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DISTRIBUTIVE PRODUCTION SETS AND EQUILIBRIA WITH INCREASING RETURNS

by

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We are grateful to Martine Quinzii for having drawn our attention to the work of Scarf on the core of a productive economy and its relation with our work on competitive equilibria with increasing returns; and to Bernard Cornet for his very helful comments and suggestions on the manuscript. Needless to say, responsibility for remaining errors is entirely ours.

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Abstract

We give a necessary and sufficient condition on production sets under which average cost pricing is compatible with voluntary trading. This condition defines a class of production sets which is related to supportable cost functions and to Aumann-Shapley average cost prices.

1. INTRODUCTION.

In the present paper, we investigate the properties of a class of production sets which exhibit increasing returns to scale. The concept of distributive production set was originally introduced by Scarf in a paper which has been circulating since 1963 and very often quoted. It has now been published among the *Contributions to Mathematical Economics* edited by W. Hildenbrand and A. Mas Colell in Honor of Gérard Debreu (1986). Scarf's paper did fit within his quest of a solution to "the problem of incorporating increasing returns in an analytical framework with the generality of the Walrasian model of equilibrium", to quote him (p. 401).

In his paper, Scarf considers economies with an aggregate production set in which some inputs, "*producer commodities*", do no enter into consumers' preferences. The relevance of the concept of distributive sets for the core follows from the fact that, for such economies, distributivity is a **necessary and sufficient** condition for the non-emptiness of the core.

To prove existence of a core allocation, Scarf introduces a concept of equilibrium which he defines as an allocation and a price system at which :

- consumers maximise utility subject to the budget constraint implied by the value of their initial endowments;
- profit is zero but maximum over the feasible production plans *using no greater amounts of producer commodities* than available.

This defines what Scarf calls a "*social equilibrium*", which is a kind of competitive equilibrium, profit being maximum over the production set truncated by the available quantities of producer commodities. The corresponding allocation is then shown to be in the core of the

economy and the existence of an equilibrium is proven for economies with distributive production sets.

The present paper extends a previous paper (1986) in which we suggest a concept of equilibrium of a competitive type for economies in which production technologies are not necessarily convex. The idea is to redefine the behavior of the firms in a perfectly competitive environment in such a way as to allow for non-convex technologies. At an equilibrium, each firm maximises profit, given the prices, over the feasible production plans whose quantities of output do not exceed the observed levels of demand, a condition of *voluntary trading*, and output prices are *minimal* among the prices at which the firm accepts to meet the demand for its products. In the convex case, this is shown to be equivalent to profit maximisation. The outcome of this behavior is not necessarily average cost pricing, i.e. profit could be positive at an equilibrium, like at a competitive equilibrium. The initial purpose of the present paper is to define the class of non-convexities for which a similar behavior leads to a situation of zero profit.

Inspired by the definition of distributive production set of Scarf, we suggest a symmetric definition which applies to situations where producers are constrained in their supply of output, instead of being constrained in their demand for input. As a slight departure from our previous paper, we shall assume here that profits instead of output prices are minimised. We show that indeed, for such technologies, voluntary trading together with minimisation of profit gives rise to average cost prices which, in addition, are anonymously equitable and subsidy free.

The paper is organised as follows. The concept of distributive production set is introduced in Section 2. We use the term "*input*-distributive" to cover the definition of Scarf, and the term "*output*-distributive" for our definition. The properties of distributive production sets are reviewed in Section 3, with emphasis on output-distributive sets. In particular, the relations with the concept of supportable cost function and Aumann-Shapley average cost prices are studied. Finally, in Section 4, the idea of voluntary trading and minimality of profit are formalised and introduced in a general equilibrium framework.

2. DEFINITIONS.

Let $\mathbb{R}^{\ell}$ be the commodity space where ℓ is the number of commodities. A *production set* is defined as a subset Y of $\mathbb{R}^{\ell}$ and we shall use the following assumptions :

A.1 Y is a closed set

A.2 $0 \in Y$ (feasibility of inaction)

A.3 $Y + \mathbb{R}^{\ell}_{-} \subset Y$ (free disposal)

The first two assumptions are standard and universally accepted. The third assumption says that Y is a *comprehensive* set. To define distributivity, we must assume that the input-output configuration is a priori fixed[1] :

A.4 There is a number k, $1 \leq k \leq \ell-1$, such that any production plan $y \in Y$ can be written as $y = (a,b)$, where $a \in \mathbb{R}^{k}_{-}$ and $b \in \mathbb{R}^{\ell-k}$; if $(a,b) \in Y$, then $(a,b^{+}) \in Y$.

For all $h = k+1,\ldots,\ell$ there exists $y \in Y$ such that $y_h > 0$.

Consequently, the outputs never appear as inputs and could all be produced, not necessarily simultaneously, in some positive quantity. There are k inputs and ℓ-k outputs. We denote by $I = \{1,\dots,k\}$ the set of inputs and by I^c, its complement, the set of outputs. It is to be noticed that the commodities which are not related to the production process described by Y are included in the set of inputs.[2]

The assumptions A.1 to A.4 will be assumed to hold throughout this section. For any finite collection $(y^1,\dots,y^n)$ of points in $\mathbb{R}^\ell$, we denote by $C(y^1,\dots,y^n)$ the convex cone with vertex 0 generated by these points : $C(y^1,\dots,y^n) = \{y \in \mathbb{R}^\ell \mid y = \Sigma\, \alpha^j y^j,\ \alpha^j \geq 0\}$.

A production set Y is *input-distributive* if for any finite collection $(y^1,\dots,y^n)$ of production plans, $y^j = (a^j, b^j) \in Y$ for all j, the following inclusion is satisfied :

$$C(y^1,\dots,y^n) \cap \{y \in \mathbb{R}^\ell \mid y = (a,b),\ a \leq a^j \text{ for all } j\} \subset Y.$$

In words, any (non-negative) weighted sum of feasible production plans must be feasible if it does not use less inputs than any of the original plans. An illustration is provided in Figure 1.

This is essentially the definition introduced by Scarf in 1963. However, closedness of Y was not included in the definition and a weaker form of free disposal was assumed, namely $\mathbb{R}^\ell_- \subset Y$. We use here the term "input-distributive" to distinguish his definition from the one we shall introduce now, for which the stronger version of free disposal as given in A.3 is necessary.

A production set Y is *output-distributive* if for any finite collection $(y^1,\ldots,y^n)$ of production plans, $y^j = (a^j, b^j) \in Y$ for all j, the following inclusion is satisfied :

$$C(y^1,\ldots,y^n) \cap \{y \in \mathbb{R}^\ell \mid y = (a,b),\ b \geq b^j \text{ for all } j\} \subset Y.$$

In words, any (non-negative) weigthed sum of feasible production plans must be feasible if it involves more outputs than any of the original plans. An illustration is provided in Figure 2.

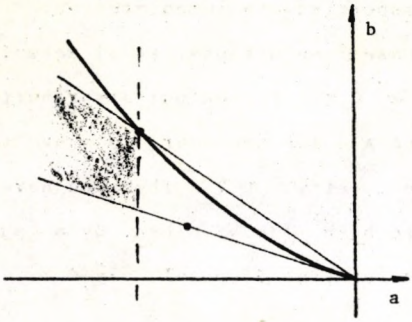

Figure 1

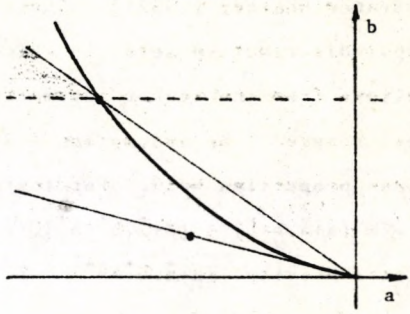

Figure 2

Obviously, the two definitions cover the constant returns case where Y is a cone, and they are equivalent in the one input-one output case. The properties of distributive sets, some of which are common to both types, will now be reviewed.

3. PROPERTIES.

In this section, we shall study the basic properties of distributive production sets. We shall however concentrate our attention on output-distributive sets. Throughout this section, we consider a given production set Y satisfying the assumptions A.1 to A.4.

3.1 Common Properties

Distributivity is stronger than *increasing returns to scale* (sometimes called "*global economies of scale*") in the sense that, if Y is distributive, the following property is satisfied :

$$y \in Y \text{ and } \lambda \geq 1 \text{ implies } \lambda y \in Y.$$

Furthermore, Y is *superadditive*:

$$y \text{ and } y' \in Y \text{ implies } y + y' \in Y.$$

Superadditivity implies that a firm whose technology is described by a distributive production set is a *natural monopoly* [see for instance Sharkey (1982)]. These properties are immediate for input-distributive sets, in which case free disposal (A.3) actually follows from the weaker assumption $\mathbb{R}^{\ell}_{-} \subset Y$. For output-distributive sets however, the assumption A.3 and A.4 are necessary to prove that these properties hold. For instance, let $y, \bar{y} \in Y$. Then, we have $y+\bar{y} = (a+\bar{a}, b+\bar{b}) \leq (a+\bar{a}, b^{+}+\bar{b}^{+})$, where $b^{+}+\bar{b}^{+} \geq b, \bar{b}$. Then, by output-distributivity, $(a+\bar{a}, b^{+}+\bar{b}^{+}) \in Y$, and by free disposal, $y+\bar{y} \in Y$.

Let us denote price systems $p = (p_a, p_b)$, where $p_a \in \mathbb{R}^{k}_{+}$ and $p_b \in \mathbb{R}^{\ell-k}_{+}$ denote the input prices and the output prices respectively. The associated cost function is defined by

$$c(b, p_a) = \inf_{(a,b)\in Y} (-p_a . a),$$

and we denote by $B = \{ b \in \mathbb{R}^{\ell-k}_{+} \mid \exists\, a \in \mathbb{R}^{k}_{-} \text{ such that } (a,b) \in Y \}$ the set of producible output vectors. Clearly, $\lambda B \subset B$ for all $\lambda \geq 1$ and $B + B \subset B$. Hence, the assumptions A.3 and A.4 ensure that, whenever Y is a distributive production set, B coincides with $\mathbb{R}^{\ell-k}_{+}$ and the cost function is then defined for all $b \in \mathbb{R}^{\ell-k}_{+}$ and all $p_a \in \mathbb{R}^{k}_{+}$. Furthermore, as a consequence of the above properties, the cost function is *subhomogeneous* and *subadditive*, i.e. :

(i) for all $b \in \mathbb{R}_+^{\ell-k}$, $p_a \in \mathbb{R}_+^k$ and $\lambda \geq 1$, $c(\lambda b.p_a) \leq \lambda c(b.p_a)$.

(ii) for all $b^o, b^1 \in \mathbb{R}_+^{\ell-k}$ and $p_a \in \mathbb{R}_+^k$,

$$c(b^o+b^1.p_a) \leq c(b^o.p_a) + c(b^1.p_a).$$

Let ∂Y denote the boundary of Y. Because Y is a closed and comprehensive set, ∂Y coincides with the set of (weakly) efficient production plans : $\partial Y = \{y \in Y \mid \nexists\, y' \in Y,\ y' \gg y\}$.

3.2 Properties of input-distributive production sets.

Let Y be input-distributive. As a first consequence of the definition, the "*isoinput*" sets are convex : For all $a \in \mathbb{R}_-^k$, the set $Y_a = \{y \in Y \mid y = (a,b)\}$ is convex (Scarf, p.411). The importance of input-distributive sets stems from the fact that the following condition is essentially[3] a necessary and sufficient condition for input-distributivity:

For all $y^* \in Y$ for which there does not exist $y \in Y$, $y \geq y^*$ and $y_h > y_h^*$ *for some* $h \in I^c$, there exists $p \in \mathbb{R}_+^\ell$, $p \neq 0$, such that :

(1) $\quad 0 = p.y^* \geq p.y$ for all $y \in Y$, $y_h \geq y_h^*$ for all $h \in I$.

Condition (1) characterises the price systems at which y^* maximises profit when the demand for inputs is constrained by y_h^*, $h \in I$. It is a condition of voluntary trading in the sense that, at such prices, it is profitable for the firm to use the inputs in quantities y_h^* instead of using less.

Input-distributivity has implications for the associated cost function: For any input prices $p_a \in \mathbb{R}_+^k$ and outputs $b^* \in \mathbb{R}_+^{\ell-k}$,

there exist output prices $p_b \in \mathbb{R}_+^{\ell-k}$ such that :

(2) $$0 = p_b.b^* - c(b^*.p_a) \geq p_b.b - c(b.p_a)$$

for all $b \in \mathbb{R}_+^{\ell-k}$ such that $c(b.p_a) \leq c(b^*.p_a)$. This property will latter be contrasted with a related property which applies to the output-distributive case.

3.3 Properties of output-distributive productions sets.

Let Y be output-distributive. The first observation to make is that the "*isoquants*" are then non-empty and convex : For all $b \in \mathbb{R}^{\ell-k}$, the set $Y_b = \{y \in Y \mid y = (a,b)\}$ is non-empty and convex. This is an immediate consequence of the definition and of the assumptions A.3 and A.4. The interest of output-distributivity stems from the following two propositions which together provide a **necessary and sufficient** condition for output-distributivity.

<u>Proposition 1</u> Let Y be output-distributive. Then for all $y^* \in \partial Y$ there exists $p \in \mathbb{R}_+^{\ell}$, $p \neq 0$, such that :

(3) $$0 = p.y^* \geq p.y \quad \text{for all } y \in Y,\ y_h \leq \text{Max}\,(0, y_h^*),\ h \in I^c.$$

<u>Proof</u> Let us first consider $y^* \in \partial Y$ such that $y_h^* \geq 0$ for all $h \in I^c$. We define $T = \{y = \Sigma\, \alpha^j y^j \mid \alpha^j \geq 0,\ y^j \in Y,\ y_h^j \leq y_h^*\ \forall\, h \in I^c,\ \forall j\}$, the smallest convex cone with vertex 0 containing the truncated production set $\{y \in Y \mid y_h \leq y_h^*$ for all $h \in I^c\}$.

T has free disposal. This follows directly from the fact that T is a cone and Y has itself free disposal. Clearly, $y^* \in T$. Furthermore, there does not exist $y \in T$ such that $y \gg y^*$.

Otherwise, $y = \Sigma \alpha^j y^j$ for some (y^j, α^j) such that for all j, $y^j \in Y$, $\alpha^j \geq 0$ and $y^j_h \leq y^*_h$ for all $h \in I^c$. Then, $y^*_h \leq y_h$ for all h implies $y \in Y$, contradicting $y^* \in \partial Y$. We can then conclude that $y^* \in \partial T$ and that there exists a supporting hyperplane passing through 0 and y^*: There exists $p \in \mathbb{R}^\ell_+$, $p \neq 0$, $0 = p.y^* \geq p.y$ for all $y \in T$ and in particular for all $y \in Y$ such that $y_h \leq y^*_h$, $h \in I^c$.

We shall now show that if $y^*_h > 0$ for some $h \in I^c$, there exists a vector p satisfying (3) such that $p_i = 0$ for all $i \in I^c$ with $y^*_i = 0$. Without loss of generality, let us assume that for some m, $y^*_{k+1}, \ldots, y^*_m > 0$ and $y^*_{m+1} = \ldots = y^*_\ell = 0$. One easily verifies that $Z = \{z \in \mathbb{R}^m | (z,0) \in Y\}$ is an output-distributive production set and that $z^* = (y^*_1, \ldots, y^*_m) \in \partial Z$. Hence there exits $q \in \mathbb{R}^m_+$, $q \neq 0$, such that $0 = q.z^* \geq q.z$ for all $z \in Z$ with $z_h \leq z^*_h$, $h = k+1, \ldots, m$ and the vector $p = (q,0) \in \mathbb{R}^\ell_+$ satisfies condition (3).

Let us now consider the general case. If $y^*_h \leq 0$ for all $h \in I^c$, any element of the normal cone to $\mathbb{R}^\ell_-$ at y^* satisfies condition (3) and the normal cone is well-defined and contains a non-zero element. In order to deal with the case where $y^*_i > 0$ for some $i \in I^c$, we consider the vector $\bar{y}$ defined by $\bar{y}_h = y^*_h$, $h \in I$, and $\bar{y}_h = \text{Max}\,(0, y^*_h)$, $h \in I^c$. Then, there exists a $p \neq 0$ such that $0 = p.\bar{y} \geq p.y$ for all $y \in Y$ with $y_h \leq \text{Max}(0, y^*_h)$, $h \in I^c$. If now $y^*_i > 0$ for some $i \in I^c$, $\bar{y}_i > 0$ and p can be choosen different from the origin and such that $p_h = 0$ for all $h \in I^c$ with $\bar{y}_h = 0$. Hence, p can be choosen in such a way that $p.y^* = p.\bar{y} = 0$. ∎

<u>Remark</u> Note from the proof that, if for some $i \in I^c$ $y^*_i = 0$ in (3), the i-th coordinate of the price vector p can be choosen freely in $\mathbb{R}_+$ subject only to the restriction $p \neq 0$. Furthermore, if for some $i \in I^c$, $y^*_i < 0$ in (3), then $p_i = 0$.

<u>Proposition 2</u> If for all $y^* \in \partial Y$, there exists $p \in \mathbb{R}^{\ell}_{+}$, $p \neq 0$, satisfying condition (3), then Y is output-distributive.

<u>Proof</u> Let us fix some (y^j) and (α^j) such that for all j, $y^j \in Y$, $\alpha^j \geq 0$ and $\bar{y}_h \geq y^j_h$ for all $h \in I^c$ where $\bar{y} = \Sigma\, \alpha^j y^j$. Assume that $\bar{y} \notin Y$. Then, because the output vector defined by $\bar{y}$ is producible and free disposal applies to the isoquants, there exists $y^* \in \partial Y$ such that $y^*_h < \bar{y}_h$ for all $h \in I$, and $y^*_h = \bar{y}_h$ for all $h \in I^c$.

Therefore, there exists $p \in \mathbb{R}^{\ell}_{+}$, $p \neq 0$, satisfying (3) and in particular, $p.y^j \leq 0$ for all j, because $y^j_h \leq \bar{y}_h = y^*_h$ for all $h \in I^c$. Hence, $p.\bar{y} \leq 0$ and, because $p.y^* = 0$, this is possible only if $p_i = 0$ for all $i \in I$. From the remark made after Proposition 1, we know that $p_h y^*_h \geq 0$ for all $h \in I^c$. Hence, $p_h y^*_h = 0$ for all $h \in I^c$. But p_h can be choosen equal to zero whenever $h \in I^c$ with $y^*_h \leq 0$. Hence $p \neq 0$ implies that for some $h \in I^c$, $p_h > 0$ and $y^*_h > 0$, contradicting $p_h y^*_h = 0$. ∎

Output distributivity has an interesting implication for the cost function, beyond subhomogeneity and subadditivity:

<u>Proposition 3</u> The cost function associated with an output-distributive set is *supportable*, i.e. for any given input prices $p_a \in \mathbb{R}^k_{+}$ and outputs $b^* \in \mathbb{R}^{\ell-k}_{+}$, there exist output prices $p_b \in \mathbb{R}^{\ell-k}_{+}$ such that :

$$(4) \qquad 0 = p_b.b^* - c(b^*.p_a) \geq p_b.b - c(b.p_a).$$

for all $b \in \mathbb{R}^{\ell-k}_{+}$, $b \leq b^*$.

This Proposition derives from the following lemma :

<u>Lemma 1</u> Let Y be output-distributive. Then, for any input prices $p_a \in \mathbb{R}^k_+$ and any finite collection of points (α^j, b^j) in $\mathbb{R}^{l+\ell-k}_+$ such that $\Sigma\, \alpha^j b^j \geq b^i$ for all i, the following condition is satisfied :

$$(5) \qquad c(\Sigma\alpha^j b^j, p_a) \leq \Sigma\alpha^j c(b^j, p_a).$$

<u>Proof</u> Let $a^j \in \{\, a \in \mathbb{R}^k_- \mid -p_a.a = c(b^j, p_a) \,\}$ and define $\bar{y} = (\bar{a}, \bar{b})$, $\bar{y} = \Sigma\, \alpha^j(a^j, b^j)$. By output-distributivity, $\bar{y} \in Y$ and therefore, we have $(-p_a.\bar{a}) \geq c(\bar{b}, p_a)$ where $p_a.\bar{a} = \Sigma\, \alpha^j p_a.a^j = -\Sigma\, \alpha^j c(b^j, p_a)$. ∎

The property of supportability corresponds to the definition introduced by Sharkey and Telser (1978) where they show that condition (5) is actually a necessary and sufficient condition for supportability. Condition (4) has to be contrasted with condition (2) which applies to input-distributive production sets. There, the alternative output vectors considered were those with a smaller cost of production. Here instead, we consider those with smaller quantities of outputs.[4]

A converse to Proposition 3 holds, namely :

<u>Proposition 4</u> Let us assume that Y is a production set *with convex isoquants*. If the resulting cost function is supportable at **all** output vectorsand input prices, then Y is output-distributive.

<u>Proof</u> Let us fix some (y^j) in Y and define $\bar{y} = \Sigma\alpha^j y^j$, $\bar{y} = (\bar{a}, \bar{b})$, where $b^j \geq 0$ for all j and the α^j's are choosen in such a way that $\bar{b} \geq b^j$ for all j. If c is a supportable cost function, then we have $c(\bar{b}, p_a) \leq \Sigma\alpha^j c(b^j, p_a)$ for all $p_a \in \mathbb{R}^k_+$. Furthermore, $y^j \in Y$ implies $c(b^j, p_a) \leq (-p_a.a^j)$. Hence, we also have $c(\bar{b}, p_a) \leq (-p_a.\bar{a})$ for all $p_a \in \mathbb{R}^k_+$. Because the set $Y_{\bar{b}}$ is convex and has the free disposal property, this last inequality implies that $\bar{a} \in Y_{\bar{b}}$ and $\bar{y} \in Y$. ∎

Figures 3 and 4 illustrate why convexity and free disposal are necessary to prove this result which would not hold at $\bar{y}$ there, since these points are compatible with cost-supportability but not with output-distributivity.

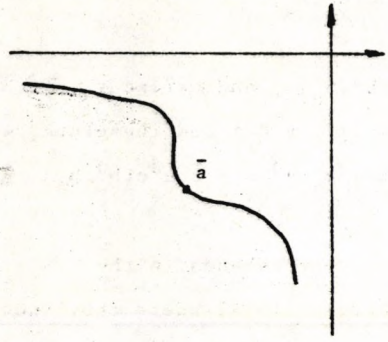

Figure 3

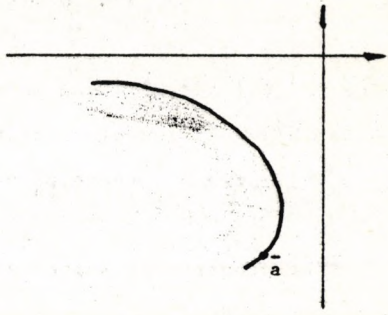

Figure 4

To conclude this section, let us consider the output prices defined by equitable cost sharing à la Aumann-Shapley [see Mirman and Tauman (1982)]. These are average cost prices which support the cost function for a class of output-distributive technologies :

<u>Proposition 5</u> Let us fix the prices of the inputs at some $p_a \in \mathbb{R}^k_+$ and assume that the cost function $c(.,p_a)$ is quasi-convex, homogeneous of degree $r \leq 1$ and differentiable on Int $\mathbb{R}^{\ell-k}_+$. Then, for any given $b^* \in$ Int $\mathbb{R}^{\ell-k}_+$, condition (4) is satisfied by the Aumann-Shapley prices defined by :

$$p_h = \int_0^1 c'_h(\lambda b^*, p_a)d\lambda \qquad (h \in I^c)$$

where c'_h denotes the marginal cost of producing output h.

The proof of this result is a straightforward transposition of the proof given by Greenberg and Shitovitz (1984). Their result, which applies in the input-distributive case with a single output, says that Aumann-Shapley input prices define a Social Equilibrium in the

sense of Scarf. The assumptions they make applies to the revenue function, in particular quasi-concavity and homogeneity of degree $r \geqslant 1$. As suggested by these authors, Proposition 5 could be generalised to the case where the cost function is not necessarily quasi-convex but can be decomposed as a sum of quasi-convex, homogeneous and differentiable functions.[5]

4. GENERAL EQUILIBRIUM.

4.1 Voluntary trading and minimal profit.

When production sets are non-convex, the behavior of producers is best represented by pricing rules which define the acceptable prices corresponding to every possible efficient production plan. This idea can be found in Dierker, Guesnerie and Neuefeind (1985) and has been elaborated further since [see Bonnisseau and Cornet (1986) for the most recent contribution]. More precisely, a pricing rule is a correpondence $\psi : \partial Y \to \mathbb{R}^{\ell}_{+}$ and, at an equilibrium, the condition " $p \in \psi(y)$ " holds for all firms. For exemple, ψ could define marginal or average cost pricing. More sophisticated pricing behaviors may also be covered by using pricing rules which depend on additionnal variables like market prices and the plans of other agents. To be able to prove the existence of a general equilibrium, the pricing rules should be closed correspondences and their values should be non-degenerated convex cones with vertex zero. Furthermore, the losses, if any, implied by the use of these pricing rules should be bounded in one way or another to avoid bankruptcy of the economy.

The purpose of this section is to construct a particular pricing rule which incorporates voluntary trading and minimality of profit, and to apply it to firms whose technology are described by

output-distributive production sets. It is inspired by our previous work on competitive equilibria with increasing returns (1986) where we also use the pricing rule approach.

Let Y be a production set satisfying the assumptions A.1 to A.3. We first define the set of price systems which, for any given production plan $y \in \partial Y$, satisfy *voluntary trading* :

$$\varphi(y) = \{p \in \mathbb{R}^{\ell}_{+} \mid p.y \geq p.y' \text{ for all } y' \in Y,\ y' \leq y^{+}\}.$$

At $p \in \varphi(y)$, it is profitable for the firm to produce the quantities y_h, $h \in I^c$, *instead of producing less*. As a consequence, at $p \in \varphi(y)$, the cost of producing the output vector y^{+} is minimised.

Clearly, for all $y \in \partial Y$, $\varphi(y)$ is a non-degenerated convex cone with vertex 0 and it defines a correspondence $\varphi: \partial Y \to \mathbb{R}^{\ell}_{+}$ whose graph is closed.[6]

For any given $y \in \partial Y$, the set $\varphi(y)$ is typically large (in particular, it does not place any upperbound on output prices). It is therefore not an appropriate pricing rule. In our previous paper, our attention was concentrated on the subset of $\varphi(y)$ in which output prices were minimal (vectorwise). Here instead we shall restrict our attention to those output prices in $\varphi(y)$ at which profit is minimal. This defines the correspondence $\varphi^{*}: \partial Y \to \mathbb{R}^{\ell}_{+}$ by :

$$\varphi^{*}(y) = \{p \in \varphi(y) \mid \nexists\, p' \in \varphi(y),\ p'_h = p_h,\ h \in I(y),\ p'.y < p.y\}$$

where $I(y) = \{h \mid y_h < 0 \text{ or } y'_h \leq 0 \text{ for all } y' \in Y\}$ denotes the set of "inputs at y".

When the input-output configuration is fixed, $I(y)$ is larger than I because it includes outputs which are disposed of. However, this does not affect the definition of φ^* because in $\varphi(y)$ these commodities have a zero price. It is to be noted that the price systems in $\varphi^*(y)$ involve minimal prices for those outputs which are **actually** produced. If $y_h = 0$, $h \in I^c$, the h-th coordinate of the price vectors in $\varphi(y)$, and in $\varphi^*(y)$, can be freely choosen in $\mathbb{R}_+$. By contrast, in our previous paper where minimality applies to the prices of **all** outputs, $p_h = 0$ for all $h \in I^c$ with $y_h = 0$.

Clearly, $\varphi^*(y)$ is a convex cone with vertex zero. But it may be degenerated and the graph of the correspondence φ^* is not necessarily closed. It is therefore necessary to enlarge it to obtain a well defined pricing rule. Let us define :

$$(6)\quad \psi(y) = \text{co}\ \{p \in \mathbb{R}^{\ell}_{+} \mid \exists\ (y^{\upsilon}, p^{\upsilon}) \to (y, p),\ y^{\upsilon} \in \partial Y/\{0\},\ p^{\upsilon} \in \psi^*(y^{\upsilon})\ \}$$

where the correspondence ψ^* is given by:

$$\psi^*(y) = \begin{cases} \varphi^*(y) & \text{if } \varphi^*(y) \neq \{0\}, \\ \varphi(y) \cap \mathbb{N}_Y(y) & \text{if } \varphi^*(y) = \{0\} \text{ and } \varphi(y) \cap \mathbb{N}_Y(y) \neq \{0\}, \\ \varphi(y) & \text{otherwise.} \end{cases}$$

Here, $\mathbb{N}_Y(.)$ denotes the cone of Clarke [see Clarke (1975) and Rockafellar (1979)]. By construction, ψ is a closed correspondence whose values are non-degenerated convex cones with vertex zero. Because φ is a closed correspondence, ψ satisfies $\psi(y) \subset \varphi(y)$ for all $y \in \partial Y$. Furthermore, **if Y is convex**, $\psi(y) \equiv \mathbb{N}_Y(y)$ for all $y \in \partial Y$, where in this case $\mathbb{N}_Y(y)$ is the cone of normals. Hence, in the convex case, voluntary trading **and** minimality of profit actually define profit maximisation.

These results are obtained by transposing the arguments developed in our previous paper. In the output-distributive case, where a given input-output configuration is assumed, we have the following result :

<u>Lemma 2</u> When Y is an output-distributive production set, φ^* is a closed correspondence whose values are non-degenerated convex cones with vertex zero.

<u>Proof</u> By Proposition 1, for all $y \in \partial Y$ there exists $p \in \varphi(y)$, $p \neq 0$ such that $p.y = 0$. Therefore, $\varphi^*(y) \neq \{0\}$. By Proposition 3, $p.y = 0$ for all $p \in \varphi^*(y)$ and all $y \in \partial Y$. Therefore, we have :

$$\varphi^*(y) = \varphi(y) \cap \{ p \in \mathbb{R}^\ell \mid p.y = 0 \},$$

and φ^* is a closed correspondence, being the intersection of closed correspondences. ∎

As a consequence, in the output-distributive case, the pricing rule ψ satisfies: $\psi(y) = \varphi^*(y)$ for all $y \in \partial Y$, $y \neq 0$. At the origin, $\varphi^*(0) = \mathbb{R}^\ell_+$ and the case where $\psi^*(0) = \mathbb{R}^\ell_+$ corresponds to the existence of *set-up costs*, a situation in which ψ does not place any restriction on prices at the origin. Our conjecture is that $\psi(0)$ collects the marginal cost at the origin : $\mathbb{N}_Y(0) = \psi(0)$.

Let us fix $y^* \in \partial Y$, $p \in \psi(y^*)$ and define (a^*, b^*) by $a^*_h = y^*_h$, $h \in I$ and $b^*_h = \mathrm{Max}(0, y^*_h)$, $h \in I^c$. If $b^* \neq 0$, then cost are minimised, i.e. $(- p_a.a^*) = c(b^*, p_a)$, and p_b supports the cost function at b^*, i.e. condition (4) is satisfied. In the case where $b^* = 0$, our conjecture is that $p_b \in \partial_b c(0, p_a)$, where $\partial_b c$ denotes Clarke's generalised gradient of the cost function with respect to b [see Clarke (1975)].

4.2 A general equilibrium concept.

Let us consider an economy in which there are m consumers and n firms. Consumers are indexed by i and characterized by a consumption set X_i, a preference relation $\succsim_i$, an initial endowment vector ω_i and shares in profits (θ_{ij}). On these characteristics we make the following assumptions :

C.1 X_i is a closed subset of $\mathbb{R}^\ell$, convex and bounded below.

C.2 $\succsim_i$ is a complete, continuous, convex[7] and non-satiated preordering of X_i.

C.3 There exists $\hat{x}_i \in X_i$ such that $\hat{x}_i \ll \omega_i$.

The shares in profits satisfy $\theta_{ij} \geq 0$ for all i,j and $\Sigma_i \theta_{ij} = 1$ for all j. These are the assumptions of Debreu's "Theory of Value" (1959), and they do not require particular comments. They could of course be weakened but the emphasis is here on the production side.

Firms are indexed by j and characterised by a production set Y_j which satisfies the assumptions A.1, A.2 and A.3. The set of feasible allocations is given by:

$$\mathcal{A} = \{ (y_1, \ldots, y_n, x_1, \ldots, x_m) \in \Pi Y_j \times \Pi X_i \mid \Sigma x_i \leq \Sigma y_j + \Sigma \omega_i \}.$$

It is a non-empty subset of $\mathbb{R}^{\ell(n+m)}$ and we assume that

B. the set $\mathcal{A}$ is compact in $\mathbb{R}^{\ell(n+m)}$.

The assumptions made so far ensure the existence of a competitive equilibrium in the case where the aggregate production set ΣY_j is convex [see Debreu (1959)].

The behavior of consumers is standard : they optimise according to their preferences under a budget constraint in which the value of their initial endowment and their shares in profits appear. The behavior of the firms is assumed to be described by the pricing rule ψ_j, as defined in (6).

Formally, an *equilibrium* is defined by a price system p in $\mathbb{R}^{\ell}_{+}$, $p \neq 0$, a set of consumption plans $(x^*_1, \ldots, x^*_m)$ and a set of production plans $(y^*_1, \ldots y^*_n)$ satisfying the following conditions :

E.1 $\quad \Sigma\, x^*_i = \Sigma\, y^*_j + \Sigma\, \omega_i$.

E.2 For all i, x^*_i is $\succsim_i$-maximal in the budget set

$$\{x_i \in X_i \mid p.x_i \leq p.\omega_i + \Sigma_i \theta_{ij} p.y^*_j\}.$$

E.3 For all j, $y^*_j \in \partial Y_j$ and $p \in \psi_j(y^*_j)$.

The first two conditions are part of the definition of competitive equilibrium. Condition E.3 implies that $p \in \varphi(y^*_j)$ i.e. y^*_j maximises $p.y_j$ on the subset $\{ y_j \in Y \mid y_j \leq (y^*_j)^+ \}$, implying that costs are minimised. This condition defines profit maximisation if Y_j is a convex set. In the case where Y_j satisfies the assumption A.4 and is output-distributive, condition E.3 implies $p.y^*_j = 0$. Output prices are then average costs.[8] In addition, they are anonymously equitable and subsidy free.[9]

<u>Proposition 6</u> Under the assumptions C.1 to C.3, A.1 to A.3 and B, there exists an equilibrium.

<u>Proof</u> Several existence proofs for models with general pricing rules are available, in particular by Kamya (1984) and by Bonnisseau and Cornet (1986). Our previous paper contains also an existence proof for loss-free pricing rules. However, because it allows to prove the existence of an equilibrium where demand **equals** supply, instead of a free disposal equilibrium, we shall use here the result of Bonnisseau and Cornet (1986) which applies to pricing rules whose losses are uniformly bounded below.

The assumption C.2 implies that consumers' preferences can be represented by continuous, quasi-concave and locally nonsatiated utility functions. Let S denote the unit simplex of $\mathbb{R}^{\ell}_{+}$. Then, by Lemma 2, the pricing rules μ_j's defined by $\mu_j(y) = \psi_j(y) \cap S$ are upper hemicontinuous correspondences with convex and compact values. Because $p.y \geq 0$ for all $y \in \partial Y_j$ and $p \in \psi_j(y)$, the pricing rules μ_j's entail no loss and therefore losses are uniformly bounded below. Together with the assumption C.3, this ensures that the wealth of the consumers always exceeds their minimal wealth. This completes the verification of the assumptions introduced by Bonnisseau and Cornet, and their result can then be used to prove the existence of an equilibrium. ∎

FOOTNOTES

(1) Notation: For any given vector x, we denote by x^+ the vector of same dimension defined by $x_h^+ = \mathrm{Max}(0, x_h)$. For vector inequalities, we adopt the following sequence of symbols: $\geq$, $>$, $>>$.

(2) Scarf introduces a distinction between "*producers commodities*" and "*consumers commodities*", the former being a subset of the set of inputs.

(3) It is not fully a necessary and sufficient condition because efficiency in a stricter sense is used in the sufficient condition [see Scarf, p. 414 and 424].

(4) It is to be noticed that input-distributivity has implications on the revenue function $r(a, p_b) = \mathrm{Max}\ p_b . b$ subject to $(a, b) \in Y$, which are symetrical to those of the cost function under output-distributivity, and vice-versa: *superhomogeneity*, *superadditivity* and a related form of *supportability*.

(5) Further results along these lines are given in Mirman, Tauman and Zang (1985). In particular, under cost complementarity, any non-decreasing cost function satisfying $c(0) \geq 0$ is supportable by Aumann-Shapley average cost prices. See also Sharkey and Telser (1973) for necessary and sufficient conditions for supportability of a cost function.

(6) See Hildenbrand (1974, p.21) for definitions and properties of closed and u.h.c. correspondences.

(7) By convex, we mean: $x \succ x'$ implies $\lambda x + (1-\lambda)x' \succ x'$ for all λ, $0 < \lambda \leq 1$.

(8) The term average cost is here used appropriately because costs are actually minimised at an equilibrium.

(9) For a given cost function $c(b)$, output prices q are *anonymously equitable* if $q.b = c(b)$, where b is compatible with demand, and $q.b' \leq c(b')$ for all b', $0 \leq b' \leq b$. They are *subsidy free* if the same conditions are satisfied for all the b' obtained from b by setting at 0 any of its coordinates [see Faulhaber and Levison (1981) and Mirman, Tauman and Zang (1985)].

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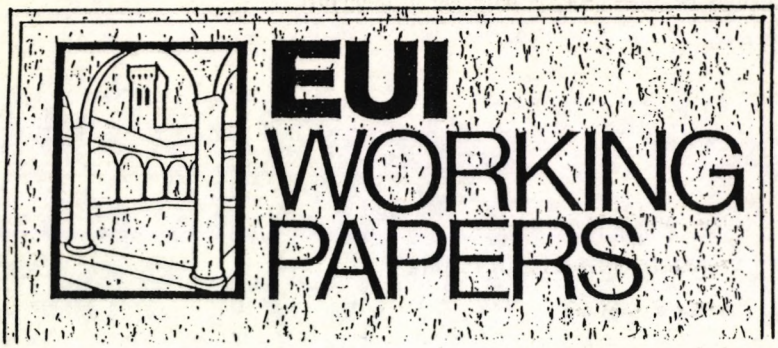

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