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MONETARY POLICY AND MISMEASURED INFLATION

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Monetary Policy and Mismeasured Inflation

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Abstract

I examine the standard New Keynesian model augmented with product entry and exit. The statistical agency in the model measures product entry with a delay. Consequently, measured inflation departs from true, utility-based inflation. I show that the gap between measured inflation and true inflation is serially correlated and varies with the state of the economy. This result contrasts with the common assumption of white-noise exogenous measurement error. True inflation is more volatile and less persistent than measured inflation, and the correlation between true inflation and true output is lower than the correlation between measured inflation and measured output. Furthermore, I analyze monetary policy given the measurement problem.

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1   Introduction

A key message of the New Keynesian model is that optimal monetary policy should stabilize inflation. In this model, inflation is defined as the change in the minimum expenditure to buy a certain amount of utility. I will refer to this inflation as "true inflation". The problem with the policy prescription is that, in practice, a central bank does not observe true inflation. In practice, the central bank observes consumer-price inflation measured by a statistical agency, or "measured inflation". Measured inflation is the expenditure to buy a certain basket of products at current prices relative to the expenditure to buy the same basket at past prices.

Broda and Weinstein (2007) study the dataset which underlies U.S. consumer-price (CPI) inflation. The dataset is maintained by the U.S. Bureau of Labor Statistics (BLS). Broda and Weinstein argue that inflation is measured with error because the BLS measures product entry with a delay. Broda and Weinstein define measurement bias (MB) in inflation as measured inflation divided by true inflation. They find that MB is serially correlated and pro-cyclical. This finding contrasts with each of the two common assumptions concerning inflation mismeasurement: constant error, and white-noise exogenous error.[1]

In this paper, I assess the effects of MB for monetary policy. I examine the standard New Keynesian model augmented with product entry and exit. The statistical agency in the model measures product entry with a delay when it compiles measured inflation. Products enter and exit the market at the same exogenous rate such that product variety remains constant. When a firm enters the market, it choses the price for its product. Thereafter, the firm adjusts its price with a given probability in each period. I show that, in line with the evidence, the gap between measured inflation and true inflation in this model is serially correlated and pro-cyclical.

Specifically, measured inflation is less volatile than true inflation. The reason is that measured inflation lacks some price changes which are contained in true inflation.

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[1] Among others, Boskin, Dulberger, Gordon, Griliches, and Jorgenson (1997) analyze constant error in inflation, and Svensson and Woodford (2003) assume white-noise exogenous error to study monetary policy under imperfect information.

In each period, true inflation records prices of all products available in the market. Therefore, true inflation routinely compares prices of products that have disappeared at the end of the previous period to prices of products that are new in the current period. Prices of new products generally differ from prices of discontinued products such that product turnover adds many price changes to true inflation. In contrast, measured inflation compares the current prices of products in a particular basket to past prices of the same products. Therefore, measured inflation does not contain the price changes that result from product turnover.

Furthermore, measured inflation is more persistent than true inflation. The reason is that measured inflation downplays price spells with short durations. Entry truncates the price spells of new products such that, by lacking new products, measured inflation underestimates the relative frequency of short price spells. Overall, measured inflation contains a smaller number and a different sample of price changes than true inflation. MB varies with the state of the economy because differences in the composition of both inflation rates matter most when price changes are large, and the size of a price change depends on the state of the economy. Technically, measured inflation depends on true inflation and lags of true inflation.

Bils (2004) argues that any MB in U.S. CPI inflation shows up as opposite bias in real output growth because U.S. National Income and Product Accounts (NIPA) deflate nominal output growth by measured inflation. I replicate this approach in the model and find that measured output equals true output plus true inflation and lags of true inflation. The MB in output is serially correlated and varies with the state of the economy like the MB in inflation.

One implication of MB in inflation, which spills over into output, is that the correlation of true inflation and true output is lower than the correlation of measured inflation and measured output. The reason is that inflation and output are not measured in the data in the same way in which inflation and output are measured in typical business-cycle models. The finding matters because business-cycle models are often evaluated by comparing the correlation of inflation and output in the model one-to-one to the

correlation of inflation and output in the data. My model suggests that this way of evaluating business-cycle models is appropriate only after accounting for endogenous MB.

In order to assess the effect of MB for monetary policy I consider the policy problem of a central bank which, by assumption, does not know true inflation and the true output gap. I find that, across monetary-policy regimes, the central bank stabilizes inflation insufficiently because of MB. When the statistical agency computes measured output, it wrongly attributes a fraction of true inflation to true output. This fraction of true inflation receives too little weight in the policy rule so long as the central bank responds more to inflation than to the output gap. Insufficient inflation stabilization deteriorates welfare.

When the central bank acts under discretion, MB leads to welfare improvements despite insufficient inflation stabilization. By targeting measured inflation the central bank effectively responds to true inflation and lags of true inflation. Furthermore, lags of true inflation enter the policy rule if the central bank targets the measured output gap. Thus, in terms of true variables, the policy rule exhibits history dependence even though the central bank is discretionary. The history dependence improves the tradeoff between stabilizing inflation or the output gap, and improves welfare.

Similar mechanisms are at work when the central bank pursues an interest-rate rule that is formulated in terms of measured output and measured inflation. By targeting measured variables, effectively, the central bank responds to true output, true inflation, and lags of true inflation. Accordingly, when true inflation is low the interest rate is expected to remain low for a longer period of time because the effective policy rule features lags of true inflation. Low expected interest rates increase current output which offsets some of the decline in inflation. Thus, the tradeoff between stabilizing inflation or the output gap improves, and so does welfare.

The literature, which studies monetary policy under imperfect information, typically assumes that the difference between true and measured variables is a white-noise exogenous process. I argue that this difference is not represented well by a white-noise

exogenous process if we take seriously the notion that mismeasurement is due to mistakes made by the statistical agency. I show that whether the difference between true and measured variables is modelled as white-noise exogenous process or as endogenous MB matters for how imperfect information on the part of the central bank affects monetary policy. Specifically, I reconsider the policy problem of a discretionary central bank which does not know true inflation and the true output gap assuming that true variables are perturbed by a white-noise exogenous process. In this case, monetary policy does not suffer from insufficient inflation stabilization, the effective policy rule does not feature additional states, and welfare deteriorates rather than improves.

With few exceptions, the empirical literature on MB in inflation has confined itself to identify constant biases and to quantify them econometrically. My analysis identifies and quantifies a time-varying and endogenous MB in inflation. My methodological contribution is to examine MB in inflation in a dynamic stochastic general-equilibrium (DSGE) model. The general-equilibrium model allows tracing out the effects of endogenous MB for monetary policy. Also, the paper contributes to the literature on the role of product entry and exit for monetary policy.

Bilbiie, Ghironi, and Melitz (2007) and Bergin and Corsetti (2008) study monetary policy in models with endogenous product entry and imperfect adjustment of product prices. Endogenous entry is a step towards realism as compared to exogenous entry. However, the formulation of imperfect price-adjustment in both papers implies a collapsed cross-section distribution of prices such that product prices coincide in equilibrium. A collapsed price distribution precludes any MB which originates a non-representative sample of price changes. In the model with exogenous entry, I derive a non-collapsed cross-section distribution of prices and show how important moments of this distribution evolve over time. Weber (2009) analyzes how endogenous firm entry affects the cross-section distribution of prices and monetary policy.

Shapiro and Wilcox (1996), Boskin, Dulberger, Gordon, Griliches, and Jorgenson (1997), Lebow and Rudd (2003), Gordon (2006), and Lebow and Rudd (2006) review and add to the empirical literature on MB in inflation but focus on constant biases.

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Broda and Weinstein (2007) argue that it is the high quality of new products which drives the MB they estimate. They derive that new products tend to be of higher quality than established products. Since the CPI records new products with a delay it lacks low quality-adjusted prices and, thus, overstates the exact index. Their MB is pro-cyclical because product entry is pro-cyclical in their data. The endogenous MB considered here is independent of product quality and pro-cyclical product entry.

Whereas I argue that it is measured CPI inflation which underestimates the number of price changes, Nakamura and Steinsson (2008b) quantify the same bias in import and export prices empirically. Orphanides (2001) shows that real-time macro data undergo major revisions over time until all statistical information has arrived. In contrast, macro data remain contaminated by the endogenous MB considered in this paper even after data are revised finally.

The paper is structured as follows. I set out the model in section 2, analyze MB in section 3, and describe the calibration of parameters in section 4. Section 5 considers the monetary-policy problem given endogenous MB, and section 6 compares the monetary-policy effects of endogenous MB with those of white-noise exogenous measurement error. Section 7 contains three applications of the model to quantify MB and its effects in U.S. data, and section 8 concludes.

2 Model

I consider a minimal setup with exogenous product entry and exit to convey implications of endogenous MB. A special case of the model is the standard New-Keynesian model without product entry and exit in Woodford (2003) and Galí (2008).

2.1 Household

The representative household maximizes expected discounted lifetime utility,

$$\max_{\{C_t(j),B_t,L_t\}_{t=0}^{\infty}} E_0 \sum_{t=0}^{\infty} \beta^t \left[u(C_t, \xi_t) - h(L_t) \right] , \quad \beta \in (0,1) . \tag{1}$$

Let C_t denote composite consumption at time t, L_t labor hours and E_0 the expectation operator conditional on information at time zero. The function $u(C_t, \xi_t)$ is twice continuously differentiable, increasing in C_t, and concave for each value of the mean-zero preference shock ξ_t. The function $h(L_t)$ is twice continuously differentiable, increasing in L_t, and convex. Utility maximization is subject to the budget constraint,

$$\int_0^1 P_t(j)C_t(j) \; dj + B_t = (1 + i_{t-1})B_{t-1} + W_tL_t + D_t \; . \tag{2}$$

The household consumes quantity $C_t(j)$ of product j at price $P_t(j)$. It receives nominal returns from past bond holdings $(1 + i_{t-1})B_{t-1}$, nominal labor income W_tL_t and nominal dividends D_t from firm ownership. W_t and i_t denote the nominal wage rate and the nominal interest rate, respectively. The household bundles all products j to composite consumption as in Dixit and Stiglitz (1977),

$$C_t = \left(\int_0^1 C_t(j)^{\frac{\theta-1}{\theta}} \; dj \right)^{\frac{\theta}{\theta-1}} , \quad \theta > 1 \; . \tag{3}$$

The parameter θ denotes the elasticity of substitution between any two products. The utility-based or true price level P_t is the minimum expenditure to buy one unit of C_t, $P_t^{1-\theta} = \int_0^1 P_t(j)^{1-\theta} dj$, and rearranges to

$$P_t = \int_0^1 \frac{C_t(j)}{C_t} \; P_t(j) \; dj \tag{4}$$

after substituting household demand for product j, $C_t(j)/C_t = (P_t(j)/P_t)^{-\theta}$. True inflation is the change in the true price level, $\pi_t = P_t/P_{t-1}$. Appendix A.1 collects the conditions which ensure optimal intertemporal household choices.

2.2 Firms

Measuring the price level P_t is a trivial task if all products sell at an identical price. In this case, observing the price of any single product reveals the aggregate price level. Identical product prices is the assumption adopted by related papers. In practice, price

level measurement is a challenging task because the cross-section distribution of prices does not collapse to a single price. I add a non-collapsed price distribution to the model by assuming that price adjustment is a variant of the mechanism in Calvo (1983). When a firm enters the market it sets a price for its product. In subsequent periods, the firm adjusts its price with probability $(1 - \alpha)$, $\alpha \in [0, 1)$, in each period until exit.

Firm entry and exit is exogenous. Each period, there is a unit mass of firms. Firms are indexed by $j \in [0, 1]$. At the beginning of each period, $\delta \in [0, 1)$ new firms enter the economy. At the end of each period, a fraction δ of firms exits the market. Exiting firms are selected randomly. Accordingly, the unit interval comprises δ new firms and $(1 - \delta)$ established firms in each period. The entry and exit setup is a special case of the setup in Bilbiie, Ghironi, and Melitz (2006).

Firm j produces quantity $Y_t(j)$ with technology $Y_t(j) = A_t f(L_t(j))$ and labor input $L_t(j)$. The function $f(.)$ is increasing and concave. Productivity $A_t > 0$ is an exogenous stochastic process with mean $\bar{A} = 1$. The firm assembles only one product such that the index j also identifies the product of the firm. The firm hires labor on a competitive factor market and sells its product on a monopolistically competitive product market.

When a firm (either new or established) sets its price, the firm solves

$$\max_{P_t(j)} E_t \sum_{s=t}^{\infty} (\kappa\beta)^{s-t} \, \Omega_{s,t} \left[P_t(j)Y_s(j) - W_s f^{-1} \left(\frac{Y_s(j)}{A_t} \right) \right] , \tag{5}$$

subject to household demand $Y_t(j)/Y_t = (P_t(j)/P_t)^{-\theta}$. The parameter $\kappa = \alpha(1 - \delta)$ is the probability to survive into the next period and to produce at old price. Therefore, the higher is the probability of exit δ, the higher is the effective rate at which the firm discounts future profits. The variable $\Omega_{s,t} = u_c(C_s, \xi_s)P_t/(u_c(C_t, \xi_t)P_s)$ denotes the stochastic household discount-factor for nominal payoffs. The solution of the maximization problem (5), stated in appendix A.2, reveals that all (re)optimizing firms set the same price $P_t^{\star}$.

2.3 Statistical Agency

Inflation as measured by the statistical agency is the change in expenditure required to buy a particular basket of products. The basket lacks new products because the statistical agency measures product entry with a delay. Measured inflation corresponds to Laspeyre's index which weights current and past prices of products in the basket by constant quantities. Let measured inflation equal the ratio of measured price levels, $\pi_t^m = P_{t,t}^m / P_{t-1,t}^m$, with

$$P_{t,t}^m = \int_{\mathcal{N}(t,\ell)} P_t(j) Q(j) \, dj \quad , \quad P_{t-1,t}^m = \int_{\mathcal{N}(t,\ell)} P_{t-1}(j) Q(j) \, dj \, . \tag{6}$$

Price $P_t(j)$ is weighted by the constant quantity $Q(j)$. The first subscript of $P_{t,t}^m$ refers to the dating of prices whereas the second subscript indicates the definition of the basket $\mathcal{N}(t,\ell)$. The basket comprises all products with a sufficiently long lifetime $\ell \geq 1$,

$$\mathcal{N}(t,\ell) = \{ \text{ all products } j \text{ available at time } t \text{ with a lifetime greater than } \ell \, \} \, .$$

Within each period, the statistical agency revises $\mathcal{N}(t,\ell)$ to prevent the coverage of the basket from shrinking over time. The revision involves phasing in newer products and phasing out discontinued ones. Whereas a product is phased in $\ell + 1$ periods after market entry, a discontinued product is phased out in the period after exit because its price is no longer observed.[2]

Between periods, the basket remains constant. Accordingly, the basket contains prices of the same product in two consecutive periods, and the statistical agency can compute product-specific inflation rates. Put differently, measured inflation is based on a matched-model index in which all price comparisons are for the same products.

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[2]Within-period revision of the basket implies that two estimates of the price level $P_{t,t}^m$ and $P_{t,t+1}^m$ exist at time t. The first estimate refers to the basket at time t, whereas the second estimate refers to the basket at time $t+1$. Numerically, the difference between both estimates is small. The standard deviation of the price-level revision equals $\text{std}(\hat{P}_{t,t}^m - \hat{P}_{t,t+1}^m) = \alpha^\ell \delta / (1 - \alpha(1-\delta)) \, \text{std}(\hat{\pi}_t)$ up to first order and the calibration adopted below implies $\alpha^\ell \delta / (1 - \alpha(1-\delta)) = 0.0172$. In practice, revisions of the CPI are minor, too.

Hence, measured inflation can be rearranged to

$$\pi_t^m \;=\; \int_{\mathcal{N}(t,\ell)} w_{t-1,t}^m(j)\pi_t(j)\;dj \;, \tag{7}$$

where $\pi_t(j) = P_t(j)/P_{t-1}(j)$ is product-specific inflation. Measured weights

$$w_{t-1,t}^m(j) = \frac{P_{t-1}(j)Q(j)}{\int_{\mathcal{N}(t,\ell)} P_{t-1}(j)Q(j)\;dj} \quad,\quad 1 = \int_{\mathcal{N}(t,\ell)} w_{t-1,t}^m(j)\;dj \;, \tag{8}$$

denote measured expenditure spent on product j and integrate to unity.

In practice, statistical agencies continuously revise the CPI basket similar to the period-by-period revision in the model. Sample rotation is the main method of the BLS to maintain coverage of the U.S. CPI basket. In every year, the CPI basket is revised for 20% to 25% of the geographic area by attaching positive instead of zero weight to prices of newer products such as to better reflect recent developments in consumer expenditure in that particular area. The entire basket rotates once every four to five years (Armknecht, Lane, and Stewart (1997)).[3]

However, updated weights are usually outdated by one, two, or more years at the time when they are employed in the CPI (Lane (2000)). One reason for delay is that sample rotation implies delayed recording of new products in those parts of the basket which have not been updated. Further delays arise from the need to estimate weights of new products by consumer-expenditure surveys; from averaging expenditure data over several periods to mitigate seasonality and idiosyncratic shocks; or, more generally, from limited funds and resources to track market trends more closely.[4] In the model, the parameter which captures the time lag between market launch and basket introduction of a new product is ℓ.

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[3]In addition, the CPI basket is updated for new products in base periods but such periods are infrequent events. In base periods, weights attached to product prices are revised. Historically, several years pass between base periods. For instance, the U.S. historical record of base-period revisions is 1940, 1953, 1964, 1978, 1987 and 1998. Moreover, until 1998 weights were usually outdated by roughly three years when they entered the base period revision. Starting with 2002 weights are updated biannually (table 1 in BLS (1997), chapter 17).

[4]New products could enter the CPI independent from sample rotation when a discontinued product must be replaced. However, Lane (2000) argues that, in practice, a discontinued product is replaced by a product almost as obsolete as the discontinued product to avoid subjective quality adjustment. CPI procedures instruct data collectors to select a product for replacement that is most comparable to the discontinued product. Such practice tends to keep new products out of the CPI.

An important feature of BLS's sample-rotation method is overlap pricing which means that the BLS collects prices for both the updated and the outdated sample in the month in which the outdated sample is replaced (Armknecht, Lane, and Stewart (1997)). In that month, CPI inflation records price changes for products in the outdated sample. In the subsequent month, however, CPI inflation records price changes for products in the updated sample. Accordingly, when the BLS computes CPI inflation it never compares current prices of updated products to past prices of outdated products. Measured inflation in the model is consistent with overlap pricing because measured inflation also matches product-models.

In setting up the model, I have assumed that firms and households know true aggregate variables whereas the statistical authority, and hence the central bank, does not know these variables. Even though this assumption is often adopted in the literature on monetary policy under imperfect information, the assumption is not realistic in that it ignores information deficiencies on the part of firms and households. I nevertheless maintain this assumption because it is useful to isolate the effects of MB, which emerge when monetary policy responds to mismeasured variables, from the effects of MB, which emerge when firms and households base their decisions on mismeasured variables.

3   Analyzing Measurement Bias

I define MB as measured inflation divided by true inflation. True inflation is a natural benchmark to judge the accuracy of measured inflation because optimal monetary policy in microfounded models suggests that true inflation follows a particular path. MB as defined here is the link which allows to extrapolate this path to measured inflation. Also, my definition of MB is compatible with the definition adopted by the empirical literature on constant MB.

3.1   Conservative Inflation

To understand MB, it is useful to introduce the concept of conservative inflation. Consider a hypothetical household which is conservative in that it consumes established products only. More precisely, conservative consumption $C_{t,t}^n$ corresponds to composite (3) but is defined over the subset $\mathcal{N}(t, \ell)$ of products rather than over all products. The conservative household minimizes costs and obtains the price of $C_{t,t}^n$ as

$$P_{t,t}^n = \left(\int_{\mathcal{N}(t,\ell)} P_t(j)^{1-\theta} \, dj \right)^{\frac{1}{1-\theta}} . \tag{9}$$

The first subscript of $P_{t,t}^n$ refers to product prices whereas the second subscript indicates the definition of $\mathcal{N}(t, \ell)$. Conservative product demand is $C_{t,t}^n(j)/C_{t,t}^n = (P_t(j)/P_{t,t}^n)^{-\theta}$.

Parallel to measured inflation, I define conservative inflation as $\pi_t^n = P_{t,t}^n/P_{t-1,t}^n$ and rearrange it as

$$\pi_t^n = \left(\int_{\mathcal{N}(t,\ell)} w_{t-1,t}^n(j) \, \pi_t(j)^{1-\theta} \, dj \right)^{\frac{1}{1-\theta}} . \tag{10}$$

Weights $w_{t-1,t}^n(j)$ equal relative conservative expenditure and integrate to unity,

$$w_{t-1,t}^n(j) = \frac{P_{t-1}(j) \, C_{t-1,t}^n(j)}{P_{t-1,t}^n C_{t-1,t}^n} \quad , \quad 1 = \int_{\mathcal{N}(t,\ell)} w_{t-1,t}^n(j) \, dj . \tag{11}$$

3.2   Partitioning Measurement Bias

I denote MB as $\mathcal{B}_t = \pi_t^m/\pi_t$ and augment it with conservative inflation,

$$\mathcal{B}_t = [\pi_t^m/\pi_t^n] \times \{\pi_t^n/\pi_t\} = [\mathcal{B}_t^{sub}] \times \{\mathcal{B}_t^{new}\} . \tag{12}$$

MB $\mathcal{B}_t$ partitions into substitution bias $\mathcal{B}_t^{sub}$ and new-product bias $\mathcal{B}_t^{new}$.

The substitution bias equals measured inflation divided by conservative inflation. Both inflation rates record price changes for products in $\mathcal{N}(t, \ell)$ only. When relative product prices change conservative inflation accounts for the fact that a household, which minimizes costs, substitutes cheap products for expensive ones.[5] Measured in-

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[5] To see the point formally rearrange $P_{t,t}^n$ as $P_{t,t}^n = \int_{\mathcal{N}(t,\ell)} (P_t(j)/P_{t,t}^n)^{-\theta} \, P_t(j) \, dj$. When $P_t(j)$ is

flation cannot capture such substitutions because quantities attached to prices remain fixed. Accordingly, the substitution bias arises because measured inflation overstates conservative inflation.

The new-product bias equals conservative inflation divided by true inflation. Both inflation rates capture substitution into cheap products when relative prices change. However, whereas conservative inflation records price changes of products in $\mathcal{N}(t, \ell)$, true inflation records price changes of all products in the market. Accordingly, the new-product bias arises because the statistical agency tracks a nonrepresentative sample of products.

I approximate MB in equation (12) up to first-order around a steady-state in which firms set flexible prices and true inflation is equal to unity. The optimality condition to the firm's problem (5) implies that all firms set the same price when prices are flexible. Symmetry in prices jointly with the assumption of homogenous quantities $Q(j) = 1/(1-\delta)^{\ell}$ implies a steady-state free of MB such that all inflation rates coincide in steady-state. Accordingly, the model abstracts from the constant MB in inflation that has been studied extensively in the empirical literature. It is straightforward to show existence and uniqueness of the steady-state. In what follows, a variable with a hat $\hat{}$ indicates percentage deviation from steady-state.

Substitution Bias

Proposition 1: The substitution bias defined in equation (12) is zero up to first order.

Proof: See appendix B.1. The first-order approximation of π_t^m in equation (7) uses the fact that, for each j in $\mathcal{N}(t, \ell)$, the term $w_{t-1,t}^m(j)\pi_t(j)$ is approximately equal to $\hat{w}_{t-1,t}^m(j) + \hat{\pi}_t(j)$ times a constant. The integral over weights $\hat{w}_{t-1,t}^m(j)$ is equal to zero because the integral over weights $w_{t-1,t}^m(j)$ (in levels) equals unity. Therefore, weights do not matter for π_t^m up to first order. The same argument applies to π_t^n in equation (10) such that the difference between the weights in measured and in conservative

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larger than $P_{t,t}^n$, the relative quantity of product j, $C_t(j)/C_{t,t}^n = (P_t(j)/P_{t,t}^n)^{-\theta}$, falls.

inflation does not matter up to first order. The remaining difference between π_t^m and π_t^n is curvature which is suppressed by the first-order approximation.

Related to the result here, Hausman (2003) demonstrates the second-order character of the substitution bias in a two-period setup without new products. Proposition 1 implies that the new-product bias $\hat{\mathcal{B}}_t^{new}$ is equal to the total bias $\hat{\mathcal{B}}_t$ up to first order.

New-Product Bias

Before deriving the new-product bias I prove a result on the true price level.

Proposition 2: The true price level (4) has the following recursive representation,

$$P_t^{1-\theta} = (1-\kappa)(P_t^\star)^{1-\theta} + \kappa P_{t-1}^{1-\theta} , \tag{13}$$

denoting with $P_t^\star$ the optimal price at date t and $\kappa = \alpha(1-\delta)$.

For a proof I employ two relationships. First, the unit mass of firms is composed of infinitely many entry cohorts,

$$1 = \sum_{s=t}^{-\infty}(1-\delta)^{t-s}\delta . \tag{14}$$

Each cohort has size δ in the entry period but diminishes in size over time due to firm exit. The second relationship describes the average price of a particular cohort of firms. Let the integer $s \leq t$ be common to all firms which entered the market at time s. Under Calvo-pricing and at time t, product prices maintained by firms in cohort s follow a truncated geometric distribution with average price

$$\Lambda_t(s) = \begin{cases} (1-\alpha)\sum_{k=t}^{s+1}\alpha^{t-k}(P_k^\star)^{1-\theta} + \alpha^{t-s}(P_s^\star)^{1-\theta} & \text{if} \quad s < t \\ (P_t^\star)^{1-\theta} & \text{if} \quad s = t . \end{cases} \tag{15}$$

Here, the size of cohort s is normalized to unity.

The true price level is equal to the sum over the average prices $\Lambda_t(s)$ of all cohorts s after weighting each average price with the size of cohort s at time t. Thus, equations (14) and (15) deliver

$$P_t^{1-\theta} = \int_0^1 P_t(j)^{1-\theta}\, dj = \sum_{s=t}^{-\infty}(1-\delta)^{t-s}\delta\, \Lambda_t(s)\ .$$

which can be rearranged as $P_t^{1-\theta} = (1-\kappa)\sum_{i=0}^{\infty}\kappa^i(P_{t-i}^\star)^{1-\theta}$. The recursive representation of $P_t^{1-\theta}$ stated in the proposition follows straight.

Proposition 2 generalizes the result that the price level evolves recursively in the New-Keynesian model without entry and exit to the case of a New-Keynesian model with entry and exit. Entry and exit $\delta > 0$ implies that the weight $(1-\alpha(1-\delta))$ attached to the current optimal price $P_t^\star$ in the true price level increases.

Proposition 3: The new-product bias defined in equation (12) is a one-sided, finite-order, invertible linear filter of true inflation up to a first-order approximation,

$$\hat{\mathcal{B}}_t^{new} \;=\; (a(L)-1)\hat{\pi}_t\ . \tag{16}$$

L denotes the lag operator. The lag polynomial $a(L)$ is defined as

$$a(L) \;=\; \begin{cases} \frac{1-\alpha}{1-\alpha(1-\delta)}L^0 + \frac{(1-\alpha)\delta}{1-\alpha(1-\delta)}\sum_{s=1}^{\ell-1}(\alpha L)^s & \text{if}\quad \ell \geq 2 \\[2mm] \frac{1-\alpha}{1-\alpha(1-\delta)}L^0 & \text{if}\quad \ell = 1\ . \end{cases}$$

The sum of coefficients fulfills $a(1) = 1$ if $\alpha = 0$, $\delta = 0$, or both. If $0 < \alpha < 1$ and $0 < \delta < 1$ then $0 < a(1) < 1$.

Proof: See appendix B.2 and appendix B.4. The new-product bias is a weighted average of true inflation and lags of true inflation. In general, the bias varies over time despite the fact that product entry and exit is exogenous and time-invariant. The time lag ℓ at which the statistical agency measures new products determines the order of $a(L)$.

I mention two special cases of a new-product bias equal to zero before delving deeper into the properties of $a(L)$. First, without entry and exit ($\delta = 0$) delayed recording of new products is irrelevant. Second, if prices are flexible ($\alpha = 0$) all products sell at an identical price such that observing any product price is sufficient to infer the true price level.

3.3 Measured Inflation and True Inflation

To obtain a mapping between measured inflation and true inflation I linearize the definition of the new-product bias $\mathcal{B}_t^{new} = \pi_t^n / \pi_t$, substitute the first-order relationship $\hat{\pi}_t^n = \hat{\pi}_t^m$ from proposition 2, and obtain $\hat{\pi}_t^m = \hat{\mathcal{B}}_t^{new} + \hat{\pi}_t$. Finally, I substitute equation (16).

Proposition 4: Up to first order, measured inflation is a one-sided, finite-order, invertible linear filter of true inflation,

$$\hat{\pi}_t^m = a(L)\hat{\pi}_t \ . \tag{17}$$

I illustrate the mapping between measured inflation and true inflation by partitioning true (measured) inflation into the fraction of price spells, which terminate in period t, times a weighted average over all terminated price spells with different durations s,

$$\hat{\pi}_t = (1 - \kappa) \left[(1 - \kappa) \sum_{s=1}^{\infty} \kappa^{s-1} (\hat{P}_t^\star - \hat{P}_{t-s}^\star) \right] ,$$

$$\hat{\pi}_t^m = (1 - \alpha) \left[(1 - \alpha) \sum_{s=1}^{\ell-1} \alpha^{s-1} (\hat{P}_t^\star - \hat{P}_{t-s}^\star) + (\alpha/\kappa)^{\ell-1} (1 - \kappa) \sum_{s=\ell}^{\infty} \kappa^{s-1} (\hat{P}_t^\star - \hat{P}_{t-s}^\star) \right] . \tag{18}$$

True inflation $\hat{\pi}_t$ records a total of $(1 - \kappa)$ terminated price spells plus κ ongoing spells which do not contribute to true inflation. The term in square brackets states that $(1 - \kappa)$ terminated spells last one period, $(1 - \kappa)\kappa$ last two periods, and so on. These

coefficients denote relative frequencies and add up to unity.[6]

Partitions (18) differ in two respects. First, measured inflation underestimates the total number of terminated spells indicating $(1 - \alpha)$ instead of $(1 - \kappa)$ terminations. To understand this difference, compare the cross-section distributions of prices of two particular product cohorts. The first cohort constitutes all products which left the market at time $t - 1$. Its price distribution replicates the market price-distribution at time $t - 1$ because exiting products are drawn randomly. The second cohort constitutes all products which are new at time t. The price distribution of this cohort collapses at the optimal price. Because many different prices (those of exiting products) switch to the same price (that of new products), true inflation records δ terminated price spells from product turnover.

In contrast, measured inflation aggregates product-specific inflation rates which compare prices of the same products. Accordingly, measured inflation ignores the price changes, which result from product turnover, and underestimates the total number of price changes. Empirically, Nakamura and Steinsson (2008b) show that the same bias occurs in import and export price-indices which link in new products on a matched-model basis.

[Figure 1 about here.]

The second respect in which partitions (18) differ is that measured inflation downplays the relative frequencies of price spells with short durations. The top panel of figure 1 plots relative frequencies of price spells with durations s (the coefficients in square brackets of (18)) for both true and measured inflation. Evidently, measured inflation shifts weight to price spells with long durations when compared to true inflation. The reason is that entry truncates the price spells of new products. Accordingly, when the statistical agency misses new products it overemphasizes price spells with long durations.

---

[6]All price spells with a duration s display the same price change $\hat{P}_t^\star - \hat{P}_{t-s}^\star$ because firms adjust to a single price. In practice, price spells with the same duration and the same termination date need not display the same price change. The partition of measured inflation applies for $\ell \geq 2$. If instead $\ell = 1$ the first sum in measured inflation disappears and $\hat{\pi}_t^m \propto \hat{\pi}_t$.

Naturally, the filter $a(L)$ reflects both differences among inflation rates. True inflation affects measured inflation less than one-to-one (recall that $a(1) < 1$) because measured inflation records less terminated price spells. Yet, $a(L)$ attaches positive coefficients to past true inflation rates because measured inflation overemphasizes price spells with long durations.

The spectrum of measured inflation, S_{π^m}, and the spectrum of true inflation, S_π, are related according to

$$S_{\pi^m}(\varpi) = a(e^{-i\varpi})a(e^{i\varpi}) \ S_\pi(\varpi) \ ,$$

because $a(L)$ is invertible (Hamilton (1994), chapter 6). Here ϖ denotes the frequency and $i = \sqrt{-1}$. The bottom panel of figure 1 plots the ratio of spectra $S_{\pi^m}/S_\pi = a(.)a(.)$ across frequencies. MB emphasizes low frequencies in true inflation. For quarterly data horizons longer than roughly three years receive more weight. Furthermore, MB dampens the variation of measured inflation across all frequencies because the ratio of spectra remains below unity. Taken together, MB increases inflation persistence and dampens inflation volatility. Accordingly, measured inflation is more persistent but less volatile than true inflation.

3.4 True Output, Measured Output, and the Output Gap

According to Bils (2004), MB in U.S. CPI inflation ends up as opposite bias in real output growth because real growth is estimated as nominal expenditure growth minus inflation. Eldridge (1999) explains that the Bureau of Economic Analysis (BEA) mainly utilizes consumer-prices, producer-prices and export/import-prices to deflate nominal output growth. Eldridge estimates the relative importance of each index with 49.7% for consumer prices, 11.8% for producer prices and 8.5% (-9.9)% for export (import) prices. Eldridge further points out that BEA primarily uses the CPI to deflate personal consumption expenditure.

Analog to the BEA approach, I define measured output as nominal expenditure

deflated by the measured price level, $Y_t^m = P_t Y_t / P_{t,t}^m$. Here Y_t denotes true real output which bundles all products $j \in [0, 1]$ according to a composite identical to equation (3). The true GDP deflator equals the true price level because output equals consumption. Up to first order, $\hat{Y}_t^m - \hat{Y}_t = -(\hat{P}_{t,t}^m - \hat{P}_t)$, and MB in the price level triggers the opposite MB in measured output.

Proposition 5: Up to a first-order approximation, the bias between the true price level and the measured price level is a one-sided, finite-order, invertible linear filter of true inflation,

$$\hat{P}_t - \hat{P}_{t,t}^m = b(L)\hat{\pi}_t , \quad b(L) = \tfrac{\alpha\delta}{1-\alpha(1-\delta)} \sum_{s=0}^{\ell-1} (\alpha L)^s .$$

Proof: See appendix B.3 and appendix B.4. As for inflation, price levels differ because the measured price level ignores price dynamics from product turnover and overemphasizes prices with long durations.

From the proposition, measured output can be rewritten as

$$\hat{Y}_t^m = \hat{Y}_t + b(L)\hat{\pi}_t . \tag{19}$$

I convert the mapping between output levels into a mapping between output gaps to analyze monetary policy below. Denote the output gap $x_t = \hat{Y}_t - \hat{Y}_t^{na}$ as the difference between output under sticky prices and natural output Y_t^{na} under flexible prices. It should be recalled that if prices are flexible any product price equals the true price level. Hence, delayed recording of new products does not trigger any price-level bias under flexible prices, and $\hat{Y}_t^{na}$ is free of MB. Subtracting $\hat{Y}_t^{na}$ from equation (19), the measured output gap $x_t^m = \hat{Y}_t^m - \hat{Y}_t^{na}$ turns out to be a biased variant of x_t,

$$x_t^m = x_t + b(L)\hat{\pi}_t . \tag{20}$$

Orphanides (2001) shows that flash estimates of macro data undergo major revisions

due to removal of noise or arrival of news. Data revisions then trigger revisions of output-gap nowcasts with implications for monetary policy. In contrast, the MB in x_t^m does not originate from data revisions. Rather, it originates from the nonrepresentative sample of products contained in the measured price level, and surfaces in measures of real activity because such measures derive from nominal expenditure deflated by measured prices. Hence, x_t^m remains biased even after data are revised finally.

4   Equilibrium and Parametrization

In equilibrium, the statistical agency compiles measured inflation, the representative household maximizes lifetime utility (1) subject to the budget constraint (2) and composite (3), firms set prices according to equation (36), product markets clear $C_t(j) = Y_t(j)$, the labor market clears $L_t = \int_0^1 L_t(j) \, dj$, the bond market clears $B_t = 0$, and the government conducts monetary policy specified below.

Aggregate supply relates inflation to inflation expectations and the output gap,

$$\hat{\pi}_t = \beta E_t \hat{\pi}_{t+1} + \phi x_t + u_t , \quad \phi = \frac{[1-\alpha(1-\delta)\beta][1-\alpha(1-\delta)]}{\alpha(1-\delta)}\zeta , \quad \zeta = \frac{\omega+\sigma^{-1}}{1+\theta\omega_p} . \tag{21}$$

See appendix A.2 for derivation. I introduce the u_t shock which reflects variation in inflation triggered by neither inflation expectations nor the output gap. Natural output $\hat{Y}_t^{na}$ is equal to the flexible-price output absent u_t shocks. Parameters in ζ are defined in appendix A.2.

The slope ϕ increases in δ. When the rate of firm entry and exit δ is high, inflation reacts more sensitive to the output gap, which is proportional to marginal costs, because more firms set prices as a function of current marginal cost. Furthermore, the slope ϕ decreases in α. A higher value of α implies that firms look further into the future. Hence, firms weigh current marginal costs less when setting their price.[7]

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[7]To highlight implications of firm entry and exit for the duration of price contracts consistent with a particular estimate of ϕ, totally differentiate

$$d\phi = \frac{\partial \phi}{\partial \delta} d\delta + \frac{\partial \phi}{\partial \alpha} d\alpha = 0 \quad \text{or} \quad \frac{d\alpha}{d\delta} = -\frac{\partial \phi}{\partial \delta} \Big/ \frac{\partial \phi}{\partial \alpha} = \frac{\alpha}{1-\delta} \geq 0 .$$

A high entry and exit rate requires a large uncensored price duration $(1 - \alpha)^{-1}$ because entry and exit

Aggregate demand relates the output gap to its expectation and the gap between the ex-ante real interest rate and the natural real rate of interest,

$$x_t = E_t x_{t+1} - \sigma(\hat{i}_t - E_t \hat{\pi}_{t+1} - \hat{r}_t^{na}) . \tag{22}$$

The natural real rate $\hat{r}_t^{na} = -\sigma^{-1} E_t (1 - L^{-1})(\hat{Y}_t^{na} - g_t)$ equals the flexible-price real interest rate absent u_t shocks. The parameter $\sigma = -\frac{u_c}{\bar{Y} u_{cc}} > 0$ governs the intertemporal elasticity of substitution. The term u_c is the marginal utility of consumption in steady-state, $\bar{Y}$ is steady-state output, and $g_t = -\frac{u_{c\xi}}{\bar{Y} u_{cc}} \hat{\xi}_t$ represents a shock to the marginal utility of consumption.

The linear model comprises the two measurement equations (17) and (20), aggregate supply and demand (21) and (22), two exogenous processes $\hat{r}_t^{na}$ and u_t, and initial conditions for true inflation. I solve for model dynamics by the numerical method of Sims (2002). I resort to a numerical solution because reasonable choices of ℓ create a sizeable state vector. For all the monetary policies considered below equilibrium is determinate and unique.

I calibrate the model to quarterly data and set the time lag ℓ to twelve quarters taking into account that the U.S. CPI basket rotates once every four to five years, and that further delays emerge in the process of obtaining the weights attached to prices. The product entry and exit rate δ is set to 0.0625 which corresponds to 25% of product turnover each year. The number is taken from Broda and Weinstein (2007) who report a median entry rate (number of new products relative to all products) of 25% per year for consumer products. As robustness check, I also report main numerical results for a value of δ equal to 0.03125.[8]

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provides an additional reason for price contracts to terminate. To offset the additional terminations α must increase. Bils and Klenow (2004) find that, for a large array of consumer products, the rate of product turnover predicts more frequent price changes. The model replicates this finding in that the mean censored price duration $(1 - \alpha(1 - \delta))^{-1}$ falls with product turnover δ.

[8]Midrigan (2007) argues that measuring the extensive margin by the entry rate overestimates its importance because new products tend to have small market value. Broda and Weinstein (2007) report that the value of new products relative to the value of all products is 9% such that my benchmark value of δ seems high. However, the benchmark value seems low along another dimension. In the model, the statistical agency samples $(1 - \delta)^\ell$ or 46% of all products. The evidence in Broda and Weinstein (2007) suggests that this number significantly overstates the fraction of products actually sampled by the BLS.

The probability α of not adjusting the price is set to 0.8 which implies an uncensored mean price-duration of five quarters. Five quarters are long compared to micro evidence in Nakamura and Steinsson (2008a), but short when compared to estimates from medium-scale DSGE models such as Smets and Wouters (2003). Jointly, α and δ imply a censored mean price-duration $(1 - \alpha(1 - \delta))^{-1}$ of four quarters, close to micro evidence.

A subjective discount rate β equal to 0.99 produces a steady-state interest rate of about three percent per year. The intertemporal elasticity of substitution σ is set to unity which corresponds to logarithmic utility of consumption. I set the steady-state markup of firms to 25% having θ equal to 5. Similar to Giannoni and Woodford (2005), ω_w is set to 0.3 and ω_p equals 0.5. Both numbers are consistent with a Cobb-Douglas technology with labor coefficient of 2/3 and a labor supply elasticity with respect to the real wage ν equal to 0.2. Slope ϕ then equals 0.044 in line with estimates in Linde (2005) and Altig, Christiano, Eichenbaum, and Linde (2005) for U.S. data.

The productivity shock $a_t = \hat{A}_t$ is AR(1) with AR-coefficient equal to 0.95. Preference shock g_t and inflation shock u_t are AR(1) with AR-coefficient 0.475.[9] Standard deviations of residuals of shocks are $0.672\%, 0.551\%$, and 0.514% for the a_t, g_t, and u_t shock, respectively. The monetary-policy shock μ_t introduced below is white noise with standard deviation 0.12%. I obtain standard deviations of shock residuals from the unconditional covariance matrix estimated for U.S. data in section 7.2.

5 Monetary Policy and Mismeasured Inflation

What are the effects of MB for monetary policy? More precisely, what happens to welfare if a central bank applies policies, which are designed for variables measured without bias, to variables that are measured with bias? To assess the effects of MB I assume that the central bank does not know true inflation and the true output gap. This

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[9]The productivity shock is more persistent than the inflation shock in line with estimates from DSGE models and to overcome observational equivalence of the two shocks conditional on observing $\hat{i}_t$, $\hat{\pi}_t$, and $\hat{Y}_t$. Equivalence of both shocks emerges because the model rearranges to $\hat{\pi}_t = \beta E_t \hat{\pi}_{t+1} + \phi \hat{Y}_t + u_t - \frac{1+\omega}{\omega + \sigma^{-1}} a_t$ and $\hat{Y}_t = E_t \hat{Y}_{t+1} - (\hat{i}_t - E_t \hat{\pi}_{t+1})$ under my calibration and ignoring g_t shocks.

assumption amounts to assuming that the central bank does not know the mapping from measured to true inflation. This assumption is reasonable because uncertainty about the model and its parameters make the mapping uncertain in practice.[10]

I find that, across monetary-policy regimes, policies designed for true variables but applied to measured variables imply insufficient inflation stabilization and exhibit unintended history dependence. Welfare effects depend on the policy regime. Welfare deteriorates for optimal monetary policy under commitment. Welfare improves for optimal monetary policy under discretion, and for a central bank that commits to an estimated interest-rate rule. Thus, MB works to the benefit of the central bank if the central bank does not commit to fully optimal monetary policy.

To derive these results, I compare two economies which differ only with respect to monetary policy. Whereas the central bank in Economy I applies a policy rule to true variables, the central bank in Economy II applies the same policy rule to measured variables. The welfare loss in Economy I is

$$\mathcal{L} = E_0 \sum_{t=0}^{\infty} \beta^t [\hat{\pi}_t^2 + \lambda x_t^2] . \tag{23}$$

The parameter $\lambda > 0$ denotes the weight attached to stabilizing the true output gap. I set $\lambda = \phi/\theta$ which corresponds to the weight of the output gap in the utility-based loss function.[11] For my calibration λ equals 0.0088. The welfare loss in Economy II is

$$\underline{\mathcal{L}} = E_0 \sum_{t=0}^{\infty} \beta^t [\underline{\hat{\pi}}_t^2 + \lambda \underline{x}_t^2] . \tag{24}$$

I define underlined variables as variables pertaining to Economy II. To assess the effects of MB, it is useful to evaluate the loss in Economy II relative to the loss in Economy I, $\mathcal{L}_R = (\underline{\mathcal{L}} - \mathcal{L})/\mathcal{L}$, for different values of the entry and exit rate δ. With a zero entry

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[10]If the central bank knows the mapping from measured to true inflation, the central bank easily maps measured inflation and the measured output gap into true inflation and the true output gap. Inverting $a(L)$ and applying it to measured inflation delivers true inflation, and equation (20) delivers the true output gap. With true variables in hand MB is irrelevant for monetary policy, and the optimal monetary-policy rules derived in Woodford (2003) or Galí (2008) apply.

[11]With $\lambda = \phi/\theta$ the loss $\mathcal{L}$ is proportional to household utility up to second order. The derivation of the utility-based loss is analog to the derivation in Woodford (2003), chapter 6, replacing α there by κ here and imposing homogenous labor.

and exit rate there is no MB such that losses in Economy I and II coincide and $\mathcal{L}_R = 0$. Raising δ above zero triggers MB in inflation and the output gap. Accordingly, monetary policies, and hence losses, in both economies diverge. I now assess the relative loss for three different monetary-policy regimes.

5.1 Optimal Commitment

When the central bank in Economy I can commit to future action, the policy rule

$$\hat{\pi}_t + \tfrac{\lambda}{\phi}(x_t - x_{t-1}) = 0 \tag{25}$$

minimizes the loss $\mathcal{L}$ in Economy I over $\{x_t, \hat{\pi}_t\}$ subject to equation (21). The central bank in Economy II applies the same policy rule to variables that are measured with bias,

$$\hat{\underline{\pi}}_t^m + \tfrac{\lambda}{\phi}(\underline{x}_t^m - \underline{x}_{t-1}^m) = 0 \ . \tag{26}$$

The left panel of figure 2 plots the relative loss $\mathcal{L}_R$ for increasing degree of MB and three different values of the time lag ℓ. For each value of ℓ, the relative loss increases in MB which indicates that welfare in Economy II is lower than welfare in Economy I.

[Figure 2 about here.]

MB affects the optimal policy rule along two dimensions, and both dimensions deteriorate welfare. To see this, rewrite the policy rule in Economy II in terms of true variables by substituting measurement equations (17) and (20),

$$
\begin{aligned}
\hat{\underline{\pi}}_t + \tfrac{\Lambda}{\phi}(\underline{x}_t - \underline{x}_{t-1}) - \tfrac{\Lambda}{\phi}b_0\hat{\underline{\pi}}_{t-1} = 0 \qquad &\text{if} \quad \ell = 1 \ , \\
\hat{\underline{\pi}}_t + \tfrac{\Lambda}{\phi}(\underline{x}_t - \underline{x}_{t-1}) + \sum_{i=1}^{\ell} \tilde{q}_i \hat{\underline{\pi}}_{t-i} = 0 \qquad &\text{if} \quad \ell \geq 2 \ .
\end{aligned}
\tag{27}
$$

I define the effective weight on stabilizing the output gap, $\Lambda = \lambda \left[a_0 + \tfrac{\lambda}{\phi}b_0\right]^{-1}$, as intended weight λ times the inverse of the factor $[a_0 + \tfrac{\lambda}{\phi}b_0]$. Parameters a_0 and b_0 denote the coefficients in $a(L)$ and $b(L)$ attached to L^0, respectively. The factor is less than

one if MB is present such that, in general, the effective weight Λ exceeds the intended weight λ. Moreover, I define $\tilde{q}_i = [a_i + \frac{\lambda}{\phi}(b_i - b_{i-1})]/[a_0 + \frac{\lambda}{\phi}b_0]$. Parameters a_i and b_i denote the coefficients in $a(L)$ and $b(L)$ attached to L^i, respectively, with $a_\ell = b_\ell = 0$.

The first effect of MB on policy is most apparent for a time lag ℓ equal to one quarter. In this case, measurement equations (17) and (20) reduce to $\hat{\underline{\pi}}_t^m = (1 - b_0)\hat{\underline{\pi}}_t$ and $\underline{x}_t^m = \underline{x}_t + b_0\hat{\underline{\pi}}_t$ having $a_0 + b_0 = 1$. The statistical agency wrongly attributes a fraction $b_0 > 0$ of true inflation to $\underline{x}_t^m$ rather than attributing it to $\underline{\pi}_t^m$. Accordingly, the central bank attaches the coefficient $\lambda/\phi < 1$ rather than unity to $b_0\hat{\underline{\pi}}_t$ when it targets measured instead of true variables. Thus, true inflation is stabilized insufficiently in the policy rule (27) because the effective weight Λ on stabilizing the output gap exceeds the intended weight λ.[12]

The second effect of MB is that targeting measured instead of true variables introduces additional states into the policy rule (27), namely the term $\frac{\Lambda}{\phi}b_0\hat{\underline{\pi}}_{t-1}$ when ℓ is equal to one quarter or the entire lag polynomial $\sum_{i=1}^{\ell} \tilde{q}_i\hat{\underline{\pi}}_{t-i}$ when ℓ exceeds one quarter. The additional history dependence interferes with the optimal history dependence ($\underline{x}_{t-1}$). The optimal history dependence ensures that increases in the price level are offset by lower inflation rates later on. Forward-looking firms anticipate the price-level reduction and increase their prices less, so that inflation increases less to start with. Therefore, the output gap falls less initially, and the tradeoff between stabilizing inflation or the output gap improves. The past true inflation rates that enter the policy rule via MB disturb the working of optimal history dependence, and deteriorate welfare.

5.2 Optimal Discretion

When the central bank in Economy I cannot commit to future action, the policy rule

$$\hat{\underline{\pi}}_t + \tfrac{\lambda}{\phi}x_t = 0 \tag{28}$$

---

[12]If $\lambda/\phi > 1$ true inflation is stabilized too much. MB does not distort stabilization outcomes if $\lambda = \phi$. Notably, policy rules which respond to nominal output growth overcome the MB considered here because MB wrongly subdivides nominal output growth into real output growth and inflation but leaves nominal output growth unaffected. Rudebusch (2002) assesses nominal output rules for monetary policy.

minimizes $\mathcal{L}$ over $\{x_t, \hat{\pi}_t\}$ subject to equation (21) and conditional on private-sector expectations. The central bank in Economy II applies the same policy rule to variables that are measured with bias,

$$\underline{\hat{\pi}}_t^m + \frac{\lambda}{\phi}\underline{x}_t^m = 0 \ . \tag{29}$$

In the middle panel of figure 2, the relative loss $\mathcal{L}_R$ increases in MB when the time lag of the statistical agency amounts to one quarter, as under commitment. Different from commitment, however, the relative loss falls when the time lag is equal to four quarters or one year. To understand this, again rewrite the policy rule in Economy II in terms of true variables using equations (17) and (20),

$$
\begin{aligned}
\hat{\underline{\pi}}_t + \frac{\Lambda}{\phi}\underline{x}_t &= 0 && \text{if} \quad \ell = 1 \ , \\
\hat{\underline{\pi}}_t + \frac{\Lambda}{\phi}\underline{x}_t + \sum_{i=1}^{\ell-1} q_i \hat{\underline{\pi}}_{t-i} &= 0 && \text{if} \quad \ell \geq 2 \ .
\end{aligned}
\tag{30}
$$

The effective weight Λ is as defined before, and $q_i = [a_i + \frac{\lambda}{\phi}b_i]/[a_0 + \frac{\lambda}{\phi}b_0]$. As before, a central bank that targets measured rather than true variables distorts the weight attached to stabilizing the output gap, and introduces a reference to past true inflation rates into the policy rule. The history dependence, which is absent in Economy I, occurs despite the fact that the central bank operates with discretion.[13]

When ℓ is equal to one quarter, the effective weight Λ implies insufficient inflation stabilization which triggers the welfare loss in Economy II relative to Economy I in figure 2. When ℓ exceeds one quarter, insufficient inflation stabilization still triggers welfare losses in Economy II. However, the history dependence in form of past true inflation rates affects inflation expectations and improves the tradeoff between stabilizing inflation or the output gap in Economy II. As evident from figure 2, the benefit of history dependence overcompensates the loss from insufficient inflation stabilization.[14]

---

[13]History dependence in equation (30) is displayed in terms of inflation. However, it is straightforward to display history dependence as $\underline{\pi}_t + \frac{\lambda}{\phi}g(L)\underline{x}_t = 0$ having $g(L) = [a(L) + \frac{\lambda}{\phi}b(L)]^{-1}$.

[14]In Nimark (2005), a discretionary central bank gets the tradeoff between stabilizing inflation or the output gap wrong when it has imperfect information. Pearlman (1992) shows that welfare may improve when a discretionary central bank has imperfect information because imperfect information reduces the costs from time inconsistency. Both papers stick to the assumptions that variables are

5.3 Interest-Rate Rule

The last policy regime I consider is the case when the central bank in Economy II pursues an estimated interest-rate rule with interest-rate smoothing,

$$\hat{\underline{i}}_t = 0.92\hat{\underline{i}}_{t-1} + (1 - 0.92)(1.5\hat{\underline{\pi}}_t^m + 0.625\hat{\underline{Y}}_t^m) + \mu_t \ . \tag{31}$$

The policy rule is of the same functional form as the one estimated in Orphanides (2001). Coefficients are OLS estimates (rounded) for quarterly U.S. data from 1984:1 to 2008:1, and μ_t is a zero-mean white-noise residual. For estimation, I equate $\hat{\underline{\pi}}_t^m$ to U.S. CPI inflation and $\hat{\underline{Y}}_t^m$ to U.S. real personal consumption expenditure (section 7.1 describes the data). Interest-rate smoothing is estimated to be high presumably because interest rates exhibit a pronounced downward trend in the sample. The central bank in Economy I pursues the same rule formulated in terms of true variables,

$$\hat{i}_t = 0.92\hat{i}_{t-1} + (1 - 0.92)(1.5\hat{\pi}_t + 0.625\hat{Y}_t) + \mu_t \ . \tag{32}$$

The right panel of figure 2 shows the relative loss that emerges when monetary policy pursues the estimated interest-rate rule. Qualitatively, the relative loss falls for realistic values of the time lag ℓ similar to the case of optimal discretion. Quantitatively, the relative loss is large in absolute terms when monetary policy commits to the estimated interest-rate rule.[15] Again, I illustrate the finding by rewriting the policy rule in Economy II in terms of true variables,

$$\hat{\underline{i}}_t = 0.92\hat{\underline{i}}_{t-1} + (1 - 0.92)([1.5a_0 + 0.625b_0]\hat{\underline{\pi}}_t + 0.625\hat{\underline{Y}}_t) + \mu_t \qquad \text{if} \quad \ell = 1 \ ,$$
$$\hat{\underline{i}}_t = 0.92\hat{\underline{i}}_{t-1} + (1 - 0.92)([1.5a(L) + 0.625b(L)]\hat{\underline{\pi}}_t + 0.625\hat{\underline{Y}}_t) + \mu_t \qquad \text{if} \quad \ell \geq 2 \ . \tag{33}$$

First, when ℓ is equal to one quarter the coefficient on true inflation is smaller in Econ-

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perturbed by white-noise exogenous error, and that the central bank engages into optimal filtering of exogenous error. In contrast, I assume that variables are contaminated by endogenous MB, and that the central bank disregards endogenous MB.

[15]To compute the relative loss, which now requires me to solve equations (21), (22) and the policy rule jointly, I set the variance-covariance matrix of shock residuals to the unconditional variance-covariance matrix of shock residuals that I estimate in section 7.2. Imposing a diagonal variance-covariance matrix of shock residuals amplifies $\mathcal{L}_R$.

omy II than in Economy I because $1.5a_0 + 0.625b_0 < 1.5$. Second, when ℓ exceeds unity lags of true inflation enter the policy rule because measured inflation and measured output depend on lags of true inflation.

To see how both differences map into welfare, figure 3 plots impulse responses to a positive productivity shock for ℓ equal to three years. Below, I find that productivity shocks are the main driver of the difference between variables in Economy II and I. The top row of the figure shows variables in Economy II whereas the bottom row shows variables in Economy I. Evidently, inflation and the output gap in Economy II react less to the shock. Thus, a monetary policy that weighs current true inflation less, but features lags of true inflation, delivers higher welfare.

Consider the dynamics in figure 3 in more detail. When productivity grows fast, optimizing firms set low prices and true inflation declines. By the lags of true inflation in the policy rule, the interest rate in Economy II is expected to remain below steady-state over a longer period of time than the interest rate in Economy I. By aggregate demand, the low expected interest rate drags output in Economy II above output in Economy I. By aggregate supply, higher output offsets some downward pressure on inflation such that true inflation in Economy II declines less than in Economy I. By the policy rule, the interest rate decline less in Economy II because of both inflation declining less and output increasing more. High sticky-price output also narrows the output gap. In sum, the lags of true inflation in the policy rule lead to greater stabilization of inflation and the output gap, and to higher welfare.[16]

In figure 3, the effects of the lags of true inflation overlay the effects of the smaller effective coefficient on current true inflation in the policy rule. The case $\ell = 1$ (not shown) isolates the effects of the smaller coefficient on true inflation in Economy II. In this case, monetary policy in Economy II boosts output less in response to low inflation such that true inflation is stabilized less. Moreover, the output gap widens

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[16]In order to assess whether welfare effects are large or small I compute the permanent reductions in consumption, which are equivalent to the loss in Economy I and to the loss in Economy II, and compare both. The household would sacrifice 0.0053 percent of steady-state consumption in Economy I to prevent inefficient variation of inflation and the output gap. The corresponding numbers in Economy II are 0.0030 percent. Thus, the benefit of a central bank that targets measured rather than true variables amounts to 0.0023 percent of steady-state consumption.

because output in Economy II remains below output in Economy I. Thus, a smaller effective coefficient on true inflation, when considered in isolation, leads to inflation and the output gap being stabilized less, and to lower welfare.

[Figure 3 about here.]

6   Measurement Bias versus Measurement Error

The literature recognizes that mismeasurement in inflation and output can be important for the conduct of monetary policy. For simplicity, the literature models the difference between true and measured variables as white-noise exogenous process.[17] I showed that, if we take seriously the notion that mismeasurement is due to mistakes made by the statistical agency, then the difference between true and measured variables is not a white-noise exogenous process. I refer to a white-noise exogenous process, which perturbs true variables, as measurement error (ME).

In this section, I show that assuming ME instead of MB matters for how disregarded mismeasurement affects monetary policy. That is, if a discretionary central bank applies its policy rule, which is designed for correctly-measured variables, to variables that are measured with white-noise exogenous error (as opposed to endogenous MB) then inflation is not stabilized insufficiently nor does the effective policy rule feature additional states. Furthermore, welfare always deteriorates in case of ME.

As before, I analyze the effects of ME by comparing two economies. The central bank in Economy I implements optimal discretionary policy $\hat{\pi}_t + \frac{\lambda}{\phi} x_t = 0$ which minimizes the loss (23). The central bank in Economy II applies the same policy rule to measured inflation and the measured output gap, $\hat{\pi}_t^{\xi} + \frac{\lambda}{\phi} x_t^{\xi} = 0$. Measured variables equal true variables plus error,

$$
\begin{aligned}
\hat{\pi}_t^{\xi} &= \tilde{\pi}_t + \xi_{\pi t} \ , \quad \xi_{\pi t} \sim (0, \sigma_\pi^2) \ , \\
x_t^{\xi} &= \tilde{x}_t + \xi_{xt} \ , \quad\ \ \xi_{xt} \sim (0, \sigma_x^2) \ .
\end{aligned}
\tag{34}
$$

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[17]Among others, Pearlman (1992), Aoki (2003), Svensson and Woodford (2003), Nimark (2005), Aoki (2006) study optimal monetary policy when monetary-policy indicators are perturbed by white-noise exogenous error.

Here, $\tilde{\pi}_t$ and $\tilde{x}_t$ denote true variables in Economy II, and $\xi_{\pi t}$ and ξ_{xt} are white-noise exogenous processes which are mutually independent and independent of u_t. The parameters which govern the degree of ME are $\sigma_\pi^2, \sigma_x^2 \geq 0$ such that the policy rule in Economy I obtains as special case $\sigma_\pi^2 = \sigma_x^2 = 0$ of the one in Economy II.

Substituting equations (34) into the policy rule in Economy II delivers

$$\tilde{\pi}_t + \tfrac{\lambda}{\phi}\tilde{x}_t = -(\xi_{\pi t} + \tfrac{\lambda}{\phi}\xi_{xt}) \ . \tag{35}$$

First, ME does not imply insufficient inflation stabilization. Rather, ME is converted into an unsystematic policy control-error. Second, no additional states enter the policy rule when the central bank targets variables measured with ME.

Analog to MB I evaluate the loss in the case of ME by $\mathcal{L}^\xi = E_0 \sum_{t=0}^\infty \beta^t[\tilde{\pi}_t^2 + \lambda\tilde{x}_t^2]$. The equilibrium loss as a function of σ_u^2, σ_π^2, and σ_x^2 rearranges to[18]

$$\mathcal{L}^\xi = \tfrac{\lambda+\phi^2}{\lambda}\left(\tfrac{f_\pi^2\sigma_u^2}{1-\rho_u^2}\right) + \tfrac{1}{\lambda+\phi^2}[\phi^2\sigma_\pi^2 + \lambda^2\sigma_x^2] \ .$$

Parameters ρ_u and σ_u denote the AR-coefficient of the u_t shock and the standard deviation of the shock residual, respectively. The first term on the right-hand side corresponds to the loss in Economy I having $\sigma_\pi^2 = \sigma_x^2 = 0$. The second right-hand term indicates the additional loss from ME in Economy II. The loss in Economy II increases in ME, and thus cannot undercut the loss in Economy I contrary to what happens for MB.

7   Applications

I quantify the effects of MB in historical U.S. data along three dimensions. I start with the correlation of inflation and output because business-cycle models are often evaluated by comparing the correlation of inflation and output in the model one-to-one

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[18]The system (35) and (21) can be easily solved by the method of undeterminate coefficients. The equilibrium paths of inflation and the output gap are $\tilde{\pi}_t = f_\pi u_t + \gamma_\pi\xi_{\pi t} + \gamma_x\xi_{xt}$ and $\tilde{x}_t = \tfrac{-\phi f_\pi}{\lambda}u_t + \tfrac{\gamma_\pi}{\phi}\xi_{\pi t} + \tfrac{\gamma_x}{\phi}\xi_{xt}$ with coefficients $f_\pi = \lambda/(\lambda(1-\beta\rho_u)+\phi^2)$, $\gamma_\pi = -\phi^2/(\lambda+\phi^2)$ and $\gamma_x = -\lambda\phi/(\lambda+\phi^2)$. In the limit $\beta \to 1$ the loss becomes $\mathcal{L}^\xi = \text{var}(\tilde{\pi}_t) + \lambda\text{var}(\tilde{x}_t)$, and equilibrium inflation and output gap serve to express the loss in terms of the variances of shock residuals.

to the correlation of inflation and output in the data (e.g. Smets and Wouters (2007)). I map inflation and output in the model to inflation and output in the data, and show that the correlation of inflation and output in the model is lower than the correlation of inflation and output in the data because of MB. Thus, comparing the correlation of inflation and output in the model to its correspondent in the data is appropriate only after accounting for endogenous MB.

7.1   Correlation of Inflation and Output

To compute correlations I equate measured inflation $\hat{\pi}_t^m$ with data on U.S. CPI inflation, and measured output $\hat{Y}_t^m$ with data on U.S. real personal consumption expenditure (PCE).[19] Then, I transform measured output and measured inflation into true output and true inflation, respectively, using equations (17) and (19). True inflation equals the inverse of $a(L)$ applied to measured inflation. True output equals measured output minus the MB in price levels $b(L)\hat{\pi}_t$. Neither transformation relies on assumptions about monetary policy.[20]

The top-left panel of figure 4 shows business-cycle correlations of true output and lags and leads of true inflation for the sample 1960:1 – 2008:1 (circled lines). The same panel contains correlations of measured output and measured inflation (crossed lines) jointly with a 95% confidence band (dashed lines).

[Figure 4 about here.]

A main finding is that the correlation of true inflation and true output is lower than the correlation of measured inflation and measured output at all leads and lags. Converting

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[19]I prefer PCE data to GDP data because, strictly speaking, the model of MB applies to private consumption rather than to government consumption or investment. Below, I confirm that the same conclusions emerge when I equate measured output in the model with GDP data.

[20]PCE is the Quantity Index Real Personal Consumption Expenditures. GDP is in billions of chained 2000 dollars. Both series are quarterly, seasonally adjusted, and obtained from BEA. I compute population as product of Employment Level (16 years and over, LNS12000000Q) times the inverse Employment-Population Ratio (LNS12300000Q). Both series are quarterly, seasonally adjusted, and obtained from BLS. I use population to compute PCE (GDP) per person and denote with $\hat{Y}_t^m$ ($\hat{Y}_{GDPt}^m$) the percentage deviation of log level PCE (GDP) from HP trend. Monthly data on the Fed Funds rate (FEDFUNDS, averages of daily figures) is from the Board of Governors of the Federal Reserve System. Monthly data on the CPI (U.S. city average, all items, CUSR0000SA0, seasonally adjusted) is from BLS. For monthly data, I use the first observation per quarter. Pre-sample values required to apply filters $a(L)^{-1}$ and $b(L)$ are set to zero.

measured output into true output amounts to subtracting $b(L)\hat{\pi}_t$. The transformation injects negative correlation of output and inflation.

As a first robustness check, the top-right panel of figure 4 shows correlations of inflation and output when I equate measured output in the model to GDP data instead of PCE data. I denote measured output by $\hat{Y}^m_{GDPt}$ in this case. Again, the correlation of true inflation and true output remains below the correlation of measured inflation and measured output. As second robustness check, I cut the calibrated value of δ half to 0.03125. In this case (not shown), the shift in the correlations of inflation and output for data from 1960:1–2008:1 is less pronounced but remains statistically significant.

As final robustness check, I split the sample into the "Great Inflation" and the "Great Moderation" period, and compare correlations of inflation and output for each subsample. The two bottom panels of figure 4 show that transforming measured variables into true variables affects the correlation of inflation and output less during the Great Moderation (right) than during the Great Inflation (left). The reason is that the relative standard deviation $\mathrm{std}(\hat{\pi}^m_t)/\mathrm{std}(\hat{Y}^m_t)$ is about 0.5 during the Great Inflation but falls to 0.35 during the Great Moderation. Accordingly, spillover effects (represented by $b(L)\hat{\pi}_t$) of inflation mismeasurement into output are large during the Great Inflation but small during the Great Moderation. More generally, the model predicts that spillover effects of inflation mismeasurement into output matter most when inflation is volatile relative to output.

7.2   Measurement Bias and U.S. Productivity Growth

Trehan (1999) argues that historical U.S. interest rates exceeded optimal levels during the period of strong productivity growth in the 1990s because policy makers missed a shift in the average growth rate of productivity and output at that time. Accordingly, policy makers interpreted most growth in productivity and output as cyclical expansion and tightened monetary policy more than done otherwise to prevent the economy from overheating.

[Figure 5 about here.]

31

I quantify a related but slightly different possibility of policy mistake, namely, that U.S. monetary policy in the 1990s reacted to measured output and measured inflation. I evaluate this possibility by a counterfactual in which monetary policy reacts to true output and true inflation. Specifically, I describe historical U.S. monetary policy by the interest-rate rule (31). I solve the model under that policy rule and use the model solution to partition historical data into historical shocks by means of the Kalman filter. Then, I solve the model under the counterfactual interest-rate rule (32), and retrieve counterfactual data using the new model solution but historical shocks.[21]

The main finding from the top-left panel of figure 5 is that counterfactual interest rates remain below historical interest rates except before 1992 and for a short episode around 2000. Between 1992:2 and 1999:2 counterfactual interest rates are lower by 70 basis points on average. The interest-rate differential is large enough that counterfactual rates hit the zero-lower bound several quarters in a row between 2003 and 2005. Overall, there would have been lower interest rates in the 1990s had the Fed accounted for endogenous MB.

The interest-rate differential comoves inversely but tightly with productivity shocks a_t shown in the bottom-right panel of figure 5. The tight comovement suggests that MB matters most in the wake of productivity shocks. Indeed, when partitioning the interest-rate differential between 1992:2 and 1999:2 into contribution per shock, productivity shocks imply a mean difference of 72.5 basis points compared to -3.33, -0.63, and 1.20 basis points for u_t, g_t, and μ_t shocks, respectively. Productivity shocks also dominate the inflation and output differentials shown in the figure.

Partly, the prevalence of productivity shocks derives from their large sample standard deviation of 3.9% compared to 0.77%, 0.89%, and 0.12% for u_t, g_t, and μ_t shocks, respectively. It matters further that inflation reacts less than output to g_t and μ_t shocks. Therefore, spillover of inflation mismeasurement into output is small, and MB affects

---

[21]I run the Kalman filter on data from 1960:1 to 2008:1, applying the policy rule estimated for the subsample 1984:1 to 2008:1, to mitigate the influence of initial values. To obtain historical shocks initial values of unobserved states are set to zero. To obtain counterfactual data initial values of unobserved states are set to unobserved states estimated for historical policy. The initial forecast-error covariance matrix is set to the unconditional covariance matrix implied by the model's recursive law of motion in both runs. Statistic are computed for the subsample 1984:1 to 2008:1 unless noted otherwise.

interest rates little. In contrast, inflation reacts more than output to u_t and a_t shocks such that spillover of inflation mismeasurement into output is large. Accordingly, the effect of MB on interest rates is large for these shocks. Finally, u_t shocks die out faster than a_t shocks and thus matter less. Therefore, in response to u_t shocks, firms and households adjust prices and quantities less, and inflation and output respond less.

Recall from section 5.3 that, after a productivity shock, historical interest rates $\hat{\underline{i}}_t$ decline less than counterfactual rates $\hat{i}_t$ because MB introduces lags of true inflation into the historical policy rule. The additional history dependence implies that measured inflation declines less whereas measured output increases more. By the policy rule, historical interest rates then decline less. Furthermore, recall from section 5.3 that welfare is higher for the historical policy than for the counterfactual policy. This ranking remains true in the sample at hand with a ratio of standard deviations $\text{std}(\hat{\underline{\pi}})/\text{std}(\hat{\pi})$ equal to 0.61, and the corresponding ratios for the output gap and the interest rate equal to 0.98 and 0.70, respectively.

The top-right panel of figure 5 shows two consequences of MB for inflation. First, historical measured inflation overstates counterfactual true inflation when productivity grows fast. That is, the correlation of productivity and historical measured inflation is less negative (-0.30) than the correlation of productivity and counterfactual true inflation (-0.59). Second, MB can hide true deflation if measured inflation is low and productivity grows fast. True deflation may arise because the inflation differential $\hat{\underline{\pi}}_t^m - \hat{\pi}_t$ is very volatile with standard deviation 1.10%, evolves fairly persistent with autocorrelation 0.97, and correlates strongly negative with historical measured inflation (-0.66). Both consequences of MB reinforce believes of policy makers that MB is a matter of particular concern when rapid changes in productivity escort low measured inflation. Issing (2001), p.2, stresses that "The present scenario of rapid changing technology combined with low inflation makes the issue of measurement biases in price indices of the utmost relevance for monetary policy."

The bottom-left panel of figure 5 shows that historically measured output is less volatile than counterfactual output with $\text{std}(\hat{\underline{Y}}^m)/\text{std}(\hat{Y})$ equal to 0.79. Moreover, mea-

sured output growth (not shown) correlates positively (0.08) with the MB in inflation $\underline{\hat{\pi}}_t^m - \hat{\pi}_t$. Accordingly, the model is consistent with the pro-cyclical MB documented by Broda and Weinstein (2007) though pro-cyclicality is weak. It is reassuring that measured output growth also correlates positively (0.14) with the inflation differential $\hat{\underline{\pi}}_t^m - \hat{\pi}_t$. Both correlations increase marginally when measured output growth is computed from data that is not HP-filtered.

As robustness check, I reduce the value of δ to 0.03125. The average interest-rate differential between 1992:2 and 1999:2 falls from 70 to 35 basis point but counterfactual interest rates continue to hit the zero lower bound between 2003 and 2005. The standard deviation of the inflation differential $\hat{\underline{\pi}}_t^m - \hat{\pi}_t$ falls from 1.10% to 0.54% whereas both its autocorrelation and its correlation with historical measured inflation remain essentially unchanged. The ratio $\text{std}(\underline{\hat{Y}}^m)/\text{std}(\hat{Y})$ increases from 0.79 to 0.89, and the correlation of measured output growth and MB in inflation $\hat{\underline{\pi}}_t^m - \hat{\underline{\pi}}_t$ increases slightly. Overall, correlations remain mostly unchanged despite the fact that differentials between historical measured and counterfactual true variables turn less variable when δ is low.

7.3 Does Policy Amplify Effects of Measurement Bias?

MB affects the economy via two channels. The first channel is that inflation and output are wrongly measured. The second channel is that the central bank loops MB back into the economy by responding to wrongly-measured variables. Here, I return to the counterfactual exercise of the previous section to estimate how much of the difference between historical measured variables and counterfactual true variables derives from a central bank which loops MB back into the economy.

Let z equal $\hat{Y}_t$ or $\hat{\pi}_t$ and denote historical measured variables as $\underline{z}^m$ and counterfactual true variables as z like before. Then, partition the difference $\underline{z}^m - z$ as $\underline{z}^m - z = (\underline{z}^m - z^m) + (z^m - z)$. The first right-hand term isolates the "loop-back" effect of MB. It is the difference between historical and counterfactual measured variables that emerges when the central bank either responds to measured variables (looping MB

back) or responds to true variables (not looping MB back). The second term reflects "pure MB", namely the difference between counterfactual measured and counterfactual true variables when monetary policy responds to true variables. The variance of the differential equals $\mathrm{var}(\underline{z}^m - z) = \mathrm{var}(\underline{z}^m - z^m) + \mathrm{var}(z^m - z) + 2\mathrm{cov}(\underline{z}^m - z^m, z^m - z)$. Table 1 collects respective sample moments.

[Table 1 about here.]

Consider output first. The table shows that the "loop-back" effect of MB and "pure MB" contribute about the same share to the variation in $\hat{\underline{Y}}_t^m - \hat{Y}_t$. Individual terms vary more than their sum because their covariance is negative and implies a large correlation of -0.63. For inflation, the "loop-back" effect accounts for the major bulk of variation in $\hat{\underline{\pi}}_t^m - \hat{\pi}_t$, whereas "pure MB" contributes little. The covariance between the "loop-back" effect and "pure MB" is positive and implies a moderate correlation of 0.26.

When reducing the value of δ to 0.03125, the most significant change in table 1 is that the variance of $\underline{z}^m - z$ relative to the variance of $\underline{z}^m$ in the last column falls by a factor of three to four. However, the relative importance of each term in the partition of $\mathrm{var}(\underline{z}^m - z)$ remains almost identical. It thus seems a robust feature of the model that the "loop-back" effect of MB is fairly important compared to "pure MB".

However, I somewhat discount the importance of the "loop-back" effect because, by assumption, the model attributes too much variation to the "loop-back" effect and too little to "pure MB". First, central banks monitor many indicators of inflation and output. To the extent that endogenous biases in these indicators are not perfectly correlated, central banks may sidestep at least some endogenous bias by responding to averages of measured indicators. Second, the model precludes by assumption that new products differ from established products with respect to quality, demand, or production efficiency. Such factors are likely to amplify price differentials between new and established products, and hence would amplify "pure MB".

8   Conclusion

Econometric analysis of U.S. product-level data points to MB in CPI inflation because the statistical agency measures product entry with a delay. Estimated MB varies over time, is serially correlated, and pro-cyclical. However, the scarcity of product-level data imposes limits to estimating MB econometrically. Greenwood and Uysal (2005) argue that, complementary to econometric analysis, economic models constitute a useful laboratory for comparing different price indices. Along these lines, I analyze a New-Keynesian model that features product entry and exit and a statistical agency which measures product entry with a delay.

The model predicts that MB in inflation is serially correlated and pro-cyclical, as in the data. Specifically, measured inflation is more persistent than true, utility-based inflation because measured inflation lacks the short price spells of new products; measured inflation is less volatile than true inflation because it lacks the price changes which result from product turnover. Furthermore, because MB in inflation transmits into output, the correlation of true inflation and true output is significantly lower than the correlation of measured inflation and measured output.

I apply the model to study monetary-policy effects of MB. MB implies that the central bank stabilizes inflation insufficiently, and that the policy rule becomes more history dependent. Whereas insufficient inflation stabilization leads to lower welfare across monetary-policy regimes, more history dependence leads to higher welfare for discretionary monetary policy. Quantitatively, the benefit from history dependence overcompensates the loss from insufficient inflation stabilization.

I conclude that certain properties of aggregate data on inflation and output at business-cycle frequency reflect measurement rather than the behavior of economic agents. Validating business-cycle models and assessing their implications for monetary policy without taking into account this fact is likely to deliver biased conclusions. Broadly speaking, endogenous MB constitutes a nontrivial link between theory and data, and matters for analysts and policymakers alike.

Future research could extend my analysis along several dimensions. First, the as-

sumption of perfect information on the part of firms and households could be modified. The modification is likely to affect the dynamics of individual product prices and, hence, the dynamics of true inflation but should not alter the conclusion that measured inflation is less volatile but more persistent than true inflation. Second, one could consider how accounting for MB alters the fit of the New-Keynesian model to data and the estimates of deep parameters. Third, it is interesting to explore how monetary policy optimally takes into account the mapping from measured to true inflation when this mapping is uncertain. Finally, endogenous MB and exogenous ME are likely to coexist in practice, and future research should study optimal monetary policy when the central bank cannot tell apart bias from error.

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A   Equilibrium Conditions

A.1   Household Optimality Conditions

Optimal household choices require

$$1 = \beta E_t \left[\frac{u_c(C_{t+1}, \xi_{t+1})}{u_c(C_t, \xi_t)} \frac{(1+i_t)P_t}{P_{t+1}} \right] \ , \quad \frac{W_t}{P_t} = \frac{h_L(L_t)}{u_c(C_t, \xi_t)} \ , \quad P_t C_t = W_t L_t + D_t \ ,$$

where transversality conditions hold and bond market clearing has been used to simplify the budget constraint (Woodford (2003), chapter 3, provides a detailed derivation). Notation $u_c(C_t, \xi_t)$ abbreviates $\partial u(C_t, \xi_t)/\partial C_t$. Notation $h_L(L_t)$ abbreviates $\partial h(L_t)/\partial L_t$.

A.2   Aggregate Supply Relationship

The optimal pricing condition to problem (5) is

$$0 = E_t \sum_{i=0}^{\infty} (\kappa\beta)^i \Omega_{t+i,t} Y_{t+i}(j) \left[(1-\theta) + \theta S_{t+i}(j)/P_t^\star(j) \right] \tag{36}$$

with real marginal costs $\frac{S_t(j)}{P_t} = \frac{h_L(L_t)}{u_c(Y_t,\xi_t)} \frac{1}{A_t f'(f^{-1}(Y_t(j)/A_t))}$. I linearize the optimal pricing condition and marginal costs and combine both equations with the linearized recursive law of motion of the true price level. I obtain

$$\hat{\pi}_t = \frac{(1-\kappa\beta)(1-\kappa)}{\kappa} (1 + \theta\omega_p)^{-1} \hat{s}_t + \beta E_t \hat{\pi}_{t+1} \tag{37}$$

denoting with $\hat{s}_t$ average real marginal costs. Parameters are defined analog to Woodford (2003) as $\nu = \frac{h_{LL}\bar{L}}{h_L}$, $\chi = \frac{f}{\bar{L}f'}$, $\omega_p = -\frac{\bar{Y}f''}{(f')^2}$, $\omega_w = \nu\chi$ and $\omega = \omega_p + \omega_w$. Natural output equals $\hat{Y}_t^{na} = \frac{1+\omega}{\omega+\sigma^{-1}} a_t + \frac{\sigma^{-1}}{\omega+\sigma^{-1}} g_t$ with $\hat{A}_t = a_t$. Real average marginal costs $\hat{s}_t = (\omega + \sigma^{-1})x_t$ are proportional to the output gap $x_t = \hat{Y}_t - \hat{Y}_t^{na}$ such that equation (37) reformulates to equation (21) in the main text.

B Measurement Bias

B.1 Substitution Bias

To show $\hat{\mathcal{B}}_t^{sub} = \hat{\pi}_t^m - \hat{\pi}_t^n = 0$, linearize equation (7) as $\hat{\pi}_t^m = \int_{\mathcal{N}(t,\ell)} w_j^m \hat{\pi}_t(j)\ dj$ exploiting steady-state relationships $\pi^m = \pi = \pi_j = 1$, and the fact that linearized measured weights integrate to zero by equation (8), $0 = \int_{\mathcal{N}(t,\ell)} w_j^m \hat{w}_{t-1,t}^m(j)\ dj$. Here w_j^m denotes the measured steady-state weight identical for all products j in a symmetric steady-state. Equivalently, linearize equation (10) as $\hat{\pi}_t^n = \int_{\mathcal{N}(t,\ell)} w_j^n \hat{\pi}_t(j)\ dj$ accounting for the fact that weights (11) integrate to zero once linearized. Because $w_j^m = w_j^n = 1/(1-\delta)^\ell$ in symmetric steady-state measured and conservative inflation are identical up to first order, $\hat{\pi}_t^m = \hat{\pi}_t^n$, and the result follows.

B.2 New-Product Bias

The proof applies the same strategy of the proof to proposition 2 to express measured price levels as functions of current and past optimal product prices.

The $(1-\delta)^\ell$ products in $\mathcal{N}(t,\ell)$ are composed out of infinitely many entry cohorts s, each of size δ in the entry period, $(1-\delta)^\ell = \sum_{s=t-\ell}^{-\infty}(1-\delta)^{t-s}\delta$. Thus, $P_{t,t}^n$ is a weighted sum of average prices in each cohort $s \leq t$,

$$(P_{t,t}^n)^{1-\theta} \quad = \quad \int_{\mathcal{N}(t,\ell)} P_t^{1-\theta}(j)\ dj \quad = \quad \sum_{s=t-\ell}^{-\infty}(1-\delta)^{t-s}\delta\ \Lambda_t(s)\ .$$

The sum puts zero weight to the ℓ most recent cohorts. Equation (15) in the main text defines $\Lambda_t(s)$. For comparison, $(P_{t-1,t}^n)^{1-\theta} = \sum_{s=t-\ell}^{-\infty}(1-\delta)^{t-s}\delta\ \Lambda_{t-1}(s)$. Rearrange $(P_{t,t}^n)^{1-\theta}$ as

$$\frac{(P_{t,t}^n)^{1-\theta}}{(1-\delta)^\ell} \quad = \quad (1-\alpha)\sum_{k=0}^{\ell-1}\alpha^k (P_{t-k}^\star)^{1-\theta} + \alpha^\ell\ P_{t-\ell}^{1-\theta}$$

employing the recursive representation of the true price level. The sum vanishes if $\ell = 1$.

Analog transformation of $(P_{t-1,t}^n)^{1-\theta}$ delivers

$$\frac{(P_{t-1,t}^n)^{1-\theta}}{(1-\delta)^\ell} = (1-\alpha)\sum_{k=1}^{\ell-1}\alpha^{k-1}(P_{t-k}^\star)^{1-\theta} + \alpha^{\ell-1} \, P_{t-\ell}^{1-\theta} \, .$$

The sum vanishes if $\ell = 1$. Linearizing $(P_{t,t}^n)^{1-\theta}$ and $(P_{t-1,t}^n)^{1-\theta}$ and exploiting $\bar{P} = \bar{P}^\star$, $\bar{P}^n/\bar{P}^\star = (1-\delta)^{\ell/(1-\theta)}$ and $(1-\alpha)\sum_{k=0}^{\ell-1}\alpha^k + \alpha^\ell = 1$ delivers

$$\hat{P}_{t,t}^n = (1-\alpha)\sum_{k=0}^{\ell-1}\alpha^k\hat{P}_{t-k}^\star + \alpha^\ell \, \hat{P}_{t-\ell} \, , \quad \hat{P}_{t-1,t}^n = (1-\alpha)\sum_{k=1}^{\ell-1}\alpha^{k-1}\hat{P}_{t-k}^\star + \alpha^{\ell-1} \, \hat{P}_{t-\ell} \, . \tag{38}$$

Thus, π_t^n linearizes as

$$\hat{\pi}_t^n = \hat{P}_{t,t}^n - \hat{P}_{t-1,t}^n = (1-\alpha)\Big[\hat{P}_t^\star - (1-\alpha)\sum_{k=1}^{\ell-1}\alpha^{k-1}\hat{P}_{t-k}^\star - \alpha^{\ell-1} \, \hat{P}_{t-\ell}\Big] \, . \tag{39}$$

Linearizing $\mathcal{B}_t^{new}$ then produces

$$\hat{\mathcal{B}}_t^{new} = (1-\alpha)\Big[\hat{P}_t^\star - (1-\alpha)\sum_{k=1}^{\ell-1}\alpha^{k-1}\hat{P}_{t-k}^\star - \alpha^{\ell-1} \, \hat{P}_{t-\ell}\Big] - \hat{\pi}_t \, .$$

The recursive law of motion of the true price level implies $\hat{P}_{t-k}^\star = \hat{P}_{t-k} + \frac{\kappa}{1-\kappa}\hat{\pi}_{t-k}$. Combine with $\hat{\mathcal{B}}_t^{new}$ to obtain

$$\hat{\mathcal{B}}_t^{new} = \Big(\frac{1-\alpha}{1-\alpha(1-\delta)} - 1\Big)\hat{\pi}_t + \frac{(1-\alpha)\delta}{1-\alpha(1-\delta)}\sum_{s=1}^{\ell-1}\alpha^s\hat{\pi}_{t-s} \, ,$$

which corresponds to $\hat{\mathcal{B}}_t^{new}$ in proposition 3.

B.3 Bias in the Price Level

Obtain $\hat{P}_{t-s}^\star = \frac{\kappa}{1-\kappa}\hat{\pi}_{t-s} + \hat{P}_{t-s}$ from the recursive law of motion of P_t. Equations (6) and (9) deliver $\hat{P}_{t,t}^m = \hat{P}_{t,t}^n$ and $\hat{P}_{t-1,t}^m = \hat{P}_{t-1,t}^n$ once linearized. Combine with (38) to obtain

$$\hat{P}_{t,t}^m = (1-\alpha)\sum_{s=0}^{\ell-1}\alpha^s\hat{P}_{t-s}^\star + \alpha^\ell \, \hat{P}_{t-\ell} = \frac{(1-\alpha)\kappa}{1-\kappa}\sum_{s=0}^{\ell-1}\alpha^s\hat{\pi}_{t-s} + (1-\alpha)\sum_{s=0}^{\ell-1}\alpha^s\hat{P}_{t-s} + \alpha^\ell \, \hat{P}_{t-\ell}$$

$$= \frac{(1-\alpha)\kappa}{1-\kappa}\sum_{s=0}^{\ell-1}\alpha^s\hat{\pi}_{t-s} + \hat{P}_t - \alpha\sum_{s=0}^{\ell-1}\alpha^s\hat{\pi}_{t-s} = \hat{P}_t - \frac{\alpha\delta}{1-\alpha(1-\delta)}\sum_{s=0}^{\ell-1}\alpha^s\hat{\pi}_{t-s} \, .$$

Thus $\hat{P}_t - \hat{P}_{t,t}^m = b(L)\hat{\pi}_t$.

B.4   Properties of $a(L)$ and $b(L)$

For $\ell = 1$, $0 < a(1) \leq 1$ follows immediately from proposition 3 if $\alpha, \delta \in [0,1)$. For $\ell = 2, 3, \ldots$ I show that (i) $a(1) = 1$ if $\alpha = 0$, $\delta = 0$ or both, and (ii) $0 < a(1) < 1$ if $\alpha, \delta \in (0,1)$. Invertibility of $a(L)$ requires that coefficients of $a(L)$ are absolutely summable. From proposition 3 it follows immediately that all coefficients of $a(L)$ are non-negative if $\alpha, \delta \in [0,1)$ and $\ell = 1, 2, \ldots$ Therefore, it suffices to show that coefficients are summable, $a(1) < \infty$, which follows from step (i) and (ii).

(i) Rewrite $a(L)$ in proposition 3 as $a(L) = \frac{1-\alpha}{1-\alpha(1-\delta)}\left((1-\delta)L^0 + \delta\frac{1-(\alpha L)^\ell}{1-\alpha L}\right)$ or

$$a(1) = \frac{1-\alpha}{1-\alpha(1-\delta)}\left((1-\delta) + \delta\frac{1-\alpha^\ell}{1-\alpha}\right) = 1 - \frac{\delta\alpha^\ell}{1-\alpha(1-\delta)} \ . \tag{40}$$

Insert $\alpha = 0$, $\delta = 0$ or both to find $a(1) = 1$.

(ii) Equation (40) implies $a(1) < 1$ if $-\frac{\delta\alpha^\ell}{1-\alpha(1-\delta)} < 0$ which is true for all $\alpha, \delta \in (0,1)$. Also, (40) implies $0 < a(1)$ if $\frac{\delta\alpha^\ell}{1-\alpha(1-\delta)} < 1$. Equivalently, $\alpha\delta(1-\alpha^{\ell-1}) > -(1-\alpha)$ which is true for all $\alpha, \delta \in (0,1)$.

Similarly, all coefficients of $b(L)$, $b_s = \frac{\alpha^{s+1}\delta}{1-\alpha(1-\delta)}$, $s = 0, \ldots, \ell - 1$ are nonnegative. Rewrite $b(L) = \frac{\delta}{1-\alpha(1-\delta)}\left(\frac{1-(\alpha L)^{\ell+1}}{1-\alpha L} - L^0\right)$ from which one finds $b(1) < \infty$. Thus, $b(L)$ is invertible.
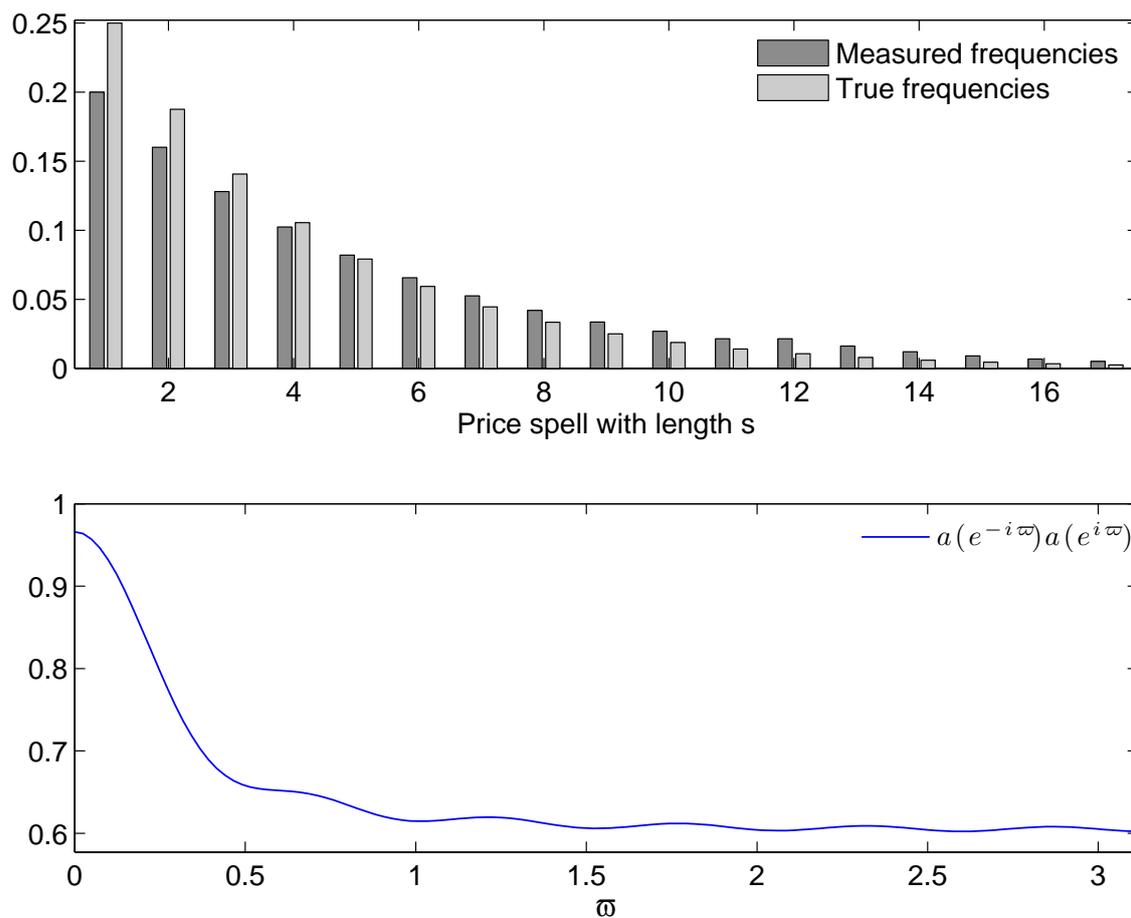
Figure 1: The top panel shows relative frequencies of terminated price spells with durations s for measured and true inflation. The bottom panel shows the spectrum of measured inflation divided by the spectrum of true inflation. Section 4 describes the calibration.
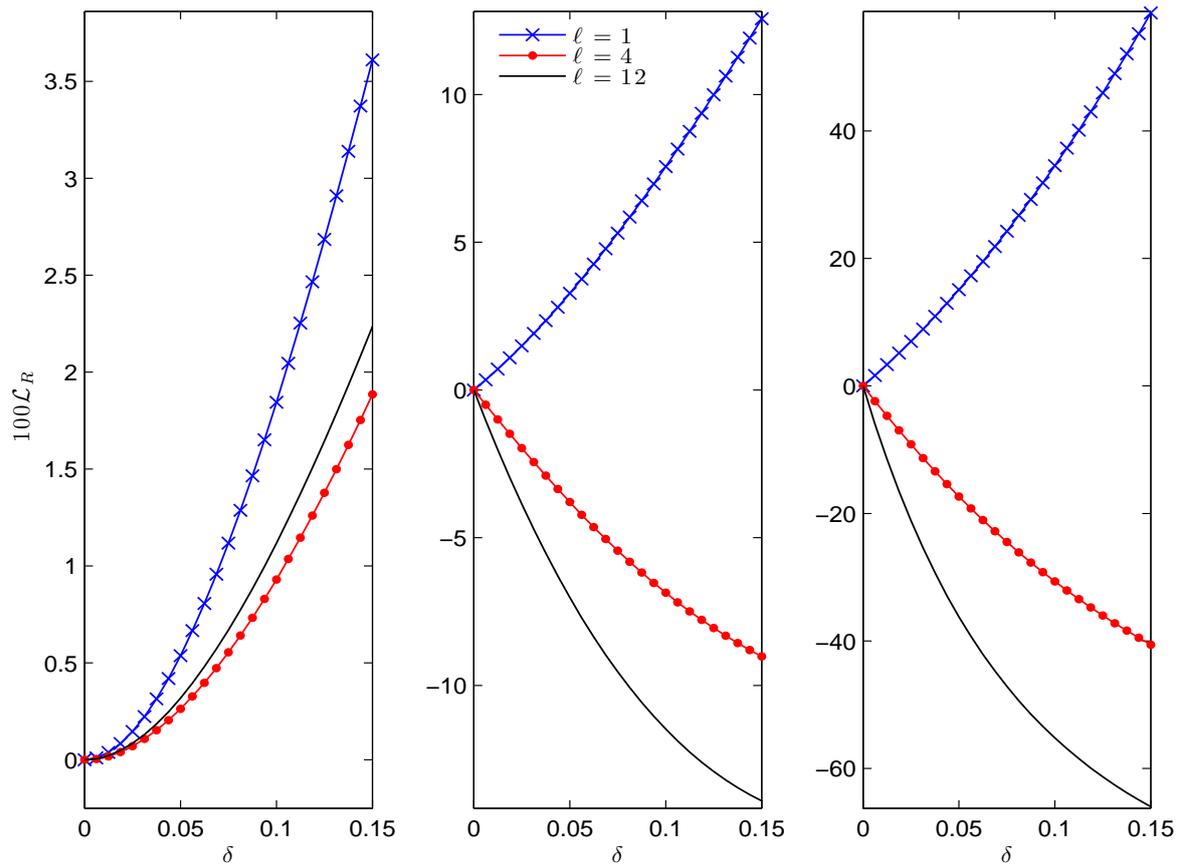
Figure 2: Welfare loss in Economy II relative to the welfare loss in Economy I, 100 $\mathcal{L}_R$, for optimal monetary policy under commitment (left), for optimal monetary policy under discretion (middle), and for monetary policy which commits to a simple interest-rate rule (right).
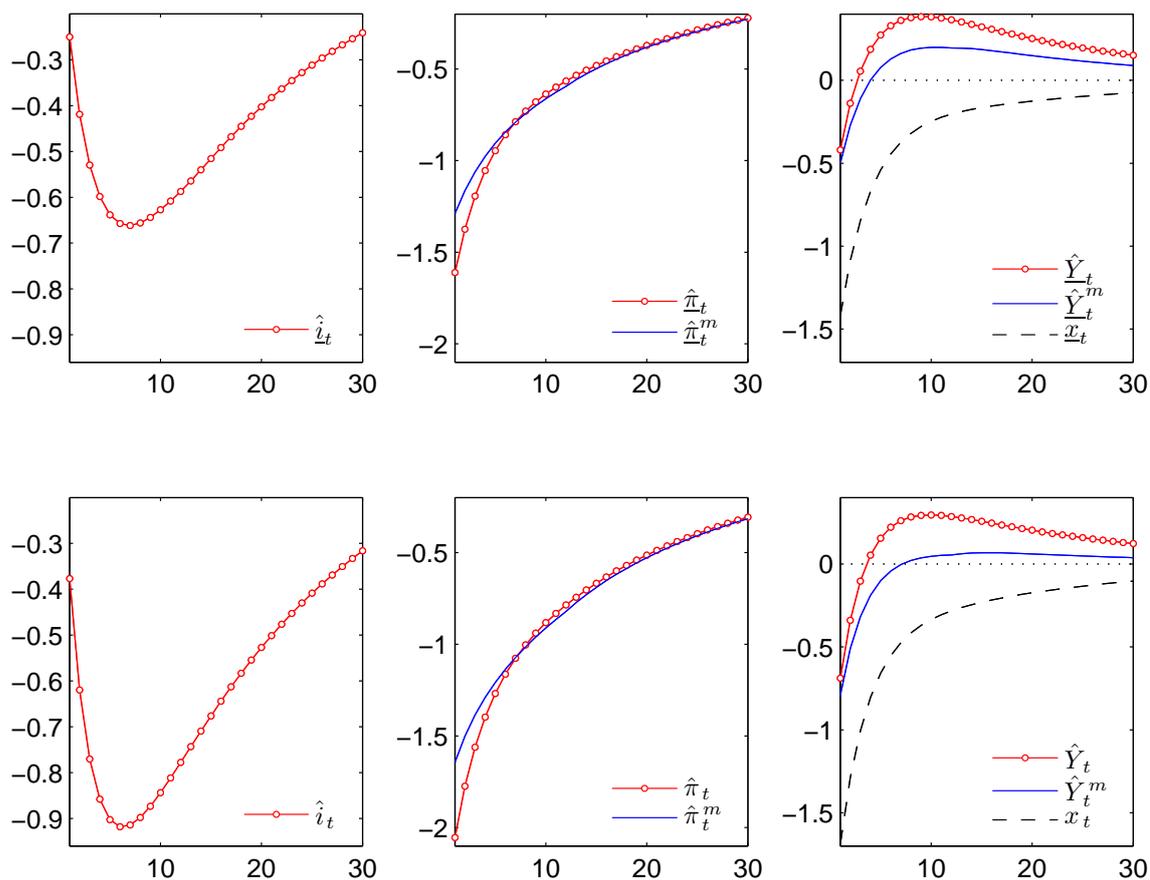
Figure 3: Impulse responses of true variables (circled lines) and measured variables (solid lines) to a positive productivity shock of one percent. Top row shows the nominal interest rate, inflation, output, and the true output gap (dashed line) in Economy II. Bottom row shows the same variables in Economy I. Interest rates and inflation rates are annual rates. Output and output gap are percentage deviations from trend.
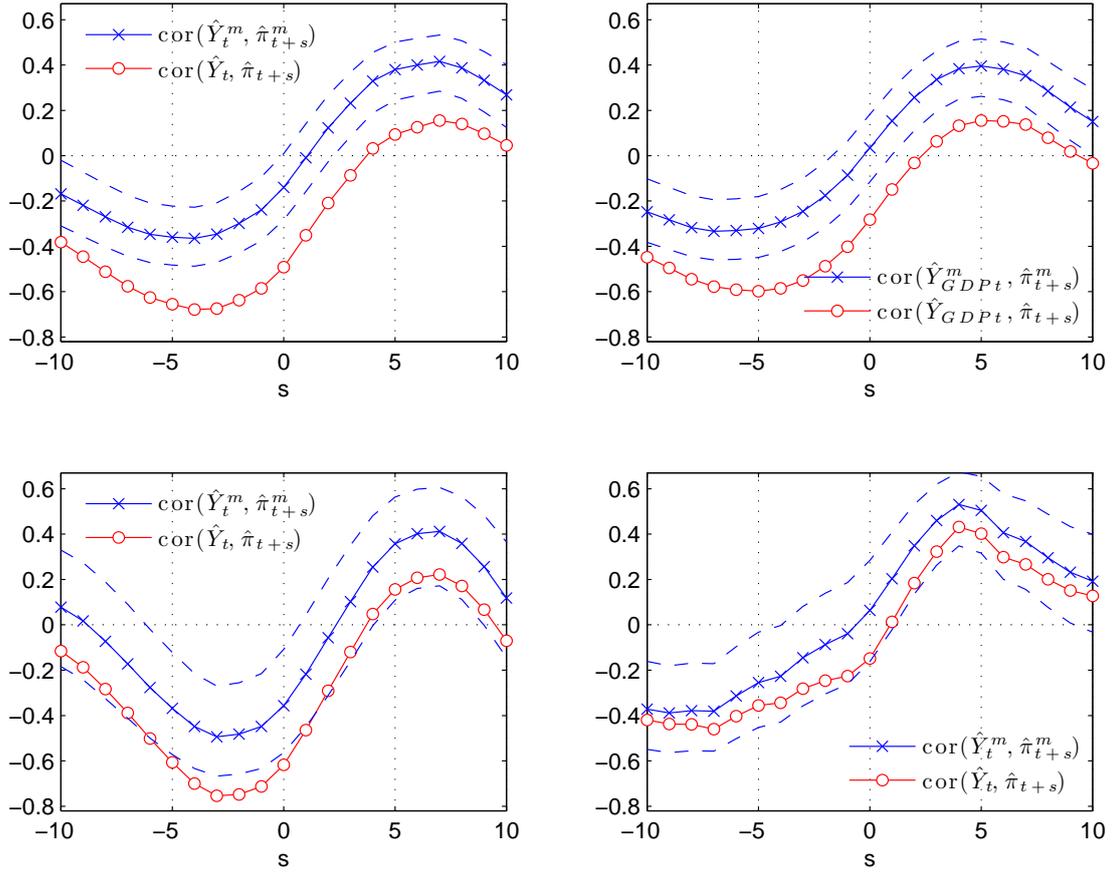
Figure 4: The top-left panel shows correlation of $\hat{Y}_t, \hat{\pi}_{t+s}$ (circled line) and $\hat{Y}_t^m, \hat{\pi}_{t+s}^m$ (crossed line) for 1960:1–2008:1. Dashed lines correspond to a 95% confidence band. The top-right panel shows correlation of $\hat{Y}_{GDPt}, \hat{\pi}_{t+s}$ and $\hat{Y}_{GDPt}^m, \hat{\pi}_{t+s}^m$ for the same period. The bottom-left panel shows correlation of $\hat{Y}_t, \hat{\pi}_{t+s}$ and $\hat{Y}_t^m, \hat{\pi}_{t+s}^m$ for the Great Inflation period 1960:1–1979:2. The bottom-right panel shows the same correlation for the Great Moderation period 1984:1–2008:1.
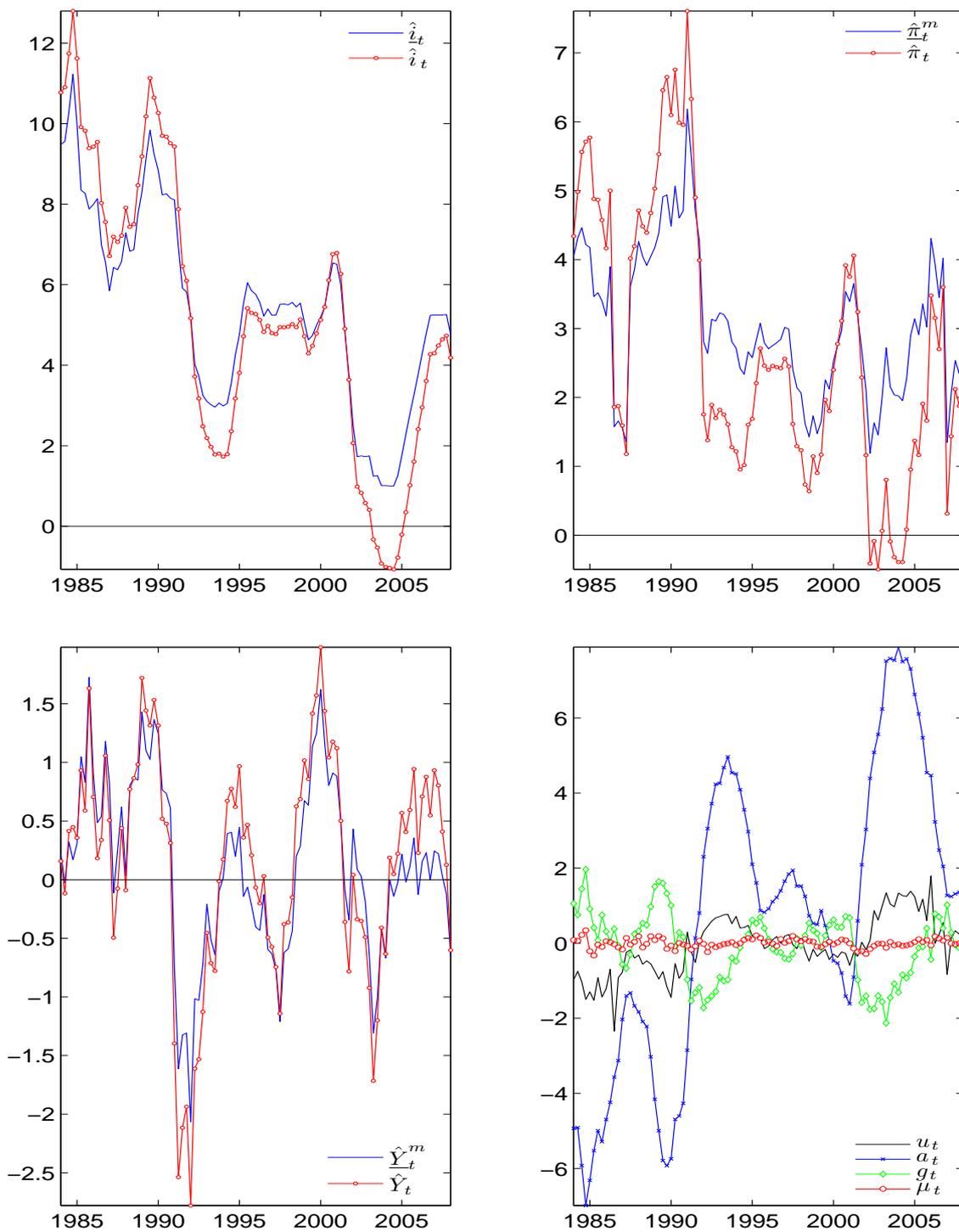
Figure 5: Top-left panel: Historical nominal interest rate (solid line) and counterfactual nominal rate (circled line). Top-right panel: Historical measured inflation (solid line) and counterfactual true inflation (circled line). Bottom-left panel: Historical measured output (solid line) and counterfactual true output (circled line). Bottom-right panel: Historical sequences of u_t, a_t, g_t and μ_t shocks. Interest rates and inflation rates are annual rates. Output and shocks are percentage deviation from trend.

z	$\mathrm{var}(\underline{z}^m - z)$	$\mathrm{var}(\underline{z}^m - z^m)$	$\mathrm{var}(z^m - z)$	$2\mathrm{cov}(\underline{z}^m - z^m, z^m - z)$	$\frac{\mathrm{var}(\underline{z}^m - z)}{\mathrm{var}(\underline{z}^m)}$
$\hat{Y}_t$	0.126	0.179	0.160	-0.212	0.226
$\hat{\pi}_t$	1.191	0.992	0.065	0.134	1.084

Table 1: Output and Inflation Differentials.
Output is percentage deviation from steady-state. Inflation is annual rate, and all numbers are rounded. The alternative partition, $\underline{z}^m - z = (\underline{z}^m - \underline{z}) + (\underline{z} - z)$, delivers similar results.

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