EUROPEAN UNIVERSITY INSTITUTE
Department of Economics

Three Essays on Labor Markets

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Thesis submitted for assessment with a view to obtaining the degree of Doctor of Economics of the European University Institute

Florence
December 2007

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Acknowledgements

It is definitely a tremendous joy to reach this point of thanking to someone who somehow helped to reach the end. Next to institutional setting and financial support by the European University Institute, the "Ministerio de Educación y Ciencia" and the "Ministerio de Asuntos Exteriores", it has been the presence of individual people what has allowed me to accomplish this complex task.

I am specially grateful for the support I received from my supervisor Morten O. Ravn. He threw me the rope that got me out of the well. Without his demanding advises and guidance this would never had been more than a lonely fruitless journey. It is very difficult to find an economist with such an economic intuition, with such a broad knowledge of so many areas of economics, and with such a capacity of passing them to you.

I also benefited from the experience of Andrea Ichino in the research related to labor markets. Omar Licandro and Giancarlo Corsetti contributed in important ways to my research, their fast comprehension and knowledge of the literature helped me substantially. The network provided by the European University Institute, allowed me to benefit from excellent researchers such as Christopher Pissarides and Ken Judd in two areas specially relevant in my research: labor markets and numerical methods. Thanks to them as well.

Four years of one's life is definitely an unforgettable period, where a lot of things happen, both good and bad. Elvira Prades and Irina Balteanu shared all of them with me and were always there for me. They were at the same the comforting shoulder in the end of the day and the daily economics discussion group.

Along this time I interact with a large number of colleges. I am grateful for the discussions I had with Pontus Rendahl and Karel Mertens. But I am also strongly indebted to Stephan Fahr and Matthias Hertweck for critical and thorough examinations of my research. Thank you also to Marta Arespa, Judith Ay, Aitor Erce, Alain Gabler, Andrea Gallice, Juan Gonzalez, Aysen Isaoglu, Christian Kascha, Tomasz Kozluk, Christopher Milde, Katrin Rabitsch, Claudia Trentini and Sanne Zwart.

Most importantly, thank you to my entire family. To my parents, Miguel and Marivi, for their love and discipline, for their advice in overcoming problems and difficulties. Special thanks also to Cristina, María and Juampa. They specially encouraged me and contributed to make my life happier and easier. I wish they know how much I benefited from them and how pleased I am to have them.

Finally, I am deeply thankful to Jorge Santos for his patience and support during all this period. Unfortunately he saw less and worse of me than he deserved. Fortunately, we have the rest of our lives to compensate those difficult moments.

Introduction

The three chapters presented in the following apply the concept of general equilibrium to topics related to labor markets.

The first chapter "Decentralization of wage bargaining" focuses on the changes in collective bargaining institutions that a number of European countries have witnessed in the last decades, with a tendency towards more decentralized wage negotiations, especially in the Scandinavian countries. In particular, this first chapter analyzes the reason why centralized systems of wage bargaining that have performed very well in terms of macroeconomic variables, collapse. We construct a general equilibrium model with matching in the labor market and include a federation of unions, which are coalitions of heterogeneous workers, whose role is to bargain wages for the workers with the firms taking into account redistributive issues, so as to create a framework able to replicate the Scandinavian labor markets for the seventies. We show that the collapse of this system is a consequence of a skill-biased technical change that increases the differences across workers making this kind of coalition unsustainable.

The second chapter "An estimated DSGE-matching model for the US economy" estimates via maximum likelihood a DSGE model using US data. The theoretical model is an extended version of the RBC Andolfatto (1996) model of frictional labor markets, in which beyond the standard neutral technology shock we have introduced a preference shock in the utility function, an investment-specific technology shock and a job-separation shock. Once estimated, we perform a variance decomposition analysis to identify which shocks are driving the cyclical fluctuations of the main variables of the model. The results show that the neutral and the investment-specific technology shocks explain most of the fluctuations of the variables of the model; and that the shock to job destruction is successful in explaining the variance of tightness.

The third chapter "A RBC model with unemployed loss of skills" proposes a model of frictional labor markets with two types of workers: high-skilled and low-skilled workers, where high-skilled workers may suffer from a depreciation of their human capital while unemployed. We estimate the parameters of the model via maximum likelihood and analyze the its cyclical properties. We also contribute to the literature that tries to explain the different performance of European and US unemployment reconciling the macro and micro evidence.

Chapter 1

Decentralization of wage bargaining

1.1 Introduction

Wage bargaining can take place at different levels. At one extreme we find decentralized wage bargaining systems in which workers and employers negotiate over wages and working conditions at the firm level while at the other extreme, national unions and employers' associations bargain for the whole country in what is called centralized wage bargaining. An intermediate case is the sectoral, branch or industry-level.

During the 1980s, there was a growing interest in explaining the macroeconomic consequences of different wage-bargaining systems. Highly centralized systems of wage bargaining lead to low unemployment and inflation rates and, in general, to a good economic performance. The well known Calmfors and Driffill (1988) "bell-shaped" curve summarizes very well the conclusion of this research: highly centralized and highly decentralized systems of wage bargaining outperform intermediate ones in terms of macroeconomic variables.

The explanation this literature gives for the macroeconomic success of economies like the Nordic ones or Austria with centralized levels of wage bargaining relies on the cooperative behavior of the negotiations. Centralized systems have the advantage of encompassing all workers and firms in the economy and this allows them to take different macroeconomic considerations into account, in particular, they can internalize bargaining externalities (for a complete survey Calmfors (1993)). In general, real wage increases for a certain group of workers have negative externalities on other groups of agents in the economy. The cooperative behavior means that the effects on others of claims of higher wages are considered and the incentives for real wage restraints are strengthened.

But last decades have witnessed substantial changes in the patter of unionization and wage bargaining in the OECD countries. According to the 1997 OECD's Employment Outlook, "*recent years have seen quite substantial changes in some countries' collective bargaining institutions*". And although the pattern has not been uniform across all OECD countries, during the 1980s the main level of interaction in industrial relations shifted from national to industrial level and from industries to individual firms. In most continental Europe several indeces of coordination and centralization in bargaining institutions show a trend towards more decentralized wage negotiations, especially in the Scandinavian countries, where the level of centralization was indeed the highest.

It is therefore, very surprising, that bargaining systems that are superior to others in terms of macroeconomic performance collapse. In this paper we analyze the determinants of the collapse of those centralized bargaining systems.

Recently, new hypothesis for deunionization and decentralization in union's wage setting based on skill biased technical change have been advanced by Acemoglu, Aghion and Violante (2001) and Ortigueira (2004). Their arguments rely on the view that unions are coalitions of heterogeneous workers which extract rents form employers and only exist as far as members have an incentive to stay in the coalition and continue bargaining in a centralized fashion. The hypothesis these authors present is that a skill-biased technical change can dramatically alter such incentives[1].

Acemoglu, Aghion and Violante (2001) explain the shift from centralization towards decentralization in the wage setting through the impact that this skill-bias has on wage compression in the sense that "*skill-biased technical change increases the outside option of skilled workers, undermining the coalition among skilled and unskilled workers in support of unions*"

We construct a general equilibrium model with matching in the labor market and heterogeneous workers. In our benchmark model there is a federation of unions whose role is to bargain worker's wages with firms taking into account redistributive issues. The fact that the federation of unions encompasses all workers allows it to take any macroeconomic consideration into account[2]. In this paper we assume that a centralized union can internalize the search externality generated in the labor market, in contrast to other papers that have modellized Nordic labor markets before see e.g. Cukierman and Lippi (1999) and Ortigueira (2004). To internalize this externality means that we assume that unions can potentially generate an efficient-enhancing role in the economy.

We introduce skill biased technical change shock into the economy and analyze its consecuences for bargaining and for welfare. To do this exercise we examine three different scenarios: (1) one scenario with decentralized levels of wage bargaining in which there are no unions and individual workers barging their wages directly with the firm, (2) another scenario with intermediate levels

[1] For a survey of the impact of technical change on labor market see Hornstein, Krusell and Violante (2005)

[2] Calmfors and Drifill (1988) are able to identify different externalities that can be internalized when negotiating wages in a centralized way. The most common ones are the effects that wage increases in one part of the economy generate on price rises of intermediate or final products and the effects that wage increases have on the unemployment rate, and consequently the tax burden to maintain the welfare system.

of wage bargaining in which the wages of homogeneous workers are bargained by a union or institution that takes into account the congestion that an additional worker creates over other workers of the same type but not over the whole working labor force and (3) the already mentioned economy with centralized levels of wage barging with a federation of unions that encompass all types of workers and bargain their wages taking into account the congestion generated in the market and redistributive issues.

The main result of the paper is that after a skill biased technical change (or SBTC) has affected the economy, there are other systems for negotiating wages different from the centralized one, that result more appealing for certain types of workers, in particular for those who benefit more from the technical change. Thus, this simple model help us to understand why centralized systems of wage bargaining collapse in spite of their excellent economic performance.

The structure of the paper is the following: in section 2, we present and justify the main assumptions of our model. In section 3 we present the model and define the equilibria under three different scenarios depending on the level of coordination in the wage setting process: decentralized, intermediate and centralized. In section 4 we will calibrate the model and in section 5 present our results, and we conclude in section 6.

1.2 Main assumptions

Our paper is based on Acemoglu, Aghion and Violante (2001). We adopt the main assumptions of their model and we embed them into the search matching framework. Those assumptions are that (1) Unions exist and they provide some benefits either to the society or to some group of workers, (2) wage compression across workers with different skills is a characteristic of unions and (3) there is a skill-biased technical change.

The assumption that unions provide some benefits to the society can be justified in economies with high levels of unionization and centralization. There are two main streams in the literature of trade unions. The traditional one, focus on the view of unions as rent-seeking institutions, i.e., as organizations that coordinate workers in order to extract rents from the employers. In this framework, unions control the labor supply and end up distorting relative prices and reducing employment (see eg. McDonald and Solow (1981), Johnson (1990), Farber (1986)). From this perspective, unions generate a bad economic performance and cause efficiency losses. The second stream of the literature on trade unions starts from the work of Hirschman (1970) who questioned whether unions are a source of inefficiency given their presence in so many countries and the empirical evidence suggesting that high levels of unionization lead to a lower rates of unemployment. These caveats lead to another approach in which unions are seen as efficiency-enhacing entities, which arise as a response to a particular form of market imperfection or an inadequate insurance against labor risks.

Freeman and Medoff (1984) provide empirical support to the fact that on net, unions are beneficial for the society because although it is true that they exert some monopoly power, this negative aspect can be outweighed by the beneficial effects they have on efficiency such as income distribution, social organization, reduction of labor turnover, etc. Other authors justify the existence of unions as a response to an inadequate insurance against labor risks, see e.g. Malcomson (1983), Agell (2000) and Hogan (2001). In a sense, the union is seen as a substitute for legal contractual enforcement and can be used to promote more efficient levels of employment when legal contractual enforcement is unavailable. Checchi and Lucifora (2002) view unions as economic agents that supply *private and collective services to their members and perform useful roles, not fulfilled by markets or government institutions.* These services are substitutes for state's provision or certain labor market institutions.

In our set-up, we introduce a potential for efficiency gains from unions by assuming that they can internalize the search externalities.

In models of search and matching, firms post vacancies and unemployed workers search for jobs, and the outcome of a match between a vacancy and a searcher is a productive job. Firms and workers behave uncoordinatedly, dedicating time and effort to the search of a partner. The probability that a firm or an unemployed worker find a partner depends on the relative number of vacancies and searchers. For example, an increase in the number of vacancies relative to the number of searchers increases the probability that an unemployed worker finds a job but reduces, at the same time, the probability that a vacancy get filled. This example shows that there is an externality in the market. Due to the fact that this externality is generated by the search activity, it is normally called a search externality[3].

Secondly, we introduce intra-union redistribution which leads to less wage inequality the more centralized is the bargaining in the economy (see eg. Freeman (1988) and Rowthorn (1992)). In

[3]Definition extracted from Bagliano and Bertola (2004)

general, collective bargaining agreements limit the ability of the firm to remunerate individual workers differently and, therefore, this form of setting wages called union "rate standardization policy" reduce wage dispersion considerably.

Furthermore, very centralized systems like the Scandinavian ones are clear examples of how high degree of centralization and low wage dispersion go hand in hand. Especially in Sweden where egalitarian wage policies were explicitly adopted by the central union confederation (see eg. Flanagan, Soskice and Ulman (1983), Flanagan (1987) or Siven (1987)). This tendency of reducing wage dispersion existed until the early eighties when wage negotiations became more decentralized

Although no fully-fledged theory about the impact of centralization on wage differentials has been built, several arguments have explained this wage compression. Among the most relevant we find Freeman (1980) and Agell and Lommerud (1992). The former explanation focuses on the political economy theory and suggests that if the union wage policy is decided by the median member, when there are differences in productivity across members and most of them are at the bottom of the distribution, one should expect a compression of wage differentials; whereas the latter explanation relies on the rawlsian "ignorance veil" to explain that if workers are risk averse and do not know their future level of skill, then they are willing to trade some low skill unemployment against reduced wage differentials. Nevertheless, as Calmfors (1993) points out, the most common argument why higher degree of centralization should reduce wage dispersion is that the distribution of wages enters the utility function of unions and members.

Finally, skill-biased technical change is defined by a change in productivity that is biased by favouring workers with higher levels of education and skills over those with lower levels. This bias occurs because the introduction of a new technology will increase the demand for workers whose skills and knowledge complement that technology. It is generally accepted that OECD countries have suffered this type of shock.

1.3 The model

In this section we present a simple model of frictional unemployment and define the equilibrium under different levels of centralization in the wage bargaining process Unions are coalitions of workers whose main role is to negotiate wages with firms taking into account the congesting effect that an additional searcher generates over the set of searchers already existing.

1.3.1 Description of the model

Workers and preferences

Workers are heterogeneous, in particular we assume that they differ in skills. We assume that there are two skill groups: skilled (s) and unskilled (u). The measure of type-j workers is denoted by x_j, for $j = s, u$ and the total measure of workers is normalized to one ($x_u + x_s = 1$). Workers are risk neutral. We assume the existence of two representative households of size x_s and x_u each. A household j, for $j = s, u$ solves the following problem,

$$Max \sum_{t=0}^{\infty} \beta_t c_{j,t} \tag{1.1}$$

where β lies between zero and one and consumption, $c_{j,t}$ equals the total wage bill $w_{j,t} n_{j,t}$. Employment, $n_{j,t}$ is a predetermined variable whose law of motion is given by

$$n_{j,t+1} = n_{j,t} - \lambda_j n_{j,t} + m_{j,t} u_{j,t} \tag{1.2}$$

where $u_{j,t}$ denotes the measure of type-j searchers, $\lambda_j > 0$ is the rate of job destruction and $m_{j,t}$ is the perceived probability that an unemployed worker of type j be matched in period t. This probability is defined as the total number of matches of type j over the set of searchers of the same type:

$$m_{j,t} = \frac{M_{j,t}}{u_{j,t}} \tag{1.3}$$

Capitalists

The owners of capital and firms are called capitalists. We assume that they are risk neutral and their only decision is to split current income between consumption, c_t, and investment, i_t. Their objective is to maximize the discounted lifetime consumption of the aggregate good. Capitalists' income is made up of capital income and firm's profits. Thus, capitalists' consumption at time t, is determined by the budget constraint,

$$c_t + i_t = r_t k_t + \pi_t \tag{1.4}$$

where i_t denotes, more specifically gross investment and π_t denotes firms' profits. Capital depreciates at rate δ, and the law of motion for capital is:

$$k_{t+1} = (1-\delta)k_t + i_t \tag{1.5}$$

Hence, it is straightforward to show that the optimal investment policy for the capitalists calls for the standard:

$$1 + r_t - \delta = \frac{1}{\beta} \tag{1.6}$$

where r_t denotes the rental price of capital.

Firms

The production sector is made up of a large number of identical competitive firms. There is a representative firm which uses capital and the two types of labor to produce the aggregate good. The production technology is represented by $F(k_t, n_{st}, n_{ut})$, where F is strictly jointly concave, twice continuously differentiable and increasing. Further assumptions on the elasticity of substitution between the two types of labor will be imposed below.

Since the labor market is frictional, the law of motion of the firm's stock of employment is given by

$$n_{j,t+1} = n_{j,t} - \lambda_j n_{j,t} + \mu_{j,t} v_{j,t} \tag{1.7}$$

where $\mu_{j,t}$ is the perceived probability that a vacancy of type-j be matched with an unemployed worker of the same type and λ_j is the exogenous destruction rate for type-j workers

The firm hires capital and open vacancies to maximize the present value of cash flows,

$$\sum_{t=0}^{\infty} \frac{1}{\prod_{\tau=0} R_\tau} \left[F(k_t, n_{st}, n_{ut}) - r_t k_t - \sum_{j=s,u} w_{j,t} n_{j,t} - \sum_{j=s,u} a_{j,t} v_{j,t} \right] \tag{1.8}$$

subject to equation (7). $R_\tau = 1 + r_\tau - \delta$ is the gross rate of return, and $a_{j,t} v_{j,t}$ denotes the cost of opening $v_{j,t}$ vacancies of type-j.

The firm's demand for capital obeys the standard optimality condition,

$$F_k = r_t \tag{1.9}$$

where F_k denotes the marginal productivity of capital and r_t denotes the rental price of capital. The condition that determines the optimal number of vacancies of type-j at period t is given by

$$\mu_{j,t} J_{j,t} = a_{j,t} \tag{1.10}$$

where $J_{j,t}$ is the income value of type-j employment to the firm. This latter value satisfies the following arbitrage condition

$$(r_{t+1} - \delta) J_{j,t} = J_{j,t+1} - J_{j,t} - \lambda_j J_{j,t+1} + (Fn_{j,t+1} - w_{j,t+1}) \tag{1.11}$$

where F_{n_j} denotes the marginal productivity of type-j labor. This arbitrage equation establishes that the capital cost of the job, $(r_{t+1} - \delta)J_{j,t}$, must equal the job's net profit flow, $F_{n_{j,t+1}}(k_t, n_{st}, n_{ut}) - w_{j,t+1}$, plus capital gains, $J_{t+1} - J_t$, net of the risk of losing the job, $\lambda_j J_{t+1}$.

Matching

The total number of matches for a type of worker j taking place per unit of time is given by the matching function:

$$M_{j,t} = m(u_{j,t}, v_{j,t}) \tag{1.12}$$

where $u_{j,t}$ represents the total number of type-j searchers and $v_{j,t}$ the total number of vacancies of type-j.

We assume that the matching function is increasing in both arguments, concave and homogeneous of degree one, and that the total number of type-j matches satisfy the following condition

$$M_{j,t} < \min(u_{j,t}, v_{j,t})$$

which means that it cannot be greater than the number of type-j searchers in the economy or the number of type-j vacancies posted by firms.

1.3.2 The equilibrium

We analyze three types of equilibria: equilibrium in an economy with decentralized levels of wage bargaining, equilibrium in an economy with intermediate levels of wage bargaining and equilibrium in an economy with centralized levels of wage bargaining. The difference across them relies on the existence or not of collective bargaining and the redistribution or not among types of workers.

Collective bargaining is a process of decision making between parties representing the employer's and the employee's interests. Depending on whether the union, which by assumption internalizes the search externality, represents the interests of one type of worker or instead encompasses both types, we will be under the intermediate or the centralized case, respectively. The decentralized equilibrium implies that workers bargain over wages directly with the firms.

Decentralized equilibrium: the economy without unions

In the economy without unions, when a worker decides to engage in looking for a job, he does not take into consideration the effects that his search exerts on the probability of other searches of being matched. This means that workers of type j take the probability of finding a job or arrival rate to a job, $m_{j,t}$, parametrically. Therefore, under this equilibrium, individual workers do not take into account that their own search congest the market and prejudice other workers.

We follow the standard literature on frictional unemployment and assume that wages are the solution to Nash-bargaining. The Nash solution maximizes the weighted product of the worker's and the firm's income values of employment. Hence, if we use p to denote the worker's bargaining power, the wage rate is,

$$w_{j,t} = \arg\max \left\{ W_{j,t}^{p} J_{j,t}^{(1-p)} \right\} \tag{1.13}$$

The first order condition to this maximization problem is

$$W_{j,t} = p(W_{j,t} + J_{j,t}) \tag{1.14}$$

which states that the worker will get a share p of the total income generated by the match.

The value of employment for the household j, $W_{j,t}$ solves the following arbitrage condition

$$(r_{t+1} - \delta)W_{j,t} + m_{j,t+1}W_{j,t+1} = w_{j,t+1} + W_{j,t+1} - W_{j,t} - \lambda_j W_{j,t+1} \tag{1.15}$$

This arbitrage equation establishes that the capital cost of a job,$(r_{t+1} - \delta)W_{j,t}$, plus the opportunity cost ,$m_{j,t+1}W_{j,t+1}$, must equal the yield of holding the job, which is made up of the wage rate, $w_{j,t+1}$ plus capital gains, $W_{j,t+1} - W_{j,t}$ net of the risk of losing the job, $\lambda_j W_{j,t+1}$.

The income value of type-j employment for a firm is given by equation (1.11). Therefore, the optimal wage rate at time t is given by:

$$w_j = pFn_j + pa_j \frac{v_j}{u_j} \text{ for } j = s, u \tag{1.16}$$

which means that workers are rewarded for their contribution to output and for the saving of hiring costs that the representative firm enjoys when a job of type-j is formed.

We can define the equilibrium for this economy, the decentralized bargaining equilibrium, as a set of infinite sequences for the rental price of equipment $\{r_t\}$, wage rates $\{w_{ut}, w_{st}\}$, employment levels $\{n_{ut}, n_{st}\}$, capital $\{k_t\}$, vacancies $\{v_{u,t}, v_{s,t}\}$, arrival rate $\{m_{u,t}, m_{s,t}\}$ and matching rates $\{\mu_{ut}, \mu_{st}\}$ such that,

(i) Taking the rental prices and matching rates as given, $\{k_t\}$ and $\{v_{u,t}, v_{s,t}\}$ maximize the firms' profits.

(ii) Taking the rental price of equipment as given, $\{k_t\}$ maximizes capitalists' lifetime utility.

(iii) Wages are the Nash solution to uncoordinated bargaining problems.

(iv) Taking wages and matching rates, $\{n_{jt}\}$ and $\{c_{jt}\}$ solve the workers' optimization problem.

(v) Matching rates and arrival rates are given by the matching function.

Intermediate equilibrium: the economy with unions

Here we assume the existence of two unions, one for each collar line. Thus, each union is formed by homogeneous workers. The assumption that unions internalize the search externality generated in the labor market means, under this equilibrium, that unions internalize the congestion that the search for a job of a particular agent has on the other searchers's behavior of the same type. This means that now, in contrast to the decentralized equilibrium, the probability for a worker of type j finding a job, $m_{j,t}$ is not taken parametrically. Instead the union has the capacity of setting wages considering the effect that an additional worker provoke on the probability of the others of finding a job.

The income value of employment for the union of workers of type j is now:

$$(r_{t+1} - \delta)W_{j,t} + \eta \cdot m_{j,t+1} W_{j,t+1} = w_{j,t+1} + W_{j,t+1} - W_{j,t} - \lambda_j W_{j,t+1} \tag{1.17}$$

where again the capital cost of a job,$(r_{t+1} - \delta)W_{j,t}$, plus the opportunity cost ,$\eta m_{j,t+1} W_{j,t+1}$, must equal the yield of holding the job, which is made up of the wage rate, $w_{j,t+1}$ plus capital gains, $W_{j,t+1} - W_{j,t}$ net of the risk of losing the job, $\lambda_j W_{j,t+1}$.

The difference between this condition and the one obtained under the decentralized equilibrium comes from the opportunity cost and the wages. The opportunity cost is lower, given that $0 < \eta < 1$ because the union internalizes the search externality and makes easier to find a job, at the cost of a lower wage, which can be obtained by substituting the value functions into the first order condition of the wage maximization problem. This yields to the optimal wage rate for the type-j worker:

$$w_j = pFn_j + p\eta a_j \frac{v_j}{u_j} \text{ for } j = s, u \tag{1.18}$$

which means, that in this scenario the type-j worker is rewarded for his contribution to output and for the saving in the hiring costs the firm enjoys when the match is created. The difference in wages with respect to the decentralized equilibrium relies on the parameter η.

We can define the equilibrium for this economy or intermediate bargaining equilibrium, as a set of infinite sequences for the rental price of equipment $\{r_t\}$, wage rates $\{w_{ut}, w_{st}\}$, employment levels $\{n_{ut}, n_{st}\}$, capital $\{k_t\}$, vacancies $\{v_{u,t}, v_{s,t}\}$, arrival rates $\{m_{ut}, m_{st},\}$ and matching rates $\{\mu_{ut}, \mu_{st}\}$ such that,

(i) Taking the rental prices and matching rates as given, $\{k_t\}$ and $\{v_{u,t}, v_{s,t}\}$ maximize the firms' profits.

(ii) Taking the rental price of equipment as given, $\{k_t\}$ maximizes capitalists' lifetime utility.

(iii) Wages are the Nash solution to uncoordinated bargaining problems.

(iv) Taking wages and matching rates, $\{n_{jt}\}$ and $\{c_{jt}\}$ solve the representative households' optimization problem.

(v) Matching rates and arrival rates are given by the matching function.

Centralized equilibrium: the economy with a union federation

Now we assume the existence of a union federation that encompasses all sectoral unions in the economy. The role of the federation is to negotiate wages for both types of workers taking into account the congestion that the search activity generates. The union federation is also worried about the distribution and redistribution of income, therefore in its objective function we not find the sum of utilities of each type of household but their weighted utilities according to the following specification of the welfare function:

$$Max \sum_{t=0}^{\infty} \beta^t \left[c_{u,t}^{\alpha} c_{s,t}^{1-\alpha} \right] \tag{1.19}$$

The income value of employment for the household $j, W_{j,t}$ solves the following arbitrage condition:

$$(r_{t+1} - \delta)W_{j,t} + m_{j,t+1}\eta W_{j,t+1} = \theta_{t+1} w_{j,t+1} + W_{j,t+1} - W_{j,t} - \lambda_{,j} W_{j,t+1} \tag{1.20}$$

where again the capital cost of a job,$(r_{t+1} - \delta)W_{j,t}$, plus the opportunity cost ,$\eta m_{j,_{t+1}} W_{j,_{t+1}}$, equals the job's yield made up of the wage rate, $w_{j,t+1}$ plus capital gains, $W_{j,t+1} - W_{j,t}$ net of the risk of losing the job, $\lambda_j W_{j,t+1}$ The difference with the intermediate equilibrium relies in the wages, there the wages are set so as redistribute income from the high-skilled workers to the unskilled according to the parameter θ_{t+1} is the shadow price of consumption.

$$\theta_{t+1} = \alpha \left(\frac{c_{s,t+1}}{c_{u,t+1}} \right)^{1-\alpha} = (1-\alpha) \left(\frac{c_{u,t+1}}{c_{s,t+1}} \right)^{\alpha} \tag{1.21}$$

The optimal wage rate for a type-j worker is given by the following expression:

$$w_j = \frac{1}{\theta(1+p) + p} \left(pFn_j + p\eta a_j \frac{v_j}{u_j} \right) \text{ for } j = s, u \tag{1.22}$$

We can define the equilibrium for this economy or centralized bargaining equilibrium, as a set of infinite sequences for the rental price of equipment $\{r_t\}$, wage rates $\{w_{ut}, w_{st}\}$, employment levels $\{n_{ut}, n_{st}\}$, capital $\{k_t\}$, vacancies $\{v_{u,t}, v_{s,t}\}$, arrival rates $\{m_{ut}, m_{st}\}$ and matching rates $\{\mu_{ut}, \mu_{st}\}$ such that,

(i) Taking the rental prices and matching rates as given, $\{k_t\}$ and $\{v_{u,t}, v_{s,t}\}$ maximize the firms' profits

(ii) Taking the rental price of equipment as given, $\{k_t\}$ maximizes capitalists' lifetime utility

(iii) Wages are the Nash solution to uncoordinated bargaining problems

(iv) Taking wages, matching rates and weights, $\{n_{jt}\}$ and $\{c_{jt}\}$ solve the representative households' optimization problem

(v) Matching rates and arrival rates are given by the matching function

1.4 Calibration

We calibrate our model under the centralized equilibrium, which is the one that correspond to the situation presented by the Scandinavian countries during the period in which wages were bargained in a centralized way. Once we will have all the parameters, we will use them to compute the other scenarios. We will use them then as a laboratory able to analyze the impact of a skill-biased technical change.

The first step is to choose the functional forms for the matching function and the production technology. The total number of matches at time t, M_t is given by a Cobb-Douglas matching function in the total number of searchers $u_{j,t}$, and vacancies, $v_{j,t}$

$$M_{j,t} = M_{j,o} \left(u_{j,t}\right)^{\eta} \left(v_{j,t}\right)^{(1-\eta)} \tag{1.23}$$

where η determines the elasticity of the matching technology with respect to unemployment and with respect to vacancies $(1-\eta)$. The reason for this choice is the empirical literature on frictional labor markets which finds that the Cobb-Douglas specification of the matching function fits the well the data.

The production function is the one proposed by Heckman, Lochner and Taber (1998), which is a CES function in capital and labor with a CES function on the two types of labor:

$$F(k, n_s, n_u) = \left[a_2 k^{\rho_2} + (1-a_2)(a_1 n_s^{\rho_1} + (1-a_1) n_u^{\rho_1})^{\rho_2/\rho_1}\right]^{1/\rho_2} \tag{1.24}$$

where $\frac{1}{1-\rho_2}$ denotes the elasticity of substitution between capital and aggregate labor, and $\frac{1}{1-\rho_1}$denotes the elasticity of substitution between skilled and unskilled labor. With this specification, the skill-biased technical change is represented by changes in the parameter a_1.

The second step is to assign values to all the parameters in the model. We set in advance as many parameters as possible using a priory information and data for the Swedish economy for the period 1970-1980, period that corresponds to the functioning of a fully fledge centralized system of wage bargaining. We set the discount factor equal to 0.95. The rate of depreciation of capital, δ, is calibrate so that it corresponds to an annual interest rate of 13 per cent. Following Heckman, Lochner and Taber (1998) the estimated elasticity of substitution between capital an labor is not statistically significantly different from 1, which implies a value for ρ_2 approximately equal to zero and a value for ρ_1 that corresponds to an elasticity of substitution between the two types of labor of 1.25, these values will help us to calibrate the other two parameters in the production function, a_1 and a_2 The remaining parameters, which correspond to the cost of posting a vacancy for the unskilled workers, a_u, and for the skilled workers a_s, the exogenous destruction rate for unskilled and skilled workers, λ_u and λ_s, the elasticity of the matching technology with respect to unemployment, η, the bargaining power of the federation, p, and the weight given to the unskilled workers by the federation, α_u are calibrated so as to match the following average values in equilibrium: an unemployment rate for the skilled and unskilled workers of 0.5% and 2.3%, a capital share on income of 30 per cent and a log-wage differential of 0.5. The number of unskilled workers is set in such a way that the fraction of labor force with university degree would be 5 per cent of the total population. Thus, we assume that $x_s = 0.05$ and $x_u = 0.95$.

Thus, the parameter values used in the model are presented in the following table:

Workers	Capitalists	Technology	Matching
$x_u = 0.95$	$\beta = 0.95$	$\rho_1 = 0.209$	$a_u = 0.072$
$x_S = 0.05$	$\delta = 0.08$	$\rho_2 = 0.002$	$a_s = 0.104$
$p = 0.6$	$r = 0.13$	$a_1 = 0.14$	$\lambda_u = 0.02$
$\alpha_j = 0.93$		$a_2 = 0.04$	$\lambda_s = 0.05$
			$\eta = 0.5$

The fact that the parameter of the matching function $\eta = 0.5$ differs from the bargaining power $p = 0.6$, imply there is no symmetry between unions and firms, and therefore is consistent with our assumption that unions extract rents from the firms. It implies as well that the Hosios condition for efficiency does not hold and yields room for the possibility of talking about efficient unions as entities which internalize the externality that searchers generate among themselves.

1.5 Results

We now examine the consequences of a skill-biased technical change that favor workers with higher levels of education, so that firms will increase the demand for this specific type of worker. In our model, this will be materialized by an increase in the marginal productivity of skilled workers relative to the unskilled, equivalent to an increase in the parameter a_1 of the production function. To properly choose the values of this parameters, we focus again on the Swedish economy but for a different period, in particular the late eighties when the technical change has already affected the economy. The new values for x_s and x_u will be 0.15 and 0.85, respectively. This means that now 15 per cent of the labor force possesses a university degree, in contrast to the 5 per cent before the skill-biased technical change shocked the economy. The parameter a_1, which measures the intensity of the shock moves from 0.14 to 0.23.

With these new values and keeping constant the remainder parameters, we calculate the welfare for each type of worker under the three different scenarios and also before and after the skill-biased technical change has shocked the economy to analyze whether there is any difference or substantial change. What we observe is that after the shock the skilled workers have an incentive to leave the union federation and to move towards more decentralized systems of wage bargaining. But let us explain it in more detail. We use the present discount values of consumption to compute welfare, Wf. The following table presents the welfare values Wf for each type of worker.

	Level of centralization in the wage setting process		
	Centralized	Intermediate	Decentralized
Before the SBTC	$Wf_s = 0.96$ $Wf_u = 0.57$	$Wf_s = 0.88$ $Wf_u = 0.55$	$Wf_s = 0.93$ $Wf_u = 0.56$
After the SBTC	$Wf_s = 0.97$ $Wf_u = 0.13$	$Wf_s = 0.91$ $Wf_u = 0.12$	$Wf_s = 0.98$ $Wf_u = 0.128$

Before the introduction of the SBTC, both skilled and unskilled workers prefer centralized systems of wage bargaining to any others. This preference is not surprising for the unskilled workers for whom the two forces interacting in this model go in the same direction. On the one hand there is an increase in welfare generated by the efficiency gain when internalizing the search externality and on the other hand, the redistribution from the skilled to unskilled workers increase their welfare as well. What happens for the skilled workers is that, the gain in efficiency generated by the internalization of the search externality is able to compensate the lose generated by the redistribution.

Nevertheless, the situation changes after the introduction of the SBTC. The welfare of the unskilled workers decreases dramatically. In spite of it, they still prefer the centralized option. But the lose that now the redistributive effect generates over the skilled worker's welfare is not compensated by the gain generated with the internalization of the search externality, so they are the ones who have an incentive to leave the coalition.

We can say that the underlying explanation is that skilled workers cross subsidize unskilled workers through the bargaining decisions. This situation is sustainable in the case in which the productivity gap among these two types of workers is not very high and the relative sizes differ substantially. The presence of a skill-biased technical change that increase the productivity gap and also the relative number of skilled workers over the unskilled undermine the coalition.

Acemoglu, Aghion and Violante (2001) find that when unions play an efficiency-enhancing role, deunionization may happen inefficiently in the sense that skilled workers ignore the positive effect that they are generating on the unskilled through the redistribution and tend to deunionize too soon. Similar results can be extracted from our work because there is room for a reduction in the rate of redistribution from skilled to unskilled workers which could have generated increases in welfare for both types of workers.

It is worth noting that our result is also in line with the theories of endogenous formation of coalitions in which two groups of heterogeneous workers may form either a joint union or two separate unions depending on their relative size and productivity of the two groups. Since, in these models, the workers' bargaining power comes from the loss that they can impose on the firm by refusing to work, they will form a single union when the two types of workers are substitutes because separate unions will have less bargaining power. In this sense, we can see a skill-biased technical change as a way of "heterogeneize" workers and reduce the substitutability among them.

There are other interesting observations we can extract from the table above. The first one is that both the very decentralized and the very centralized case work better than the intermediate case. We can interpret this result in line with the hump-shaped curve proposed by Calmfors and Driffill (1988) who, as we stated in the introduction of the paper, find that intermediate levels of centralization yield the worst outcome in terms of macroeconomic performance. The idea behind their study, stated as well by Olson (1982) is that under intermediate levels of wage bargaining, "*organized interest are strong enough to cause major disruptions but not sufficient to take into account the costs of their actions for the society*".

Furthermore, wage compression arises in the presence of unions. This result is interesting in the sense that we have not imposed it but results as an outcome of the way in which we have modellized unions.

An interesting extension of the model could rely on the modelization of the second scenario, the intermediate level of wage bargaining. Usually intermediate levels are identified with sectoral or industrial wage negotiations. In this paper, we have adopted a particular intermediate level: a horizontal unionization, and although this systems existed in practice, for example UK, they are not the main representation of intermediate levels, sectoral bargaining could represent most of the central and southern European bargaining negotiations. Modelization of sectoral matching model with unions can be found in Delacroix(2004). Nevertheless, this intermediate level is a secondary issue in our work because we just use it for comparative reasons. The same result will hold even if we eliminate this scenario and use only the two extremes.

1.6 Conclusion

With this paper we contribute to the recent literature on deunionization and decentralization in the wage setting process. During the eighties several OECD countries witnessed a process of decentralization in their wage setting negotiations. This fact could seem surprising if we take into account that the macroeconomic performance of economies with highly centralized systems of wage bargaining have benefit from it.

This paper first calls the attention of this fact and tries to look for an explanation of why a system that perform so well in terms of macroeconomic variables and result so appealing from outside collapses. We answer this question through a model in which unions act as coalitions of workers that bargain wages with the firms. Unions extract rents from the firms along the bargain process at the same time as they play an efficiency enhancing role taking into consideration the congesting effect that searchers generate over each other.

We embed these unions into a simple search-matching framework to show that, for the values obtained under the calibration, a skill-biased technical change increases the productivity gap across heterogeneous workers and generate the collapse of very centralized systems of wage bargaining characterized by high levels of redistribution across workers.

This result is in line with the evolution of different labor systems with highly levels of centralization in their wage setting processes. If we follow the performance of the Swedish labor market we see that along the centralized period, most of the high skilled workers were slowly leaving the coalition. The skill biased technical change contributes to the end of a system that, although very appealing from outside was generating some tensions inside.

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Chapter 2

An estimated DSGE-matching model for the US economy

2.1 Introduction

In this chapter we present and estimate via maximum likelihood a dynamic stochastic general equilibrium model (DSGE) with frictional labor markets for the US economy. The labor market is modelled adopting the Mortensen-Pissarides (1994) matching framework.

We extend the Andolfatto (1996) model with a number of additional structural shocks in order to evaluate how each of these help accounting for cyclical variations in labor market indicators and other key macroeconomic aggregates. Following Bencivenga (1992) we introduce a preference shock in the utility function; an investment-specific technology shock like the one in Fisher(2006); and a job separation shock in the spirit of Shimer (2005). We estimate the parameters of the model via maximum likelihood. This alternative to the more conventional calibration offers some potential advantages. First of all, when models have a large number of parameters, standard calibration does not seem the best technique given that neither the focus on a limited set of moments nor the transfer of microeconomic estimates from one model to another would provide the discipline to quantify the behavior of the model. Second, a DSGE model, once estimated can be used to generate forecasts and can be used to decompose the k-step-ahead forecast error variances of the variables into different orthogonal components attributables to each of the structural shocks already mentioned. This second part is very useful to identify which shocks are driving the cyclical fluctuations of the different variables.

We estimate the parameters of the model and the stochastic processes governing the structural shocks using six key macro-economic time series in the US economy: real GDP, consumption, investment, total hours worked, productivity and tightness. Following standard mechanisms for maximum likelihood, we estimate the model by maximizing a numerical approximation of the likelihood function based on the application of the Kalman filter to the linearized state-space representation of the theoretical DSGE model.

Several results are worth highlighting. First, the estimation procedure yields a plausible set of estimates for the structural parameters of the model. An important parameter is the one that corresponds to the power of the workers in the Nash-bargaining process, which instead of taking the standard one half, takes a much smaller value of about 0.103, close to the value proposed by Hagedorn and Manovskii (2006). Other important parameters are those governing the structural shocks. In all cases, they are quite persistent and more volatile than what is standard in the literature.

Second, we analyze the effects of the already mentioned structural shocks on the main variables on the model using impulse-responses functions. Overall, we find that qualitatively those effects are in line with the existing evidence.

Finally, there are two structural shocks that explain a significant fraction of output: the neutral and the investment specific technology shocks. In addition to these technology shocks, the preference shock is an important determinant of consumption and investment mainly in the short run, whereas the shock to job destruction is more successful in explaining the variance of tightness

The rest of the paper is structured as follows: Section 2 presents the theoretical model. Section 3 we discuss the estimation methodology and present the main results. Section 4, analyzes the impulse responses of the various structural shocks and their contribution to the fluctuation of the variables. Finally, Section 5 presents some of the main conclusions we can draw from our analysis.

2.2 The model

The economy is populated by a continuum of infinitely lived agents. Workers and firms engage in employment relationships. As in Andolfatto (1996), we assume that each household is populated by many individuals who can be either employed or unemployed and insure each other completely against idiosyncratic risks.

The labor market is frictional. We adopt the basic Mortensen-Pissarides (1994) matching framework. Workers and firms search for a partner to generate a productive job. The total number of matches in the economy per unit of time M_t is given by the following Cobb-Douglas function

$$M_t = \chi V_t^{\alpha} (eU_t)^{(1-\alpha)} \tag{2.1}$$

where V_t represents the total number of vacancies open by the firms per unit of time t, U_t denotes the amount of searchers per unit of time t and the constant e implies that search effort is constant in this economy.

We assume that all unemployed workers search for a partner. The probability for a searcher of finding a vacant job can be obtained from the matching function as $\mu_t = \frac{M_t}{u_t}$. Equally the probability that a vacancy gets filled per unit of time is given by $q_t = \frac{M_t}{v_t}$.

Employment is a predetermined variable, whose law of motion is given by the following equation.

$$N_{t+1} = N_t - \sigma_t N_t + M_t \tag{2.2}$$

i.e. the stock of employment in a certain period is equal to the stock of employment in the period before minus the flow of workers who have lost their jobs with the exogenous probability σ_t plus the flow of searches that have been matched and thus move from unemployment to employment.

Households and preferences - The representative household has preferences over consumption and leisure defined through the following expected utility function

$$E\left\{\sum_{t=0}^{\infty}\beta^t\left[d_t\log(C_t) + N_t\phi_1\frac{(1-H_t)^{(1-\eta)}}{(1-\eta)} + U_t\phi_2\frac{(1-e)^{(1-\eta)}}{(1-\eta)}\right]\right\} \tag{2.3}$$

where the discount factor satisfies $0 < \beta < 1$ and $\phi_i > 0$, for $i = 1, 2$. C_t denotes consumption, H_t denotes the number of hours worked and d_t represents the preference shock whose law of motion follows this AR process:

$$\ln(d_{t+1}) = \rho_d \ln(d_t) + (1-\rho_d)\ln(\overline{d}) + \varepsilon^d_{t+1} \tag{2.4}$$

The representative household will maximize utility function above subject to the following budget constraint:

$$C_t + I_t = w_t N_t H_t + r_t K_t + \Pi_t \tag{2.5}$$

this means that the household has to decide how to split current income, made up of capital income $r_t K_t$, profits Π_t and the wage bill $w_t N_t H_t$, between consumption C_t and investment I_t. The law of motion for capital is given by the following equation:

$$K_{t+1} = (1-\delta)K_t + X_t I_t \tag{2.6}$$

where δ denotes the depreciation rate. Now in contrast to more standard specifications there is an investment-specific technology shock represented by X_t that is defined by the following specification:

$$\ln \frac{X_{t+1}}{X_t}=\rho_x \ln \frac{X_t}{X_{t-1}}+(1-\rho_x)\ln \gamma_x+\varepsilon^x_{t+1}$$

Production sector and technology - The representative firm produces an aggregate good Y_t with capital and labor according to a constant returns to scale technology described by a Cobb-Douglas production function

$$Y_t = A_t K_t^{\theta}(z_t T_t)^{1-\theta} \tag{2.7}$$

where $0 < \theta < 1$. K_t denotes capital and T_t denotes labor input and is equal to the number of workers N_t times the number of hours worked by each of them H_t. z_t represents the deterministic growth rate of the economy, which is equal to equal to γ^t, where $\gamma > 1$[1]. The technology level is represented by A_t, which follows a first order autorregresive process:

$$\ln(A_{t+1}) = \rho \ln(A_t) + (1-\rho)\ln(\overline{A}) + \varepsilon^a_{t+1} \tag{2.8}$$

where $\overline{A} > 0$ and $0 < \rho < 1$. The serially uncorrelated innovation ε^a_t is normally distributed with mean zero and standard deviation σ_a.

The firm hires capital and opens vacancies to maximize the expected present value of cash flows

$$\sum_{t=0} \Delta^t[F(A_t, K_t, z_t T_t) - w_t N_t H_t + r_t K_t - aV_t] \tag{2.9}$$

where a denotes the cost of opening a vacancy, and Δ represents the discount factor for the firm, which can be stated as where $\Delta = \beta \frac{U_{c_{t+1}}}{U_{c_t}}$.

When maximizing this problem, the firm takes into account the law of motion of employment specified in equation (2) considering that $M_t = q_t v_t$. In our specification the job destruction probability σ becomes stochastic. In particular, it is defined through the following AR process:

$$\ln \sigma_{t+1}= \rho_\sigma \ln \sigma_t+(1-\rho_\sigma)\ln \overline{\sigma}+\varepsilon^\sigma_{t+1}$$

[1]This implies that our variables, except those related to hours and vacancies, grow at the common rate γ in steady state. Therefore, data is automatically detrended as part of the estimation process and we do not need to filter it with other methods such as H-P filter or B-P filter, etc.

Optimal contract - Wages are obtained through Nash bargaining. Denoting as p the bargaining power of the workers, the optimal contract is obtained as

$$w_t l_t = p\left(F_2 l_t + a\frac{v_t}{u_t}\right) - (1-p)\left(\frac{\phi_1}{\lambda_t}\frac{(1-l_t)^{(1-\eta)}}{1-\eta} - \frac{\phi_2}{\lambda_t}\frac{(1-e)^{(1-\eta)}}{1-\eta}\right) \tag{2.10}$$

this means, that the optimal contract is a weighted sum of two elements, the first one is the labor's yields plus the savings in vacancy posting for the firm and the second is the reservation wage of the households.

The optimal number of hours per worker is given by the following equation

$$\frac{\partial S_t}{\partial l_t} = F_2 - \frac{\phi_1}{\lambda_t}(1-l_t)^{-\eta} = 0 \tag{2.11}$$

where S_t denotes the surplus of the match and $F_2 = (1-\theta)\frac{y_t}{n_t l_t}$

Equilibrium - To obtain the equilibrium allocation for this economy, we first need to detrend the growing variables of this economy defined as $y_t = Y_t/z_t, c_t = C_t/z_t, i_t = I_t/z_t, k_{t+1} = K_{t+1}/z_t, h_t = H_t, n_t = N_t, m_t = M_t$.

When the vector $\varepsilon_t = 0$ the economy converges to a steady state in which all the detrended variables remain constant. Appendix A describes the equilibrium for this economy. It also contains the log-linearizations of the non linear system around this steady-state. We apply the "*Toolkit*" method proposed by Uhlig (1997) to the log-linearized system to obtain a solution of the form:

$$x_t = Px_{t-1} + Qz_t$$
$$y_t = Rx_{t-1} + Sa_t$$
$$a_{t+1} = Na_{t-1} + \varepsilon_{t+1}$$

This system describes the recursive equilibrium law of motion of the real business cycle model where x_t and y_t represent vectors of logarithmic deviations of the endogenous states and the control variables from their steady-state levels. a_t represents the logarithmic deviation of the exogenous states. More precisely,

$$x_t = \left[\ln\left(\frac{k_t}{k^*}\right) \quad \ln\left(\frac{n_t}{n^*}\right) \quad \ln\left(\frac{u_t}{u^*}\right)\right]'$$
$$y_t = \left[\ln\left(\frac{c_t}{c^*}\right) \quad \ln\left(\frac{i_t}{i^*}\right) \quad \ln\left(\frac{w_t}{w^*}\right) \quad \ln\left(\frac{v_t}{v^*}\right) \quad \ln\left(\frac{\theta_t}{\theta^*}\right) \quad \ln\left(\frac{m_t}{m^*}\right) \quad \ln\left(\frac{h_t}{h^*}\right) \quad \ln\left(\frac{t_t}{t^*}\right)\right]'$$
$$a_t = [at\ dt\ \sigma t]$$

The system above can be easily written in a more general form:

$$s_{t+1} = As_t + B\varepsilon_{t+1} \quad (2.12)$$

$$y_t = Cs_t \quad (2.13)$$

where $s_{t+1} = \begin{pmatrix} x_t & a_{t+1} \end{pmatrix}'$ $A = \begin{pmatrix} P & Q \\ 0 & N \end{pmatrix}$ $B = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$ and $C = \begin{pmatrix} R & S \end{pmatrix}$

The matrices A, B and C depend on the structural parameters of the model $\beta, \phi_1, \phi_2, \eta, a,$ $\xi, \theta, \sigma, \rho_a, \varepsilon^a, \rho_d, \varepsilon^d, \rho_x, \varepsilon^x, \rho_\sigma, \varepsilon^\sigma, \chi$ and α.To estimate those parameters, we have to transform the theoretical model into a state-space empirical representation. As we will see, we do not need to estimate all of them given that some of them can be obtained as combinations of the others and the steady-state variables. In the following section, we transform the theoretical model into a state-space empirical representation and estimate those parameters.

2.3 Maximum Likelihood Estimation of the Model

There are various ways of giving values to the parameters of a DSGE model. According to Geweke (1999), there is a *weak econometric interpretation* and a *strong econometric interpretation.* Under the former case, the parameters of the model are calibrated in a way that some selected theoretical moments of the model match those of the data[2]. These methods allow the researcher to focus on the characteristics in the data for which the DSGE model is more relevant but have the problem of focusing on a limited set of moments instead of using all the information contained in the data. The strong interpretation case attempts to provide a full characterization of the observed data series and when successful it allows for proper specification testing and forecasting. In this paper, we follow this strong interpretation and as in Altug (1989), Sargent (1978) or Ireland (2001, 2004) among others, we apply Maximum Likelihood Estimation.

Maximum Likelihood provides a systematic procedure to estimate the parameters of interest. Except in a few cases, there is no analytical or numerical procedure to directly evaluate the likelihood function of our DSGE, but we can transform the theoretical model into a state-space econometric model and assuming that the shocks to the economy are normally distributed and the

[2]A number of papers have estimated the parameters of DSGE models by these moment-matching estimation. See, for instance, Hansen (1997), Rotenberg and Woodford (1998) or Christiano et. al. (2001)

policy functions of the model are approximated linearly, we can look for a numerical approximation of the likelihood function with the help of the Kalman filter. In what follows we explain this in more detail.

2.3.1 Transformation of the theoretical model

State-space Representation - As Ireland (2004) and Ingram et.al.(1994) explain, the fact there are less shocks in the economy that number of time series used in the estimation, makes the model stochastically singular.

There are two common approaches to deal with this problem. The first one consists of increasing the number of structural shocks in the model until we have the same number of shocks as time series used in the estimation; the second approach, which is the one we use here, consists of augmenting the equation (2.13) of the system above with an error term or measurement error proposed in Altug(1989) or Ireland (2004). These residuals represent the movements in the data that the theory does not explain (those movements that are not generated by the shocks specified in the model).

The state-space representation of the theoretical model above is given by the following equations (2.14)-(2.18)

$$s_{t+1} = Fs_t + V\varepsilon_{t+1} \tag{2.14}$$

$$y_t = Gs_t + me_t \tag{2.15}$$

where me_t is a vector of *measurement errors* uncorrelated across variables, s_t represents the *state* vector, f_t denotes the vector of *observable* variables at time t, F and G are again matrices of parameters. The first equation of the system is known as the state equation and the second called the observation equation. Vectors ε_t and me_t are white noise vectors with:

$$E[\varepsilon_t \varepsilon_t'] = Q. \tag{2.16}$$

$$E[me_t me_t'] = R. \tag{2.17}$$

$$E[\varepsilon_t me_t'] = 0 \tag{2.18}$$

State-space econometric models allow for the evaluation of the likelihood function using the Kalman filter algorithm explained in detail in Hamilton (1994, Chapter 13). The Kalman filter takes the observations of f_t for $t = 1, ..T$ as inputs and works recursively to construct implied series of forecast errors. The application of the Kalman filter lets us calculate the numerical approximation of the log-likelihood function of the model as follows

$$ln(L) = -\frac{3T}{2}\ln(2\pi) - \frac{1}{2}\sum_{t=1}^{T}\ln\left|G\Sigma_t G'\right| - \frac{1}{2}\sum_{t=1}^{T} w_t' \left(G\Sigma_t G'\right)^{-1} w_t \tag{2.19}$$

where G comes directly from equation (14) and Σ is obtained from the application of the Kalman filter.

This likelihood function can be evaluated for any given set of parameter values. Making use of a numerical search algorithm one can find the set of parameters that maximize the likelihood function. Usually, maximum likelihood estimations are criticized because it is very difficult to be sure whether we are in the global maximum or on the contrary we are just in a local one, given that the likelihood function displays a quite sinuous pattern. To avoid this criticism with our estimations we borrow from physics another algorithm called *"simulated annealing"* . This is a generic probabilistic meta-algorithm for the global optimization problem, i.e. it looks for a good approximation to the global optimum of a function in a large search space (as is the case of the likelihood function). Each step of the simulated annealing algorithm replaces the current solution by a random "nearby" solution. The allowance for these movements saves the method from becoming stuck at a local minimum.

In principle, this algorithm is allowed to select values of the parameters lying anywhere between the positive and the negative infinity. But to ensure that our parameters satisfy the theoretical restrictions of the model listed in section 2, we have to impose additional constraints[3].

Computing standard-errors - Often, algorithms for computing maximum likelihood estimates have the drawback that they do not produce standard errors. This means that we should also look for numerical approximations of the derivatives of the likelihood function so as to compute the information matrix and from it, the standard errors.

[3] In particular, some of ours parameters are constrained to be positive, so we constraint the algorithm to work with absolute values. Many of ours parameters are probabilities that should lie between zero and one, so we again constraint the algorithm to work with the logistic transformation of these parameters.

Fortunately, if certain regularity conditions hold[4], the maximum likelihood estimates are consistent and asymptotically normal. Under these circumstances, the information matrix for a sample of size T can be calculated from the second derivatives of the maximized log-likelihood function as

$$I_T = -\frac{1}{T}\left\{\sum_{t=1}^{T}\frac{\partial^2 \log L(y_t,\theta)}{\partial\theta\,\partial\theta'}\right\} \tag{2.20}$$

Standard errors are then the square roots of the diagonal elements of $\frac{1}{T}(I_T)^{-1}$. This matrix has elements of very different magnitudes and therefore, the reported standard errors should be interpreted with caution.

2.3.2 The Data

Data is taken from the Federal Reserve Bank of St. Louis' FRED database, data for gross domestic income and wages is taken from the Bureau of Economic Analysis. Appendix B presents a table with detailed information about each series. Monthly data has been transformed into quarterly data using averages. The sample period is 1964-1 to 2005-4.

When the model takes the form of a state-space representation, it can be estimated via maximum likelihood once analogs to the model's variables are found in the data. Therefore, C_t is defined as real personal consumption on non durables and services plus government expenditure. Investment, I_t is defined as the sum of purchases of durable consumption plus private sector fixed investment. Vacancies, V_t are proxied by a widely used index which reflects the number of "*help-wanted*" advertisement registered in US newspapers. N_t comes directly from the number of civilian employment, and thus unemployment can be computed as $1 - N_t$. All the variables have been divided by the civilian population aged 16 or over, so as to have them in per capita terms.

Making the data comparable with the variables of the model - To properly evaluate the likelihood function, one more transformation of the data is needed. Data series should be

[4]These conditions include that the model must be identified, the eigenvalues of A lie inside the unit circle, the true values of the estimations do not fall on a boundary of the allowable parameter space and that variables x_t behave asymptotically like a full-rank linearly indeterministic covariance-stationary process

comparable with the vectors of logarithmic deviations of the variables from their steady-state levels. Thus, we have to use the definitions:

$$\widehat{c_t} = \log(c_t) - \log(\overline{c})$$
$$\widehat{i_t} = \log(i_t) - \log(\overline{i})$$
$$\widehat{\Pr_t} = \log(\Pr{}_t) - \log(\overline{\Pr}) - t\log(\gamma)$$
$$\widehat{t_t} = \log(t_t) - \log(\overline{t})$$
$$\widehat{\theta_t} = \log(\theta_t) - \log(\overline{\theta})$$

for all $t = 1, 2...T$. Remember that to solve the problem we have normalized output to 1, so consumption and investment here are defined as $\widehat{c_t} = \widehat{\left(\frac{c_{d,t}}{y_t}\right)}$. and $\widehat{i_t} = \widehat{\left(\frac{i_{d,t}}{y_t}\right)}$. Productivity, Pr, is growing at rate γ in steady state. Once those transformations are made, the vector of observables is given by:

$$f_t = \left[\begin{array}{ccccc} \widehat{c_t} & \widehat{i_t} & \widehat{t_t} & \widehat{Pd_t} & \widehat{\theta_t} \end{array} \right]$$

2.3.3 Parameter estimates

As in Altug(1989) and Ireland (2004), we keep fixed the value of some parameters of the model for which the estimation yields unreasonable results. Those parameters are the representative household discount factor β, the capital share of output θ, and the depreciation rate δ.

As Hornstein, Krusell and Violante (2005) point out there are some aspects of the calibration of the standard matching model that are relatively uncontroversial whereas some other aspects deserve special attention.

We directly fix those "uncontroversial values" that correspond to those mentioned above. We set the discount factor β equal to 0.99. The depreciation rate of capital, δ, is calibrated so that it corresponds to an annual depreciation of around 10 percent. And the capital share of output, θ, is set equal to 0.36.

The estimated values for the "controversial parameters": $\alpha, \rho, \varepsilon, \eta, p, \sigma, \gamma$ and their standard errors are presented in the Table 1.

As one can observe, the estimates of the DSGE model's parameters seem quite reasonable. The parameter α from the matching function decreases its value from 0.6 in Andolfatto (1996) to

0.2. The parameters driving the technology shock adopt a value of 0.914 instead of the traditional 0.95, which means that in this model the TFP shock shows a smaller persistence. In contrast the variance is larger, in particular the parameter ε^a moves from 0.007 to 0.009. The remaining shocks of the model are also quite persistent, with values of ρ around 0.9. The less volatile shock is the investment-specific technology shock whose ε^x takes a value of 0.005.

Table 1: Results from the estimation

Parameter	Definition	Estimated Value	Standard Error
α	elasticity of vacancies	0.200	0.0274
ρ_a	persistence of the TFP shock	0.914	0.0155
ε_a	variance of the TFP shock	0.009	0.0001
ρ_d	persistence preference shock	0.897	0.0160
ε_d	variance preference shock	0.010	0.0013
ρ_σ	persistence job destruction shock	0.900	0.0037
ε_σ	variance job destruction shock	0.010	0.0003
ρ_i	persistence inv-spec shock	0.885	0.0091
ε_i	variance inv-spec shock	0.005	0.0001
p	workers' bargaining power	0.103	0.0020
η	parameter utility function	3.398	0.0025
γ_z	rate of growth - labor augmenting	1.005	0.0001
γ_i	rate of. growth - investment	1.005	0.0001
me_c	measurement error	0.0001	0.0004
me_i	measurement error	0.0001	0.0003
me_t	measurement error	0.051	0.0137
me_p	measurement error	0.050	0.0192
me_θ	measurement error	0.049	0.0001

The estimate $\gamma_z = 1.005$ corresponds to an annualized growth rate of real per-capita output in the model around 2%. Nevertheless, more interesting for our purposes are the estimations of the variables related to the labor market. The value of the worker's bargaining power takes a value of 0.103 instead of the 0.5 proposed by Nash (1959). This means that the power of an isolated worker in the bargaining of his wage is smaller than the power of the hiring firm. It goes also in the direction proposed by Hagedorn and Manovskii (2006). The value of the parameter of the utility function η is estimated to 3.398 and is larger than the one calibrated in Andolfatto (1996) that corresponds to 2.197. This parameter is important in the sense that drives the trade-off between the intensive and the extensive margin of labor input.

Measurement errors for investment and tightness take larger values than the others. This is consistent with the fact that those series are the most volatile. Not surprisingly, productivity and total hours worked measurement errors have similar values. Finally and in spite of the fact that standard errors have had to be approximated numerically, they take very small values making significant all the values of our estimation.

The remainder of the parameters can be obtained directly from the estimations above and the steady-state values of the variables. They are shown in the table below:

Table 2: Other parameters of the model

Parameter	Value	Explanation
$1-\alpha$	0.80	prob. of downgrade of skills
χ	1.137	efficiency paramet. matching function
ξ	1.359	parameter production function
a	0.105	cost of posting a vacancy
ϕ_1	1.221	parameter in the utility function
ϕ_2	0.325	parameter in the utility function

2.4 Which shocks are important?

In this section we analyze which shocks are important in driving the empirical business cycle fluctuations of the main variables of the model according to the US data used in the estimation of the parameters of the model.

2.4.1 Impulse response analysis

Graphs 1 to 4 of Appendix C, plot the impulse-response functions to the structural shocks included in the model, i.e. a preference shock; a job-separation shock; an investment-specific technology shock and a productivity shock.

Graph 1 shows that following a positive productivity shock, consumption, investment and employment rise. Wages and vacancies rise as well. Positive productivity shocks raise output in all matches but do not affect the rate at which employed workers lose their jobs. Interesting is the response of tightness which shows a more satisfactory performance than what is usual in this type of models. On the contrary, the response of the variable hours worked fluctuates around its steady-state value more than expected.

Graph 2 shows the effects of a job separation shock. An increase in the separation rate, increases separations and reduces employment duration. As a direct consequence, the unemployment rate increases and vacancies increase as well. The final effect is an increase in tightness. Consumption and output fall while the number of hours worked by the survival matches increase, counterweighting only partially the negative effect that the job destruction shock has on the extensive margin. The total effect on labor input is negative as one can see from the fall in the impulse-response function of total hours worked.

Graph 3 shows that a positive preference shock, while increasing consumption and output significantly, has an initial crowding-out effect on investment. Nevertheless, to satisfy this higher demand it is necessary to increase capacity and consequently, it generates an increase in employment and a recuperation of investment via capital.

Finally, graph 4 shows, the effects of an investment-specific technology shock. Following a positive shock, investment increases whereas consumption decreases substantially, given that a larger fraction of output is initially devoted to investment. Vacancies increase but in a smaller extent than the increase experimented by the other three shocks above. Both employment and hours worked increase substantially. As a consequence, labor input increase. Wages fall initially and slightly recover afterwards.

2.4.2 Variance decomposition

Maximum Likelihood estimation and the state-space representation of the model allow to perform a variance decomposition analysis to identify which shocks are more important in driving the empirical business cycle fluctuations of the variables of the model presented in section 2.

The contribution of each of the structural shocks to the forecast error variance of the main variables of the model at various horizons: short run: one quarter ahead and 1 year - 4 quarters-; medium term: 2 years -8 quarters- and long run: 5 years -20 quarters- and their standard errors are shown in Table 3 of the Appendix D.

Standard errors also appear in Table 3 and show the statistical uncertainty surrounding the model's ability to explain the observed data. In our model, this statistical uncertainty is smaller than what has been previously suggested by Altug (1989) or Ireland (2004), this might be due to the introduction of other shocks different to the standard technology shock proposed by the real business cycle model.

Let us first focus on the determinants of output. At the one-year horizon, output variations are driven primarily by the neutral technology shock. In the medium term, the two technology shocks (neutral and investment-specific) together account for almost all of the forecast error variance. In the long-run, the neutral technology shock dominates, but the investment-specific technology shock accounts for about 40% of the forecast error in output. This pattern is somehow surprising given that standard RBC models usually explain very high and very low frequency movements in output, but is less successful in explaining quarter-to-quarter movements. The introduction of additional shocks changes this behavior.

With respect to the determinants of consumption, the investment-specific technology shock accounts for more than 80% of the unconditional variance in detrended consumption and almost 54% of the one-quarter-ahead forecast error. In the short run the preference shock plays an important role accounting for more than 35% of the one-quarter-ahead forecast error variance in output but it looses importance over time, showing a pattern opposite to the technology shock. The shock to job destruction plays an almost negligible impact.

Focusing on investment, the investment-specific technology shock accounts for about 30% on average of the k-step-ahead forecast error variances for values of k ranging from 4 to 20. The neutral technology shock accounts for almost another 50% (48.36%) of the unconditional variance in detrended investment, although its importance increases in the medium term. Finally, the preference shock can also account for part of the short run variation in investment, losing its importance over time.

The unconditional variance in total hours is mainly explained by the preference and the investment-specific technology shocks. Nevertheless, in the short run there must be other forces, different from the shocks aforementioned driving the fluctuations of this variable.

Finally, we focus on the determinants of tightness. The shock to job destruction, accounts for almost 10% of the unconditional variance, almost 20% of the one-quarter-ahead forecast error variance and around 23% of the k-step-ahead for k ranging from 4 to 12. The neutral technology shock explains almost 50% of the one quarter ahead forecast error variance but its importance decreases over time. The fact that the technology shock does not play an important role, shows that there are other forces driving the tightness' cyclical fluctuations different from the one proposed in the standard RBC model. The investment-specific technology shock accounts for most of the unconditional variance.

Summarizing these results, there are two structural shocks that explain a most of the cyclical fluctuations of the variables in the model: the neutral and the investment specific technology shocks.

In addition, the preference shock is an important determinant of consumption and investment. This preference shock explains a significant portion of the fluctuations of the variables in the short run, while is less successful in the long run.

It is worth noting that in explaining the variance of tightnes, in addition to the technology shocks, both neutral and investment-specific, the shock to job destruction plays a significant role mainly at the business cycle frequency. This could explain why standard RBC models are not satisfactory when replicating the volatility of unemployment and vacancies.

2.5 Conclusion

In this chapter, we propose and estimate a dynamic stochastic general equilibrium model (DSGE) with frictional labor markets and four structural shocks. Beyond the standard neutral technology shock, we introduce a preference shock in the utility function, an investment-specific technology shock and a job separation shock.

We estimate the parameters of the model via maximum likelihood for the US economy, using six key macro-economic time series. Overall, the estimates seem quite reasonable, although in some cases differ from the values traditionally considered as standard.

The variance-decomposition analysis show that there are two structural shocks that explain a significant fraction of the variance of the main variables of the model: the neutral and the investment specific technology shocks. In addition, the preference shock explains a portion of consumption and investment in the short run.

It is worth noting that in explaining the variance of tightnes, in addition to the technology shocks, both neutral and investment-specific, the shock to job destruction plays a significant role mainly at the business cycle frequency. This could explain why standard RBC models are not satisfactory when replicating the volatility of unemployment and vacancies.

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2.A Appendix: Solving for the equilibrium of the model

2.A.1 Households

Households maximize the following problem with respect to $\{C_t, K_{t+1}\}_{t=0}^{\infty}$

$$Max \left\{ \sum_{t=0}^{\infty} \beta \left[d_t \ \log C_t + N_t \phi_1 \frac{(1-L_t)^{(1-\eta)}}{1-\eta} + (1-N_t)\,\phi_2 \frac{(1-e_t)^{(1-\eta)}}{1-\eta} \right] \right\}$$

subject to (1) the resource constraint

$$\begin{aligned} C_t + I_t &= g_t w_t N_t L_t + r_t K_t + \Pi_t \\ K_{t+1} &= (1-\delta) K_t + X_t I_t \\ \ln d_{t+1} &= \rho_d \ln d_t + (1-\rho_d) \ln \overline{d} + \varepsilon_{t+1}^d \\ \ln \frac{x_{t+1}}{x_t} &= \rho_x \ln \frac{x_t}{x_{t-1}} + (1-\rho_x) \ln \frac{x}{x} + \varepsilon_{t+1}^x \end{aligned}$$

and an initial condition (k_0)

The growth rates of the different variables along the balanced growth path can be derived as follows. First the exogenous variables Z_t and X_t grow at the gross rates γ_z and γ_x, respectively. From the resource constraint and the accumulation of capital equation, it follows that Y_t, C_t, I_t and A_t all have to grow at the same rate g_t. Capital, K_t, however grows faster, at a rate of $g\gamma_x$. Finally, the production function implies that $g_t = \gamma_z \gamma_x^{\frac{\theta}{1-\theta}}$. Thus, the following conditions hold for balanced growth:

$$g_t = \gamma_z \gamma_x^{\frac{\theta}{1-\theta}}$$

and

$$g_{kt} = \gamma_z \gamma_x^{\frac{\theta}{1-\theta}} \gamma_x = \gamma_z \gamma_x^{\frac{1}{1-\theta}}$$

Detrended variables may be defined as $c_t = \frac{C_t}{g_t}, i_t = \frac{I_t}{g_t}, l_t = L_t, n_t = N_t, k_{t+1} = \frac{K_{t+1}}{g_{kt}}$ and the detrended maximization problem could be written as shown in eq.

subject to (1) the detrended resource constraint

$$c_t + i_t = w_t n_t l_t + r_t k_t x_{t-1} + \pi_t$$

the law of motion for capital

$$\gamma_z \gamma_x^{\frac{\theta}{1-\theta}} k_{t+1} = (1-\delta) k_t \gamma_x^{-1} + i_t$$

and the autorregresive processes for the shocks

$$\ln d_{t+1} = \rho_d \ln d_t + (1-\rho_d) \ln \overline{d} + \varepsilon_{t+1}^d$$

$$\ln \frac{x_{t+1}}{x_t} = \rho_x \ln \frac{x_t}{x_{t-1}} + (1-\rho_x) \ln \frac{x}{x} + \varepsilon_{t+1}^x$$

where $\gamma_{x,t} = \frac{x_t}{x_{t-1}}$

Exploiting the recursive structure of the problem one may equivalently reformulate it in terms of a dynamic program. Let $s = (k)$ denote the current period capital stock and employment rate or state vector of this economy system. Let s_0 denote an arbitrary initial condition and let primed variables denote "next period" values. The value function W satisfies the following Bellman equation

$$W(s) = \max_{c,k'} \left\{ d_t \log c_t + n_t \phi_1 \frac{(1-l_t)^{(1-\eta)}}{1-\eta} + (1-n_t)\, \phi_2 \frac{(1-e)^{(1-\eta)}}{1-\eta} + \beta E\left[W(s')\right] \right\}$$

subject to the constraint

$$c_t + \gamma_z \gamma_x^{\frac{\theta}{1-\theta}} k_{t+1} - (1-\delta) k_t \gamma_x^{-1} = w_t n_t l_t + r_t k_t x_{t-1} + \pi_t$$

letting λ denote the multiplier associated the constraint above. Therefore, the first order conditions, assuming interior solution, can be expressed as follows:

- w.r.t. consumption (c)

$$\frac{d_t}{c_t} - \lambda_t = 0$$

- w.r.t. capital ($k\prime$)

$$\beta E\left[W_k(s_{t+1})\right] - \lambda_t = 0$$

- constraint () holding with equality

$$c_t + \gamma_z \gamma_x^{\frac{\theta}{1-\theta}} k_{t+1} - (1-\delta) k_t \gamma_x^{-1} = w_t n_t l_t + r_t k_t x_{t-1} + \pi_t$$

From the envelope theorem, one derives

$$W_k(s_t) = \lambda_t \left[(1-\delta)\,{\gamma_x}^{-1} + r_t x_{t-1}\right]$$

Therefore the optimal decision for the household comes from:

$$\frac{d_t}{c_t}\gamma_z {\gamma_x}^{\frac{\theta}{1-\theta}} = \beta E\left\{\frac{d_{t+1}}{c_{t+1}}\left[(1-\delta)\,{\gamma_x}^{-1} + r_{t+1}x_t\right]\right\}$$

2.A.2 Firms

Firms maximize the following

$$Max\ \sum_{t=0}^{\infty}\Delta^t\left[F(K_t, N_t L_t, z_t) - g_t w_t N_t L_t - r_t K_t - g_t a_t V_t\right]$$

subject to the law of motion for employment

$$N_{t+1} = (1-\sigma_t)N_t + q_t V_t$$

where

$$F(K_t, N_t L_t, z_t) = \zeta A_t K_t^{\theta}(z_t N_t L_t)^{(1-\theta)}$$

and the law of motion for the productivity shock follows the following AR process

$$\ln a_{t+1} = \rho_a \ln a_t + (1-\rho_a)\ln\overline{a} + \varepsilon_{t+1}^{a}$$

$$\ln \sigma_{t+1} = \rho_\sigma \ln \sigma_t + (1-\rho_\sigma)\ln\overline{\sigma} + \varepsilon_{t+1}^{\sigma}$$

and where $z_t = \gamma_z^t$

As before, detrended variables can be defined as $l_t = L_t, n_t = N_t, k_{t+1} = \frac{K_{t+1}}{g_{k,t}}, v_t = V_t, y_t = \frac{Y_t}{g_t}$

And exploiting the recursive structure we can reformulate it as a dynamic program

$$J(s) = \max_{k,n',v}\left\{\zeta a_t k_t^{\theta} t_t^{(1-\theta)} - w_t n_t l_t - r_t k_t x_{t-1} - a v_t + \beta E\left[\frac{\lambda_{t+1}}{\lambda_{t+1}} J(s')\right]\right\}$$

subject to

$$n_{t+1} = (1 - \sigma_t)n_t + q_t v_t$$

$$\ln a_{t+1} = \rho_a \ln a_t + (1 - \rho_a) \ln \overline{a} + \varepsilon^a_{t+1}$$

$$\ln \sigma_{t+1} = \rho_\sigma \ln \sigma_t + (1 - \rho_\sigma) \ln \overline{\sigma} + \varepsilon^\sigma_{t+1}$$

where μ is the multiplier associated with the constraint above.

Therefore, the first order conditions, assuming interior solution, can be expressed as follows:

- w.r.t. capital (k)

$$F_k = \theta \frac{y_t}{k_t} = r_t x_{t-1}$$

- w.r.t. vacancies (v)

$$-a + \mu_t q_t = 0$$

- w.r.t. employment ($n\prime$)

$$\beta E \left[W_n(s_{t+1}) \right] - \mu_t = 0$$

- constraint () holding with equality

$$n_{t+1} = (1 - \sigma)n_t + q_t v_t$$

From the envelope theorem, one derives

$$W_n(s_t) = F_n l_t - w_t l_t + \mu_t \left[1 - \sigma_t\right]$$

Therefore the optimal decision for the firms are specified by:

$$F_k = \theta \frac{y_t}{k_t} = r_t x_{t-1}$$

$$\frac{a v_t}{m_t} = \beta E \left[\frac{\lambda_t}{\lambda_{t+1}} \left((1 - \theta) \frac{y_t}{t_t} l_t - w_t l_t + \frac{a v_{t+1}}{m_{t+1}} \left[1 - \sigma_{t+1}\right] \right) \right]$$

$$n_{t+1} = (1 - \sigma_t)n_t + q_t v_t$$

2.A.3 Matching

The number of matches per unit of time is given by the following technology

$$m_t = \chi v_t^{\alpha} \left(e_t u_t\right)^{(1-\alpha)}$$

2.A.4 Nash-bargaining

The income value of employment for a household and a firm can be represented as follows:

$$J_{n,t} = F_2 l_t - w_t l_t + \beta E \left[\frac{\lambda_{t+1}}{\lambda_t} J_{n,t+1}\right] (1-\sigma_t)$$

$$W_{n,t} = \phi_1 \frac{(1-l_t)^{(1-\eta)}}{1-\eta} - \phi_2 \frac{(1-e)^{(1-\eta)}}{1-\eta} + \lambda_t w_t l_t + \beta E \left[W_{n,t+1}\right] (1-\sigma_t-\phi_t)$$

Optimal contracts are obtained through Nash bargaining as

$$w_t l_t = \arg\max W^p J^{(1-p)}$$

Therefore the optimal contract is given by the following equation:

$$w_t l_t = p\left(F_2 l_t + a\frac{v_t}{u_t}\right) - (1-p)\left(\frac{\phi_1}{\lambda_t}\frac{(1-l_t)^{(1-\eta)}}{1-\eta} - \frac{\phi_2}{\lambda_t}\frac{(1-e)^{(1-\eta)}}{1-\eta}\right)$$

The optimal number of hours per worked is obtained as the partial derivative of the surplus of the match with respect to hours worked

$$\frac{\partial S_t}{\partial l_t} = F_2 - \frac{\phi_1}{\lambda_t}(1-l_t)^{-\eta} = 0$$

where $F_2 = (1-\theta)\frac{y_t}{n_t l_t}$

2.A.5 Non-stochastic equilibrium

The unknown policy functions for $\{k_t, l_t, v_t, c_t, i_t, y_t, n_t, m_t, w_t\}$ are characterized by the following system of equations:

$$\gamma_z {\gamma_x}^{\frac{\theta}{1-\theta}} = \beta E\left\{\frac{c_t}{c_{t+1}}\frac{d_{t+1}}{d_t}\left[(1-\delta)\,{\gamma_{x,t+1}}^{-1} + \theta\frac{y_{t+1}}{k_{t+1}}\right]\right\}$$

$$\phi_1(1-l_t)^{(-\eta)} = \frac{d_t}{c_t}(1-\theta)\frac{y_t}{n_t l_t}$$

$$\frac{av_t}{m_t} = \beta E\left\{\frac{c_t}{c_{t+1}}\frac{d_{t+1}}{d_t}\left[(1-\theta)\frac{y_{t+1}}{n_{t+1}} - w_t l_t + \frac{av_{t+1}}{m_{t+1}}(1-\sigma_{t+1})\right]\right\}$$

$$c_t + i_t + \kappa v_t = y_t$$

$$\gamma_z {\gamma_{x,t}}^{\frac{\theta}{1-\theta}} k_{t+1} = (1-\delta)k_t {\gamma_{x,t}}^{-1} + i_t$$

$$y_t = \zeta a_t k_t^{\theta} {t_t}^{1-\theta}$$

$$n_{t+1} = (1-\sigma_t)n_t + m_t$$

$$m_t = \chi v_t^{\alpha}\left(e_t u_t\right)^{(1-\alpha)}$$

$$w_t l_t = p\left(F_2 l_t + a\frac{v_t}{u_t}\right) - (1-p)\left(\frac{\phi_1}{\lambda_t}\frac{(1-l_t)^{(1-\eta)}}{1-\eta} - \frac{\phi_2}{\lambda_t}\frac{(1-e)^{(1-\eta)}}{1-\eta}\right)$$

$$\ln a_{t+1} = \rho_a \ln a_t + (1-\rho_a)\ln \overline{a} + \varepsilon^a_{t+1}$$

$$\ln \sigma_{t+1} = \rho_\sigma \ln \sigma_t + (1-\rho_\sigma)\ln \overline{\sigma} + \varepsilon^\sigma_{t+1}$$

$$\ln d_{t+1} = \rho_d \ln d_t + (1-\rho_d)\ln \overline{d} + \varepsilon^d_{t+1}$$

$$\ln \gamma_{x,t+1} = \rho_x \ln \gamma_{x,t} + (1-\rho_x)\ln \overline{\gamma_x} + \varepsilon^x_{t+1}$$

2.A.6 Log-linearized model

The deterministic equations are

$$0 = y(t) - n(t) - l(t) + d(t) - c(t) + \eta f(t)$$

$$0 = \overline{c}c(t) + \overline{i}i(t) + \kappa\overline{v}v(t) - \overline{y}y(t)$$

$$0 = \gamma_z {\gamma_x}^{\frac{\theta}{1-\theta}}\overline{k}\left[\frac{\theta}{1-\theta}\gamma_x(t) + k(t+1)\right] - (1-\delta)\overline{k}\,\overline{\gamma_x}^{-1}(k(t) - \gamma_x(t)) - \overline{i}i(t)$$

$$y_t = \zeta\left(k_t g_t\right)^{-1\theta}\left(n_t l_t\right)^{(1-\theta)}$$

$$0 = y(t) - \theta k(t) + \theta g(t) - (1-\theta)(n(t) + l(t))$$

$$0 = \overline{n}n(t+1) - (1-\sigma)\overline{n}n(t) + \overline{m}m(t)$$

$$0 = m(t) - \alpha v(t) - (1-\alpha)u(t)$$

$$\begin{aligned} 0 \;=\; & \overline{we}\,(w(t) + l(t)) - p(1-\theta)\frac{\overline{y}}{\overline{n}}(y(t) - n(t)) - pa\frac{\overline{v}}{\overline{u}}\,(v(t) - u(t)) + \\ & (1-p)\phi_1 \frac{\overline{c}}{\overline{d}} \frac{\overline{f}^{(1-\eta)}}{1-\eta}\,(c(t) - d(t) + (1-\eta)f(t)) - (1-p)\phi_2 \frac{\overline{c}}{\overline{d}} \frac{(1-e)^{(1-\eta)}}{1-\eta}\,(c(t) - d(t)) \end{aligned}$$

and the expectational equations:

$$0 = E\left[\begin{array}{c} \frac{\gamma_z \overline{\gamma_x}^{\frac{\theta}{1-\theta}}}{\beta}\,(c(t) - c(t+1) + d(t+1) - d(t+1)) \\ -(1-\delta)\overline{\gamma_x}^{-1}\gamma_x(t+1) + \theta\frac{\overline{y}}{\overline{k}}\,(y(t+1) - k(t-1)) \end{array}\right] - \frac{\gamma_z \overline{\gamma_x}^{\frac{\theta}{1-\theta}}}{\beta}\frac{\theta}{1-\theta}\gamma_x(t)$$

$$0 = E\left\{\begin{array}{c} \frac{1}{\beta}\frac{a\overline{v}}{\overline{m}}\,(c(t) - c(t+1) + d(t+1) - d(t+1)) + (1-\theta)\frac{\overline{y}}{\overline{n}}\,(y(t+1) - n(t+1)) \\ -\overline{w}\overline{l}\,(w(t+1) + l(t+1)) + \frac{a\overline{v}}{\overline{m}}(1-\overline{\sigma})\,(v(t+1) - m(t+1)) - \frac{a\overline{v}}{\overline{m}}\overline{\sigma}\sigma(t+1) \end{array}\right\} - \frac{1}{\beta}\frac{a\overline{v}}{\overline{m}}\,(v(t) - m(t))$$

2.B Appendix: Macro-data

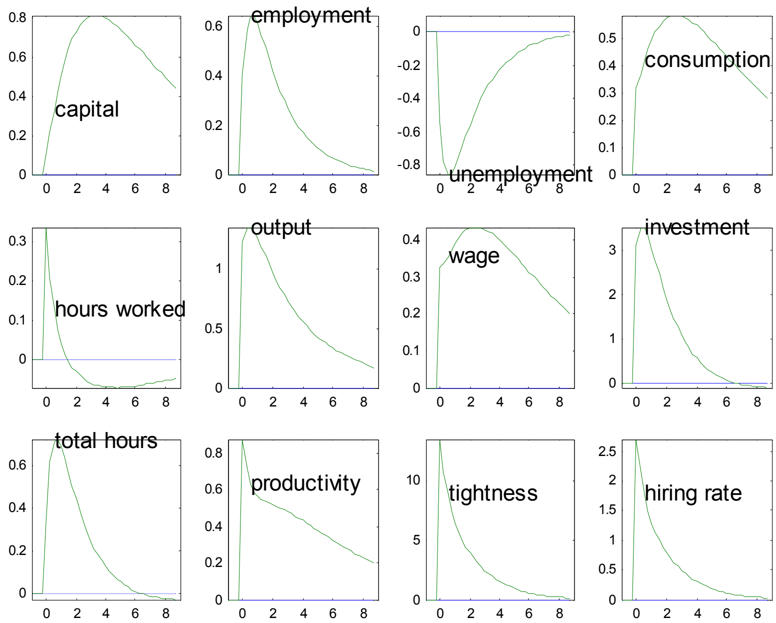
The macro-data used in this study is real aggregate data of the United States for the period 1964:Q1-2005:Q4. The source is the Federal Reserve Economic Data (FRED) from the Federal Reserve Bank of Saint Louis.

- Consumption: (real consumption of non durables + real consumption of services + government expenditures)/(population +16)
- Investment = (real consumption of durable goods+ real fixed private investment)/(population +16)
- output = consumption + investment + vacancies*cost per vacancy
- vacancies = help wanted advertising in newspapers / (population+16)

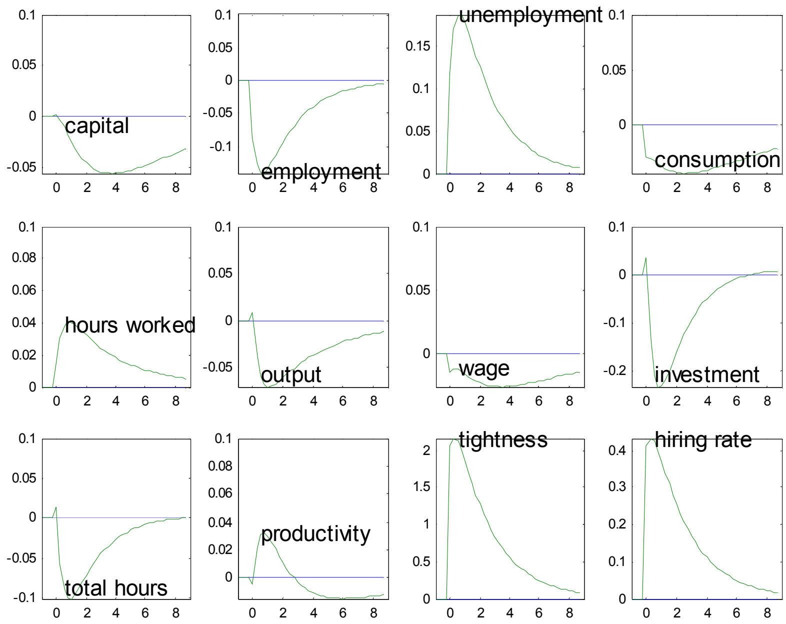
- employment = (civilian employment +16) / (population+16)
- unemployment = 1 - employment
- tightness = vacancies/unemployment
- total hours = employment*average weekly hours / (population+16)
- labor productivity = output / total hours

2.C Impulse-response functions

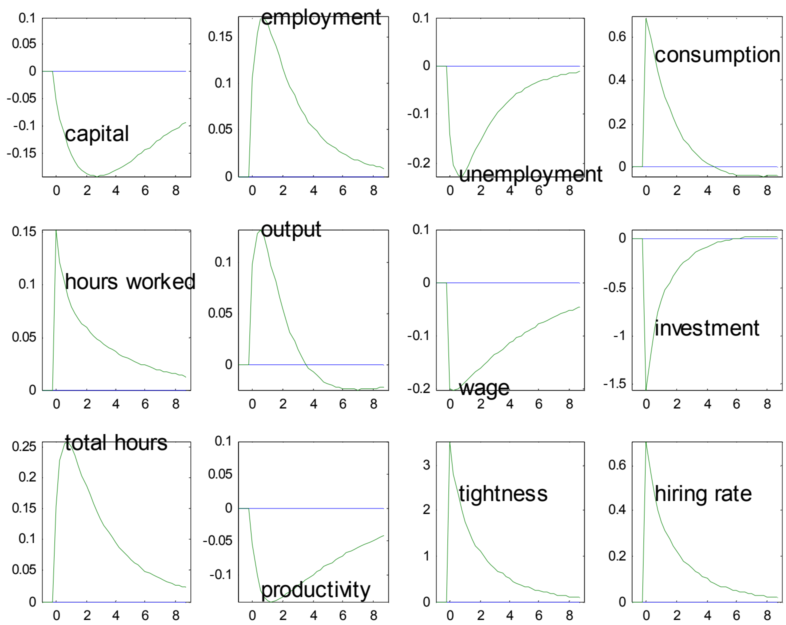
Impulse responses to neutral technology shocks



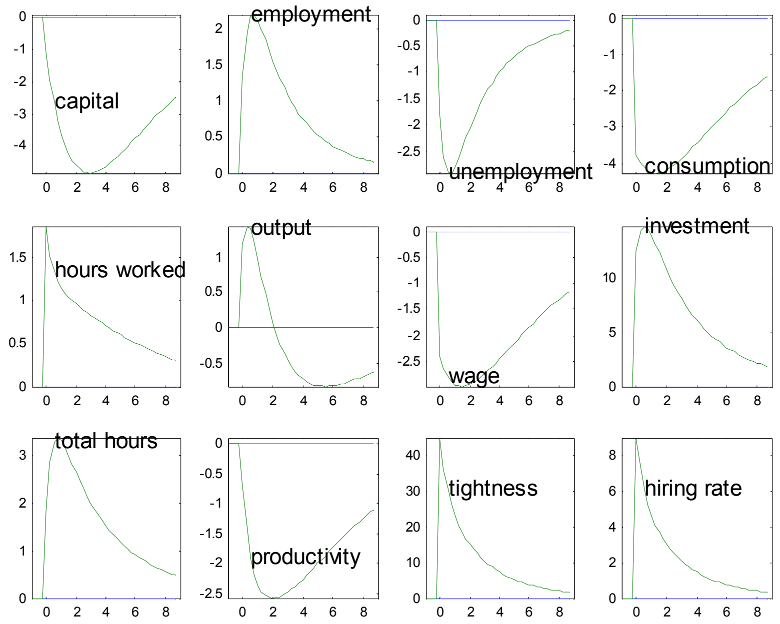
Impulse responses to shocks to job separations



Impulse responses to preference shock



Impulse responses to investment-specific technology shocks



2.D Variance decomposition

TABLE 3: VARIANCE DECOMPOSITION ANALYSIS

Quarters ahead	Percentage due to technology	Percentage due to job destruction	Percentage due to preference	Percentage due to Invst-spcf.
(A) Output				
1	94.7719 (0.0031)	0.0062 (0.0001)	0.7634 (0.0031)	4.4514 (0.0001)
4	81.5334 (0.0032)	0.1048 (0.0001)	1.5592 (0.0031)	16.8012 (0.0002)
8	72.6304 (0.0031)	0.1601 (0.0002)	1.5618 (0.0028)	25.6466 (0.0002)
20	61.5615 (0.0027)	0.1704 (0.0003)	1.2211 (0.0024)	37.0465 (0.0004)
∞	57.5115 (0.0027)	0.1641 (0.0003)	1.0989 (0.0022)	41.2251 (0.0005)
(B) Consumption				
1	8.2348 (0.0251)	0.0754 (0.0001)	37.8825 (0.0258)	53.7989 (0.0009)
4	22.6212 (0.0337)	0.1605 (0.0001)	45.1705 (0.0340)	32.0441 (0.0003)
8	23.4532 (0.0340)	0.1522 (0.0001)	22.3989 (0.0344)	53.9943 (0.0006)
20	14.5434 (0.0244)	0.0912 (0.0002)	5.4398 (0.0244)	79.9253 (0.0018)
∞	12.2991 (0.0205)	0.0769 (0.0002)	3.4240 (0.0200)	83.8704 (0.0024)
(C) Investment				
1	48.8137 (0.0078)	0.0032 (0.0001)	12.3811 (0.0076)	38.8014 (0.0006)
4	58.5699 (0.0051)	0.1496 (0.0001)	6.5485 (0.0049)	34.7319 (0.0004)
8	65.4239 (0.0044)	0.2642 (0.0003)	5.3245 (0.0041)	28.9873 (0.0003)
20	59.2955 (0.0042)	0.2707 (0.0003)	4.3548 (0.0039)	36.0789 (0.0003)
∞	48.3641 (0.0042)	0.2210 (0.0003)	3.5449 (0.0038)	47.8698 (0.0005)
(D) Total Hours				
1	2.8348 (0.0222)	0.0010 (0.0001)	1.3383 (0.0221)	1.8957 (0.0007)
4	8.9659 (0.0224)	0.0604 (0.0008)	2.7896 (0.0223)	2.3073 (0.0003)
8	22.0435 (0.0241)	0.1319 (0.0013)	3.4541 (0.0240)	5.5941 (0.0002)
20	52.6153 (0.0266)	0.1508 (0.0015)	3.6112 (0.0267)	15.4644 (0.0005)
∞	64.2666 (0.0266)	0.1325 (0.0014)	3.2774 (0.0267)	26.5599 (0.0008)

TABLE 3: VARIANCE DECOMPOSITION ANALYSIS (Con't.)

Quarters ahead	Percentage due to technology	Percentage due to job destruction	Percentage due to preference	Percentage due to Invst-spcf.
(E) Tightness				
1	47.4503 (0.0121)	18.2118 (0.0120)	2.9180 (0.0003)	18.2564 (0.0000)
4	50.4378 (0.0122)	24.0603 (0.0121)	6.2366 (0.0003)	12.2850 (0.0000)
8	39.1042 (0.0124)	22.7985 (0.0121)	7.2138 (0.0006)	25.9582 (0.0000)
20	18.8518 (0.0125)	13.0137 (0.0123)	4.5369 (0.0008)	43.3472 (0.0000)
∞	14.9616 (0.0126)	9.9506 (0.0123)	3.4933 (0.0008)	61.2816 (0.0000)

Chapter 3

A RBC model with unemployed loss of skills

3.1 Introduction

In this chapter, we propose a real business cycle model with matching in the labor market and two types of workers, high-skilled and low-skilled, in which high-skilled workers might suffer from a decapitalization of their human capital while unemployed.

Related to the determinants of the persistence in unemployment, Pissarides (1992) proposes an overlapping generation model to show that when unemployed workers lose some of their skills, the effects of a negative temporary shock to employment can persist for a long time. The key mechanism that drives the result is a variant of the "thin market externality" that reduces the demand of jobs when duration of unemployment increases. A similar underlying idea we find in Blanchard and Diamond (1994) who study the relationship between "ranking" -or the preference of employers for short-term unemployed workers- wages and unemployment. The hypothesis of loss of skills during unemployment has also been used in the literature to explain the differences between unemployment rates in Europe and US. Ljungqvist and Sargent (1998) is the first paper that introduces this "turbulence" shock in the literature.

We propose a model that emcompasses the Pissarides and Ljungqvist-Sargent models of skill loss. Given that none of the papers above study the cyclical behavior of unemployment and other macro-variables, it seems sensible, once we have understood which are the key problems of labor markets nowadays (i.e. the steadily increase of unemployment since the late 70s and the large fraction of long-term unemployed), to try to embed them into a standard real business cycle model so as to construct a suitable framework for policy making.

Our starting point would be the seminal papers that introduce frictional labor markets into a RBC framework (Merz (1995) and Andolfatto (1996)). These two papers outperform previous studies in terms of explaining the performance of the macroeconomic variables along the business cycle. However, as Hall (2005) and Shimer (2005) pointed out, there is still room for improvement, mainly in terms of volatility and persistence of vacancies and unemployment, and therefore of the labor market tightness. Shimer suggests that this deficiency could be overcome by introducing sticky wages. We will analyze, as well, how the assumption introduced in this model, i.e., the loss of skills, can contribute or not to better understand the propagation mechanism of unemployment, and consecutively, of labor market tightness.

We estimate via maximum likelihood the structural parameters of the model using key macroeconomic time series for the US economy. We find sensible values for the structural parameters of the model and an improvement in the performance of the model in terms of volatilities of the main variables of the model with respect to a more standard specification in which unemployed workers do not suffer from this loss of skills. On the contrary, although not surprisingly, unemployment shows larger persistence in the model than in the data. Finally, our model allows us to test whether there is empirical evidence in favor of an increase in "turbulence" as proposed by Ljunqvist and Sargent. We do not find strong evidence in favor of an increase in turbulence shock for the US.

The structure of the paper is the following: in Section 2, we present the theoretical model. Section 3 estimates the structural parameters of the model . Section 4 evaluates the model by analyzing the second-order moments both in the model and in the data. Section 5 analyzes the stability of the parameters of the model and relates the result with the literature that explains the rise of unemployment in the European labor markets. Finally, Section 6 concludes.

3.2 The model

The economy is populated by a continuum of infinitely lived agents with measure one. Workers are assumed to be either high-skilled, h, or low-skilled, l. High skilled workers who have just lost their jobs retain their skill for a certain period of time. The loss of skill occurs over time and is modelled as a random process following a Poisson distribution with parameter γ. i.e. with probability γ a high-skilled unemployed worker is going to suffer a depreciation of his human capital and be transformed into a low-skilled unemployed. On the other hand, there is a probability η that a low skilled employed worker upgrades his human capital and become a high-skilled worker.

Both types of workers can exogenously lose their jobs with probabilities σ_h for the high skilled and σ_l for the low-skilled. The probabilities of leaving unemployment or equivalently, the probabilities of finding a job are ϕ_h for the high-skilled and ϕ_l for the low-skilled.

Unemployed workers receive an unemployment insurance when losing their jobs which is a fraction, ψ of the wage they had while working. In this way we have three types of unemployed workers: U_h, which represent the pool of unemployed workers with high skills and high unemployment benefit, U_t, which represent the pool of unemployed workers who have suffered a depreciation of their human capital but still receive high unemployment benefits and U_b, that represents the pool of unemployed workers with low skills and low unemployment benefits.

When looking for a job, we pool the low-skilled searchers with low and high unemployment benefits, and create a group $U_l = U_t + U_b$, so that firms opening vacancies for workers with low-skills face the supply U_l.

Therefore, our model can be summarized through the flows represented in the next figure, where E_h and E_l represent the pool of employed workers with high and low skills, respectively.

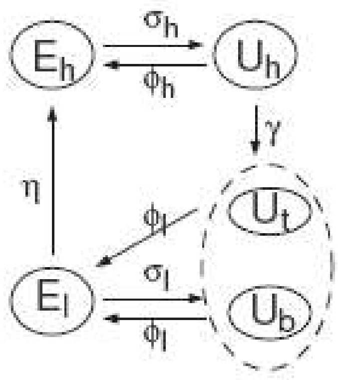


Figure 1: Flows of workers

Firms and Technology - The production sector is made up of a large number of identical competitive firms. The production technology is represented by a constant returns to scale Cobb-Douglas production function. Therefore, there exists a representative firm which uses capital, K_t, and labor, T_t, to produce the aggregate good, Y_t, according to the following technology

$$Y_t = \xi A_t K_t^{\theta} (\ell^t T_t)^{1-\theta}$$

where $\ell > 1$ measures the gross rate of labor-augmenting technological progress. The fact that we have a deterministic growth rate, would make necessary to detrend the variables in such a way that in equilibrium the economy would converge to a steady state in which the detrended variables of the model would remain constant. We define the detrended variables, which will be represented in small letters, as: $y_t = Y_t/\ell^t$, $k_t = K_t//\ell^t$, $t_t = T_t$, $a_t = A_t$, $u_{j,t} = U_{j,t}$, $n_{j,t} = N_{j,t}$, $h_{j,t} = H_{j,t}$, $w_{j,t} = \ell^t w_{j,t}/\ell^t$, $v_{j,t} = V_{j,t}$, $a_{j,t} = \ell^t a_{j,t}/\ell^t$, $m_{j,t} = M_{j,t}$, $i_{j,t} = I_{j,t}/\ell^t$. In what follows we will work with the stationary model. This means that the production function above can be stated as:

$$y_t = \xi a_t k_t^{\theta} (t_t)^{1-\theta}$$

The TFP shock a_t follows a first order autorregresive process

$$\ln\left(a_t\right) = \rho \ln\left(a_{t-1}\right) + (1-\rho)\ln\left(\overline{a}\right) + \varepsilon_t^a$$

where $\overline{a} > 0$ represents the steady state value, and $-1 < \rho < 1$. The serially uncorrelated innovation ε_t^a is assumed to be normally and independently distributed over time with mean 0 and variance σ_ε.

The firm hires capital, k_t, and labor, t_t and opens vacancies for high and low-skilled workers, v_h and v_l, to maximize the expected present value of cash flows,

$$\sum_{t=0}^{\infty} \Delta^t \left[F(a_t, k_t, t_t) - w_{h,t} n_{h,t} h_{h,t} - w_{l,t} n_{l,t} h_{l,t} - r_t k_t - a(v_s + v_l)\right]$$

subject to the laws of motion of employment (1) and (2) specified below. a denotes de cost of opening a vacant v_j, with $j = h, l$ and Δ^t is the discount factor for the firm, with $\Delta^t = \frac{\beta Uc(s')}{Uc(s)}$. The amount of labor included in the production function is defined in efficiency units as follows

$$t_t = n_{h,t} h_{h,t} + \tau n_{l,t} h_{l,t}$$

where $n_{j,t}$ denotes the number of workers of type j in period t and $h_{j,t}$ denotes the number of hours worked by each type of worker. $\tau < 1$.

Since the labor market is frictional, the laws of motion for the two types of workers (high and low skill) are defined as:

$$n_{h,t+1} = n_{h,t}(1 - \sigma_s) + q_{h,t} v_{h,t} + \eta n_{l,t} \tag{3.1}$$

$$n_{l,t+1} = n_{l,t}(1 - \sigma_l) + q_{l,t} v_{l,t} - \eta n_{l,t} \tag{3.2}$$

where $q_{j,t}$ represents the perceived probability that a vacancy of type j gets matched with an unemployed worker of the same type. $\eta n_{l,t}$ represents the fraction of low-skilled unemployed workers that suffer an upgrade of skills every period. Thus, upgrading follows a Poisson process with η rate which is independent of other processes in the model.

The labor market - The labor market is modellized as a frictional market in which firms and workers engage in employment relationships. The total number of matches per unit of time is represented by the following technology

$$m_{j,t} = m(v_{j,t} \ , \ u_{j,t})$$

where $u_{j,t}$ represents the total number of type j searchers and $v_{j,t}$ the total number of vacancies of type j. This matching function is increasing in both arguments, concave and homogeneous of degree one.

The job vacancies and the unemployed workers that are matched at any point in time t, are randomly selected from the sets v and u. Therefore, the process that changes the state of vacant jobs to filled vacant is a Poisson with rate $q_{j,t}=\frac{m_{j,t}}{v_{j,t}}$. Similarly, unemployed workers move into employment with probability $\phi_{j,t}=\frac{m_{j,t}}{u_{j,t}}$.

The empirical literature has further found that a log-linear Cobb-Douglas approximation of the matching function fits the data well[1]. So, in our model, the total number of matches at time t is given by a Cobb-Douglas matching function in the total number of searchers and vacancies of type j.

$$m_{j,t} = \chi_j v_{j,t}^{\alpha j} u_{j,t}^{1-\alpha_j}$$

where χ_j is called the "efficiency parameter" of the matching function. Under the Cobb-Douglas specification above, the probability of finding a job, ϕ_j increases with the tightness ratio $(\frac{v}{u})$ with elasticity $1 - \alpha_j < 1$.

Households and Preferences - The economy is populated by identical, infinitely-lived households. In each household there are high and low skilled workers. The measure of type j workers is denoted by e_j, for $j = h, l$ and the total measure of workers is normalize to one. We assume a complete set of insurance markets such that the worker's saving choices do not depend on its state on the labor market. Thus there is a representative household that solves the following problem:

$$Max \ E \left\{ \sum_{t=0}^{\infty} \beta^t U(c_t, h_t) \right\} \tag{3.3}$$

where c_t denotes consumption and h_t denotes time spent at the work place. The specification of the utility function adopted in our model is the following:

[1] see Pissarides (1990), ch.1 and Petrongolo and Pissarides (2001) for a survey.

$$U(c_t, h_t) = \log(c_t) + n_{h,t}\Gamma_t^{n_h} + n_{l,t}\Gamma_t^{n_l} + u_{h,t}\Gamma_t^{u_h} + u_{l,t}\Gamma_t^{n_l}$$

where $\Gamma_t^{n_j} = \phi_{1j}\frac{(1-h_{i,t})^{1-\eta^u}}{1-\eta^u}$ and $\Gamma_t^{u_j} = \phi_{2j}\frac{(1-e_t)^{1-\eta^u}}{1-\eta^u}$. This function, is the one used in Andolfatto (1996). Although is not the standard specification in RBC models, it would allow us to analyze straightforward the implications of introducing the assumption of loss of skills during unemployment with respect to this "reference model"

The household has to decide how to split current income between consumption and investment. Its income is made up of capital income, unemployment benefits and the wage bill net of the lump sum, Ψ_t, they have to pay to the government to finance the unemployment insurances, $b_{j,t}$. Therefore, the household's budget constraint in period t is

$$c_t + i_t + \Psi_t \leq w_{h,t}n_{h,t}h_{h,t} + w_{l,t}n_{l,t}h_{l,t} + u_{h,t}b_{h,t} + u_{l,t}b_{l,t} + r_t k_t + p_t$$

and investment is defined as follows:

$$i_t = \ell k_{t+1} - (1-\delta)k_t$$

The dynamics of unemployment of the high skilled and the low skilled workers are given as:

$$u_{h,t+1} = u_{h,t} - m_{h,t} + \sigma_h n_{h,t} - \gamma u_{h,t}$$

$$u_{l,t+1} = u_{l,t} - m_{l,t} + \sigma_l n_{l,t} + \gamma u_{h,t}$$

where $u_{j,t}$ denotes the measure of type j searchers, σ_j is the exogenous rate of job destruction and ϕ_j is the perceived probability that an unemployed worker be matched in period t. $\gamma u_{h,t}$ represents the fraction of workers that suffer from a loss of skill while unemployed. As we said, this process follows a Poisson distribution with parameter γ. Since we have normalized the measure of the population to one, this means that every period a fraction γ of the high skilled workers suffers a "decapitalization" of their human capital while becoming long term unemployed.

Optimal contract - Following the standard literature on frictional unemployment, we assume that wages are the solution to a Nash-bargaining problem. Hence, if we denote as p_j the worker's bargaining power, the optimal contract is given by

$$w_{j,t}h_{j,t}{=}\arg\max\left\{W_{nj,t}^{p}J_{nj,t}^{(1-p)}\right\},\quad \text{for } j=h,l$$

where $W_{n_j,t}$ and $J_{n_j,t}$ represent the income value of employment of type j to the household and the firm respectively. p_j will be treated as a constant parameter moving strictly between 0 and 1.

The income value of high-skilled employment to the household in units of the consumption good is given by:

$$W_{n_h}(\Omega_t^H)=\left\{\begin{array}{l}(\Gamma^{n_h}-\Gamma^{u_h})+\lambda_t w_{h,t}n_{h,t}h_{h,t}-\lambda_t u_{h,t}b_{h,t}+\\ +\beta E_t\left[W_{n_h}(\Omega_{t+1}^H)\right](1-\phi_h-\sigma_h-\gamma)+\gamma\beta E\left[W_{nl}(\Omega_{t+1}^H)\right]\end{array}\right\}$$

and is made up of three components: (i) the household gain in terms of wages because an additional high-skilled agent starts working (ii) the utility losses in terms of leisure that this new job generates and (iii) the expected present value of this job in the future. This expected present value is formed by the continuation value of the job, that is the net probability of keeping a high-skilled employment, minus the probability γ that the high-skilled worker suffer from a depreciation of his human capital and become a low-skilled worker

The minimum wage the worker is going to accept comes from $W_{n_h}(\Omega_t^H)=0$. Notice that the high skilled worker takes into account the possibility of becoming long term unemployed and with probability γ losing some of his skills, therefore he is willing to accept a lower wage than in the standard case in which $\gamma=0$.

Similarly, the firm's marginal benefit from employment is made up of the job's yields, i.e., the contribution to output of this marginal worker minus the returns to his work, plus the expected present value of this job in the future.

$$J_{n_h}(\Omega_t^F)=F_{n_h,t}-w_{h,t}h_{h,t}+\beta E_t\left[\frac{\lambda_{t+1}}{\lambda_t}J_{ns}(\Omega_{t+1}^F)\right](1-\sigma_h)$$

For low-skilled workers, the household's marginal value of low skilled employment is given by the equation below, which includes the net gain in utility for an additional low skilled worker plus the expected present value of this job in the future.

$$W_{n_l}(\Omega_t^H)=(\Gamma^{n}-\Gamma^{u})+\lambda_t w_{l,t}h_{l,t}-\lambda_t u_{l,t}b_{l,t}+\beta E_t\left[W_{nl}(\Omega_{t+1}^H)\right](1-\phi_{l,t}-\sigma_l)$$

The firms' marginal value of low skilled employment equals the job's yields plus the expected present value of this position in the future.

$$
\begin{aligned}
J_{n_l}(\Omega_t^F) &= F_{nl,t} - w_{l,t} h_{l,t} + \beta E_t \left[\frac{\lambda_{t+1}}{\lambda_t} J_{nl}(\Omega_{t+1}^F) \right] (1-\sigma_l) \\
&\quad + \eta \beta E_t \left\{ \frac{\lambda_{t+1}}{\lambda_t} \left[J_{ns}(\Omega_{t+1}^F) - J_{nl}(\Omega_{t+1}^F) \right] \right\}
\end{aligned}
$$

notice that this expected present value includes the probability η of transforming the low quality match into a high quality one. $J_{nh}(\Omega_t^F) = 0$, gives the maximum compensation that the employer is willing to pay[2].

From the first order condition of this maximization problem, we get the optimal contract for each type of worker in this economy is given by the reservation wage and a fraction p of the net surplus they create by accepting the job offer. Net surplus means the product value net of what workers give up in terms of leisure and reservation wage. These optimal contracts can expressed as follows:

$$
\begin{aligned}
w_{h,t} h_{h,t} &= p_h F_{nh,t} + p_h a_h \frac{v_{h,t}}{u_{h,t}} + p_h \gamma \left(\frac{a_h}{q_{h,t}} - \frac{a_l}{q_{l,t}} \right) + (1-p_h) b_{h,t} \\
&\quad - (1-p_h) c_t \left(\phi_{1,h} \frac{(1-h_{h,t})^{(1-\eta_h)}}{(1-\eta_h)} - \phi_{2,h} \frac{(1-e)^{(1-\eta_h)}}{(1-\eta_h)} \right)
\end{aligned}
$$

The reservation wage of a high-skilled worker is given by the unemployment insurance plus the leisure in terms of utility enjoyed by the potential worker. The net surplus is given by the contribution of the worker to the output, which is his marginal productivity plus the savings in terms of posting vacancies cost and the opportunity cost for the firm of not hiring the high skilled worker given that with probability γ he can suffer a depreciation of skills net of what the worker gives up which is his reservation wage.

Similarly, the optimal contract for the low-skilled worker is given by the following equation:

$$
\begin{aligned}
w_{l,t} h_{l,t} &= p_l F_{nl,t} + p_l a_l \frac{v_{l,t}}{u_{l,t}} + p_l \eta \left(\frac{a_h}{q_{h,t}} - \frac{a_l}{q_{l,t}} \right) + (1-p_l) b_{l,t} \\
&\quad - (1-p_l) c_t \left(\phi_{1,l} \frac{(1-h_{l,t})^{(1-\eta_l)}}{(1-\eta_l)} - \phi_{2,h} \frac{(1-e)^{(1-\eta_h)}}{(1-\eta_h)} \right)
\end{aligned}
$$

[2] Note that if the firm offers this maximum compensation to the worker, it would generate negative profits in the steady state, because it does not take into account the fact that posting vacancies is a costly activity.

The only difference with respect to the optimal contract for the other type of workers is that now, the firm takes into account that when it hires a low-skilled worker he can become high-skilled type with probability η.

To disentangle wages and hours worked we need two additional equations. We can compute the optimal number of hours for each type of worker differentiating the total surplus of each type of match $S_{j,t}=\frac{1}{\lambda_t}W_{n_j,t}+J_{n_j,t}$ with respect to the hours $\frac{\partial S_{j,t}}{\partial h_{j,t}}$, so that the optimal number of hours worked for each type of worker can be represented as:

$$\phi_1 \frac{1}{\lambda_t}(1-h_{j,t})^{(-\eta_j)}=(1-\theta)\frac{y_t}{l_t}$$

with $j=h,l$.

Definition of the recursive equilibrium - We can define the equilibrium of this economy as a set of infinite sequences for the rental price of capital $\{r_t\}$, wage rates $\{w_{h,t},w_{l,t}\}$, employment and unemployment levels $\{n_{h,t},n_{l,t},u_{h,t},u_{l,t}\}$, capital $\{k_t\}$, consumption $\{c_t\}$, vacancies $\{v_{h,t},v_{l,t}\}$, hazard rates for workers $\{\phi_{h,t},\phi_{l,t}\}$ and vacancies $\{q_{s,t},q_{l,t}\}$, such that,

(i) Taking the rental prices and matching rates as given, $\{k_t\}$, $\{n_{h,t},n_{l,t}\}$ and $\{v_{h,t},v_{l,t}\}$ maximize the firms' profits

(ii) Wages are the solution to the Nash bargaining problem

(iii) Taking wages, the rental price of capital and hazard rates, $\{c_t\}$ and $\{k_t\}$ solves the household optimization problem

(iv) Hazard rates are given by the matching function

(v) The government constraint holds

3.3 Estimation of the model

The model presented above has a large number of parameters. This rises the problem of assigning values to all of them. Standard calibration does not seem the best technique when models are richly parameterized given that neither the focus on a limited set of moments of the model nor the transfer of microeconomics estimates from one model to another will provide the discipline to quantify the behavior of the model; so we have to rely in alternative methods that allow us to properly estimate the parameters of the model. We will estimate the parameters of our model via Maximum Likelihood.

Maximum Likelihood provides a systematic procedure to give values to all the parameters of interest. This means that we have to evaluate the likelihood function of our DSGE model. Except in a few cases, there is no analytical or numerical procedure to directly do it. But we can transform the theoretical model into a state-space econometric model and under the assumptions that the shocks to the economy are normally distributed and that the policy functions of the model are linearly approximated, we can look for a numerical approximation of the likelihood function with the help of the Kalman filter. In what follows we explain this in more detail.

State-space representation of the model - Appendix A describes the competitive solution to the model above, so that when $\varepsilon_t^a = 0$, the economy converges to a steady state in which each of the detrended variables remain constant. This steady state, depends on the structural parameters of the model describing tastes, technologies and matching. Appendix B contains the log-linearizations around the steady state from which we will implement the method proposed by Uhlig (1997) that when applied to a linear system yields the approximate solution or policy rules of the form:

$$x_t = Ax_{t-1} + B\varepsilon_t$$
$$y_t = Cx_t$$

where x_t and y_t represent vectors of logarithmic deviations of the states and the control variables from their steady-state levels. The elements of the matrices A, B and C depend on some of the model structural parameters.

The solution above considers that there is only one shock, the technology shock, driving the business cycle. This makes the model stochastically singular, i.e., the model predicts that certain combinations of the endogenous variables will hold with equality, and if in the data these exact relationships do not hold, maximum likelihood estimation will not be a valid method for the estimation.

Therefore, we should do any transformation in the model that allow us to overcome this problem. As Ireland (2004) explains, there are two common approaches to face the stochastic singularity problem. The first one consists in increasing the number of shocks in the model until we have the same number of shocks as number of time series used in the estimation; whereas the second approach first proposed by Altug (1989), which is the one we use here, consists in augmenting last equation of the system above with an error term or measurement error,

me_t. These errors represent the movements in the data that the theory does not explain (those movements that are not generated by the TFP shock, in our case) and are uncorrelated across variables. Then we have a system of the form:

$$s_t = Fs_{t-1} + V\varepsilon_t$$
$$f_t = Gs_t + me_t$$

where f_t denotes a vector of variables observed at date t, and s_t is the state vector, F and G are again matrices of parameters. The first equation of the system is known as the state equation and the second is known as the observation equation. Vectors ε_t and me_t are white noise vectors with $E[\varepsilon_t \varepsilon_t'] = Q$ and $E[me_t me_t'] = R$. Also $E[\varepsilon_t me_t'] = 0$

Thus, once we have included this measurement errors, our theoretical DSGE model takes the form of a state-space econometric model whose parameters can now be estimated via maximum likelihood.

Kalman filter and approximation of the likelihood function - The empirical model written as a state-space econometric model, allows for the evaluation of the likelihood function using the Kalman filter algorithm explained in detail in Hamilton (1994, Chapter 13).

The ultimate objective is to estimate the values of the unknown parameters in the system on the basis of these observations $f_1, f_2, ..., f_T$. The Kalman filter works as a recursive estimator that takes initial values for the state-vector $\widehat{s}_{t|t-1}$ and its associate mean squared error $P_{t|t-1}$, to calculate linear least square forecast of the state-vector for subsequent periods t=2,3,...T. This forecasts are of the form. $\widehat{s}_{t|t-1} = \widehat{E}[s_{t+1} | f_t]$, where $\widehat{E}[s_{t+1} | f_t]$ is the linear projection of s_{t+1} on f_t and a constant. The Kalman filter has two main phases: prediction and update.

In the prediction phase, using the law of iterated projections, it plugs the forecast $\widehat{s}_{t|t-1}$ into the observable equation to yield a forecasting of f_t

$$\widehat{f}_{t|t-1} = G\widehat{s}_{t|t-1}$$

the error of this forecast is defined as $w_t = f_t - \widehat{f_{t|t-1}} = Gs_t + me_t - G\widehat{s}_{t|t-1} - me_t$ with MSE

$$E[(f_t - \widehat{f_{t|t-1}})(f_t - \widehat{f_{t|t-1}})'] = FP_{t|t-1}F' + R.$$

In the second phase, the inference about the current value of s_t is updated on the basis of the observation of f_t to produce $\widehat{s}_{t|t.}$. Introducing it into the state equation produces a forecast of $\widehat{s}_{t+1|t}$

$$\widehat{s}_{t+1|t} = F\widehat{s}_{t|t} + 0$$

evaluating $\widehat{s}_{t|t}$ by using the formula of updating a linear projection, and substituting it above, we get the best forecast of s_{t+1} based on a constant and a linear function of the observable vector f_t

$$\widehat{s}_{t+1|t} = F\widehat{s}_{t|t-1} + K_t(f_t - \widehat{f}_{t|t-1})$$

where K_t is the optimal Kalman gain matrix, which depends on the matrices F, G, R and the stationary variance Σ_t. Those matrices are not function of the data but entirely determined by the population parameters of the process. $\widehat{s}_{t+1|t}$ denotes the best forecast of s_{t+1} based on a constant and a linear function of the observables f_t if and only if K_t is the optimal gain matrix.

The application of the Kalman filter let us calculate the log-likelihood function of the hybrid model as

$$ln(L) = -\frac{3T}{2}\ln(2\pi) - \frac{1}{2}\sum_{t=1}^{T}\ln\left|G\Sigma_t G'\right| - \frac{1}{2}\sum_{t=1}^{T} w_t'\left(G\Sigma_t G'\right)^{-1} w_t$$

Using a numerical search algorithm one can find the set of parameters contained in the matrices F, G, Q and R that maximize the likelihood function. Usually, maximum likelihood estimations of this type are criticized because it is very difficult to be sure whether we are in the global maximum or on the contrary we are just in a local one, given that the likelihood function displays a quite sinuous pattern. To avoid this criticism with our estimations we borrow from physics another algorithm called "*simulated annealing*". This is a generic probabilistic meta-algorithm for the global optimization problem, i.e. it looks for a good approximation to the global optimum of a function in a large search space. Each step of the simulated annealing algorithm replaces the current solution by a random "nearby" solution. The allowance for these movements saves the method from becoming stuck at a local minimum.

In principle, this numerical algorithm is allowed to select values of the parameters that lie anywhere between the positive and the negative infinity. But to ensure that our parameters satisfy the theoretical restrictions listed in section 2, additional constraints have been imposed[3].

The data - Data is taken from the Federal Reserve Bank of St. Louis' FRED database. Data for gross domestic income and wages is taken from the Bureau of Economic Analysis. In Appendix C, we present detailed information about each series. Monthly data has been transformed into quarterly data using averages. The period selected goes from 1964-1 to 2005-4.

When the model takes the form of a state-space representation, it can be estimated via maximum likelihood once analogs to the model's variables are found in the data. Therefore, C_t is defined as real personal consumption on non durables and services plus government expenditure. Investment, I_t is defined as the sum of consumption on durable goods plus investment. Vacancies, V_t are proxied by a widely used index which reflects the number of "*help-wanted*" advertisement registered in US newspapers. N_t comes directly from the number of civilian employment, and thus unemployment can be computed as $1 - N_t$. Unemployment duration, D_t ,also comes directly from the median duration of unemployment series.

All the variables have been divided by the civilian population aged 16 or over, so as to have them in per capita terms. On top of that we have taken logarithms of all the variables and calculated the growth rates when necessary. Series have not been filtered in any other way.

Finally, to make them comparable with the vectors of logarithmic deviations of the variables from their steady-state levels, we have to use the definitions:

$$\begin{aligned}
\widehat{c_t} &= \log(c_t) - \log(\overline{c}) \\
\widehat{i_t} &= \log(i_t) - \log(\overline{i}) \\
\widehat{\Pr_t} &= \log(\Pr{}_t) - \log(\overline{\Pr}) - t\log(\gamma) \\
\widehat{t_t} &= \log(t_t) - \log(\overline{t}) \\
\widehat{\theta_t} &= \log(\theta_t) - \log(\overline{\theta}) \\
\widehat{d_t} &= \log(d_t) - \log(\overline{d})
\end{aligned}$$

for all $t = 1....T$. Remember that to solve the problem we have normalized output to 1. Therefore, consumption and investment here are defined as $\widehat{c_t} = \widehat{\left(\frac{c_{d,t}}{y_t}\right)}$. and $\widehat{i_t} = \widehat{\left(\frac{i_{d,t}}{y_t}\right)}$. Productivity is

[3] In particular, some of our parameters are constrained to be positive, so we constraint the algorithm to work with absolute values. Many of our parameters are probabilities that should lie between zero and one, so we again constraint the algorithm to work with the logistic transformation of the parameter.

growing at rate ℓ in steady state. Once those transformations are made, the vector of observable is given by:

$$f_t = \left[\begin{array}{cccccc} \widehat{c_t} & \widehat{i_t} & \widehat{t_t} & \widehat{Pd_t} & \widehat{\theta_t} & \widehat{dur_t} \end{array} \right]$$

Parameter estimates - Usually algorithms for computing maximum likelihood estimates have the drawback that they do not produce standard errors. This means that we should look for numerical approximations of the derivatives of the likelihood function so as to compute the information matrix and then the standard errors.

Fortunately, we know that if certain regularity conditions hold[4], the maximum likelihood estimates are consistent and asymptotically normal. Under these circumstances, the information matrix for a sample of size T can be calculated from the second derivatives of the maximized log-likelihood function as

$$I_T = -\frac{1}{T}\left\{\sum_{t=1}^{T}\frac{\partial^2 \log L(y_t,\theta)}{\partial\theta\ \partial\theta'}\right\} \tag{3.4}$$

Standard errors are then the square roots of the diagonal elements of $\frac{1}{T}(I_T)^{-1}$. This matrix has elements of very different magnitudes and therefore, the reported standard errors should be interpreted with caution.

Results of the estimation - In the next table we report the maximum likelihood estimations for the parameters and their standard errors. The values of the parameters estimated constitute all of them sensible results.

It is quite common to obtain low estimates for the discount factor β, showing the preference of the household for consumption today. What we have done is to estimate the parameters of the model keeping β fixed and equal to 0.99.

The first thing worth noting is the slow upgrade of skills. This result is in line with the assumption made by the turbulence hypothesis presented in Ljungqvist and Sargent's series of

[4]These conditions include that the model must be identified, the eigenvalues of A are inside the unit circle, the true values of the estimations do not fall on a boundary of the allowable parameter space and that variables x_t behave asymptotically like a full-rank linearly indeterministic covariance-stationary process

papers and has been widely challenged by Den Haan, Ramey and Watson (2001 and 2005) in different papers because what they think is a too low value for upgrading of skills. What we see is that the data does not give support to this criticism and supports the value proposed by LS.

Data does not fulfill the efficiency condition or Hosios condition which states that the bargaining power or the workers p_j should equal the parameter $(1-\alpha_j)$ of the matching function so as to have efficient results. As we can see from our estimations, there is no evidence in favor of this condition neither in support of Hagedorn and Manovskii's proposal. Another important difference with respect to the values proposed by Andolfatto (1996) are those related to the job separation rates, σ_i, which adopt values around 0.08 instead of the 0.15. Nevertheless, the 0.15 proposed by Andolfatto (1996) is somehow higher to what is common in the literature that corresponds to a value of 0.05.

Table 2: Estimated parameters and standard deviations

Parameter	Value	Explanation	Std. error
α_h	0.8001	elasticity vacancies type h	0.0010
α_l	0.8012	elasticity vacancies type l	0.0021
η	0.0186	upgrade of skills	0.0040
ρ	0.9492	technology persistence	0.0119
ε	0.0098	volatility	0.0001
p_h	0.1067	bargaining power workers	0.0009
p_l	0.3552	bargaining power workers	0.0007
γ	1.0057	deterministic growth rate	0.0001
θ	0.583	elasticity of capital	0.0027
δ	0.033	depreciation of capital	0.0059
τ	0.901	efficiency units of low product workers	0.0124
σ_h	0.0781	exog. destruc rate for h	0.0027
σ_l	0.0897	exog. destruc rate for l	0.0020
η^u	3.3341	parameter utility function	0.0009

The persistence of the technology shock adopts a sensible value, close to the standard value of 0.95. A bit higher is the volatility ε^a which usually adopts a value of 0.007 but here that is almost 0.01.

Finally and in spite of the fact that standard errors have had to be approximated numerically, the take very small values making significant all the values of our estimation.

The deterministic growth rate of the economy takes a value close to one. This means that we could have worked with a stationary model. The least satisfactory results are those values driving the investment and capital accumulation of the economy. We obtain a value of θ close to 0.6 whereas the standard value in the literature is close to 0.4. Also the depreciation of capital, δ, takes a higher value than what is standard. This has led us to estimate a second version of the model, in which we have constrained the values of those parameters to take these standard values. The performance of variables such as consumption and investment improves with this new specification as we will see afterwards.

The remainder parameters of the model can be obtained through the estimations above and the steady state values of the variables. Table 3 describes those parameter, which again seem to be quite reasonable

Table 3: Parameters that can be obtained through the estimations

Parameter	Value	Explanation
γ	0.0265	prob. of downgrade of skills
χ^h	0.8499	efficiency paramet. matching function
χ^l	0.8526	efficiency paramet matching function
ξ	0.5585	parameter production function
a	0.1856	cost of posting a vacancy
$\phi_{1,h}$	1.4260	parameter in the utility function
$\phi_{1,l}$	0.7130	parameter in the utility function
$\phi_{2,h}$	2.9669	parameter in the utility function
$\phi_{2,h}$	0.8157	parameter in the utility function

The important parameter above is mainly the probability for high-skilled workers from suffering from a depreciation of skills, which takes a value of 0.0265, very close to the Ljunqvist and Sargent's value. The remainder parameters take sensible values.

3.4 Dynamic implications

This section is divided in two parts. In the first one we present the quantitative properties of our model whereas in the second we analyze the contribution of our model to the literature that tries to explain the rise in European unemployment and its persistence over time.

3.4.1 Evaluation of the model

Table 4 presents the volatilities of the main variables of the model and compares them with the values for the US economy and the Andolfatto (1996) model. We have two columns of results: DSGE(1) and DSGE (2). The first one corresponds to the values of the parameters presented in Table 2 above whereas DSGE(2) represents the more constrained model in which we have fixed the values of β, θ and δ fixed.

What we can see from these values is that the volatility of total hours worked, employment, hours per worker and tightness increase substantially with respect to the Andolfatto model. Our model increases also the volatility of both unemployment and vacancies with respect to the search economy or the Andolfatto results. We can see this as a success given the large literature dealing with it nowadays.

The model has some difficulty in explaining the volatility of the variable hours per worker, which results substantially larger in the model than in the data. Accordingly, the volatilities of the wage bill and the labor share of output also result more volatile than in the data. This result depends up to some extent on the parameter η^u of the utility function and the replacement ratio used to compute the unemployment benefit. Probably, a more standard specification of the utility function could yield more satisfactory results concerning the performance of this variable or even allowing for this parameter to take larger values can partially solve the problem.

The DSGE(2) yields overall better results in terms of volatilities than DSGE(1). We do not see any inconvenient in dealing with this model instead with the less restricted one. The performance of investment, consumption and tightness improves significantly when allowing for those constraints while the remaining values are pretty similar to those obtained under the DSGE(1).

Table 4: Volatilities

Variable	US Economy	RBC Search	DSGE (1)	DSGE (2)
Consumption	0.56	0.32	0.27	0.32
Investment	3.14	2.98	2.08	3.12
Total hours	0.93	0.59	0.80	0.75
Employment	0.67	0.51	0.72	0.70
Hours\Worker	0.34	0.22	1.06	1.07
Tightness	9.12	3.30	4.61	6.78
Wage bill	0.97	0.94	1.86	1.89
Productivity	0.64	0.46	0.27	0.32
Real Wage	0.44	0.39	0.26	0.27

One of the major interests of the theoretical model presented in this paper is that if satisfactory, it can be used to analyze European labor markets. One common characteristic of those markets is the existence of long-term unemployment or, in other words, high persistence of unemployment. The standard RBC model with frictional labor markets, although satisfactory in replicating the persistence of US unemployment, have major problems when replicating the persistence of European labor markets (see Esteban-Pretel, 2004). When we introduce the assumption of loss of skills in combination with unemployment benefits, the persistence of unemployment increases substantially. For example, under the Andolfatto model, 86% of the unemployed workers that lose their jobs remain unemployed one quarter apart whereas only 18% of them remain unemployed within a year. Under our specification almost half of them remain unemployed within a year, which constitutes almost 70% of the unemployed workers who have suffered from a depreciation of skills.

Table 5: Persistence of unemployment

Variable (x)	$x(t)$	$x(t+1)$	$x(t+2)$	$x(t+3)$	$x(t+4)$
Search Economy					
Total unemployment	1	0.86	0.63	0.39	0.18
Turbulent workers	–	–	–	–	–
DSGE (1)					
Total unemployment	1	0.93	0.80	0.63	0.45
Turbulent workers	1	0.98	0.91	0.80	0.67
DSGE (2)					
Total unemployment	1	0.94	0.80	0.63	0.45
Turbulent workers	1	0.98	0.91	0.80	0.67

3.4.2 Implications for the labor market performance

Nowadays we rely on at least three possible potential explanations for the high European unemployment: the combination of aggregate macro shocks and labor market institutions, the combination of micro-shocks and labor market institutions and the impact of the evolution of labor taxation on labor supply. But to disentangle the exact effect of labor market institutions on unemployment is still an issue we need to resolve[5].

The turbulence hypothesis is at the core of the second explanation. In particular, it relies on the combination of microeconomic shocks to unemployed workers' human capital and labor market institutions to explain the steadily increase in European unemployment from the late 1970s onwards while the American unemployment fluctuate around its II World Ward's value. Ljungqvist and Sargent (1998) central assumption is that "*the last couple of decades saw an increased probability of human capital loss at the time of an involuntary job displacement*" both in Europe and US.

Given that the values of the estimated parameters are in line with the values proposed by LS, we can split the initial sample into two sub-samples and analyze whether the data gives support to the hypothesis of an increase in turbulence or not. Therefore, we split the sample into two disjoint sub-samples. The first one covers the period 1964Q1-1980Q1, and the second runs the

[5] See Appendix D for a brief summary of this literature.

rest of the sample, i.e. 1980Q2-2005Q1. The breakpoint corresponds to the date around which we consider the European unemployment started rising[6].

Then we re-estimate the parameters and check for their stability. Across the board, the test rejects the null hypothesis of parameter stability. This result is in line with previous findings of Ireland (2004) and Altug (1989) and Stock and Watson (1996) who also found evidence of instability in parameters in RBC and VAR models respectively.

But what is more interesting for our analysis, is that the parameter γ that measures the probability of human capital loss remains constant along the two sub-samples and for both specifications of the model - with and without constraints on some of the parameters-.

Period	Parameter
t_1= $1964Q1 - 1980Q1$	γ_1=0.0265
t_2= $1980Q2 - 2005Q4$	γ_2=0.0265
Conclusion	$\gamma_1 = \gamma_2$

We see this result very interesting as it contributes to the literature on unemployment in the following way: according to our model and the estimation of its parameters for the US economy, we do not observe evidence in favor of an increase in the probability of human capital loss. This does not mean that we find the LS explanation wrong because, in spite of the fact that we do not observe an increase in the value of this parameter, the turbulence hypothesis is necessary to improve the performance of the RBC. It is able to increase the persistent unemployment and consequently generates long-term unemployment. This is a characteristic that previous RBC models embedding matching models in the labor market were unable to produce.

Stated in other words, we explain the increase in unemployment in the following terms: We believe that there is a negative aggregate shock that increases the pool of unemployed workers. This makes the fraction of turbulent workers increase- not because the probability of losing skills increases but because the initial pool has become bigger due to an aggregate shock-. At this point we have an increase in the number of turbulent unemployed workers who have loss their initial skills but receive unemployment benefits that correspond to the high skilled level. The rest of the story is the LS one. In a way, our results conciliate the macro and micro explanations proposed in the literature to explain the increase in unemployment and its persistence over time.

[6]We have tried different breakpoints running from 1975Q4 to 1982Q4, giving all of them similar results.

3.5 Conclusion

In this paper we have proposed a DSGE model in which the labor market is explicitly modellized as a frictional market where firms and workers engage in productive job matches. There are two types of workers, high-skilled and low-skilled workers. Furthermore, high-skilled workers might suffer from a depreciation of their human capital while unemployed.

We have estimated the parameters of the model for the US economy via maximum likelihood and we have seen that the values obtained are in line with those proposed by LS. In particular, we find very low probability of upgrading skills for the already matched workers and similar value for the turbulence parameter. We have also analyzed the quantitative properties of our model and we have studied the stability of the probability of suffering from a depreciation of human capital.

We find that our model seems to fit the data well, increases the persistence of unemployment and increases the volatility of tightness. We also find that our model estimated with US data does not give evidence of an increase in the probability of losing skills around the eighties, contrary to the central assumption of LS's explanation. Nevertheless, allowing for this depreciation of worker's human capital, generates a very strong persistence of the unemployment for all workers in the economy and mainly for those who have suffered the loss of skills.

We conclude that according to our model we cannot accept the assumption that a general increase in turbulence occurred at the end of the 70s both in Europe and US. Instead we find evidence on the existence of a negative aggregate shock that increased the pool of unemployed workers. We do not see this as a rejection of subsequent explanation given by LS, i.e. that we need the combination of the loss of skills assumption and generous unemployment benefits to produce long-term unemployment and be able to explain the steadily increase in unemployment over time.

In a way, our conciliate the micro and macro-shock based explanations in the following way: according to our model, what we find is that is a macro shock that generates the initial increase in the pool of unemployment but that is the micro shock in combination with generous unemployment benefits what contributes to its persistence.

Nevertheless, further research is needed in this area. It would be desirable to re-estimate it for some European countries to have a broader perspective of the problem. Unfortunately, we will have to wait a few years until this data would be available.

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3.A Appendix: solving for the non-stochastic equilibrium

3.A.1 Households

The household maximizes the following problem

$$W(\Omega_t^H) = \max_{c,k'} \left\{ \begin{array}{c} \log(c_t) + n_{h,t}\phi_{1,h}\frac{(1-h_{h,t})^{(1-\eta_h)}}{(1-\eta_h)} + u_{h,t}\phi_{2,h}\frac{(1-e)^{(1-\eta_h)}}{(1-\eta_h)} \\ + n_{l,t}\phi_{1,l}\frac{(1-h_{l,t})^{(1-\eta_l)}}{(1-\eta_l)} + u_{l,t}\phi_{2,l}\frac{(1-e)^{(1-\eta_l)}}{(1-\eta_l)} + \beta E_t W(\Omega_{t+1}^H) \end{array} \right\}$$

subject to the budget constraint and the law of motion for capital

$$c_t + i_t + \Upsilon_t = w_{h,t}n_{h,t}h_{h,t} + w_{l,t}n_{l,t}h_{l,t} + u_{h,t}b_{h,t} + u_{l,t}b_{l,t} + r_t k_t + \Pi_t$$

$$k_{t+1} = (1-\delta)k_t + i_t$$

where $u_{l,t} = u_{t,t} + u_{b,t}$ and $b_{l,t} = \frac{u_{t,t}b_{h,t} + u_{b,t}b_{b,t}}{u_{l,t}}$

The first order conditions of the problem and the envelope theorem are presented below:

w.r.t. consumption $\quad \frac{1}{c_t} = \lambda_t$

w.r.t. capital $_{t+1}$ $\quad \beta E_t\left[W_k(\Omega_{t+1}^H)\right] = \lambda_t$

budget constraint $\quad c_t + k_{t+1} - (1-\delta)k_t = w_{h,t}n_{h,t}h_{h,t} + w_{l,t}n_{l,t}h_{l,t} + u_{h,t}b_{h,t} + u_{l,t}b_{l,t} + r_t k_t + \Pi_t$

envelope theorem $\quad W_k(\Omega_t^H) = \lambda_t\left[(1-\delta) + r_t\right]$

and yield the optimal behavior of the household. Therefore, the **optimal decisions for the households** are fully summarized by the following equations:

$$1 = \beta E_t\left\{\frac{\lambda_{t+1}}{\lambda_t}\left((1-\delta) + r_{t+1}\right)\right\}$$

$$c_t + i_t + \Upsilon_t = w_{h,t}n_{h,t}h_{h,t} + w_{l,t}n_{l,t}h_{l,t} + u_{h,t}b_{h,t} + u_{l,t}b_{l,t} + r_t k_t + \Pi_t$$

$$k_{t+1} = (1-\delta)k_t + i_t$$

3.A.2 Firms

The Bellman equation that represents the problem of the firm can be stated as follows:

$$J(\Omega_t^F) = \max_{k,n'_h,n'_l,v_h,v_l} F(\xi a_t k_t^\theta L_t^{(1-\theta)} - w_{h,t}n_{h,t}h_{h,t} - w_{l,t}n_{l,t}h_{l,t} - r_t k_t - a_h v_{h,t} - a_l v_{l,t} + \beta E\left[\tfrac{\lambda_{t+1}}{\lambda_t} J(\Omega_{t+1}^F)\right]$$

subject to the laws of motion for employment and the AR process defining the technology

$$n_{h,t+1} = n_{h,t}(1-\sigma_h) + q_{h,t}v_{h,t} + \eta n_{l,t}$$

$$n_{l,t+1} = n_{l,t}(1-\sigma_l) + q_{l,t}v_{l,t} - \eta n_{l,t}$$

$$\log a_{t+1} = \rho \log a_t + (1-\rho)\log \overline{a} + \varepsilon_{t+1}^a$$

The first order conditions of the problem are presented below:

w.r.t capital	$(1-\theta)\frac{y_t}{k_t} = r_t$
w.r.t vacancies for high-skilled workers	$a_h = \Theta_{h,t}\ q_{h,t}$
w.r.t vacancies for low-skilled workers	$a_l = \Theta_{l,t}\ q_{l},_t$
w.r.t high-skilled employment t+1	$\Theta_{h,t} = \beta E_t\left[\frac{\lambda_{t+1}}{\lambda_t} J_{n_h}(\Omega_{t+1}^F)\right]$
w.r.t low-skilled employment t+1	$\Theta_{l,t} = \beta E_t\left[\frac{\lambda_{t+1}}{\lambda_t} J_{n_l}(\Omega_{t+1}^F)\right]$
law of motion for high-skilled employment	$n_{h,t+1} = n_{h,t}(1-\sigma_h) + q_{h,t}v_{h,t} + \eta n_{l,t}$
law of motion for high-skilled employment	$n_{l,t+1} = n_{l,t}(1-\sigma_l) + q_{l,t}v_{l,t} - \eta n_{l,t}$

And applying the envelope theorem we have:

for high-skilled employment	$J_{nh}(\Omega_t^F) = F_{n_h,t} - w_{h,t}h_{h,t} + \Theta_{h,t}(1-\sigma_h)$
for low-skilled employment	$J_{nl}(\Omega_t^F) = F_{n_l,t} - w_{l,t}h_{l,t} + \Theta_{l,t}(1-\sigma_l) + \eta(\Theta_{h,t} - \Theta_{l,t})$

Thus, **optimal decisions for the firms** are fully summarized by:

$$(1-\theta)\frac{y_t}{k_t} = r_t$$

$$\frac{a_h}{q_{h,t}} = \beta E_t \left\{ \frac{\lambda_{t+1}}{\lambda_t} \left(F_{n_h,t+1} - w_{h,t+1} h_{h,t+1} + \frac{a_h}{q_{h,t+1}}(1-\sigma_h) \right) \right\}$$

$$\frac{a_l}{q_{l,t}} = \beta E_t \left\{ \frac{\lambda_{t+1}}{\lambda_t} \left(F_{n_l,t+1} - w_{l,t+1} h_{l,t+1} + \frac{a_l}{q_{l,t+1}}(1-\sigma_l) + \eta \left(\frac{a_h}{q_{h,t+1}} - \frac{a_l}{q_{l,t+1}} \right) \right) \right\}$$

$$n_{h,t+1} = n_{h,t}(1-\sigma_h) + q_{h,t} v_{h,t} + \eta n_{l,t}$$

$$n_{l,t+1} = n_{l,t}(1-\sigma_l) + q_{l,t} v_{l,t} - \eta n_{l,t}$$

3.A.3 Matching

The standard specification of the matching function is the following:

$$m_{j,t} = \chi_j v_{j,t}^{\alpha_j} u_{j,t}^{1-\alpha_j}$$

and the companion probabilities of matching for workers and vacancies are respectively:

$$\phi_{j,t} = \frac{m_{j,t}}{u_{j,t}} \quad \text{and} \quad q_{j,t} = \frac{m_{j,t}}{v_{j,t}}$$

3.A.4 Optimal contract:

To compute the optimal contract we just have to apply the first order condition of the Nash bargaining problem, which can be stated as follows

$$(1-p_j)\frac{1}{\lambda_t} W_{n_j} = p_j J_{n_j}$$

The marginal value of high-skilled and low-skilled employment for the household comes from the following expressions

$$W_{n_h}(\Omega_t^H) = \left\{ \begin{array}{l} \phi_{1,h}\frac{(1-h_{h,t})^{(1-\eta_h)}}{(1-\eta_h)} - \phi_{2,h}\frac{(1-e)^{(1-\eta_h)}}{(1-\eta_h)} + \lambda_t w_{h,t} n_{h,t} h_{h,t} - \lambda_t u_{h,t} b_{h,t} \\ +\beta E_t\left[W_{n_h}(\Omega_{t+1}^H)\right](1-\phi_h-\sigma_h) + \gamma\beta\left(E\left[W_{nl}(\Omega_{t+1}^H)\right] - E_t\left[W_{n_h}(\Omega_{t+1}^H)\right]\right) \end{array} \right\}$$

$$W_{n_l}(\Omega_t^H) = \left\{ \begin{array}{l} \phi_{1,l}\frac{(1-h_{l,t})^{(1-\eta_l)}}{(1-\eta_l)} - \phi_{2,h}\frac{(1-e)^{(1-\eta_h)}}{(1-\eta_h)} + \lambda_t w_{l,t} h_{l,t} - \lambda_t u_{l,t} b_{l,t} \\ +\beta E_t\left[W_{n_l}(\Omega_{t+1}^H)\right](1-\phi_{l,t}-\sigma_l) \end{array} \right\}$$

which together with the income values of employment for the firms, J_{n_h} and J_{n_l}, yield the following contracts:

$$w_{h,t}h_{h,t}=p_h F_{nh,t}+p_h a_h \frac{v_{h,t}}{u_{h,t}}+p_h\gamma\left(\frac{a_h}{q_{h,t}}-\frac{a_l}{q_{l,t}}\right)+(1-p_h)b_{h,t}-(1-p_h)c_t\left(\phi_{1,h}\frac{(1-h_{h,t})^{(1-\eta_h)}}{(1-\eta_h)}-\phi_{2,h}\frac{(1-e)^{(1-\eta_h)}}{(1-\eta_h)}\right)$$

$$w_{l,t}h_{l,t}=p_l F_{nl,t}+p_l a_l \frac{v_{l,t}}{u_{l,t}}+p_l\eta\left(\frac{a_h}{q_{h,t}}-\frac{a_l}{q_{l,t}}\right)+(1-p_l)b_{l,t}-(1-p_l)c_t\left(\phi_{1,l}\frac{(1-h_{l,t})^{(1-\eta_l)}}{(1-\eta_l)}-\phi_{2,h}\frac{(1-e)^{(1-\eta_h)}}{(1-\eta_h)}\right)$$

the optimal values for the hours worked of each type of worker are obtained via the mutual surplus of the match

$$S_{j,t}=\frac{1}{\lambda_t}W_{n_j,t}+J_{n_j,t}$$

and yield the following results

$$\frac{\partial S_{h,t}}{\partial h_{h,t}}= -\phi_1\frac{1}{\lambda_t}(1-h_{h,t})^{(-\eta_h)}+(1-\theta)\frac{Y_t}{L_t}\left(1-\theta\frac{n_{h,t}h_{h,t}}{L_t}\right)=0$$

$$\frac{\partial S_{l,t}}{\partial h_{l,t}}= -\phi_1\frac{1}{\lambda_t}(1-h_{l,t})^{(-\eta_l)}+(1-\theta)\frac{Y_t}{L_t}\tau\left(1-\theta\tau\frac{n_{l,t}h_{l,t}}{L_t}\right)=0$$

3.A.5 Government

$$u_{h,t}b_{h,t}+u_{l,t}b_{l,t}=\Upsilon_t$$

3.A.6 Non-stochastic general equilibrium

The general equilibrium is defined as a set of functions$\{c, i, v_h, v_l, u_h, u_l, n_h, n_l, t, k,$ $w_h, w_l, h_h, h_l, m_h, m_l, y_t, \tau_t, b_{h,t}, b_{l,t}\}$, solution of the following system formed by the optimal decisions above plus the following equations:

- the resource constraint:

$$c_t + i_t + a_h v_{h,t} + a_l v_{l,t} = y_t$$

where

$$y_t = \xi A k^{\theta} (t_t)^{(1-\theta)}$$

and

$$t_t = n_{h,t} h_{h,t} + \tau n_{l,t} h_{l,t}$$

- the laws of motion for unemployment

$$u_{h,t+1} = u_{h,t} + \sigma_h n_{h,t} - m_{h,t} - \gamma u_{h,t}$$

$$u_{l,t+1} = u_{l,t} + \sigma_l n_{l,t} - m_{l,t} + \gamma u_{h,t}$$

- the unemployment benefits

$$b_{h,t} = rr_h w_{h,t-1}$$

$$b_{l,t} = rr_l w_{l,t-1}$$

Given that we also impose that $1 = n_{h,t} + n_{l,t} + u_{h,t} + u_{l,t}$ one of the laws of motion above is a linear combination of the others plus the condition above and we have to take this into account

3.B Log-linearized equations

3.B.1 Deterministic equations

$$0 = c^* c(t) + i^* i(t) + a_h v_h^* v_h(t) + a_l v_l^* v_l(t) - y^* y(t)$$

$$0 = k^* k(t) - (1-\delta) k^* k(t-1) - i^* i(t)$$

$$0 = n_l^* n_l(t) - (1 - \sigma_l - \eta) n_l^* n_l(t-1) - m_l^* m_l(t)$$

$$0 = n_h^* n_h(t) - (1 - \sigma_h) n_h^* n_h(t-1) - m_h^* m_h(t) - \eta n_l^* n_l(t-1)$$

$$0 = n_h^* n_h(t) + n_l^* n_l(t) + u_h^* u_h(t) + u_l^* u_l(t)$$

$$0 = u_h^* u_h(t) - (1-\gamma) u_h^* u_h(t-1) - \sigma_h n_h^* n_h(t-1) + m_h^* m_h(t)$$

$$y(t) - a(t) - \theta k(t-1) - (1-\theta) l(t)) = 0$$

$$l^* l(t) = n_h^* h_h^* [n_h(t) h_h(t)] + \tau n_l^* h_l^* [n_l(t) h_l(t)]$$

$$m_h(t) - \alpha_h v_h(t) - (1-\alpha_h) u_h(t) = 0$$

$$m_l(t) - \alpha_l v_l(t) - (1-\alpha_l) u_l(t) = 0$$

$$\begin{aligned} 0 \;=\; & p_h (1-\theta) \frac{y^*}{t_h{}^*} h_h{}^* \left(y(t) - t(t) - h_h(t)\right) + p_h a_h \frac{v_h^*}{u_h^*} \left(v_h(t) - u_h(t-1)\right) + (1-p_h) b_h^* b_h(t) \\ & + p_h \gamma \frac{a_h v_h^*}{m_h^*} \left(v_h(t) - m_h(t)\right) - p_h \gamma \frac{a_l v_l^*}{m_l^*} \left(v_l(t) - m_l(t)\right) + (1-p_h) \phi_{2,h} c^* \frac{(1-e)^{(1-\eta_h)}}{(1-\eta_h)} c(t) \\ & - (1-p_h) \phi_{1,h} c^* \frac{(f_{h,t}^*)^{(1-\eta)}}{(1-\eta)} (c(t) + (1-\eta) f_h(t)) - w_h^* h_h^* (w_h(t) + h_h(t)) \end{aligned}$$

$$\begin{aligned} 0 \;=\; & p_l (1-\theta) \frac{y^*}{l_l{}^*} \tau h_l^* \left(y(t) - l(t) + h_l(t)\right) + p_l \frac{a_l v_l^*}{u_l^*} \left(v_l(t) - u_l(t-1)\right) + (1-p_l) b_l^* b_l(t) \\ & + p_l \eta \frac{a_h v_h^*}{m_h^*} \left(v_h(t) - m_h(t)\right) - p_l \eta \frac{a_l v_l^*}{m_l^*} \left(v_l(t) - m_l(t)\right) + (1-p_l) \phi_{2,l} c^* \frac{(1-e)^{(1-\eta_l)}}{(1-\eta_l)} c(t) - \\ & - (1-p_h) \phi_{1,l} c^* \frac{(f_{l,t}^*)^{(1-\eta_l)}}{(1-\eta_l)} (c(t) + (1-\eta) f_l(t)) - w_l^* h_l^* (w_l(t) + h_l(t)) \end{aligned}$$

$$0 = u_h^* b_h^* (u_h(t-1) + b_h(t)) + u_l^* b_l^* (u_l(t-1) + b_l(t)) - \Upsilon_t \Upsilon(t)$$

$$b_h(t) - w_h(t-1) = 0$$

$$b_l(t) - w_l(t-1) = 0$$

3.B.2 Expectational equations

$$0 = E_t \left[c(t) - c(t+1) + \beta\theta \frac{y^*}{k^*} (y(t+1) - k(t+1)) \right]$$

$$0 = E_t \left[\begin{array}{c} \frac{a_h V_h^*}{\beta M_h^*} (c(t) - c(t+1)) + \theta_h \frac{Y^*}{N_h{}^*} h_h^* (y(t+1) - n_h(t) + h_h(t+1)) \\ -w_h^* h_h^* (w_h(t+1) + h_h(t+1)) + (1-\sigma_h) \frac{a_h V_h^*}{M_h^*} (v_h(t+1) - m_h(t+1)) \end{array} \right] - \frac{a_h V_h^*}{\beta M_h^*} (v_h(t) - m_h(t))$$

$$0 = E_t \left[\begin{array}{c} \frac{a_l V_l^*}{\beta M_l^*} (c(t) - c(t+1)) + \theta_l \frac{Y^*}{N_l{}^*} \tau h_l (y(t+1) - n_l(t) + h_l(t+1)) + \eta \frac{a_h V_h^*}{M_h^*} (v_h(t+1) - m_h(t+1)) \\ -w_l^* h_l^* (w_l(t+1) + h_l(t+1)) + (1-\sigma_h-\eta) \frac{a_l V_l^*}{M_l^*} (v_l(t+1) - m_l(t+1)) \end{array} \right] - \frac{a_l V_l^*}{\beta M_l^*} (v_l(t) - m_l(t))$$

3.C Appendix: Macro-data

The macro-data used in this study is real aggregate data of the United States for the period 1964:Q1-2005:Q4. The source is the Federal Reserve Economic Data (FRED) from the Federal Reserve Bank of Saint Louis.

- vacancies = help wanted advertising in newspapers / (population+16)
- employment = (civilian employment +16) / (population+16)
- unemployment = 1 - employment
- tightness = vacancies/unemployment
- total hours = employment*average weekly hours / (population+16)

- labor productivity = output / total hours
- consumption: (real consumption of non durables + real consumption of services + government expenditures)/(population +16)
- investment = (real consumption of durable goods+ real fixed private investment)/(population +16)
- output = consumption + investment + vacancies*cost per vacancy
- duration = median duration of unemployment/ (population+16)
- Normalization of output
  - output =1
  - consumption = Consumption / output
  - investment = Investment / output

3.D Appendix: Explanations to the European unemployment

It is well known in the literature of unemployment that until the second half of the seventies, the European unemployment was significantly lower than the American unemployment, and that since the late seventies and during the eighties the tendency changed and the European unemployment started to steadily rise while the American unemployment continue to fluctuate around its post-World War II value.

The increase in European unemployment was largely caused by a lengthening of the average duration of unemployment spells. So although many Europeans leave unemployment relatively quickly, a significant fraction of workers become trapped in long-term unemployment and have little chance of finding the jobs they want.

The first attempts to explain this increase in unemployment relied on the role played by labor market institutions such as employment protection legislation, both the duration and generosity of unemployment insurances (see Martin, 1996) and the role of firing costs (see Bentolila and Bertola, 1990). The problem with this explanation is that also during the sixties and seventies, when the unemployment in Europe was lower than in the US those labor market institutions existed already (see Krugman, 1987).

Another early attempt to explain this rise in unemployment focused on the negative effect that some macro-shocks could have had on unemployment. Among this macro shocks we find the oil-price shock of 1973 and 1979, the TFP growth slowdown since the early 1970s and other shifts in labor demand experienced since the 1980s. This interpretation was also challenged by Phelphs (1994) who saw improbable that these initial shocks, which indeed have been largely reversed lately, could still be responsible for high unemployment more than fifteen years later. Phelps, for example, emphasized factors that increased the real interest rate and consequently the rate of unemployment.

The stability of European labor market institutions before and after the late seventies and the difficulty of aggregate shocks to explain the persistence of unemployment, lead to another stream of explanations that consider the possibility that changes in the economic environment, in particular aggregate macroeconomic shocks, interacted with labor market institutions to unleash persistently high unemployment. This hypothesis blamed adverse shocks for the initial increase in the rate of unemployment, and labor market institutions for the persistence of this rate.

The explanation based on the interaction of adverse shocks with adverse labor market institutions has been studied in detail by Blanchard and Wolfers (1999). They call the attention about the potential to explain not only the increase in unemployment over time through adverse shocks and the fact that some institutions may affect its persistence but they can also explain cross country differences[7]. In a companion paper Blanchard and Wolfers, (2000) look, through panel data specifications, at the empirical evidence about the role of macro shocks, the role of institutions and the role of the interaction between shocks and institutions in accounting for the European unemployment. Their results suggest that specifications that allow for shocks, institutions and interactions can account both for much of the rise and much of the heterogeneity in the

[7] Recently, Nickell et. al. (2005) consider a plausible story the fact that in response to the initial increase in unemployment, governments reacted by taking the wrong measures. They explain how governments in order to alleviate the pain of unemployment increased the generosity and duration of unemployment or in order to limit the increase in unemployment, they tried to prevent firms from laying off workers through tougher employment protection or even . To better share the burden of low employment, they used early retirements and work sharing to better share the burden of low employment. All these measures then in turn increased unemployment even as the initial shocks disappeared.

evolution of unemployment in Europe.

The second big stream of explanations given to the high European unemployment focus on the interaction of micro-shocks and labor market institutions rather than focussing on the interactions of those institutions and aggregate shocks. The two main interpretations of these findings come from Bertola and Ichino(1995) and Ljungqvist and Sargent (1998). Bertola and Ichino show that given the rigid wages and the high firing costs that prevail in Europe during the 80s, a higher likelihood of negative shocks in the near future decreases labor demand by hiring firms. And as long as the wage rate does not fall, the equilibrium unemployment rate would rise. This explanation remind us, the "thin market externality" reasoning proposed by Pissarides (1992).

Ljungqvist and Sargent's series of papers, LS from now onwards, advocate for the interaction of shocks to individual worker's human capital, *turbulence* in their words, and generous unemployment benefits to produce long-term unemployment in Europe. In particular, they assume that in the late 70s and during the 80s, the probability of suffering from a depreciation of human capital increased and unleash the following mechanism: Imagine a worker who suddenly loss his job. Once unemployed he receives an unemployment benefit proportional to his former wage and become a low-skilled worker. If any, he is going to receive job offers corresponding to this low-skilled level and accordingly, he is going to be offered low wages. It easily could be that those low-wages do not cover the reservation wage of the worker which we can identify with the high-skilled unemployment insurance. If this is the case, he is going to reject the offer and will become trapped in unemployment.

More recently, a new hypothesis come up to the fore. Prescott (2004), advocates for the role of tax rates, in particular the effective marginal tax rate on labor income, in accounting for the changes in the relative labor supply across time and across countries. Interesting findings of this study are that when European and US tax rates were comparable, European and US labor supplies were comparable and that the low labor supplies of Germany, France and Italy during the nineties are largely due to high tax rates.

Therefore, nowadays we rely on at least three possible potential explanations for the high European unemployment: the combination of aggregate macro shocks and labor market institutions, the combination of micro-shocks and labor market institutions and the impact of the evolution of labor taxation on labor supply. But to disentangle the exact effect of labor market institutions on unemployment is still an issue we need to resolve.