{N-1}, i = M+1, .., N$$

---

[36]In particular, it will be at $\bar{J}-1$ when c is sufficiently closer to $\frac{1}{K}\left(\frac{K-1}{K}\right)^{N-\bar{J}+1}$ than to $\frac{1}{K}\left(\frac{K-1}{K}\right)^{N-\bar{J}}$ and at $\bar{J}$ otherwise.

Payoffs with no sale of information The last possibility is configuration No, where no buyer acquires information, hence all buyers stay uninformed and make a bid equal to their expected valuation, $1/K$. The object is then randomly allocated to one buyer, who pays for it an amount equal to his expected value for the good and hence gets no surplus and the payoff of every buyer is:

$$\pi_{B_i}^{No} = 0 \quad \text{for all } i = 1, .., N$$

Proof of Claim 2. From Claim 1 it follows that no information is gathered in equilibrium when:

$$\pi_{B_i}^{No} = 0 \geq \pi_{B_1}^1.$$

This condition can be rewritten as

$$
\begin{aligned}
c \;\geq\; & \max\left\{ \frac{1}{N-2}\frac{1}{K^2}, \frac{1}{K}\left(\frac{K-1}{K}\right) + (N-2)\min\left[c, \frac{1}{K}\left(\frac{K-1}{K}\right)^{N-1}\right]\right\} \\
=\; & \frac{1}{K}\left(\frac{K-1}{K}\right) + (N-2)\min\left[c, \frac{1}{K}\left(\frac{K-1}{K}\right)^{N-1}\right].
\end{aligned}
$$

Such condition may only be satisfied if $c > \frac{1}{K}\left(\frac{K-1}{K}\right)^{N-1}$, in which case it reduces to:

$$c \geq \frac{1}{K}\left(\frac{K-1}{K}\right) + (N-2)\frac{1}{K}\left(\frac{K-1}{K}\right)^{N-1} = c^I. \tag{21}$$

■

Proof of Claim 3. To get an information duopoly at an equilibrium of the overall game we need:

$$
\begin{aligned}
\pi_{B_2}^{OL(2)} \;\geq\; & \pi_{B_i}^1, \text{ for } i = 2, .., N-1, & (22) \\
\pi_{B_i}^{OL(2)} \;\geq\; & \pi_{B_3}^{OL(3)}, \text{ for } i = 3, .., N & (23)
\end{aligned}
$$

Condition (22) can be rewritten as

$$\frac{1}{K}\left(\frac{K-1}{K}\right)^{N-1} - c \geq \frac{1}{K}\left(\frac{K-1}{K}\right)^{N-1} - p(1) = \max\left\{\frac{1}{K}\left(\frac{K-1}{K}\right)^{N-1} - c, 0\right\}.$$

which holds if and only if $\frac{1}{K}\left(\frac{K-1}{K}\right)^{N-1} - c = c^D - c \geq 0$. Finally, it is immediate to see that condition (23) always holds.■

This completes the analysis of the possible configurations which may arise at equilibrium and hence the proof of Theorem 1. ■

Proof of Proposition 1

The result follows immediately by comparing the threshold for efficient information acquisition, found in (2), with the threshold found in Theorem 1 for information acquisition not to take place in equilibrium, given by (21). We show next that the interval of values of c identified in condition (3) is non empty, i.e.:

$$\frac{1}{K}\left(\frac{K-1}{K}\right) + (N-2)\frac{1}{K}\left(\frac{K-1}{K}\right)^{N-1} < \left(\frac{K-1}{K}\right)\left(1 - \left(\frac{K-1}{K}\right)^{N-1}\right).$$

It is easy to verify that this inequality is equivalent to:

$$\left(1 - \frac{1}{K}\right)^{N-3}\left(1 + \frac{N-3}{K}\right) < 1. \tag{24}$$

Since the term on the left hand side approaches one as $K \to \infty$, it suffices to show that this term is always strictly increasing in K to be able to conclude that (24) holds for all K, N. Notice that a term is increasing if its logarithm is increasing. Taking then the logarithm of the left hand side of (24) and differentiating it with respect to K yields:

$$\frac{(N-3)}{K^2}\left(\frac{1}{\left(1-\frac{1}{K}\right)} - \frac{1}{\left(1+\frac{N-3}{K}\right)}\right) = \frac{(N-3)}{K^2}\left(\frac{\frac{N-2}{K}}{\left(1-\frac{1}{K}\right)\left(1+\frac{N-3}{K}\right)}\right),$$

which is strictly positive since we always have $K > 1$ and $N > 3$. ∎

Proof of Proposition 2

The maximal price a buyer is willing to pay for information when a total number J of buyers stay uninformed is obtained as in the proof of Theorem 1 and again given by $\min\left\{c, (K-1)^{N-J}/K^{N-J+1}\right\}$. By a similar argument we can also show that a result analogous to Claim 1 still holds when information is sold by an agent different from a potential buyer: the revenue from the sale of information of a monopolist seller is maximal when information is sold to all buyers except one, i.e. in this case to $N-1$ buyers. Hence, if information is acquired and sold by a disinterested trader his payoff, equal to this revenue, is:

$$\pi_{Dis} = (N-1)\min\left\{c, \frac{1}{K}\left(\frac{K-1}{K}\right)^{N-1}\right\} - c \geq 0. \tag{25}$$

It is immediate to see from (25) that an equilibrium exists with information acquisition by a disinterested trader if:

$$c \leq (N-1)\frac{1}{K}\left(\frac{K-1}{K}\right)^{N-1}.$$

Hence to prove the result it suffices to show that the threshold found in (21) for information acquisition to take place at an equilibrium when information is sold by a potential buyer is higher:

$$\frac{1}{K}\left(\frac{K-1}{K}\right) + (N-2)\frac{1}{K}\left(\frac{K-1}{K}\right)^{N-1} \quad > \quad (N-1)\frac{1}{K}\left(\frac{K-1}{K}\right)^{N-1}$$

$$\Longleftrightarrow \quad \frac{1}{K}\left(\frac{K-1}{K}\right) > \frac{1}{K}\left(\frac{K-1}{K}\right)^{N-1}$$

which is clearly always true. ∎

Proof of Proposition 3

The payoff of the seller of the good when he acquires and sells information is given by the sum of his information and auction revenues. As already argued in the proof of Proposition 2, the information revenue is maximal when information is sold to all buyers except one. The auction revenue, when the number of buyers remaining uninformed is $J \geq 2$, is either $1/K$ (if no informed buyer likes the object) or 1 (if more than one informed buyer likes the object). Thus, when $J \geq 2$ the auction revenue decreases with J, as the probability that more than one informed buyer likes the object is decreasing in J. Putting this together with the fact that the revenue from the sale of information is also decreasing in J for $J > 0$, we get that the payoff of the seller of the object is higher in configuration $J = 2$ than in any other configuration $J > 2$. To find the optimal choice of the seller it remains so to compare his payoff at $J = 2$ with that at configurations $J = 0$ and $J = 1$.

The payoff when $J = 2$ is[37]

$$(N-2)\frac{1}{K}\left(\frac{K-1}{K}\right)^{N-2} + \left(1 - \left(\frac{K-1}{K}\right)^{N-2} - (N-2)\frac{1}{K}\left(\frac{K-1}{K}\right)^{N-3}\right) +$$
$$\frac{1}{K}\left(\left(\frac{K-1}{K}\right)^{N-2} + (N-2)\frac{1}{K}\left(\frac{K-1}{K}\right)^{N-3}\right) - c = 1 - \left(\frac{K-1}{K}\right)^{N-1} - c.$$

When $J = 1$ the seller's revenue from the auction is either 0 (if no buyer who purchases information likes the object), $1/K$ (if exactly one buyer who purchases information likes the object), and 1 otherwise. Summing the information revenue the seller's payoff is:

$$(N-1)\frac{1}{K}\left(\frac{K-1}{K}\right)^{N-1} + \left(1 - \left(\frac{K-1}{K}\right)^{N-1} - (N-1)\frac{1}{K}\left(\frac{K-1}{K}\right)^{N-2}\right) +$$
$$+\frac{1}{K}\left((N-1)\frac{1}{K}\left(\frac{K-1}{K}\right)^{N-2}\right) - c = 1 - \left(\frac{K-1}{K}\right)^{N-1} - c,$$

identical to the payoff when $J = 2$. When $J = 0$, since the price at which information is sold is zero, the seller's payoff is simply his auction revenue (equal to 0 in this case if no buyer,

---

[37]This expression holds when the cost c is not too low (the case of interest when investigating the threshold for information acquisition), or $c > \frac{1}{K}\left(\frac{K-1}{K}\right)^{N-2}$, in which case information is sold at a price $\frac{1}{K}\left(\frac{K-1}{K}\right)^{N-2}$.

or exactly one, likes the object, and 1 otherwise):

$$1 - \left(\frac{K-1}{K}\right)^N - N\frac{1}{K}\left(\frac{K-1}{K}\right)^{N-1} - c$$

It is then immediate to see that

$$1 - \left(\frac{K-1}{K}\right)^{N-1} > 1 - \left(\frac{K-1}{K}\right)^N - \frac{N}{K}\left(\frac{K-1}{K}\right)^{N-1} \iff \frac{N}{K}\left(\frac{K-1}{K}\right)^{N-1} > \frac{1}{K}\left(\frac{K-1}{K}\right)^{N-1}$$

always holds. Thus, the seller's payoff is maximized by setting the price of information so that $J = 1$ or $J = 2$. Information is acquired in equilibrium if the seller's payoff with $J = 1$ (or 2) exceeds his payoff without information (which for the seller is not 0 but $1/K$):

$$1 - \left(\frac{K-1}{K}\right)^{N-1} - c \geq \frac{1}{K}. \tag{26}$$

The claim of the Proposition follows by comparing the threshold for efficient information acquisition, found in (2), with the one implicitly defined by (26) and verifying the latter is strictly smaller:

$$\left(\frac{K-1}{K}\right)\left(1 - \left(\frac{K-1}{K}\right)^{N-2}\right) < \left(\frac{K-1}{K}\right)\left(1 - \left(\frac{K-1}{K}\right)^{N-1}\right),$$

always true. ∎

CLAIM 4 *When $K - N$ is sufficiently large, or $K < 6$, efficiency is higher if information is provided by a potential buyer rather than by the owner of the object (condition (11) holds). Otherwise, the opposite holds.*[38]

Proof of Proposition 4

Denote by P the price paid to gain the object in the auction and by p the price of information. Let then B_i^{Win} be the event in which buyer B_i wins the auction and E_{B_i} the expectation conditional on B_i's information at stage 2 of the game. The general expression of the payoff of a buyer of information is:

$$\pi_{B_i} = \Pr(B_i^{Win}|v = \theta_i)\Pr(v = \theta_i) - \mathbb{E}_{B_i}(P|B_i^{Win})\Pr(B_i^{Win}) - p,$$

---

[38]The proofs of this claim as well as of Claims 5 and 6 in the proof of Proposition (4) involve some straightforward computations are then relegated to Appendix B, available online at http://www.eco.uc3m.es/acabrales/research/IMAppB.pdf.

where the probabilities clearly depend on the number $J > 0$ of buyers who remain uninformed, on who sells information and his reporting strategy. When there is a monopolist seller of information its price is then:

$$p = \Pr(B_i^{Win}|v = \theta_i)\Pr(v = \theta_i) - \mathbb{E}_{B_i}(P|B_i^{Win})\Pr(B_i^{Win}) - \max\{\pi_{IC}^J, \pi_U^J\}, \qquad (27)$$

where π_{IC}^J and π_U^J are as in the proof of Theorem 1.

Using (27), the equilibrium payoff of the seller of the good when he is also the monopolist seller of information and there are $J > 0$ uninformed buyers (say B_i, $i = N - J + 1, .., N$), both when he knows and does not know buyers' types, is given by the following expression:

$$
\begin{aligned}
\pi_S^{S,J} &= \mathbb{E}_S(P) + \sum_{i=1}^{N-J} \left(\mathbb{E}_S\left(\Pr(B_i^{Win}|v = \theta_i)\Pr(v = \theta_i)\right) - \mathbb{E}_S\left(\mathbb{E}_{B_i}(P|B_i^{Win})\Pr(B_i^{Win})\right)\right) \\
&\quad -(N-J)\max\{\pi_C^J(S), \pi_U^J(S)\} - c = \sum_{i=N-J+1}^{N} \mathbb{E}_S\left(P|B_i^{Win}\right)\Pr(B_i^{Win}) \qquad (28) \\
&\quad + \sum_{i=1}^{N-J} \mathbb{E}_S\left(\Pr(B_i^{Win}|v = \theta_i)\Pr(v = \theta_i)\right) - (N-J)\max\{\pi_C^J(S), \pi_U^J(S)\} - c,
\end{aligned}
$$

where $\mathbb{E}_S$ is the expectation conditional on the seller's information. When $c > \frac{1}{K}\left(\frac{K-1}{K}\right)^{N-J}$, if the seller has no information on buyers' types, as argued in the proof of Proposition 3, $\max\{\pi_C^J(S), \pi_U^J(S)\} = 0$ and the equilibrium payoff of buyers of information is zero, for any $J \geq 1$. Also, whenever there is at least one (indirectly) informed buyer who likes the object one of them always gets it, hence the surplus extracted by the seller of the object in that event equals the total surplus $\Pr(v = \theta_i$, for some $i = 1, .., N - J)$ and (28) reduces to:

$$
\begin{aligned}
\pi_S^{S,J,no\inf o} &= \sum_{i=N-J+1}^{N} \mathbb{E}_S\left(P|B_i^{Win}\right)\Pr(B_i^{Win}) \qquad (29) \\
&\quad + \Pr(v = \theta_i, \text{ for some } i = 1, .., N - J) - c
\end{aligned}
$$

Whenever $J \geq 2$, since $\mathbb{E}_S\left(P|B_i^{Win}\right) = 1/K = \Pr(\theta_i = v)$ for any $i > N - J$, the term $\sum_{i=N-J+1}^{N} \mathbb{E}_S\left(P|B_i^{Win}\right)\Pr(B_i^{Win})$ has the same value whether or not the seller of information has any information over buyers' types. Since, as argued above, the maximal value of $\sum_{i=1}^{N-J} \mathbb{E}_S\left(\Pr(B_i^{Win}|v = \theta_i)\Pr(v = \theta_i)\right) - (N-J)\max\{\pi_C^J(S), \pi_U^J(S)\}$ is $\Pr(v = \theta_i$, for some $i = 1, .., N - J)$ it follows that $\pi_S^{S,J,no\inf o} \geq \pi_S^{S,J,\inf o}$.

We show next that the same is true when $J = 1$. In particular, we will show that, when the seller knows the type of buyers, in equilibrium the uninformed buyer never gets the object, $\Pr(B_N^{Win}) = 0$. Hence the first term of (28) is zero. Recalling that, as argued in the

previous paragraph, the other two terms of (28) are maximal at the equilibrium where the seller has no information over buyers' types, the result follows.

The message subgame is an in Section 2. At an equilibrium with maximal degree of truthfulness of agents reporting the seller adopts the following reporting strategy:

i) if there is at least one pair $i, j \in (1, ..N-1)$ such that $\theta_i = \theta_j$ (i.e. there is a 'tie') :
$$m_S = \begin{cases} v \text{ if } v = \theta_i = \theta_j \text{ for some } i, j \in (1, ..N-1), \\ \theta_i \neq v \text{ if } v = \theta_k \text{ for at most one } k \in (1, .., N-1) \end{cases}$$

ii) if $\theta_i \neq \theta_j$ for all $i, j \in (1, .., N-1)$ (there are no 'ties'):
$$m_S = \begin{cases} v \text{ if } v = \theta_i \text{ for some } i \in (1, ..N-1), \\ \theta_i \neq v \text{ with probability } 1/(N-1) \text{ if } v \neq \theta_i \text{ for all } i = 1, .., N-1 \end{cases}$$
(30)

Thus the seller tells the truth only when at least two buyers of information like the object (there is a tie on the truth) or all buyers like a different type (there are no ties) and one of them likes the object.

To complete the argument for the case $J = 1$ it remains then to verify that

CLAIM **5** *Given the seller's reporting strategy in (30), there is always at least one indirectly informed buyer whose bid is strictly greater than $1/K$, and thus $\Pr(B_N^{Win}) = 0$.*

We are then left with establishing the result for $J = 0$. In such case, from (28) we get

$$\pi_S^{S,0,\inf o} \leq \sum_{i=1}^{N} \mathbb{E}_S \left(\Pr(B_i^{Win}|v = \theta_i) \Pr(v = \theta_i) \right) - c.$$

The equilibrium reporting strategy of the seller is analogous to the one described in equation (30). Thus the only possible misallocation of the object is when only one of the N buyers likes the object and there is a tie on a different type, so that

$$\sum_{i=1}^{N} \mathbb{E}_S \left(\Pr(B_i^{Win}|v = \theta_i) \Pr(v = \theta_i) \right) =$$
$$\Pr(v = \theta_i, \text{ for some } i = 1, .., N) - \Pr(v = \theta_i, \text{ for exactly one } i = 1, .., N \text{ and there is a tie)}$$

As shown in the proof of Proposition 3, in the equilibrium where the seller has no information over buyers' types we have $J = 1$ and the seller's payoff is:

$$\pi_S^{S,no\inf o} = \pi_S^{S,1,no\inf o} = \Pr(v = \theta_i, \text{ for some } i = 1, .., N-1) - c.$$

Hence the result is established by showing:

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$$\Pr(v \;=\; \theta_i, \; \textit{for some } i = 1, .., N - 1) >$$
$$\Pr(v \;=\; \theta_i, \; \textit{for some } i = 1, .., N) - \Pr(v = \theta_i, \; \textit{for exactly one } i = 1, .., N \; \textit{and there is a tie})$$

∎

Proof of Proposition 5 (missing details)

1. It remains here to verify that, when the disinterested trader is the monopolist seller of information, the change in the auction revenue of the owner of the object if he were to enter the market for the sale of information is zero. Entry only changes the bidding behavior of the single buyer (say B_N) who in the monopoly equilibrium is uninformed and in the case of entry also gets a truthful report. Thus the changes in auction revenue are limited to the states where the bid of B_N is pivotal, i.e. where only one of the first $N - 1$ buyers likes the object and B_N, by becoming informed, raises the price from $1/K$ to 1 if he learns that he likes the object and lowers it to 0 if he learns that he doesn't. The expected difference in revenue of the owner of the object is then

$$(N-1)\frac{1}{K}\left(\frac{K-1}{K}\right)^{N-2}\frac{1}{K}\left(1-\frac{1}{K}\right)+(N-1)\frac{1}{K}\left(\frac{K-1}{K}\right)^{N-2}\frac{K-1}{K}\left(0-\frac{1}{K}\right)=0,$$

   Note that the same is true for the duopoly equilibrium where two potential buyers sell information at a zero price since in that case all buyers are fully informed.

2. The change in the auction revenue of the owner of the good if he were to enter (when a potential buyer is an information monopolist) is positive (goes from $1/K$ to 1) when the directly informed buyer and at least another buyer like the object and negative (goes from $1/K$ to 0) when only one directly or indirectly informed buyer likes the object. Thus the expected change in revenue is lower than the cost of entry (and thus entry is not profitable) if:

$$\frac{1}{K}\left(1-\left(\frac{K-1}{K}\right)^{N-1}\right)\left(1-\frac{1}{K}\right)-(N-1)\frac{1}{K}\left(\frac{K-1}{K}\right)^{N-1}\frac{1}{K} < c.$$

∎

Proof of Proposition 6

Most of this proof involves routine computations which are similar in nature to those of Theorem 1 and have then been relegated to Appendix B.[39] The only distinctive aspects are two.

First, buyers of information have now to report their types to the sellers of information, and we have to verify that they choose to report truthfully.

Consider an arbitrary buyer B_i, purchasing a report of type l. If B_i reports something different from his true type his payoff is not affected when the seller of information or any buyer purchasing a report of higher quality $j > l$ like the object, because in that case they always bid 1 and gain the object. On the other hand, when no buyer who has information of higher quality than B_i likes the object, a lie of B_i may affect the report sent to the buyers purchasing reports of lower quality; in particular in the event where B_i likes the object some of the buyers below him in the hierarchy may still be told the truth and may then make a higher bid. It thus follows that the buyer of information cannot gain, and may actually lose, by misreporting his type.

Next, we need to establish the following:

LEMMA **1** *The optimal choice of the seller of information concerning the degree of differentiation of the information sold is always to have as many types of reports as the number of buyers of information.*

Proof. Suppose there are two buyers purchasing the same type of report, say l. To establish the result we show that the seller's payoff always increases by introducing some differentiation in the report sold to each of them, that is if layer l of the hierarchy is split into two adjacent ones, $l' < l''$:

1. The price paid by the seller in the auction does not change.

2. Buyers' willingness to pay for reports of a quality different from l does not vary, since the payoff of a buyer in some layer i only depends on the total number of other buyers in his same layer or above it, not on their distribution across such layers, and the first one is not affected by the split.

3. By the same argument, the willingness to pay for the lower quality report l'' is the same as the one for report l before the split, while that for report l' is strictly higher.∎

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[39]Available online at: http://www.eco.uc3m.es/acabrales/research/IMAppB.pdf.

Proof of Proposition 7

To establish the result we only need to show that there is always an equilibrium of the subgame with two sellers of information where they both make strictly positive profits. Suppose the first seller offers a complete hierarchy of reports, one for each of the $N-2$ buyers of information, and charges a price equal to zero for the lowest quality report and a positive price equal to half the maximal willingness to pay of a buyer for every other report. Then we claim that the best response of the second seller is to do exactly the same. If he does that, any buyer who considers purchasing a report of any quality (except the lowest one) will buy it from both sellers. Purchasing the report only from one seller does not yield in fact a priority level over all the buyers who purchase reports of lower quality, but is equivalent to purchasing the lowest quality report. Such priority level is only attained if the same type of report is bought from both sellers. By replicating the strategy of the first seller, the second seller shares so all buyers with him and obtains a positive revenue from the sale of information, approximately equal to half the monopolist revenue, and a positive payoff from the auction (as he can get the object at a zero price whenever he likes it and the other seller does not like it). Any other strategy meant to attract buyers only to the second seller is clearly less profitable.

Having shown that the profits of the two sellers are strictly positive it follows that, provided c is not too high, so will be their total payoff, net of c.■

Proof of Proposition 8

If the claim does not hold there must be at least two messages m' and m'' such that $\Pr(k=i|m')=1=\Pr(k=i|m'')$ and $\Pr(q=H|m')>\frac{1}{2}>\Pr(q=H|m'')$. That is, both m' and m'' truthfully reveals the variety of the object is i and, in addition, they reveal something concerning the quality of the object. In this situation the owner of the good could induce a higher bid from the Se buyers, and hence increase his profits[40], by always announcing m' rather than m'', without affecting the bid of the In types, who do not care about quality. Since the seller of information chooses always to lie when he can profit by doing so, m'' would never be sent in equilibrium, a contradiction.

This establishes the first part of the claim. It is then immediate to see that, if $\Pr(k=i|m)=1$ for some $i \in \mathcal{K}$ and $\Pr(q=H|m)=\frac{1}{2}$, when $V>2$ all Se buyers who like variety i always bid more for the good than In buyers who like the same variety, and vice versa when $V<2$. ■

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[40]More precisely, the seller's profits strictly increase if $\Pr(q=H|m')V>1$. When this condition does not hold the properties of the equilibrium are the same as when $\Pr(q=H|m')=\Pr(q=H|m'')=1/2$.