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LOCAL BIFURCATIONS AND STATIONARY SUNSPOTS

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A B S T R A C T

This paper analyses the relations between deterministic intertemporal equilibria with perfect foresight, and stationary sunspot equilibria, near a stationary state. The study takes place within the framework of an overlapping generations model, where the deterministic dynamics is described by a one-dimensional difference equation, and employs elementary geometrical arguments. One verifies that a stationary sunspot equilibrium exists in <u>every</u> neighbourhood of the Golden Rule if, and in general only if, it is stable in the deterministic dynamics. One shows also, by looking at what happens when a local bifurcation occurs, that a stationary sunspot equilibrium can exist in <u>some</u> neighbourhood of the Golden Rule, even when it is unstable in the deterministic dynamics.

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The purpose of this work is to make progress toward a better understanding of the relationships that may exist between the local behaviour of deterministic intertemporal equilibria and stationary Markov sunspot equilibria near a stationary state, in economies the characteristics of which are constant over time.

We shall use a simple version of the overlapping generations model, in which the state variable is one dimensional, that was already employed in this context by Azariadis (1981), Azariadis and Guesnerie (1986). If we stay close enough to the stationary state, things are almost linear. Stationary Markov sunspot equilibria exist then in every, however small, neighborhood of the stationary state if, and in general only if, it is deterministically locally indeterminate, and the set of such sunspot equilibria is infinite dimensional. Similar results were obtained in the study of various specific examples (Guesnerie (1986), Woodford (1984,1986), Laitner (1986), Grandmont (1985b,1986)).

Matters are further complicated if we look at what happens still near the stationary state, but a little farther away from it, since the deterministic dynamics may then involve small but significant nonlinearities. We shall show that in the simple context under consideration, one can get more information on that issue through the theory of bifurcations. Suppose that we look at a one parameter family of economies having a stationary state that changes stability at some point. Bifurcation theory describes exactly what deterministic intertemporal equilibria look like in the vicinity of the stationary state, immediately before and after the bifurcation takes place. One can then characterize the set of stationary Markov sunspot equilibria, and its relationships with the set of deterministic intertemporal equilibria, near the stationary state. It will be shown in particular that stationary Markov sunspot equilibria may exist in <u>some</u> neighborhood of the stationary state, even though it is deterministically locally determinate.

As was said earlier, our study is carried out in a one dimensional framework that permits a simple geometrical analysis. It remains to be seen whether the methods of the present paper can be extended to a more realistic multidimensional framework.

1. HOUSEHOLDS BEHAVIOUR

We consider a discrete time model with overlapping generations (OLG) of households. There are no bequests and no population growth. Households live two periods. We assume that they are all alike, or equivalently, that there is a single representative household in each generation. There is a single good. When young, households supply labour and save their wage earnings by holding money balances ; they consume when they are old.

The tastes of a generation are represented by the separable utility function $V_1(\ell^* - \ell) + V_2(c)$, where $0 \leqq \ell^* - \ell \leqq \ell^*$ is leisure (ℓ is labor supply) and $c \geqq 0$ is consumption of the good. We assume throughout

(1.a) $\ell^* > 0$.

(1.b) <u>The utility functions</u> V_1 <u>and</u> V_2 <u>are continuous on</u> $[0,\ell^*]$ <u>and</u> $[0,+\infty]$.

They are k-times continuously differentiable with $k \geqq 4$, on $(0,\ell^*]$ and $(0,+\infty)$ respectively, with $V'_\tau(a) > 0$, $V''_\tau(a) < 0$, $\lim_{a\to 0} V'_1(a) = +\infty$.

The deterministic case

We look first at the behavior of a representative household under certain conditions. Consider a young household at any date, who observes the current money wage $w \geqq 0$ and expects the money price $p^e > 0$ of the good to prevail at the next date. He has to choose his current labor supply $0 \leqq \ell \leqq \ell^*$, his current money demand $m \geqq 0$ and his future consumption $c \geqq 0$, so as to maximize his utility function subject to $w\ell = m$ and $p^e c = m$. The solution is unique, and c, ℓ, m are positive if and only if $\theta = w/p^e$ exceeds

$\bar{\theta} = V'_1(\ell^*)/\lim_{c\to 0} V'_2(c)$.

It is easily seen from the first order conditions of the problem that if $\theta > \bar{\theta}$, the optimum pair (ℓ,c) is the unique solution satisfying $\ell > 0$, $c > 0$, of the system

(1.1) $v_1(\ell) = v_2(c)$ and $\theta\ell = c$,

in which v_1 and v_2 are defined, for $0 \leqq \ell < \ell^*$ and $c > 0$, by

(1.2) $v_1(\ell) = \ell V'_1(\ell^* - \ell)$ and $v_2(c) = cV'_2(c)$.

The function v_1 is increasing and maps $[0,\ell^*)$ onto $[0,+\infty)$. Thus we may define the household's offer curve by the equation $\ell = \chi(c)$, where

(1.3) $\chi(c) = v_1^{-1}[v_2(c)]$ for $c > 0$ and $\chi(0) = 0$.

Then, for any $\theta > \bar{\theta}$, as shown in Fig. 1, the optimum values of c and ℓ are obtained as the unique intersection verifying $c > 0$, $\ell > 0$, of the line

$\theta \ell = c$ with the offer curve. When $0 \leqq \theta \leqq \bar{\theta}$, this intersection reduces to the origin.

Fig. 1

The case $\lim_{c \to 0} cV_2'(c) = 0$ is described in Fig. 1a, while Fig. 1.b represents the case in which this relation does not hold [(1)]. In all cases, the functions v_1 , v_2 and χ are (k-1) times continuously differentiable for $0 \leqq \ell < \ell^*$ or $c > 0$. Moreover χ has a positive fixed point, which is then unique, if and only if $\bar{\theta} < 1$; it corresponds to $\theta = 1$.

It is useful to note at this stage that the elasticity of the function χ , i.e. $\varepsilon_\chi = c\chi'(c)/\chi(c)$, for $c > 0$, is given by

$$(1.4) \quad \varepsilon_\chi = [1 - R_2(c)]/[1 + \frac{\ell}{\ell^* - \ell} R_1(\ell^* - \ell)]$$

with $\ell = \chi(c)$, where R_τ is the coefficient of relative risk aversion of V_τ , i.e. $R_\tau(a) = - aV_\tau''(a)/V_\tau'(a) > 0$, for $a > 0$ and each $\tau = 1,2$.

Uncertainty

In preparation to the analysis of sunspot equilibria, we look at the household's behaviour under uncertainty. Consider a young agent at any date, who observes the current money wage $w \geqq 0$, and expects the prices $p^e = (p_1^e,\ldots,p_r^e)$ to prevail at the next date, with probability $q_j \geqq 0$, $j = 1,\ldots,r$, where $p_j^e > 0$ for all j . He has to choose his current labor supply ℓ , his current money demand $m \geqq 0$, and his future contingent consumption c_j, so as to maximize his expected utility $V_1(\ell^* - \ell) + \Sigma_j q_j V_2(c_j)$ subject to

$$(1.5) \quad w\ell = m \text{ and } p_j^e c_j = m \text{ , for } j = 1,\ldots,r .$$

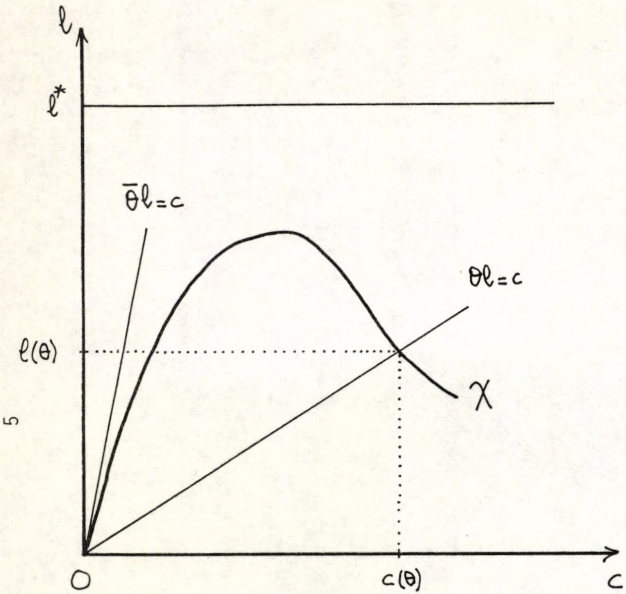


Fig. 1.a

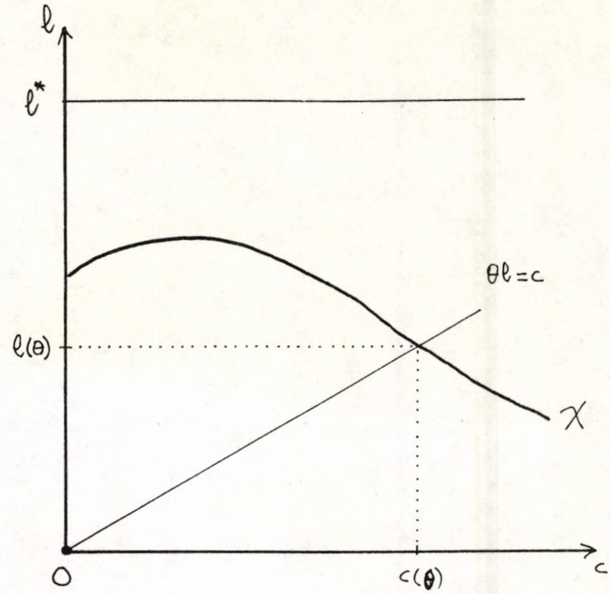


Fig. 1.b

Again, the solution is unique, and from the first order conditions, each of the quantities ℓ, m, c_j is positive if and only if $\bar{\theta} < \Sigma_j q_j \theta_j$, where $\theta_j = w/p^e_j$. When this relation holds, labor supply and future contingent consumption is the unique solution verifying $\ell > 0$, $c_j > 0$, of the system

(1.6) $v_1(\ell) = \Sigma_j q_j v_2(c_j)$ and $\theta_j \ell = c_j$, for all j.

2. *INTERTEMPORAL EQUILIBRIA AND SUNSPOTS*

We consider a simple version of the overlapping generations model, in which 1) the money stock is constant and equal to $M > 0$, and 2) the good is perishable and produced from labor in each period, with no production lag, one unit of labor yielding one unit of output.

There are competitive markets for labor, output and money at each date. At a monetary equilibrium, consumption of the old trader at date t, i.e. c_t, is equal to his real money balance $\mu_t = M/p_t > 0$, in which $p_t > 0$ is the equilibrium money price of the good. Clearly c_t is also equal to the current equilibrium output y_t and labor supply ℓ_t. Profit maximization implies then that the equilibrium real wage is unity, so that we can identify equilibrium money wages and prices at all dates, $w_t = p_t$ for all t.

<u>Deterministic intertemporal equilibria</u>

We first look at the case in which the households are certain about future prices and have perfect foresight. A deterministic <u>intertemporal (monetary) equilibrium</u> is an infinite sequence of money prices $p_t > 0$, $t \geq 1$, such that markets clear at all dates. As noted earlier, one has $\mu_t = c_t = \ell_t$ along such an equilibrium. Thus, from the first order condition (1.1), one may define equivalently such an intertemporal equilibrium as a sequence of positive real balances $\mu_t = M/p_t$ that satisfy for all t $v_1(\mu_t) = v_2(\mu_{t+1})$, or

(2.1) $\mu_t = \chi(\mu_{t+1})$.

Fig. 2.a describes the backward perfect foresight (b.p.f.) dynamics on real balances implied by (2.1) by means of the offer curve, as well as the dynamics of the corresponding gross real interest rates $\theta_t = p_t/p_{t+1}$. The same figure can be used to generate intertemporal equilibria, in which time goes forward. The trick is to reverse the direction of the arrows : start from the offer curve, go vertically to the 45° line, and horizontally back to the offer curve, see Fig. 2.b. There exists a monetary stationary state, which is unique, if and only if $\bar{\theta} < 1$. It corresponds to $\theta_t = 1$ or to $\mu_t = \bar{\mu}$ for all t , where $\bar{\mu}$ is the unique positive fixed point of χ .

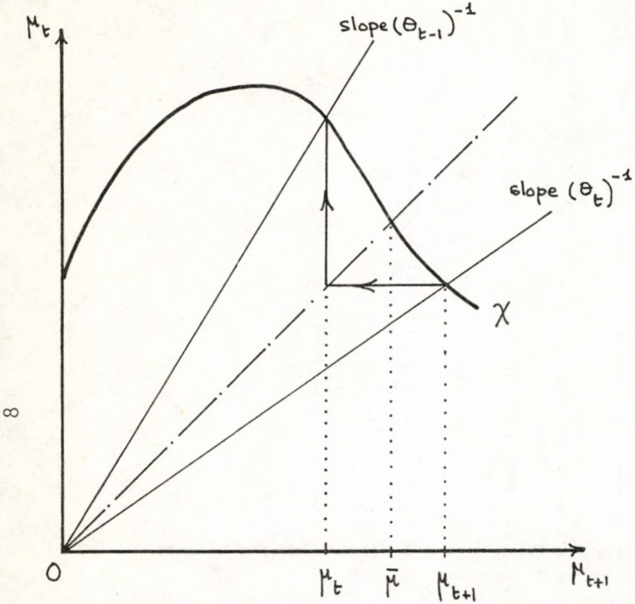
Fig. 2

Stationary sunspot equilibria

We consider now the same economy and look at the case where traders believe that prices and quantities are affected by random factors (sunspots), although they do not influence the "fundamental" characteristics of the economy, and where this belief turns out to be self-fulfilling, in equilibrium.

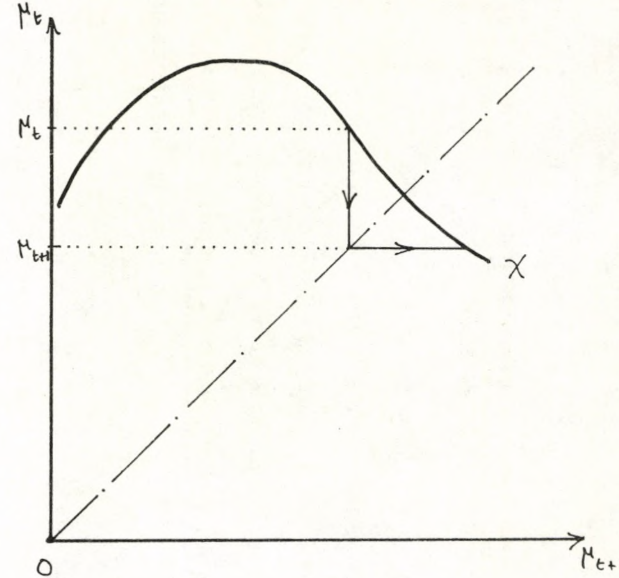
Assume that the traders observe a Markov process of signals s_t (sunspots) that belong to $S = \{s_1,\ldots,s_r\}$, with known transition probabilities $q_{ij} > 0$. Assume further that households believe these signals to be perfectly correlated with equilibrium prices through the relation

(2.2) $p_i = f(s_i)$, all i , with $0 < p_r < \ldots < p_{i+1} < p_i < \ldots < p_1$.



μ_t
slope $(\theta_{t-1})^{-1}$
slope $(\theta_t)^{-1}$
χ
0
μ_t
$\bar{\mu}$
μ_{t+1}
μ_{t+1}

Fig. 2.a



μ_t
μ_t
μ_{t+1}
χ
0
μ_{t+1}

Fig. 2.b

Note that here, as in the previous section, the equilibrium real wage is unity at all dates, so that we do not need to distinguish between money wages and prices.

The belief (2.2) generates a stationary Markov sunspot equilibrium (s.m.s.e.) , with r states, corresponding to the given sunspot process, if it is self-fulfilling, i.e. if $p = p_i$ clears indeed the market when the sunspot s_i is observed, for each $i = 1,...r$ and at each date.

Note that here also, every date, and in any equilibrium corresponding to the observation of an arbitrary value of the sunspot, the real balance M/p is equal to the equilibrium values of output, of consumption and of the labour supply. Thus if the households have the belief (2.2), from the first order condition (1.6), the price p that achieves equilibrium at some date when the sunspot s_i is observed is given by

$$v_1(M/p) = \Sigma_j q_{ij} v_2(M/p_j)$$

A s.m.s.e. may therefore be equivalently defined by real balances $\mu_i = M/p_i$ that satisfy

(2.3) $0 < \mu_1 < ... < \mu_r$ and $v_1(\mu_i) = \Sigma_j q_{ij} v_2(\mu_j)$, all i .

Equilibrium real balances follow then a Markov chain on $(\mu_1,...,\mu_r)$ with transition probabilities $q_{ij} > 0$. Conversely, any such Markov chain of real balances can be interpreted as a stationary sunspot equilibrium, by constructing a Markov process on S with transition probabilities $q_{ij} > 0$, provided that the underlying probability space is rich enough.

Studying the set of stationary Markov sunspot equilibria, with r states, amounts therefore to analyzing the set of real balances (μ_i) and of transition probabilities $q_{ij} > 0$ that verify (2.3).

<u>A constructive Global Characterization</u>

The set of s.m.s.e. is easily characterized in this particular case, as shown in Grandmont (1985b, 1986). Consider real balances μ_i and transition probabilities $q_{ij} > 0$ that satisfy (2.3). Let m,n be integers such that $v_2(\mu_n) \leq v_2(\mu_i) \leq v_2(\mu_m)$ for all i . It follows then from (2.3) that $v_2(\mu_n) < v_1(\mu_i) < v_2(\mu_m)$ for all i , or equivalently

$$(2.4) \quad \chi(\mu_n) < \mu_1 < \ldots < \mu_r < \chi(\mu_m)$$

One cannot have $\mu_n < \mu_m$ in this model, for there would exist then more than one monetary stationary state. Thus, if one considers a s.m.s.e., there are two real balances $\mu_m < \mu_n$ that verify (2.4).

Consider, conversely, real balances $0 < \mu_1 < \ldots < \mu_r$, with $\mu_m < \mu_n$ that satisfy (2.4). Then there are transition probabilities verifying (2.3). They are given by

$$(2.5) \quad q_{im} = [v_1(\mu_i) - v_2(\mu_n) - \sum_{j\neq m,n} q_{ij}(v_2(\mu_j) - v_2(\mu_n))]/[v_2(\mu_m) - v_2(\mu_n)]$$

$$(2.6) \quad q_{in} = [v_2(\mu_m) - v_1(\mu_i) - \sum_{j\neq m,n} q_{ij}(v_2(\mu_m) - v_2(\mu_j))]/[v_2(\mu_m) - v_2(\mu_n)]$$

in which the $q_{ij} > 0$ are chosen arbitrarily for $j \neq m,n$, subject to the constraints that $q_{im} > 0$ and $q_{in} > 0$, i.e.

$$(2.7) \quad \sum_{j\neq m,n} q_{ij}(v_2(\mu_j) - v_2(\mu_n)) < v_1(\mu_i) - v_2(\mu_n) \quad \text{and}$$

$$\sum_{j\neq m,n} q_{ij}(v_2(\mu_m) - v_2(\mu_j)) < v_2(\mu_m) - v_1(\mu_i)$$

Clearly, given the real balances (μ_i) , the set of such transition probabilities q_{ij} , $i = 1,\ldots,r$, $j \neq m,n$, is non empty (think of the case where all

q_{ij} in (2.7) are close to 0), convex, open and thus of dimension $r(r - 2)$.

To sum up this discussion, the real balances (μ_i) define a stationary Markov sunspot equilibrium, with r states - in the sense that there are corresponding transition probabilities $q_{ij} > 0$ that verify (2.3) - if and only if there exist $\mu_m < \mu_n$ that satisfy (2.4). The corresponding transition probabilities are given (non uniquely) by (2.5), (2.6), (2.7). Obviously, the stationary state $\bar{\mu}$ must belong to the interval (μ_m, μ_n).

Local sunspots

We apply now the foregoing characterization to ε-local sunspot equilibria, i.e. stationary Markov sunspot equilibria such that the corresponding real balances stay within ε of the stationary state $\bar{\mu}$, for ε small.

Assume that $\chi'(\bar{\mu}) \neq 0$, and choose ε small enough to ensure that $\chi'(\mu)$ keeps the same sign on the interval $(\bar{\mu}-\varepsilon, \bar{\mu}+\varepsilon)$, as well as on its image by the map χ. If an ε-local sunspot equilibrium is to exist, then in view of (2.4), the offer curve must be decreasing on these intervals, and one may require $m = 1$, $n = r$ in the previous characterization. It follows that if ε has been chosen in this way, an ε-local sunspot equilibrium is characterized by real balances that satisfy $|\mu_i - \bar{\mu}| < \varepsilon$ and

(2.8) $$\chi^2(\mu_1) < \chi(\mu_r) < \mu_1 < \ldots < \mu_r < \chi(\mu_1) < \chi^2(\mu_r)$$

in which $\chi^2 = \chi o \chi$ is the second iterate of the map χ.

Any such ε-local sunspot equilibrium can thus be generated through the following procedure. First one looks for μ_1 close to and on the left of $\bar{\mu}$, such that $\chi^2(\mu_1) < \mu_1$. Since χ is decreasing near $\bar{\mu}$, there is a unique μ' in $(\bar{\mu},\chi(\mu_1))$ such that $\chi(\mu') = \mu_1$. Then μ_r must be chosen in the interval $(\mu',\chi(\mu_1))$, see Fig. 3 (of course μ_r has to be also within ε of $\bar{\mu}$, which will be always verified if μ_1 itself is close enough to the stationary state). Finally, the other values of μ_i , $i \neq 1,r$, can be distributed arbitrarily in the interval (μ_1,μ_r) , while the corresponding transition probabilities can be generated (again, nonuniquely) through (2.5), (2.6), (2.7).

Fig. 3

3. LOCAL DETERMINATENESS AND SUNSPOTS

An immediate consequence of the preceding analysis is that, if we assume away the exceptional cases $\chi'(\bar{\mu}) = 0$ and $\chi'(\bar{\mu}) = -1$, there are ε-local sunspot equilibria for every $\varepsilon > 0$ if and only if the stationary state is deterministically locally indeterminate.

The notion of local determinateness pertains to the behavior of deterministic intertemporal equilibria near the stationary state $\bar{\mu}$. To be formal, we say that a deterministic intertemporal equilibrium is an ε-local equilibrium if μ_t stays within ε of $\bar{\mu}$, for all $t \geq 1$. The stationary state is then locally determinate if there exists $\varepsilon > 0$ such that there is no ε-local equilibrium other than the stationary state itself. It is locally indeterminate otherwise.

Local determinateness is governed by the behaviour of the trajectories generated by the difference equation that is obtained by inverting locally the equation (2.1) (which is always possible when $\chi'(\bar{\mu}) \neq 0$) . Note that in view of (1.4),

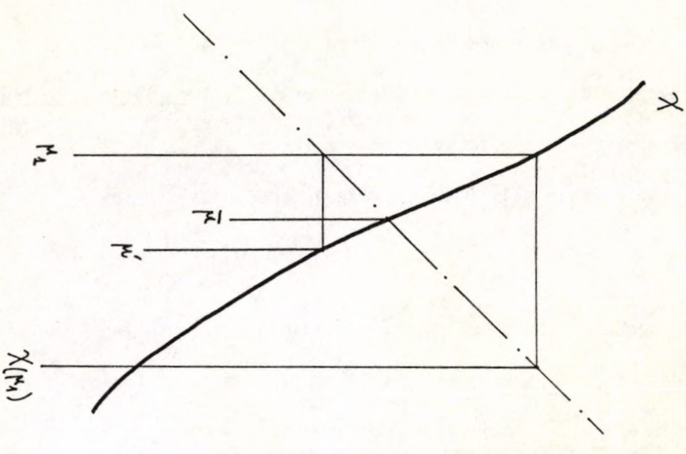


Fig. 3

(3.1) $\chi'(\bar{\mu}) = [1 - R_2(\bar{\mu})]/[1 + \frac{\bar{\mu}}{{}^{*} - \bar{\mu}} R_1(\ell^{*} - \bar{\mu})]$

so that $\chi'(\bar{\mu})$ is always less than 1. Intuitively, everything else being equal, $\chi'(\bar{\mu})$ decreases as the utility function V_2 becomes more concave, i.e. as $R_2(\bar{\mu})$ gets larger. If $\chi'(\bar{\mu}) > -1$, intertemporal equilibria starting close enough to $\bar{\mu}$ have to move away from it : the stationary state is <u>locally determinate</u>, see Fig. 4.a. If $\chi'(\bar{\mu}) < -1$ and if ε is small enough, for <u>every</u> μ_1 close enough to $\bar{\mu}$, there exists an ε-local equilibrium $(\mu_t)_{t \geqq 1}$ with initial condition μ_1 . Given μ_1 , such an intertemporal equilibrium is in fact unique, and it converges eventually to $\bar{\mu}$: the stationary state is <u>locally indeterminate</u>, see Fig. 4.b. Thus if we assume away the cases $\chi'(\bar{\mu}) = 0$, and $\chi'(\bar{\mu}) = -1$, the stationary state is locally indeterminate if and only if $\chi'(\bar{\mu}) < -1$.

<u>Fig. 4</u>

The characterization of sunspot equilibria given in the previous section implies immediately that ε-local sunspot equilibria exist for all ε if and only if the stationary state is locally indeterminate, whenever $\chi'(\bar{\mu}) \neq 0$ and $\chi'(\bar{\mu}) \neq -1$. If $\chi'(\bar{\mu}) \neq 0$,we can apply the constructive characterization given in (2.8) and Fig. 3. Then there are ε-local sunspot equilibria for every ε , if and only if there exists a sequence μ^k that tends to $\bar{\mu}$ such that $\chi^2(\mu^k) < \mu^k < \bar{\mu}$ for all k. If one remarks next that $(\chi^2)'(\bar{\mu}) = [\chi'(\bar{\mu})]^2$, and if one assumes away the case $\chi'(\bar{\mu}) = -1$, such a sequence exists if and only if $\chi'(\bar{\mu}) < -1$.

This implies (2)

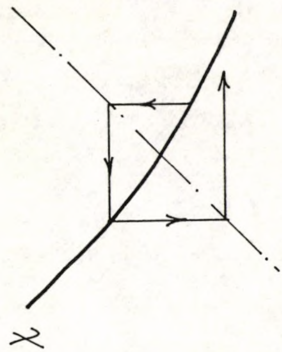


Fig. 4.a

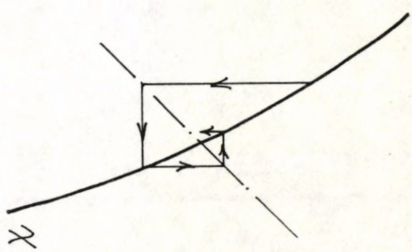


Fig. 4.b

Proposition 3.1. Assume $\chi'(\bar{\mu}) \neq 0$ and $\chi'(\bar{\mu}) \neq -1$. Then there exist ε-local sunspot equilibria for every $\varepsilon > 0$ if and only if $\chi'(\bar{\mu}) < -1$.

Remark : The above results can be refined in the cases where $\chi'(\bar{\mu}) = 0$ and $\chi'(\bar{\mu}) = -1$, by looking at higher order derivatives. The flavour of the results is the same.

4. LOCAL BIFURCATIONS AND SUNSPOTS

Asking for the existence of local sunspot equilibria in every, however small, neighborhood of the stationary state is very demanding. There may be cases where stationary sunspot equilibria do exist in the vicinity of $\bar{\mu}$, but not arbitrarily near it. To get information about the circumstances under which this may obtain, we look at what happens when there is a local bifurcation of the deterministic dynamics implied by (2.1). We may also hope to get in the process a better understanding of the relations that exist between deterministic and sunspot equilibria.

Deterministic bifurcations

We wish first to analyse the local behaviour of deterministic intertemporal equilibria when the stationary state changes stability in the dynamics implied by (2.1). To this effect, we index the characteristics of the economy by a real number λ in some open neighborhood U of 0 , say $(\ell^*_\lambda, V_{\tau\lambda})$, and assume that the slope of the associated offer curves at the stationary state goes through -1 at $\lambda = 0$.

(4.a) ℓ^*_λ (resp. $V_{\tau\lambda}(a)$) is four-times continuously differentiable on U (resp. $U \times (0,+\infty)$ and $\bar{\theta}_{1\lambda} = V'_{1\lambda}(\ell^*_\lambda)/\lim_{c\to 0} V'_{2\lambda}(c) < 1$ for all λ in U .

Assumption (4.a) implies that the corresponding offer curves χ_λ are 3-times continuously differentiable on U x $(0,+\infty)$, and that for each λ in U, there is a unique monetary stationary state $\bar{\mu}_\lambda > 0$. The next assumption states that the map χ_λ undergoes a <u>flip bifurcation</u> at $\lambda = 0$.

(4.b) $\chi_0(\mu_0) = -1$ <u>and</u> $\frac{d}{d\lambda}[\chi'_\lambda(\bar{\mu}_\lambda)]_{\lambda=0} < 0$

The theory tells us that we should expect a cycle of period 2 to appear near $\bar{\mu}_\lambda$ for λ small enough. Such a cycle can be identified with a positive fixed point of $\chi_\lambda^2 = \chi_\lambda o \chi_\lambda$ that differs from the stationary state $\bar{\mu}_\lambda$. For each λ, of course, $\bar{\mu}_\lambda$ is a fixed point of χ_λ^2. By differentiation, one gets immediately that $(\chi_\lambda^2)'(\bar{\mu}_\lambda) = [\chi'_\lambda(\bar{\mu}_\lambda)]^2$,which implies in view of (4.b)

(4.1) $(\chi_0^2)'(\bar{\mu}_0) = +1$ and $\frac{d}{d\lambda}[(\chi_\lambda^2)'(\bar{\mu}_\lambda)]_{\lambda=0} > 0$

Further differentiation shows that the second derivative of χ_0^2 at $\bar{\mu}_0$ vanishes. The shape of χ_0^2 near $\bar{\mu}_0$ is thus governed by the sign of the third derivative of χ_0^2.

We consider the "generic" case where $(\chi_0^2)'''(\bar{\mu}_0) \neq 0$. It is easily seen that this expression is equal to $-[2\chi_0'''(\bar{\mu}_0) + 3(\chi_0''(\bar{\mu}_0))^2]$, and that it is closely related to the value of the Schwarzian derivative of χ_0 at $\bar{\mu}_0$. Indeed the Schwarzian derivative of a thrice continously differentiable map f from the real line into itself is defined as

$$Sf(x) = \frac{f'''(x)}{f'(x)} - \frac{3}{2}\left[\frac{f''(x)}{f'(x)}\right]^2$$

whenever $f'(x) \neq 0$. Since $\chi_0'(\bar{\mu}_0) = -1$, we have therefore

(4.2) $2S\chi_0(\bar{\mu}_0) = (\chi_0^2)'''(\bar{\mu}_0)$

We consider two cases

Case 1 : $S\chi_0(\bar{\mu}_0) < 0$

The corresponding graph of χ_0^2 near $\bar{\mu}_0$ is pictured in Fig. 5.b. Remark that the assumption implies that $\bar{\mu}_0$ is asymptotically stable in the b.p.f. dynamics (2.1), although $\chi_0'(\bar{\mu}_0) = -1$. The graphs of χ_λ^2 near $\bar{\mu}_\lambda$ are represented in Fig. 5.a and 5.c for λ small, negative and positive. If $\lambda \leqq 0$, there is a no cycle of period 2 near $\bar{\mu}_\lambda$, and no intertemporal equilibrium other than $\mu_t = \bar{\mu}_\lambda$ for all $t \geqq 1$, that stays near the stationary state (Fig. 5.a, 5.b). If $\lambda > 0$, there is a unique cycle of period 2, $\bar{\mu}_{1\lambda},\bar{\mu}_{2\lambda}$, near the stationary state. Moreover, there exists an intertemporal equilibrium corresponding to the initial condition μ_1 , that stays near $\bar{\mu}_\lambda$ for all $t \geqq 1$, if and only if μ_1 lies in $[\bar{\mu}_{1\lambda},\bar{\mu}_{2\lambda}]$. Given such a μ_1 , the associated intertemporal equilibrium is unique, is contained in $[\bar{\mu}_{1\mu},\bar{\mu}_{2\lambda}]$, and converges to $\bar{\mu}_\lambda$ whenever μ_1 does not belong to the period 2 orbit.

Fig. 5

Case 2 : $S\chi_0(\bar{\mu}_0) > 0$.

The corresponding graphs of χ_λ^2 near the stationary state, for small λ , are represented in Fig. 6. There, the unique cycle $\bar{\mu}_{1\lambda},\bar{\mu}_{2\lambda}$, appears for λ negative (Fig. 6.a). In all cases, for any initial condition μ_1 near $\bar{\mu}_\lambda$, there is a unique intertemporal equilibrium that stays near $\bar{\mu}_\lambda$. If $\lambda \geqq 0$, all such intertemporal equilibria converge to the stationary state. When $\lambda < 0$, they converge to the cycle of period 2 , provided that μ_1 differs from $\bar{\mu}_\lambda$.

Fig. 6

Case 1: $S\chi_0(\bar{\mu}_0) < 0$

(Arrows indicate intertemporal equilibria, every two periods)

$\lambda < 0$

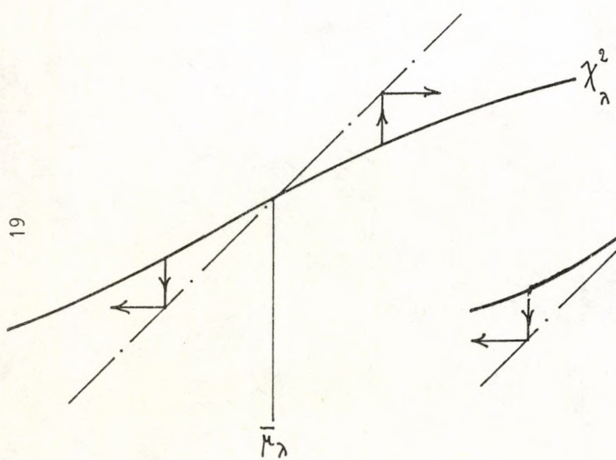


Fig. 5.a

$\lambda = 0$

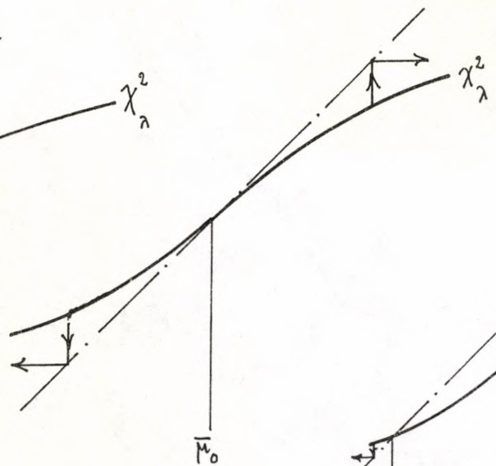


Fig. 5.b

$\lambda > 0$

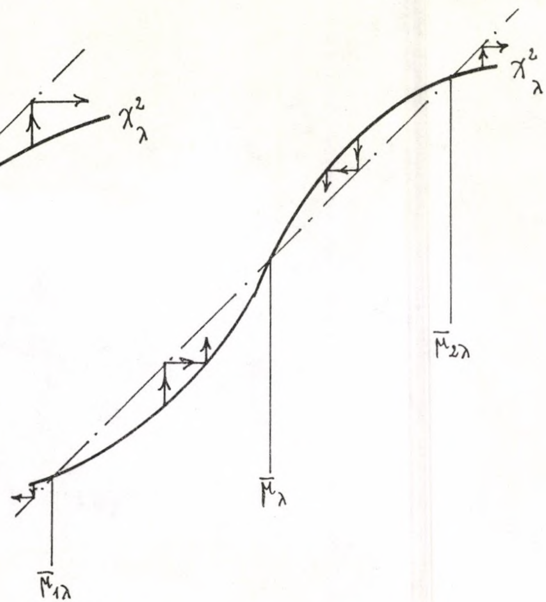


Fig. 5.c

Case 2 : $SX_{\lambda}(\bar{\mu}_0) > 0$

(Arrows indicate intertemporal equilibria, every two periods)

$\lambda < 0$

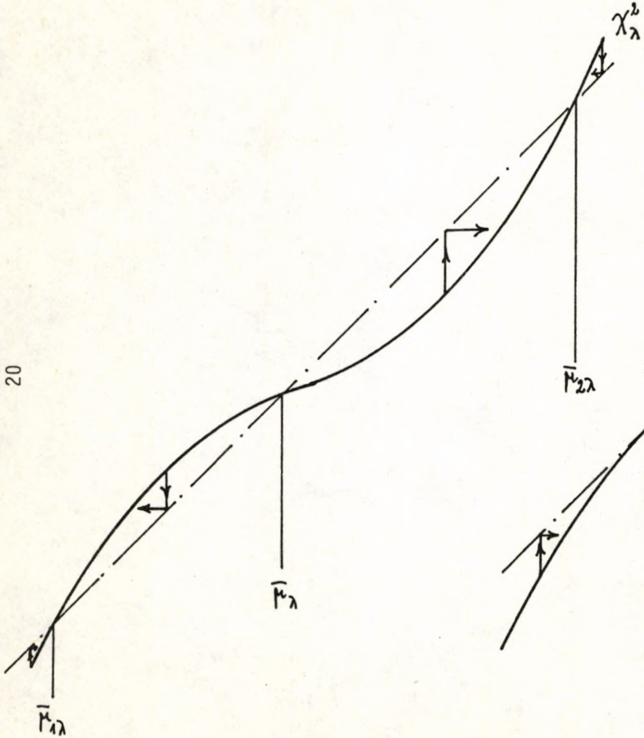


Fig. 6.a

$\lambda = 0$

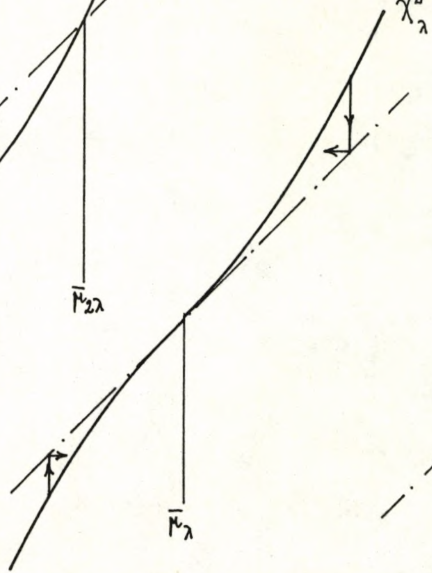


Fig. 6.b

$\lambda > 0$

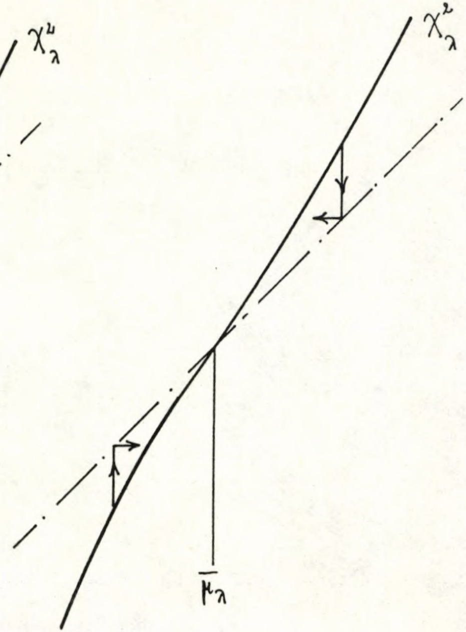


Fig. 6.c.

The following formal result summarizes the preceding discussion. Given λ in U , and $\varepsilon > 0$, we say that the intertemporal equilibrium $(\mu_t)_{t \geq 1}$ is an ε-local equilibrium if μ_t stays within ε of the stationary state $\bar{\mu}_\lambda$ for all $t \geq 1$.

Theorem 4.1 : Consider a one parameter family of economies satisfying (4.a), (4.b).

1) Let $S\chi_0(\bar{\mu}_0) < 0$. Then there exist $\lambda_1 < 0$, $\lambda_2 > 0$ and $\varepsilon > 0$ such that

1.a) If $\lambda_1 < \lambda \leq 0$, there is no ε-local equilibrium other than the stationary state $\bar{\mu}_\lambda$.

1.b) If $0 < \lambda < \lambda_2$, there is a unique cycle of period 2, i.e. $\bar{\mu}_{1\lambda}, \bar{\mu}_{2\lambda}$, within ε of the stationary state, and all ε-local equilibria lie in $[\bar{\mu}_{1\lambda}, \bar{\mu}_{2\lambda}]$. Given μ_1 in that interval, there is a unique ε-local equilibrium with initial condition μ_1 . It is contained in $[\bar{\mu}_{1\lambda}, \bar{\mu}_{2\lambda}]$, and converges to the stationary state when $\mu_1 \neq \bar{\mu}_{1\lambda}, \bar{\mu}_{2\lambda}$.

2) Let $S\chi_0(\bar{\mu}_0) > 0$. There exist $\lambda_1 < 0$, $\lambda_2 > 0$ and $\varepsilon > 0$ such that for any λ in (λ_1, λ_2) and any μ_1 within ε of the stationary state $\bar{\mu}_\lambda$, there is a unique ε-local equilibrium with initial condition μ_1 . Moreover,

2.a) If $\lambda_1 < \lambda < 0$, there is a unique cycle of period 2, i.e. $\bar{\mu}_{1\lambda}, \bar{\mu}_{2\lambda}$ within ε of the stationary state. Any ε-local equilibrium other than the stationary state converges to the period 2 orbit.

2.b) If $0 \leq \lambda < \lambda_2$, all ε-local equilibria converge to the stationary state $\bar{\mu}_\lambda$.

Proof. The existence of cycles and their asymptotic stability are a local version of Whitley (1983, Proposition 1.2), by applying it to the family of offer curves χ_λ and reversing time, or more directly to the family of local inverses χ_λ^{-1} near the stationary states $\bar{\mu}_\lambda$. The statements concerning ε-local equilibria are then straightforward consequences of these stability properties.

Q.E.D.

Fig. 7 below translates the preceding result into a qualitative picture, in the plane (λ,μ) , within a band of $\pm\varepsilon$ on each side of the stationary state $\bar{\mu}_\lambda$. Shaded areas describe the regions filled by deterministic ε-local equilibria. Arrows show where they converge, as $t \to +\infty$, for each λ .

Fig. 7

Local sunspots

We investigate now the relationships between ε-local sunspot equilibria and cycles in the above bifurcating family. The principle of the approach is straightforward : We assume that λ_1 , λ_2 and ε in Proposition 4.1 have been chosen small enough to enable us to apply the constructive characterization given in (2.8) and Fig. 3.

It is clear that in Case 1, i.e. $S\chi_0(\bar{\mu}_0) < 0$, as described in Fig. 5, no local sunspot equilibrium exists for $\lambda \leqq 0$, since $\chi_\lambda^2(\mu) > \mu$ for all μ close to but less than $\bar{\mu}_\lambda$. By contrast, for $\lambda > 0$, there are infinitely many ε-local sunspot equilibria. Each of them can be generated from an arbitrary μ_1 in the interval $(\bar{\mu}_{1\lambda},\bar{\mu}_\lambda)$. The support $(\mu_1,\ldots,\mu_r)$ of every ε-local sunspot equilibrium is contained in the interval $(\bar{\mu}_{1\lambda},\bar{\mu}_{2\lambda})$ determined by the orbit of period 2, and the union of these supports is the whole interval.

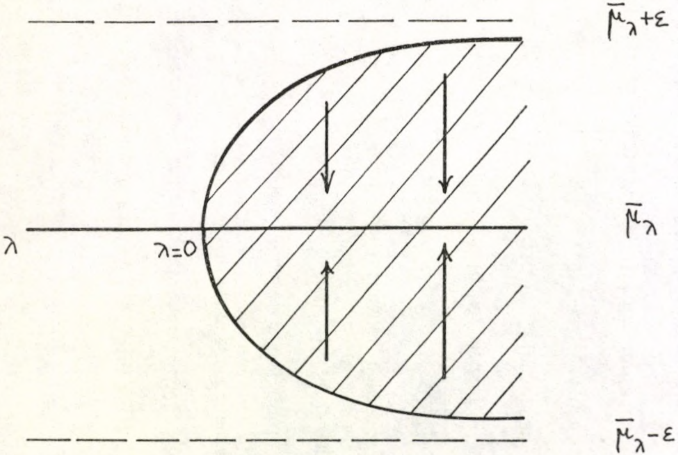


Fig. 7.a. Case 1.

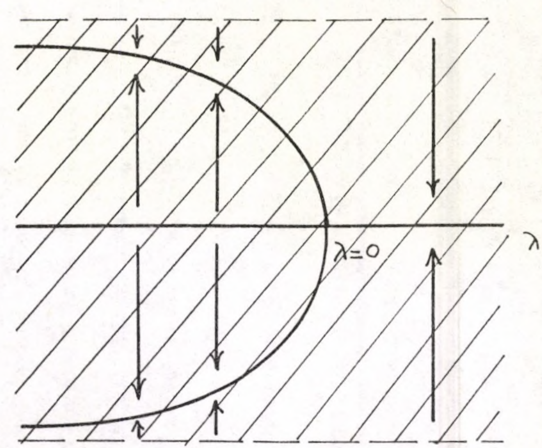


Fig. 7.b. Case 2.

In this case, ε-local sunspot equilibria exist only for $\lambda > 0$, and there are then infinitely many of them in every neighborhood of the stationary state. The qualitative picture is much different in the second case, i.e. when $S\chi_0(\bar{\mu}_0) > 0$ (Fig. 6). There, stationary local sunspot equilibria exist in the vicinity of the stationary state for <u>all</u> values of the parameter λ , and the union of the supports $(\mu_1,\ldots,\mu_r)$ of local sunspot equilibria fills the whole neighborhood of the stationary state. For $\lambda \geqq 0$, (Fig. 7.b, 7.c), stationary local sunspot equilibria exist in every neighborhood of $\bar{\mu}_\lambda$. But for $\lambda < 0$ (Fig. 7.a), there are <u>no</u> stationary sunspot equilibria that lie arbitrarily near the stationary state : for every $\lambda < 0$, the closure of the support $(\mu_1,\ldots,\mu_r)$ of every ε-local sunspot equilibrium contains in its interior the interval determined by the period two cycle $[\bar{\mu}_{1\lambda},\bar{\mu}_{2\lambda}]$.

One notices a fact that may be of potential generality, namely that in each of the above two cases, the union of the supports $(\mu_1,\ldots,\mu_r)$ of ε-local sunspot equilibria coincides with the union of deterministic ε-local intertemporal equilibria. Thus the shaded areas in the bifurcation diagram of Fig. 7 represent also the regions filled by the supports $(\mu_1,\ldots,\mu_r)$ of stationary local sunspot equilibria in each type of bifurcation.

The following formal result summarizes the preceding discussion.

Theorem 4.2 : <u>Consider a one parameter family of economies satisfying</u> (4.a), (4.b), <u>and let the parameters</u> λ_1 , λ_2 , ε <u>appearing in Theorem</u> 4.1 <u>be small enough</u>.

1) <u>Let</u> $S\chi_0(\bar{\mu}_0) < 0$. <u>Then if</u> $\lambda_1 < \lambda \leqq 0$, <u>there is no</u> ε-<u>local sunspot equilibrium. If</u> $0 < \lambda < \lambda_2$, <u>there are infinitely many</u> ε-<u>local sunspot equilibria in every neighborhood of the stationary state, and the union of the</u>

<u>supports</u> $(\mu_1,\ldots,\mu_r)$ <u>of these sunspot equilibria is</u> $(\bar{\mu}_{1\lambda},\bar{\mu}_{2\lambda})$.

2) <u>Let</u> $S\chi_0(\bar{\mu}_0) > 0$. <u>Then, for each</u> λ , <u>the union of the supports</u> $(\mu_1,\ldots,\mu_r)$ <u>of</u> ε-<u>local sunspot equilibria is the whole interval</u> $(\bar{\mu}_\lambda - \varepsilon, \bar{\mu}_\lambda + \varepsilon)$. <u>If</u> $\lambda_1 < \lambda < 0$, <u>for every</u> ε-<u>local sunspot equilibrium one has</u> $\mu_1 < \bar{\mu}_{1\lambda} < \bar{\mu}_{2\lambda} < \mu_r$. If $0 \leqq \lambda < \lambda_2$, <u>there are infinitely many stationary local sunspot equilibria in every neighborhood of</u> $\bar{\mu}_\lambda$.

<u>Remarks</u> 4.1. In Case 1 (resp. Case 2), the family χ_λ undergoes a socalled supercritical (resp. subcritical) bifurcation. Had we considered instead the local inverses χ_λ^{-1} , the family χ_λ^{-1} would have undergone a subcritical flip bifurcation in Case 1, a supercritical one in Case 2. Fig. 7 can be interpreted as the "bifurcation diagram" of χ_λ^{-1} .

4.2. In view of (3.1), the sort of bifurcation discussed here will occur if one increases the concavity of the utility function V_2 . It follows from Footnote 1 and Grandmont (1985a, Lemma 4.6), that with the specification $V_1(a) = a^{1-\alpha_1}/(1-\alpha_1)$ and $V_2(c) = (\ell_2^* + c)^{1-\alpha_2}/(1-\alpha_2)$, the map χ has a negative Schwarzian derivative whenever $\alpha_1 \leqq 1$, $\alpha_2 \geqq 2$, at all $\mu > 0$ such that $\chi'(\mu) \neq 0$ if $\ell_2^* \geqq 0$, and also at $\mu = 0$ if $\ell_2^* > 0$.

FOOTNOTES

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(1) The specification of the households' sector is taken from Reichlin (1986), Woodford (1986). See also Benhabib and Laroque (1986). The specification used in Grandmont (1985a) is isomorphic to the case considered here, up to a reinterpretation of the variables, when $V_2(c) = V(\ell_2^* + c)$ with $\ell_2^* > 0$. This corresponds to Fig. 1.a.

(2) The result was proved in a similar model in Grandmont (1986). For related results, see Azariadis and Guesnerie (1986), Guesnerie (1986), Woodford (1984, 1986), Laitner (1986).

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