Department of Economics

# Three Essays in Competition and Banking

Agnese Leonello

Thesis submitted for assessment with a view to obtaining the degree of
Doctor of Economics of the European University Institute

Florence, July 2011

EUROPEAN UNIVERSITY INSTITUTE
**Department of Economics**

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Leonello, Agnese (2011), Three Essays in Competition and Banking
European University Institute
DOI: 10.2870/30719

*To my secret reader...my Dad.*


DOI: 10.2870/30719

## Acknowledgements

Doing the PhD has not been easy. Writing good acknowledgements is almost as complicated...

Those years in Florence have been special and I wish to express my deepest gratitude to all who made it possible.

Most of all, I am indebted to my supervisor Elena Carletti. I am grateful to her for the invaluable guide and support during the preparation of the thesis. I have benefited a lot from working with her and our discussions have produced the first chapter of this thesis and have contributed to shape my approach to research. They did, indeed! I also have to thank her for being always present and encouraging, well beyond the thesis supervision.

I am also deeply grateful to Massimo Motta. My first attempt to research was under his supervision and his comments and suggestions have been extremely important for my PhD. I wish to thank also Franklin Allen and Fernando Vega Redondo for their precious comments and suggestions. Very useful comments on the three chapters of this thesis came from Monica Andini, Claudio Calcagno, Lapo Filistrucchi and Emanuele Tarantino. Thank you, guys!

A special mention goes to all the wonderful people I met during this time in Florence, to Monica, Emanuela, Agnieszka and Claudia for their presence and support throughout this journey. To Emanuela Iancu I grateful not only as friend but as coauthor. I had the pleasure of working with her on the third chapter of this thesis and it was a great experience.

My final thought goes to Gianluca and my family for being such an important part of my life.


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# 1

# Introduction

The effect of competition on financial stability has been extensively discussed in the academic literature but the results are inconclusive. A strand of the literature supports the idea that competition is detrimental for financial stability, while other contributions suggest that competition is beneficial. So far, the focus of the analysis has been on the effect that competition has on banks' risk-taking incentive. The existing literature considers risk taking as the credit risk of the assets banks invest in. However, as the recent crisis has shown, there is another important source of risk taking for financial institutions linked to the amount of liquidity they hold and the functioning of the interbank market. The **first chapter** analyzes the effect of competition on financial stability. It shows that competition is beneficial for financial stability as it induces banks to invest in more liquid portfolios and thus, makes them less vulnerable to a liquidity shock. In a simple theoretical model in which banks can trade assets on the interbank market to meet their stochastic liquidity demand, the paper shows that, depending on the degree of credit market competition, two equilibria can emerge. When competition in very intense, all banks keep enough liquid asset (reserves) to face their demand for liquidity and default never occurs. When competition is low, instead, banks make different initial portfolio choices. Some banks invest mostly in the illiquid asset (loans to entrepreneurs) while others choose to hold a large amount of the liquid asset. The former sell their loans to meet their liquidity demands, when the liquidity shock materializes. In the state where their liquidity demand is low, they sell part of their loans and make positive profits. In the other state where their liquidity demand is high, they must sell all their loans and default. The existence of the two equilibria described above depends on the degree of competition in the credit market.



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The intuition is that in order for banks not to default, they have to hold a large fraction of their portfolios in reserves. This entails a cost in terms of foregone return on the loans. Such a cost is larger as competition decreases, as granting loans becomes more profitable for banks. Thus holding a large amount of reserves and avoid default is optimal for banks only when competition is intense. As competition decreases, it becomes optimal to reduce reserves and invest more in loans and, thus, defaulting with positive probability.

The **second chapter** investigates another factor that might have contributed to the recent financial crisis affecting the ability of the banks to hedge again the liquidity shock. The focus of the analysis in this chapter is on the functioning of interbank market. It shows that incomplete information may induce banks not to participate in interbank market. In a simple theoretical model of interconnected banks where they can borrow liquidity on the interbank market to meet their liquidity needs, the analysis shows that interbank market breakdown may emerge in equilibrium as consequence of the uncertainty on the counterparty risk. In this case, banks in distress have to sell their asset to outside investors at fire sale prices. Mark-to-market accounting may, thus, turn a simple illiquidity problem into a solvency issue and generate a wave of bankruptcies. Interbank market freeze and contagion arise as consequence of the intertwining of incomplete information and mark-to-market accounting. When the perceived risk of contagion is low, banks prefer not to participate in the interbank market. This leads to fire sales and, in some circumstances, to systemic crisis. The results presented in the chapter are consistent with some features of the recent financial turmoil. During the crisis, the uncertainty of the counterparty risk caused a prolonged malfunctioning of the interbank market as banks in distress were no longer able to obtain the liquidity they needed. This triggered a downward spiral as the fear of contagion and the interbank market frictions were reinforcing each other. Banks were forced to sell assets at resale prices to meet their liquidity demand. The depressed valuations propagated in the banking system through mark-to-market accounting, setting the ground for contagion.

The **third chapter** analyzes the effects of a merger in two-sided markets. In two-sided

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markets, two distinct groups of agents (sides) interact on one or more platforms. The main feature of these markets is the existence of indirect network externalities. Consumers' reservation value depends on the number of agents that patronize the same platform on the other side of the market. The interest in these markets among practitioners and academics has grown in the last decade. Moreover, in the last few years, several mergers between two-sided markets took places in different industries. The newspaper industry, for example, has seen an increased concentration as consequence of several mergers, in several countries. In the chapter, in a simple setting of horizontal differentiation and price competition, I analyze the effect of a merger on platforms' profitability and consumers' welfare. The main result of the paper is that, even without efficiency gains, the merger may be welfare enhancing. The existence of indirect network externalities induces merging platforms to keep their price low at least on one side of the market. This in turn produces a positive effect on consumers' welfare.

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European University Institute DOI: 10.2870/30719

DOI: 10.2870/30719

2

# Credit Market Competition and Interbank Liquidity

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***Joint with Elena Carletti***

## 2.1 Introduction

There is a long standing debate in the academic literature whether competition is beneficial or detrimental to financial stability. The key issue is how competition affects banks' risk taking behavior. One strand of the literature (see, e.g., Keeley, 1990, and subsequent papers) argues that by reducing banks' profits, credit market competition reduces the incentives for bank managers to behave prudently. On the opposite side, another strand of the literature (e.g., Boyd and De Nicoló, 2005) argues that market power on the credit market is detrimental for financial stability. The idea is that a higher market power increases borrowers' costs and thus their incentives to take risks. This in turn increases the risk of banks' portfolios.

All this literature considers risk taking as the credit risk of the assets banks invest in. However, as the recent crisis has shown, there is another important source of risk taking for financial institutions linked to the amount of liquidity they hold and the functioning of the interbank market. This allows banks to reshuffle liquidity and meet their uncertain liquidity demands. When liquidity is abundant on the market, the interbank market works smoothly and liquidity risk does not represent a source of instability for financial



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European University Institute DOI: 10.2870/30719

institutions. However, when liquidity is scarce on the market, financial institutions may be unable to cover their liquidity needs and go bankrupt. Crucial to the stability of the financial institutions is then the amount of total liquidity on the market as determined by banks' initial portfolio allocations. Competition in the credit market affects such allocations as it determines the returns on the longer term investments and thus the opportunity cost of holding liquidity.

So far there has not been research on the role of credit market competition on liquidity provision in the interbank market. The purpose of this paper is to develop a simple theoretical framework for analyzing how credit market competition affects banks' liquidity risk in the interbank market. Our analysis is based on a standard banking model developed in Allen and Gale (2004a, 2004b, 2007) and Allen, Carletti and Gale (2009). As usual, there are two periods. Banks raise deposits from risk averse consumers and can hold a one-period liquid asset, that we define as reserves, or grant a two-period risk free loan to entrepreneurs with a return that depends on the degree of competition in the credit market. Banks face aggregate uncertainty relative to their demand for liquidity at the interim date as a stochastic fraction of their consumers want to consume early. To meet their liquidity demands, banks can sell their loans on the interbank market at a price that is endogenously determined by the amount of supply and demand of liquidity in the market. The former is determined by the banks' initial choice of reserve holding, which depends on the degree of competition in the credit market. The latter is determined by realization of the liquidity shock and the promised repayments to consumers in the deposit contract. In contrast to the models mentioned above, banks choose their initial portfolio allocation and the deposit contract to maximize their expected profits and are monopolist on the deposit market.

We show that two types of equilibria can emerge. In the first, that we call no default equilibrium, banks keep enough reserves to face their liquidity demands in all states. No bank defaults, and since there is only aggregate uncertainty, there is no trade on the interbank market. In the second equilibrium, which we define mixed equilibrium, banks make different portfolio choices at the initial date. Some banks, that we call risky, choose to invest all their deposits in loans and none in reserves. At the interim date, they sell their loans to meet their liquidity demands. In the state where their liquidity demand is low, they sell part of their loans and make positive profits. In the other state where their liquidity demand is high, they must sell all their loans and default. The remaining banks, that we call safe, choose to hold a large amount of reserves at the initial date to acquire the loans sold by the risky banks on the interbank market at date 1. The existence of this mixed equilibrium is due to the price volatility across the two states of the world. Since the supply of liquidity at date 1 is fixed and equal to the total initial investment of the safe banks in reserves, prices must adjust in the two states to accommodate the fluctuations in the aggregate demand for liquidity. This clears the market at any date and state. The price volatility depends on the degree of competition in the credit market, since the banks' portfolio allocations as well as the promised deposit contract depend on it.

The main result of the paper is to show that the existence of the two equilibria described above depends on the degree of competition in the credit market. When competition is intense, only the no default equilibrium exists. As competition reduces, only the mixed equilibrium exists. The intuition is that in order for banks not to default, they have to hold a large fraction of their portfolios in reserves. This entails a cost in terms of foregone return on the loans. Such a cost is larger as competition decreases, as granting loans becomes more profitable for banks. Thus holding a large amount of reserves and avoid default is optimal for banks only when competition is intense. As competition decreases, it becomes optimal to reduce reserves and invest more in loans. Some banks find it optimal then to become risky and default with positive probability. Although risky banks default in the state when the aggregate liquidity demand is high, the loan rate is high enough to ensure that they make enough profit in the other state. The result shows that the lack of competition in the credit market is detrimental to financial stability.

The paper is related to various others. A few studies (e.g., Matutes and Vives, 1996; see also Carletti,2008, and Vives, 2010, for a survey) have analyzed the effect of deposit

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European University Institute DOI: 10.2870/30719

market competition on banks' vulnerability to runs. The results are inconclusive but tend to suggest that a higher degree of competition in the deposit market makes banks more unstable. A number of papers have analyzed the functioning of the interbank market. Allen, Carletti and Gale study the functioning of the interbank market when banks cannot hedge against liquidity shocks. In contrast to our analyzes, they analyze only the case where the no default equilibrium exists. Other papers analyze various market failures in the interbank markets. For example, Heider, Hoerova (2009) focus on asymmetric information; Acharya, Gromb and Yorulmazer (2008) on market power. By contrast, we consider competitive interbank markets and focus on banks' inability to fully hedge against liquidity shocks.

The paper proceeds as follows. Section 2 describes the model. Sections 3 and 4 characterize the no default and the mixed equilibrium, respectively. Section 5 analyzes the existence of the two equilibria. Section 6 discusses the implications of the model in terms of welfare. Section 7 concludes.

## 2.2 The model

The model is based on Allen and Gale (2004b). Consider a three date (t = 0, 1, 2) economy with three classes of agents: banks, entrepreneurs and investors. Banks act as intermediaries between consumers and entrepreneurs. At date 0 they raise one unit of funds from investors in exchange for a deposit contract and provide loans to entrepreneurs. Consumers can deposit their funds at one bank only and entrepreneurs can obtain loans from one bank only. Banks exercise monopoly power over consumers while they need to compete to attract entrepreneurs. The idea is that banks operate in distinct regions. Investors are locked in and can only deposit their funds at the bank in their region. Entrepreneurs are instead mobile across regions.

There is a continuum of mass one of consumers in each region. Consumers have one unit of endowment each at date 0 and nothing thereafter. They are all ex ante identical

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but can be of either early or late type at date 1. Early consumers value consumption only at date 1 while late consumers value consumption only at date 2. Each consumer has a probability $\lambda_\theta$ of being an early consumer given by

$$\lambda_\theta = \begin{cases} \lambda_L & \text{w. pr. } \pi \\ \lambda_H & \text{w. pr. } (1-\pi), \end{cases}$$

with $\lambda_H > \lambda_L$. As usual, we adopt the Law of Large Numbers so that $\lambda_\theta$ represents also the fraction of early consumers at each bank.

The uncertainty about time preferences generates a preference for liquidity and a role for banks as liquidity providers. Consumers deposit their funds in exchange for a demand deposit contract that offers them $c_1$ at date 1 and $c_2$ at date 2 and have an ex ante expected utility equal to

$$E[u(c_1, c_2, \lambda_\theta)] = E[\lambda_\theta u(c_1) + (1-\lambda_\theta)u(c_2)].$$

The utility function is twice differentiable and satisfies all the usual neoclassical assumptions: $u'(c) > 0$, $u''(c) < 0$. Given that banks are assumed to be monopolists in the deposit market, consumers deposit their funds at date 0 if they expect to have an utility at least equal to the one they would obtain from storing their endowment.

Each bank invests a fraction $R_i$ of the funds raised from consumers in reserves and a fraction $L_i$ in loans to entrepreneurs. Reserves are a storage technology: one unit invested at date $t$ produces one unit at date $t+1$. Lending to entrepreneurs is a longer investment: one unit lent in date 0 gives a return $r_i$ to the bank at date 2. Such return depends on the degree of competition in the credit market. Entrepreneurs invest the loan obtained by the bank in a (safe) project which yields $V > 1$ at $t = 2$ and gives the bank a (gross) interest rate equal to

$$r_i = \gamma V, \tag{2.1}$$

where $\gamma \epsilon [\frac{1}{V}, 1]$ represents the degree of competition in the credit market. The higher $\gamma$ the lower the competition in the credit market and the higher the return to the bank

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from granting loans. When $\gamma = \frac{1}{V}$, the credit market is perfectly competitive. The bank obtains $r_i = 1$ and the entrepreneur retains the whole surplus $V$ generated by the project. The other extreme, when $\gamma = 1$, represents the case of a monopolistic credit market when the bank obtains the whole surplus $V$ from the projects. Entrepreneurs are just indifferent between taking the loan or not as they are assumed to have a zero opportunity cost. Intermediate values of $\gamma$ measure intermediate degrees of competition in the credit market, when banks and entrepreneurs share the surplus generated by the project. In this sense, (2.1) can be interpreted as the result of a Nash bargaining process between the bank and the entrepreneurs. While being a shortcut, (2.1) captures the general idea that the return for the bank reduces as competition increases. In what follows, for simplicity we refer to $r_i$ as being set by the bank according to (2.1).

Although they are illiquid, loans can be sold on an interbank market at date 1 for a price $P_\theta$. Participation in this market is limited in that banks can buy and sell loans in this market but consumers cannot. The price $P_\theta$ is endogenously determined in equilibrium by the aggregate demand and supply of liquidity in the market, as explained further below.

The timing of the model is as follows. At date 0, banks choose the deposit contract $(c_1, c_2)$ to offer to consumers and how to split their funds between reserves and loans so to maximize their expected profits. In doing so, they anticipate that at date 1, after the liquidity shocks are realized, they may buy or sell loans on the market at the price $P_\theta$.

All uncertainty is resolved at the beginning of date 1. In particular, depositors learn whether they are early or late consumers and the value of $\theta$ is determined. The type of each consumer remains, however, private information. An early consumer cannot misrepresent his type because he needs to consume at date 1; but a late consumer can claim to be an early consumer, withdraw $c_1$ at date 1, store it until date 2 and then consume it. To avoid this, the deposit contract that the bank offers to consumers must be *incentive compatible* and guarantee that the residual payment to late consumers at date 2 is at least $c_1$. This

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requires the following constraint to be satisfied:

$$\lambda_\theta c_1 + (1-\lambda_\theta) c_1 \frac{P_\theta}{r_i} \leq R_i + P_\theta L_i. \tag{2.2}$$

The left hand side is the present value of the consumers' claims on bank $i$ at date 1 when the late consumers receive at least $c_1$. The first term represents the consumption given to the $\lambda_\theta$ early consumers. The second term $(1-\lambda_\theta)c_1$ is the consumption given to the late consumers discounted at its present value $\frac{P_\theta}{r_i}$. The right hand side is the value of the bank's portfolio. The bank has $R_i$ units of reserves and $L_i$ units of loans worth $P_\theta$ per unit. Thus, condition (2.2) is necessary and sufficient for the deposit contract $c_1$ to satisfy incentive compatibility and the budget constraint simultaneously. If (2.2) is satisfied, the bank does not default. The early consumers withdraw at date 1 and the late consumers wait till date 2. If, instead, (2.2) is not satisfied a run occurs and the bank is liquidated at date 1. The late consumers find it optimal to withdraw at date 1 as, if they left their funds in the bank till date 2, they would receive less than early consumers. The bank has to sell all its loans, consumers receive a *pro rata share* and the bank obtains nothing. We assume that this is the only case where a run occurs in the model. That is, (sunspot) runs do not occur when (2.2) is satisfied.

This discussion suggests that there can be different equilibria of the model, depending on whether (2.2) is satisfied or not. In particular, the model has two equilibria. In the first, that we define as no default equilibrium, (2.2) is satisfied for all banks so that none defaults at date 1. In the second, that we define as mixed equilibrium, (2.2) is satisfied for some banks, that we call safe, but not for others, that we call risky. The safe banks do not default, but the risky default in some state of the world. The two types of banks hold a different portfolio allocation between reserves and loans. This implies that, although there is no idiosyncratic liquidity shock in the model, there is trade in the interbank market at date 1, as we describe in detail below. In what follows we derive the two equilibria in turn.

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European University Institute DOI: 10.2870/30719

## 2.3 The no default equilibrium

The no default equilibrium exists when consumers' incentive compatibility constraint (2.2) is always satisfied so that no bank defaults at date 1. Given that all banks remain active till date 2 and they are all identical, they will choose the same initial allocation of reserves and loans. There is no trade on the interbank market so that the price $P_\theta$ does not have real effects on the equilibrium allocation. Given all this, we can simply analyze a representative bank in the economy. In what follows we also assume that depositors have a logaritmic utility function, that is $u(c_t) = \ln(c_t)$ with $t = 1, 2$. This simplifies the determination of the deposit contract without affecting the results.

The bank chooses the deposit contract $(c_1, c_2)$ and the portfolio allocation $(R_i, L_i)$ simultaneously so to maximize its expected profit at $t = 0$. The bank's maximization problem is given by

$$\underset{c_1,c_2,R_i,L_i}{Max}\ \Pi_i = r_iL_i+\pi(R_i-\lambda_Lc_1)+(1-\pi)(R_i-\lambda_Hc_1)-[\pi(1-\lambda_L)+(1-\pi)(1-\lambda_H)]c_2 \quad (2.3)$$

subject to

$$R_i + L_i = 1$$

$$\lambda_\theta c_1 \leq R_i \quad (2.4)$$

$$(1-\lambda_\theta)c_2 \leq r_iL_i + R_i - \lambda_\theta c_1 \quad (2.5)$$

$$E[u(c_1, c_2, \lambda_\theta)] = [\pi\lambda_L + (1-\pi)\lambda_H]\, u(c_1) + [\pi(1-\lambda_L) + (1-\pi)(1-\lambda_H)]\, u(c_2) \geq 0 \quad (2.6)$$

$$c_2 \geq c_1 \quad (2.7)$$

 DOI: 10.2870/30719

for any $\theta = L, H$ and taking into account that the loan rate $r_i$ is given by (2.1). Given there is no trade at date 1 and no default in equilibrium, the expression for the bank's profit $\Pi_i$ is given by sum of the returns from the loans $r_i L_i$ and the expected excess of liquidity $\pi(R_i - \lambda_L c_1) + (1-\pi)(R_i - \lambda_H c_1)$ minus the expected payments $[\pi(1-\lambda_L) + (1-\pi)(1-\lambda_H)]c_2$ to depositors at date 2. The first constraint represents the budget constraint at date 0. The next two constraints represent the budget constraints at date 1 and 2. Constraint (2.4) requires that at date 1 the bank needs enough reserves to satisfy its demand for liquidity as given by $\lambda_\theta c_1$ for any $\theta = L, H$. Constraint (2.5) requires that at date 2 the promised amount to the late consumers $(1-\lambda_\theta)c_2$ must not exceed the resources available to the bank $r_i L_i + R_i - \lambda_\theta c_1$. Constraint (2.6) is consumers' participation constraint. It requires that the utility $E[u(c_1, c_2, \lambda_\theta)]$ that they receive when depositing their endowment at the bank must be at least equal to the one they would obtain from storing their initial endowment. This is simply equal to 0 given the assumption of logaritmic utility function. Constraint (2.7) is the consumers' incentive compatibility constraint. It requires that the promised repayments $c_1$ and $c_2$ at date 0 are such that the late consumers do not run at date 1. Note that (2.7) together with (2.4) and (2.5) imply that (2.2) is satisfied for any $P_\theta$.

We have the following result.

**Proposition 1** *In the no default equilibrium, each bank invests an amount* $R^{ND} = \lambda_H c_1^{ND}$ *in reserves and* $L^{ND} = 1 - R^{ND}$ *in loans, and offers consumers a deposit contract*

$$c_1^{ND} = \left(\frac{\pi\lambda_L + (1-\pi)\lambda_H}{\pi\lambda_L + (r-\pi)\lambda_H}\right)^{\pi(1-\lambda_L)+(1-\pi)(1-\lambda_H)} < 1 \tag{2.8}$$

*and*

$$c_2^{ND} = \left(\frac{\pi\lambda_L + (r-\pi)\lambda_H}{\pi\lambda_L + (1-\pi)\lambda_H}\right)^{\pi\lambda_L+(1-\pi)\lambda_H} > 1. \tag{2.9}$$

**Proof.** See appendix A. ■

The intuition behind Proposition 1 is simple. In the no default equilibrium all banks behave alike. They have the same initial portfolio allocation and offer consumers the same deposit contract. In order to avoid default, they need to hold an amount of reserves large enough to satisfy the highest liquidity demand by early consumers at date 1 in state $H$. Since the loan rate is $r \geq 1$ and there is no trade on the interbank market, they do not find it optimal to hold reserves in excess of the amount needed. Since banks do not default, deposits are safe and consumers always received their promised repayments. The deposit contract maximizes the bank's expected profit while satisfying consumers' participation with equality. The bank offers $c_2^{ND} > 1 > c_1^{ND}$ except in perfect competition where $r = 1$ and thus $c_1^{ND} = c_2^{ND} = 1$. The reason is that banks need to hold a high level of reserves to ensure themselves against the higher liquidity shock in state $H$. Holding reserves entails an opportunity cost as represented by the foregone return $r$ on the loans. Such a cost is higher the lower is competition in the credit market. Choosing $c_1^{ND} < 1$ and $c_2^{ND} > 1$ allows the bank to reduce its reserve holding while still being able to satisfy the early liquidity demand at date 1 in state $H$ and consumers' participation constraint. The promised repayments $c_1^{ND}$ and $c_2^{ND}$ move in opposite direction as competition decreases. As the opportunity cost of holding reserves increases as competition decreases, $c_1^{ND}$ decreases with $r = \gamma V$ while $c_2^{ND}$ increases.

From Proposition 1, the bank's expected profit in the no default equilibrium becomes

$$\Pi^{ND} = \gamma V - c_2^{ND}. \tag{2.10}$$

The bank's profit is simply equal to the difference between the return on loans and the promised repayment $c_2^{ND}$ to the late consumers. This suggests that the reserves holdings and the liquidity demand by the early consumers affect the bank's profits only to the extent that they affect $c_2^{ND}$. When the credit market is perfectly competitive so that $\gamma = \frac{1}{V}$, the bank makes zero expected profit. In such case, the budget constraint (2.5) is binding for any $\theta = L, H$.

Although there is no trade in the no default equilibrium, we need to determine the price $P_\theta$ in order to complete the characterization of the equilibrium. Given that there are two states of the world $\theta = L, H$, the price $P_\theta$ is volatile and takes two values, $P_L$ and $P_H$. These prices have to be such that the market clears at both dates 0 and 1. This means that the intertemporal returns of the two assets have to be equal. In state $L$ banks have an excess of liquidity as $\lambda_L c_1^{ND} < \lambda_H c_2^{ND}$. Thus, as banks have to be indifferent between using their excess of liquidity to buy loans and storing it for one additional period, it must hold that

$$P_L = r. \tag{2.11}$$

A lower value of $P_L$ would imply that all banks would want to hold only the long asset, while a higher value of $P_L$ would imply that they would be willing to hold only the short asset. It then follows that $P_H$ must be such that banks are willing to hold both reserves and loans between dates 0 and 1. In particular, $P_H$ must satisfy

$$\pi \frac{r}{P_L} + (1-\pi)\frac{r}{P_H} = r. \tag{2.12}$$

This implies that $P_H < 1$ as $P_L > 1$. Otherwise loans would dominate reserves at date 0.

## 2.4 The mixed equilibrium

So far we have assumed that (2.2) is always satisfied so that banks find it optimal not to default at date 1. However, avoiding default is costly as to do so banks have to keep a high proportion of reserves and forego the higher return on the loans. If the costs of avoiding default are large, banks may find it optimal to choose a deposit contract and an initial amount of reserves such that they default at date 1 with positive probability. This is a consequence of the presence of aggregate uncertainty on the demand for liquidity at date 1 and the consequent price volatility across states. As we will see, the cost of avoiding default depends on the parameter $\gamma$ representing the degree of competition in the credit market. When $\gamma$ is low, the cost of avoiding default is low as the foregone return on the

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European University Institute DOI: 10.2870/30719

loan from keeping reserves is low. However, as $\gamma$ increases, such a cost increases and banks may find it optimal to default. In this section, we characterize the equilibrium when some banks find it optimal to default at date 1 in state $H$ while some others remain always solvent. We start by looking at the banks' problem. Then, we analyze the market clearing conditions that must hold in equilibrium.

### 2.4.1 The banks' problem

Given that there is no uncertainty on the loan return, a bank defaults at date 1 only if it violates (2.2) in some state. In equilibrium not all banks can default simultaneously. If all banks made the same investments at date 0 and all default at date 1 there would be no bank left on the other side of the market to buy the loans of the defaulting banks so that $P_\theta = 0$. This cannot be an equilibrium since it would be optimal for a bank to remain solvent and keep excess liquidity to buy up all the loans at the price $P_\theta = 0$. This implies that in order to have default, the equilibrium must be mixed. Despite being ex ante identical, banks must make different initial portfolio allocation decisions and offer different deposit contracts to consumers. Some banks, that we define safe, invest enough in reserves at date 0 to remain solvent at date 1 in either state $\theta = L, H$ for any $P_\theta$. Others, that we define as risky, choose instead an initial allocation of reserves and loans such that they default at date 1 with positive probability. Given that they cannot default in all states and there are only two states of the world in our model, at date 1 risky banks remain solvent in state $\theta = L$ and default in state $\theta = H$ as their (2.2) is violated. Safe and risky banks have therefore different maximization problems at date 0 when they choose the deposit contract and the initial portfolio allocation between loans and reserves. However, in equilibrium safe and risky banks must have the same expected profits as they have to be indifferent between being either of the two types.

As in the no default equilibrium, at date 0 banks choose the deposit contract and the portfolio allocation simultaneously. Consumers know the type of banks they deposit their endowment in so that the safe and the risky banks must offer different deposit contracts

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European University Institute DOI: 10.2870/30719

to consumers. Either contract has to guarantee that consumers have an expected utility at least as large as the utility they could receive from storing their endowment.

We start by characterizing the problem for the safe banks. This is similar to the one in the no default equilibrium, with the difference that the safe banks have now the possibility to buy loans from the risky banks on the interbank market at date 1. Given the market prices $P_H$ and $P_L$, each safe bank chooses simultaneously the deposit contract $(c_1^S, c_2^S)$ and the initial allocation of funds between reserves $R^S$ and loans $L^S$ so to solve the following problem:

$$\max_{c_1^S, c_2^S, R^S, L^S} \Pi^S = rL^S + \pi(\frac{R^S - \lambda_L c_1^S}{P_L})r + (1-\pi)(\frac{R^S - \lambda_H c_1^S}{P_H})r - [\pi(1-\lambda_L) + (1-\pi)(1-\lambda_H)]c_2^S \tag{2.13}$$

subject to

$$R^S + L^S = 1$$

$$\lambda_\theta c_1^S \leq R^S \tag{2.14}$$

$$(1-\lambda_\theta)c_2^S \leq r\left(L^S + \frac{R^S - \lambda_\theta c_1^S}{P_\theta}\right) \tag{2.15}$$

$$E[u(c_1^S, c_2^S, \lambda_\theta)] = [\pi\lambda_L + (1-\pi)\lambda_H]\, u(c_1^s) + [\pi(1-\lambda_L) + (1-\pi)(1-\lambda_H)]\, u(c_2^s) \geq 0 \tag{2.16}$$

$$c_2^S \geq c_1^S \tag{2.17}$$

In contrast to the no default equilibrium, there is now trade on the interbank market at date 1. The expression for the bank's profit $\Pi^S$ is given by the sum of the returns from the loans $rL^S$ and from the expected excess of liquidity $\pi(\frac{R^S - \lambda_L c_1^S}{P_L})r$ and $(1-\pi)(\frac{R^S - \lambda_H c_1^S}{P_H})r$ in states $L$ and $H$ minus the expected payments $[\pi(1-\lambda_L) + (1-\pi)(1-\lambda_H)]c_2^S$ to depositors at date 2. Safe banks can use any excess liquidity at date 1 to acquire loans from the risky

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banks. With probability $\pi$ the safe bank has $R^S - \lambda_L c_1^S$ units of liquidity in excess and can use them to buy $\frac{R^S-\lambda_L c_1^S}{P_L}$ units of loans from the risky banks yielding a per-unit return of $r$. Similarly for state $H$. The first constraint represents the budget constraint at date 0, which is always satisfied with equality to indicate that the bank invests all its funds at date 0. Constraint (2.14) requires that the safe bank has enough reserves $R^S$ to satisfy the liquidity demand $\lambda_\theta c_1^S$ of the early consumers at date 1 in either state. Constraint (2.15) requires that the bank has enough resources at date 2 to repay the promised amount $(1-\lambda_\theta)c_2^S$ to the late consumers. Constraint (2.16) guarantees that the consumers receive an expected utility from the deposit contract at least equal to the utility $u(1) = 0$ that they would obtain from storing their endowment. Constraint (2.17) is simply the consumers' incentive compatibility constraint. Together with (2.15), this guarantees that also the incentive compatibility constraint for the late consumers at date 1 as in (2.2) is satisfied so that the safe banks never experience a run at date 1.

The risky banks solve a similar problem with the difference that they default in state $H$ at date 1. In state $L$ they may have to sell loans but have enough resources to repay the promised amounts $(c_1^R, c_2^R)$ to consumers and make non negative expected profits. In state $H$ they sell all the loans and go bankrupt. Early and late consumers receive the proceeds from the sale as given by $R^R + P_H L^R < c_1^R$. Each risky bank chooses the deposit contract $(c_1^R, c_2^R)$ and the initial allocation of funds between reserves $R^R$ and loans $L^R$ to solve the following problem

$$\max_{c_1^R, c_2^R, R^R, L^R} \Pi^R = \pi\left(rL^R - r\left(\frac{\lambda_L c_1^R - R^R}{P_L}\right) - (1-\lambda_L)c_2^R\right) \tag{2.18}$$

subject to

$$R^R + L^R = 1$$

$$\lambda_L c_1^R \leq R^R + P_L L^R \tag{2.19}$$

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$$(1-\lambda_L)c_2^R \leq r\left(L^R - \frac{\lambda_L c_1^R - R^R}{P_L}\right) \tag{2.20}$$

$$E[u(c_1^R, c_2^R, \lambda_\theta)] = \pi[\lambda_L u(c_1^R) + (1-\lambda_L)u(c_2^R)] + (1-\pi)[u(R^R + P_H L^R)] \geq 0 \tag{2.21}$$

$$c_2^R \geq c_1^R \tag{2.22}$$

The risky banks have positive expected profits only when with probability $\pi$ state $L$ occurs. These are equal to the returns from the initial investment in loans $rL^R$ minus the foregone return $r$ on the $(\frac{\lambda_L c_1^R - R^R}{P_L})$ units of loans that they sell at date 1 at the price $P_L$ and the expected repayments $(1-\lambda_L)c_2^R$ to the late consumers. The first constraint is the usual budget constraint at date 0, which always binds. The second constraint is the resource constraint at date 1 in state $L$. It requires that the maximum possible resources $R^R + P_L L^R$ that the risky bank obtains from its reserve holdings and the proceeds from the sale of all its loans are enough to satisfy the promised repayments $\lambda_L c_1^R$ to the early consumers at date 1. Constraint (2.20) ensures that in state $L$ the bank has enough resources at date 2 to repay the late consumers their promised repayment $(1-\lambda_L)c_2^R$. Note that constraint (2.20) implies that (2.19) is also satisfied. Constraint (2.21) requires that the deposit contract satisfies the constraint that the consumers' expected utility $E[u(c_1^R, c_2^R, \lambda_\theta)]$ is at least equal to the utility $u(1) = 0$ they would obtain from storing their endowment. Differently from before, the depositors of a risky bank receive the promised repayment $(c_1^R, c_2^R)$ only in state $L$, while they receive the proceeds $R^R + P_H L^R$ of the bank's portfolio in state $H$. Finally, the deposit contract has to satisfy the incentive constraint for the late consumers at date 0 as represented by the last constraint.

### 2.4.2 Market clearing

To complete the characterization of the equilibrium it remains to be determined the prices $P_H$ and $P_L$, and the fractions $\rho$ and $1-\rho$ of safe and risky banks. To be an equilibrium the solutions to the banks' maximization problems as described above must be consistent with the market clearing conditions determining $P_H$ and $P_L$. Moreover, the equilibrium requires that the expected profit of a risky bank and of a safe bank are equal so that banks are initially indifferent between becoming either type.

We start by characterizing the market clearing conditions. Consider first state $L$. Market clearing requires that the total demand for liquidity equals the excess supply of liquidity at date 1. Define as $\rho$ the number of safe banks and as $1-\rho$ that of risky banks. Then, it must hold that

$$(1-\rho)\lambda_L c_1^R = \rho(R^S - \lambda_L c_1^S), \tag{2.23}$$

where the left hand side represents the total liquidity demand of the $1-\rho$ risky banks in state $L$ and the right hand side the total net supply of liquidity of the $\rho$ safe banks after repaying $\rho\lambda_L c_1^S$ to their early depositors. Condition (2.23) implicitly requires $P_L < r$ so that the short asset is dominated between dates 1 and 2 and the safe banks are willing to use all their excess liquidity to buy loans from the risky banks.

Now consider state $H$. The safe banks have a total excess of liquidity equal to $\rho(R^S - \lambda_H c_1^S)$ while risky banks must sell all their $(1-\rho)L^R$ loans. Market clearing requires the price $P_H$ to satisfy

$$(1-\rho)P_H L^R = \rho(R^S - \lambda_H c_1^S). \tag{2.24}$$

Condition (2.24) implies that there is cash-in-the-market pricing in state $H$ as $P_H$ has to equalize the total supply and the total demand for liquidity.

As mentioned above, in equilibrium banks have to be indifferent between being a safe or a risky bank. This requires the expected profits of safe and risky banks to be the same, that is

$$\Pi^S = \Pi^R. \tag{2.25}$$

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We can now characterize the mixed equilibrium as defined by the vector $\{R^S, L^S c_1^S, c_2^S, R^R, L^R, c_1^R, c_2^R, \rho, P_L, P_H\}$. We have the following.

**Proposition 2** *The mixed equilibrium is characterized as follows:*

*1. The safe banks invest an amount $R^S$ in reserves and $L^S = 1 - R^S$ in loans, and offer consumers a deposit contract $(c_1^S, c_2^S)$ as follow*

$$R^S = \lambda_H c_1^S + \frac{1-\rho}{\rho} P_H, \tag{2.26}$$

$$c_1^S = \left( \frac{P_L}{r} \frac{\pi\lambda_L + (1-\pi)\lambda_H}{\pi\lambda_L + (P_L - \pi)\lambda_H} \right)^{\pi(1-\lambda_L)+(1-\pi)(1-\lambda_H)} < 1 \tag{2.27}$$

$$c_2^S = \left( \frac{r}{P_L} \frac{\pi\lambda_L + (P_L - \pi)\lambda_H}{\pi\lambda_L + (1-\pi)\lambda_H} \right)^{\pi\lambda_L+(1-\pi)\lambda_H} > 1. \tag{2.28}$$

*2. The risky banks invest an amount*

$$R^R = 0$$

*in reserves and $L^R = 1$ in loans, and offer consumers a deposit contract $(c_1^R, c_2^R)$ such that*

$$c_1^R = \frac{P_L}{r} c_2^R \tag{2.29}$$

$$c_2^R = \frac{c_2^S - r(1-\pi)}{\pi} > 1. \tag{2.30}$$

*3. The price $P_L > 1$ is the solution to (2.21), while $P_H$ is given by*

$$P_H = \frac{P_L(1-\pi)}{P_L - \pi} < 1. \tag{2.31}$$

*4. The fraction of safe banks is*

$$\rho = \frac{\lambda_L c_1^R}{\lambda_L c_1^R + R^S - \lambda_L c_1^S} < 1. \tag{2.32}$$

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European University Institute DOI: 10.2870/30719

**Proof.** See appendix A. ■

The proposition shows that safe and risky banks behave quite differently. Safe banks hold more reserves than the amount they need to satisfy their own early liquidity demand in state $H$. The amount in excess that each safe bank holds, $\frac{1-\rho}{\rho}P_H$, has to be enough to absorb the supply of loans of the risky banks in state $H$ as given by $(1-\rho)P_H$.

As in the no default equilibrium, the safe banks do not default and always repay their consumers the promised repayments. They still offer $c_2^S > 1 > c_1^S$ except in perfect competition where $r = 1$ and thus $c_1^S = c_2^S = 1$. As $\gamma$ increases, $c_1^S$ and $c_2^S$ move in opposite direction. The only difference with the no default equilibrium is that $c_2^S$ and $c_1^S$ depend on the price $P_L$ as well as on $r$ as there is now trade on the interbank market.

The risky banks do not hold any reserves and default in state $H$. As they anticipate their default in state $H$, risky banks find it optimal to invest everything in loans at date 0. At date 1 they sell $\frac{\lambda_L c_1^R}{P_L}$ units of loans to satisfy the liquidity demand $\lambda_L c_1^R$ of the early depositors in state $L$ while they sell everything and default in state $H$. Depositors at risky banks receive the promised repayments $c_1^R$ and $c_2^R$ only in state $L$. Their ratio is given by $\frac{c_2^R}{c_1^R} = \frac{r}{P_L} \geq 1$ as long as $P_L \leq r$.

The prices $P_L > 1$ and $P_H < 1$ adjust so to sustain the mixed equilibrium. This means that the interbank market clears in each state and safe banks are willing to hold the excess of reserves needed at date 0. The proportion of safe banks is always smaller than one, and safe and risky banks make the same expected profits in equilibrium. The price volatility crucially depends on the aggregate uncertainty of the liquidity demand. Given that there is excess liquidity in state $L$ relatively to state $H$, it must be the case that $P_L > 1 > P_H$.

## 2.5 Existence of equilibria

So far we have characterized two types of equilibria. In the no default equilibrium all banks hold enough reserves to satisfy their liquidity demand at date 1 for any $\theta = L, H$ and do not default. In the mixed equilibrium, banks are heterogenous. A fraction $\rho$ hold

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enough reserves and do not default. The remaining fraction $(1-\rho)$ chooses optimally to invest only in loans and default at date 1 in state $H$. There is trade on the interbank market, and the prices clear the market in both states at date 1 and at date 0.

In deriving the two equilibria we have so far abstracted from their existence. The no default equilibrium, as characterized in Proposition 1, exists if and only if no bank has an incentive to hold a lower amount of reserves and default at date 1 in state $H$. A deviation entailing higher reserves than in equilibrium is not profitable as it implies foregoing the higher returns from the loans without giving the possibility to use the excess liquidity on the interbank market. Similarly, for the mixed equilibrium in Proposition 2 to exist, the safe banks must be willing to buy the loans from the risky banks at date 1. This implies to verify that the equilibrium market prices $P_L$ and $P_H$ are within the feasible ranges.

We now turn to characterizing the existence of the equilibria as a function of the parameters of the model. In particular, we analyze how this depends on the degree of competition in the credit market. We start with the no default equilibrium.

### 2.5.1 Existence of the no default equilibrium

In the no default equilibrium all banks hold an amount of reserves $R^{ND} = \lambda_H c_1^{ND}$. This allows all banks to satisfy their early liquidity demand in any state of the economy and avoid default. However, it entails an opportunity cost in that banks forego the higher return $r$ on the loans on each unit of reserves that they hold. Such opportunity cost is higher the lower is the level of competition on the credit market. Thus, as $\gamma$ increases, it may be optimal for banks to reduce their amount of reserves so to appropriate the higher returns of the loans. To see whether this becomes profitable, we consider unilateral deviations from the equilibrium.

A deviating bank chooses the amount of reserves to hold and the deposit contract $(c_1^d, c_2^d)$ to offer to the consumers to solve the following maximization problem

$$\max_{c_1^d, c_2^d, R^d, L^d} \Pi^d = \pi \left[ rL^d - r\left(\frac{\lambda_L c_1^d - R^d}{P_L}\right) - (1-\lambda_L)c_2^d \right] \tag{2.33}$$

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subject to

$$R^d + L^d = 1$$

$$\lambda_L \leq R^d + P_L L^d$$

$$(1-\lambda_L)c_2^d \leq r\left(L^d - \frac{\lambda c_1^d - R^d}{P_L}\right)$$

$$E[u(c_1^d, c_2^d, \lambda_\theta)] = \pi[\lambda_L u(c_1^d) + (1-\lambda_L)u(c_2^d)] + (1-\pi)[u(R^d + P_H L^d)] \geq 0 \qquad (2.34)$$

The maximization problem of the deviating bank is similar to that of the risky banks in the mixed equilibrium. Since the deviating bank holds $R^d < R^{ND}$, it makes positive expected profit in state $L$ and defaults in state $H$. With probability $\pi$ it obtains the return $rL^d$ from the initial investment in loans minus the losses $r(\frac{\lambda_L c_1^d - R^d}{P_L})$ from the sale of the loans on the interbank market needed to satisfy all the demands of the early consumers at date 1 if $\lambda_L c_1^d - R^d > 0$ and the repayments $(1-\lambda_L)c_2^d$ to the late consumers at date 2. The first constraint is the usual resource constraint at date 0. The next two constraints are the resource constraints at dates 1 and 2 in state $L$. The last constraint is consumers' participation constraint at date 0. The difference relative to the mixed equilibrium is that the market prices $P_L$ and $P_H$ are still given by (2.11) and (2.12) as in the no default equilibrium, as the behavior of a deviating bank does not influence them. This implies that, given that it can sell loans at the price $P_L = r$ in state $L$ at date 1, the deviating bank finds it profitable to invest all in loans at date 0.

The no default equilibrium exists as long as deviating is not profitable, that is as long as $\Pi^{ND} \geq \Pi^d$. We have the following result.

**Proposition 3** *For* $\pi[\frac{V-\pi}{1-\pi}]^{1-\pi} + V(1-\pi) < 1$*, there exists a degree of credit market competition* $\gamma^* \in (\frac{1}{V}, 1)$ *such that the no default equilibrium exists for any* $\gamma \leq \gamma^*$.

**Proof.** See appendix A. □ ■

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The proposition shows that the no default equilibrium exists only for high degrees of competition in the credit market. In equilibrium banks hold enough reserves to satisfy the liquidity demands by the early consumers in state $H$. This ensures that they do not default in either state, but it leads to an excess of liquidity in state $L$. Such an excess is inefficient at date 1 as banks can only store between dates 1 and 2 and it may make a deviation profitable. As competition decreases, it becomes increasingly profitable for a bank to reduce reserves and holding more loans. This entails higher profits than in equilibrium in state $L$ but it also implies that banks default at date 1 in state $H$. When competition is intense, banks do not have an incentive to deviate from the equilibrium as the returns from investing in loans are too low to compensate them for defaulting in state $H$. By contrast, as competition decreases, the loan rate $r$ becomes high enough to make the deviation profitable. The condition $\pi[\frac{V-\pi}{1-\pi}]^{1-\pi} + V(1-\pi) < 1$ requires that the probability $\pi$ of the occurrence of state $L$ and the return $V$ on the loans are high enough to ensure that deviating is profitable when the credit market is monopolistic, that is when $\gamma$ is equal to 1. Thus, it also represents a sufficient condition for the existence of $\gamma^*$ in the interval $(\frac{1}{V}, 1)$.

### 2.5.2 Existence of the mixed equilibrium

The mixed equilibrium as characterized in Proposition 2 exists when the safe banks are willing to buy the loans from the risky banks at date 1. This implies to verify that the equilibrium market prices $P_L$ and $P_H$ are within the feasible ranges. The price $P_L$ must be in the interval

$$1 < P_L < r. \tag{2.35}$$

The lower bound guarantees that there is enough price volatility so that the safe banks are willing to invest in both reserves and loans at date 0. The upper bound ensures that they are willing to use any excess of liquidity to buy the loans in state $L$ at date 1. From (2.31), the price $P_H$ adjusts with $P_L$ so that the market always clears. This implies that


DOI: 10.2870/30719

as long as (2.35) is satisfied, $P_H$ is also in the admissible range. Thus, (2.35) is the only condition that matters for the existence of the mixed equilibrium. We have the following result.

**Proposition 4** *For any $\gamma > \gamma^* \in (\frac{1}{V}, 1)$, it holds $1 < P_L < r$ and the mixed equilibrium exists.*

**Proof.** See appendix A. □ ■

Proposition 4 states that the mixed equilibrium does not exist for high values of competition in the credit market. The intuition is analogous to the one for the existence of the no default equilibrium. The risky banks can make the same expected profits as the safe banks only if they are able to compensate the default in state $H$ with high profits in state $L$. When competition is intense, the return $\gamma V$ from the loans is low. Thus, the risky banks can make the same profit as the safe banks only if they can sell their loans at a high price in state $L$. For $\gamma < \gamma^*$, the required price for $\Pi^R = \Pi^S$ is $P_L > r$. However, at this price, the safe banks are unwilling to buy loans and thus the equilibrium does not exist. For $\gamma > \gamma^*$, the loan return $\gamma V$ is high enough so to ensure that the condition $\Pi^R = \Pi^S$ is satisfied for a price $P_L < r$.

Taken together, Propositions 3 and 4 show that the existence of the two equilibria is continuous in the parameter $\gamma$ representing the degree of competition in the credit market. The two equilibria converge at the level $\gamma = \gamma^*$. The intuition is that starting with the no default equilibrium, as $\gamma$ reaches $\gamma^*$, it becomes profitable for a bank to deviate by lowering its reserve holdings. As all banks are alike, more banks have an incentives to do the same. The equilibrium jumps to the mixed one with $1-\rho$ banks becoming risky. Thus, at $\gamma = \gamma^*$ the two equilibria coexist with $P_L = r$ and all banks have the same profits, that is $\Pi^{ND} = \Pi^d = \Pi^R = \Pi^S$.

Figure 1 shows the deposit contract that banks offer to depositors.

The figure shows that all the promised consumption levels to the late consumers increase with $\gamma$ while those to the early consumers decrease. The intuition is that as $\gamma$

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European University Institute DOI: 10.2870/30719

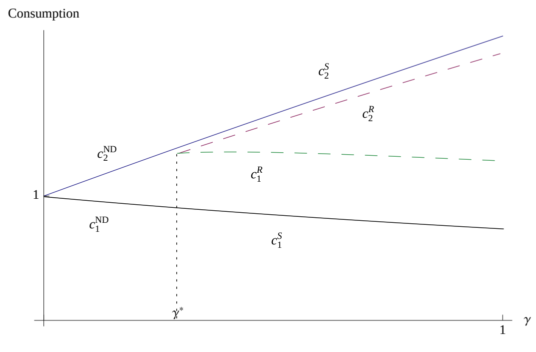


Figure 2.1: *Deposit contract*

increases, the opportunity cost of holding reserves increases and banks find it optimal to reduce the promised repayments to the early consumers so to reduce their reserve holdings. Thus, the promised repayments to the late consumers have to increase so to satisfy the consumers' participation constraint. The deposit contracts offered by the banks in the no default equilibrium and by the safe ones in the mixed equilibrium are equal at $\gamma = \gamma^*$. In the mixed equilibrium, the safe banks offer a more volatile deposit contract than the risky banks. The ratio between the consumption levels is indeed given by

$$\frac{c_2^S}{c_1^S} = \frac{r}{P_L} \frac{\pi\lambda_L + (P_L - \pi)\lambda_H}{\pi\lambda_L + (1-\pi)\lambda_H} > \frac{c_2^R}{c_1^R} = \frac{r}{P_L}$$

for any $r$ and $1 < P_L \leq r$ as $\frac{\pi\lambda_L + (P_L - \pi)\lambda_H}{\pi\lambda_L + (1-\pi)\lambda_H} > 1$. Both banks promise different amounts to the early and late depositors to reduce the liquidity needed at date 1 to satisfy the demand of the early depositors. To have more liquidity at date 1 safe banks need to hold a greater amount of reserves initially and forego the higher return $r$ on the loans. This effect is more important as $\gamma$ increases as the opportunity cost of holding reserves increases. Thus, $c_1^S$ decreases with $\gamma$ and $c_2^S$ increases as to allow depositors to break even in expectation. Risky banks obtain instead liquidity at date 1 by selling their loans. This also entails giving up the return $r$ on the loans sold but, given $P_L > 1$, it is less costly than holding reserves initially. The risky banks promise then different amounts to early and late consumers but the volatility of the contract they offer is lower than that of the


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safe banks.

Figure 2 shows the price volatility in the two equilibria.

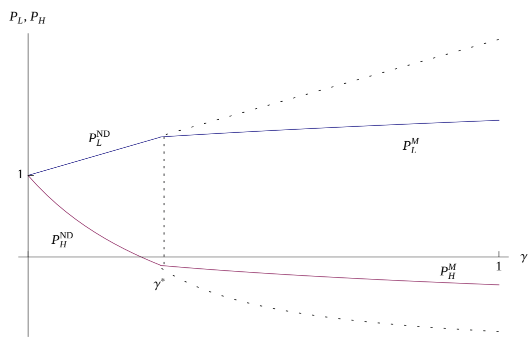


Figure 2.2: *Price volatility*

The figure shows that price volatility increases with $\gamma$ in both equilibria. The price $P_L$ increases with $\gamma$ as it is linked to the fundamental value $r$ of the loans, which increases with $\gamma$. Thus, as prices must move in opposite directions to make banks willing to hold both assets at date 0, $P_H$ decreases.

## 2.6 Discussion

Propositions 3 and 4 have important implications in terms of the effects of credit market competition on the stability of the banking system. When competition is intense, only the no default equilibrium exists. As competition decreases and $\gamma$ reaches the level $\gamma^*$, only the mixed equilibrium exists. Thus, the banking system becomes riskier as the credit market becomes less competitive. Even if it entails default, the functioning of the interbank market in the mixed equilibrium reduces the opportunity cost of holding reserves for the safe banks and allows the risky banks not to keep reserves and still obtain enough liquidity not to default in state $L$. This may have a positive effect on welfare as it may expand the initial investment in loans. To see this, consider the welfare in the two equilibria. In both cases, the welfare is given by the sum of banks' expected profits, entrepreneurs' surplus and consumers' expected utility. Given that banks are monopolist on the deposit side,

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consumers' expected utility is simply equal to zero. Welfare is then given by

$$W^{ND} = r - c_2^{ND} + (V - r)(1 - R^{ND})$$

in the no default equilibrium, and by

$$\begin{aligned} W^M &= r - c_2^S + (V - r)\left[(1 - \rho) + \rho(1 - R^S)\right] \\ &= r - c_2^S + (V - r)\left[(1 - \rho R^S)\right]. \end{aligned}$$

in the mixed equilibrium. At $\gamma = \gamma^*$ where the two equilibria coexist, $c_2^{ND} = c_2^S$ and $W^M > W^{ND}$ if $\rho R^S < R^{ND}$. Thus, the mixed equilibrium is welfare improving if it allows the banking system to hold fewer reserves and more loans in aggregate.

This result seems to suggest that banks' default is optimal in our model. However, it hinges upon the fact that entrepreneurs are always willing to borrow from banks as long as $r \leq V$ and default is costless in that it does not entail any deadweight costs. If default was costly, there would be a trade-off in the mixed equilibrium between the expansion of loans and the bankruptcy costs. For example, if consumers would only appropriate a fraction $\alpha$ of the proceeds $P_H$ of the sale of loans in state $H$, the risky banks would need to increase the promised repayments $c_2^R$ and $c_2^R$ to consumers to compensate them for the lower amounts they would obtain in state $H$. This would reduce the expected profits of the risky banks and thus the range of values for $\gamma$ in which the mixed equilibrium exists. The presence of bankruptcy costs would also negatively affect the social welfare, which could then be lower in the mixed equilibrium.

## 2.7 Concluding remarks

In this paper we have developed a simple model where banks trade loans to meet their uncertain liquidity demands and the returns on loans depend on the degree of credit market competition. We have shown that two types of equilibria can exist. In the no

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European University Institute DOI: 10.2870/30719

default equilibrium, banks are self sufficient in that they hold enough reserves to meet their liquidity demands in all states of the world. In the mixed equilibrium, banks make very different initial allocative choices. A group of banks does not keep any reserves and sell loans on the interbank market to satisfy their liquidity demands. In the state where the aggregate liquidity demand is low, they sell a part of their loans and make positive profits. In the other state, where the aggregate liquidity demand is high, they sell all their loans and default. Thus, in the mixed equilibrium default is observed with positive probability. The existence of the two equilibria depends on the degree of competition in the credit market. When this is intense, then only the no default equilibrium exists, while when it is low, only the mixed equilibrium exists. For intermediate values both equilibria may coexist. This implies that the lack of competition is detrimental to financial stability.

In our model banks are monopolist on the deposit market. An interesting extension is to consider the case where there is perfect competition on the deposit market so that banks choose their portfolio allocations in order to maximize consumers' expected utility rather than their expected profits. This would have implications for the amounts promised to consumers, banks' portfolio allocations and thus welfare.

We have modeled competition in the credit market as representing a Nash bargaining game between the bank and the entrepreneurs. Our qualitative results hold for a more general specification of competition as long as the returns from loans accruing to the bank decrease in the degree of competition in the credit market.

## 2.8 Appendix A

**Proof of Proposition 1:** The bank's maximization problem has a simple solution. In order to avoid default the bank chooses to keep enough reserves to cover its demand for liquidity at date 1 in every state of the world. Given $\lambda_H > \lambda_L$, in equilibrium it must then hold

$$R^{ND} = \lambda_H c_1. \tag{2.36}$$

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This implies that (2.4) is satisfied with equality in state $H$ and with strict inequality in state $L$, where there is an excess of liquidity. It is to show that the only other binding constraint in equilibrium is the consumers' participation constraint as given by (2.6). Solving it with respect to $c_2$, we obtain

$$c_2 = c_1^{-[\frac{\pi\lambda_L+(1-\pi)\lambda_H}{\pi(1-\lambda_L)+(1-\pi)(1-\lambda_H)}]}. \tag{2.37}$$

Substituting the expression for $c_2$ and (2.36) into (2.3) gives

$$\Pi_i = r(1-\lambda_H c_1) + \pi(\lambda_H - \lambda_L)c_1 - [\pi(1-\lambda_L) + (1-\pi)(1-\lambda_H)]c_1^{-\frac{\pi\lambda_L+(1-\pi)\lambda_H}{\pi(1-\lambda_L)+(1-\pi)(1-\lambda_H)}}.$$

Differentiating this with respect to $c_1$, we obtain

$$-r\lambda_H + \pi(\lambda_H - \lambda_L) + [\pi\lambda_L + (1-\pi)\lambda_H]c_1^{-\frac{1}{\pi(1-\lambda_L)+(1-\pi)(1-\lambda_H)}} = 0,$$

from which $c_1^{ND}$ as (2.8) in the proposition. Substituting (2.8) into (2.37) gives $c_2^{ND}$ as in (2.9) in the proposition. □

**Proof of Proposition 2:** We derive the vector $\{R^S, L^S, c_1^S, c_2^S, R^R, L^R, c_1^R, c_2^R, \rho, P_L, P_H\}$ characterizing the mixed equilibrium as the solution to the maximization problem of the safe and risky banks, the market clearing conditions and the equality between the expected profit of risky and safe banks. In the banks' maximization problems the only binding constraints are the consumers' participation constraints given by (2.16) and (2.21). All other constraints representing the resources constraints at dates 1 and 2 are satisfied with strict inequality, as shown in Appendix B.

Consider first the maximization problem for the safe banks. Using the Lagrangian, this can be written as

$$^S = \Pi^S - \mu^S\left[[\pi\lambda_L + (1-\pi)\lambda_H]\,u(c_1^s) + [\pi(1-\lambda_L) + (1-\pi)(1-\lambda_H)]\,u(c_2^s)\right].$$

The first order conditions with respect to the reserves $R^S$, $c_1^S$, $c_2^S$ and $\mu^S$ are as follow:

$$\frac{\pi}{P_L} + \frac{1-\pi}{P_H} = 1, \tag{2.38}$$

$$\left[\frac{\pi\lambda_L}{P_L} + \frac{(1-\pi)\lambda_H}{P_H}\right] r = -\frac{\mu^S}{c_1^S}\,[\pi\lambda_L + (1-\pi)\lambda_H] \tag{2.39}$$

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$$c_2^S = -\mu^S \tag{2.40}$$

$$[\pi\lambda_L + (1-\pi)\lambda_H]\, u(c_1^S) + [\pi(1-\lambda_L) + (1-\pi)(1-\lambda_H)]\, u(c_2^S) = 0. \tag{2.41}$$

Consider now the maximization problem of the risky banks. Using the Lagrangian, this becomes

$$^R = \Pi^R - \mu^R \left[\pi[\lambda_L u(c_1^R) + (1-\lambda_L)u(c_2^R)] + (1-\pi)[u(R^R + P_H(1-R^R))]\right].$$

The first order conditions with respect to the reserves $R^R$, $c_1^R$, $c_2^R$ and $\mu^R$ are as follow:

$$-\pi r + \frac{\pi r}{P_L} = \frac{\mu^R(1-\pi)(1-P_H)}{R^R + P_H(1-R^R)} \tag{2.42}$$

$$\frac{r}{P_L} = -\frac{\mu^R}{c_1^R} \tag{2.43}$$

$$c_2^R = -\mu^R \tag{2.44}$$

$$\pi[\lambda_L u(c_1^R) + (1-\lambda_L)u(c_2^R)] + (1-\pi)[u(R^R + P_H(1-R^R))] = 0. \tag{2.45}$$

The equilibrium is the solution to the system of all the first order conditions together with (2.23), (2.23) and (2.25).

A convenient way to solve the system is to start by using (2.38) to derive $P_H$ as in (2.31). Then, we derive $R^S$ as in (2.26) from (2.23). Using (2.40), (2.39) and (2.41) after substituting (2.31) gives $c_1^S$ and $c_2^S$ as in (2.27) and (2.28) in the proposition.

Using (2.43) and (2.44), we can express $c_1^R$ as in (2.29) in the proposition and $R^R$ from (2.42) as

$$R^R = \frac{(1-\pi)c_1^R}{\pi(P_L-1)} - \frac{P_H}{1-P_H}. \tag{2.46}$$

Substituting $P_H$ from (2.31) into (2.46) and rearranging it gives

$$R^R = \frac{(1-\pi)}{\pi(P_L-1)}(c_1^R - P_L) \leq 0$$

for any $P_L - 1 > 1$ and $c_1^R - P_L \leq 0$. The former follows from (2.31), as otherwise

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$P_H > 1 > P_L$. This contrasts with the equilibrium where $P_L > P_H$ must hold as there is more excess of liquidity in state $L$ than in state $H$. The latter, $c_1^R - P_L \leq 0$, follows from the fact that the profits of the risky banks must be non-negative in equilibrium. To see this, we rewrite (2.18) as follows:

$$\Pi^R = \pi\left[r + \left(\frac{1}{P_L} - 1\right)R^R r - (1-\lambda_L)c_2^R - \frac{\lambda_L c_1^R}{P_L}r\right].$$

As $\left(\frac{1}{P_L} - 1\right)R^R r < 0$ for $P_L > 1$, $\Pi^R \geq 0$ requires

$$r - (1-\lambda_L)c_2^R - \frac{\lambda_L c_1^R}{P_L}r > 0.$$

Rewriting $r$ as $\lambda_L r + (1-\lambda_L)r$ and rearranging the terms gives

$$(1-\lambda_L)(r - c_2^R) + \lambda_L r\left(\frac{P_L - c_1^R}{P_L}\right). \tag{2.47}$$

This is positive if $P_L - c_1^R > 0$ as this implies also that $r - c_2^R > 0$. Consider $P_L - c_1^R < 0$. Then, from (2.44), it is $c_2^R > r$ and (2.47) is negative. Then, in equilibrium $P_L - c_1^R > 0$ must hold. It then follows that $R^R = 0$ as in the proposition.

To find $c_2^R$ as in the proposition we first rearrange (2.25) as

$$\begin{aligned}\Pi^S - \Pi^R =& R^S[-1 + \frac{\pi}{P_L} + \frac{1-\pi}{P_H}]r - \left[\frac{\pi\lambda_L}{P_L} + \frac{(1-\pi)\lambda_H}{P_H}\right]rc_1^S + \\ &+ r(1-\pi) + \pi c_2^R - [\pi(1-\lambda_L) + (1-\pi)(1-\lambda_H)]c_2^S = 0.\end{aligned}$$

From (2.38) $[-1 + \frac{\pi}{P_L} + \frac{1-\pi}{P_H}] = 0$. From (2.39)

$$\left[\frac{\pi\lambda_L}{P_L} + \frac{(1-\pi)\lambda_H}{P_H}\right]rc_1^S = [\pi\lambda_L + (1-\pi)\lambda_H]c_2^S.$$

Substituting these into the expression above for $\Pi^S - \Pi^R$, we have $c_2^R$ as in (2.30) in the proposition.

Finally, from (2.21) and (2.23), we have $P_L$ and $\rho$ as in (2.32) in the proposition. □

**Proof of Proposition 3:** The maximization problem of the deviating bank is the same as the one of the risky banks in the proof of Proposition 2. The first order conditions with respect to the reserves $R^d$, $c_1^d$, $c_2^d$ and $\mu^d$ are as (2.42), (2.43), (2.44) and (2.45). The only difference is that $P_L = r$ and thus $P_H = \frac{r(1-\pi)}{r-\pi}$. The solutions to the first order

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conditions are:

$$R^d = 0$$

$$c_1^d = c_2^d = c^d = (\frac{r-\pi}{1-\pi})^{1-\pi}. \tag{2.48}$$

Substituting these expressions into (2.33) gives

$$\Pi^d = \pi(r - c^d).$$

To see when deviating is profitable, it remains to compare $\Pi^d$ and $\Pi^{ND}$ from (2.10) as a function of $\gamma$. In perfect competition when $\gamma = \frac{1}{V}$ and $r = 1$, $\Pi^{ND} = \Pi^d = 0$ since from (2.8), (2.9) and (2.48), $c_1^{ND} = c_2^{ND} = c^d = 1$. Differentiating $\Pi^{ND}$ and $\Pi^d$ with respect to $\gamma$ and rearranging the expressions, we have

$$\frac{\partial \Pi^{ND}}{\partial \gamma} = V(1 - \lambda_H c_1^{ND}) > 0$$

$$\frac{\partial \Pi^d}{\partial \gamma} = \pi V \left(1 - (\frac{1-\pi}{r-\pi})^{\pi}\right) > 0.$$

At $\gamma = \frac{1}{V}$, $\frac{\partial \Pi^{ND}}{\partial \gamma}\Big|_{\gamma=\frac{1}{V}} = V(1-\lambda_H) > \frac{\partial \Pi^d}{\partial \gamma}\Big|_{\gamma=\frac{1}{V}} = 0$. At $\gamma = 1$, $\Pi^{ND} = V - \left(\frac{\pi\lambda_L + (V-\pi)\lambda_H}{\pi\lambda_L + (1-\pi)\lambda_H}\right)^{\pi\lambda_L + (1-\pi)\lambda_H}$ and $\Pi^d = \pi\left(V - (\frac{V-\pi}{1-\pi})^{1-\pi}\right)$. Rearranging these expression, we can express the difference $\Pi^{ND} - \Pi^d$ at $\gamma = 1$ as

$$\begin{aligned}\Pi^{ND} - \Pi^d\Big|_{\gamma=1} \quad &= \quad (\pi\lambda_L + (1-\pi)\lambda_H)^{\pi\lambda_L + (1-\pi)\lambda_H}\left[(1-\pi)^{2-\pi}V + \pi(V-\pi)^{1-\pi}\right] + \\ &\quad -(1-\pi)^{1-\pi}(\pi\lambda_L + (V-\pi)\lambda_H)^{\pi\lambda_L + (1-\pi)\lambda_H}.\end{aligned}$$

Since $\pi\lambda_L + (V-\pi)\lambda_H > \pi\lambda_L + (1-\pi)\lambda_H$, $\Pi^{ND} - \Pi^d < 0$ if $(1-\pi)^{1-\pi} > (1-\pi)^{2-\pi}V + \pi(V-\pi)^{1-\pi}$, from which $\pi[\frac{V-\pi}{1-\pi}]^{1-\pi} + V(1-\pi) < 1$ as in the proposition. Thus, it exists $\gamma^* \in (\frac{1}{V}, 1)$ such that $\Pi^{ND} - \Pi^d \geq 0$ for any $\gamma \leq \gamma^*$ and $\Pi^{ND} - \Pi^d < 0$ otherwise. The proposition follows. □

**Proof of Proposition 4:** Recall that in the mixed equilibrium $\Pi^S = \Pi^R$ and note from (2.18) that $\Pi^R$ increases with $P_L$ for any given $\gamma$. Moreover, $\Pi^R = \Pi^d$ and $\Pi^S = \Pi^{ND}$ when $P_L = r$. Thus, as $\Pi^d = \Pi^{ND}$ at $\gamma = \gamma^*$ and $\Pi^R = \Pi^S$ in the mixed equilibrium, it must also hold $\Pi^d = \Pi^{ND} = \Pi^R = \Pi^S$ at $\gamma = \gamma^*$. Consider now a value of $\gamma < \gamma^*$, where $\Pi^d < \Pi^{ND}$ from Proposition **3**. It follows that $\Pi^R\big|_{P_L=r} < \Pi^S\big|_{P_L=r}$. For $\Pi^R = \Pi^S$ to hold as required in the mixed equilibrium, it must then be $P_L > r$. Consider now a value


DOI: 10.2870/30719

of $\gamma > \gamma^*$, where $\Pi^d > \Pi^{ND}$ from Proposition **3**. It follows that $\Pi^R\big|_{P_L=r} > \Pi^S\big|_{P_L=r}$. Thus, it must be $P_L < r$ for $\Pi^R = \Pi^S$. It follows that the mixed equilibrium exists only for $\gamma \geq \gamma^*$. $\square$

## 2.9 Appendix B

In this Appendix we prove that the resource constraints in the banks maximization problems are non binding.

**Proof Constraint (2.15)**

We start from the resources constraint state $L$ at date 2.

$$(1-\lambda_L)c_2^S \leq r\left(L^S + \frac{R^S - \lambda_L c_1^S}{P_L}\right)$$

Substitute $L^S = 1 - R^S$ so to have

$$(1-\lambda_L)c_2^S \leq r\left(1 - R^S + \frac{R^S - \lambda_L c_1^S}{P_L}\right)$$

Multiply both sides by $P_L$ and add and subtract $r$ on the right hand side. The expression simplifies to

$$P_L(1-\lambda_L)c_2^S \leq r(P_L - 1)\left(1 - R^S\right) + r\left(1 - \lambda_L c_1^S\right).$$

Substituting $c_1^S = c_2^S \frac{P_L}{r} \frac{(\pi\lambda_L + (1-\pi)\lambda_H)}{(\pi\lambda_L + (P_L - \pi)\lambda_H)}$ from (2.39) into the expression above, after some algebraic manipulations;, it gives:

$$P_L\left[\lambda_H P_L - \pi(\lambda_H - \lambda_L) - (P_L - 1)\lambda_H\lambda_L\right]c_2^S \leq r\left(\pi\lambda_L + (P_L - \pi)\lambda_H\right)\left(P_L(1 - R^S) + R^S\right).$$

It can be easily seen that the right hand side is increasing in $R^S$ as $P_L > 1$. Thus, if the condition holds for $R^S = 1$,it also holds for any $R^S < 1$. Taking $R^S = 1$ and multiplying both sides by $\frac{(\pi\lambda_L + (1-\pi)\lambda_H)}{(\pi\lambda_L + (P_L - \pi)\lambda_H)^2}$we obtain

$$c_1^S - \frac{c_1^S(P_L - 1)}{(\pi\lambda_L + (1-\pi)\lambda_H)} \leq \frac{(\pi\lambda_L + (1-\pi)\lambda_H)}{(\pi\lambda_L + (P_L - \pi)\lambda_H)}.$$

Adding and subtract $\lambda_H$ on the left hand side, the expression above can be rewritten as follows:

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$$c_1^S\left(\pi\lambda_L + (1-\pi)\lambda_H\right) - c_1^S(P_L - 1)(1-\lambda_H \leq \left(\pi\lambda_L + (1-\pi)\lambda_H\right)$$

that holds as the second term on the left hand side is negative and $c_1^S < 1.\square$

Consider now the resources constraint of the safe banks in state $H$.

$$(1-\lambda_H)c_2^S \leq r\left(1 - R^S + \frac{R^S - \lambda_H c_1^S}{P_H}\right)$$

Multiply both sides by $P_H$ and substitute the expression $R^S = \lambda_H c_1^S + \frac{1-\rho}{\rho} P_H$ from (2.38). The expression above simplifies to

$$P_H\left[(1-\lambda_H)c_2^S - r(1-\lambda_H c_1^S)\right] \leq rP_H\left(\frac{1-\rho}{\rho}\right)(1-P_H).$$

The right hand side is positive for any $P_H < 1$, while the left hand side is negative as $r > c_2^S$ and $c_1^S < 1$. Thus, the condition always holds.$\square$

**Proof Constraint(2.22)**

We start proving that the resources constraint in state $L$ at date 2 is satisfied in equilibrium.

$$(1-\lambda_L)c_2^R \leq r\left(L^R - \frac{\lambda_L c_1^R - R^R}{P_L}\right)$$

In equilibrium, $R^R = 0$, thus the constraint can be rewritten as follows

$$(1-\lambda_L)c_2^R \leq r\left(1 - \frac{\lambda_L c_1^R}{P_L}\right).$$

Substituting $c_2^R = \frac{r}{P_L} c_1^R$ from (2.43), the expression simplifies to

$$(1-\lambda_L)c_1^R \leq \left(P_L - \lambda_L c_1^R\right)$$

and then to

$$c_1^R \leq P_L.$$

thus the constraint is always satisfied in equilibrium. $\square$

It remains to show that in state $H$, risky banks are not able to pay to the late consumers the promised repayment. This implies that

$$(1-\lambda_H)c_2^R > r\left(L^R - \frac{\lambda_H c_1^R - R^R}{P_H}\right)$$

Using $R^R = 0$ and $c_2^R = \frac{r}{P_L} c_1^R$ in the condition above we have

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$$P_H(1-\lambda_H)c_1^R > \left(P_L P_H - P_L \lambda_H c_1^R\right).$$

Substituting the expression for$P_H = \frac{P_L(1-\pi)}{P_L-\pi}$ from (2.31) and rearranging the terms we have

$$\lambda_H(P_L-1)c_1^R > (P_L - c_1^R)(1-\pi).$$

Given $c_1^R > 1$, a sufficient condition for the constraint to hold is $\lambda_H > (1-\pi)$.$\square$

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Leonello, Agnese (2011), Three Essays in Competition and Banking
European University Institute DOI: 10.2870/30719

3

# Financial Networks, Interbank Market Breakdowns and Contagion

***Joint with Emanuela Iancu***

## 3.1 Introduction

The recent financial crisis unveiled the fragility of the financial system and the inadequacy of the regulatory framework to prevent the crisis and contain its negative effects. Having started in the subprime mortgage market, the turmoil spread to the whole financial industry and to the real sector.

The high interconnectedness of the financial system facilitated contagion. The distress of few financial intermediaries spilled over to the whole sector, through direct and indirect linkages. The existence of these interdependencies amplified the effects of the shock in the financial system and hindered a proper assessment of individual risk in the global context. Connections between financial institutions stem from both the asset and the liability side of their balance sheet (see Allen and Babus, 2009 for a survey). On one hand, connections allow for better diversification so reducing the risk at individual bank level. On the other hand, they prompt a higher correlation of the portfolios of financial intermediaries and, as the recent financial crisis showed, in case of a negative shock, make the system less



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resilient.

One major source of instability during the recent financial crisis was the liquidity risk. Banks with a shortage of liquidity were not able to obtain the liquidity they needed. Uncertainty about the counterparty risk played an important role in prompting the malfunctioning of interbank markets, making difficult for the banks in distress to meet their liquidity demand. Being not able to obtain the liquidity on the interbank market, those banks were forced to sell assets at fire-sale prices. Due to the high interconnectedness, the distress of few institutions propagated in the banking system through mark-to-market accounting, setting the ground for contagion. Marking-to-market requires firms to book their assets and liabilities at fair values. When prices reflect the fundamentals, this accounting standard guarantees a true measure of the value of the company. This does not hold when other factors affect the prices (i.e. liquidity) as mark-to-market induces firms to underestimate (or overestimate) profits. The recent financial crisis highlighted the drawbacks of this accounting principle as several banks sold their assets at fire-sale prices, bringing banks with correlated exposures on the brink of insolvency.

This paper aims at capturing these features and at highlighting the mechanism through which contagion rises. We develop a simple two periods theoretical model in which profit-maximizing banks are subject to both credit and liquidity risk. Banks raise deposits from risk-averse consumers and invest the proceeds in a safe asset and a risky project. In order to hedge against the credit risk, at the initial date a bank exchanges shares of its project with other banks, further referred to as neighbors. As a result, a financial network is formed. Banks face idiosyncratic uncertainty relative to their demand for liquidity, as a stochastic fraction of their depositors are early consumers. All uncertainty is resolved at the interim period: each bank receives perfect information about the return of its portfolio and the distribution of idiosyncratic demand for liquidity becomes common knowledge.

At the interim period, for a given initial investment in liquidity, some banks face a shortage and some an excess of liquidity. Banks facing a shortage of liquidity have two possibilities to meet their liquidity needs: i) they can borrow from their neighbors with an excess of

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liquidity; ii) they can sell assets to outside investors. We assume the existence of a specific market for each asset. Investors have no information about the return of the assets the banks sell. The asymmetric information gives rise to adverse selection, forcing the banks to sell the assets at fire-sale prices.

Banks sell assets only if they can not borrow from their neighbors with an excess of liquidity. Resorting to the interbank market has the benefit of avoiding fire-sales and therefore a temporary depression of the portfolio value. Due to mark-to-market accounting, fire-sale prices affect the balance sheets of all banks holding that particular asset under distress and hence their solvency. When becoming insolvent, a bank has to liquidate its portfolio, triggering the drop in value of further assets. The domino effect prompted by fire-sales emerges eventually into a wave of bankruptcies.

The main result of the paper is that incomplete information may lead to interbank market freeze and this, in turn, may trigger contagion through asset sales and mark-to-market accounting. Under incomplete information, banks have different perceived probabilities of bankruptcy. When the banks with excess of liquidity underestimate the risk of contagion and expect those with a shortage to attach a higher probability to it, the former are induced to demand excessively high interest rates. Such offers are rejected by the banks in distress that prefer to resort to outside investors. As consequence, fire sales trigger contagion with positive probability.

The paper is related to various others. Several papers (Allen and Gale (2000), Cifuentes, Ferrucci and Shin (2005), Caballero and Simsek (2010)) have analyzed contagion in financial networks. In a network in which banks are connected through interbank deposits, Allen and Gale (2000) discuss the optimality of different network structures in terms of financial stability. They show that complete networks are more resilient to contagion as banks' exposure to other institutions are smaller. Cifuentes *et al.* (2005) analyze the interplay of mark-to-market accounting and regulatory requirements. They show that it may negatively affect financial stability in a network in which banks are connected through direct and indirect linkages. As in our paper, contagion mainly arises through asset sales

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and fire sale prices. The main difference with our paper is that uncertainty plays no role as the network is complete. In Caballero and Simsek (2010), instead, uncertainty plays a crucial role. Banks do not have information about the soundness of the other players. As they are part of a financial network, this information is crucial to assess the actual risk of being hit indirectly by a shock. Complexity generates a large uncertainty and reduces the incentive of banks to provide liquidity to the market having a negative effect on financial stability. In this respect, this paper differs from ours. In our model, interbank market freeze does not occur because banks are reluctant to lend but rather because they demand an excessively high interest rate so to exploit the informational rent in the transaction with the borrowers.

Our paper is also related to the literature on the functioning of interbank market. Various papers have analyzed the inefficiency in the interbank market. Acharya, Gromb and Yorulmazer focus on market power; Heider, Hoerova (2009), instead, consider asymmetric information. In Gale and Yorulmazer (2010) frictions on the interbank market are due to incompleteness of the market. They show that the liquidity hoarding emerged during the recent financial crisis was a consequence of speculative and precautionary motives. Uncertainty about the counterparty risk as well as the possibility of using the excess liquidity to buy assets at lower future prices induced banks to hold liquidity in excess rather than provide it to the interbank market. The precautionary and speculative motives are also important features in our model to explain the interbank market freeze and financial contagion.

The paper proceeds as follows. Section 2 describes the model. Section 3 and 4 characterize the equilibria in the complete and incomplete information case, respectively. In Section 5 we provide a numerical example and section 6 concludes.

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European University Institute DOI: 10.2870/30719

## 3.2 The Model

### 3.2.1 The Environment

Consider a three date economy $t = 0, 1, 2$ with four regional banks $i \in \mathbf{N} = \{A, B, C, D\}$, depositors and outside investors.

At date 0, banks raise one unit of resources from depositors and invest a fraction $x$ into a safe asset and $1 - x$ in a risky project.

*Assets*

Each unit invested in the safe asset returns $\rho$ units, $\rho > 1$ in the subsequent period. The risky project produces a return

$$R_i = \begin{cases} R_L & \text{w. pr. } \pi \\ R_H & \text{w. pr. } (1-\pi) \end{cases} \tag{3.1}$$

at date 2, with $R_H > 1 > R_L$, $\forall i \in \mathbf{N}$. The expected return of the risky project is higher than the one of the safe asset

$$\pi R_L + (1-\pi) R_H \geq \rho^2. \tag{3.2}$$

Returns are independently distributed across banks.

To hedge against the possibility that the risky project returns $R_L$, at date 0 banks exchange equal shares of their risky projects with two other banks, henceforth called neighbors.

A financial network is formed (Figure 1). Each bank holds a share $\frac{(1-x)}{3}$ of its neighbors' projects in its portfolio.

The network is represented by a matrix $\mathbf{g} = (g_{ij})_{i,j \in \mathbf{N}}$, where $g_{ij} = \begin{cases} 1 & \text{if } i \text{ and } j \text{ exchanged assets} \\ 0 & \text{otherwise} \end{cases}$


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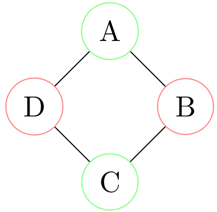


Figure 3.1: The Financial Network

$\forall i \in \mathbf{N}, i \neq j$,

Since each bank retains a portion of its own project, we have that $g_{ii} = 1$. Let $\mathcal{N}_i$ be the set of agents other than $i$ holding a portion of $i$'s long asset: $\mathcal{N}_i = \{j \in \mathbf{N} \backslash \{i\}, | \, g_{ij} = 1\}$. We further define $i$'s *neighborhood* as himself and his neighbors, i.e. $\widetilde{\mathcal{N}_i} = \mathcal{N}_i \cup \{i\}$

Given the network described above, we can formally describe the set of possible distributions of portfolio returns as follows

$$\mathcal{G}(\mathbf{R}) = \frac{(1-x)}{3} (\mathbf{g} \times \mathbf{R}) = \frac{(1-x)}{3} \begin{pmatrix} 1 & 1 & 0 & 1 \\ 1 & 1 & 1 & 0 \\ 0 & 1 & 1 & 1 \\ 1 & 0 & 1 & 1 \end{pmatrix} \times \begin{pmatrix} R_A \\ R_B \\ R_C \\ R_D \end{pmatrix} \tag{3.3}$$

*Depositors and Uncertainty*

There is a continuum of mass one of depositors in each region. Depositors are ex ante identical but ex post two types can be distinguished: *early* and *late*. Early type only wants to consume at date 1, while the others prefer to consume at date 2. Their initial endowment of one unit is deposited into a bank in exchange for the contract $(1, c_2)$. To avoid a run, late depositors are promised $c_2 \geq 1$.

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The probability of being an *early* depositor is given by

$$\lambda_i = \begin{cases} \lambda_L & \text{w. pr. } \frac{1}{2} \\ \lambda_H & \text{w. pr. } \frac{1}{2} \end{cases} \tag{3.4}$$

$\forall i \in \mathbf{N}$, with $0 < \lambda_L < \lambda_H < 1$.

The fraction of early depositors is bank specific. There is no aggregate liquidity shock and the total fraction of early depositors is one. For simplicity, we replace $\lambda_L$ and $\lambda_H$ with $(1-\lambda)$ and $\lambda$, respectively. All uncertainty is resolved at date 1. Banks observe the realization of $\lambda_i$ and this information becomes common knowledge. Due to the idiosyncratic liquidity shock, two banks have an excess of liquidity and two a shortage. Without loss of generality, we assume that the banks with excess of liquidity are A and C, whereas the ones with a shortage are B and D. Banks with an excess of liquidity have always enough resources to provide liquidity to one bank in distress. It follows:

$$\mid x\rho - (1-\lambda) \mid \geq \mid x\rho - \lambda \mid.$$

*Interbank Market*

Banks in distress can meet their liquidity need borrowing from the banks with an excess of liquidity on the interbank market at an interest rate $r$. The borrowing-lending game starts at date 1 and has the following timeline:

I.a Banks with excess decide between non-lending and lending to either bank in distress. The decisions become public knowledge.

I.b In case they lend, they choose the interest rate.

II.a Banks in distress decide whether to accept the offer or not.

II.b In case they reject, they resort to the asset market.

After stage I.a, four situations may arise: i) A and C lend to the same bank, ii) A and C make offers to different banks, iii) only one lender makes an offer, iv) no offers are made

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to B and D. In the first situation, A and C compete in offering the loan. In contrast, banks behave as a monopolist in two circumstances: i) when they make offers to two different banks; ii) in the case they are the only bank to make an offer.

Borrowing on the interbank market does not affect the value of banks' portfolio. No bank has to book the assets at fire sale price so contagion is avoided.

For a transaction to take place both lenders and borrowers must be willing to participate. When no transaction takes place, banks with a shortage of liquidity resort to the asset market.

*Asset Market*

Banks can sell a share of one of the long assets they hold to outside investors. If the transaction takes place, investors pay the price $P$ and get $\frac{(1-x)}{3}$ of the asset put for sale. The investors have no information on the realization of the risky project and have a profitable outside investment opportunity returning $R_0$ at date 2. They are willing to provide liquidity to the banks as long as it is more profitable than investing in the outside opportunity. For a sufficiently profitable investment opportunity, it holds $P < R_L$. This captures the idea of *fire sales* and ensures that only bank in distress eventually sell an asset on the market.[1]

We assume that at price $P$, selling one asset is enough to cover the shortage of liquidity:

$$P\frac{(1-x)}{3} = (\lambda - x\rho). \tag{3.5}$$

*Mark-to-market accounting and bankruptcy*

According to mark-to-market accounting, banks must book their asset and liabilities at the market values. At date 1, a bank is insolvent if the value of its assets is lower than

[1] An equivalent formalization is to consider $P$ as an exogenous liquidation value.


DOI: 10.2870/30719

the value of the outstanding debt.

$$\mathcal{V}_i = x\rho + \frac{(1-x)}{3} \sum_{j \in \widetilde{\mathcal{N}}_i} R_j < 1 \tag{3.6}$$

The value of a bank's portfolio $\mathcal{V}_i$ is given by the sum of the return of the safe asset $x\rho$ plus the market value of the bank's investment in the risky projects. The right hand side is the value of bank's liabilities at date 1 as the bank has to pay one unit to all potential consumers. The market value of bank's $i$ risky assets is determined according to the following rules:

- If the market is active, then the asset is booked at the market price $P$
- If the market is not active, the asset is booked at the " the price that would be received by the holder of the financial asset in an orderly transaction" i.e. fundamental value.

When an asset is sold, its market value equals the price $P$ paid by outside investors. As consequence of mark-to-market accounting, the portfolio value of all banks holding that asset is affected.

A bank is declared insolvent when

- It has three low return asset in its portfolio

$$x\rho + [R_L + R_L + R_L] \frac{(1-x)}{3} < 1 \tag{3.7}$$

- Two assets are sold

$$x\rho + [R_i + P + P] \frac{(1-x)}{3} < 1. \tag{3.8}$$

When two assets are sold in the economy there is at least one bank insolvent. Insolvency forces the bank to liquidate its portfolio on the market at date 1, making zero profits. Due to high correlation of banks' portfolios, this causes the bankruptcy of all other banks,

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as each of them has at least two out of three assets in portfolio to book at price $P$. In this case, the distress of a single financial institution spreads to the whole economy (contagion).

*Information*

The liquidity and credit shocks realize at date 1 and their distributions are stochastically independent. The realization of liquidity shocks are common knowledge to all the banks, the consumers and the investors. The return of the risky projects can be either common knowledge to the banks- the *complete information* $\mathcal{CI}$- or they can be private information to the banks which hold the respective assets- the *incomplete information* $\mathcal{II}$. Depositors can perfectly monitor their banks.

We focus our attention on the cases in which the banks with excess of liquidity are not direct neighbors of each other. Thus, sixteen cases can be identified. We focus on the specific distribution of returns represented by the return vector $\mathbf{R} = (R_A, R_B, R_C, R_D,) = (R_L, R_H, R_L, R_H)$, henceforth referred to as the true state of the economy.

In the next two sections, we characterize the equilibria in the complete and incomplete information framework. Then we compare the two scenarios so to show that interbank market freeze may emerge as an equilibrium when uncertainty is introduced.

## 3.3 Complete Information Case

In the complete information case, each bank observes the realization of all the returns in the economy. There is no uncertainty once the shocks realize at the interim period and the banks' decisions are taken in a context of complete information. The state the economy is described by the return vector and is represented in Figure 2.

As described in the previous section, banks A and C decide sequentially to whom to lend and at which interest rate. Then, in case no transaction in the interbank market takes

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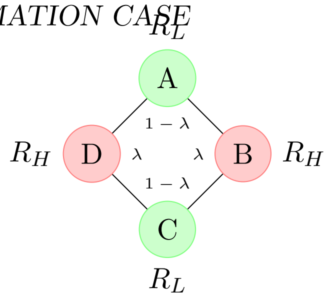


Figure 3.2: True state of the economy.

place, $B$ and $D$ have to decide which asset to sell. We solve the problem by backward induction starting from the subgame in which B and D sell their assets to outside investors to repay their early consumers.

*Stage 2*

In the complete information case, the type spaces of banks are singleton sets

$$\mathcal{T}_{\mathcal{B}}^{\mathcal{CI}} = \{R_H\} \qquad\qquad \mathcal{T}_{\mathcal{D}}^{\mathcal{CI}} = \{R_H\} \tag{3.9}$$

The action spaces are

$$\begin{aligned} \mathcal{A}_B &= \{\text{sell } A, \text{sell } B, \text{sell } C\}, \qquad (3.10)\\ \mathcal{A}_D &= \{\text{sell } A, \text{sell } D, \text{sell } C\}. \end{aligned}$$

The payoff functions of the two players are symmetric and the one for B takes the following values

$$\Pi_B(t_B, t_D, a_B, a_D) = \begin{cases} 0 & \text{if } a_B \neq a_D \\ \frac{(1-x)}{3}(2R_L + R_H) - \frac{(1-x)}{3}a_B - (1-\lambda)c_2 & \text{if } a_B = a_D \end{cases} \tag{3.11}$$

Both expressions represent the profits that bank $i$ accrues at date 2. In case two different assets are sold on the market, all banks in the economy are forced into early liquidation at date 1 and they earn zero profits. When, instead, banks B and D sell the same asset, all the banks in the economy survive to date 2 and B and D make positive profits. Their profits are given by the return on the investment in the risky assets $\frac{(1-x)}{3}(2R_L + R_H)$

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minus the forgone return on the asset they sell to cover their liquidity demand $\frac{(1-x)}{3}a_i$ and the promised repayment to the late depositors $(1-\lambda)c_2$. The following proposition establishes the equilibrium of the subgame between $B$ and $D$.

**Proposition 5** *The B-D subgame has three pure strategies and one mixed strategy Nash equilibria.*

- *The pure strategies Nash Equilibria are* $(a_B^*, a_D^*) \in \{$*(sell A, sell A), (sell C, sell C) and (sell B, sell D)*$\}$
- *The mixed strategy Nash Equilibrium is :* $\gamma^* = (\gamma_A^*, \gamma_{B/D}^*, \gamma_C^*) = (1/2, 0, 1/2)$

*where* $\gamma_k^*$ *is the optimal weight that players B and D put on playing the pure strategy* $k \in \{A, B/D, C\}$.

**Proof.** See Appendix. ■ The intuition behind Proposition 5 is simple. Banks prefer to coordinate and survive until date 2 as non-coordination triggers bankruptcy at date 1. Despite miscoordination being a weakly dominated action, (sell B, sell D) emerges though as a Nash equilibrium. In the pure strategy equilibria, bankruptcy occurs either with probability 1 or 0.

Selling either low-asset (A or C) yields the same payoffs to both B and D. Hence, there exists a mixed equilibrium, in which the two banks randomize between the two assets. In the mixed equilibrium, there is a probability $\frac{1}{2}$ of miscoordination and, in turn, of bankruptcy.

*Stage 1*

Given the equilibria in the $B$ and $D$'s subgame, we move backward to the first stage. In what follows, we will only consider the mixed strategy equilibrium in which miscoordination and contagion emerge with positive probability.

As described in the previous section, the game at stage 1 between A and C (henceforth called the *A-C game*) consists of two decisions: first, A and C decide simultaneously upon

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a loan offer to either B or D. These decisions become common knowledge. When they decide to lend, $A$ and $C$ engage in an interest-rate bidding process by submitting a take-it-or-leave-it interest rate to the bank of their choice. If B and/or D accept the offer, the game ends here. In case either of them declines, they enter the subgame in which they sell assets to investors.

In the complete information case, the type spaces of A and C are singleton sets

$$\mathcal{T}_{\mathcal{A}}^{\mathcal{CI}} = \{R_L\} \qquad \mathcal{T}_{\mathcal{C}}^{\mathcal{CI}} = \{R_L\}$$

and the action spaces are

$$\mathcal{A}_A = \{\text{offer to } D, \text{offer to } B, \text{no offer}\} \times \{[0, \infty[ \cup \{\emptyset\}\} \tag{3.12}$$

$$\mathcal{A}_C = \{\text{offer to } D, \text{offer to } B, \text{no offer}\} \times \{[0, \infty[ \cup \{\emptyset\}\}. \tag{3.13}$$

We define $r^{max}$ and $r^{min}$ as the maximum interest rate that the borrower is willing to accept and the lowest interest rate demanded by the lender, respectively. For a transaction on the interbank market to occur, it must be that the interest rate offered by the lender is in an admissible range. The interest rates $r^{max}$ and $r^{min}$ represent the upper and lower bound of the admissible range. The interest rate $r^{max}$ is the solution to the following equation:

$$(R_H + 2R_L)\frac{(1-x)}{3} - (1-\lambda)c_2 - (\lambda - x\rho)r^{max} = \frac{1}{2}\left[(R_H + R_L)\frac{(1-x)}{3} - (1-\lambda)c_2\right] + \frac{1}{2} * 0.$$

The left hand side represents the borrower profits in case it borrows. They are given by the date 2 return on its portfolio $(R_H + 2R_L)\frac{(1-x)}{3}$ minus the promised repayment to the late depositors $(1-\lambda)c_2$ and the face value of the loan $(\lambda - x\rho)r^{max}$. The right hand side is, instead, its date 2 profits in case it met its liquidity needs selling an asset to outside investors. With probability $\frac{1}{2}$, B and D sell two different assets being forced into early liquidation. With probability $\frac{1}{2}$, instead, B and D sells the same asset. In this case, they

 DOI: 10.2870/30719

survive to the final date forgoing $R_L\frac{(1-x)}{3}$ on the asset they put on sale. Thus, $r^{max}$ is given by

$$r^{max} = \frac{1}{(\lambda - x\rho)}\left[\frac{(R_H + R_L)\frac{(1-x)}{3} - (1-\lambda)c_2}{2} + \frac{R_L(1-x)}{3}\right]. \tag{3.14}$$

Given that there is a positive probability of going bankrupt as consequence of miscoordination, B and D are willing to pay a higher interest rate than the forgone return $\frac{R_L(1-x)}{3}$ on the asset they sell on to outside investors. The additional amount they are willing to pay for the loan $\frac{1}{2}\left[(R_H + R_L)\frac{(1-x)}{3} - (1-\lambda)c_2\right]$ equals the expected losses in case of miscoordination. The interest rate that makes the lender indifferent between lending and not lending, $r^{min}$, is the solution to the following equation

$$\begin{aligned}&(2R_H + R_L)\frac{(1-x)}{3} + (2x\rho - 1)\rho + (\lambda - x\rho)r^{min} - \lambda c_2 = \\ =&\frac{1}{2}\left[\frac{(1-x)}{3}[2R_H + R_L] + (x\rho - (1-\lambda))\rho - \lambda c_2\right] + \frac{1}{2} * 0\end{aligned}$$

The left hand side represents the date 2 profits in case lending occurs. They are given by the return on the investment in the risky asset $(2R_H + R_L)\frac{(1-x)}{3}$ plus the return on the investment of the excess liquidity $(2x\rho - 1)\rho$ plus the the return on the loan $(\lambda - x\rho)r^{min}$ minus the expected repayment to the late depositors $\lambda c_2$. The right hand side are the profits in case A and C refrain from offering a loan, facing bankruptcy with probability $\frac{1}{2}$. In this case, A and C profits are given by the return on the risky projects $\frac{(1-x)}{3}[2R_H + R_L]$ plus the return on the safe asset $(x\rho - (1-\lambda))\rho$ minus the promised repayment to late depositors $\lambda c_2$. Solving the expression above with respect to $r^{min}$, we have

$$r^{min} = \frac{1}{(\lambda - x\rho)}\left[\frac{(\lambda - x\rho)\rho}{2} - \frac{(2R_H + R_L)\frac{(1-x)}{3} + (2x\rho - 1)\rho - \lambda c_2}{2}\right]. \tag{3.15}$$

In case lending does not occur, A and C are forced into early liquidation with probability $\frac{1}{2}$. The risk of bankruptcy is reflected in the premium the lenders are willing to forgo relatively to a bankruptcy-free equilibrium.

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For lending to occur, the following condition must hold

$$r^{max} > r^{min}. \tag{3.16}$$

If this is the case, there exists a feasible range for the interest rate in which both lenders and borrowers are willing to participate on the interbank market. Restricting our attention to the parameter space in which (3.16) holds, the following proposition states the Nash equilibria in the *A-C game*.

**Proposition 6** *When $r^{max} > r^{min}$, in equilibrium at least one bank receives a loan offer. The offer is always accepted. Lending occurs and no bank is forced into early liquidation.*

**Proof.** See Appendix. ∎

Under complete information, lending occurs as long as $r^{max} > r^{min}$. The intuition behind this result is very simple. For $r^{max} > r^{min}$, there exists an interest rate on which both parties agree. The game has multiple equilibria in which in at least one loan offer is accepted by the banks in distress. As long as the demanded interest rate is higher than $R_L \frac{(1-x)}{3(\lambda - x\rho)}$, only one bank would take the offer. This is the case because, when an offer is accepted, there is no longer risk of contagion. Thus, conditional on the other bank having accepted the offer, a bank with a shortage of liquidity is better off resorting to outside investors giving up the amount $R_L \frac{(1-x)}{3}$ of its future profits.

If $R_L \frac{(1-x)}{3} < \rho(\lambda - x\rho)$, lenders have no incentive to demand an interest rate $r < \rho$, once an offer has been accepted. Thus, in equilibrium, only one bank in distress receives an offer. In contrast, when $R_L \frac{(1-x)}{3} < \rho(\lambda - x\rho)$, the best response to a bank's successful offer $r^{max}$ is to demand $R_L \frac{(1-x)}{3(\lambda - x\rho)}$. In this case, both banks B and D receive an offer. Offering a loan to the same bank in distress emerges as an equilibrium only in the case $R_L \frac{(1-x)}{3} < \rho(\lambda - x\rho)$ and it is a weakly dominated one.

 DOI: 10.2870/30719

## 3.4 Incomplete Information

Under incomplete information, each player only observes the returns of its project and the ones of its neighbors.

Like under complete information, the following distribution of returns $\mathbf{R} = (R_L, R_H, R_L, R_H)$ describes the true state of the economy. Unlike in the complete information case, each bank assumes two possible states of the world. From the perspective of bank A,[2] with probability $\pi$ the distribution of returns is the one described above; with probability $1-\pi$, instead, A expects the economy to be in the following state $\mathbf{R} = (R_A, R_B, R_C, R_D,) = (R_L, R_H, R_H, R_H)$.

In other words, in bank A's perspective

$$R_C = \begin{cases} R_L & \text{w. pr. } \pi \\ R_H & \text{w. pr. } (1-\pi) \end{cases} \tag{3.17}$$

The game has the same structure as in the complete information case. First, banks A and C decide about lending and the interest rate to charge. Then B and D decide to accept or reject the offer. In case they reject, they have to sell an asset to outside investors to meet the liquidity demand. We solve the game by backward induction.

*Stage 2*

We start by deriving the continuation equilibria of the B-D subgame. The information sets of the two banks are:

$$\mathcal{T}_{\mathcal{B}}^{\mathcal{II}} = \{R_H, R_L\} \qquad \mathcal{T}_{\mathcal{D}}^{\mathcal{II}} = \{R_H, R_L\}$$

[2] Given the symmetry of their portfolios, the same analysis applies to bank C. From now on we proceed through the analysis only considering the problem from A's perspective.

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Consider first the case in which $R_C = R_L$. When B and D resort to outside investors, B's payoff is given by:

$$\Pi_B(t_B, t_D, a_B, a_D) = \begin{cases} 0 & \text{if } a_B \neq a_D \text{ or } t_D = R_L \\ \frac{(1-x)}{3}(2R_L + R_H) - a_B - (1-\lambda)c_2, & \text{if } a_B = a_D \text{ and } t_D = R_H. \end{cases} \tag{3.18}$$

D's payoffs are symmetric.

Whenever B and D miscoordinate, contagion occurs at time 1. The same outcome emerges when $R_D = R_L$. In this case, D has three low-return assets in its portfolio and is forced into early liquidation according to (3.7). When B and D choose to sell a common asset, they retain some positive profits at date 2, after the payment of the promised $c_2$ to late depositors.

The following proposition establishes the equilibria in the B and D subgame

**Proposition 7** *In B-D game with incomplete information in which the true state of the world is represented by the vector* $\mathbf{R} = (R_L, R_H, R_L, R_H)$*:*

- *The pure strategies Bayesian Nash Equilibria are* $(a_B^*, a_D^*) \in \{$*(sell A, sell A), (sell C, sell C) and (sell B, sell D)*$\}$
- *The mixed strategy Bayesian Nash Equilibrium is :* $\gamma^* = (\gamma_L^*, \gamma_H^*) = ((\gamma_L^{*A}, \gamma_L^{*B}, \gamma_L^{*C}),$
  $(1/2, 0, 1/2))$ *where* $\gamma_L^{*A} + \gamma_L^{*B} + \gamma_L^{*C} = 1$ *and* $\gamma^{*k}$ *is the optimal weight that player B respectively D puts on playing the pure strategy* $k \in \{A, B/D, C\}$ *whenever B or D is of low type*[3]*.*

**Proof.** See Appendix. ■

The equilibria in the incomplete information case are the same as in the complete information. The uncertainty about other bank's type does not affects the equilibrium actions.

[3]Henceforth, low and high type correspond to the case in which bank i's asset has returns $R_i = R_L$ and $R_i = R_H$, respectively.


DOI: 10.2870/30719

This is due to the fact that when $t_D = R_L$, bank B expects to go bankrupt independently of the undertaken actions. When $t_D = R_H$, the same argument as in the complete information applies.

Using the same arguments, we can derive the equilibria in the B and D's subgame in the state of the economy corresponding to the vector of returns $\mathbf{R} = \{R_L, R_H, R_H, R_H\}$ as described in Figure 3.

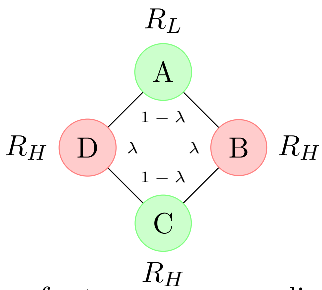


Figure 3.3: Distribution of returns corresponding to $\mathbf{R} = \{R_L, R_H, R_H, R_H\}$.

In this case, B's payoffs are given by

$$\Pi_B(t_B, t_D, a_B, a_D) = \begin{cases} 0 & \text{if } a_B \neq a_D \text{ or } t_D = R_L \text{ and} \\ & a_B = a_D = \text{sell C} \\ \frac{(1-x)}{3}(2R_H + R_L) - a_i - (1-\lambda)c_2, & \text{if } a_B = a_D = \text{sell C and } t_D = R_H \\ & \text{if } a_B = a_D = \text{sell A and } t_D \in \{R_H, R_L\} \end{cases} \tag{3.19}$$

Selling two different assets causes the bankruptcy of one bank and contagion. Banks expect to make positive profits when they put on sale a common asset, unless the $R_D = R_L$ and they sell C. In this case they expect to be forced into early liquidation and to make zero profits at date 1.

The equilibria for the B and D subgame are described in the following proposition:

**Proposition 8** *In B-D game with incomplete information in which the state of the world*

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*is represented by the vector* $\mathbf{R} = (R_L, R_H, R_H, R_H)$*, the pure strategies Bayesian Nash Equilibria are* $(a_B^*, a_D^*) \in \{$*(sell A, sell A), (sell C, sell C) and (sell B, sell D)*$\}$

**Proof.** See Appendix. ■

As in the complete information case, early liquidation emerges as a consequence of banks' miscoordination. Whenever B and D sell the common asset A, no bank is forced into early liquidation. However, the occurrence of bankruptcy cannot be only limited to the situation in which the two banks miscoordinate. B expects to be forced into early liquidation with probability $\pi$ when $R_D = R_L$ and both banks sell asset C. In this state of the economy corresponding to the vector of return $\mathbf{R} = (R_L, R_H, R_H, R_H)$, the return on assets A and C are no longer identical. This means that selling either asset has different implications in terms of probability of going bankrupt as well as on the value of banks' portfolios.

*Stage 1*

Given the continuation equilibria in the B-D subgame, we move backward to the first stage in which banks $A$ and $C$ decide first upon lending followed by a take-it-or-leave-it interest rate offer. As stated in Propositions 3 and 4, we have a multiplicity of equilibria in the continuation game. We focus our analysis on the following:

- **Case 1**:
  - for $\mathbf{R} = (R_L, R_H, R_L, R_H)$, A anticipates that B and D will play the mixed strategy equilibrium as described in Proposition 7.
  - for $\mathbf{R} = (R_L, R_H, R_H, R_H)$, A anticipates that B and D will play the pure strategy equilibrium $(\text{sell A}, \text{sell A})$ as described in Proposition 23.
- **Case 2**:
  - for $\mathbf{R} = (R_L, R_H, R_L, R_H)$, A anticipates that B and D will play the mixed strategy equilibrium as described in Proposition 7.
  - for $\mathbf{R} = (R_L, R_H, R_H, R_H)$, A anticipates that B and D will play the pure strategy equilibrium $(\text{sell C}, \text{sell C})$ as described in Proposition 23.

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The type spaces of A and C are :

$$\mathcal{T}_A^{II} = \{R_L, R_H\} \qquad \mathcal{T}_C^{II} = \{R_L, R_H\} \tag{3.20}$$

Analogously to the complete information case, we define the admissible range $[\tilde{r}^{min}, \tilde{r}^{max}]$ in which both borrowers and lenders are willing to participate in a transaction on the interbank market.

We consider first the problem of the lender. In order for bank A to provide resources to a bank in distress, lending must yield higher expected profits.

Bank A's profit when lending does not occur, in both case 1 and 2, are

$$\Pi_A^{nl} = \frac{\pi}{2} * 0 + \left(1 - \frac{\pi}{2}\right)\left(\frac{(1-x)}{3}(2R_H + R_L) + (x\rho - (1-\lambda))\rho - \lambda c_2\right). \tag{3.21}$$

Bank A accrues zero profits when the state of the economy is $\mathbf{R} = (R_L, R_H, R_L, R_H)$ and B and D miscoordinate. This occurs with probability $\frac{\pi}{2}$. Otherwise, bank A is not forced into early liquidation and makes positive profits with probability $\left(1 - \frac{\pi}{2}\right)$. A's profits are given by the return on the investment in the risky projects $\frac{(1-x)}{3}(2R_H + R_L)$ plus the return $(x\rho - (1-\lambda))\rho$ it obtains investing its excess of liquidity in the safe asset between date 1 and 2 minus the promised repayment to the late depositors $\lambda c_2$. The profits in case of lending are given by:

$$\Pi_A^{l} = \frac{(1-x)}{3}(2R_H + R_L) + (2x\rho - 1)\rho + (\lambda - x\rho)\tilde{r} - \lambda c_2. \tag{3.22}$$

where $\tilde{r} \in [\tilde{r}^{min}, \tilde{r}^{max}]$. Equalizing (3.21) and (3.22) and solving with respect to $\tilde{r}$ we obtain the lowest bound of the admissible interest rate range.

$$\tilde{r}^{min} = \frac{1}{(\lambda - x\rho)}\left[\left(1 - \frac{\pi}{2}\right)(\lambda - x\rho)\rho - \frac{\pi}{2}\left(\frac{(1-x)}{3}(2R_H + R_L) + (2x\rho - 1)\rho - \lambda c_2\right)\right]. \tag{3.23}$$

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It is important to notice that $\tilde{r}^{min} > r^{min}$ as defined in (3.15) and (3.23). The intuition is that in the incomplete information case bank A underestimates the probability of bankruptcy. A expects that, with probability $(1-\pi)$, $R_C = R_H$ and bankruptcy does not emerge in equilibrium. This implies that in the complete information case the lender is willing to demand a lower interest rate than in the incomplete information framework. In other words, it is willing to forgo some returns so to prevent B and D from going to the asset market and miscoordinating with probability $\frac{1}{2}$. As $\pi$ decreases, so that bankruptcy and contagion become less likely, $\tilde{r}^{min}$ increases.

Turning to the problem of the borrowers, two different $\tilde{r}^{max}$ must be defined depending whether $R_C = R_L$ or $R_C = R_H$. We refer to them as $\tilde{r}_1^{max}$ and $\tilde{r}_2^{max}$. $\tilde{r}^{max}$ is the interest rate that makes the borrowers indifferent between borrowing on the interbank market and selling to investors.

When A expects $R_C = R_L$ and the continuation equilibrium is the mixed strategy equilibrium as defined in Proposition 7, then the borrower' s profits [4] are

$$\Pi_B^{nb} = \pi * 0 + \frac{(1-\pi)}{2} * 0 + \frac{(1-\pi)}{2}\left((R_H + R_L)\frac{(1-x)}{3} - (1-\lambda)c_2\right). \tag{3.24}$$

in case no bank borrows and

$$\Pi_B^{b} = \pi * 0 + (1-\pi)\left((R_H + 2R_L)\frac{(1-x)}{3} - (1-\lambda)c_2 - (\lambda - x\rho)\tilde{r}_1\right). \tag{3.25}$$

in case it borrows. Equalizing (3.24) and (3.25), we derive

$$\tilde{r}_1^{max} = \frac{1}{(\lambda - x\rho)}\left[\frac{R_L(1-x)}{3} + \frac{1}{2}\left((R_H + R_L)\frac{(1-x)}{3} - (1-\lambda)c_2\right)\right]. \tag{3.26}$$

[4] We consider bank B but as they have the same portfolios, the same analysis applies to D.

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Symmetrically we find $\tilde{r}_2^{max}$ when A expects $R_C = R_H$ and the continuation equilibria are $(\text{sell A}, \text{sell A})$ and respectively $(\text{sell C}, \text{sell C})$ as defined in Proposition 23.

$$\tilde{r}_2^{max/(AA)} = \frac{R_L(1-x)}{3(\lambda - x\rho)} \tag{3.27}$$

and

$$\tilde{r}_2^{max/(CC)} = \frac{1}{(\lambda - x\rho)}\left[\frac{R_H(1-x)}{3} + \pi\left((R_H + R_L)\frac{(1-x)}{3} - (1-\lambda)c_2\right)\right]. \tag{3.28}$$

When the continuation equilibrium is $(\text{sell A}, \text{sell A})$, bankruptcy never occurs. This implies that B and D are not willing to pay more than the forgone return $\frac{R_L(1-x)}{3}$ from selling the lowest return asset to outside investors. When, instead, the continuation equilibrium is $(\text{sell C}, \text{sell C})$, B and D are willing to pay significantly more as there is a non negligible probability, $\pi$, of going bankrupt in case they do not borrow. This explains why $\tilde{r}_2^{max/(CC)} > \tilde{r}_2^{max/(AA)}$.

As in the previous case, it is worth noticing that for $\pi < \frac{1}{2}$, the interest rate $r^{max}$ as defined in (3.14) is never lower than the interest rates that make borrowers to break-even in the incomplete information framework.

The occurrence of interbank market freeze and contagion depends on how $\tilde{r}^{min}$, $\tilde{r}_1^{max}$, $\tilde{r}_2^{max/(AA)}$ and $\tilde{r}_2^{max/(CC)}$ are ordered. The following proposition summarizes the results.

**Proposition 9** *In equilibrium interbank market freeze occurs when*

$$\tilde{r}_2^{max/(CC)} > \tilde{r}^{min} > \tilde{r}_1^{max}$$

*As consequence, contagion arises in equilibrium with probability $\frac{1}{2}$ in the state of the economy described by the vector of return $\mathbf{R} = (R_L, R_H, R_L, R_H)$.*

**Proof.** See Appendix. ∎

The proposition shows that interbank market freeze occurs in equilibrium when banks are uncertain about the value of their neighbors' portfolios. When this is the case lenders

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A and C underestimate the probability of contagion. This induces them to demand an excessively high interest rate that borrowers are only willing to accept in the state of the economy described by $\mathbf{R} = (R_L, R_H, R_H, R_H)$, in which the borrowers' portfolios consist of three high return assets. Interbank market freeze emerges in equilibrium for a sufficiently small $\pi$ and sufficiently high $R_H$.

Uncertainty affects borrowers' willingness to pay. When $\mathbf{R} = (R_L, R_H, R_L, R_H)$, borrowers attach a high probability to bankruptcy once they resort to the asset market. With probability $\pi$, each borrower expects to be forced into early liquidation independently of the asset sold on the market. Contagion also occurs with probability $\frac{(1-\pi)}{2}$ as consequence of B and D miscoordination. In the state corresponding to $\mathbf{R} = (R_L, R_H, R_H, R_H)$, borrowers expect to go bankrupt with probability $\pi$, that is when the continuation equilibrium is (sell C, sell C) and the opponent's project returns $R_L$. In this case, lenders expect that borrowers are willing to pay for a loan more than the forgone return of the asset they sell to outside investors. A and C have no incentive to demand an interest rate lower than $\tilde{r}^{min}$ and they know that borrowers are willing to pay $\tilde{r}_2^{max}$ when they have three high return assets in their portfolio. Thus, the lenders demand in equilibrium $r_2^{max}$ knowing that the bank in distress will reject it in the state of the economy $\mathbf{R} = (R_L, R_H, R_L, R_H)$. This thus lead to interbank market freeze and contagion spreads through all the economy with probability $\frac{1}{2}$. This case in which $\tilde{r}_2^{max/(CC)} > \tilde{r}^{min} > \tilde{r}_1^{max}$ occurs when $\pi$ is sufficiently low and $R_H$ sufficiently high.

In the same parameter space, interbank market freeze would occur when the continuation equilibrium is (sell A, sell A) as described in **Case 1**. In the two cases illustrated above, interbank market freeze is *ex post* inefficient as it may lead to contagion.

*Comparison between complete and incomplete information case*

Proposition 6 and Proposition 9 state the equilibria of the game under complete and incomplete information.

**Proposition 10** *There exists a parameter space in which lending occurs under complete*

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*information case but not in the game with incomplete information.*

The intuition behind proposition 10 is simple. In the complete information case, all market participants know that bankruptcy occurs with probability $\frac{1}{2}$ as consequence of the miscoordination of B and D. Under incomplete information, borrowers and lenders attach different probabilities to bankruptcy and contagion. In the borrowers' perspective, in the state of the economy $\mathbf{R} = (R_L, R_H, R_L, R_H)$, the probability of early liquidation is $\frac{1+\pi}{2}$ and $\pi$ in the state described by the return vector $\mathbf{R} = (R_L, R_H, R_H, R_H)$. Lenders expect bankruptcy to arise with probability $\frac{\pi}{2}$ when the state of the economy is $\mathbf{R} = (R_L, R_H, R_L, R_H)$ and they do not expect to be forced to liquidate their portfolio at date 1 otherwise. As shown before, this affects their willingness to participate on the interbank market. Lenders attach lower probability to the event of bankruptcy than the borrowers. The interest rate that makes the lenders to break-even is higher under incomplete information than under complete. This means that, ceteris paribus, A and C are less willing to lend in the incomplete information case.

$$\tilde{r}^{min} > r^{min}$$

In contrast, the interest rate that makes the borrowers to break-even is higher under incomplete than under complete information, implying that the borrower is willing to accept a higher interest rate under incomplete information.

$$\tilde{r}_1^{max} > r^{max}$$

This allows the lender to exploit an informational rent leading him to demand a high interest rate. It anticipates that, in the state $\mathbf{R} = (R_L, R_H, R_H, R_H)$, the borrower attaches a positive probability to bankruptcy and, thus, it is willing to pay more in order to avoid to resort to outside investors. Thus, it exists a parameter space in which

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| Complete information | Incomplete information |
|---|---|
| $r^{max} > r^{min}$ | $\tilde{r}_2^{max/(CC)} > \tilde{r}^{min} > \tilde{r}_1^{max}$ |

Due to the uncertainty on the realization of the return on the asset they do not hold, lenders underestimate the probability of contagion and expect to be able to extract informational rents from the borrowers in the state $\mathbf{R} = (R_L, R_H, R_H, R_H)$. For a low $\pi$, this induces lenders to behave *greedily* and to find it optimal to choose $\tilde{r}_2^{max/(CC)}$. As $\tilde{r}_2^{max/(CC)} > \tilde{r}_1^{max}$, the loan offer will be turned down in state $\mathbf{R} = (R_L, R_H, R_L, R_H)$ and interbank market freeze occurs. Figure 4 illustrates the proposition by showing the banks lending

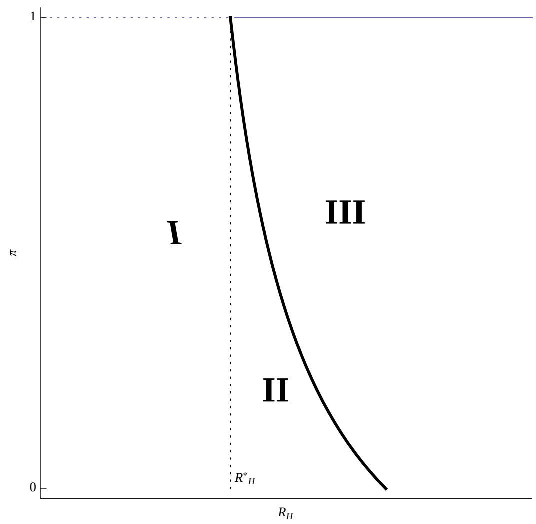


Figure 3.4: *Interbank market freeze in complete and incomplete information case.*

decisions under complete and incomplete information as function of the return $R_H$ and the probability of a low return $\pi$. Lending is not observed in equilibrium either under complete or incomplete information in region I. In region II, lending occurs in the complete information case but not in the incomplete one while in region III, lending occurs under both complete and incomplete information.

The complete information case corresponds to the horizontal line $\pi = 1$. In that scenario, lending occurs only for a sufficiently high $R_H$, $R_H > R_H^*$. As $R_H$ increases, banks are more willing to participate on the interbank market. The interest rate $r^{max}$, as defined in

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(3.14), increases with $R_H$.

$$\frac{\delta r^{max}}{\delta R_H} = \frac{(1-x)}{6(\lambda - x\rho)} > 0.$$

This is due to the fact that the borrower's forgone profits, when forced to liquidate its portfolio at date 1, are higher. In contrast, from the lenders' perspective $r^{min}$ decreases with $R_H$.

$$\frac{\delta r^{min}}{\delta R_H} = -\frac{(1-x)}{3(\lambda - x\rho)} > 0.$$

This means that lenders are more willing to lend. The effect of a change in $R_H$ is to extend the admissible range $\left[r^{min}, r^{max}\right]$.

Under incomplete information, the solid line represents all the combination of $R_H$ and $\pi$ for which $\tilde{r}^{min} = \tilde{r}_1^{max/(CC)}$. On the left of the solid line, where $\tilde{r}^{min} > \tilde{r}_1^{max/(CC)}$, lending does not occur. When uncertainty is introduced in the model, $\pi \neq 1$, banks are less willing to lend than under complete information (Region II). Lenders underestimate the probability of contagion for any $\pi \neq 1$ and this implies that they have less incentive to lend. For them to be willing to lend, $R_H$ must to be sufficiently high. When this is the case they expect to incur high losses in case of bankruptcy at date 1. For the same reasons, lenders expect borrowers to be willing to give up a larger amount of resources to obtain the loans and, thus, to be able to extract larger rent in the state of the economy in which the borrowers' portfolio consists of three high return assets (i.e. R$_C = R_H$).

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## 3.5 An example

To illustrate the consequences of uncertainty on interbank market transactions and contagion we consider a numerical example with the following parameters:

$$\lambda = 0.7; \quad \tilde{R} = \begin{cases} 0.5 & \text{w. pr. } 0.05 \\ 1.5 & \text{w. pr. } 0.95 \end{cases}; \quad c_2 = 1.01; \quad x = 0.5454; \quad \rho = 1.155.$$

At date 1, bank B and D have a shortage of liquidity $(\lambda - x\rho) = 0.0700$ while banks A and C' s excess of liquidity is given by $(x\rho - (1-\lambda)) = 0.3299$. In case B and D decide to resort to outside investors to cover their liquidity demand, the price paid by the investors is

$$P = \frac{3(\lambda - x\rho)}{1-x} = 0.46236 < R_L = 0.5. \tag{3.29}$$

The price P is lower than the lowest return. This capture the idea of *fire sale* prices and implies that only banks with a shortage of liquidity eventually sell an asset to outside investors. Given the price P, $R_L$ and $R_H$ as defined above, the bankruptcy rules as defined in (3.7) and (3.8) hold

$$x\rho + [R_L + R_L + R_L]\frac{(1-x)}{3} = 0.8572 \text{ and } x\rho + [R_H + P + P]\frac{(1-x)}{3} = 0.9973.$$

No bank, instead, is forced into early liquidation at date 1 when only one asset is sold to investors. In this case, even when only one asset returns $R_H$ at date 2, the market value of bank's portfolio is

$$x\rho + [R_L + R_H + P]\frac{(1-x)}{3} = 1.003$$

*Complete Information Case*

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When no transaction occurs on the interbank market, the two banks in distress sell an asset to the outside investors. One equilibrium in the B-D game is the mixed strategy equilibrium in which banks miscoordinate and sell two different asset with probability 0.5. In this case the expected profits are

$$\frac{1}{2}\left[(R_H + R_L)\frac{(1-x)}{3} - (1-\lambda)c_2\right] = 0.00003$$

for B and D and

$$\frac{1}{2}\left[\frac{(1-x)}{3}[2R_H + R_L] + (2x\rho - 1)\rho + (\lambda - x\rho)\rho - \lambda c_2\right] = 0.1022.$$

for A and C. The interest rates that make a bank in distress indifferent between borrowing or not and a lender indifferent between lending or not are, respectively

$$r^{max} = 1.0818 \text{ and } r^{min} = -0.304$$

The negative interest rate $r^{min}$ means that it is always profitable for the lender to lend. Therefore, lending always occurs in the complete information case.

*Incomplete Information Case*

Bank A does not observe the return on bank's project $R_C$. It expects $R_C = 0.5$ with probability $\pi = 0.05$. In case bank A expects this, it anticipates that in equilibrium banks B and D miscoordinate and sell two different assets with probability 0.5. In this case it expects to be forced into early liquidation and to earn zero profits. When, instead, A expects $R_C = 1.5$, then it anticipates that in case no transaction takes place on the interbank market, both B and D would optimally choose to sell their shares of asset C. In this case, no bank is forced into early liquidation and makes positive profits. For bank A to be willing to lend then the interest rate should be such that, by lending, it would make

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at least the same profits as

$$\Pi_A^{nl} = \frac{\pi}{2} * 0 + \left(1 - \frac{\pi}{2}\right)\left(\frac{(1-x)}{3}(2R_H + R_L) + (2x\rho - 1)\rho + (\lambda - x\rho)\rho - \lambda c_2\right) = 0.1993.$$

Thus, the minimum interest rate bank A is willing to charge is $\tilde{r}^{min} = 1.08205$. Unlike, the complete information, lending is not always profitable. This depends on the fact that the probability of being forced into early liquidation is $\frac{\pi}{2} = 0.025 < 0.5$. The lender expect, instead, borrowers to attach probabilities $\frac{\pi}{2} = 0.525$ and $\pi = 0.05$ to bankruptcy when $R_C = R_L$ and $R_C = R_H$, respectively. As early liquidation and contagion are less likely in the incomplete information case than under complete information, lenders have more bargaining power so that the lowest interest rate at which they are willing to lend is higher than the one in the complete information case.

Uncertainty also changes the value of the alternative funding opportunity for the borrowers. Bank A anticipates that, depending on the realized return on the share of asset C they hold, the maximum interest rate borrowers are willing to pay are

$$\tilde{r}_1^{max} = \frac{1}{(\lambda - x\rho)}\left[\frac{R_L(1-x)}{3} + \frac{1}{2}\left((R_H + R_L)\frac{(1-x)}{3} - (1-\lambda)c_2\right)\right] = 1.0818$$

when $R_C = 0.5$ and

$$\tilde{r}_2^{max/(CC)} = \frac{1}{(\lambda - x\rho)}\left[\frac{R_H(1-x)}{3} + \pi\left((R_H + R_L)\frac{(1-x)}{3} - (1-\lambda)c_2\right)\right] = 3.24427 \tag{3.30}$$

when $R_C = 1.5$. Given

$$\tilde{r}_2^{max/(CC)} > \tilde{r}^{min} > \tilde{r}_1^{max} = r^{max} > r^{min}$$

lending occurs in the complete information case but not in the incomplete information one. Banks A and C demand the interest rate $\tilde{r}_2^{max/(CC)}$ and B and D reject the lender's offer when $R_C = 0.5$. In that case, interbank market freeze may induce contagion as,

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with probability 0.5, banks miscoordinate selling two different assets when they resort to outside investors to meet their liquidity demand. In this circumstance, all banks are forced into early liquidation and earn zero profits.

It can be easily shown that, in the same parameter space, lending does not occur also when the considered equilibrium in the B-D subgame when $R_C = 1.5$ is $(\text{sell A}, \text{sell A})$ instead of $(\text{sell C}, \text{sell C})$ we have just focused on. In this case, the admissible range for the interest rate changes as follows. The borrowers are willing to pay at most

$$\tilde{r}_1^{max} = \frac{1}{(\lambda - x\rho)}\left[\frac{R_L(1-x)}{3} + \frac{1}{2}\left((R_H + R_L)\frac{(1-x)}{3} - (1-\lambda)c_2\right)\right] = 1.0818$$

when $R_C = 0.5$ and

$$\tilde{r}_2^{max/(AA)} = R_L\frac{(1-x)}{3} = 1.08141 \tag{3.31}$$

when $R_C = 1.5$. The lowest interest rate demanded by the lender is, as before,

$$\tilde{r}^{min} = \frac{1}{(\lambda - x\rho)}\left[\left(1 - \frac{\pi}{2}\right)(\lambda - x\rho)\rho - \frac{\pi}{2}\left(\frac{(1-x)}{3}(2R_H + R_L) + (2x\rho - 1)\rho - \lambda c_2\right)\right] = 1.08205. \tag{3.32}$$

Given

$$\tilde{r}^{min} > \tilde{r}_2^{max/(AA)} > \tilde{r}_1^{max} = r^{max} > r^{min}$$

no transaction takes place on the interbank market. This implies that, when $R_C = 0.5$, contagion may occurs as consequence of interbank market freeze. However, as before the interbank market freeze is a consequence of the uncertainty on the counterparty risk. In the first case, interbank market freeze occurs because lenders and borrowers have different perceived probabilities of bankruptcy. Lenders expect borrowers to attach a higher probability to bankruptcy. This prompt a "greedy" behavior of the lenders resulting in the demand of an excessively high interest rate. In the second case, instead, lending does

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not occur as it is not profitable. The lender anticipates that as, bankruptcy never occurs when $R_C = 1.5$, the borrowers are not willing to accept any profitable (for the lender) interest rate.

## 3.6 Concluding remarks

In this paper we have developed a simple model where banks can borrow on the interbank market to meet their uncertain liquidity demands. Alternatively they can resort to the asset markets selling the assets at fire sale price. The intertwining of fire sale prices and mark-to-market accounting may trigger a wave of bankruptcies and a systemic failure. We have shown that, despite the risk of contagion, banks may not find it optimal to participate on the interbank market. This occurs when banks do not have complete information about the distribution of returns in the economy. Under incomplete information lenders underestimate the risk of contagion and expect banks in distress to be willing to pay higher interest rate than the one that makes them indifferent between borrowing and selling the asset. Thus, lenders demand an excessively high interest rate that borrowers reject. Due to this *greedy* behavior of the lenders, interbank market freeze occurs and this may lead to contagion with positive probability.

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 DOI: 10.2870/30719

## Appendix

**Proof of Proposition 5**

We start from the derivation of the pure strategy Nash equilibria.

If banks B and D sell two different assets, all banks in the economy are forced into early liquidation and earn zero profits as the market value of bank's assets $\mathcal{V}_i$ is lower than the value of bank's liabilities at date 1 as stated in (3.8). In the cases, instead, both banks sell either asset A or C, no bank will go bankrupt at date 1. As assets A and C have the same return at date 2, we can only focus on one of the two cases (i.e. they sell asset A), without loss of generality. When asset A is sold on the market, the market value of banks' portfolio becomes:

- $\mathcal{V}_A = (P + R_H + R_H)\frac{(1-x)}{3} + x\rho > 1$
- $\mathcal{V}_B = (P + R_H + R_L)\frac{(1-x)}{3} + x\rho > 1$
- $\mathcal{V}_C = (2R_H + R_L)\frac{(1-x)}{3} + x\rho > 1$
- $\mathcal{V}_D = (P + R_H + R_L)\frac{(1-x)}{3} + x\rho > 1.$

and no bank is forced into early liquidation. Thus, B and D's profits at date 2 are:

$$\Pi_B = \Pi_D = (R_H + R_L)\frac{(1-x)}{3} - (1-\lambda)c_2 > 0 \tag{3.33}$$

and the pure strategy Nash equilibria are as in the proposition.

Regarding the mixed strategy equilibrium. Selling its own asset is for both B and D a weakly dominated strategy as banks accrue zero profits independently of the action chosen by the other bank in distress. As the game is symmetric for both B and D, to derive the mixed strategy equilibrium we can only consider one of the two (say B). Define $(p, q, 1-p-q)$ as the probabilities B attaches to D playing actions $(\text{sell A}, \text{sell C}, \text{sell D})$, respectively. The B's payoffs from selling A are

$$p\left[(R_H + R_L)\frac{(1-x)}{3} - (1-\lambda)c_2\right] + q * 0 + (1-p-q) * 0. \tag{3.34}$$

The payoffs from selling B are

$$p * 0 + q * 0 + (1-p-q) * 0 \tag{3.35}$$


DOI: 10.2870/30719

and those from selling C are

$$p * 0 + q * \left[ (R_H + R_L)\frac{(1-x)}{3} - (1-\lambda)c_2 \right] + (1-p-q) * 0. \tag{3.36}$$

Thus, solving with equality

$$q * \left[ (R_H + R_L)\frac{(1-x)}{3} - (1-\lambda)c_2 \right] = p * \left[ (R_H + R_L)\frac{(1-x)}{3} - (1-\lambda c_2) \right] \tag{3.37}$$

we have

$$p = q = \frac{1}{2} \tag{3.38}$$

The same analysis applies to D and the proposition holds.

**Proof of Proposition 6**

We solve the A-C game by backwards induction. After the first stage of the A-C game, four outcomes are possible: A and C have chosen to offer a loan to the same bank (henceforth *the competition case*), A and C have decided to make offers to different banks, each holding monopoly power in the bargaining process (*the monopolist case 1*), only one bank (A or C) makes an offer while the other bank withholds (*the monopolist case 2*) and no bank makes an offer (*non-lending case*).

To prove the proposition, we have to distinguish three possible intervals.

1. $r^{min} < R_L \frac{1-x}{3(\lambda - x\rho)} < \rho < r^{max}$

   (a) *The competition case*

   When A and C are both offering a loan to the same bank, they compete à la Bertrand. The Nash equilibrium of the interest rates bidding game is $(r_A^{comp}, r_C^{comp}) = (\rho, \rho)$. Offering $\rho$ is a weakly dominant strategy. Any deviation from this interest rate, would ensure its rejection from the side of the borrower. Since the borrower is indifferent between the offers, each offer is accepted with probability $\frac{1}{2}$. The profits of A and C are

$$\Pi_A^{liq} = \Pi_C^{liq} = \left[ \frac{(1-x)}{3}[2R_H + R_L] + (2x\rho - 1)\rho \right] + \underbrace{(\lambda - x\rho)\rho}_{\text{Loan face value}} - \lambda c_2. \tag{3.39}$$

   (b) *The monopolist case 1*

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DOI: 10.2870/30719

Without loss of generality, assume that A makes an offer to B and C makes an offer to D. The equilibrium interest rate at this stage is not straightforward. The first guess would be to assume that whenever lenders have monopolistic power, they will be able to extract the whole surplus from the borrower, requesting $r^{max}$. In the simultaneous decision game of B and D, only one offer is accepted, as, given that one of the banks accepts, the other is better off rejecting the take-it-or-leave-it offer. Resorting to outside investors, the bank would give up only $R_L\frac{(1-x)}{3} < (\lambda - x\rho)r^{max}$ of his future profits. Moreover, conditional on one bank accepting $r^{max}$, the other bank would reject all interest rates offer above $R_L\frac{(1-x)}{3(\lambda-x\rho)}$. When $R_L\frac{(1-x)}{3} < \rho(\lambda - x\rho)$, as in this case, conditional on bank D having accepted the C's take-it-or-leave-it offer $r^{max}$, bank A will always find it optimal not to offer anything lower or equal to $\rho$. Hence, the equilibrium interest rates of this stage are: $(l, r^{max})$, with $l \in ]R_L\frac{(1-x)}{3(\lambda-x\rho)}, \infty[$. In equilibrium, only one offer is accepted.

In the equilibrium in which C's offer $r^{max}$ to D is accepted, and any offer above $R_L\frac{(1-x)}{3(\lambda-x\rho)}$ made by A is rejected, A and C's profits are:

$$\Pi_A^{liq} = \left[\frac{(1-x)}{3}[2R_H + R_L] + (2x\rho - 1)\rho\right] + (\lambda - x\rho)\rho - \lambda c_2. \tag{3.40}$$

$$\Pi_C^{mon} = \left[\frac{(1-x)}{3}[2R_H + R_L] + (2x\rho - 1)\rho\right] + (\lambda - x\rho)r^{max} - \lambda c_2. \tag{3.41}$$

(c) *The monopolist case 2*

In the case in which there is only one lender making an offer, the bargaining power is entirely on the lender's side. The lender is able to appropriate the entire surplus from the borrower. Hence, the equilibrium interest rates at this stage are: $(r^{max}, \emptyset)$. In the equilibrium in which A is the lender and C has opted out, A and C's profits are:

$$\Pi_A^{mon} = \left[\frac{(1-x)}{3}[2R_H + R_L] + (2x\rho - 1)\rho\right] + (\lambda - x\rho)r^{max} - \lambda c_2. \tag{3.42}$$

$$\Pi_C^{liq} = \left[\frac{(1-x)}{3}[2R_H + R_L] + (2x\rho - 1)\rho\right] + (\lambda - x\rho)\rho - \lambda c_2. \tag{3.43}$$

(d) *The non lending case*

When neither A nor C intend to lend, their date 2 expected profits are:

$$\Pi_A^{nl} = \frac{1}{2}\left[\frac{(1-x)}{3}[2R_H + R_L] + (x\rho - (1-\lambda))\rho - \lambda c_2\right] \quad (3.44)$$

$$\Pi_C^{nl} = \frac{1}{2}\left[\frac{(1-x)}{3}[2R_H + R_L] + (x\rho - (1-\lambda))\rho - \lambda c_2\right] \quad (3.45)$$

Bank A and C's payoffs described above can be easily ordered as follows:

$$\Pi^{nl} < \Pi^{liq} < \Pi^{mon}$$

Under these circumstance, the A-C game has five subgame perfect Nash Equilibria:

- $\{(\text{lend to D}, \text{lend to D}), (\rho, \rho)\}$
- $\{(\text{lend to D}, \text{lend to B}), (r^{max}, l)\}\text{with}\, l \in ]R_L\frac{(1-x)}{3(\lambda-x\rho)}, \infty[$
- $\{(\text{lend to B}, \text{lend to D}), (l, r^{max})\}\text{with}\, l \in ]R_L\frac{(1-x)}{3(\lambda-x\rho)}, \infty[$
- $\{(\text{lend to D}, \text{non lending}), (r^{max}, \emptyset)\}$
- $\{(\text{non lending}, \text{lend to D}), (\emptyset, r^{max})\}$

2. $r^{min} < \rho < R_L\frac{1-x}{3(\lambda-x\rho)} < r^{max}$

 (a) *The competition case*
 As long as $\rho < r^{max}$, the potential lenders will always compete á la Bertrand. The result of this stage remains unchanged compared to the previous stage. The Nash equilibrium of the interest rates bidding game is
 $(r_A^{comp}, r_C^{comp}) = (\rho, \rho)$.
 A and C's profits are therefore:

$$\Pi_A^{liq} = \Pi_C^{liq} = \left[\frac{(1-x)}{3}[2R_H + R_L] + (2x\rho - 1)\rho\right] + \underbrace{(\lambda - x\rho)\rho}_{\text{loan face value}} - \lambda c_2. \quad (3.46)$$

 (b) *The monopolist case 1*
 Assume now that A decides to lend to B and C to D. As argued before, only one borrower will accept $r^{max}$. Let this borrower be B. When B accepts $r^{max}$, D's best reply is to reject any offer above $R_L\frac{(1-x)}{3(\lambda-x\rho)}$. Since $R_L\frac{(1-x)}{3} > \rho(\lambda - x\rho)$, C's best reply to A's successful offer $r^{max}$ is to offer


DOI: 10.2870/30719

$R_L\frac{(1-x)}{3(\lambda-x\rho)}$ to D. Hence, the equilibrium interest rates when A and C are monopolists are $\{(r^{max}, R_L\frac{(1-x)}{3(\lambda-x\rho)}), (R_L\frac{(1-x)}{3(\lambda-x\rho)}, r^{max})\}$ and both offers are accepted. Assuming that the equilibrium at the interest rate stage is $(r^{max}, R_L\frac{(1-x)}{3(\lambda-x\rho)})$, A and C's monopolists profits are:

$$\Pi^{mon}_{r^{max}} = \frac{(1-x)}{3}[2R_H + R_L] + (2x\rho - 1)\rho + (\lambda - x\rho)r^{max} - \lambda c_2 \tag{3.47}$$

$$\Pi^{mon}_{\frac{R_L(1-x)}{3}} = \frac{(1-x)}{3}[2R_H + R_L] + (2x\rho - 1)\rho + \frac{R_L(1-x)}{3} - \lambda c_2 \tag{3.48}$$

(c) *The monopolist case 2*

As in the previous case, when only one bank lends, it is able to appropriate the entire surplus and thus charge the borrower $r^{max}$. The equilibrium interest rates of the stage are: $(r^{max}, \emptyset)$ The profits of A and C are as before:

$$\Pi^{mon}_{A} = \frac{(1-x)}{3}[2R_H + R_L] + (2x\rho - 1)\rho + (\lambda - x\rho)r^{max} - \lambda c_2 \tag{3.49}$$

$$\Pi^{liq}_{C} = \frac{(1-x)}{3}[2R_H + R_L] + (2x\rho - 1)\rho + \rho(\lambda - x\rho) - \lambda c_2 \tag{3.50}$$

Bank A and C's payoffs described above can be easily ordered as follows:

$$\Pi^{nl} < \Pi^{liq} < \Pi^{mon}_{\frac{R_L(1-x)}{3}} < \Pi^{mon}_{r^{max}}$$

Under these circumstance, the A-C game has two subgame perfect Nash Equilibria:

- $\{(\text{lend to D}, \text{lend to B}), (r^{max}, R_L\frac{(1-x)}{3(\lambda-x\rho)})\}$
- $\{(\text{lend to B}, \text{lend to D}), (R_L\frac{(1-x)}{3(\lambda-x\rho)}, r^{max})\}$

3. $R_L\frac{(1-x)}{3(\lambda-x\rho)} < r^{max} < r^{min} < \rho$

In this case the analysis is much simpler than in the previous ones. As $r^{min} > r^{max}$, there is not profitable interest rate for the lender that will be accepted by the borrower. Thus, no transaction take ever place on the interbank market.

The proposition follows.

**Proof of Proposition 7**

The proof is similar to the one we derive in the complete information case. We start from the derivation of the pure strategy Nash equilibria. Given the symmetry between B and D's portfolios, we derive the equilibrium only from the perspective of B.

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If banks B and D sell two different assets, all banks in the economy are forced into early liquidation and earn zero profits independently of the D's type. In the cases, instead, both banks sell either asset A or C, no bank will go bankrupt at date 1 unless $B$ expects $D$ to have a low asset: $R_D = R_L$. With probability $\pi$, $D$ has a portfolio comprising three low assets and, according to (3.7), it is forced into early liquidation.
When B and D sell the same asset and B expects $R_D = R_H$, the market values of banks' portfolios are:

- $\mathcal{V}_A = (P + R_H + R_H)\frac{(1-x)}{3} + x\rho > 1$

- $\mathcal{V}_B = (P + R_H + R_L)\frac{(1-x)}{3} + x\rho > 1$

- $\mathcal{V}_C = (2R_H + R_L)\frac{(1-x)}{3} + x\rho > 1$

- $\mathcal{V}_D = (P + R_H + R_L)\frac{(1-x)}{3} + x\rho > 1.$

and no bank is forced into early liquidation. When coordinating B and D's expected profits at date 2 are:

$$\Pi_B = \Pi_D = \pi * 0 + (1-\pi)\left[(R_H + R_L)\frac{(1-x)}{3} - (1-\lambda)c_2\right] > 0. \tag{3.51}$$

Deviating from selling a common asset triggers the immediate bankruptcy of at least one bank and through the dynamics induced by mark-to-market accounting, all banks will go bankrupt at date 1. Putting on sale a common asset is a weakly dominant strategy. $(\text{sell A}, \text{sell A})$ and $(\text{sell C}, \text{sell C})$ emerge therefore as payoff dominant Bayesian Nash equilibria.
The Bayesian-Nash equilibrium $(\text{sell B}, \text{sell D})$ arises as a payoff dominated equilibrium. When either bank sells its own asset early liquidation occurs independently of what the opponent decides to sell.

We now derive the mixed strategy equilibrium.
Selling its own asset is for both B and D a weakly dominated strategy as players accrue zero profits independently of the action chosen by the other bank in distress and its type.From the perspective of bank B the game develops as follows.
Define $(p, q, 1-p-q)$ and $(\tilde{p}, \tilde{q}, 1-\tilde{p}-\tilde{q})$ as the probabilities B attaches to D playing actions $(\text{sell A}, \text{sell C}, \text{sell D})$ when $R_D = R_H$ and $R_D = R_L$, respectively. Bank B's

payoff from selling A are

$$\begin{aligned}
&\pi * [\tilde{p} * 0 + \tilde{q} * 0 + (1 - \tilde{p} - \tilde{q}) * 0] + \\
&+(1-\pi)\left\{p\left[(R_H + R_L)\frac{(1-x)}{3} - (1-\lambda)c_2\right] + q * 0 + (1-p-q) * 0\right\} = \\
&=(1-\pi)p\left[(R_H + R_L)\frac{(1-x)}{3} - (1-\lambda)c_2\right].
\end{aligned}$$

The payoffs from selling B are

$$\begin{aligned}
&\pi * [\tilde{p} * 0 + \tilde{q} * 0 + (1 - \tilde{p} - \tilde{q}) * 0] + \\
&+(1-\pi)[p * 0 + q * 0 + (1-p-q) * 0] = 0
\end{aligned}$$

and those from selling C are

$$\begin{aligned}
&\pi * [\tilde{p} * 0 + \tilde{q} * 0 + (1 - \tilde{p} - \tilde{q}) * 0] + \\
&+(1-\pi)\left\{p * 0 + q * \left[(R_H + R_L)\frac{(1-x)}{3} - (1-\lambda)c_2\right] + (1-p-q) * 0\right\} = \\
&=(1-\pi) * q * \left[(R_H + R_L)\frac{(1-x)}{3} - (1-\lambda)c_2\right].
\end{aligned}$$

Thus, solving with equality

$$q * \left[(R_H + R_L)\frac{(1-x)}{3} - (1-\lambda)c_2\right] = p * \left[(R_H + R_L)\frac{(1-x)}{3} - (1-\lambda)c_2\right] \tag{3.52}$$

we have

$$p = q = \frac{1}{2} \tag{3.53}$$

The same analysis applies to D and, thus, the proposition follows.

**Proof of Proposition 23**

As in the case $R_C = R_L$, when B and D sell two different assets they are forced into early liquidation together with all the other banks in the economy. There is another circumstance in which banks earn zero profits: when the rival bank is expected to have a low return asset and banks sell asset C. The expected payoff of B and D in case they sell

A is instead given by:

$$\Pi_B = \Pi_D = \left[2R_H \frac{(1-x)}{3} - (1-\lambda)c_2\right] > 0. \tag{3.54}$$

In case banks B and D coordinate on selling asset C, the expected payoffs are

$$\Pi_B = \Pi_D = \pi * 0 + (1-\pi)\left[(R_H + R_L)\frac{(1-x)}{3} - (1-\lambda)c_2\right] > 0. \tag{3.55}$$

Thus, the proposition follows.

**Proof of Proposition 9** We proceed as in the proof of Proposition 6. We solve the A-C game by backward induction. After the first stage of the A-C game, four outcomes are possible: A and C have chosen to offer a loan to the same bank (henceforth *the competition case*), A and C have decided to make offers to different banks, each holding monopoly power in the bargaining process(*the monopolist case 1*), only one bank (A or C) makes an offer while the other bank withholds (*the monopolist case 2*) and no bank makes an offer (*non-lending case*).

We start our proof by showing that whenever $\tilde{r}^{min} < \tilde{r}_1^{max}$, we observe lending in both scenarios, which will prevent bankruptcy and contagion.

1. *The competition case*
   When A and C offer a loan to the same bank, they compete à la Bertrand. The offers made by the two lenders in equilibrium depend on the return on investment in the safe asset between date one and two, relative to $\tilde{r}_1^{max}$. We first consider $\rho < \tilde{r}_1^{max}$. In this parameter space, the Bertrand-type competition will drive both interest rates down to $\rho$. In contrast, when $\rho > \tilde{r}_1^{max}$, lenders will make differentiated offers in which only one of them proposes $\tilde{r}_1^{max}$, while the other one will charge anything above this level. The lowest interest is going to be accepted in both scenarios and since there is going to be only one bank liquidating an asset, bankruptcy and contagion are prevented.

2. *The monopolist case 1*
   When lenders make offers to different banks, at least one lender will offer $\tilde{r}_1^{max}$. It is easy to prove that the situation in which both lenders' offers are strictly higher than $\tilde{r}_1^{max}$ cannot be part of an equilibrium.. Whenever both interest rates offered are above the $\tilde{r}_1^{max}$ threshold, both offers will be rejected in the true state of the economy ($R_C = R_L$) and only one of them will be accepted in the alternative state of the economy. Anticipating this equilibrium actions on the side of the borrowers,

Leonello, Agnese (2011), Three Essays in Competition and Banking
European University Institute DOI: 10.2870/30719

the lender who expects its offer to be rejected in both states realizes the same profit as in the non-lending case. Since we are under the assumption $\tilde{r}^{min} < \tilde{r}_1^{max}$, this lender finds a profitable deviation in the range $\left[\tilde{r}^{min}, \tilde{r}_1^{max}\right]$. It must be then the case that at least one of the offers made is less or equal to the threshold value $\tilde{r}_1^{max}$. This implies that at least one loan offer is accepted in equilibrium.

3. *The monopolist case 2*

In the case in which there is only one lender making an offer, the bargaining power is entirely on the lender's side. The lender is able to appropriate the entire surplus from the borrower. Hence, the equilibrium interest rates at this stage are either $(\tilde{r}_2^{max}, \emptyset)$ or $(\tilde{r}_1^{max}, \emptyset)$. The lender chooses $\tilde{r}_2^{max}$ instead of $\tilde{r}_1^{max}$, if the profits it accrues in the first case are higher than the one it obtains demanding $\tilde{r}_1^{max}$:

$$\begin{aligned}
&\frac{\pi}{2}\left[\frac{(1-x)}{3}[2R_H+R_L]+(2x\rho-1)\rho+(\lambda-x\rho)\rho-\lambda c_2\right]+\\
&+(1-\pi)\left[\frac{(1-x)}{3}(2R_H+R_L)+(\lambda-x\rho)\tilde{r}_2^{max}+(2x\rho-1)\rho-\lambda c_2\right]> \qquad (3.56)\\
&\left[\frac{(1-x)}{3}[2R_H+R_L]+(2x\rho-1)\rho\right]+(\lambda-x\rho)\tilde{r}_1^{max}-\lambda c_2
\end{aligned}$$

The left hand side are the profits the lender accrues when it demands $\tilde{r}_2^{max}$, while the right hand side are the profits in case it chooses $\tilde{r}_1^{max}$. When the condition (3.56) holds one lender offers $r_2^{max}$ and the offer is rejected in the true state of the economy. In the equilibrium in which A is the lender and C has opted out, A and C's profits are:

$$\begin{aligned}
\Pi_A^{mon_2}&=\frac{\pi}{2}\left[\frac{(1-x)}{3}[2R_H+R_L]+(2x\rho-1)\rho+(\lambda-x\rho)\rho-\lambda c_2\right]+\\
&+(1-\pi)\left[\frac{(1-x)}{3}(2R_H+R_L)+(\lambda-x\rho)\tilde{r}_2^{max}+(2x\rho-1)\rho-\lambda c_2\right]\\
\Pi_C^{mon_2}&=\Pi^{nl}.
\end{aligned}$$

4. *Non lending case*

Leonello, Agnese (2011), Three Essays in Competition and Banking
European University Institute DOI: 10.2870/30719

When neither A nor C intend to lend, their date 2 expected profits are:

$$\Pi_A^{nl} = \left(1 - \frac{\pi}{2}\right)\left[\frac{(1-x)}{3}[2R_H + R_L] + (x\rho - (1-\lambda))\rho - \lambda c_2\right] \quad (3.57)$$

$$\Pi_C^{nl} = \left(1 - \frac{\pi}{2}\right)\left[\frac{(1-x)}{3}[2R_H + R_L] + (x\rho - (1-\lambda))\rho - \lambda c_2\right] \quad (3.58)$$

On the one hand, in both the competition and monopoly case 1 lending always occurs in equilibrium. On the other hand, it can never be a Bayesian-Nash equilibrium a situation in which only one bank makes an offer $r > \tilde{r}_1^{max}$, as shown in the *monopolist case 1.*

Now we turn our attention to the case in which $\tilde{r}_1^{max} < \tilde{r}^{min}$. Here we distinguish two cases:

1. When $\tilde{r}_2^{max} < \tilde{r}^{min}$ the same analysis applies as in the case $r^{min} > r^{max}$ under complete information. As $\tilde{r}^{min}$ is higher than the maximum interest rate borrowers are willing to pay in both states of the economy, no lending will take place. Thus, contagion arises in equilibrium with positive probability.

2. When $\tilde{r}_1^{max} < \tilde{r}^{min} < \tilde{r}_2^{max}$

   (a) *The competition case*
   The equilibrium interest rate is $(\rho, \rho)$. This offer is going to be accepted only in the case both lenders expect the other lender to have a high return asset. No bank has an incentive to understate this offer, as offering $\tilde{r}_1^{max}$ yields less expected profits than under non-lending. Overstating yields the same profit as the the equilibrium.

   (b) *The monopolist case 1*
   In equilibrium, one bank offers $\tilde{r}_2^{max}$ and this offer is accepted while the other bank's offer is always rejected. The equilibrium of this subgame is: $(\tilde{r}_2^{max}, l)$ where $l \in ]\tilde{r}_1^{max}, \infty[$. In the state of the economy in which A expects $R_C = R_H$, the only offer accepted is $\tilde{r}_2^{max}$, while in the true state of the economy borrowers reject the offer.

   (c) *The monopolist case 2*
   The only lender demands in equilibrium an interest rate $\tilde{r}_2^{max}$ which is rejected in the true state of the economy and accepted in the other state. It does not exist a profitable interest rate for the lender that will be accepted in both states.

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European University Institute DOI: 10.2870/30719

The profits can easily be ordered:
$\Pi^{mon1} = \Pi^{mon2} > \Pi^{nl}$

In equilibrium only one offer is accepted and only in the state of the economy described by $\mathbf{R} = (R_L, R_H, R_H, R_H)$.

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European University Institute DOI: 10.2870/30719

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# Horizontal Mergers in Two-Sided Markets

## 4.1 Introduction

The interest in two-sided markets has grown in the last decade. There are many markets, in several different industries, that are two or multi sided. Two-sided platforms are common in traditional industries (i.e. supermarket, credit card, video games) as well as in new-economy industries (operating system producers, web portals) and advertising supported media (newspapers, radio, TV) (Rysman, 2009).

Although type-specific features can be detected, there are common attributes identifying a two-sided market. First, there exist two or more distinct customer groups that want to interact with each other but, due to transaction costs, they need a platform that allows and facilitates their interaction. Second, the platform internalizes the indirect network externalities between the groups and allows the members of the two groups to capture the benefits from having access to each other. This has relevant consequences on the pricing decisions, as well as on other platforms' strategic choices. The volume of transactions does not only depend on the aggregate price level, as it is the case in one-sided markets, but it is crucially affected by the price structure (how the price is allocated between the two sides of the market).

All these features are summarized in the following definition minted by Rochet and Tirole (2004):



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European University Institute DOI: 10.2870/30719

*"A market is two-sided if the platform can affect the volume of transactions by charging more to one side of the market and reducing the price paid by the other side by an equal amount; in other words the price structure matters, and platforms must design it so as to bring both sides on board"*

In other words, pricing their goods, platforms have to take into account that raising the price on one side of the market implies, as usual, a reduction in the number of the consumers served on that side. However, the total effect is much broader: due to the indirect network externalities, the decrease in demand implies a further decrease in the demand on the other side and this proceeds iteratively from one side of the market to the other, generating a feedback loop.

In the last few years, several mergers between two-sided platforms took place in different industries. For example, the newspaper industry has seen an increased concentration as consequence of several mergers, in several countries. This has contributed to an increase in interest in this topic among practitioners and antitrust authorities.[1] Nonetheless, there has been very little work examining the effects of mergers in two-sided markets.

In this paper, I analyze the effects of a merger between two platforms on consumers' welfare. I show that, under some conditions, a merger between two-sided platforms is not only profitable for platforms but it is also beneficial for consumers.

The possibility of having welfare-enhancing mergers is due to the fact that platforms internalize the effects that a change in prices has on the demand for both products on both sides of the markets. After the merger, the two-sided platforms face a trade off. On one hand, they would like to increase prices because of the lower competition. On the other hand, indirect network externalities create an incentive for keeping the price low, at least on one side of the market, to induce an increase in demand on the other side of the market and get higher revenue on the inframarginal units. When indirect network

[1]Recently, the OFT was seeking views and advice in order to revise the merger regime regarding local and regional printed media. The need of a review came after several cases regarding mergers between local and regional newspapers that OFT investigated in the last few years. The Dutch antitrust authority also recently commissioned a study on mergers in two-sided markets.

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externalities are sufficiently high, the latter effect dominates the former. In this case, the platforms' pricing strategy positively affects consumers' surplus. The crux of the analysis is to evaluate which effect dominates.
This result contrasts sharply with traditional merger analysis performed in one-sided markets. In traditional industries, in case of price competition (*strategic complements*), mergers are welfare-detrimental as they only cause an increase in price due to the lower competition. The opposite result (a merger is welfare-enhancing) may be obtained if the merger entails efficiency gains.[2]

The paper is related to various others. Evans and Noel (2007) show how the traditional techniques used to analyze the effects of mergers are biased if applied to the two-sided market case. They, thus, modify the traditional techniques in order to incorporate the two-sided nature of these markets and apply them to the analysis of the merger between Google and Doubleclick. The closest paper to mine is a paper by Chandra and Wexler (2009). They develop a model to study the effects of a merger in the newspaper industry and test the predictions using data relative to the mergers between Canadian newspapers. They show that the effects of the merger on consumers' welfare are ambiguous. The merger may lead to lower prices on either side of the market under specific circumstances. A necessary condition for the merged platforms to charge a lower price is that the newspaper is sold at a price below marginal cost to readers. Their result depends on specific assumptions of the model. The main difference with my paper is that they assume that there is price competition only on one side of the market. This implies that the merger does not have a direct effect on the price on that side. It only indirectly affects the price through the change in the number of consumers patronizing the platform on the other side of the market. This assumption has important implications for the results. They find that, depending on the circumstances, the prices increase or decrease on both sides of the market, while in my paper, the post-merger price on the side of the market where the indirect network

[2]See Deneckere and Davidson (1985) for the analysis of mergers when firms compete in prices (*strategic complements*).

externalities are the highest, always increases after the merger.

The paper proceeds as follows. Section 2 describes the model. The pre-merger equilibrium is derived in Section 3. Section 4 presents the post-merger equilibrium and compares the pre- and post-merger equilibria. Section 5 concludes.

## 4.2 The Model

The model is based on D'Aspremont and Motta (1994) and Armstrong (2006). Consider a market with two competing platforms, A and B. Each platform serves two groups of consumers (or sides), 1 and 2, and it enables them to interact. For example, an operating system producer (Windows, Apple etc.) is a platform serving both clients and application developers. Similarly, credit cards (Visa, MasterCard, Amex etc.) deal with both card-holders and merchants and newspapers serve both readers and advertisers.

A real line represents each side of the market. Platforms A and B are located in 0 and 1, respectively. For simplicity, I assume that the locations are fixed before and after the merger. This means that the choice of the products is taken as given by the platforms.

In supplying the product, each platform incurs a cost

$$C_i^k = \frac{1}{2}(n_i^k)^2$$

on each side of the market. Denote with $n_i^k$ the number of consumers served by platform $k$, with $k = A$, $B$, on the side $i$ of the market. The convexity of the cost function ensures the concavity of the profit function.

Consumers have an inelastic demand. Each of them buys one unit of the good if the price is less than their reservation value, zero otherwise. On each side of the market, there is single-homing. This means that consumers can join only one platform.

The utility of a consumer on side $i$ joining platform $k$ is

$$u_i^k = S + \alpha_i n_j^k - p_i^k - t_i x_i$$

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European University Institute DOI: 10.2870/30719

$\forall\, i = 1, 2$ and $k = A, B$. Consumer' s utility is given by net value the consumer attaches to the good $S - p_i^k - t_i x_i$ plus the benefit $\alpha_i n_j^k$ the consumer derives from the fact that there are $n_j^k$ agents patronizing the platform on the other side of the market. The net surplus he/she obtains from buying the good is given, in turn, by the difference between the intrinsic value of the good $S$, with $S > 0$, the price $p_i^k$ and the transportation costs $t_i x_i$. As in the traditional Hotelling model, $t_i x_i$ can be interpreted as the physical distance or, alternatively, it may represent a measure of the implicit cost to the consumer from purchasing a good deviating from his "ideal" product. For simplicity, unit transportation costs are normalized to one on both sides. Thus,

$$t_1 = t_2 = 1.$$

The term $\alpha_i n_j^k$ measures the indirect network externalities. An additional consumer patronizing platform k on side $j$ induces an increase in side $i$ consumers' utility exactly equal to $\alpha_i$, with $\alpha_i \geq 0$

Throughout the analysis, I assume, for simplicity, that $\alpha_1 = 0$ and $\alpha_2 > 0$. Recall that the main idea of the paper is to show that in a two-sided markets a merger may be welfare enhancing, because of the indirect network externalities. Thus, the higher they are, the larger are the incentives for the platform to keep the price relatively low. Therefore, setting $\alpha_1 = 0$ is a conservative assumption that does not affect the results. With any $\alpha_1 > 0$ the results would be even stronger. Moreover, this assumption is consistent with the newspaper market. Newspapers are two-sided platforms. On one side there are the readers, on the other the advertisers. Unlike advertisers, readers do not usually attach any value to the presence of advertising on the newspaper. Readers, as well as advertisers, do not represent a homogeneous group; for example, they can be distinguished on the basis of their political connotation: moving from the left to the right of the line, we have left-wing activists, centrist and right-wingers. Another possible source of heterogeneity is geographic location. In this case, the two platforms would represent two local newspapers

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DOI: 10.2870/30719

for neighboring towns.

On each side of the market, platforms serve two groups of consumers: each platform supplies potential consumers on its left and those on its right (Figure 1). Therefore, on each side, three segments can be distinguished: the interval $[0, 1]$ and two *wings*.[3] Platforms cannot price discriminate. They charge the same price to all consumers on each side of the market.

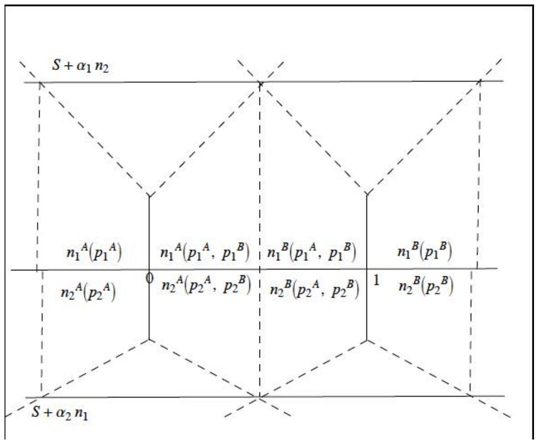


Figure 4.1: *The market with overlapping market areas*

On the *wings*, the platforms always enjoy the position of local monopoly, but, in the interval $[0, 1]$, depending on the value of $S$ and $\alpha_i$, the platform can either enjoy a position of local monopoly or it can share that segment with the rival firm. I define below the constraints on the parameters such that, on each side of the market, the interval $[0, 1]$ is always covered. The case in which platforms' market areas do not overlap is not very interesting as in that case, the post-merger platform's maximizing strategy would be exactly the same as in the pre-merger situation.

Define $\tilde{n}_i^k$ and $\bar{n}_i^k$ as the demand when the platform enjoys a position of local monopoly

[3]From now on, I define *wings* the segments, on each side, that are on the platform A's left side and on platform B' s right, respectively.

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and when it competes with its rival, respectively.

On the *wings*, the demand depends only on the price $p_i^k$ chosen by the platform and the gross surplus $S + \alpha_i n_j^k$. It can be derived as follows

$$\tilde{n}_i^k = S + \alpha_i n_j^k - p_i^k \tag{4.1}$$

$\forall\, i = 1, 2$ and $k = A, B$.

In the middle segment, instead, the demand is also a function of the price set by the rival platform.

$$\bar{n}_i^k = \frac{1 + \alpha_i(n_j^k - n_j^{-k}) + (p_i^{-k} - p_i^k)}{2} \tag{4.2}$$

$\forall\, i = 1, 2$ and $k = A, B$.

Platform k's total demand on each side of the market is given by the sum of (4.1) and (4.2) and is equal

$$n_1^k(p_1^k, p_1^{-k}) = \frac{(1 + 2S) + (p_1^{-k} - 3p_1^k)}{2} \tag{4.3}$$

$\forall\, k = A, B$ and

$$n_2^k(p_1^k, p_1^{-k}, p_2^k, p_2^{-k}) = \frac{(1 + 2S)(1 + \alpha_2) + (p_2^{-k} - 3p_2^k) + \alpha_2(3p_1^{-k} - 5p_1^k)}{2} \tag{4.4}$$

$\forall\, k = A, B$.

On side 1, the demand depends only on the prices charged by the two platforms on that


DOI: 10.2870/30719

side of the market. On side 2, instead, the demand depends also on the prices that the two platforms charge on the other side of the market. The lower the price a platform charges to consumers on side 1, the larger the number of consumers patronize that platform and thus the larger is the demand on side 2.

## 4.3 Pre-Merger Equilibrium

Before the merger, the platforms compete in prices on each side of the market. The platforms' problem consists of choosing simultaneously the prices. Platforms choose prices in order to maximize their profits as given by

$$\max_{p_1^k,p_2^k} \Pi^k = p_1^k n_1^k(p_1^k,p_1^{-k}) - \frac{1}{2}[n_1^k(p_1^k,p_1^{-k})]^2 + p_2^k n_2^k(p_1^k,p_1^{-k},p_2^k,p_2^{-k}) - \frac{1}{2}[n_2^k(p_1^k,p_1^{-k},p_2^k,p_2^{-k})]^2 \tag{4.5}$$

The platforms' profits are given by the sum of the profits on each side of the market. On each side the profits are simply equal to the difference between the total revenue $p_i^k n_i^k(p_i^k,p_i^{-k})$ minus the total cost $\frac{1}{2}[n_i^k(p_i^k,p_i^{-k})]^2$.

**Proposition 11** *In the pre-merger equilibrium both platforms charge prices* $p_1^* = \frac{5(1+2S)(4-\alpha_2-\alpha_2^2)}{64-10\alpha_2^2}$ *and* $p_2^* = \frac{5(1+2S)(8+3\alpha_2^2)}{2(64-10\alpha_2^2)}$ *and serve* $n_1^* = \frac{(1+2S)(12-5\alpha_2)}{(64-10\alpha_2^2)}$ *and* $n_2^* = \frac{3(1+2S)(8+3\alpha_2)}{2(64-10\alpha_2^2)}$ *consumers on side 1 and 2 respectively. The equilibrium profits are* $\Pi^* = \frac{(1+2S)^2(2688+848\alpha_2-591\alpha_2^2-200\alpha_2^3)}{32(32-5\alpha_2^2)^2}$.

***Proof.*** *See Appendix.* ■

The equilibrium variables, as defined in Proposition 11 are consistent with the two-sided market literature. The size of the indirect network externalities affects the platforms' pricing strategy. The price $p_1$decreases as $\alpha_2$ increases while $p_2$ increases with $\alpha_2$.

$$\frac{\delta p_1^*}{\delta \alpha_2} < 0 \text{ and } \frac{\delta p_2^*}{\delta \alpha_2} > 0.$$

The derivative $\frac{\delta p_1^*}{\delta \alpha_2}$ simplifies to

$$\frac{\delta p_1^*}{\delta \alpha_2} = -\frac{5(1+2S)\left(32+24\alpha+5\alpha^2\right)}{2\left(32-5\alpha^2\right)^2}$$

It is always negative. This means that as the indirect network externalities increase, platforms optimally choose to lower the price on side 1 so to attract more consumers. Doing so, they can to extract a larger share of consumers' surplus on side 2. Thus, as the indirect network externalities become larger, the price charged on side 2 increases. The derivative $\frac{\delta p_2^*}{\delta \alpha_2}$ is equal to

$$\frac{\delta p_2^*}{\delta \alpha_2} = \frac{(1+2S)\left(96+80\alpha_2+15\alpha_2^2\right)}{4\left(32-5\alpha_2^2\right)^2}$$

and it is always positive for any value of $\alpha_2$ and $S$. As the indirect network externalities increase, it becomes optimal for the platform to reduce the price on side 1 so to attract more consumers. This in turn increase consumers' gross surplus $S + \alpha_2 n_1$ on side 2, thus, allowing the platform to charge a higher price on that side of the market.

Moreover, the price on side 1 is negative for high value of the network externalities. This is the case for

$$1.56 < \alpha_2 < 4\sqrt{\frac{2}{5}},$$

This means that, for large enough network externalities, platforms have an incentive to give the good out for free to the consumers on side 1 in order to attract more consumers on the other side and charge them a higher price. This strategy implies that side 1 is subsidized with the profits the platforms make on the other side. Such a situation is common in the newspaper industry. Some newspapers are given for free to the readers and newspapers only make profits from the advertisers' side.

In order for the equilibrium described in Proposition 11 to be an equilibrium, it must be that profits $\Pi^*$ and quantities $n_1^*$ and $n_2^*$ are non negative. Moreover, we have also to check that the equilibrium described in Proposition 11 is consistent with the assumption that platforms' market areas overlap on both sides of the market. The following result holds.

**Proposition 12** *In the equilibrium described in Proposition 11 platforms have overlapping market areas on each side of the market. Such an equilibrium exists for any* $S > \frac{13}{6}$ *and* $0 < \alpha_2 < 2.10$.

**Proof.** See Appendix. ■

Finally, I compute the pre-merger consumers' welfare. This is equal to the sum of the consumers' surplus on side 1 and that on side 2 patronizing both platforms. The two expressions for consumers' surplus are, respectively

and

$$CS_1 = 2\int_0^{\frac{1}{2}} (S - p_1^* - x)\, dx + 2\int_1^{z} (S - p_1^* - (1 - x))\, dx \tag{4.6}$$

$$CS_2 = 2\int_0^{\frac{1}{2}} (S + \alpha_2 n_1^* - p_2^* - x)\, dx + 2\int_1^{w} (S + \alpha_2 n_1^* - p_2^* + x - 1)\, dx \tag{4.7}$$

with $z$ and $w$ are the marginal consumers on the wings on side 1 and 2, respectively. In each of the two expressions above, the fist integral represents the utility of consumers lying in the interval $[0, 1]$, while the second one is that of agents being on the *wings*.

## 4.4 Post-Merger Equilibrium

To focus on the role of indirect network externalities, I assume that no efficiency gains result from the merger. I also assume that platforms' locations do not change, meaning that, after the merger, platforms continue to supply the same products as before.

I model the merger as a non-consolidating merger. This means that the number of active platforms does not change. The only difference between the pre- and post-merger situation is that, post-merger, the merging platforms become a single entity in the negotiation

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European University Institute DOI: 10.2870/30719

with the consumers on both sides of the market. The assumption of a non-consolidating merger is convenient for the newspaper case. Mergers between newspapers are usually non-consolidating mergers. In this industry, a merger often results in an editorial group printing different newspapers.
A non-consolidating merger captures the same effects of a consolidating merger reducing the number of active platforms on the market. One effect refers to the market power of the merged entity: it increases as a consequence of the lower competition. The other effect is the increase in the potential number of consumers patronizing the platforms and, thus, in the potential value of joining the platform and in the price they can charge. To capture such effect in the framework of a non-consolidating merger, I assume that the new entity offers consumers the possibility to interact with the customers patronizing either platform on the other side of the market. The merged platforms have an incentive to do so as they can increase consumers' reservation value and, thus, the price they can charge on the market.

Referring to the newspaper example, this can be implemented by the newly merged entity offering a bundle to advertisers: independently of the platform they patronize, advertisers can place advertising on both newspapers. This allows advertisers to have access to all the agents patronizing either platform. In other industries, this assumption would represent the introduction of compatibility between the platforms' products.

The introduction of compatibility/ sale bundle of advertising captures the increase in the number of potential consumers on each side of the market and thus the potential increase in the value of joining the platform.

Considering the newspaper case, an increase in the number of readers that an advertiser can reach implies an increase in the gross surplus of advertisers and this allows platforms to charge a higher price.

In this framework, it is reasonable to think that consumers do not attach the same value to the agents joining the two platforms on the other side of the market. In the newspapers example, advertisers do not attach the same value to all the readers but value differently

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the readers depending on the newspaper they read. Reasonably, advertisers attach a higher value to the readers buying the newspaper closer to their original platform.

A new parameter $\chi$, with $0 \leq \chi \leq 1$, is introduced into the model. It represents the weight the consumers on side 2 attach to the consumers patronizing a different platform on side 1. The utility of a consumer on side 2 joining platform $k$ is now

$$u_i^k = S + \alpha_i n_j^k + \alpha_i \chi n_j^{-k} - p_i^k - t_i x_i$$

$\forall\ i = 1, 2$ and $k = A, B$.

The utility of a consumer on side 1 is the same as before. As in the pre merger equilibrium, the demand functions can be derived as follows

$$n_1^k = \frac{(1+2S) + (p_1^{-k} - 3p_1^k)}{2} \tag{4.8}$$

and

$$n_2^k = \frac{(1+2S)(1+\alpha_2(1+\chi)) + (p_2^{-k} - 3p_2^k) + \alpha_2(3-5\chi)p_1^{-k} - \alpha_2(5-3\chi)p_1^k}{2} \tag{4.9}$$

$\forall k = A, B$ on side 1 and 2, respectively.

As in the pre-merger case, the platforms' problem consists of choosing the price to charge on each side of the market. Merged platforms choose the price in order to maximize their joint profits. Platforms' profits are now given by the sum of the profits of the two platforms on the two sides of the market. The profit maximization problem for the merged platforms is

$$\max_{p_1^A, p_2^A p_1^B, p_2^B} \Pi^M = \sum_{k=A}^{B} p_1^k n_1^k(p_1^k, p_1^{-k}) - \frac{1}{2}[n_1^k(p_1^k, p_1^{-k})]^2 + p_2^k n_2^k(p_1^k, p_1^{-k}, p_2^k, p_2^{-k}) - \frac{1}{2}[n_2^k(p_1^k, p_1^{-k}, p_2^k, p_2^{-k})]^2 \tag{4.10}$$

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Unlike in the pre merger equilibrium, there are now two effects that merged platforms trade off when they choose the prices. On one hand, they would like to increase the price on both side of the market because of lower competition. On the other hand, there is also a stronger incentive to keep the price lower at least on one side of the market than in the pre merger case due to indirect network externalities. After the merger, the number of potential consumers increases and thus the value of joining the platforms. Referring to the newspaper example, the merged platforms have an incentive to lower the price on the readers' side in order to attract more of them and thus being able to charge a higher price to the advertisers. In equilibrium the two effects balance. The following results hold.

**Proposition 13** *In the post merger equilibrium both platforms charge prices* $p_1^A = p_1^B = p_1^M = \frac{(1+2S)(2-\alpha_2(1+\chi))}{2(3-\alpha_2(1+\chi))}$ *and* $p_2^A = p_2^B = p_2^M = \frac{(1+2S)}{3-\alpha_2(1+\chi)}$ *and serve* $n_1^M = n_2^M = \frac{(1+2S)}{2(3-\alpha_2(1+\chi))}$ *consumers on side 1 and 2. The equilibrium profits are* $\Pi^M = \frac{(1+2S)^2}{4(3-\alpha_2(1+\chi))}$.

As in the pre merger equilibrium, the price on side 1 is a decreasing function of the externality parameter $\alpha_2$ while the price charged to side 2 consumers is increasing in $\alpha_2$. The larger $\chi$, the larger the magnitude of the two effects.

$$\frac{\partial p_1^M}{\partial \alpha_2} < 0 \text{ and } \frac{\partial p_2^M}{\partial \alpha_2} > 0.$$

The first derivative simplifies to

$$\frac{\partial p_1^M}{\partial \alpha_2} = \frac{-(1+2S)(1+\chi)}{2(3-\alpha_2(1+\chi))^2}$$

. It is always negative for any $S > 0$. Regarding, the second derivative, it simplifies to

$$\frac{\partial p_2^M}{\partial \alpha_2} = \frac{(1+2S)(1+\chi)}{(3-\alpha_2(1+\chi))^2}$$

is always positive for any $S > 0$ and $\alpha_2 > 0$.

The higher the network externalities parameter on side 2, the lower the price the merged platforms set on side 1. Below I check that in the admissible parameters space, as defined in Proposition 12, the merger is profitable, the post merger equilibrium quantities are positive and platforms' market areas overlap.

I start considering the profitability of the merger. For the merger to take place it must be that

$$\Pi^M \geq \Pi^* \tag{4.11}$$

As the post merger profits increase with $\chi$, to prove that the merger is profitable it is enough to show that platforms have incentive to merge even if $\chi = 0$. Substituting the expressions for $\Pi^M$ and $\Pi^*$, as in Propositions 11 and 13 and choosing $\chi = 0$, the condition (4.11) simplifies to

$$\frac{128 + 144\alpha_2 + 61{\alpha_2}^2 + 9{\alpha_2}^3}{32\,(3-\alpha_2)\,(32-5{\alpha_2}^2)^{\,2}}(1+2S)^2 > 0 \tag{4.12}$$

which is always positive for any admissible $\alpha_2$ and $S$. As in the pre-merger equilibrium, it must be the case that platforms have overlapping market areas after the merger. For the market to be covered, the following conditions must be satisfied:

$$S - p_1^M - \frac{1}{2} > 0 \tag{4.13}$$

and

$$S + \alpha_2(1+\chi)n_2^M - p_2^M - \frac{1}{2} > 0. \tag{4.14}$$

where (4.13) and (4.13) state that the marginal consumer in the interval $[0, 1]$, on both sides of the market, is willing to buy the good from either platform.

Substituting the expressions for the equilibrium quantities and prices as defined in Proposition 13, both inequalities (4.13) and (4.14) simplify to

$$\frac{2S + 2\alpha_2(1+\chi) - 5}{2(3-\alpha_2(1+\chi))} \tag{4.15}$$

which is positive for $S > \frac{5}{2}$, $\alpha_2$ in the admissible parameter space and $\frac{3}{7} \leq \chi \leq 1$.

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Given the characterization of the pre- and post-merger equilibrium, the next step is to show that the merger may be welfare enhancing.
Unlike in the one-sided markets, in the two-sided one, platforms face a trade off after the merger. On one hand, they would like to increase the price because of weaker competition; on the other, they have an incentive to keep the price low so to benefit from the indirect network externalities. The bundle sales of advertising/ compatibility reinforces this effects. Therefore, whether the merger is welfare enhancing or not depends only on the parameters $\alpha_2$ and $\chi$. To evaluate the welfare properties of the merger, it is important to compare pre- and post-merger prices and quantities. The results of the comparison between the pre and post-merger equilibria are summarized in the following propositions.

**Proposition 14** *Given $\dot{\chi} > 0$, there exist a value of the indirect network externalities $\alpha_2^T$ such that, after the merger,the price on side 1 decreases and the demand increases for $\alpha_2 > \alpha_2^T$.*
***Proof.*** *See Appendix* ■

The intuition behind Proposition 14 is simple. When indirect network externalities are large enough, the merged platforms find it profitable to reduce the price on side 1 so to attract more consumers on that side. As the number of consumers patronizing either platform increases, the reservation value of consumers on side 2 increases so that they are willing to pay a higher price. This pricing strategy is profitable only when the lower gains platforms make from choosing a lower price on side 1 are compensated with the larger revenue they obtain on side 2. For low value of the indirect network externalities this strategy is not profitable as the increase in the reservation value of consumers on side 2 is not large enough to offset the forgone revenue on side 1. The parameter $\chi$ also plays an important role. It goes in the same direction of the indirect network externalities. Figure 2 shows the difference between post- and pre-merger prices on side 1 as function of $\alpha_2$. The dashed line represents the difference between post- and pre-merger prices when $\chi = 1$. The solid line is price difference for $\chi = 0.5$.

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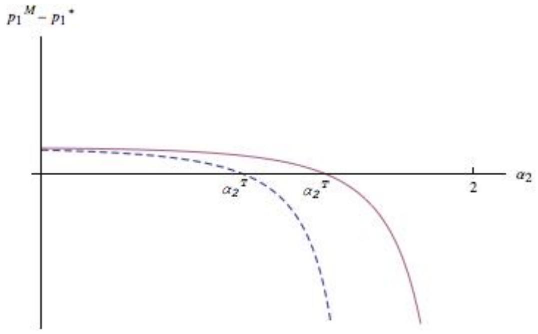


Figure 4.2: *Comparison between pre- and post-merger prices on side 1.*

The graph shows that there exists a threshold, $\alpha_2^T$ such that for any $\alpha_2 > \alpha_2^T$, the post-merger price on side 1 after the merger is lower than the one charged in the pre-merger equilibrium. $\alpha_2^T$ is a decreasing function of $\chi$. The higher $\chi$, the lower is the value of the network externalities for which after the merger the platforms charge a lower price than in the pre-merger equilibrium.

In the range where $\alpha_2 > \alpha_2^T$, the price on side 1 is lower than the one charged in the pre-merger equilibrium. This implies that $n_1^M < n_1^*$ as it can be seen from (4.3) and (4.8).

The higher the $\chi$, the lower the size of the indirect network externalities that induces the merging platforms to lower the price charged on side 1. As stated above, even for relatively small values of $\alpha_2$ and $\chi$, the platforms have incentives to set a lower price on side 1 so as to attract more consumers on that side of the market.

A larger demand on side 1 would imply a higher gross surplus for side 2 consumers. This means that the merged platform can set a higher price on that side of the market and thus get higher revenue on the inframarginal units. Moreover, for the same values of $\alpha_2$ and $\chi$ there are customers on side 2 (those on the *wings* that were not patronizing any platform before the merger) now willing to buy the good even if the price has increased. Thus, the following result holds

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**Proposition 15** *There exists a subset of the parameter space defined in Proposition 14, in which the demand on side 2 increases post-merger equilibrium.*

**Proof.** See Appendix. ■

The existence of a range of values for $\alpha_2$ for which the demand on side 2 increases after the merger is shown in Figure 3.

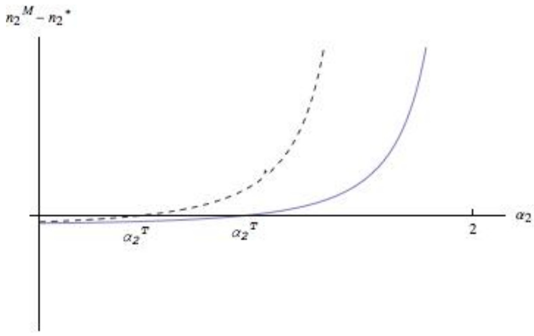


Figure 4.3: *Comparison between pre- and post-merger quantities on side 2.*

Figure 3 shows the difference between the quantity sold on side 2 by each platform in the post- and pre-merger equilibrium. The dashed line is drawn for $\chi = 0.5$ while the solid line is drawn for $\chi = 1$. In both cases, the difference is negative for low values of the network externalities. As $\alpha_2$ increases, the difference becomes smaller. For $\alpha_2 > \alpha_2^T$, the quantity sold in the post-merger equilibrium becomes larger than the one sold in the pre-merger one. The value for $\alpha_2^T$ depends on the parameter $\chi$. The larger $\chi$, the lower the threshold is.

The increase in demand on side 2 and the lower price charged on side 1 imply an increase in consumers' surplus. The consumers' welfare is computed as in the pre-merger equilibrium. It is given by the sum of (4.6) and (4.7). The only difference with the pre-merger case is that now the marginal consumers on the *wings* $z$ and $w$ are different.

Figure 4 shows the effect of the merger on consumers' surplus.

It shows how the change in consumers' surplus varies with the parameter measuring the indirect network externalities for several values of $\chi$. The relation is positive. For


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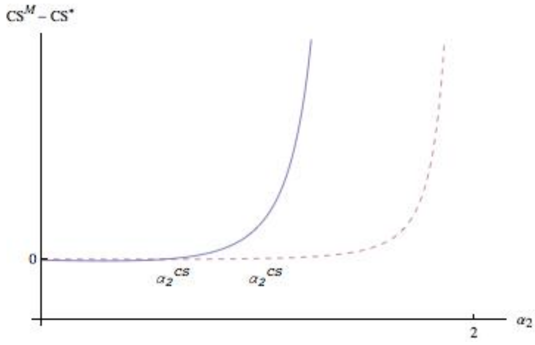


Figure 4.4: *Comparison between pre- and post-merger consumers' surplus.*

low values of $\alpha_2$ the post-merger consumers' surplus is smaller than the pre-merger one, thus implying that the merger is welfare detrimental. By contrast, when $\alpha_2$ is above the threshold level $\alpha_2^{CS}$ the merger is pro competitive. As in the other graph, the dotted line is drawn for $\chi = 0.5$ while the solid line for $\chi = 1$.

The intuition for this result is simple. When the price on side 1 decreases, two effects are produced:

- The demand on side 1 increases
- Given the increases in demand, side 2 consumers' gross surplus also increases.

These effects are very significant after the merger. Unlike in the pre- merger equilibrium, in choosing the optimal pricing strategy, the merged platforms internalize the effects that a change in price has on *both* products on the two sides of the market. This explains why the merged platforms have a strong incentive to set a lower price (even lower than in pre-merger situation) on side 1 and simultaneously to set a higher price on side 2. However, in the interval of values for the externalities and the compatibility parameters described above, the demand increases also on side 2 meaning that the gross surplus grows more than the price. Thus, all these forces together produce the increase in consumers' surplus. As shown in Figure 4, the higher the degree of compatibility, the lower the value of the network externalities for which the merger is welfare enhancing. The graph also shows that $\chi$ is important for the merger to be welfare enhancing: the higher the value the consumers on side 2 patronizing platform $k$ attach to the consumers on side 1 patronizing the platform $-k$, the lower is the value of the network externalities for which the merger has a positive effect on consumers' welfare.

## 4.5 Conclusion

In this paper I discussed the effects of mergers in two-sided markets. The main results of the paper is that the existence of indirect network externalities drives the platforms' pricing strategy, after the merger, toward a reduction of price, rather than the usual increase that we observe in one-sided markets due to increased concentration (absent efficiency gains). In two-sided markets, mergers may be welfare enhancing, under some specific conditions. This results is due to the existence of indirect network externalities: consumers value all the consumers patronizing the platforms on the other side of the market. The merger strengthens the incentives for the merged platforms to set a low price on one side of the market. Charging a low price on one side, platforms can gain from the other side of the market where more consumers buy the good at a higher price. A lower price on one side 1 attracts more consumers and, thus, it implies an increase in the gross surplus of the consumers on the other side of the market. For sufficiently high value of the network externalities, a lower price on side 1 and higher demand and price on side 2 than in the pre merger equilibrium are observed. The higher the network externalities, the more profitable this pricing strategy is. Another factor influencing the welfare effects of the merger is compatibility. A higher degree of compatibility amplifies the positive welfare effects of the merger. Indeed, ceteris paribus, a higher value of $\chi$ corresponds to a larger drop in the post merger price on side 1. Consistently, the effect on the demand on side 2 and thus, on welfare is also larger. The positive welfare effects of the merger, thus, depend on the size of the externality. When the indirect network externalities are small, the traditional merger analysis applies: the prices increase on both sides of the market

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and the merger is welfare detrimental.

The analysis proposed in the paper suits media platforms but it can be also extended to other industries. In a situation in which on both sides of the markets there are positive indirect network externalities (i.e. credit cards, operating system etc.), this result would be even stronger. The existence of positive indirect network externalities on both sides of the market would create an incentive for the platforms to keep the price low after the merger in order to increase the demand on both sides. This would imply lower revenues from the infra-marginal units but, at the same time, a larger demand and, overall, higher profits for the platforms. Thus, this strategy would also be beneficial for consumers.

The paper offers relevant policy implications for antitrust authorities. It suggests that dealing with two-sided platforms, the authorities should apply a particular attention in examining mergers in such markets. In fact, the criteria and techniques applied in the case of mergers in the traditional one-sided markets might not apply in the two-sided ones.

## Appendix

**Proof of Proposition 11** Platforms choose simultaneously $p_1^k$ and $p_2^k$ in order to maximize (4.5). As they are identical, $p_1^A = p_1^B = p_1^*$ and $p_2^A = p_2^B = p_2^*$. The equilibrium prices are given by the solution to the following system of equations

$$\frac{(1+2S)+(p_1^{-k}-6p_1^k)}{2}+\frac{3[(1+2S)+(p_1^{-k}-3p_1^k)]}{4}-\frac{5p_2^k\alpha_2}{2}+\frac{5\alpha_2[(1+2S)(1+\alpha_2)+(p_2^{-k}-3p_2^k)+\alpha_2(3p_1^{-k}-5p_1^k)]}{4}=0$$

and

$$\frac{(1+2S)(1+\alpha_2)+(p_2^{-k}-3p_2^k)+\alpha_2(3p_1^{-k}-5p_1^k)}{2}-\frac{3p_2^k}{2}+\frac{3[(1+2S)(1+\alpha_2)+(p_2^{-k}-3p_2^k)+\alpha_2(3p_1^{-k}-5p_1^k)]}{4}=0.$$

The equilibrium quantities, $n_1^*$ and $n_2^*$ are obtained substituting $p_1^*$ and $p_2^*$ into (4.3) and (4.4). □

**Proof of Proposition 12** First I check that profits and quantities are non negative. Given $S > 0$, it can be easily seen that this is the case for any $0 < \alpha_2 < 2.10$.

In order for the platforms to have overlapping market areas, it must be that in equilibrium, on both sides of the market, all the consumers in the interval $[0, 1]$ patronize one platform. To prove this, it is enough to show that for the consumers located exactly at $\frac{1}{2}$ the gross surplus from the good is higher than the sum of the price and the transportation costs.

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European University Institute DOI: 10.2870/30719

This must be the case on both sides of the market. Thus, to have overlapping market areas it must be that

$$S > p_1^* + \frac{1}{2}. \tag{16}$$

and

$$S + \alpha_2 n_1^* > p_2^* + \frac{1}{2}. \tag{17}$$

Substituting the expression for $p_1^*$, into (16) and those for $n_1^*$ and $p_2^*$ into (17), we obtain

$$\frac{(24+10\alpha_2^2)S - (52 - 5\alpha_2 - 10\alpha_2^2)}{64 - 10\alpha_2^2} > 0. \tag{18}$$

and

$$\frac{-72 + 21\alpha_2 + 20\alpha_2^2 + 14S\,(8+3\alpha_2)}{4\left(32 - 5\alpha_2^2\right)} > 0. \tag{19}$$

respectively, for side 1 and 2. Given the parametric restriction on $\alpha$ stated above the denominator is always positive. To prove, thus, that the inequality (18) and (19) hold, it must be shown that the numerators of either conditions are positive.
I start from the first inequality (18). Solving the numerator with equality I can find the lowest value of $S$, $S_1^{MIN}$, such that for $S > S_1^{MIN}$ the numerator is positive. The same reasoning applies to the inequality (19). The two values $S_1^{MIN}$ and $S_2^{MIN}$ are given by

$$S_1^{MIN} = \frac{(52 - 5\alpha_2 - 10\alpha_2^2)}{(24+10\alpha_2^2)} \tag{20}$$

and

$$S_2^{MIN} = \frac{71 - 21\alpha_2 - 20\alpha_2^2}{14\,(8+3\alpha_2)} \tag{21}$$

Both $S_1^{MIN}$ and $S_2^{MIN}$ are decreasing in $\alpha_2$.
When $\alpha_2 = 4\sqrt{\frac{2}{5}}$, $S_1^{MIN} \simeq -\frac{1}{2}$ and when $\alpha_2 = 0$, $S_1^{MIN} = \frac{13}{6}$. Similarly it can be found that a sufficient condition for the second inequality to hold is $S > 0.6$. Putting together all the conditions, the proposition follows. $\square$

**Proof of Proposition 14** Pre and post merger prices, $p_1^*$ and $p_1^M$, are both monotonically decreasing in $\alpha_2$. When $\alpha_2 = 0$,

$$p_1^M = \frac{(1+2S)}{3} \text{ and } p_1^* = \frac{5(1+2S)}{16}.$$

Thus, at $\alpha_2 = 0$, $p_1^M > p_1^*$. Given that both $p_1^*$ and $p_1^M$ are both decreasing in $\alpha_2$, to show that $\alpha^T$exists so that the post-merger price is lower than the pre-merger one, it is enough to show that the price function in the post merger equilibrium is steeper than the one in the pre merger equilibrium that is

$$|\frac{\partial p_1^M}{\partial \alpha_2}| > |\frac{\partial p_1^*}{\partial \alpha_2}| \tag{22}$$

where

$$\frac{\partial p_1^M}{\partial \alpha_2} = \frac{-(1+2S)(1+\chi)}{2(3-\alpha_2(1+\chi))^2} \text{ and } -\frac{\partial p_1^*}{\partial \alpha_2} = -\frac{5(1+2S)\left(32+24\alpha+5\alpha^2\right)}{2\left(32-5\alpha^2\right)^2}$$

For $\chi > 0$, solving (22) with equality, it gives $\alpha_2^T(\chi)$.

**Proof of Proposition 15** After the merger the platforms have overlapping market areas. This means that all the consumers in the interval $[0, 1]$, on both side of the market, patronize either platform. Thus, if the merger produces an increase in the demand on side 2, it must be the case that more consumers on the wings decide to join a platform. It must be that

$$S + \alpha_2(1+\chi)n_1^M - p_2^M > S + \alpha_2 n_1^* - p_2^*$$

The price charged on side 2 after the merger is higher than that in the pre-merger equilibrium.

Therefore, if an increase in demand is observed on side 2, it is due to an increase in the gross surplus of consumers located on side 2 that compensate the negative effect of an increase in price. This means that

$$S + \alpha_2(1+\chi)n_1^M - S + \alpha_2 n_1^* > 0.$$

Substituting the values for $n_1^M$ and $n_1^*$ as defined in Propositions 11 and 13 , the inequality above simplifies to

$$\frac{(1+2S)\left({\alpha_2}^2(44+9\chi)+\alpha_2(29+56\chi)-104-15{\alpha_2}^3(1+\chi)\right)}{4\left(32-5{\alpha_2}^2\right)\left(3-\alpha_2(1+\chi)\right)}. > 0 \tag{23}$$

For any $S > 0$, there always exists a combination of $\alpha_2$ and $\chi$ in the admissible parameter space such that (23) is greater than zero meaning that, after the merger, the demand on side 2 increases.

Leonello, Agnese (2011), Three Essays in Competition and Banking
European University Institute DOI: 10.2870/30719

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European University Institute DOI: 10.2870/30719