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QUANTITY GUIDED PRICE SETTING

by

Egbert DIERKER * and Wilhelm NEUEFEIND **

* University of Vienna

** Washington University in St. Louis

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European University Institute
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Abstract

We consider an economy with two sectors. The first sector consists of competitively behaving consumers and producers; the second, non-competitive, sector, the P-sector, consists of firms (P-firms) producing commodities (P-goods) that are not produced in the competitive sector. The P-firms receive their gross output levels and the market prices of their inputs as decision parameters. They minimize costs and set prices for their outputs according to a specific pricing rule. There is also a planning agency that ensures that a certain net production (gross production minus the intra-consumption in the P-sector) of the P-goods is reached.

We give assumptions assuring the existence of equilibrium which requires market clearing, meeting the production aspirations of the planning agency and setting prices for the P-goods which are compatible with market prices in the sense that the market prices cannot be higher than the prices to be charged by the P-firms, and if the target for a P-good is exceeded, the price charged by the P-firm equals the market price.

1. Introduction

During the past ten years there has been a large number of papers analyzing the conditions for the existence of equilibria when firms with possibly non-convex technologies set prices according to some rules such as marginal cost pricing (called MCP henceforth). The theorems available by now are quite general in scope and cover a wide range of possible set-ups.[1]

The reason why economists are interested in MCP or, in a second best framework, in Boiteux-Ramsey pricing, is that these rules are supposed to incorporate goals of optimality or efficiency. However, as we will indicate later in this introduction, focusing on pricing rules may lead to undesirable results due to the neglect of the quantity range in which firms should operate.

This point is perhaps best illustrated in the case of the MCP-rule, but applies to other rules as well. Consider the case of a firm which incurs a large fixed cost. The firm's technology is illustrated in Figure 1.[2]

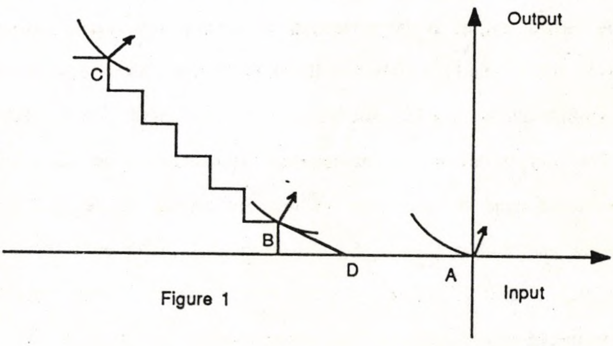

Figure 1

[1]We will refrain from giving a survey of the existing literature and refer the reader to e.g. Bonnisseau and Cornet (1986).

[2]The reader may, without altering the nature of the example, round off the corners if he prefers marginal costs to be well-defined in a traditional sense.

There are three MCP equilibria in this example, denoted by A, B, and C. We have drawn parts of the (Scitowski) indifference curves and the equilibrium prices corresponding to A, B, and C.

Point A represents a rather trivial MCP equilibrium. The firm charges prices which are so high that there is no demand at all, and correspondingly, the firm does not operate. Observe that the literature on the existence of MCP equilibria does not distinguish between such a trivial equilibrium and other equilibria.

At B, the situation is even worse than at A. The firm is active in this case but produces a rather small quantity of output which is sold at a high price. But although the price is high, there remains a substantial deficit, which is measured by the distance AD. The firm produces a small quantity accompanied by a very large deficit, not a desirable situation. But the MCP-rule can lead the economy to this equilibrium.

The equilibrium we would prefer for the economy is given by C. The size of the firm's output is large enough to justify the corresponding deficit. However, there is nothing embodied in the MCP-rule which would insure that an MCP equilibrium with a large output quantity, like in C, will be realized.

One may be tempted to object that the choice of the above example is unfair, since, due to the kinks in the production frontier, all prices can appear at any kink. Hence, the restriction that prices equal marginal costs has no bite. This is correct, but one cannot expect a first order condition such as the MCP condition or the Boiteux-Ramsey condition, to have so much bite as to lead the firm to a global optimum when the technologies are non-convex. Why should a property of local importance lead to a global optimum? We exaggerate in the example -- to make our point in a more pronounced fashion. We could have used a classical production function, too.

We are used to thinking in terms of a downward sloping demand curve and an upward sloping supply curve. Note that the "supply" curve (MC-curve) of the firm with the extremely smooth and regular technology illustrated in Figure 2 is sloping downward rather than upward as a standard supply curve does. Since we know from the literature that MCP equilibria do exist, we might expect a situation as the one depicted in Figure 3.

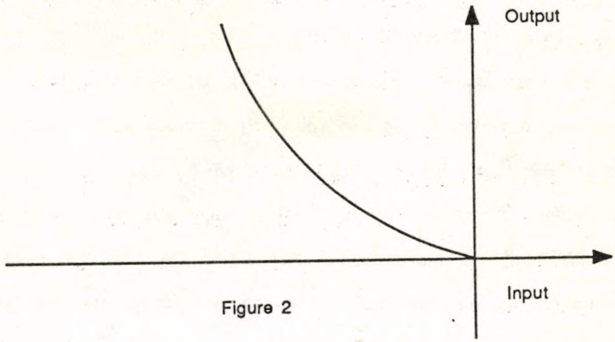

Figure 2

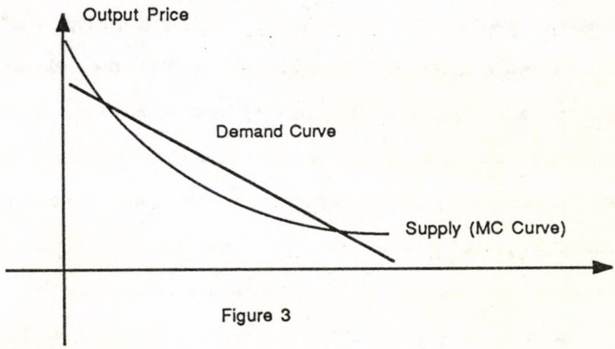

Figure 3

Examples like this make is clear that there is a built-in tendency towards a multiplicity of MCP equilibria, inspite of the downward sloping demand curve -- a situation which substantially differs from the competitive case. Different MCP equilibria generally have different degrees of inefficiency. The example presented in Figure 1, though perhaps somewhat unfair to the MCP-rule, seems appropriate to us, since it poses most sharply the unavoidable question of how we have to proceed when we want to steer production into a desirable range since the MCP rule is ill-suited to assist in the actual choice. It appears natural to us to take recourse to quantity targets when pricing rules fail to discriminate between equilibria.

Led by the traditional fundamental theorems of welfare economics one is, at first, tempted to search for a mechanism, prescribing quantities and/or prices, that leads to Pareto optimality. This goal, however, is too ambitious. It has been shown by Guesnerie (1975) that a Pareto efficient MCP equilibrium need not exist if the rule according to which wealth and deficit are shared are specified a priori. In a recent paper by Beato and Mas-Colell (1985), an economy is exhibited where all three MCP equilibria are not even production efficient, let alone Pareto efficient. One of the equilibria Pareto dominates the second and the second and third are not Pareto comparable.

Furthermore, even if a Pareto efficient MCP equilibrium happens to exist, there may be many others and it seems to us that the informational requirements for a mechanism to distinguish between efficient and inefficient MCP equilibria are simply too large. If one would imagine a central agency responsible for such a mechanism, it would require the agency to be extremely well-informed. Otherwise it would never induce the economy to reach a Pareto efficient equilibrium if there are inefficient equilibria possibly nearby.

We prefer to think of a central agency which has limited information on production possibilities and consumers' demand and which uses this information to formulate some minimal requirements. An example of such a requirement may be that the size of the deficit caused by the production of some output the agency wants to influence has to be in line with the quantity produced. In particular, a deficit of a certain size appears untolerable unless the amount produced (and also consumed in equilibrium) exceeds a certain target. Note that the price of the product may also influence the decision.

To take a simple example, the subsidy given to the opera in Vienna would be considered untolerably large if there were only few performances per week due to insufficient demand at the ticket prices currently charged. We are convinced that it would be too difficult a task for the authorities to fine tune the budget, out of which the deficit is paid, to an extent that the production of culture in Vienna takes Pareto efficient levels. But we are also convinced that the amount actually produced is not simply the outcome of a pricing rule. There seems to be an essential element of direct quantity control through some public agency in many areas which are not completely left to the forces of competitive markets.

We believe that the direct control of quantities will be based on a satisficing rather than optimizing behavior of the central agency which chooses aspiration levels in the form of minimal quantity targets for the production of outputs. This is particularly appropriate for an environment in which optimality may be out of reach. Unfortunately, there is no theory at hand to justify the size of the aspiration levels. They should, however, depend on such things as prices and deficits. The agency's view of consumer demand could also be considered one of the determinants of the minimal quantity targets.

Minimal quantity targets are used to avoid situations such as A or B in Figure 1. Since no distinction is made between production levels beyond the required minimum, there may be room left for an additional criterion to determine the outcome within the admissible range. Thus one could imagine that an equilibrium such as C in Figure 1 has been obtained by following the MCP-rule subject to minimal quantity restrictions. Minimal quantity restrictions being violated at A and B, the economy is bound to reach a point such as C under this arrangement.

If only one publicly controlled firm is considered or, more generally, if the output of any such firm is never the input of another one, then minimal targets are simply in terms of the outputs of these firms. The situation, however, is different if the publicly controlled firms produce intermediate goods which can be used as inputs by other firms in this sector. Due to this intra-consumption within the public sector, minimal targets will be in terms of net rather than gross outputs of the public sector. Since large deficits within the public sector can only be justified by a large net output rather than a large intra-consumption, we shall net out the latter when prescribing minimal quantity targets.

Since the information about production possibilities and aggregate demand on which quantity targets are based has to be incomplete and vague, we will face the following type of difficulty. It may happen that the public agency has a technology such as in Figure 1 in mind and wants to reach point C. It does so by an apparently suitable choice of minimal quantity targets and by requiring the firm to adopt the MCP-rule. In reality, however, there is no point such as C although the agency is led to believe there is. In this case the simultaneous fulfillment of the pricing rule and the quantity restrictions turns out to be impossible. We will examine in this paper to what extent the contradictory goals can be reconciled.

It was for ease of exposition and for illustrative purposes only that we mainly alluded to the MCP-rule in the above discussion. There are many other pricing rules of interest, as Boiteux-Ramsey pricing, full cost pricing and Aumann-Shapley pricing. Hence the analysis in our paper will be formulated in terms of abstract pricing rules, a framework which was presented in Dierker, Guesnerie, and Neuefeind (1985). These pricing rules could have been derived from some neoclassical optimization principle but need not. The present paper

thus extends the model of that paper to the case in which general pricing rules are supplemented by minimal quantity requirements. Since the quantity aspect itself has previously been discussed in Dierker, Fourgeaud, and Neuefeind (1976) and Neuefeind (1975), the model to be presented here is also an extension of the models used in these papers and we will freely borrow from all of them.

2. Model and Result

For simplicity's sake we will assume some properties, e.g. of demand and supply functions, directly without deriving them from assumptions on more fundamental concepts, e.g. preferences and production sets.

We distinguish between the competitive sector of the economy, which we call C-sector, and the public sector, P-sector for short, which is partially controlled by a planning agency and partially organized by the use of prices which have to be set according to specific rules. The role of the planning agency is to make sure that the global economic performance is, if not optimal, so at least satisfactory according to the agency's view. If the agency is alarmed by the outcome resulting from the use of prices, it interferes by setting minimal quantity targets for the goods to be provided by the P-sector to the C-sector. Such may be the case if the P-sector's deficit appears large in comparison to the goods and services made available to the C-sector.

There are $\ell \geq 1$ P-goods,[3] i.e. goods produced by the P-sector, and $k \geq 1$ C-goods. The commodity space is $R^{\ell+k}$. Note that this characterization of the goods allows P-goods to be produced in the C-sector whereas C-goods, by definition, cannot be produced in the P-sector. Consumers are described by their aggregate demand $d(p,w)$. More precisely, $d(p,w)$ is the sum of the quasi demands of all consumers at price system p if total wealth equals w. The prefix quasi is used to reflect the fact that the function is also defined if total wealth is insufficient to let all consumers survive. Moreover, we implicitly assume that total wealth is distributed among consumers according to some scheme which lets nobody starve as long as total wealth allows to do so.

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[3]We will sometimes consider P to be set of P-goods and will use $h \epsilon P$ or $i \epsilon P$ to denote that h or i are P-goods.

Assumption (C.1): a) The aggregate consumption set $X \subset R^{\ell+k}$ is compact.[4]

b) d: $R_+^{\ell+k}\backslash\{0\}) \times R \rightarrow X$ is a continuous function, homogeneous of degree 0 in (p,w). It fulfills Walras' Law in the sense that $pd(p,w) \leq w$ if $\min pX < w$ and $pd(p,w) \geq w$ if $w < \max pX$.

The compactness of X is directly assumed here in order to avoid a cumbersome truncation procedure. Also, for convenience, we shall work with functions instead of correspondences; generalizations should be a routine matter. The last part of our paper's version of Walras' Law simply states that the consumers are not satiated.

Assumption (C2): The aggregate production set $Y \subset R^{\ell+k}$ of the competitive producers is compact. Their total supply $s: R_+^{\ell+k}\backslash\{0\} \rightarrow Y$ is a continuous function homogeneous of degree 0, fulfilling $ps(p) = \max pY$.

P-producers are denoted by $j \in J$. P-goods produced by different P-producers are treated as different goods. Therefore, the set of P-goods can be partitioned into #J subsets, each corresponding to one producer. If a_j denotes firm j's output vector, the gross output vector of the P-sector takes the form $a = (a_1,...,a_j,...,a_{\#J}) \in R_+^{\ell}$.

Since P-firms may use P-goods as input, we have to distinguish between the gross and the net output of the P-sector. Intra-consumption of P-goods depends, of course, on the P-sector's activity as described by $a = (a_1,...,a_j,...,a_{\#J})$ and on prices p. The net supply of the P-sector is

[4]Since we are using the excess demand function d as the primitive concept to denote the consumption sector, we would, strictly speaking, not need introduce the consumption set X (and the production sets Y and Z later on). We nevertheless introduce X for convenience of notation and comparability with earlier papers on similar subjects.

denoted by $z(p,a) \in R^{\ell+k}$. By definition of C-goods, $z(p,a)|_C \leq 0$.[5] The components of $z(p,a)|_P$ may have any sign (unless we are in equilibrium). The P-sector's technology is denoted by Z.

<u>Assumption</u> (P): The net supply $z: (R_+^{\ell+k} \setminus \{0\}) \times R_+^{\ell} \to Z \subset R^{\ell+k}$ is a continuous function, homogeneous of degree 0 in p, and fulfilling $z(p,a)|_P \leq a$.

The vector of <u>aggregate</u> <u>initial</u> <u>endowments</u> in the economy is called $e \in R^{\ell+k}$. An allocation $(x,y,z) \in X \times Y \times Z$ is called <u>feasible</u> if $x \leq y + z + e$. Let $\underline{x} \in R^{\ell+k}$ be a lower bound of the consumption set X.

<u>Assumption</u> (B): For all prices $p \in R^{\ell+k} \setminus \{0\}$, all $y \in Y$, and all $z \in Z$ with $\underline{x} \leq y + z + e$, the inequality $p(y + z + e) < \max pX$ holds.

Assumption (B) states that the truncation X of the aggregate consumption set is chosen large enough to include all consumption bundles which belong to feasible allocations.

In a similar context (see Dierker, Fourgeaud, and Neuefeind (1976) and Neuefeind (1975)) it is assumed that the P-sector is asymptotically productive in the sense that the net output of at least one P-good becomes large if $\|a\|$ becomes large. This property, which holds for productive Leontief models in the linear case, has been derived from assumptions about asymptotic cones of technologies exhibiting increasing returns. Asymptotic productivity

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[5]The image of a set $S \subset R^{\ell+k}$ under the projection $R^{\ell+k} \to R^k$ is denoted by $S|_C$. Similarly, the image of S under the projection $R^{\ell+k} \to R^{\ell}$ is denoted by $S|_P$. The same notation applies to vectors in $R^{\ell+k}$.

implicitly involves some bound for input substitution of P-goods for C-goods.[6] Here we will use a similar assumption.

A net output vector $b \in R^{\ell}_{+}$ of the P-sector is called feasible if there is a feasible vector (x,y,z) such that the P-sector's net supply of P-goods, $z|_P$ equals b. A gross output vector $a \in R^{\ell}_{+}$ will be called feasible if there exists a price vector p such that the resulting net output vector $z(p,a)|_P$ is feasible. We assume that, disregarding C-inputs, the P-sector is productive in the following sense.

<u>Assumption</u> (PROD): There exists $(\alpha_1,\ldots,\alpha_h,\ldots,\alpha_{\ell}) \gg 0$ such that for every feasible net output $b \in R^{\ell}_{+}$ of the P-sector and for every $(p,a) \in (R^{\ell+k}_{+}\backslash\{0\}) \times R^{\ell}_{+}$ the following holds: If $a_h > \alpha_h$ for some P-good h, then there exists a P-good i such that $z_i(p,a) > b_i$.

This assumption implies that both, the set of feasible net output vectors and the set of feasible gross output levels are bounded.

A <u>pricing rule</u> is a function $q\colon (R^{\ell+k}_{+}\backslash\{0\}) \times R^{\ell}_{+} \rightarrow R^{\ell}_{+}$. Intuitively, q(p,a) describes the prices for the P-goods which are to be charged by the P-firms at prices p and gross activity levels a according to some economic principle such as pricing according to marginal costs. If a P-firm follows the MPC-rule, it considers only those prices in the price vector p which correspond to the commodities it uses as inputs. A broader interpretation, however, is possible: A P-firm could take into account the current market

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[6] If such a bound is not naturally given by technological reasons, one may employ an internal price system within the P-sector, which may be different from the market price system p, to control intraconsumption in the P-sector. To avoid unnecessary complexity of our model, we refrain from introducing P-internal prices here.

prices for its outputs and, out of equilibrium, set its prices differently as e.g. in pricing à la Boiteux.

Although such principles are often designed to achieve a welfare theoretic goal, there is, in general, no guarantee that they actually achieve the goal they are tailored for. As we pointed out in the introduction, there can be an economy with an MPC equilibrium in which deficits are high and output is low, although there is another one with a high output and a tolerable deficit. A multiplicity of MPC equilibria is to be expected, in particular if there are wiggly returns to scale. Some of these equilibria may appear more satisfactory than others. To rule out undesirable equilibria, a planning agency sets minimum quantity targets. That is to say that allocations are ruled out as an equilibrium if the P-sector's set output does not meet the target for a P-good, even if the pricing rule applies. It is natural to let minimal targets depend on economic indicators such as deficits or prices. In our model these indicators are determined by the basic variables (p,a). A deficit of the P-sector, for instance, would read $|pz(p,a)|$. Thus we define <u>minimal quantity targets</u> to be a function $b: (R_+^{\ell+k}\setminus\{0\}) \times R_+^{\ell} \to R_+^{\ell}$, of prices and gross output levels.

We suppose that the planning agency knows the economic possibilities well enough to choose minimal targets which will not strain the economy too much. In particular, we will require that possible losses of the P-sector will not drive the economy into or even near bankruptcy; i.e. if the targets are reached for a pair (p,a) of prices and activity levels then total value of supply, $pz(p,a) + ps(p) + pe$, exceeds the minimal wealth needed to survive, $\min pX$, with some slack.

In specifying a pricing rule q for a P-firm, the planning agency is aware of possibly disastrous consequences of setting prices which are too low: If prices p and activities a are such that the deficit $|pz(p,a)|$ becomes untolerably large, then the price p_h of one of the P-goods, which is supplied in an amount exceeding $b_h(p,a)$, is raised.

It turns out that the two restrictions on the quantity targets and the pricing rule can be conveniently phrased in one assumption.

<u>Assumption</u> (QT-PR): a) The minimum quantity targets $b: (R_+^{\ell+k}\backslash\{0\}) \times R_+^{\ell} \to R_+^{\ell}$ and the pricing rule $q: (R_+^{\ell+k}\backslash\{0\}) \times R_+^{\ell} \to R_+^{\ell}$ are continuous functions, homogeneous of degree 0 in p. Moreover, all values of b represent feasible net output vectors.

b) There exists a threshold[7] $\vartheta > 0$ such that $pz(p,a) \leq \min p(X-Y-e) + \vartheta(\sum_i p_i)$ implies that there exists a P-good h with $z_h(p,a) > b_h(p,a)$ and $q_h(p,a) > p_h$.

Note that part (a) of the assumption assures that the set of feasible net output vectors is non-empty.

Now, we are ready to formulate our existence result. Clearly, one cannot expect that there generally exists a price system $\bar{p}$ and a vector of activity levels $\bar{a}$ such that all markets clear, the net supply of P-goods is at or above the required level, i.e. $z(\bar{p},\bar{a})|_P \geq b(\bar{p},\bar{a})$, <u>and</u> the pricing rule applies, i.e. $\bar{p}|_P = q(\bar{p},\bar{a})$, since minimal targets may be set too high to allow for a fulfillment of the pricing rule. As for solutions of the general complementarity problem as treated, e.g. in Saigal and Simon (1972), one would expect trade-offs between the fulfillment of the quantity constraint and the pricing rule. Our problem here differs since the non-negativity constraints of the complementarity problems are replaced by the requirement that the

[7]Strictly speaking, we do not need $\vartheta > 0$ here. This would be different if we would not have started out with compact sets X and Y. The reason why we do not drop ϑ here is that its interpretation as a particular level of wealth which must not be touched by the activities of the P-sector is economically meaningful.

target has to be fulfilled.

The following statement is true.

Theorem: Assume (C1), (C2), (B), (P), (PROD), and (PR-QT). Then there exists a pair of prices and activity levels $(\bar{p},\bar{a}) \in (R_{+}^{\ell+k}\setminus\{0\}) \times R_{+}^{\ell}$ such that

(i) $d(\bar{p},\bar{a}) \leq s(\bar{p}) + z(\bar{p},\bar{a}) + e$ with equality for all commodities h with a positive price $\bar{p}_h$,

(ii) $z(\bar{p},\bar{a})|_P \geq b(\bar{p},\bar{a})$;

(iii) $q(\bar{p},\bar{a}) \geq \bar{p}|_P$;

(iv) $[q(\bar{p},\bar{a}) - \bar{p}|_P\,]\,[z(\bar{p},\bar{a})|_P - b(\bar{p},\bar{a})] = 0$ (complementary slackness).

Condition (iv) may be interpreted as saying that too ambitious a quantity target $b_h(\bar{p},\bar{a})$ for good h can make it necessary to set the price $\bar{p}_h$ below its level $q_h(\bar{p},\bar{a})$ suggested by the pricing rule. In the particular case where $\bar{a}_h = b_h(\bar{p},\bar{a}) = 0$ and, thus, $z_h(\bar{p},\bar{a}) = 0$, the complementary slackness condition reduces to the boundary assumption PR(α) of Dierker, Guesnerie, and Neuefeind (1985), p. 1383, which allows inactive P-firms to charge prices lower than q_h. This boundary assumption has been motivated there.

The Theorem may also be read as giving a sufficient conditon for the existence of a more desirable equilibrium at which the pricing rule applies, i.e. $\bar{p}|_P = q(\bar{p},\bar{a})$, and the minimal quantity requirement is satisfied, i.e. $z(\bar{p},\bar{a})|_P \geq b(\bar{p},\bar{a})$. Suppose, for example, that the planning agency, which can be uninformed about the true intra-consumption within the P-sector, knows the following for all market clearing pairs (p,a). If, for some P-good h, the excess demand of the C-sector for h equals $b_h(p,a)$, then $q_h(p,a)$ is below p_h. In this case, the agency can be assured, according to the Theorem, that there exists an equilibrium $(\bar{p},\bar{a})$ satisfying $z(\bar{p},\bar{a})|_P \geq b(\bar{p},\bar{a})$.

It is possible to give various other sets of conditions guaranteeing the existence of an equilibrium satisfying the pricing rule and the quantity restrictions. These statements basically rely on the complementary slackness condition (iv).

Observe that the conclusion of the Theorem treats the quantities of the P-goods and their prices in a rather symmetric way. If one replaces q by b and p by z, one only has to reverse the inequalities to obtain the original result. We found this surprising in view of the debate about constraints versus a planner's objective in the literature about planning (see, e.g., Johansen (1977), chapter 3.11). In line with Johansen (1977, p. 254) we have associated the "satisficing" or "aspiration level" approach with quantity targets, whereas the pricing rule is being considered an offspring of neoclassical maximization. The conclusion of our Theorem, however, does in no way formally reflect this difference.

3. Proof of the theorem:

All demand and supply functions and the price setting rule are homogeneous of degree zero in prices. Hence we can work with normalized prices. Let $\|.\|$ be the summation norm and put

$S = \{p \epsilon R_+^{\ell+k} \mid \|p\| = 1\}$.

Next choose $\alpha \epsilon R_+$ large enough to ensure that, first, no activity level $a \epsilon R_+^{\ell}$ is feasible if $a_h = \alpha$ for some P-good h and, second, $\alpha \geq \alpha_h$ for all P-goods h, where α_h is one of the numbers in the productivity assumption (PROD). Put

$$A = \prod_{h=1}^{\ell} [0,\alpha].$$

Define $g^0: S \times A \rightarrow R^{\ell}$ componentwise for each P-good h by

$$g_h^0(p,a) = a_h + \max\{0, b_h(p,a) - z_h(p,a), p_h - q_h(p,a)\}$$
$$+ \max\{0, z_h(p,a) - b_h(p,a)\} \cdot \min\{0, p_h - q_h(p,a)\}.$$

Also, define $g^1: S \times A \rightarrow R^{\ell}$ componentwise by

$$g_h^1(p,a) = a_h + \max\{0, \max_{i \epsilon P}\{b_i(p,a) - z_i(p,a)\}\} \cdot (b_h(p,a) - z_h(p,a))$$
$$+ \max\{0, z_h(p,a) - b_h(p,a)\} \cdot \min\{0, p_h - q_h(p,a)\}.$$

The function g^0 is designed to control the gross output levels in the "feasible part" of A, whereas g^1 does so on the "upper boundary" of A. To combine the two functions, let $\lambda: A \rightarrow [0,1]$ be a continuous function with

$\lambda(a) = 0$ if a belongs to a feasible allocation and

$\lambda(a) = 1$ if $a_h = \alpha$ for some P-good h.

Define $g^2: S \times A \rightarrow R^{\ell}$ by $g^2(p,a) = \lambda(a) \cdot g^1(p,a) + (1-\lambda(a)) \cdot g^0(p,a)$. Then $g^2(p,a) = g^0(p,a)$ whenever a belongs to a feasible allocation and $g^2(p,a) = g^1(p,a)$ whenever a is in the upper boundary of A.

To obtain a mapping[8] into A instead of R^{ℓ}, define $g: S \times A \rightarrow A$ componentwise by

[8]This mapping and the mapping f are taken from Dierker, Guesnerie, and Neuefeind (1985), p. 1387.

$$g_h(p,a) = \begin{cases} 0 & , \text{ if } g_h^2(p,a) \le 0 \\ g_h^2(p,a), & \text{if } 0 \le g_h^2(p,a) \le \alpha \\ \alpha & , \text{ if } \alpha \le g_h^2(p,a) \end{cases}$$

Since g^0, g^1 and λ are continuous, g is continuous.

Next we construct a mapping $f: S \times A \to S$, the task of which is to ensure market clearing at fixed points. Put

$$w(p,a) = ps(p) + pz(p,a) + pe.$$

Then

$$\hat{f}(p,a) = d(p,w(p,a)) - s(p) - z(p,a) - e$$

is the quasi excess demand of the economy. Let

$\frac{1}{\gamma} = (\ell+k)\,(1 + \max_{(p,a)\in S\times A} \|\hat{f}(p,a)\|_\infty) > 0$, where $\|.\|_\infty$ is the maximum norm.

for each $(p,a) \in S \times A$ there exists a commodity i such that

$$p_i \ge \frac{1}{\ell + k} > \gamma\ \|\hat{f}(p,a)\|_\infty \ge \gamma\ |f_i(p,a)|.$$

Therefore, at least one component of $p + \gamma\hat{f}(p,a)$ is positive and hence $(p + \gamma\hat{f}(p,a))_+ \ne 0$ where $_+$ indicates the positive part of a vector. Put

$$f(p,a) = \frac{(p + \gamma\hat{f}(p,a))_+}{\|(p + \gamma\hat{f}(p,a))_+\|}.$$

f clearly is continuous.

According to Brouwer's Fixed Point Theorem the mapping

$$(f,g): S \times A \to S \times A$$

possesses a fixed point, say $(\bar{p},\bar{a})$. We will show that $(\bar{p},\bar{a})$ indeed is an equilibrium for the economy.

To simplify notation we will write $\bar{b}$ instead of $b(\bar{p},\bar{a})$, $\bar{z}$ instead of $z(\bar{p},\bar{a})$, etc.

Step 1: The Quantity Constraints Hold, i.e. $\bar{b} \leq \bar{z}|_P$

Assume that this does not hold. We will show that

- there exists a P-good h with $\bar{a}_h = \alpha$, and (*)

•• $\bar{b} \geq \bar{z}|_P$. (**)

Since $\bar{b}$ is feasible by assumption (QT-PR), these two statements cannot both hold because of assumption (PROD) and the choice of α. Hence the quantity constraints must hold.

In fact, since the quantity constraints $\bar{b} \leq \bar{z}|_P$ are assumed to be violated, there exists a P-good h with $\bar{b}_h > \bar{z}_h$. Thus we obtain, by construction of g^0 and g^1, that $\bar{g}^0_h, \bar{g}^1_h > \bar{a}_h$ and, hence, $\bar{g}^2_h > \bar{a}_h$, which implies $\bar{g}_h = \alpha$. Since $(\bar{p},\bar{a})$ is a fixed point of (f,g), we get $\bar{a}_h = \alpha$; i.e. $\bar{a}$ is in the upper boundary of A which implies statement (*).

It also implies $\bar{g}^2 = \bar{g}^1$ and we obtain, recall that $\bar{b}_h > \bar{z}_h$, that

$$\text{sign}(\overline{g}^2_i - \overline{a}_i) = \text{sign}(\overline{b}_i - \overline{z}_i) \text{ for all } i \in P.$$

For $0 < \bar{a}_i < \alpha$, we get $\bar{g}^2_i - \bar{a}_i = 0$; hence, $\bar{b}_i = \bar{z}_i$.

For $\bar{a}_i = \alpha$, we get $\bar{g}^2_i - \bar{a}_i \geq 0$; hence $\bar{b}_i \geq z_i$.

For $\bar{a}_i = 0$, we get $\bar{z}_i \leq 0$ by assumption (P) and hence, $\bar{b}_i \geq 0 \geq \bar{z}_i$.

Thus $\overline{b} \geq \overline{z}|_P$ and statement (**) also follows. Thus we are done with step 1.

Step 2: Survival at a Fixed Point

Note for the sequel that, because of $\bar{b} \leq \bar{z}|_P$,

$$\bar{g}^0_h - \bar{a}_h = \max\{0,\bar{p}_h - \bar{q}_h\} + (\bar{z}_h - \bar{b}_h) \cdot \min\{0, \bar{p}_h - \bar{q}_h\},$$

and

$$\bar{g}^1_h - \bar{a}_h = (\bar{z}_h - \bar{b}_h) \cdot \min\{0, \bar{p}_h - \bar{q}_h\}.$$

Consider a P-good h for which $\bar{b}_h < \bar{z}_h$. Note that this implies $\bar{a}_h > 0$.

If $\bar{a}_h = \alpha$, then $\bar{g}^2_h - \bar{a}_h = \bar{g}^1_h - \bar{a}_h \geq 0$.

Because of $\bar{g}^1_h - \bar{a}_h = (\bar{z}_h - \bar{b}_h) \cdot \min\{0, \bar{p}_h - \bar{q}_h\}$ we get $\bar{p}_h - \bar{q}_h \geq 0$ or $\bar{p}_h \geq \bar{q}_h$.

If $0 < \bar{a}_h < \alpha$, then $\bar{g}^2_h - \bar{a}_h = 0$. If $\bar{p}_h - \bar{q}_h$ were negative, $\bar{g}^1_h - \bar{a}_h$ as well as $\bar{g}^0_h - \bar{a}_h$ were negative, hence $\bar{g}^2_h - \bar{a}_h$, too. This, however, is impossible and, hence, $\bar{p}_h \geq \bar{q}_h$.

Summarizing, $\bar{b}_h < \bar{z}_h$ implies $\bar{p}_h \geq \bar{q}_h$ for any P-good h. (***)

Assume now that the economy is bankrupt. This means

$$\bar{w} = \bar{p}\,(\bar{s} + \bar{z} + e) \leq \min \bar{p}X + \vartheta.$$

Then, by part (b) of Assumption (QT-PR), there exists a P-good h with $\bar{b}_h < \bar{z}_h$ and $\bar{p}_h < \bar{q}_h$ which we have just shown to be impossible, and we obtain survival.

Step 3: Market Clearing

Because $\bar{w} \geq \min \bar{p}X + \vartheta > \min \bar{p}X$, the inequality $\bar{p}d(\bar{p},\bar{w}) \leq \bar{w}$, holds by Assumption (C1) and we obtain $\bar{p}\hat{f}(\bar{p},\bar{a}) \leq 0$. Now, one can use the same argument as in Dierker, Guesnerie, and Neuefeind (1985, p. 1388) to obtain $\hat{f}(\bar{p},\bar{a}) \leq 0$. Hence the gross output level $\bar{a}$ belongs to a feasible allocation. Therefore, $\underline{x} \leq d(\bar{p},\bar{w}) \leq \bar{s} + \bar{z} + e$ and by assumption (B), $\bar{w} = \bar{p}(\bar{s} + \bar{z} + e) < \max \bar{p}X$. Hence, by Walras' Law, $\bar{p}\hat{f}(\bar{p},\bar{a}) = 0$ and $\hat{f}_h(\bar{p},\bar{a}) = 0$ whenever $\bar{p}_h > 0$, i.e. we have shown market clearing in the sense of part (i) of our Theorem.

Step 4: Pricing Rule and Complementary Slackness

Since the gross output level $\bar{a}$ belongs to a feasible allocation, $\bar{g}^2 = \bar{g}^0$ holds. Feasibility also implies that $\bar{a}_h < \alpha$ for all P-goods h. This implies $\bar{g}^0_h - \bar{a}_h \leq 0$ by construction of g and, hence $\bar{q}_h \geq \bar{p}_h$ by construction of g^0. This shows that (iii) holds. To show (iv), we note that nothing is contributed to the sum in (iv) if $\bar{z}_h = \bar{b}_h$. If $\bar{z}_h > \bar{b}_h$, we have obtained the reverse inequality $\bar{p}_h \geq q_h$ for the prices, (see (***) above), i.e. $\bar{p}_h = \bar{q}_h$. This shows that zero is contributed to the sum in (iv) in this case, too.

This concludes the proof of the Theorem.

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