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THE SECOND WELFARE THEOREM
IN NONCONVEX ECONOMIES

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The Second Welfare Theorem in Nonconvex Economies*

ABSTRACT

The purpose of this paper is to report an extension of the second welfare theorem when both convexity and differentiable assumptions are violated.

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1. INTRODUCTION

The purpose of this paper is to report an extension of the second welfare theorem when both convexity and differentiable assumptions are violated. This theorem, which has a well-known long history back to Allais [1], Hotelling [16], Lange [19], Samuelson [25], has first asserted that the marginal rates of substitution and the marginal rates of transformation must hold at a Pareto optimal allocation. However, it was not until Arrow [2] and Debreu [9], that this assertion was rigorously formulated and proved by means of convex analysis, i.e., a separation theorem. In [15], in the absence of convexity and differentiable assumptions, Guesnerie proved a theorem asserting that, at a Pareto optimal allocation, it can be associated a nonzero price vector such that each consumer satisfies the first order necessary conditions for expenditure minimization and each firm satisfies the first order necessary conditions for profit maximization.

Our results, which extend the (finite dimensional version of the) results of Kahn and Vohra [17], [18], Quinzii [23], Yun [28], are in the spirit of the above assertion of Guesnerie [15], but differ from it in the way the "necessary" conditions and the "marginal" rule are mathematically formalized. Guesnerie uses the concept of normal cone of Dubovickii and Miljutin [12], whereas we use, as in Cornet [8], Brown et al. [5] and the above papers, the normal cone of Clarke [6], which allows to extend significantly the class of economies which are considered. It allows also to use new mathematical techniques developed during the past ten years and reported in the books [7], [26].

The next section presents the model we consider in which the preferences of the consumers may be noncomplete and nontransitive, as in Gale and Mas-Colell [14] (see also Fon and Otani [13]) in the convex case. The main results are stated in Section 3; they assert that the second welfare theorem will hold under a nonsatiation assumption of the preferences of the consumers and if either all commodities can be freely disposed of (Theorems 3.1 and 3.3) or some consumer has convex (or monotonic) preferences and all the production sets are closed (Theorem 3.4). In Section 4, we give the proofs of these theorems, which
are directly deduced from a more general result (Lemma 4.1), also of interest for itself, but rather technical. The proof of it, hence of all the results of the paper, relies on a (nonsmooth) generalization of John-Kuhn-Tucker's theorem by Clarke [7, 6.1.1]. This paper will remain in the setting of a Euclidean space of commodities and we refer to [4], [18], for infinite dimensional formulations of the second welfare theorem in the spirit of Debreu [10], in the convex case.

2. THE MODEL

We consider an economy $E$ with $l$ goods, $m$ consumers and $n$ firms. We let $X_i \subset \mathbb{R}^l$ be the consumption set of the $i$-th consumer and, for $x = (x_1, \ldots, x_m)$ in $X_1 \times \ldots \times X_m$, we let $P_i(x)$ be the set of elements in $X_i$ which are preferred to $x$ by the $i$-th consumer. In other words, the preferences of the $i$-th consumer are described by a correspondence $P_i$ from $X_1 \times \ldots \times X_m$, to $X_i$. We let $Y_j \subset \mathbb{R}^l$ be the production set of the $j$-th firm and $\omega$ in $\mathbb{R}^l$ be the vector of total initial endowments.

DEFINITION 2.1 A $(m+n)$-tuple $((x^*_i), (y^*_j))$ of elements of $\mathbb{R}^l$ is said to be a feasible allocation of $E$ if (a) for all $i$, $x^*_i \in X_i$ , (b) for all $j$, $y^*_j \in Y_j$ and (c) $\sum_i x^*_i - \sum_j y^*_j - \omega = 0$ . A $(m+n)$-tuple $((x^*_i), (y^*_j))$ of elements of $\mathbb{R}^l$ is said to be a Pareto optimum (resp. a weak Pareto optimum) if it is a feasible allocation of $E$ and if there exists no other feasible allocation $((x'_i), (y'_j))$ of $E$ such that, for all $i$, $x'_i \in \text{cl } P_i(x^*_1, \ldots, x^*_m)$, the closure of $P_i(x^*_1, \ldots, x^*_m)$, and, for some $i$, $x'_i \in P_i(x^*_1, \ldots, x^*_m)$ (resp., for all $i$, $x'_i \in P_i(x^*_1, \ldots, x^*_m)$) . In the following, we shall simply denote $P_i(x^*_1, \ldots, x^*_m)$ by $P^*_i$.

Our next definition of price quasi-equilibrium involves the concept of normal cone in the sense of Clarke ([6], [7]), that we now present. Firstly, for $x = (x_h)$, $y = (y_h)$ in $\mathbb{R}^l$, we let $x \cdot y = \sum_h x_h y_h$ be the scalar product of $\mathbb{R}^l$, $\|x\| = \sqrt{x \cdot x}$ be its Euclidean norm, $B(x, \varepsilon) = \{x' \in \mathbb{R}^l ; \|x - x'\| < \varepsilon\}$ be the open ball of center $x$ and radius $\varepsilon > 0$ , we let $E^l_+ = \{x = (x_h) ; x_h > 0$ , $h = 1, \ldots, l\}$ and the notation $x \geq 0$ means that $x \in E^l_+$. If $C$ is a
subset of $\mathbb{R}^k$, we let $\text{cl} \, C$ and $\text{int} \, C$ be, respectively, the closure and the interior of $C$ and we say that a nonzero vector $v$ is perpendicular to $C$ at an element $x$ in $\text{cl} \, C$, denoted $v \perp C$ at $x$, if there exists $x'$ in $\mathbb{R}^k$ such that $v = x' - x$ and $\|x' - x\| = \inf\{\|x' - c\|; c \in \text{cl} \, C\}$, i.e., $x$ is the closest element to $x'$ in $\text{cl} \, C$. Then, Clarke's normal cone to $C$ at an element $x$ in $\text{cl} \, C$, denoted by $N^C_C(x)$, is the closed convex cone generated by the origin and the set:

$$\{v = \lim_{q \to q} v / \|v\|; v \perp C \, \text{at} \, x, \{x\} \subseteq \text{cl} \, C, \, x + x \, \text{and} \, v \to 0\}.$$  

(We recall that a subset $N$ of $\mathbb{R}^k$ is said to be a cone if $\lambda x \in N$ for all $\lambda > 0$ and $x \in N$.) We refer to Clarke's book [7, 2.5.7] for the above definition but also for properties of the normal cone $N^C_C(x)$ and an equivalent definition of it which takes the tangent cone as the primary concept. Incidentally, we notice that the above definition of Clarke's normal cone is unaffected if $C$ is replaced by $\text{cl} \, C$, i.e., $N^C_C(x) = N^{\text{cl} \, C}_C(x)$, for all $x$ in $\text{cl} \, C$.

**DEFINITION 2.2** A $(m+n)$-tuple $((x^*_i),(y^*_j))$ of elements of $\mathbb{R}^k$ is said to be a quasi-equilibrium with respect to a price system, or, simply, a price quasi-equilibrium, if there exists a nonzero price vector $p^*$ in $\mathbb{R}^k$ satisfying:

1. For all $i$, $x^*_i \in \text{cl} \, P^*_i$ and $-p^* \in N^*_i(x^*_i)$ [where $P^*_i = P_i(x^*_1, \ldots, x^*_m)$];
2. For all $j$, $y^*_j \in Y^*_j$ and $p^* \in N^*_j(y^*_j)$;
3. $\Sigma_i x^*_i - \Sigma_j y^*_j - \omega = 0$.

The above definition is essentially Guesnerie's definition of P.A. equilibrium, with the only difference that we use here the concept of Clarke's normal cone, instead of Dubovickii-Miljutin's one as in Guesnerie ([15]). (See Remark 2.2 for comments on our terminology.)

**REMARK 2.1** The above conditions (a) and (b) are the rigorous formalization of the assertion in the introduction that "each consumer satisfies the first order necessary condition for expenditure minimization and each firm satisfies the first order necessary condition for profit maximization." This
follows from [7, 2.4.3 and 2.2.4] since, if C is an arbitrary subset of $\mathbb{R}^\ell$ and $p^*$ is a vector in $\mathbb{R}^\ell$, then:

$x^*$ minimizes $p^* \cdot x$ over C implies $-p^* \in N_C(x^*)$.

REMARK 2.2 If $E = \{(X_i, P_i), (Y_j), \omega\}$ is a convex economy, in the sense that, for all $i$, $P_i$ is convex-valued, and, for all $j$, $Y_j$ is convex, then the conditions (α) and (β) are both necessary and sufficient. Indeed, by [7, 2.4.4], if $C$ is a convex subset of $\mathbb{R}^\ell$ and if $x$ is an element in $\text{cl } C$, then $N_C(x)$ coincides with the cone of normals in the sense of convex analysis ([24]), i.e., $N_C(x) = \{p \in \mathbb{R}^\ell; p^* \cdot x \geq p^* \cdot c \text{ for all } c \in C\}$, hence, $x^*$ minimizes $p^* \cdot x$ over $C$ if and only if $-p^* \in N_C(x^*)$. Consequently, if, for all $i$, $P_i^*$ is convex and, for all $j$, $Y_j^*$ is convex, the conditions (α) and (β) hold if and only if:

(α') for all $i$, $x_i^*$ minimizes $p_i^* \cdot x_i$ over the preferred set $P_i^*$,

(β') for all $j$, $y_j^*$ maximizes $p_j^* \cdot y_j$ over the production set $Y_j^*$.

Hence, for convex economies, the above notion of "price quasi-equilibrium" coincides exactly with the conclusion of Debreu [11, Thm. 6.4], the notion of "price quasi-equilibrium" in Mas-Colell [20], [21] and the one of "compensated equilibrium" in Arrow and Hahn [3]. For the (standard) way to go, from price quasi-equilibria to equilibria, i.e., from expenditure minimization to preference maximization by the consumers, we refer to the three above books. Here, we shall only consider the notion of price quasi-equilibrium.

3. STATEMENT OF THE RESULTS

Our first main result states:

THEOREM 3.1 Let $((x_i^*, y_j^*))$ be a weak Pareto optimum (resp. a Pareto optimum and assume $m > 1$), then it is a price quasi-equilibrium if, for all $i$, $x_i^* \in \text{cl } P_i^*$, and the following condition is satisfied either, for some $i$, by $C = P_i^*$ (resp. $\text{cl } P_i^*$) at $c^* = x_i^*$ or, for some $j$, by $C = -Y_j$ at $c^* = -y_j^*$:

(D) $\exists e \in \mathbb{R}^\ell$, $\exists \theta > 0$, $\forall t \in (0, \varepsilon)$, $te + \text{cl } C \cap B(c^*, \varepsilon) \subset \text{int } C$. 

The nonsatiation assumption that, for all $i$, $x^*_i \in \text{cl } P^*_i$, is a standard one in convex economies with non-transitive, non-complete preferences (see Gale and Mas-Colell [14] and Fon and Otani [13]). We interpret the vector $\mathbf{e}$ in (D) as a desirable (monotonic) "direction" at $x^*_i$ if $C = P^*_i$ (resp. $\text{cl } P^*_i$) and $-\mathbf{e}$ as an admissible "direction" at $y^*_j$ if $C = -Y^*_j$. The vector $\mathbf{e}$, in (D), is uniformly defined on a neighborhood of $c^*$ and this uniform property cannot be dispensed with (see Remark 3.1, hereafter). We also point out that, for $m = 1$, the notions of Pareto optimality and of weak Pareto optimality are identical.

The next proposition gives several cases under which (D) will be satisfied. The proofs of the proposition and of Theorem 3.1 are given in Section 4.

**Proposition 3.2** Condition (D) is satisfied by $C \subset \mathbb{R}_+^n$, $c^* \in \text{cl } C$, if one of the following assertions holds:

1. $C$ is convex with a nonempty interior;
2. $C + \text{int } \mathbb{R}_+^n \subset C$;
3. $C$ is epi-Lipschitzian at $c^*$, [25], i.e.,
   \[ \exists \epsilon \in \mathbb{R}_+, \exists \epsilon > 0, \forall \epsilon' \in (0,\epsilon), \forall \epsilon' \in B(\epsilon,\epsilon), \text{te'} + C \cap B(c^*,\epsilon) \subset C; \]
4. $N_C(c^*) \cap -N_C(c^*) = \{0\}$ and $C$ is closed;
5. int $T_C(c^*) \neq \emptyset$ and $C$ is closed,
   where $T_C(c^*) = \{ v \in \mathbb{R}_+^n; v \cdot p \leq 0 \text{ for all } p \in N_C(c^*) \}$.

Condition (D.1) needs not special comments but it is worth pointing out that the noninteriority assumption cannot be dispensed with on the production side (see Remark 4.1, hereafter); it will be the purpose of Theorem 3.4, however, to weaken it (and also (D)) on the consumption side. We interpret (D.2) as a free-disposal assumption if $C = -Y_j$; if $C = P^*_i$ (resp. $\text{cl } P^*_i$) then (D.2) is satisfied if the preferences are transitive and monotonic. Condition (D.2') is just a weakening of (D.2). Finally we point out that, by a theorem of Rockafellar [25] (see also [7, Cor. 1 of 2.5.8]), conditions (D.3') and (D.3''),
which are equivalent, both imply the $\varepsilon$-Lipschitzian condition (D.3) and the converse is true if $C$ is closed.

The following Corollary shows that, if all commodities can be freely disposed of, the nonsatiation assumption is sufficient for the second welfare theorem to hold, i.e., the $P^*(i=1,\ldots,m)$ and $Y^*_j(j=1,\ldots,n)$ may be arbitrary nonempty subsets of $\mathbb{R}^2$. To make this statement precise, we modify the notion of Pareto optimality of Definition 2.1, to allow "free-disposition," as follows. Formally, the $(m+n)$-tuple $((x^*_1),(y^*_j))$ of elements of $\mathbb{R}^2$ is said to be a free disposal weak Pareto optimum of $E$ if (a) for all $i$, $x^*_i \in X_i$, (b) for all $j$, $y^*_j \in Y_j$, (c') $\Sigma_i x^*_i - \Sigma_j y^*_j - \omega \leq 0$ and if there exists no other $(m+n)$-tuple $((x'_1),(y'_j))$ of elements of $\mathbb{R}^2$ satisfying (a), (b), (c') and such that, for all $i$, $x'_i \in P_i(x^*_1,\ldots,x^*_m) = P^*_i$. The only difference with Definition 2.1 lies in the feasibility condition (c') which allows free-disposition. Clearly, $((x^*_1),(y^*_j))$ is a free disposal weak Pareto optimum of $E$ if and only if the $(m+n+1)$-tuple $((x^*_1),(y^*_j),\ldots,y^*_n,y^*_n+1)$, where $y^*_{n+1} = \Sigma_{i=1}^m x^*_i - \Sigma_{j=1}^n y^*_j - \omega$ is a weak Pareto optimum of the economy $E' = \{(x'_1,P'_1),(Y_1,\ldots,Y_n,Y_{n+1}),\omega\}$, obtained from the original economy $E$, by the addition of a $(n+1)$-th firm with production set $Y_{n+1} = -\mathbb{R}^2_+$, which satisfies both assumptions (D.1) and (D.2). From Remark 2.2, one easily deduces that, for $y^*_{n+1}$ in $Y_{n+1} = -\mathbb{R}^2_+$, $N_{Y_{n+1}}(y^*_{n+1}) = \{p^*_n \geq 0; p^*_n y^*_{n+1} = 0\}$. Hence, from Theorem 3.1, one deduces the:

**THEOREM 3.3** Let $((x^*_1),(y^*_j))$ be a free-disposal weak Pareto optimum such that, for all $i$, $x^*_i \in cl P^*_i$. Then there exists a nonzero price vector $p^* \in \mathbb{R}^2$ satisfying: (a) for all $i$, $-p^*_i \in N_{P^*_i}(x^*_i)$, (b) for all $j$, $p^*_j \in N_{Y_j}(y^*_j)$ and (c') $p^*_i(E^*_i x^*_i - E^*_j y^*_j - \omega) = 0$ and $E^*_i x^*_i - E^*_j y^*_j - \omega \leq 0$.

At this stage, we are now able to discuss the link between our results and the other ones on this subject. As already said, the main difference with Guesnerie ([15]) lies in the different concepts of normality which are used. An important justification of the choice of Clarke’s normal cone (see Cornet [8]) is that it allows us to consider production sets with "inward kinks,"
such as $Y = \{(y_1, y_2) \in \mathbb{R}^2 ; y_1 \leq 0 \text{ and } y_2 \leq \max\{0, -1-y_1\}\}$, of particular economic importance, which are ruled out by Guesnerie in [15]. For a more detailed discussion on this fact and the relation between Theorem 3.1 and Guesnerie's result, we refer to Bonnisseau-Cornet [4]. The above Theorem 3.3 extends significantly the results of Kahn and Vohra [17], and [18] (in the finite dimensional setting), Quinzii [23] and Yun [28], who all consider free disposal Pareto optima, but assume additionally that, for all consumers and, for all firms, the monotonicity-free disposal condition (D.2) is satisfied ([17], [23], [28]) or the epi-Lipschitzian condition (D.3) is satisfied ([18]).

At the present stage, we cannot deduce, from Theorem 3.1, the second welfare theorem for convex economies ([2], [9]) in its more general version in Euclidean space [11, Thm. 6.4], where no interiority assumption is made on the sets $P_i^*$ or $Y_j$. For this purpose, we have to weaken condition (D) (which clearly implies that the set $C$ has a nonempty interior) as in the next theorem.

**THEOREM 3.4** A (resp. weak) Pareto optimum $((x^*_i), (y^*_j))$ is a price quasi-equilibrium if, for all $i$, $x^*_i \in \overline{P}_i^*$, if, for all $j$, $Y_j$ is closed or convex, and, if, for some $i$ (resp. for all $i$) $C = P_i^*$, $c^* = x^*_i$ satisfies:

\[(W.D.) \exists \epsilon \in \mathbb{R}^+, \exists \varepsilon > 0, \forall t \in (0, \varepsilon), t e + \overline{c} \cap B(c^*, \epsilon) \subseteq C.\]

The proofs of Theorem 3.4 and Theorem 3.1 will be given in Section 4, as a direct consequence of a more general (but less intuitive) result (Lemma 4.1).

It is worth pointing out however that neither Theorem 2.1 implies Theorem 3.4, nor the converse is true and that we have lost in Theorem 3.4 the symmetry between the consumer and the producer sides of the economy that we had before in Theorem 3.1. This latter fact will be further stressed in Remark 3.1, hereafter.

Assumption (W.D.), which is clearly weaker than (D), needs no additional comments to the ones made before to (D). The next proposition gives conditions under which (W.D.) is satisfied. The proof of it is given in Section 4.
PROPOSITION 3.5  Condition (W.D.) is satisfied by $C \subseteq \mathbb{R}^d$, $c^* \in \text{cl } C$, if, either (i) $C$ is closed, or (ii) $C$ is convex, or (iii) $C$ and $c^*$ satisfy condition (D).

In view of Proposition 3.5 we shall prove later a slightly stronger result than Theorem 3.4, by only assuming that, for all $j$, $C = Y_j$, $c^* = y^*_j$ satisfy (W.D.). We are now able, however, to deduce from Theorem 3.4, the second welfare theorem for convex economies (Arrow [2], Debreu [9]) as modified by Gale and Mas-Collell [14] (see also Fon and Otani [13] in a pure exchange economy) to the case of noncomplete nontransitive preferences.

COROLLARY 3.4  Let $((x^*_i), (y^*_j))$ be a weak Pareto optimum such that, for all $i$, $x^*_i \in \text{cl } P^*_i$, $P^*_i$ is convex and $\sum_j Y_j$ is convex. Then, there exists a nonzero price vector $p^*$ in $\mathbb{R}^d$ satisfying: (a') for all $i$, $x^*_i$ minimizes $p^* x^*_i$ over $P^*_i$; (B') for all $j$, $y^*_j$ maximizes $p^* y^*_j$ over $Y_j$ and

$\sum_i x^*_i - \sum_j y^*_j - \omega = 0$.

The proof goes as follows. The $(m+1)$-tuple $((x^*_i), \sum_j y^*_j)$ of elements of $\mathbb{R}^d$ is clearly a weak Pareto optimum of the economy $E = \{(X^*_i, Y^*_j), \omega\}$. Hence, by Theorem 3.4, Proposition 3.5 and Remark 2.2, there exists a nonzero price vector $p^*$ such that (a') is satisfied, together with (B): $\sum_j y^*_j$ maximizes $p^* y$ over $\sum_j Y_j$, which clearly implies (B').

We end the section by a remark.

REMARK 3.1. The second welfare theorem does not hold, in general, if we only assume that the preference of the consumers are nonsatiated and that, for some $j$, $Y_j$ is convex. In other words, the assumption that $C = Y_j$ has a nonempty interior in (D.1) of Theorem 3.1 cannot be dispensed with, in general. We consider the following economy with two goods, $\omega = (1,1)$ as the vector of initial endowments, one convex firm with production set $Y = \{(y_1, y_2) ; y_2 = -y_1\}$ and one (nonconvex) consumer with complete, transitive preferences as represented on Figure 1.
Then, if \( x_1^* = \omega^* = 0 \), \((x_1^*, y_1^*)\) is a Pareto optimum but \( \{0\} = N_{P_1}(\omega) \cap N_y(0) \) since \( N_y(0) = Y \) and one easily sees that \( N_{P_1}(\omega) = Y \).

One may object to the above counterexample that it satisfies the nonsatiation assumption, \( x_1^* \in \text{cl } P_1^* \), at the optimum but not on a neighborhood of \( x_1^* \) (i.e., the segment \([x_1^*, \omega]\) on Figure 1). However, the preferences can be modified, as in Figure 2, for the nonsatiation assumption to hold everywhere, and the above analysis remains valid.

The above example also shows that the conditions (D) (and also (W.D.)) need to be assumed on a neighborhood of \( x_1^* \) and not only at \( x_1^* \) (Figure 1) and that the vector \( e \) in (D) need to be chosen uniformly on the neighborhood \( \text{cl } P_1^* \cap B(x_1^*, e) \) (Figure 2) for the second welfare theorem to hold, in general.

4. PROOFS

The proofs of the Theorems 3.1 and 3.4 will be a direct consequence of the following lemma, also of interest for itself, which gives a general (but rather technical) condition under which the second welfare theorem holds.

**Lemma 4.1** The (resp. weak) Pareto optimum \((x_1^*, y_1^*)\) is a price quasi-equilibrium if, for all \( i \), \( x_1^* \in \text{cl } P_1^* \) and if there exists \( e \) in \( \mathbb{R}^k \), \( e > 0 \) and \( i_0 \in \{1, \ldots, m\} \) such that, for all \( t \in (0, e) \)
(Δ) \( t e + \Sigma_{i} \, cl \, P_{i}^{*} \cap B(x_{i}^{*}, e) - \Sigma_{j} \, cl \, Y_{j} \cap B(y_{j}^{*}, e) \subseteq P_{i}^{*} + \Sigma_{i \neq i_{0}} \, cl \, P_{i}^{*} - \Sigma_{j} \, Y_{j} \),

(resp. (WA) \( t e + \Sigma_{i} \, cl \, P_{i}^{*} \cap B(x_{i}^{*}, e) - \Sigma_{j} \, cl \, Y_{j} \cap B(y_{j}^{*}, e) \subseteq \Sigma_{i} \, P_{i}^{*} - \Sigma_{j} \, Y_{j} \)).

Moreover, the nonzero price vector \( p^{*} \) in (a),(b) satisfies \( p^{*} \cdot e > 0 \).

PROOF. We first claim that, for all \( t \in (0, e) \),

\[ \omega - te \notin \Sigma_{i} \, cl \, P_{i}^{*} \cap B(x_{i}^{*}, e) - \Sigma_{j} \, cl \, Y_{j} \cap B(y_{j}^{*}, e) . \]

Indeed, if the claim is not true, there exists \( t \in (0, e) \) such that \( \omega - te \notin \Sigma_{i} \, cl \, P_{i}^{*} \cap B(x_{i}^{*}, e) - \Sigma_{j} \, cl \, Y_{j} \cap B(y_{j}^{*}, e) \) and, from condition (Δ) (resp. (W.A)), we have \( \omega - P_{i}^{*} \cap B(x_{i}^{*}, e) - \Sigma_{j} \, cl \, Y_{j} \cap B(y_{j}^{*}, e) \) (resp. \( \omega - \Sigma_{i} P_{i}^{*} - \Sigma_{j} Y_{j} \)), which contradicts that \( (x_{i}^{*}, y_{j}^{*}) \) is a Pareto optimum (resp. a weak Pareto optimum).

From the above claim, we now notice that \( (0, (x_{i}^{*}), (y_{j}^{*})) \) is a solution of the following maximization problem:

maximize \( t \)

subject to: \( \Sigma_{i} x_{i} - \Sigma_{j} y_{j} - \omega + te = 0 \),

\[ x_{i} \in cl \, P_{i}^{*} , \quad i = 1, \ldots, m, \]

\[ y_{j} \in cl \, Y_{j} , \quad j = 1, \ldots, n, \]

\[ t \in \mathbb{R} , \]

\( t, (x_{i}^{*}), (y_{j}^{*}) \in U \),

where \( U = (-e, +e) \times \prod_{i} B(x_{i}^{*}, e) \times \prod_{j} B(y_{j}^{*}, e) \) is an open subset of \( \mathbb{R}^{1+2m+2n} \).

From the Lagrange multiplier rule, in Clarke [7, 6.1.1 and 6.1.2 (iv)] there exist \( \lambda \geq 0 \) and a vector \( p \) in \( \mathbb{R}^{2} \) such that \( (\lambda, p) \neq (0, 0) \) and, if we let:

\[ L(t, (x_{i}^{*}), (y_{j}^{*})) = -\lambda t + p^{*} (\Sigma_{i} x_{i} - \Sigma_{j} y_{j} - \omega + te) , \]

\[ \nabla L(t, (x_{i}^{*}), (y_{j}^{*})) = (-\lambda + p^{*} e, p, \ldots, p, -p, \ldots, p) \in \mathbb{R} \times \mathbb{R}^{2m} \times \mathbb{R}^{2n} , \]

denote, respectively, the Lagrangian of the problem and its gradient at \( (t, (x_{i}^{*}), (y_{j}^{*})) \), one must have:

\[ 0 \in \nabla L(0, (x_{i}^{*}), (y_{j}^{*})) + \sum_{i=1}^{m} x_{i}^{*} \times Y_{i}^{*} \times \ldots \times Y_{n}^{*} (0, (x_{i}^{*}), (y_{j}^{*})) . \]

But, since the normal cone to a Cartesian product sets is the Cartesian product of the normal cones [7, Cor. of 2.4.5], and since \( N_{\mathbb{R}}(0) = \{0\} \), one deduces that:
\[ \lambda - p^* e = 0 , \quad -p \in N_{p_i}(x^*)(i = 1, \ldots, m) , \quad p \in N_{y_j}(y^*)(j = 1, \ldots, n) . \]

The proof will then be complete if we show that the vector \( p \) is nonzero. But, from the above equality, \( p = 0 \) implies \( \lambda = 0 \), a contradiction with \((\lambda, p) \neq (0, 0)\). This ends the proof of Lemma 4.1.

We shall use later several times the following simple fact, the proof of which is immediate.

**Lemma 4.2** Let \( U \) be an open subset of \( \mathbb{R}^\ell \) and let \( C_1, \ldots, C_k \) be arbitrary subsets of \( \mathbb{R}^\ell \). Then

\[ U + \text{cl} \ C_1 + \ldots + \text{cl} \ C_k = U + C_1 + \ldots + C_k \]

is an open set.

**Proof of Theorem 3.1** follows from Lemma 4.1 by checking that the weak Pareto optimum (resp. Pareto optimum) \((x^*_1, y^*_j)\) satisfies condition \((W\Delta)\) (resp. \((\Delta)\)). Indeed, let us assume that, for some \( i_0 \in \{1, \ldots, m\} \), say \( i_0 = 1 \), the desirability condition \((D)\) is satisfied by \( P^*_{i_1} \) (resp. \( \text{cl} \ P^*_{i_1} \)) for some \( e \in \mathbb{R}^\ell \) and some \( \varepsilon > 0 \), then, for all \( t \in (0, \varepsilon) \),

\[ te + \text{cl} \ P^*_{i_1} \cap B(x^*_1, \varepsilon) - \Sigma_j \text{cl} \ Y_j \cap B(y^*_j, \varepsilon) \subseteq \text{int} \ P^*_{i_1} + \Sigma_i \neq 1 \text{cl} \ P^*_{i_1} - \Sigma_j \text{cl} \ Y_j \]

(resp. \( \text{int} \ P^*_{i_1} + \Sigma_i \neq 1 \text{cl} \ P^*_{i_1} - \Sigma_j \text{cl} \ Y_j \)),

which, by Lemma 4.2, is a subset of \( \text{int} \ P^*_{i_1} - \Sigma_j Y_j \) (resp. \( \text{cl} \ P^*_{i_1} + \Sigma_i \neq 1 P^*_{i_1} - \Sigma_j Y_j \)), hence \((W)\) is satisfied (resp. hence \((\Delta)\) is satisfied, recalling that \( m > 1 \)).

We leave the reader adapt the above proof to the case where \((D)\) is satisfied by some firm.

**Proof of Theorem 3.4** We first notice that if, for all \( j \), \( Y_j \) is closed or convex, then, by Proposition 3.5, \( C = Y_j , c^* = y_j^* \) satisfy condition \((W.D.)\) for some vector \( e_j \) in \( \mathbb{R}^\ell \). Under this latter (and weaker) assumption, the theorem follows from Lemma 4.1 by checking that the (resp. weak) Pareto optimum \((x^*_1, y^*_j)\) satisfies the condition \((\Delta)\) (resp. \((W\Delta)\)) for the vector \( e = e_1 - \Sigma_j e_{i_1} \) (resp. \( e = e_{i_1} - \Sigma_j e_{i_1} \)) where \( e_1 \) is the vector for which, for some \( i \) (resp. for all \( i \)), \( P^*_{i_1} \) satisfies \((\Delta)\) (resp. \((W\Delta)\)).

We now give the proofs of Propositions 3.2 and 3.5, for which we need the following lemma.
LEMMA 4.3 Let $C$ be a convex subset of $\mathbb{R}^n$ and let $c^*$ be in $\text{cl } C$, then, there exist a vector $e$ in $\mathbb{R}^n$ and $\varepsilon > 0$ such that, for all $t \in (0, \varepsilon)$,
\[ te + \text{cl } C \cap B(c^*, \varepsilon) \subset \text{ri } C \], the relative interior of $C$.

PROOF. Let $A$ be the affine space spanned by $C$ and let $x$ be in $\text{ri } C$. Then, there exists $\varepsilon > 0$ such that $B(x, \varepsilon) \cap A \subset \text{ri } C$ and we can further suppose that $\varepsilon < 1$. We now check that $e = x - c^*$ and $\varepsilon$ satisfy the above property. Indeed, for all $c$ in $\text{cl } C \cap B(c^*, \varepsilon)$ and all $t \in (0, \varepsilon) \subset (0,1)$, one has $te + c = t(e+c) + (1-t)c$. But $c \in \text{cl } C$ and $e + c = x + c - c^* \in B(x, \varepsilon) \cap A \subset \text{ri } C$, hence [24, Thm. 6.1], $te + c \in \text{ri } C$.

PROOF OF PROPOSITION 3.2 Condition (D.1) implies (D) by Lemma 4.3 and the fact that $\text{ri } C = \text{int } C$, since the convex set $C$ has a nonempty interior [24, p. 46]. Condition (D.2') implies (D) for any $e$ in $Q$. It suffices to notice that, by Lemma 4.2, for all $t > 0$, $te + \text{cl } C \in Q \cap \text{cl } C = Q \cap C$, an open set, contained in $C$ by (D.2'), hence also contained in $\text{int } C$. (D.3) also implies (D) by the same type of argument (with $Q$ replaced by the open ball $B(e, \varepsilon)$). Finally, (D.3') and (D.3''), which are equivalent, both imply (D.3), hence also (D), by a theorem of Rockafellar [25] (see also [7, Cor. 1 of 2.5.8]).

PROOF OF PROPOSITION 3.5 follows from Lemma 4.3.

We end the paper by the following remarks.

REMARK 4.1 The proof of Lemma 4.1 and the Lagrange multiplier rule of [7, 6.1.1] yields, in fact, to the (slightly) stronger conclusion that:
\[(a'')\] for all $i$, $-p^* \in \partial d_{p_i}(x^*)$; \[(b'')\] for all $j$, $p^* \in \partial d_{y_j}(y^*_j)$,
where, for $C \subset \mathbb{R}^n$, $x \in \text{cl } C$, we denote by $\partial d_{C}(x)$ the convex hull of the origin and the set
\[ \{v = \lim q \|v\| v \in C \text{ at } x, \{x\} \subset \text{cl } C, x \rightarrow x \text{ and } v \rightarrow 0\}; \]
hence, $\partial d_{C}(x)$ is clearly a subset of $N_{C}(x)$ and, by Clarke [7, 2.5.6], $\partial d_{C}(x)$ is exactly the generalized gradient of the distance function $d_{C}$ at $x$. This remark allows a strengthening of the definition of price quasi-equilibria...
(by replacing the conditions (a) and (β) by the above ones (α') and (β'))
which allow us to get a property which may be of interest. Indeed, from its
definition, the correspondences \( \exists \mathcal{d}_p^* (\cdot) \), \( \exists \mathcal{d}_y (\cdot) \) are of closed graph (and,
in fact, upper semicontinuous, since they are bounded), a property not uni-
sally possessed by the correspondences \( N_p^* (\cdot) \), \( N_y (\cdot) \) (see however [25]).

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</thead>
<tbody>
<tr>
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