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A Volatility-based Theory of Fiscal Union Formation*

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Abstract

In a setting where heterogeneous jurisdictions share a common policy, an increase in volatility breaks unanimous support for the status-quo and may prompt a subset of jurisdictions to favor either deepening the agreement through a fiscal union, or revert to autarky. A reassignment of political weights in the fiscal union may restore unanimous support for the common policy. We show how the bargaining space depends on relative country income, size, and cross country correlation of shocks. Finally, we discuss the future of a common currency in Europe and the different emphasis member countries have put on the choice of fiscal federalism versus autarky.

Keywords: Fiscal Union; Common currency; Autarky; Bargaining space; Voting Weights; Heterogeneous countries.

JEL classification numbers: D70, D78, E62, F15, H77.

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1 Introduction

This paper provides a political economy model that identifies some important determinants to obtain a consensus agreement for a fiscal union, in presence of a common policy. We start with a set of $M$ countries, heterogeneous in average income and in population size, who have agreed on a common policy, and are subject to different income shocks. We think of the common policy as deriving from a deep institutional agreement of the type exemplified by the European Union. Below we will illustrate the common policy with reference to the monetary union in Europe, and its common currency, the Euro. We consider an upward shift in income volatilities, so that country preferences regarding participation in the union change. There are three alternatives: to remain in the status quo, with the common policy; to abandon the common policy, and revert to autarky; or to add a fiscal union dimension to the initial institutional agreement. By a fiscal union we understand a mechanism that involves some measure of counter-cyclical transfers among jurisdictions.\footnote{This concept is substantially different from the idea of a “fiscal stability union” debated by the EU leaders. As Paul Krugman in New York Times (December 10, 2011) puts it: “Rather than creating an inter-regional insurance mechanism involving counter-cyclical transfers, the version on offer would constitutionalize pro-cyclical adjustment in recession-hit countries, with no countervailing measures to boost demand elsewhere in the eurozone. Describing this as a “fiscal union,” as some have done, constitutes a near-Orwellian abuse of language”.} Our goal in this paper is to highlight that a fiscal union may be essential to sustain the benefits of a common currency in a scenario of increased volatility due, for instance, to an economic and financial crisis.

We focus on the interesting scenario where, after the increase in volatility, there is no agreement in favour of adding the fiscal union dimension.\footnote{As put forward by de Grauwe (2010), “it can be said that the government debt crisis in the eurozone is the result of a failure of economic governance”.
} In fact, we show that some countries may favour deepening the union and adding a fiscal dimension but, if that is not possible, they would rather revert to autarky. Furthermore, we show how the addition of institutional design and a change of decision weights can, under some conditions, re-establish consensus on the desirability of a fiscal union. We highlight the role of heterogeneity in
income per capita and population size, both in terms of positive analysis of country preferences, and in terms of normative analysis of the type of reallocations of decision power that would make a fiscal union consensus feasible. We find that, given their autarky threat point, countries with a large relative income and a large relative size will demand a higher decision weight in the fiscal union.

In order to provide insights on the scope of agreement for the formation of a fiscal union among heterogenous countries, we characterize the “bargaining space” of a set of countries that decide between adding the fiscal union dimension or reverting to autarky. The Consensus Bargaining Space (CBS) consists of the set of all vectors of country weights that assign a higher utility to all countries in the fiscal union, relative to autarky. After this characterization, we present some simulations that illustrate how the bargaining space of a two-country union changes when we change the correlation between shocks, and the countries’ relative income and size. We relate our findings to the existing literature, in particular Alesina and Perotti (1998), Alesina and Barro (2002), Casella (1992b), and Gordon (1983). We find that the likelihood of consensus for a fiscal union decreases in the degree of heterogeneity between countries’ income and size, and increases the more negative is the correlation between countries’ shocks. We also illustrate that, for a union formed by countries heterogeneous in income and size, and with different shock correlations, there are voting weights in the non-empty bargaining space that make all countries better off when adding a fiscal union, rather than reverting to autarky. Our contribution can be viewed as a volatility-based theory of the formation of fiscal unions.

As a way of illustration our model can be relevant to analyzing the case of the common currency in the European Union. Our theory of volatility suggests that, in a context of low volatility, it is natural for a monetary union to be formed, without the simultaneous

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3Although we think that the consensus rule is the most likely to be considered, one might think of an alternative scenario where only a fraction $\alpha$ of countries is required to form the fiscal union. Then, if the $\alpha$-majority is achieved, all countries must form the union (provided some form of constitutional requirement or penalties for the implementation of the fiscal union). This alternative scenario is discussed in the Appendix below.
creation of a fiscal union. This was the case when the Euro was adopted in 1999. In fact, when volatilities are relatively low for all countries in a union, there is no unanimity in favor of a fiscal union. Any proposal in this sense would be defeated. Recently, as documented in Table 1 in Section 6, volatility increased dramatically for most of the European countries, and their preferences towards maintaining the common currency shifted. Our model will allow us to discuss issues like: given the shift in volatilities, which countries would be unhappy with the common currency (that is, the status quo); predict how these countries rank the other options, namely fiscal union and autarky; the extent to which these countries are willing to lose political weight political weight if the fiscal union is anonymously adopted; given the realizations of different country volatilities, whether a low volatility group would accept a fiscal union with high political weights, or rather break up on and enter in an institutional agreement on their own.

**Relationship with the literature:**

The pioneering work of Gordon (1983) presented a now classic argument highlighting the insurance benefits of a common fiscal policy. The ensuing literature concentrated on the possible negative co-movement of output across jurisdictions, as it created incentives to share a fiscal policy. See for instance, Bolton and Roland (1997) and Alesina and Spolaore (1997) for pure interregional redistributive models where the threat of secession by the rich imposes a binding constraint on federal fiscal policy. Persson and Tabellini (1996a) investigate the trade-off between risk sharing and redistribution when jurisdictions are asymmetric as far as aggregate risk parameters are concerned.

Acquiring access to a more stable tax base became the linchpin of the discussion on

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4Sachs and Sala-i-Martin (1992) discuss how monetary integration was not accompanied by fiscal federalism. Furthermore, they present evidence that national transfers to the EU budget are not responsive to local income fluctuations, unlike what occurs among states in the United States.

5See also Persson et al. (1997) for a discussion of different preferences over deepening of European policy-making in diverse policy areas.


7In a companion paper, Person and Tabellini (1996b), the authors have focused on the trade-off between interregional risk sharing and the presence of moral hazard in local government behaviour. See Casella (1992a, 2001) for different formalizations of the main issues at stake, and Ruta (2005) for a survey.
fiscal unions. Alesina and Perotti (1998) have labelled such an incentive “economic risk”,
but have then raised the possibility that a reduction in the volatility of the tax base, which
decreases economic risk, might be associated with higher volatility of the tax rate for a
given allocation of decision power. The latter might actually raise what the authors call
“political risk”, discouraging the establishment of a common fiscal policy. The mechanism
is simple: faced with non-synchronous fluctuations in output over time, countries or regions
decrease economic risk by sharing budgetary decisions and stabilizing the tax base; however,
the non-synchronous shocks may lead the larger country – which holds decision power - to
respond to a negative shock by imposing a higher tax rate on the union. In sum, in fiscal
unions with heterogeneous jurisdictions, economic insurance and political risk may be part
of the same bundle. Relative to the traditional literature, the model in Alesina and Perotti
(1998) reduces the range of parameters for which a fiscal union is desired. Even with shocks
that are negatively and perfectly correlated, the country with less decision power may want
to avoid the fiscal union.

Our model examines how the allocation of voting power across jurisdictions interacts
with heterogeneity in population size and income per capita to determine the likelihood
of unanimous adhesion to a fiscal union. This is in the tradition of constitutional design
exemplified by Buchanan and Tullock (1967) and Curtis (1972). We incorporate economic
and political fundamentals and show that there are allocations of voting rights that enlarge
the set of parameters for which fiscal unions are formed. In a sense, we enlarge the par-
parameter set so that Gordon (1983) and Alesina and Perotti (1998) can be seen as particular
cases of a broader discussion.

The voting weights in collective decision making are always a central part of treaties
(e.g. the Nice Treaty, the Lisbon Treaty, etc) - see Felsenthal and Machover (2001) and
references therein-. One can find work on voting weights reallocations for the prospect of
widening of the Union, see e.g. Sutter (2000) and Barsan-Pipu and Tache (2009). However,
the issue of the weights ascribed to countries of differing size and economic conditions
has never been explicitly related to the implications for fiscal unification. We contribute to the literature on fiscal federalism by explicitly discussing the relation between voting arrangements on the one hand, and the decision between embarking on a joint fiscal policy and abandoning the common policy (currency) on the other.

A related strand of literature focuses on the distribution of voting weights among the countries of a monetary union. Casella (1992b) examines the range of feasible weights between two different countries in a monetary union. Casella’s main conclusion is that a sufficient small country will require and obtain a weight in the common monetary policy decision making larger than its size if deviation is allowed and cooperation is to be sustained. In contrast, our analysis of fiscal union formation in presence of a common policy asserts that the country with a larger relative size will demand a higher decision weight given its autarky threat point. The reason is that, when countries become dissimilar in size, the larger country is discouraged from imposing heavy taxes on its richer partners in the union, as it would be imposing deadweight losses on its own, large population (which forms a large fraction of the tax base).

Alesina and Barro (2002), on the other hand, analyze the trade-off between volume of trade and price stability for the formation of a currency union and for the optimal number of countries in a currency union. In a two-countries economy, Alesina and Barro identify the country that has more to gain from giving up its monetary policy as a small country with a history of high inflation and high correlation with its trading partner. On the other hand, our paper takes the country’s benefits of belonging to a monetary union as a parameter, whereas we open the black box for the formation of a fiscal union. In contrast to the result of Alesina and Barro, we show that the likelihood of a fiscal union increases the smaller is the correlation between shocks, and the smaller the heterogeneity in income and size between countries. This finding suggests that the effects of country size and

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8 See Cooper and Kempf (2004) for the question when the correlation of shocks is a useful metrix for evaluating the welfare gains from a monetary union. A related model is Dixit and Lambertini (2003), where in addition to a monetary union the authors also consider separate fiscal policies of the member countries.
shocks correlation move in opposite directions when comparing the formation processes of a monetary union and a fiscal union.

In this paper we do not consider that countries can share risks using financial instruments. But it should be clear that the financial markets are not a substitute for a fiscal union. This point was made by Celentani, Conde-Ruiz, and Desmet (2004). In that paper the authors show how a set of decentralized fiscal entities (“autarky” in our terminology) can manipulate relative prices, leading to an inefficient risk sharing, even if countries have access to a sequentially complete financial structure of assets. These authors point out that the creation of a fiscal union can recover the efficiency of risk sharing. Thus, they argue, a fiscal union plays an important role and is necessary even if countries have access to complete markets.

The remaining of this paper is as follows. Section 2 presents the model set-up. Section 3 examines country’s preferences for each of the three possible alternatives: the status quo, autarky, and fiscal union. Section 4 presents our theory of volatility for the formation of a fiscal union. Section 5 characterizes the bargaining space, and shows how proportional proposal rights enlarge the feasibility set to support a fiscal union, for different parameter sets as far as the correlation between shocks and countries’ asymmetries in size and income. Also in Section 5, we discuss how to accommodate the Nash bargaining solution. Section 6 discusses the implications of our model for the recent Euro crisis and the future of monetary union. Section 7 concludes.

2 Model

Let us consider a set of $M$ countries - or, more generally, jurisdictions- with fixed frontiers and population. In our model, we think of an abstract measure of the country’s economic activity - for simplicity we shall call such a measure “income” - as a summary of the its

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9This issue has recently arisen in the current discussion of the benefits of the European Union issuing Euro bonds.
economic performance to which economic shocks affect.\textsuperscript{10} Countries may differ in terms of population size and income, which are treated as exogenous parameters in this model. We denote country $i$’s pre-tax income by $Y_i \in R_+$. We assume that all individuals in country $i$ have the same income, excluding therefore the issue of internal redistribution.\textsuperscript{11} Country $i$’s population is denoted by $N_i \in R_+$. Total population is therefore $N = \sum_{i=1}^{M} N_i$. The relative population size of country $i$ in the union is $n_i = N_i/N$ with $n_i \neq 0$.\textsuperscript{12}

All countries are subject to idiosyncratic random productivity shocks that change their income levels. We interpret shocks as medium to long term shocks to productivity, not yearly cyclical fluctuations. Country $i$’s after shock income level is denoted by $X_i = Y_i(1 + \varepsilon_i)$. The triplet shock-income-size $(\varepsilon_i, Y_i, N_i)$ characterizes country $i$’s type. Let $\varepsilon = (\varepsilon_1, \ldots, \varepsilon_M)$ denote a vector of shocks for this economy. The vector $\varepsilon$ is drawn by Nature according to an $M$-dimensional distribution $\text{Pr}(\varepsilon)$. We denote by $C$ the space of symmetric matrices $[\rho_{ij}]_{M \times M}$ of pair-wise correlation coefficients with generic element $\rho_{ij}$. A state $\omega = (C, (Y_i, N_i)_{i=1, \ldots, M})$ is a vector of shock correlations and country incomes and sizes. The state space is denoted by $\Omega = C \times \mathbb{R}_+^M \times \mathbb{R}_+^M$. We say that, given $\omega \in \Omega$, there is possibility of reversal for a country $i$ if there exists a vector of shocks $\varepsilon = (\varepsilon_i)_{i=1}^M$ such that $Y_i > y$ and $X_i < x$, where $y = \sum_{i=1}^{M} n_i Y_i$ denotes the average income per capita in the fiscal union, before shocks are realized, and $x = \sum_{i=1}^{M} n_i X_i$ denotes the pre-tax, after-shock average income of the fiscal union. In other words, a reversal turns a pre-shock relatively

\textsuperscript{10}We are thinking of an expost measure of revenues minus costs, that should incorporate GDP, private debt, and public debt in the revenue side, and debt payments (interest plus principal, unmodeled in this paper) and other measures alike in the the cost side. To see why, let us consider the case of Italy in year 2011. This country is richer than many other euro countries, but it has an important debt component, very sensible to variations in interest rates, and this makes Italy more vulnerable to negative economic shocks, and hence a natural candidate to receive transfers from its Euro-zone counterparts.

\textsuperscript{11}As pointed out in the Introduction, the purpose of the current study is to analyze the “country incentives” to add a fiscal union dimension to an existing set of international institutions. The “class incentives” in countries with heterogeneous internal income levels have been studied, e.g. by Casella (2001), Person and Tabellini (1996a,b), Barberà and Jackson (2006), and Morelli et al. (2011), and references therein. There is a basic feasibility trade-off: when allowing for internal heterogeneous incomes it is difficult to allow for asymmetric population sizes and country incomes in the analysis of strategic institutional choice. In this paper the elimination of internal heterogeneities allows us to introduce the relevant heterogeneities across countries.

\textsuperscript{12}Migration issues among countries are absent here. For a fiscal union model with migration see Ortúñor-Ortín and Sempere (2006).
rich country into a relatively poor country after the shock, when compared to the union’s average income.\textsuperscript{13} A reversal can be due either to the large size of the shocks, given the differences in pre-shock incomes, or to a low correlation coefficient among country shocks, but large differences in pre-shock incomes.

We think of the status quo as the situation where all countries are part of an international common agreement where countries share the benefits of a common policy, but decide fiscal policies at the country level.\textsuperscript{14} Country \textit{i}’s benefits of the common policy are associated with the scalar \( g_i > 1 \). The absence of the common policy corresponds to \( g_i = 1 \). For the case of a common currency the benefits are associated with a reduction in transaction costs, price transparency, and the like. See Mundel (1961) and Alesina and Barro (2002). One can think of \( g_i \) as a reduced form that reflects both interjurisdictional spillovers, as well as the different local benefits of the commonly provided good, as exploited in Besley and Coate (2003).

We start from a status quo situation where the common policy is present, and then work out the conditions under which the policy is kept or abandoned. In status quo the utility of the representative agent is logarithmic,\textsuperscript{15} and increases in the consumption of the private good (\( X_i \)) and the benefits of the common policy (\( g_i \)).\textsuperscript{16}

\[
V_i^{SQ} = \ln(g_i X_i)
\]  

(1)

In functional form (1), the parameter \( g_i \) associated with the common policy benefits always

\textsuperscript{13}In the basic two country case, this just means that the countries switch positions in terms of relative income. Reversals between pairs of EU countries were relatively frequent in the case of Belgium, France, Germany, Netherlands, U.K. and Denmark. Reversals with respect to the EU average income also occur: such was the case for Belgium (1973, 1977, 1979), Germany (1974), Finland (1995), Ireland (1997), and Italy (2007). de Grauwe (2010) documents how country relative unit labor costs, an indicator of competitiveness, suffer reversals in a ten year period leading up to 2010. This evidence is supportive of our view that there is scope for income reversals in the medium to long-run.

\textsuperscript{14}In Section 4 below we characterize the parameter region where all countries indeed choose the status quo.

\textsuperscript{15}Any concave utility function would serve our purposes of representing risk aversion.

\textsuperscript{16}\( g_i \) can differ from \( g_j \) for some pair of countries \((i, j)\). In the common currency interpretation, \( g_i \) is greater for countries that trade more with their monetary union counterparts.
increases the after-shock income. We denote by $\sigma_i^C$ the standard deviation of shocks in presence of the common policy $g$, with $\sigma_i^C < 1$.

The timing is as follows. Date -1 accounts for the inherited status quo, where countries unanimously decided to adopt a common policy. At date 0, there is a change in fundamentals, so that countries expectations on future shocks change. Uncertainty is resolved at date 1, when idiosyncratic random productivity shocks change countries’ income levels.

At date 0, some countries want to revise the institution agreement. There are three options to choose from. Either all countries agree to remain in the status quo (the benchmark case); all countries agree to add a common fiscal policy dimension; or they revert to autarky, the situation where countries do not share the benefits of the common policy, nor form a fiscal union. The three alternative alternatives that need to be compared are:

**(SQ)** Remaining in the “status quo” (SQ), characterized by the same functional form as (1), with a shock standard deviation $\sigma_i^C$.

**(A)** Reversion to autarky (A), which means abandoning the benefits of the common policy. In the common currency illustration, reverting to individual currencies lowers the variance of after-shock disposable incomes. The argument derives from standard monetary theory.\textsuperscript{17} The notation for the standard deviation of shocks in autarky is $\sigma_i^A$. We consider that there is at least one country $i$ for which $\sigma_i^A < \sigma_i^C < 1$. Country $i$ representative agent’s utility of consumption in autarky is given by

$$V_i^A = \ln X_i$$

**(U)** Adding a fiscal union to the status quo. In this case countries will jointly decide the tax rate of a linear tax system after the unanimous decision to form the fiscal union.

Joint taxation acts as a form of insurance before the realization of shocks at date

\textsuperscript{17}This lower shock variance is due to the possibility of local adjustments made possible by independent monetary policy instruments. Roughly speaking, with its own monetary policy, a country is able to smooth the shock, making the shock volatility ex-post smaller.
1. Given that the common policy benefit is present in both the fiscal union and the status quo alternatives, we assume that the standard deviations of the shock in both regimes are the same, i.e., $\sigma_i^U = \sigma_i^C$ for all countries, and also that the common policy parameter is also present in the fiscal union, with $g_i > 1$.

The utility of an individual in country $i$ in the fiscal union is

$$V_i^U (T(\varepsilon)) = \ln g_i + \ln \left( (1 - T(\varepsilon)) X_i + (T(\varepsilon) - \frac{1}{2} T(\varepsilon)^2) x \right)$$

where $T(\varepsilon)$ denotes the tax rate that is chosen in the union after the realization of shocks $\varepsilon$. The term $(1 - T(\varepsilon)) X_i$ is the after-tax income after the shock is realized and the common tax is imposed. The term $(T(\varepsilon) - \frac{1}{2} T(\varepsilon)^2)x$ corresponds to the amount received after tax rebates, which depends on the average income $x$ in the fiscal union, as well as on the deadweight loss $(-\frac{1}{2} T(\varepsilon)^2)$ generated by the tax system.\(^\text{18}\) To determine the benefit of the alternative U, we first need to compute the preferred tax rate of each country in the fiscal union. Let us take the first order condition of $V_i^U (T(\varepsilon))$ with respect to $T(\varepsilon)$. Country $i$‘s preferred tax rate is given by

$$T_i(\varepsilon) = \begin{cases} 
1 - \frac{X_i}{x} & \text{if } X_i < x \\
0 & \text{otherwise}
\end{cases}$$

The lower the country $i$‘s after shock income relative to the average income, the higher its preferred tax rate in the fiscal union. Observe that the term $X_i/x$ makes the preferred tax rate for a given country depend on the income and population of all other countries in the union. Figure 1 plots the preferred tax rate $T_i(\varepsilon)$ for a generic country $i$ in a two-country fiscal union composed by countries $i$ and $j$ with negative correlated shocks. $T_i(\varepsilon)$ is a function of both the relative income $Y_i/Y_j$ and relative country size $N_i/N_j$.

\(^\text{18}\)The quadratic deadweight loss prevents the poor individual from imposing a tax rate that fully expropriates the rich. This is a standard assumption in the literature. If there were no deadweight loss, then every country’s aftershock income would equal the union average after-shock income.
Lemma 1: Country i’s desired tax rate is (weakly) decreasing in its relative size and relative income.

Proof: See the Appendix.

Figure 1: In this graph the shocks are assumed to be \((\varepsilon_i, \varepsilon_j) = (-0.2, 0.2)\). Axes \(Y_i/Y_j\) and \(N_i/N_j\) are measured in a logarithmic scale. A log scale makes the distance between two points \(b/a\) and \(a/b\) equal. For example, the distance between point \(1/2\) (where country \(i\) has half income than country \(j\)) and \(2/1\) (where country \(i\) has double income than country \(j\)) is the same since \(\log(2/1) = -\log(1/2)\). Observe that point 0 in the \(Y_i/Y_j\) axis represents the case where both countries have the same income.

We make the following two assumptions:

A1: For the fiscal union to be formed, all countries must agree.

Assumption A1 can be relaxed in different ways. We could think on some subset of countries having veto power, so the fiscal union can only form if it has these countries’ approval. Alternatively, we could consider some type of majority rule \(\alpha \in [1/2, 1]\), where
3 The effect of income and size on voting weights

At date 0 countries face the uncertainty of shock realizations at date 1, and decide among the three alternatives: SQ, A, or U. This is particularly important for the fiscal union regime, since we assume that the decision of joining the fiscal union occurs at date 0, before shocks are realized. Thus, countries’ utilities must be written in expectation. In order to obtain simple close form solutions for the expected utility, let us assume that, for each country \( i \), the random productivity shock \( \varepsilon_i \) follows a discrete distribution\(^{19} \) and can take two possible values, \( \bar{\varepsilon}_i \in [0, 1) \) and \( -\bar{\varepsilon}_i \).\(^{20} \) For any pair of countries \( (i, j) \) with shocks \( (\varepsilon_i, \varepsilon_j) \), there are four possible realizations \( (\bar{\varepsilon}_i, \bar{\varepsilon}_j) \), \( (\bar{\varepsilon}_i, -\bar{\varepsilon}_j) \), \( (-\bar{\varepsilon}_i, -\bar{\varepsilon}_j) \) and \( (-\bar{\varepsilon}_i, \bar{\varepsilon}_j) \). We assume that, for any pair of countries, asymmetric shocks, \( (\bar{\varepsilon}_i, -\bar{\varepsilon}_j) \) and \( (-\bar{\varepsilon}_i, \bar{\varepsilon}_j) \), occur with the same probability, denoted as \( q^a_{ij} \equiv \Pr(\bar{\varepsilon}_i, -\bar{\varepsilon}_j) = \Pr(-\bar{\varepsilon}_i, \bar{\varepsilon}_j) \). Likewise for symmetric shocks, \( (\bar{\varepsilon}_i, \bar{\varepsilon}_j) \) and \( (-\bar{\varepsilon}_i, -\bar{\varepsilon}_j) \), which occur with probability \( q^s_{ij} \equiv \Pr(\bar{\varepsilon}_i, \bar{\varepsilon}_j) = \Pr(-\bar{\varepsilon}_i, -\bar{\varepsilon}_j) \). Clearly, we must have that \( q^s_{ij} + q^a_{ij} = 1/2 \). Lemma A.1 in the Appendix shows that \( E(\varepsilon_i) = 0, \sigma_i = \bar{\varepsilon}_i \) and \( q^A_{ij} = (1 - \rho_{ij})/4 \). We consider that \( \Pr(\varepsilon) \) is one of the possible \( M \)-dimension  

\(^{19}\)Similar results and figures are obtained when using a continuous distribution, such as the multivariate normal. However, in the latter case, closed form solutions cannot be obtained.

\(^{20}\)We require after shock income to be non-negative and so take the lower bound of \( \varepsilon_i \) to be \(-1\). By symmetry we take the upper bound of \( \varepsilon_i \) to be \( 1 \).
distributions that is consistent with the two-dimensional distributions proposed for all pairs.\footnote{Notice that there are many $M$-dimensional distributions consistent with a single set of two-dimensional distributions. See Stoyanov (1996, p. 53) for examples showing that pairwise independence does not imply joint independence.}

We find that:

**Lemma 2:** Country $i$’s expected utility in status quo, under the institutional agreement, is increasing in the level of benefits from the common policy and in the pre-shock income per-capita, and decreasing in the volatility of its own shock, as follows:

$$E[V_i^{SQ}(\varepsilon_i)] = \ln g_i + \ln Y_i + \frac{1}{2} \ln(1 - (\sigma_i^C)^2)$$  \hspace{1cm} (5)

**Proof:** See the Appendix.

**Lemma 3:** Country $i$’s expected utility in autarky is increasing in the pre-shock income per-capita and decreasing in the volatility of its own shock, as follows:

$$E[V_i^A(\varepsilon_i)] = \ln Y_i + \frac{1}{2} \ln(1 - (\sigma_i^A)^2)$$  \hspace{1cm} (6)

We now proceed to compute the expected utility of a country if a fiscal union is formed. For this, we will first write the country $i$’s expected utility in the fiscal union, for the cases when either country $i$ or country $j$ decide the tax rate. These are, respectively,\footnote{Expression (7) is obtained by substituting the desired tax rate (4) into country $i$’s utility in the fiscal union regime (3), and then taking the expected value of the resulting form. Expression (8) follows a similar procedure in computation: we substitute the desired tax rate (4) into country $j$’s utility in the fiscal union regime (3), and then write the function in expectation form.}

$$E[V_i^U(T_i(\varepsilon)) = \sum_{\varepsilon \in \{\varepsilon_i, -\varepsilon_i\}^M} \Pr(\varepsilon) \left( \ln g_i + \ln \left( \frac{X_i(2 - X_i)}{2x} + \frac{x}{2} \right) \right)$$  \hspace{1cm} (7)

$$E[V_j^U(T_j(\varepsilon)) = \sum_{\varepsilon \in \{\varepsilon_i, -\varepsilon_i\}^M} \Pr(\varepsilon) \left( \ln g_j + \ln \left( \frac{X_j(2X_i - X_j)}{2x} + \frac{x}{2} \right) \right)$$  \hspace{1cm} (8)

We consider that, if a fiscal union is formed, the tax rate is chosen by a random dictator.
mechanism. In a collective decision policy by a random dictator mechanism what matters is the frequency with which country $i$ decides, or the “weight” $p_i$ country $i$ has in the decision system. The weights must obviously satisfy $p_i \in [0, 1]$ for all $i$, and $\sum_{i=1}^{M} p_i = 1$. For a given system $(p_1, \ldots, p_M)$ of proportional weights, expected utility in a fiscal union under such weights is given by

$$E[V_i^U(\varepsilon)] = p_i E[V_i^U(T_i(\varepsilon))] + \sum_{j \neq i} p_j E[V_j^U(T_j(\varepsilon))]$$  \hspace{1cm} (9)$$

We now change the relative country incomes and relative country sizes to illustrate how expected utilities (7) and (8) change. We then provide some intuition on how the decision weights in a two countries fiscal union should be assigned, for different relative incomes and sizes, so that a country prefers the fiscal union (9) to autarky (6).

![Graph](image)

Figure 2: We take $\sigma_i^A = \sigma_j^A = 0.6$, $\sigma_i^g = \sigma_j^g = 0.815$, $\rho = -0.5$ and $g_i = g_j = 1.45$.

Figure 2 depicts expected utilities (7), (8) and (6) as a function of relative incomes, for
equal country sizes. When country $i$ decides on the tax rate, its expected utility in the fiscal union is always above its expected utility in autarky because country $i$ will never be expropriated. Country $i$ chooses a positive tax rate when it is after-shock poorer, and a zero tax rate when it is after-shock richer, as the optimality condition (4) shows. When country $i$’s relative income increases, its expected utility in the fiscal union, as country $i$ decides on the tax rate, decreases but converges to a value $\ln g_i$ above country $i$’s expected utility in autarky. If it is the case that country $j$ is the one that decides on the tax rate, then country $i$’s expected utility in the fiscal union is closer to autarky levels when is relatively very poor, and thus not expropriated by $j$. However, country $i$ becomes worse off than in autarky as its relative income increases.

Figure 3: We take $\sigma^A_i=\sigma^A_j=0.6$, $\sigma^B_i=\sigma^B_j=0.815$, $\rho=-0.5$ and $g_i=g_i=1.45$.

Figure 3 shows how, for different country relative sizes but same incomes, country $i$’s expected utilities in the fiscal union change when both country $i$ or country $j$ decide. Country $i$’s expected utility in the fiscal union when $i$ decides the tax rate is above autarky
levels, and decreases the larger is country $i$’s relative size, as the tax base of the union becomes more and more composed of country $i$’s income. This makes country $i$ choose a lower and lower tax rate in order to avoid the deadweight loss of taxation. In the limit, country $j$ has a negligible size in the union, and the tax rate chosen by country $i$ is 0. Its expected utility converges to a value $\ln g_i$ above expected utility in autarky. In Figure 3 we also can see that country $i$’s expected utility when country $j$ decides on the tax rate decreases from $n_i = 0$ to a scenario when $n_i$ gets closer to 1, and then it increases\(^2\) and converges to a line that is below but parallel to country $i$’s expected utility in autarky, as $n_i \to 1$ ($T_j \to 1 - \frac{X_j}{X}$). Notice that when country $i$ is very small ($n_i$ close to 0), the tax base is almost entirely composed by country $j$’s income, so country $j$ chooses an almost zero tax rate. This makes country $i$’s utility in the fiscal union close to but slightly above its autarky level (where the benefit of the common policy is absent). On the other hand, when country $i$’s relative size is very large ($n_i$ close to 1), all tax base is composed by country $i$’s after-shock income, and optimality tells us that country $j$ expropriates at a non-negative tax rate $T_j = 1 - \frac{X_j}{X}$.

The previous analysis reveals that:

**Lemma 4:** Countries with a large relative income and a large relative size will demand a higher decision weight in the fiscal union given their autarky threat point.

The result follows a standard convex analysis. Country $i$ prefers the fiscal union to autarky if the convex combination $p_i E[V_i^U(T_i(\varepsilon))] + (1-p_i) E[V_j^U(T_j(\varepsilon))]$ at a given point (relative income or relative size) is above the autarky level $E[V_i^A(\varepsilon_i)]$. Figure 2 shows that the larger is country $i$’s relative income, the closer must be country $i$’s weight to 1 for country $i$ to be better off in the fiscal union than in autarky. In Figure 3 we can see that $E[V_i^U(T_i(\varepsilon))]$ is always decreasing in size and approaching the expected utility level in autarky, requiring the weight $p_i$ to be closer to 1 the larger is country $i$’s relative size.

\(^2\)It is easy to see that $\frac{dV_i^U(T_i(\varepsilon))}{dn_i} < 0$ for all $n_i \in [0,1]$, whereas $\frac{dV_j^U(T_j(\varepsilon))}{dn_i} < 0$ if $2X_i < X_j$, and $\frac{dV_j^U(T_j(\varepsilon))}{dn_i} > 0$ if $2X_i > X_j$. For the parameters chosen in Figure 3, we see that $2X_i < X_j$ occurs when $(\varepsilon_i, \varepsilon_j) = \{(\bar{\varepsilon}, \bar{\varepsilon}), (\bar{\varepsilon}, -\bar{\varepsilon})\}$ and that $2X_i > X_j$ occurs when $(\varepsilon_i, \varepsilon_j) = \{(-\bar{\varepsilon}, \bar{\varepsilon}), (-\bar{\varepsilon}, -\bar{\varepsilon})\}$. 

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4 A volatility-based theory of fiscal union

In this section we focus on the role of the shock volatility parameter for a country to discriminate among the three regimes (A, SQ, or U). This analysis follows a sequence of events similar to those experienced by the European countries before and after the recent economic crisis. At an initial date, all countries agreed to share a common policy, namely the Euro. This agreement came in a context of low volatility. At some later date, some of these countries experienced an episode of high uncertainty, that is, $\sigma_i$ jumped for some $i$. Countries came to face the choice between abandoning the status quo and reverting to autarky, or adding the fiscal policy dimension to the monetary union, with an “appropriate” distribution of voting weights. We start our formal analysis by characterizing the possible regimes. Let us define at date 0, for each country $i$, three thresholds: $\sigma_i$, denoting the volatility of shocks that makes country $i$ indifferent between the status quo and the fiscal union; $\bar{\sigma}_i$, at which country $i$ is indifferent between status quo and autarky; and $\tilde{\sigma}_i$, which makes country $i$ indifferent between fiscal union and autarky.\textsuperscript{24} Heterogenous countries may have different volatility thresholds. All volatility thresholds depend in a continuous manner on the weights, and on the relative size and income. Figure 4 illustrates the three thresholds for a country $i$ in a two countries economy. For simplicity, in this figure, we assume that both countries have the same income and size, and the same weight in the fiscal union.

The following proposition establishes a theory of how volatility affects preferences over the different regimes: autarky, status quo and fiscal union.

**Proposition 1:** For a set of realizations $(\varepsilon_i, Y_i, N_i)_{i \in \mathbf{I}}$, there are three thresholds $\hat{\sigma}_i$, $\bar{\sigma}_i$, and $\tilde{\sigma}_i$.\textsuperscript{24} The expected utility in autarky is given by (6), which is a horizontal straight line for all $\sigma_i$, portraying the fact that country $i$ can use its own monetary policy to smooth its shock volatility to some constant level $\sigma_i^\dagger$. On the other hand, country $i$’s expected utility in status quo, given by (5), is decreasing and concave in $\sigma_i$ (see Lemma App.2 in the Appendix). These two facts guarantee that $\sigma_i^\dagger$, at which country $i$ is indifferent between status quo and autarky, is uniquely defined (see Lemma App.2 in the Appendix). In order to have $\hat{\sigma}_i$ and $\sigma_i^\dagger$, we need the single crossing property to hold between, respectively, functions (9) and (5), and (9) and (6). This occurs when $\frac{dE[V^c(c)]}{d\sigma_i} < 0$ and $\frac{d^2E[V^c(c)]}{d(\sigma_i)^2} < \frac{d^2E[V^c(c)]}{d(\sigma_i^\dagger)^2} < 0$, for all $\sigma_i \in [0, 1]$. 

\textsuperscript{24}The expected utility in autarky is given by (6), which is a horizontal straight line for all $\sigma_i$, portraying the fact that country $i$ can use its own monetary policy to smooth its shock volatility to some constant level $\sigma_i^\dagger$. On the other hand, country $i$’s expected utility in status quo, given by (5), is decreasing and concave in $\sigma_i$ (see Lemma App.2 in the Appendix). These two facts guarantee that $\sigma_i^\dagger$, at which country $i$ is indifferent between status quo and autarky, is uniquely defined (see Lemma App.2 in the Appendix). In order to have $\hat{\sigma}_i$ and $\sigma_i^\dagger$, we need the single crossing property to hold between, respectively, functions (9) and (5), and (9) and (6). This occurs when $\frac{dE[V^c(c)]}{d\sigma_i} < 0$ and $\frac{d^2E[V^c(c)]}{d(\sigma_i)^2} < \frac{d^2E[V^c(c)]}{d(\sigma_i^\dagger)^2} < 0$, for all $\sigma_i \in [0, 1]$. 

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\( \bar{\sigma}_i \) and \( \hat{\sigma}_i \) that can be represented as in Figure 4 so that, for each country \( i \) subject to shock \( \varepsilon_i \):

a) the thresholds \( \hat{\sigma}_i \), \( \bar{\sigma}_i \) and \( \hat{\sigma}_i \) can be ordered so that \( \hat{\sigma}_i < \bar{\sigma}_i < \hat{\sigma}_i \).

b) if \( \sigma_i < \hat{\sigma}_i \), the status quo is preferred to the fiscal union, which in turn is preferred to autarky. Formally, \( E[V_i^{SQ}(\varepsilon_i)] > E[V_i^{U}(\varepsilon_i)] > E[V_i^{A}(\varepsilon_i)] \).

c) if \( \hat{\sigma}_i < \sigma_i < \bar{\sigma}_i \), the fiscal union is preferred to the status quo, which in turn is preferred to autarky. Formally, \( E[V_i^{U}(\varepsilon_i)] > E[V_i^{SQ}(\varepsilon_i)] > E[V_i^{A}(\varepsilon_i)] \).

d) if \( \sigma_i < \bar{\sigma}_i < \bar{\sigma}_i \), the fiscal union is preferred to autarky, which in turn is preferred to the status quo. Formally, \( E[V_i^{U}(\varepsilon_i)] > E[V_i^{A}(\varepsilon_i)] > E[V_i^{SQ}(\varepsilon_i)] \).

e) if \( \hat{\sigma}_i < \sigma_i < 1 \), autarky is preferred to the fiscal union, which in turn is preferred to the status quo. Formally, \( E[V_i^{A}(\varepsilon_i)] > E[V_i^{U}(\varepsilon_i)] > E[V_i^{SQ}(\varepsilon_i)] \).

![Different scenarios as a function of \( \varepsilon_i \) and \( \sigma_i \)](image)

Figure 4

As pointed out above, initially \( \sigma_i < \hat{\sigma}_i \) for all \( i \) and all countries agree to share a common policy. We then consider a upwards shift in some of the \( \sigma_i \) that makes at least
one of the countries have \( \sigma_i > \hat{\sigma}_i \). Now, given \((\varepsilon_i, Y_i, N_i)\) and the new \( \sigma_i \), for all \( i \), we can characterize four different relevant scenarios, in the absence of additional economic or political incentives, as far as the distribution of country preferences over regimes is concerned.

**Proposition 2:** Assume that Assumptions A1 and A2 hold. Then, we can identify the following regions:

**Region 1:** Countries remain in status quo if all countries have \( \sigma_i < \hat{\sigma}_i \) and there is at least two countries \( i \) and \( k \) with \( \sigma_i \in (\hat{\sigma}_i, \bar{\sigma}_i) \) and \( \sigma_k < \hat{\sigma}_k \), respectively.

**Region 2:** Countries form a fiscal union if all countries have \( \sigma_i \in (\hat{\sigma}_i, \bar{\sigma}_i) \).

**Region 3:** The union reverts to autarky if there is at least two countries \( i \) and \( k \) with \( \sigma_i < \hat{\sigma}_i \) and \( \sigma_k \in (\hat{\sigma}_k, \bar{\sigma}_k) \).

**Region 4:** The union reverts to autarky if there is at least one country with \( \sigma_i > \bar{\sigma}_i \).

In Region 1, at least one country prefers a fiscal union, but there is no unanimity in support of that move. Since all countries prefer status quo to autarky, the regime remains in status quo. Notice that, if sufficient countries are in region \( \sigma_i \in (\hat{\sigma}_i, \bar{\sigma}_i) \), where they prefer fiscal union to the status quo, it may be in their interest to compensate countries with \( \sigma_i < \hat{\sigma}_i \) through economic and political incentives, and make them approve the creation of the fiscal union.

In Region 2, the fiscal union is formed since there is unanimity. Notice that even if all countries have \( \sigma_i \in (\hat{\sigma}_i, \bar{\sigma}_i) \), so that all prefer autarky to the status quo, they unanimously prefer the fiscal union.

In Region 3, country \( i \) prefers to remain in status quo, and country \( k \) prefers to move to the fiscal union. Thus, without further compensation, no unanimity is attained and the union reverts to autarky. Similarly to region 1 above, if enough countries have \( \sigma_i \in (\hat{\sigma}_i, \bar{\sigma}_i) \), they could use economic and political incentives to convince countries with \( \sigma_i < \hat{\sigma}_i \) to vote in favor of the fiscal union.

In Region 4, country \( i \) has \( \sigma_i > \bar{\sigma}_i \), so it prefers autarky to both status quo and fiscal
union. Again, by a similar argument as above, only if there are sufficiently many countries with \( \sigma_i \in (\bar{\sigma}^i, \tilde{\sigma}^i) \) to compensate countries with \( \sigma_i > \tilde{\sigma}^i \), and eventually countries with \( \sigma_i < \bar{\sigma}^i \), will the unanimous creation of the fiscal union be possible.

In Regions 3 and 4 we observe the interesting trade-off pointed out in the Introduction: Without a fiscal union, high volatility kills the common currency. Next, we pose a simple example that illustrates this trade-off:

**Example:** Let three countries, namely 1, 2 and 3. For simplicity we assume that all countries have the same volatility, \( \sigma \), but individual thresholds are different. These are \( \hat{\sigma}_1 = 0.6, \hat{\sigma}_2 = 0.35, \hat{\sigma}_3 = 0.1, \tilde{\sigma}_1 = 0.8, \tilde{\sigma}_2 = 0.7 \) and \( \tilde{\sigma}_3 = 0.45 \). Now, consider the situations “before” and “after” bad news arrive and volatility \( \sigma^C \) increases. “Before” the news, \( \sigma = 0.05 \) and the three countries prefer the fiscal union to the status quo, and the status quo to autarky. “After” the news, with \( \sigma = 0.5 \), the situation changes. Country 1 still has the same preferences as “before”, but countries 2 and 3 preferences change. Country 2 now prefers the fiscal union to status quo, and status quo to autarky. Now, country country 3 prefers fiscal union to the status quo, but if the former regime is not unanimously supported, country country 3 reverts to autarky, and makes status quo collapse. \( \Box \)

The question that we shall address in the next section is: Are there political incentives that the countries in favor of the fiscal union can use to convince those other countries that prefer a reversion to autarky. In terms of our previous example, the question can be reformulated as follows: What is the nature of the institutional design that provides the political incentives that bring country 1 to support the fiscal union?

## 5 Bargaining space

As shown in the previous section there are instances (namely Regions 3 and 4) where an interesting trade-off arises between moving unanimously to fiscal union or reverting to autarky. Institutional design comes into the scene precisely at this point. By choosing the
appropriate proposal weights, countries can provide the necessary incentive to convince all
countries to adhere to the fiscal union. The mechanism proposed in this paper consists on
the assignment of a proposal weight to each of the countries in the fiscal union, so that the
tax rate is decided in line with these specific weights. As indicated above, these weights (or
probabilities) are described by a vector \((p_i)_{i=1}^M\). Weights must be such that, for all countries
(Assumption A1), the expected utility in the fiscal union is at least equal to the expected
utility in autarky. We are thus incorporating institutions into the picture, in the form of
proportional representation rights. In other words, we bring into our model an element
of political insurance where the literature has so far concentrated on economic insurance.
Institutional design is at the core of economic unions: it corresponds to the creation of
new institutions and voting rules that sustain the agreement and preserve the benefits of
common policies.

We now proceed to characterize the bargaining space in which a fiscal union is formed.
Let us define by \(p_i\) the country \(i\)’s minimum weight compatible with country \(i\) joining the
fiscal union. Following the assumption of unanimity for the formation of the fiscal union,
we denote by \(\bar{p}_i\) the country \(i\)’s maximum weight that is compatible with all the other
countries joining the fiscal union. In the Appendix we characterize the expressions \(\bar{p}_i\) and
\(p_i\) as functions of countries’ expected utilities in autarky and in the fiscal union. The
consensus bargaining space (CBS) then corresponds to the difference between these upper
and lower bounds, that is, \(\bar{p}_i - p_i\). A non-empty CBS (i.e., \(\bar{p}_i - p_i > 0\)) implies that the fiscal
union improves upon autarky for all countries in the union. Observe that if \(\bar{p}_i - p_i > 0\)
holds for one country, then it must hold for all countries, so the fiscal union improves upon
the autarky regime for all countries.\(^{25}\)

Let us now concentrate in the case of a two countries union. Our goal is to understand
the role that income and size heterogeneities and the correlation of shocks in the formation

\(^{25}\)One could depart from Assumption 1, which imposes the unanimity rule to form the fiscal union, and
consider instead that only a qualified majority \(\alpha \in [0, 1]\) of countries are necessary in order to implement
the fiscal union. In the Appendix we characterize the bargaining space corresponding to this alternative
qualified majority rule.
of a fiscal union. This analysis is useful because we can easily compare our results to
existing work, in particular, Alesina and Perotti (1998). Moreover, we can explore when
there exist voting weights that preserve the benefits of a common policy while limiting the
political risk of entering a fiscal union.

**Correlation of shocks and voting weights:** Our analysis extends both Gordon
(1983) and Alesina and Perotti (1998) to the whole range of possible shocks correla-
tions $\rho$ and voting weights $(p_i)_{i=1}^{M}$ parameters. Gordon (1983) develops the classical one-
dimensional analysis highlighting the role of negative cross-correlation of shocks on risk
sharing possibilities between countries. He showed that when $\rho < 0$ a fiscal union is a way
to provide economic insurance. The more negative the value of $\rho$, the higher the benefits
of common insurance.\(^{26}\) Alesina and Perotti (1998) extend Gordon’s framework by letting
one country, which they assume marginally larger than the other, to always decide on the
fiscal instrument, thus introducing what these authors term as "political risk”.

Figure 5 illustrates how the bargaining space between two countries $i$ and $j$ changes
when the correlation between country’ shocks change. This figure illustrates the argument
in Alesina and Perotti (1998) that “political risk” may overwhelm “economic risk”. This
reverses the classical result in Gordon (1982), where a perfectly negative cross-correlation
between shocks favors the formation of a fiscal union. Also, in our paper, the lower is
the correlation between shocks, the larger is the bargaining space.\(^{27}\) However, we have
non-empty bargaining space even with $\rho > 0$. In the context of Alesina and Perotti (1998),
where it was always the same country that decided on the common tax rate (corresponding,
in our setting, to $p_i = 1$), the fiscal union might not be possible due to the implicit political
risk, even if $\rho = -1$ and economic insurance is most attractive. In contrast to Alesina and
Perotti (1998), where institutional design was absent, our setup shows how voting weights

\(^{26}\) In the same vein of Bolton and Roland’s (1997) political economy model of integration, Fidrmuc (2004)
considers the impact of region-specific shocks in a dynamic setting, and shows that negatively correlated
temporary shocks allow the greatest gains from inter-regional risk sharing.

\(^{27}\) At $\rho = -1$ the fiscal union is sustainable when $p_i \in [0.45, 0.55]$. The bargaining space becomes negative
when $\rho \geq 0.7$. 

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need to be reassigned for the two countries to accept joining the union. In other words, what this picture brings forward is how a sensible redistribution of voting power, which decreases the likelihood of country \( i \) deciding, guarantees ex-ante unanimity in favor of the fiscal union. Thus, the addition of the institutional dimension countered the “political risk effect” in Alesina and Perotti (1998). In a sense, by allowing for variable voting weights, countries in our setup insure against political risk as in Alesina and Perotti (1998).

![Graph showing bargaining space as a function of \( \alpha \), when \( \alpha = \frac{M}{N_i} \) and \( \alpha \leq \alpha \).](image)

Figure 5: We set \( Y_i/Y_j=1, N_i/N_j=1, \) and \( g_i=g_j=1.45 \).

It is easy to see that the threshold at which the bargaining space becomes positive (that is, non-empty) changes with relative income.\(^{28}\) For a given correlation of shocks, increasing country \( i \)'s relative income makes both the minimum and maximum country \( i \)'s voting weights to increase. The intuition is simple. A country \( j \) that becomes relatively poorer demands a lower weight in the union since in this new situation the chances to expropriate country \( i \) through transfers increase. On the other hand, given the higher probability of being partially expropriated, the richer country \( i \) demands a higher weight in the union in

\(^{28}\)See Luque, Morelli, and Tavares (2011).
order to keep its expected utility above the autarky level.

**Voting weights and asymmetries in income and size:** Previous results in the literature have focused on the correlation coefficient between country shocks to analyze the benefits of forming a union, for equal countries’ ex-ante income and population. Here, instead, we start with countries that are different in income and size, and characterize the bargaining space for different possible parameters. We pose the following question: given the unanimous voting rule, what are the income and size parameters that make a fiscal union possible?

![Bargaining Space in differences as a function of relative incomes and sizes, in log scale](image)

**Figure 6:** We set $\rho = -0.5$.

Figure 6 shows how the bargaining space changes when relative incomes and population sizes change. We assume $\rho = -0.5$. The three dimensional graph shows that the less heterogeneous the countries, the larger the scope for agreement. The reasoning is as follows: when countries become dissimilar in income, the country with higher income demands higher weight in the union to avoid expropriation through the common tax. This in turn
decreases the bargaining space. On the other hand, when countries become dissimilar in size, the country with a larger population is discouraged from imposing heavy taxes on its richer partners in the union, as this implies imposing deadweight losses on its own, large population.

**Bargaining procedure and Nash equilibrium:**

Any agreement on adding a fiscal union dimension among a set of countries should belong to the previous characterized bargaining space. We now present a bargaining procedure, which has its origins in the seminal work of Nash (1953) on cooperative two-person games. We focus on the simplest scenario, where two countries negotiate whether to form a fiscal union in order to keep the benefits of the common policy, or abandon the common policy and revert to autarky. Let us consider two countries, namely countries $i$ and $j$. As it is common in the literature on bargaining, a threat acts as common device to come to a solution in any negotiation process. The elements under negotiation in our context are the weights that each of the two countries will have in a fiscal union, and the country’s threat. The threat consists in making the union revert to autarky and abandoning the benefits of the common policy, if the proposed weights give the country a expected utility that is lower than its autarky level. We assume that there is an enforceable mechanism with which the two countries commit to the bargain, once an agreement is achieved. Such a mechanism may consist on penalties, or constitutional enforcement. The formal negotiation model is described in the Appendix. An equilibrium of this two-country fiscal union bargaining game consists of a pair of weights $(p_i, p_j)$, with $p_j = 1 - p_i$, such that $p_i$ maximizes the product $E[V_i^U(p_i, p_j)] \cdot E[V_j^U(p_i, p_j)]$.

The extension to a set of $M > 2$ countries would require a more sophisticated theory of bargaining and cooperation among the set of countries, and is left for future work. This would be in line with Compte and Jehiel (2010), Hart and Mas-Colell (1997, 2010), Harsanyi (1959, 1963), and Thomsom (1994), among others. In such theory of coalition formation we should have the benefits of the common policy and the shocks volatility parameterized in the
group of countries that form a mutually beneficial institutional agreement with the common policy. That is, for a set \( S \) of countries with a joining common policy, these parameters would be \( g_i(S) \) and \( \sigma_i^C(S) \), respectively. We expect \( g_i(\cdot) \) and \( \sigma_i^C(\cdot) \) to increase with the size of the union \( S \). This is similar to the well known trade-off between size and heterogeneity, present in Alesina and Spolaore (1997) and Casella (2001), among others. In the case of common currency, we think of \( g_i(S) \) as the benefit associated with the decrease in transaction cost that comes with the adoption of a common currency. On the other hand, a high \( \sigma_i^C(S) \) captures the cost of accommodating heterogeneous economies with a common monetary policy. Alesina and Barro (2002) explore a parallel trade-off in the decision whether a country joins a currency union, where a decrease in transaction cost is weighed against the difficulty of having a common credible policy responding simultaneously to heterogeneous shocks.

We expect a break down among the set of all \( M \) countries if volatilities change in such a way that for some group of countries, say group 1 \( (S^1) \), their volatilities decrease, while for another group of countries, say group 2 \( (S^2) \), their volatilities increase. In terms of Figure 4, Group 2 of countries may be between thresholds \( \tilde{\sigma}_i \) and \( \bar{\sigma}_i \), while group 1 have their volatilities below \( \tilde{\sigma}_i \). Group 2 now wants to go to fiscal union, or revert to autarky. Group 2 is willing to reassign weights. Group 1 now considers whether to accept this offer and go to fiscal union, or to break apart into their own status quo. If \( g_i(S^1) \) is much below \( g_i(M) \), where \( M \) stands for the set of all \( M \) countries, then group 1 may prefer the latter.\(^{29}\) In our model, there is a conflict between the number of members in the union and the scope of policies undertaken centrally, just as in Alesina et al. (2005): when we move towards centralizing fiscal policy, it may be harder to sustain the union. Here political weights can partially circumvent the tradeoff between union size and policy deepening.

\(^{29}\)One could interpret the break-up outcome, following Lockwood (2002), with a situation where shared benefits between the two groups are small and regions are heterogeneous. See also Oates (1972).
6 Implications for Europe

The origins of the European Union can be traced back to a project of economic integration initiated in 1957 which has progressively expanded to wider and deeper policy areas. This project culminates with the creation of the Economic and Monetary Union (EMU) in 1999, when several countries adopted a common currency, the Euro. Until the recent economic and financial crisis, the EU project was seen as the longest lived and deepest institutional agreement among heterogeneous countries. In short: the most successful. That is no longer the case. In the wake of the crisis, an intense debate arose with the options of abandoning the euro and reverting to the old currencies squarely before some of the countries that had adopted the common currency. The institutional framework of the Euro and even the European Union is seen as in flux. Interestingly, the options on the table are extreme: either all or part of the countries move towards fiscal union; or the Euro might collapse, and at least some countries will have to revert to autarky. As our theory suggests, and illustrates in Figure 4, it is precisely in the high volatility countries where we expect the fiscal union or autarky options to become more salient. The countries most affected by economic crisis will rank an unanimous move to fiscal integration as the most preferable option. If that is not possible, autarky becomes an attractive second option. The status-quo may no longer be viable.\textsuperscript{30}

As our theory suggests, when volatility jumps for some of the countries in a monetary union, a fiscal union may be a necessary and sufficient institution to sustain the common currency and avoid reversion to autarky. When unanimity in favour of the fiscal union is not possible, some reassignment of political weights may be in order to convince the low volatility countries to join in. Current developments suggest a clear shift in political weights in favour of lower volatility countries such as Germany. A discussion of a two-

\textsuperscript{30}Bordo et al. (2011) analyse the history of several monetary unions in history and highlight that the EMU has one singular feature: unlike all other monetary unions, the adoption of a common currency and common monetary policy in Europe was not accompanied by moves toward fiscal integration of a comparable nature.
speed European Unions has re-emerged, with the least afflicted countries going their own way towards greater fiscal integration. Table 1 below documents the volatility of GDPpc growth in the twelve years before and the twelve years after the adoption of the Euro. With the exception of Germany and Finland, all countries experienced increases in volatility. This may be due to the fact that, subject to the same monetary policy, EMU countries lost the possibility of using devaluation to smooth adverse shocks to their economies. As Table 1 documents EMU countries can be roughly divided into a high volatility group, which includes countries such as Greece, Ireland, Italy, Portugal, and Spain, and a low volatility group, which includes Germany, Finland, and others. As our theory points out, and was discussed at the end of the previous section, if the low volatility countries do not loose much in terms of the benefit from lower transaction costs, when going it alone, they may create new institutional arrangements that exclude high volatility countries.

<table>
<thead>
<tr>
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<tbody>
<tr>
<td>Austria</td>
<td>1.84</td>
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</tr>
<tr>
<td>Belgium</td>
<td>3.18</td>
<td>4.94</td>
</tr>
<tr>
<td>Finland</td>
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<tr>
<td>France</td>
<td>3.79</td>
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<tr>
<td>Germany</td>
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<tr>
<td>Greece</td>
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<td>16.07</td>
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<tr>
<td>Ireland</td>
<td>18.10</td>
<td>62.27</td>
</tr>
<tr>
<td>Italy</td>
<td>7.53</td>
<td>11.46</td>
</tr>
<tr>
<td>Luxembourg</td>
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<tr>
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<tr>
<td>Portugal</td>
<td>13.67</td>
<td>18.56</td>
</tr>
<tr>
<td>Spain</td>
<td>6.29</td>
<td>16.15</td>
</tr>
</tbody>
</table>

Table 1: Volatilities of Gross Domestic Product per capita (GDPpc), relative to trend, in periods 1987-1998 and 1999-2011.
7 Final remarks

- We analyse the set of parameters that supports a fiscal union in the case of countries that are heterogeneous in average income and population and subject to medium to long-term income shocks with a given correlation across economies. Our set-up relates to the classic literature on fiscal federalism, but substantially enlarges the parameter space under scrutiny, allowing us to highlight the role of heterogeneities, both in terms of the positive analysis of country preferences for a fiscal union, and in terms of the normative analysis of the reallocations of decision power necessary to sustain a consensus in favour of such a union. Countries with a large relative income and a large relative size will demand a higher decision weight in the fiscal union given their autarky threat point.

- This paper provides volatility-based theory for the formation of a fiscal union when a common policy (such as a common currency) is already present. We uncover a scenario where countries subject to high income volatility favour either a fiscal union or autarky, the status-quo no longer a desirable option. These high-volatility countries may be willing to redistribute political weights in favour of low-volatility countries, so that all can unanimously move towards a fiscal union.

- We characterize the bargaining space of a set of countries that decide between adding the fiscal union dimension or reverting to autarky. We show how the correlation of income shocks, relative population size, and relative income interact with decision weights to characterize the feasibility set. In addition to the classical issues of economic risk and, more recently, political risk, the sustainability of a fiscal union is substantially enhanced when political weights are brought to the fore. The existence of a feasible consensus can be guaranteed for a large set of shocks-distribution scenarios and heterogeneities among countries, far beyond what could be achieved without political reforms.
• Some issues related to our framework and that are of interest to future research are left unexplored. One is the effect of non-linear taxation, as in Morelli, Yang and Ye (2011), on the feasibility of a fiscal union.\footnote{For example, one could consider a Laffer curve, where the term \((T - \frac{1}{2}T^2)x\) in (3) is replaced by \(Tx(T)\), being the after-shock average income endogenously depending on the tax rate.} A second issue is how to characterize the efficient voting weights once the fiscal union (with the common policy) has been formed, in line with Barberá and Jackson (2006). The choice of such weights could be influenced, besides efficiency, by issues of dynamic stability and equity, as well as their interaction. We see our framework as a first step towards jointly incorporating issues of volatility and heterogeneity into the assessment of the feasibility of fiscal union in the presence of a common policy.

8 Appendix

8.1 Proofs

Proof of Lemma 1: Lemma 1 immediately follows if we rewrite country \(j\)’s preferred tax rate when \(X_j < x\) as follows

\[
T_i(\varepsilon) = \frac{(1 - n_i)((1 + \varepsilon_j) - (Y_i/Y_j)(1 + \varepsilon_i))}{1 + \varepsilon_j + n_i((Y_i/Y_j)(1 + \varepsilon_i) - (1 + \varepsilon_j))} \quad \text{if } X_i < x
\]

Proof of Lemma 2: First, notice that the expected value is \(E(\varepsilon_i) = \bar{\varepsilon}_i \Pr(\varepsilon_i) + (-\bar{\varepsilon}_i) \Pr(-\varepsilon_i) = \bar{\varepsilon}_i(1/2) + (-\bar{\varepsilon}_i)(1/2) = 0\). The variance is \(Var(\varepsilon_i) = E(\varepsilon_i^2) - E(\varepsilon_i)^2 = \frac{1}{2} \varepsilon_i^2 + \frac{1}{2}(-\varepsilon_i)^2 = \varepsilon_i^2\), so \(\sigma_i = \varepsilon_i\). Now, since \(\sigma_i = \bar{\varepsilon}_i\) and \(\Pr(\varepsilon_i = \bar{\varepsilon}_i) = \Pr(\varepsilon_i = -\bar{\varepsilon}_i) = 1/2\), we have that \(E[V_i^{SQ}(\varepsilon_i)] = \frac{1}{2} \ln(Y_i(1 + \sigma_i)) + \frac{1}{2} \ln(Y_i(1 - \sigma_i))\), which can be rewritten as \(E[V_i^{SQ}(\varepsilon_i)] = \ln(Y_i) + \frac{1}{2} \ln(1 - \sigma_i^2)\). ■

Proof of Lemma 3: Similar procedure to Proof of Lemma 2. Notice that the function (5) is now, by assumption, evaluated at \(\sigma_i^A\) (a constant). ■
Lemma App.1: $E(\varepsilon_i) = 0$, $\sigma_i = \bar{\varepsilon}_i$ and $\rho_{ij} = 1 - 4g_{ij}^A$.

Proof of Lemma App.1: In the proof of Lemma 2 above we show that $E(\varepsilon_i) = 0$ and $\sigma_i = \bar{\varepsilon}_i$. Consider now shocks to two countries, whose covariance is given by $Cov(\varepsilon_i, \varepsilon_j) = E(\varepsilon_i\varepsilon_j) - E(\varepsilon_i)E(\varepsilon_j) = q_{ij}^A[\bar{\varepsilon}_i(\bar{\varepsilon}_j) + (-\bar{\varepsilon}_i)(-\bar{\varepsilon}_j)] + \frac{1}{2-g_{ij}^A}[\bar{\varepsilon}_i\bar{\varepsilon}_j + (-\bar{\varepsilon}_i)(-\bar{\varepsilon}_j)] = \bar{\varepsilon}_i\bar{\varepsilon}_j(1 - 4q_{ij}^A)$. From the above it follows that the correlation is $\rho_{ij} = \frac{Cov(\varepsilon_i, \varepsilon_j)}{\sqrt{Var(\varepsilon_i)Var(\varepsilon_j)}} = 1 - 4q_{ij}^A$. Alternatively, given a correlation parameter $\rho_{ij} \in [-1, 1]$, we have $g_{ij}^A = \frac{1-\rho_{ij}}{4}$.

Lemma App.2: Country $i$ ’s expected utility in status quo, given by (5), is decreasing and concave in $\sigma_i^C$.

Proof of Lemma App.2: Formally, the threshold are defined as $\tilde{\sigma}_i = \{\sigma_i^C : E[V_i^{SQ}] = E[V_i^U(T(\varepsilon))]\}$, $\bar{\sigma}_i = \{\sigma_i^C : E[V_i^{SQ}] = E[V_i^A]\}$ and $\hat{\sigma}_i = \{\sigma_i^C : E[V_i^U(T(\varepsilon))] = E[V_i^A]\}$. Furthermore, we can obtain the following closed solution for $\tilde{\sigma}_i$, by making $E[V_i^{SQ}] = E[V_i^A]$ (using expressions (6) and (5)): $\tilde{\sigma}_i = \sqrt{1 - (1 - (\sigma_i^A)^2)/g_i^2}$. Since $\sigma_i^C = \bar{\varepsilon}_i \in (0, 1)$, we have $dE[V_i^{SQ}(\varepsilon_i)]/d\sigma_i = -\frac{\sigma_i}{1 - \bar{\varepsilon}_i^2} < 0$ and $d^2E[V_i^{SQ}(\varepsilon_i)]/d(\sigma_i)^2 = -\frac{1 + \sigma_i^2}{(1 - \bar{\varepsilon}_i^2)^2} < 0$.

Proof of Proposition 1: Formally, the threshold in status quo is defined as All items immediately follow using the intermediate value theorem, making use of the single crossing property, which guarantees that the three thresholds are uniquely defined. In particular, to find each threshold, we need to find two values for $\sigma_i$, for which the difference between the two functional forms that determine it takes different signs. For example, for $\tilde{\sigma}_i$, take $\sigma_i = \sigma_i^A < 1$, so $E^o[V_i^{SQ}(\varepsilon_i)] - E[V_i^A(\varepsilon_i)] > 0$, and $\sigma_i = 1 - \delta$, with $\delta > 0$ sufficiently small, for which $E^o[V_i^{SQ}(\varepsilon_i)] - E[V_i^A(\varepsilon_i)] < 0$. Recall that $E[V_i^A(\varepsilon_i)]$ is constant in $\sigma_i$ (see (6), and that $E^o[V_i^{SQ}] \rightarrow -\infty$ as $\delta \rightarrow 0$ (see (5). The other two thresholds can be found using the same procedure.

8.2 Characterization of the bargaining set

Recall that the weights $(p_i)_{i=1}^M$ must satisfy $p_i \in [0, 1]$ for all $i$, and $\sum_{i=1}^M p_i = 1$. Now, in the instances where country $i$ does not decide, which occurs with probability $1 - p_i$, the other
$M-1$ countries will decide given some weights, which we denote by $(p'_{ij})_{j \neq i}$, such that
\[ \sum_{j \neq i} p'_{ij} = 1. \]
Observe that $p_j = (1 - p_i)p'_{ij}$. We thus have that $p_i + (1 - p_i)(\sum_{j \neq i} p'_{ij}) = 1$. Thus, weights (or probabilities) are described by a vector $(p_i, (p'_{ij})_{j \neq i})_{i=1}^M$.

- **Minimum weight of country $i$ compatible with country $i$ joining the fiscal union**: We denote by $p_i$ the minimum weight (or probability) with which country $i$ decides the tax rate, given the vector of weights $(p'_{ij})_{j \neq i}$, that is compatible with the same country $i$ being at least as well off in the union as in autarky. Formally,

$$ p_i(\omega, (p'_{ij})_{j \neq i}) \in \arg \min p_i \text{ such that }$$

\[ p_i E[V^U_i(T_i(\varepsilon))] + (1 - p_i) \sum_{s \neq i} p'_{is} E[V^U_s(T_s(\varepsilon))] \geq E[V^A_i] \quad (10) \]

Using equality in (10), we find that

$$ p_i(\omega, (p'_{ij})_{j \neq i}) = \frac{E[V^A_i] - \sum_{s \neq i} p'_{is} E[V^U_s(T_s(\varepsilon))] - \sum_{s \neq i} p'_{is} E[V^U_i(T_i(\varepsilon))]}{E[V^U_i(T_i(\varepsilon))] - \sum_{s \neq i} p'_{is} E[V^U_i(T_i(\varepsilon))]} \quad (11) $$

- **Maximum weight of country $i$ compatible with country $j$ joining the fiscal union**: We denote by $p_{ij}$ the maximum value of country $i$’s weight ($p_i$), such that country $j$ is at least as well off in the union as in autarky, for given state $\omega$ and other country weights $(p'_{ij})_{j \neq i}$. Formally,

$$ p_{ij}(\omega, (p'_{ij})_{j \neq i}) \in \arg \max p_{ij} \text{ such that }$$

\[ p_{ij} E[V^U_j(T_i(\varepsilon))] + (1 - p_{ij}) \sum_{s \neq i} p'_{is} E[V^U_j(T_s(\varepsilon))] \geq E[V^A_j] \quad (12) \]

Let us denote by $\lambda_{ij}$ the country $i$’s shadow value associated with constraint (12).

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Footnote: \footnote{Here, $p'_{ij}$ denotes the weight with which country $j$ chooses the tax rate when country $i$ does not choose it.}
\[ \bar{p}_{ij}(\omega, (p'_{ij})_{j \neq i}) - p_i(\omega, (p'_{ij})_{j \neq i}) > 0. \] Notice that unanimity is necessary in a two-countries scenario for the bargaining space to be non-empty. However, when there are more than two countries in the union, formation of the fiscal union by some majority rule might be considered. Let us now characterize the bargaining space of a set of \( M > 2 \) countries, given some qualified majority of votes.

- **Qualified majority:** Let us recall our previous notation. We denote the threshold of votes by letter \( \alpha \in [0, 1] \). If the percentage of votes in favor of the fiscal union is above this threshold \( \alpha \), then the fiscal union will be formed. For instance, if \( \alpha = 1 \), then the fiscal union requires unanimity, whereas if \( \alpha = 0.5 \) the fiscal union requires simple majority.\(^{33}\) Let us denote by \( \hat{S}_i(\gamma) \) the set of countries composed by country \( i \) and those countries \( j \neq i \) with a shadow value \( \lambda_{ij} \) on their respective participation constraint (12) smaller or equal than a given threshold \( \gamma \). Formally, \( \hat{S}_i(\gamma) \equiv \{ j \neq i : \lambda_{ij} \leq \gamma \} \cup \{ i : (10) \text{ holds} \} \). Since constraints (12) and (10) hold for the constructed set \( \hat{S}_i(\gamma) \), all countries in \( \hat{S}_i(\gamma) \) will prefer ex-ante a fiscal union to autarky. By construction, it is easy to see that

**Remark:** A fiscal union is formed given the majority \( \alpha \) if there exists a threshold \( \gamma \) and a country \( i \) for which \( \frac{\| \hat{S}_i(\gamma) \|}{M} \geq \alpha \).

For each \( \alpha \) there exists a minimum \( \hat{\gamma} \) such that the inequality in the previous remark hold. Let us denote by \( S_i(\alpha) \) the set of countries, including country \( i \), that satisfy the qualified majority \( \alpha \). By construction this set is \( S_i(\alpha) \equiv \hat{S}_i(\hat{\gamma}) \).

- **Bargaining space for an \( M>2 \) countries union, given a qualified majority rule:** Let us now consider the more general case with \( M > 2 \) countries. We will see how the majority rule \( \alpha \) will affect the design of the bargaining space. For a given majority \( \alpha \), state \( \omega \), and weights \( (p'_{ij})_{j \neq i} \), let us denote by \( \bar{p}_i(\alpha, \omega, (p'_{ij})_{j \neq i}) \) the maximum value of the proportional proposal right that the union can assign to country \( i \), providing that

\(^{33}\)We do not address the issue of choosing the majority rule since we think that it would make this paper too cumbersome. Future research on this should be related to the work by Barbera and Jackson (2004).
the set $S_i(\alpha)$ is non-empty (recall that all countries in the set $S_i(\alpha)$ are least as well off in the fiscal union as in autarky). Then, $\bar{p}_i(\alpha, \omega, (p'_{ij})_{j\neq i})$ equals $\min_{j \in S_i(\alpha)} \bar{p}_{ij}$, and is the value of $\bar{p}_{ij}$ associated with the country $j$ that faces the highest shadow value attached to restriction (12) among all countries in $S_i(\alpha)$. Thus, it might be that, at the resulting $\bar{p}_i$, some countries in $S_i(\alpha)$ find their constraint (12) not binding. We find that $\bar{p}_i(\alpha, \omega, (p'_{ij})_{j\neq i})$ is given

$$\bar{p}_i(\alpha, \omega, (p'_{ij})_{j\neq i}) = \min_{j \in S_i(\alpha)} \left\{ \frac{E[V_j^A] - \sum_{s \neq i} p_{is} E[V_s^U(T_s(\varepsilon))]}{E[V_j^U(T_i(\varepsilon))] - \sum_{s \neq i} p_{is} E[V_j^U(T_s(\varepsilon))]} \right\}$$  \hspace{1cm} (13)

Conditionally on the qualified majority rule that all countries in $S_i(\alpha)$ must have an expected utility level above their value in autarky, we can assert the following:

**Remark:** If $\bar{p}_i(\alpha, \omega, (p'_{ij})_{j\neq i}) - p_i(\omega, (p_{ij}')_{j\neq i}) > 0$ occurs for one country $i \in S_i(\alpha)$, then the fiscal union forms given majority $\alpha$, at realization $\omega$.

We denote by $P_i(\alpha, \omega, (p'_{ij})_{j\neq i})$ the space of all vectors $(p_i(\alpha, \omega), (p'_{ij})_{j\neq i})$ such that $p_i(\alpha, \omega) \in [p_i, \bar{p}_i] \neq \emptyset$. Then, for a given realization $\omega$ and a majority $\alpha$ of countries in favor of the fiscal union, the fiscal union forms if there exists at least one country $i$ with $P_i(\alpha, \omega, (p'_{ij})_{j\neq i}) \neq 0$. Since, in that case, (10) is satisfied for all countries in $S_i(\alpha)$, the union is ex-ante Pareto improving (in the usual sense) only for those countries in the set $S_i(\alpha)$. Those countries outside $S_i(\alpha)$ prefer autarky. Secession could be avoided by some constitutional commitment, or by voting on the formation of the fiscal union under unanimity requirement (choosing $\alpha = 1$). Finally, notice that if the space $P(\alpha, \omega) = \cup_i P_i(\alpha, \omega, (p'_{ij})_{j\neq i})$ is empty, there is no set of assigned proportional proposal rights that satisfy the incentive compatibility constraint for every country given the threshold $\alpha$.

### 8.3 Characterization of the Nash bargaining procedure

The formal negotiation model consist on the following two steps:
Step 1: Each country announces its minimum expected utility level \( E[V_i^A] \) for country \( i \) and \( E[V_j^A] \) for country \( j \), which act as the players’ threats in this game. Here we avoid to deal with information problems.

Step 2: Each country decides independently upon its weight in the fiscal union, which has associated a level of expected utility. Country \( i \)’s action variable is \( p_i \in [0,1] \) and the associated utility is \( E[V_i^U(p_i,p_j)] = p_i E[V_i^U(T_i(\varepsilon))] + p_j E[V_i^U(T_j(\varepsilon))] \). Country \( j \)’s action variable is \( p_j \in [0,1] \) and the associated utility is \( E[V_j^U(p_i,p_j)] = p_i E[V_j^U(T_i(\varepsilon))] + p_j E[V_j^U(T_j(\varepsilon))] \).

If demands \( (p_i,p_j) \) can be simultaneously satisfied (i.e., \( p_i + p_j \leq 1 \)), then each country gets what it demanded. Otherwise, the threats are executed and each country gets its expected utility in autarky. The notion of equilibrium in Definition 1 is consistent with the idea of one country maximizing its expected utility, given the other country’s strategy. Notice that each point in the bargaining space (characterized in the previous section) corresponds to a couple of utility levels \( (E[V_i^U(p_i,p_j)], E[V_j^U(p_i,p_j)]) \). Let us denote the corresponding space of utility levels that is isomorphic to the bargaining space by \( B \). The threat point in this topological representation corresponds to the autarky point \( (E[V_i^A], E[V_j^A]) \), which is the worst point of the set \( B \) in the two dimensional space consisting of countries \( i \) and \( j \)’ utilities (in the \((x,y)\)-axis). The topological representation of the bargaining equilibrium solution described above then corresponds to the tangent point between the set \( B \) and the product function \( E[V_i^U(p_i,p_j)] \cdot E[V_j^U(p_i,p_j)] \). It is worth to notice that the solution is continuous in the threat point. Existence of equilibrium for this two-person bargaining game is analogous to Nash (1953), and therefore we refer to the original paper for the details.
References


