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Product and Process Innovation in a Growth Model of Firm Selection*

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Abstract

Recent empirical evidence based on firm level data emphasizes firm heterogeneity in innovation activities and the different effects of process and product innovations on the productivity level and productivity growth. To match this evidence, this paper develops an endogenous growth model with two sources of firm heterogeneity: production efficiency and product quality. Both attributes evolve endogenously through firms' innovation choices. Growth is driven by innovation and self-selection of firms and sustained by entrants who imitate incumbents. Calibrating the economy to match the Spanish manufacturing sector, the model enables to quantify the different effects of selection, innovation, and imitation as well as product and process innovation on growth. Compared to single attribute models of firm heterogeneity, the model provides a more complete characterization of firms' innovation choices explaining the partition of firms along different innovation strategies and generating consistent firm size distributions.

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Keywords: endogenous growth theory, firm dynamics, heterogeneous firms, productivity, quality, innovation

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1 Introduction

Globalization and the rise of new technologies have challenged firms' abilities in developing innovation strategies to face increasing market competition. Innovation has become a fundamental source of firm survival and growth.[1] The literature has widely analyzed the relationship between innovation and economic growth. However, little attention has been paid to the relationship between firm heterogeneity and innovation activities and even less to the relationship between firm heterogeneity and different innovation strategies as well as to their impact on firms' competitiveness and productivity growth. The channel between firm growth and aggregate growth is still comparatively unexplored. Understanding the determinants of firms' innovation strategies and the mechanism of resource reallocation through which they impact on aggregate growth is therefore crucial and can also contribute to enhance the effectiveness of policies aimed at fostering economic growth and welfare.

This paper proposes a new framework to analyze the effects of process and product innovation on firm dynamics and growth, highlighting the importance of product quality in the growth process. For this purpose, an endogenous growth model with two sources of firm heterogeneity, production efficiency and product quality is developed. Calibrating the model to match the Spanish manufacturing sector, it generates moments and a firm size distribution consistent with recent empirical evidence on innovation and firm dynamics. The interplay between the two sources of firm heterogeneity and costly innovation results in a non-monotonic relation between firm size and innovation strategies. Small firms undertake product innovation, medium firms both process and product innovation while large firms specialize only in process innovation. Moreover, it is emphasized the importance of the reallocation of resources not only from less efficient firms to more efficient ones, but also among different innovation strategies. In this respect, the model yields interesting predictions that can be empirically tested. In particular, innovation appears to be the main factor in explaining aggregate growth: 91.87% of growth is due to innovation while only 8.13% is due to the firms' turnover. Moreover, process innovation explains 69.8% of aggregate growth

[1] For instance, Huergo and Jaumandreu (2007) estimate that the contribution of firms that perform R&D explains between 45% and 85% of productivity growth in the industry with intermediate or high innovation activity. Moreover, Bartelsman and Doms (2000) report evidence of a self-reinforcing mechanism between productivity and innovation. Profitable firms have a higher propensity to innovate and innovation is positively related with productivity and productivity growth.

while product innovation contributes only for the remaining 30.2%. Additionally, this model contributes to the literature that tries to undertand why firm heterogenity is persistent endogenizing the evolution of firm technology.

Existing growth literature distinguishes between only two types of innovations: horizontal innovation which expands the number of varieties in the market, and vertical innovation which increases the quality or reduces the production cost of existing goods. In these models quality improvement (product innovation) or cost reduction (process innovation) are seen as interchangeable and yield the same prediction. Contrastingly, recent surveys at firm-level allow to distinguish when firms undertake process or product innovation highlighting that firms perceive in a different way product quality improvement or cost reduction innovations.[2] Firms not only have different incentives to invest either in product or process innovation, or even in both simultaneously, but also their impact on firms' pricing strategies, productivity, and TFP growth is different. Three main pieces of evidence arise from these surveys: innovations are *heterogeneous*, *asymmetric*, and *complementary*.

Firstly, innovation are *heterogeneous* in the sense that some firms do not innovate, some firms specialize in process innovation, others in product innovation and some in both types of innovations. Table 1 shows the share of firms across the different innovation strategies for four European countries.[3] Jaumandreu (2003) in a sample of Spanish firms in the manufacturing sectors finds that half of the firms never innovate, 30% undertake either process or product innovation and 20% of the firms undergo both types of innovations. Similar statistics are also available for Germany and Great Britain (Harrison et all, (2008)) and the Netherlands (Cefis and Marsili, (2005)).

[2]The European Commission has developed a program aimed at studying the innovation systems of the states member of the European Union with the scope of promoting innovation and growth. The core of the program is based on firm-level surveys (Community Innovation Surveys) which ask detailed questions about the innovation investments of firms distinguishing between cost reducing innovations and quality improving innovations. This information is then merged with structural and macroeconomic data drawn from OECD surveys. Additionally, some European Countries carry out nation-specific surveys. For instance, in Spain there is the *Encuestas Sobre Estrategias Empresariales* that is issued every three years. The same analysis becomes more difficult with American data where innovation is measured as patents and therefore the two innovations cannot be distinguished. However, for a concise summary Klette and Kortum (2004) report a list of stylized facts concerning firm R&D, innovation, and productivity.

[3]It should be noticed that the data sets are not homogeneous. Hence table 1 does not allow comparisons across countries but only the ability to observe the stated heterogeneity in the innovation choices.

Table 1: Heterogeneity in Innovation Strategies

Country	Share of Innovative Firms			
	No Innovation	**Process**	**Product**	**Process and Product**
Spain	55.4%	12.2%	12.4%	20%
Germany	41%	10.2%	21%	27.4%
Great Britain	60.5%	11%	14.2%	14.3%
Netherlands	36.6%	5.8%	18.8%	42.7%

Secondly, the innovation strategies are *asymmetric*. Huergo and Jamandreu (2004) estimate that process innovation increases productivity by 14% and product innovation by 4% over a three year period. As expected, innovating firms are characterized by a productivity distribution that stochastically dominates the productivity distribution of non-innovators. But in the case of product innovation the distribution becomes more skewed to the right.

Thirdly, innovations are *complements*. Process innovation is more frequent than product innovation, while the probability of introducing a product innovation is higher for firms that also introduce a process innovation in the same period. However process innovation does not necessarily imply product innovation. Firms innovate on their existing products, aiming at increasing product differentiation and hence prices, in the hope of exploiting consumers' willingness to pay for a higher quality good. Instead process innovation increases the firms' production efficiency. This leads to higher firm productivity, lower prices and a larger scale of production. Complementarity between process and product innovation then arises: product innovation allows new product designs but these new designs become profitable only when they are affordable for the consumers.

When talking about firm dynamics it is important not to abstract from entry and exit. They play an important role in explaining the reallocation of resources from less productive firms to more productive firms and therefore growth. In addition, exit is associated with a lower level of pre-exit innovations, while entrants present a high probability of innovation.

Standard models with only one source of innovation cannot explain all these pieces of evidence. The literature on heterogeneous firms is usually based only on one factor of heterogeneity, either cost efficiency or the ability of producing quality. In these models a single attribute monotonically predicts firms' revenue, competitiveness, and innovation. This characteristic then implies a threshold firm size above which all firms innovate and below none do. This paper takes the gap between the existing theoretical literature and the pieces of empirical evidence as a starting point and links firm level growth due to different innovation choices, the process of resources reallocation, and aggregate growth. For this scope, a general equilibrium heterogeneous firms model with endogenous product and process innovation is developed. The industry structure is taken from Hopenhayn (1992) and the competitive structure from Melitz (2003), using monopolistic competition instead of perfect competition. Firms produce differentiated goods and are heterogeneous in their production efficiency and in their product quality. The evolution of both efficiency and quality is given by a stochastic permanent component and by an endogenous component proportional to the optimal investment decision taken by the firm. In each period non profitable incumbents exit the industry, implying that the average productivity of the remaining firms increases. New firms enter in the market as in Gabler and Licandro (2005) and Luttmer (2007). They try to imitate the average incumbent, they do not succeed completely but on average entrants are more productive than exiting firms increasing the average productivity of the industry. Growth arises due to firms' innovation and firms' self-selection and is sustained endogenously by entrants' imitation.

In this model the relationship between firm size and innovative strategies is more articulate in explaining why different firms choose optimally different innovation strategies. Additionally, comparing industries that differ for innovation costs or for entry barriers allows for a better understanding of the growth rate composition and how it is affected by changes in the industry structure. Hence this model provide a suitable framework for the analysis of policy implications aimed at fostering growth.

1.1 Related Literature

This paper attempts to link the literature on firm dynamics and endogenous growth theory by explicitly modeling different types of firm-level innovations. As in the seminal models of Romer (1990), Grossman and Helpman (1991) and Aghion and Howitt (1992), innovation is firm-specific and it is motivated by the appropriation of the revenues associated with a successful R&D investment. In Romer (1990) growth is driven by two elements. The first one is the invention of new inputs which make the production of the final good sector more efficient. In this sense and from the point of view of the final good firm it can be seen as process innovation. The second one is knowledge spillovers from past R&D: the higher the stock of knowledge, the easier the invention of new varieties. In this paper there is a similar spillover, which is the imperfect imitation of incumbent firms by entrants. Grossman and Helpman (1991) introduce growth through quality improving innovation of existing products. However, in their model, different qualities are perceived as perfect substitutes and hence the representative consumer buys only the cheapest variety (adjusted by quality). Instead, in my model each variety is perceived as different by the consumer and higher quality varieties give higher utility. In Aghion and Howitt (1992) growth is based on the idea of Schumpeterian creative destruction in which new innovations replace the previous ones driving the incumbent monopolist out of the industry. The creative destruction mechanism is not far from the idea of firm selection. Successful firms grow and drive out of the market unsuccessful ones. Based on these general features my work adds firm heterogeneity, permanent idiosyncratic shocks that hit both production efficiency and product quality, and endogenous investment choices made by incumbent firms. These new elements endogenously link aggregate growth with firm-specific growth and hence with the mechanism of resource reallocation from non-innovators to innovators and from exiting to active firms. The resulting distribution of firm size is consistent with the data.

The idea of firm selection was already present in Jovanovic (1982). He introduces the first model with firm-specific stochastic productivities with unknown mean but known variance. As time goes by firms learn their productivity and the inefficient firms exit. As firms learn their productivity the effects of selection on firms evolution dies out and eventually the industry converges to a stationary equilibrium without entry and exit. For this reason, this paper takes the industry structure from Hopenhayn (1992), who develops a partial dynamics stochastic heterogeneous firms' model which generates a stationary equilibrium with entry and exit that is capable of studying the effects of structural changes in the industry on the distribution of firm size and age. Hopenhayn and Rogerson (1993) analyze the general equilibrium of the Hopenhayn model focusing on the process of labor reallocation. Both papers study the stationary equilibrium in which each firm is hit by shocks characterized by a stationary AR(1) process. However, both papers focus only on firm productivity growth between cohorts and disregard the effects on aggregate growth.

The link between the process of resource reallocation due to selection at the firm level and economic growth is studied in Gabler and Licandro (2005) and in Luttmer (2007). In both papers firm technology is hit by permanent shocks which together with firm selection and entrant imitation generates endogenous growth. The resulting stationary distribution is a consequence of the knowledge spillover that links the distribution of entrants productivities to the distribution of incumbents productivities. This assumption is necessary to generate endogenous growth. In fact without imitation, as incumbent firms become more productive through selection, the incentives to enter the industry diminish and eventually vanish. In the end no new firms enter into the industry and the equilibrium is characterized by the absence of entry and exit similarly as Jovanovic (1982). Gabler and Licandro (2005) model a competitive equilibrium with heterogeneous firms using both labor and capital as inputs. When calibrating their model on US data they show that selection and imitation account for a fifth of productivity growth. This represents a lower bound. Luttmer (2007) instead considers a monopolistic competition market in which each firm produces a different variety and it is subjected to shocks to both productivity and demand. Calibrating his model to US data he finds that half of output growth can be attributed to selection and imitation. This can be seen as an upper bound.

This paper attempts to extend Gabler and Licandro (2005) and Luttmer (2007) by considering alongside their models the role of innovation in linking firm level growth to aggregate growth. Modeling endogenously firm innovation investments in both firm efficiency and product quality can help to distinguish the differing contributions of selection and imitation versus innovation in process and product when explaining economic growth.

The other papers that shed light on the relationship between innovation, firm heterogeneity and the role of resource reallocation of the growth process are Klette and Kortum (2004) and Lenz and Mortensen (2008). The former, building on Grossman and Helpman (1991), introduces firms that exogenously differ in the profits earned by selling their own products. Endogenous growth is then generated through innovation investments aimed at increasing the number of goods produced by each firm and firms adjust the production lines in response to their own and competitors' investment in R&D. However they posit permanent exogenous differences across firm profitability and hence across the size of the innovative step. This simplification results in a distribution of innovative firms that have the same volatility as the distribution of the firms that do not innovate. This model, defining innovation as an endogenous drift into the stochastic evolution of firm productivity and quality, can account for the differing variances of the distribution of innovators and non-innovators. Lenz and Mortensen (2008) relate to Klette and Kortum (2004) introducing heterogeneity in the expected productivity of the new variety produced. But as in both models the engine of growth is a mechanism of creative destruction on the numbers of goods existing in the economy at a given point in time, they can analyze only one channel of innovation.

More recently, Atkeson and Burstein (2007) address the relation between the decision of heterogeneous firms to innovate and engage in international trade by introducing two types of stochastic innovation activities. Though their model abstracts from endogenous growth, they define as process innovation the decision to increase the stock of firm-specific factors that then translates in higher profits opportunities. This is analogous to process innovation defined in this model. They define as product innovation the creation of a new firm and hence a new product. This is the analogous to firm entry discussed in this model. In fact, this model defines differently from them as product innovation the decision of firms to improve the quality of an exiting variety. Moreover, the jump in the efficiency and/or quality scale are, in this paper, proportional to the research intensity.

Finally two other papers of note, Melitz (2003) and Hallak and Sivadasan (2008). Melitz (2003) proposes a static model with heterogeneous firms in which the exposure to international trade increases firm selection and generates a partition among firms such that the more productive firms are the ones who gain access foreign markets. Hallak and Sivadasan (2008), building on Melitz (2003), introduce a partial and static equilibrium model in which firms differ in two attributes: labor efficiency and ability to produce high quality varieties. Under the assumption of minimum quality requirements they study how openness affects firm distribution. In their model as in Melitz (2003) the partition of firms between domestic producers and exporters is generated by the presence of a fixed cost to enter the foreign market. Here the same mechanism is used to generate the partition of firms among the different innovation strategies. However, the firm partition and the effects on the size distribution of firms is not the result of a one-shot change but it is the result of the combination of permanent shocks on both states and inter-temporal innovation decisions.

2 The Model

This section develops a general equilibrium model in discrete time with infinite horizon.

2.1 Consumer Problem

The representative consumer maximizes his utility choosing consumption and supplying labor inelastically at the wage rate w. Its lifetime utility is assumed to take the following form:

$$U = \sum_{t=0}^{\infty} \beta^t \ln(U_t) \tag{1}$$

where $\beta < 1$ is the discount factor and t is the time index. In every period the consumer faces the problem of maximizing his current consumption across a continuum of differentiated products indexed by $i \in I$ where I is a measure of the available varieties in the economy. Specifically, the preferences are represented by an augmented Dixit-Stiglitz utility function with constant elasticity of substitution between any two goods $\sigma = 1/(1-\alpha) > 1$ with $\alpha \in (0,1)$. Hence, the utility function at time t is:

$$U_t = \left(\int_{i \in I} (q_t(i)x_t(i))^{\alpha} d\mathrm{i} \right)^{\frac{1}{\alpha}}. \tag{2}$$

where $x(i)$ is the quantity of variety $i \in I$ and $q(i)$ is the quality of variety $i \in I$. This utility function is augmented to account for quality variation across products and quality acts as a utility shifter: for a given price the consumer prefers products with high quality rather than products with low quality.

The representative consumer maximizes his utility subject to the budget constraint $E_t = \int_{i \in I} p_t(i)x_t(i)di$ where E_t is total expenditure at time t and $p_t(i)$ is the price of variety $i \in I$ at time t. Solving the intra-temporal consumer problem yields the demand for each variety $i \in I$,

$$x_t(i) = \left(\frac{P_t q_t^{\alpha}(i)}{p_t(i)} \right)^{\frac{1}{1-\alpha}} X_t = \left(\frac{P_t^{\alpha} q_t^{\alpha}(i)}{p_t(i)} \right)^{\frac{1}{1-\alpha}} E_t \tag{3}$$

with:

$$P_t = \left(\int_{i \in I} \left(\frac{p_t(i)}{q_t(i)} \right)^{\frac{\alpha}{\alpha-1}} \mathrm{d}i \right)^{\frac{\alpha-1}{\alpha}} \quad \text{and} \quad X_t = U_t. \tag{4}$$

P_t is the price quality index at time t of all the bundle of varieties consumed and X_t is the aggregate set of varieties consumed.

Finally, the optimal inter-temporal allocation of consumption yields the standard Euler equation:

$$\frac{X_{t+1}}{X_t} = \beta(1 + r_t). \tag{5}$$

where r_t is the return on asset holding.

2.2 Firms

This section outlines a dynamic two factor heterogeneous firm model. The first source of heterogeneity is production *efficiency*, $a(i) \in \mathbb{R}_{++}$, which increases the marginal productivity of labor, as in the seminal paper of Hopenhayn (1992), and the second source is *quality* of the firm's variety, $q(i) \in \mathbb{R}_{++} \setminus (0,1)$, which decreases the marginal productivity of labor. In this respect, a higher quality variety has a higher variable cost as in Verhoogen (2008). Firms are distributed over productivity and quality. $\widetilde{\mu}(a,q) = \mu(a,q)I$ is defined as the measure of firm with state (a,q) at time t, where I is the number of firms in the industry and $\mu(a,q)$ is a density function. It is assumed that each firm produces only one variety so that the index i identifies both the firm and the corresponding variety produced by that firms and I represents both the set of varieties and the mass of incumbent firms active in the industry. The following definition are used, A is the set of all production efficiencies, Q is the set of all product qualities, and $\Omega \equiv A \times Q$ is the state space.

2.2.1 Production Decision

After paying a fixed operational cost, c_f, expressed in terms of labor, active firms receive their new technology level, (a,q). Firms produce and price their own products under the assumption of monopolistic competition. The production decision is particularly simple since it involves only an intra-temporal dimension of profit maximization given consumer demand and firm technology. Close to Hallak and Sivadasan (2008), the production function is assumed to be linear in labor, n, which is the unique input, increasing in firm efficiency, a, and decreasing in firm product quality, q. That is, $x_t(i) = a_t(i)q_t(i)^{-\eta}n_t(i)$ with $\eta \in (0,1)$. The parameter η introduces asymmetry between firm efficiency and product quality and measures the difficulties in producing a higher quality variety: the higher η, the more difficult and costly it becomes to produce a high quality product. This particular functional form is justified by empirical evidence: it generates a price distribution consistent with the estimates of Smolny (1998) and moreover complementarity between process and product innovation is obtained.

The profit maximization problem, faced by each firm, is formulated as:

$$\pi_t(a(i), q(i)) = \max_{p(i)} p_t(i)x_t(i) - w_t n_t(i) - c_f \tag{6}$$

where w_t is the wage rate at time t common to all firms. The first order condition with respect to price yields the optimal pricing rule:

$$p_t(a(i), q(i)) = \frac{w_t q_t^{\eta}(i)}{\alpha a_t(i)}. \tag{7}$$

$1/\alpha$ is the constant mark-up associated with the CES demand function. In contrast to the standard models with a single factor of firm heterogeneity, firms' prices depend on both firms' efficiency and quality of their products. Consistent with both the theoretical predictions and the empirical estimates, the price schedule is increasing in the quality of the variety produced by the firms and decreasing in firms' efficiency.[4] As in Melitz (2003) the nominal wage is normalized to one. Using the monopolistic price to solve for the optimal demand for each variety yields:

$$x_t(a(i), q(i)) = \left(\frac{\alpha a_t(i) P_t^{\alpha}}{q_t(i)^{\eta-\alpha}}\right)^{\frac{1}{1-\alpha}} E_t. \tag{8}$$

Firm output is an increasing function of both the aggregates and of the efficiency level of firms. The relationship between product quality and output is ambiguous and depends on the comparison between α, related to consumer preferences, and η, coming from firm production function. If $\eta > \alpha$ then firm output is decreasing in the product quality: high quality varieties are characterized by a relatively lower market share. In this case, the positive effect of quality on consumer utility is completely offset by the related high market price. The opposite is true when $\alpha > \eta$, though this last scenario appears to be counterfactual.

The optimal labor demand is given by:

$$n_t(a(i), q(i)) = \left(a_t(i) q_t(i)^{1-\eta}\right)^{\frac{\alpha}{1-\alpha}} \left(\alpha P_t^{\alpha}\right)^{\frac{1}{1-\alpha}} E_t. \tag{9}$$

Labor input is an increasing function of both firms' state variables. Consequently, firms with more advanced technology demand more labor input. Finally, the net per period profit of firm i is given by:

$$\pi_t(a(i), q(i)) = \left(a_t(i) q_t(i)^{1-\eta} \alpha\right)^{\frac{\alpha}{1-\alpha}} (1-\alpha) P_t^{\frac{\alpha}{1-\alpha}} E_t - c_f. \tag{10}$$

[4] Smolny (1998), studying a panel of West German firms in the manufacturing sector in the period 1980-1992, estimates that product innovation increases the probability and the frequency of positive net prices increases by more than 18% while process innovation does not reveal a conclusive effect on firm pricing strategies. However, he clearly estimates that process innovations increases the probability of employment and especially output increases. Making increases in output and employment without a lower price is difficult. Hence the effects on output and employment support the relevance of price effects and of the complementarity between the two forms of innovation.

Although product quality has an ambiguous effect on the optimal output of firms, profits are increasing in both labor efficiency and product quality. This provides incentives for firms to improve endogenously their position in the technology distribution via firms' innovation policies. In this respect, the model predicts that a change in efficiency impacts more a firm's profit than a change in quality.

The different effects of firm efficiency and quality on the monopolistic price, on the output, and on the profits provide a suitable framework in which to study the interplay among different innovation choices taken by a firm and their effects on a firm's competitiveness.[5]

2.2.2 Innovation Decision

Firms receive idiosyncratic permanent shocks on both states. That is, firms' log efficiency and log quality follow a random walk. This is a way of capturing the role of firm-specific characteristics and the persistence of firm productivity which is established in the empirical literature.[6] Besides the exogenous random walks, firms can endogenously affect the evolution of their states through private innovation activities. In line with the terminology used in the surveys at the firm-level, this paper identify two different types of innovation: *process innovation* and *product innovation*. Process innovation refers to the decision of firms to invest labor, with the aim of lowering firm production costs, while product innovation refers to the decision of firms to direct labor investment at increasing the quality of the varieties produced.

According to the theoretical growth literature, the benefits derived by firms' innovation investments are proportional to the amount of resources spent. In particular, it is assumed that innovation introduces an endogenous drift in the random walk processes which reflects the amount of variable labor that firms optimally invest in R&D. The innovation choice is history dependent as today investment in process or product innovation results in tomorrow higher firm production efficiency and/or product quality. In addition, firms have to pay also a fixed cost of innovation, c_a and c_q, for process and product innovation, respectively. This

[5]An innovation in product, aimed at increasing product quality, results in a higher market price for the given variety and, for appropriate parameters, in a contraction of the market quota. This then determines an incentive to invest also in process innovation and hence to increase firm efficiency. That in turn leads to a lower market price and to an unambiguous larger market share.

[6]For instance, the idiosyncratic shocks can capture factors as absorption techniques, managerial ability, gain and losses due to the change in the labor composition and so on.

is a way of capturing the costs necessary to set up an R&D department, to conduct market analysis and technically it determines the partition of firms among different innovation strategies. Depending on the firms' technology state, some firms decide to innovate either in process or in product or in both types of innovation. In whichever form innovation comes, it represents a first source of endogenous growth since it shifts the bivariate firms' distribution to the right.

Specifically, log efficiency is assumed to evolve according to:

$$\log a_{t+1} = \begin{cases} \log a_t + \varepsilon^a_{t+1} & \text{when } \; z_t = 0 \\ \log a_t + \lambda^a \log z_t(a,q) + \varepsilon^{az}_{t+1} & \text{otherwise} \end{cases} . \tag{11}$$

Shocks are firm-specific and distributed as $\varepsilon^a_{t+1} \sim N(0, \sigma^2_a)$, $\varepsilon^{az}_{t+1} \sim N(0, \sigma^2_{az})$ where σ^2_a is the variance of the random walk when innovation does not occur and σ^2_{az} is the variance of the process when innovation takes place. $z_t(a, q) > 0$ is the labor that a firm with states (a, q) decide optimally to invest in process innovation. $\lambda^a > 0$ is a parameter that, together with the log form of the innovation drift, scales the effects of innovation. The log functional form chosen for the innovation drift is important as together with firm selection assure a bounded growth and hence the existence of a stationary distribution. Similarly log quality evolves as:

$$\log q_{t+1} = \begin{cases} \log q_t + \varepsilon^q_{t+1} & \text{when } \; l_t = 0 \\ \log q_t + \lambda^q \log l_t(a,q) + \varepsilon^{ql}_{t+1} & \text{otherwise} \end{cases} . \tag{12}$$

Again $\varepsilon^q_{t+1} \sim N(0, \sigma^2_q)$, $\varepsilon^{ql}_{t+1} \sim N(0, \sigma^2_{ql})$ where σ^2_q and σ^2_{ql} are the two variances without and with innovation. $l_t(a, q)$ is the variable labor devoted to product innovation and $\lambda^q > 0$ is the related scale parameter. The means of the efficiency and quality shocks are normalized to zero eliminating exogenous sources of growth. In fact, abstracting from innovation and firm selection, in expectation firms do not grow.

The random component ε is independent both across firms and over time. Moreover, the two processes, efficiency and quality, are independent.[7] Define the density function of a_{t+1} conditional on a_t as $f(a_{t+1}|a_t)$, and the density functions of q_{t+1} conditional on q_t as $p(q_{t+1}|q_t)$.

The transition of the two state variables depends on the firms' innovation decisions and the idiosyncratic shocks. Considering jointly the two transition functions, $\Phi : \Omega \to \Omega$ can

[7] This simplification does not affect qualitatively the model predictions, but has the advantage to narrow the set of parameters necessary to calibrate since it is possible to ignore the covariances of the two processes.

be defined as the joint transition function, which moves firms' quality and efficiency states. The corresponding transition probability function is defined as $\phi : \Omega \times \Omega \to [0, 1]$, which gives the probability of going from state (a, q) to state (a', q'). The transition probability takes different forms depending on the innovation decisions and on the exit decision defined below. If the two processes are independent then $\phi(\cdot) = f(\cdot)p(\cdot)$.

2.2.3 Firm Value Function

Incumbent firms face a dynamic optimization problem of maximizing their expected value. Once abstracted from the innovation decision this is a particularly simple problem since it is a sequence of static optimizations. With the innovation scheme, current investments in innovation affect the transition probabilities and thus the value of future technology. This generates a dynamic interplay between firm technology and the innovative position taken by the firm. This is summarized by the following value function:

$$v(a, q) = \max\{v^P(a, q), v^A(a, q), v^{AQ}(a, q), v^Q(a, q)\}. \tag{13}$$

The max operator indicates that in each period firms face different discrete choices which depend on the current level of production efficiency and product quality. $v^P(a, q)$ is the value when no innovation investments occurred, $v^A(a, q)$ when a firm produces and innovates in process, $v^{AQ}(a, q)$ when both process and product innovation are undertaken and $v^Q(a, q)$ when a firm specializes only in product innovation.

Using $J = \{P, A, Q, AQ\}$ and defining with prime the next period variables, the Belman equation for each choice is given by:

$$v^J(a, q) = \max_p \left\{ \pi^J(a, q) + \frac{1}{1+r} \max \left\{ \int_\Omega v(a', q')\phi(a', q'|a, q)\mathrm{d}a'\mathrm{d}q', 0 \right\} \right\}. \tag{14}$$

where $\pi^P(a, q)$ is given by equation (11), $\pi^A(a, q) = \pi(a, q) - z(a, q) - c_a$, $\pi^{AQ}(a, q) = \pi(a, q) - (z(a, q) + l(a, q)) - c_a - c_q$, and $\pi^Q(a, q) = \pi(a, q) - l(a, q) - c_q$.

These value functions characterize a partition of firms among the different decisions (only produce or produce and innovate, and in the latter case if process, or product or both at the same time) which depends on the relation between the technological state of each firm and the fixed costs. In fact, given the specific position of a firm inside the bivariate distribution of technology, the fixed costs of innovation generate different firms decisions consistently with equation (14). Two sources of firm heterogeneity implies that the thresholds, characterizing the border among the different innovation strategies, are given by infinite combinations of (a,q) couples. For this reason, it becomes convenient to express the reservation values in terms efficiency as a function of quality, $a(q)$ and to obtain *cutoff functions* rather than cutoff values as in one factor heterogeneous firm models. For given $q \in Q$ it is possible to define the following cutoff functions: $a_A(q)$ delimits the area in which process innovation is optimal, $a_Q(q)$ delimits the area in which product innovation is optimal, and $a_{AQ}(q)$ delimits the area in which both innovations are chosen by the firms.[8] Appendix A provides a formal definition of these cutoff functions.

The cutoff functions are decreasing in q, highlighting a non-monotonic relation between the innovation strategies and firms efficiency. In contrast with one factor heterogeneous firm models, also less efficient firms but characterized by a product with high quality innovate. Notice that firm profits, $\pi(a,q)$, are increasing in both efficiency and quality generating the incentives to innovate which are slowed down by the log form in which the innovation drift is modeled. Abstracting from the discontinuity in the value function due to the fixed costs of innovation, the more advanced the firm technology, the higher the innovation investment but the lower the benefit due to the diminishing returns of innovation.

2.2.4 The Exit Decision

Firms exit the industry after a bad technological draw such that the expected value of continuing is lower than the exit value which has been normalized to zero.[9] Since firm value is increasing in both states the exit reservation value is decreasing in both of them. Again a cutoff function $a_x(q)$ can be defined such that:

$$E[v(a'(q), q')|(a_x(q), q)] = 0. \tag{15}$$

For each quality level, there is a maximum efficiency level such that below this maximum firm value is negative and therefore firms find optimally to exit the industry. Interestingly, the cutoff function $a_x(q)$ is decreasing in quality: for given efficiency firms with a high quality product can survive longer in the market when hit by a bad efficiency shock.

[8] It is equivalent to express product quality as a function of efficiency, $q(a)$. Using a specific formulation for the cutoff function does not affect the implications of the model.

[9] Notice that exit is triggered by the assumption of fixed operational costs, c_f, paid by active firms in each period. Without fixed operational costs, firms hit by bad shocks instead of exiting the market could temporary shut down their production and just wait for better periods when positive shocks hit their technology and then start again producing.

Firms innovation decisions, exit and the law of motion of (a, q) define the transition function $\Phi_{xI} : A \setminus A_x \times Q \rightarrow (A_p \cup A_A \cup A_Q \cup A_{AQ} \cup A_x) \times Q$ where the support of efficiency is partitioned into $A_x = \{(a, q) : a \in A, q \in Q : a(q) < a_x(q)\}$ (exit support), $A_P = \{(a, q) : a \in A, q \in Q \wedge v(a, q) = v^P(a, q)\}$ (production support), $A_A = \{(a, q) : a \in A, q \in Q \wedge v(a, q) = v^A(a, q)\}$ (process innovation support), $A_Q = \{(a, q) : a \in A, q \in Q \wedge v(a(q), q) = v^Q(a(q), q)\}$ (product innovation support) and $A_{AQ} = \{(a, q) : a \in A, q \in Q \wedge v(a(q), q) = v^{AQ}(a(q), q)\}$ (process and product innovation support). These partitions differ across different elements of Q. The corresponding transition probability of going from state $(a, q) \in (A_p \cup A_A \cup A_Q \cup A_{AQ}) \times Q$ to $(a', q') \in (A_p \cup A_A \cup A_Q \cup A_{AQ} \cup A_x) \times Q$ is given by a function $\phi_{xI}(\cdot)$.

2.2.5 Firms Entry

Every period there is a mass of potential entrants in the industry which are *a priori* identical. To enter firms have to pay a sunk entry cost, c_e, expressed in terms of labor. This cost can be interpreted as an irreversible investment into setting up the production facilities. After paying the initial cost, firms draw their initial efficiency level, a, and their initial product quality, q, from a common bivariate density function, $\gamma(a, q)$. The associated distribution is denoted by $\Gamma(a, q)$ and has support in $\mathbb{R}_+ \times \mathbb{R}_+$. Define $\overline{\gamma}_e$ the mean of the joint distribution and σ^2_{ea} and σ^2_{eq} the variances of the entrants efficiency and quality processes (the covariance is zero given the current assumption of independence between the evolution of the two states). In equilibrium the free entry condition holds: potential entrants enter until the expected value of entry is equal to the entry cost:

$$v^e(a, q) = \int_{\Omega_e} v(a, q) d\Gamma(a, q) = c_e, \tag{16}$$

M_t is the mass of firms that enter in the industry at time t. At the stationary equilibrium also a stability condition needs to be satisfied: the mass of new entrants exactly replaces the mass of unsuccessful incumbents who are hit by a bad shock and exit the market: $M = \int_{a_x(q)} \int_Q I\mu(a, q)$.

The average technology of surviving firms grow due to randomness and innovation. This implies that the demand of labor grows over time at a positive rate and, given a fixed exogenous supply, the wage rate rises. Hence, if the joint distribution of entrants efficiency and quality, $\gamma(a, q)$, was completely exogenous and constant, the expected value of entry would be driven to zero and no firms would eventually enter the market. To avoid this scenario the entrants technology is linked to incumbent firms technology through an imitation mechanisms related to Luttmer (2007) and used also in Gabler and Licandro (2005). Entrants imitate incumbent firms: the mean of the entrant distribution is a constant fraction $\psi_e \in (0, 1)$ of the mean of the joint distribution of incumbents defined as $\overline{\mu}$. That is, $\overline{\gamma}_e = \psi_e \overline{\mu}$. Consistently with empirical evidence, entrants are on average less productive than incumbents. However, as the distribution of incumbents shifts to the right due to growth, so does the distribution of entrants due to imitation. Imitation is then needed to guarantee a positive measure of new firms in every period and hence, together with selection, to assure the existence of a stationary distribution. This knowledge spillover, that goes from incumbent firms to entrants, is the only externality present in the model and combined with firm selection and together with innovation generates endogenous growth.[10]

2.3 Cross Sectional Distribution and Aggregates

All firms' choices and the processes for the idiosyncratic shocks yield the low of motion of firms distribution across efficiencies and qualities, $\mu(a, q)$. That is:

$$\mu'(a', q') = \int_{A_P} \int_Q \mu(a, q)\phi(a', q'|a, q)\mathrm{d}q\mathrm{d}a + \int_{A_A} \int_Q \mu(a, q)\phi(a', q'|a, q, z)\mathrm{d}q\mathrm{d}a + \int_{A_{AQ}} \int_Q \mu(a, q)\phi(a', q'|a, q, z, l)\mathrm{d}q\mathrm{d}a + \int_{A_Q} \int_Q \mu(a, q)\phi(a', q'|a, q, l)\mathrm{d}q\mathrm{d}a + \int_A \int_Q \frac{M}{I}\gamma(a, q)\mathrm{d}q\mathrm{d}a \tag{17}$$

Tomorrow density is given by the contribution of all surviving firms (the domain of the integrals is restricted to surviving firms only) and of entrants. The contribution of new firms is represented by the last term of (21). The first integral represents the share of surviving firms that only produce and do not invest neither in process nor in product innovation, the second integral shows the contribution of the firms that successfully produce and invest in process innovation. The third one instead represents the firms that produce and undertake both types of innovation and finally the forth one highlights the share of producers that

[10]Eeckhout and Jovanovic (2002) used a wider mechanisms of knowledge spillover in which all firms and not only entering firms, can imperfectly imitate the whole population of firms.

specialize in product innovation only.[11]

To summarize the information about the average firm efficiency and product quality, a weighted mean of firm technology can be introduced. That is:

$$\overline{\mu} = \left(\int_{a_x(q)} \int_Q \left(aq^{1-\eta}\right)^{\frac{\alpha}{1-\alpha}} \mu(a,q)\mathrm{d}q\mathrm{d}a \right)^{\frac{1-\alpha}{\alpha}}. \tag{18}$$

Notice that $aq^{1-\eta}$ is an index of firm level technology that maps one to one to firms' profits and size. Differing from Melitz (2003), this weighted mean not only depends on two states, efficiency and quality of the firm variety, but also the weights reflect the relative quality adjusted output shares of firms with different technology levels rather than the simple output shares. Moreover, the weighted mean can be also seen as the aggregate technology incorporating all the information contained in $\mu(a,q)$. In fact, it has the property that the aggregate variables can be expressed as a function of only $\overline{\mu}$ disregarding the technology distribution, $\mu(a,q)$.[12]

2.4 Equilibrium Definition

In equilibrium the representative consumer maximizes its utility, firms maximize their discounted expected profit and markets clear. The stationary equilibrium of this economy is a sequences of prices $\{p_t\}_{t=0}^{\infty}$, $\{P_t\}_{t=0}^{\infty}$, real numbers $\{I_t\}_{t=0}^{\infty}$ $\{M_t\}_{t=0}^{\infty}$, $\{X_t\}_{t=0}^{\infty}$ functions $n(a,q;\mu)$, $z(a,q;\mu)$, $l(a,q;\mu)$ $v(a,q;\mu)$, cutoff functions $a_x(q)$, $a_A(q)$, $a_{AQ}(q)$, and $a_Q(q)$ and a sequence of probability density function $\{\mu_t\}_{t=0}^{\infty}$ such that:

- the representative consumer chooses asset holding and consumption optimally so that to satisfy the Euler Equation (6),
- all active firms maximize their profits choosing a price that satisfies (8) and employment and innovation policies that satisfy $n(a,q;\mu)$, $z(a,q;\mu)$, and $l(a,q;\mu)$ yielding the value function $v(a,q)$ as specified by equation (14) and its components,

[11] Since the industry is populated by a continuum of firms and only independent idiosyncratic shocks occurs the aggregate distribution evolves deterministically. As a consequence, though the identity of any firms i associated with a couple (a,q) is not determined, their aggregate measure is deterministic. For the same reason the other aggregate variables evolve deterministically.

[12] See Appendix B for more details.

- innovation is optimal such that the cutoff functions $a_A(q)$, $a_{AQ}(q)$, and $a_Q(q)$ satisfy the previous conditions,
- exit is optimal such that $a_x(q)$ is given by equation (19) and firms exit if $a(q) < a_x(q)$,
- entry is optimal: firms enter until equation (20) and the aggregate stability condition are satisfied,
- the number of active firms I adjusts till the labor market clears: the aggregate demand of labor is equal to the exogenous labor supply:

$$\int_A \int_Q \big(n(a,q) + l(a,q) + z(a,q)\big) I\mu(a,q)\mathrm{d}q\mathrm{d}a + I\int_{A_A}\int_Q \mu(a,q)c_a \mathrm{d}q\mathrm{d}a$$
$$+I\int_{A_Q}\int_Q \mu(a,q)c_r \mathrm{d}q\mathrm{d}a + I\int_{A_{AQ}}\int_Q \mu(a,q)(c_a + c_r)\mathrm{d}q\mathrm{d}a + Ic_f + Mc_e = N^s,$$

- the stationary distribution of firms evolves accordingly to (21) given μ_0, I, M and the cutoff values,
- the stability condition, $M = \int_{a_x(q)}\int_Q I\mu(a,q)$, holds.

In equilibrium a_x, a_A, a_{AQ} a_Q, I and M are such that the sequence of firms distribution is consistent with the law of motion generated by the entry and exit rules.[13]

3 Endogenous Growth

3.1 Balanced Growth Path

In general, on the Balanced Growth Path output, consumption, real wage, prices and the aggregate technology grow at a constant rate, the bivariate distribution of efficiency and quality shifts to the right by constant steps, its shape is time invariant, and the interest rate, the aggregate expenditure, the aggregate profit, the profit and the labor demand distributions, the number of firms, the firm turnover rate, and the other characteristics of the firms' distribution are constant.

[13] Hopenhayn (1992)'s paper proves the existence of equilibrium for similar economies.

Define g as the average growth rate of firm productivity. That is the growth rate of the mean of the joint distribution of efficiency and quality. It is given by a combination of the growth rate of the efficiency state, denoted by g_a, and of the growth rate of the product quality state, indicated by g_q. Intuitively, growth arises because in every period the log of the joint aggregate technology shifts to the right by a factor g, meaning that the average productivity and the average product quality of the industry grow. Defining the growth factors of firm efficiency and product quality by $G_A = \frac{a_{t+1}}{a_t} = 1 + g_a$ and $G_Q = \frac{q_{t+1}}{q_t} = 1 + g_q$, the Balanced Growth Path can be found as follows. From the labor market clearing condition, given the assumption of a constant labor supply, N_s, also the number of incumbent firms, I, and the number of entrants, M, have to be constant as well as the share of labor allocated to production and innovation.[14] Aggregate expenditure, E, has to be equal to the aggregate labor income, N_s, given the wage normalization. This in turn implies that E is constant and hence also Π has to be constant. The profit distribution, equation (11), shows that $\pi(a, q)$ has to be constant because of constant fixed operational costs. Given a constant expenditure, profits are constant only if $aq^{1-\eta}P$ is constant. For positive growth rate of the technology, the previous condition holds if the price index growth factor is inversely related to the average technology growth factor, $G_P = (G_A G_Q^{1-\eta})^{-1}$. In other words, as the industry grows and the average technology advances, the price index diminishes. With the same reasoning also the distribution of manufacturing labor, equation (10), is time invariant, which together with the labor market clearing condition implies that also the distributions of the labor hired for the innovation activities, $z(a, q)$ and $l(a, q)$, are constant. From the consumer problem $E = PX$, which holds only if the aggregate consumption X grows at a constant factor $(G_A G_Q^{1-\eta})$. This results in a constant interest rate as shown by the Euler equation, $r = (1 + g)\beta - 1$. The price distribution, $p(a, q)$, decreases at a factor equal to $\frac{G_Q^\eta}{G_a}$ which is lower than the growth rate of the price index. This is a consequence of the fact that the price index is adjusted to consider the growth in the product quality. Finally, $x(a, q)$ grows at a factor of $\frac{G_A}{G_Q^\eta}$.

A Balanced Growth Path equilibrium exists if there is a g_a and a g_q consistent with the stationary equilibrium. To find these growth rates and to characterize the equilibrium itself and the stationary firms' distribution it is necessary to transform the model such that all the

[14]If there was population growth then the number of varieties, and the number of entrant firms would grow at the same rate as population grows.

variables are constant along the Balanced Growth Path. Hence, all growing variables need to be divided by the corresponding growth factor, $\widetilde{s} = s/G_s^t$ and the stochastic processes in efficiency and quality need to be de-trended by the respective growth rates, $\log \widetilde{a}_t = \log a_t - g_a t$ and $\log \widetilde{q}_t = \log q_t - g_q t$, where "$\sim$" denotes the stationarized variables. In expected terms both average firm efficiency and average quality increase and thus in expectation in every period each firm falls back relative to the distribution. This transformation affects also the transition functions and hence log efficiency and log quality, in the stationarized economy, which evolve according to:

$$\log \widetilde{a}_{t+1} = \begin{cases} \log \widetilde{a}_t - g_a + \varepsilon_{t+1}^a \\ \log \widetilde{a}_t - g_a + \lambda^a \log \widetilde{z}_t + \varepsilon_{t+1}^{az} \end{cases} \tag{19}$$

$$\log \widetilde{q}_{t+1} = \begin{cases} \log \widetilde{q}_t - g_q + \varepsilon_{t+1}^q \\ \log \widetilde{q}_t - g_q + \lambda^q \log \widetilde{l}_t + \varepsilon_{t+1}^{ql}. \end{cases} \tag{20}$$

For positive growth rates firm efficiency and quality follows a random walk with negative drifts. This negative drift determines a finite expected lifetime for any level of technology and hence the existence of a stationary distribution in the de-trended economy is guaranteed.

The previous discussion leads to the following proposition:

Proposition 1: *Given G_a and G_q growth factors of firms efficiency and quality the economy admits a Balanced Growth Path along which the mean of the joint distribution of incumbent firms and of entrant firms and the aggregate consumption grow at a rate $G_a G_q^{1-\eta}$, the price index decreases at a rate $G_a G_q^{1-\eta}$, the output distribution grows at a rate G_a/G_q^{η}, the price distribution grows at a rate G_q/G_a^{η} and the number of firms, the number of entrants, the aggregate expenditure, the aggregate profits, the profit distribution, and the labor distributions are constant.*

3.2 Growth Rate Determinants

Firms' *Selection* and *Innovation* drive endogenous growth which is then sustained by entrants' *Imitation*. Firm selection results from the assumption of a random walk process for both the evolution of labor efficiency and product quality together with firm exit. Abstracting from the endogenous drift introduced by firms' innovations, the random walk process, for a given set of firms, is characterized by a constant expectation and by a variance that grows over time. However, firms at the bottom of the distribution exit the industry truncating the joint distribution from below and allowing the distribution to grow only to the right towards higher level of efficiency and quality. The selection of firms with low efficiency and low quality and the consequent reallocation of resources from these firms to more technological advanced ones increases the average efficiency and quality of the surviving set of firms.

When firms' innovation with the related endogenous drift is introduced into the random walk process it reinforces growth. For a given set of innovative firms, not only the variance grows over time but also the expectation. The expectation of a firm technology depends on the initial state and on the sequence of resources invested in innovation. After every successful innovation the average technology shifts upwards due to the endogenous drift. However, innovation has decreasing returns through the log form in which the innovation drift is modeled. For this reason the resource reallocation effect from non-innovators to innovators is controlled by the selection effect and the result is that growth is reinforced but still bounded. Consequently, as time goes by the distribution of incumbent firms shrinks as exit is an absorbing state and firms keep exiting the industry.

Hence, entrants' imitation is needed to sustain growth and assure the existence of a stationary distribution with entry and exit. In equilibrium the mass of entrants has to be equal to the mass of firms exiting the market. However entrants are on average more productive than exiting firms otherwise they would not find optimal to enter the market. Since exiting firms are replaced by entrants with on average better efficiency and quality levels, the resulting firm distribution moves every period upwards towards higher technological levels.[15] Notice that innovation affects growth also allowing for a better imitation.

In the de-trended economy a stationary firm size distribution arises because the average technology of the incumbent firms improves at a rate that it is not too high relative to the rate at which the technology available to entrants firms improves. Technically, a stationary distribution exists because firm lifetime is finite for any (a, q). This is assured by the combination of decreasing return on innovation and by the downward drift in the random walk. Any successful firm which performs innovation will not be an innovator forever but eventually it will exit the market, leading to a finite expectation and to a finite variance of

[15] Selection and innovation are important to emphasize the fundamental role of reallocation of resources in the growth process. Growth could still be generated without selection and innovation assuming that the joint mean of the entrants distribution shifts every period exogenously by g. However in this way growth would just result from entry and exit.

the incumbent firm distribution.

When innovation occurs the efficiency and quality processes have also higher variances of the stochastic component. This increases the probability of a bad shock hitting the innovative firms and the dispersion of the innovator distribution against the distribution of non-innovators and exiting firms. On the one hand, selection results in a higher average technology for innovators because relatively bad firms fall among the pool of non-innovators resulting in a scenario where only relatively low cost and high quality firms keep innovating. On the other hand, the pool of non-innovators becomes larger, implying a higher weight to the distribution of non-innovators which has a lower average technology. The final effect of higher variances of the innovation random walks on the mean of the joint distribution is ambiguous.[16] However, calibrating the model to match the Spanish data shows that the positive effect of innovation always outweighs the negative effect.

3.3 Growth Rate Decomposition

On the Balanced Growth Path the growth rate of aggregate and average consumption can be rewritten and approximated (the derivation are in the Appendix) as:

$$g \approx \frac{1}{\alpha \bar{X}^{\alpha}} \left\{ \int_A \int_Q \hat{x}(a,q)^{\alpha} \left[T_{xI}\mu(a,q) - \left(1 - \frac{M}{I}\right)\mu(a,q) + \frac{M}{I}\big(\gamma(a,q) - \mu(a,q)\big) \right] \mathrm{d}q \mathrm{d}a \right\}, \quad (21)$$

where $\bar{X}$ is the average consumption, $\hat{x}(a,q)$ is the firm's quality weighted output, T_{xI} is the transition function with the exit and innovation rules and M/I is the entry/exit equilibrium rate. The first difference into the squared bracket represents the growth contribution of selection and innovation. That is, the difference between the average quality weighted output of surviving firms (both innovators and non innovators) and the one of the previous period incumbents. The more significant the innovation investments, the larger $T_{xI}\mu$ and the tougher selection, the smaller $(1 - M/I)\mu$. The second difference instead represents the contribution of entrants' imitation. The easier or cheaper the imitation mechanism (the smaller the distance between the entrants' and incumbents' distributions) the larger the contribution of entrants to the aggregate growth.

[16] The negative effect is then reinforced by the fact that the value function is convex in both states. Thus, a higher variance impacts the continuation values for the innovation strategies such that they are higher, relaxing the general cutoff function between innovate or not to innovate. This reduces selection and therefore growth.

Adopting the terminology introduced by Poschke (2008), μ can be divided into μ_{con}, continuing firms, and μ_{exit}, exiting firms. This allows for a further disaggregation of the aggregate growth rate:

$$\begin{aligned} g \;\approx\; & \frac{1}{\alpha \bar{X}^{\alpha}} \left\{ \int_A \int_Q \hat{x}(a,q)^{\alpha} \big[T\mu_{con}(a,q) - \mu_{con}(a,q) \big] \mathrm{d}q \mathrm{d}a \, + \right. \\ + & \left. \int_A \int_Q \hat{x}(a,q)^{\alpha} \left[\frac{M}{I} \gamma(a,q) - \mu_{exit}(a,q) \right] \right\}. \end{aligned} \tag{22}$$

The first integral catches the share of growth due to firms' innovation activities and due to the idiosyncratic shocks hitting surviving firms' level technology.[17] The second integral instead represents the share of growth due to net entry. It is clear that the selection of inefficient firms exiting the market and the imitation of new entrants generate positive growth only if entrants are on average more productive than exiting firms. This condition holds in the stationary equilibrium with positive entry. Furthermore, splitting the density of continuing firms between the densities of firms that only produce, μ_p, and of firms that innovate and produce, μ_i, the first integral in equation (30) can be further disaggregated in:

$$\begin{aligned} & \int_A \int_Q \hat{x}(a,q)^{\alpha} \big[T\mu_{con}(a,q) - \mu_{con}(a,q) \big] \mathrm{d}q \mathrm{d}a = \\ & \int_A \int_Q \hat{x}(a,q)^{\alpha} \big[(T\mu_p(a,q) - \mu_p(a,q)) + (T\mu_i(a,q) - \mu_i(a,q)) \big] \mathrm{d}q \mathrm{d}a. \end{aligned} \tag{23}$$

Among surviving firms it is now possible to calculate the share of growth that is due to only firms' experimentation based on the random walk processes without drift and the share of growth due to both experimentation and firms' innovation. The numerical analysis of the model will then quantify the share of growth due to net entry, innovation, and firms' experimentation.

The innovation investments of firms affect aggregate growth both directly and indirectly through a better imitation. In fact, innovation results in a higher joint mean of the incumbents' distribution and hence on entrants that can draw their initial technology from a distribution that stochastically dominates the distribution of entrants in an economy without

[17]Without weighting the firm distribution by the share of quality weighted output the resulting expected growth rate of the average technology of continuing firms would be zero. However, given that the optimal consumption is a convex function of the technology index $aq^{1-\eta}$, by Jensen inequality, the average growth rate of the output weighted technology is positive.

innovation. Given that $\bar{\mu}$ is the key variable in the imitation process, the contribution of innovation on a better imitation can be assessed using the following equation:

$$
\begin{aligned}
1 &= \frac{1}{\bar{\mu}^{\frac{\alpha}{1-\alpha}}}\left(\int_{A_P}\int_Q (aq^{1-\eta})^{\frac{\alpha}{1-\alpha}} u_p(a,q)\mathrm{d}q\mathrm{d}a + \int_{A_I}\int_Q (aq^{1-\eta})^{\frac{\alpha}{1-\alpha}} u_i(a,q)\mathrm{d}q\mathrm{d}a + \right.\\
&+ \left.\int_0^{a_x(q)}\int_Q (aq^{1-\eta})^{\frac{\alpha}{1-\alpha}} u_{exit}(a,q)\mathrm{d}q\mathrm{d}a\right), \qquad (24)
\end{aligned}
$$

where A_P is the support of surviving firms that produce but do not innovate while $A_I = A_A \cup A_Q \cup A_{AQ}$ is the support of firms that produce and innovate. The second integral captures the contribution of innovation in determining the joint mean of the incumbent firms. It is clear that the larger this term, the higher the indirect growth contribution of innovation via a better imitation.

4 Numerical Analysis

The algorithm, used to solve the model in the stationary equilibrium, is explained in Appendix D.

4.1 Calibration

Sixteen parameters, linked to firm dynamics characteristics, firms specific innovation behavior and to the general economic environment, need to be chosen. Since all of them interact with each other to determine the stationary equilibrium only four of them are parametrized while twelve are jointly calibrated to match the Spanish manufacturing sector.[18] The parameters *a priori* chosen are the discount factor, β, the preference parameter, α, the imitation parameter, ψ_e, and the growth rate of labor productivity, g. β is set equal to 0.95 to analyze a yearly time span. Accordingly to Ghironi and Melitz (2003), α is set equal to 0.73,

[18] The Spanish economy has been empirically widely studied in both the dimensions object of this paper: the new dimension related to firm innovation behavior and the traditional dimension related to firm dynamics. Hence, from the Spanish data it is possible to obtain enough information to calibrate successfully the model. Similar studies are available also for other European countries (Bartelsman et al. (2004), Bartelsman et al. (2003) for OECD countries; Cefis and Marsili (2005) for the Netherlands, Smolny (2003) and Fritsch and Meschede (2001) for Germany).

so that the price mark-up charged by the monopolistic firm is of 36% over the marginal cost. This high mark-up could be seen at odds with the macro literature that delivers a standard mark-up of around 20% over the marginal/average cost. In this model, a higher mark-up is justified by the presence of the fixed costs. In fact, given the free entry condition, firms on average break even. On average, firms price at the average cost leading to reasonaby high mark-ups over the average cost. Since the aim of this paper is to provide a model able to disentangle the contribution of efficiency and quality improvments in explaining the economy growth rate and not to test the ability of the model in matching the aggregate growth rate, g is set equal to 0.042. This number is taken from European Innovation Scoreboard (2001) and represents the labor productivity growth measured in terms of value added per workers as average over the nineties. The last parameter chosen is ψ_e which relates the joint mean of the entrants distribution with the joint mean of the incumbents. Given the importance of this parameter in determining the growth rates of the economy it is set individually to match its empirical counterpart. That is, ψ_e is chosen such that the average size of entrants is 38% of the size of incumbent firms as estimated by Gracia and Puente (2006).

The other twelve parameters are calibrated using a genetic algorithm as described by Dorsey and Mayer (1995).[19] These are: the ratio among the fixed costs, c_e/c_f, c_a/c_f, and c_q/c_f, the quality parameter η, the four variances of the incumbent random walks σ_a, σ_{az}, σ_q, and σ_{ql}, the two variances of the entrant random walks, σ_{ea} and σ_{eq}, and finally the two parameters that scale the innovation drifts into the stochastic processes, λ_a and λ_q. These parameters jointly determine the shape, the truncation functions of the stationary distribution of firms, and the partition of firms among the different innovation strategies. They are calibrated, using as targets, static and dynamic empirical moments that are informative and related to the main objective of the paper. It is possible to distinguish between two sets of targets.

The first group refers to a set of moments traditionally used as targets in the firm dynamic literature. These are firms' survival rates after two and five years upon entry, firms' yearly turnover rate, the job creation rate due to entry, the fraction of firms below average productivity, and the productivity spread, which calibrate the six variances of the model and

[19]The object of the algorithm is to jointly calibrate the parameters in order to minimize the mean relative squared deviation of twelve model moments with respect to the corresponding moments in the data. Since the problem is highly non-linear, the minimization can be characterized by many local minima and the genetic algorithm used has the nice feature to increase the probability of choosing the global minimum.

the size of entrants with respect to exiting firms which gives information about the entry cost. Accordingly to Garcia and Puente (2006), the two and five year survival rates for Spanish manufacturing firms are estimated to be 82% and 58%, respectively.[20] They report also a yearly firm turnover rate of 9% and a job creation rate due to entry equal to 3%.[21] Garcia and Puente (2006), estimate that entrants firms are 23% bigger than exiting firms in terms of employment. Bartelsman et al. (2004) estimate that the fraction of Spanish firms below average productivity is equal to 83%, highlighting a right skewed firm size distribution. The last moment is the productivity spread between the 85^{th} and 15^{th} percentile which is estimated to be between 3 and 4.

The second set of empirical moments used in the calibration gives information related to the innovation behavior of firms. The targets used are the share of Spanish manufacturing firms performing process innovation, product innovation and the share of firms that do not innovate and the intensity of the innovation investments in process and product, respectively. Given the novelty of these statistics, these moments have not been used before in the literature. However, in the scope of this paper these are relevant moments that help to calibrate the fixed cost of process and product innovation, η, λ_a, and λ_q. Harrison et al. (2008) working on data derived from the CIS report that 12.2% of Spanish firms in the manufacturing sector declared a process innovation between 1998 and 2000, while 12.4% declare a product innovation and more than half of the firms do not innovate in the time span considered. This numbers are very close to the one published by the National Statistics Institute (www.ine.es) using the ESEE. The innovation intensity of the Spanish manufacturing sector, computed as the aggregate innovation expenditure over the aggregate sales, in the 1998 is of 1.71%, process innovation intensity accounts for 1.26% while product innovation intensity accounts for the remaining 0.44%.[22]

Table 2 shows the values assigned to the parameters characterizing the economy. The

[20] Those numbers are aligned to the one reported by other developed countries as UK, Germany and Nederland (Bartelsman et al. (2003)).

[21] Firms' turnover is computed as the sum of the number of entering and exiting firms over the total number of firms while job creation rate is computed as the total amount of labor employed by entering firms in a year divided by the total employment in the same year.

[22] The European Innovation Scoreboard 2001 reports and innovation intensity for the Spanish manufacturing sector in the 1998 of 2.4% of aggregate sales. This number has been computed on the basis of the CIS which restricts its sample to firms with more than ten employees. This can explain the different numbers between the Euroean Commission survey and the INE statistics.

Table 2: Calibration

Parameter	Value	Description
Calibrated Parameters		
c_e	142.28%	Entry cost, % of average firm size
c_f	3.85%	Fixed cost, % of average firm size
c_a	31.96%	Process innovation cost, % of average firm size
c_q	16.29%	Product innovation cost, % of average firm size
η	0.74	Quality parameter
σ_a	0.15	Variance of productivity shock
σ_{az}	0.9	Variance of productivity shock with innovation
σ_q	0.32	Variance of quality shock
σ_{ql}	1.2	Variance of quality shock with innovation
σ_{ea}	0.40	Variance of log productivity distribution of entrants
σ_{eq}	0.32	Variance of log quality distribution of entrants
λ_a	0.083	Scale coefficient for process innovation
λ_q	0.025	Scale coefficient for product innovation
Parametrization		
β	0.95	Discount factor
α	0.73	Preference parameter
θ	0.38	Relative entrant mean
g	0.042	Growth rate of labor productivity

fixed costs are expressed in relation to the average employment devoted to production. As expected the entry cost, which represents a sunk entry investment, is the highest, more than ten times the operational cost. Reasonable values are attributed to the fixed cost of both process and product innovation. The parameter associated with the difficulty to produce high quality, η, is just above α and hence such that the optimal output produced by each firm is decreasing in the quality dimension. When new firms enter the market there is high uncertainty on their profitability, and the probability of surviving the market

Table 3: Empirical Targets and Model Statistics

Targets	**Data**	**Model**
Targets for Calibration		
Share process innovation	12.2%	13.4%
Share no innovation	55.4%	60.92%
Share product innovation	12.4%	11.1%
Product innovation intensity	0.44%	0.5%
Process innovation intensity	1.26%	1.29%
2 year survival rate	0.8	0.74
5 year survival rate	0.58	0.6
Firm turnover rate	0.09	0.086
Firm below average productivity	0.83	0.78
Job creation due to entry	0.03	0.02
Size entrants wrt exiting firms	1.23	1.31
Productivity spread	[2, 3]	2.48
Targets for Parametrization		
Entrant size/incumbent size	0.38	0.38
Mark-up over marginal cost	0.37	0.37

competition is low. However, once a firm survives the first selection the growth rate of young firms is on average higher than the growth rate of incumbents. This fragility is represented by a variance of the entrants distribution that is higher than the variance of the random walk process associated with the evolution of the states when firms optimally decide to only produce. Innovation also introduces uncertainty, reflected by higher variances of the corresponding random walk processes. In particular, a very high variance is associated with product innovation.

Table 3 reports the empirical targets used and the corresponding model moments. Despite the large number of parameters to calibrate, the model statistics match closely the data in both sets of targets. Hence, the innovation choices of firms, the shape of the distribution, its dynamic characteristics, and entrants' behavior seem to reproduce accurately the Spanish manufacturing sector.

4.2 The Role of Innovation

After setting g equal to 4.2%, the model predicts an annual growth rate of firms' production efficiency, g_a, of 2.93% and of product quality, g_q, of 4.64%. Using that $g \approx g_a+(1-\eta)g_q$, it is fair to conclude that 69.8% of the aggregate growth is due to the growth in firms' level efficiency and that only 29.81% is due to the growth in product quality.[23] These figures are very close to the estimates reported by Huergo and Jamandreu (2004) confirming the validity of the model in explaining the dynamics of the Spanish manufacturing sector.

Equations (23) and (24) are used to distinguish the effect of innovation, selection, and imitation in determining the aggregate growth rate. The model predicts that 8.63% of the growth is due to entry (10.61%) and exit (−1.98%) and the remaining 91.37% is due to both experimentation and innovation of the firms that remain active in the industry.[24] Deconstracting further this last term into the contribution of the sole firm's experimentation and into the contribution of a firm's innovation helps to asses the important role played by innovation in determining the aggregate growth rate. In fact, the growth contribution of firms that are and remain only producers is negative (−8.34% of the 91.37%). These firms are characterized by a low level of technology and are destined to exit the market after a series of bad shocks. The high likelihood of receiveing a bad shock and the firm's powerlesseness to escape exit explains their negative contribution to growth. However, this negative effect is more than compensated by the growth contribution of innovative firms that develops to

[23]In equilibrium $(1+g) = (1+g_a)(1+g_q)^{(1-\eta)}$ holds. Approximating it using a logarithmic transformation yields $g \approx g_a + (1-\eta)g_q$.

[24]Puente and Garcia (2006) estimate that entry and exit account only for 5% of the productivity growth of Spanish firms. This number is much lower with respect to the ones that are typically found in the literaute. For istance, Bartelsman et al. (2004) working on a panel of 24 OECD countries over the nineties find that between 20% and 50% of aggregate productivity growth is due to entry and exit of firms. This numbers are in line with the US data. Foster et al. (2001) find that in the U.S. Census Manufactures, more than a quarter of the increase in aggregate productivity between 1997 and 1978 was due to entry and exit. Moreover, Lenz and Mortensen (2008) estimating their model on a panel of Danish firms find that entry and exit of firms can account for 20% of the aggregate growth while within firm growth account for 55%.

be the leading force of aggregate growth.

Additionally, innovative firms have a higher weighted mean of their technology index than firms that do not innovate. This implies that innovation increases the weighted mean of the technology distribution of active firms, that is used as reference by the entering firms. Hence innovation also means better imitation and therefore higher growth. Applying equation (25), it is possible to conclude that 84.31% of the joint mean is due to the average technology level reached by the innovative firms.

4.3 Firms Partition and Cutoff Functions

Figure 1 displays how the two attributes of firm heterogeneity together with the fixed operational and innovation costs determine the partition of firms between those exiting and remaining, and among process innovators, product innovators, and both types of innovators or non-innovators. Hence, it illustrates the equilibrium cutoff functions and the combinations of efficiency (x-axis) and quality (y-axis) for which the different choices faced by firms are optimal. The firm distribution over the two dimensions of technology (Figure 2, left) shows that the firm distribution is right skewed in both states and that the largest mass of firms is concentrated in the bottom-left corner. This information complements the partition of firms and strengthens the subsequent interpretation.

The first area on the left represents the firms with production efficiency and product quality lower than $a_x(q)$ which optimally exit the market. These area represent about 9% of the total mass of firms. The exit cutoff function is the border between the exit region and the region where firms remain active and only produce. Due to the trade-off between quality and efficiency this cutoff function is decreasing in quality: relatively high cost firms can survive longer in the market when the quality of the variety they produce is high. In the second region, for slightly higher level of efficiency and quality, firms are sufficiently profitable to stay in the market but not enough to innovate, $v(a,q) = v^P(a,q)$. These are firms with relatively high level of cost but with all the possible levels of quality. In fact, product quality has a lower impact on firm profitability than productivity.

Moving along the efficiency dimension, for relatively small level of quality, it is optimal for firms to pay c_a and undertake process innovation while for relatively high level of quality it is optima to pay c_q and undertake product innovation. This is the result of the interplay between the fixed costs of innovation and the convexity of the profit function in a. The higher

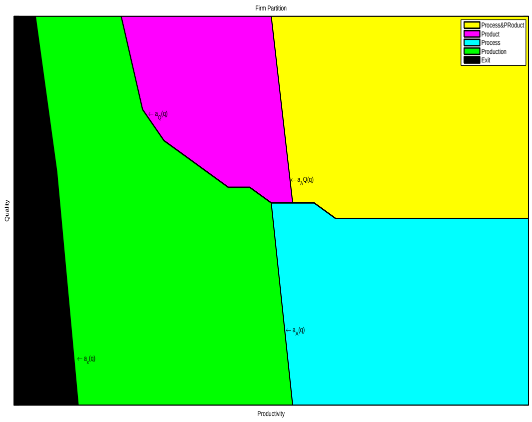

Figure 1: Firms Partition

the efficiency level reached by the firm the higher the gain in terms of profitability resulting from a marginal reduction of the production cost. This explains why it is optimal for firms to innovate in process when their efficiency has already reached a minimum level. The same is true for the quality dimension, though the profit function is concave in q. However this disadvantage is compensated by the lower fixed cost of product innovation. The last region is represented by firms with high efficiency and high quality that optimally innovate in both process and product.

Table 4: Conditional Probabilities

	Exit	**No Innovation**	**Process**	**Product**	**Both**
No Innovation	5.1%	87.84%	0.84%	5.6%	0.21%
Process	0	4.5%	75.9%	0.95%	18.65%
Product	0	34.65%	1.22%	51.84%	12.3%
Both	0	1.83%	33.26%	3.3%	61.61%

Table 4 shows the equilibrium conditional probabilities of switching actions after a one-year period given the current decision of the firms.[25] The first column lists the current action

[25] This information is contained in the optimal transition function T_{XI} and the derivations are in the Appendix.

of the firms and the rows give the transition probabilities of each future decision. Due to the persistence of the random walk process a high probability is attached to the repetition of the current action. Interestingly, consistent with empirical evidence, this persistence appears less strong in the case of product innovators: 34% of product innovators today will not innovate tomorrow while 15% will switch to process innovation, both alone and with product innovation, and only 51% will repeat an innovation in product quality. The relative low persistence in quality enhancing innovation is due to the high variance associated with this decision. A high variance implies that the probability of receiving a bad shock is high as well as the probability of switching to a different strategy. Empirical evidence emphasises that exit is associated with a low level of pre-exit innovation (Huergo and Jamandreu (2004) for evidence on Spanish firms). This model predicts that a firm exits the market with 5% of probabilty only if in the current year no innovation has been introduced. This also implies that an innovative firm, before exiting the market, has to receive a bad shock and become a non-innovator.

4.4 Firms Distribution

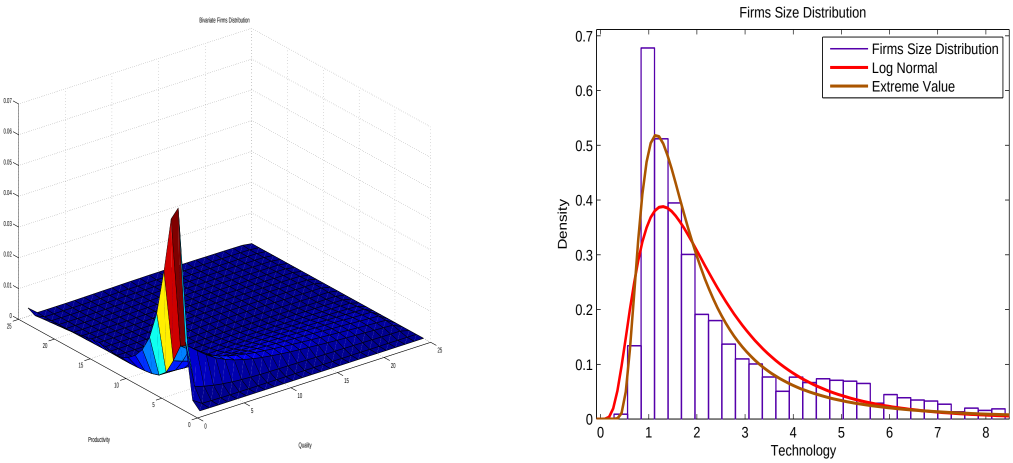

Figure 2: Bivariate and Univariate Firms Distribution

The equilibrium distribution of firms is determined endogenously and it is shaped by the static and dynamic decisions of incumbent firms together with entrants imitation. Figure

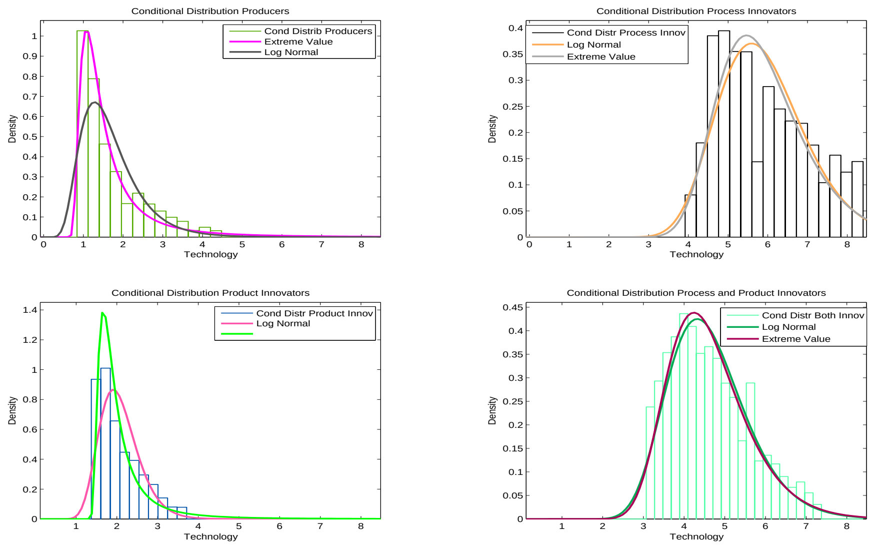

Figure 3: Conditional Firms Size Distributions

2, left panel, shows the bivariate firms distribution over the two attributes of firm heterogeneity. However, empirical studies are not able to distinguish these two dimensions and hence Figure 2, right panel, displays the corresponding univariate firm size distribution over a technological index that summarizes the information contained in a and q. That is, $aq^{1-\eta}$. Notice that this is the equivalent of the employment distribution of firms which is observed in the data. The univariate firm distribution looks right skewed and hence with a right thick tail (the moments of the distribution are reported in Table 5).[26] A Generalized Extreme Value distribution fits it best, though the more widely used log-normal distribution is not inadequate. Empirically there is not much information about the moments of the size distribution of the Spanish manufacturing sector and only few empirical works such as Doraszelski and Jaumandreu (2007) have analyzed it. They conclude that the distribution of Spanish firms in the manufacturing sectors is right skewed.

The conditional distribution of firms that only produce and do not innovate is concentrated at lower levels of the technological index $aq^{1-\eta}$ than the conditional distributions of innovators (Figure 3 and Table 5). Consistently with the empirical evidence innovative firms have a higher labor productivity and are bigger than firms that do not innovate. The

[26]The underlying distribution used to compute the skewness is a log-normal distribution.

comparison among innovators is more interestingly: on average small firms do product innovation, medium and large firms do both product and process innovation and large firms do process innovation. Finally, the conditional distribution of product innovators is more right skewed than the distribution of firms that do process innovation or do not innovate. Also this last feature is confirmed by empirical estimations of the firm size distribution in the Spanish manufacturing sector.[27]

Table 5: Descriptive Statistics of Firms Distributions

	Mean	**Variance**	**Coef. of Variation**	**Skewness**
Size Distribution	2.41	3.05	0.72	0.95
Cond. on Process Innov.	5.9	1.26	0.19	0.89
Cond. on Product Innov.	2.08	0.24	0.23	2.32
Cond. on Both Innov.	4.63	0.98	0.21	1.1
Cond. on No innovation	1.67	3.05	0.44	0.95

5 Comparative Statics

This section analyzes how changes in the key parameters of the model, which characterize the industry structure, affect the process of labor reallocation among firms and hence the equilibrium growth rates of the economy. In particular, changes in the innovation costs, c_a and c_q, as well as changes in the entry cost, c_e, are analyzed. Both types of costs are directly linked to growth: changes in c_a and c_q bring changes in the composition of the pool of innovative firms and changes in c_e affect the imitation process of entrants firms. High entry cost are seen as barrier to enter the industry and they are often regarded as a protection of incumbent firms and hence as a stimulus to innovation. On the other hand, high innovation costs are seen as detrimental of innovation. Hence, it becomes important to understand how the economy responds to changes in these key features in order to design

[27]Doraszelski and Jaumandreu (2007) find that the distribution of process innovators have a higher mean than the distribution of firms that do not innovate and that the distribution of product innovators are also more skewed.

policy recommendations aimed at fostering growth. The first part of this section discusses changes in the dynamic and static characteristics of the industry as well as changes in the growth rates of efficiency, g_a, and quality, g_q, for fixed g, while the second part considers how the aggregate growth rate, g, changes as the key parameters changed, fixing the relation between g_a and g.

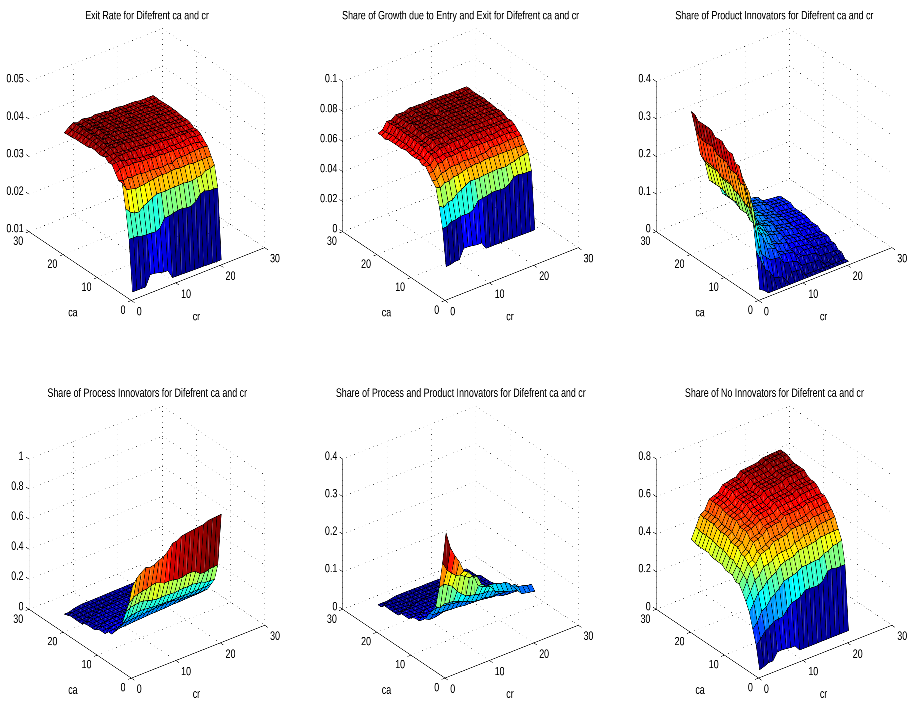

Figure 4: Comparative statics for different c_a and c_r, given g

5.1 Comparative Statics for Given g

Figure 4 and 5 show how the exit rate, the share of growth due to net entry, the firms partition, and the growth rate of efficiency and quality change as the innovation costs, c_a (y-axis) and c_q (x-axis) change. As the innovation costs decline g_a increases which, given a constant g, implies that g_q declines. Everything else equals a reduction in the innovation costs mainly benefits process innovators. Hence, the share of aggregate growth explained by g_a increases. This is the result of a higher impact of the production efficiency in determining

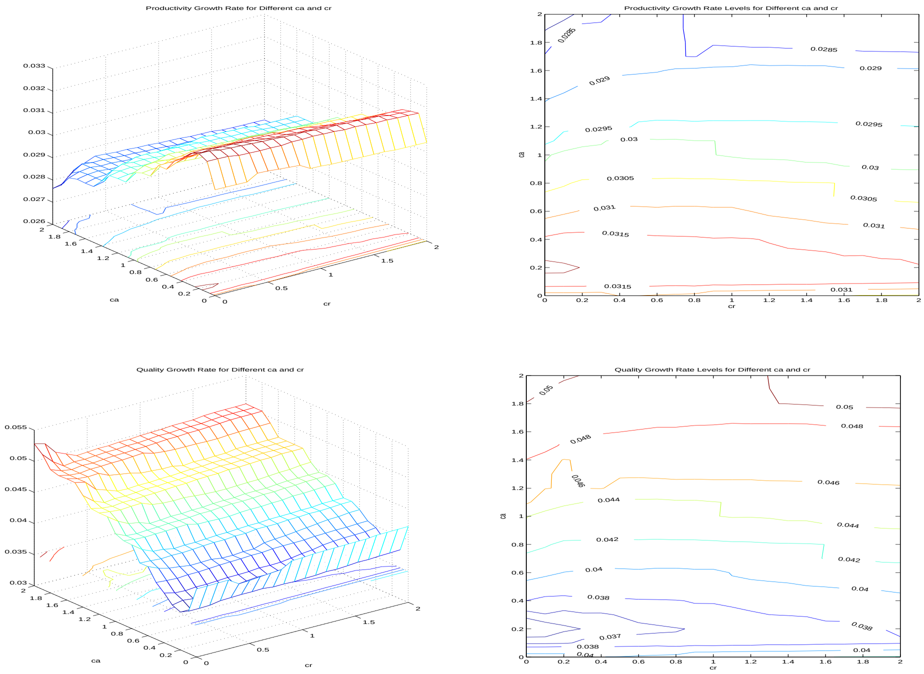

Figure 5: g_a (top) and g_q (bottom) for different c_a and c_r, given g

firms' profitability. Moreover, both g_a and g_q are more sensitive to changes in the cost of undertaking process innovation than product innovation. This last feature can be clearly observed plotting the isoquants of the growth rates (right pictures in Figure 5). However, the highest level of g_a (equal to 3.21%) is reached by a zero cost of doing product innovation and a small but positive cost of doing process innovation. In fact, a positive c_a not only allows for a higher share of innovators but also for a sizeable selection as can be shown in the first two pictures of Figure 4. Hence, both growth channels are strong. The same is not true when both innovation costs are equal to zero. In this scenario, innovation is very cheap and many firms innovate. However, being c_a and c_q equal to zero, the labor demand decreases and hence the wage rate decrases reducing the exit rate and hence firms' turnover. For zero cost of innovation the selection mechanism is not at work. The last feature to consider is the different reaction of the share of process and product innovators to changes in the innovation costs shaped by the comparative advantages of one type of innovation with respect to the other.

An increase in the entry cost leads in equilibrium to a higher expected value of entry

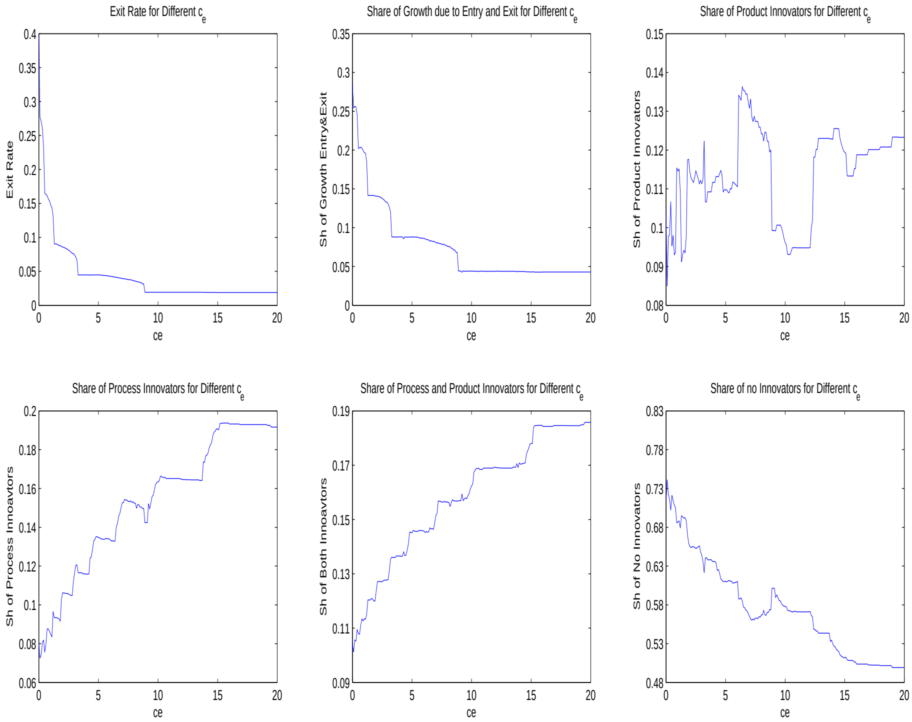

Figure 6: Comparative Statics for different c_e, given g

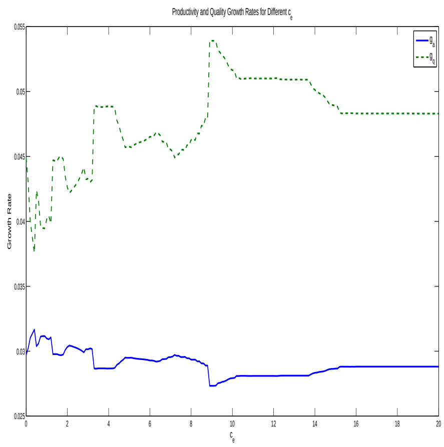

Figure 7: g_a and g_q for different c_e, given g

which in turn implies that the discounted expected profits of incumbent firms need to be higher. The exit cutoff function shifts to the left and hence firms survival becomes easier. The turnover rate reduces and as a consequence the share of growth that is due to entry and exit progressively declines in favor of the share of growth due to innovation. Indeed higher expected profits lead to more innovators and hence the share of growth due to incumbents firms increases to meet the fixed growth rate g (Figure 6). Interestingly, as the entry cost increases, the growth rates of efficiency reduces slightly and the growth rate of quality increases (Figure 7). Hence, a more concentrated industry favors product quality innovations. When a firm's market share is already large, firms benefit from increasing the quality of their variety.

5.2 Comparative Statics for Varying g

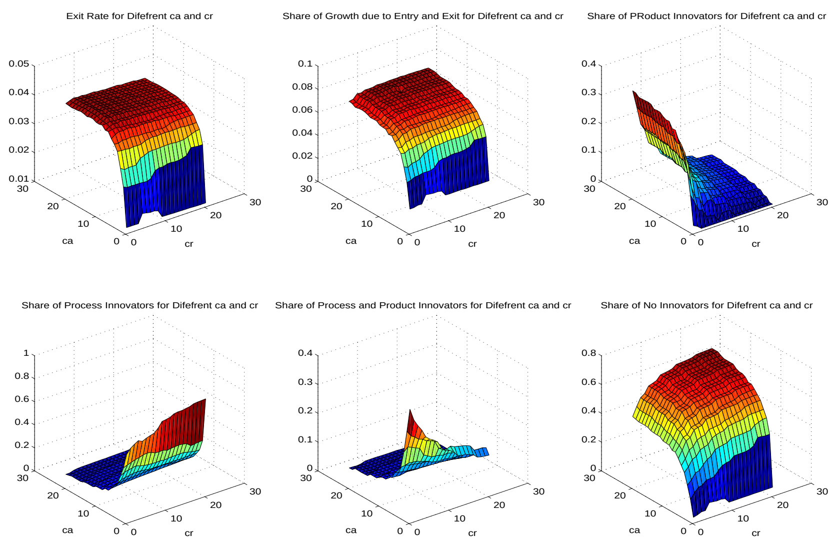

Figure 8: Comparative statics for different c_a and c_r, varying g

The quantitative analysis in Section 4.3 permits the conclusion that efficiency growth explains 69.8% of the aggregate growth. Here this information is used to fix the ratio between g_a and g and to use the calibrated algorithm to determine endogenously g.

Figure 9, left panel, plots the equilibrium growth rate for different values of the fixed

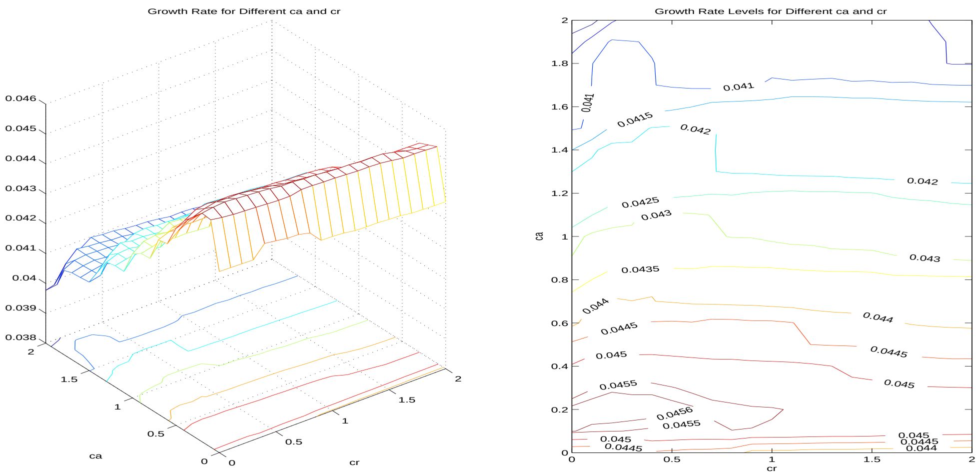

Figure 9: g for different c_a and c_r

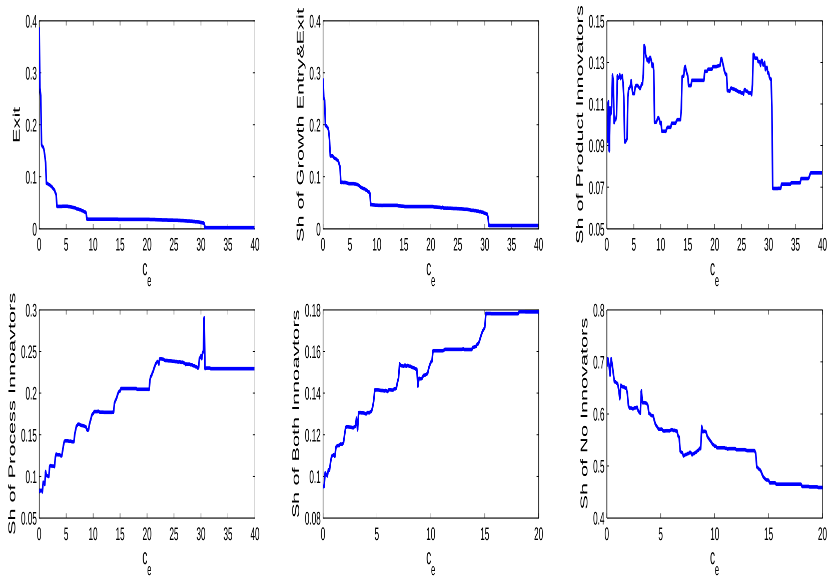

Figure 10: Comparative Statics for different c_e, varying g

costs of innovation: on the x-axis the cost of doing product innovation, c_q, while on the y-axis the cost of doing process innovation, c_a. As both the innovation fixed costs decline two opposite effects arise. On the one hand, innovation becomes cheaper and more firms

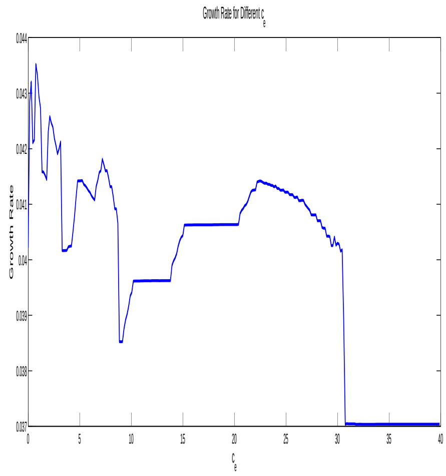

Figure 11: g for different c_e

find it profitable. Hence the pool of innovative firms increases and this affects positively and directly the growth rate of the economy (Figure 8). This positive effect is then reinforced by an indirect effect. If the mass of innovators is larger, more firms will pay the fixed costs. This sustains the demand of labor and hence the wage rate, thus assuring a strong selection. On the other hand, if the innovation costs are reduced, less labor is demanded by the individual innovative firm. Consequently, the demand of labor by an innovative firm declines and hence the real wage declines to satisfy the labor market clearing condition. A lower wage translates into a weaker selection and hence in a lower effect on the economy growth rate. The final response of the growth rate to the changes in the innovation costs results from the combination of these two effects. Generally, the positive effect prevails. The lower the innovation costs, the higher the growth rate. This holds true for all the values of the fixed cost of undertaking product innovation but only for high and intermediate value of the fixed cost of doing process innovation. The maximum growth rate is obtained for $c_q = 0$ but small and positive c_a, showing that for very low levels of c_a the negative effect offsets the positive one. Additionally, the economy growth rate is more sensitive to changes in c_a than to changes in c_q. Hence, a policy aimed at promoting only growth would be more successful when used to address an increase in process innovation.

Finally, when entry cost are low, imitation is cheap, and many firms enter and exit the market, which results in a high growth rate (Figure 11). As the entry cost increases, firms' selection and imitation become weaker and the growth rate declines. For relatively low level of c_e this decline is reinforced by a reduction in the number of innovators: new entrants pay a higher cost and this increases the labor demand and wage rate, and hence innovation becomes more expensive. This negative trend in the number of innovators remains for process innovators while it becomes positive for product innovators as c_e increases. The industry becomes more and more concentrated due to costly imitation, and the market share of each incumbent increases leading to more product innovation. Product innovation has a lower impact on g than process innovation, and though the number of innovators is higher, the growth rate is lower than in a industry characterized by lower barriers to enter and higher competition. However, though the share of process innovators declines as the industry becomes more concentrated it is still higher than the share of product innovators (Figure 10).

6 Final Remarks

This paper proposes an endogenous growth model with heterogeneous firms where firms differ in two dimensions: production efficiency and product quality. Both dimensions are subject to idiosyncratic permanent shocks but firms can affect endogenously their evolution through process, product or both types of innovations. Growth arises due to incumbent firms' innovation and selection and is sustained by entrants' imitation. Selection eliminates the inefficient firms from the market, thereby increasing the average productivity of incumbents. Innovation amplifies this not only increasing directly the average technology of firms but also increasing selection. Entrants imitate the average incumbent and are, on average, more productive than exiting firms. The result is that the firm distribution shifts upwards, generating growth.

The economy is calibrated to the Spanish manufacturing sector and closely matches static and dynamic moments related to the firms' distribution and new moments related to the innovation behavior of firms. Hence, the model provides an accurate representation of the Spanish economy and an explanation of the heterogeneity in the innovation activities among firms. Firms' process innovation explains 69.8% of the aggregate growth while product innovation contributes only for the remaining 30.2%. When decomposing the aggregate growth rate between the contribution of innovative and surviving firms and the net contribution of entering and exiting firms, the role of innovation is substantive: 91.87% of growth is due to innovation. Innovation is also necessary to survive market competion: only non-innovative firms exit the industry. An unanswered question is to identify which type of innovation, between process and product innovation, allows for a greater period of firms' longevity.

The endogenous firm size distribution is right skewed and approximated well by a log-normal distribution. The conditional distributions of innovators are consistent with the data: innovators are larger than non-innovators and in the case of product innovators also more right skewed. Additionally, small firms do product innovation, intermediate firms do both product and process innovation and large firms do process innovation only. Hence, there is a non-monotonic relation between firm size and innovation though firm size is still an indicator of the type of innovation undertaken by firms. For given aggregate growth rate, industries characterized by lower innovation costs have a higher contribution of the growth rate of firm efficiency. Hence, when innovation is cheap there are more process innovators driving the growth of the industry. On the other hand, when the entry barriers are increased the share of all types of innovators increases and the growth contribution of product quality becomes more important. The industry growth rate reacts positively to reductions in the innovation costs, however the model predicts that its maximum is reached for a positive but small cost of process innovation. Though entry barriers protect and stimulates innovation, growth is maximized for relatively low entry costs which are accompanied by a more dynamic industry with a high turnover and a higher share of process innovators. As the industry becomes more concentrated, the aggregate share of innovators increases but in favor of product innovators and both types of innovators which impact growth less strongly as process innovators.

These considerations leads to attractive policy recommendations aimed at fostering growth and welfare. The next step is therefore to compute the optimal allocation and design innovation policies that can implement the first best in the decentralized economy.

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Appendix

A Innovation Cutoff Functions

Define $A_P = \{(a,q) : a \in A, q \in Q \wedge v(a,q) = v^P(a,q)\}$ the production support, $A_A = \{(a,q) : a \in A, q \in Q \wedge v(a,q) = v^A(a,q)\}$ the process innovation support, $A_Q = \{(a,q) : a \in A, q \in Q \wedge v(a,q) = v^Q(a,q)\}$ the product innovation support and $A_{AQ} = \{(a,q) : a \in A, q \in Q \wedge v(a,q) = v^{AQ}(a,q)\}$ the process and product innovation support. Moreover, let $B = \{(a+\epsilon, q+\epsilon)\}$ for $|\epsilon| > 0$ arbitrarily small. The innovation cutoff function are defined as $a_A = \{(a,q) : (a,q) \in A_A \wedge (A_P \cup A_Q \cup A_{AQ}) \setminus A_A \neq \emptyset\}$, $a_Q = \{(a,q) : (a,q) \in A_Q \wedge (A_P \cup A_A \cup A_{AQ}) \setminus A_Q \neq \emptyset\}$ and $a_{AQ} = \{(a,q) : (a,q) \in A_{AQ} \wedge (A_P \cup A_A \cup A_Q) \setminus A_{AQ} \neq \emptyset\}$.

B Aggregate Variables

Using the information contained in equation (19), the price index, the aggregate consumption, and the aggregate profits can be rewritten as:

$$P = \left(\int_{a_x(q)} \int_Q \left(\frac{p(a,q)}{q(a,q)} \right)^{\frac{\alpha}{\alpha-1}} I\mu(a,q)\mathrm{d}q\mathrm{d}a \right)^{\frac{\alpha-1}{\alpha}} = I^{\frac{\alpha-1}{\alpha}} p(\overline{\mu}), \tag{25}$$

$$X = \left(\int_{a_x(q)} \int_Q \big(qx(a,q)\big)^{\alpha} I\mu(a,q)\mathrm{d}q\mathrm{d}a \right)^{\frac{1}{\alpha}} = I^{\frac{1}{\alpha}} x(\overline{\mu}). \tag{26}$$

$$\Pi = \left(\int_{a_x(q)} \int_Q \pi(a,q) I \mu(a,q) \mathrm{d}q \mathrm{d}a \right) = I\pi(\overline{\mu}). \tag{27}$$

C Growth Rate Disaggregation

On the Balanced Growth Path, given that the number of firms is constant, the growth factor of aggregate (X) and average ($\bar{X}$) consumption coincides:

$$G = \frac{X'}{X} = \frac{\bar{X}'}{\bar{X}}. \tag{28}$$

Defining the firm's quality weighted output with $\hat{x}(a,q)$, the growth factor can be rewritten as:

$$G = \frac{\left(\int_{a_x(q)} \int_Q \hat{x}(a,q)^\alpha \mu'(a,q) \mathrm{d}q \mathrm{d}a \right)^{\frac{1}{\alpha}}}{\bar{X}}. \tag{29}$$

Rewrite μ' using its law of motion yields:

$$G = \left(\frac{\int_A \int_Q \hat{x}(a,q)^\alpha \big(T_{xI}\mu(a,q) + \frac{M}{I}\gamma(a,q) \big) \mathrm{d}q \mathrm{d}a}{\bar{X}^\alpha} \right)^{\frac{1}{\alpha}}, \tag{30}$$

where T_{xI} is the optimal transition function with the exit and innovation rules. Adding and subtracting $\bar{X}^\alpha = \int_{a_x(q)} \int_Q \hat{x}(a,q)^\alpha ((1 - M/I)\mu(a,q) + M/I\mu(a,q))$ to the numerator and rearranging the equation gives:

$$G = \left(\frac{\int_A \int_Q \hat{x}(a,q)^\alpha \big(T_{xI}\mu(a,q) - \big(1 - \frac{M}{I}\big)\mu(a,q) + \frac{M}{I}\big(\gamma(a,q) - \mu(a,q)\big)\big) \mathrm{d}q \mathrm{d}a}{\bar{X}^\alpha} + 1 \right)^{\frac{1}{\alpha}}. \tag{31}$$

The last step to obtain the growth rate decomposition consists in taking the logarithm of both terms of the equation and approximating them using the rule $\ln(G) \approx g$, given that g is a small number. This results in:

$$g \approx \frac{1}{\alpha \bar{X}^\alpha} \left\{ \int_A \int_Q \hat{x}(a,q)^\alpha \left[T_{xI}\mu(a,q) - \left(1 - \frac{M}{I}\right)\mu(a,q) + \frac{M}{I}\big(\gamma(a,q) - \mu(a,q)\big) \right] \right\}, \tag{32}$$

which is equation (29) in the main body of the paper.

D Algorithm

The state space $A \times Q$ is discretized. The grid chosen is of 30 points for each state yielding 900 technology combinations, (a, q).[28] Firms' value function is computed through value function iteration. The unknown variables are the growth rates g_a and g_q which combines in the growth rate of the aggregate technology g and the aggregate expenditure and price index summarized by $k = P^{\frac{\alpha}{1-\alpha}} E$. The growth rate of labor productivity, g, is fixed exogenously. For given g_a, $g_q = (G/G_a)^{\frac{1}{1-\eta}} - 1$, and k compute the stationary profit $\widetilde{\pi}(a, q; g_a, k)$ and then the firm value function $\widetilde{v}(a, q; g_a, k)$.[29] While iterating the value function, the optimal policies for the investment in process and product innovation, $\widetilde{z}(a, q; g_a, k)$ and $\widetilde{l}(a, q; g_a, k)$, are computed and the random walk processes, that govern the transition of firm productivity and product quality, are approximated using the method explained by Tauchen (1987). This step is time consuming since each firm's problem has to be solved via first order conditions for each single couple of states, (a, q), till convergence is reached. Once the value function is approximated the algorithm computes the cutoff functions $a_x(q; g_a, k)$, $a_A(q; g_a, k)$, $a_Q(a; g_a, k)$, and $a_{AQ}(q; g_a, k)$. Then the transition matrix Φ_{xI} is computed. This is the final transition matrix which takes into account the exit and the innovation decisions. After guessing an initial distribution for entrant firms and normalizing its initial joint mean to zero, the expected value of entry is computed. The free entry condition is used to pin down the equilibrium value of k resulting from the first iteration of the algorithm. Using the equilibrium k, the firm value, the cutoff functions, and the transition matrix can be found for given initial g_a. The binomial firm distribution is then determined using the formula for the ergodic distribution $\widetilde{\mu} = (I - T_{xI})^{-1} G$ as proved by Hopenhayn (1992). The algorithm is closed using the condition on the mean of the entrant distribution, $\overline{\gamma}_e = \psi_e \overline{\mu}$, and pinning down the equilibrium growth rate, g_a, that satisfies this equation. Once g_a is determined, g_q

[28] The choice of 30 grid points for each state is due to the fact that the algorithm is computationally heavy given the presence of two states and the endogenization of the dynamic choice of the innovation investment. On the one hand, increasing the grid size would improve the precision of the calibration but would not affect qualitatively the results. On the other hand, the technology combination (a, q) available to firms would increase quadratically in the grid size and the code would eventually become unfeasible. Hence, given that the results are not qualitatively affected by the grid size, a quality and productivity grid of 30 points is a reasonable restriction.

[29] Notice that all the variables depend on both g_a and g_q. However for notational convenience g_q is omitted since it is a function of g_a.

is determined as well. All these steps are repeated until all conditions are jointly satisfied and convergence is reached.

E Conditional Probabilities

The final transition function $T_{XI}(a', q'|a, q)$ contains all the information to compute the probability that tomorrow a firm will optimally decide to do action $Y \in A'$ given that today it chose action $X \in A$ where A' ={Exit, Not to Innovate, Do Process Innovation, Do Product Innovation, Do Both Innovations} and A ={Not to Innovate, Do Process Innovation, Do Product Innovation, Do Both Innovations}. Weighting these probabilities by the firm density in each state allows to calculate the fraction of firms that today chose action X and tomorrow will switch to action Y. Simplify the notation and define a vector of states, s, of all the possible combinations of a and q couples. Indicating with "$'$" the next period variables the conditional probabilities are computed as follows

$$P(Y|X) = \frac{1}{\int_{s:A=X} \mu(s)\mathrm{d}s} \int_{s':A'=Y} \int_{s:A=X} \phi(s'|s)\mu(s)\mathrm{d}s\mathrm{d}s'. \tag{33}$$