



European  
University  
Institute

ROBERT SCHUMAN CENTRE FOR ADVANCED STUDIES

# EUI Working Papers

RSCAS 2010/76

ROBERT SCHUMAN CENTRE FOR ADVANCED STUDIES

BANK MARKET STRUCTURE, SYSTEMIC RISK,  
AND INTERBANK MARKET BREAKDOWNS

Marcella Lucchetta



**EUROPEAN UNIVERSITY INSTITUTE, FLORENCE**  
**ROBERT SCHUMAN CENTRE FOR ADVANCED STUDIES**

*Bank Market Structure, Systemic Risk,  
and Interbank Market Breakdowns*

**MARCELLA LUCCHETTA**

This text may be downloaded only for personal research purposes. Additional reproduction for other purposes, whether in hard copies or electronically, requires the consent of the author(s), editor(s). If cited or quoted, reference should be made to the full name of the author(s), editor(s), the title, the working paper, or other series, the year and the publisher.

ISSN 1028-3625

© 2010 Marcella Lucchetta

Printed in Italy, October 2010

European University Institute

Badia Fiesolana

I – 50014 San Domenico di Fiesole (FI)

Italy

[www.eui.eu/RSCAS/Publications/](http://www.eui.eu/RSCAS/Publications/)

[www.eui.eu](http://www.eui.eu)

[cadmus.eui.eu](http://cadmus.eui.eu)

## **Robert Schuman Centre for Advanced Studies**

The Robert Schuman Centre for Advanced Studies (RSCAS), directed by Stefano Bartolini since September 2006, is home to a large post-doctoral programme. Created in 1992, it aims to develop inter-disciplinary and comparative research and to promote work on the major issues facing the process of integration and European society.

The Centre hosts major research programmes and projects, and a range of working groups and ad hoc initiatives. The research agenda is organised around a set of core themes and is continuously evolving, reflecting the changing agenda of European integration and the expanding membership of the European Union.

Details of this and the other research of the Centre can be found on:

<http://www.eui.eu/RSCAS/Research/>

Research publications take the form of Working Papers, Policy Papers, Distinguished Lectures and books. Most of these are also available on the RSCAS website:

<http://www.eui.eu/RSCAS/Publications/>

The EUI and the RSCAS are not responsible for the opinion expressed by the author(s).

## **Abstract**

This paper explores theoretically the implications of bank market structure and banking system risks concentration for the functioning of interbank markets. It employs a simple model where banks are exposed to both credit and liquidity risk, there is no asymmetric information, no market power, no friction in secondary markets and deposit contracts are fully contingent. We show that (a) the concentration of risks induced by changes in bank market structure makes interbank market breakdowns more likely; (b) welfare monotonically decreases in risk concentration; and (c) risk concentration and a high probability of interbank market breakdowns can be driven by risk control diseconomies of scale and scope and increases in financial firms' size. As banking systems become more concentrated, improvement of risk control technologies in financial institutions and in regulatory bodies appear as important as other policies considered in the literature to minimize the probability of interbank market breakdowns.

## **Keywords**

Bank Market Structure, Systemic Risk, Interbank Markets

JEL: E42, G01, G21

*“Evidence suggests that [risk] interdependencies between large and complex banking organisations have increased over the last decade in the United States and Japan, and are beginning to do so in Europe. Although a causal link has not been established, these increases are positively correlated with measures of consolidation. Areas of increased interdependency that are most associated with consolidation include interbank loans, market activities such as OTC derivatives, and payment and settlement systems.”*

Group of Ten, Report on Consolidation in the Financial Sector, January 2001

## **I. Introduction**\*

One prominent feature of the recent financial crisis has been the disruption and prolonged malfunctioning of interbank markets. This has come as a surprise to most observers, since interbank markets have been functioning smoothly historically, even in the face of severe stress episodes such as the LTCM failure (Furfine, 2000).

Recent theoretical research addressing why interbank markets may not function properly has provided explanations based on: asymmetric information (e.g. Flannery, 1996, Freixas and Jorge, 2008, Heider et al., 2008); market power (e.g. Acharya, Gromb and Yorulmazer, 2008, and Cai and Thakor, 2008); malfunctioning secondary asset markets (e.g. Gorton and Huang, 2004 and 2006, and Diamond and Rajan, 2005 and 2008). An exception is Allen, Carletti and Gale (2009), who show that the incompleteness of markets can be at the root of the breakdown of interbank markets' during periods of stress. In their model, such incompleteness arises only from the absence of tradable state contingent securities *and* from the use of debt contracts modeled with no state-contingency, since there is no asymmetric information, no market power and secondary asset markets function smoothly. Their results can be viewed as showing that asymmetric information, market power, and dysfunctional secondary asset markets are not necessary, although they are sufficient, for interbank markets breakdowns.

Since the Group of Ten study cited above, however, there has been increasing evidence of a positive relationship between systemic risk in major banking systems and bank concentration, as well as evidence of increases in systemic risk associated with changes in bank market structure.<sup>1</sup> This evidence raises the question of whether changes in bank market structure could have a significant impact on the functioning of interbank markets, and if so, why. To our knowledge, this question has not been addressed in the literature.

In this paper we explore the implications of banking system risk concentration for the functioning of interbank markets. This is accomplished by building a simple model along the lines of Diamond and Dybvig (1983) and Battacharya and Gale (1987) models. Differing from these papers and from that of Allen, Carletti and Gale (2009), banks are exposed to *both* credit and liquidity risk, there is no asymmetric information, markets are perfectly competitive, there is no friction in secondary markets. Importantly banks offer fully contingent debt contracts to their financiers/depositors, and use interbank markets to smooth their credit and liquidity shocks.

---

\* I thank without implications Gianni De Nicolò, Florian Heider, Marie Hoerova, Cornelia Holthausen, Philipp Hartmann, Jose-Luis Peydro, Lorian Pelizzon, Bruno Maria Parigi, Maurizio Murgia, Goetz Von Peter, Kostas Tsatsaronis and seminar participants at the European Central Bank, the International Monetary Fund, the University of Bozen, University of Venice and the Bank for International Settlements for comments and suggestions. (First draft: September 29, 2009. This draft: July 8, 2010)

<sup>1</sup> Evidence on the positive relationship between bank concentration and measures of bank systemic risk is in Boyd, De Nicolò and Loukoianova (2009) and Boyd, De Nicolò and Jalal (2009). During the 1990s and early 2000s, De Nicolò and Kwast (2002) found increased risk interdependencies among U.S. large and complex banking organizations. Risk profiles of large and complex U.S. and European banks were also found to have increased in the U.S. in Europe, and globally in De Nicolò et al (2004), De Nicolò, Hayward, and Vir Bhatia (2004), Stiroh (2004), Hartmann, Straetmans, and de Vries (2005), Stiroh and Rumble (2006), and Houston and Stiroh (2006).

We show that an increase in the concentration of risks—possibly arising from concentrated market structure—makes interbank markets breakdowns more likely. Differing from the previous literature, our results are not driven by asymmetric information, market power, dysfunctional secondary markets or contracts incompleteness, rather they can be explained by *risk control diseconomies* associated with large sizes of financial institutions and the wide scope and complexity of their activities. Indeed, large and complex banking groups (LCBG) are seen as institutions to be carefully monitored by regulator as a potential source of instabilities (ECB Financial Stability Review, December 2009).<sup>2</sup> Problems related to the functioning of the interbank markets and the attitude of banks to merge to solve the liquidity needs is highlighted in Carletti, Hartmann and Spagnolo (2007) who show that mergers aimed at raising liquidity may also increase the concentration of risks.

In section II we describe the model. The model set-up captures in stylized form an important distinction is made between diversification of a banking firm and diversification of the financial system along the lines of De Nicolò and Kwast (2002). Banks can be perfectly diversified individually, but aggregate risk in the banking system can be either perfectly diversified or concentrated *across* banks. As the system becomes composed of fewer and larger banks, each bank will be more diversified individually, but the banking system will be less diversified, since a larger fraction of banks is exposed to the same aggregate shocks. In the model, the degree of *banking system diversification* is parameterized between the two extremes of perfect diversification and maximal concentration.

Section III defines the interbank equilibrium. Interbank market breakdowns are simply defined as situations in which the interbank equilibrium does not exist and is replaced by the autarkic equilibrium, where each bank is disconnected from each other.

In section IV, existence of interbank equilibriums is established for the two extreme cases of a perfectly diversified and a maximally concentrated banking system. Then, it is shown that for a large set of economies, the size of the set of interbank equilibriums under a diversified banking system is always strictly larger than that of a concentrated banking system for any level of credit and liquidity risk. This result indicates that in the presence of aggregate risk, a diversified banking system is likely to be less prone to interbank market breakdowns.

Section V defines the welfare properties of interbank and autarkic allocations. When an interbank equilibrium exists, it clearly gives a higher expected utility to depositors than the autarkic equilibrium, but importantly, agents' welfare is monotonically decreasing in the degree of risk concentration.

Section VI provides an explanation of why banks may choose risk so that a banking system may take on concentrated risk. Risk concentration and a high probability of interbank market breakdowns may be due to *risk control diseconomies of scale and scope* arising *within* large and complex financial firms.

Our explanation is *technological*, although it can be easily complemented by all other explanatory factors emphasized in the literature. Specifically, we show that when our model is modified by introducing a risk control technology with decreasing returns over a certain investment threshold, then banks will choose a level of risk concentration that increases in size. Such level is higher the larger is the cost of risk control arising from the internal organization of large and complex firms. This result suggests improvement of risk control technologies in large and complex financial institutions and in regulatory bodies may be policy concerns as important as other policies considered in the literature to minimize the probability of interbank market breakdowns.

Section V concludes. Proofs of all propositions are in the Appendix.

---

<sup>2</sup> In a related contribution, Hartmann, Carletti and Spagnolo (2007) examine the impact of changes in bank market structure on the distribution of liquidity shocks and the recourse to interbank markets. Unlike this paper, however, they assume that the interbank rate is exogenous and, therefore, there are no breakdowns.



## II. The Model

There are three periods,  $t = 0, 1, 2$ , and one risky asset that yields a random return at date 2 per unit invested at date 0. It can assume two values,  $R = R^h$ , and  $R = R^l$ . If a portion of the investment in this asset is liquidated at  $t = 1$ , it yields a certain return of  $\lambda$  per unit invested. The fraction of the asset that is liquidated is denoted  $\alpha$ . It is assumed that  $R^h > \lambda > R^l \geq 1$  so that if storage is available, this technology will be dominated in rate of return in both dates and will be never used.

At date 0 consumers are endowed with one unit of the date 0 consumption good, which is assumed to be invested all in one bank. Consumers are uncertain about their time preferences: with probability  $\mu$  they are *early* consumers, who want to consume at date 1 only, and with probability  $1 - \mu$  they are *late* consumers, who want to consume at date 2 only.

Their preferences are represented by a utility function  $U(c)$ , twice continuously differentiable, increasing, and strictly concave. The fraction of early consumers is also random and can assume two values:  $\mu = \mu^h$ , and  $\mu = \mu^l$ , with  $1 > \mu^h > \mu^l > 0$ .

The banking sector is composed of a continuum of banks that invest consumer's endowments at date 0. The banking sector is perfectly competitive, so that banks' objective is to maximize depositors' expected utility.

Each bank is exposed to liquidity and credit risk shocks. The realization of both shocks is observed by a bank at date 1. We assume there is no aggregate uncertainty, so that the fraction of banks which is exposed to a given combination of credit and liquidity shocks is deterministic. However, the probabilities (and hence the distribution) of these banks at date 0 depends on an exogenous parameter  $\sigma \in (0, 1)$  that defines the market structure with respect to the *concentration of liquidity and credit risk* in the banking system.

Specifically, the fraction of banks that experience a given pair of realizations of credit and liquidity risk  $(R, \mu)$  is given by the following table, where  $p \in (0, 1)$  and  $q \in (0, 1)$ :

Returns \ $\mu$	Low	High
High	$\sigma p$	$(1 - \sigma)q$
Low	$(1 - \sigma)(1 - q)$	$\sigma(1 - p)$

Thus, as of date 1, there are four types of banks:

type1	type2
type3	type4

Thereafter we refer to parameter  $\sigma$  as an index of “market structure”.

A fraction of banks  $\sigma p$  (type 1) experiences a *low* liquidity shock and a *high* final date return on the asset; by contrast, a fraction of banks  $\sigma(1 - p)$  (type 4) experiences the reverse, that is, a *high* liquidity shock and a *low* final date return on the asset. Thus, credit risks and liquidity risks are perfectly *positively correlated* for type 1 and 4 banks.

Conversely, a fraction of banks  $(1 - \sigma)q$  (type 2) experiences a *high* liquidity shock but also a *high* final date return on the asset, whereas a fraction of banks  $(1 - \sigma)(1 - q)$  (type 3) a *low* liquidity shock but also a *low* final date return on the asset. Thus, credit risk and liquidity risks are perfectly *negatively correlated* for type 2 and 3 banks..

The exogenous parameter  $\sigma \in (0, 1)$  indexes the degree of *concentration of liquidity and credit risk* in the banking system. If  $\sigma = 0$ , then the banking system is made of banks that have chosen to *diversify* their credit and liquidity risks, while if  $\sigma = 1$ , banks have chosen to *concentrate* their credit

and liquidity risks. As  $\sigma$  increases, the banking system exhibits a higher concentration of risks for any given values of  $(p, q)$ . This can be the result either of consolidation and/or incentives for banks to take on correlated risks.

### III. Interbank Equilibrium

Here, we examine how well an *interbank* market works for a given market structure.

There is an interbank market where liquidity can be traded at the intermediate date. The amount of funds that each bank trade in the interbank market is denoted  $b$  and the interbank rate is denoted by  $r$ .

At date 1, competitive banks maximize the expected utility of depositors. They choose the amount of borrowing  $b$  (if positive, borrowing, if negative, lending) and the amount of asset to liquidate,  $\alpha$ , to solve:

$$\underset{\alpha, b}{\text{Max}} \Pi_1 = \mu U(c_1) + (1 - \mu)U(c_2) \quad (1)$$

Subject to

$$\mu c_1 = \alpha \lambda + b \quad (2)$$

$$(1 - \mu)c_2 = R(1 - \alpha) - rb \quad (3),$$

where  $r$  is the “interbank” rate, to be determined in the  $t = 1$  credit equilibrium.

Assume an interbank equilibrium exists. Substituting (3) in (2) through  $b$ ,

$$\mu c_1 + (1 - \mu) \frac{c_2}{r} = \alpha \lambda + \frac{R}{r}(1 - \alpha) \quad (4).$$

The solution  $\alpha^*$  will be the one that makes the bank’s budget constraint the largest, i.e. that makes the right hand side of (4) the largest. Hence, the solution is given by:

$$\alpha^* = 0 \text{ if } \frac{R}{r} > \lambda \text{ and } \alpha^* = 1 \text{ if } \frac{R}{r} < \lambda \quad (5).$$

A necessary condition for the existence of an interbank equilibrium is that

$$r \in \left( \frac{R^l}{\lambda}, \frac{R^h}{\lambda} \right) \quad (6).$$

This is because if  $r \leq R^l / \lambda$ , by (4) all banks will not liquidate the investment in the risky technology, and by (2) they will wish to finance all date 1 consumption by borrowing. Thus, there would be no lenders, hence no interbank equilibrium. Likewise, if  $r \geq R^h / \lambda$ , by (5) all banks will liquidate the investment in the risky technology, and by (3) they will wish to finance all date 2 consumption by the repayments on lending at date 2. But at date 1 there would be no borrowers, hence no interbank equilibrium.

Using (2), (3) and (5) in (1), bank  $(R, \mu)$  solves:

$$\text{Max}_b \Pi_1 = \mu U \left( \frac{\alpha^* \lambda + b}{\mu} \right) + (1 - \mu) U \left( \frac{R(1 - \alpha^*) - rb}{1 - \mu} \right) \quad (1a),$$

the FOC with respect to  $b$  is

$$U' \left( \frac{\alpha^* \lambda + b}{\mu} \right) = r U' \left( \frac{R(1 - \alpha^*) - rb}{1 - \mu} \right) \quad (7).$$

Thus, the solution of the bank problem is given by (5) and (7). Note that banks optimal choices are the liquidation decision  $\alpha^*(R)$ , which does not depend on  $\mu$ , and  $b(R, \mu)$ . The liquidation decision in response to the credit risk realization does not depend on the liquidity shock (in (5) nothing depends on  $\mu$ ), but the borrowing decision depends on both shocks (by (7)).

We characterize equilibriums for log utility preferences, i.e.  $U(c) = \ln(c)$ . The choice of this specification is motivated by simplicity and by the fact that for these preferences, date 1 spot market allocations and optimal banking allocations generally coincide in liquidity preference frameworks such as ours (see Allen and Gale 2007). Hence, these preferences can be viewed as a useful benchmark to judge differences in equilibriums and associated welfare properties in diversified and concentrated risk economies, as these comparisons are unlikely to be affected by efficiency wedges between market and banking allocations.

Equation (7) yields:

$$\frac{\mu}{\alpha^* \lambda + b} = \frac{r(1 - \mu)}{\rho(1 - \alpha^*) - rb} \quad (8).$$

Solving (8), we have

$$b(R, \mu) = \frac{1}{r} (\mu R(1 - \alpha^*) - r(1 - \mu)\alpha^* \lambda) \quad (9).$$

Since  $r \in (R^l / \lambda, R^h / \lambda)$ , by (5), optimal asset's liquidation is  $\alpha^*(R^l) = 1$  and  $\alpha^*(R^h) = 0$ .

In sum, the four bank types have the following borrowing/lending positions

$$b(R^l, \mu^l) = -(1 - \mu^l)\lambda \quad (9a)$$

$$b(R^l, \mu^h) = -(1 - \mu^h)\lambda \quad (9b)$$

$$b(R^h, \mu^l) = \frac{1}{r} (\mu^l R^h) \quad (9c)$$

$$b(R^h, \mu^h) = \frac{1}{r} (\mu^h R^h) \quad (9d),$$

and equilibrium in the interbank market requires

$$\sigma p \frac{1}{r} \mu^l R^h + (1 - \sigma) q \frac{1}{r} \mu^h R^h - (1 - \sigma)(1 - q)(1 - \mu^l)\lambda - \sigma(1 - p)(1 - \mu^h)\lambda = 0 \quad (10).$$

The above equation (10) is linear with respect to  $r$  and has the unique solution:

$$r^* = \frac{R^h (\sigma p \mu^l + (1-\sigma)q\mu^h)}{[(1-\sigma)(1-q)(1-\mu^l) + \sigma(1-p)(1-\mu^h)]\lambda} \quad (11).$$

Equation (11) says that the interbank equilibrium rate raises as the liquidity needs and the opportunity cost of holding the asset,  $R^h$ , increase.

#### IV. Comparisons of Equilibriums

In this section we identify conditions ensuring existence of equilibriums for the extreme values of  $\sigma$ , and compare the set of parameters for which equilibriums exists for such values.

Using (11) and  $r^* \in (\frac{R^l}{\lambda}, \frac{R^h}{\lambda})$ , we obtain

$$1 \geq \frac{\sigma p \mu^l + (1-\sigma)q\mu^h}{(1-\sigma)(1-q)(1-\mu^l) + \sigma(1-p)(1-\mu^h)} \geq \frac{R^l}{R^h} \quad (12).$$

We use (12) to assess the existence of interbank equilibriums under two extreme cases, that of banking system's *perfect diversification* ( $\sigma = 0$ ), and that of *maximal risk concentration* in the banking system ( $\sigma = 1$ ). The main result is summarized in the following proposition.

**Proposition 1.** For the **perfect diversified economy** ( $\sigma = 0$ ) and for the **perfect concentrated economy** ( $\sigma = 1$ ), the set of interbank equilibriums is non-empty for any parameter configuration of credit and liquidity risk ( $\mu^h, \mu^l, R^h, R^l$ ).

Now, we wish to compare the *size* of the set of economies, indexed by  $p$  and  $q$ , for which equilibriums exist for  $\sigma = 0$  and for  $\sigma = 1$ .

Using the right and the left hand side of (12), the equilibrium domains for  $\sigma = 0$  and for  $\sigma = 1$  are respectively

$$\left[ a_0 = \frac{\frac{R^l}{R^h}(1-\mu^l)}{\mu^h + \frac{R^l}{R^h}(1-\mu^l)}, b_0 = \frac{1-\mu^l}{1+\mu^h-\mu^l} \right] \quad (13)$$

and

$$\left[ a_1 = \frac{\frac{R^l}{R^h}(1-\mu^h)}{\mu^l + \frac{R^l}{R^h}(1-\mu^h)}, b_1 = \frac{1-\mu^h}{1-\mu^h+\mu^l} \right] \quad (14).$$

The larger is the interval for which equilibriums exists under diversification or concentration, the larger is the set of economies that may benefit from the risk sharing opportunities offered by the interbank market.

Consider the difference between the equilibriums interval when  $\sigma = 0$  and when  $\sigma = 1$ , defined as:

$$G \equiv \Delta_0 - \Delta_1 = (b_0 - a_0) - (b_1 - a_1) \quad (15).$$

Computations give:

$$\Delta_0 = \frac{\mu^h (1 - \mu^l) \left(1 - \frac{R^l}{R^h}\right)}{(1 + \mu^h - \mu^l) \left(\mu^h + \frac{R^l}{R^h} (1 - \mu^l)\right)} \quad (16),$$

and

$$\Delta_1 = \frac{\mu^l (1 - \mu^h) \left(1 - \frac{R^l}{R^h}\right)}{(1 - \mu^h + \mu^l) \left(\mu^l + \frac{R^l}{R^h} (1 - \mu^h)\right)} \quad (17).$$

Of particular interest is the comparison of the set of equilibriums for relatively large credit and liquidity shocks. This comparison is made clearer by substituting  $\mu^h = \gamma\mu^l$ , where  $1/\mu^l \geq \gamma \geq 1$ , and  $R^l = \beta R^h$  where  $\beta \leq 1$ . Parameter  $\gamma$  is a measure of the *liquidity risk* and  $\beta$  is the *credit risk*. The smaller is  $\beta$ , the larger is the difference between high and low return. The larger is  $\gamma$ , the larger is the difference between low and high liquidity shock.

Thus, (15) can be expressed as

$$G(\mu^l, \gamma, \beta) = \frac{\gamma\mu^l (1 - \mu^l)}{(1 + \gamma\mu^l - \mu^l)(\gamma\mu^l + \beta(1 - \mu^l))} - \frac{\mu^l (1 - \gamma\mu^l)}{(1 - \gamma\mu^l + \mu^l)(\mu^l + \beta(1 - \gamma\mu^l))} \quad (18).$$

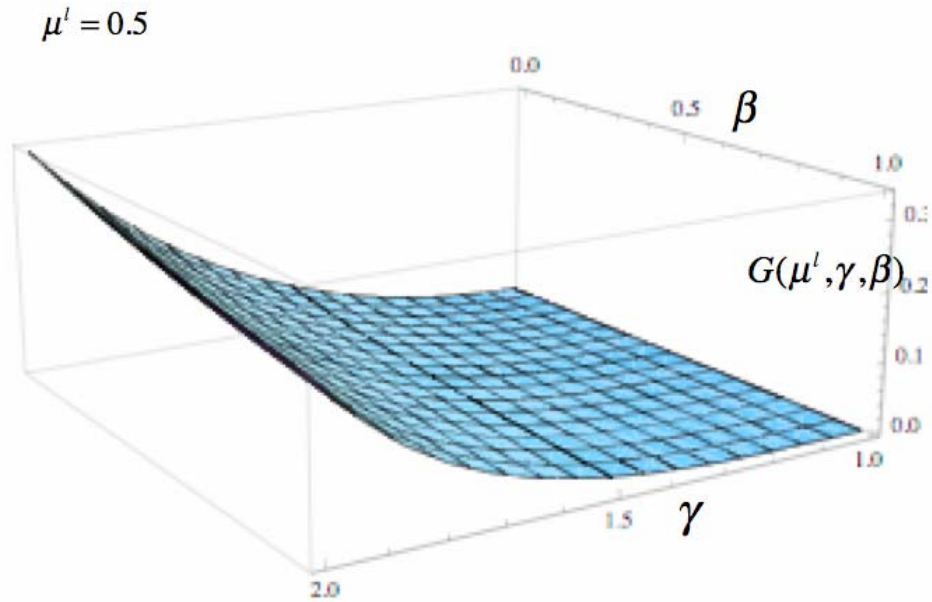
The following proposition establishes a ranking of the size of equilibriums under a diversified economy and a risk-concentrated economy.

**Proposition 2.** There exists a  $\bar{\mu}^l$  such that for  $\mu^l \geq \bar{\mu}^l$  and any  $(\gamma, \beta)$  the set of interbank equilibrium under the diversified economy is always strictly larger than for the concentrated economy.

When liquidity shocks are large, the diversified economy has interbank equilibriums for a larger set of economies than the risk-concentrated economy. In other words, when shocks are large, the interbank equilibrium breaks down if the banking structure exhibits concentration of risks. Therefore, a diversified economy offers a better insurance against high liquidity and credit risk. Finally note that by (18), for  $\beta = \gamma = 1$ , which amounts to absence of credit and liquidity risk,  $G(1,1) = 0$ , then the two sets are equivalent. This means that bank market structure is important especially when the shocks are high.

Figure 1 shows graphically the result of proposition 2 for a set of economies. The surface representing the function  $G(\mu^l, \gamma, \beta)$  is increasing in liquidity risk.

**Figure 1. Behavior of  $G(\mu^l, \gamma, \beta)$**



As figure 1 highlights, the set of equilibriums is increasing in the degree of banks' risk diversification for a liquidity risk greater than  $\bar{\mu}^l$ . This indicates that an interbank is less likely to break-down when the economy is more diversified.

## V. Welfare Comparisons

We want to compare agents' welfare in economies with banking systems differing according to market structure. To do that, we compute depositors' expected utility.

Using (2) and (3), the consumption in states 1 and 2 is

$$c_1 = \frac{\alpha\lambda + b}{\mu} \quad (19)$$

and

$$c_2 = \frac{R(1-\alpha) - rb}{1-\mu} \quad (20).$$

Substituting equilibrium values  $\alpha$ ,  $b$  and  $r$  in (19) and (20), we obtain the consumption allocation offered by different bank types,  $C^{type(i)} = (c_1^{type(i)}, c_2^{type(i)})$  for  $i = 1, 2, 3, 4$ :

$$C^{type(1)} = \left( \frac{\left[ (1-\sigma)(1-q)(1-\mu^l) + \sigma(1-p)(1-\mu^h) \right] \lambda}{\sigma p \mu^l + (1-\sigma)q \mu^h}, R^h \right) \quad (21a)$$

$$C^{type(2)} = \left( \frac{\left[ (1-\sigma)(1-q)(1-\mu^l) + \sigma(1-p)(1-\mu^h) \right] \lambda}{\sigma p \mu^l + (1-\sigma)q \mu^h}, R^h \right) \quad (21b)$$

$$C^{type(3)} = \left( \lambda, \frac{R^h (\sigma p \mu^l + (1-\sigma)q \mu^h)}{(1-\sigma)(1-q)(1-\mu^l) + \sigma(1-p)(1-\mu^h)} \right) \quad (21c)$$

$$C^{type(4)} = \left( \lambda, \frac{R^h (\sigma p \mu^l + (1-\sigma)q \mu^h)}{(1-\sigma)(1-q)(1-\mu^l) + \sigma(1-p)(1-\mu^h)} \right) \quad (21d).$$

The ex-ante the expected utility of a representative consumer is therefore

$$W \equiv \sum_{i=1}^4 P(type(i))U(c_1^{type(i)}, c_2^{type(i)}) \text{ for } i = 1, 2, 3, 4 \quad (22).$$

Equivalently:

$$W \equiv \sigma p U(C^{type(1)}) + (1-\sigma)q U(C^{type(2)}) + (1-\sigma)(1-q)U(C^{type(3)}) + \sigma(1-p)U(C^{type(4)}) \quad (23).$$

Unfolding (23) we get:

$$\begin{aligned} W \equiv & \sigma p \left( \mu^l \log \left( \frac{\left[ (1-\sigma)(1-q)(1-\mu^l) + \sigma(1-p)(1-\mu^h) \right] \lambda}{\sigma p \mu^l + (1-\sigma)q \mu^h} \right) + (1-\mu^l) \log(R^h) \right) + \\ & (1-\sigma)q \left( \mu^h \log \left( \frac{\left[ (1-\sigma)(1-q)(1-\mu^l) + \sigma(1-p)(1-\mu^h) \right] \lambda}{\sigma p \mu^l + (1-\sigma)q \mu^h} \right) + (1-\mu^h) \log(R^h) \right) + \\ & (1-\sigma)(1-q) \left( \mu^l \log(\lambda) + (1-\mu^l) \log \left( \frac{R^h (\sigma p \mu^l + (1-\sigma)q \mu^h)}{(1-\sigma)(1-q)(1-\mu^l) + \sigma(1-p)(1-\mu^h)} \right) \right) + \\ & \sigma(1-p) \left( \mu^h \log(\lambda) + (1-\mu^h) \log \left( \frac{R^h (\sigma p \mu^l + (1-\sigma)q \mu^h)}{(1-\sigma)(1-q)(1-\mu^l) + \sigma(1-p)(1-\mu^h)} \right) \right) \end{aligned} \quad (24).$$

Figure 2 shows  $W$  for given return parameters. Note that  $W$  is decreasing both in  $\sigma$  and in the liquidity risk parameter  $\gamma$ .

**Figure 2. The behavior of  $W$  as a function of  $\sigma$  and liquidity risk  $\gamma$**

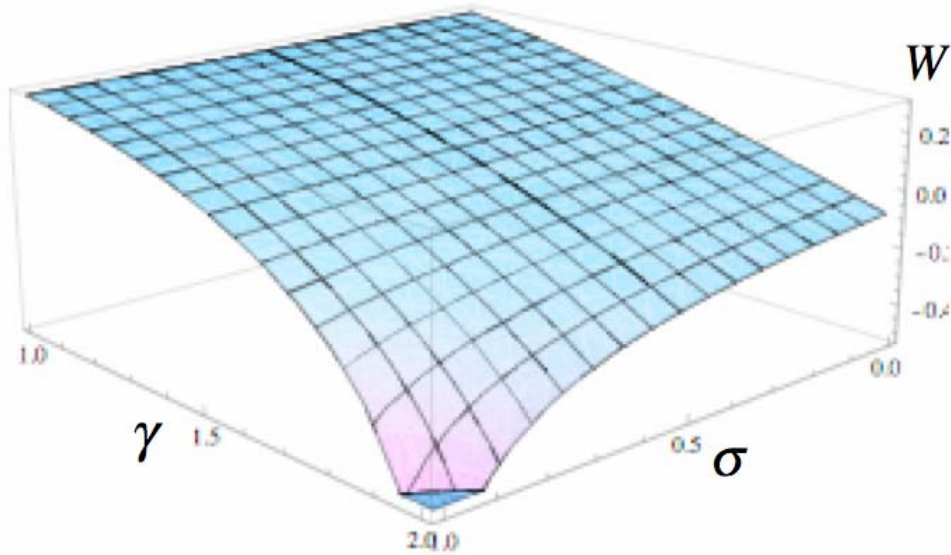


Figure 2 shows that welfare decreases when market structure is concentrated and liquidity risk is high. The corner representing high levels of  $\gamma$  and  $\sigma$  can be viewed as a “crisis” realization with high welfare losses.

When the interbank equilibrium does not exist, the autarkic allocation will prevail. Depositors’ expected utility under autarky is given by the solution of the following problem:

$$\text{Max}_{\alpha} W^A = \mu U(c_1) + (1 - \mu)U(c_2) \quad (1b)$$

Subject to

$$\mu c_1 = \alpha \lambda \quad (2b)$$

$$(1 - \mu)c_2 = R(1 - \alpha) \quad (3b).$$

The optimal solution is  $\alpha^* = \mu$ . Therefore, substituting in (2b) and (3b), the consumption allocations for each bank type at an autarkic equilibrium are:

$$C^{\text{type}(1)} = C^{\text{type}(2)} = (\lambda, R^h) \quad (25a)$$

$$C^{\text{type}(3)} = C^{\text{type}(4)} = (\lambda, R^l) \quad (25b).$$



Correspondingly, the expected utility of an agent at the initial date is:

$$W^A \equiv \sigma p[\mu^l \log(\lambda) + (1 - \mu^l) \log(R^h)] + (1 - \sigma)q[\gamma\mu^l \log(\lambda) + (1 - \gamma\mu^l) \log(R^h)] + (1 - \sigma)(1 - q)[\mu^l \log(\lambda) + (1 - \mu^l) \log(R^l)] + \sigma(1 - p)[\gamma\mu^l \log(\lambda) + (1 - \lambda\mu^l) \log(R^l)] \quad (26).$$

Figure 3 shows  $W$ , the expected utility when the interbank equilibrium exists, and  $W^A$ , the expected utility under autarky, as functions of  $\sigma$  and  $\gamma$  for two sets of economies. For high values of risk concentration and liquidity risk the interbank equilibrium does not exist, as  $W^A > W$ . When the interbank surface lies over the autarky plane, the interbank equilibrium exists since  $W > W^A$  for all parameter values, with  $W$  strictly decreasing in the degree of risk concentration  $\sigma$ .

**Figure 3. Behavior of  $W$  and  $W^A$  with different liquidity shocks**

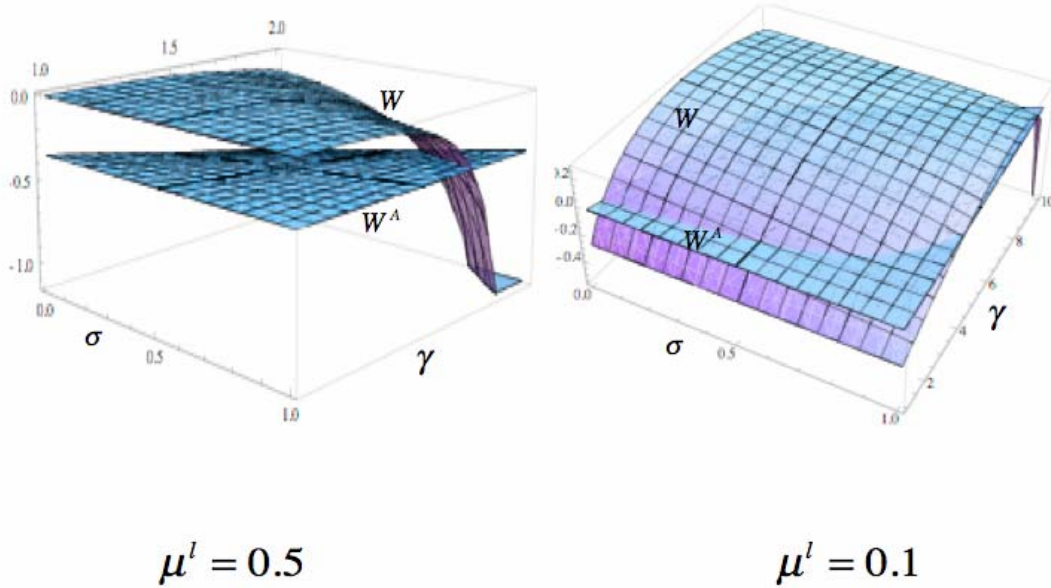


Figure 3 illustrates a general finding summarized in the following proposition 3. The figure describes the existence of the interbank market and the welfare behavior for different levels of market structure and liquidity shocks. The corner with high liquidity shocks and high concentration is a crisis. In this case the interbank market will not exist since the autarky equilibrium (the plane  $W^A$ ) gives higher welfare than the interbank equilibrium.

**Proposition 3.**

- a. There exists  $(\bar{\sigma}, \bar{\gamma})$  such that  $W^A > W$  for all  $(\sigma, \gamma) > (\bar{\sigma}, \bar{\gamma})$ ;
- b. When  $W > W^A$  the interbank equilibrium exists, and  $W$ , is strictly decreasing in  $\sigma$  for any  $\gamma$ .

It is useful to illustrate examples for different set of economies. Figure 4 clearly shows that when  $\sigma$  is high and liquidity risk,  $\gamma$ , is sufficiently high, the interbank allocation is dominated by the autarky allocation.

**Figure 4.  $W$  and  $W^A$  with relatively high  $p$  and  $q$ .**

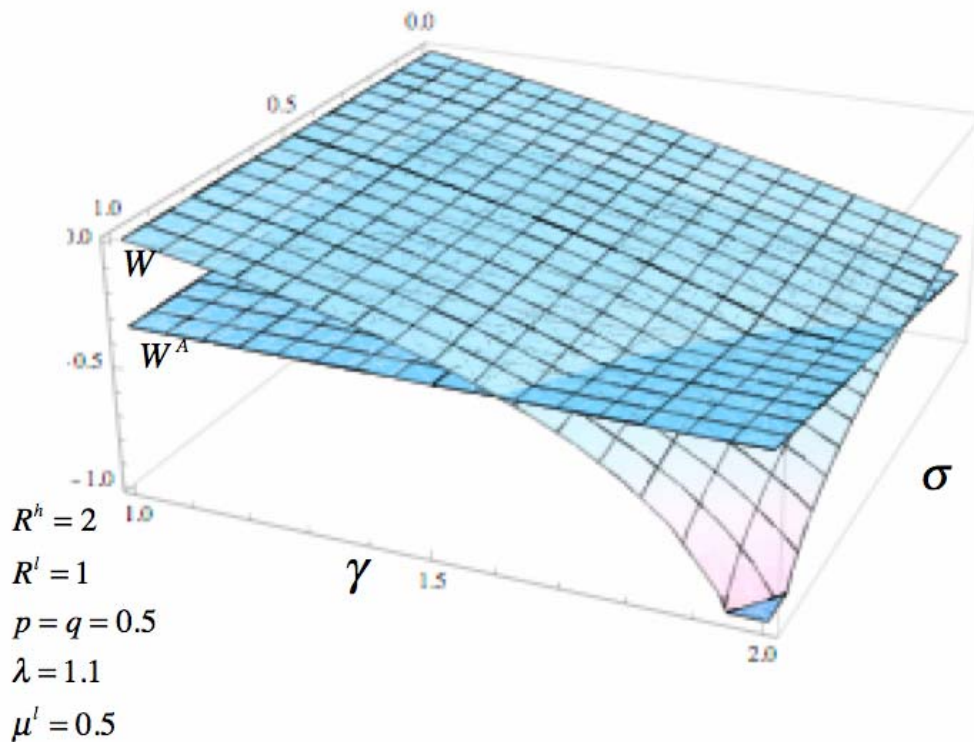
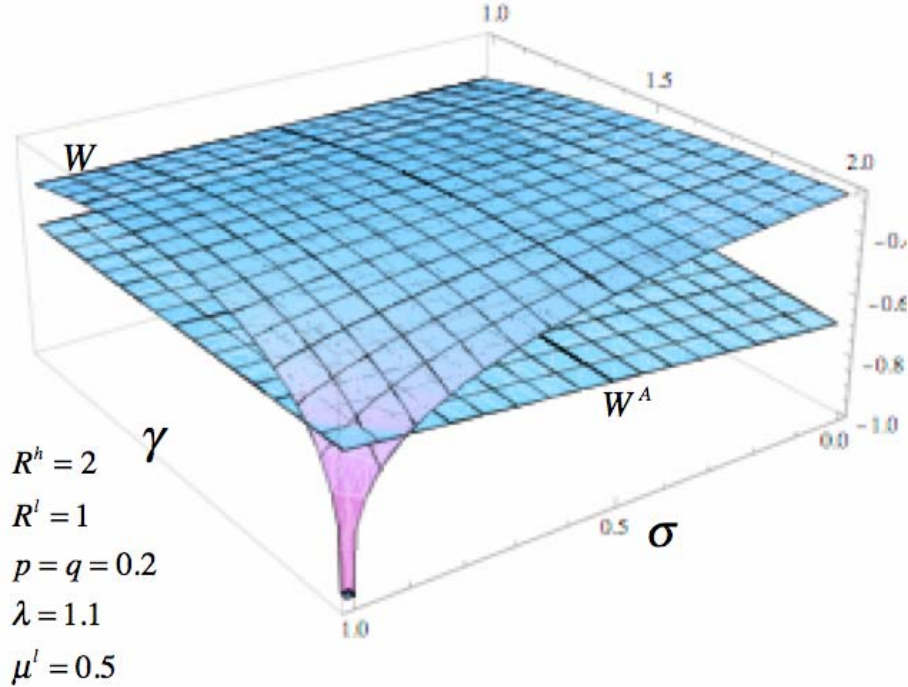


Figure 5 shows that with high credit risk, i.e. low probability of realization of the high return, the interbank market is more likely to exist than the autarky allocation. This suggests that interbank markets insure against credit risk also, while  $W$  remains decreasing with respect to  $\sigma$ .

Figure 5.  $W$  and  $W^A$  with relatively low  $p$  and  $q$ .



The pictures 3, 4 and 5 highlight that welfare always decreases when the economy exhibits a high risks concentration and high liquidity exposure.

## VI. Endogenous Degree of Risk Concentration

So far the degree of risk concentration in the banking system (parameter  $\sigma$ ) has been treated as exogenous. We have shown that welfare decreases in the degree of market concentration. For all examples shown, the highest depositors' expected utility is reached at the minimum level of risk concentration ( $\sigma = 0$ ).

Recall that  $\sigma$  indexes probabilities as of date 0. Therefore, a choice of  $\sigma$  can be viewed as a bank choice of credit and liquidity risk. If achieving perfect diversification at a system level is costless, then in a perfectly competitive banking system, banks would choose the minimum level of risk concentration and the probability of interbank market breakdowns would be minimized.

In reality, achieving diversification and controlling risk is costly, since risk control can be viewed as a *technology* available to firms, similar to the technology underlying credit risk models. The documented impact of increased risks among large financial institutions associated with increased concentration in market structure mentioned previously suggests that firms' internal risk control systems are likely to be costlier. The potential for negative externalities is severe for larger and complex financial firms since the size and scope of their operations expands, financial firms' span of control over their many units can become less effective.

In this light, a lack of sufficient banking system diversification and a higher probability of interbank market breakdowns may be ultimately due to *risk control diseconomies of scale and scope*.

The potential for first order effects of risk control diseconomies of scale and scope on risk concentration in the banking system, and their relationship with bank size, can be illustrated by the following modification of our model.

Suppose at date 0 banks have a size  $S \geq 0$ , and choose  $\sigma$  employing part of date 0 resources. Specifically, they invest a fraction  $x$  of date resources in the technology and choose  $\sigma$  incurring a cost  $z(\sigma)S^\phi$  as a fraction of date 0 resources, where  $\phi > 0$  is the scale cost parameter. Their resource constraint at date 0 is therefore:

$$x + z(\sigma)S^\phi = S \quad (27)$$

Assume  $z(\sigma) = \frac{a_0}{\sigma} + a_1\sigma$ , with  $a_0$  and  $a_1$  as positive coefficients. This function can be interpreted as a cost function of a risk control technology that exhibits decreasing returns to investment over a certain thresholds. Its parameters could depend on size and scope of financial firms operations, as well as on incentives arising from asymmetric information, market power and other factors pointed out in the literature, which may in turn be affected by firms' size.

To compute  $W(\sigma)$ , we replace 1 with  $S(1 - z(\sigma)S^\phi)$  in the consumption allocations,

$$c_1 = \frac{\alpha\lambda S(1 - z(\sigma)S^\phi) + b}{\mu} \quad (28)$$

and

$$c_2 = \frac{R(1 - \alpha)S(1 - z(\sigma)S^\phi) - rb}{1 - \mu} \quad (29).$$

Then, optimal borrowing/lending choices in the interbank market are:

$$b(R, \mu) = \frac{S(1 - z(\sigma)S^\phi)}{r} (\mu R(1 - \alpha^*) - r(1 - \mu)\alpha^*\lambda) \quad (30),$$

With the liquidation choices  $\alpha$  unchanged, the equilibrium in the interbank market is the solution of the following equation:

$$\begin{aligned} \sigma p \frac{S(1 - z(\sigma)S^\phi)}{r} \mu^l R^h + (1 - \sigma)q \frac{S(1 - z(\sigma)S^\phi)}{r} \mu^h R^h \\ - (1 - \sigma)(1 - q)S(1 - z(\sigma)S^\phi)(1 - \mu^l)\lambda - \sigma(1 - p)S(1 - z(\sigma)S^\phi)(1 - \mu^h)\lambda = 0 \end{aligned} \quad (31),$$

which yields

$$r^* = \frac{R^h (\sigma p \mu^l + (1 - \sigma)q \mu^h)}{[(1 - \sigma)(1 - q)(1 - \mu^l) + \sigma(1 - p)(1 - \mu^h)]\lambda} \quad (32).$$

Note that (32) is equivalent to (11), meaning that the equilibrium interbank rate does not depend on the function  $S(1 - z(\sigma)S^\phi)$ . This means that in the market for liquidity is the excess of demand on supply

that determines interbank rate. Multiplying both demand and supply for the size function would not affect the equilibrium interbank rate.

The consumption allocations for the four bank types which allow us to compute depositors' expected utility are:

$$C^{type(1)} = \left( \frac{\left[ (1-\sigma)(1-q)(1-\mu^l) + \sigma(1-p)(1-\mu^h) \right] \lambda S(1-z(\sigma)S^\phi)}{\sigma p \mu^l + (1-\sigma)q \mu^h}, R^h S(1-z(\sigma)S^\phi) \right) \quad (33a)$$

$$C^{type(2)} = \left( \frac{\left[ (1-\sigma)(1-q)(1-\mu^l) + \sigma(1-p)(1-\mu^h) \right] \lambda S(1-z(\sigma)S^\phi)}{\sigma p \mu^l + (1-\sigma)q \mu^h}, R^h S(1-z(\sigma)S^\phi) \right) \quad (33b)$$

$$C^{type(3)} = \left( \lambda S(1-z(\sigma)S^\phi), \frac{R^h (\sigma p \mu^l + (1-\sigma)q \mu^h) S(1-z(\sigma)S^\phi)}{(1-\sigma)(1-q)(1-\mu^l) + \sigma(1-p)(1-\mu^h)} \right) \quad (33c)$$

$$C^{type(4)} = \left( \lambda S(1-z(\sigma)S^\phi), \frac{R^h (\sigma p \mu^l + (1-\sigma)q \mu^h) S(1-z(\sigma)S^\phi)}{(1-\sigma)(1-q)(1-\mu^l) + \sigma(1-p)(1-\mu^h)} \right) \quad (33d).$$

Figure 6 shows (with different angles) an example of the expected utility function  $W$  as a function of  $\sigma$  and the liquidity risk parameter  $\gamma$ . It is apparent that function  $W$  is strictly concave, with a maximum for  $\sigma$  as an interior point. We plot the surface for size parameter  $S = 4$  and  $\phi = 1$ .

**Proposition 4.** For any value of  $\phi$  and for a given level of liquidity risk  $\bar{\gamma}$ , the optimal bank risk concentration level  $\sigma^*$  is increasing in  $S$ .

Proposition 4 states that the scale parameter  $\phi$  is the determinant of the degree of diversification that a bank chooses, and bank diversification is inversely related to its assets size.

Figure 6 shows the concavity property of the welfare function in the case of costly diversification. This implies the choice of a  $\sigma$  different from 0.

Figure 6. Expected utility  $W$  with costs of risk control  $\phi = 1$  and  $S = 4$ .

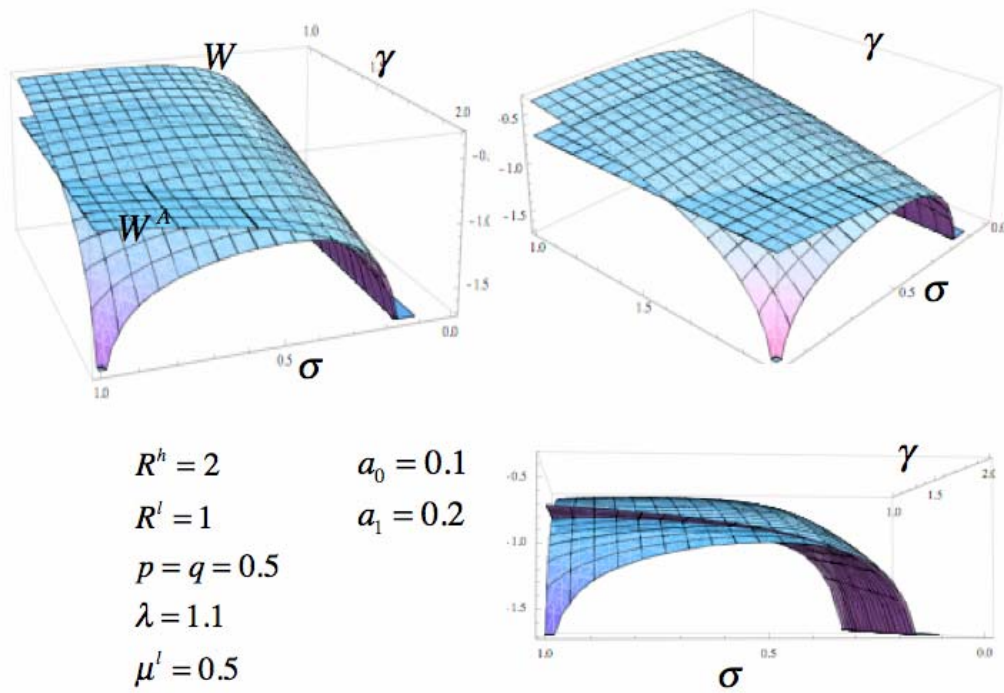
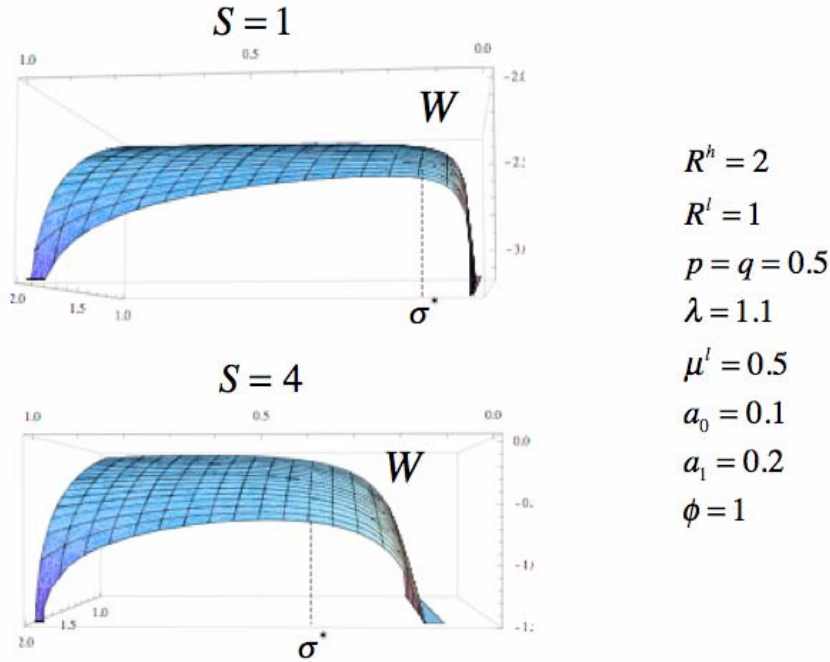


Figure 7 shows the optimal  $\sigma$  for different bank size. The maximum of the expected utility is different according to  $S$ . The concentration is greater for a bank size bigger than one. While, for a small bank size, the concentration approaches zero.

Figure 7. Expected utility  $W$  for  $S = 1$  and  $S = 4$ .



Therefore, the optimal level of risk concentration might be larger than the minimum feasible. In turn, the concentration is larger for a greater bank size. The level of  $a_0$  and  $a_1$  may depend fundamentally on market structure, that is on the distribution of firms by size. As a result, any incentive to increase financial firms' size, such as too-big-to-fail incentives, may carry higher risk control costs, which in turn could result in a higher level of risk concentration in the banking system.

## V. Conclusion

Our model shows that the interaction of market structure and risk control diseconomies of scale seems of first order importance for the smooth functioning of interbank markets, as risk concentration increases in bank size. Our results are obtained in a model with no asymmetric information, no market power, no dysfunctional secondary markets or contracts' incompleteness since deposit contracts are fully contingent. While incentives can be important in affecting bank managers' choices of risk, the evidence reported by Fahlenbrach and Stulz (2009) suggests that incentive effects are perhaps less important than previously thought.

With regard to policy, the literature has prominently focused on the important role of the Central Bank as lender of last resort when interbank market breakdowns occur (see e.g. Freixas, Parigi and Rochet (2000), Repullo (2005), Goodhart and Illing (2002), Allen, Gale and Carletti (2009), and Freixas, Martin and Skeie (2009)). Our model suggests that the improvement of risk control technologies in large and complex financial institutions as well as in regulatory bodies may be as important to minimize the probability of interbank market breakdowns.

## Appendix

**Proposition 1.** For the **perfect diversified economy** ( $\sigma = 0$ ) and for the **perfect concentrated economy** ( $\sigma = 1$ ), the set of interbank equilibriums is non-empty for any parameter configuration of credit and liquidity risk  $(\mu^h, \mu^l, R^h, R^l)$ .

Proof: The left-hand side inequality of (12) can be expressed as:

$$(1-\sigma)(1+\mu^h-\mu^l)q \leq (1-\sigma)(1-\mu^l) + \sigma(1-\mu^h) - p\sigma(1-\mu^h+\mu^l) \quad (\text{A1}),$$

$$\text{If } \sigma = 0, \text{ then} \quad (1+\mu^h-\mu^l)q \leq (1-\mu^l) \quad (\text{A2})$$

$$\text{If } \sigma = 1, \text{ then} \quad 0 \leq (1-\mu^h) - p(1-\mu^h+\mu^l) \quad (\text{A3})$$

The right-hand side inequality of (12) can be expressed as:

$$q(1-\sigma)(\mu^h + \frac{R^l}{R^h}(1-\mu^l)) \geq \frac{R^l}{R^h}[\sigma(1-\mu^h) + (1-\sigma)(1-\mu^l)] - p\sigma[(1-\mu^h)\frac{R^l}{R^h} + \mu^l] \quad (\text{A4})$$

$$\text{If } \sigma = 0, \text{ then} \quad q\left(\mu^h + \frac{R^l}{R^h}(1-\mu^l)\right) \geq \frac{R^l}{R^h}(1-\mu^l) \quad (\text{A5})$$

$$\text{If } \sigma = 1, \text{ then} \quad 0 \geq \frac{R^l}{R^h}(1-\mu^h) - p[(1-\mu^h)\frac{R^l}{R^h} + \mu^l] \quad (\text{A6})$$

Thus, the set of economies for which an interbank equilibrium exists is indexed by  $q \in [0,1]$  if  $\sigma = 0$ , and by  $p \in [0,1]$  if  $\sigma = 1$ .

$$\text{If } \sigma = 0, \text{ using (A2) and (A5), we get: } \frac{\frac{R^l}{R^h}(1-\mu^l)}{\mu^h + \frac{R^l}{R^h}(1-\mu^l)} \leq q \leq \frac{1-\mu^l}{1+\mu^h-\mu^l} \quad (\text{A7}).$$

Therefore, if  $\sigma = 0$ , the set of interbank equilibriums is non-empty if

$$\frac{1-\mu^l}{1+\mu^h-\mu^l} \geq \frac{\frac{R^l}{R^h}(1-\mu^l)}{\mu^h + \frac{R^l}{R^h}(1-\mu^l)} \quad (\text{A8}).$$

Inequality (A8) implies that  $(1-\mu^l)(\mu^h + \frac{R^l}{R^h}(1-\mu^l)) \geq \frac{R^l}{R^h}(1-\mu^l)(1+\mu^h-\mu^l)$ , which can be

further simplified to  $\mu^h \geq \frac{R^l}{R^h}\mu^h$ , which in turn is always verified since  $\frac{R^l}{R^h} < 1$ .



If  $\sigma = 1$ , using (A3) and (A6) we get:  $\frac{1 - \mu^h}{1 - \mu^h + \mu^l} \geq p \geq \frac{\frac{R^l}{R^h}(1 - \mu^h)}{\mu^l + \frac{R^l}{R^h}(1 - \mu^h)}$  (A9).

Inequality (A9) is always satisfied, since it reduces to  $\mu^l \geq \frac{R^l}{R^h} \mu^l$ , which holds as

$$\frac{R^l}{R^h} < 1.$$

QED.

**Proposition 2.** There exists a  $\bar{\mu}^l$  such that for  $\mu^l \geq \bar{\mu}^l$  and any  $(\gamma, \beta)$  the set of interbank equilibrium under the diversified economy is always strictly larger than for the concentrated economy.

Proof. Differentiating (18) with respect to  $\mu^l$ , and evaluating the derivative at  $\mu^l = 1$ :

$$G_{\mu^l}(0, \gamma, \beta) \square \frac{(\gamma - 1)(4(1 + \beta)^2 + \gamma^2 \beta(1 + 5\beta) - \gamma^3 \beta^2 - \gamma(1 + 7\beta + 8\beta^2))}{(\gamma - 2)^2 \gamma (\beta(\gamma - 1) - 1)^2} \quad (\text{A10}),$$

since  $(\gamma - 1) > 0$  and  $(\gamma - 2)^2 \gamma (\beta(\gamma - 1) - 1)^2 > 0$ , (A10) is positive if

$$4(1 + \beta)^2 + \gamma^2 \beta(1 + 5\beta) > \gamma^3 \beta^2 + \gamma(1 + 7\beta + 8\beta^2) \quad (\text{A11}).$$

Observe that for  $\mu^l \rightarrow 1$ ,  $\gamma \rightarrow 1$ , then (A11) reduces to

$$3 + 2\beta > 0 \quad (\text{A12}).$$

Therefore, there exist values of  $\bar{\mu}^l$  and  $\bar{\gamma}$  such that for  $\mu^l \geq \bar{\mu}^l$  and for  $\gamma \geq \bar{\gamma}$  the function  $G(\mu^l, \gamma, \beta)$  is increasing. QED

**Proposition 3.**

c. There exists  $(\bar{\sigma}, \bar{\gamma})$  such that  $W^A > W$  for all  $(\sigma, \gamma) > (\bar{\sigma}, \bar{\gamma})$ ;

d. When  $W > W^A$  the interbank equilibrium exists, and  $W$ , is strictly decreasing in  $\sigma$  for any  $\gamma$ .

Proof. We prove part a) and b) separately.

a. There is a  $(\sigma, \gamma) > (\bar{\sigma}, \bar{\gamma})$  such that the autarky allocation dominates the interbank equilibrium.

The expected utility under autarky for  $\sigma = 0$  and for  $\sigma = 1$  are

$$W^{A0} \equiv q[\gamma \mu^l \log(\lambda) + (1 - \gamma \mu^l) \log(R^h)] + (1 - q)[\mu^l \log(\lambda) + (1 - \mu^l) \log(R^l)] \quad (\text{A13}),$$

and

$$W^{A1} \equiv p[\mu^l \log(\lambda) + (1 - \mu^l) \log(R^h)] + (1 - p)[\gamma \mu^l \log(\lambda) + (1 - \lambda \mu^l) \log(R^l)] \quad (\text{A14}).$$

Hence, we compare the expected utility within a diversify economy between interbank allocation and autarky allocation

$$\begin{aligned}
 W^0 - W^{A0} &\equiv q \left( \gamma \mu^l \log \left( \frac{(1-q)(1-\mu^l)\lambda}{q\gamma\mu^l} \right) + (1-\gamma\mu^l) \log(R^h) \right) + \\
 &(1-q) \left( \mu^l \log(\lambda) + (1-\mu^l) \log \left( \frac{R^h q \gamma \mu^l}{(1-q)(1-\mu^l)} \right) \right) - \\
 &\{q[\gamma\mu^l \log(\lambda) + (1-\gamma\mu^l) \log(R^h)] + (1-q)[\mu^l \log(\lambda) + (1-\mu^l) \log(R^l)]\} \text{ (A15),} \\
 &\Rightarrow q \left( \gamma \mu^l \left( \log \left( \frac{(1-q)(1-\mu^l)\lambda}{q\gamma\mu^l} \right) - \log(\lambda) \right) \right) + \\
 &(1-q) \left( (1-\mu^l) \left( \log \left( \frac{R^h q \gamma \mu^l}{(1-q)(1-\mu^l)} \right) - \log(R^l) \right) \right)
 \end{aligned}$$

The difference of expected utility in a risk concentrated economy between the interbank allocation and autarky allocation is

$$\begin{aligned}
 W^1 - W^{A1} &\equiv p \left( \mu^l \log \left( \frac{(1-p)(1-\gamma\mu^l)\lambda}{p\mu^l} \right) + (1-\mu^l) \log(R^h) \right) + \\
 &(1-p) \left( \gamma \mu^l \log(\lambda) + (1-\gamma\mu^l) \log \left( \frac{R^h p \mu^l}{(1-p)(1-\gamma\mu^l)} \right) \right) - \\
 &\{p[\mu^l \log(\lambda) + (1-\mu^l) \log(R^h)] + (1-p)[\gamma\mu^l \log(\lambda) + (1-\lambda\mu^l) \log(R^l)]\} \text{ (A16).} \\
 &\Rightarrow p \left( \mu^l \left( \log \left( \frac{(1-p)(1-\gamma\mu^l)\lambda}{p\mu^l} \right) - \log(\lambda) \right) \right) + \\
 &(1-p) \left( (1-\gamma\mu^l) \left( \log \left( \frac{R^h p \mu^l}{(1-p)(1-\gamma\mu^l)} \right) - \log(R^l) \right) \right)
 \end{aligned}$$

Computing (A15) and (A16) for  $\gamma \rightarrow \frac{1}{\mu^l}$ ,  $W^0 - W^{A0} > 0$ , and  $W^1 - W^{A1} < 0$ . Then, we conclude that there exist a couple  $(\sigma, \gamma) > (\bar{\sigma}, \bar{\gamma})$  such that the autarky allocation gives a higher expected utility than the interbank market equilibrium.

b. When we have an interbank equilibrium, the expected utility decreases in  $\sigma$ . Take the derivative of (24) with respect to  $\sigma$  and compute it for  $\sigma \rightarrow 1$  and for  $\gamma \rightarrow \frac{1}{\mu^l}$ ,

$$W_{\sigma \rightarrow 1} \left( \gamma \rightarrow \frac{1}{\mu^l} \right) \rightarrow -\gamma < 0 \text{ (A17).}$$

Compute the derivative also for  $\mu^l \rightarrow 0$ ,

$$W_{\sigma \rightarrow 1}(\mu^l \rightarrow 0) < 0 \quad (\text{A18}).$$

Therefore, as risk concentration increases, the expected utility in the interbank economy is decreasing.

Moreover, the expected utility for  $\sigma = 0$  and for  $\sigma = 1$  are

$$\begin{aligned} W^0 \equiv & q \left( \gamma \mu^l \log \left( \frac{(1-q)(1-\mu^l)\lambda}{q\gamma\mu^l} \right) + (1-\gamma\mu^l) \log(R^h) \right) + \\ & (1-q) \left( \mu^l \log(\lambda) + (1-\mu^l) \log \left( \frac{R^h q \gamma \mu^l}{(1-q)(1-\mu^l)} \right) \right) \end{aligned} \quad (\text{A19})$$

and

$$\begin{aligned} W^1 \equiv & p \left( \mu^l \log \left( \frac{(1-p)(1-\gamma\mu^l)\lambda}{p\mu^l} \right) + (1-\mu^l) \log(R^h) \right) + \\ & (1-p) \left( \gamma \mu^l \log(\lambda) + (1-\gamma\mu^l) \log \left( \frac{R^h p \mu^l}{(1-p)(1-\gamma\mu^l)} \right) \right) \end{aligned} \quad (\text{A20}).$$

Let  $\gamma \rightarrow \frac{1}{\mu^l}$ , the difference of expected utilities is

$$\begin{aligned} W^0 - W^1 = & q \left( \log \left( \frac{(1-q)(1-\mu^l)\lambda}{q} \right) \right) + (1-q) \left( \mu^l \log(\lambda) + (1-\mu^l) \log \left( \frac{R^h q}{(1-q)(1-\mu^l)} \right) \right) \\ & - \left( p \left( \mu^l \log(\lambda) + (1-\mu^l) \log(R^h) \right) + (1-p) \log(\lambda) \right) > 0 \end{aligned} \quad (\text{A21}),$$

since  $W^1 \left( \gamma \rightarrow \frac{1}{\mu^l} \right) < 0$  and  $W^0 \left( \gamma \rightarrow \frac{1}{\mu^l} \right) > 0$ .

Let  $\mu^l \rightarrow 0$ ,

$$W^0 - W^1 \rightarrow 0 \quad (\text{A22}).$$

We conclude that for any  $\gamma$  the expected utility decreases in concentration,  $\sigma$ . QED

**Proposition 4.** For any value of  $\phi$  and for a given level of liquidity risk  $\bar{\gamma}$ , the optimal bank risk concentration level  $\sigma^*$  is increasing in  $S$ .

Proof. Take the optimal  $\sigma^*$  solving  $W$  for a given level of liquidity risk  $\bar{\gamma}$  and for  $p = q = 0.5$ . For  $S \rightarrow 0$ , we have that  $\sigma^* \rightarrow 0$  and for  $S \rightarrow \infty$   $\sigma^* \rightarrow 1$ . QED

## References

- Acharya, Viral V., Denis, Gromb and Tanju, Yorulmazer, 2008. "Imperfect Competition in the Inter-Bank Market for Liquidity as a Rationale for Central Banking," working paper, London Business School.
- Allen, Franklin, Douglas Gale, and Elena Carletti, 2009. "Interbank Market Liquidity and Central Bank Intervention", *Journal of Monetary Economics*, forthcoming.
- Allen, Franklin, and Douglas Gale, 2007. *Understanding Financial Crises*, Oxford University Press, Clarendon Lectures in Finance, Oxford, U.K.
- Bhattacharya, Sudipto, and Douglas Gale, 1987. "Preference Shocks, Liquidity and Central Bank Policy". *New Approaches to Monetary Economics*, Edited by William Barnett and Kenneth Singleton, Cambridge University Press.
- Boyd, John H., Gianni De Nicolò and Elena Loukoianova, 2009, "Banking Crises and Crisis Dating: Theory and Evidence", IMF Working Paper 09/141.
- Boyd, John H., Gianni De Nicolò and Abu M. Jalal, 2009, "Bank Competition, Risk and Asset Allocations", IMF Working Paper 09/143.
- Cai, Jian, and Anjan V. Thakor, 2008. "Liquidity Risk, Credit Risk, and Interbank Competition", Working paper Olin Business School, W.D.C.
- Carletti, Elena, Philipp Hartmann and Giancarlo Spagnolo, 2007. "Bank Mergers, Competition, and Liquidity", *Journal of Money, Credit, and Banking*, 39 (5), pp. 1067-1105.
- De Nicolò, Gianni, and Myron L. Kwast, 2002. "Systemic Risk and Financial Consolidation: Are They Related?", *Journal of Banking & Finance*, 26, 861-880.
- De Nicolò, Gianni, Peter Hayward, and Ashok Vir Bhatia, 2004, "U.S. Large Complex Banking Groups: Business Strategies, Risks and Surveillance Issues," IMF Country Report 04/228, July, pp. 72-86 (Washington: International Monetary Fund).
- De Nicolò, Gianni, Phillip Bartholomew, Jahanara Zaman and Mary Zephirin, 2004, "Bank Consolidation, Internationalization and Conglomeration: Trends and Implications for Financial Risk," *Financial Markets, Institutions & Instruments*, Vol. 13, No. 4, pp. 173 -217.
- Diamond, Douglas W., and Phillip H. Dybvig, 1983. "Bank Runs, Deposit Insurance, and Liquidity". *The Journal of Political Economy*, 91 (3), pp. 401-419.
- Diamond, Douglas W., and Raghuram G. Rajan, R., 2005. "Liquidity shortages and banking crises". *Journal of Finance*, 60, 615-647.
- Diamond, Douglas W, and Raghuram G. Rajan, R., 2008. "Illiquidity and interest rate policy". Working paper, University of Chicago.
- ECB, Dember 2009. "Financial Stability Review". European Central Bank, Frankfurt Am Main.
- Fahlenbrach, Rudiger, and Renè M. Stulz, 2009. "Bank CEO incentives and the Credit Crisis", NBER Working Paper n. 15212.
- Flannery, Mark J, 1996. "Financial Crises, Payment System Problems and Discount Window Lending," *Journal of Money, Credit and Banking*, 28, 804-824.
- Freixas, Xavier, Parigi, Bruno M., and Jean-Charles Rochet, 2000. "Systemic Risk, Interbank Relations, and Liquidity Provision by the Central Bank". *Journal of Money, Credit and Banking*, 32 (3), 611-638.

- Freixas, Xavier, Martin, Antoine, and David Skeie, 2009. "Bank Liquidity, Interbank Markets, and Monetary Policy". Federal Reserve Bank of New York Staff Report N. 371.
- Freixas, Xavier, and José Jorge, 2008. "The Role of Interbank Markets in Monetary Policy: A Model with Rationing". *Journal of Money, Credit and Banking*, 40 (6), 1151-1176.
- Furfine, Craig H., 2000. "Interbank Payments and The Daily Federal Funds Rate". *Journal of Monetary Economics*, 46, 535-553.
- Gorton, Gary, and Lixin Huang, L., 2004. "Liquidity, efficiency and bank bailouts". *American Economic Review* 94, 455-483.
- Gorton, Gary, and Lixin Huang, L., 2006. "Banking Panics and Endogeneity Coalition Formation". *Journal of Monetary Economics* 53, 1613-1629.
- Goodhart, Charles, and Gerhard Illing, 2002. "Financial Crises, Contagion, and the Lender of Last Resort". Oxford University Press.
- Group of Ten, 2001, Report on Consolidation in the Financial Sector, January, Basel.
- Hartmann, Philipp, Stefan Straetsman, and Casper de Vries, 2005, "Banking System Stability: A Cross-Atlantic Perspective," Working Paper No. 527, September (Frankfurt: European Central Bank).
- Heider, Florian, Maria Hoerova and Cornelia Holthausen, 2009. "Liquidity Hoarding and Interbank Markets Spreads: The Role of Counterparty Risk". ECB Discussion Paper, 2009-11S.
- Houston, Joel F., and Kevin J Stiroh, 2006, "Three Decades of Financial Sector Risk," U.S. Financial Holding Companies," July (unpublished; New York: Federal Reserve Bank of New York).
- Repullo, Rafael, 2005. "Liquidity, Risk Taking, and the Lender of Last Resort". *International Journal of Central Banking*. Vol. 2, pp. 47-80.
- Stiroh, Kevin J, 2004, "Diversification in Banking: Is Noninterest Income the Answer?" *Journal of Money, Credit and Banking*, Vol. 36, No. 5, pp. 853-882.
- Stiroh, Kevin J, and Adrienne Rumble, 2006, "The Dark Side of Diversification: The Case of U.S. Financial Holding Companies," *Journal of Banking and Finance*, Vol. 30, No. 8, pp 2131-2161.

**Author contacts:**

**Marcella Lucchetta**

European University Institute, Florence

Robert Schuman Centre for Advanced Studies

Villa La Pagliaiuola

Via delle Palazzine, 17/19

I-50014 San Domenico di Fiesole

Email: [Marcella.Lucchetta@eui.eu](mailto:Marcella.Lucchetta@eui.eu)

