

Block Order Restrictions in Combinatorial Electric Energy Auctions

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Abstract

In Europe, the auctions organized by “power exchanges” one day ahead of delivery are multi-unit, double-sided, uniformly priced combinatorial auctions. Generators, retailers, large consumers and traders participate at the demand as well as at the supply side, depending on whether they are short or long in electric energy. Because generators face nonconvex costs, in particular startup costs and minimum run levels, the exchanges allow “block orders” that are all-or-nothing orders of a given amount of electric energy in multiple consecutive hours, while the standard order consists of an amount for a single hour that can be curtailed. All exchanges restrict the size (MWh/h), the type (span in terms of hours) or the number (per participant per day) of blocks that can be introduced. This paper discusses the rationale of block order restrictions. Based on simulations with representative scenarios, it is argued that the restrictions could be relaxed, which some exchanges have already started doing.

Keywords OR in energy, E-commerce, Combinatorial Auctions/bidding, Pricing, Integer programming

1. Introduction

In Europe, the auctions organized by “power exchanges” one day ahead of delivery are an increasingly important part of the wholesale market (Meeus et al., 2005). Although participation is voluntary and the average traded volume is only about 10% of consumption, the hourly auction price is an important reference price for all contract negotiations. Generators, retailers, large consumers and traders increasingly participate at the demand as well as at the

supply side, depending on whether they are long or short in electric energy.

The orders that can be introduced at these auctions are for the delivery or off-take of electric energy during an hour of the next day. The exchanges also allow “block orders” that are all-or-nothing orders of a given amount of electric energy in multiple consecutive hours. An auction with block orders can therefore be called a combinatorial auction. Combinatorial auctions have in common that orders can be placed on combinations of heterogeneous

items, called packages or bundles, rather than just on individual items. An inspiring and comprehensive work on this topic is the book edited by Cramton, Shoham and Steinberg (2005). Combinatorial auctions have recently been employed in a variety of industries. De Vries and Vohra (2003) provide a comprehensive survey.

The advantage of combinatorial auctions is that participants can more fully express their preferences, such as complementarities between heterogeneous items. In electricity markets, there are complementarities between deliveries of electric energy in consecutive periods, for instance because of start-up costs of power plants. Block orders can indeed be seen as a combination of hourly orders. Blocks allow participants to provide an average price for a combination of hours. On average generators can offer cheaper prices for delivery in multiple consecutive hours as this allows them to spread out the start-up cost.

Both exchanges and participants consider blocks as important. On some exchanges up to 20% of total traded volume consists of block orders. Still, all exchanges restrict the size (MWh/h), the type (span in terms of hours) or the number (per participant per day) of blocks that can be introduced. This paper therefore analyses the rationale of block order restrictions.

Limiting the allowable combinations is known to be effective in reducing computational complexity (Pekec and Rothkopf, 2003; Park and Rothkopf, 2005). This and other reasons to restrict the use of block orders on exchanges are investigated by solving to optimality representative scenarios, based on the historical aggregated order curves of APX, to which sets of block order are added with various degrees of restrictions.

Section 2 explains how the representative scenarios have been constructed. Section 3 introduces the model that is used for the simulations. It therefore also introduces the auction optimization problem with blocks and the pricing approach applied by exchanges to clear their markets. Section 4 then discusses the effect of restrictions, based on the simulation results. Section 5 finally evaluates the restrictions imposed by exchanges.

2. Representative scenarios

The power exchanges with blocks are APX (Netherlands), Belpex (Belgium), Borzen (Slovenia), EEX (Germany), EXAA (Austria), Nord Pool (Norway, Sweden, Denmark and Finland) and Powernext (France). As illustrated in Table 1, the kind of blocks that can be introduced to these exchanges differ substantially.

Table 1: Block order restrictions on APX, Belpex, Powernext and EEX

	Nr block types	Max nr blocks / day / participant	Max size (MWh/h)
APX	354 ¹	50	50
Powernext	10	INF ²	100 ³
EEX	11	6	250

1 All combinations of consecutive periods are allowed
2 Per portfolio it is possible to submit every type once, but participants can submit several portfolios
3 Before 2005 it was 50 MWh

Powernext for instance does not restrict the number of block orders that can be submitted per participant per day, while the size is for instance more restricted on APX (50MWh/h) than on EEX (250MWh/h). On APX, any combination of consecutive hours is allowed so that 354 types of block orders can be traded. Powernext and EEX on the other hand restrict blocks to 10 or 11 types. Table 2

illustrates the 10 block types that can be traded on Powernext.

Table 2: Block products on Powernext

Contract name	Time interval
Block Bid 1-4	00.00h – 04.00h
Block Bid 5-8	04.00h – 08.00h
Block Bid 9-12	08.00h – 12.00h
Block Bid 13-16	12.00h – 16.00h
Block Bid 17-20	16.00h -20.00h
Block Bid 21-24	20.00h – 24.00h
Block Bid 1-24	00.00h – 24.00h
Block Bid 9-20	08.00h – 20.00h
Block Bid 1-6	00.00h – 06.00h
Block Bid 1-8	00.00h – 08.00h

The scenarios used in this paper are based on the historical aggregated order curves of the Dutch power exchange APX. Their order curves are publicly available, which is not the case for most other exchanges. The 19 days illustrated in Table 3 have been randomly selected. APX launched their day-ahead auction in 1999 and its liquidity has since steadily increased as can be seen from the table.

Table 3: Days used for scenarios

Date (DD/MM/YY)	Average price (€/MWh)	Maximum price (€/MWh)	Total traded volume (MWh)
15/01/03	32	108	32636
27/03/03	30	41	31240
20/05/03	33	91	32874
04/07/03	33	100	27691
22/11/03	36	96	34102
22/02/04	20	26	34474
19/04/04	29	41	35864
15/06/04	35	70	31357
18/08/04	31	44	35279
21/10/04	32	42	38886
10/12/04	36	75	46350

29/01/05	33	44	50146
10/02/05	36	45	42239
25/03/05	39	60	46373
03/04/05	26	50	40843
07/05/05	32	42	42964
25/05/05	43	80	35119
26/06/05	31	46	47448
20/07/05	45	63	47792

These days are from different years, seasons, week-weekend. The hourly orders are extracted from these curves. Every scenario includes the hourly orders of one of these days. To simulate the effect of adding blocks to these representative days, sets of blocks are generated with various degrees of restrictions as follows:

- To study the effect of a type restriction, in half of all scenarios blocks can be of any type, as on APX, while in the other half, block are restricted to the 10 types found on Powernext (Table 2). Note that the Powernext types have been chosen because they are most restrictive.
- To study the effect of a size restriction, every scenario has a maximum block size between 10 and 300MWh/h. The blocks in a scenario can therefore have different sizes, but all are smaller than the determined scenario size limit. Note that the size limit considered in the analysis is higher than the largest allowed blocks of 250MWh/h on EEX. Blocks larger than 300MWh/h are not considered because such large capacity plants are base load and typically scheduled outside the exchanges.
- To study the effect of an number restriction, the number of blocks in a scenario ranges between 0 and 200. Note that if 200 blocks would be submitted, their share in total traded volume in the scenarios would be larger as it currently is on the exchanges. As mentioned in the introduction,

blocks are said to represent up to 20% on some exchanges. Given an average block size of 150MWh/h, 200 blocks correspond to 30000MWh/h. For a block that on average spans 8 hours (1/3 of a day), this corresponds to a total volume of 1000MWh/day, which is up to 35% of the total traded volume on the days used to construct scenarios (Table 3).

Additionally, the following assumptions in line with what can be observed on exchanges, have been made:

- Blocks are as likely to be introduced at the demand and supply side
- Blocks are price-setting orders, meaning that their prices are significantly different from zero and close to the market prices. Their price limits have been generated so that they deviate less than 10%, from the average price of the day (Table 3).
- The maximum admissible order price limit (P_{max}) is 2500€/MWh, as on APX. Note that this is not intended to be a price cap but rather to protect against human error.

A batch of 200 scenarios has been created in the manner explained above. The results are presented in Section 4. Increasing the batch size to 200 has proved to be sufficient to present results that are not batch specific. The next Section explains how the scenarios are solved to optimality.

3. Auction optimization problem with blocks

Combinatorial auctions are typically difficult to solve optimization problems (Xia et al., 2005). This is also the case for the auction problem with blocks. The all-or-nothing constraint of block orders means that binary variables are necessary to model the auction problem. Models with binary variables for blocks and constrained continuous variables for hourly orders are

Mixed Integer Linear Problems (MILP), which are difficult to solve.

With,

- hourly orders characterized by the hour (h) in which they are introduced, whether they are supply (i) or demand (j) and by a price (€/MWh) and quantity (MWh) limit (P_h, Q_h);
- block orders characterized by the hours included in the block ($h \in H$), whether they are supply (k) or demand (l) and by an average price (€/MWh) and quantity (MWh/h) limit (P, Q);
- nH the number of hours included in a block;
- block orders having a binary variable to implement the all-or-nothing constraint ($b=1$ if block is accepted; $b=0$ otherwise);
- block orders having a quantity limit for every hour to simplify the notation, which is zero for the hours not included in the block ($Q_h=0$ if $h \notin H$);
- the accepted order quantities ($q_{ih}, q_{jh}, q_{kh}, q_{lh}$) as the decision variables;

The auction optimization problem with blocks is as follows: maximize total gains from trade (or trade efficiency),

$$\text{Max} \sum_h \left(\sum_j q_{jh} P_{jh} + \sum_l q_{lh} P_{lh} - \sum_i q_{ih} P_{ih} - \sum_k q_{kh} P_{kh} \right) \quad (1)$$

subject to market clearing constraints, equalizing demand and supply in every hour:

$$\forall h: \sum_i q_{ih} + \sum_k q_{kh} = \sum_j q_{jh} + \sum_l q_{lh} \quad (2)$$

and the order constraints:

$$q_{ih} \leq Q_{ih} \quad (3)$$

$$q_{jh} \leq Q_{jh} \quad (4)$$

$$q_{kh} = b_k Q_{kh} \quad (5)$$

$$q_{ih} = b_l Q_{ih} \quad (6)$$

Combinatorial auctions are non-convex. This means that linear market clearing prices do not necessarily exist (see for instance Scarf, 1994 and Elmaghraby, 2004). If there are no hourly prices at which demand equals supply, one possibility is to resort to nonlinear pricing (see O'Neill et al., 2005 for a discussion on how shadow prices can be used to implement nonlinear pricing). Nonlinear pricing means that the optimal solution to (1)-(6) in terms of traded volumes (q, MWh) would be settled at hourly prices (p, €/MWh) in combination with a side payment (A, €) which can be different for all orders, i.e. resulting in a “pq + A” settlement.

Exchanges in Europe however have in common that they do not use side payments to clear their day-ahead auction markets (A=0). Instead, they equalize demand and supply at hourly prices by rejecting blocks that should be accepted looking at the hourly prices, i.e. Paradoxically Rejected Blocks (PRB). Note that blocks are however only accepted when they should be and hourly orders are cleared (accepted and rejected) completely in accordance with the hourly prices. To get the optimal solution with the above characteristics, the following constraints including the hourly prices (p_h) need to be added to the auction problem (1)-(6):

First, if a supply block is accepted ($b_k = 1$), the average market price should be at least as high as the price limit of the block, with nH the number of hours included in a block:

$$\forall k : b_k nH_k P_k \leq \sum_{k \in H_k} p_k \quad (7)$$

Equally, if a demand block is accepted ($b_l = 1$), the average market price should not be higher than the price limit of the block, with P_{\max} the maximum

admissible price for an order:

$$\forall l : \sum_{l \in H_l} p_h \leq nH_l (P_l + P_{\max} (1 - b_l)) \quad (8)$$

Second, if an hourly supply order or offer is accepted ($b_{ih} = 1$), the hourly price (p_h) needs to be at least as high as the price limit of the offer (P_{ih}), with b_h a binary variable equal to one if the hourly order is accepted:

$$\forall i, h : b_{ih} P_{ih} \leq p_h \quad (9)$$

Equally, if an hourly demand order or bid is accepted ($b_{jh} = 1$), the hourly price (p_h) cannot be higher than the price limit of the bid (P_{jh}):

$$\forall j, h : p_h \leq P_{jh} + P_{\max} (1 - b_{jh}) \quad (10)$$

Third, partially rejected or curtailed hourly orders should set the price. Therefore, if an offer is partially rejected ($b_{ih} = d_{ih} = 1$) or completely ($b_{ih} = d_{ih} = 0$), the hourly price cannot be higher than the price limit of the offer, with d_h a binary variable equal to one if the hourly order is partially rejected:

$$\forall i, h : p_h \leq P_{ih} + P_{\max} (b_{ih} - d_{ih}) \quad (11)$$

Equally, if a bid is partially rejected ($b_{jh} = d_{jh} = 1$) or completely ($b_{jh} = d_{jh} = 0$), the hourly price needs to be at least as high as the price limit of the bid:

$$\forall j, h : P_{jh} - P_{\max} (b_{jh} - d_{jh}) \leq p_h \quad (12)$$

All exchanges impose linear prices, which means that every day they solve the optimization problem (1)-(12). If they would drop constraints (7)-(12), they would increase gains from trade (and avoid PRBs), but trade would have to be settled by using side-payments.

As mentioned earlier, exchanges have however chosen to avoid the complexities of a settlement with side payments. Simplicity can indeed be considered as

an important design feature of the exchanges in their role of fine tuning market of which the reference price is more important than the volume they clear directly.

4. Effect of block order restrictions

A batch of 200 scenarios has been solved to optimality according to the MILP model (1)-(12) on a Pentium® IV, using the CPLEX v11.0® solver software called from Matlab® using the Tomlab® interface.

In two scenarios, the optimal solution was not yet found after 2.5 days so that the solver was stopped. For all other scenarios, the solver calculation time is 4 minutes on average. The minimum and maximum calculation time is respectively a few seconds and 3.5 hours. 50% of the scenarios solve in less than one minute and 95% less than 10 minutes. This is typical for the performance of commercial MILP solvers.

The optimal solution to the MILP model (1)-(12) yields 4.15 PRBs per day on average, with a maximum of 27 in a day. In total, there are 829 PRBs for 19619 blocks in these scenarios. Therefore, the likelihood of blocks to be paradoxically rejected is only 4.36%. It is important to note that almost 40% of these PRBs are actually not losing any money, i.e. their price limit is equal to the average market price, but other blocks lose up to 18€/MWh/h.

In the remainder of this Section, the effects of restricting the use of blocks on calculation time, the number of PRBs and trade efficiency are considered based on the simulation results.

4.1 Calculation time

Pekec and Rothkopf (2003) discuss non-computational approaches to mitigating computational problems in combinatorial auctions. Limiting the combinations participants are allowed to bid is

described as an effective way to reduce the computational complexity of combinatorial auctions. Park and Rothkopf (2005) even propose an auction with bidder-determined allowable combinations.

Also in combinatorial electric energy auctions this is true. As discussed in the Section 2, in 50% of the scenarios every combination of consecutive hours is allowed, while in the other 50% of scenarios only have the 10 combinations that are allowed at Powernext. The difference in calculation time between these scenarios is illustrated in Figure 1.

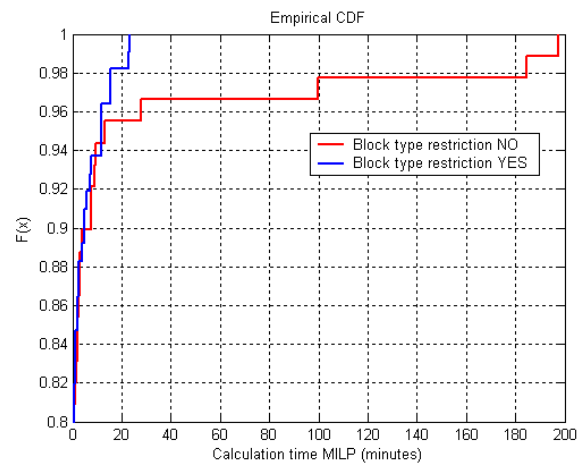


Figure 1: Calculation time MILP model (1)-(12) in minutes with and without a block type restriction

As illustrated in the figure, the group of scenarios in which the allowed combinations or block types are not restricted has more extreme outliers. Indeed, also the two scenarios not indicated in the figure that were stopped after 2.5 days of calculation are scenarios without a type restriction.

Significant coherence between calculation time and the number or size of blocks in the scenarios could not be found. One could expect a correlation between the number of blocks and the solver calculation time, as the number of blocks increases the problem size in terms of binary decision variables, but

such a correlation could not be found. The correlation in the batch of 200 scenarios is only 0.041 and not significant. This can be partly explained by the fact that binary variables are also assigned to hourly orders and the number of hourly orders differs more between scenarios than the number of blocks.

Note that if linear prices are not imposed on the clearing, the calculation time significantly reduces to 0.6 seconds on average with a maximum of 1.4 seconds. This clearly indicates that the most significant computational complexity comes from constraints (7)-(12) and the binary variables that need to be assigned to the hourly orders to implement these constraints and therefore not from the number of blocks.

4.2 Paradoxically Rejected Blocks (PRB)

On average 4.36% of the blocks are paradoxically rejected. This indicates that it is not that big of an issue for the auction participants, which has been confirmed by talking to traders. Still, this paragraph will respectively consider whether block type, size and number restrictions are an effective way of reducing the number or likelihood of PRBs.

Table 4 compares the PRBs of the scenarios with and without a type restriction. There is no significant difference in the number of PRBs between these categories of scenarios. The null hypothesis that the means are equal, assuming a normal distribution for both samples and equal standard deviations cannot be rejected for a 5% significance (p-value is 0.1585).

Table 4: Effect block type restriction on PRB

Nr PRB	All types	Powernext types
Mean	3.6	4.5
Standard deviation	3.6	5.2

From the combinatorial nature of blocks, it can be expected that small blocks are less likely to become paradoxically rejected. Indeed, for instance only 1% of blocks smaller than 50MWh/h are paradoxically rejected, which is four time less than the average for blocks. However, as indicated in Table 5, there is no significant correlation between the likelihood of PRB and the maximum block size. Such a correlation would appear if all blocks in the scenarios are taken equal to the maximum block size, but what these results indicate is the presence of large blocks does not increase the likelihood that small blocks are paradoxically rejected.

It can also be expected that the number of PRBs increases with the number of blocks. The results in Table 5 confirm this, but also indicate that the increase is more or less proportional, as there is no significant correlation between the likelihood of PRB and the number of blocks in a scenario.

Table 5: Linear effect size and number of blocks on PRB throughout the whole range of that data

Correlations (linear regression R^2)	Nr blocks	Maximum block size
Nr PRB	0.6407 (41.4%)	0.3053 (9.3%)
Likelihood PRB (Illustrated in Figure 2)	-0.0362 (0.13%)	0.2139 (4.6%)
Likelihood PRB blocks < 50MWH/h	0.103 (1%)	0.181 (2.2%)

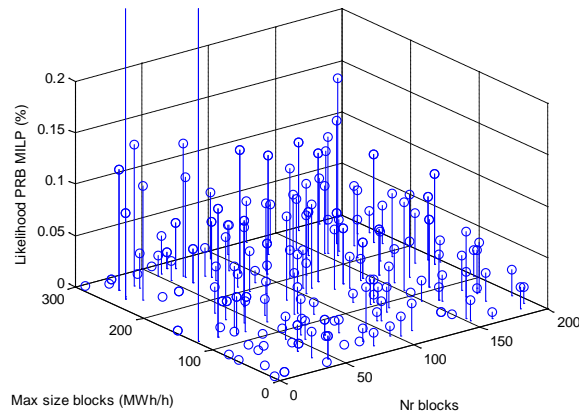


Figure 2: Likelihood PRB in MILP model (1)-(12)

4.3 Trade efficiency

The value of the objective function (1) is largely driven by the hourly orders because there are many price taking hourly orders. This does not mean that power exchanges should simply stop using block orders and thereby avoid the complexity of dealing with them. On the contrary, blocks are important for market parties and represent up to 20% of traded volume on the exchanges.

This does however explain why restricting the number, size or types does not have a statistically significant effect on the total gains from trade. This also explains why imposing linear prices only results in a loss of .0.05% in terms of gains from trade.

Note that the lost value is linked to paradoxically rejected blocks and can therefore be avoided by applying nonlinear pricing. However, this would also mean that side payments would have to be made. Applying the nonlinear pricing approach introduced in O'Neill et al. (2005) to the 200 scenarios, would for instance mean that 317393€ side payments need to be made in total. This is almost 9 times more than the total gains from trade that can be won by making these

side payments. Note that only blocks would receive side payments, the average payment being 502€

5. Evaluation of restrictions

From the previous section can be concluded that a block type restriction is an interesting option to consider. The results indicate that a type restriction has a clear effect on the solver calculation time and reducing this time can be of interest to exchanges that typically have only between 15 and 30 minutes to clear their day-ahead auctions. A type restriction is also not necessarily binding for the auction participants as blocks are mainly introduced for base load, peak load, etc and the allowed combinations typically match these periods.

From the previous section could also be concluded that the number of blocks and their size should not be restricted. The simulations clearly indicate that these restrictions have no significant impact on calculation time, the likelihood of PRB or trade efficiency. Still, it can be explained why all exchanges have such restrictions. One possible explanation is that participants were not used to trade blocks under the linear pricing regime introduced by power exchanges, which has been introduced in this paper and which is very different from the pricing approaches in other combinatorial auctions, so that every PRB is a potential complaint for starting exchanges. Note however that restricting the use of blocks is an artificial way of reducing PRBs. The real solution would be to avoid PRBs by resorting to nonlinear pricing.

It is also sometimes said that the unrestricted use of blocks would increase price volatility. For immature or illiquid markets with a lack of hourly orders, the lumpiness of blocks can indeed be an issue for the formation of prices. The scenarios used in this

paper are based on APX from 2003 to 2005, which is more than 4 years after the exchange started in 1999. The results indicate that for mature markets the impact on prices of adding blocks is limited. In other words, there are ways to explain why exchanges have introduced these restrictions, but as these markets have matured it is time for them to omit or at least relax them.

Note that the size restrictions are currently clearly binding for traders. Generation units are easily larger than 50 MW and even larger than 250 MW. Because blocks can be paradoxically rejected, submitting 5 blocks of 50 MWh/h is not the same as submitting a block of 250 MWh/h.

7. Conclusions

The simulation results presented in this paper argue against restricting the use of blocks in the day-ahead auctions organized by exchanges. It is in the benefit of exchanges and auction participants to omit or at least relax these restrictions. Some exchanges have already starting doing that. The French Powernext has for instance doubled the allowed block size from 50 to 100 MWh/h and more recently also allows more combinations of hours in a block order.

The simulations are based on representative scenarios using actual order data from the Dutch exchange APX. Block sets with various degrees of block restrictions are added to these scenarios to study the rationale of these restrictions. The results clearly argue against block size restrictions and also against restrictions on the number of blocks a participant can submit per day. Inline with existing combinatorial auction literature (Pekec and Rothkopf, 2003; Park and Rothkopf, 2005), the results however do confirm that limiting the allowable combinations that can be included in a block reduces the solver calculation

time. This could therefore justify a block type restriction.

It has also been explained that order restrictions in general can be justified for starting or illiquid exchanges. For instance the Austrian exchange EXAA introduced blocks in 2003 after one year of operation when the market had somewhat matured. More recently also the Belgian exchange BELPEX started without blocks in 2006, but introduced them after a few months of operation.

Apart from providing guidelines to exchanges on how to deal with blocks, this paper also discusses their particular approach of imposing linear prices in a nonconvex auction. An interesting extension to this work could therefore be to consider this pricing approach for other combinatorial auction settings (see Xia et al. 2004 for an overview of pricing approaches in combinatorial auctions). Specifically towards power exchanges, this work could be extended by considering other combinatorial products. A block in itself is also a restricted product. The auction participants might for instance be interested to combine hours without having to offer the same amount of electric energy in every hour. Note that some exchanges have already started to introduce more flexible combinatorial products and other are looking into this issue.

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