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Abstract

In an otherwise unique-equilibrium model, agents are segmented into a few informational islands according to the signal they receive about others' expectations. Even if agents perfectly observe fundamentals, rational-exuberance equilibria (REX) can arise as they put weight on expectational signals to refine their forecasts. Constant-gain adaptive learning can trigger jumps between the equilibrium where only fundamentals are weighted and a REX. This determines regime switching in aggregate volatility despite unchanged monetary policy and time-invariant distribution of exogenous shocks. In this context, a thigh inflation-targeting policy can lower expectational complementarity preventing rational exuberance, although its effect is non-monotone.

Keywords

Non-fundamental volatility; perpetual learning; comovements in expectations; professional forecasters.

JEL codes: E3, E5, D8.

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Max Weber Fellow, 2010-2011

1 Introduction

What are the determinants of switches in the volatility of macro-variables? In principle, a persistent reduction in the amplitude of business fluctuations can be thought to be either the result of *good policy*, namely a change of policy by some major actor within the economy, or of *good luck,* that is, a decrease of volatility of the exogenous shocks hitting the economy. It is not always easy to distinguish between the two. An example is provided by the intense debate on the sources of the "great moderation" in the 80s' (Stock and Watson, 2002; Mc Connell M.M. and Perez-Quiros, 2000).

This paper presents a simple model where the introduction of signals about expectations of others jointly with adaptive learning can generate shifts in aggregate volatility with unchanged monetary policy and time-invariant distribution of exogenous shocks. Still, it assigns to monetary policy an important but ambiguous role. Policies of tight targeting on inflation can in fact kill the possibility of regimes of high volatility, even if, only marginal hardening is counter productive once high volatility occurs.

I consider a monopolistic competition economy where only a fraction of producers have to set their price before knowing the aggregate price. A policy maker enforces a flexible targeting rule according to his preferred trade-off between output gap and inflation volatility. In this model the actual output gap responds to actual inflation that in turns responds to uninformed producers' aggregate expectation about current inflation. Under homogenous information a unique rational expectation equilibrium exists. In this context two main twists are introduced.

First, the economy is split into two symmetrical islands. On each island, the expectation of each uninformed producer is just a private noisy signal of the forecast of an island-specific type of professional forecaster. The latter is intended as a medium or a statistical office that releases reports on the future course of inflation. Thus, there is an information transmission channel that maps professional forecasts into naive producers' expectations depending on the distribution of the observational noises across the population.

The professional forecasters perfectly observe all the fundamental determinants of inflation, but they also receive a private signal of the other professional forecaster's expectation. That is, each professional forecaster can anticipate the forecasts of the other with some uncertainty. Expectational signals are the only sources of heterogeneity between the professional forecasters since each one observes a signal from the other one.

The introduction of heterogeneous expectational signals can give origin to a multiplicity of rational expectation equilibria. A fundamental equilibrium always exists in which experts just use fundamental information and do not put weight on expectational signals. Then, two rational exuberance equilibria can arise in which experts put weight on expectational signals self-fulfilling rational exuberance. In particular, a multiplicity of equilibria exist under two conditions: i) the monetary policy is not aggressive enough about the inflation target and ii) the map from professional forecasts to producers' expectations entails an amplification of the non-fundamental component. The sum of these two effects can provide the degree of complementarity needed to self-fulfil the predictive power of expectational signals. In the case of a rational exuberance equilibrium, non-fundamental volatility driven by observational noises transmits to actual inflation entailing a regime of higher volatility.

The concept of rational exuberance equilibria is developed in Gaballo (2011). As in a typical sunspot equilibrium, at a rational exuberance equilibrium it is optimal to put weight on some non-fundamental signal if everybody does the same. Nevertheless the two equilibrium concepts are essentially different[1]. The latter require that a commonly understood exogenous signal drives the coordination of agents' beliefs, the former instead originate with *heterogeneous* signals that are *endogenous* to the forecasting rule. Expectational signals are not simple coordination devices, but they entail a signal extraction problem that sustains a multiplicity of equilibria. In fact, as with the model at hand, rational exuberance equilibria can exist where typical sunspots do not.

The second twist is to explore the consequences of professional forecasters acting like econometricians, that is using linear regressions on observables to form their forecasts (Evans and Honkapohja, 2001). In particular, I explore the possibility that they learn with a constant gain, so that exponentially decreasing weights are given to later data. This class of learning algorithms is particularly suited to learn about stochastic processes that are potentially open to sudden structural changes.

The paper proves that whenever rational exuberance equilibria exist at least one of them is learnable under adaptive learning[2]. Moreover, the learnability of rational exuberance equilibria and of the fundamental equilibrium coexists in a large region of the parameter space. Therefore, in this region, constant gain learning selects among them and potentially triggers unpredictable and endogenous jumps among learnable equilibria following few lucky or unlucky expectational aggregate shocks. This is possible as long as a small number of types is considered, so that, when expectational signals are weighted, the impact of observational noises do not vanish into the aggregation. In this way different regime switching in volatility can occur despite unchanged monetary policy and a time-invariant distribution of exogenous shocks.

Nevertheless, the monetary authority still has an important role. By implementing a

[1] For comprehensive reviews on sunspots see Benhabib and Farmer (1999) and Guesnerie (2001).

[2] This is an other difference with typical sunspot equilibria that instead are learneable only in some limited cases under special representations (see Evans and McGhough (2011))

flexible targeting rule the central bank can amplify or dampen the impact of aggregate expectation on actual inflation. In particular, a sufficiently high focus on price stability can prevent a multiplicity of equilibria. Still, a more aggressive monetary policy is *locally* not beneficial whenever rational exuberance is already in play. This means that the change of monetary policy must be drastic. Only a gradual focus on price stability instead is fated to generate periods of even higher volatility. In this sense, *good policy* is not strictly necessary, but it is sufficient to prevent *bad luck* if appropriately conducted.

The dynamic system is finally simulated for several calibrations in the cases of two and more-than-two informational islands. With a finite number of islands the same qualitative results obtain even if the quantitative dimension becomes less important as the number of islands increases.

2 Related literature

Angeletos and Werning (2006) and Hellwig and others (2006) have emphasized the importance of endogenous signals in restoring multiplicity in the static version of the benchmark currency attack model when agents are privately uncertain about the fundamentals. As shown in more detail in Gaballo (2011), the effect of expectational signals goes beyond their fundamental content. They do not just restore multiplicity, but they can give origin to a multiplicity of equilibria in otherwise unique-equilibrium static models even when agents are perfectly informed on fundamentals.

The attempt to reconcile macro-volatility regime switches with constant gain learning is shared with Branch and Evans (2007). In their model, stochastic volatility is the result of an evolutionary competition among different misspecified predictors. Sargent and others (2008) also use misspecified constant-gain learning to explain the rise and fall of South American inflation. The results in this paper do not rely on misspecified forecasting rules, but rather on informational frictions.

Other works have proved that a very aggressive monetary policy stabilizes macro-volatility when agents learn. Orphanides and Williams (2005) show that when agents use perpetual adaptive learning schemes then reduced price fluctuations are reflected in a low volatility of expectations, which in turn stabilizes output. In Branch and others (2009) agents choose a level of attention by balancing the effect of active monetary policy on output volatility. Adam (2009) proves the local uniqueness of an equilibrium where fully rational inattentive firms can more easily process data when prices are stable, contributing to overall stability. In the present model there are two important differences: the change in monetary policy has to be drastic to be beneficial when exuberance is already in play, and *good policy* is not necessary for the stability of the system.

3 Baseline model

3.1 Aggregate supply with uninformed suppliers

This section introduces a simple model to address the issue of regime switching in volatility driven by heterogeneous expectations. The basic framework (details in appendix) is a textbook monopolistic competition model populated by a continuum of suppliers with unit mass along the lines of Woodford (2003).

To allow for heterogeneous expectations, I assume there is a fraction $1-\tau$ of firms that price their product before knowing the current aggregate price. The log-linearized optimal pricing rule of uninformed producers is

$$p_{i,t} = E^i_{t-1}p_t + \omega_y y_t + \omega_c c_{i,t} + \widetilde{z}_{t-1}, \tag{1}$$

where, $E^i_{t-1}p_t$ denotes the expectation of producer i of the aggregate price p_t, y_t is the output gap, $c_{i,t}$ is the consumption by workers type i, $\widetilde{z}_{t-1}$ is a a predetermined technology shock drawn from a normal distribution centred on zero with finite variance σ_z, finally ω_y and ω_c are deep parameters. The rest of the producers are perfectly informed of the aggregate price, so their price is given by (1) with $E^i_{t-1}p_t = p_t$. In other words, each firm knows the aggregate quantity supplied and the consumption of its own workers, but only a fraction τ sets price before knowing others' current pricing[3]. After aggregation we have

$$p_t = \tau p_t + (1-\tau)\, E_{t-1}p_t + (\omega_y + \omega_c)\, y_t + \widetilde{z}_{t-1}$$

where

$$E_{t-1}p_t \equiv (1-\tau)^{-1} \int_0^{1-\tau} E^i_{t-1}p_t \; di \tag{2}$$

denotes the average expectation across uninformed firms. After simple manipulation, the aggregate supply (AS) relation is written as

$$y_t = \kappa\, (p_t - E_{t-1}p_t) + z_{t-1}, \tag{3}$$

where z_{t-1} is an appropriate re-scaling of $\widetilde{z}_{t-1}$and $\kappa \equiv ((\omega_y + \omega_c))^{-1}\, (1-\tau) > 0$ captures the degree of strategic complementarity in price setting determined by the degree of market power, the technology process, the elasticity of consumption and the fraction of uninformed producers. The AS is a kind of new classical Phillips curve encompassing that in Lucas (1973), and Kydland and Prescott (1977).

[3]This informational assumption implies a weaker departure from full knowledge with respect to the one originally postulated in Woodford (2003).

3.2 Monetary policy

A monetary authority has the instruments to successfully implement the following flexible targeting rule

$$\pi_t + \phi y_t = \pi^* + \widetilde{\eta}_t, \tag{4}$$

where $\pi_t \equiv p_t - p_{t-1}$ is the inflation rate, $\widetilde{\pi}$ is the publicly announced inflation target, $\widetilde{\eta}_t$ are i.i.d. white noise shocks with finite variance interpreted as monetary transmission frictions and $\phi \geq 0$ represents the degree of flexibility of the targeting rule. Specifically, $\phi = 0$ entails the most restrictive monetary regime with actual inflation being on average equal to the target. As ϕ increases, the response to inflation becomes weaker during recessions and stricter in expansionary periods. This specification is a simple and general way to embody the implications of different degrees of policy-maker tolerance to deviations from the inflation target conditional on output deviations from the steady state.

3.3 The equilibrium under homogeneous expectations

The policy rule (4) together with (3) yields the following reduced form:

$$\pi_t = \boldsymbol{\alpha}' \mathbf{z}_{t-1} + \beta E_{t-1}\pi_t + \eta_t, \tag{5}$$

where

$$\boldsymbol{\alpha} \equiv \begin{bmatrix} \frac{\phi^{-1}}{\kappa+\phi^{-1}} \\ -\frac{1}{\kappa+\phi^{-1}} \end{bmatrix}, \ \mathbf{z}_{t-1} \equiv \begin{bmatrix} \pi^* \\ z_{t-1} \end{bmatrix}, \ \beta \equiv \frac{\kappa}{\kappa+\phi^{-1}}, \ \eta_t \equiv \frac{\phi^{-1}}{\kappa+\phi^{-1}} \widetilde{\eta}_t,$$

which describes the course of actual inflation given producers' expectations. Notice β measures the impact of aggregate expectations of the current course of inflation. It depends on the structural parameters of the economy and the policy of the monetary authority.

Definition 1 *The fundamental rational expectation equilibrium of the model (FREE) is characterized by*

$$\begin{aligned} \pi_t &= \pi^* - \phi z_{t-1} + \eta_t, \\ y_t &= \psi_{t-1} + \kappa \eta_t \\ E^i_{t-1}\pi_t &= \overline{\pi}_t \equiv \pi^* - \phi z_{t-1} \end{aligned}$$

for each i, that is, the unique stationary sequence of inflation rates, output gaps and individual expectations that satisfies (1), (2), (3) and (5) under the restriction of homogeneous expectations, namely $E_{t-1}\pi_t = E^i_{t-1}\pi_t$ for each i.

The fundamental equilibrium arises when every uninformed producer expects the fundamental inflation rate $\overline{\pi}_t$, that is, the one predicted by the fundamentals of the economy: the inflation target and the predetermined technology shock. The mere existence of the FREE cannot explain macro-volatility switches unless there are structural changes in the distribution of the exogenous shocks or shifts in the preferences of the monetary authority.

4 Introducing heterogeneous expectations

In the following I will introduce heterogeneity in expectations. I will develop the simplest (and more transparent) case when uninformed producers are symmetrically segmented into two informational islands, namely $\imath$ and $\jmath$. Extensions to a finite number of islands can be easily obtained and will be briefly discussed through numerical simulations at the end of the paper.

Professional Forecasters. On each island there is an island-specific type of professional forecasters whose only aim is to truthfully provide the best projections of the inflation course (in the mean square error sense) to uninformed producers inhabiting their own island. Professional forecasters neither produce nor consume. They can be thought of as media or statistical offices that serve a certain industrial district, as well as institutional agencies that release reports on the expected course of the economy based on available information.

The information set of the professional forecaster type ι is

$$\Omega_{t-1}^{\iota} \equiv [\{\mathbf{z}_{\tau}\}_{\tau=-\infty}^{t-1}, \{s_{\iota,\tau}\}_{\tau=-\infty}^{t-1}], \tag{6}$$

with

$$s_{\iota,t-1} \equiv E_{t-1}^{\jmath}\pi_t + \eta_{\iota,t-1},$$

where $\eta_{\iota,t-1}$ is a type-specific white noise measurement error drawn from a normal distribution $N(0,\sigma)$ with zero mean and finite variance σ. The noises in expectational signals are orthogonal to fundamental variables and independently distributed in time. The information set of professional forecaster type $\jmath$ is a mirror image. In other words, professional forecasters observe fundamentals (the inflation target and the predetermined technology shock) and a signal of the simultaneous expectation of the professional forecaster on the other island. Expectational signals captures uncertainty about others' expectations measured by the size of σ. The case of perfect information obtains in the limit of $\sigma \to 0$, whereas the case of no information about others' forecasts arises in the limit of $\sigma \to \infty$.

Individual-specific noises in the expectational signals are the only source of heterogeneity in the information set of the professional forecasters. This feature allow us to divide their forecasting problem into two sequential tasks: estimating the fundamental rate of inflation and estimating non-fundamental fluctuations from the fundamental. In fact, the professional forecasters form expectations about the fundamental rate $\overline{\pi}_t$ conditioning on the commonly observed fundamental variables according to

$$E_{t-1}^{\iota}\overline{\pi}_t = E_{t-1}^{\jmath}\overline{\pi}_t = \overline{\pi}_t^e \equiv \mathbf{a}'\mathbf{z}_{t-1}, \tag{7}$$

where $\mathbf{a}$ is a vector of real weights. The expected fundamental rate is the same since both types of professional forecasters use the same fundamental information.

Even in the case that the professional forecasters correctly estimate the fundamental inflation rate, unexpected fluctuations are caused by the exogenous unobserved disturbance η_t. However, the professional forecasters are uncertain whether such departures are truly exogenous or partly due to expectations of non-fundamental fluctuations held on the other island. In this case the expectational signal would be a useful predictor of course of the actual inflation. Therefore, the professional forecasters estimate non-fundamental fluctuations around the estimated fundamental rate using the linear rule

$$\begin{aligned} E_{t-1}^{\iota}\pi_t - \overline{\pi}_t^e &= b\ \left(E_{t-1}^{\jmath}\pi_t + \eta_{\iota,t-1} - \overline{\pi}_t^e\right), \\ E_{t-1}^{\jmath}\pi_t - \overline{\pi}_t^e &= c\ \left(E_{t-1}^{\iota}\pi_t + \eta_{\jmath,t-1} - \overline{\pi}_t^e\right), \end{aligned}$$

where b and c are weights to be determined. Following this rule, each professional forecaster expects a departure of the actual inflation from the fundamental rate proportional to his own noisy perception of the other's mirror-like expectation. Solving the equations above for the professional forecasts, we can rewrite them as functions of the expected fundamental rate and observational errors. We have

$$E_{t-1}^{\iota}\pi_t = \overline{\pi}_t^e + \frac{bc}{1-bc}\eta_{\jmath,t-1} + \frac{b}{1-bc}\eta_{\iota,t-1}, \tag{8a}$$

$$E_{t-1}^{\jmath}\pi_t = \overline{\pi}_t^e + \frac{bc}{1-bc}\eta_{\iota,t-1} + \frac{c}{1-bc}\eta_{\jmath,t-1}, \tag{8b}$$

with $bc \neq 1$. Notice that both types of professional forecasts are determined by both b and c. In particular they collapse to (7) if and only if both b and c are equal to zero.

Uninformed producers. Actual inflation reacts to the aggregate expectation of uninformed agents, and not directly to that of professional forecasters. I assume that uninformed producers do not have a particular theory of how the economy works, but rather they rely on the expectations of a more sophisticated agent. Imagine that although the reports of the professional forecasters are public they can be perceived or interpreted in different ways. For the sake of simplicity, I assume producer i on the island ι holds the following naive expectation:

$$E^{i}_{t-1}\pi_t = s_{i,t-1} \equiv E^{\imath}_{t-1}\pi_t + \eta_{i,t-1},$$

where $\eta_{i,t-1}$ is an individual-specific white noise measurement error drawn from a normal distribution with zero mean and finite variance. In other words, producers' expectations are just their own noisy perceptions of the professional forecast on their own island. The aggregate expectation of the uninformed producers (2) is equal to

$$E_{t-1}\pi_t = \frac{1}{2}\left(E^{\imath}_{t-1}\pi_t + E^{\jmath}_{t-1}\pi_t\right) + (1-\tau)^{-1}\int_0^{1-\tau} \eta_{i,t-1}\, di. \tag{9}$$

The relation above entails a map from the average professional forecast to the average expectation across uninformed producers. The properties of this map depends on the aggregation of observational noises.

The transmission channel. This structure shapes a relation between experts and the private sector, which is illustrated in figure 1.

[figure 1 about here]

Two types of professional forecasters equally affect the aggregate expectation calculated over a continuum of uninformed suppliers. The aggregate expectation yields the actual inflation rate as implied by (5). The professional forecasters observe fundamentals and have noisy perceptions of each others' expectations.

Notice that the last term in (9) shapes the degree of neutrality of the transmission channel from experts to the private sector: it is zero if and only if the cross-sectional correlation across uninformed producers of expectational signals is zero . Here instead I want to allow for a particular case, that in which the observational noises of uninformed producers have a correlation with the non-fundamental component of the professional forecast they rely on. To capture this effect one can express $\eta_{i,t} \sim N\left(0, \sigma_{i,t}\right)$ in the following way:

$$\eta_{i,t-1} = \gamma(E^{\imath}_{t-1}\pi_t - \overline{\pi}^e_t) + \epsilon_{i,t},$$

where $\epsilon_{i,t}$ is a i.i.d. shock normally distributed according to $N(0,\ \sigma_\epsilon)$ across agents and time. With this form, the aggregate expectation can be written simply as

$$E_{t-1}\pi_t = \overline{\pi}^e_t + (1+\gamma)\left(\frac{E^{\imath}_{t-1}\pi_t + E^{\jmath}_{t-1}\pi_t}{2} - \overline{\pi}^e_t\right), \tag{10}$$

where γ measures the effect of the transmission channel from professional to naive forecasts. Notice that the impact of the non-fundamental component of the professional forecasts on the aggregate expectation is amplified or dampened according to the sign and size of γ.

I focus on this particular assumption for two reasons. First, the relations (7), (8) and (10) together are able to capture well-documented stylized facts recovered from the examination of expectation surveys. Mankiw, Reis and Wolfers (2004) prove important claims about the relation between disagreement in the Survey of Professional Forecasters (SPF) and the Michigan Survey of Consumers' expectations: i) both cross-sectional distributions are closely centred on the right value of inflation, that is, average expectations are almost rational expectations; ii) both present similar slow reactions to news about fundamental macroeconomic data that only account for a small fraction of the overall volatility[4]; iii) both types of expectations present substantial correlation, so they seem to react to similar fundamental *and* non-fundamental shocks. On top of that, the observation that the expectations of the private sector are systematically more volatile than professional ones, although they exhibit a strong correlation but a similar slow reaction to fundamental news[5], supports a positive value of γ. That is, the private sector is more sensitive than professional forecasters to non-fundamental shocks, which in this model are captured by the noises in expectational signals.

A second more pedagogical reason for this ad-hoc assumption is that it allows a simpler understanding of the dynamics. In fact, β and γ will be the only parameters determining the expectational complementarity between the professional forecasters' expectations, which is ultimately what matters for the emergence of a multiplicity of equilibria. With different assumptions on the correlation of expectational signals one can still obtain a multiplicity of equilibria but at the cost of much more cumbersome conditions. One other possibility is to assume a correlation between fundamentals and observational noises as shown in Gaballo (2011), but this would also make the mechanism less transparent. I refer the interested reader to that work. I choose here to prioritize the very purpose of this paper which is to show how constant gain learning can trigger endogenous regime switching in volatility with unchanged monetary policy while keeping the distribution of exogenous shocks time-invariant.

[4]In particular, this can reflect the fact that expectations adaptively incorporates news concerning fundamental data.

[5]Carrol (2003) presents an epidemiological model where the private sector's expectations exhibit even slower reactions to news of macoreconomic fundamentals than do those of professional forecasters.

5 Multiple Learnable Equilibria

5.1 Rational Expectations Equilibria

It is possible now to recover the actual low of motion (ALM) as a function of exogenous shocks only, parameterized by the weights of the forecasting rules used by the two types of professional forecasters. Plugging (8) and (10) into (5), the ALM for inflation is

$$\pi_t = \boldsymbol{\alpha}'\mathbf{z}_{t-1} + \beta\overline{\pi}_t^e + \frac{\beta^*}{2}\left(\frac{b\,(1+c)}{1-bc}\eta_{\imath,t-1} + \frac{c(b+1)}{1-bc}\eta_{\jmath,t-1}\right) + \eta_t \tag{11}$$

where $\beta^* \equiv \beta(1+\gamma)$ measures the final impact of professional forecasts of non-fundamental fluctuations. Equation (11) features *temporary equilibria* of the model, that is, it describes the course of actual inflation for *given* coefficients ($\mathbf{a}$,b, c) of the forecasting rules (7)-(8). The rational expectation equilibrium values of ($\mathbf{a}$,b, c) imply instead that such forecasting rules are optimal linear projections of available information, so that the professional forecasters' mistakes are orthogonal to available information collected in (6).[6] Orthogonality restrictions can be expressed in terms of the so called T-map $(T_{\mathbf{a}}, T_b, T_c)$ as

$$\begin{aligned}
\mathbf{E}[\mathbf{z}_{t-1}\left(\pi_t - T_{\mathbf{a}}\mathbf{z}_{t-1}\right)] &= 0,\\
\mathbf{E}[\left(E_{t-1}^{\jmath}\pi_t + \eta_{\imath,t-1} - \overline{\pi}_t^e\right)\left(\pi_t - \overline{\pi}_t^e - T_b\left(E_{t-1}^{\jmath}\pi_t + \eta_{\imath,t-1} - \overline{\pi}_t^e\right)\right)] &= 0,\\
\mathbf{E}[\left(E_{t-1}^{\imath}\pi_t + \eta_{\jmath,t-1} - \overline{\pi}_t^e\right)\left(\pi_t - \overline{\pi}_t^e - T_c\left(E_{t-1}^{\imath}\pi_t + \eta_{\jmath,t-1} - \overline{\pi}_t^e\right)\right)] &= 0,
\end{aligned}$$

where $(T_{\mathbf{a}}, T_b, T_c)$ represent the optimal weights (in the mean square error sense) for a given calibration ($\mathbf{a}$,b, c) of the inflation process (11). In practice, we can obtain the T-map by working out the orthogonal restrictions above to obtain the explicit form

$$\begin{aligned}
T_{\mathbf{a}}\left(\mathbf{a}\right) &= \boldsymbol{\alpha} + \beta\mathbf{a},\\
T_b\left(b,c\right) &= \frac{\beta^*}{2}\left(\frac{b\,(1+c)\,(1+c\rho) + c(1+b)\,(c+\rho)}{1+c^2+2c\rho}\right),\\
T_c\left(b,c\right) &= \frac{\beta^*}{2}\left(\frac{b\,(1+c)\,(b+\rho) + c(1+b)\,(1+b\rho)}{1+b^2+2b\rho}\right),
\end{aligned}$$

where $\rho \equiv$ corr$[\eta_{\imath,t}, \eta_{\jmath,t}]$ is the correlation among the simultaneous observational errors of the professional forecasters. Notice that the T-map does not depend on the variance of observational errors.

[6] When shocks are normally distributed, this restriction is enough to pin down the forecasting rule that identifies the correct conditional distribution of the variable to be forecasted. Also note that the forecast error of each uninformed producer is $1+\gamma$ times that of the professional forecaster on the same island plus a white noise component. Therefore, the former is also orthogonal to the available information whenever the latter is.

Definition 2 *Rational expectation equilibria are a sequence of actual inflation rates, output gaps and individual expectations as defined by (3)-(8)-(11) for a triple* $(\hat{\mathbf{a}}, \hat{b}, \hat{c}) = (T_{\mathbf{a}}, T_b, T_c)$ *that is a fix point of the T-map.*

The following proposition states the existence of multiple determinate equilibria.

Proposition 3 *The system has one or three rational expectation equilibria. The fundamental rational expectation equilibrium (FREE) with* $(\hat{\mathbf{a}}, \hat{b}, \hat{c}) = \left((1-\beta)^{-1} \boldsymbol{\alpha}, 0, 0\right)$ *always exists. A high rational exuberance equilibrium (hREX) with* $(\hat{\mathbf{a}}, \hat{b}, \hat{c}) = \left((1-\beta)^{-1} \boldsymbol{\alpha}, b_+, c_+\right)$*, and a low rational exuberance equilibrium (lREX) with* $(\hat{\mathbf{a}}, \hat{b}, \hat{c}) = \left((1-\beta)^{-1} \boldsymbol{\alpha}, b_-, c_-\right)$*, where*

$$b_\pm = c_\pm = \frac{\beta^* - (2-\beta^*)\,\rho \pm 2\sqrt{(\beta^*-1)\,(1-\rho^2)}}{2-\beta^*\,(1+\rho)},$$

both exist provided $\beta^* > 1$.

Proof. In the appendix. ■

The signal extraction problem entailed by the expectational signals is crucial to the existence of REX. This occurs whenever $|\mathrm{corr}[s_{\imath,t}, s_{\jmath,t}]| \neq 1$, that is the correlation between the expectational signals is not perfect. In particular, notice that the correlation between expectational signals is endogenous to the given weights b and c. At the REX values $b_\pm = c_\pm$ is

$$\mathrm{corr}[s_{\imath,t}, s_{\jmath,t}] = \frac{2b_\pm + \left(1+b_\pm^2\right)\rho}{2\rho b_\pm + 1 + b_\pm^2},$$

whose absolute value is one only at $b_\pm = c_\pm = 1$ obtained for $\beta^* = 1$ or $\rho = \pm 1$. These are the limit points at which heterogeneity of the information sets vanishes. In such a case expectations are homogeneous and so the FREE is the only equilibrium. For all the other cases in which $b_\pm = c_\pm$ exist and are different from one - that is whenever $\beta^* > 1$ - two REX exist.

Figure 2 plots T_b for four different calibrations. Given the symmetric nature of the problem, REX lie at the intersections of T_b with the bisector. Line a) is obtained for $\beta^* = 0.8$ and $\rho = 0$. In this case there is a unique intersection at the FREE values $\hat{b} = \hat{c} = 0$. For β^* greater than one, two rational exuberance equilibria (REX) emerge. Ceteris paribus, increasing values of ρ widen the distance between the low and the high REX values (contrast b) with c)). Finally, an extreme calibration such as that entailing line d) gives rise to negative low REX values.

[figure 2 about here]

The intuition for the result is simple. A multiplicity of equilibria is due to the non-linearity of the T-map originated by the endogeneity and heterogeneity of the expectational signals. These two features create externalities to the individual forecasting problem and REX as coordination failures. In fact, both types of professional forecasters could obtain a lower variance of their forecast errors if both coordinated on the FREE values. But suppose type $\imath$ puts weight on his expectational signal. Now $E^{\imath}_{t-1}\pi_t$ reacts to $\eta_{\imath,t-1}$, which therefore affects the course of inflation π_t. Then $s_{\jmath,t-1}$ is now partly informative of actual inflation fluctuations away from the estimated fundamental rate so that type $\jmath$ wants to condition on it, coming in full circle. In other words, if type $\imath$ deviates from the fundamental rate then this creates an incentive for type $\jmath$ to do the same. When the expectational complementarity is strong enough ($\beta^* > 1$) then this mechanism is self-reinforcing and a multiplicity of determinate equilibria - consisting of two REX and a FREE - exists; otherwise putting zero weight on the signals is the only equilibrium forecasting rule.

5.2 Adaptive learning

This section explores the learnability of equilibria, and in particular the possibility of the professional forecasters being stuck in a REX when they learn adaptively with a constant-gain algorithm. The concept of learnability refers to the nature - stable or unstable - of the learning dynamics around the equilibria. Consider that $(\hat{\mathbf{a}},\hat{b},\hat{c})$ are recursively estimated with constant stochastic gradient (CSG) according to

$$\mathbf{a}_t = \mathbf{a}_{t-1} + g\ \mathbf{z}_{t-1}\left(\pi_t - \mathbf{a}'_{t-1}\mathbf{z}_{t-1}\right), \tag{12a}$$

$$b_t = b_{t-1} + g\ \left(E^{\jmath}_{t-1}\pi_t + \eta_{\imath,t-1} - \overline{\pi}^e_t\right)\ (\pi_t - E^{\imath}_{t-1}\pi_t), \tag{12b}$$

$$c_t = c_{t-1} + g\ \left(E^{\imath}_{t-1}\pi_t + \eta_{\jmath,t-1} - \overline{\pi}^e_t\right)\ (\pi_t - E^{\jmath}_{t-1}\pi_t), \tag{12c}$$

where g is a fixed gain, $(\pi_t - E^{\imath}_{t-1}\pi_t)$ is the forecast error and $\left(E^{\jmath}_{t-1}\pi_t + \eta_{\imath,t-1} - \overline{\pi}^e_t\right)$ is the noisy observed displacements of others' expectations from the estimated fundamental rate. If both estimates b_t and c_t are close to zero, the professional forecasters discard non-fundamental information and forecast the fundamental rate given by (12a). If this is not the case, considering noisy observations actually improves the accuracy of forecasts and the system can possibly converge on a learnable REX.

Definition 4 *An equilibrium entailed by* $(\hat{\mathbf{a}},\hat{b},\hat{c})$ *is locally learnable under a constant-gain learning algorithm if and only if there exists a sufficiently small gain* $\widetilde{g}$ *and some neighborhood* $\Im(\hat{\mathbf{a}},\hat{b},\hat{c})$ *of* $(\hat{\mathbf{a}},\hat{b},\hat{c})$ *such that for each initial condition* $(\mathbf{a}_0,b_0,c_0) \in \Im(\hat{\mathbf{a}},\hat{b},\hat{c})$ *and positive gain* $g \leq \widetilde{g}$ *the estimates converge almost surely in distribution to the equilibrium values.*

The particular constant-gain algorithm (12) is similar to the recursive version of OLS where the estimated correlation matrix is kept equal to the identity matrix and the gain is fixed[7]. CSG converges to an ergodic distribution centred on a fixed point of the T-map whenever recursive OLS asymptotically converges (to a point). CSG like any constant gain learning rule, exhibits perpetual learning since more weight is given to more recent data. This makes this class of algorithms particularly suitable for learning structural changes. In this model, CSG has the advantage of showing convergence to the equilibria and, at the same time, the possibility of endogenous and unpredictable shifts from the FREE to a REX. Recursive OLS on the contrary converge at the cost of a huge stickiness of the dynamics after relatively few repetitions.

To check learnability one needs to investigate the Jacobian of the T-map. If the Jacobian of the T-map computed at the equilibrium values has all eigenvalues lying inside the unit circle, the equilibrium is stable under learning (Marcet and Sargent, 1989; Evans and Honkapohja, 2001). The Jacobian for the T-map takes the form

$$JT(\mathbf{a},b,c) = \begin{pmatrix} \beta & 0 & 0 & 0 \\ 0 & \beta & 0 & 0 \\ 0 & 0 & \frac{dT_b(b,c)}{db} & \frac{dT_b(b,c)}{dc} \\ 0 & 0 & \frac{dT_c(b,c)}{db} & \frac{dT_c(b,c)}{dc} \end{pmatrix}$$

where the explicit forms of the partial derivatives are recovered in the appendix.

Figure 3 shows a numerical analysis for the whole parameter range spanned by β^* and ρ. Remember that a necessary condition for the learnability of equilibria is always $\beta < 1$, given that β is the largest eigenvalue associated with the learning dynamics of the fundamental component of actual inflation. This is assumed in the following discussion.

[figure 3 about here]

The FREE is the only learnable equilibrium in the region $\beta^* < 1$. In the white area a learnable high REX arises besides the FREE. In this case the learning algorithm selects between them. As ρ increases in modulus, FREE and hREX are both learnable for lower values of β^*. Specifically, as ρ approaches unity for a sufficiently high value of β^* the low REX becomes the unique learnable equilibrium. In contrast, as ρ decreases for sufficiently high value of β^* the system presents no learnable equilibria.

[7] The CSG algorithm is derived as the optimal solution to the minimization of the forecast error variance provided agents are "sensitive" to risk in a particular form. For details see Evans, Honkapohja and Williams (2005). The CSG formula is obtained by setting the covariance matrix equal to the unitary matrix in the recursive expression for the constant gain OLS. In order to obtain adjustments comparable with constant gain OLS the gain has to be rescaled by the covariance matrix of the regressors.

Necessary condition for arising and learnability of REX in this model is that $\gamma > \beta^{-1} - 1$, that is, the private sector commits observational errors that positively covary with the estimated departures of actual inflation from the fundamental rate. In that respect, the transmission channel in the economy plays an essential role in that it provides the degree of expectational complementarity needed for the emergence of REX.

Whenever learnable REX exist, the distances between the equilibria measured on the bisector in figure 2 are indicative of the size of the basins of attraction. In particular, at least in the range considered, the T-map behaves like a cubic yielding three dynamic equilibria. As usual, the equilibrium in the middle is either unstable and works as the threshold between two basins of attraction or it is the only stable equilibrium with a basin of attraction lying between the other two. In the case of lines b) and c) in figure 2, the low REX divides the basins of attraction of the FREE and of the high REX. In particular, as β^* increases the latter enhances whereas the former shrinks.

Constant gain algorithms trigger continuous temporary escapes from the equilibrium values. Such escapes can displace the system from one basin of attraction to another. This can occur since only two (or a small number of) islands are considered so that expectational shocks do not vanish in the aggregation and affect current inflation. In this way regime switching in volatility can alternate despite unchanged monetary policy and time-invariant distribution of exogenous shocks.

5.3 Aggregate volatility and monetary policy

From a qualitative point of view non-fundamental volatility generated by rational exuberance has the features of demand-side volatility. The model implies that as long as a decrease in the overall variance is due to a shift from a REX to a FREE, output gaps and hours should exhibit a sudden decrease in volatility whereas labor productivity should be unaffected[8]. This effect is in sharp contrast with a pure good luck theory for which all these variables should present an equal decrease in variation resulting from a weakening of technological shocks.

[figure 4 about here]

Figure 4 plots excess volatility at the learnable REX measured in observational error variance units. For β^* values close to unity excess volatility triggered by the high REX is huge but it initially decreases very soon and then slowly increases again as β^* increases. This turns out to be crucial in the evaluation of the monetary policy conduct. A more aggressive monetary policy, that is a lower ϕ, lowers β and so β^*. For values of ϕ such

[8] Part of the evidence uncovered by Galì and Gambetti (2009) seems to confirm these predictions relative to the "great moderation" puzzle.

that $\beta^* < 1$ rational exuberance cannot emerge. Nonetheless, when rational exuberance is already in play, a gradual decrease in ϕ enhances volatility as β^* approaches unity. In other words, more focus on price stability is not locally optimal. Therefore, the monetary authority must implement a drastic change of policy to avoid high-volatility periods, whereas partial adjustments are harmful whenever rational exuberance is already in play.

6 Simulations

Examples of endogenous and unpredictable regime switching in volatility are provided in this section. I choose calibrations to allow a simple comparison with the analytical results. Without loss of generality, I set $\pi^* = 0$ and $\psi_t = 0$ so that the fundamental rate of inflation is equal to zero at all times. The exogenous shocks are all Gaussian white noises with $\sigma = 0.01$ variance. As baseline setup I start by setting a quite flexible monetary targeting policy with ϕ close to unity. I assume only a rather small fraction of firms τ can adjust their prices instantaneously. The values of the benchmark calibration are $\tau = 0.05$; $\phi = 1$; $\zeta = 6$; $\theta = 20$ so that $\beta = 0.94$. In all the figures the following conventions hold. From top to bottom the three boxes in each figure respectively display the series for: actual inflation, the forecast of the fundamental rate, and the evolution of the weights on expectational signals. Whenever a multiplicity of equilibria exist, their values are indicated by dotted lines in the last two boxes.

6.1 Two islands

Figure 5 displays the benchmark case of convergence in distribution to the FREE values for $\rho = \gamma = 0$. The gain is settled at $g = 0.01/\sigma$. The factor σ has been included in the gain so that the adjustments of both b_t and c_t are substantially equal to those obtained with constant gain recursive least squares around the FREE for 0.01 (see footnote 7). Notice how constant-gain learning generates continuous small displacements away from the fundamental equilibrium values. Such displacements are temporary escapes and do not substantially affect the variance of actual inflation displayed in the first box.

[figure 5 about here]

[figure 6 about here]

Figure 6 shows the occurrence of an endogenous structural change affecting the volatility of actual inflation in a persistent and substantial way. The calibration of figure 5 is modified only by setting $\gamma = 0.14$, so that $\beta^* = 1.08$. Therefore the learnable high REX arises for $\hat{b} = \hat{c} = 1.78$ as shown by figure 3 in correspondence with curve b). For about 1300, periods the dynamics of figures 5 and 6 are indistinguishable. Nevertheless, as estimates approach the low REX values, the course of actual inflation changes dramatically even if the economy is perturbed by the same series of exogenous shocks. In particular, as estimates overcome the low REX values (this happens after about 2000 periods) the dynamics enters in the basin of attraction of the high REX. The extra non-fundamental volatility is about three times the FREE one. This change could be easily misidentified by an external observer as a truly exogenous increase in non-fundamental volatility. Notice also that expectational volatility affects the learning process of the estimated fundamental rate contributing to the overall volatility.

[figure 7 about here]

Figure 7 displays an example of endogenous and unpredictable switches from the FREE to high REX and vice versa. Several features contribute to the result. Firstly γ is cut by half at 0.07 (which makes $\beta^* = 1.01$ very close to unity), but ρ is now fixed at 0.4. For this calibration the two basins of attraction have almost the same size. Moreover, I choose a bigger gain, namely $g = 0.02/\sigma$ to make the estimates more volatile and hence jumps more likely. The next examples will show that similar dynamics can be generated with a less extreme calibration by considering more than two islands.

6.2 More-than-two islands

The generalization of the model to cases with more than two islands is quite straightforward even if analytically a bit cumbersome. In this section I show two simulations with four and eight symmetrical types of professional forecasters. The only modification is that now the professional forecasters receive a signal about the average expectation of all the others. Figures 8 and 9 make clear that similar qualitative results occur. Figure 8 is generated with the same calibration and shock series as figure 6 but with a slightly bigger gain ($g = 0.0125/\sigma$). The high volatility regime originates after 500 periods and a few periods after time 2400 a sufficiently more aggressive monetary policy makes the FREE the unique equilibrium, and so globally stable. The extent of the excess non-fundamental volatility turns out to be only slightly smaller with respect the previous exercises.

[figure 8 about here]

The two jumps in the inflation dynamics in figure 9 are both endogenous. They are obtained with the same calibration as figure 8 but with a much smaller gain $g = 0.005/\sigma$ and a smaller correlation $\rho = 0.2$. As the number of islands increases the high REX values decrease. This relation is intuitive as in the limit one expects the standard case in which REX collapse to the FREE.

[figure 9 about here]

All the same, the dynamics with more than two islands are much more sensitive to changes in correlation among observational errors. In both the above examples a decrease of 0.1 in ρ is enough to prevent the first endogenous unlucky jump. This comes from the fact that decentralized coordination with more islands is in principle more demanding. Nonetheless, increasing the number of islands also has the effect of shrinking both basins of attraction and so jumps are more likely.

7 Conclusion

This study has identified a rational for reconciling the "good luck or good policy" explanations of macro-volatility switches. The concept of rational exuberance equilibria was introduced in a simple monetary model where agents have heterogeneous expectations and, in particular, they are segmented into a few types according to the signals they receive regarding other agents' expectations of inflation.

On the one hand, equilibria entailing non-fundamental volatility can occur when agents put weight on the expectational signals as predictors of business fluctuations. Moreover, when expectations are adaptively formed using constant-gain algorithms, self-fulfilling rational exuberance can arise endogenously as the economy jumps from a fundamental equilibrium, where expectational signals are ignored, to a rational exuberance equilibrium, where agents put weight on expectational signals.

On the other hand, a decrease in non-fundamental volatility can be (even if not necessarily) the result of a drastic tightening of inflation-targeting policy that reduces the impact of expectations in the economy and prevents a multiplicity of equilibria. Nevertheless, timid "improvements" in monetary conduct are counter-productive when exuberance is already in play. The discontinuity in the effect of monetary policy could partly illuminate the difficulties in the implementation of a gradual recovery towards full stability and, on the contrary, the success of a big policy change such as that made possible by the arrival of a new governor.

The paper has focused on the importance of uncertainty about others' expectations as one of the possible sources of non-fundamental volatility. It has pointed out the role of monetary policy in affecting the impact of expectations on the real economy beyond fundamentals, and provided a new argument in favor of tight inflation-targeting.

Appendix A

Households

An index $j \in (0,1)$ denotes a continuum of representative households and $i \in (0,1)$ a continuum of different goods. Each household j consumes a basket of all the goods produced in the economy, and supplies a type j labor input specific to the production of the good $i = j$. Households type j solve

$$\max_{\{C_{j,t};N_{j,t};B_{j,t}\}} \sum_{t=0}^{\infty} \beta^t \left(\log C_{j,t} - \frac{N_{j,t}^{1+\zeta}}{1+\zeta} \right),$$

subject to

$$P_t C_{j,t} + P_t B_{j,t} = P_t W_{j,t} N_{j,t} + \int \Pi_{i,t}\ di + (1 + r_{t-1}) P_{t-1} B_{j,t-1},$$

where $B_{j,t}$ is a bond stock, $W_{j,t}$ is the real wage, $\int \Pi_{i,t}$ is profit deriving from equally distributed ownership of firms[9], r_t is the rate of interest,

$$C_{j,t} \equiv \left(\int C_{j,i,t}^{\frac{\theta-1}{\theta}}\ di \right)^{\frac{\theta}{\theta-1}} \quad \text{and} \quad P_t \equiv \left(\int P_{i,t}^{1-\theta}\ di \right)^{\frac{1}{1-\theta}}$$

are CES indexes with $C_{j,i,t}$ and $P_{i,t}$ being respectively consumption by agent j of the good type i, and the price of the good type i. No-ponzi conditions apply. Trade is frictionless. The optimal supply of labor type j is determined by

$$W_{j,t} = N_{j,t}^{\zeta} C_{j,t}, \tag{13}$$

by combining first-order conditions on labor and consumption. The individual cost-minimizing demand for good i by agent j, namely $C_{j,i,t}$, is given by $C_{j,i,t} = (P_{i,t}/P_t)^{-\theta} C_{j,t}$, where θ is the CES coefficient. The total demand function for good i is given by

$$Y_{i,t} = \int \left(\frac{P_{i,t}}{P_t} \right)^{-\theta} C_{j,t}\ dj = Y_t \left(\frac{P_{i,t}}{P_t} \right)^{-\theta}. \tag{14}$$

In equilibrium the market clears so that the total supply

$$Y_t = \int Y_{i,t}\ di = \int \int C_{j,i,t}\ dj\ di\ , \tag{15}$$

[9] This assumption guarantees that wealth across agents is uniformly distributed irrespective of possibly different ex-post profitability.

is equal to the total demand across agents for all different goods. This is assumed to be known by the firms, which independently decide their production as explained in the following.

Firms

Each firm produces a differentiated good and sells it in a monopolistic competitive market. Firms set their prices to maximize profits. The technology for the production of the good i is given by the following

$$Y_{i,t} = e^{\psi_{t-1}} N_{i,t}, \tag{16}$$

where ψ_t is a withe noise stochastic disturbance with finite variance, and $N_{i,t}$ is quantity of labour type $j = i$ hired. Firm i solves the problem $\max_{\{P_{i,t}\}} \Pi_{i,t}$ where $\Pi_{i,t} \equiv P_{i,t} Y_{i,t} - W_{i,t} P_t N_{i,t}$. Using (16) and clearing conditions, and then substituting for (13), one can write the expression for the real marginal cost $mc_{i,t}$ as

$$mc_{i,t} = \left(e^{\psi_{t-1}}\right)^{-(\zeta+1)} \; Y_{i,t}^{\zeta} C_{i,t}.$$

Notice that $C_{i,t}$ labels consumption by agent $j = i$, and not cumulative consumption of good i. The seller's desired mark-up is determined by the usual Lerner formula as $P_{i,t} = (\theta/(\theta-1))\, mc_{i,t} P_t$ yielding the condition for optimal pricing,

$$\left(\frac{P_{i,t}}{P_t}\right)^{1+\theta\zeta} = \frac{\theta}{\theta-1} \left(e^{\psi_{t-1}}\right)^{-(\zeta+1)} \; Y_t^{\zeta} C_{i,t} \tag{17}$$

obtained after substituting for (14) followed by some trivial manipulations. Now, let us consider a log-linear approximation of (17) around the deterministic steady state at $\overline{P}_{i,t} = \overline{P}_t = 1$ written as

$$p_{i,t} = p_t + \omega_y y_t + \omega_c c_{i,t} + \widetilde{z}_{t-1},$$

where $\omega_y = \zeta/(1+\theta\zeta)$, $\omega_c = 1/(1+\theta\zeta)$ and $\widetilde{z}_{t-1} = -\left(\left(\zeta+1\right)/\left(1+\theta\zeta\right)\right)\psi_{t-1}$. Lower case denotes log deviations from the steady state, that is $x_t \equiv \log X_t - \log \overline{X}$. The latter corresponds to (1) in the main text in the case of perfectly informed producers.

Appendix B

Existence of REX

Equilibria are given by the system of equations:

$$\begin{aligned}
\hat{\mathbf{a}}' &= \boldsymbol{\alpha} + \beta\hat{\mathbf{a}}' \\
\hat{b} &= \frac{(\beta^*/2)\,\hat{c}(\hat{c}+\rho)}{(1-(\beta^*/2)\,(1-\rho)\,)\hat{c}^2+((2-\beta^*)\rho-\beta^*/2)\hat{c}+(1-(\beta^*/2)\,)} \\
\hat{c} &= \frac{(\beta^*/2)\,\hat{b}(\hat{b}+\rho)}{(1-(\beta^*/2)\,(1-\rho)\,)\hat{b}^2+((2-\beta^*)\rho-\beta^*/2)\hat{b}+(1-(\beta^*/2)\,)}
\end{aligned}$$

remembering that $bc \neq 1$. It is easily proved by substitution that the fundamental rational expectation solution is always a rest point of the T-map. Other equilibria values are in correspondence with $\hat{b} = \hat{c}$ and are obtained as solutions to

$$\hat{c}\left(\hat{c}^2\left((1-\beta^*/2)-(\beta^*/2)\,\rho\right)-\left(\beta^*-(2-\beta^*)\,\rho\right)\hat{c}+(1-\beta^*/2)-(\beta^*/2)\,\rho\right)=0$$

featuring respectively the high REX values (b_+, c_+) and the low REX values (b_-, c_-).

Computing the Jacobian of the T-map

The partial derivatives of the Jacobian of the T-map are given by

$$\begin{aligned}
\frac{\mathrm{d}T_b(b,c)}{\mathrm{d}b} &= \frac{\beta^*}{2}\frac{(1+c)(1+c\rho)+c(c+\rho)}{1+c^2+2c\rho}, \\
\frac{\mathrm{d}T_b(b,c)}{\mathrm{d}c} &= \frac{\beta^*}{2}\frac{2bc^2\rho^2+c^2\rho-bc^2+2bc\rho+2c+\rho+b}{(c^2+2\rho c+1)^2}, \\
\frac{\mathrm{d}T_c(b,c)}{\mathrm{d}b} &= \frac{\beta^*}{2}\frac{2cb^2\rho^2+b^2\rho-cb^2+2bc\rho+2b+\rho+c}{(b^2+2\rho b+1)^2}, \\
\frac{\mathrm{d}T_c(b,c)}{\mathrm{d}c} &= \frac{\beta^*}{2}\frac{(1+b)(1+b\rho)+b(b+\rho)}{1+b^2+2b\rho}.
\end{aligned}$$

To analyze the learnability of equilibria we have to investigate the sign of the eigenvalues of the matrix $K \equiv JT - I$ (where I is the identity matrix) evaluated at the equilibrium values $\hat{\mathbf{a}}$ and $\hat{c} = \hat{b}$. The equilibrium values $(\hat{\mathbf{a}}, \hat{b}, \hat{c})$ are learnable if and only if the matrix $K_{(\hat{\mathbf{a}}, \hat{b}, \hat{c})}$ has all negative eigenvalues. The matrix $K \equiv JT - I$ at the equilibrium values $\hat{\mathbf{a}}$ and $\hat{c} = \hat{b}$ is written as

$$K_{(\hat{\mathbf{a}},\hat{b},\hat{c})} = \begin{pmatrix} \beta - 1 & 0 & 0 & 0 \\ 0 & \beta - 1 & 0 & 0 \\ 0 & 0 & \left[\frac{dT_b(b,c)}{db}\right]_{(\hat{b},\hat{c})} - 1 & \left[\frac{dT_b(b,c)}{dc}\right]_{(\hat{b},\hat{c})} \\ 0 & 0 & \left[\frac{dT_c(b,c)}{db}\right]_{(\hat{b},\hat{c})} & \left[\frac{dT_c(b,c)}{dc}\right]_{(\hat{b},\hat{c})} - 1 \end{pmatrix}, \tag{19}$$

with

$$\begin{aligned}
\left[\frac{dT_b(b,c)}{db}\right]_{(\hat{b},\hat{c})} - 1 &= \frac{\left(\left(\beta^*/2\right)\left(1+\rho\right) - 1\right)\hat{b}^2 + \left(\left(\beta^*/2\right)\left(1+2\rho\right) - 2\rho\right)\hat{b} + \left(\beta^*/2\right) - 1}{1 + \hat{b}^2 + 2\hat{b}\rho}, \\
\left[\frac{dT_b(b,c)}{dc}\right]_{(\hat{b},\hat{c})} &= \frac{\beta^*}{2}\frac{(2\rho-1)\hat{b}^3 + 3\rho\hat{b}^2 + 3\hat{b} + \rho}{\left(1 + \hat{b}^2 + 2\hat{b}\rho\right)^2}, \\
\left[\frac{dT_c(b,c)}{dc}\right]_{(\hat{b},\hat{c})} &= \left[\frac{dT_b(b,c)}{db}\right]_{(\hat{b},\hat{c})} \quad \text{and} \quad \left[\frac{dT_c(b,c)}{db}\right]_{(\hat{b},\hat{c})} = \left[\frac{dT_b(b,c)}{dc}\right]_{(\hat{b},\hat{c})}.
\end{aligned}$$

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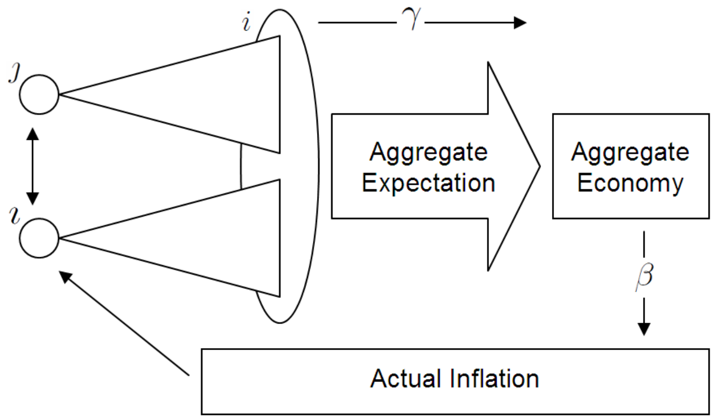

Figure 1: Information flow in the economy.

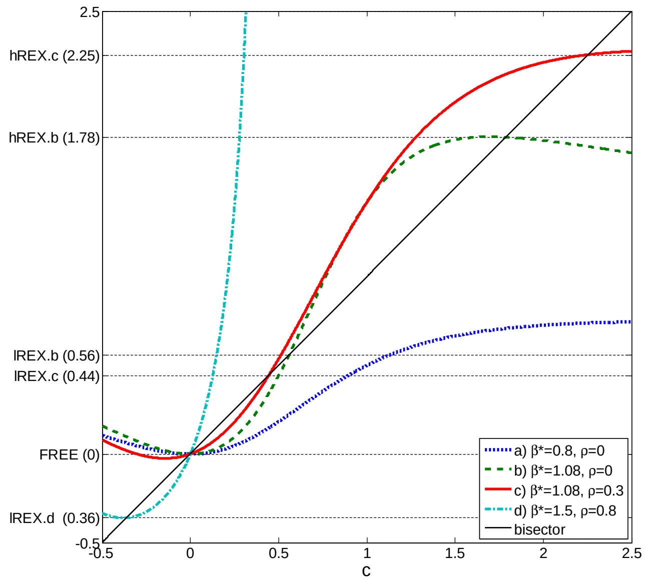

Figure 2: T-map representation for different calibrations. Equilibria lie at the intersections of the T-map with the bisector.

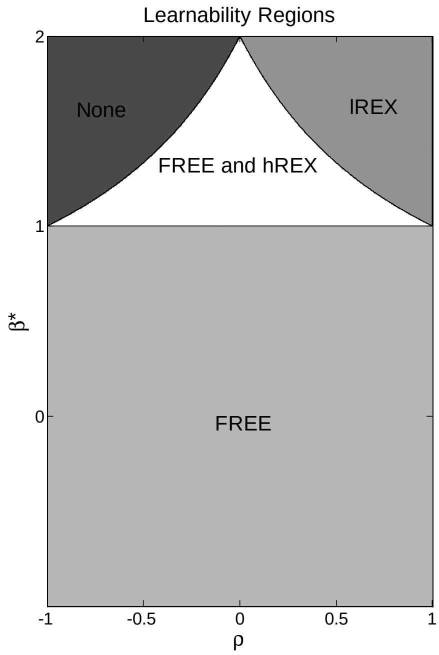

Figure 3: Numerical learnability analysis in the whole parameter space.

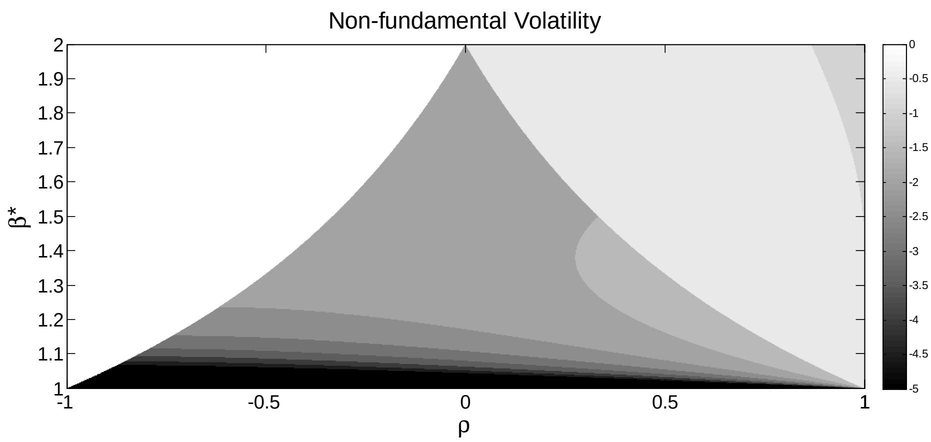

Figure 4: Numerical analysis of non-fundamental volatility. The picture shows the volatility of the aggregate expectation obtained for β^* values for which learnable REX arise. The variance of the observational errors is the unit of the measurement.

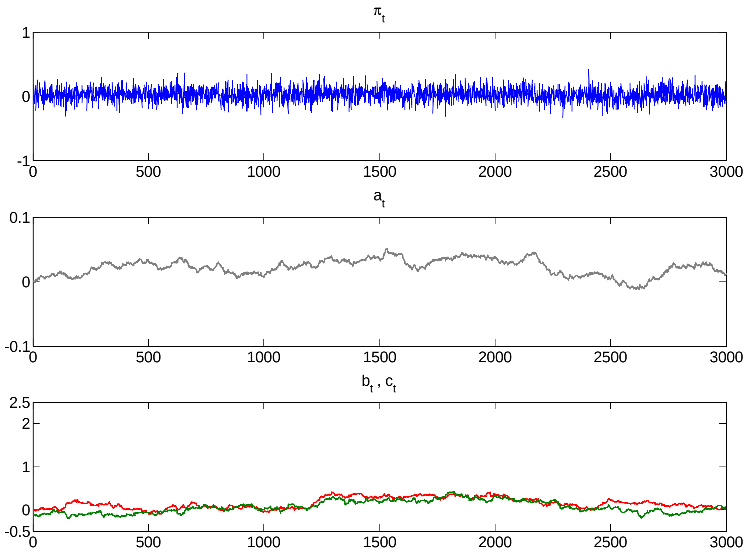

Figure 5: Benchmark case ($\beta = 0.94$, $\gamma = 0$, $\rho = 0$).

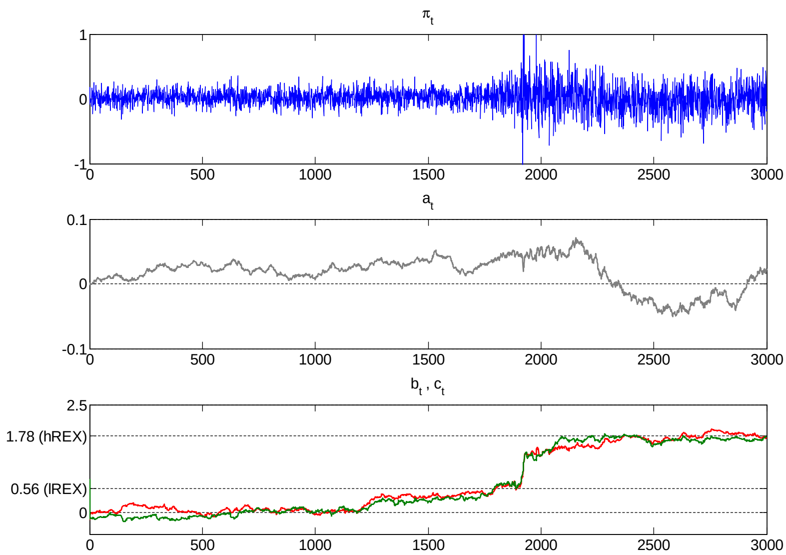

Figure 6: A case of bad luck ($\beta = 0.94$, $\gamma = 0.14$, $\rho = 0$): curve b) in figure 2.

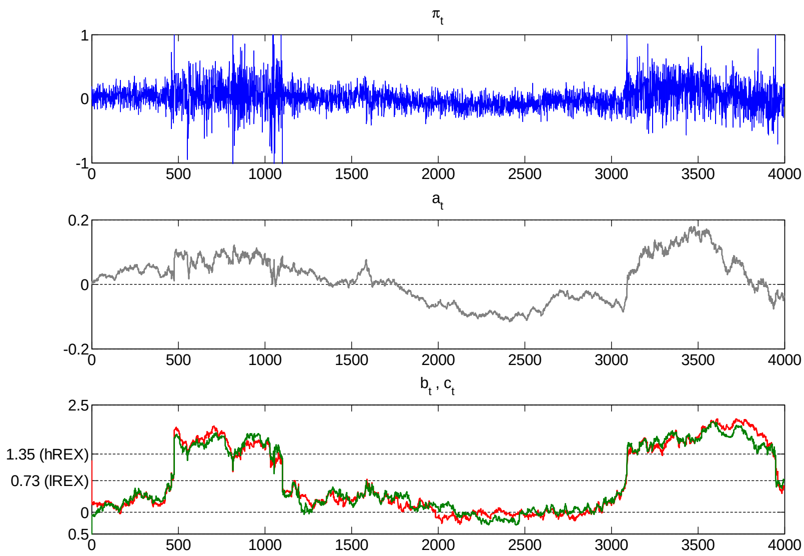

Figure 7: Alternance of bad and good luck ($\beta = 0.94$, $\gamma = 0.07$, $\rho = 0.4$).

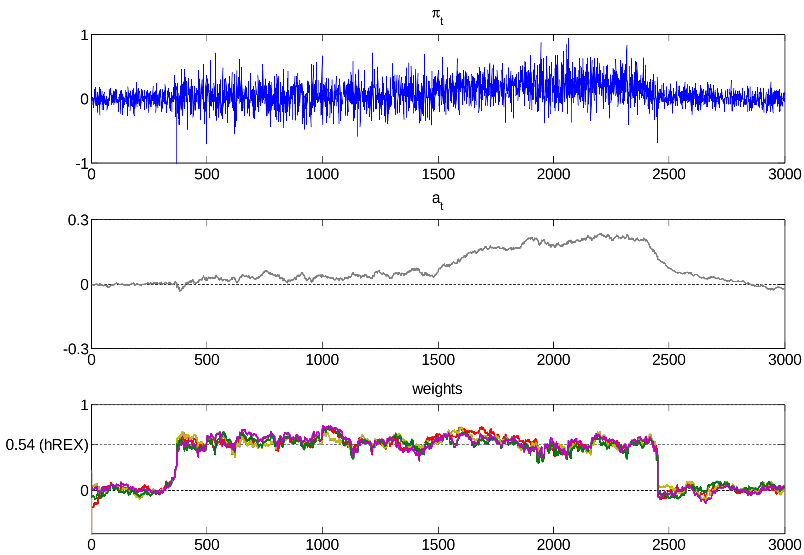

Figure 8: Good policy after bad luck with four islands ($\beta = 0.94$, $\gamma = 0.14$, $\rho = 0.3$). At time 2400 a drastic change in monetary policy makes β falls around 0.5.

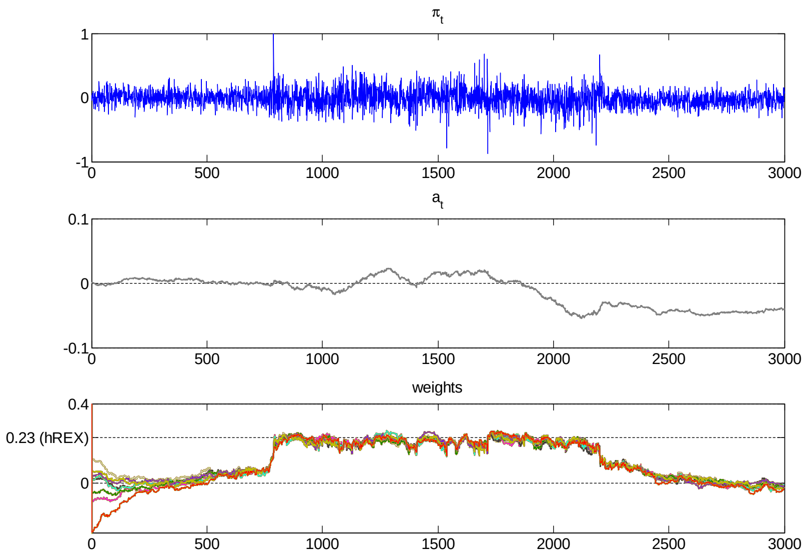

Figure 9: Good luck after bad luck with eight islands ($\beta = 0.94$, $\gamma = 0.14$, $\rho = 0.2$).

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