

# Legal unbundling and auctions in vertically integrated (utilities) markets.

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## Abstract

This paper addresses the effectiveness of auctions and legal unbundling as regulatory measures to tender a vertically integrated industry more competitive. Specifically, I analyze if implementing auctions and legal unbundling can counter market power in an industry where a Vertically Integrated Corporation (VIC) has a monopoly position in an essential, scarce upstream activity and also owns one of the firms active in the competitive downstream activity. In an earlier paper, Van Koten (2011), I showed that in this configuration the VIC, by having its downstream firm bid more aggressively, can – through increased auction revenue – increase its profit, while disadvantaging downstream competitors and lowering efficiency. Here I analyze the regulatory measure of also legally separating the downstream firm from the VIC. I show that such a measure may only be partially effective; the VIC can formulate a simple compensation scheme that does not violate restrictions typically imposed by legal separation but induces the manager of the VIC-owned downstream firm to bid more aggressively. This increases the profits of the VIC, decreases efficiency, and disadvantages downstream competitors.

*Keywords: regulation, vertical integration, competition, electricity markets, strategic delegation.*

*JEL classification code: L22, L43, L51, L94, L98.*

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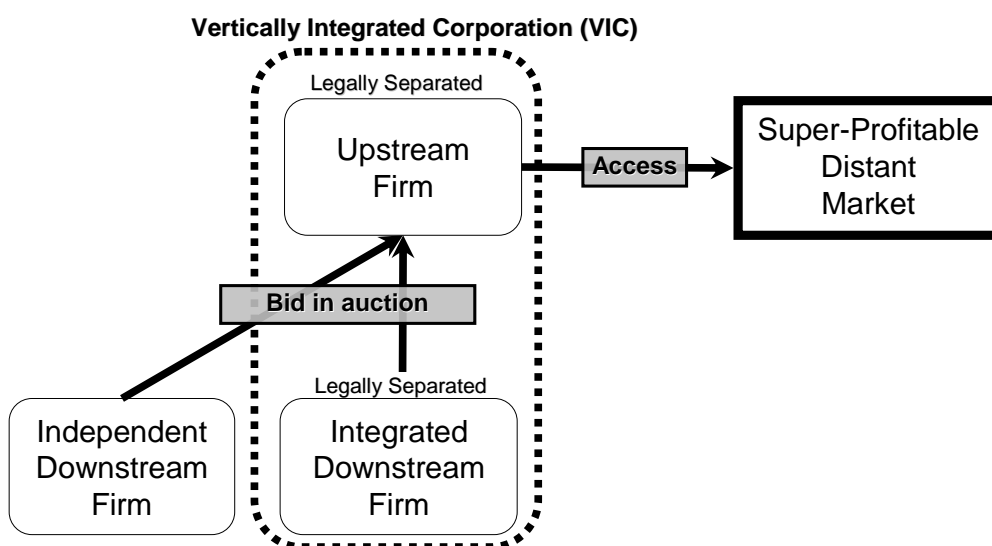
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## 1. Introduction

This paper addresses the effectiveness of auctions and legal unbundling as regulatory measures to make a vertically integrated activity more competitive. I analyze a setup in which an unregulated upstream producer sells scarce access rights to a bottleneck network that connects to a distant, super-profitable market. The access rights are scarce in the sense that they are in excess demand. The upstream producer is vertically integrated into a Vertically Integrated Corporation (VIC) with a downstream firm (henceforth “the downstream integrated firm”) that competes with the other independent downstream firms in the auction for the scarce access rights. Figure 1 shows the basic setup.

**Fig. 1 Competition for access rights by independent and integrated downstream firms**



An example of such a setup can be found in the EU electricity market.<sup>1</sup> The upstream producer is a network firm that operates an unregulated cross-border transmission line that can export electricity to a foreign country. Such transmission lines are called interconnectors. Downstream firms are domestic electricity generation firms that compete for access to the interconnector. The electricity market in such a foreign country is, from the point of view of domestic firms, a super-profitable, exclusive market when the expected clearing price is high and the interconnector capacity scarce. There is indeed a dramatic shortage of interconnector capacity between the EU countries, which, most of the time, prevents price convergence between different EU countries (European Commission 2007; European Climate Foundation 2010). As a result, access rights for export on the interconnector mostly have a positive value. Electricity generators may also be able to produce for a domestic downstream market where they do not need to obtain scarce access rights to

<sup>1</sup> This part draws on Van Koten (2011) and Van Koten and Ortmann (2008), where a more detailed account can be found.

interconnector capacity, but instead to gain access to a regulated national transmission network. My analysis does not address the allocation of capacity on national lines. For such analysis see, amongst others, Vickers (1995), Biglaiser and DeGraba (2001), Höfler and Kranz (2007), and Foros, Kind & Sjørgard (2007)..

Because of the shortage of interconnector capacity between the EU countries, new EU laws allow firms, conditional on approval by the national regulators, to build and operate unregulated interconnectors for profit. Such for-profit lines are called merchant interconnectors. EU laws will probably in most cases require merchant interconnectors to allocate capacity non-discriminatorily (European Commission 2004, art. 19 and art. 34; 2009; CRE 2010, p.4). An auction is the most straightforward manner by which to implement a non-discriminatory allocation of capacity in the electricity industry (ERGEG 2009). At the moment, not many merchant interconnectors have been built yet, but probably many more will be built in the future, especially by corporations that are also active in electricity generation (de Hauteclocque and Rious 2009; Van Koten 2011). As a result, a vertically integrated corporation may own a merchant interconnector (the upstream producer) and an electricity generation firm (the downstream integrated firm), and thus be competing in an auction for access to its own interconnector (the scarce upstream good).

Burkart (1995) and Van Koten (2011) show that in such an auction the downstream integrated firm will bid more aggressively, leading to inefficient and discriminatory outcomes. The integrated firm is more likely to win the auction and the profitability of the competing downstream firms is decreased. Van Koten (2011) shows that the legal unbundling of the upstream producer, while guaranteeing that the auction is fair, does not remediate the negative effects of inefficiency and discrimination. The negative effects are caused by the downstream integrated firm maximizing the joint profit made by itself and the upstream producer, and thus bidding more aggressively to increase the auction revenue. I therefore examine the effectiveness of an additional remedy that aims to neutralize the incentive of the downstream integrated firm to bid more aggressively: the remedy of legally separating the downstream integrated firm from the Vertically Integrated Corporation (VIC) that owns both the upstream producer and downstream firm.<sup>2</sup> I use legal separation as specified in the EU (European Commission 2009, article 14). While the VIC retains the ownership of the downstream integrated firm, and is thus the residual claimant of its profit, it is not allowed to intervene in the day-to-day decision making of the downstream integrated firm. And while the VIC has the right to periodically (e.g. bi-annual) set performance criteria and bonus

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<sup>2</sup> Legal separation is sometimes also called legal unbundling.

schemes, these criteria and schemes may depend only on the outcomes of the downstream firm (and not on the outcomes of other — upstream — activities of the VIC).<sup>3</sup>

The assumption that legal separation can be enforced by the regulator and that the VIC can thus be prevented from giving day-to-day instructions to its affiliated downstream firm is central to this analysis. If legal separation cannot be enforced, then the VIC can instruct the downstream firm directly to maximize the total VIC profits. Such a setting has been analyzed in Van Koten (2011). However, legal separation often figures as a policy measure, which suggests that – at least in policy circles – there is a strong belief in its effectiveness. This study may also be of value to those who are more skeptical of the effectiveness of legal separation, as the results show that *even if* legal separation can be enforced, it will likely not be very effective: The VIC has alternative ways to influence its affiliated downstream firm to act in a way that approximates the maximization of the VIC profits.

When intervention in day-to-day decision making is outlawed, the VIC must delegate decision power to the manager of the now legally independent downstream firm. The VIC can, however, still influence the manager's decisions by setting an ex-ante compensation scheme. The literature on strategic managerial delegation has shown that when an owner must commit to a compensation scheme, he has incentives to set the compensation for his manager as a linear combination of profit and revenue (Vickers 1985; Fershtman and Judd 1987; Sklivas 1987). I modify such a compensation scheme for use in an auctions setting and show that it would be profitable to offer this compensation scheme to the manager of the legally separated downstream firm.<sup>4</sup> The compensation scheme I consider respects the legal independence of the downstream firm; compensation is based on performance indicators of the downstream firm only. I assume that the other competing independent downstream firms are maximizing profits, as their owners are not able to commit credibly to a compensation scheme other than that of maximizing profits.

Earlier papers examine the effects of the vertical integration of an upstream monopolist with a firm in the competitive downstream market. Vickers (1995), Biglaiser and DeGraba (2001), Øystein, Kind and Sjørgard (2007), and Höfler and Kranz (2007) examine the effect of the upstream price on outcomes in the downstream market. Outcomes in the downstream market are determined by Cournot competition (Vickers 1995; Øystein, Kind and Sjørgard 2007), by competition on a Hotelling line (Biglaiser and DeGraba, 2001), or are — for maximum generality — left unspecified (Höfler and Kranz 2007). Øystein, Kind and Sjørgard (2007) assume that a non-discrimination

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<sup>3</sup> This is a realistic requirement: the European Commission, for example, inspects whether performance schemes for managers in legally separated electricity and gas network companies (both upstream activities) are independent of the other activities of the VIC (Gómez-Acebo & Pombo Abogados, S.L. and Charles Russell LLP, 2005, 25 -27).

<sup>4</sup> It is possible to consider more general forms of compensation schemes, e.g. non-linear ones, but the compensation scheme under consideration suffices to show that the holding company can increase its profits this way. Moreover, linear compensation schemes seem to be widely used (Dixit, 2002).

regulation could affect the internal organization of the integrated downstream firm in such a way that the downstream firm acts as if it were an independent firm that faces the same net costs of purchasing the upstream input as other downstream firms. Øystein, Kind and Sørgard (2007) do not specify the mechanisms that implement non-discrimination and how these mechanisms may affect or interact with the behavior of the firms. In contrast, I examine the specific mechanisms of legal separation: outlawing day-to-day instructions and requiring compensation schemes to be based on the performance of the downstream integrated firm only, and I show how these mechanisms may interact with the behavior of the VIC and the downstream integrated firm. Legal separation is the most rigorous remedy — short of ownership separation — for implementing non-discrimination.

In the above papers, it is shown that the effect of vertical integration is that downstream integrated firms have a cost advantage over their independent competitors. When purchasing the upstream inputs, downstream integrated firms face a net cost equal to the (low) marginal cost of production and not the higher regulated price. As a result of this cost advantage, downstream integrated firms produce more than their independent competitors. My paper is similar in that it is shown that the downstream integrated firm, due to the vertical integration, does not face the same net costs as other firms when buying the upstream input. In the above papers, outcomes for the upstream market are trivial: the price of the upstream input is given ex-ante by a regulator and the upstream input is in abundant supply: The focus of the analyses in the above papers is on the outcomes in the downstream market. In contrast, my focus is on outcomes in the upstream market. The price of the upstream input is determined by the competing downstream firms bidding in the auction. I further assume that the downstream firms form a rational expectation of the price in the distant, super-profitable market and that they have the same expectation of the price. The price in the distant, super-profitable market is determined in a rational way, taking into account the extra supply resulting from the bottleneck network. For ease of exposure, I will assume that is done by perfect competition. In the example of merchant interconnectors, the price in the foreign country (the distant, super-profitable market) is determined by competition among the foreign downstream producers, taking into account the given, fixed import of electricity over the interconnector. See Joskow and Tirole (2005) for an example of such a setup. These assumptions ensure that both downstream firms will value access to the distant market and will thus make positive bids in the auction for access.

The compensation scheme I use for the manager of the downstream integrated firm — a linear combination of profit and revenue — was originally proposed by Vickers (1985), Fershtman and Judd (1987) and Sklivas (1987) and is here modified for application in an auctions setting. In Vickers (1985), Fershtman and Judd (1987), and Sklivas (1987), the incentive for the VIC to offer his manager a compensation scheme is to create a strategic interaction effect. This effect is absent in

second-price auctions and plays only a minor role in first-price auctions. In this paper the main incentive to offer a compensation scheme is to have the downstream integrated firm internalize (at least a part of) the positive effect of higher auction revenues on the profit of the upstream firm.

Øystein, Kind and Sjørgard (2007) also apply the strategic delegation framework of Vickers (1985), Fershtman and Judd (1987), and Sklivas (1987) and assume that owners of all firms, both integrated and independent, can implement compensation schemes.<sup>5</sup> This is, however, not a realistic assumption. Owners may have the incentive to announce a compensation scheme, but they also have the incentive to secretly instruct their manager to maximize profits. Without a commitment device to “tie their hands”, independent firms cannot commit to a compensation scheme (Dewatripont 1988; Katz 1991; Williamson 1983). The legal separation of firms, verified and enforced by a regulator, is a commitment device.<sup>6</sup> I therefore assume that only the VIC, the owner of the legally separated downstream integrated firm, can commit to a compensation scheme.

The remainder of this paper is organized as follows. In the next sections, I analyze the effects of compensation schemes on the outcomes of auctions. I first describe the general setup of the model, then determine the equilibrium bidding functions of bidders and the equilibrium compensation scheme in second-price auctions (for any number of competing downstream firms), and show the effects on profits and welfare. I then determine the equilibrium bidding functions of bidders and the equilibrium compensation scheme in first-price auctions (for one competing downstream firm) and again show the effects on profits and welfare. I conclude by discussing the implications of my results for regulation policy.

## 2. The model

I assume that a downstream firm does not know the value of its competitors for the good to be auctioned: it thus treats it as a random variable, drawn from a distribution which, for the sake of tractability, I will assume to be uniform.<sup>7</sup> This assumption allows me to derive closed-form expressions. I further assume that a VIC fully owns the downstream firm and a part  $\gamma$  of the upstream firm that organizes the auction.

Two types of bidders participate in the auction. The first is a downstream firm owned by the VIC, labeled as integrated bidder V. In the analyses there is only one, unique integrated bidder V.

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<sup>5</sup> I use the term “compensation scheme”, for compensation schemes with a strictly positive weight on revenues. I use the term “maximizing profit” for compensation schemes with a zero weight on revenues and a positive weight on profits.

<sup>6</sup> This assumption may be disputed. If the VIC can circumvent the legal separation and give – illegally – day-to-day instructions to its affiliated downstream firm, then the VIC cannot credibly commit to a cost weight. However, the VIC can then instruct the downstream firm to maximize the total profits of the VIC, a setting that has been analyzed in Van Koten (2011).

<sup>7</sup> See Van Koten (2011) for examples of the EU and US electricity industry where bidders have randomly distributed values in transmission capacity. See also Schöne (2009) and Parisio & Bosco (2008).

The second type is a downstream independent firm, labeled as independent bidder X. When analyzing outcomes in the second price auction, I will allow for any number  $n$  of independent bidders X. In the first price auction, which is mathematically more complex than the second price auction, I will allow for only one independent bidder X, so as to be able to derive a closed-form solution. The bidding function of integrated bidder V is determined by its manager, referred to as manager  $V^m$ . Manager  $V^m$  receives remuneration according to a compensation scheme set by the VIC. The other type of firm, X, is independent and the firm owner cannot credibly offer its manager (“ $X^m$ ”) incentives that differ from profits maximization (Dewatripont 1988; Katz 1991; Williamson 1983). As a result, the bidding incentives of a manager “ $X^m$ ” and his firm X are identical, and I will thus not distinguish between the two and will refer to the independent firm type as an independent bidder X.

In line with the literature, I assume that there exists a differentiable, strictly increasing bidding strategy  $b_v[\cdot]$  ( $b_x[\cdot]$ ), that maps a bidder’s realized value  $u_v \in [0,1]$  ( $u_x \in [0,1]$ ) into its bid  $b_v[v_v]$  ( $b_x[x]$ ).<sup>8</sup> It follows from this that the bidding strategy  $b_v[\cdot]$  ( $b_x[\cdot]$ ) has an inverse  $v[\cdot]$  ( $x[\cdot]$ ) such that  $v[b_v[u_v]] = u_v$  ( $x[b_x[u_x]] = u_x$ ).

The VIC wants to maximize the joint profit from its downstream and upstream firms. Because of legal separation, the VIC cannot influence the day-to-day decision-making of its integrated downstream firm V. It therefore offers its manager  $V^m$  a compensation scheme that serves its interests best, while respecting the rules for legal unbundling.<sup>9</sup> For the compensation schemes to be effective, it must be credible: it should be a part of a Nash equilibrium.

One possibility, as considered by Vickers (1985), Fershtman and Judd (1987) and Sklivas (1987), is to give manager  $V^m$  a compensation  $w$  proportional to a linear combination of profits and revenue.<sup>10</sup> Sklivas (1987) shows that such a compensation scheme  $w$  is equal to a proportion of the revenues minus costs, where the costs are weighted by a factor  $a$ .

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<sup>8</sup> The strategies  $b_v[\cdot]$  and  $b_x[\cdot]$  (and their respective inverses  $v[\cdot]$  and  $x[\cdot]$ ) are dependent on the ownership share  $\gamma$ . For notational convenience I will not include the variable “ $\gamma$ ” in the derivation to follow. I allow for a bidding function  $b[\cdot]$  to be strictly increasing on an interval  $[0, \bar{u}]$  with  $\bar{u} : 0 < \bar{u} < 1$  and then to be flat on  $[\bar{u}, 1]$ . In this case the inverse is only defined on  $[0, \bar{u}]$ .

<sup>9</sup> For a compensation scheme not to violate legal independence, it ought to be based on performance indicators of downstream firm V only, and not on profit indicators of the VIC or the upstream firm.

<sup>10</sup> Fershtman and Judd (1987) and Sklivas (1987) considered the effect of compensation schemes in the context of two competing firms who each have a manager that makes the crucial output and pricing decisions. They found that, due to an interactive effect, the optimal compensation scheme has a cost weight  $a$  such that  $a > 1$  ( $a < 1$ ) for Bertrand competition (Cournot competition); the optimal compensation scheme exaggerates (understates) part of the costs and makes the firms competing weaker (stronger). The firms become “fat cats” (“top dogs”) in the sense of Fudenberg and Tirole (1984).

$$1) \quad w = i \cdot (a\pi + (1 - a)R),$$

(where  $w$  is the compensation,  $\pi$  is the profit,  $R$  is the revenue,  $a$  is the linear weight, and  $i$  denotes the proportion),

$$= i \cdot (a(R - c) + (1 - a)R),$$

$$= i \cdot (R - ac).$$

From here on, I will refer to factor  $a$  as the cost weight. Setting  $a > 1$  gives the manager the incentive to be more concerned about costs and less about revenue. Such a manager can thus be expected to be less focused on expansion and more on cost-cutting. In contrast, setting  $a < 1$  gives the manager the incentive to be less concerned about costs and more about revenue. Such a manager can thus be expected to be more aggressive and more focused on expansion in the market. From this perspective, normal profit maximization is the special case where the cost weight is set equal to unity:  $a = 1$ .

Proportion  $i$  is determined endogenously in the model. As the expected compensation for manager  $V^m$  must equal his reservation wage  $w^0$ , proportion  $i$  is determined by

$E[w] = E[i \cdot (R - ac)] = w^0$ . In an auction, the costs and returns are expected values that are endogenously determined by the bids. In this case, the expected compensation for manager  $V^m$  is  $E[w] = i \cdot x[b_V](u_V - ab_V)$ . The expected value of the auction,  $x[b_V]u_V$ , corresponds to the revenue<sup>11</sup> and the expected payment,  $x[b_V]ab_V$ , is the expected cost of realizing the “revenue”.

### 3. Results

#### 3.1 Second-price auctions

It is a well-known result that in second-price auctions, bidders have a weakly dominant strategy to set their bid equal to their value, regardless of the number of bidders in the auction or their bidding strategies (e.g., see Krishna, 2002). Therefore, the independent bidders  $X$  will bid their values. Manager  $V^m$  effectively only pays proportion  $a$  of his bid, and thus set  $a$  times his bid equal to his value:  $ab_V = u_V$ . As a result,  $V^m$  will thus bid  $b_V[u_V] = \frac{u_V}{a}$ . Proposition 1 summarizes the result.

**Proposition 1:** *In a second-price auction with  $n+1$  bidders, of which  $n$  are independent and one is integrated, the independent bidders bid their values,  $b_X[u_X] = u_X$ , and the integrated bidder bids*

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<sup>11</sup> We will see shortly that  $x[b_V]$  is the probability of manager  $V^m$  winning the auction.



$$b_V[u_V] = \frac{u_V}{a},^{12} \text{ for any } n \geq 1.$$

The result in proposition 1 is general and holds for any distribution of values. Cost weight  $a$  modulates the aggressiveness of bidding of manager  $V^m$ ; a cost weight smaller than one induces him to bid more aggressively, and one larger than one to bid less aggressively. This is an intuitive result: a cost weight smaller (larger) than one makes the manager less (more) concerned about costs and more (less) about revenues. The VIC sets the cost weight so as to maximize its profit function. This profit function can be characterized as follows:

$$\begin{aligned} 2) \quad \pi_{VIP, u_V \leq a}^{(n)}[u_V] &= \Pr_{V \text{ wins, } u_V \leq a}[b_V] \cdot u_V - (1 - \gamma) \cdot m_{V, u_V \leq a}[b_V] + \gamma \cdot m_{\sum X_i, u_V \leq a}[b_V], \text{ with} \\ P_{V \text{ wins, } u_V \leq a}[b_V] &= b_V^n, \\ m_{V, u_V \leq a}[b_V] &= E[\text{highest bid from } n \text{ bidders} \mid V \text{ wins and } u_V \leq a], \\ m_{\sum X_i, u_V \leq a}[b_V] &= \Pr[V \text{ has } 2^{\text{nd}} \text{ highest bid}] \cdot b_V + \\ &\quad \sum_{i=3}^{n+1} \Pr[V \text{ has } i^{\text{th}} \text{ highest bid}] \cdot E[2^{\text{nd}} \text{ highest bid from } n-1 \text{ bidders} \mid V \text{ has } i^{\text{th}} \text{ highest bid}]. \end{aligned}$$

The first term,  $\Pr_{V \text{ wins, } u_V \leq a}[b_V] \cdot u_V$ , is the expected value of the good for the VIC; the probability that  $V$  wins times the value of the good. The second term,  $(1 - \gamma) \cdot m_{V, u_V \leq a}[b_V]$ , is the net expected auction revenue that  $V$  pays; this is equal to  $1 - \gamma$  times the highest expected bid from the  $n$  competing independent bidders conditional on  $V$  winning. The third term,  $\gamma \cdot m_{\sum X_i, u_V \leq a}[b_V]$ , is equal to the proportion of ownership by the VIC,  $\gamma$ , times the expected payment of all the independent bidders  $X_i$ , conditional on  $V$  losing.  $V$  can lose either by having the  $2^{\text{nd}}$  highest bid or by having a lower bid. When  $V$  has the  $2^{\text{nd}}$  highest bid, it loses the auction and sets the price to be paid by the winner of the auction; the winning independent bidder  $X_i$  must thus pay the bid of  $V$ ,  $b_V$ . When  $V$  has a bid lower than the  $2^{\text{nd}}$  highest bid, it loses the auction and does not set the price; the expected payment by a winning independent bidder is the  $2^{\text{nd}}$  highest bid from the  $(n-i)$  independent bidders that have a higher bid than  $V$ .

Assuming that values are drawn from independent and uniform distributions, the VIC can set an optimal cost weight that maximizes the profit function. Proposition 2 presents the result.

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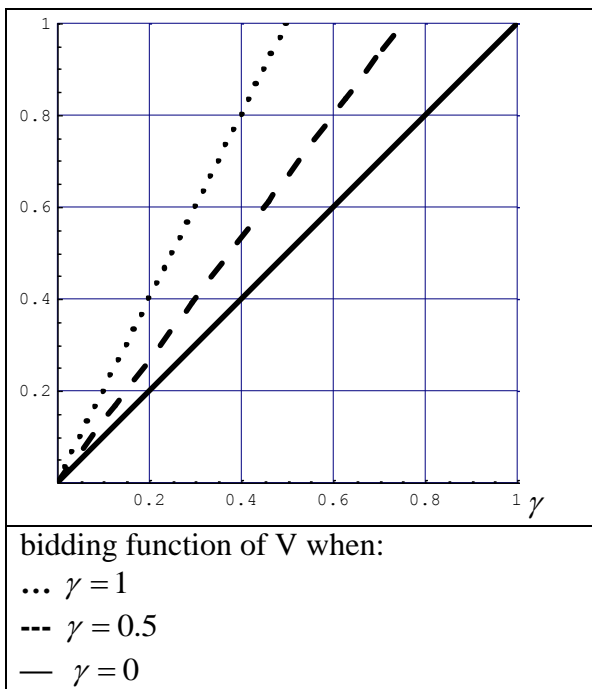
<sup>12</sup> Strictly spoken, when  $u_V \geq a$ , any bidding strategy  $b_V[u_V] \geq 1$  is a Nash-Equilibrium. All auction outcomes and profits are the same for these strategies (the integrated bidder wins and pays the second highest bid), and I will thus restrict my treatment to the strategy  $b_V[u_V] = \frac{u_V}{a}$  without loss of generality.

**Proposition 2:** *In a second-price auction with  $n+1$  bidders, of which  $n$  are independent and one is integrated, the VIC sets the cost weight for the integrated bidder equal to  $a^{(n)}[\gamma] = 1 - \frac{\gamma}{n+1}$ , for any  $n \geq 1$ .*

**Proof:** *See the Appendix.*

Proposition 2 has two interesting implications. Firstly, note that for the ownership proportion  $0 < \gamma < 1$ :  $\frac{1}{2} < a^{(n)}[\gamma] < 1$ , and that the cost weight decreases in the ownership share: the VIC wants the integrated bidder to bid more aggressively, and increasingly so as the VIC owns a larger share  $\gamma$  of the upstream firm. Bidding more aggressively makes independent bidders X pay more when they win the auction, which increases the profits of the VIC. Also, the higher the number  $n$  of independent firms, the higher the expected auction revenue and the smaller the relative gain of bidding aggressively. Figure 2 shows the bidding functions of integrated bidder V for different ownership shares when V competes with one independent bidder X.

**Fig. 2 The bidding function of integrated bidder V in second-price auctions**



Secondly, note that  $a^{(n)}[0] = 1$ : a VIC that has no ownership share in the upstream firm prefers its bidder to maximize profits in second-price auctions. This explains why the owner of independent bidder X has no incentive to offer its manager “X<sup>m</sup>” a similar compensation scheme — he has no

ownership share in the upstream firm and thus does not receive a share of the auction outcomes. The effects on auction outcomes are summarized by Proposition 3.

**Proposition 3:** *In a second-price auction with  $n+1$  bidders, of which  $n$  are independent and one is integrated and has cost weight  $a^{(n)}[\gamma]=1-\frac{\gamma}{n+1}$ , where values are distributed independently and uniformly on  $[0,1]$ , the independent bidders bid their value, and the integrated bidder bids*

$$b_V[u_V] = u_V \left(1 + \frac{\gamma}{n+1-\gamma}\right), \text{ for any } n \geq 1. \text{ As a result, with increasing } \gamma :$$

- a) *The ex ante expected auction revenue is increasing in  $\gamma$ .*
- b) *The ex ante expected profit of the VIC is increasing in  $\gamma$ .*
- c) *The strategic profit, the increase in profits relative to not setting a cost weight, is increasing in  $\gamma$ .*
- d) *The ex ante expected profit of  $X_i$  is decreasing in  $\gamma$ . The relative loss in profit for each independent bidder is increasing in  $\gamma$ .*
- e) *The ex ante efficiency is decreasing in  $\gamma$ .*

**Proof:** *See the Appendix.*

When the ownership share increases, the auction revenue increases as well (Prop. 1a). Notably, for an auction with two bidders (thus with one competing independent bidder), the auction revenue is equal to  $\frac{4+\gamma}{12}$ , which is shown below to be different from auction revenues in first-price auctions. Also, the total profit of the VIC (the profit of its downstream firm plus its share of the auction revenue) increases (Prop. 1b). Also the strength of the incentive for V to bid more aggressively increases.<sup>13</sup> The strength of this incentive, which I call the “strategic profit”, is the relative increase in profits by setting the optimal cost weight. It can be calculated by taking the difference in profits between using a strategy of maximizing total profits (downstream firm profits and  $\gamma$  times auction revenue) and of using a strategy (which I call the naïve strategy) of maximizing the profit of only the downstream firm. The profit of independent bidders  $X_i$  decrease; they are less likely to win, and if they win, they pay a higher price (Prop. 1c). The efficiency of the auction decreases; now, in some cases, V wins without having the highest value (Prop. 1d).

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<sup>13</sup> This is an important indicator for external validity of the model; experimental evidence has shown that the strength of incentives is important for theoretical predictions to show in real settings (Hertwig and Ortmann, 2001).

**Fig. 3 Outcomes in second-price auctions with one independent bidder**

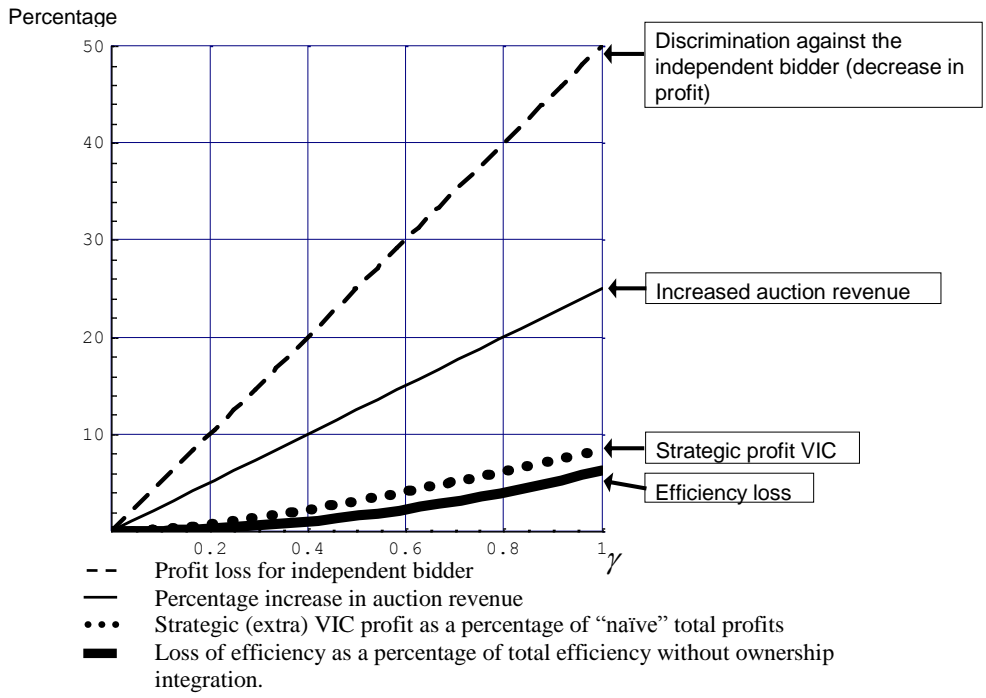


Figure 3 shows the effect of ownership share on auction outcomes when the integrated bidder competes with one independent bidder. There is a considerable efficiency loss,<sup>14</sup> up to 6.25%. The gain for the VIC given by the strategic profit<sup>15</sup> is also considerable; a VIC can, by bidding more aggressively, increase its profit by up to 8.3%.<sup>16</sup> The price of the good (the auction revenue) is strongly affected; it can increase by up to 25%. The strongest effect is a discrimination effect against the independent bidder: the expected profit of the independent bidder is decreased by up to 50%. Also at low levels of ownership integration the discrimination effect is considerable; even with an ownership share of only 20%, the profit of the independent bidder is decreased by 10%.

<sup>14</sup> The efficiency loss percentage is calculated as  $\frac{w[0]-w[\gamma]}{w[0]} = \frac{25\gamma^2}{(1+\gamma)^2}$ , with  $w[\gamma]$  equal to the total welfare.

<sup>15</sup> The strategic profit percentage is calculated as  $\frac{\pi_{VIP}(a=a(\gamma))}{\pi_{VIP}(a=1)}$ , where  $\pi_{VIP}(a=a(\gamma))$  is the profit maximizing strategy, and  $\pi_{VIP}(a=1)$  the “naïve” strategy.

<sup>16</sup> For comparison: the increase in profit without legal unbundling of the downstream firm is up to 16.7%, almost twice as much (Van Koten, 2011).

**Fig. 4 Outcomes in second-price auctions with 1, 2, 3, 4, and  $\infty$  independent bidders**

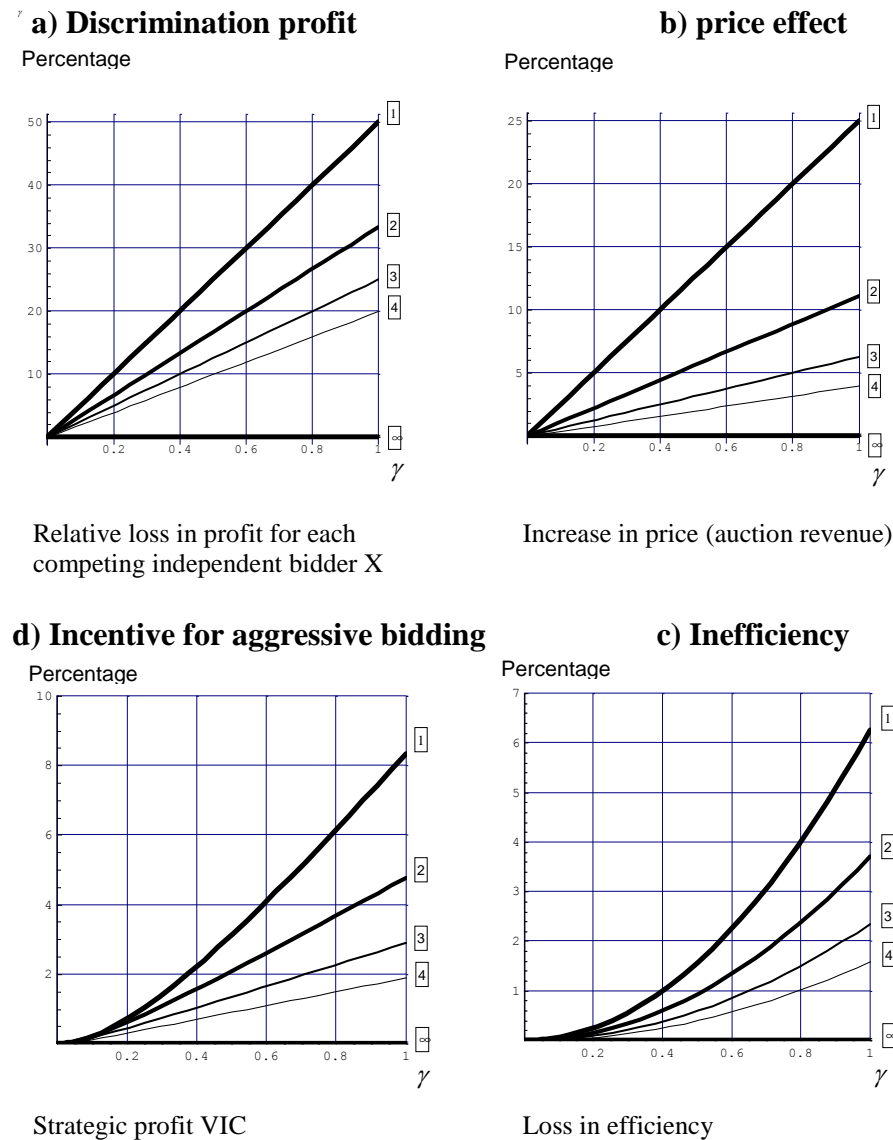


Figure 4 shows that effects are strong for low numbers and converge to zero when the number of independent bidders goes to infinity. The discrimination effect of integrated ownership is remarkably strong. Graph (a) shows the loss in expected profits for each competing independent firm, which can be as high as 50%. With two competing independent firms, each of them has a decrease in profits of up to 33%. Even with as many as three competing independent firms, each has a decrease in profits of up to 25%. Graph (b) shows the strength of incentives for V to bid more aggressively, as given by the strategic profit as a percentage of the naïve profit. This can be as high as up to 8.3% with one competing bidder, up to 4.7% with two competing bidders, and up to 2.8% with three competing bidders. Graph (c) shows the loss in efficiency, which represents a considerable social loss.

### 3.2 First-price auctions

#### 3.2.1 The VIC with a first mover's advantage

For first-price auctions I restrict the analysis to the case with one independent bidder X. While in second-price auctions the implementation of a compensation scheme for manager  $V^m$  does not affect the bidding of an independent bidder X, this is not so in first-price auctions. The bidding schedule of X is affected by the compensation scheme for manager  $V^m$ , an effect I will call the “interaction effect”. The VIC can use the interaction effect to strategically influence the bidding schedule of X. For such an interaction effect to occur, X needs to know the value of the cost weight. In the main analysis I will assume that X is rational and X is thus able to calculate the optimal cost weight for the VIC. I will also assume that the rules on legal separation forbid the VIC from spreading false information about the compensation scheme. As a result the VIC can be sure that the compensation scheme it announces is known and believed by independent bidder X; this gives the VIC a first mover's advantage. It is ironic that precisely legal separation – meant to increase competition – is a means of credible commitment that gives the VIC a first mover's advantage. Below, I will relax these two assumptions.

Figure 5 depicts the timeline of events in the auction. At time 1, the VIC implements a compensation scheme for manager  $V^m$  with cost weight  $a$ , and X is informed of its compensation scheme (or deduces it by calculating the profit maximizing choice of compensation scheme for the VIC). Note that the owner of independent bidder X can only credibly instruct X to maximize profits (Williamson (1983)).<sup>17</sup> Manager  $V^m$  and X, anticipating each other's reactions, simultaneously determine the bidding functions  $b_V[u_V]$  and  $b_X[u_X]$ . At time two,  $V^m$  and X, plugging in their respective values, determine their bids in the auction and the highest bidder wins.

**Fig. 5 Timeline of key decisions**

	$t_1$	$t_2$
VIC:	The VIC implements a compensation scheme with cost weight $a$	The VIC, bounded by the rules of legal unbundling, sticks to the compensation scheme as announced in $t_1$
V & X:	Manager $V^m$ is informed about cost weight $a$ . X is informed about cost weight $a$ (or deduces it). Manager $V^m$ and X simultaneously determine the bidding functions $b_V[u_V]$ and $b_X[u_X]$ .	Plugging in their respective values $V^m$ and X determine their bids. The highest bidder wins the auction.

<sup>17</sup> See the Appendix for a numeric example based on the model.

Given the bidding strategy of X,  $b_X[u_X]$ , V wins the auction when the bid of the independent bidder,  $b_X[u_X]$ , is smaller than its bid  $b_V$  :

$$3) \quad b_X[u_X] < b_V \Leftrightarrow \\ u_X < b_X^{-1}[b_V] \equiv x[b_V].$$

The probability of V winning the auction is thus  $F[x[b_V]]$ , which is equal to  $x[b_V]$ , as values are drawn from the uniform distribution on  $[0,1]$ . The expected profit of V with value realization  $u_V$ , bidding  $b_V$ , is therefore:

$$4) \quad \pi_V = x[b_V](u_V - b_V) - w^0.$$

Likewise, the expected profit of independent bidder X with value realization  $u_X$ , bidding  $b_X$ , is

$$5) \quad \pi_X = v[b_X](u_X - b_X).$$

The expected compensation for manager  $V^m$  is:

$$6) \quad \pi_V = i \cdot x[b_V](u_V - ab_V).$$

To calculate the reaction function of manager  $V^m$ , differentiate equation 5 with respect to  $b_V$ , set it equal to zero and solve for  $x'[b]$ :

$$7) \quad x'[b] = \frac{a \cdot x[b]}{u_V - ab} = \frac{a \cdot x[b]}{v[b] - ab}.$$

To calculate the reaction function of independent bidder X, differentiate equation 6 with respect to  $b_X$ , set it equal to zero and solve for  $v'[b]$ :

$$8) \quad v'[b] = \frac{v[b]}{u_X - b} = \frac{v[b]}{x[b] - b}.$$

Equations 7 and 8 form a system of differential equations that can be solved for  $x[b]$  and  $v[b]$ . After taking inverses, this gives us the bidding functions of X and V. Proposition 4 presents the result.

**Proposition 4:** *In a first-price auction with two bidders, of which one is independent and one is integrated and has cost weight  $a$ , where values are distributed independently and uniformly on  $[0,1]$ , the bidding functions of  $X$  and  $V$  are given by:*

$$9) \quad b_V[u_V] = \frac{\sqrt{u_V^2 + a^2(1-u_V^2)} - a}{(1-a^2)u_V} \text{ for } a \neq 1 \text{ and } b_V[u_V] = \frac{1}{2}u_V \text{ for } a = 1.$$

$$10) \quad b_X[u_X] = \frac{1 - \sqrt{a^2u_X^2 + (1-u_X^2)}}{(1-a^2)u_X} \text{ for } a \neq 1 \text{ and } b_X[u_X] = \frac{1}{2}u_X \text{ for } a = 1.$$

The maximum bid  $\bar{b}$  is equal to  $\bar{b} = \frac{1}{(1+a)}$ .

**Proof:** *See the Appendix.*

Cost weight  $a$  modulates the aggressiveness of bidding of manager  $V^m$  in a comparable way as in second price auctions: a cost weight smaller than one induces him to bid more aggressively, and one larger than one to bid less aggressively. However, a new effect is that independent bidder  $X$  will now accommodate the aggressive bidding of  $V$  and also bid more aggressively; the above-mentioned interaction effect.

The profit function of the VIC in first-price auctions is equal to the profit of firm  $V$  plus the ownership share  $\gamma$  times the total auction revenue:

$$11) \quad \pi_{VIP}[a, \gamma] = \pi_V[a] + \gamma \cdot (m_V[a] + m_X[a]), \text{ with}$$

$$\pi_V[a] = E[(u_V - b_V) | b_V \text{ is highest bid}] = \int_0^1 x[b_V[u_V]] \cdot (u_V - b_V[u_V]) du_V, \text{ the profit of firm } V,$$

$$m_V[a] = E[b_V | b_V \text{ is highest bid}] = \int_0^1 x[b_V[u_V]] \cdot b_V[u_V] du_V, \text{ the auction revenue paid by } V,$$

$$m_X[a] = E[b_X | b_X \text{ is highest bid}] = \int_0^1 v[b_X] \cdot b_X[u_X] du_X, \text{ the auction revenue paid by } X.$$

Where  $x[\cdot]$  and  $v[\cdot]$  are the inverse bidding functions, and  $x[b_V]$  ( $v[b_X]$ ) is the probability that  $V$  ( $X$ ) wins the auction with bid  $b_V$  ( $b_X$ ). Maximizing this profit function gives the optimal cost weight  $a[\gamma] = \text{ArgMax}_a [\pi_{VIP}[a, \gamma]]$ . Proposition 5 presents the effect of  $a[\gamma]$  on auction outcomes.

**Proposition 5:**

*In a first-price auction with two bidders, of which one is independent and one is integrated and has cost weight  $a$ , where values are distributed independently and uniformly on  $[0,1]$ , the optimal cost weight,  $a[\gamma]$ , is strictly decreasing in the ownership share  $\gamma$ ; it reaches a maximum at  $\gamma = 0$  equal*



to  $a[0] \approx 1.39$ , a minimum at  $\gamma = 1$  equal to  $a[1] \approx 0.32$ , and at  $\gamma \approx 0.30$  it is equal to unity. This affects auction outcomes as follows:

- a) The ex ante expected auction revenue,  $m[a]$ , is increasing in  $\gamma$ .
- b) The expected profit of X,  $\pi_X[\gamma]$ , is decreasing in  $\gamma$ .
- c) The ex ante expected profit of the VIC,  $\pi_{VIP}[\gamma]$ , is increasing in  $\gamma$ .
- d) The ex ante efficiency,  $W[\gamma]$  is decreasing (increasing) in  $\gamma$  for  $\gamma > 0.3$  ( $\gamma < 0.3$ )

**Proof:** See the Appendix.

**Figure 6 Optimal cost weight,  $a[\gamma]$**

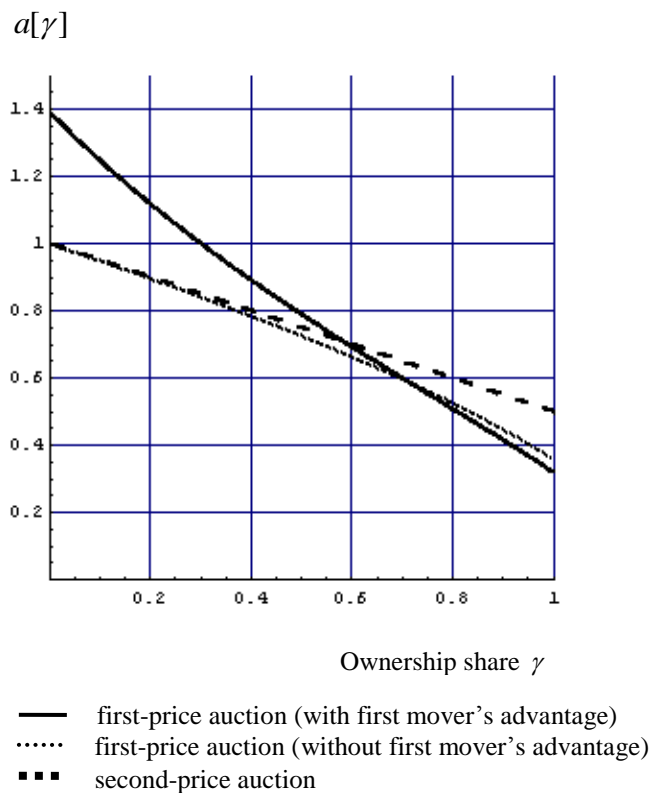


Figure 6 shows the optimal cost weight  $a[\gamma]$  as a function of the ownership share  $\gamma$  for first and second-price auctions (also for first-price auctions without first mover's advantage – details to be explained below). When the ownership share is small,  $\gamma < 0.3$ , the VIC with a first mover's advantage in first-price auctions sets the cost weight *higher* than unity to make V bid *less* aggressively and to *lower* the auction revenue. This is profitable because of the interaction effect in first-price auctions; overstating the costs of bidding makes V a “fat cat” (Fudenberg and Tirole 1984), and the competing independent bidder reacts by also bidding less aggressively which lowers

the bidding costs for both bidders.<sup>18</sup> The negative effect this has through lower auction revenues is of little importance as the VIC has a low ownership share,  $\gamma < 0.3$ . When the ownership share of the auctioneer is larger,  $\gamma > 0.3$ , the VIC sets the cost weight lower than unity to make V bid more aggressively and to increase the auction revenue. For large ownership shares,  $\gamma > 0.6$ , the VIC sets a lower cost weight in first-price auctions than in second-price auctions. This is a result of the interaction effect in first-price auctions, which makes the independent bidder also bid more aggressively which decreases the asymmetry of the auction and thereby makes lowering the cost weight less costly for the VIC.

**Fig. 7 Bidding functions of V and X**

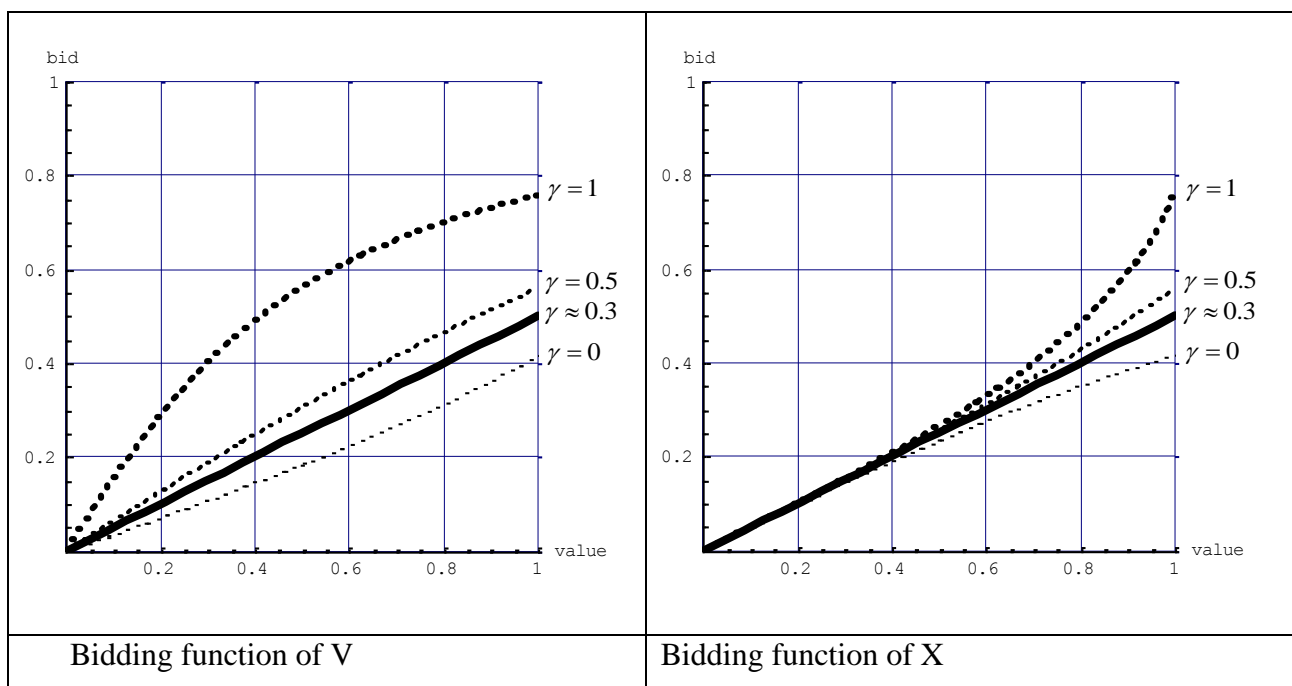


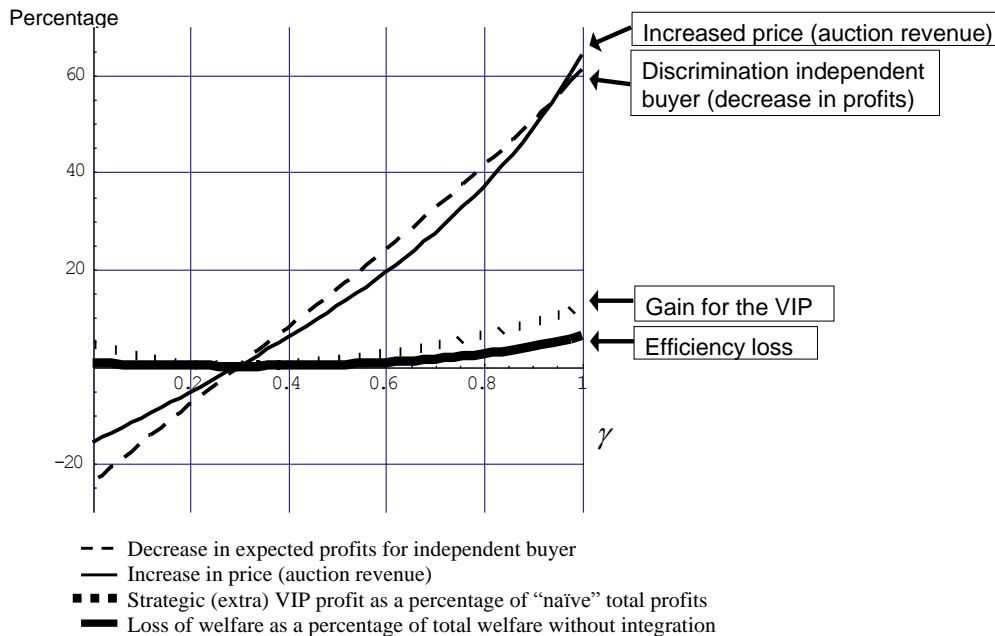
Figure 7 shows the effect of the ownership share  $\gamma$  on the bidding functions. When  $\gamma \approx 0.3$ , the VIC sets the cost weight equal to one, and X and V bid as in a standard symmetrical auction with uniform values  $b[u] = \frac{1}{2}u$  (the thick straight bidding function). For  $\gamma < 0.3$  V bids less aggressively, and for  $\gamma > 0.3$  more aggressively. X accommodates the bidding of V.

Figure 8 shows the effect of integration and legal separation on auction outcomes. The VIC gains from legal separation when the ownership share is smaller or larger than 0.3. When the ownership share is smaller than 0.3, the weaker bidding of V and X lowers the expected price by up to 16%, and increases the profit of the VIC by up to 5%, and increases the profit of X by up to 24%. The asymmetry between X and V (V bids weaker than X) leads to a small efficiency loss up to

<sup>18</sup> This effect is comparable to the “fat cat” effect in Bertrand competition found in Fershtman and Judd (1987) and Sklivas (1987).

0.6%. When the ownership share is larger than 0.3, the aggressive bidding of V increases the expected price by up to 64%, increases the profits of the VIC by up to 12%,<sup>19</sup> and decreases the profits of X by up to 61%. The asymmetry between X and V (V bids stronger than X) leads to an efficiency loss up to 6%.

**Fig. 8 Outcomes in first-price auctions with one independent bidder**



Comparing first-price and second-price auctions, restricting the focus on markets with one competing independent downstream firm, the VIC prefers first-price auctions above second-price auctions when it has either a low or a high ownership share. The VIC has a higher profit in first-price auctions than in second-price auctions when its ownership share is lower than  $\gamma \approx 0.18$ , or higher than  $\gamma \approx 0.79$ .<sup>20</sup> When the ownership share is lower than  $\gamma \approx 0.18$  (higher than  $\gamma \approx 0.79$ ), the interaction effect lowers (increases) the expected winning price which lowers (increases) the expected auction revenue. For ownership shares in between, the VIC prefers second-price auctions.

### 3.2.2 The VIC without a first mover's advantage

One of the assumptions in the preceding model is that the rules on legal separation forbid the VIC from spreading false information about the compensation scheme. As a result the VIC can be sure that the compensation scheme it announces is known and believed by independent bidder X;

<sup>19</sup> For comparison: without legal unbundling of the downstream firm the increase in profits is up to 8.3%. A VIC would thus welcome legal unbundling of its downstream firm.

<sup>20</sup> Determined by numerical approximation.

this gives the VIC a first mover's advantage, which enables the VIC to take advantage of the interaction effect as shown above.

Once this assumption is relaxed and the VIC is allowed, or otherwise able, to provide false information about the compensation scheme, the VIC cannot credibly commit to just any compensation scheme. In the second step in the timeline in Figure 4,  $t_2$ , the VIC now is able to change the compensation scheme with a different cost weight  $a$ ; as a result the VIC has no longer a first mover's advantage and the auction outcomes are less favorable for the VIC.

I calculate the Nash equilibrium cost weight by first supposing that the VIC announces a compensation scheme with cost weight  $a$ , and then, assuming that independent bidder X believes the announcement, maximizes its profits  $\pi_{VIP}[a^{NE}, q]$  with a (possibly different) cost weight  $q$ . A Nash equilibrium exists if and only if the VIC announces a compensation scheme with cost weight  $a^{NE}$  for which  $q = \text{ARGMAX}_q(\pi_{VIP}[a^{NE}, q]) = a^{NE}$ .

For any announced compensation scheme with cost weight  $a$  the bidding function of X is:

$$b_X[u_X; a] = \frac{1 - \sqrt{a^2 u_X^2 + (1 - u_X^2)}}{(1 - a^2)u_X}.$$

Manager  $V^m$  then maximizes his profit given the bidding function of X,  $b_X[u_X; a]$ , and  $q$ , which results in:

$$b_V[u_V; a, q] = \frac{\sqrt{u_V^2 + q^2 - a^2 u_V^2} - q}{(1 - a^2)u_V}.$$

The VIC then sets  $q$  to maximize its compound profit:  $q = \text{ARGMAX}_q(\pi_{VIP}[a, q])$ .

Figure 6 shows numerical approximations<sup>21</sup> for the optimal cost weight  $a^{NE}[\gamma]$  as a function of the ownership share  $\gamma$  for first-price auctions without first mover's advantage (also for first-price auctions with a first mover's advantage and second-price auctions). The VIC without a first mover's advantage cannot strategically use the interaction effect in first-price auctions and therefore sets the cost weight equal to unity for no ownership. Interestingly, the cost weight in first-price auctions without first mover's advantages is close to the cost weight in second-price auctions, but lower and increasingly so when the ownership share increases.

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<sup>21</sup> I used a Mathematica program for approximation. The precise code (with comments) can be downloaded as a Mathematica file from [http://home.cerge-ei.cz/svk/Legally\\_separated](http://home.cerge-ei.cz/svk/Legally_separated).

**Fig. 9 First-price auctions without first mover's advantage**

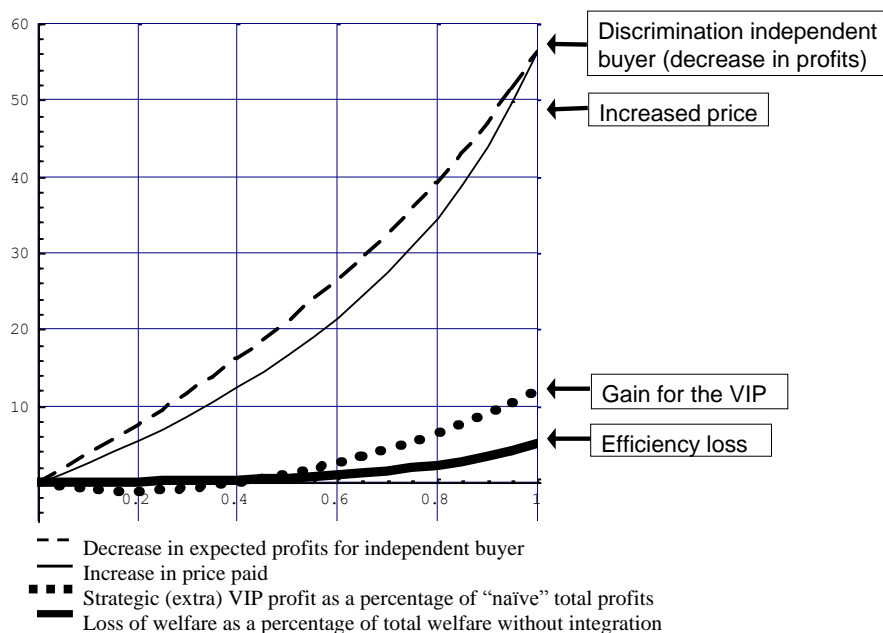


Figure 9 shows the effects on auction outcomes. Note that a VIC *without* a first mover's advantage receives a negative strategic profits for  $0 < \gamma < 0.4$ . Legal separation of the firm from the VIC without a ban on spreading false information about the compensation scheme becomes a burden for a VIC that has a relatively small ownership share of the upstream firm. For higher ownership shares, the strategic profit is very close to that in the first-price auction with a first mover's advantage.

### 3.3 Revenue equivalence

The previous exposition shows that revenue equivalence does not generally hold for these types of auctions. Restricting the focus on markets with two bidders, of which one is independent and one is integrated with full ownership, the auction revenue is approximately equal to 0.42 in second price auctions, to 0.55 in first-price auctions with a first mover's advantage, and to 0.52 in first-price auctions without a first mover's advantage.

This should not be surprising: one of the sufficient assumptions for revenue equivalence, symmetry, does not hold, and the asymmetry that is introduced in this model, a cost weight, affects the auction revenue different for different auction formats. In first-price auctions independent bidders accommodate the bidding of the integrated bidder due to the interaction effect, but do not do so in second-price auctions.

One of the assumptions in the above model is that X is perfectly informed about the value of the cost weight. Relaxing this assumption can reinstate revenue equivalence: if independent bidders in first-price auctions are not informed about the strategic delegation used by the VIC and thus assume

that the integrated bidder maximizes profit (a cost weight equal to unity), then independent bidders do not change their bidding and as a result revenue equivalence between first and second-price auctions holds. Proposition 6 formalizes this intuition.

**Proposition 6:** *When independent bidders  $X_i$  (incorrectly) believe integrated bidder  $V$  to maximize downstream profits, and this belief of independent bidders  $X_i$  is known to  $V$  and the VIC, then the auction revenue is identical in first-price and second-price auctions.*

**Proof:** *See the Appendix.*

The bidding behavior of independent bidders in the auction sketched in Proposition 6 is not equilibrium, and independent bidders are likely to update their belief and to adapt their bidding schedule to accommodate the aggressive bidding of the integrated bidder, thus again upsetting revenue equivalence.

### 3.4 Does the VIC want legal unbundling?

As the analyses in Sections 3.1 and 3.2 show, the VIC is always better off under legal unbundling than under ownership unbundling.

The VIC is also better off under partial legal unbundling (of only the upstream firm) than under legal unbundling (of both the upstream and downstream firm), except in first-price auctions when ownership share are small. In second price auctions the VIC earns the highest profit when the bidding function of its integrated downstream firm maximizes the combined profits of its downstream and upstream firms (see Van Koten 2011), and under partial legal unbundling the VIC can order its downstream firm to do so. The VIC cannot order its downstream firm to do so under legal unbundling, but instead designs a compensation scheme that motivates the integrated bidder to choose a bidding function that imperfectly approximates such a maximizing bidding function. As a result, the profit of the VIC is lower under legal unbundling than under partial legal unbundling. For the same reason the VIC is better off with partial legal unbundling in first-price auctions for high ownership shares. The interaction effect, however, has a small effect on profits that becomes positive when the ownership share is zero or very small. In this case the VIC sets the cost weight larger than one to make the integrated bidder act as a “fat cat” to lower its expected payment when it wins the auction. As a result a VIC prefers legal unbundling when its ownership share is smaller than  $\gamma \approx 0.13$ .<sup>22</sup>

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<sup>22</sup> Determined by numerical approximation.

## 4. Conclusions

I modeled a Vertically Integrated Corporation (VIC) that owns both a monopoly upstream firm and a downstream firm in a competitive market. As the monopoly upstream firm provides an essential, scarce input, the VIC has been forced by regulation to allocate its products or services by auction. In an earlier paper, Van Koten (2011), I showed that a VIC could increase nonetheless its profits by having its downstream firm bid more aggressively; which increases the profits of the VIC, lowers welfare, and lowers the profits of competing downstream firms. In the present paper, I explored in a similar setup to which extent the additional legal separation of the downstream firm from the VIC could improve auction outcomes. When the downstream firm is legally separated, the VIC can no longer implement a compensation scheme to maximize the profits of the overall VIC.<sup>23</sup> However, the compensation scheme analyzed in this paper mimics maximizing of the total VIC profits to a considerable degree, while respecting the legal separation. By implementing this compensation scheme, the VIC increases its profits, increases the auction revenue, decreases efficiency and decreases the profits of independent downstream firms.

My model suggests that ownership separation is a solution: once the VIC is not the residual claimant of the auction revenue any more, it loses the incentive to have its integrated firm bid excessively aggressively. Applied to the electricity market this remedy implies outlawing VICs to have their generator firms bid for capacity on its merchant interconnectors. Another possible remedy is to strictly regulate the auction revenue and prevent VICs from receiving the auction revenues of their upstream firm, and instead use rate-of-return regulation. Applied to the electricity market this remedy implies outlawing the building and operating of merchant interconnectors by VICs that own generation firms. Alternatively, regulators might forbid use of the compensation scheme. However, this requires regulators to have a very good understanding of the operations of the downstream firm; they have to be able to determine (and, likely, to defend in court) to which extent a compensation scheme maximizes the profit of the downstream firm as opposed to the profit of the VIC.

## 5. Appendix

**Proposition 2:** *In a second-price auction with  $n+1$  bidders, of which  $n$  are independent and one is integrated, the VIC sets the cost weight for the integrated bidder equal to  $a^{(n)}[\gamma] = 1 - \frac{\gamma}{n+1}$ , for any*

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<sup>23</sup> I assume here that legal separation is effective. In cases where it is likely that violations of the restrictions imposed by legal separation go unpunished, the VIC can freely instruct its downstream firm to maximize the profits of the VIC. I show in Van Koten (2011) that the qualitative outcomes remain unchanged (the VIC profits, welfare and competing downstream firms suffers).

$n \geq 1$ .

**Proof:** Using  $V$  for the integrated bidder and  $X$  for the independent bidders, the profit function for the VIC has been derived by Van Koten (2011) as:

$$2) \pi_{VIP, u_V \leq a}^{(n)}[u_V] = \Pr_{V \text{ wins, } u_V \leq a}[b_V] \cdot u_V - (1 - \gamma) \cdot m_{V, u_V \leq a}[b_V] + \gamma \cdot m_{\sum X, u_V \leq a}[b_V], \text{ with}$$

$$m_{V, u_V \leq a}[b_V] = \Pr[V \text{ wins}] \cdot E[\text{highest bid from } n \text{ bidders} \mid V \text{ wins and } u_V \leq a],$$

$$m_{\sum X, u_V \leq a}[b_V] = \Pr[V \text{ has } 2^{\text{nd}} \text{ highest bid}] \cdot b_V +$$

$$\sum_{i=2}^n \Pr[V \text{ has } (i+1)^{\text{th}} \text{ highest bid}] \cdot E[2^{\text{nd}} \text{ highest bid from } n-1 \text{ bidders} \mid V \text{ has } (i+1)^{\text{th}} \text{ highest bid}].$$

The form of equation 1) depends on the size of  $u_V$ ; if  $u_V \leq a$  equation 1) takes the following form:

$$12) m_{V, u_V \leq a}[b_V] = b_V^n \cdot \frac{1}{b_V^n} \int_0^{b_V} n z^{n-1} z dz,$$

$$= \frac{n}{n+1} b_V^{n+1},$$

and

$$13) m_{\sum X, u_V \leq a}[b_V] = (n b_V^{n-1} (1 - b_V) b_V) + \sum_{i=2}^n \left( \frac{n!}{(n-i)! i!} b_V^{n-i} (1 - b_V)^i \int_{b_V}^1 \frac{i(i-1)(1-z)(z-b_V)^{i-2}}{(1-b_V)^i} z dz \right),$$

$$= \frac{1}{n+1} (n-1 + b_V^n (n+1 - 2n b_V)).$$

Thus substituting  $b_V[u_V] = \frac{u_V}{a}$  into equation 1 and simplifying gives:

$$14) \pi_{VIP, u_V \leq a}^{(n)}[u_V] = \frac{1}{a^{n+1}(n+1)} (a^{n+1} j(n-1) + u_V^n (-1-j) n u_V + a(1+n)(j + u_V)).$$

If  $u_V > a$ , then manager  $V$  wins with probability one (as  $b_V[u_V] = \frac{u_V}{a} > 1 > \text{MAX}(b_X)$ ), and as

$$15) m_{V, u_V > a}[b_V] = \int_0^1 n z^{n-1} z dz = \frac{n}{n+1},$$

the expected profit of the VIC is then:

$$16) \pi_{VIP, u_V > a}^{(n)}[u_V] = u_V - (1 - \gamma) \cdot m_{V, u_V > a}[b_V] = u_V - (1 - \gamma) \frac{n}{1+n},$$

The ex ante expected profit of the VIC is thus equal to:

$$17) E\pi_V^{(n)}[a, \gamma] = \int_0^a \pi_{VIP, u_V \leq a}^{(n)} du_V + \int_a^1 \pi_{VIP, u_V > a}^{(n)} du_V,$$



$$= \frac{2 - n(a^2(n+1) + 2a(\gamma - (n+1)) + n + 1 - 2\gamma(n+2))}{2(n+2)(n+1)}.$$

Differentiating the expected profit of the VIC to  $a$  gives the first order condition:

$$18) \frac{dE\pi_V^{(n)}[a, \gamma]}{da} = (1-a) \frac{n}{n+2} - \gamma \frac{n}{2+3n+n^2}.$$

Setting the first order condition equal to zero and solving for  $a$  gives:

$$19) a^{(n)}[\gamma] = 1 - \frac{\gamma}{n+1}.$$

**Proposition 3:** In a second-price auction with  $n+1$  bidders, of which  $n$  are independent and one is integrated and has cost weight  $a^{(n)}[\gamma] = 1 - \frac{\gamma}{n+1}$ , where values are distributed independently and uniformly on  $[0,1]$ , the independent bidders bid their value, and the integrated bidder bids  $b_V[u_V] = u_V(1 + \frac{\gamma}{n+1-\gamma})$ , for any  $n \geq 1$ . As a result, with increasing  $\gamma$  for all  $n \geq 1$ :

a) The ex ante expected auction revenue,  $Em^{(n)}[\gamma] = \frac{n(\gamma + (n+1)^2)}{(n+2)(n+1)^2}$ , is increasing in  $\gamma$ .

b) The ex ante expected profit of  $V$ ,  $E\pi_V^{(n)}[\gamma] = \frac{2+n(2+\gamma^2+2\gamma(n+1)^2)}{2(n+2)(n+1)^2}$ , is increasing in  $\gamma$ .

The strategic profit, the increase in profits relative to not setting a cost weight, is equal to

$$\frac{E\pi_{VIP}^{(n)}[\gamma] - (E\pi_{VIP}^{(n)}[0] + \gamma m^{(n)}[0])}{(E\pi_{VIP}^{(n)}[0] + \gamma m^{(n)}[0])} = \frac{1}{2} \frac{n}{(n+1)} \gamma(\gamma + 2n^2 + 4n + 2).$$

c) The expected profit of  $X_i$ ,  $\pi_{X_i}^{(n)}[\gamma] = \frac{n+1-\gamma}{(n+1)^2(n+2)}$ , is decreasing in  $\gamma$ . The relative loss in

profit for each independent bidder,  $\frac{\pi_{X_i}^{(n)}[\gamma] - \pi_{X_i}^{(n)}[0]}{\pi_{X_i}^{(n)}[0]} = \frac{\gamma}{n+1}$ , is increasing in  $\gamma$ .

d) Efficiency,  $W^{(n)}[\gamma] = \frac{2n^3 + 6n^2 + 6n + 2 - n\gamma^2}{2(n+1)^2(n+2)} W^{(n)}[\gamma]$ , is decreasing in  $\gamma$ .

**Proof:**

a) The ex ante expected auction revenue,  $Em^{(n)}[\gamma] = \frac{n(\gamma + (n+1)^2)}{(n+2)(n+1)^2}$ , is increasing in  $\gamma$ . The ex ante expected payment by  $V$  is equal to

$$\begin{aligned}
20) \quad Em_V^{(n)}[\gamma] &= \int_0^a \left( m_{V, v_Y \leq a} \left[ \frac{z}{a} \right] \right) dz + \int_a^1 \left( m_{V, v_Y > a} \left[ \frac{z}{a} \right] \right) dz \Bigg|_{a=1-\frac{\gamma}{n+1}} \\
&= \int_0^a \left( \frac{n}{n+1} \left( \frac{z}{a} \right)^{n+1} \right) dz + \int_a^1 \left( \frac{n}{n+1} \right) dz \Bigg|_{a=1-\frac{\gamma}{n+1}}, \text{ (using equation 2 and 4)} \\
&= \frac{(1+\gamma)n}{n^2+3n+2}
\end{aligned}$$

The ex ante expected payment by all  $X$  is equal to

$$\begin{aligned}
21) \quad Em_{\sum X}^{(n)}[\gamma] &= \int_0^a m_{\sum X, z \leq a} \left[ \frac{z}{a} \right] dz \Bigg|_{a=1-\frac{\gamma}{n+1}}, \\
&= \int_0^a \left( n \frac{z^{n-1}}{a^{n-1}} \left( 1 - \frac{z}{a} \right) \frac{z}{a} \right) + \sum_{i=2}^n \left( \frac{n!}{(n-i)!i!} \frac{z^{n-i}}{a^{n-i}} \left( 1 - \frac{z}{a} \right)^i \int_{\frac{z}{a}}^1 \frac{i(i-1)(1-q)(q-\frac{q}{a})^{i-2}}{(1-\frac{q}{a})^i} q dq \right) dz \Bigg|_{a=1-\frac{\gamma}{n+1}}, \\
&= \frac{n^2(n+1-\gamma)}{(n+1)^2(n+2)}.
\end{aligned}$$

Thus:

$$\begin{aligned}
22) \quad Em^{(n)}[\gamma] &= Em_V^{(n)}[\gamma] + Em_{\sum X}^{(n)}[\gamma] \\
&= \frac{(1+\gamma)n}{n^2+3n+2} + \frac{n^2(n+1-\gamma)}{(n+1)^2(n+2)}, \\
&= \frac{n(\gamma+(n+1)^2)}{(n+2)(n+1)^2}.
\end{aligned}$$

b) The ex ante expected profit of the VIC,  $E\pi_{VIP}^{(n)}[\gamma] = \frac{2+n(2+\gamma^2+2\gamma(n+1)^2)}{2(n+2)(n+1)^2}$ , is increasing in  $\gamma$ .

The relative increase in profits by setting a cost weight is equal to

$$\frac{E\pi_{VIP}^{(n)}[\gamma] - (E\pi_{VIP}^{(n)}[0] + \gamma m^{(n)}[0])}{(E\pi_{VIP}^{(n)}[0] + \gamma m^{(n)}[0])} = \frac{1}{2} \frac{n}{(n+1)} \gamma(\gamma + 2(n+1)^2).$$

Using equation 6,  $E\pi_{VIP}^{(n)}[a, \gamma] = \frac{2-n(a^2(n+1)+2a(\gamma-(n+1))+n+1-2\gamma(n+2))}{2(n+2)(n+1)}$ , substituting for  $a$

with  $a[\gamma] = 1 - \frac{\gamma}{n+1}$  gives  $E\pi_{VIP}^{(n)}[\gamma] = \frac{2+n(2+\gamma^2+2\gamma(n+1)^2)}{2(n+2)(n+1)^2}$ . Differentiating to  $\gamma$  shows that

$E\pi_{VIP}^{(n)}[\gamma]$  is increasing.

The relative increase in profits by setting a cost weight is equal to

$$\frac{E\pi_{VIP}^{(n)}[\gamma] - (E\pi_{VIP}^{(n)}[0] + \gamma m^{(n)}[0])}{(E\pi_{VIP}^{(n)}[0] + \gamma m^{(n)}[0])} = \frac{1}{2} \frac{n}{(n+1)} \gamma (\gamma + 2(n+1)^2).$$

c) The expected profit of  $X_i$ ,  $\pi_{X_i}^{(n)}[\gamma] = \frac{n+1-\gamma}{(n+1)^2(n+2)}$ , is decreasing in  $\gamma$ . The relative loss in

profit for each independent bidder,  $\frac{\pi_{X_i}^{(n)}[\gamma] - \pi_{X_i}^{(n)}[0]}{\pi_{X_i}^{(n)}[0]} = \frac{\gamma}{n+1}$ , is increasing in  $\gamma$ .

The ex ante expected payment of all independent bidders  $X_i$  is equal to  $Em_{\sum X}^{(n)}[\gamma] = \frac{n^2(n+1-\gamma)}{(n+1)^2(n+2)}$ ,

(see calculations under a). Using symmetry, the ex ante expected payment of an independent bidder  $X_i$  is thus equal to this number divided by  $n$ :

$$Em_{X_i}^{(n)}[\gamma] = \frac{1}{n} Em_{\sum X}^{(n)}[\gamma] = \frac{n(n+1-\gamma)}{(n+1)^2(n+2)}$$

The ex ante expected gross profit of the auction (excluding payment),  $E\pi_{X_i}^{Gross}[\gamma]$ , for an

independent bidder  $X_i$  is equal to  $E\pi_{X_i}^{Gross}[\gamma] = \int_0^1 P_{X_i \text{ wins}}[u_X] \cdot u_X du_X$ , and

$P_{X_i \text{ wins}}[u_{X_i}] = \left(1 - \frac{\gamma}{n+1}\right) u_{X_i}^{n-1}$  as the probability of an independent bidder to win the auction is equal to the probability that his bid is higher than that of the integrated bidder (which is the case when  $v_{X_i} \geq \left(1 - \frac{\gamma}{n+1}\right) u_V$ ), and higher than that of the other  $n-1$  independent bidders. Thus

$$E\pi_{X_i}^{Gross}[\gamma] = \frac{n+1-\gamma}{n^2+3n+2}$$

The expected profit of an independent bidder  $X_i$  is thus equal to the ex ante expected gross profit minus the ex ante expected payment:

$$\begin{aligned} E\pi_{X_i}^{(n)}[\gamma] &= E\pi_{X_i}^{Gross}[\gamma] - Em_{X_i}^{(n)}[\gamma], \\ &= \frac{n+1-\gamma}{n^2+3n+2} - \frac{n(n+1-\gamma)}{(n+1)^2(n+2)}, \\ &= \frac{n+1-\gamma}{(n+1)^2(n+2)}. \end{aligned}$$

The relative loss in profit for an independent bidder is then:

$$\frac{E\pi_{X_i}^{(n)}[\gamma] - E\pi_{X_i}^{(n)}[0]}{E\pi_{X_i}^{(n)}[0]} = \frac{\gamma}{n+1}.$$

d) Efficiency,  $W^{(n)}[\gamma] = \frac{2n^3 + 6n^2 + 6n + 2 - n\gamma^2}{2(n+1)^2(n+2)}$ , is decreasing in  $\gamma$ .

Efficiency is equal to the profit of the  $n$  independent bidders, the profit of the VIC, and the auction revenue that has not been received by the VIC:

$$W^{(n)}[\gamma] = nE\pi_{x_i}[\gamma] + E\pi_{VIP}[\gamma] + (1-\gamma)Em[\gamma].$$

Substituting for  $\pi_{VIP}[\gamma]$  from a),  $m[\gamma]$  from b), and  $\pi_{x_i}[\gamma]$  from c) gives:

$$W^{(n)}[\gamma] = \frac{2n^3 + 6n^2 + 6n + 2 - n\gamma^2}{2(n+1)^2(n+2)}.$$

**Proposition 4:** In a first-price auction with two bidders, of which one is independent and one is integrated and has cost weight  $a$ , where values are distributed independently and uniformly on  $[0,1]$ , the bidding functions of  $X$  and  $V$  are given by:

$$9) \quad b_v[u_v] = \frac{\sqrt{u_v^2 + a^2(1-u_v^2)} - a}{(1-a^2)u_v}$$

$$10) \quad b_x[u_x] = \frac{1 - \sqrt{a^2u_x^2 + (1-u_x^2)}}{(1-a^2)u_x} \text{ for } a \neq 1 \text{ and } b_x[u_x] = \frac{1}{2}u_x \text{ for } a = 1.$$

The maximum bid  $\bar{b}$  is equal to  $\bar{b} = \frac{1}{(1+a)}$ .

**Proof:** The conditions that the inverse bidding functions  $x[b]$  and  $v[b]$  should fulfill are:

$$23) \quad x'[b] = \frac{a \cdot x[b]}{u_v - ab} = \frac{a \cdot x[b]}{v[b] - ab}.$$

$$24) \quad v'[b] = \frac{v[b]}{u_x - b} = \frac{v[b]}{x[b] - b}.$$

Furthermore, a solution should fulfill the following additional constraint:

$$25) \quad x[0] = y[0] = 0 \quad (\text{a bidder with value zero bids zero}).$$

$$26) \quad x[\bar{b}] = y[\bar{b}] = 1, \text{ where } \bar{b} \text{ is the maximum bid } 0 < \bar{b} < 1 \\ (\text{a bidder with value 1 bids a unique maximal bid}).$$

Rewriting 25) and 26) gives

$$27) \quad x'[b] \cdot (v[b] - ab) = ax[b] \Leftrightarrow$$

$$28) \quad (x'[b] - 1) \cdot (v[b] - ab) = ax[b] - v[b] + ab$$

$$29) \quad v_y[b] \cdot (x[b] - b) = v[b] \Leftrightarrow$$

$$30) \quad (v'[b] - a) \cdot (x[b] - b) = v[b] - ax[b] + ab.$$

Adding up 27) and 29) gives

$$31) \quad (x'[b]-1) \cdot (v[b]-ab) + (x[b]-b) \cdot (v'[b]-a) = 2ab \Leftrightarrow$$

$$32) \quad \frac{\partial}{\partial b} (x[b]-b) \cdot (v[b]-ab) = 2ab.$$

Integrating 32) over 0 until the maximum bid  $\bar{b}$  using  $x[0] = v[0] = 0$  gives

$$33) \quad (1-\bar{b}) \cdot (1-a\bar{b}) = a\bar{b}^2 \Leftrightarrow$$

$$34) \quad 1+a\bar{b}^2 - (1+a)\bar{b} = a\bar{b}^2.$$

Therefore the maximum bid  $\bar{b}$  is given by

$$35) \quad \bar{b} = \frac{1}{1+a}.$$

Integrating 32) over 0 until  $b$  using  $x[0] = v[0] = 0$  gives

$$36) \quad (x[b]-b) \cdot (v[b]-ab) = ab^2.$$

Applying 36) to 25) and 26) gives

$$37) \quad x'[b] = \frac{x[b](x[b]-b)}{b^2},$$

$$38) \quad v'[b] = \frac{v[b](v[b]-ab)}{ab^2}.$$

Using 12) substituted into the condition  $x[\bar{b}] = v[\bar{b}] = 1$ , 37) and 38) can be shown to have the solution:

$$39) \quad x[b] = \frac{2b}{1+b^2-a^2b^2}.$$

$$40) \quad v[b] = \frac{2ab}{1-b^2+a^2b^2}.$$

Taking inverses gives us the optimal pure bidding strategies:

$$9) \quad b_v[u_v] = \frac{\sqrt{u_v^2 + a^2(1-u_v^2)} - a}{(1-a^2)u_v}$$

$$10) \quad b_x[u_x] = \frac{1 - \sqrt{a^2 u_x^2 + (1-u_x^2)}}{(1-a^2)u_x} \text{ for } a \neq 1 \text{ and } b_x[u_x] = \frac{1}{2}u_x \text{ for } a = 1.$$

Differentiating the bidding functions to  $a$  gives, for  $a > 0, a \neq 1$  and  $0 < u < 1$ :

$$41) \quad \frac{db_Y[u_V]}{da} = \frac{au}{u_V - a^2 u_V} \left( \frac{1 + a^2 + u_V^2(1 - a^2)}{\sqrt{u_V^2 + a^2(1 - u_V^2)}} - (1 + a^2) \right) < 0,$$

$$42) \quad \frac{db_X[v_X]}{da} = \frac{a}{(1 - a^2)^2 u_V} \left( \frac{-2 + u_V^2(1 - a^2) + 2\sqrt{1 - (1 - a^2)u^2}}{\sqrt{1 - (1 - a^2)u_V^2}} \right) < 0.$$

**Proposition 5:** In a first-price auction with two bidders, of which one is independent and one is integrated and has cost weight  $a$ , where values are distributed independently and uniformly on  $[0,1]$ , the optimal cost weight,  $a[\gamma]$ , is strictly decreasing in the ownership share  $\gamma$ ; it reaches a maximum at  $\gamma = 0$  equal to  $a[0] \approx 1.39$ , a minimum at  $\gamma = 1$  equal to  $a[1] \approx 0.32$ , and at  $\gamma \approx 0.30$  it is equal to unity. This affects auction outcomes as follows:

- a) The ex ante expected auction revenue,  $m[a]$ , is increasing in  $\gamma$ .
- b) The expected profit of  $X$ ,  $\pi^X[\gamma]$ , is decreasing in  $\gamma$ .
- c) The ex ante expected profit of the VIC,  $\pi_{VIP}[\gamma]$ , is increasing in  $\gamma$ .
- d) The ex ante efficiency,  $W[\gamma]$  is decreasing (increasing) in  $\gamma$  for  $\gamma > 0.3$  ( $\gamma < 0.3$ )

**Proof:** I first prove that the optimal cost weight,  $a[\gamma]$ , is decreasing in the ownership share  $\gamma$ :

$$\text{sign} \left[ \frac{da[\gamma]}{d\gamma} \right] = -1.$$

The profit function of the VIC is given by:

$$\pi_{VIP}[a, \gamma] = \int_0^1 \left( x[b_V] \cdot (u_V - (1 - \gamma)b_V(a, u_V)) + \gamma \int_{x[b_V[u_V]]}^1 b_X(a, u_X) du_X \right) du_V.$$

Using the above functions, it can be shown that  $\pi_{VIP}[a, \gamma]$  is twice continuously differentiable for  $a > 0, a \neq 1$  and  $0 < v < 1$ .

Differentiating the profit function with respect to  $a$  at the optimal cost weight  $a(\gamma)$  gives:

$$\frac{d\pi_{VIP}[a(\gamma), \gamma]}{da} \equiv 0.$$

Differentiating again to the ownership share  $\gamma$  gives:

$$\frac{d^2\pi_{VIP}[a(\gamma), \gamma]}{(da)^2} \frac{da(\gamma)}{d\gamma} + \frac{d^2\pi_{VIP}[a(\gamma), \gamma]}{da \cdot d\gamma} = 0.$$

$$\text{Thus } \text{sign} \left[ \frac{da[\gamma]}{d\gamma} \right] = \text{sign} \left[ \frac{d^2\pi_{VIP}[a[\gamma], \gamma]}{da \cdot d\gamma} \right], \text{ as } \frac{d^2\pi_{VIP}[a(\gamma), \gamma]}{(da)^2} < 0.$$

As  $\pi_{VIP}[a, \gamma]$  is twice continuously differentiable,  $\frac{d^2 \pi_{VIP}[a[\gamma], \gamma]}{da \cdot d\gamma} = \frac{d^2 \pi_{VIP}[a[\gamma], \gamma]}{d\gamma \cdot da}$ .

Using the envelope theorem,

$$\begin{aligned} \frac{d\pi_{VIP}[a[\gamma], \gamma]}{d\gamma} &= \int_0^1 \left( x[b_V[a, u_V]] \cdot b_V[a, u_V] + \int_{x[b_V[a, u_V]]}^1 b_X[a, u_X] du_X \right) du_V \\ &= (1-a^2)^{-\frac{3}{2}} \left( (1-a)a\sqrt{1-a^2} + \text{ArcCos}[a] + \text{Ln}\left[\frac{a}{1+\sqrt{1-a^2}}\right] + a^2 \left( -2\text{ArcCsch}\left[\frac{a}{\sqrt{1-a^2}}\right] + i\text{Ln}\left[\frac{a-i\sqrt{1-a^2}}{a+i\sqrt{1-a^2}}\right] \right) \right) \end{aligned}$$

Numerical inspection of the above expression shows that it decreasing in  $a$  for all  $a > 1, a \neq 1$ .

And thus  $\text{sign}\left[\frac{da[\gamma]}{d\gamma}\right] = \text{sign}\left[\frac{d^2 \pi_{VIP}[a[\gamma], \gamma]}{da \cdot d\gamma}\right] = -1$ .

a) The ex ante expected auction revenue is increasing in  $\gamma$ .

The bidding functions are strictly decreasing in  $a$ , thus the auction revenue

$$\begin{aligned} \tilde{m}[a] &= \int_0^1 \int_0^1 \text{Max}[b_V[u_V, a], b_X[u_X, a]] du_V du_X \\ &= \frac{1}{1+a} - a(1-a^2)^{-\frac{3}{2}} \left( \text{ArcCsch}\left(\frac{a}{\sqrt{1-a^2}}\right) + \text{ArcSinh}\left(\sqrt{a^2-1}\right) \right) \text{ is strictly decreasing in } a, \text{ and} \end{aligned}$$

thus  $m[\gamma] = \tilde{m}[a] \Big|_{a=a[\gamma]}$  strictly increasing in  $\gamma$ .

b) The ex ante expected profit of  $X$  is decreasing in  $\gamma$ .

The profit of independent bidder  $X$  as a function of  $a$ ,

$$\tilde{\pi}^X[a] = \frac{a\left((a-2)\sqrt{a^2-1} + \text{ArcSinh}(\sqrt{a^2-1})\right)}{2(a^2-1)^{\frac{3}{2}}}, \text{ is strictly increasing in cost weight } a, \text{ and thus}$$

$\pi^X[\gamma] = \tilde{\pi}^X[a] \Big|_{a=a[\gamma]}$  is strictly decreasing in  $\gamma$ .

c) The ex ante expected profit of the VIC is strictly increasing in  $\gamma$ .

The profit of the VIC is given by:

$$\pi_{VIC}[\gamma] = \int_0^1 \left( x[b_V] \cdot (u_V - (1-\gamma)b_V(a, u_V)) + \gamma \int_{x[b_V[u_V]]}^1 b_X(a, u_X) du_X \right) du_V \Big|_{a=a[\gamma]}.$$

As the bidding functions  $b_V$  and  $b_X$  are strictly increasing  $\gamma$ , the profit is strictly increasing in  $\gamma$  for any fixed value of  $a$ . Allowing  $a$  to change to maximizing the profit weakly increases the profit.

Inspecting the explicit expression of the profit of the VIC verifies this:

$$\tilde{\pi}_{VIP}[\gamma] = \frac{\sqrt{1-a^2} - (2-a)a \operatorname{ArcCsch}\left(\frac{a}{\sqrt{1-a^2}}\right)}{2(1-a^2)^{\frac{3}{2}}} - w^0 + \gamma \left( \frac{1}{1+a} - a(1-a^2)^{-\frac{3}{2}} \left( \operatorname{ArcCsch}\left(\frac{a}{\sqrt{1-a^2}}\right) + \operatorname{ArcSinh}\left(\sqrt{a^2-1}\right) \right) \right) \Big|_{a=a[\gamma]}.$$

e) The ex ante efficiency,  $W[\gamma]$  is decreasing (increasing) in  $\gamma$  for  $\gamma > 0.3$  ( $\gamma < 0.3$ )

The welfare as a function of  $a$ ,

$$\tilde{W}[a] = \frac{1}{2} \left( 1 - a(1-a^2)^{-\frac{3}{2}} \left( a \operatorname{ArcCsch}\left(\frac{a}{\sqrt{1-a^2}}\right) + \operatorname{ArcSinh}\left(\sqrt{a^2-1}\right) \right) \right), \text{ is maximized at the cost}$$

weight  $a = 1$ , which is reached at  $\gamma \approx 0.3$  and thus  $W[\gamma]$  is decreasing (increasing) in  $\gamma$  for  $\gamma > 0.3$  ( $\gamma < 0.3$ ).

**Proposition 6:** When independent bidders  $X_i$  (incorrectly) believe integrated bidder  $V$  to maximize downstream profits, and the belief of independent bidders  $X_i$  is known by  $V$  and the VIC, then the auction revenue is identical in first-price and second-price auctions.

**Proof:** Independent bidders  $X_i$  believing integrated bidder  $V$  to maximize downstream profits implies  $X_i$  believing  $V$  to maximize a compensation scheme with a cost weight set equal to one. Independent bidders  $X_i$  then bid as in the symmetrical mode:

$$b_{X_i} = \frac{n}{n+1} u_{X_i}, \text{ and the highest bid is equal to } \bar{b} = \frac{n}{n+1}.$$

The VIC and  $V$  are informed of the bidding functions of the independent bidders. Given value  $u_V$  and bid  $b_V$ , the expected profit of  $V$  will be:

$$43) \quad \pi_V[u_V, b_V] = \left( \frac{n+1}{n} b_V \right)^n (u - a b_V).$$

The first part is the probability that  $V$  wins the auction; this follows from

$$b_{X_i} < b_V \Leftrightarrow \frac{n}{n+1} u_{X_i} < b_V \Leftrightarrow u_{X_i} < \frac{n+1}{n} b_V. \text{ The last inequality holds for all } n \text{ independent bidders}$$

with probability  $\left( \frac{n+1}{n} b_V \right)^n$ . The second part is the value minus the cost weight,  $a$ , times the

payment when winning,  $b_V$ .

Maximizing  $\pi_V[u, b_V]$  for  $u < a$  gives the bidding function for  $V$ :



$$44) \quad b_V[u_V] = \frac{n}{(n+1)a} u_V, \quad \text{when } u < a$$

$$45) \quad b_V[u_V] = \frac{n}{(n+1)}, \quad \text{when } u > a.$$

When  $u < a$ ,  $V$  wins the auction when  $u_{X_i} < \frac{n+1}{n} b_V \Leftrightarrow u_{X_i} < \frac{n+1}{n} \frac{n}{(n+1)a} u_V = \frac{u_V}{a}$ .

When  $u > a$ ,  $V$  bids the highest bid  $\bar{b} = \frac{n}{n+1}$  and wins the auction with probability one.

The VIC has profit function:

$$\pi_{VIP}[a, \gamma] = \int_0^a \left( \frac{u_V}{a} \right) \cdot \left( u_V - (1-\gamma) \frac{n}{n+1} \frac{u_V}{a} \right) du_V + \int_a^1 \left( u_V - (1-\gamma) \frac{n}{n+1} \right) du_V + \gamma \int_0^a \left\{ \int_{\frac{u_V}{a}}^1 \left( \frac{nz}{n+1} nz^{n-1} \right) dz \right\} du_V$$

The first two integrals give the expected profit when  $V$  wins the auction, the first integral when  $u_V < a$ , the second when  $u_V > a$ . The third integral gives the expected payment of the independent bidders. This is equal to the expected highest bid of  $n$  bidders conditional on  $V$  losing the auction (thus conditional on  $\frac{u_V}{a}$  not being the highest value). Calculating the integrals gives

$\pi_{VIP}[a, \gamma] = \frac{1}{30} (-10a^2 + 4(5-\gamma)a - 9 + 24\gamma)$ , and maximizing this function for  $a[\gamma]$  gives

$a[\gamma] = 1 - \frac{\gamma}{n+1}$ , the same cost weight as in second-price auctions.

The auction revenue is equal to:

$$m[a, \gamma] = \int_0^a \left( \frac{u_V}{a} \right) \cdot \left( \frac{n}{n+1} \frac{u_V}{a} \right) du_V + \int_a^1 \left( \frac{n}{n+1} \right) du_V + \int_0^a \left\{ \int_{\frac{u_V}{a}}^1 \left( \frac{nz}{n+1} nz^{n-1} \right) dz \right\} du_V.$$

The first two integrals give the expected payment of  $V$ , the first integral when  $u_V < a$ , the second when  $u_V > a$ . The third integral gives the expected payment of the independent bidders. Calculating

the integrals and substituting  $a[\gamma] = 1 - \frac{\gamma}{n+1}$  gives  $Em^{(n)}[\gamma] = \frac{n(\gamma + (n+1)^2)}{(n+2)(n+1)^2}$ , which is identical to

the auction revenue in second-price auctions.

### Example: a compensation scheme for “Manager X”?

In the text it is shown that owner  $X$  will not give its manager incentives different from profit maximizing as he has no ownership share in the upstream firm running the auction. In other words, owner  $X$  will always provide a compensation scheme with cost weight  $s = 1$ . For an illustration of this general principle, suppose that both the VIC and owner  $X$  had the opportunity to offer their managers compensation schemes and commit to it. Furthermore, the VIC has full ownership of the upstream firm and  $X$  has no ownership. Assuming that the optimal choices of cost weights for both

managers are determined simultaneously, owner X would offer cost weight  $\tilde{s} \approx 1.431$ , which makes both bidders bid less aggressively, and the VIC would offer cost weight  $\tilde{a} \approx 0.308 < 0.319 \approx a^{1*}$ , which makes both bidders bid more aggressively ( $a^{1*}$  is the cost weight the VIC would have chosen for  $\tilde{s} = 1$ ). The bidding functions of independent bidder X and integrated bidder V would be:

$$b_X[u_X; \tilde{a}, \tilde{s}] = \frac{\sqrt{\tilde{s}^2(1-u_X^2) + \tilde{a}^2 u_X^2} - \tilde{s}}{(\tilde{a}^2 - \tilde{s}^2)u_X}, \quad b_V[u_V; \tilde{a}, \tilde{s}] = \frac{\sqrt{\tilde{a}^2(1-u_V^2) + \tilde{s}^2 u_V^2} - \tilde{a}}{(\tilde{s}^2 - \tilde{a}^2)u_V}. \quad 24$$

The maximum bid would be  $\bar{b} \approx 0.575$ . In this case, the profits of X would increase to  $\pi_X[\tilde{a}, \tilde{s}] \approx 0.071 > 0.065 \approx \pi_X[a^{1*}, s = 1]$ .<sup>25</sup>

The result above can be generalized for any ownership share holding of the VIC. Owner X offers a cost weight of  $s \approx 1.37$  when the ownership share of the VIC is zero, and increases the cost weight when the ownership share of the VIC increases up to a maximum of  $s \approx 1.431$ , when the VIC has a full ownership share.

Owner X cannot, however, credibly commit to these compensation schemes without a legal requirement to publicly announce his compensation scheme; he has the possibility to provide a (secret) side contract that sets  $s = 1$  (maximizing profits). Independent bidder X then finds its bidding function by maximizing his profits, given the above bidding function of V;  $b_V[u_V; \tilde{a}, \tilde{s}]$ . While V would believe that X chooses the bidding function  $b_X[u_X; \tilde{a}, \tilde{s}]$  as described above, X chooses instead the bidding function:

$$\frac{\sqrt{1-u_X^2 + \tilde{a}^{*2} u_X^2} - 1}{(\tilde{a}^{*2} - \tilde{s}^2)u_X} \quad \text{for } u_X < 0.699$$

$$0.575 \quad \text{for } u_X > 0.699.$$

X then earns a profit of  $\pi_X[\tilde{a}, s = 1] \approx 0.105 > 0.071 \approx \pi_X[\tilde{a}, \tilde{s}]$ . As this deviation is profitable for owner X, him setting  $\tilde{s} > 1$  cannot be part of a Nash equilibrium.

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<sup>24</sup> These formulas are obtained by solving 4) and a likewise equation for the manager of allied bidder X with cost weight  $s$ .

<sup>25</sup> Other interesting auction outcomes would be that the profits of the holding company would fall,  $\pi_{\text{Holding Company}}^Y[\tilde{a}] \approx 0.530 - w^0 < 0.560 - w^0 \approx \pi_{\text{Holding Company}}^Y[a^{1*}]$ , and that, as the auction would be more asymmetric, the welfare loss would increase,  $WL[\tilde{a}, \tilde{s}] \approx 0.065 > 0.041 \approx WL[a^{1*}, s = 1]$ .

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