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# DUOPOLY UNDER DEMAND UNCERTAINTY

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# DUOPOLY UNDER DEMAND UNCERTAINTY*

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**Abstract**

This paper examines how the major results of the duopoly literature are altered when firms operating in the market face exogeneous demand uncertainty. Under the usual assumption of risk aversion one would expect them to react by cutting back their production level. The fact that firms interact in the market influences the outcome ; we analyse how each firm and the market react to mean or spread preserving shifts in demand and/or changes in the degree of risk aversion. The results are proved analytically and shown diagrammatically, facilitating in this way the analysis of the market within both an uncertain demand environment and the strategic interaction framework. By taking into account the market perspective under different types of behaviour, we are able to show that the usual result of a negative reaction of firms to uncertainty may not hold any longer; this arises because their interaction in the market place affects profitability and risk borne by both firms. We also analyse the conditions upon which the duopoly may turn into a monopoly or conversely.

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## 1. Introduction

Uncertainty is an inherent feature of duopoly and oligopoly markets, resulting to a great extent from the way the decision-maker's rivals are acting. This type of uncertainty, arising from strategic considerations, could be labelled "endogeneous" uncertainty in the sense that it emerges from the functioning of the market. There exists already a sizeable literature on the subject (See references in Friedman (1977, 1981), Hey (1979) and Varian (1984)). The purpose of this paper is to examine how, in the case of a duopoly, the major results of the above mentionned literature are altered when we take into account an additional source of uncertainty, truly exogeneous to the firms operating on the market. In this case, firms face simultaneously two types of uncertainty: the first arising from strategic considerations and the second due to demand randomness beyond their control (we assume they are unable to affect the realizations of the random demand variable).

In his survey of the oligopoly literature, Friedman (1981) writes:

*"Uncertainty and incomplete information are beyond the scope of this book; indeed, not a great deal has been done in either direction. One wonders whether introducing uncertainty merely means that everything carries through after being restated in expected value terms or whehter fundamental changes in behavior result.(...) It is probably necessary to include intrinsic uncertainty in the model."* (page 135)

In addition to Friedman's view, it has to be aknowledged that firms usually spend non negligible resources to analyze the behaviour of their rivals, but also to access the likely trends in the economic environment and the future state of demand. Depending on the sector or industry, the variability of demand may be high or low, but there are probably very few instances where firms pay less attention to the overall development than to the behaviour of their competitors. Therefore, it seems that this issue has to be addressed.

The introduction of uncertainty rises the question of what should be the objective function of the firm, because profits are now stochastic from an *ex-ante* point of view. One possibility is to assume that firms maximize expected profits. However, we believe that more attention has to be paid to the variability of profits, since the behaviour of firms is characterized by an element of risk aversion.

We relate risk aversion to the variance of demand. Thus, the objective function of the firm is assumed to be the difference between the expectation and the variance of profits, the latter being multiplied by a parameter mesuring the rate at which the firm trades–off (expected) profits for risk — thus, defining the degree of risk aversion of the firm. The results derived in the paper depend on this particular objective function, which is a simple way of introducing risk aversion in the analysis, and on the assumption of a linear demand function, which on its turn is close to its analogue in the literature on oligopolistic behaviour under certainty. We believe that this formulation preserves the main elements in real life (uncertain) situations, providing a useful approximation to the decision process of the firm.

We adopt a presentation technique similar to that of Katz *et al.* (1982). Most of the results are shown diagrammatically, facilitating in this way both the analysis of the type of uncertainty here introduced and the conjectural variations unified framework of this oligopolistic behaviour. The algebraic proofs of the results are left to the appendix.

This paper should be seen as a step toward the analysis of imperfectly competitive markets in the presence of exogeneous uncertainty. It is structured as follows: Section 2 introduces the model and explains how exogeneous demand shocks appear. In Section 3 we solve the model under the Cournot-Nash behaviour the effects of changes in the various parameters and variables of the model and analyze by comparative statics methods. In Section 4 we turn our attention to Stackelberg behaviour. The fifth section is devoted to the analysis of "degenerated" cases where entry in and exit out of the market is linked to the degree of demand uncertainty.

## 2. The Model

Consider two firms selling a homogeneous product on one market. Market demand takes the following form:

$$\tilde{p} = \mathrm{M} - x_1 - x_2 + \tilde{\alpha}, \tag{1}$$

where $p$ denotes the price of the good, $x_i$ is the production of firm $i$, M is a constant, and $\tilde{\alpha}$ is a random variable capturing demand shifts (a $\tilde{}$ denotes a random variable). We assume that $\tilde{\alpha}$ may be written as:

$$\tilde{\alpha} = \bar{\alpha} + \sqrt{\beta}\,\epsilon \quad , \quad \mathrm{E}(\epsilon) = 0, \sigma_\epsilon^2 = 1 \quad , \quad P(-\gamma < \epsilon < \gamma) = 1, \tag{2}$$

where $\gamma$ is a constant which determines the width of demand fluctuations around its mean.[1] Hence $\bar{\alpha}$ is the expected value of $\tilde{\alpha}$ and $\beta$ denotes its variance. Thus a spread-preserving increase in demand may be represented by an increase in $\bar{\alpha}$ and a mean-preserving increase in the variability of demand may be represented by an increase in $\beta$, as defined by Rothschild and Stiglitz (1970).

For the sake of simplicity we assume that each firm's cost function exhibits constant marginal costs $C_i'$, which are not necessarily equal across firms. The profit function of firm $i$ is therefore given by:

$$\tilde{\Pi}_i = (\tilde{p} - C_i')x_i = (\mathrm{M} - x_i - x_j + \tilde{\alpha} - C_i')x_i, \qquad i,j = 1,2, \text{ and } i \neq j. \tag{3}$$

It is assumed that firms maximize the expected utility of profits and have a quadratic utility function, where $\theta_i$ measures firm $i$'s risk aversion. The optimization problem becomes:

$$\max_{x_i} U_i(\tilde{\Pi}_i) = \mathrm{E}(\tilde{\Pi}_i) - \theta_i \mathrm{V}(\tilde{\Pi}_i), \qquad \theta_i > 0, \tag{4}$$

where $\mathrm{E}(\cdot)$ is the expectation operator and $\mathrm{V}(\cdot)$ denotes the variance.

As it is very often the case in the literature on behaviour under uncertainty, we assume that the firms must decide on the level of their control variable (production, in this

[1] It is assumed that $\gamma < \min(C_1', C_2')$. This condition ensures that the equilibrium will not lie in the area where the probability function of $\epsilon$ is truncated, i.e., to the right of $Q_\epsilon$ in Figure 1.

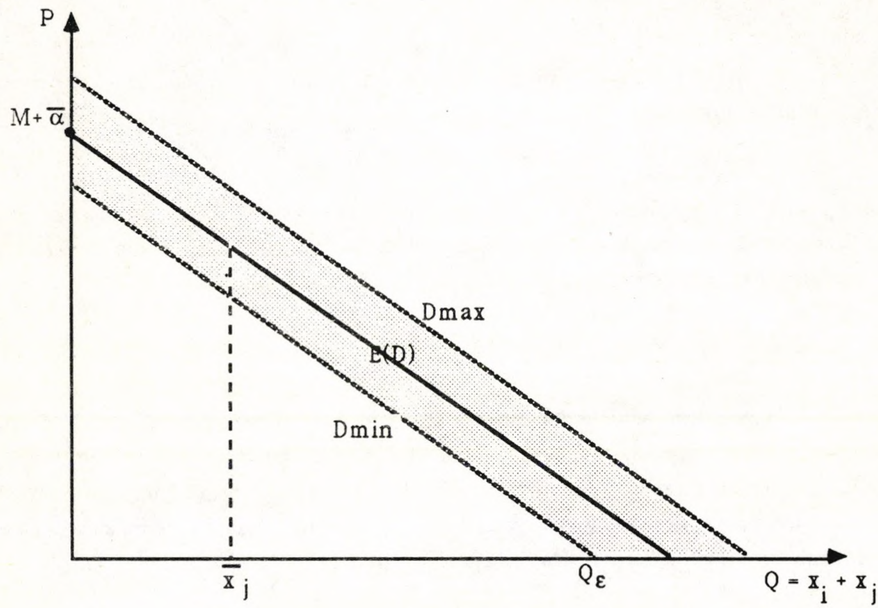


Figure 1: The market demand schedule

case) before the state of the world is revealed; hence, their objective functions can be rewritten as follows:

$$U_i(\tilde{\Pi}_i) = (\mathrm{M}_i - x_i - x_j)x_i - \beta\theta_i x_i^2 , \qquad i \neq j, \tag{5}$$

where $\mathrm{M}_i = \mathrm{M} + \bar{\alpha} - C_i'$.

Graphically, the market can be represented as in Figure 1. The demand curve $D$ is downward sloping; uncertainty, introduced here in an additive form, leads to movements of the demand curve within a band $[D_{min}, D_{max}]$.

Let us abstract for a while from strategic behaviour and consider that output of firm $j$ is fixed at $\bar{x}_j$; firm $i$ is now facing the remaining demand (vertical difference $D - \bar{x}_j$). This situation is depicted in Figure 2.

In the certainty case, firm $i$ produces the amount of output $x_i^*$, where marginal cost $C_i'$ is equal to marginal revenue $R'$. Under uncertainty, firm $i$ ignores the precise location

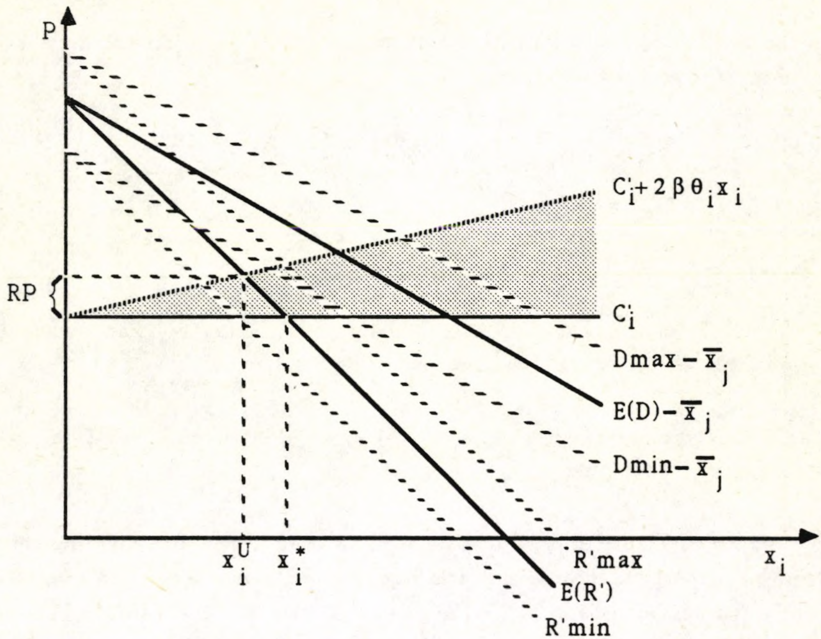


Figure 2: The firm demand schedule

of the demand curve, knowing only that it will be somewhere in the $[D_{min}-\bar{x}_j, D_{max}-\bar{x}_j]$ interval. Consequently, the marginal revenue curve is, as well, somewhere inside a band $[R'_{min}, R'_{max}]$. As it is shown in point 1 of the appendix, the firm no longer equates marginal revenue to marginal cost to determinate its output level. Instead, it determines its output level by the intersection of the marginal revenue curve and the vertical sum of marginal cost with a risk-premium. Graphically, this is depicted in Figure 2 where $RP$ is the risk-premium. Output is then equal to $x_i^U$. It is readily seen that the risk averse firm supplies less under uncertainty than under certainty. As the risk-premium is a positive function of all of its arguments $(\theta_i, \beta, x_i)$, the absolute difference between production in these two cases is larger whenever any of these increases.[2]

[2] Remark that under the more usual hypothesis of an increasing concave cost function,

## 3. The Cournot-Nash Equilibrium

From the first order conditions of the maximization of equation (5) we obtain the reaction functions of the two firms:

$$RF_1: \qquad x_1(x_2) = \frac{M_1 - x_2}{2(1+\beta\theta_1)}$$

$$RF_2: \qquad x_2(x_1) = \frac{M_2 - x_1}{2(1+\beta\theta_2)}, \tag{6}$$

Both reaction functions can be drawn in a diagram in the $(x_1,x_2)$ plane where $x_1$ and $x_2$ are quantities sold by the two firms.

Lemma 1. *In the plane $(x_1,x_2)$, slope of $RF_1 < -1 <$ slope of $RF_2 < 0$.*
(Proof is given in the appendix).

As it is shown in Figure 3, it is possible to draw a 45 degree line dividing the space between both reaction curves and crossing them at their intersection point. Along this line $Q$ remains constant, hence the expected price is also constant. Therefore, it may be called the "iso-price curve", and any movement along this curve indicates a redistribution of output between the two firms ($dx_1 = -dx_2$).

In the rest of the paper we often resort to displacements of the "iso-price curve" to measure changes in market size following exogeneous changes in the various parameters of the model. When this line moves upwards (resp. downwards), the total quantity supplied increases (resp. decreases), and consequently the expected price falls (resp. increases).

Under Cournot strategies, equilibrium output levels are given by:

$$\begin{pmatrix} x_1^N \\ x_2^N \end{pmatrix} = \frac{1}{\Delta} \begin{pmatrix} 2(1+\beta\theta_2) & -1 \\ -1 & 2(1+\beta\theta_1) \end{pmatrix} \begin{pmatrix} M_1 \\ M_2 \end{pmatrix}, \tag{7}$$

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the output reduction due to uncertainty is smaller than the one depicted in Figure 2 where $C'' = 0$.

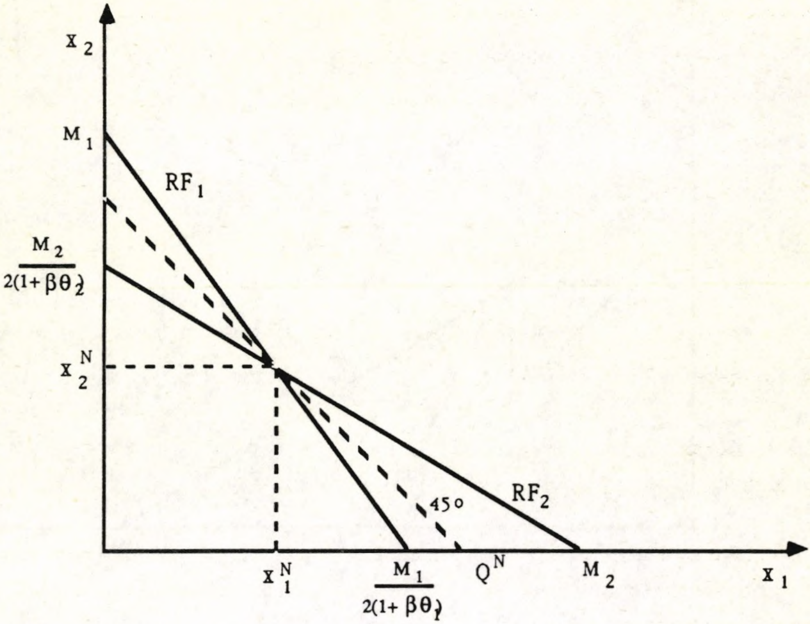



Figure 3: The reaction functions

where $\Delta = 4(1+\beta\theta_1)(1+\beta\theta_2) - 1 > 0$. Diagrammatically, the intersection of the two reaction schedules gives us the equilibrium point of production $(x_1^N, x_2^N)$ as in Figure 3.

Proposition 1. *The equilibrium is always unique. Both firms have strictly positive output levels provided that* $\mathrm{M}_1/2(1+\beta\theta_1) < \mathrm{M}_2 < 2(1+\beta\theta_2)\mathrm{M}_1$.
(Proof is given in the appendix).

This proposition follows from Lemma 1. The reaction functions are straight lines with different slopes, therefore intercepting only once. The equilibrium (intersection) point associates positive values to $x_1^N$ and $x_2^N$ only if (see Figure 3): (i) the reaction function of firm 1 (the steepest) intercepts the vertical axis above firm 2's reaction function, implying that $\mathrm{M}_1 > \mathrm{M}_2/2(1+\beta\theta_2)$; (ii) the reaction function of firm 1 intercepts the horizontal axis to the left of firm 2's, implying that $\mathrm{M}_2 > 2(1+\beta\theta_2)\mathrm{M}_1$. Adding up

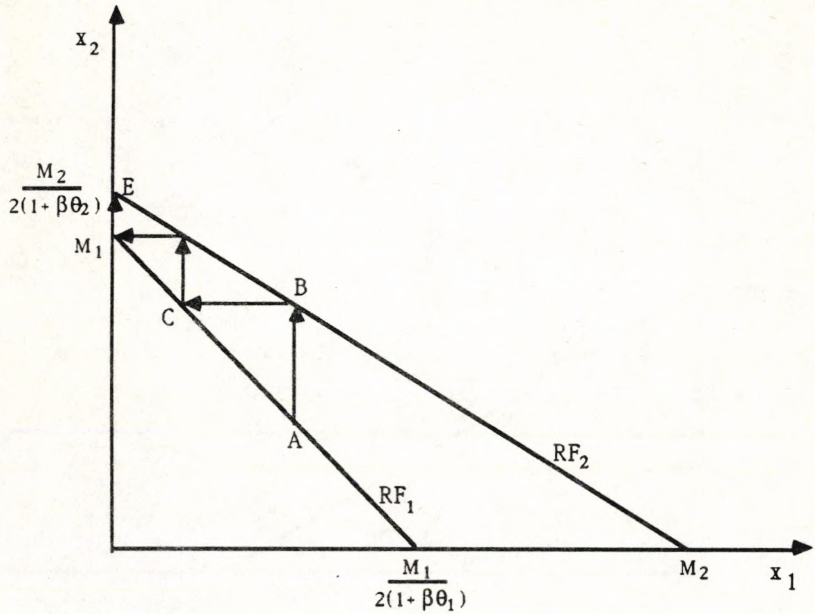



Figure 4: Equilibrium output level
when $M_2 > 2(1+\beta\theta_2)M_1$

these two conditions we get the one presented in Proposition 1. If these conditions were not to hold it would be optimal for the most competitive firm to behave as if it were a monopolist; at the current expected price the other firm would make losses and therefore would exit the market.

The situation is depicted in Figure 4, for the case when firm 2 is more competitive than firm 1.[3] Point $A$ in this figure cannot be an equilibrium, since firm 2 is not behaving optimally; for the same level of $x_1$, firm 2 would rather produce at $B$. However, at this point firm 1 is not acting optimally, and therefore would prefer to

[3] $RF_2$ associates higher output levels to $x_2$ than $RF_1$ for the whole range of $x_1 \geq 0$ and $x_2 \geq 0$. This means that for all positive levels of firm 1's output the optimal quantity supplied by firm 2 is higher than that consistent with optimal behaviour of firm 1, thus rendering the price too low for the latter.

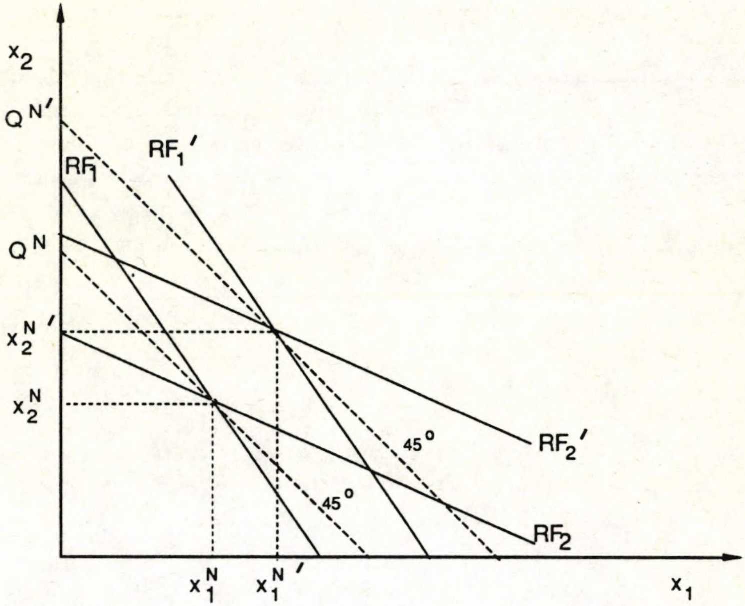



Figure 5: Effects of a spread-preserving increase in demand ($\Delta\bar{\alpha}$) on output

cut its production to $C$. The only equilibrium point is $E$, where firm 2 is a monopolist. Similarly, if firm 1 is more competitive than firm 2, it supplies the monopoly quantity to the market, forcing $x_2 = 0$.

Proposition 2. *A spread-preserving increase in demand* ($\Delta\bar{\alpha}$) *leads to an increase of production of both firms and, consequently, to an increase in total output.*
(Proof is given in the appendix).

An increase in $\bar{\alpha}$ shifts both reaction curves upwards as depicted in Figure 5. The new equilibrium is such that $x_1^{N'} > x_1^N$ and $x_2^{N'} > x_2^N$. This is due to the fact that, at the initial equilibrium point the expected profitability of both firms has increased while the variance of profits (the negative term in the utility function) remained the same. Each firm has then an incentive to increase production until the expected profitability $(R' - C')$ equals the risk-premium $(2\beta\theta_i x_i)$. As the expected profitability is decreasing in production and the risk-premium is a positive function of output, the firm always increases its production by less than the exogeneous increase in $\bar{\alpha}$. Accordingly, the total quantity supplied to the market rises unambiguously ($Q^{N'} > Q^N$).

As it is shown mathematically in the appendix (A7), the output expansion is distributed among the two firms according to their relative risk aversion coefficients. Under the assumption of constant marginal costs, an increase in $\bar{\alpha}$ does not affect the cost of any additional unit of output and, therefore, the only elements that matter for the distribution of the output increase between the two firms are the risk aversion coefficients. In other words, the relative profitability of both firms does not affect the way they react to an exogenous change in $\bar{\alpha}$. For example, it may well be that the less efficient firm (the one having the highest unit cost) expands its production by a larger amount of units of output than the most efficient firm.

Proposition 3. *An increase in firm i's risk aversion ($\Delta\theta_i$) leads to a reduction of its own production level, an increase in output of firm j, and a reduction in the total quantity supplied to the market.*
(Proof is given in the appendix).

In Figure 6 we consider an increase in $\theta_1$. Because of the symmetrical structure of the model under Cournot behaviour, the analysis equally applies for an increase in $\theta_2$. An increase in $\theta_1$ moves the reaction function of firm 1 around the point $(0, M_1)$ towards the origin. As it can be seen in Figure 6, $(x_1^N - x_1^{N'}) > (x_2^{N'} - x_2^N)$, given the fact that the slope of $RF_2$ is larger than $-1$ (the 45 degree dashed line is steeper than $RF_2$). Similarly, for any increase in $\theta_2$, $(x_2^N - x_2^{N'}) > (x_1^{N'} - x_1^N)$.

An increase in the risk aversion coefficient $\theta_1$ means that firm 1 pays now relatively more attention to the risk term vis-a-vis its expected profitability, or, alternatively, it requires now an increase in expected profitability to bear the same amount of risk than before. In fact, at the initial equilibrium point, an increase in $\theta_1$ has neither affected the expected profit nor the variance. Therefore, the only way for the firm to improve its expected utility (find a new balance between expected profitability and uncertainty) is to cut back production.

For firm 2 the argument runs as follows: at the initial production level $x_2^N$, expected profitability has now increased due to the reduction in $x_1$. By the same reasoning used in Proposition 2, firm 2 always increases its output by less than the initial increase in profitability. In all circumstances, the reduction in firm 1's output exceeds the

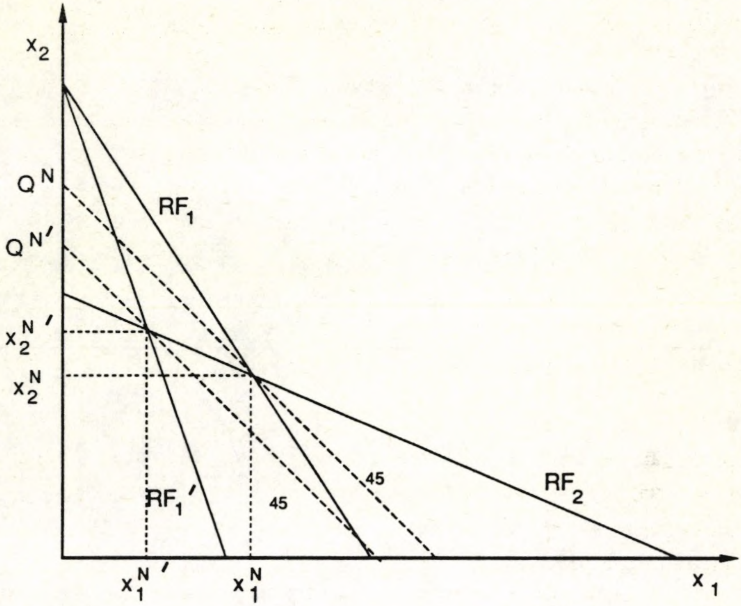



Figure 6: Effects of an increase in firm 1's risk aversion ($\Delta\theta_1$) on output

production expansion of the other firm. As a consequence total output decreases.

Proposition 4. *A mean-preserving increase in the variability of demand* ($\Delta\beta$) *leads to an output reduction of at least one firm, and reduces the total amount of output.* (Proof is given in the appendix).

As illustrated in Figure 7, an increase in $\beta$ (which has similar effects than an increase in $\theta_i$ in which concerns the movements of the reaction function curves) might have three different outcomes. Figure 7a shows the case where both firms decrease output and Figures 7b and 7c the cases where only one firm decreases output. In all three cases, and by the same reasoning used in the discussion of Proposition 2, the total output supplied in the market decreases.

Again, for a given expected profitability, firms are obliged, at the initial equilibrium

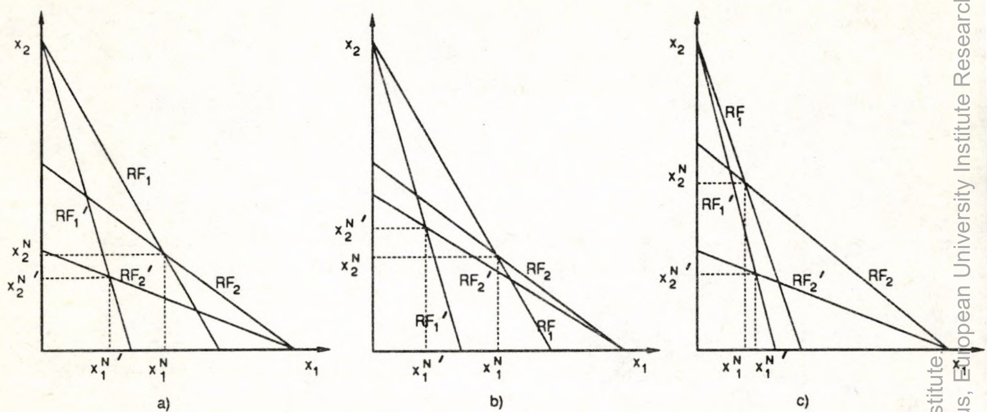



Figure 7: Effects of a mean-preserving increase
in the variability of demand ($\Delta\beta$) on output

$(x_1^N, x_2^N)$, to bear more risk than desired, having then, as we discussed already, an incentive to cut back production. The fact that one of the duopolists may decrease its production by a very large amount (because he cares relatively more about risk than expected profitability as compared to its rival) may lead to an increase in output by the other firm (until its expected marginal profitability equals again its risk premium). Total output falls.

This proposition encompasses both the certainty and the uncertainty cases. The market size is larger under certainty than under uncertainty ($Q^* > Q^U$). The reduction of total output sold in the market induced by the introduction of uncertainty may be obtained by different combinations of $x_1$ and $x_2$. When the two firms have a similar attitude toward risk ($\theta_i \approx \theta_j$), the result of a reduced production associated with the introduction of risk obtained in the literature at the firm level carries over in the present framework.[4] However, when the risk aversion coefficients are sufficiently different, there is a threshold beyond which one firm reacts to uncertainty by expanding

[4] Under imperfect competition firms do not always react to an increase in uncertainty by scaling down their output levels. But in the case of additive uncertainty , like the one considered here, they always do that. See discussion and additional references in Aiginger (1987).

its production as compared to the certainty case.

## 4. The Stackelberg Equilibrium

So far we have analyzed the market when both firms behave symmetrically. Let us now turn our attention to the case of leadership. Consider that firm 1 behaves as a Stackelberg leader and firm 2 as the follower. That is, firm 2 believes that firm 1 does not react to his own decision, while firm 1 takes into account the reaction function of firm 2.

Mathematically and graphically, this means that any equilibrium point $(x_1^L, x_2^F)$ will always belong to the reaction curve of firm 2; firm 1 selects one point on this reaction curve by maximizing its utility. While firm 2's reaction curve remains the same than in equation (6), firm 1's optimization problem becomes:

$$\max_{x_1} U_1 = (\mathrm{M}_1 - x_1 - x_2(x_1))x_1 - \beta\theta_1 x_1^2, \tag{8}$$

Lemma 2. *The iso–utility curves of the leader are concave and have their maximum along its reaction function, reaching the highest value (absolute maximum) at the monopoly point* $(x_1 = \mathrm{M}_1/2(1+\beta\theta_1), x_2 = 0)$.
(Proof is given in the appendix).

$U_1$ defines the family of iso–utility curves of the leader. Firm 1 chooses its production level by selecting the point on firm 2's reaction curve where it reaches the highest expected utility of profit. Substituing $RF_2$ of (6) into the first order condition of its maximization problem, we obtain the production functions:

$$x_1^L = \frac{2(1+\beta\theta_2)\mathrm{M}_1 - \mathrm{M}_2}{\Delta - 1}, \tag{9}$$

$$x_2^F = \frac{\Delta\mathrm{M}_2 - 2(1+\beta\theta_2)\mathrm{M}_1}{2(1+\beta\theta_2)(\Delta - 1)}. \tag{10}$$

Comparing the new supply function of firm 1 $(x_1^L)$, given by (9), with the one obtained under a Cournot strategy $(x_1^N)$ given by (7), we can write:

$$x_1^L = \frac{\Delta}{\Delta - 1} x_1^N,$$

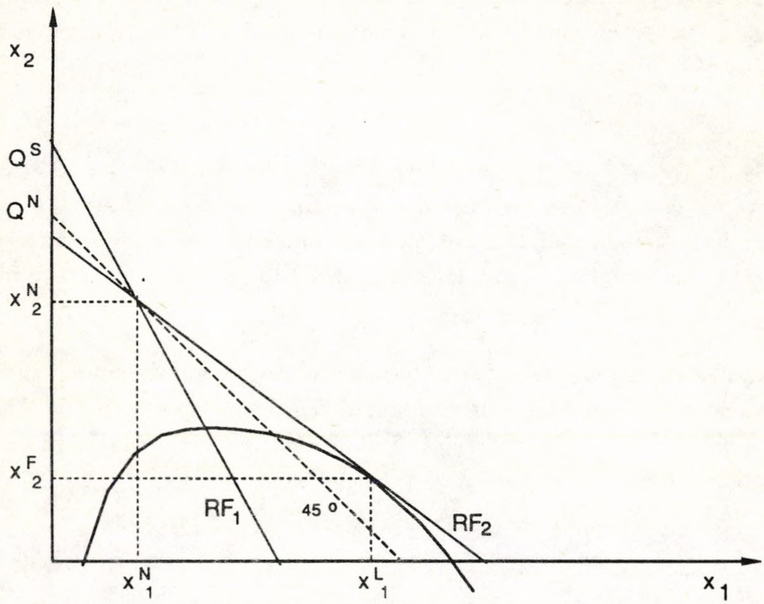



Figure 8: Stackelberg equilibrium

which proves that $x_1^L > x_1^N$. We can then use Lemma 1 in order to prove that $x_2^F < x_2^N$ and $Q^S > Q^N$, or else direct comparison of (7) and (10) proves the result.

The new equilibrium point satisfies by definition — see Friedman (1977) — the Cournot reaction function of the follower and the first order condition of the maximization problem of the leader. The conjectural variations of the follower and the leader are zero and the slope of the follower's reaction function, respectively. This is obtained in Figure 8 where the iso-utility curves are drawn in the output space.

Production levels of both firms are different under Cournot and Stackelberg behaviour. Let us define the gain from leadership and the loss from followership, originating two different sets of benchmarks.

On one hand, the advantage of leadership can be measured by the additional ouptut produced by the leader over the Cournot output: $x_1^L - x_1^N$. On the other hand, in case of an asymmetric behaviour, the gain from leadership can be defined by the difference between output levels under leadership and followership: $x_1^L - x_1^F$. From (7), (8) and (10), these two measures are:

$$GL_1 = x_1^L - x_1^N = \frac{2(1+\beta\theta_2)\mathrm{M}_1 - \mathrm{M}_2}{\Delta(\Delta-1)} = \frac{1}{\Delta} x_1^L > 0\,,$$
$$GL_2 = x_1^L - x_1^F = \frac{\mathrm{M}_1}{2(1+\beta\theta_1)(\Delta-1)} > 0\,. \tag{11}$$

The loss from followership can be expressed in the same way:

$$LF_1 = x_2^N - x_2^F = \frac{2(1+\beta\theta_2)\mathrm{M}_1 - \Delta\mathrm{M}_2}{2(1+\beta\theta_2)\Delta(\Delta-1)} > 0\,,$$
$$LF_2 = x_2^L - x_2^F = \frac{\mathrm{M}_2}{2(1+\beta\theta_2)(\Delta-1)} > 0\,. \tag{12}$$

Given the relationship between $x_1^L$ and $x_1^N$ and between $x_2^N$ and $x_2^F$, it can be shown that $GL_1 < GL_2$ and $LF_1 < LF_2$. As total output under Stackelberg exceeds that under Cournot, $GL_1$ is always larger than $LF_1$.

One crucial question in the analysis of Stakelberg equilibria is which of the two firms will be the leader. The traditional answer is that the most profitable (competitive) firm assumes this role. In this paper we are able to present an alternative to this answer.

It is clear from (11) and (12) that both firms benefit from leadership: the leader always reaches an higher level of utility than that of the corresponding Nash outcome.[5] Notice also that the leader produces more than the follower and faces the same price. Therefore, the situation of reluctant leadership, which arises in some models of non-cooperative behaviour (*v.g.*, the literature on macroeconomic policy coordination) is not present here — both firms are willing to be the leader. One may argue that it is the firm with the highest gain from leadership that will perform this role, since this

[5] Otherwise, this outcome would be the solution of (8).

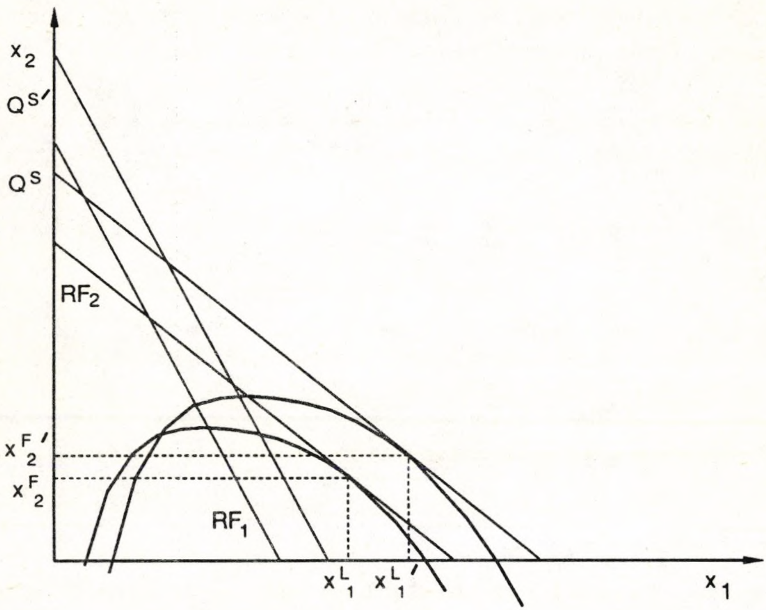



Figure 9: Effects of a spread-preserving increase in demand ($\Delta\bar{\alpha}$) on output

firm is readier to struggle for it. Using the above definitions, this condition implies that firm 1 is leader if and only if:

$$GL_1 > LF_2 \iff \frac{2(C_2 - C_1) + 2\beta(\theta_2 - \theta_1)(\mathrm{M} + \bar{\alpha}) + 2\beta(\theta_1 C_2 - \theta_2 C_1)}{\Delta^2 - 1} > 0\,. \quad (13)$$

If there is no uncertainty or if the risk aversion coefficients are equal, we get the usual result that there is an incentive for the most efficient firm to assume leadership. However, when the degrees of risk aversion are different and $\beta \neq 0$, this incentive depends on the interplay of costs and attitudes toward risk. In the limit, attitudes toward risk may outweigh cost considerations.

Proposition 5. *A spread-preserving increase in demand ($\Delta\bar{\alpha}$) leads to an increase of production of both firms and, consequently, to an increase in total output.*
(Proof is given in the appendix).

An increase in $\bar{\alpha}$ shifts the Cournot reaction functions upwards (see Proposition 2). The iso-utility curves change as depicted in Figure 9; at the new equilibrium point production of both firms increases and, accordingly, total ouput rises. An increase in expected demand raises the profitability of both firms and leads to higher output levels. Reasoning in the same way as in the case of Proposition 2, one can explain why the increase in ouput is smaller than the initial increase in profitability.

Comparing the results obtained here to those derived under Cournot behaviour, it can be noticed that an identical increase in demand leads to a larger expansion of production for the leader and a smaller expansion for the follower (*i.e.*, $\partial GL_1/\partial\bar{\alpha} > 0$ and $\partial LF_1/\partial\bar{\alpha} > 0$ — remember that $LF_1$ was defined as $x_2^N - x_2^F$), and a larger output expansion. In addition, the benefits accruing to firm 1 (leader) from the increase in output are more likely to be higher than those accruing to firm 2 than under Cournot. As equation (A15) shows, firm 2's output increases by more than firm 1's only under a more stringent condition than that given by (A7).

The alternative definitions $GL_2$ and $LF_2$ are increasing in $\bar{\alpha}$, that is, as expected demand increases the incentive to be the leader rises, as does the desincentive to accept followership. One can therefore reasonably argue that each firm seeks to be the leader (we noted above a condition linking cost and risk aversion of firms to the attribution of a role of leadership). A change in $\bar{\alpha}$ alters the relative strength of the two firms in their struggle for leadership; as shown by equation (A15), it leads to gains and losses that depend only on the relative magnitude of the risk aversion coefficients (under constant marginal costs). If the current leader (firm 1) is more risk averse than the follower (firm 2), an increase in $\bar{\alpha}$ tends to destabilize the market hierarchy because it rises the cost of followership borne by firm 2; moreover, output of firm 2 increases by more than that of firm 1, as shown in (A16). This outlines a framework for the analysis of the stability of leadership.

Proposition 6. *(i) An increase in the Stackelberg leader's risk aversion* ($\Delta\theta_1$) *leads to a reduction of its own production, an increase in output of the follower, and a reduction in the total quantity supplied to the market, (ii) an increase in the Stackelberg follower risk aversion* ($\Delta\theta_2$) *leads to a reduction of its own production, an ambiguous effect on the leader's production, and a reduction on the total output supplied to the market.* (Proof is given in the appendix).

In Figure 10 we consider graphically the effects of an increase in $\theta_1$, the leader's risk

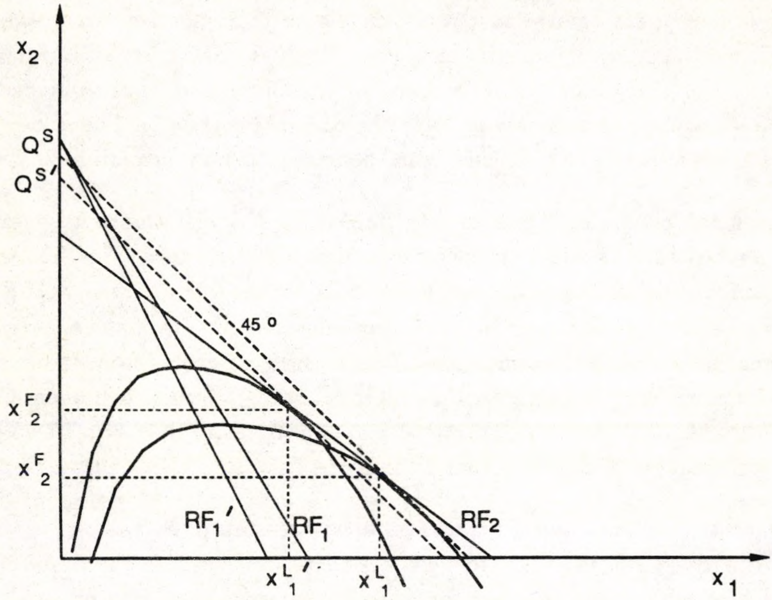



Figure 10: Effects of an increase in firm 1's risk aversion ($\Delta\theta_1$) on output

aversion coefficient. While $RF_2$ is left unaltered, there is a displacement and a change in the shape of the leader's iso–utility curves. An increase in $\theta_1$ moves these curves, that become more lump–shaped, to the left since firm 1 is now more cautious about the volatility of its profits than before. In other words, at any level of utility the firm prefers to cut sales. This induces an increase in the follower's profitability and, therefore, in its output level, dampening the effects of the output reduction of firm 1 on the market price and forcing the latter to cut production even further.

At the new equilibrium, the follower's output is higher and both the market output and the leader's are smaller, a pattern of adjustment that is similar to that observed in the previous section. Using (11) and (12), we may compare the results obtained on $GL$ and $LF$. When firm 1 becomes more risk averse, its gains from being the leader decrease ($GL_1$ becomes smaller) — in other words, the output reduction following an increase in $\theta_1$ is larger under Stackelberg than under Cournot — as do its losses from followership ($LF_1$ becomes smaller, too). Under the alternative definitions $GL_2$ and $LF_2$, an increase in $\theta_1$ reduces the asymmetry arising from the Stackelberg behavioural assumptions.

Figure 11 depicts the effects of an increase of the follower's risk aversion coefficient, $\theta_2$; Figure 11a presents the case where the leader's output rises, while Figure 11b presents the case where it decreases. The effect on the leader's production is ambiguous, which can be explained as follows. Two opposing mechanism are at work: loosely speaking, they can be labelled (i) substitution effect and (ii) income effect. An increase in $\theta_2$ unambiguously increases the market price because it lowers firm 2's output. Firm 1 is therefore "richer" and has an incentive to raise its output — this is the income effect (IE in Figure 11), which is always positive. However, the equilibrium now is on the new reaction function of firm 2, leading to a displacement along a new utility curve of firm 1 (closer to the monopoly production point) — this is the substitution effect (SE), which is always negative.[6] Depending on the magnitude of these two effects, firm 1's production may rise or fall; however, industry output decreases.

Proposition 7. *A mean-preserving increase in the variability of demand ($\Delta\beta$) leads to an output reduction of at least one firm, and reduces total output supplied in the market.*
(Proof is given in the appendix).

Figure 12 depicts the various possibilities that arise when the level of uncertainty increases. In Figure 12b both firms reduce their output levels, while in Figures 12a and 12c only one cuts production. Independently of firms' reactions, total output decreases.

At the initial equilibrium, both firms bear more risk than they want to, having an incentive to scale down production. Depending on the relative magnitude of risk aversion, three cases may arise. If both firms exhibit similar risk aversion, the output levels of both are reduced. However, if they put very different weights on the risk term of their utility functions, the more risk averse firm reduces output by such an amount that the corresponding market price increase leads the other firm to rise output. In

[6] Notice that this displacement is always to the left because the slope of firm 2's reaction function is now smaller in absolute value. At the equilibrium point, the slope of the utility curve of the leader has also to decrease.

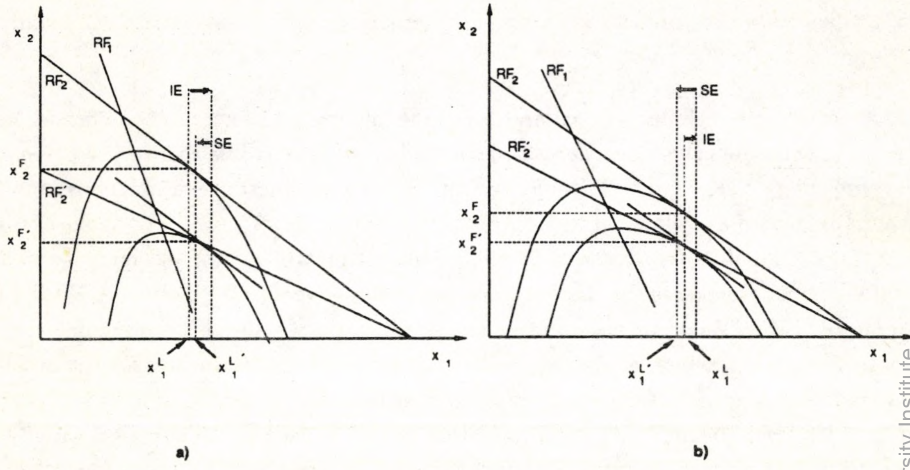


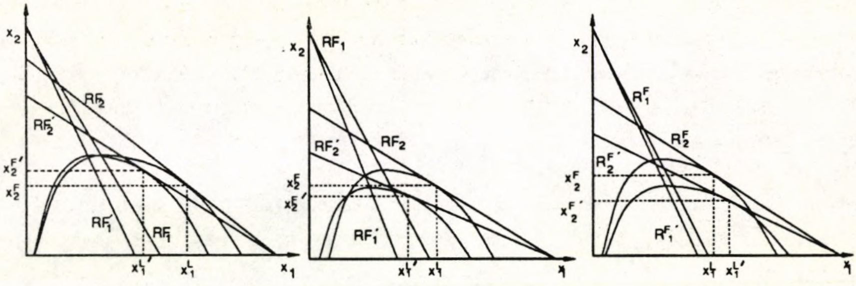

Figure 11: Effects of an increase in
firm 2's risk aversion ($\Delta\theta_2$) on output

this case, for the latter firm the increase in uncertainty is more than offset by a rise in profitability, leading to a rise in output.

## Market structure analysis

As we saw in Proposition 1 there are situations where only one firm participates in the market, acting as a monopolist. This was illustrated in Figure 4: if $M_2 \geq 2(1+\beta\theta_2)M_1$, firm 2 is the sole supplier. Conversely, it is out of the market (produces a zero output if $M_1 \geq 2(1+\beta\theta_1)M_2$.

The factors determining which firm stays in and which firm goes out of the market are their relative profitability and risk aversion, and also the magnitude of uncertainty affecting the market. One firm will be the sole producer if its profitability and/or its risk aversion is sufficiently higher/smaller than that of the other firm. In this case, it is not worthwhile for its competitor to produce at any level of output. However, a




a) b) c)

Figure 12: Effects of a mean-preserving increase in the variability of demand ($\Delta\beta$) on output

change in market conditions and attitudes toward risk may bring about a change in market structure with the entry of a second firm. In other words, the monopoly may be replaced by a duopoly.[7]

This is often the case in emerging markets, which are generally caracterised by a high level of demand uncertainty. In such cases firms are likely to hold different views on market conditions and this may lead to differences in risk aversion and behaviour. One firm (the most competitive or the less risk averse) is present in the market and behaves as a monopolist. Afterwards, as market conditions are more thoroughly known and as production turns out to be profitable, the established firm faces the potential entry of competitors. This firm might assume some kind of leadership when others join the market.[8] We therefore concentrate our attention on a Stackelberg framework, if entry does occur.

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[7] We assume that there is only one potential entrant. Considering more than one entrant increases analytical complexity without significantly changing the analysis.

[8] This assumption is reasonable at least for small changes in the parameters. Early treatments of entry assumed Stackelberg leadership of the established firm on the basis of the Bain–Sylos postulate or preentry commitments, such as investment decisions (see Dixit, 1980).

Moreover, due to the static nature of the model used in this paper, we restrict our attention to "static entry", allowing the number of firms in the market (either one or two) to be endogenous.[9] The assumption of Stackelberg leadership by the established firm creates, as we shall see later, an "innocent entry barrier" (Salop, 1979), in the sense that this barrier results from profit maximization and not from some type of strategic behaviour.

Consider for instance an increase in expected demand ($d\bar{\alpha} > 0$), when firm 1 is the monopolist. Firm 2's profitability goes up, and therefore it may have an incentive to enter the market.

Proposition 8. *A spread-preserving increase in demand* ($\Delta\bar{\alpha}$) *leads to a change in market structure from a monopoly to a duopoly provided that* $\Delta M_2 > 2(1+\beta\theta_2)M_1$ *holds for the new value of* $\bar{\alpha}$.
(proof is given in the appendix)

Obviously, firm 1 always prefers to keep its monopoly position, benefiting from the high price. However, this affects positively the expected profitability of a potential entrant. Deterring entry may turn out to be too costly because it may require a significant reduction in price, lowering the leader's profits below its optimal (pre-deterrence) level. In this case, firm 1 prefers a situation of duopoly, instead of a monopoly.

Defining the concept of "$\alpha$–expansion path" as the equilibrium output levels of both firms for the various values of $\bar{\alpha}$, it can be shown that:

Proposition 9. *The* $\bar{\alpha}$*–expansion path is a positively sloped straight line for positive output levels of both firms.*
(proof is given in the appendix)

The $\alpha$–expansion path is the set of all equilibrium points, under leadership of firm 1, for the relevant values of $\bar{\alpha}$. If $\bar{\alpha}$ is small enough, firm 2 has a null output level,

9 As argued in Friedman (1981), this is not studying entry but rather determining wether two firms can coexist or not. Thus, we are analysing here the effects of changes in the random demand process or in risk aversion on the number of firms in the market.

a sufficiently high increase in $\bar{\alpha}$, increasing the profitability of this firm, leads to a strictly positive output level. Further increases in $\bar{\alpha}$ bring about rises in both firms' output levels (see Proposition 5), and therefore define a direct relationship between $x_1$ and $x_2$.

**Appendix**

**1.** Consider that the output level of firm $j$ is fixed at $\bar{x}_j$. The optimization problem of firm $i$ then becomes:

$$\max_{x_i} (\mathrm{M} - x_i - \bar{x}_j + \bar{\alpha})x_i - C_i(x_i) - \beta\theta_i x_i^2. \tag{A1}$$

Under certainty the last term drops out and the output level is obtained when

$$\mathrm{M} - 2x_i - \bar{x}_j + \bar{\alpha} = C_i'. \tag{A2}$$

The term on the LHS of the equality is the marginal revenue ($R'$) and the one on the RHS is the marginal cost ($C'$). The optimum output level is reached at the point where marginal revenue equals marginal cost.

Under uncertainty, maximization leads to:

$$(\mathrm{M} - 2x_i - \bar{x}_j + \bar{\alpha}) = C_i' + 2\beta\theta_i x_i. \tag{A3}$$

The firm no longer equates marginal revenue to marginal cost. The last term on the RHS of (A3) is the risk-premium required by the firm to produce $x_i^U$ (as represented in Figure 2 in the text). Given risk aversion, this risk-premium is strictly positive and, therefore, production under uncertainty is strictly smaller than production under certainty.

**2. Proof of Lemma 1.**

The slopes of the reaction curves are given by:

$$\left.\frac{\mathrm{d}x_2}{\mathrm{d}x_1}\right|_{RF_2} = -\frac{1}{2(1+\beta\theta_2)} \quad \text{and} \quad \left.\frac{\mathrm{d}x_2}{\mathrm{d}x_1}\right|_{RF_1} = -2(1+\beta\theta_1). \tag{A4}$$

It is readily seen that :

$$\left.\frac{\mathrm{d}x_2}{\mathrm{d}x_1}\right|_{RF_1} < -1 < \left.\frac{\mathrm{d}x_2}{\mathrm{d}x_1}\right|_{RF_2} < 0,$$

which follows from the non-negativity of $\beta$ and $\theta_i$.

**3. Proof of Proposition 1.**

The equilibrium is determined by (6), which in matrix form reads:

$$\begin{pmatrix} 2(1+\beta\theta_1) & 1 \\ 1 & 2(1+\beta\theta_2) \end{pmatrix} \begin{pmatrix} x_1^N \\ x_2^N \end{pmatrix} = \begin{pmatrix} \mathrm{M}_1 \\ \mathrm{M}_2 \end{pmatrix}.$$

Since the matrix on the LHS has a non-zero determinant ($\Delta > 0$) and the system is nonhomogeneous, it has a unique non-trivial solution. $x_1^N$ and $x_2^N$ are indeed uniquely defined (see equation (7)). Moreover, $x_1^N$ and $x_2^N$ must also be non-negative, which happens if and only if:

$$x_1^N = \Delta^{-1}[2(1+\beta\theta_2)\mathrm{M}_1 - \mathrm{M}_2] \geq 0 \iff \mathrm{M}_1 \geq \mathrm{M}_2/2(1+\beta\theta_2)$$

$$x_2^N = \Delta^{-1}[2(1+\beta\theta_1)\mathrm{M}_2 - \mathrm{M}_1] \geq 0 \iff \mathrm{M}_2 \geq \mathrm{M}_1/2(1+\beta\theta_1)$$

If any of these conditions is not met we are in presence of a monopoly (in which case we would have as well a unique equilibrium). Assume that $\mathrm{M}_1 < \mathrm{M}_2/2(1+\beta\theta_2)$. If firm 2 were alone in the market, it would produce at the level that solves the following optimization problem:

$$\max_{x_2} \mathrm{M}_2 x_2 - (1+\beta\theta_2)x_2^2$$

yielding:

$$x_2^M = \frac{\mathrm{M}_2}{2(1+\beta\theta_2)} .$$

Producing at this level yields the highest possible value for firm 2's profits, and can only be achieved if $x_1 = 0$. When $x_2^M$ is supplied to the market, the expected price becomes:

$$p = \mathrm{M}_1 - \frac{\mathrm{M}_2}{2(1+\beta\theta_1)} + C_1 < C_1 ,$$

and since for firm 1 marginal revenue is smaller than marginal cost, $x_1 = 0$. The same argument applies when $\mathrm{M}_2 < \mathrm{M}_1/2(1+\beta\theta_1)$, in which case firm 2 is out of the market.

## 4. Proof of Proposition 2.

Using equations (6), we obtain:

$$\frac{\partial RF_1}{\partial \bar{\alpha}} = \frac{1}{2(1+\beta\theta_1)} > 0 \text{ and } \frac{\partial RF_2}{\partial \bar{\alpha}} = \frac{1}{2(1+\beta\theta_2)} > 0 . \qquad (A5)$$

A spread-preserving increase in demand leads, therefore, to an upward shift of both reaction schedules. Output effects are derived from equation (7):

$$\frac{\partial x_i^N}{\partial \bar{\alpha}} = \frac{1}{\Delta}(1+2\beta\theta_j)\left\{\begin{matrix} > 0 \\ < 1 \end{matrix}\right. \text{ and } \frac{\partial Q^N}{\partial \bar{\alpha}} = \frac{2}{\Delta}[1+\beta(\theta_1+\theta_2)]\left\{\begin{matrix} > 0 \\ < 1 \end{matrix}\right. . \qquad (A6)$$

Starting from a situation where production of both firms is positive, an increase in $\bar{\alpha}$ leads to an increase of their production and, consequently, to an increase in total output. Moreover, these derivatives are less than unity. We can, therefore, conclude that an increase in $\bar{\alpha}$ leads to a less than proportional increase of either firm's production and of total output.

From (A6), we get:

$$\frac{\partial(x_i^N - x_j^N)}{\partial\bar{\alpha}} = \frac{2\beta}{\Delta}(\theta_j - \theta_i). \qquad (A7)$$

A spread preserving increase in demand leads to an output expansion of both firms. The share of each firm in this expansion depends only on the respective degree of risk aversion. Independently of their marginal costs (assumed constant throughout the paper), the firm with a lower risk aversion coefficient increases its production by more than its rival.

## 5. Proof of Proposition 3.

Using equation(6), we obtain:

$$\frac{\partial \left.\frac{\mathrm{d}x_2}{\mathrm{d}x_1}\right|_{RF_1}}{\partial\theta_1} = -2\beta < 0\,, \qquad \frac{\partial \left.\frac{\mathrm{d}x_2}{\mathrm{d}x_1}\right|_{RF_1}}{\partial\theta_2} = 0 \quad \text{and}$$

$$\frac{\partial \left.\frac{\mathrm{d}x_2}{\mathrm{d}x_1}\right|_{RF_2}}{\partial\theta_1} = 0\,, \qquad \frac{\partial \left.\frac{\mathrm{d}x_2}{\mathrm{d}x_1}\right|_{RF_2}}{\partial\theta_2} = \frac{\beta}{2(1+\beta\theta_2)^2} > 0. \qquad (A8)$$

An increase in firm $i$'s risk aversion coefficient leaves its rival's reaction function unchanged and moves its own reaction curve toward the origin by changing its slope. It is readily seen that the point $(x_i = 0, x_j = \mathrm{M}_i)$ belongs to the new and the old reaction functions.

Starting from a position where production of both firms is positive, an increase in $\theta_i$ leads to a decrease in firm $i$'s output, an increase in firm $j$'s output, and a decrease in total output. By using (7), we get:

$$\frac{\partial x_i^N}{\partial\theta_i} = -\frac{4\beta(1+\beta\theta_j)}{\Delta}x_i^N < 0\,, \quad \frac{\partial x_j^N}{\partial\theta_i} = \frac{2\beta}{\Delta}x_j^N > 0\,, \quad \text{and}$$

$$\frac{\partial Q^N}{\partial\theta_i} = -\frac{2\beta(1+2\beta\theta_j)}{\Delta}x_i^N < 0. \qquad (A9)$$

## 6. Proof of Proposition 4.

Using equation (6), we obtain:

$$\frac{\partial \left.\frac{\mathrm{d}x_2}{\mathrm{d}x_1}\right|_{RF_1}}{\partial\beta} = -2\theta_1 < 0 \quad \text{and} \quad \frac{\partial \left.\frac{\mathrm{d}x_2}{\mathrm{d}x_1}\right|_{RF_2}}{\partial\beta} = \frac{\theta_2}{2(1+\beta\theta_2)^2} > 0.$$

An increase in $\beta$ alters the two reaction functions by changing their slopes. As it was the case with changes in $\theta_i$, an increase in $\beta$ moves the reaction schedules toward the origin around the points $(x_i = 0, x_j = \mathrm{M}_i)$.

By using (7) and Lemma 1, we get:

$$\frac{\partial x_i^N}{\partial \beta} = \frac{2}{\Delta}[\theta_j x_j^N - 2\theta_i(1+\beta\theta_j)x_i^N] \lesseqgtr 0 \ , \quad \frac{\partial x_j^N}{\partial \beta} = \frac{2}{\Delta}[\theta_i x_i^N - 2\theta_j(1+\beta\theta_i)x_j^N] \gtreqless 0 \ ,$$

$$\text{and } \frac{\partial Q^N}{\partial \beta} = -\frac{2}{\Delta}.[(1+2\beta\theta_i)\theta_j x_j^N + (1+2\beta\theta_j)\theta_i x_i^N] < 0 \,. \qquad (A10)$$

The sign of the first two expressions is ambiguous and depends on the relative size of risk aversion and costs of the two firms; nevertheless at least one of the partial derivatives is negative. Total output is unambiguously reduced.

## 7. Proof of Lemma 2.

From (8), we can write:

$$\left.\frac{\mathrm{d}x_2}{\mathrm{d}x_1}\right|_{\bar{U}} = \frac{\mathrm{M}_1 - x_2 - 2(1+\beta\theta_1)x_1}{x_1} \gtrless 0 \,.$$

iso–utility curves $\bar{U}$ are increasing (resp. decreasing) in the output space $(x_1, x_2)$ when

$$\frac{\mathrm{M}_1 - x_2}{2(1+\beta\theta_1)} > x_1 \text{ (resp. <)} \,, \qquad (A11)$$

and

$$\left.\frac{\mathrm{d}}{\mathrm{d}x_1}\left(\frac{\mathrm{d}x_2}{\mathrm{d}x_1}\right)\right|_{\bar{U}} = -\frac{\mathrm{M}_1 - x_2}{x_1^2} < 0 \,, \qquad (A12)$$

which proves that the iso–utility curves are concave.

If we take $\mathrm{d}x_2/\mathrm{d}x_1 = 0$ (the necessary condition for a maximum) we get the expression of firm 1's reaction curve as in (6). The lower the iso–utility curves, the higher the level of utility. This comes from the following reasoning: if we take firm 1's output as fixed, say at $\bar{x}_1$, then the lower the iso–utility curve, the lower the value of $x_2$ corresponding to $\bar{x}_1$, hence the higher the remaining demand and marginal revenue curves of firm 1 and, consequently, the higher its profits. As the risk term does not change (the quantity of $x_1$ remains unchanged) the higher is the utility.

The highest level of utility is achieved when $x_2 = 0$, where the remaining demand is equal to total market demand and, hence, firm 1 behaves as a monopolist.

**8. Proof of Proposition 5.**

A spread-preserving increase in demand affects firm 2's (follower) reaction function in the same way than in proposition 2: the $RF_2$ curve shifts upwards. For firm 1 (leader) the points where the iso-utility curves reached their maximum are moving rightwards. From (9) and (10), we can compute the output effects:

$$\frac{\partial x_1^L}{\partial\bar{\alpha}} = \frac{(1+2\beta\theta_2)}{\Delta-1}\begin{cases}>0\\<1\end{cases}, \quad \frac{\partial x_2^F}{\partial\bar{\alpha}} = \frac{2(1+\beta\theta_2)(1+2\beta\theta_1)-1}{2(1+\beta\theta_2)(\Delta-1)}\begin{cases}>0\\<1\end{cases} \quad \text{and}$$

$$\frac{\partial Q^S}{\partial\bar{\alpha}} = \frac{4(1+\beta\theta_2)(1+\beta(\theta_1+\theta_2))-1}{2(1+\beta\theta_2)(\Delta-1)}\begin{cases}>0\\<1\end{cases}. \qquad (A13)$$

An increase in $\bar{\alpha}$ leads to an increase of production of both firms and, consequently, to an increase in total output. Changes in production resulting from an incrase in $\bar{\alpha}$ are stronger than under Cournot competition, as can be seen in (A14):

$$\frac{\partial(Q^S-Q^N)}{\partial\bar{\alpha}} = \frac{(1+2\beta\theta_2)^2}{2(1+\beta\theta_2)\Delta(\Delta-1)} > 0\,. \qquad (A14)$$

The shares of the two firms in the output expansion are determined by the following expression:

$$\frac{\partial(x_1^L-x_2^F)}{\partial\bar{\alpha}} = \frac{4\beta(1+\beta\theta_2)(\theta_2-\theta_1)+1}{2(1+\beta\theta_2)(\Delta-1)}\,. \qquad (A15)$$

Hence, the condition for an increase in the follower's output to be larger than that of the leader is:

$$\frac{\partial x_2^F}{\partial\bar{\alpha}} > \frac{\partial x_1^L}{\partial\bar{\alpha}} \iff \theta_1 > \theta_2 + \frac{1}{4\beta(1+\beta\theta_2)}\,. \qquad (A16)$$

**9. Proof of Proposition 6.**

As shown in (A8), the follower's reaction curve is left unchanged when $\theta_1$ increases. The output effects are given by the following equations:

$$\frac{\partial x_1^L}{\partial\theta_1} = -\frac{4\beta(1+\beta\theta_2)}{\Delta-1}x_1^L < 0\,, \quad \frac{\partial x_2^F}{\partial\theta_1} = \frac{2\beta}{\Delta-1}x_1^L\begin{cases}>0\\<1\end{cases}, \quad \text{and}$$

$$\frac{\partial Q^S}{\partial\theta_1} = -\frac{2\beta(1+2\beta\theta_2)}{\Delta-1}x_1^L < 0. \qquad (A17)$$

The effects on the gains from leadership and loss from followership are as follows:

$$\frac{\partial GL_1}{\partial\theta_1} = -\frac{4\beta(2\Delta-1)(1+\beta\theta_2)}{\Delta-1}x_1^L < 0\,, \quad \frac{\partial GL_2}{\partial\theta_1} = \frac{\beta\Delta M_1}{(1+\beta\theta_1)^2(\Delta-1)^2} < 0 \quad \text{and}$$

$$\frac{\partial LF_1}{\partial \theta_1} = -\frac{2\beta(2\Delta-1)x_1^N}{\Delta(\Delta-1)^2}x_1^L < 0 \ , \ \ \frac{\partial LF_2}{\partial \theta_1} = -\frac{2\beta \mathrm{M}_2}{(\Delta-1)^2} < 0 \qquad (A18)$$

An increase in $\theta_2$ moves the follower's reaction function toward the origin around the point $(\mathrm{M}_2, 0)$ like in Proposition 3. The output effects are:

$$\frac{\partial x_1^L}{\partial \theta_2} = \frac{2\beta}{\Delta-1}.\left[x_2^L - \frac{M_1}{\Delta-1}\right] \gtrless 0$$

$$\frac{\partial x_2^F}{\partial \theta_2} = \frac{\beta}{(1+\beta\theta_2)(\Delta-1)}.\left(\frac{\Delta \mathrm{M}_2}{(1+\beta\theta_2)(\Delta-1)} - (\Delta+1)x_2^L\right) < 0 \qquad (A19)$$

$$\frac{\partial Q^S}{\partial \theta_2} < -\frac{\partial \Delta}{\partial \theta_2}\frac{1}{(\Delta-1)^2}\left(\frac{\mathrm{M}_2}{\Delta+1}\left[1 + \frac{4\beta\theta_2(1+\beta\theta_1)}{\Delta+1}\right]\right) < 0$$

## 10. Proof of Proposition 7.

From (9) and (10), the following partial derivatives can be computed:

$$\frac{\partial x_1^L}{\partial \beta} = -\frac{4}{\Delta-1}\theta_1(1+\beta\theta_2)x_1^L + \frac{4}{(\Delta-1)^2}\theta_2(1+\beta\theta_1)\mathrm{M}_2 \gtrless 0 \, ,$$

$$\frac{\partial x_2^F}{\partial \beta} = -\left(\frac{\Delta^2+1}{2(1+\beta\theta_2)^2}\theta_2 + 2\theta_1\right)\mathrm{M}_2 + (\Delta+1)\left(\frac{\theta_1}{1+\beta\theta_1} + \frac{\theta_2}{1+\beta\theta_2}\right)\mathrm{M}_1 \lesseqgtr 0 \, ,$$

$$\frac{\partial Q^S}{\partial \beta} < -\frac{1}{(\Delta-1)^2}\left(4\beta\theta_1\theta_2\Delta Q^N + \frac{\theta_2 \mathrm{M}_2}{(1+\beta\theta_2)^2}\left[1+\beta\theta_2(\Delta+1)\right]\right) < 0,$$

which proves Proposition 7.

## 11. Proof of Proposition 8.

Initially, $x_2^F = 0$, *i.e.*, $\mathrm{M}_2 \leq 2(1+\beta\theta_2)\mathrm{M}_1/\Delta$; firm 1 is in a monopolistic situation, producing (according to Lemma 2) $\mathrm{M}_1/2(1+\beta\theta_1)$. However, if the change in $\bar{\alpha}$ reverses the above inequality, $x_2^F > 0$ by (10). This completes the proof. Notice that firm 1 can only be a duopoly leader if:

$$\frac{2(1+\beta\theta_2)}{\Delta} < \frac{\mathrm{M}_2}{\mathrm{M}_1} < 2(1+\beta\theta_2) \, . \qquad (A20)$$

## 12. Proof of Proposition 9.

The points in the $\alpha$–expansion path are characterized by the equality between the slopes of $RF_2$ and the iso–utility curves of firm 1. A change in $\bar{\alpha}$ does not affect the former, but the slope of the iso–utility curve changes.

To remain on $RF_2$ after the change in $\bar{\alpha}$, changes in $x_1$ and $x_2$ must obbey:

$$dx_2 = \frac{d\bar{\alpha} - dx_1}{2(1+\beta\theta_2)} \; ;$$

optimallity (equality of slopes), on its turn, implies that:

$$dx_2 = d\bar{\alpha} - \frac{\Delta dx_1}{2(1+\beta\theta_2)} \; .$$

Solving this system and expressing $dx_2$ in terms of $dx_1$ yields:

$$\frac{dx_2}{dx_1} = \frac{1 + 2\beta\theta_2 + 4\beta\theta_1(1+\beta\theta_2)}{2(1+\beta\theta_2)(1+2\beta\theta_2)} > 0 \; .$$

This is the slope of the $\alpha$–expansion path. Notice that it is constant (independent of $x_1$, $x_2$ and $\bar{\alpha}$) and positive.

Furthermore, if (A20) holds both firms have non-zero output levels. In particular, 1 belongs to that interval. Let us assume that $\bar{\alpha}$ increases. Before the increase one of the following three cases has to hold:

(i) $M_1 = M_2$, which implies $M_1/M_2 = 1$. No matter how much $\bar{\alpha}$ increases, $M_1/M_2 = 1$ always holds and therefore both firms have always positive levels of ouptut.

(ii) $M_2 > M_1$; in this case, $M_1/M_2 < 2(1+\beta\theta_2)$ since firm 1 is the leader, which only makes sense if $x_1^L \neq 0$. $M_1/M_2$ tends to 1 by above as $\bar{\alpha} \to \infty$ because:

$$\frac{d(M_2/M_1)}{d\bar{\alpha}} < 0 \quad ; \quad \frac{d^2(M_2/M_1)}{d\bar{\alpha}^2} > 0 \quad .$$

Again, both firms always have positive levels of output.

(iii) $M_1 > M_2$, implying that initially $M_1/M_2 < 1$. $M_1/M_2$ tends to 1 by below as $\bar{\alpha} \to \infty$ since the derivatives computed above are now symmetric in sign to those presented above. Therefore if at the outset we have a duopoly, no matter the increase in $\bar{\alpha}$, the market remains a duopoly. If initially firm 1 is a monopolist, the market becomes a duopoly for a sufficient large increase in $\bar{\alpha}$. Only in this case the market structure changes.

**References**

AIGINGER, Karl, 1987, *Production and Decision Theory under Uncertainty*, (Basic Blackwell: London).

DIXIT, Avinash, 1979, "A Model of Duopoly Suggesting a Theory of Entry Barriers," *The Bell Journal of Economics*, 10(1):20–32.

DIXIT, Avinash, 1980, "The Role of Investment in Entry Deterrence," *The Economic Journal*, 90:95–106.

FRIEDMAN, James, 1977, *Oligopoly and the Theory of Games*, (North-Holland: Amsterdam).

FRIEDMAN, James, 1981, *Oligopoly Theory*, (Cambridge University Press: Cambridge).

HEY, John D., 1979, *Uncertainty in Microeconomics*, (Martin Robertson: Oxford).

KATZ, Eliakim, Jacob PAROUSH, and Nava KAHANA, 1982, "Price Uncertainty and the Price Discriminating Firm in International Trade," *International Economic Review*, 23( ):389–400.

ROTHSCHILD, Michael, and Joseph E. STIGLITZ, 1970, "Increasing Risk: 1. A Definition," *Journal of Economic Theory*, 2(3),225–243.

SALOP, Steven C., 1979, "Strategic Entry Deterrence," *American Economic Review*, Papers and Proceedings, 335–338.

VARIAN, Hal R., 1984, *Microeconomic Analysis*, 2nd edition, (Norton: New York).

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