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# Credit Market Competition and Liquidity Crises*

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**Abstract**

We develop a two-period model where banks invest in reserves and loans, and are subject to aggregate liquidity shocks. When banks face a a shortage of liquidity, they can sell loans on the interbank market. Two types of equilibria emerge. In the no default equilibrium, banks keep enough reserves and remain solvent. In the mixed equilibrium, some banks default with positive probability. The former equilibrium exists when credit market competition is intense, while the latter emerges when banks exercise market power. Thus, competition is beneficial to financial stability. The effect of default on welfare depends on the exogenous risk of the economy as represented by the probability of the good state of nature.

JEL Classifications: G01, G21.

Keywords: Interbank market, default, price volatility.

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# 1 Introduction

There is a long and wide standing debate both among academics and policymakers on the nexus between competition and financial stability. The key issue is how competition affects banks' and borrowers' risk taking behavior. One view is that by reducing banks' franchise value, competition reduces the incentives for banks to behave prudently (see, Keeley, 1990, and the subsequent papers surveyed in Carletti, 2008, and Carletti and Vives, 2009). An opposite view is that competition is beneficial to financial stability since low loan rates induce borrowers to take less risk thus reducing the risk of banks' portofolio (Boyd and De Nicoló, 2005). Yet, by narrowing lending margins, competition reduces banks' buffers and thus their ability of withstanding loan losses (Martinez-Miera and Repullo, 2010). Along with the theoretical literature, the empirical evidence is inconclusive on whether competition is beneficial or detrimental to financial stability. Results differ across samples and time periods and very much depend on the estimates used to measure competition and stability (see the discussion in Carletti, 2010).

Recently, the debate has centered around the question of whether competition has contributed to the recent financial crisis. One view is that the increased competition deriving from the process of deregulation of the 1980s and 1990s has exacerbated banks' risk taking behavior. Another view, supported by the observation that countries with similar market structures (such as Australia, Canada and the UK) have been affected very differently by the recent crisis, suggests that other factors like a proper regulatory framework and institutional environment can insulate banking systems from risk taking problems (Carletti, 2010, and Beck et al., 2011).

One issue that has not been explored so far is the link between competition and liquidity as a source of risk. As the recent crisis has shown, liquidity is a crucial source of risk in the banking industry because of the maturity transformation function that banks perform in the economy. When faced with large liquidity demands, banks need to access financial markets to raise additional liquidity at short notice by either borrowing or selling illiquid assets. Market liquidity determines the level of asset prices and thus banks' ability to withstand liquidity shocks. When liquidity is scarce, asset prices may be too low for banks to remain solvent and a liquidity crisis can turn into a solvency problem. Key to the emergence of liquidity crises are then the amount of reserves that banks hold and the total supply of liquidity on the market. The former affects individual banks' need of additional liquidity; the latter determines market liquidity and thus the level of asset prices.

In this context, we develop a novel theory where credit market competition affects the emergence of crises as it determines banks' incentives to hold reserves, and thus market liquidity and asset prices. The theory provides numerous new insights into the relationship between competition and stability. In contrast to the charter value hypothesis described above, we show that competition is beneficial to financial stability as it induces banks to behave prudently and hold more liquidity. However, avoiding a liquidity crisis is not always efficient as it may require the system to hold large amounts of liquid reserves and reduce credit availability excessively. These results contrast with those in Boyd and De Nicoló (2005) in that we focus on liquidity risk rather than credit risk as a source of instability, and we characterize the welfare properties of financial stability.

We build on a standard banking model as developed in Allen and Gale (2004a, 2004b) and Allen, Carletti and Gale (2009). There are two periods. Banks raise funds from risk-averse consumers in the form of deposits. On the asset side, they hold a one-period liquid asset (reserves) or grant a two-period loan to entrepreneurs with a return that depends on the degree of competition in the credit market. Banks face aggregate uncertainty relative to their demand for liquidity at the interim date as a stochastic fraction of their consumers need to consume early. There is a good state with a small fraction of early depositors, and a bad state where the fraction of early depositors is larger. Banks can meet their liquidity demands by holding reserves initially or selling loans on a (competitive) interbank market at the interim period. Holding reserves is costly in terms of foregone return on the loans. Asset prices are endogenously determined and are volatile across the two states of nature, as they depend on the amount of supply and demand for liquidity in the market. The former is fixed by banks' total reserve holdings and is thus inelastic at the interim date. The latter depends on the realization of the liquidity shock and the terms of the deposit contract. Credit market competition affects liquidity demand and supply as it affects banks' portfolio allocation and the terms of the deposit contract. Banks make their initial investment choices to maximize expected profits subject to the constraint that consumers are willing to deposit their funds initially.

We first show that two types of equilibria can emerge, depending on the degree of competition in the credit market. A no default equilibrium emerges when competition is intense. As loans are not very profitable, the opportunity cost for banks of holding reserves is low. All banks find it optimal to keep enough reserves to repay the early depositors in both states of nature. As competition decreases, holding reserves becomes increasingly more costly and the no default equilibrium ceases to exist. In the new equilibrium, defined as mixed, banks behave differently despite being ex ante alike. Some banks, which we call risky, invest only in loans and default in the bad state of nature when all consumers withdraw and a bank run occurs. Banks sell then all their loans, asset prices drop significantly and consumers obtain the liquidation proceeds instead of the promised repayments. The remaining banks, defined as safe, hold enough liquidity to always meet their commitments and acquire the loans of the risky banks. In equilibrium, safe and risky banks make the same expected profits and consumers are indifferent between the two types of banks.

We then show that the degree of competition for which default starts to emerge and the number of defaulting banks crucially depend on the level of exogenous risk in the economy as represented by the probability of the bad state of nature. When such probability is low, default is unlikely to occur and more banks have incentives to reduce their reserve holdings. Thus, in normal times when the economy is characterized by a more stable environment, crises are less frequent but are more severe in that they involve a larger number of banks and emerge in more competitive credit markets. In contrast, in economies characterized by greater exogenous risk, banks prefer to behave prudently. Fewer banks behave risky and default only occurs when banks exercise enough market power. These results suggest that credit market competition and exogenous risk are substitutes in terms of their impact on banks' risk taking behavior.

A final important insight of the model concerns the optimality of crises. We show that default is socially optimal when the exogenous risk in the economy is low, and it is inefficient when such a risk is high. Default introduces some contingency in the repayments to depositors and some elasticity in the demand for liquidity at the interim period. Consumers at the safe banks always receive the promised consumption, where those at the risky banks receive the promised repayments in the good state and the liquidation proceeds in the bad state. In this state, risky banks need to sell all their loans, and their demand for liquidity becomes elastic to the price. Despite going default, risky banks have to make the same expected profits as the safe banks in equilibrium. When the probability of the bad state is high, the total demand for liquidity at the interim date is greater with default than without and the system must hold more reserves to satisfy it. This leads to a lower supply of loans in aggregate and thus to lower welfare. The opposite happens when the probability of the bad state is low.

The key feature of the model is that there is a wedge between the loan return accruing to banks and the return from holding reserves. The magnitude of such a wedge is determined by the level of competition in the credit market. The less competitive the credit market, the more profitable loans are and the more costly holding reserves is. Any other factor affecting the difference in the profitability of loans and reserves is consistent with our story. For example, banks granting loans to more profitable industries have a higher opportunity cost of holding reserves and are therefore more prone to behave risky. Similarly, highly leveraged banks are able to obtain higher returns from their investments and have therefore lower incentives to insure themselves against liquidity shocks.

The paper has a number of empirical implications. First, it predicts that banks in competitive banking systems behave more prudently than banks in less competitive systems. Second, systems with similar levels of competition are more likely to be unstable when banks are less subject to large liquidity shocks. Third, crises occuring in systems with low expectations of large liquidity shocks are more severe in terms of number of defaulting banks but also more efficient as they allow the economy to provide a larger supply of loans. Fourth, economies with a small probability of high liquidity shocks are more efficient than economies with a high probability of large liquidity shocks, even when they entail default. Finally, the model predicts that default leads to greater credit availability, except in banking systems with high exogenous risk.

The novelty of the paper is to analyze the relationship between competition and liquidity risk, and to show that liquidity crises can be efficient. In this sense, it is linked to various strands of literature. A few papers have looked at the effect of competition on bank instability in terms of runs (see also Carletti, 2008, and Carletti and Vives, 2009, for a survey). The analysis of Rochet and Vives (2004) and Goldstein and Pauzner (2005) suggests that when banks offer higher repayments to early depositors (as would be the case with more intense competition on the deposit market), bank runs are more likely to occur as a result of coordination failures. Matutes and Vives (1996) show that deposit market competition does not have a clear effect on banks' vulnerability to runs, but higher promised repayments to depositors tend to make banks more unstable. Carletti et al. (2007) analyze the impact of credit market competition on banks' incentives to hold liquidity after a merger. They show that an increase in market power as after a merger among large banks increases banks' liquidity needs and thus the probability of liquidity crises. In contrast to these papers, we focus on the impact of credit market competition and banks' holdings in a framework where runs are due to deterioration of asset prices rather than to depositors' coordination failures.

Our paper shows that competition is beneficial to financial stability but not necessarily to efficiency. The reason is that default is socially desirable if it leads to a decrease in the amount of reserves in the system and thus to greater credit availability. The idea that crises can be efficient is related to that in Allen and Gale (1998) that bank runs can be efficient as they improve risk sharing between early and late depositors. Similarly, Boyd, De Nicoló and Smith (2004) shows that competitive banking systems are more esposed to crises than monopolistic ones, but are more efficient as they provide better inter-temporal insurance to depositors. This contrasts with the result that competition exacerbates risk taking and thus lowers welfare by either leading to excessive deposit rates (Matutes and Vives, 2000) or by worsening the average quality of banks' borrowers (Freixas et al., 2011).

Several recent contributions on financial stability have focused on crises generated from asset price volatility and fire sales losses. Examples are Acharya and Yorulmazer (2008), Acharya, Shin and Yorulmazer (2011) Diamond and Rajan (2011) and, in particular, Allen and Gale (1994, 2004a, 2004b), and Allen and Carletti (2006, 2008). We contribute to this literature by analyzing how competition affects asset prices and thus the emergence of liquidity crises.

We show that the presence of competitive interbank markets supports the existence of a mixed equilibrium where some banks default in one state of nature and sell their loans to other banks at a price that is endogenously determined by demand and supply of liquidity. This mixed equilibrium can be efficient or inefficient depending on the amount of total liquid reserves that are needed to clear the market. In this sense, the paper is related to some contributions that focus on the interbank market such as Flannery (1996), Freixas and Jorge (2008) and Acharya, Gromb and Yorulmazer (2011).

The paper proceeds as follows. Section 2 describes the model. Section 3 derives the no default and the mixed equilibria. Section 4 looks at the efficiency properties of the two equilibria. Section 5 discusses the main implications of the model. Section 6 concludes. All proofs are in the appendix.

## 2 The model

Consider a three date (t = 0, 1, 2) economy with three types of agents: banks, consumers and entrepreneurs. Banks raise funds from consumers in exchange for a deposit contract and provide loans to entrepreneurs. Banks enjoy monopoly power in the deposit market while they compete to attract entrepreneurs. The idea is that banks operate in distinct regions. Consumers can only deposit their funds at one bank in their region. Entrepreneurs obtain loans from one bank only but can move across regions.

Each bank raises funds at date 0 from a continuum of mass one of consumers endowed with one unit at date 0 and nothing thereafter. Consumers are all ex ante identical but are either early or late types ex post. The former value consumption only at date 1; the latter value consumption only at date 2. Each consumer has a probability of being an early type $\lambda_\theta$ given by

$$\lambda_\theta = \begin{cases} \lambda_L & \text{w. pr. } \pi \\ \lambda_H & \text{w. pr. } (1-\pi), \end{cases}$$

with $\lambda_H > \lambda_L$. From the Law of Large Numbers, $\lambda_\theta$ represents the fraction of early types at each bank. As there is only aggregate uncertainty, the realization of $\lambda_\theta$ is the same for all banks. Thus, there are two states of nature, $L$ and $H$, which we refer to as the good and the bad state respectively.

The ex ante uncertainty about consumers' types generates a role for banks as liquidity providers. Consumers are offered a deposit contract allowing them to withdraw a (non-contingent) amount $c_1$ at date 1 or $c_2$ at date 2, and have an expected utility equal to

$$E[u(c_1, c_2, \lambda_\theta)] = E[\lambda_\theta u(c_1) + (1-\lambda_\theta)u(c_2)].$$

The utility function is twice differentiable and satisfies the usual neoclassical assumptions: $u'(c) > 0$, $u''(c) < 0$ and $\lim_{c\to 0} u'(0) = \infty$. For the consumers to deposit their endowment at a bank at date 0, the contract has to guarantee them an expected utility at least equal to the one they would obtain from storing.

Each bank invests a fraction $R$ of its funds in reserves and a fraction $L$ in loans to entrepreneurs at the initial date. Reserves are a storage technology: one unit invested at date $t$ produces one unit at date $t+1$. Loans are a long technology: one unit invested at date 0 gives a return $r$ to the bank at date 2. Such a return depends on the degree of competition in the credit market. Entrepreneurs invest the amount obtained by the bank in a (safe) divisible project yielding $V > 1$ at date 2 and pay the bank a (gross) interest rate equal to

$$r = \gamma V. \tag{1}$$

The parameter $\gamma \epsilon (\frac{1}{V}, 1)$ measures the intensity of competition in the credit market. The higher $\gamma$ the lower the degree of competition and the higher the return $r$ accruing to the bank. In the limit as $\gamma \rightarrow \frac{1}{V}$, the credit market is perfectly competitive. The bank receives $r = 1$ and the entrepreneur retains $V$. At the other extreme, when $\gamma \rightarrow 1$, the credit market is monopolistic and the bank obtains $r \rightarrow V$. Entrepreneurs are still willing to take the loan as they are assumed to have a zero opportunity cost. Values of $\gamma$ between $\frac{1}{V}$ and 1 represent intermediate levels of competition in the credit market, when banks and entrepreneurs share the surplus generated by the project. In this sense, the parameter $\gamma$ can be seen as capturing the bank's bargaining power over entrepreneurs.

Loans can be sold on a (competitive) interbank market at date 1 for a price $P_\theta$. Participation in this market is limited in that only banks can buy and sell loans. The price $P_\theta$ is endogenously determined in equilibrium by the aggregate demand and supply of liquidity in the market, as explained further below. As there are only two states $\theta = H, L$, the price $P_\theta$ can take at most two values.

The timing of the model is as follows. At date 0, banks choose the deposit contract $(c_1, c_2)$ and the initial portfolio allocation between reserves and loans in order to maximize their expected profits. At the beginning of date 1, consumers learn privately their type and the state $\theta = H, L$ is realized. Early consumers always demand $c_1$ at date 1 to meet their consumption needs. In contrast, late consumers can either wait and demand the promised consumption $c_2$ at date 2, or claim to be early types and demand $c_1$ at date 1, thus precipitating a run. In the absence of runs, a fraction $\lambda_\theta$ of consumers are paid $c_1$ at date 1 and the remaining fraction $1 - \lambda_\theta$ are paid $c_2$ at date 2. In the presence of a run, the bank has to sell all its loans and it goes bankrupt, and consumers receive a pro rata share of the bank's resources. A run occurs in the model only when the value of the bank's portfolio at date 2 is not enough to repay at least $c_1$ to the late consumers. That is, (sunspot) runs do not occur.

# 3 Equilibrium

Two equilibria arise endogenously in the model. In the first, that we define as no default equilibrium, runs do not occur and all banks remain solvent in both states $\theta = H, L$. In the second, defined as mixed equilibrium, some banks experience a run and go bankrupt in some state, while some others always remain solvent. In what follows we characterize the two equilibria in turn. We first solve the bank's problem in each equilibrium. Then, we analyze for which parameter space, and in particular for which level of competition in the credit market, the two equilibria exist. We start with the no default equilibrium.

## 3.1 The no default equilibrium

The no default equilibrium exists when all consumers withdraw according to their time preferences so that runs do not occur and all banks remain solvent. As they are all ex ante identical and there is no default, banks behave alike at the initial date concerning both their portfolio allocation and the terms of the deposit contract. Each bank chooses the deposit contract $(c_1, c_2)$ and the portfolio allocation $(R, L)$ simultaneously so as to maximize its expected profit at $t = 0$. The bank's maximization problem is then given by

$$\underset{c_1,c_2,R,L}{Max}\ \Pi = rL + \pi(R - \lambda_L c_1) + (1-\pi)(R - \lambda_H c_1) - [\pi(1-\lambda_L) + (1-\pi)(1-\lambda_H)]c_2 \quad (2)$$

subject to

$$R + L = 1 \quad (3)$$

$$\lambda_\theta c_1 \leq R \quad (4)$$

$$(1-\lambda_\theta)c_2 \leq rL + R - \lambda_\theta c_1 \quad (5)$$

$$c_2 \geq c_1 \tag{6}$$

$$E[u(c_1, c_2, \lambda_\theta)] = [\pi\lambda_L + (1-\pi)\lambda_H]\, u(c_1) + [\pi(1-\lambda_L) + (1-\pi)(1-\lambda_H)]\, u(c_2) \geq u(1) \tag{7}$$

$$0 \leq R \leq 1, c_1 \geq 0.$$

for any $\theta = L, H$. Bank's profit $\Pi$ is given by sum of the returns from the loans $rL$ and the expected excess of liquidity $\pi(R - \lambda_L c_1) + (1-\pi)(R - \lambda_H c_1)$ minus the expected payments $[\pi(1-\lambda_L)+(1-\pi)(1-\lambda_H)]c_2$ to depositors at date 2. Constraint (3) represents the budget constraint at date 0. The next two constraints are the resource constraints at dates 1 and 2. Constraint (4) requires that the bank has enough resources at date 1 to satisfy the demands $\lambda_\theta c_1$ by the early consumers for any $\theta = L, H$. Constraint (5) requires that the resources $rL + R - \lambda_\theta c_1$ available to the bank at date 2 are enough to repay the promised amount $(1-\lambda_\theta)c_2$ to the late consumers. Constraint (6) ensures that at date 0 the late consumers are offered a repayment $c_2$ at least equal to $c_1$. Taken together, (5) and (6) imply that the deposit contract is incentive compatible both at dates 0 and 1 so that no run occurs. Constraint (7) is consumers' participation constraint at date 0. It requires that the utility $E[u(c_1, c_2, \lambda_\theta)]$ that they receive from the deposit contract is at least equal to the utility $u(1)$ that they would obtain from storing their endowment. Finally, the last constraint is simply a non-negative requirement for reserves and consumption bundles.

In what follows, we assume that depositors have a logaritmic utility function, that is $u(c_t) = \ln(c_t)$ with $t = 1, 2$. This simplifies the analysis and allows us to obtain closed form solutions, without affecting our qualitative results. We have the following.

**Proposition 1** *There exists a unique (symmetric) no default equilibrium, in which each bank invests an amount $R^{ND} = \lambda_H c_1^{ND}$ in reserves and $L^{ND} = 1 - R^{ND}$ in loans, and it offers consumers a deposit contract*

$$c_1^{ND} = \left(\frac{\pi\lambda_L + (1-\pi)\lambda_H}{\pi\lambda_L + (r-\pi)\lambda_H}\right)^{\pi(1-\lambda_L)+(1-\pi)(1-\lambda_H)} < 1 \tag{8}$$

*and*

$$c_2^{ND} = \left(\frac{\pi\lambda_L + (r-\pi)\lambda_H}{\pi\lambda_L + (1-\pi)\lambda_H}\right)^{\pi\lambda_L + (1-\pi)\lambda_H} > 1. \tag{9}$$

The intuition behind Proposition 1 is simple. In the no default equilibrium all banks behave alike. Each bank finds it optimal to hold an amount of reserves just enough to satisfy its highest liquidity demand $\lambda_H c_1^{ND}$ by early consumers at date 1 in state $H$. The deposit contract maximizes the bank's expected profit while satisfying consumers' participation constraint with equality. Depositors always receive the promised repayments $c_2^{ND} > c_1^{ND}$.

Holding reserves entails an opportunity cost for banks as represented by the foregone return $r$ on loans. Such a cost is higher the more intense is competition in the credit market. This implies that the consumption $c_1^{ND}$ falls with $\gamma$ (and thus with $r$) while $c_2^{ND}$ increases. The ratio between $c_2^{ND}$ and $c_1^{ND}$, as given by $\frac{\pi\lambda_L + (r-\pi)\lambda_H}{\pi\lambda_L + (1-\pi)\lambda_H}$, increases as competition decreases in a way to guarantee that consumers' participation constraint remains satisfied.

Substituting (8) and (9) into the expression for the bank's expected profit as in (2), we obtain

$$\Pi^{ND} = \gamma V - c_2^{ND}. \tag{10}$$

The bank's profit is simply equal to the difference between the return on the loans and the promised repayment $c_2^{ND}$ to the late consumers. This means that the reserve holdings and the liquidity demand by the early consumers affect the bank's profits only to the extent that they affect $c_2^{ND}$.

Since all banks hold enough reserves to self-insure themselves against liquidity shocks and there is only aggregate uncertainty in the model, no loans are traded on the interbank market at date 1. Still, the equilibrium allocation must be supported by a vector of prices that satisfies the market clearing conditions. These require that the total demand for liquidity does not exceed the total supply of liquidity for any state $\theta$. Both demand and supply are inelastic at date 1. The demand for liquidity is inelastic since it is determined solely by consumers' preferences. The supply is fixed by the bank's portfolio decisions at date 0. Shocks to the demand cause price volatility across states. Since $\lambda_H > \lambda_L$ and $R^{ND} = \lambda_H c_1^{ND}$, there is an excess of liquidity in state $L$ and date 1. Thus, it must hold that

$$P_L = r \tag{11}$$

for banks to be indifferent between buying loans and storing the excess liquidity between dates 1 and 2. With $P_L < r$, loans would dominate storage between dates 1 and 2, while $P_L > r$ would imply the opposite.

The price $P_H$ must ensure that banks are willing to hold both reserves and loans between dates 0 and 1. This means that $P_H$ must satisfy

$$\pi \frac{r}{P_L} + (1-\pi)\frac{r}{P_H} = r. \tag{12}$$

Given $P_L > 1$, this implies $P_H < 1$. Otherwise loans would dominate reserves at date 0. The equilibrium is characterized by price volatility as a consequence of the aggregate uncertainty of the demand for liquidity and the inelasticity of supply at date 1.

### 3.2 The mixed equilibrium

So far we have considered the equilibrium where no banks default. However, avoiding default is costly as it requires banks to hold a large enough amount of reserves and forego the higher return on the loans. As competition in the credit market decreases, the opportunity cost of holding reserves becomes high, and banks can find it optimal to reduce their reserve holdings and default at date 1 with positive probability. In this section, we characterize the equilibrium when default becomes optimal. We start by looking at the banks' problem. Then, we analyze the market clearing conditions supporting the equilibrium.

A bank defaults when its late consumers run at date 1 and the price $P_\theta$ drops enough to generate insolvency. In equilibrium not all banks can default simultaneously. If all banks made the same investments at date 0 and all defaulted at date 1, there would be no bank willing to buy the loans of the defaulting banks so that $P_\theta = 0$. This cannot be an equilibrium since it would be optimal for a bank to remain solvent and buy the loans at the price $P_\theta = 0$. This implies an equilibrium with default must be mixed.

Despite being ex ante identical, banks differ in terms of initial portfolio allocations and deposit contracts. A fraction $\rho$ of banks, that we define as safe, invest enough in reserves at date 0 to remain solvent at date 1 in either state $\theta = L, H$ for any $P_\theta$. The remaining $1 - \rho$ banks, defined as risky, invest so much in loans that they may not have enough reserves to satisfy consumers' liquidity demands at date 1. When this is the case, risky banks sell their loans on the interbank market at the price $P_\theta$ and default with positive probability. In equilibrium safe and risky banks must have the same expected profits as they have to be indifferent between being either of the two types. This implies that the risky banks can default only in one state.

Given the structure of the model, they remain solvent in state $\theta = L$ and default in state $\theta = H$. Even if they sell part of their loans, the price $P_L$ is high enough for them to meet their commitments. In state $H$, they are unable to do so as there is a self-reinforcing drop in the price $P_H$. Anticipating the default, late consumers at the risky banks run. This forces the risky banks to sell all their loans. The larger demand for liquidity relative to state $L$ coupled with the inelasticity of the supply drives down the price $P_H$ to a level that is too low for the risky banks to remain solvent. This means that default occurs as a consequence of the endogenous determination of market prices. Consumers know the type of banks they deposit their endowment in. Safe and risky banks offer different deposit contracts so as to satisfy consumers' participation constraint.

We start by characterizing the problem for the safe banks. This is similar to the one in the no default equilibrium, with the difference that banks have now the possibility to buy loans on the interbank market at date 1. Given the market prices $P_H$ and $P_L$, each safe bank chooses simultaneously the deposit contract $(c_1^S, c_2^S)$, the amount of reserves $R^S$

and of loans $L^S$ so as to solve the following problem:

$$\max_{c_1^S, c_2^S, R^S, L^S} \Pi^S = rL^S + \pi(\frac{R^S - \lambda_L c_1^S}{P_L})r + (1-\pi)(\frac{R^S - \lambda_H c_1^S}{P_H})r - [\pi(1-\lambda_L) + (1-\pi)(1-\lambda_H)]c_2^S \tag{13}$$

subject to

$$R^S + L^S = 1$$

$$\lambda_\theta c_1^S \leq R^S \tag{14}$$

$$(1-\lambda_\theta)c_2^S \leq r\left(L^S + \frac{R^S - \lambda_\theta c_1^S}{P_\theta}\right) \tag{15}$$

$$c_2^S \geq c_1^S \tag{16}$$

$$E[u(c_1^S, c_2^S, \lambda_\theta)] = [\pi\lambda_L + (1-\pi)\lambda_H]\, u(c_1^s) + [\pi(1-\lambda_L) + (1-\pi)(1-\lambda_H)]\, u(c_2^s) \geq 0. \tag{17}$$

$$0 \leq R^S \leq 1, c_1^S \geq 0.$$

The expression for the bank's profit $\Pi^S$ is given by the sum of the returns from the loans $rL^S$ and from the expected excess of liquidity $\pi(\frac{R^S - \lambda_L c_1^S}{P_L})r$ and $(1-\pi)(\frac{R^S - \lambda_H c_1^S}{P_H})r$ in states $L$ and $H$ minus the expected payments $[\pi(1-\lambda_L) + (1-\pi)(1-\lambda_H)]c_2^S$ to late consumers at date 2. Safe banks use any excess liquidity at date 1 to acquire loans from the risky banks. With probability $\pi$ the safe bank has $R^S - \lambda_L c_1^S$ units of excess liquidity and buys $\frac{R^S - \lambda_L c_1^S}{P_L}$ units of loans from the risky banks yielding a per-unit return of $r$. The same happens in state $H$. The first constraint is the budget constraint at date 0, which is always satisfied with equality to indicate that the bank invests all its funds. Constraint (14) states that the safe bank must have enough reserves $R^S$ to satisfy the demand $\lambda_\theta c_1^S$ by the early consumers at date 1 in either state $\theta$. Constraint (15) requires that the bank has enough resources at date 2 to repay the promised amount $(1-\lambda_\theta)c_2^S$ to the late consumers. Constraint (16) ensures that the deposit contract is incentive compatible at date 0. Together with (15), it guarantees that the safe banks never experience a run at date 1. Constraint (17) guarantees that consumers are willing to deposit their funds at date 0.

The last constraint is the usual non-negative requirement on reserves and consumption.

The risky banks solve a similar problem except that they default in state $H$. In state $L$ they may have to sell loans at the price $P_L$ to satisfy consumers' demands, but remain solvent and make positive profits. In state $H$, they sell all loans at the price $P_H$. As this price is low, they go bankrupt. Early and late consumers share the proceeds $R^R + P_H L^R < c_1^R$ of the liquidated portfolio. Thus, anticipating default when $\theta = H$, each risky bank offers the deposit contract $(c_1^R, c_2^R)$ and chooses the amounts of reserves $R^R$ and loans $L^R$ to solve the following problem:

$$\max_{c_1^R, c_2^R, R^R, L^R} \quad \Pi^R = \pi \left( rL^R - r(\frac{\lambda_L c_1^R - R^R}{P_L}) - (1 - \lambda_L) c_2^R \right) \tag{18}$$

subject to

$$R^R + L^R = 1$$

$$\lambda_L c_1^R \leq R^R + P_L L^R \tag{19}$$

$$(1 - \lambda_L) c_2^R \leq r \left( L^R - \frac{\lambda_L c_1^R - R^R}{P_L} \right) \tag{20}$$

$$c_2^R \geq c_1^R \tag{21}$$

$$E[u(c_1^R, c_2^R, \lambda_\theta)] = \pi[\lambda_L u(c_1^R) + (1 - \lambda_L) u(c_2^R)] + (1 - \pi)[u(R^R + P_H L^R)] \geq 0 \tag{22}$$

$$0 \leq R^R \leq 1, c_1^R \geq 0.$$

The risky banks make positive profits only with probability $\pi$ when state $L$ occurs. These are equal to the returns from the initial investment in loans $rL^R$ minus the foregone return $r$ on the $(\frac{\lambda_L c_1^R - R^R}{P_L})$ units of loans sold at date 1 to cover the shortage of liquidity $\lambda_L c_1^R - R^R$ and the expected repayments $(1 - \lambda_L) c_2^R$ to the late consumers. The first constraint is the usual budget constraint at date 0, which always binds. The second constraint is the resource constraint in state $L$ at date 1. It states that the maximum amount $R^R + P_L L^R$ of available resources from reserves and all liquidated loans is enough to satisfy the demands $\lambda_L c_1^R$ by the early consumers. Constraint (20) ensures that at date 2 the bank has enough resources in state $L$ to honor the promised repayments $(1-\lambda_L)c_2^R$ to the late consumers. These two constraints must hold with strict inequality in order for the risky banks to make positive profits in state $L$. As usual, the deposit contract has to satisfy the incentive constraint for the late consumers at date 0 as indicated by (21). Constraint (22) requires the deposit contract to satisfy consumers' participation constraint. As the risky banks default in state $H$, consumers receive $(c_1^R, c_2^R)$ only in state $L$ and the proceeds $R^R + P_H L^R$ of the bank's liquidated portfolio in state $H$. The last constraint is the usual non-negative requirement.

As mentioned above, in equilibrium banks have to be indifferent between being safe or risky. This requires the expected profits of safe and risky banks to be the same, that is

$$\Pi^S = \Pi^R. \tag{23}$$

It remains to determine the prices $P_H$ and $P_L$, and the fractions $\rho$ and $1-\rho$ of safe and risky banks. The solutions to the banks' maximization problems must be consistent with the market clearing conditions determining $P_H$ and $P_L$.

Consider first state $L$. Market clearing requires that at date 1 the demand for liquidity equals the supply of liquidity in aggregate. Thus, it must be the case that

$$(1-\rho)(\lambda_L c_1^R - R^R) = \rho(R^S - \lambda_L c_1^S). \tag{24}$$

The left hand side represents the aggregate liquidity demand as given by the liquidity shortage $\lambda_L c_1^R - R^R$ of each of the $1-\rho$ risky banks. The right hand side is the aggregate liquidity supply as determined by the excess liquidity $R^S - \lambda_L c_1^S$ of each of the $\rho$ safe banks. Condition (24) requires $P_L \leq r$ so as to guarantee that the safe banks are willing to use their excess liquidity to purchase loans from the risky banks.

Now consider state $H$. The risky banks sell their $(1-\rho)L^R$ loans at the price $P_H$, while the safe banks have $\rho(R^S - \lambda_H c_1^S)$ excess of liquidity in total. Market clearing requires the supply and demand to be equal at the price $P_H$. Thus, it must be the case that

$$(1-\rho)P_H L^R = \rho(R^S - \lambda_H c_1^S). \tag{25}$$

Conditions (24) and (25) imply that there is cash-in-the-market pricing in the model. The prices $P_L$ and $P_H$ vary endogenously across the two states and depend on the supply and demand of liquidity in the market.

The mixed equilibrium is characterized by the vector $\{R^S, L^S c_1^S, c_2^S, R^R, L^R, c_1^R, c_2^R, \rho, P_L, P_H\}$. We have the following result.

**Proposition 2** *The mixed equilibrium is characterized as follows:*

*1. The safe banks invest an amount $R^S$ in reserves and $L^S = 1 - R^S$ in loans, and offer consumers a deposit contract $(c_1^S, c_2^S)$ as follow*

$$R^S = \lambda_H c_1^S + \frac{1-\rho}{\rho} P_H, \tag{26}$$

$$c_1^S = \left( \frac{P_L}{r} \frac{\pi\lambda_L + (1-\pi)\lambda_H}{\pi\lambda_L + (P_L - \pi)\lambda_H} \right)^{\pi(1-\lambda_L)+(1-\pi)(1-\lambda_H)} < 1 \tag{27}$$

$$c_2^S = \left( \frac{r}{P_L} \frac{\pi\lambda_L + (P_L - \pi)\lambda_H}{\pi\lambda_L + (1-\pi)\lambda_H} \right)^{\pi\lambda_L+(1-\pi)\lambda_H} > 1. \tag{28}$$

*2. The risky banks invest an amount*

$$R^R = 0$$

*in reserves and $L^R = 1$ in loans, and offer consumers a deposit contract $(c_1^R, c_2^R)$ such that*

$$c_1^R = \frac{P_L}{r} c_2^R \tag{29}$$

$$c_2^R = \frac{c_2^S - r(1-\pi)}{\pi} > 1. \tag{30}$$

*3. The price* $1 \leq P_L \leq r$ *is the solution to (22), while* $P_H$ *is given by*

$$P_H = \frac{P_L(1-\pi)}{P_L - \pi} < 1. \tag{31}$$

*4. The fraction of safe banks is*

$$\rho = \frac{\lambda_L c_1^R - P_H}{\lambda_L c_1^R - P_H + (\lambda_H - \lambda_L) c_1^S} < 1. \tag{32}$$

The proposition shows that safe and risky banks behave quite differently. Each safe bank holds an amount $\frac{1-\rho}{\rho} P_H$ of reserves in excess of the early liquidity demand $\lambda_H c_1^S$ in state $H$, and uses it to purchase the loans $(1-\rho)P_H$ sold by the risky banks. As in the no default equilibrium, the safe banks offer $c_2^S > 1 > c_1^S$ and always remain solvent. Both repayments depend now on the loan return as well as the market prices since the interbank market is active.

The risky banks do not hold any reserves and default at date 1 in state $H$. As default is anticipated and $P_L > 1$, they find it optimal to invest everything in loans at date 0. At date 1 in state $L$ the risky banks sell $\frac{\lambda_L c_1^R}{P_L}$ units of loans to satisfy the liquidity demand $\lambda_L c_1^R$ of the early depositors but remain solvent. In state $H$ they liquidate their entire portfolio and default. Depositors at the risky banks receive the promised repayments $c_1^R$ and $c_2^R$ in state $L$ only. These repayments, together with the amount $P_H$ that consumers receive in state $H$, have to satisfy their participation constraint.

Default introduces volatility in consumption across banks. The ratio of the consumption levels offered by the two types of banks is given by

$$\frac{c_2^S}{c_1^S} = \frac{r}{P_L} \frac{\pi\lambda_L + (P_L - \pi)\lambda_H}{\pi\lambda_L + (1-\pi)\lambda_H} > \frac{c_2^R}{c_1^R} = \frac{r}{P_L}$$

since $P_L > 1$ and $\frac{\pi\lambda_L + (P_L-\pi)\lambda_H}{\pi\lambda_L + (1-\pi)\lambda_H} > 1$. This means that the safe banks offer a more volatile deposit contract than the risky banks. Both banks find it optimal to reduce the liquidity needed at date 1. The cost of holding liquidity is the foregone return $r$ on the loans for the safe banks and $\frac{r}{P_L}$ for the risky banks, as they obtain liquidity at date 1 by selling their loans. Given $P_L > 1$, obtaining liquidity in the market at date 1 is less costly than holding reserves initially. Thus, the risky banks find it optimal to offer a less volatile deposit contract than the safe banks.

The prices $P_L$ and $P_H$ satisfy the market clearing conditions in each state. The expression (31) for $P_H$ is equivalent to (12) in the no default equilibrium. It ensures that the safe banks are willing to hold both reserves and loans between dates 0 and 1. Given that the aggregate of supply of liquidity $\rho(R^S - \lambda_\theta c_1^S)$ is greater in state $L$ than in state $H$, it must be the case that $P_L > 1 > P_H$. As before, the price volatility crucially depends on the aggregate uncertainty of the liquidity demand and the inelasticity of the supply at date 1. The difference is that the demand for liquidity is no longer driven entirely by consumers' preferences. In state $H$, when a run occurs, the total demand for liquidity is $(1 - \rho)P_H$ as all consumers at the risky banks withdraw and receive the proceeds of the liquidated portfolio as given by $P_H$.

Finally, the proportion $\rho$ of safe banks is always positive and smaller than one given that $\lambda_H > \lambda_L$. Thus, the model generates partial default in that only a group of banks experience a run and go bankrupt.

### 3.3 Existence of equilibria

Now that we have characterized the two equilibria of the model, we analyze the parameter space in which they exist. The key element for the existence of the equilibria is whether default is optimal. This depends on the opportunity cost of holding reserves, and thus on the parameter $\gamma$ representing the degree of competition in the credit market. The no default equilibrium, as characterized in Proposition 1, exists if and only if no bank finds it optimal to choose a different portfolio allocation and deposit contract that results in default in state $H$. The mixed equilibrium, as characterized in Proposition 2, exists if and only if neither safe banks nor risky banks prefer portfolio allocations and deposit contracts that do not support default. For default to be sustained as an equilibrium, safe banks must be willing to hold excess liquidity at date 0 and use it to buy loans in the interbank market at date 1. This is verified if and only if the price $P_\theta$ is admissible.

*Existence of the no default equilibrium*

In the no default equilibrium banks hold an amount of reserves $R^{ND} = \lambda_H c_1^{ND}$. This allows them to remain solvent in any state $\theta$ but at the cost of foregoing the higher return $r$ on the loans. Such a cost is higher the lower is the level of competition in the credit market. As $\gamma$ increases, it may become optimal for a bank to lower its reserves so as to appropriate the higher returns on the loans. Thus, the no default equilibrium exists if no bank finds it optimal to default in state $H$.

A deviating bank chooses reserves $R^D$, loans $L^D$ and a deposit contract $(c_1^D, c_2^D)$ so as to maximize

$$\max_{c_1^D, c_2^D, R^D, L^D} \Pi^D = \pi \left[ rL^D - r\left(\frac{\lambda_L c_1^D - R^D}{P_L}\right) - (1-\lambda_L)c_2^D \right] \tag{33}$$

subject to

$$R^D + L^D = 1$$

$$\lambda_L c_1^D \leq R^D + P_L L^D$$

$$(1-\lambda_L)c_2^D \leq r\left( L^D - \frac{\lambda_L c_1^D - R^D}{P_L} \right)$$

$$c_2^D \geq c_1^D$$

$$E[u(c_1^D, c_2^D, \lambda_\theta)] = \pi[\lambda_L u(c_1^D) + (1-\lambda_L)u(c_2^D)] + (1-\pi)[u(R^D + P_H L^D)] \geq 0 \tag{34}$$

$$0 \leq R^D \leq 1, c_1^D \geq 0.$$

The deviating bank makes positive profit only in state $L$. With probability $\pi$, it obtains the return $rL^D$ from the initial investment in loans minus the foregone return $r$ on the $(\frac{\lambda_L c_1^D - R^D}{P_L})$ loans sold at date 1 if $\lambda_L - R^D > 0$ and the repayments $(1-\lambda_L)c_2^D$ to the late consumers at date 2. The first constraint is the usual resource constraint at date 0. The next two constraints are the resource constraints at dates 1 and 2 in state $L$. They both must hold with strict inequality for the deviating bank to make positive profits in state $L$. The next constraint is the usual incentive compatibility constraint at date 0. Constraint (34) is consumers' participation constraint at date 0. The last constraint is the usual non-negative requirement.

The maximization problem has a simple solution as summarized in the following lemma.

**Lemma 1** *The deviating bank invests an amount* $L^D = 1$ *in loans and offers all consumers a repayment* $c^D = (P_H)^{-(\frac{1-\pi}{\pi})}$.

The deviating bank behaves similarly to a risky bank in the mixed equilibrium in that it chooses not to hold any reserves. The difference is that the market prices are still $P_L = r$ and $P_H$ as in (12) given that unilateral deviations do not affect them. This implies that obtaining liquidity on the market in state $L$ is costless and the deviating bank offer the same repayment to early and late consumers as this minimizes its costs of funds.

The no default equilibrium exists as long as deviating is not profitable, that is as long as $\Pi^{ND} \geq \Pi^D$. We have the following result.

**Proposition 3** *If the probability* $\pi$ *of state* $L$ *is greater than some cutoff value* $\underline{\pi}$, *there exists a degree of credit market competition* $\gamma^* \in (\frac{1}{V}, 1)$ *such that the no default equilibrium exists for any* $\gamma \leq \gamma^*$.

The proposition shows that the no default equilibrium exists when competition is intense. The reason is that a high degree of competition implies a low cost of avoiding default and it makes it optimal for banks to hold a high level of reserves. In other words, when $\gamma$ is low, the returns on loans in state $L$ are too low to compensate banks for the default in state $H$. By contrast, as $\gamma$ increases, the loan rate $r$ becomes high enough to make the deviation profitable. The high enough value of $\pi$ required in the proposition ensures that deviating is profitable at least when $\gamma \to 1$ and thus implies the existence of $\gamma^*$ in the interval $(\frac{1}{V}, 1)$.

*Existence of the mixed equilibrium*

The mixed equilibrium as characterized in Proposition 2 exists if and only if neither safe banks nor risky banks choose portfolio allocations and deposit contracts that are not consistent with the occurrence of default in state $H$. This requires that the price $P_\theta$ is admissible. The price $P_L$ must lie in the interval

$$1 < P_L \leq r. \tag{35}$$

The lower bound is consistent with $P_H < 1$, while the upper bound ensures that the safe banks are willing to buy loans at date 1. From (31), the price $P_H$ is always admissible as it adjusts with $P_L$ so as to guarantee that safe banks hold reserves at date 0. Thus, only (35) matters for the existence of the mixed equilibrium. We have the following result.

**Proposition 4** *The mixed equilibrium exists for any $\gamma \geq \gamma^*$, with $\gamma^* \in (\frac{1}{V}, 1)$.*

Proposition 4 states that the mixed equilibrium only exists when competition is not intense. A level of $\gamma \geq \gamma^*$ makes it no longer optimal to avoid default as the foregone return on loans is high. The risky banks choose to default in state $H$. The high returns on loans in state $L$ are enough to ensure that they are able to make the same expected profits as the safe banks for an admissible value of $P_L$. For $\gamma < \gamma^*$, loans are not profitable enough to guarantee the existence of the two groups of banks for an admissible value of $P_L$.

Taken together, Propositions 3 and 4 show that the existence of the two equilibria is continuous in the parameter $\gamma$ representing the degree of competition in the credit market. The two equilibria coexist at $\gamma = \gamma^*$. The intuition is that as $\gamma$ reaches $\gamma^*$, it becomes profitable for a bank to deviate and lower its reserves. As all banks are alike, some other banks have an incentive to do the same. Then the mixed equilibrium with $\rho$ safe banks and $1-\rho$ risky banks arises. This implies that at $\gamma = \gamma^*$ all banks have the same profits ($\Pi^{ND} = \Pi^D = \Pi^S = \Pi^R$) and the deposit contract is the same for the banks defaulting and for those not defaulting ($c^D = c_1^R = c_2^R$, $c_1^{ND} = c_1^S$ and $c_2^{ND} = c_2^S$). The equality in the promised consumption $c_1^{ND} = c_1^S$ implies that each safe bank keeps a larger amount of reserves than a bank in the no default equilibrium, as it appears from comparing (26) and (40).

*Comparative statics*

The range of $\gamma$ characterizing the existence of the two equilibria of the model depends on the probability $\pi$ of state $L$, which can be interpreted as a measure of risk in the economy in that higher values of $\pi$ correspond to a lower probability of the bad state. To see this, we analyze the threshold value $\gamma^*$ as a function of the parameter $\pi$. We have the following.

**Proposition 5** *The threshold $\gamma^*$ decreases with $\pi$ (i.e., $\frac{d\gamma^*}{d\pi} < 0$).*

The proposition states that the range of $\gamma$ in which default is observed in equilibrium becomes larger as $\pi$ increases. The reason is that as $\pi$ increases, the good state is more likely and thus deviation becomes more profitable for any given $\gamma$. This has also an implication on the fraction of banks in the economy taking risk, as the following proposition illustrates.

**Proposition 6** *The number of non defaulting banks $\rho^*$ at $\gamma = \gamma^*$ decreases with $\pi$ (i.e., $\frac{d\rho^*}{d\pi} < 0$).*

As an increase in the probability of the good state makes deviation more profitable, it will lead to a higher fraction of risky banks in the economy at the level of competition at which default starts to emerge in equilibrium.

# 4 Welfare

Propositions 3 and 4 have important implications for the relationship between credit market competition and bank stability. When competition is intense, only the no default equilibrium exists. As competition decreases and $\gamma$ reaches the level $\gamma^*$, the mixed equilibrium starts to exist. The range of $\gamma$ in which default occurs increases with the probability $\pi$ of the low liquidity shock. As $\pi$ increases, default will be observed less frequently but at more intense levels of competition.

The number $1-\rho$ of banks defaulting also changes with $\pi$. Since behaving as a risky bank becomes more profitable as $\pi$ increases, more banks will optimally choose to default. Thus, when the economy becomes less exposed to bad states, default will occur less often but it will involve a larger number of banks.

One important question concerns the impact of default on welfare. The interbank market allows banks in need of liquidity to obtain it at date 1 by selling loans, and those with liquidity in excess to use it to purchase loans at the price $P_\theta$. The risky banks can invest more in loans initially and the safe banks have a lower opportunity cost of holding reserves. Default introduces some elasticity in the demand for liquidity at date 1. Without default, all banks offer non-contingent contracts to consumers and both the demand for and the supply of liquidity at date 1 are inelastic. The former depends only on the realization of the liquidity shocks. The latter is fixed by the initial holding of reserves. With default, the demand for liquidity becomes elastic and the repayment to consumers are state contingent. Safe banks pay the non-contingent amount $c_1^S$ to the early consumers in both states. The risky banks pay the promised repayment $c_1^R$ to the early consumers in state $L$ but only the value of the liquidated portfolio $P_H$ to all consumers in state $H$. This implies that the total demand for liquidity in state $H$ becomes elastic as it depends on the market price $P_H$.

The state contingency of the total demand for liquidity with default has a crucial effect on welfare. To see why consider the expression for welfare as given by

$$W^{ND} = r - c_2^{ND} + (V - r)(1 - R^{ND}) \tag{36}$$

in the no default equilibrium, and by

$$\begin{aligned} W^{M} &= r - c_2^{S} + (V - r)\left[(1-\rho) + \rho(1 - R^{S})\right] \\ &= r - c_2^{S} + (V - r)\left[(1 - \rho R^{S})\right] \end{aligned} \tag{37}$$

in the mixed equilibrium. In both cases, welfare equals the sum of banks' expected profits and entrepreneurs' surplus only, since consumers always have zero expected utility.

To evaluate the impact of default, we focus on the level of competition $\gamma = \gamma^*$ where the no default and the mixed equilibria coexist. To simplify notation, in what follows we use the subscript $nd$ to refer to a non- defaulting bank at $\gamma = \gamma^*$ in either the no default or the mixed equilibrium; and the subscript $d$ to refer to either a deviating bank in the default equilibrium or a risky bank in the mixed equilibrium at $\gamma = \gamma^*$. We have the following result.

**Lemma 2** *At $\gamma = \gamma^*$, welfare is lower with default if*

$$P_H > \lambda_H c_1^{nd}, \tag{38}$$

*and it is higher otherwise.*

The lemma states that the impact of default on welfare depends on the comparison between the repayment $P_H$ accruing to all consumers at a risky bank and that to the early consumers at a non-defaulting bank in state $H$ at date 1. The reason is that at $\gamma = \gamma^*$ banks make the same expected profits with and without default and therefore the comparison in welfare is exclusively determined by the amount of loans granted to entrepreneurs at date 0 and thus by the aggregate reserves in the two equilibria. In equilibrium aggregate reserves must equal total demand for liquidity as given by $\rho\lambda_H c_1^S + (1-\rho)P_H$ and $\lambda_H c_1^{ND}$ with and without default, respectively. Thus, since $c_1^S = c_1^{ND} = c_1^{nd}$ at $\gamma = \gamma^*$, only the difference between $P_H$ and $\lambda_H c_1^{nd}$ matters for welfare. When $P_H > \lambda_H c_1^{nd}$, the system needs more reserves in the mixed equilibrium than in the no default one to repay all consumers withdrawing at date 1 in state $H$. This implies lower loans in aggregate, and thus lower welfare.

The sign of inequality (38) depends on the condition (23) that the expected profit of safe and risky banks must be the same in equilibrium. Rearranging the expressions for $\Pi^S$ and $\Pi^R$ as in (13) and (18) after substituting $R^S$ as in (26) and $P_L = r$ at $\gamma = \gamma^*$ gives

$$(1-\pi)(1-\frac{\lambda_H c_1^{nd}}{P_H})r = \pi\lambda_L c_1^{nd} + [\pi(1-\lambda_L) + (1-\pi)(1-\lambda_H)]\, c_2^{nd} - \pi c_2^d. \tag{39}$$

The left hand side can be interpreted as the difference in the loan returns between a risky and a safe bank. With probability $1-\pi$ the bad state occurs and the risky bank loses the return $r$ on the loans while the safe bank loses $r$ on the $\frac{\lambda_H c_1^{nd}}{P_H}$ units of loans that it holds to meet the commitments to the early consumers in state $H$. The right hand side represents the difference in the repayments to consumers between a safe and a risky bank other than those at date 1 in state $H$. The first two terms are the expected repayments of a safe bank to the early consumers in state $L$ and to the late types in both states. The last term is the expected repayment of a risky bank to early and late consumers in the good state given that $c_1^R = c_2^R = c^d$ at $\gamma = \gamma^*$. For (39) to hold, if the risky bank suffers a net loss in terms of loan returns relative to a safe bank, it must benefit in terms of consumers' repayments. It follows that the difference $P_H - \lambda_H c_1^{nd}$ is positive if the risky bank has a cost advantage relative to the safe bank and it is negative if instead the risky bank has higher net returns on loans.

The probability $\pi$ of the good state affects all terms in the expression (39) and thus the sign of the difference $P_H - \lambda_H c_1^{nd}$, as illustrated in the following proposition.

**Proposition 7** *Define $\overline{\pi}$ as the cutoff value of the probability of state L such that $P_H -$*

$\lambda_H c_1^{nd} = 0$ *at* $\gamma^* \to 1$. *Then, if the difference* $P_H - \lambda_H c_1^{nd}$ *is decreasing in* $\pi$ *(i.e.,* $\frac{d(P_H - \lambda_H c_1^{nd})}{d\pi} < 0$*), it is the case that:*

*i) If* $\underline{\pi} \leq \overline{\pi}$*, there exists a value* $\widehat{\pi} \in (\underline{\pi}, 1)$ *such that at* $\gamma = \gamma^*$ *default leads to lower welfare for* $\pi < \widehat{\pi}$ *and to higher welfare otherwise.*

*ii) If* $\underline{\pi} > \overline{\pi}$*, then at* $\gamma = \gamma^*$ *default leads to higher welfare for any* $\pi \in (\underline{\pi}, 1)$*.*

Insert Figure 1

The proposition highlights the importance of the probability of the good state, as representing the inverse of the exogenous risk in the economy, for welfare. The results are illustrated in Figure 1, which plots the welfare in the no default and mixed equilibrium as a function of $\gamma$ for different levels of $\pi$. As the figure shows, the occurence of default is more likely to lead to lower welfare in riskier economies, i.e., when $\pi_0 < \widehat{\pi}$. In this case, at $\gamma = \gamma^*$, the welfare is higher in the no default equilibrium than in the mixed one because in the latter more reserves are needed in aggregate to satisfy the higher consumers' repayments in state $H$. By contrast, in economies characterized by low risk, i.e., when $\pi_1 > \widehat{\pi}$, welfare is higher in the mixed equilibrium where default occurs.

The proposition requires that the difference $P_H - \lambda_H c_1^{nd}$ is decreasing in $\pi$ to ensure the uniqueness of the cutoff values $\overline{\pi}$ and $\widehat{\pi}$. Unfortunately, it is not easy to prove the monotonicity analytically. The consumption $c_1^{nd}$ increases with $\pi$, but $P_H$ is not monotonic in $\pi$. To see this, consider the derivative of $P_H$ with respect to $\pi$ as given by

$$\frac{dP_H}{d\pi} = -\frac{r(r-1)}{(r-\pi)^2} - \frac{\pi(1-\pi)}{(r-\pi)^2} V \frac{d\gamma^*}{d\pi}.$$

The first term represents the (negative) direct effect of a change in $\pi$, while the second term is the indirect one through a change in $\gamma^*$. Since $\frac{d\gamma^*}{d\pi} < 0$, the second term becomes positive and, depending on the value of $\pi$, may dominate so that $\frac{dP_H}{d\pi}$ is not monotonic in $\pi$. However, even when this is the case, as long as the indirect effect is small enough, the monotonicity of $P_H - \lambda_H c_1^{nd}$ is guaranteed.

The previous result has implications for the comparison across different equilibria, as shown in the following proposition.

**Proposition 8** *Take two (sufficiently close) values $\pi_0$ and $\pi_1$ with $\pi_1 > \pi_0$ and define $W^{nd}(\pi_0)$ and $W^d(\pi_1)$ as the welfare without default for $\pi_0$ and with default for $\pi_1$. Then,*

*(i) if $\pi_1 \in (\underline{\pi}, \widehat{\pi})$, $W^{nd}(\pi_0) \geq W^d(\pi_1)$ for $\gamma \in [\gamma^*(\pi_1), \gamma^*(\pi_0)]$;*

*(ii) if $\pi_0 \in (\widehat{\pi}, 1)$, $W^d(\pi_1) > W^{nd}(\pi_0)$ for $\gamma \in [\gamma^*(\pi_1), \gamma^*(\pi_0)]$.*

Insert Figure 2

The proposition illustrates how the probability of the good state $\pi$ influences the relationship between competition, stability and welfare. The results are illustrated in Figure 2, which plots the welfare in the two equilibria as a function of $\gamma$ for different values of $\pi$. For a given $\gamma$, the value of $\pi$ determines whether the equilibrium features default or whether all banks behave safely. The higher $\pi$ is, the smaller the threshold value $\gamma^*$ at which default starts occuring. As the figure shows, economies with a higher probability of the good state are not always more efficient. As long as $\pi$ is below $\widehat{\pi}$, increasing the probability of the good state from $\pi_0$ to $\pi_1$ reduces welfare as it induces some banks to default. By contrast, when $\pi$ is above $\widehat{\pi}$, default is efficient and thus a higher likelihood of the good state increases welfare further.

# 5 Implications

The model has several important implications in terms of the relationship between competition, stability and efficiency. The first insight is that competitive banking systems are more stable. When competition is intense, banks behave prudently. Each bank holds enough reserves to insure itself against the risk of experiencing large liquidity shocks. This is individually optimal as the opportunity cost of holding reserves is low when the credit market is competitive. This result is consistent with the findings in Berger and Bouwman (2009) that banks enjoying greater market power as a result of a process of mergers and acquisitions are more likely to hold fewer reserves and grant more loans; and with the finding in Petersen and Rajan (1995) that banks grant more loans as competition becomes less intense.

The second insight of the model is that the cutoff value of credit market competition at which default emerges in equilibrium crucially depends on the level of the exogenous risk in the economy. A low probability of experiencing large liquidity shocks increases the opportunity cost of holding reserves and thus induces banks to behave imprudently, even in more competitive credit markets. This implies a negative relationship between exogenous and endogenous risk in the model in terms of the effect of the magnitude of the exogenous risk on banks' incentives to default. The implication is that economies with similar levels of competition may differ in terms of stability depending on the level of exogenous risk. The result may provide an explanation for the mixed empirical evidence on the relationship between competition and stability.

The model also delivers some implications concerning the optimality and severity of crises in terms of number of risky banks in the economy. When large liquidity shocks are unlikely, more banks find it optimal to reduce their reserve holdings and bear the consequences of default in the bad state. This implies less frequent but more severe crises, and greater efficiency due to less market liquidity in the system. These findings are consistent with the results in Acharya and Viswanathan (2011) that when expectations of fundamentals are good banking systems are characterized by more severe crises due to greater system-wide leverage, and in Acharya, Shin and Yorulmazer (2011) that bank liquidity is countercyclical. Extending our reasoning to a more dynamic framework, our results are also consistent with the observation, as reported in Castiglionesi, Feriozzi and Lorenzoni (2010), that market liquidity has decreased over time during the boom phase preceding the recent crisis.

Finally, the model has implications for credit availability. It shows that the amount of loans granted depends on both the degree of competition and the exogenous risk in the economy. The relationship between competition and provision of loans is not clear-cut. In risky economies with less competitive credit markets, stable banking systems where all banks behave prudently exhibit greater credit availability than systems where some banks default. In contrast, in economies that are less exposed to large liquidity shocks and have a more competitive credit market, the opposite is true in that banking systems with defaulting banks ensure greater loan provision than safe ones.

## 6 Concluding remarks

In this paper we have developed a simple model where banks face liquidity shocks and can invest in liquid reserves and safe loans. The latter can be sold on an interbank market at a price that depends on the demand and supply of liquidity. We have shown that two types of equilibria exist, depending on the degree of credit market competition. In the no default equilibrium, all banks are self sufficient as they hold enough reserves to always meet their liquidity demands. In the mixed equilibrium, banks make very different initial investment choices. A group of banks, defined as risky, does not keep any reserves and sell loans on the interbank market to satisfy their liquidity demands. In the state with low liquidity shocks, they sell a part of their loans and make positive profits. In the other state with large liquidity shocks, they sell all their loans and default. Thus, in the mixed equilibrium default is observed with positive probability. The no default equilibrium exists when competition in the credit market is intense, while the mixed equilibrium exists in more monopolistic credit markets. This implies that competition is beneficial for financial stability, but it is not necessarily welfare-enhancing. The mixed equilibrium is efficient when the economy is characterized by a sufficiently low probability of large liquidity shocks, as it allows the system to economize on reserves and increase credit availability.

The analysis is based on the assumption that banks compete for loans but are monopolist on the deposit market. An interesting extension is to consider deposit market competition as well. This would induce banks to provide better risk sharing to depositors by increasing the consumption promised to early types.

We have also considered that in the mixed equilibrium consumers can observe the type of bank they deposit at so that safe and risky banks offer different deposit contracts. If this assumption was removed, there would be a pooling in deposit contracts and consumers would be promised the same deposit terms irrespective of the type of bank. Despite guaranteeing depositors' participation, this would lead to ex post differences among consumers as those at the safe banks would enjoy a positive rent while those at the risky banks would suffer a loss. This would in turn lower the desirability of default.

The assumption that banks' type is observable to consumers guarantees also that the safe banks remain solvent when the risky banks default. Removing this assumption would also lead to the possibility of contagion across types of banks, in particular in the presence of a large number of risky banks in the economy as this would cast further doubts on the solvency of the remaining banks. This may lead banks to hold greater reserves initially and to develop strategies to signal their types to their depositors. Modelling an economy with unobservable types of banks would constitute an interesting future research topic.

We have assumed that default is costless as it does not entail any bankruptcy costs. Despite this, we have shown that default is not always welfare efficient as in some circumstances it can lead to lower credit availability. Introducing bankruptcy costs that are not internalized by the individual banks would lower the desirability of default for the economy as a whole. Introducing bankruptcy costs affecting the recovery rate of consumers in the case of default would increase the costs of default for the risky banks and would therefore lower the value of competition beyond which default is observed.

A final remark regards the way we have modeled competition in the credit market and in particular the fact that the demand by entrepreneurs is inelastic. An interesting alternative specification would be to consider that the demand for loans decreases as the interest rate increases. This would generate an additional trade-off between liquidity and credit availability as it would limit the profitability for banks to extend loans depending on the degree of competition.

## Appendix

**Proof of Proposition 1:** The bank's maximization problem is convex and since the profit function is concave, it has a unique solution. In order to avoid default the bank chooses to keep enough reserves to cover its demand for liquidity at date 1 in either state. Given $\lambda_H > \lambda_L$, in equilibrium it must hold

$$R^{ND} = \lambda_H c_1. \tag{40}$$

This implies that (4) is satisfied with equality in state $H$ and with strict inequality in state $L$. It is easy to show that the only other binding constraint in equilibrium is the consumers' participation constraint as given by (7). Solving it with respect to $c_2$, we obtain

$$c_2 = c_1^{-[\frac{\pi\lambda_L+(1-\pi)\lambda_H}{\pi(1-\lambda_L)+(1-\pi)(1-\lambda_H)}]}. \tag{41}$$

Substituting the expression for $c_2$ and (40) into (2) gives

$$\Pi_i = r(1-\lambda_H c_1) + \pi(\lambda_H - \lambda_L)c_1 - [\pi(1-\lambda_L) + (1-\pi)(1-\lambda_H)]c_1^{-\frac{\pi\lambda_L+(1-\pi)\lambda_H}{\pi(1-\lambda_L)+(1-\pi)(1-\lambda_H)}}.$$

Differentiating this with respect to $c_1$, we obtain

$$-r\lambda_H + \pi(\lambda_H - \lambda_L) + [\pi\lambda_L + (1-\pi)\lambda_H]c_1^{-\frac{1}{\pi(1-\lambda_L)+(1-\pi)(1-\lambda_H)}} = 0,$$

from which $c_1^{ND}$ as (8) in the proposition. Substituting (8) into (41) gives $c_2^{ND}$ as in (9) in the proposition. □

**Proof of Proposition 2:** We derive the vector $\{R^S, L^S, c_1^S, c_2^S, R^R, L^R, c_1^R, c_2^R, \rho, P_L, P_H\}$ characterizing the mixed equilibrium as the solution to the maximization problem of the safe and risky banks, the market clearing conditions and the equality between the expected profit of risky and safe banks. In the banks' maximization problems the only binding constraints are the consumers' participation constraints given by (17) and (22). It can be shown that all other constraints representing the resources constraints at dates 1 and 2 are satisfied with strict inequality.

Consider first the maximization problem for the safe banks. Using the Lagrangian, this can be written as

$$\mathcal{L}^S = \Pi^S - \mu^S \left[ [\pi\lambda_L + (1-\pi)\lambda_H]\, u(c_1^s) + [\pi(1-\lambda_L) + (1-\pi)(1-\lambda_H)]\, u(c_2^s) \right].$$

The first order conditions with respect to the reserves $R^S$, $c_1^S$, $c_2^S$ and $\mu^S$ are as follow:

$$\frac{\pi}{P_L} + \frac{1-\pi}{P_H} = 1, \tag{42}$$

$$\left[ \frac{\pi\lambda_L}{P_L} + \frac{(1-\pi)\lambda_H}{P_H} \right] r = -\frac{\mu^S}{c_1^S} [\pi\lambda_L + (1-\pi)\lambda_H] \tag{43}$$

$$c_2^S = -\mu^S \tag{44}$$

$$[\pi\lambda_L + (1-\pi)\lambda_H]\, u(c_1^S) + [\pi(1-\lambda_L) + (1-\pi)(1-\lambda_H)]\, u(c_2^S) = 0. \tag{45}$$

Consider now the maximization problem of the risky banks. Using the Lagrangian, this becomes

$$\mathcal{L}^R = \Pi^R - \mu^R \left[ \pi[\lambda_L u(c_1^R) + (1-\lambda_L) u(c_2^R)] + (1-\pi)[u(R^R + P_H(1-R^R))] \right].$$

The first order conditions with respect to the reserves $R^R$, $c_1^R$, $c_2^R$ and $\mu^R$ are as follows:

$$-\pi r + \frac{\pi r}{P_L} = \frac{\mu^R (1-\pi)(1-P_H)}{R^R + P_H(1-R^R)} \tag{46}$$

$$\frac{r}{P_L} = -\frac{\mu^R}{c_1^R} \tag{47}$$

$$c_2^R = -\mu^R \tag{48}$$

$$\pi[\lambda_L u(c_1^R) + (1-\lambda_L) u(c_2^R)] + (1-\pi)[u(R^R + P_H(1-R^R))] = 0. \tag{49}$$

The equilibrium is the solution to the system of all the first order conditions together with (24), (25) and (23).

We solve the system by first using (42) to derive $P_H$ as in (31). Then, we derive $R^S$ as in (26) from (24). Using (44), (43) and (45) after substituting (31) gives $c_1^S$ and $c_2^S$ as in (27) and (28) in the proposition.

Using (47) and (48), we can express $c_1^R$ as in (29) in the proposition and $R^R$ from (46) as

$$R^R = \frac{(1-\pi)c_1^R}{\pi(P_L - 1)} - \frac{P_H}{1-P_H}. \tag{50}$$

Substituting $P_H$ from (31) into (50) and rearranging it gives

$$R^R = \frac{(1-\pi)}{\pi(P_L-1)}(c_1^R - P_L) \leq 0$$

for any $P_L - 1 > 1$ and $c_1^R - P_L \leq 0$. The former follows from (31), as otherwise $P_H > 1 > P_L$. This contrasts with the equilibrium where $P_L > P_H$ must hold as there is more excess of liquidity in state $L$ than in state $H$. The latter, $c_1^R - P_L \leq 0$, follows from the fact that the profits of the risky banks must be non-negative in equilibrium. To see this, we rewrite (18) as follows:

$$\Pi^R = \pi\left[r + \left(\frac{1}{P_L} - 1\right)R^R r - (1-\lambda_L)c_2^R - \frac{\lambda_L c_1^R}{P_L} r\right].$$

As $\left(\frac{1}{P_L} - 1\right) R^R r < 0$ for $P_L > 1$, $\Pi^R \geq 0$ requires

$$r - (1-\lambda_L)c_2^R - \frac{\lambda_L c_1^R}{P_L} r > 0.$$

Rewriting $r$ as $\lambda_L r + (1-\lambda_L)r$ and rearranging the terms gives

$$(1-\lambda_L)(r - c_2^R) + \lambda_L r(\frac{P_L - c_1^R}{P_L}). \tag{51}$$

This is positive if $P_L - c_1^R > 0$ as this implies also that $r - c_2^R > 0$. Consider $P_L - c_1^R < 0$. Then, from (48), it is $c_2^R > r$ and (51) is negative. Then, in equilibrium $P_L - c_1^R > 0$ must hold. It follows that t then follows that $R^R = 0$ as in the proposition.

To find $c_2^R$ as in the proposition we first rearrange $\Pi^S = \Pi^R$ from (23) as

$$R^S[-1+\frac{\pi}{P_L}+\frac{1-\pi}{P_H}]r - \left[\frac{\pi\lambda_L}{P_L} + \frac{(1-\pi)\lambda_H}{P_H}\right] r c_1^S + r(1-\pi) + \pi c_2^R - [\pi(1-\lambda_L)+(1-\pi)(1-\lambda_H)]c_2^S = 0.$$

From (42) it is $[-1+\frac{\pi}{P_L}+\frac{1-\pi}{P_H}]=0$. From (43) it holds

$$\left[\frac{\pi\lambda_L}{P_L}+\frac{(1-\pi)\lambda_H}{P_H}\right]rc_1^S=[\pi\lambda_L+(1-\pi)\lambda_H]c_2^S.$$

Substituting these into the expression above for $\Pi^S-\Pi^R$, we have $c_2^R$ as in (30) in the proposition.

Finally, from (22), (24) and (26), we have $P_L$ and $\rho$ as in (32) in the proposition. □

**Proof of Lemma 1:** The maximization problem of the deviating bank is the same as the one of the risky banks in the proof of Proposition 2. The first order conditions with respect to the reserves $R^D$, $c_1^D$, $c_2^D$ and $\mu^D$ are as (46), (47), (48) and (49). The only difference is that $P_L=r$ and thus $P_H=\frac{r(1-\pi)}{r-\pi}$. The solutions to the first order conditions are:

$$R^D=0 \tag{52}$$

$$c_1^D=c_2^D=c^D=(P_H)^{-\frac{(1-\pi)}{\pi}}=\left(\frac{r(1-\pi)}{r-\pi}\right)^{-\frac{(1-\pi)}{\pi}}. \tag{53}$$

The lemma follows. □

**Proof of Proposition 3:** Substituting (52) and (53) into (33) gives

$$\Pi^D=\pi(r-c^D). \tag{54}$$

Deviating is profitable if and only if $\Pi^D\geq\Pi^{ND}$, where $\Pi^{ND}$ is as in (10). Define $f(\pi,\gamma)=\Pi^{ND}-\Pi^D$. When $\gamma\to\frac{1}{V}$ and $r\to 1$, $f(\pi,\gamma)\to 0$ since from (8), (9) and (53), $c_1^{ND}=c_2^{ND}=c^D\to 1$. Differentiating $f(\pi,\gamma)$ with respect to $\gamma$ gives

$$\frac{\partial f(\pi,\gamma)}{\partial\gamma}=\frac{\partial\Pi^{ND}}{\partial\gamma}-\frac{\partial\Pi^D}{\partial\gamma}$$

where

$$\frac{\partial\Pi^{ND}}{\partial\gamma}=V(1-\lambda_H c_1^{ND})>0$$

$$\frac{\partial\Pi^D}{\partial\gamma}=\pi V\left(1-\frac{(1-\pi)c^D}{(r-\pi)r}\right)>0.$$

The profits $\Pi^{ND}$ and $\Pi^D$ are monotonically increasing in $\gamma$. For $\gamma\to\frac{1}{V}$, $\frac{\partial f(\pi,\gamma)}{\partial\gamma}\to V(1-\lambda_H)>0$. Thus, there exists a unique threshold $\gamma^*\in(\frac{1}{V},1)$ such that $f(\pi,\gamma)=$

$\Pi^{ND} - \Pi^{D} = 0$ if and only if $f(\pi, \gamma) \leq 0$ for $\gamma \to 1$. A sufficient condition is that $\pi$ is sufficiently high. To see this, we first show that $f(\pi, \gamma)$ is monotonically decreasing in $\pi$. Differentiating $f(\pi, \gamma)$ with respect to $\pi$ gives

$$\frac{\partial f(\pi, \gamma)}{\partial \pi} = -(r - c^{D}) - \frac{\partial c_2^{ND}}{\partial \pi} + \pi \frac{\partial c^{D}}{\partial \pi}, \tag{55}$$

where

$$\frac{\partial c_2^{ND}}{\partial \pi} = (\lambda_H - \lambda_L) c_2^{ND} \left[ \ln \left( \frac{\pi \lambda_L + (1 - \pi)\lambda_H}{\pi \lambda_L + (r - \pi)\lambda_H} \right) + \frac{\lambda_H (r - 1)}{\pi \lambda_L + (r - \pi)\lambda_H} \right],$$

$$\frac{\partial c^{D}}{\partial \pi} = -\frac{c^{D}}{\pi} \left[ \frac{1}{\pi} \ln \left( \frac{r - \pi}{r(1 - \pi)} \right) - \frac{r - 1}{(r - \pi)} \right].$$

For $\lambda_H \to \lambda_L$, $\frac{\partial c_2^{ND}}{\partial \pi} \to 0$. The sign of $\frac{\partial c^{D}}{\partial \pi}$ is negative if the difference in the square bracket is positive. It is easy to see that such a difference is increasing in $r$ and it equals zero for $r \to 1$. This implies $\frac{\partial c^{D}}{\partial \pi} < 0$ for any $r > 1$. Also, it then holds that $\frac{\partial f(\pi, \gamma)}{\partial \pi} < 0$ for any $\gamma$. Given the monotonicity of $f(\pi, \gamma)$ in $\pi$, there exists a value $\underline{\pi}$ such that $f(\pi, \gamma) = \Pi^{ND} - \Pi^{D} = 0$ for $\gamma \to 1$ and $f(\pi, \gamma) < 0$ for $\gamma \to 1$ for any $\pi > \underline{\pi}$. The proposition follows. □

**Proof of Proposition 4:** Recall that in the mixed equilibrium $\Pi^{S} = \Pi^{R}$ and note from (18) that $\Pi^{R}$ increases with $P_L$ for any given $\gamma$. Moreover, $\Pi^{R} = \Pi^{D}$ and $\Pi^{S} = \Pi^{ND}$ when $P_L = r$. Thus, as $\Pi^{D} = \Pi^{ND}$ at $\gamma = \gamma^*$ and $\Pi^{R} = \Pi^{S}$ in the mixed equilibrium, it must also hold $\Pi^{D} = \Pi^{ND} = \Pi^{R} = \Pi^{S}$ at $\gamma = \gamma^*$. Consider now a value of $\gamma < \gamma^*$, where $\Pi^{D} < \Pi^{ND}$ from Proposition 3. It follows that $\Pi^{R}\big|_{P_L = r} < \Pi^{S}\big|_{P_L = r}$. For $\Pi^{R} = \Pi^{S}$ to hold as required in the mixed equilibrium, it must then be $P_L > r$. Consider now a value of $\gamma > \gamma^*$, where $\Pi^{D} > \Pi^{ND}$ from Proposition 3. It follows that $\Pi^{R}\big|_{P_L = r} > \Pi^{S}\big|_{P_L = r}$. Thus, it must be $P_L < r$ for $\Pi^{R} = \Pi^{S}$. It follows that the mixed equilibrium exists only for $\gamma \geq \gamma^*$. □

**Proof of Proposition 5:** Recall that $\gamma^*$ is the solution to $f(\pi, \gamma) = \Pi^{ND} - \Pi^{D} = 0$, where $\Pi^{ND}$ and $\Pi^{D}$ are given respectively by (10) and (54). The solution depends on $\pi$ and, from the implicit function theorem, $\frac{d\gamma^*}{d\pi} = -\frac{\partial f(\pi, \gamma)/\partial \pi}{\partial f(\pi, \gamma)/\partial \gamma}$. The numerator is the same as (55), which is negative for $\lambda_H \to \lambda_L$. So the sign of $\frac{d\gamma^*}{d\pi}$ is given by the sign of the denominator $\partial f(\pi, \gamma)/\partial \gamma$. As shown in the proof of Proposition 3, $\partial f(\pi, \gamma)/\partial \gamma > 0$ for $\gamma \to \frac{1}{V}$, and $\partial f(\pi, \gamma)/\partial \gamma < 0$ in the range of $\gamma$ where $\gamma^*$ exists. Thus, $\frac{d\gamma^*}{d\pi} < 0$, as in the proposition. □

**Proof of Proposition 6:** Differentiating the expression for $\rho$ in (32) with respect to

$\pi$ and rearranging it gives

$$\frac{d\rho}{d\pi} = (\lambda_H - \lambda_L)\left[c_1^S \frac{d(\lambda_L c_1^R - P_H)}{d\pi} - \frac{dc_1^S}{d\pi}(\lambda_L c_1^R - P_H)\right].$$

Differentiating $c_1^S$ as in (27) with respect to $\pi$ after substituting $P_L = r$ at $\gamma = \gamma^*$ gives

$$\begin{aligned}\frac{dc_1^S}{d\pi} &= (\lambda_H - \lambda_L)c_1^S\left[\ln\left(\frac{\pi\lambda_L + (1-\pi)\lambda_H}{\pi\lambda_L + (r-\pi)\lambda_H}\right) - \frac{[\pi(1-\lambda_L) + (1-\pi)(1-\lambda_H)]\,\lambda_H(r-1)}{[\pi\lambda_L + (r-\pi)\lambda_H]\,[\pi\lambda_L + (1-\pi)\lambda_H]}\right] + \\ &\quad - \frac{[\pi(1-\lambda_L) + (1-\pi)(1-\lambda_H)]}{[\pi\lambda_L + (r-\pi)\lambda_H]}\frac{d\gamma^*}{d\pi}c_1^S,\end{aligned}$$

which is positive for $\lambda_H \to \lambda_L$. Thus, a sufficient condition to have $\frac{d\rho}{d\pi} < 0$ is that $\frac{d(\lambda_L c_1^R - P_H)}{d\pi} < 0$, which holds because of the concavity of consumers' utility functions. □

**Proof of Lemma 2:** Substituting the expressions for $R^{ND}$ and $R^S$ from (40) and (26) into (36) and (37), we have

$$W^{ND} - W^M = c_2^S - c_2^{ND} + (V - r)\left[\rho\lambda_H c_1^S + (1-\rho)P_H - \lambda_H c_1^{ND}\right]. \tag{56}$$

At $\gamma = \gamma^*$ when $P_L = r$, with $c_1^{ND} = c_1^S = c_1^{nd}$ and $c_2^{ND} = c_2^S = c_2^{nd}$, (56) simplifies to

$$W^{nd} - W^d = (V - r)(1-\rho)(\lambda_H c_1^{nd} - P_H).$$

The lemma follows. □

**Proof of Proposition 7:** We first define the cutoff value $\overline{\pi}$. From (31) and (8), differentiating the difference $P_H - \lambda_H c_1^{nd}$ with respect to $\pi$ at $\gamma = \gamma^* = 1$ gives:

$$\begin{aligned}\frac{d(P_H - \lambda_H c_1^{nd})}{d\pi} &= -\frac{V(V-1)}{(V-\pi)^2} - \lambda_H(\lambda_H - \lambda_L)c_1^{nd}\left[\ln\left(\frac{\pi\lambda_L + (1-\pi)\lambda_H}{\pi\lambda_L + (V-\pi)\lambda_H}\right) + \right. \\ &\quad \left. -\lambda_H\frac{[\pi(1-\lambda_L) + (1-\pi)(1-\lambda_H)]\,(V-1)}{[\pi\lambda_L + (1-\pi)\lambda_H]\,[\pi\lambda_L + (V-\pi)\lambda_H]}\right].\end{aligned}$$

For $\lambda_H \to \lambda_L$, it is $\frac{\partial(P_H - \lambda_H c_1^{nd})}{\partial\pi} < 0$. Thus it exists a unique solution for the equation $P_H - \lambda_H c_1^{nd} = 0$ at $\gamma = \gamma^*$ defined as $\overline{\pi}$ such that $P_H - \lambda_H c_1^{nd} \geq 0$ for $\pi \leq \overline{\pi}$ and $P_H - \lambda_H c_1^{nd} < 0$ for $\pi > \overline{\pi}$.

(i) If $\underline{\pi} \leq \overline{\pi}$, then $P_H - \lambda_H c_1^{nd} \geq 0$ at $\gamma = \gamma^* = 1$. For $\pi \to 1$, $P_H - \lambda_H c_1^{nd} < 0$. From

(31) and (8), differentiating $P_H - \lambda_H c_1^{nd}$ with respect to $\pi$ at any $\gamma^* < 1$ gives:

$$\begin{aligned}\frac{d(P_H - \lambda_H c_1^{nd})}{d\pi} &= -\frac{r(r-1) + \pi(1-\pi)V\frac{d\gamma^*}{d\pi}}{(r-\pi)^2} - \lambda_H(\lambda_H - \lambda_L)c_1^{nd}\left[\ln\left(\frac{\pi\lambda_L + (1-\pi)\lambda_H}{\pi\lambda_L + (r-\pi)\lambda_H}\right) + \right.\\ &\left. -\lambda_H \frac{[\pi(1-\lambda_L) + (1-\pi)(1-\lambda_H)](r-1)}{[\pi\lambda_L + (1-\pi)\lambda_H][\pi\lambda_L + (r-\pi)\lambda_H]}\right] + \\ &-\lambda_H c_1^{nd}\frac{[\pi(1-\lambda_L) + (1-\pi)(1-\lambda_H)](r-1)}{[\pi\lambda_L + (r-\pi)\lambda_H]}V\frac{d\gamma^*}{d\pi},\end{aligned}$$

where $\frac{d\gamma^*}{d\pi} < 0$ from Proposition 5. For $\lambda_H \to \lambda_L$, the expression simplifies to

$$\frac{d(P_H - \lambda_H c_1^{nd})}{d\pi} = -\frac{r(r-1)}{(r-\pi)^2} + \left(-\frac{\pi(1-\pi)}{(r-\pi)^2} + \lambda_L(1-\lambda_L)\frac{c_1^{nd}}{r}\right)V\frac{d\gamma^*}{d\pi}.$$

Assuming $-\frac{\pi(1-\pi)}{(r-\pi)^2}V\frac{d\gamma^*}{d\pi}$ is sufficiently small, $\frac{d(P_H - \lambda_H c_1^{nd})}{d\pi} < 0$. Then, there exists a value $\widehat{\pi} \in (\underline{\pi}, 1)$ such that $P_H - \lambda_H c_1^{nd} \geq 0$ for any $\pi \leq \widehat{\pi}$ and $P_H - \lambda_H c_1^{nd} < 0$ otherwise. From Lemma 2, part (i) of the proposition follows.

ii) If $\underline{\pi} > \overline{\pi}$, then $P_H - \lambda_H c_1^{nd} < 0$ at $\gamma^* = 1$, and, given the monotonicity assumption, it remains negative for all admissible values of $\pi$. Part (ii) of the proposition follows. □

**Proof of Proposition 8:** We proceed in steps. We first show that the welfare $W^{ND}$ in the no default equilibrium is increasing in $\pi$ and in $\gamma$. Then, we compare the welfare without default with the one with default in the interval $[\gamma^*(\pi_1), \gamma^*(\pi_0)]$ depending on whether $\pi_1 \in (\underline{\pi}, \widehat{\pi})$ or $\pi_0 \in (\widehat{\pi}, 1)$.

Differentiating the expression for welfare without default as in (36) with respect to $\pi$ gives

$$\frac{\partial W^{ND}}{\partial \pi} = -\frac{\partial c_2^{ND}}{\partial \pi} - (V-r)\lambda_H \frac{\partial c_1^{ND}}{\partial \pi}.$$

Using the expressions for $c_1^{ND}$ and $c_2^{ND}$ as in Proposition 1, we obtain

$$\frac{\partial c_1^{ND}}{\partial \pi} = (\lambda_H - \lambda_L)c_1^{ND}\left[\ln\left(\frac{\pi\lambda_L + (1-\pi)\lambda_H}{\pi\lambda_L + (r-\pi)\lambda_H}\right) - \frac{[\pi(1-\lambda_L) + (1-\pi)(1-\lambda_H)]\lambda_H(r-1)}{[\pi\lambda_L + (r-\pi)\lambda_H][\pi\lambda_L + (1-\pi)\lambda_H]}\right] < 0$$

and

$$\frac{\partial c_2^{ND}}{\partial \pi} = -(\lambda_H - \lambda_L)c_2^{ND}\left[\ln\left(\frac{\pi\lambda_L + (r-\pi)\lambda_H}{\pi\lambda_L + (1-\pi)\lambda_H}\right) - \frac{\lambda_H(r-1)}{\pi\lambda_L + (r-\pi)\lambda_H}\right].$$

The sign of $\frac{\partial c_2^{ND}}{\partial \pi}$ is negative if the difference in the square bracket is positive. It is easy to see that such a difference is increasing in $r$ and it equals zero for $r \to 1$. This implies $\frac{\partial c_2^{ND}}{\partial \pi} < 0$ for any $r > 1$. It follows that $\frac{\partial W^{ND}}{\partial \pi} > 0$.

Differentiating (36) with respect to $\gamma$ and rearranging it gives

$$\frac{\partial W^{ND}}{\partial \gamma} = V(V-r)\lambda_H^2 \frac{\pi(1-\lambda_L)+(1-\pi)(1-\lambda_H)}{\pi\lambda_L+(r-\pi)\lambda_H} c_1^{ND} > 0.$$

(i) Given that $\frac{\partial W^{ND}}{\partial \gamma} > 0$, we only need to compare $W^{ND}(\pi_0)$ and $W^M(\pi_1)$ at $\gamma = \gamma^*(\pi_1)$. At $\gamma = \gamma^*(\pi_1)$, $c_1^S(\pi_1) = c_1^{ND}(\pi_1) = c_1^{nd}(\pi_1)$ and $c_2^S(\pi_1) = c_2^{ND}(\pi_1) = c_2^{nd}(\pi_1)$ so that the difference between welfare without and with default simplifies to

$$W^{ND}(\pi_0) - W^M(\pi_1) = c_2^{nd}(\pi_1) - c_2^{ND}(\pi_0) + (V-r)\left[\lambda_H\left(\rho c_1^{nd}(\pi_1) - c_1^{ND}(\pi_0)\right) + (1-\rho)P_H\right].$$

For $\pi_1 \to \pi_0$, $c_2^{nd}(\pi_1) \to c_2^{ND}(\pi_0)$ and $c_1^{nd}(\pi_1) \to c_1^{ND}(\pi_0)$. Then, the sign of $W^{ND}(\pi_0) - W^M(\pi_1)$ is given by the difference $P_H - \lambda_H c_1^{ND}(\pi_0)$, which is positive if $\pi_1 \in (\underline{\pi}, \widehat{\pi})$ from Proposition 7. Part (i) of the proposition follows.

(ii) It holds from Proposition 7 that $W^M(\pi_1) > W^{ND}(\pi_1)$ for $\pi \in (\widehat{\pi}, 1)$. This, together with $\frac{\partial W^{ND}}{\partial \pi} > 0$, implies part (ii) of the proposition. □

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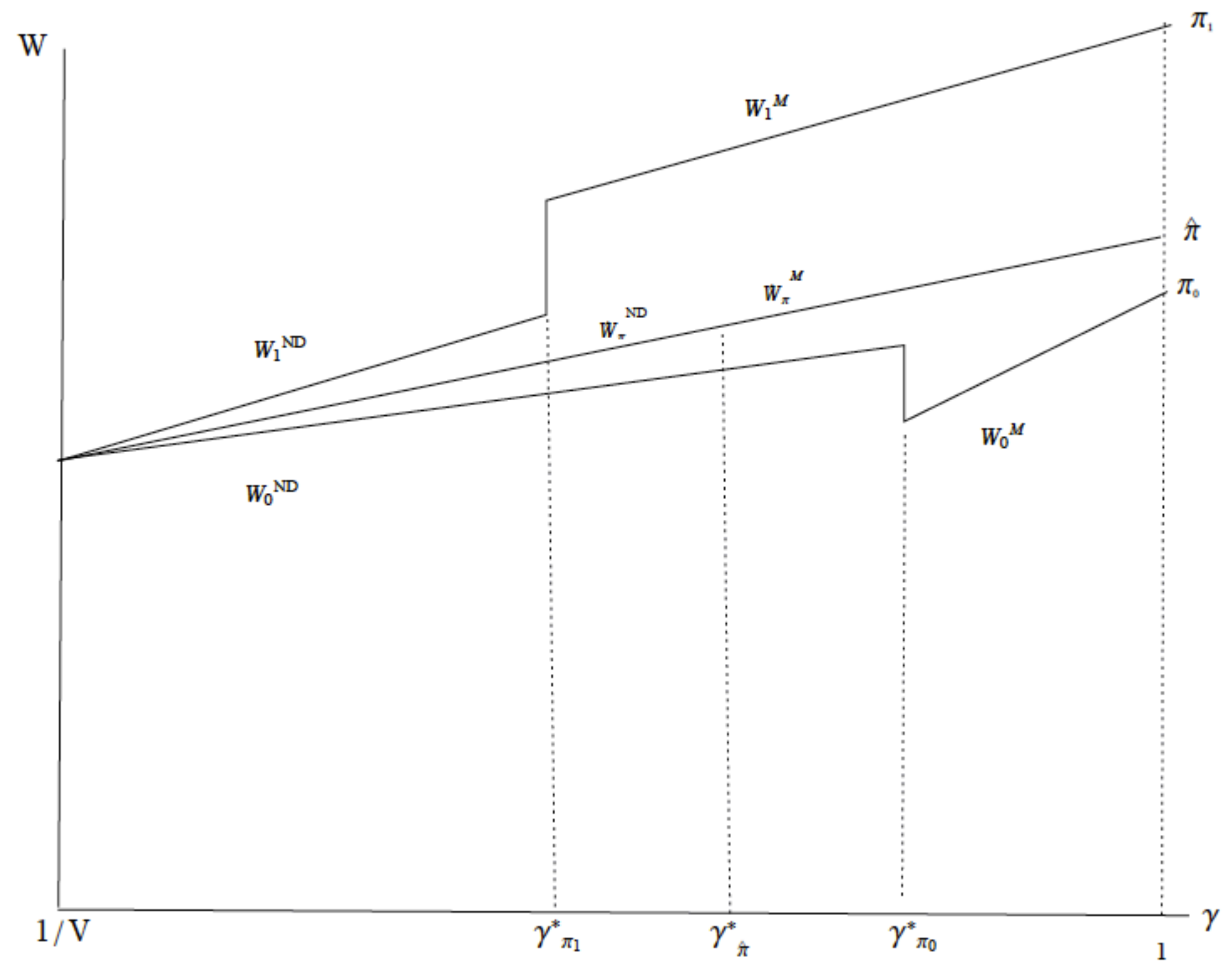


**Figure 1: Welfare as a function of the degree of credit market competition and of the probability of the good state.** The figure plots the welfare function in the no default equilibrium ($W^{ND}$) and in the mixed equilibrium ($W^M$) for different values of the probability of the good state ($\pi_0$, $\hat{\pi}$ and $\pi_1$) as a function of the degree of competition $\gamma$. For each value of $\pi$, the figure shows the threshold value $\gamma^*$ below which the no default equilibrium exists and above which the mixed equilibrium exists.

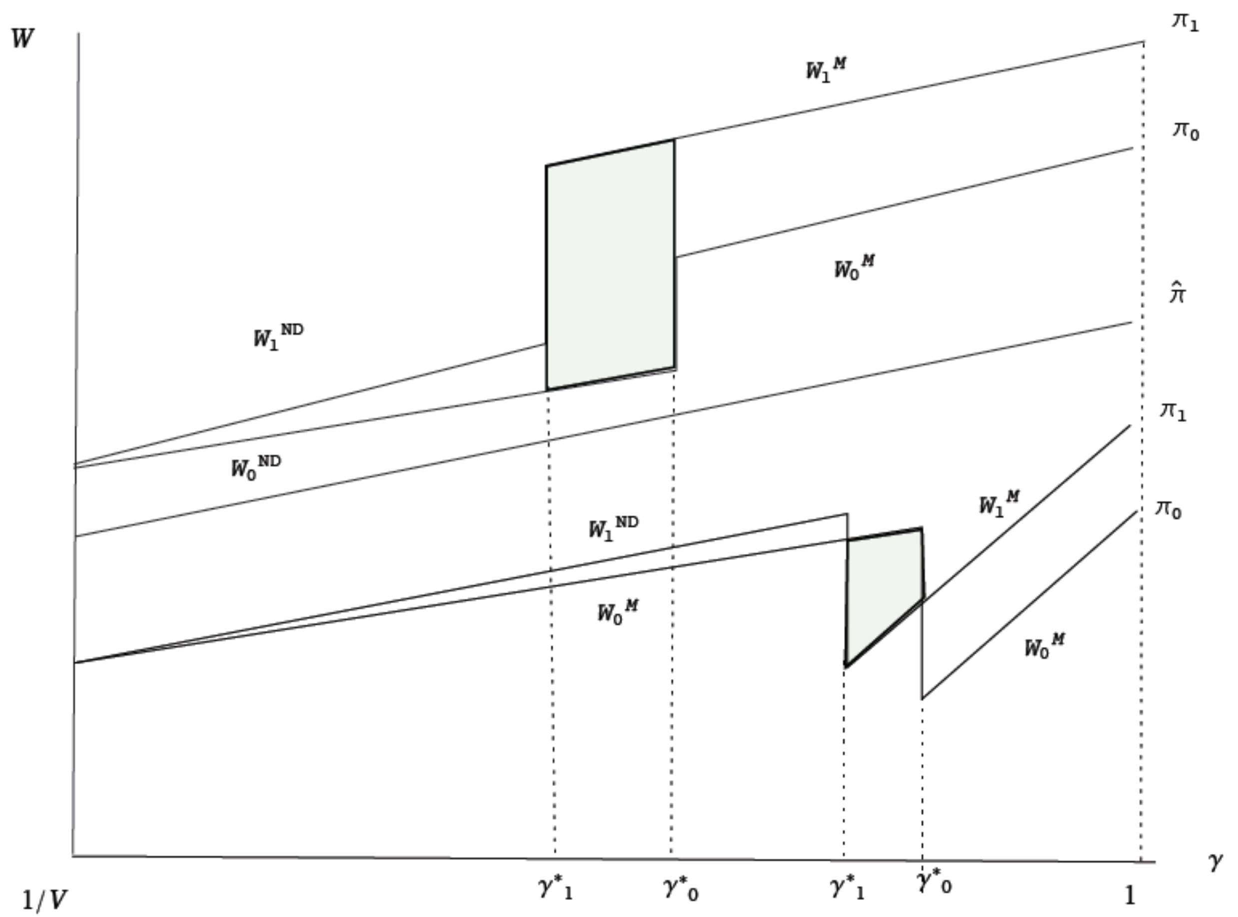


**Figure 2: Comparison of welfare across different values of the probability of the good state as a function of the degree of credit market competition.** The figure plots the welfare function in the no default equilibrium ($W^{ND}$) and in the mixed equilibrium ($W^M$) for different values of the probability of the good state ($\pi_0$, $\hat{\pi}$ and $\pi_1$) as a function of the degree of competition $\gamma$. The figure highlights the areas where welfare is compared for different values of $\pi$.

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