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Measuring the Electoral Barrier
Problems and Solutions to Estimation of the
Threshold(s) at the National Level

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Measuring the Electoral Barrier - Problems and Solutions to Estimation of the Threshold(s) at the National Level

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Abstract

This paper reviews different approaches to measuring the barrier or 'strength' of electoral systems and offers specific solutions to problems of estimation. Particularly a method for measuring the impact of the geographical electoral structure on the barrier is presented here for the first time. The structure of the paper is the following;

It is firstly argued that the 'threshold-approach' invented by Rokkan (1968) and recently extended to national level by Taagepera (2002) has clear advantages over other methods, including classification schemes and proportionality indices, for capturing the degree of representational openness.

Secondly, conceptual and methodological problems pertinent to threshold estimation at the national level are addressed. Regarding methodology, errors are identified in the technique used by Taagepera to estimate the mid point between the thresholds of inclusion and exclusion. Moreover his ignorance based method for including concern for the impact of the geographical vote concentration is substituted by one incorporating actual vote concentration values. At the conceptual level, it is argued that the barrier at the national level can be described by two thresholds, one defined in terms of the attainment of 'a seat' and the other in terms of 'proportional shares'.

Finally, a new indicator defined in terms of the attainment of proportional representation, the Threshold of Proportionality, is proposed to capture the average barrier effect of electoral systems, and it is discussed how indicators defined in terms of attainment of 'one seat' (such as Taagepera's empirical and nationwide thresholds) can serve as an indicator of openness to localized political interests. The new Threshold of Proportionality is then compared to other indicators and evaluated in light of empirical evidence.

1 Choosing an Indicator of the Electoral Barrier

Electoral systems invariably influence political processes and outcomes. The structure they impose on the choice offered to voters and the mechanism they employ to translate electoral support for candidates and parties into parliamentary representation are widely recognized as important determinants of static as well as dynamic features of the party systems that evolve within their framework. Arguably, the most important property of electoral systems is that they pose a barrier to the entry of smaller parties. By excluding – wholly or partially – new and smaller parties from representation the fragmentation of the party system is reduced and parliamentary parties shielded from new competition. To capture the size of this barrier posed by electoral systems three main types indicators have been employed in comparative research¹.

The simplest and probably the most widely used is the dichotomous classification of PR versus plurality-systems, initially proposed by Duverger. A more recent, but also problematic addition, to this classification scheme is the insertion of the class of mixed-systems as an intermediate category². As many have pointed out, however, significant variation between the systems remains hidden when they are lumped together in a few categories. More finely tuned indicators have therefore frequently substituted these in comparative studies.

One such set of indicators are based on the observed deviance from proportionality between votes and seats obtained by parties³. Disproportionality indicators offer a continuous measure of the distortions introduced by the electoral systems and have on occasion served as proxies of the electoral barrier in comparative research (e.g. Strøm, 1989; Bartolini and Mair, 1990; Lijphart, 2000). However, the underlying assumption that the size of the barrier of electoral systems consistently co-varies with observed disproportionality is not a valid one. Disproportionality

¹The term 'strength' of the electoral system coined by Sartori is also frequently used to refer to this property of electoral systems (Sartori, 1997)

²P. Norris as well as Woldendorp, Keman and Budge use 'combined'/'mixed' systems as an intermediary category between PR and majoritarian systems. Norris uses this ordinal classification is then used as an explanatory variable to account for various phenomena including the level of party fragmentation, while Woldendorp et al. uses it to explain government duration (Norris, 2003: chapter 3; Woldendorp et al.,2000). Implicit in this scheme is the assumption that these systems combining features found in proportional and plurality systems also fall between the two in terms of effects. The reality is, however, that they differ greatly with respect to their constraining effects some resembling PR-systems and others majoritarian ones (Massicotte and Blais, 1999; Shugart and Wattenberg, 2001).

³See Lijphart (1994) and Pennisi (1998) for a review of the different indicators of disproportionality.

is caused by a conjunction of factors of which electoral rules constitute but one set. A source of constraint, whether it stems from the district magnitude, legal threshold or allocation formula, is necessary but not sufficient on its own to produce disproportionality. The party system, that is the number of parties running and their relative electoral success, co-determines outcomes. And like electoral rules, the electoral party system is a necessary but not sufficient cause of disproportionality. Even strong increases in the number of parties and the degree of fragmentation will have only a marginally deflating effect on the proportionality of outcomes when the constraining properties of the electoral system are weak, as evidenced by the consistently proportional results observed in for instance the Netherlands despite significant changes in the party system. The lower the district magnitude or the higher legal thresholds, however, the greater the scope for the party system to influence outcomes. In single-member district systems, for example, the number of parties and their relative vote shares explain the great variation in disproportionality observed across countries and time. Likewise for systems using legal thresholds significant variation can be traced back to the degree of fragmentation - even if the thresholds are relatively low, if many run but fail to pass them, the recorded disproportionality will be high⁴. Since we know that increased fragmentation leads to a lowering of the vote share necessary to win votes, and that more parties may be encouraged to participate when the barrier is perceived to be lower or deterred where it is higher, the co-variation of disproportionality indices with the fragmentation of the party system in stronger systems brings on problems of validity if used as a proxy of the barrier⁵.

A third and more promising approach to capturing the electoral barrier, used by an increasing number of scholars in the field, consists in constructing a measure on basis of the characteristics electoral system itself. Indicators of this type rely mainly on the constraining effect of the district magnitude, which at times is also simply used directly⁶, but some also include consideration for the potential impact of the electoral formula, the number of parties running and lately also the number of districts. Following this method scores are produced which are independent of the concrete electoral outcomes and thus less subject to random fluctuations. Among these the Effective Threshold used by Lijphart in his

⁴See Penadés,1997; Lijphart, 1994; Rose, 1987 for a discussion of the causes of disproportionality.

⁵Sartori remarks on this fact that 'this least-proportional proportionality may not show up in our measures, and this for the simple reason that poor propotionality penalizes the smaller parties and eventually wipes them out'. (Sartori, 1997)

⁶For instance, Ordeshook and Shvetsova (1994) and Monroe and Rose (2002) both employ the district magnitude to investigate the effects of electoral systems.

large comparative study of electoral systems has emerged as a particular popular indicator of the electoral barrier (Lijphart,1994)⁷.

The Effective Threshold, which is defined as ‘*the vote share with which parties have a 50-50 chance of winning their first seat*’, builds on and incorporates earlier work on electoral thresholds. Operationally it is identified as the mid point between the thresholds of inclusion, originally invented by Rokkan (1968)⁸, and its logical complementary the threshold of exclusion proposed by Rae, Hanby and Loosemoore (1971). Apart from the fact that the Effective Threshold is straightforward to interpret, it shares the advantage with disproportionality based measures in being continuous. At the same time it avoids the main pitfalls associated with the latter. It yields more stable scores over time and similar scores for countries with identical institutions. Recent criticism pertaining to its measurement in practice has, however, challenged its position as the standard measure of the electoral barrier.

1.1 From Definition to Measurement

Given the widespread use of the Effective Threshold in comparative research it is naturally important that it is estimated correctly. The comprehensive critique recently delivered by Taagepera of the method used by Lijphart to estimate the Effective Threshold calls therefore for careful consideration (Taagepera, 2002). Essentially Taagepera introduces two revisions that both have significant impact on the calculation of the threshold.

Firstly, he rightly points out that the way in which the Effective Threshold was estimated by Lijphart implied a failure to distinguish between the district and the national levels. In spite of the fact that the formulas used to calculate the thresholds of inclusion and exclusion were initially developed for the district level, they are used in the Effective Threshold to estimate the size of the threshold at the national level. The criticism applies not only to the Effective Threshold but also to similar measures such as the Effective Magnitude that use the same approach (Shugart and Taagepera, 1989). The failure of the Effective Threshold to take the difference between district and nationwide levels into account has, according to Taagepera, led to consistently misleading estimates. He therefore proposes a method for including national level factors to enhance the precision of the scores.

The second and, as shall be argued, less convincing argument pre-

⁷A number of recent comparative studies use the Effective Threshold as an indicator (e.g. Hug, 2001; Powell and Vanberg, 2000; Anckar,1997).

⁸The threshold of inclusion was called the threshold of representation by Rokkan (1968).

sented by Taagepera contains a critique of the method for estimating the mid point between the thresholds of inclusion and exclusion used to calculate the Effective Threshold. Referring to earlier work (Taagepera, 1998 and 1999), Taagepera argues that instead of taking the arithmetic mean (average) of the two thresholds we should take the geometric mean which has the effect of producing significantly lower scores.

Finally, Taagepera proposes a revised formula which is launched under the name of the Nationwide Threshold of Representation. It shares definition with the Effective Threshold, but is 'simply' calculated differently: including national level factors in the formula(s) and using the geometric mean to estimate the mid point between inclusion and exclusion threshold. However, the new method of calculation overturns established conventions about the constraining properties of electoral systems. The scores assigned to the majoritarian systems fall in the same range, and often below, those of the PR-systems. To support the claim that these scores are more accurate, Taagepera evaluates them in light of empirical evidence and concludes that they provide superior estimates to those of their predecessor.

Taagepera's work therefore leaves us with a clear dilemma. Either we abandon firmly established conventions concerning the constraining effects of electoral systems in the light of the fresh evidence presented or we reject the Nationwide Threshold Representation as a measure of this property. In the following it shall be argued that at least partly the latter rather than the former route should be taken. A number of objections can namely be raised to Taagepera's arguments and method. In the following a detailed critique of the steps taken and final results reached shall therefore be offered, and new indicator of the electoral barrier presented and tested against empirical evidence. The structure of the article is the following:

Firstly, it is demonstrated that the argument presented for establishing the midpoint between the thresholds of inclusion and exclusion contains significant errors. At the root of this is a fallacy consisting in a confusion between estimating the centre of a distribution informing us of the probabilities of parties of varying sizes obtaining seats and a frequency distribution of party sizes. This confusion is closely related to the choice of a misleading benchmark, the Empirical Threshold, for testing the accuracy of the both district level and national level threshold estimates. *Secondly*, it is argued that moving the locus of threshold estimation from the district to national level not only requires inclusion of new factors in the formula to calculate the threshold values, but forces us to reconsider what the national threshold can be used to measure. It is shown that a threshold defined in terms of 'winning a seat'

captures but one aspect of the electoral barrier and not necessarily the most important one for the most common research agendas. To capture the general constraining effect across different types of systems, it is argued that we need a threshold defined in terms of attaining proportionality. Such an indicator is therefore proposed under the name of the Threshold of Proportionality. *Thirdly*, it is shown that Taagepera's method for considering the impact of the vote concentration on the national threshold value entails assuming an electoral geography which is unrealistic for most countries. A method is therefore suggested for how knowledge of the real vote concentration can be included when estimating the threshold. *Finally*, estimates of the Threshold of Proportionality are evaluated in light of empirical evidence and found to give accurate estimates of the vote share at which we can expect parties to be proportionally represented.

2 The Elusive Mid Point between the Thresholds of Inclusion and Exclusion

The method used to Estimate the Effective Threshold (T_{eff}) as well as the Nationwide Threshold of Representation (T_{nat}), proceeds by establishing the boundary conditions for the range possible outcomes. As Taagepera points out if we don't know particular value, but know that it cannot exceed a given level nor fall below another, then we can estimate it to be somewhere in the centre of this range (Taagepera, 1999). The T_{eff} , as well as the T_{nat} , is determined in this way by establishing the boundary conditions of attaining parliamentary representation, namely the threshold of inclusion (T_i) and threshold of exclusion (T_x). The T_i is defined as the minimum share of votes that could win a party at least one seat under the most favorable circumstances, while its counterpart, the T_x , is defined as the maximum share of votes a party could gain but still fail to win a seat with under the most unfavorable of circumstances. In other words, if a party obtains less than the T_i share of votes, it will certainly fail to obtain seats, while if its vote share exceeds the T_x , it cannot fail to obtain a seat⁹.

To determine the 'vote share with which parties have a 50-50 chance of winning their first seat' (defining the T_{eff} and T_{nat}) Lijphart takes the average of the T_i and the T_x , so that:

$$T_{eff} = \frac{T_i + T_x}{2} \quad (1)$$

⁹The formulas for calculating the two thresholds for different formulaic structures have been developed by different scholars and are presented in appendix I.

Taagepera argues, however, that the geometric mean of the two extremes T_i and T_x , rather than the arithmetic mean (average) should be used. The formulaic expression is thus:

$$T_{nat} = (T_i \cdot T_x)^{0.5} \quad (2)$$

While this distinction may sound very technical and inherently uninteresting, it is nonetheless important, since it strongly influences the value of the Nationwide Threshold, especially when the difference between the thresholds of inclusion and exclusion is high¹⁰. This will particularly be the case in systems with high number of districts and low district magnitudes (as particularly is the case in plurality-majoritarian systems). Taagepera advances theoretical arguments in favour of this choice and subsequently furnishes empirical evidence to back it up.

The theoretical argument made by Taagepera favouring the geometric mean over the arithmetic mean is related to assumptions concerning the distribution of the data, as he explains in some detail in a research note of an earlier date (Taagepera, 1999). It is here stated that while the arithmetic mean or average gives the centre (median) of a normal distribution, the geometric mean yields the centre of a lognormal distribution. The question would therefore seem to be which type of distribution is the more suitable one for the data we wish to analyze.

Taagepera expresses strong disapproval of the tendency among social scientists to use the normal distribution uncritically and argues that it should under certain circumstances be replaced by the lognormal. Regarding the properties of the two types of distribution, it is explained that whereas the normal distribution extends to both positive and negative infinity the lognormal distribution has its lower boundary at zero and only extends into positive infinity. Using data with a conceptual lower limit of zero (e.g. one cannot have negative vote shares) makes the lognormal distribution more conceptually appropriate. As he points out, however, when the mean of a distribution is many times larger than its standard deviation the normal distribution can be used (and is very similar to the lognormal) since the probabilities of getting the conceptually impossible positive or negative extremes of the distribution are extremely low. When this is not the case, however, the lognormal should be used instead. Finally, when seeking to estimate the centre (median) of a distribution, Taagepera claims that the geometric mean is *'always advisable when the ratio of the largest to the smallest entry is large (say, over 10) – even when the best fit deviates from the lognormal. In such a situation the arithmetic mean would basically depend on the*

¹⁰For example, the average of 2 and 50 is 26, while the geometric mean of the two is 10.

largest entries, regardless of the precise size of the smallest' (Taagepera, 1999:424). In the research note two very different examples are given of how the geometric mean yields more accurate predictions of the median. In the first, the minimum and maximum weights of mammals are presented and the geometric mean used to indicate the median. In the second example, it is the median size of states in the US which is sought predicted given our knowledge of the sizes of the largest and smallest respectively. In both cases the geometric mean gives estimates much closer to reality than the arithmetic mean.

In the article on the Nationwide Threshold, Taagepera's backs up his theoretically founded choice of mean for estimating the centre between the T_i and T_x by empirical evidence. In order to test the calculated estimates of the Nationwide Threshold, he compares them to an observed value, the Empirical Threshold (T_{em}). The T_{em} is defined as '*the vote share below and above which an equal number of parties have won and failed to win seats*' (Taagepera, 1989). It can be identified both at the national and district levels, but was initially proposed by Taagepera to compensate for the failure of theoretical threshold calculations for the national level as well as a test for the theoretical estimates¹¹. The Empirical Threshold is determined for a number of countries and it is demonstrated that the Nationwide Threshold calculated using the geometric mean between the T_i and T_x yield values closer to the Empirical Threshold than values estimated by using the arithmetic mean (Taagepera, 2002)¹².

There are, however, problems in the method suggested. Firstly, Taagepera's argument that the lognormal distribution should be used when the chance of obtaining conceptually invalid scores assuming a normal distribution is too high, appears to overlook that such problems can be overcome simply by truncating the normal distribution so that the chances of predicting non-existing values is reduced to a minimum. Secondly, it is difficult to see any inherent reason why it would be advisable to use the lognormal distribution simply because the ratio from

¹¹As Taagepera writes 'When one proceeds beyond a single district which uses a standard allocation formula (such as d'Hondt), theoretical calculations bog down. Thresholds in terms of nation-wide vote shares depend on local concentrations of these votes and cannot be calculated, unless one introduces knowledge about such geographical distribution of votes. Therefore, theoretical threshold formulas up to now have been restricted to a single district (Taagepera, 1989:106).

¹²Using the geometric mean is not the only cause of estimates close to the T_{em} , the method for including concern for the vote concentration also plays an important role. However, as will be shown in section 4, the estimates made on basis Taagepera's method for including consideration of the vote concentration, but using the arithmetic mean to estimate the mid point, fall around 25 pct. Using the geometric mean instead results in scores of just a few percentage points which is in the vicinity of the observed T_{em} .

highest to lowest observation is high. Whether the main bulk of cases fall towards the lower end of the distribution, as in the lognormal, or around the mean, as in the normal, can only be determined in one of the two following ways; Knowledge about the nature of the data can induce an expectation that the underlying distribution is congruous with a particular shape or empirical sampling can demonstrate that the cases fall in pattern that resembles a particular distribution. The range in the data set alone offers no help in this respect.

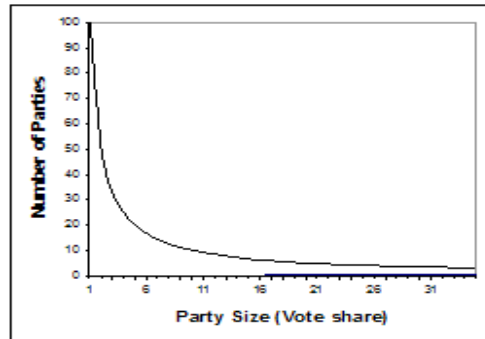
It remains to be explained, however, why the estimates using the geometrical mean of the T_i and T_x to find the T_{nat} , predicts the T_{em} much more accurately than those using the average.

2.1 Distributions, means and the nature of the data

The first method for determining which distribution is the more appropriate consists in considering the nature of the data we wish to analyze. For some types of data, the value of one case does not affect that of another. A typical example of this would be the height of people. For others, like party vote shares, they are intimately related. In defining the empirical threshold as he does, Taagepera implies that the data is the sizes of parties in terms of vote shares and the frequency of their occurrence. If we want to estimate the vote share of the party that below it and above it has an equal number of parties have won and failed to win a seat ($=T_{em}$) then we are looking for a distribution of the frequency of parties with particular vote shares. And it is clear that the vote shares of parties within a country are intimately connected simply because they are drawn from the same pool of votes: The higher the percentage obtained by one, the lower the share available to others. Within a given party system we have the logical possibility of finding a relatively high number of parties with low vote shares (say less than 5 pct.), while we logically can find very few with high vote shares (say more than 25 pct.). A graphic illustration of this point is presented below.

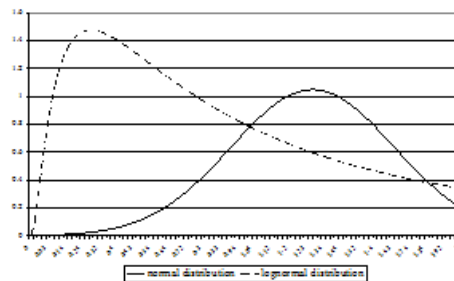
The graph shows the relationship between the sizes of parties and the maximum frequency for each particular size. The line is drawn on basis of the formula $frequency = \frac{1}{party-voteshare}$. That is, if we know the only occurring party size is 50 %, then only $\frac{1}{50\%}=2$ parties can be contained in a system, and if the only party size is 25%, then the number is $\frac{1}{25\%}=4$. Of course no party system contains parties with only one size, but it is reasonable to expect that the graph drawn according to this logic will share properties of a graph showing frequency of sizes appearing if we were to draw all possible samples a party system. It therefore provides us with an indication of the frequency with which parties of various sizes can be expected to appear - given of course that no other factors

Figure 1: Expected Frequency of Party Sizes



influence their sizes. What we can see from the graph above is then that the probability mass is strongly skewed to the left. This is also the case for the lognormal distribution as opposed to the normal distribution which is symmetric around its mean (see figure below).

Figure 2: The Normal and Lognormal Probability Density Functions

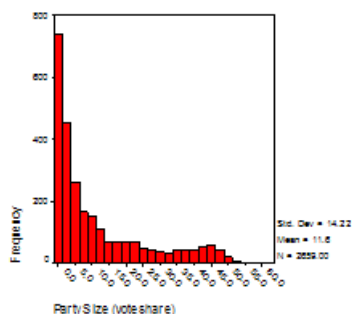


Empirical support for this theoretical argument concerning the distribution of party sizes can be found in observing the actual frequencies of parties of different sizes. The frequency with which various party sizes appear in the party systems of 20 countries covering the time period 1945-2000, encompassing 2659 cases, is presented in the histogram below¹³.

The first observation that can be made is that the shape of the histogram bears a striking similarity to the graph presented above. It is

¹³The countries the parties are taken from include: Australia, Austria, Belgium, Canada, Denmark, Finland, France, Germany, Greece (1974-2000), Ireland, Italy, Japan, Netherlands, New Zealand, Norway, Portugal (1975-2000), Spain (1977-2000), Sweden, Switzerland and United Kingdom (Source: Mackie and Rose, 1990 and 1997; European Journal of Political Research various issues 1995-2000)

Figure 3: Observed frequency of party sizes



clear that the proposition that frequency is inversely proportional to size has a firm hold in reality. The vast majority of cases lie on the left side of the distribution and indeed the mode of the distribution is the very low 0,2 pct. However, there are also a higher number of cases above 15-20 pct than we would conjecture and the line is not smoothly decreasing. This is, however, only to be expected inasmuch as the sizes of parties is not a random phenomenon. Several factors such as the advantages of size in campaigning, obtaining seats and access to government clearly provide advantages for larger parties.

This fact is, I would argue, an important part of the explanation for why superior estimates (i.e. closer to the observed T_{em}) are made using the geometric mean of the T_i and T_x . The geometric mean gives much lower estimates of the median than the average does and thus reflects that that the main bulk of the cases lie to the left of the distribution. That the median of party vote shares would be accurately predicted by the geometric mean is then explained by the fact that as is the case for parties we are dealing with distribution of unit-sizes which are intimately related inasmuch as their sum is equal to 100 pct. That the median size of mammals is predicted by taking the geometric mean of the highest and the lowest known sizes, as Taagepera uses as an example, would seem purely fortuitous, however.

Finally, it should also be observed that if we were to accept to take the geometric mean between the T_i and T_x because we assume an underlying lognormal distribution (which as noted resembles the real distribution), an methodological inconsistency would be involved. Since the extremes T_i and T_x are given by characteristics of the electoral system, we would be assuming a lognormal distribution not for the whole data set, but only for the points between the T_i and the T_x . This has the somewhat absurd consequence that if our T_i and T_x is 1 and 5 pct respectively, we expect the median party size to be 2,2 pct., whereas if the T_i and T_x

are 2 and 15 pct., we can expect 5,5 pct to be the median size. In other words the assumed median party size changes with the features of the electoral system instead of being external to it.

2.2 The Nationwide Threshold and the Empirical Threshold: Equivalent concepts?

While it is then clear that the geometric mean provides a better method than the average for predicting the median of a distribution of party sizes, the question is whether this indeed is what we are looking for. This essentially depends on whether the Empirical and the Nationwide Thresholds are equivalent concepts as Taagepera claims they are.

To answer this question it necessary to revisit the definitions of the thresholds provided by Taagepera. Both the Effective and the Nationwide Thresholds are defined as: *'the vote share, with which parties have a 50-50 chance of winning their first seat'* (Lijphart, 1994:25; Taagepera, 2002:384). The observable Empirical Threshold, on the other hand, is operationally defined as *'the vote share below and above which an equal number of parties have won and failed to win votes respectively'*(Taagepera, 2002). Taagepera claims that the procedure used to identify this vote share implies that a party has a fifty-fifty chance of winning a seat at the Empirical Threshold (Taagepera, 1989:107)¹⁴. However in the process of providing an operational definition of the threshold, the focus is subtly changed from a probability statement of the vote shares with which parties have equal chances of winning or losing a seat, to a statement of the frequency of parties with certain vote shares we can expect to observe. And this is not the same. Only if we had an equal number of parties of all sizes running at elections could we trust that the vote share above and below which an equal number of parties have succeeded and failed to enter were representative of the point of equal probabilities. But as shown above, we can ceteris paribus expect to find a higher number of smaller parties than larger ones. This is, I would argue, one part of the reason that the Empirical Threshold is not a precise test for the Nationwide Threshold scores. We simply cannot expect an equal frequency of attempts to enter for all party sizes between the T_i and the T_x but rather a sharply declining one as shown above, and

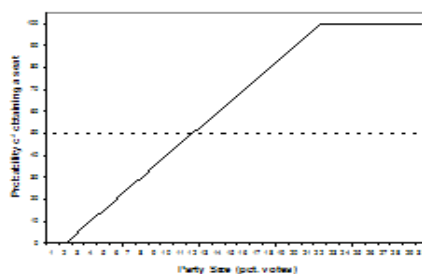
¹⁴The procedure for identifying the empirical threshold is described as follows: *'find the vote shares for all those cases where a party obtained one seat but no more. Rank these votes by increasing size. Also find the vote shares for cases where parties with non-negligible vote shares failed to win a seat, and rank these shares by decreasing size. The empirical representation threshold (T) is defined as the vote share (v) such that the number of cases where a party fails to get a seat with $v > T$ equals the number of cases where a party with $v < T$ does win a seat.'* (Taagepera, 1989:106).

therefore the observations of entry or failed entry are not representative of the chances of entering alone.

Taagepera's own analysis of district level thresholds in Finland, where it is found that '*the empirical threshold is not halfway between the exclusion and inclusion threshold but tends to be at or even below the inclusion threshold*' lends support to this hypothesis (Taagepera, 1989:113). Taagepera suggests that the phenomenon is caused by electoral alliances, but it is highly likely that we are also observing an effect caused by the higher frequency of smaller parties and would indeed find a similar pattern in systems that do not accommodate such alliances.

The question is whether rejecting the T_{em} as a litmus test for estimates of the vote share giving even odds of winning a seat implies simply reverting to taking the arithmetic mean between the T_i and the T_x . A distinction arises on this point between the district and national levels. The assumption of perfect proportionality between increase in vote share and increase in probability of winning a seat between the T_i and T_x *at the district level* remains uncontested and it is therefore still valid to use the arithmetic mean to estimate the mid point. This is then not because we are assuming that probabilities follow a normal distribution but simply because the arithmetic mean gives the centre of any distribution symmetric around its mean - in this case a linear increase in probabilities per unit increase in size between the T_i and the T_x , as shown below.

Figure 4: Size and the Probability of Winning a Seat



However, for national level thresholds the effect of the frequency distribution on the position of the T_{em} is only a small part of the story. The additional factors of geographical vote concentration and party strategy that emerge at this level intervene and provide strong explanations for why the national T_{em} is closer to T_i than the T_x . As shall be discussed below, this added complication forces us to reconsider the concept we want to measure prior to deciding on which estimation-technique to use.

2.3 Conceptions of the Electoral Barrier - 'a seat' or a proportional share?

At the national level we have strong reasons to expect a non-linear relationship between vote shares and the probability of winning a seat between the T_i and the T_x . In fact, we would expect, a sharp increase in the probability of winning a seat after the T_i and then a strongly diminishing rate of increase the closer we get to the T_x . This is caused by a combination of geographically heterogenous electorate and party strategy.

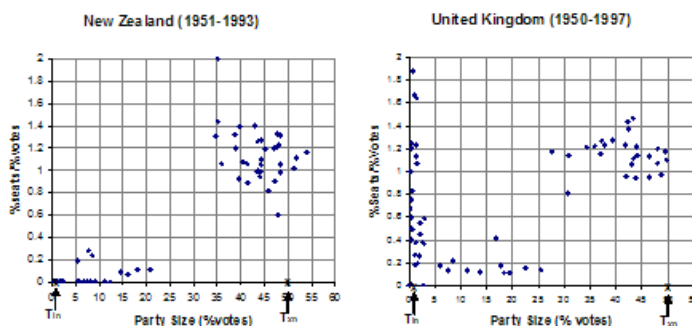
Considering the impact of the vote concentration, Taagepera explains that the threshold for winning a seat at the national level is given by the district level T_i divided by the share of votes held in one district. For instance, in a plurality single member districts system, a party might win a seat with 35 pct of the votes in a given district. But if this party's total vote were concentrated in that district and it held just 1 percent of the total electorate, then the seat can logically be obtained with only 0,35 percent of the national vote. Based on this simple observation Taagepera delivered a sharp critique of Effective Threshold as estimated by Lijphart as well as similar estimates published prior to his article¹⁵. While he claims that the thresholds estimated were never intended to predict the nationwide vote share necessary to gain a single seat (as was in fact implied by the definitions), he does point out that the way in which national legal thresholds were equated with those calculated on basis of district level factors, indicated a confusion of the two levels (Taagepera, 2002:386).

This scenario of total vote concentration in one district of course depicts an extreme situation, but the norm is that parties' vote shares vary across the national territory along with differences in social-economic, ethnic, religious etc. composition of the population. A party that has strong appeal to voters in one constituency may, as a consequence, hardly have foothold in another. Given variation in electoral structure across districts, it is obvious that smaller parties running have powerful incentives to concentrate their campaigning efforts in districts where the chances of winning representation are better. And the other side of the coin is that parties can economize on their resources by refraining from presenting candidates and/or campaign less vigorously in districts where they are unlikely to win seats anyhow. The combination of uneven electoral geography on the one hand and strategic participation and campaigning efforts on the other therefore creates a situation where

¹⁵The critique also applies to the Effective Magnitude proposed by Shugart and Taagepera (1989).

parties with very small vote shares are only slightly less likely to win a seat than much larger parties are. All that is needed to pass a one-seat-threshold is to have a relatively strong appeal to the electorate in a single district. It is therefore not strange that we find quite a number of parties in single-member district systems who in spite of tiny national vote shares nonetheless succeed in winning seats. The question is, however, how interesting the fact that parties may win a seat somewhere is for the attempt to capture the electoral barrier or in Sartori's terminology the 'strength' of the electoral system?

A glance at the electoral results and seat allocations of the Liberal party in the UK illustrate the problem of using 'one seat' as the yardstick of representation; At its absolutely worst election in the post war period, the Liberal party received a mere 2,6 pct of the national vote, but managed even so to win 1 pct of the seats. It thus appeared on that occasion to be only marginally disadvantaged by the electoral system. 20 years later, in the election of 1974, the same party won an impressive 19,6 pct of the popular vote, but this time the barrier of the electoral system was tangible: the impressive electoral support translated into a mere 2,2 pct. of the seats. And this is not an outstanding case, as the proportionality profiles of the United Kingdom and New Zealand presented below clearly demonstrate. Proportionality profiles show the advantage ratio of parties ($\%seats/\%votes$) as a function of their size ($\%votes$) (Shugart and Taagepera, 1989).



The first observation that can be made from looking at the profiles of the two countries is that all observations with a zero-score on the Seats/Votes-ratio (where no seats are obtained) are crowded close to the national T_i (as Taagepera explains: $T_{in} = \frac{T_i - district}{districts - total}$) - there is not a single observation close to the T_x or even at the midpoint between the two. In the British case there are 41 cases of failed entry of 129 observations in the period and none obtained more than 2,5 percent of the vote – and in fact 40 of the cases obtained 0,6 pct of the votes. UK would

thus appear equivalent to a country like Denmark with a legal threshold of just 2 pct in terms of its electoral barrier. In New Zealand 33 failed entries of 74 cases can be observed. 22 of the failures obtained below 5 pct, while the rest received between 5-12 pct. of the vote. Similar observations can be made about the other countries with plurality or majoritarian electoral systems. In the Canadian case, out of 116 observations there are 48 cases of failed entry in parliament and 90 pct of these obtained shares of less than 1,5 pct. The largest share of votes obtained by any party in the system that failed to obtain seats was a mere 4,5 pct. If obtaining a seat is the measure of the threshold Canada then would clearly seem to present a lower threshold than Germany's 5 pct. In Australia and France the main bulk of those not succeeding to win a seat are indeed very small parties, although here like for New Zealand we do find a few cases where parties as large as 10-13 percent fail to gain access to parliament. In the case of France for example there is but a single case of a larger party being excluded, namely the National Front in 1993 that obtained 12,7 pct of the vote, but no seats. Apart from this, however, only parties with less than 4 pct ever experienced such total exclusion. The implication of this would seem to be that the conventional proposition that proportional systems are more open to small parties than versus plurality-majoritarian systems should be fundamentally revised, as Taagepera in fact suggests. However, if we instead look at parties that are strongly disfavoured it is clear from the profiles of both New Zealand and the UK that strong negative disproportionality is experienced by parties with up to 30 percent of the vote.

We clearly miss a big part of what we want to capture with respect to the electoral barrier by using a one seat criteria as the yardstick of representation. In fact, we end up with a measure that conceives of representation in formal rather than substantial terms. While gaining access to parliament may lend some political credibility to new parties and possibly strengthen their chances for survival in the long term, it is usually not very important for national politics that small parties obtain a seat here and there. What is, instead, of overriding importance for the dynamics of party competition and the fragmentation of the party system is whether minor parties are likely to be strongly disadvantaged in the process of seat allocation or simply receive roughly proportional shares. Whether expansion beyond a few token seats is likely to be hampered or not. In countries where dominant cleavages are not geographically based, the only parties that can threaten the political establishment are those that solicit the support of a 'naturally' dispersed electorate. Parties with localized appeals might win a seat here and there, but they cannot pose a credible threat to cut into the electoral base of the major

parliamentary parties. Only if a new party were successful in changing the policy space by inserting a new territorially based cleavage would it be able to obtain a larger share of votes and benefit from systems that give better access to geographically concentrated electorates. But such transformations of the political space occur only rarely. As has been demonstrated the territorial base of cleavages is mostly quite stable at least in the longer standing democracies (Caramani, 2001).

Constructing a comparative measure of the electoral barrier requires sensitivity to how constraints are imposed across systems. In systems with national legal thresholds the electoral barrier operates like a threshold that can be scaled only by those above a certain size and then introduces no impediments to expansion beyond that size. In other systems, as those with small district magnitudes, the picture is more complex. Some parties enter with a small but concentrated share of the national vote while, some with much larger but dispersed votes fail to win seats. The barrier is neither a fixed vote share nor does passing it one district entail escaping its effects in others. If we choose winning one seat as the criteria of representation, we inevitably put systems that on average penalize smaller parties on a par with systems that over a certain size treat them the same as larger parties. A PR-system such as Sweden with a legal threshold of 4 pct where winning a seat is intrinsically bound up with being proportionally represented would, according to Taagepera's estimates, be equated with plurality New Zealand.

The only method which would allow us to measure the average barrier imposed by a system is to pitch the measure around the vote share likely to result in proportional seat shares. It means setting a higher standard for representation, but in this way the average disadvantage imposed on smaller parties could be captured. We would then aim to capture the electoral barrier understood as the *vote share with which parties escape being penalized by the electoral system*. This implies, however, producing scores that are not sensitive to the fact that smaller vote shares may give parties access to representation. The bottomline is then that whichever criteria is chosen problems of comparability will arise. The determining factor for choosing a definition for a we want a national level comparative measure of the barrier is the research question. We have to *decide what type of representational constraint we are interested in capturing*.

2.3.1 Estimating the National 'One Seat' Barrier

If we are looking for a comparative measure of *how difficult it is for localized political interests to gain access to parliament*, we could use an indicator defined in terms of attaining 'one seat' such as the Nationwide Threshold. The question of estimation is then a purely technical one.

I would, however, have some hesitations in picking up the formula proposed by Taagepera for the following reasons. Firstly, as shall be further explained below, the method suggested for considering the potential impact of vote concentration implies assuming that parties's vote shares are concentrated in half (or even less) of a countries districts. This is clearly unrealistic for most countries (see further section 3.) , and serves only to artificially deflate scores so that they fall closer to the T_{em} . Secondly, there is no reason to assume that the vote share giving even odds to win or fail to win a seat can be estimated by the geometric mean. In fact, I would argue that no model can really be developed to predict this vote share since it is *neither* random *nor* determined by factors we can claim knowledge about ex ante. We know that the lower limit is given by the T_i and that there is a clear incentive structure inducing smaller parties to focus their efforts in real or potential strongholds, but that is all. We have no theoretical tools enabling us to predict how widely parties might spread their net, how many votes they could muster or how these might be dispersed across districts.

One option in this theory void might simply be to use Taagepera's Empirical Threshold, which he initially proposed to compensate for the lack of theoretically based national measures (Taagepera, 1989). With this we do not get a barrier measure that tells us the vote share that gives even odds of winning or failing to win a seat, but one that gives us the *typical vote share of parties winning their first seat in a given system*. Identifying the T_{em} requires, however, following a rather cumbersome procedure. Furthermore it varies considerable across systems with identical electoral rules, from 0.3 in U.K. (1918-1979) to 8 pct in New Zealand (1880-1981). This is a natural consequence of the fact that the T_{em} reflects a mix of objective constraint and the fortunes of parties that participate. Finally, we can expect the T_{em} for countries that only have a short electoral history to yield unstable scores as the vote share is determined on the basis of only few 'trials'.

Perhaps a better option would then be to simply use the T_i . We know that the T_x only has negligible traction in terms of influencing the sizes at which parties win seats. The T_i is institutionally the determining factor and while we would get a vote share below the typical size of parties entering, we would have a stable measure which makes sense theoretically and furthermore is very easy to compute¹⁶. However, the effect of the actual electoral structure would be completely left out.

¹⁶It should be noted that if we, in spite of the lack of theoretical validity, were to use the formula proposed by Taagepera, we would instead get too high estimates of the typical vote share with which parties win seats as Taagepera himself points out (Taagepera, 2002).

2.3.2 Defining and Estimating a Barrier of Proportionality

If our research question were instead to require an indicator which captures the national level representational constraints then we are, as discussed, forced to consider the issue of proportionality. Without an eye to proportionality we lose the ability to compare the strength of the electoral barrier across different types of systems. I therefore propose a new national threshold measure defined as *'the vote share with which parties have a 50-50 chance of winning a share of seats proportional to their share of votes'*. This indicator can be called *'National Threshold of Proportionality'* (T_{pro}). The definition of the T_{pro} resembles the interpretation Lijphart gives of the Effective Threshold as a national measure. Lijphart writes: *'all effective thresholds except national legal thresholds are not only rough estimates but also midpoints in a range between no representation and full representation. Hence, falling short of such an effective threshold does not necessarily entail getting no representation at all – as it does when the threshold is a national legal barrier – but being substantially underrepresented'* (Lijphart, 1994: 29)¹⁷. The issue of proportional representation is clearly brought into play although it is not entirely clear what Lijphart means by 'full representation'.

The question is then how a threshold of proportionality can be estimated. To do this, I would suggest simply following the logic indicated by Lijphart in the citation above. That is, using estimated district level thresholds and relying on a process of cancelling out over- and under-representation across districts. If we use the district level measure of 50-50 probability to win or fail to win a seat, it seems reasonable to expect that the wins will evenly compensate for losses resulting in an overall proportional representation. The issue of how to deal with the impact of vote concentration on party representation persists, however. To produce a truly national level indicator, as opposed to the district level Effective Threshold, we need a method for dealing with this. Such a method shall therefore be proposed in the next section.

3 The National Threshold of Proportionality and the Vote Concentration

That the geographical vote concentration impacts the representation of small parties hardly constitutes news. As Sartori discusses the constrain-

¹⁷Lijphart also brings the issue proportionality to bear on the operational decisions necessary to make for calculation of the Effective Threshold (1994). To decide which district magnitude should be included in the threshold formula in systems using more than one tier, Lijphart argues that it is the tier (and district magnitude of that tier) that is decisive for the proportionality of the results that matters.

ing properties of electoral systems on the party system depends on the electoral structure. Discussing the circumstances under which plurality electoral systems will lead to a two party system he writes: 'a two party system is impossible if minorities are concentrated in above-plurality proportions in particular constituencies or geographical pockets' (Sartori, 1997:46). The question is just how to deal theoretically with the impact of the geographical electoral structure on our measures of the electoral barrier. Logically it can be dealt with in three different ways.

The first is the approach 'semi-consciously' taken by Lijphart (1994)¹⁸, which Taagepera criticizes. The underlying assumption of Lijphart's approach is that parties' vote shares are evenly distributed among the districts. This allows him to make estimates of the Effective Threshold on the basis of the same factors that are necessary to estimate it at the district level. The lack of realism in this assumption, however, leads Taagepera to suggest another way of dealing with the issue. The method he proposes instead entails active inclusion of ignorance of its real value. The formulaic expression used estimates the national T_i under assumption of complete concentration and the national T_x under the assumption of complete dispersion of the vote. The most and least favourable circumstances for representation. The problem in his method is, however, that when we then take the average of the two we get values of the threshold that reflect the situation where the votes of parties are typically concentrated in half its districts¹⁹. Assuming a concentration in half of the districts would be close to the truth in a country like Belgium, which in electoral terms is split down the middle, but very far off the mark for most other countries that have more geographically homogenous electorates. The result of using an approximation far from the political realities in most countries we investigate is of course that the estimates we get are strongly misleading. In fact, as will be show later, they are mostly more misleading than the scores based on the assumption of even distribution.

A third approach, which shall be shown below, entails measuring the actual vote concentration of countries and including its value in calculation of the threshold.

¹⁸In presenting the Effective Threshold, Lijphart does not draw attention to the fact that an even distribution of the vote must be assumed in order to make the step from district level thresholds to national ones. However, in discussing how to transform complex legal thresholds in two-tier systems into effective thresholds, Lijphart recognises that an extrapolation from district to national threshold requires the assumption of complete homogeneity of the vote (Lijphart, 1994: 37).

¹⁹Taking the geometric mean between the two would reflect a much stronger heterogeneity with parties typically collecting in just a a small part of the national territory.

3.1 The Factors: individual versus systemic features

Calculation of the threshold requires knowledge of factors internal and external to the electoral system. The internal factors are the district magnitude and the electoral formula, while the external factors are the number of parties and the vote concentration. Except for the electoral formula, the factors often vary across the territory. The district magnitude typically varies within the same system and frequently we find a different number of parties running in each of these. However, while these variations affect the value of electoral threshold equally for all parties running, the vote concentration is party specific and influences the threshold individually. For instance the electorates of the Labour and the Conservative Parties in the UK are highly dispersed, while those of the Scottish National Party and the Welsh Plaid Cymru are highly concentrated. The result of this is that the latter two parties face much lower National Proportional Thresholds than do Labour and the Conservatives. So if we are interested in getting an average estimate of the electoral barrier in a particular country, the individual party scores must be transformed into a systemic value taking the relative importance of the respective party thresholds into account. That is party competition on the left-right cleavage typically cuts across electoral districts, so that parties competing on such platforms face highly dispersed electorates. Other cleavages like linguistic or religious and certainly regional cleavages tend to have geographically concentrated electorates. If we want a systemic threshold value, we therefore need to take into account the relative electoral strength of the parties, and thus the cleavages, and weight the scores accordingly.

3.2 The National Proportional Threshold with the Vote Concentration

The question is how real values of the vote concentration can be included in the formula for calculating the T_{pro} . To explain how this can be done it would be useful to briefly review how the vote concentration exerts an influence on its value. On the one hand, if the vote of a party is completely homogenous, it means that both T_i and T_x can simply be calculated without consideration of the number of districts. If, on the other hand, the vote of a party is completely concentrated in one district, both will be lowered in proportion to the number of districts in the system as Taagepera has explained (Taagepera, 2002). Let the national level and district level thresholds be written T_{in} and T_{xn} and T_{id} and T_{xd} respectively and let D_{es} signify the number of districts in a

system. Then the relationship can be expressed as follows:

$$T_{in} = \frac{T_{id}}{D_{es}} \quad (3)$$

$$T_{xn} = \frac{T_{xd}}{D_{es}} \quad (4)$$

What we need therefore is a measure of the vote concentration, which can replace the number of districts (D_{es}) in the denominator in such a way that when the vote moves toward total dispersion it comes close to 1 and when it moves towards total concentration it equals the number of districts used in the particular electoral system we want to estimate the threshold for. Among the existing measures of national vote concentrations, however, none can be found which fulfils these criteria (see review of these in Caramani, 2002). So a different approach to measuring vote concentration has to be taken. I suggest the following steps are taken;

First, the Herfindalh-Hirshman concentration index (HH) is applied to distribution of parties' vote shares across districts. This means taking the number of votes a party has obtained in a given district and divide it by its total vote. Each district vote fraction thus obtained is then squared for all districts. Let v be the fraction of a party's total vote and i be each district, the formula can be stated as follows:

$$HH = \sum (v_i)^2 \quad (5)$$

The second step is then to estimate the number of districts that the party's vote is mainly concentrated in. This can be found by taking the inverse of the HH. The measure thus found can be called the effective number of districts (D_{eff}) since it is similar to the effective number of parties (Laakso and Taagepera, 1979).

$$D_{eff} = \frac{1}{\sum (v_i)^2} \quad (6)$$

D_{eff} gives us a measure of how many districts a given party's vote is dispersed over. If a party obtains 100% of its votes in one district only, we get the value 1. On the other hand, if its votes are equally distributed on all districts, we get the number of districts used in the system. This score then gives us the opposite of what we need. But if we take the number of districts in the system and divide it by the D_{eff} we will get a measure that can be used in the manner suggested above. This measure

can be called the effective vote concentration, V_{eff} , and the formula for calculating is written as follows:

$$V_{eff} = \frac{D_{es}}{D_{eff}} \quad (7)$$

It should be observed that V_{eff} as a measure of concentration is equivalent to the variance. Although the scores obtained fall within widely different ranges, V_{eff} and S^2 scores were found to correlate perfectly (pearson's $r = 1$). The perfect correlation is explained by the fact that V_{eff} is in fact equal to the variance plus a constant²⁰

To get a score for each country that reflects its electoral structure, as discussed above, a weighted V_{eff} value is calculated. This can be done by taking the sum of the D_{eff} for each party multiplied by the party's share of the total vote. The number of districts is then divided by this system- D_{eff} to get the system- V_{eff} . This has the advantage over a similar weighting directly of the V_{eff} scores of each party, that it prevents very small parties with very high V_{eff} scores (in systems with high number of districts) from exerting undue influence on the systemic value.

$$SystemV_{eff} = \frac{D_{es}}{\sum D_{eff_i} \cdot v_i} \quad (8)$$

The formula for the National Proportional Threshold using the systemic Effective vote concentration and the district threshold of inclusion and exclusion can then be expressed as follows:

$$T_{pro} = \frac{T_{id} + T_{xd}}{SystemV_{eff} \cdot 2} \quad (9)$$

The appropriate formulas for calculating the thresholds of inclusion and exclusion can then be inserted. Whether a standard formula is used to calculate the district T_i and T_x for all systems as is done by Lijphart and Taagepera, or whether one lets the formula vary according the allocation rules used and the number of parties running is then up to the user (the formulas for different formulaic structures and the short cut formulas are listed in appendix I.).

3.3 The effect of vote concentration on the threshold – some further considerations

In order to avoid a misinterpretation of the T_{pro} , it is necessary to consider a bit more carefully how V_{eff} interacts with party size in influencing its value. While the argument presented by Taagepera regarding the

²⁰The relationship between the V_{eff} and S^2 can be expressed as follows: $S^2 = V_{eff} - \bar{x}^2$

lowering of the threshold according to increases in vote concentration holds for the example given, there is not a straightforward relationship between the two.

Firstly, it should be noted that V_{eff} scores have a natural upper limit for parties depending on their size. Given that all constituencies have the same size, a party obtaining 30 pct cannot exceed a V_{eff} score of 3.3, while one of 10 percent cannot exceed 10, since this would signify that the total vote is concentrated in 30 and 10 pct of all the districts respectively. Very high V_{eff} scores are consequently reserved for very small parties, whose vote can actually be contained in just one or a few districts.

Secondly, one should be aware that a higher V_{eff} for a party does not always translate into more advantageous seat allocation²¹. The optimal vote concentration for a given party, that is the vote concentration that will maximize its seat share, depends on the share of votes obtained. To illustrate how this works a graph is presented which shows the V_{eff} scores that would lead to the maximum share of seats possible for a party running in a system with 100 single member districts and with only two parties running.

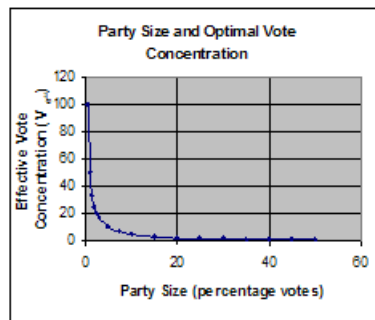


Figure 5: Vote Concentration and Maximizing Representation

A party winning just over 25 pct of the vote could potentially win 50 pct of the seats if its votes were dispersed evenly over 50 districts and thus had a $V_{eff} = 2$, while a party of just over 1 pct of the vote would need a V_{eff} of 50 to obtain a maximum of 2 seats.

Moreover, the effect the vote concentration not only varies according to size, but each unit increase in V_{eff} for a party of the same size not only has a non-linear effect on its seat share, but can have directly

²¹The tendency for larger parties to be less optimally represented due to strong concentration of votes has already been pointed out (Taylor and Johnston, 1979; Grofman et al., 1997)

opposite effects. A party receiving 25 percent of the vote running the same system as described above would get a proportional seat share if its entire vote were contained in just 25 of its districts ($V_{eff}=4$). If its votes are dispersed completely evenly in half the districts ($V_{eff}=2$) it will increase its seat share to 50 pct, but a further decrease in vote concentration leading it to receive the same share of votes in each district ($V_{eff}=1$), would have the opposite effect, since it shall receive no seats at all. There are many combinations possible of vote shares the different districts and the same vote concentration score for a party may cover a very fortunate situation where it just gains enough votes to win a seat in many districts and loses just a few or a situation where it comes close in many districts but only wins a few. The V_{eff} measure cannot capture such situations accurately. As is the case for the effective number of parties, the same V_{eff} score can be produced by several vote concentration scenarios and some of these may be more opportune for the party in question than others²².

A good illustration of the dilemma is found in the New Zealand election of 1990. The Labour party gained 47,8 pct of the total vote but just 28,9 pct of the seats, while the National Credit Party with just 35,1 pct of the vote took 70,1 pct of the seats. The V_{eff} scores of the two parties do not help us to explain this blatant misallocation of seats, since the values were very similar: Labour's V_{eff} being 1,09 while National Party's was 1,08. At the following election in 1993, the results were far more proportional; The National Party obtained the same share of votes but this time only 50,5 pct of the seats, while Labour with only 34,7 obtained 45,5 pct of the seats. Again, however, the V_{eff} scores for that election were closely similar: 1,12 for Labour and 1,14 for National Credit. The example clearly illustrates that the V_{eff} cannot be used to predict the advantage ratio (%seats/%votes) for larger parties, while it offers much more secure predictions when we are dealing with minor and very small parties. This is clear when comparing the Country Party of Australia or Bloc Quebecois of Canada with the Liberal Party in UK. The two former have V_{eff} scores in the range 4,5-6,5 and vote shares ranging from 5-15 pct typically been overrepresented, while the Liberal Party with similar vote shares but V_{eff} scores typically between 1,5-2,5 has been strongly disadvantaged under the same electoral rules.

The examples serve to illustrate that the vote concentration measure must be used with some care in connection with threshold calculations.

²²Dunleavy and Boucek demonstrate that different combinations of party numbers and party vote shares may lead to similar effective number of parties scores. As a result they recommend that the index is interpreted carefully (Dunleavy and Boucek, 2003).

However, since the threshold values primarily serve the purpose of predicting the openness of the political system to minor and parties, we are concerned with predicting what the chances for these to attain fair representation. For such parties running in systems with small magnitudes facing a heterogenous electoral structure will facilitate easier access to representation than will a homogenous one. In other words these systems will offer space for a more fragmented political representation.

4 The Estimates of the National Proportional Threshold

To show how T_{pro} compares to the T_{eff} of Lijphart and T_{nat} of Taagepera, all three types of estimates for 17 countries in the period 1950-2000 are presented in the table below (Taagepera, 2002; Lijphart, 1994). Unlike the T_{pro} , where the original formulas are used to calculate the values, the T_{nat} and T_{eff} are estimated on the basis 'short-cut' formulas in which both formulaic structure and the number of parties are disregarded. One should thus be aware that the only feature that causes the variance across systems for these two types of scores is the district magnitude. Furthermore the T_{eff} for countries using plurality-majoritarian systems are not based on calculation, but simply represents 'guesstimates' assigned by Lijphart, since the formula used was believed by him to yield unrealistically high scores (Lijphart, 1994). Including the vote concentration and the number of parties in calculated the T_{pro} solves this problem. In a nutshell, Lijphart's problem in using the number of parties in plurality systems was essentially that including all running would for some countries deflate the scores artificially. Counting parties in proportion to the share of districts they run in and using a cut-off point of 2 pct, however, solves this problem. If a party presents candidates in almost all districts, it counts as one, but if it only runs in half it only counts as half. Parties running just in a mere fraction of the districts are simply omitted. This is a logical consequence of the fact that parties only affect the threshold values in the districts where they are present and collect some minimum of votes.

As can be seen from the table below the three threshold scores do differ significantly. Taagepera's T_{nat} -scores stand out by being consistently lower and with a much narrower range than both the T_{pro} and T_{eff} scores and the ranking of the countries is also significantly different. Both T_{pro} and T_{eff} places the plurality-majoritarian systems as the ones with the highest thresholds, but T_{nat} scores rank these very differently. None of the countries are assigned a threshold higher Germany with its

Country	Veff	Tpro	Tnat	Teff
Australia (1951-1998)	1.41	37.1	3.6	35
Austria (1953-1966) #	1.17	3.8	3	8.5
Austria (1970-1990) #	1.05	1	3.3	2.6
Austria (1994-1999) #	1.09	4*	4*	4*
Belgium (1950-1991) #	1.6	2.5	0.8	4.8
Belgium (1995-1999) #	1.96	3.1	1.5	5.2
Canada (1953-1997)	1.41	27.9	2.4	35
Denmark (1950) #	1.1	5	0.8	5.5
Denmark (1953-1998)	1.1	2*	2*	2*
Finland (1951-1999)	1.3	4.7	1.3	5.4
France (1958-97(excl.86)	1.57	20	1.7	35
France (1986)	1.15	13.6	1.1	11.7
Germany (1953-1998)	1.1	5*	5*	5*
Ireland (1951-1997)	1.19	11.3	2.6	17.2
Italy (1953-1992)	1.13	2.4	0.4	2
Italy (1994-1996)	1.32	25.7	28.2	1.4
New Zealand (1951-1993)	1.16	32	4.2	35
New Zealand (1996-1999)	-	5*	5*	5*
Norway (1953-1981)	1.14	9.6	1.9	8.9
Norway (1985-1997)	1.14	4*	4*	4*
Portugal (1980-1999)	1.18	6.6	1.3	5.7
Spain (1982-1996)	1.23	9.14	1.4	10.2
Sweden (1952-1968)	1.06	6.1	1.5	8.4
Sweden (1970-1998)	1.06	4*	4*	4*
Switzerland (1951-1999)	1.86	5.2	1.7	8.5
United Kingdom (1950-1997)	1.24	31.2	1.5	35

Figure 6: * Signifies that the value is a legal threshold. #Signifies that more than one tier is used for seat allocations (see appendix for explanation of operational decisions in these cases). Information on the electoral systems and number of parties was obtained from Caramani (2000), Lijphart (1994) and Mackie and Rose (1990;1997). District level electoral results for the countries of Western Europe is from Caramani (2000), Jack Vowels kindly provided files on New Zealand, Australia was obtained from Adam Carr's Election Archive (www.adam-carr.net) and Canada from the Library of the Canadian Parliament (www.parl.gc.ca). See appendix for explanation of calculation of scores for the individual countries.

5 pct. legal threshold and U.K. and majoritarian-system France have identical values to Switzerland and Sweden before 1968. The range of calculated (excl. legal thresholds) values is from 0.4 to 4.2, while they for the two other scores vary from around 1 to 35-40. The T_{nat} scores thus makes the systems appear much more similar.

In general the T_{pro} scores tend to lend credibility to the 35 pct. arbitrarily assigned by Lijphart to the single-member district systems since it in many cases falls close to this value. The T_{pro} , however, differentiates between the countries in this group. The value for single-member-district France is only slightly higher than half of Australia's. This pronounced difference, in spite of similar electoral rules, is explained by the higher number of parties as well as by the relatively high V_{eff} values for France. The data for calculating the latter value are, however, imperfect and the difference may be smaller in reality (see appendix II for notes on the calculation).

It is clear that it matters which method is used to estimate the barrier, but it remains to be evaluated against empirical evidence which are the more accurate measures of the barrier. This question shall be addressed in the following section.

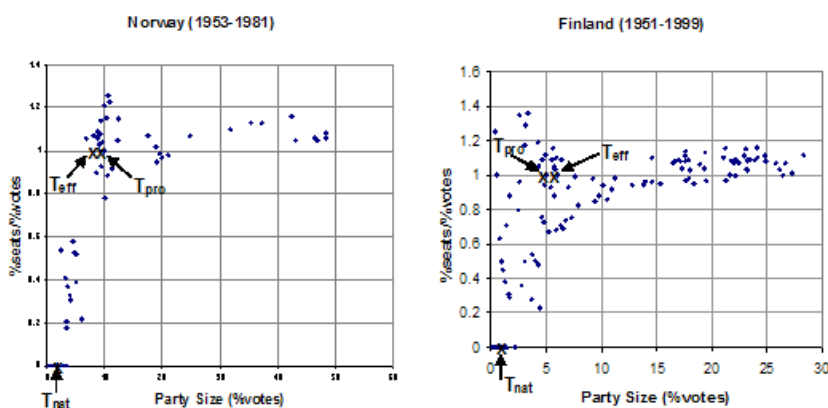
4.1 Testing the accuracy of the estimates

The question is how to test the accuracy of the estimates. A simple, albeit somewhat impressionistic method, is by visual representation. Producing proportionality profiles that show the relationship between the share of votes obtained and the advantage ratio ($\%seats/\%votes$), gives us a method for evaluating whether the T-scores give fair estimates of the size with which parties obtain proportional representation. Proportionality Profiles were in fact developed by Shugart and Taagepera to estimate the Break-Even point, which can be interpreted as the empirical counterpart of the T_{pro} . The break-even point they suggested should be identified as the point, where a line drawn through the points of the proportionality profile crosses the perfect proportionality line, that is where the advantage ratio = 1 (Shugart and Taagepera, 1989).

The proportionality profiles of 5 plurality and 4 low magnitude PR-systems are therefore presented below. These countries were selected because the three types T-scores generally differ more for them than for other systems and/or since these have a longer history with stable electoral rules and relatively stable vote concentrations allowing us to interpret the cases as a result of the interaction of the two conditions. The T_{nat} scores are positioned on the x-axis reflecting its definition as measuring the point of entry, while the T_{pro} is put on the perfect proportionality line as it seeks to identify where parties have even odds

of obtaining a share of seats proportional to their votes. Finally T_{eff} scores are also put on this line although its definition places it on the x-axis, its interpretation by Lijphart resembles the T_{pro} as do the scores. For the PR-systems the observations fall somewhat more densely and in the pattern of a line than they do for the plurality-majoritarian systems, which makes it is easier for the former to evaluate how well the calculated values captures the mechanics of the electoral systems.

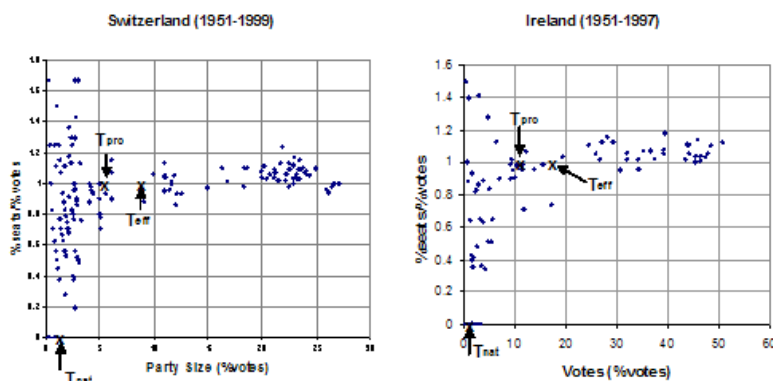
Figure 7: Proportionality Profiles (PR-systems)



As can be seen from the figures below, representing Norway and Finland, the T_{nat} score is much lower than the two others and also clearly closer to the point where parties are likely to win a seat. The T_{pro} and T_{eff} scores, on the other hand, appear to capture the point where parties typically obtain proportional representation. There is not way of assessing which value is better than the other, since they fall very close and the points are scattered obscuring the point where a line would cross the perfect proportionality line. This suggests that it is not necessary to take the vote concentration into account when we are calculating thresholds for countries characterized by homogenous electorates. The difference in the calculated values is so small that it can safely be ignored. Using Taagepera's method for considering the vote concentration would in these cases take us further from the truth rather than closer to it.

In order to assess whether the T_{pro} represents an improvement over the T_{eff} scores for countries with more heterogenous electorates, two countries with very different scores, are presented below²³.

Figure 8: Proportionality Profiles (PR-systems)



First, in the case of Switzerland the inclusion of the vote concentration leads to a lowering the T_{pro} compared to the T_{eff} , due to its heterogenous electorate²⁴. And in comparing the two scores, the T_{pro} appears to give a more accurate estimate than the T_{eff} of where proportional results are typically achieved. In the case of Ireland, the T_{pro} also appears to be more precise than the T_{eff} ²⁵.

But as can be seen in the table above, for many systems with quite homogenous electoral structures the difference between the two types of scores is not very large. Only when the electorates are more heterogenous, does it make a clear difference and the impact is potentially largest in single-member districts systems. However, the proportionality profiles of the single member district systems (SMD) unfortunately do not yield as clear a picture as those for proportional systems.

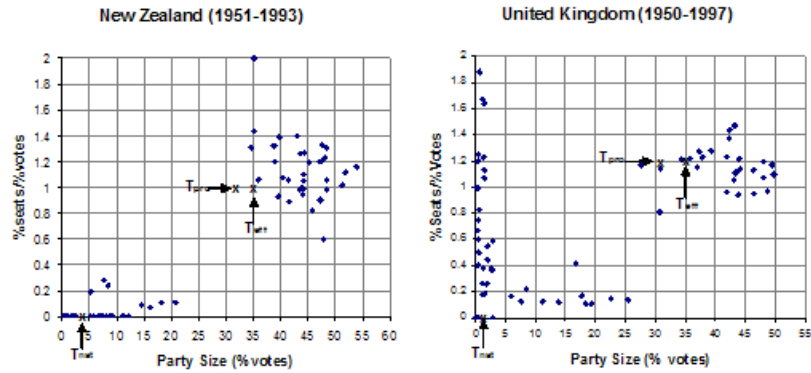
Again it can be observed that T_{nat} scores generally captures the typical entry point, although in the case of France, New Zealand and Australia (see below) cases of failed entry with much higher vote shares can be observed as discussed earlier. It is also clear that if we want

²³For both Switzerland and Ireland a few tiny parties obtaining very high advantage ratios were removed to ensure a good graphical representation.

²⁴This is also the case for Belgium, but since its V_{eff} scores have almost doubled from 1.15 to 2.03 in the post-war period, and it had an electoral reform in the early 90s, it is difficult to produce a proportionality profile with sufficient cases where the parties run under similar conditions.

²⁵In the case of Ireland, the T_{pro} is lower than the T_{eff} mainly, but not only, as a result of considering the electoral structure (V_{eff}). Also important is that different district level formulas were used. The T_{pro} is based on the T_x of STV-systems, but the T_i of Hare. The T_i of STV equal 0, which reflects a highly improbable situation.

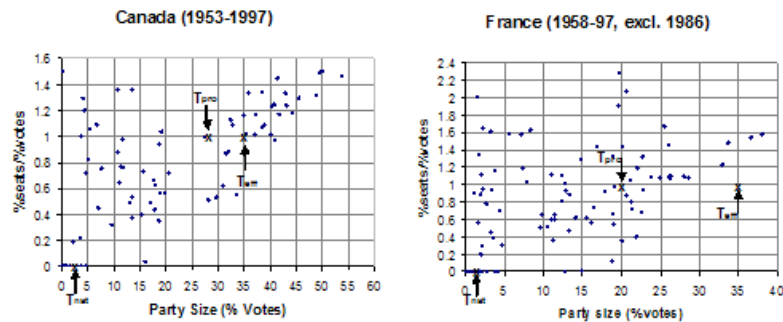
Figure 9: Proportionality Profiles (SMD)



to estimate the barrier small to medium sized parties face the T_{nat} is misleading. Comparing for example the profiles of the U.K. and Finland above, who have practically the same T_{nat} score (1.3 and 1.4), it is obvious that the identical scores do not give much information about the widely diverging mechanics of inclusion and exclusion so evident in the diagrams.

The T_{pro} values for both countries above fall quite close to the T_{eff} -guesstimates and thus lends credibility to the values assigned by Lijphart. Looking at the profiles above, however, it is impossible to see whether precision is gained from the theoretical calculation.

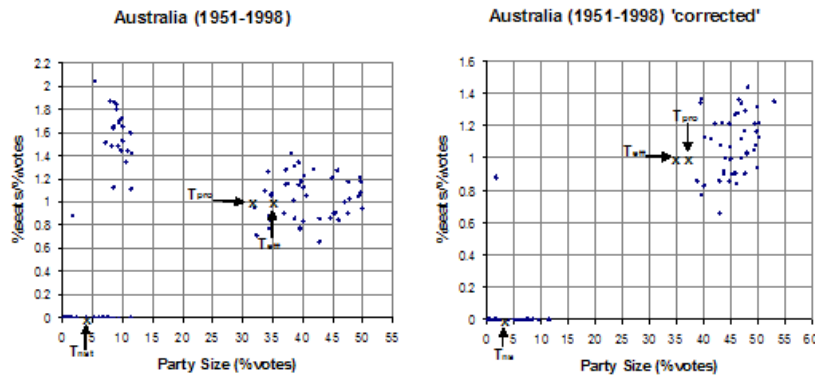
Figure 10: Proportionality Profiles (SMD)



Among the single-member-district systems analyzed here, France and Canada have the most geographically heterogenous electorates. In the case of Canada there has been an increase in vote concentration in the period examined (with the success of the Quebec party in early 90s),

while it has decreased gradually in French case (It should be noted, however, that the electoral data available to calculate the V_{eff} for France was far from perfect - see further appendix II). The conditions underlying the scores have therefore not been completely stable. It is, however, interesting to have a closer look at how the T_{pro} scores represent the cases here since they differ more from the T_{eff} . For France, taking the vote concentration into account clearly helps give a better estimate of the electoral barrier. Compared to the T_{eff} , it lies much more in the centre of where the cases fall around the perfect proportionality line. For Canada, it is more difficult to evaluate which of the two Threshold estimates is the more accurate. It is also abundantly clear that the T_{pro} in the plots does not predict where the cases fall with any great accuracy. In France particularly parties obtaining vote shares close to the T_{pro} have received both very low and very high advantage ratios. As mentioned earlier, the Threshold in this type of systems can only seek to capture an average value.

Figure 11: Proportionality Profiles (SMD)



Finally, for Australia two plots were produced; One where the Country and Liberal Parties are regarded as separate and one where they are regarded as one. Since the parties hardly compete electorally (rarely present candidates in the same districts) nor in the parliamentary arena (they follow each other in government and opposition), there are good reasons for seeing it as one party when analyzing the impact of electoral rules on party competition. The score of 37.1 is therefore the one most suited to capture the barrier for Australia. The plots illustrate that the T_{pro} is lower when the two parties are regarded as separate, but the lack of parties obtaining 10-30 percent of the votes, makes it difficult to see

whether the scores represent good estimates. A further analysis of the single-member-districts is therefore presented below.

4.2 Single-member-district systems

A method for circumventing the ‘visual-method’ and retain an empirically based evaluation of the accuracy of the T-values, would be to apply a logistic regression model. Creating dichotomous dependent variable, so that all parties being proportionally or over-represented in the process of seat allocation (advantage ratio ≤ 1) fall into the one category (category P+) and all the under-represented (advantage ratio < 1) into another (category P-). The point where there is a 50/50 chance of falling into one or the other categories would give us the empirical equivalent of the T_{pro} . Logistic regression models exactly seek to predict the probabilities of certain outcomes on basis of variation in the independent variable(s).

In addition to the average T_{pro} for the period 1950-2000, a row with $T_{pro-ign}$ - scores made under assumption of ignorance of the real value of the vote concentration is also included. The method proposed by Taagepera is followed only that the mean between the T_i and T_x is used ($T_{pro-ign} = \left(\frac{T_i}{D_{es}} + T_x \right) / 2$). Since it is somewhat cumbersome to calculate the vote concentration, it would be useful to know whether much is gained by this in terms of precision. The Logistic Regression scores reflect the size, calculated on basis of the coefficients reported in the model, at which there is a 50-50 chance of being in the P+ or P- category²⁶. It should of course be noted that this defines the T_{pro} , but not strictly speaking the T_{eff} , although the latter is interpreted in a way similar to the T_{pro} by Lijphart, as discussed earlier.

As can be seen from the table, the T_{pro} scores closely follow the values predicted by the regression model and are mostly closer than $T_{pro-ign}$. The only exception to a good fit was the United Kingdom where the regression model estimate was much lower than the expected. Suspecting that the high incidence of very small parties with high vote concentrations and advantage ratios above 1 could be the reason for this, a second regression was run where all parties obtaining less than 2 pct of the vote were excluded. As a result of excluding the very small parties the value

²⁶For all countries the constant and the b-coefficient used to predict the threshold are significant at the 1 pct. level, except for Australia where it is at the 5 pct. Level. In terms of how much the models explain, the Nagelkerke R squares vary from a high 71,9 pct for Australia-2, to a low 33,5 in UK and 33,5 in France. It is quite clear that larger number of unexplained cases is related to the impact of the high vote concentration of smaller parties that give then a higher advantage ratio than the model would predict. Examining the residuals, however, also reveals that larger parties whose advantage ratio fall just below 1 has the same effect on the model fit – there are, however, much fewer such cases in the countries examined here.

Measuring the Electoral Barrier

Country	Australia	Canada	France	N.Z.	U.K.
Log.Reg.Est.	29 (41.9*)	26.3	17.9	36.6	8.2(32.8#)
T _{pro}	32.13(37.1*)	27.3	20	32	31.2
T _{pro-ign}	25.2	25.1	25	25	25

Figure 12: * The scores in brackets cover the situation where the Liberal and the Country party are regarded as one party. Since the two parties neither compete in the electoral arena (they do not field candidates in the same districts) nor in the parliamentary-governmental arena (they are always in government or opposition together), there are good grounds for seeing them as one when analyzing the impact of electoral rules on party competition. #U.K. contains a very high number of very small regional parties that succeed in obtaining a seat share higher than their vote share. Since the logistics estimate of party size was only 8,19 compared to the theoretical estimate of 31.2 pct, the model was applied again excluding all parties obtaining less than 2 pct of the national vote and the score is written in brackets.

predicted by the model came much closer to the theoretical estimate. While France and Canada both have many parties with higher vote concentrations than the systemic value that achieve high advantage ratios, several of these are medium sized and they thus lower the calculated T_{pro} values as well as the regression estimates resulting in more congruent figures. In U.K. a large number of parties with high advantage ratios are tiny and therefore have no effect on the calculated T_{pro} , while they strongly influence the logistics regression estimates. The accuracy of the scores by considering the move to the national level and including the vote concentration has therefore been distinctly improved. Where the other indicators are constant across the different systems, the T_{pro} values vary with the national context and they are therefore better indicators of how a particular system imposes constraints within a particular context. The congruence between the logistic regression estimates and the T_{pro} values confirms that the electoral vote concentration matters to the representational conditions for parties.

The very encouraging results with respect to obtaining estimates close to the calculated values should, however, be interpreted with some care. Firstly, in constructing the dependent variable variation is lost. The model does not distinguish between advantage ratios of 0 and of 0.99, and this difference is very important for evaluating the impact of electoral systems. To investigate the importance of keeping this variation, a linear regression model was applied now using the A-ratio as the dependent variable. The point of intersect with the perfect proportionality line ($A=1$) was calculated yielding scores within very close range from those obtained by logistic regression²⁷. These scores of course do

²⁷The following values for x (party size) were predicted for the intersect with the

not represent probabilities as do the values predicted on basis of the logistics regression model. Only in the case of U.K. is the difference between the two types of scores palpable; the regression line changes only little the parties below 2 pct. are excluded and the linear regression model is thus less sensitive to the presence the many small parties with high advantage ratios, but the adj. R^2 is almost doubled from 29,9 to 57,8 pct when the small parties are removed. The explained variance as expressed by the adj. R^2 follows the same pattern as Nagelkerk R^2 of the logistics model. It varies from a high 92,2 for Australia to a low 35 pct. in France. Secondly, it is clear just from a simply viewing of the scatterplots that statistical readings based on this data set must be taken with a grain of salt. Neither independent nor dependent variable complies with the requirement of normal distribution of the data, and there are clear problems of heteroscedasticity. The latter is clearly due to the different vote concentration of the parties causing much larger variation in advantage ratio scores towards the lower end of the size range. Entering V_{eff} scores for the parties into the model would hardly do much good, however, since the effect of vote concentration is far from linear as discussed above. Size, on the other hand, can be expected to have an effect on the advantage ratio that is linear. In other words, it is a set of data which is difficult to analyze with statistical techniques.

5 Conclusion

The aptness of an indicator naturally hinges on the effect we are trying to capture. In the literature on electoral systems, however, there is a tendency to dwell precious little on what the indicators proposed actually show. Initially, the available indicators of the electoral barrier (or strength of the electoral system) were therefore briefly reviewed and it was argued that the 'Threshold-approach' has clear advantages over both disproportionality measures and simple classification schemes. These advantages are related to capturing variation, ease of interpretation as well as accuracy and consistency of the scores across a variety of systems. However, problems emerge when we apply the threshold(s) measures developed so far to capture the barrier at the national as opposed to the district level. Solutions to these problems were sought both at the level of definition and operational estimation. A number of conclusion can then be drawn from the analysis of the issues presented.

It was argued that contrary to the district level no single best threshold-measure can be identified allowing us to compare the electoral barrier

perfect proportionality line: Australia (41,44), Canada (31,7), France (19,75), New Zealand (36,6), UK (37,18) and UK-2 (38,8).

across systems for the national level. Rather we are faced with a choice between two alternative conceptions of representation: a measure of what it would take for a party to win just one seat in parliament on the one hand or a measure showing the barrier against proportional representation of parties below a certain size on the other. The two measures give us very different rank orderings of systems and therefore cannot be used inter-changeably. If we, for instance, wish to investigate how difficult it is for geographically concentrated interest to obtain political representation, we should clearly opt for the first definition. And in that case we would be well advised to use either Taagepera's empirical threshold or Rokkan's threshold of inclusion estimated for the national level. However, for the standard questions such as the degree of constraints put on party system fragmentation, the strength of the pressure parties are under to avoid splits or the extent to which parliamentary parties are shielded from new competition etc. we need to use the Threshold of Proportionality as proposed here. The method of estimation was found to give accurate scores, but it is important to emphasize that the values we get represent an average barrier effect. This means that some parties may obtain proportionality with smaller vote shares, while others fail to achieve it with larger shares.

Finally, and with relevance for practical research, it was shown that for countries which we would expect to have reasonable homogenous electorates, we are better off simply assuming dispersion than following Taagepera's method entailing estimation of the T_i and T_x under assumptions of complete concentration and dispersion respectively. So if we lack the data (district level electoral results) enabling us to calculate effective vote concentration, the district level measure can be used for such countries as it will only lead to a moderate overstatement of the proportional threshold.

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A Appendix

A.1 Formulas for calculation of district level thresholds

Below the formulas for calculating the T_i and T_x at the district level developed by Lijphart and Gibberd (1977) as well as the for plurality rule (Rae, Hanby, and Loosemore 1971) and STV (Gallagher, 1992:486) for are given (source of overview: Hug, 2001:177). The following abbreviations are used: m = district magnitude, n = number of parties, D_{es} = number of districts

	Plurality	d'Hondt	Pure Saint-Laguë	Modified Saint-Laguë	Largest Remainder	STV
T_x						
If $n-1 \geq m$	1/2	1/m+1	1/m+1	1/m+1	1/m+1	1/m+1
If $m/2 \leq n-1 \leq m$	1/2	1/m+1	1/2m-n+2	1.4/1.6m-2n+1.6	n-1/mn	1/m+1
If $n-1 \leq m/2$	1/2	1/m+1	1/2m-n+2	1.4/2m+n+2.4	n-1/mn	1/m+1
T_i	1/n	1/m+n-1	1/2m+n-2	1.4/2m+1.4-2.4	1/mn	0

The 'short-cut' formulas suggested by Lijphart, which omits the number of parties, are: $T_{id} = \frac{1}{2m}$ and $T_{xd} = \frac{1}{(m+1)}$

A.2 Notes on the Calculation of the T_{pro} and the V_{eff}

General. For PR-systems the vote concentration is calculated on basis of the percentage votes obtained by the parties within each district. This is done to correct for the large differences in the sizes of the districts (both in terms of magnitude and number of voters) that are normal in these systems. If the share of each party's total vote obtained within each district were used instead, the vote concentration would appear higher than it is and reduce the threshold artificially. This correction is not made for plurality systems, where instead the share of a party's total vote obtained in each district is used. Because here there is always the same number, namely one, candidate up for election in these systems, the variation in the electoral sizes of districts must be actively included as it affects the vote share needed to obtain seats.

1. Australia's V_{eff} score was calculated for only the 2 election years of 1955 and 1977, since district level data was not available in machine readable format and therefore had to be entered manually. However, the

V_{eff} scores for the Senate (which use the states as electoral districts) were stable in the period, and it is therefore reasonable to assume the same for the House. The values calculated with the apparent party system were 1.54 and 1.31 and an average of 1.42 was used. When the Country and Liberal parties are regarded as one, the scores were 1,17 and 1.27 and the average of 1.22 was used in the calculations.

2. The operational decisions underlying the T_{pro} of Austria (before the legal threshold was introduced) differ from those taken by Lijphart. Since one seat has to be obtained in the 1st tier to get access to the 2nd tier allocations, the threshold was calculated using the largest district magnitude in the primary tier since this is where we would expect smaller parties to be. Lijphart argued that the 2nd is decisive for the proportionality of the outcomes uses the 2nd, which is of course true, but he overlooks that winning a seat in the primary is needed to gain access to allocations here.

3. For Belgium until 1994, the 2nd tier district magnitude is used (following Lijphart, 1994)

4. Canada's V_{eff} was only calculated for the two elections of 1974 (1,34) and 1997 (1,55) since machine readable data were not available. The period until the electoral success of the Quebec party in 1994 was therefore set at 1,4 and the two elections of 1994 and 1997 at 1,55.

5. The V_{eff} scores for France are calculated on basis of electoral results aggregated over 94-99 districts, since they are not available by the electoral districts that have been used. The scores displayed the largest change among the countries examined here. Before the 1967 election it is above 3 and after it drops to below 2 and falls gradually until 1997, where the value is 1,6. Comparing to scores obtained when using scores correcting for differences in size between electoral units, it became clear that the aggregated units simply contained very different shares of the electorate. The score of 1.6. for the 1997 election, which was more congruent with the corrected scores, was therefore extended to the whole period. When the difference in size (number of voters) between the units is corrected for, as was done for 1986 elections with PR-system, the vote concentration is much lower (1.15).

6. For Germany the V_{eff} reported is if the CSU and CDU are regarded as one party, if they are not the V_{eff} score is an average of 1,3.

7. For Ireland the formula of T_i for Hare was used instead of that for STV – the latter is 0 and getting close to this seems highly unrealistic.

8. The scores for Italy after the introduction of the mixed system are calculated in the following way; A T_{pro} -value is calculated for the PR-system and one for the Plurality-system: each with its specific formula, district magnitude and number of parties. The scores are then weighted

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by the percentage of seats allocated in each (25 and 75 pct. respectively) and then summed up to give a unified score for the whole system.

References

- [1] Anckar, C., 1997. Determinants of disproportionality and wasted votes. *Electoral Studies* 16(4), 501-515.
- [2] Bartolini, S. and Mair, P., 1990. *Identity, competition and electoral availability*. Cambridge: Cambridge University Press.
- [3] Benoit, K., 2001. District magnitude, electoral formula, and the number of parties. *European Journal of Political Research* 39, 203-224.
- [4] Caramani, D., 1997. *The nationalisation of electoral politics: a comparative and historical analysis of territories elections*. Florence.
- [5] Caramani, 2002. *The Measurement of Territorial Homogeneity: A Test on Comparative Electoral Data Since 1832*. Florence: European University Institute Working Paper, RSC. No. 2002/26.
- [6] Cox, Gary W., 1997. *Making Votes Count*. Cambridge: Cambridge University Press.
- [7] Cox, Gary W. and Shugart, M. S., 1991. Comment on Gallagher's 'Proportionality, Disproportionality and Electoral Systems'. *Electoral Studies* 10(4), 348-352.
- [8] Dunleavy, P. and Boucek, F. (2003). Constructing the Number of Parties. *Party Politics* 9(3), 291-315.
- [9] Gallagher, M., 1991. Proportionality, disproportionality and electoral systems. *Electoral Studies* 10(1), 33-51.
- [10] Gallagher, M., 1992. Comparing Proportional Representation Electoral Systems: Quotas, Thresholds, Paradoxes and Majorities. *British Journal of Political Science* 22, 469-496.
- [11] Grofman, B., 2001. A note of caution in interpreting the threshold of exclusion. *Electoral Studies* 20, 299-303.
- [12] Grofman, B., Koetzle, W., Brunell, T., 1997. An integrated perspective on the three potential sources of partisan bias: malapportionment, turnout differences, and the geographic distribution of party vote shares. *Electoral Studies* 16(4), 457-470.
- [13] Hug, S., 2001. *Altering party systems - strategic behavior & the emergence of new political parties in western democracies*. Michigan : The University of Michigan Press.
- [14] Laakso, M., Taagepera, R., 1979. Effective Number of Parties: A Measure with Application to West Europe. *Comparative Political Studies* 12, 3-27.
- [15] Lijphart, A., 1994. *Electoral systems and party systems - a study of twenty-seven democracies, 1945-1990*. Oxford: Oxford University Press.
- [16] Lijphart, A. and Gibberd, R.W., 1977. Thresholds and payoffs in list systems of proportional representation. *European Journal of*

- Political Research 5, 219-244.
- [17] Lijphart, A., 1999. *Patterns of Democracy*. New Haven: Yale University Press.
 - [18] Mackie, T. and Rose, R. (1991) *The International Almanac of Electoral History*. London: Macmillan.
 - [19] Mackie, T. and Rose, R. (1997) *A Decade of Electoral Results: Updating the International Almanac*. Glasgow: Centre for the Study of Public Policy, University of Strathclyde.
 - [20] Massicotte, L., Blais, A., 1999. Mixed electoral systems: a conceptual and empirical survey. *Electoral Studies* 18, 341-366.
 - [21] Monroe, Burt L. and Rose, Amanda G. (2002) Electoral Systems and Unimagined Consequences: Partisan Effects of Districted Proportional Representation. *American Journal of Political Science* 38 (1), 100-123.
 - [22] Ordeshook, P.C. and Svetsova, Olga V. (1994) Ethnic Heterogeneity, District Magnitude and the Number of Parties. *American Journal of Political Science*, 38 (1), 100-123.
 - [23] Penadés, A., 1997. A critique of Lijphart's 'Electoral Systems and Party Systems'. *Electoral Studies* 16(1), 59-71.
 - [24] Pennisi, A., 1998. Disproportionality indexes and robustness of proportional allocation methods. *Electoral Studies* 17(1), 1-19.
 - [25] Powell, G. B., Vanberg G.S., 2000. Election Laws, Disproportionality and Median Correspondence: Implications for Two Visions of Democracy. *British Journal of Political Science* 30, 383-411.
 - [26] Rokkan, S., 1968. *Elections: Electoral Systems*. *International Encyclopedia of the Social Sciences*. New York: Crowell-Collier, MacMillan.
 - [27] Sartori, G., 1997. *Comparative Constitutional Engineering. An Inquiry into Structures, Incentives and Outcomes*. 2nd edition. Houndmills: MacMillan Press Ltd.
 - [28] Shugart, Matthew Soberg and Wattenberg, Martin P. Eds. 2001. *Mixed-Member Electoral Systems: The Best of Both Worlds?* New York: Oxford University Press.
 - [29] Strøm, K., 1989. Inter-party competition in advanced democracies. *Journal of Theoretical Politics* (3):277-300.
 - [30] Strøm, K., 1989. Party competition and the politics of economic openness and growth. *European Journal of Political Research* 17, 1-16.
 - [31] Taagepera, R., 1989. Empirical threshold of representation. *Electoral Studies* 8(2), 105-116.
 - [32] Taagepera, R., 1998. Effective Magnitude and Effective Threshold. *Electoral Studies* 17(4), 393-404.

- [33] Taagepera, R., 1998, Nationwide inclusion and exclusion thresholds of representation. *Electoral Studies* 17(4), 405-417.
- [34] Taagepera, R., 2002. Nationwide threshold of representation. *Electoral Studies* 21, 383-401.
- [35] Taagepera, R. and Shugart, M. S., 1989. *Seats and votes - the effects and determinants of electoral systems*. New Haven/London: Yale University Press.
- [36] Taylor, P.J., Johnston, R.J., 1979. *Geography of Elections*. Penguin Books. Harmondsworth.
- [37] Woldendorp, J., Keman, H. and Budge, I. (2000). *Party Government in 48 Democracies (1948-1998)*. Dordrecht: Kluwer Academic Publishers.