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ECONOMIC SYSTEMS AND THEIR REGULATION

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ECONOMIC SYSTEMS AND THEIR REGULATION

There are two aspects to regulation: there is conscious variation of parameters to attain some distinct behaviour; there are, as part of an existing system, in-built elements of self-control or auto-regulation. Optimal control is a large and growing discipline and I shall leave that to Dr. Velupillai's treatment in Part II. It is of basic importance to have a rather precise knowledge of the structure of a system, one of the best procedures is to study its dynamical behaviour.

I

It occurred to me that one of the most illuminating ways of regarding the economic system is to see it as an elaborate set of auto-controls. Many complex machines have sub-system controls to ensure satisfactory functioning. One has only to consider the human body: thus the bloodstream is regulated homeostatically as to pressure, temperature and constituents, under extreme variations of 'load' conditions. Such auto-control has been a central theme of most economic analysis since Adam Smith: it is the doctrine of laissez-faire that there are available non-conscious controls in most parts of the economy which will function better than conscious controls. Such doctrines are in fact too simplistic and their function is capitalist apologetics. Such sub-system controls can easily exhibit various forms of dys-function, e.g. Parkinson's disease, or stocks cycles.

Consider a given technology with a given set of linear, variable costs yielding an input-output structure, which we can write as an economic potential[(1)]:

$$V(p,q) = \langle p_i \rangle \left[\left[I \right] - (1+g) \left[a \right] \right] \left\{ q_i \right\} \qquad (1)$$

$$i = 1, \ldots . n$$

$\langle \quad \rangle$ represents a row vector, $\{ \quad \}$ a column vector, $[\quad]$ a square matrix, and $\lfloor \quad \rfloor$ a diagonal matrix.

g is a scalar and represents the common component of growth in all sectors - and profit rate as well, it turns out. There are 2 n variables and the gradient of the potential with respect to the n prices is:

$$\nabla_p V = \left[I - (1+g)a \right] \quad \{q\} \tag{2}$$

which gives for each sector the difference between supply and demand. Similarly grad. w.r.t. q is

$$\nabla_q V = \langle p \rangle \left[I - (1+g)a \right] \tag{3}$$

which give the difference between price and variable costo for each sector. The g here really refers to the common rate of profit r, but since

$$\Big(p \; (I - (1+r)a) \Big) \; q = p \; \Big((I - (1+g)a) \; q \Big), \quad r = g \tag{4}$$

All goods are produced by goods with the exception of 'factors'. I take homogenous labour as sole limitational factor, though in the same way we may add land (natural resources), but <u>not</u> capital. Consequently, the input matrix structure is to be partitioned thus:

$$(1+g) \; \langle p; \; w \rangle \left[\begin{array}{c:c} a & a_c \\ \hdashline a_1 & a_{11} \end{array} \right] \left\{ \begin{array}{c} q \\ \hdashline 1 \end{array} \right\} \tag{5}$$

where w is money wage rate and l, employment, giving:

$$p = (1+g) \; (pa + wa_1) \text{ and } q = (1+g)(aq + a_c 1) \tag{6}$$

along with the two scalar equations:

$$w = (pa_c + wa_{11}) \quad \text{and} \quad 1 = (1+g)(a_1 q + a_{11} \; 1) \tag{7}$$

Defining n standard commodities, we may transform to principal coordinates[2] thus separating variables and yielding scalar gradient dynamics.

$$p - (1+g)\left(p\begin{bmatrix} \lambda \end{bmatrix} + wa_1\right); \quad q - (1+g)\left(\begin{bmatrix} \lambda \end{bmatrix} q + a_c l\right) \quad (8)$$

Some of these eigenvalues, λ_i, may (almost certainly will) be complex, but occurring necessarily in conjugate complex pairs, they are reducible to the real block diagonal form, with blocks

$$\begin{bmatrix} \alpha & -\beta \\ \beta & \alpha \end{bmatrix} .$$

Our main interest, however, is not in such minor cycles, but rather in growth or cyclical movements common to all sectors.

In equilibrium, with price equal to cost, the distribution of net product becomes simply formulated:

$$1 - \lambda_i = \frac{w}{p_i} a_{li} + g\left(\lambda_i + \frac{w}{p_i} a_{li}\right) \qquad i = 1,\ldots.n \qquad (9)$$

or, set $\theta_i = \frac{w}{pi} a_{li}$,

$$= \theta_i + g\left(\lambda_i + \frac{w}{p_i} a_i\right)$$

Dividing by $1 - \lambda_i$

$$1 = u_i + (1-u_i) \quad = \quad \text{share of labour + share of capital} \qquad (10)$$

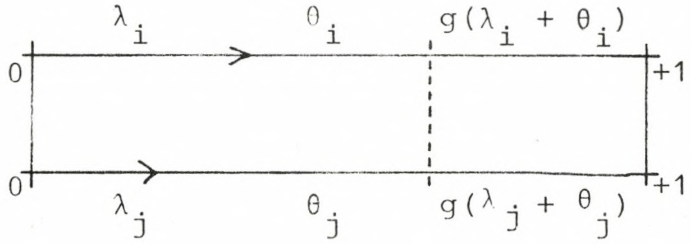
For this simplified version of the economy, it is helpful to make the assumption that all profits are saved and all wages consumed. Then, in equilibrium,

$$1 = \frac{a_{ci} \; {}^{l}/_{qi}}{1 - \lambda_i} + g \, \frac{(\lambda_i + a_{ci} \; {}^{l}/_{qi}}{1 - \lambda_i} \qquad (11)$$

= share of consumption + share of investment (in circulating capital).

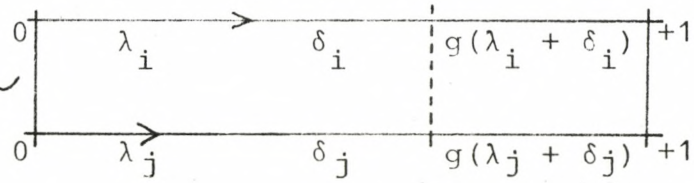
Letting $\delta_i = \dfrac{a_{ci}\ {}^{1}/q_i}{1-\lambda_i} = \theta_i$, the system is fully represented by: (12)

value ~



(13)

and output ~



(14)

since $\quad a_{ci}\dfrac{1}{q_i} = \dfrac{w}{p_i}\cdot a_{li}\ ,\quad a_{ci} = \left(\dfrac{1}{1/q_i}\right)\cdot\left(\dfrac{w/p_i}{1/a_{ci}}\right)$ (15)

$\dfrac{w}{p_i}\cdot a_{li}$ is generalized unit labour cost and, with a given technology, $1/q_i$ is constant, so that unit consumption is proportional to unit wage cost, which is the ratio of the real wage to labour productivity. Therefore, with price equal to cost and supply equal to demand in every sector,

> given g, the share of labour is determined, and
> given labour productivity, the real wage as well;
>
> conversely, given labour productivity and the real wage, the rate of growth is determined.

Proposition: Given that labour is the sole limitational input its real value is determined by relative scarcity, measured by $l/N=V$, where l is employment and N the given available labour force. This being so, there is an aggregative internal feedback or auto-control which leads alternatively to a too high growth rate g, and to a too low one, averaging to the long-run equilibrium growth rate. We may make the assumption that there is a common constant rate of growth in labour productivity, i.e. $\frac{a_{1i}}{a_{11}} = -\bar{g}_a$

This simplification has some justification because technical change is made up of many small independent changes and because we are interested in what is common to all sectors. $\dot{N}/_N$ may be taken as a constant $\bar{g}_N$

$$\frac{\dot{V}}{V} = \frac{\dot{l}}{l} - \frac{\dot{N}}{N} = g - \left(\overline{g_a + g_N}\right), \quad \text{since,}$$

$$\frac{\dot{l}}{l} = \frac{\dot{q}_i}{q_i} - g_a = g - g_a \tag{16}$$

$$g = \frac{1}{\lambda_i + \theta_i} - 1 = \frac{1}{\lambda_i + \delta_i} - 1$$

Thus

$$\frac{\dot{V}}{V} = \frac{1}{\lambda_i + \theta_i} - \left(1 + \overline{g_a + g_N}\right)$$

$$\frac{\dot{\theta}_i}{\theta_i} = \frac{\dot{w}}{w} - \frac{\dot{p}_i}{p_i} - \bar{g}_a \quad \text{, but by hypothesis} \tag{17}$$

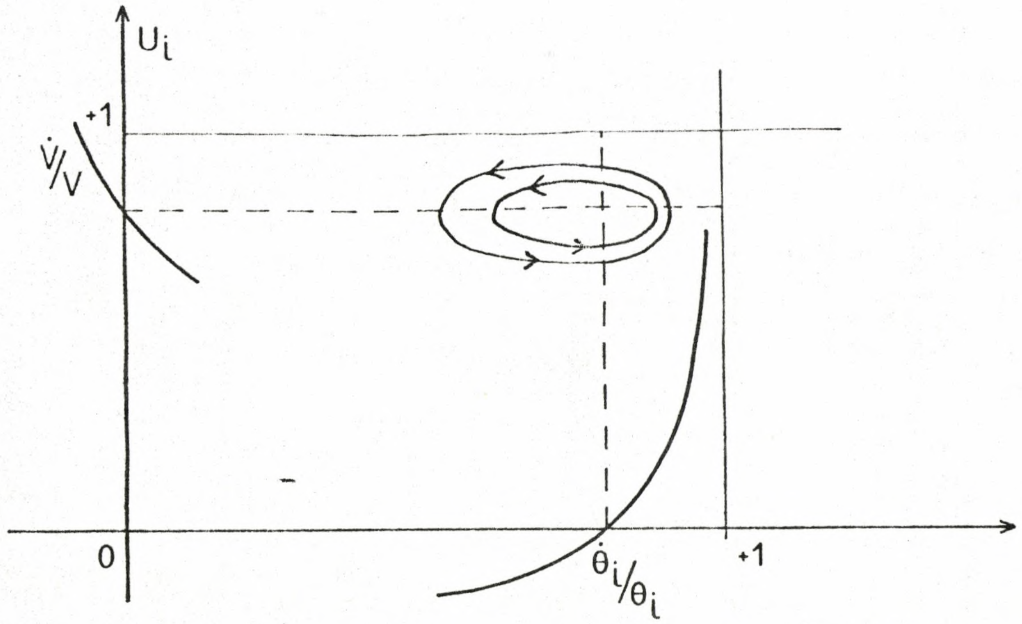
$$\frac{\dot{w}}{w} - \frac{\dot{p}_i}{p_i} = \phi(V) \quad , \quad \phi' > 0$$

Therefore our idealized economic system is represented by the pair of sets of equations:

$$\frac{\dot{\theta}_i}{\theta_i} = \phi(V) - \bar{g}_a \quad , \quad \text{common to all eigensectors;}$$

$$\frac{\dot{V}}{V} = \frac{1}{\lambda_i + \theta_i} = \left(1 + g_a + g_N\right) \text{, unique to each eigensector} \tag{18}$$

In phase space :



The system shows n constituent cycles, all of different amplitude in output and employment but similar variations in distributive shares, both fluctuating around but rarely being equal to the long-run average values. The behaviour of each actual sector is a particular linear combination of the n constituent motions. This is a malfunctioning feedback control of individual sectors through an idealized single labour market. The historical statistics of percentage unemployment in capitalist countries provide ample support for this behaviour. The model is, of course, oversimplified not only in the single labour market but also in the assumption of price-cost and supply and demand equilibrium in every market. In fact, such equality does not necessarily exist and it is only because of a variety of auto-controls that there is any approximation to it. Economists have variously noted at least four basic types.

I as $p_i \gtrless$ cost, p_i falls or rises.
II as $p_i \gtrless$ cost, q_i rises or falls.
III as $d_i \gtrless$ q_i, q_i rises or falls.
IV as $d_i \gtrless$ q_i, p_i rises or falls.

Plus, of course, various combinations of the cases, which, along with at least three common types of control routine, makes rather a lot of cases.

First there is Harrods' great contribution — the inherent instability of capitalism. Taking q-d as the error, and noting that g controls q, since $q_{t+1} = (1+g)(q_t)$,

$$\dot{g} = -\alpha\ (q - d) \tag{19}$$

which yields a monotonically unstable system.
Equally plausible and probably more realistic is <u>proportional control</u> of level:

$$q_{t+1} = d_t \tag{20}$$

giving $$\Delta q_i = -\beta\left(1 - (\lambda_i + \delta_i)\right)q_i \tag{21}$$

which is asymptotically stable about the equilibrium value.

If, on the other hand, producers use <u>integral control</u>: stocks, s, are the integral (or sum) of the error,

$$S_i = \Sigma\ \Delta\ S_i = \Sigma\ (q_i - d_i) \tag{22}$$

Therefore,

$$\begin{aligned} \Delta q_i &= -\beta\ (S_i - \bar{S}_i); \\ \Delta S_i &= q_i - (\lambda_i + \delta_i)\ q_i\ , \end{aligned} \tag{23}$$

which gives the simplest version of the well-observed stocks cycle in disaggregated form with n cycles of differing amplitude but common period.

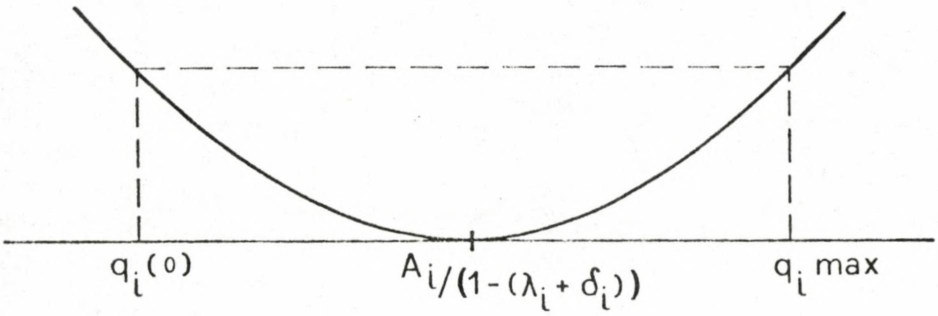
The level of the potential being arbitrary, an exogenous parameter can always be added to it. Add $\left\{A_i\right\}$ representing all other real demands, i.e. exports, investment and government outlays, and transform it to the principal axes. Then

$$S_i = q_i - (\lambda_i + \delta_i)\, q_i - A_i \tag{24}$$

and

$$\ddot{q}_i + \beta\Big(1 - (\lambda_i + \delta_i)\Big) q_i = \beta A_i \tag{25}$$

setting the minimum of V_pV equal to the equilibrium of q, we have reduced the problem to its simplest form with each



eigengood performing simple harmonic motion about its equilibrium value. Given the arbitrary initial value, the output, if undisturbed, will vibrate forever between $q_i(0)$ and q_i max.

If demand is greater than output, the latter should be increased less if demand is falling, or more if it is rising, on the assumption that demand is independent of the action of the producer. Hence derivative control is relevant to dynamical analysis. It should be noted that there are no decisions in eigensectors, but that this is merely a representation of the type of decisions made in actual sectors.

Let
$$\begin{aligned}\dot{q}_i &= -\beta\Big(1 - (\lambda_i + \delta_i)\Big) q_i - \gamma\big(\dot{q}_i - \dot{d}_i\big) \\ &= -\Big(1 - (\lambda_i + \delta_i)\Big)\Big(\beta q_i + \gamma \dot{q}_i\Big)\end{aligned} \tag{26}$$

The solution being,

$$\dot{q}_i = \frac{-\beta\Big(1 - (\lambda_i + \delta_i)\Big)}{1 + \gamma\Big(1 - (\lambda_i + \delta_i)\Big)}\, q_i \tag{27}$$

The system remains stable but is less stable than it would be with proportional control alone.

It may seem surprising, or indeed unconvincing, that a system pervaded by many negative feedback controls, is capable of such serious malfunction. The explanation is simple and highly significant: the adaptive response alters the desired goal. Thus each individual producer quite correctly takes demand as given and alters his output so as to approximate the demand. His output will, in fact, not significantly affect demand. However, given a whole economy in which many or all producers are all doing the same, then demand is thoroughly subject to the output. In such a case, the system may 'hunt' its equilibrium without ever finding it. Thus if pursuer and pursued each alter course in relation to the other, there need be no stable solution.

THE VALUE DUAL AND THE PROCESS OF INFLATION

Regardless of how or why inflation is initiated, once under way it is an endogenous dynamical process - a self-sustaining mechanism, analogous to the chain reaction in an atomic explosion, or the burning of a fire. To understand the self-generating nature of an inflation, it is necessary to disaggregate. Thus when one union or sector negotiates a wage settlement it gains a real increase in wages, but when in the course of the succeeding months, other sectors also negotiate increases, all or part of the gain will be eroded. Even though negotiators come to recognize this fact, there is nothing they can or will do about it. In each producing unit there are fixed charges - wages and salaries independent of the level of output, rents and interest cost of total invested capital, and depreciation allowances. Dividing these by some expected or normal output, gives unit fixed cost.

The potential then becomes

$$V(p,q) = \langle p;w \rangle \left[\begin{array}{c:c} a & a_c \\ \hdashline a_l & a_{ll} \end{array}\right] \left\{\begin{array}{c} q \\ \hdashline l \end{array}\right\} + \langle p \rangle \left\{ A \right\} + \langle B \rangle \left\{ q \right\} \qquad (28)$$

The gradient with respect to q is

$$\nabla_q V = \langle p \rangle a + w \langle a_l \rangle + \langle B \rangle \qquad (29)$$

With a given rate of interest r, equilibrium requires

$$\langle p \rangle = (1+r)\ (pa + wa_l) + \langle B \rangle \qquad (30)$$

The common practice is to set price by a mark-up on variable cost which, at expected output $\hat{q}$ will give a rate of return equal to r. Thus, though, in some sense $r_i = r_j$ in equilibrium, the mark-up π_i is unequal to π_j since $B_i \neq B_j$.

$$\langle p \rangle = \langle pa + wa_l \rangle \boxed{1 + \pi} = (1 + r) \langle pa + wa_l \rangle + \langle B \rangle$$

Given existing capacity and technology, the mark-ups are constant and may be absorbed into a and a_1, so that unit cost becomes pa + wa_1. Profit or loss then arises for two quite distinct reasons, i.e. as

$$p \gtrless p_t a + wa_1$$

and (31)

$$q \gtrless \hat{q}$$

Proportional control arises by the setting of price equal to cost of the previous period (it cannot be for the same period since costs are only known after prices are set).

$$p_{t+1} = p_t a + w_t a_1 \qquad (32)$$

With this model we can see how serious and how complex is the lag introduced into the process of inflation. For a single increase Δw in wages, $\Delta p_{t+1} = \Delta w_t a_1$, and hence the rise in prices is proportionately less than that in money wages. But then $\Delta p_{t+2} = \Delta p_{t+1}\, a$ and $\Delta p_{t+3} = \Delta p_{t+1}\, a^2$ and so on. Thus there is a complicated, very long lag between a single change in wages and the final consequences.

Since this is a convergent process ultimately all prices will rise in the same proportion as the increase in the money wage rate, but during the process real wages have risen and are only gradually reduced back to the original value. The error is price less marked-up cost and with constant unit labour cost, any error is graudally reduced to zero:

$$\Delta p = -\left[p - (pa + wa_1)\right] \qquad (33)$$

or more generally $= - \gamma\left[p - (pa + wa_1)\right]$

so that it is a homeostatic mechanism which, from any initial position, will produce a constellation of prices yielding a common rate of profit.

Consider, however, the realistic case in which different sectors have differing rates of growth of labour productivity

$$a_1(t) = < E_1(1-\alpha_1)^t \quad E_2(1-\alpha_2)^t \ldots . \; E_n(1-\alpha_n)^t > \quad (34)$$

arranged in descending order, $\alpha_1 > \alpha_2 > \ldots . > \alpha_n$

Assuming materials inputs constant, and ignoring all the transient states, consider the steady state of such a system. The leading sector will be expanding and hence needing labour; it can easily grant money wage increases without raising price or reducing operating profit below normal. Consider the pure case in which it has all wage increases which do not raise unit labour cost, i.e. $w(t)a_{11}(t) = Z_1$. In a perfect labour market, or for any reason, this then becomes the common wage rate.

$$w_t \; a_{11}(t) = \bar{w} \; (1+\beta)^t \; E_1(1-\alpha_1)^t \simeq \bar{Z}_1 \; (1+\beta-\alpha_1)^t = Z_1 \quad (35)$$

$$\beta = \alpha > \alpha_i \quad , \quad i = 2, \ldots . n.$$

Thus every other sector will be experiencing rising unit wage costs equal to the difference between its growth of labour productivity and that of the maximum. Not only this but there will be a set of differently rising material costs which will in turn affect most other sectors in different degrees. Thus the ith sector will have rising unit labour cost of $\bar{w}\,E_i\big(1+(\alpha_1-\alpha_i)\big)^t$ and the separate effect of this on all prices can be obtained by multiplying it by the ith row of $\left[I - a\right]^{-1}$. Adding all these positive inflation rates together gives us a steady state inflation.

This over-simplified example shows how essential disaggregation is: it can explain the global result but the converse is not so. Thus the notion of linking wage increases to labour productivity, which seems to make sense in aggregates, is seen to be quite misleading by ignoring the effect of wage increases on other sectors. The model also helps us to see why the powerful and pervasive method of negative feedback can nevertheless fail so badly to achieve a stationary equilibrium. The individual producer knows his costs and sets a price to cover them; his price does not affect his costs. Yet the moment we consider the whole economy, we see that in altering prices to equal costs, other costs are altered as well.

This aspect of the problem becomes even more interesting and important when we consider money wage and real wage. Wages are necessarily paid in money and the contract is normally for a quantity of money. So long as the rate of inflation is low enough, workers as consumers are unaware of the gentle rise in prices: they have the money illusion. There are, however, rates of inflation high enough to destroy the money illusion, as is very evident in recent times. There is thus an indistinct band where a threshold is passed: the money wage is related to prices. For the purposes of analysis, I take this band to be a point and hence it is, in the technical sense, a bifurcation or a catastrophe, since it changes the type of behaviour exhibited by the system; it is analogous to critical mass in an atomic explosion. When this happens, all variable costs, labour as well as materials, change with changes of prices; the control routine then shows an even stronger dysfunction - it is a mechanism chasing its own shadow.

In passing, I should like to note that the problem of the measure of the real wage presents almost exactly the same insoluble problem that Sraffa pointed out for the measurement of real capital. Just as a change in the rate of profit alters all prices and hence the quantity of capital measured in money, so an alteration in the real wage, changes all relative prices and hence destroys any invariant measure of the real wage. Having admitted this, I shall nonetheless use the concept of a real wage, approximately measurable.

Labour has its cost (of living) analogously to that of the producers; the price of labour becomes adjustable so as to cover costs. We have then a binary dynamical system which is naturally more complicated. In generalized coordinates there are the n separated equations

$$\langle \Delta p \rangle = - \left\langle \langle p \rangle \begin{bmatrix} 1-\lambda_i & & \\ & \ddots & \\ & & 1-\lambda n \end{bmatrix} - w \langle a_1 \rangle \right\rangle \qquad (36)$$

from which

$$\Delta p_i = - \left(p_i \, (1 - \lambda_i) - wa_{1i} \right), \quad i = 1, \ldots . n \qquad (37)$$

or

$$\frac{\Delta p_i}{p_i} = - \left((1 - \lambda_i) - \frac{w(t) a_{1i}(t)}{p_i} \right) \qquad (38)$$

as we have seen - if wa_{1i} is constant, i.e. Money wages growing with productivity, the behaviour is stable. If, however, because of high employment, high price of raw materials, or for any other reason, inflation reaches the threshold of money dis-illusion, workers bargain for and obtain, in varying degree, a constant $\frac{wa_{1i}}{p_i}$ which is higher than $1 - \lambda_i$, i.e. higher than marked-up net product, then we get a homeodynamic system in place of the homeostatic one. There will be a stable, constant rate of inflation, of a size dependent on the size of $\frac{wa_{1i}}{p_i}$. Thus a rate of inflation greater than some given size, brings about a change of structure such that the inflation becomes self-perpetuating, a point of no return - a bifurcation or a catastrophe.

The situation is one of conflict, with producers requiring one distribution of net product and the workers another, so that there is no static solution. This kind of situation arises most commonly because of low unemployment, scarcity of labour, leading to a tendency for the real wages to rise, or because of a scarcity of raw materials leading to a tendency of the real wage to fall. The latter appears in the form of rising prices, with a falling real wage, which at some point is resisted. Given a real wage, defined as a basket of goods, $\left\{\begin{matrix} a_{ci} \\ a_{11} \end{matrix}\right\}$ (39)

the wage earner acts in the same manner as the employer: he sets a price to cover his costs (generally not marked-up).

$$w_{t+1} = \langle p_t \rangle \left\{ a_c \right\} + wa_{11}$$

To simplify, ignore the difference between w_t and w_{t+1} and redefine a_c as $\frac{a_c}{1-a_{11}}$ then $w_{t+1} = p_t a_c$ so that the system becomes

$$\begin{aligned} p_{t+1} &= p_t\, a + w_t a_1 \\ w_{t+1} &= p_t\, a_c \quad , \quad \text{a scalar equation} \end{aligned} \tag{40}$$

In any situation where the real wage, defined by $\left\{a_c\right\}$, is higher than the equilibrium value,

$$\Big\langle \langle p_t \rangle \big\lfloor a \big\rceil + \langle p_{t-1} \rangle \left\{ a_c \right\} \langle a_1 \rangle \Big\rangle > \langle p_t \rangle \tag{41}$$

Therefore $p_{t+1} > p_t$ for all t so long as the conditions remain unaltered. Unfortunately the separation of variables is not possible since the money wage depends on all prices.

$$\langle \Delta p \rangle = - \langle p_t \rangle \left[I - a \right] + \langle p_{t-1} \rangle \left\{ a_c \right\} \langle a_1 \rangle \tag{42}$$

If $\langle a_1 \rangle$ is constant, this is a set of second order difference equations with at least one positive root. Hence it is very likely to exhibit accelerating rates of inflation.

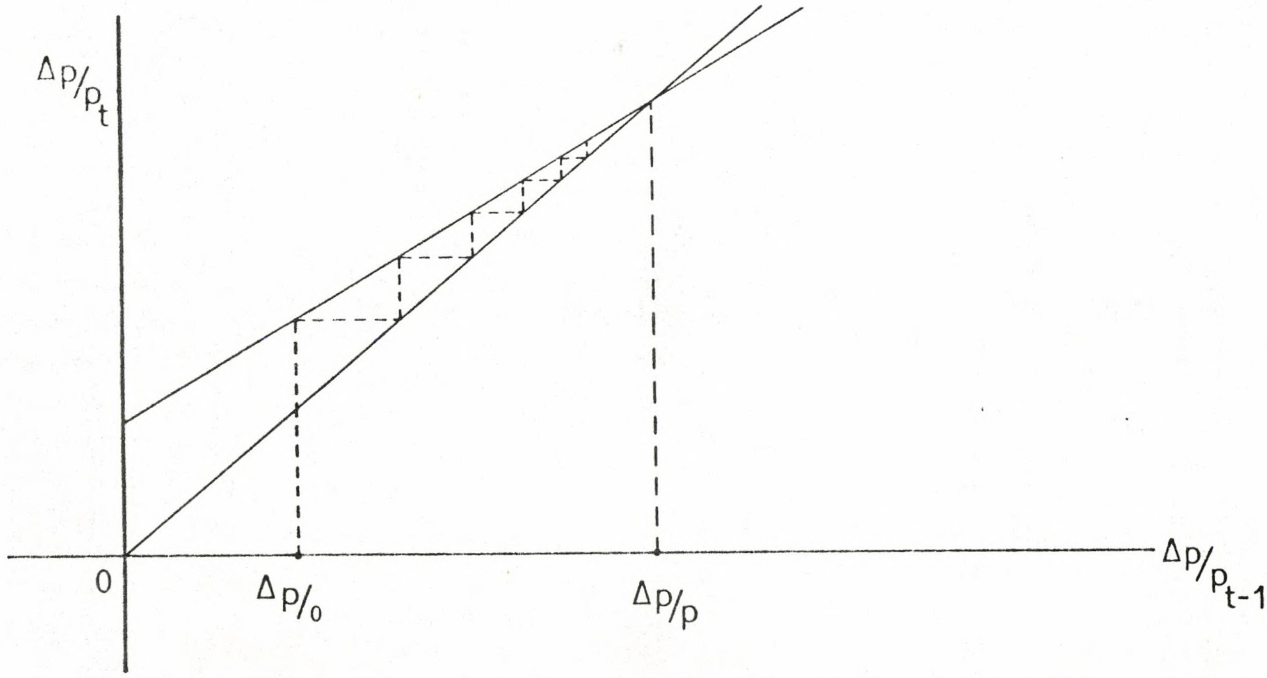
We can get some qualitative insight into its behaviour by considering an aggregative analogue.

$$\Delta p_t + (1 - a) p_t - a_c a_1 p_{t-1} = 0 \tag{43}$$

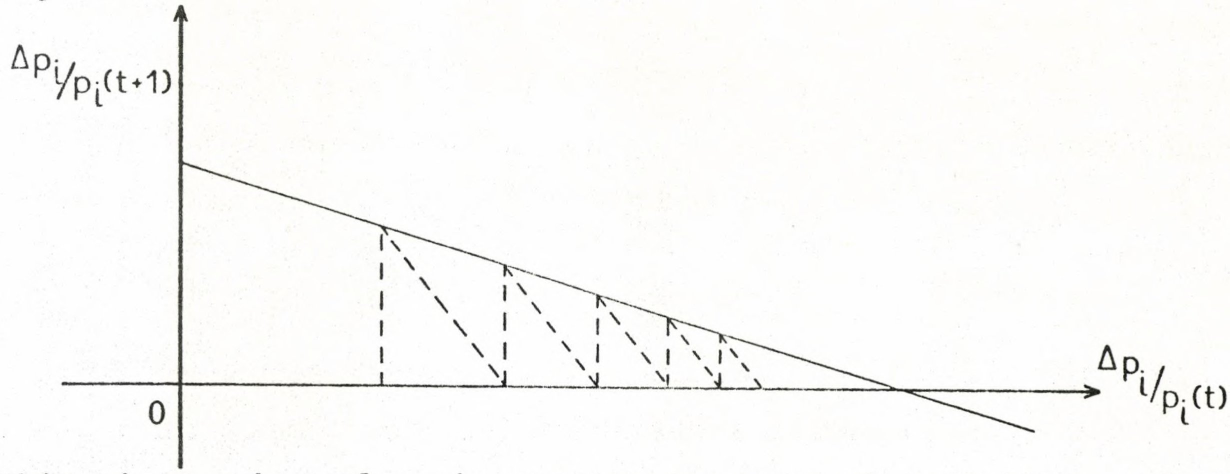
$$\frac{\Delta p}{p_t} = -(1-a) + a_c a_l \left(1 + \frac{\Delta p}{p_{t-1}}\right)$$

$$= a + \frac{c}{q} - 1 + \frac{c}{q}\,\frac{\Delta p}{p_{t-1}} \quad , \quad a_c a_l = \frac{c}{q}$$

given that $a + \frac{c}{q} > 1$,



From an initial rate of inflation, there will be an accaleration of the rate at a decelerating rate, towards a high, constant level. With a more elaborate analysis, one can find the following result:



This gives a homeodynamic equilibrium rate of inflation, different for every sector.

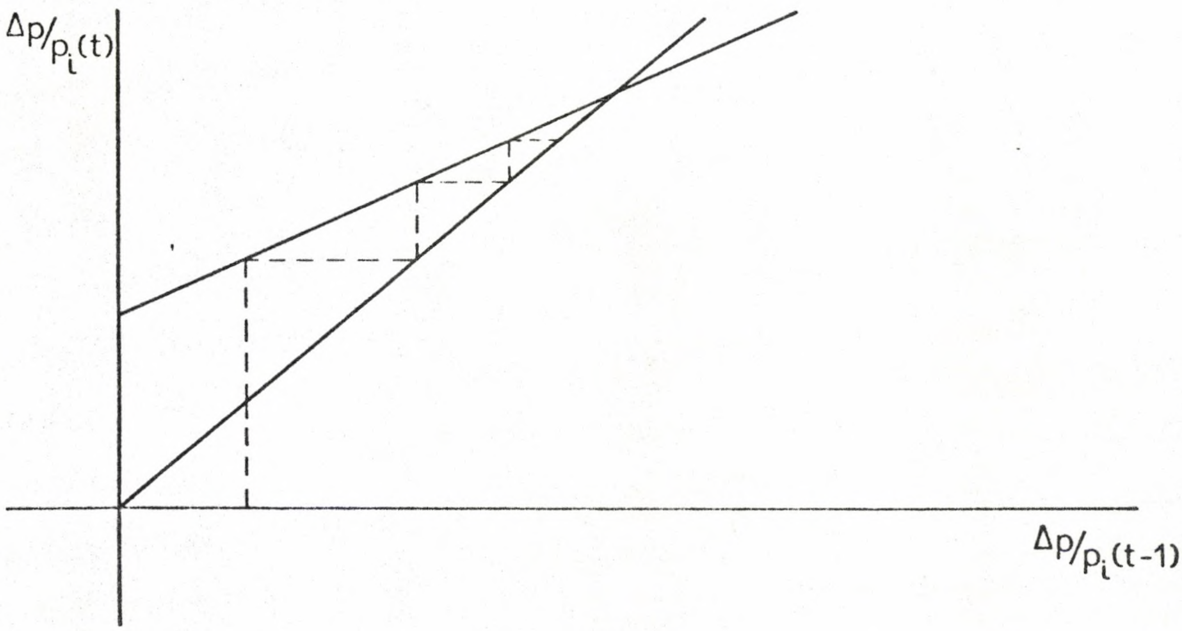
If inflation becomes rapid enough, both producers and workers want not only the level of prices to equal cost but also price to grow as fast as cost. The wage bargaining will be conducted in terms of the expected real value of wages, since the wages will be actually spent in the succeeding period. Thus suppose the expected, real unit cost of labour is kept constant (where expected $p_{t+1} = p_t + (p_t - p_{t-1})$, then in generalized co-ordinates,

$$\frac{w(t)\ a_{li}(t)}{p_i(t) + \Delta p_i(t-1)} = \bar{\theta} \quad , \qquad \text{a constant}$$

$$\frac{\Delta p_i}{p_i} = \left(\frac{w(t)\ a_{li}(t)}{p_i(t)} + \lambda_i - 1\right) \tag{44}$$

$$= \theta\left(\frac{p_i + \Delta p_i(t-1)}{p_i}\right) + \lambda_i - 1$$

$$= \theta\ \frac{\Delta p_i}{p_i}(t-1) + (\theta + \lambda_i - 1)$$



Inflation is a sequential resolution of the conflict over distribution. First wages are raised, then prices, then wages and so on. The last word is always with the producer, and that is why the working class has found it more or less impossible to achieve a durable alteration in the distribution of income. However, they achieve a transient redistribution so long as inflation is accelerating, due to the lag in the response to price. But even this may disappear. The producer is being damaged if, in inflation, he maintains a constant mark-up; he will not be able to replace either circulating or fixed capital if he maintains a fixed mark-up.

Thus $\pi_{real} = \pi_{actual} - \Delta p/_P$ so that if the producer is to maintain the real value of his mark-up, he must set $\pi_A = \pi_R + \frac{\Delta p}{p}$. There is no reason to expect entrepreneurs to be more given to the money illusion than workers. The previous models depended on the fact that, with lagging response of prices, workers gained at the expense of employers. With both sides reacting to costs, the problem becomes complicated, with a shifting of gains back and forth between them. Generalized coordinates are no help. To see the general nature of the behaviour, we may consider an aggregative model. Suppose that producers use both proportional and derivative control. $\frac{wa_1}{p} = \bar{\theta}$, a constant, and $a + \theta = \alpha$, also a constant, with values such that $\alpha(1+\pi_R) > 1$

$$\Delta p = -\left(p - \alpha p\ (1 + \pi_R + \frac{\Delta p}{p})\right) - \left(\Delta p - \Delta(\alpha p(1 + \pi_R + \frac{\Delta p}{p}\right) \qquad (45)$$

so that

$$\alpha\Delta^2 p + \left(\left((1 + \pi_R)\alpha - 1\right) - (1 - \alpha)\right) \Delta p + \left((1 + \pi_R)\alpha - 1\right) p = 0 \qquad (46)$$

This can exhibit either exponential growth or cycles, depending on the values of the parameters. In a proper disaggregated analysis, it would almost certainly contain both types of behaviour. Prices will thus accelerate in an irregular fashion without limit, as a rsult of the $\Delta^2 p$ term. This result is to be expected because employers no longer allow themselves to be damaged because of a constant mark-up. Each gains a transient advantage over the other in a succession of back and forth redistributions.

The behaviour of prices appears, in these examples, to be predominantly monotonic, whereas in fact, prices have shown a marked tendency to fluctuation. This is to be explained by the interaction of the output and value systems. Only when unemployment is tending towards zero, do real wages rise and only in the later stages do they tend to rise faster than productivity. This condition was, in the past, limited by the periodic collapse of the capitalist economies into depression and unemployment, thus easing the upward pressure of wages. Viewed in this light, one can see that capitalism needed periodic crises.

Keynesian practice offered a simple remedy for these repeated doses of unemployment, and thus appeared to be able to save capitalism from its own misbehaviour. However, the prolonged near full employment of the post-war period has revealed that this was a too facile solution. Capitalism needs repeated doses of unemployment to break the power of trade unions and restore potential profitability. Continued full employment seems to have brought into sharp focus the conflict over the distribution of income.

There are two ways to remedy the malfunction of a system. One is to add to it a sub-system which will counterbalance it in such a way as to moderate or remove the undesirable aspects: this is the familiar method of Keynesianism - manipulation of taxes, government spending, interest rate, banking policy and exchange rate management.

There is, however, a second procedure - to redesign the mechanism, so as to eliminate its faults. It is obvioulsy not easy to do for an economy, and, in some aspects, may be impossible. The construction of a centrally planned socialist economy is an extreme example. There is also another procedure which may be regarded as directed to correcting the central failing of decentralized decision control in laissez-faire capitalism. The basic source of dysfunction is that the individual producer and/or worker, does not and cannot take account of the wider effects of his decisions. Thus in altering his price and his output, the pro-

producer cannot calculate the effect of all others on price and demand for his goods. Similarly current price and current demand do not give any sound basis for estimating future demands and prices. A centralized planning commission can carry out simulations of different actions and thus discover consistent, feasible and possibly optimal solutions. Having accomplished some such analysis, the consequences can be spelled out to individual sectors of the economy in such a way as to persuade the producers and workers to alter decisions so as to fit the plan. This, in effect, is a redesigning of the decision structure of the economy, and, as a result, a variation of its functioning. Something of the sort seems to have been done in France and, in a different context, in Hungary. The labour party in England promises to have a renewed attempt at it, if returned to power. The idea, which is not new, of curing the dysfunctions of capitalism by the use of planning procedures bears a family resemblance to the currently fashionable notion of rational expectations.

A particularly instructive example, especially relevant at this moment, is furnished by the problem of balance of payments. All of the developed countries of the world are experiencing varying degrees of serious unemployment. Yet most of them can do little about it because, if they do, they will run into balance of payments deficits. There is no world government or institution which could act to improve the situation. If, however, there existed an international agency which could and would carry out the necessary analysis, it could specify the rates of expansion for each country or group of countries. If the expansions were to be in the 'world' proportions, the problem is solved. Given the necessary information as a basis for decisions, it is in the self-interest of each country to act according to the plan.

To specify the model we need the matrix, $[B]$, all marginal, systematic payments coefficients, internal and external, and in aggregative terms. The net payments vector, u, is specified, for constant prices, $\bar{p}$, by

$$u = \left[B - \lfloor \theta \rceil \right] \bar{p} \; q \tag{47}$$

where θ is the diagonal matrix of column sums. The level of expenditure is specified by

$$B\bar{p}q + z = \bar{p}q \tag{48}$$

where z(t) is the vector of all payments, public and private, which are not systematically proportional to current expenditure. Taking u - o as the error, proportional control implies

$\Delta z = \varepsilon \, (u - o)$, ε being the common proportions. Then

$$\begin{aligned} \Delta pq &= \left[I - B \right]^{-1} \Delta z \\ &= \varepsilon \left[I - B \right]^{-1} \left[B - \theta \right] \bar{p}q \\ &= \varepsilon \left[\left[I - B \right]^{-1} \lfloor 1 - \theta \rceil - I \right] \bar{p}q \end{aligned} \tag{49}$$

If ε is made small enough, the system is stable and $u \rightarrow o$. As the world is now and has been for a long time, when $u_i > o$, no corrective action is taken, and when $u_i < o$ persistently, $\Delta z_i < o$ so that we have a malfunctioning auto-control with a tendency to $\Delta z \leq o$.

Such an analysis shows how the important trading nations infect one another with their problems. If one of them develops a deficit in payments (which can be from purely financial transactions, which are ignored here), then it reduces z_i and corrects the deficit but transfers an equal deficit to the rest of the group, who then take similar actions with similar consequences and so on. Equally, expansions are transmitted in the same way.

For control purposes there are two targets, output level and desired net balance of payments. No single set of exogenous outlays can deliver both desired outputs and zero net balances. In a linear system like this, for two sets of targets, we require two sets of instruments. One helpful partial plan is to specify a desired set of increases in output, $\bar{p}q^*$, and solve for that set of δz which will achieve this target without altering the balance of payments: such a solution always exists. A more ambitious programme is to use a homeostatic feedback routine to bring all net trade balances to zero. This, of course, may not be acceptable to all participants since it may and almost certainly will mean a reduction of outputs in some countries. It can be shown that, by combining a balanced set of expansions with a zeroing exchange feedback, no region need actually contract; some simply grow more slowly than others.(3) This does still not bring all countries or regions to full employment: that requires further structural changes.

Footnotes

(1) In doing this I am developing a hint of von Neumann, who said his formulation of the economic system "appears to be similar to that of thermodynamic potentials". Thus I am using the method of gradient differential dynamics.

(2) We may apply a similarity transformation to all the variables, the effect of which is the highly beneficial result that a becomes diagonal, thus $[h]\ [a]\ [h]^{-1} = \lfloor\lambda\rceil$. I shall continue to use the same letters, p, q, a_e, a_c for the transformed elements, e.g. $p = p'h$, where p' are the observed variables and p the same set measured in generalized coordinates.

(3) cf. chapter 3 in my Essays in Linear Economic Structures, forthcoming, 1983.

II

I shall not discuss policy optimization problems as optimal control problems, but as solutions to mathematical programming problems. Mathematically of course it is clear that an optimal control problem can be shown to be equivalent to an infinite-dimensional mathematical programming problem. However, I have chosen the latter formulation because I am also able to quantify the 'pursuit-evasion' problem of inflation as discussed in part I without undue restrictions on the criterion function. Before that, it is necessary to clarify some issues of dynamics that have been discussed in part I.

§1.

The following two propositions have been formulated in part I:

(a). Disaggregation is essential in describing the global result of the process of inflation. Thus, "[disaggregation is essential]: it can explain the global result [of inflation] but the converse is not so. Thus the notion of linking wage increases to labour productivity, which seems to make sense in aggregates, is seen to be quite misleading by ignoring the effect of wage increases on other sections". (p.12, above).

(b). It is assumed, in part I, that wages are determined via a bargaining mechanism. Money wages and not real wages are the outcome of the bargaining process. However, it is then claimed that: "So long as the rate of inflation is low enough, workers as consumers are unaware of the gentle rise in prices: they have the money illusion. There are, however, rates of inflation high enough to destroy the money illusion... There is thus an indistinct region where a threshold is passed: the money wage is related to prices" (p.13, above). The proposition, then, is that this region, taken to be a point, bifurcates the system or leads to a catastrophe.

In this section, using simple aggregative models of the dynamics of functional income distribution and unemployment developed by Prof. Goodwin himself, I try to show that neither of the above propositions are necessarily true. The essential point of the first proposition is that even if wage rises are linked to growth in productivity, the differential growth rates of the latter across sectors coupled to price formation as a mark-up on unit labour (prime) costs leads to inflation; it is claimed, then, that aggregative models where wage-price dynamics are similar to the above cannot generate the dynamics of inflation. Now, it is clear that even if disaggregation is necessary for the validity of such a proposition, it is certainly not necessary to have more than two sectors. The Scandinavian - or EFO - model of inflation with two sectors, one of which is sheltered from world competition and the other not, is an example. Indeed, it is not even necessary to assume as inputs a perfect labour market - no such assumption is made in the EFO-model.

In the second proposition, the implication is that for rates of inflation greater than some critical level the system bifurcates or leads to a catastrophe (in the strict technical sense) because the functional form for growth in the wage rate changes; for a critical level of the rate of inflation, growth in (money-) wage rates depends not only on growth in productivity but also on growth in prices. It is clear that the disaggregative dynamics, in normal coordinates, in part I, are the sectoral variants of the aggregative model presented in Prof. Goodwin's celebrated Dobb Festschrift paper on 'A Growth Cycle'. I will therefore approach the discussion of the above two propositions for a suitably modified version of that model of fluctuation in distributive shares and the employment ratio. Basically, by allowing workers to save, and hence own a part of the capital stock in the economy, we combine the Goodwin model of growth cycles with a Kaldor-Pasinetti model of distribution. Then, instead of disaggregating into sectors but only separating decisions to invest and utilize capacity, it can be shown that the above two propositions are false. On the basis of behavioural and technical assumptions about money-wage dynamics, pricing, technical progress, investment and capacity utilization, Goodwin, in 'A Growth Cycle' was able to generate interesting non-linear dynamics in the wage share and unemployment. Remaining as close as possible to that model we make the following assumptions for the above relations:

(i) Money-wage dynamics:

$$\frac{\dot{m}}{m} = f\left(\frac{\dot{Y}}{Y} - \frac{\dot{L}}{L}\right) \qquad \ldots\ldots \ (1)$$

where $f' > 0$ and $f \in C'$.

and m: money-wages

L: employed labour force

Y: output level

$$u = \frac{mL}{pY} \qquad \ldots\ldots \ (2)$$

$$\text{and } V = \frac{L}{N} \qquad \ldots\ldots \ (3)$$

Therefore:

$$\frac{\dot{u}}{u} = \frac{\dot{m}}{m} - \frac{\dot{p}}{p} - \left(\frac{\dot{Y}}{Y} - \frac{\dot{L}}{L}\right) \qquad \ldots\ldots (4)$$

$$\frac{\dot{V}}{V} = \frac{\dot{L}}{L} - \frac{\dot{N}}{N} \qquad \ldots\ldots (5)$$

where p: price level

N: available labour force

u: share of labour

V: employment ratio

In (i) growth in money-wages depend only on growth in productivity.

(ii) Price dynamics:

$$\frac{\dot{p}}{p} = \lambda(\log m - \log p + \log \pi - \log Y + \log L) \quad \ldots. (6)$$

i.e., $$\frac{\dot{p}}{p} = \lambda(\log \pi - \log u) \qquad \ldots\ldots (7)$$

where $\pi > 0$ (mark-up factor)

$\lambda > 1$ (adjustment coefficient)

For simplicity we rewrite (7) as:

$$\frac{\dot{p}}{p} = g(u\, ;\, \lambda\, ,\, \pi\,) \qquad \ldots\ldots (8)$$

where $g_u' > 0$ and $g \in C'$.

Relation (6) is simply the continuous-time analogue of the discrete-time version in part I.

(iii) Production:

$$\frac{\dot{Y}}{Y} - \frac{\dot{L}}{L} = h\left(\frac{\dot{K}}{K} - \frac{\dot{L}}{L}\right) \qquad \ldots\ldots (9)$$

where $h \in C'$.

This is nothing other than the famous Kaldorian Technical Progress Function.

(iv) Investment:

In the original model, and in much of part I, it is assumed that all profits are saved and automatically invested. Contrarily, it was assumed that there were no savings from wages. If, now, savings out of wages are allowed then it is clear, following Pasinetti, that workers must be allowed to own part of the stock of capital in the economy. It is then reasonable to go beyond Pasinetti and also allow workers to influence investment and capacity utilization decisions. Thus, we assume:

$$K = Kc + Kw \qquad \ldots\ldots (10)$$

where Kc: capital stock owned by capitalists

Kw: capital stock owned by workers.

Postulating, for investment behaviour, a variant of Kaldor's investment function presented first in his growth model of 1957 (it would be possible to use, without undue complications, also either his 'Corfu' version or the 'Kaldor-Mirrlees' version) we get:

$$\dot{K}w = Kw \cdot \frac{\dot{Y}}{Y} + s_w (v,u) \cdot u \cdot Y \qquad \ldots\ldots (11)$$

$$\dot{K}c = Kc \cdot \frac{\dot{Y}}{Y} + s_c (u)(1-u) \cdot Y \qquad \ldots\ldots (12)$$

where s_w: (aggr.) savings propensity of workers

s_c: (aggr.) savings propensity of capitalists.

Relations (11) and (12) combine an accelerator principle - where 'own-capital-output ratios' are maintained - with ex-ante savings intentions. It is not assumed, contrary to standard Kaldor-Pasinetti models, that savings propensities are constant. Denote also:

$$\frac{Kw}{Y} = \lambda w \qquad \text{......} \quad (13)$$

$$\frac{Kc}{Y} = \lambda c \qquad \text{......} \quad (14)$$

From (11) and (12) we get:

$$\frac{\dot{K}}{Y} = \frac{\dot{Y}}{Y} \; \frac{1}{Y} \; (Kw + Kc) \; + s_w(v,u) \,.\, u + s_c(u)(1-u) \qquad \text{......} \quad (15)$$

Capacity utilization:

Using Desai's extension of the Goodwin model to relax the assumption of constant capital-output ratio(s) - and generalizing it, we get:

$$k_c = \frac{Kc}{Y} = q_c(v) \qquad \text{......} \quad (16)$$

where $q_c' < 0$ and $q_c \in C'$

$q_c'' > 0$

and $k_w = \frac{Kw}{Y} = q_w(v)$ ...... (17)

$q_w' < 0 \qquad q_w \in C'$

$q'' > 0,$

but $\left| q_c' \right| \gg \left| q_w' \right|$ ...... (18)

Therefore $k = kc + kw = q_c(v) + q_w(v) = q(v)$ ...... (19)

and $q' < 0, \quad q \in C'$

$q'' > 0.$

From (15), (16), (17), (18) and (19) we get:

$$\frac{\dot{K}}{K} = \frac{\dot{Y}}{Y} + s_w(v,u) \cdot u \cdot \frac{1}{q(v)} + s_c(u) \cdot (1-u) \cdot \frac{1}{q(v)} \qquad \ldots\ldots (20)$$

It is easy to show that by combining and rearranging (1) ~ (20), we can derive reduced-form equations for u and v:

$$\frac{\dot{u}}{u} = H(u,v) \qquad \ldots\ldots (21)$$

and $$\frac{\dot{v}}{v} = G(u,v) \qquad \ldots\ldots (22)$$

clearly we can assume the following:

$$\lambda_c \gg \lambda_w \qquad \ldots\ldots (23)$$

For suitable and not unrealistic assumptions on the constituent functions, savings propensities etc., it is easy to prove the existence of a stable limit cycle in the u-v plane. Assume, in (1), that proportional growth in money-wage rates is identically equal to growth in productivity. Then, from (4) it is clear that proportional growth rates in the share of wages and prices are inversely related. If now investment behaviour is such that, given (23), $\lambda = \frac{\lambda c}{\lambda w} < \lambda^*$ capitalists' decisions to create capacity (and utilize existing capcity) is blunted, then from the interaction between growth in productivity (technical progress function), investment and capcity utilization growth in the share of wages is arrested. From (4) this implies inflation <u>even if</u> growth in wages is identically equal to productivity improvements. On the other hand, for assumptions on the constituent functions and parameters not inconsistent with those made in part I such that (21) and (22) generate a stable limit cycle (or any other stable attractor) the model will be STRUCTURALLY STABLE. For such assumptions and resulting dynamics even if relation (1) is modified to include the influence of inflation, the qualitative dynamics of (21) and (22) in phase-plane for u and v will remain the same.

We omit technical details and proofs but these are available on request. The above results remain valid for a partitioning of the economy into any number of finite classes or income-groups. I conjecture that observed fluctuations can be better approximated by the switching behaviours that will be exhibited by the dynamical system as the mutual relationships between the (finite number of) λs change, for λ_i and i = 1, ..., n, paralleling the argument in Pasinetti (1974), esp. p. 140 ff.

(Professor Mario Nuti pointed out that the system for u and v to discuss via relation (4) inflation, income distribution and unemployment is overdetermined. Technically, this is partially true. Implicit in the above model is a third order system in wages, prices and unemployment. Since disequilibria are considered only for the labour market and not for the goods market I have suppressed third order dynamics. The conclusions, however, will remain the same (though different techniques of proof will be needed).)

§2.

'Optimal control' as pointed out in part I, is a large and growing discipline. No attempt will be made to survey the field; nor will any attempt be made to apply the theory of optimal control to any particular version of the models presented in part I or §1 of part II. Instead, something much more modest - and more concrete - will be attempted, within the framework not of optimal control but of mathematical programming.

The 'concrete' aim is to try to model the idea that 'an adaptive response alters the desired goal' (p. 9, above) and thus may result in an economic system hunting its equilibrium without ever finding it. Therefore, 'if pursuer and pursued each alter course in relation to the other, there need be no stable solution' (p. 9, above). At first sight it might appear as if the 'Theory of Pursuit Games' as developed by Otomar Hajek would be the ideal framework. This, however, is not correct because an essential idea in the above formulation is that the 'desired goal' is (continuously) altered. On the other hand, it is not clear, in the above formulation, whether it is assumed that an equilibrium exists around which stable or unstable oscillations take place.

I will summarize, in words, the formulation and results of this section before proceeding to formalizations. Whether in a bargaining game or in production processes, if some criterion is being optimized subject to constraints, the usual formulation of the problem is in terms of minimizing deviations from desired targets - at least in policy optimization studies. If desired values are being altered dynamically, one way to encapsulate this change explicitly would be to allow variations in the weights attached to the variables in the criterion function.

In part I, the earlier sections treat the dynamics of producers' decisions; the later sections, in normal coordinates, are about bargaining between 'eigen capitalists' and homogeneous labour. In the former case, the producer takes the system as given and keeps adapting sequentially his controls as responses to the realized values of the system dynamics; in the latter case it is almost a direct bargaining game between 'capital' and labour. In either case we have a 'two-person' game with incomplete (not just imperfect) information. For the sake of concreteness and simplicity, I will consider, in the formalization, only the 'game' between, say, capitalists and workers. However, by stacking, for n-producers n-such (simultaneous) bargaining games, the producers' problem can also be treated. In such a two-person process the optimum instruments chosen by any one agent would lead to (revised) optimum controls by the other agent. The realized first-period values of the n-variable, T-period optimizing problem, taking into account the values of the opponents' realized controls, for any agent, when compared with optimum values (computed on the basis of expected values for variables not directly under control) would show deviations from desired values, for relevant variables. If the next period controls are to be so chosen that these un-desirable deviations are to be eliminated, then, of course, the weights have to be changed. However, the weights should be changed as functions of the desired directions in which the deviations should be corrected so that the new optimal controls would achieve the (modified) desired values subject to the constraints of the model or system. These optimal values, for a new n-variable, T-period dynamic problem, are then implemented for the second period - and so on. The systematic alteration of the weights could be such that cyclic solutions appear; it is also possible that 'dys-functions' in the sense that each new weighting leads to optimum controls such that the deviation between desired values and re-alized values increase at each iteration. This, of course, cannot be unbounded so long as the system itself is stable. We proceed, now, to the formalization. We need some definitions:

Definition 1:

$$G \triangleq < I, \{S_i\}_{i\varepsilon I}, \{J_i\}_{i\varepsilon I} >$$

ard G is called a NON-COOPERATIVE GAME, where

I: finite set of all producers (and consumers) as in part I,

S_i: set of strategies of 'player' i,

J_i: the payoff function or interim function for 'player' i,

defined on:

$$S = \prod_{i\varepsilon I} S_i$$

Assume, now, that every non-cooperative game G will be preceded by a BARGAINING GAME B(G) in which the set I is partitioned into two equivalence classes according to some well-defined criterion (e.g., by defining a Decisive Set on G and calling this set 'employers' or 'capitalists'; the complement of the Decisive Set in I could then be called 'Workers' or 'Labour'). In the bargaining game B(G), 'the players will try to agree on their payoffs and on strategies for obtaining these payoffs. Only after the bargaining game has been completed will the players play the main game G itself' [cf., Harsanyi (1966)]. The game G is played at a much more disaggregated level (the stacking of n two-person games solved simultaneously). The aggregate variables agreed and decided upon in the bargaining game B(G) are taken as the bounds within which G has to operate. The variables of G would predominantly be those of microeconomics and the central theme of this game would be decisions related to production and consumption. The problem of the violation, during G, of the aggregate values decided in the bargaining game B(G) may easily be resolved by resuming B(G) and seqentially updating the bargaining strategies wheneven such a violation occurs. In the sequel only B(G) is discussed in detail. The two players of B(G) will be denoted by C and L respectively for the 'capitalists' and 'labourers'. The analysis of B(G) will be based on the assumption that C and L would like to minimize deviations of some relevant variables from specified 'desired' values for such varibales. To do this, they employ those strategies or actions under their complete control; all other variables are replaced by expected values taken over given (subjective) probability distributions.

The optimization problem, for each agent, can be expressed as:

$$\min \{J_i(Y, U). \mid Y, U \in \Omega_i, f(Y, U) = 0, g(Y, U) \geq 0\} \qquad (1)$$

$$i = C \text{ or } i = L,$$

where

$$Y^T = (y^T(1), y^T(2), \ldots, y^T(k), \ldots, y^T(N)) \qquad (2)$$

is the mN vector of endogenous variables throughout the period of optimization [1,N], with y(k) denoting the m-vector of endogenous variables at time k, $1 \leq k \leq N$, and

$$U^T = (u^T(1), u^T(2), \ldots, u^T(k), \ldots, u^T(N)) \qquad (3)$$

is the nN-vector of control and independent (policy and exogenous) variables written in the same form as (2) with $u(k)$ as an n-vector. The first e elements of $u(k)$, corresponding to period k, are taken to be completely under the control of player E. The next ℓ elements are controlled by L and the final $n - (e + \ell)$ variables are completely outside the control of the players and their expectations must be substituted before the optimisation problem (1) is solved for either player. Also, in (1) $J_i(Y, U)$ is the aggregate convex cost function for i; $f(Y, U) = 0$ is the vector valued function denoting the model equations as well as any other constraints imposed on Y and U, throughout the period $1 \leq k \leq N$; $g(Y, U) \geq 0$ is the vector valued function denoting the inquality constraints on Y and U; Ω_i is the set of admissable values of Y, U from the point of view of i.

Assume that J_i can be approximated by the second order Taylor series expansion about the unconstrained optimum of the true non-linear criterion function $J_i(Y, U)$. Ignoring the constant term of this expansion, consider the quadratic criterion function for i:

$$J_i^q(Y, U) = \frac{1}{2} \left(\begin{bmatrix} Y \\ \cdot\cdot \\ U \end{bmatrix} - \begin{bmatrix} Y \\ \cdot\cdot \\ U \end{bmatrix}_i^d \right)^T Q_i \left(\begin{bmatrix} Y \\ \cdot\cdot \\ U \end{bmatrix} - \begin{bmatrix} Y \\ \cdot\cdot \\ U \end{bmatrix}_i^d \right) \qquad (4)$$

where the superscript d denotes the desired values for i, the symmetric matrix Q_i is an $(n + m)N \times (n + m)N$ dimensional positive semi-definite matrix specified by i. The diagonal elements of these matrices penalise the departure of a variable from its desired trajectory. The off-diagonal elements indicate the measure of importance attached to the deviation in one variable versus the deviation in another. These weights reflect the priorities of i. The vector $\begin{bmatrix} Y \\ .. \\ U \end{bmatrix}$ will be referred to either as a trajectory of variables due to the time factor in (2) or (3) or, simply as a vector of variables.

Definition 2:

The set of Y, U satisfying the functional constraints of the optimization problem (1) is defined on

$$F \triangleq \{Y, U \in E^{(n+m)N} \mid f(Y,U) = 0,\ g(Y,U) \geq 0\}. \tag{5}$$

It should be noted that the set F is the same for both C and L. $E^{(n+m)N}$ denotes the $(n + m)N$ dimensional Euclidean space. It will be assumed that F is convex. In addition:

$$F \cap \Omega_C \cap \Omega_L \neq \phi \tag{6}$$

implying
$$F \cap \Omega_C \neq \phi \tag{7}$$

and
$$F \cap \Omega_L \neq \phi \tag{8}$$

It is easy to see that:

$$S_i = \Omega_i \cap F \tag{9}$$

and
$$S = \prod_{i \in I} S_i = \Omega_C \cap \Omega_L \cap F \tag{10}$$

Now, using (4) consider the ;inimization problem:

$$\min {}' \{ J_i^q (Y, U) \mid Y, U \in F \} \tag{11}$$

If the solution of (11) lies outside Ω_i, the results described in Rustem, Velupillai, Westcott (1978) may be utilized to modify J_i^q (Y, U) and generate a solution satisfying Ω_i. The desired values specified by i satisfy

$$\begin{bmatrix} Y \\ \cdots \\ U \end{bmatrix}_i^d \varepsilon \, \Omega \, i \tag{12}$$

by definition. They are also the minima of the Taylor series expansion (4). As this expansion is about the unconstrained optimum of the true non-linear cost function in (1), $\begin{bmatrix} Y \\ \cdots \\ U \end{bmatrix}_i^d$ is exactly the vector that minimises J(Y, U). The vector $\begin{bmatrix} Y \\ \cdots \\ U \end{bmatrix}_i^d$ may, therefore, be thought of as desired but possibly unfeasible considering the restriction imposed by the set F (5). The constrained optimization problem (11) can thus be interpreted as getting as near as possible to these desired values. Clearly, if these desired values satisfy F, then they are also the solution of (11) for i.

The formulation (1) and (11) are the representations of dynamic problems in terms of static optimization. The solution to the static optimization problem will be considered in the Euclidean spaces E^{mN} and E^{nN} for Y and U respectively.

Assuming, now, the results in Rustem, Velupillai, Westcott (op. cit.) and the relevant discussions therein, the adaptive response and the alteration of the desired goal results from the following mechanism summarized as an algorithm:

Step 0: Given the current weighting matrix Q_i^c and the desired values $\begin{bmatrix} Y \\ \dots \\ U \end{bmatrix}_i^d$ for i and $Q_{i'}^c$, $\begin{bmatrix} Y \\ \dots \\ U \end{bmatrix}_{i'}^d$ for i', assume each agent to be ignorant of these values for the other agent. Using their respective desired values and current weighting matrices both agents compute their current optimal solutions:

$$\begin{bmatrix} Y \\ \cdot\cdot \\ U \end{bmatrix}_i^c \tag{13}$$

to the quadratic minimization problem (11). Assume that the results of the decisions of i' are realized first. Then:

Step 1: For agent i:

(a). If (13) for i' is an element of Ω_i, then expected values and realized values have a non-empty intersection. Decisions, desired values and weights need not be revised. This is the uninteresting case. If this is not the case, i.e., if:

$$\begin{bmatrix} Y \\ \dots \\ U \end{bmatrix}_{i'}^c \not\subset \Omega_i \tag{14}$$

then expectations are violated and realized values are such that desired values deviate unduly from actual values. Given that the model constraints cannot be changed, at least in the short-run, then the only way to prevent a breakdown of the system would be to modify weights in such a way that the membership relation in (14) is not violated. This modification can be formalized as follow: agent i chooses a 'preferred' trajectory:

$$\begin{bmatrix} Y \\ \cdots \\ U \end{bmatrix}_i^P \quad \text{such that :}$$

$$\begin{bmatrix} Y \\ \cdots \\ U \end{bmatrix}_i^P \varepsilon \Gamma_i \triangleq \left\{ \begin{bmatrix} Y \\ \cdots \\ U \end{bmatrix} \varepsilon \Omega_i \;\middle|\; \left\| \begin{bmatrix} Y \\ \cdots \\ U \end{bmatrix}_{i'}^c - \begin{bmatrix} Y \\ \cdots \\ U \end{bmatrix} \right\|_{Q_i^c} \leq \left\| \begin{bmatrix} Y \\ \cdots \\ U \end{bmatrix}_{i'}^c - \begin{bmatrix} Y \\ \cdots \\ U \end{bmatrix}_i^c \right\| \right\} \tag{15}$$

If $\Gamma_i = \phi$ and hence no such preferred trajectory can be chosen, then a preferred trajectory is sought that satisfies

$$\begin{bmatrix} Y \\ \cdots \\ U \end{bmatrix}_i^P \varepsilon \bar{\Gamma}_i \triangleq \left\{ \begin{bmatrix} Y \\ \cdots \\ U \end{bmatrix} \varepsilon \Omega_i \;\middle|\; \left\| \begin{bmatrix} Y \\ \cdots \\ U \end{bmatrix}_{i'}^c - \begin{bmatrix} Y \\ \cdots \\ U \end{bmatrix} \right\|_{Q_i^c} \leq \left\| \begin{bmatrix} Y \\ \cdots \\ U \end{bmatrix}_{i'}^c - \begin{bmatrix} Y \\ \cdots \\ U \end{bmatrix}_i^c \right\|_{Q_i^c} \right\}. \tag{16}$$

where the norm $\|.\|_{Q_i^c}$ denotes $\|x\|_{Q_i^c} \triangleq x^T Q_i^c x$.

(If $\bar{\Gamma}_i$ also is empty then the decisions of the two agents imply that (6) is violated. The system breaks down in the form of strikes, lockouts and other catastrophes).

(b). Next compute the displacement vector δ_i given by:

$$\delta_i = \begin{bmatrix} Y \\ \cdots \\ U \end{bmatrix}_i^P - \begin{bmatrix} Y \\ \cdots \\ U \end{bmatrix}_i^c \qquad (17)$$

and update the current weighting matrix Q_i^c using:

$$Q_i^n = Q_i^c + \mu \frac{Q_i^c \delta_i \delta_i^T Q_i^c}{\delta_i^T Q_i^c \delta_i} \qquad (18)$$

with $\mu = 1$ to obtain the new weighting matrix. This is the crucial step in the alteration of desired values. Detailed heuristic discussions are available in Rustem, Velupillai, Westcott (op. cit.)

(c). Using Q_i^n and the desired values new optimal values are computed. If they are not contained in Ω_i, then from (19) and (20) below α can be reduced by reducing μ and returning to (b).

$$\begin{bmatrix} Y \\ \cdots \\ U \end{bmatrix}_i^c = P(Q_i^c) \begin{bmatrix} Y \\ \cdots \\ U \end{bmatrix}_i^d . \qquad (19)$$

where $P(Q_i)$ denotes the projection w.r.t. Q_i of any vector in the appropriate Euclidean space on F in (5). Then, the new optimal trajectory, using (18), may be expressed as:

$$\begin{aligned} \begin{bmatrix} Y \\ \cdots \\ U \end{bmatrix}_i^n &= P(Q_i^n) \begin{bmatrix} Y \\ \cdots \\ U \end{bmatrix}_i^d \\ &= P(Q_i^c) \begin{bmatrix} Y \\ \cdots \\ U \end{bmatrix}_i^d + \alpha P(Q_i^c)\delta_i \qquad (20) \\ &= \begin{bmatrix} Y \\ \cdots \\ U \end{bmatrix}_i^c + \alpha\, P(Q_i^c)\delta_i \end{aligned}$$

where α is an increasing function of μ and $\alpha \gtrless 0$ if $\delta_i^T \nabla J_i^q(Y_i^c, U_i^c) \lessgtr 0$.

On the other hand, if $\begin{bmatrix} Y \\ .. \\ U \end{bmatrix}_i^n \varepsilon\ \Omega_i$ $0 < \alpha < 1$, then

expectations have not been sufficiently optimistic and desired values, via the weights, are altered in, for the agent, a positive direction. In any case, such

$\begin{bmatrix} Y \\ .. \\ U \end{bmatrix}_I^n$ will be the starting point for the decision

sequence for the other agent.

Step 2: For agent i':

If

$$\left\| \begin{bmatrix} Y \\ .... \\ U \end{bmatrix}_i^n - \begin{bmatrix} Y \\ .... \\ U \end{bmatrix}_{i'}^c \right\|_{Q_{i'}^c} > \left\| \begin{bmatrix} Y \\ .... \\ U \end{bmatrix}_i^c - \begin{bmatrix} Y \\ .... \\ U \end{bmatrix}_{i'}^c \right\|_{Q_i^c}$$

then the realized results of the second period are not better than the previous period for i'. In this case one would expect i' to continue the same constellation of decision. This step, made necessary when $\Gamma_i = \phi$ in step 1 (a), results, if continued with step 1 (a), in a sort of neutrally stable cyclic process.

Step 3: For agent i':

Same as step 1 (a), (b), (c) with i replacing i' and vice versa. After this exchange of indices the current optimal trajectory of i, $\begin{bmatrix} Y \\ .... \\ U \end{bmatrix}_i^c$ should be replaced by the new optimal trajectory of i, $\begin{bmatrix} Y \\ .... \\ U \end{bmatrix}_i^n$, which is computed in step 1 itself.

Step 4: For agent i:

Identical discussion as in step 2 with indices changed.

Step 5: Reset $Q^c_{i'} = Q^n_{i'}$, $Q^c_i = Q^n_i$, $\begin{bmatrix} Y \\ \cdots \\ U \end{bmatrix}^n_{i'} = \begin{bmatrix} Y \\ \cdots \\ U \end{bmatrix}^n_{i'}$,

$$\begin{bmatrix} Y \\ \cdots \\ U \end{bmatrix}^c_i = \begin{bmatrix} Y \\ \cdots \\ U \end{bmatrix}^n_i$$ and go to step 1.

Repeated applications of the sequences of steps given by 1 ~ 5 completely captures the ideas expressed above in part I (cf. p. 9). Technical theorems about convergence of the iterative process is given in Rustem-Velupillai (1982). If, for each producer, represented by i, the results of the economic system as a whole are represented by the process for i and n such 'producers' iterative schemes are stacked and solved simultaneously the system may in fact 'hunt' its equilibrium without ever finding it. It is not even clear that there is an equilibrium to 'hunt'.

On the other hand, as a bargaining game B(G), the interpretations are straightforward. The iterative process can be applied dually, instead of to the weights, to the shadow prices. The same process can also be used in formalizing some aspects of disequilibrium theory - particularly the Benassy variant. Special cases of this process are equivalent to the standard policy optimization exercises.

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