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MARKET UNCERTAINTY: CORRELATED EQUILIBRIUM
AND SUNSPOT EQUILIBRIUM IN MARKET GAMES

by
James Peck * and Karl Shell **

* Northwestern University
** Cornell University

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BADIA FIESOLANA, SAN DOMENICO (FI)
1. Introduction and summary.

Uncertainty about the natural world is a source of uncertainty about economic outcomes. There are economic institutions — such as insurance companies — which help people to cope with their uncertainties about nature. These institutions can be thought of as dampening or even stabilizing the economic effects of uncertainty.

Uncertainty is not always dampened. In some instances, the effects of uncertainty are actually amplified by the economy. Indeed, economies can create uncertainties that have no source in the economic fundamentals (or in the natural world, for that matter). There are very good reasons when designing an economic mechanism to ask how the mechanism stands up to "shocks" and what uncertainties are generated by the mechanism itself. The macroeconomics literature on automatic stabilizers recognizes the possibility of a tradeoff between "efficiency" and "stability." There is an argument that "efficient" financial markets are not likely to promote "economic stability".

In the present study, we analyze the polar case in which economic fundamentals (such as tastes and endowments) are completely certain. If in this case, the economic outcomes are uncertain, either the economy created the uncertainty or it infinitely amplified a disturbance (such as "sunspots") from outside the economy. The first case, creation of uncertainty by the economic mechanism itself, clearly represents uncertainty endogenous to the economy. The second case, the "sunspot" case, is a little less clear. It represents both a special (or limiting) case of exogenous uncertainty and a type of endogenous uncertainty is which the economy selects "sunspot activity" as part of its allocation device.

Our analysis is based on the market-game model of Shubik and Shapley (see, Shubik (1973), Shapley (1976), Shapley and Shubik (1977), and Mas-Colell...

The present paper is clearly a son of last summer's paper (Peck-Shell (1985a)), but asymmetric information is stressed in this version. Also, more attention is paid here to the relationship between Correlated Equilibrium and Bayes-rational Equilibrium and to the dynamic aspects.

In Section 2, we present the Market Game $T$ and analyze the pure-strategy Nash equilibria to $T$. We establish the existence of an interior pure-strategy Nash equilibrium, i.e., a NE in which all markets are open, and the existence of an autarkic NE, i.e., a NE in which no market is open. These two equilibria are distinct if and only if endowments are not Pareto-optimal.

In Section 3, we define Correlated Equilibrium for the game $T$. This concept generalizes Pure Strategy Nash Equilibrium and Mixed Strategy Nash Equilibrium. The correlated equilibria to $T$ can be identified with self-fulfilling prophecies for $T$ after a hand-waving reference to Aumann (1985).

In Section 4, we define the Securities Game $\hat{T}$, an Arrow (1964) extension of $T$ to incorporate extrinsic uncertainty. It is shown that every correlated equilibrium allocation to $T$ is a pure-strategy NE allocation to $\hat{T}$. It is also shown that if and only if endowments are not Pareto-optimal,
there is a pure-strategy NE to \( \Gamma \) in which "sunspots" affect the allocation of resources.

In Section 5, we provide examples. In one example, we show that not every pure strategy NE allocation to \( \Gamma \) can be achieved as a Correlated Equilibrium allocation to \( \Gamma \): For at least some games, Correlated Equilibrium allocations to \( \Gamma \) form a proper subset of the pure-strategy NE allocations to \( \Gamma \). In a second example, we display a properly Correlated Equilibrium allocation to a market game \( \Gamma \), an equilibrium which is neither a randomization over pure-strategy NE allocations nor is a mixed strategy NE allocation.

The second example has some peculiarities. Less objectionable examples seem harder to construct. We conjecture that the difficulty in generating these examples is related to our overly severe and unrealistic bankruptcy rules.

In the remaining space, we begin consideration of important intertemporal aspects of this problem.


There are \( \ell + 1 \) goods: \( \ell \) commodities (or consumption goods), indexed by \( i = 1, \ldots, \ell \) and \( j = 1, \ldots, \ell \), and money. There are neither taxes nor transfers, so all money is "inside money", representing the private debt of the consumers. There are \( n \) consumers (or traders), indexed by \( h = 1, \ldots, n \) and \( k = 1, \ldots, n \). Consumer \( h \) is endowed with a positive amount of commodity \( i \), \( w_{h}^{i} \) for \( i = 1, \ldots, \ell \). If we denote by \( w_{h} \) the endowment vector \( (w_{h}^{1}, w_{h}^{2}, \ldots, w_{h}^{\ell}) \), then we have \( w_{h} \in \mathbb{R}_{++}^{\ell} \) for \( h = 1, \ldots, n \).

There are \( \ell \) trading posts. For each commodity, there is a single trading post on which the commodity is exchanged for money. Consumer \( h \)
supplies a nonnegative quantity of commodity $i$, $q^i_h$, at trading post $i$. He also supplies a nonnegative quantity of money, $b^i_h$, at trading post $i$. We say that $q^i_h$ is his offer (of commodity $i$) and that $b^i_h$ is his (money) bid (for commodity $i$). Let $b_h = (b^1_h, \ldots, b^i_h, \ldots, b^n_h)$ and $q_h = (q^1_h, \ldots, q^i_h, \ldots, q^n_h)$ denote (respectively) his bids and his offers. Offers must be made in terms of the physical commodities. Hence, offers cannot exceed endowments, i.e., we have $q^i_h \leq \omega^i_h$ for $i = 1, \ldots, i$. The strategy set $S_h$ of consumer $h$ is then given by

$$S_h = \{ (b_h, q_h) \in \mathbb{R}_{+}^{2i} \mid q_h \leq \omega_h \}.$$

The trading process is simple. The total amount of commodity $i$ which is offered, $\sum_{k=1}^{n} q^i_k$, is allocated to consumers in proportion to their shares of the bids for commodity $i$. Consumer $h$'s share of the bids at post $i$ is

$$\frac{b^i_h}{\sum_{k=1}^{n} b^i_k} \frac{\sum_{k=1}^{n} q^i_k}{\sum_{k=1}^{n} b^i_k}.$$

Thus, the gross receipts of commodity $i$ for consumer $h$ are

$$\frac{b^i_h}{\sum_{k=1}^{n} b^i_k} \frac{\sum_{k=1}^{n} q^i_k}{\sum_{k=1}^{n} b^i_k}$$

for $i = 1, \ldots, i$ and $h = 1, \ldots, n$. If all bids at post $i$ are zero, the ratio $(b^i_h / \sum_{k=1}^{n} b^i_k)$ is equal to $0/0$ and would appear to be indeterminate. We assume, however, that if there are no bids on post $i$ all offers on this post are "lost", i.e., no commodity is delivered. Thus, we take the fraction $(b^i_h / \sum_{k=1}^{n} b^i_k)$ to be zero if there are no positive bids at post $i$.

At trading post $i$, the money from bids, $\sum_{k=1}^{n} b^i_k$, is allocated to consumers in proportion to their offers of commodity $i$. Consumer $h$'s share
of the offers at post \(i\) is \(\left(\frac{q^i_h}{\sum_{k=1}^{k=n} q^i_k}\right)\). Thus the gross money receipts on post \(i\) for consumer \(h\) are

\[
\frac{q^i_h}{\sum_{k=1}^{k=n} q^i_k} \sum_{k=1}^{k=n} b^i_k
\]

for \(i = 1,\ldots,Jt\) and \(h = 1,\ldots,n\). If all offers of commodity \(i\) are zero, the ratio \(\left(\frac{q^i_h}{\sum_{k=1}^{k=n} q^i_k}\right)\) is equal to 0/0. We assume that if there are no offers on post \(i\), all money bids on the post are "lost". Thus, we take the fraction \(\left(\frac{q^i_h}{\sum_{k=1}^{k=n} q^i_k}\right)\) to be zero if there are no positive offers on post \(i\).

Consumers do not face liquidity constraints, i.e., constraints which restrict their debt issuance on any given market or proper subset of markets. Each consumer does face a single overall budget constraint, which he must meet or be punished. He is required to finance his bids (for commodities) by his offers (of commodities). The budget constraint for consumer \(h\) is

\[
\sum_{j=1}^{j=\ell} \left(\frac{q^j_h}{\sum_{k=1}^{k=n} q^j_k}\right) \sum_{k=1}^{k=n} b^j_k \geq \sum_{j=1}^{j=\ell} b^j_h
\]

for \(h = 1,\ldots,n\). The right-hand side of Inequality (2.1) is the sum of the dollars delivered by \(h\) in the form of bids to the trading posts. The left-hand side is the sum of the dollars delivered to \(h\) from the trading posts in payment for his commodity offers. The consumer is punished if he issues more money debt than he collects.

Let \(x^i_h\) denote the consumption of commodity \(i\) by consumer \(h\), and let \(x^h = (x^1_h,\ldots,x^i_h,\ldots,x^\ell_h)\) be his consumption vector. Assume that consumer \(k\) chooses the strategy \((b^k_k, q^k_k) \in \mathbb{R}^2_+\) for \(k = 1,\ldots,n\); then the consumption
of consumer $h$ is given by

$$x_h^i = \omega_h^i - q_h^i + \sum_{k=1}^{b_i} \frac{b_{k,n}}{k} q_k^i$$

if (2.1) is satisfied, and

$$x_h^i = 0$$

if (2.1) is not satisfied

for $i = 1, \ldots, t$ and $h = 1, \ldots, n$. Failure to meet budget constraint (2.1) leads to confiscation of all of the consumer's goods.

The consumption set of consumer $h$ is the nonnegative orthant

$$\{x_h \mid x_h \in \mathbb{R}_+^t\}.$$ His utility function, $u_h^i$, is strictly increasing, smooth and strictly concave on the strictly positive orthant $\mathbb{R}_+^t$. Also, the closure in $\mathbb{R}^t$ of each indifference surface from $\mathbb{R}_+^t$ is contained in $\mathbb{R}_+^t$. (This last assumption allows us to avoid some pesky boundary solutions.) The boundary of the consumption set, ($\mathbb{R}_+^t \setminus \mathbb{R}_+^t$), is also the indifference surface of least utility, so that (i) if we have $x_h \in \mathbb{R}_+^t \setminus \mathbb{R}_+^t$ and $y_h \in \mathbb{R}_+^t$, then we also have $u_h(x_h) = u_h(y_h) = u_h(0)$, and (ii) if we have $x_h \in \mathbb{R}_+^t \setminus \mathbb{R}_+^t$ and $y_h \in \mathbb{R}_+^t$, then we also have $u_h(x_h) > u_h(y_h) = u_h(0)$.

We have specified the strategy sets $S_h$, the outcomes $x_h$ (through Equation (2.2)), and the payoffs $u_h(x_h)$ for the Market Game $\Gamma$. We adopt the standard concept of Nash Equilibrium (NE).

Let $\sigma_h = (b_h, q_h)$ be a strategy in $S_h$. Define the set $S$ by

$$S = S_1 \times \ldots \times S_h \times \ldots \times S_n \subset (\mathbb{R}_+^t)^n.$$ Consider the strategies $\sigma = (\sigma_1, \ldots, \sigma_h, \ldots, \sigma_n) = ((b_1, q_1), \ldots, (b_h, q_h), \ldots, (b_n, q_n)) \in S$, ($\sigma | \sigma_h = (\sigma_1, \ldots, \sigma_{h-1}, \sigma_{h+1}, \ldots, \sigma_n) = ((b_1, q_1), \ldots, (b_{h-1}, q_{h-1}), (b_{h+1}, q_{h+1}), \ldots, (b_n, q_n)) \in S$, and $\sigma_h = ((b_1, q_1), \ldots, (b_{h-1}, q_{h-1}), (b_{h+1}, q_{h+1}), \ldots, (b_n, q_n)) \in S_1 \times \ldots \times S_{h-1} \times S_{h+1} \times \ldots \times S_n \subset (\mathbb{R}_+^t)^{n-1}$. From
Equation (2.2), we see that \( x_h \) is a function of the \( b' \)'s and \( q' \)'s, so that the outcome can be written as a function of the strategies \( \sigma \), namely \( x_h(\sigma) \).

2.3 Definition. A Nash Equilibrium strategy to the Market Game \( \Gamma \) is a \( \sigma \in S \) with the property

\[
u_h(x_h(\sigma)) = \max_{\sigma' \in S_h} \{u_h(x_h(\sigma|\sigma'))\}
\]

for \( h = 1, \ldots, n \). The corresponding NE allocation is \( x(\sigma) = (x_1(\sigma), \ldots, x_h(\sigma), \ldots, x_n(\sigma)) \in \mathbb{R}_n^n \).

We next establish that consumer \( h \)'s optimal response \( \sigma_h \) to the (equilibrium or disequilibrium) strategies of others is "individually rational".

2.4 Lemma. Let \( \sigma_h \) be consumer \( h \)'s best response to the strategies \( \sigma_{-h} \) in the market game \( \Gamma \), i.e.,

\[
u_h(x_h(\sigma)) = \max_{\sigma' \in S_h} \{u_h(x_h(\sigma|\sigma'))\}.
\]

Then, we have

\[
u_h(x_h(\sigma)) \geq u_h(\omega_h).
\]

Proof: If consumer \( h \) selects the trivial strategy \( \sigma_h' = (b'_h, q'_h) = (0,0) \), then we have \( x_h(\sigma|\sigma_h') = \omega_h \). Hence, if \( \sigma_h \) is the best response to \( \sigma_{-h} \), it follows that \( u_h(x_h(\sigma)) \geq u_h(x_h(\sigma|\sigma')) = u_h(\omega_h) \).
This shows that in equilibrium no consumer is punished, since we have
\[ u_h(x_h) > u_h(\omega_h) > u_h(0). \]

We next show that there is always a very simple NE for the market game \( \Gamma \). Later, we show that there are also other NE for \( \Gamma \).

2.5. Lemma. Let \( \sigma = (\sigma_1, \ldots, \sigma_h, \ldots, \sigma_n) \) be the trivial vector defined by
\[ \sigma_h = (b_{h1}, q_{h1}) = (0,0) \text{ for } h = 1, \ldots, n. \]
Then \( \sigma \) is a NE strategy for \( \Gamma \). The associated NE allocation \( x(\sigma) = (x_1(\sigma), \ldots, x_n(\sigma), \ldots, x_n(\sigma)) \) is defined by \( x_h(\sigma) = \omega_h \) for \( h = 1, \ldots, n. \)

**Proof:** Obvious.

In the autarkic NE, there are no bids and no offers. All markets are closed. We shall study existence of NE in which markets are open. We need definitions of an open (and a closed) market.

2.6. Definition. Let \( \sigma = ((b_{11}, q_{11}), \ldots, (b_{h1}, q_{h1}), \ldots, (b_{n1}, q_{n1})) \) be a NE vector of strategies in the Market Game \( \Gamma \). We say that market \( j \) is closed (resp. open) if and only if \( \sum_{k=1}^{n} b_{jk} = 0 \) (resp. \( \sum_{k=1}^{n} b_{jk} > 0 \)).

2.7. Lemma. Let \( \sigma = ((b_{11}, q_{11}), \ldots, (b_{h1}, q_{h1}), \ldots, (b_{n1}, q_{n1})) \) be a NE vector of strategies in the Market Game \( \Gamma \). Market \( j \) is closed (resp. open) if and only if \( \sum_{k=1}^{n} q_{jk} = 0 \) (resp. \( \sum_{k=1}^{n} q_{jk} > 0 \)).

Next, we define an interior NE strategy vector and then study its welfare and existence properties.
2.8. Definition. The strategy $\sigma = \{ (b_{h}, q_{h}) \}_{h=1}^{n} \in S$ is said to be an interior Nash Equilibrium to the market game $\Gamma$ if $\sigma$ is a NE for $\Gamma$ (Definition (2.3)) in which each of the $n$ markets is open (Definition (2.6)). The corresponding allocation $x(\sigma) = (x_{1}(\sigma), \ldots, x_{h}(\sigma), \ldots, x_{n}(\sigma)) \in \mathbb{R}_{++}^{n}$ is called an interior NE allocation of $\Gamma$.

2.9. Proposition. An interior NE allocation of $\Gamma$ is autarkic (i.e., $x_{h} = \omega_{h}$ for $h = 1, \ldots, n$) if and only if the endowment vector $\omega = (\omega_{1}, \ldots, \omega_{h}, \ldots, \omega_{n})$ is Pareto optimal. Furthermore, if $\omega$ is not Pareto optimal, an interior NE allocation of $\Gamma$, $x = (x_{1}, \ldots, x_{h}, \ldots, x_{n})$, must satisfy $u_{h}(x_{h}) \geq u_{h}(\omega_{h})$, with strict inequality for at least one $h$, $h = 1, \ldots, n$.

Proof: Let $x = (x_{1}, \ldots, x_{h}, \ldots, x_{n})$ be an interior NE allocation which is autarkic, i.e., $x = \omega = (\omega_{1}, \ldots, \omega_{h}, \ldots, \omega_{n})$. The interior first-order condition, which is necessary and sufficient for consumer $h$'s utility to be optimized given the strategies of the other consumers. Since $x_{h} = \omega_{h}$, we must have

\[
\frac{\partial u_{h}(x_{h})}{\partial x_{h}^i} = \sum_{k=1}^{n} \frac{b_{k}}{b_{k}} \left[ \sum_{k=1}^{n} q_{k} \right]^{2} \frac{\partial u_{h}(x_{h})}{\partial x_{h}^j} = \sum_{k=1}^{n} \frac{b_{k}}{b_{k}} \left[ \sum_{k=1}^{n} q_{k} \right]^{2} \frac{\partial u_{h}(x_{h})}{\partial x_{h}^j}
\]
for \( i = 1, \ldots, k \), because of Equation (2.2). Hence, First-Order Condition (2.10) yields

\[
\frac{\partial u_h(\omega_h)}{\partial x_h^i} = \frac{k^n}{k=1} b_i^k \frac{k^n}{k=1} q_k^i,
\]

for \( i, j = 1, \ldots, l \). Since the right-hand side of Equation (2.11) is independent of \( h \), all consumers have the same marginal rates of substitution, establishing that the allocation is Pareto optimal.

Consumer \( h \), given the strategies of others, can always guarantee his endowment by setting \( q_h^i = 0 \) and \( b_h^i = 0 \) for \( i = 1, \ldots, l \). Thus, if \( x = (x_1, \ldots, x_h, \ldots, x_n) \) is a NE allocation, we have \( u_h(x_h) > u_h(\omega_h) \) for \( h = 1, \ldots, n \).

It follows that, if \( \omega \) is Pareto optimal, we have \( u_h(x_h) = u_h(\omega_h) \) for \( h = 1, \ldots, n \). Since \( u_h \) is strictly increasing and strictly concave on \( \mathbb{R}_{++}^l \), if \( \omega \) is Pareto optimal then the NE allocation must be autarkic, i.e., we must have \( x = \omega \).

Assume next that the endowment vector, \( \omega = (\omega_1, \ldots, \omega_h, \ldots, \omega_n) \), is not Pareto optimal. Let \( x = (x_1, \ldots, x_h, \ldots, x_n) \) be an interior NE allocation for \( \Gamma \). We have already established that (i) \( x_h \neq \omega_h \) for at least one \( h \), and (ii) \( u_h(x_h) > u_h(\omega_h) \) for all \( h \). Since \( u_h \) is strictly increasing and strictly quasi-concave, we know that the strict inequality \( u_h(x_h) > u_h(\omega_h) \)
holds for at least one $h$.

2.12. Proposition. There is an interior NE strategy $\sigma = \{(b_h, q_h)\}_{h=1}^{n} \in \mathcal{S}$ for the Market Game $\Gamma$.

Proof: See the proof of Proposition (2.12), pages 17-30 in Peck-Shell (1985a).

2.13. Remark. The proof of existence of interior equilibrium given in Peck-Shell (1985a) is based on the sell-all prespecification, i.e., $q = \omega$. Since the proof also goes through for all sufficiently large $q$, we have that there are (at least) $n$ dimensions of indeterminacy in strategy space. We can take the vector $q$ as a rough measure of "market thickness" or even "market confidence". Large $q$'s mean thick markets with the potential for large trades. Small $q$'s mean thin markets with (at most) limited trades.


The game $\Gamma$ is unchanged, but a more general solution concept is analyzed. Purely extrinsic uncertainty is introduced. The fundamentals of the economy -- here tastes and endowments -- are unaffected by the random variable; call it sunspot activity. There are $r$ states of nature. The set of states of nature is

$$P = \{1, \ldots, s, \ldots, r\}.$$  

In observing sunspot activity, consumers receive differing and possibly imperfect signals about the true state. The events which consumer $h$ can observe are described by $I_h$, a partition of $P$. After receiving his signal,
consumer h knows in which element of I_h the true state lies. Because of
this restriction on his information, consumer h's actions must be
"measurable with respect to I_h."

Let \( x_h(s) = (x^1_h(s), \ldots , x^r_h(s), \ldots , x^n_h(s)) \in \mathbb{R}_+^r \) be consumer h's
consumption basket if state \( s \) occurs \((s = 1, \ldots , r \text{ and } h = 1, \ldots , n)\) and
define \( \tilde{x}_h = (x^1_h(1), \ldots , x^r_h(s), \ldots , x^r_h(r)) \in \mathbb{R}_+^r \) and
\( \tilde{x} = (\tilde{x}_1, \ldots , \tilde{x}_h, \ldots , \tilde{x}_n) \in \mathbb{R}_+^{rn} \). Consumer h has the strictly concave von
Neumann-Morgenstern utility function \( v_h \) defined by

\[
v_h(\tilde{x}_h) = \sum_{s=1}^{r} \pi(s) u_h(x_h(s)),
\]

where \( u_h \) is the utility function described in Section 2, \( \pi(s) \) is the
(objective) probability of the occurrence of state \( s \), \( 0 < \pi(s) < 1 \),
\[
\sum_{s=1}^{r} \pi(s) = 1.
\]

Let \( b_h(s) \) and \( q_h(s) \) be, respectively, the bid and the offer of
consumer h on spot market trading post i given that state of nature \( s \)
has occurred. Define \( \tilde{b}_h \) and \( \tilde{q}_h \) by
\[
\tilde{b}_h = (b^1_h(1), \ldots , b^r_h(1), \ldots , b^1_h(s), \ldots , b^r_h(s), \ldots , b^1_h(r), \ldots , b^r_h(r),
\ldots , b^1_h(r), \ldots , q^1_h(r), \ldots , q^r_h(r)) \in \mathbb{R}_+^{2r} \text{ and }
\tilde{q}_h = (q^1_h(1), \ldots , q^r_h(1), \ldots , q^1_h(s), \ldots , q^r_h(s), \ldots , q^1_h(r), \ldots , q^r_h(r)) \in \mathbb{R}_+^{2r}.\]

Also define the strategy
\( \tilde{\sigma} = (\tilde{\sigma}_1, \ldots , \tilde{\sigma}_h, \ldots , \tilde{\sigma}_n) \) by
\( \tilde{\sigma}_h = (\tilde{b}_h, \tilde{q}_h). \)

Following Aumann (1985), we next define correlated strategies and then
define Correlated Equilibrium.

**3.1. Definition.** A randomized strategy to \( \Gamma \) for consumer h is a function

\[
f_h : P \rightarrow S_h
\]
which associates the state of nature with a (pure) strategy,

\[ f_h: s \mapsto \sigma_h = (b_{h,s}, q_{h,s}) \in \mathbb{R}^2_+ \]

and is measurable with respect to \( I_h \). A correlated strategy is a function

\[ f = (f_1, \ldots, f_h, \ldots, f_n) \]

\[ f: P \rightarrow S, \]

where \( f_h \) is a randomized strategy for consumer \( h, h = 1, \ldots, n \). Hence, we have

\[ f: s \mapsto \sigma = (\sigma_1, \ldots, \sigma_h, \ldots, \sigma_n) = \{(b_{h,s}, q_{h,s})\}_{h=1}^{h=n} \in \mathbb{R}^2_{+n}. \]

Assume that consumer \( h \) plays the strategy \( \sigma_h(s) \in S_h \) if state \( s \) occurs, where

\[ \sigma_h(s) = (b_{h,1}(s), \ldots, b_{h,h}(s), q_{h,1}(s), \ldots, q_{h,h}(s)). \]

If \( s \) and \( s' \) are signals which fall in the same element of \( I_h \), then the measurability assumption entails \( \sigma_h(s) = \sigma_h(s') \).

3.2. Definition. Let \( f = (f_1, \ldots, f_h, \ldots, f_n) \) be a correlated strategy (Definition (3.1)), where \( f_h: s \mapsto \sigma_h(s) \) for \( h = 1, \ldots, n \) and

\[ s = 1, \ldots, r, \tilde{\sigma}_h = (\sigma_h(1), \ldots, \sigma_h(s), \ldots, \sigma_h(r)) \]

and \( \tilde{\sigma} = (\tilde{\sigma}_1, \ldots, \tilde{\sigma}_h, \ldots, \tilde{\sigma}_n) \). A Correlated Equilibrium to \( \Gamma \) is a correlated strategy \( f \) which satisfies

\[ v_h(\tilde{\sigma}) \geq v_h(\tilde{\sigma}'|\tilde{\sigma}_h) \]

for all randomized strategies \( \tilde{\sigma}'_h \) and for \( h = 1, \ldots, n \). The corresponding allocation \( x(\tilde{\sigma}) \in \mathbb{R}^r_+ \) is said to be a Correlated Equilibrium allocation.

3.3. Remark. There is another way to look at Correlated Equilibrium. Replace \( \Gamma \) with a related two-stage extensive-form game. Nature chooses the state
s = 1, ..., r in the first stage and asymmetric signals flow to each of the consumers. In the second stage, each consumer chooses a spot market strategy given his own information. Call this two-stage game \( \Gamma' \). Obviously, a (pure-strategy) Nash Equilibrium allocation for \( \Gamma' \) is a Correlated Equilibrium allocation for \( \Gamma \), and conversely.

3.4. Remark. (1) A (pure-strategy) NE to \( \Gamma \) is also a Correlated Equilibrium to \( \Gamma \). (1a). Take the special case \( r = 1 \). Obviously, \( x \in \mathbb{R}^{tn} \) is a NE allocation to \( \Gamma \) (Definition 2.3) if and only if \( x \) is a Correlated Equilibrium allocation to \( \Gamma \) (Definition 3.2). (1b). Let \( x = (x_1, \ldots, x_h, \ldots, x_n) \in \mathbb{R}^{tn} \) be a NE to \( \Gamma \) and let \( x_h(s) = x_h \) for \( s = 1, \ldots, r \) and \( h = 1, \ldots, n \). Then \( \tilde{x} = (x_1(l), \ldots, x_h(s), \ldots, x_n(r)) \in \mathbb{R}^{tn} \) is a Correlated Equilibrium allocation to \( \Gamma \). (2) A mixed-strategy NE to \( \Gamma \) is also a Correlated Equilibrium to \( \Gamma \), where \( P \) and \( I \), \( h = 1, \ldots, n \), are appropriately constructed.

We next establish that consumer \( h \)'s optimal (randomized) response to the (equilibrium or disequilibrium) (randomized) strategies of others is "individually rational".

3.5. Lemma. Let \( \tilde{\sigma}_h \) be consumer \( h \)'s best response to the strategies \( \tilde{\sigma} - h \) in the market game \( \Gamma \), i.e.,

\[
v_h(\tilde{x}_h(\tilde{\sigma})) = \max_{\tilde{\sigma}_h} \{ v_h(\tilde{x}_h(\tilde{\sigma})|\tilde{\sigma}_h') \}.
\]

Then, we have

\[
v_h(\tilde{x}_h(\tilde{\sigma})) \geq v_h(\omega_h, \ldots, \omega_h, \ldots, \omega_h) = u_h(\omega_h).
\]
Proof: If consumer $h$ selects the trivial strategy $\tilde{a}_h = (b_h', q_h') = (0,0)$, then we have $x_h(\tilde{a}_h, \tilde{a}_h') = \omega_h$. Hence, if $\tilde{a}_h$ is the best response to $\tilde{a}_{-h}$, it follows that $v_h(x_h(\tilde{a})) \geq v_h(x_h(\tilde{a}_h, \tilde{a}_h')) = u_h(\omega_h)$.

3.6. Remark. Under the assumption of common priors, Aumann (1985, page 10) establishes the equivalence for matrix games of Bayes-rational equilibria and correlated equilibria. If this result extends to market games (with infinite strategy sets), then we would have (assuming common priors) the equivalence of correlated equilibria and "self-fulfilling prophecies".


Spot market trading is the same as in Section 2. The equilibrium concept is (as in Section 2) that of (pure-strategy) Nash Equilibrium. The new wrinkle is the addition of securities markets in which consumers are able to hedge against the potential economic effects of sunspots. Uncertainty and the signalling of information is the same as in Section 3. Also following Section 3, the $n$ consumers are assumed to be expected-utility maximizers. The simple Market Game $\Gamma$ is replaced by the related Securities Game $\tilde{\Gamma}$.

<table>
<thead>
<tr>
<th>Sunspots Imperfectly Observed; Element in $p_h$ Revealed</th>
<th>Exact Element in $p$ Revealed; Securities Redeemed</th>
</tr>
</thead>
<tbody>
<tr>
<td>Securities Traded</td>
<td>Spot Commodities Traded</td>
</tr>
<tr>
<td></td>
<td>Commodities Consumed</td>
</tr>
</tbody>
</table>

This is our time line. Each of the $n$ consumers is alive and active during the entire period. There are $r$ states of nature indexed by $s$. Let $x_h(s) = (x_h^1(s), ..., x_h^4(s), ..., x_h^r(s)) \in \mathbb{R}_+^r$ be consumer $h$'s consumption
basket if state \( s \) occurs (\( s = 1, \ldots, r \) and \( h = 1, \ldots, n \)) and define
\[
\tilde{x}_h = (x_h(1), \ldots, x_h(s), \ldots, x_h(r)) \in \mathbb{R}_+^r \quad \text{and} \quad \tilde{x} = (\tilde{x}_1, \ldots, \tilde{x}_h, \ldots, \tilde{x}_n) \in \mathbb{R}_+^{rn}.
\]
Let \( \omega_h(s) = (\omega'_h(s), \ldots, \omega_h(s), \ldots, \omega_h(s)) \in \mathbb{R}_+^r \) be consumer \( h \)'s endowments in state \( s \), and define \( \tilde{\omega}_h = (\omega'_h(1), \ldots, \omega_h(s), \ldots, \omega_h(r)) \in \mathbb{R}_+^r \) and
\[
\tilde{\omega} = (\tilde{\omega}_1, \ldots, \tilde{\omega}_h, \ldots, \tilde{\omega}_n) \in \mathbb{R}_+^{rn}.
\]
Since uncertainty is purely extrinsic, we have
\[
\omega_h(s) = \omega_h
\]
for \( s = 1, \ldots, r \) and \( h = 1, \ldots, n \), where \( \omega_h \) is the certainty endowment vector introduced in Section 2.

The securities market is composed of \( r \) trading posts, one for each state of nature. Bids are denominated in "general monetary units", but offers are made in state-specific units of account. After consumers receive their private signals, they trade on the spot market, composed as in Section 2 of \( l \) posts, one for each commodity. Let \( b_h^i(s) \) and \( q_h^i(s) \) be, respectively, the bid and the offer of consumer \( h \) on spot market trading post \( i \) given that state of nature \( s \) has occurred. Let \( b_h^m(s) \) and \( q_h^m(s) \) be, respectively, the bid and the offer of consumer \( h \) on security market \( s \). Define \( \hat{b}_h \) and \( \hat{q}_h \) by
\[
\hat{b}_h = (b_h^1(1), \ldots, b_h^i(1), \ldots, b_h^r(1), b_h^1(s), \ldots, b_h^i(s), \ldots, b_h^r(s), \ldots, b_h^1(r), \ldots, b_h^i(r), \ldots, b_h^r(r)) \in \mathbb{R}^{r(l+1)}_+
\]
and
\[
\hat{q}_h = (q_h^1(1), \ldots, q_h^i(1), \ldots, q_h^r(1), q_h^1(s), \ldots, q_h^i(s), \ldots, q_h^r(s), \ldots, q_h^1(r), \ldots, q_h^i(r), \ldots, q_h^r(r)) \in \mathbb{R}^{r(l+1)}_+.
\]
Also define the strategy \( \hat{\sigma} = (\hat{\sigma}_1, \ldots, \hat{\sigma}_h, \ldots, \hat{\sigma}_n) \) by \( \hat{\sigma}_h = (\hat{b}_h, \hat{q}_h) \). Then the strategy set \( \hat{S}_h \) for consumer \( h \) in the securities game \( \hat{r} \) is given by
\[
\hat{S}_h = \{ \hat{\sigma}_h \in \mathbb{R}_{+}^{r(l+1)} | \tilde{\sigma}_h \text{ is measurable with respect to } I_h, \text{ and } \}
\]
\[
q_h^i(s) \leq \omega_h^i \quad \text{for } i = 1, \ldots, l \text{ and } s = 1, \ldots, r \}.
\]
We assume if $s$ and $s'$ fall in the same element in $I_i$, then
$b_h^i(s) = b_h^i(s')$ and $q_h^i(s) = q_h^i(s')$ for $i = 1, \ldots, t$. Because securities are redeemed after (or as) the precise state is revealed to everyone, there is no such restriction on $b_h^m(s)$ and $h_h^m(s')$ or on $q_h^m(s)$ and $q_h^m(s')$. In (4.1), "measurability" is to be taken in this sense. Define $\sigma$ and $\hat{S}$ by
$$\hat{\sigma} = (\hat{\sigma}_1, \ldots, \hat{\sigma}_h, \ldots, \hat{\sigma}_n) \in \hat{S}_1 \times \ldots \times \hat{S}_h \times \ldots \times \hat{S}_n = \hat{S}.$$ Also define $\hat{\sigma}^{-}_h$ by
$$\hat{\sigma}^{-}_h = (\hat{\sigma}_1, \ldots, \hat{\sigma}_{h-1}, \hat{\sigma}_{h+1}, \ldots, \hat{\sigma}_n) \in \hat{S}_1 \times \ldots \times \hat{S}_{h-1} \times \hat{S}_{h+1} \times \ldots \times \hat{S}_n$$ and $(\sigma|\sigma')$ by
$$(\sigma|\sigma') = (\hat{\sigma}_1, \ldots, \hat{\sigma}_{h-1}, \hat{\sigma}_h', \hat{\sigma}_{h+1}, \ldots, \hat{\sigma}_n) \in \hat{S}.$$

There are two markets: The securities market, which meets before consumers receive their private signals, and the spot commodities market, which meets after consumers receive their private signals but before the state $s$ is perfectly revealed. Consumer $h$ must satisfy two constraints, one for each market; if either one or both are not satisfied, consumer $h$ is punished. The securities-market constraint is:

\[
(4.2.1) \quad \sum_{s=1}^{s=r} q_h^m(s) \leq \sum_{s=1}^{s=r} \left[ \sum_{k=1}^{k=n} b_k^m(s) \right],
\]

i.e., the sum of the securities-market bids in "general dollars" (the left-hand side of Inequality (4.2.1)) must be no greater than the sum of the revenue in "general dollars" from the sales of securities (the right-hand side of Inequality (4.2.1)). Purchases of securities are financed by the sales of securities. A single unit of security $s$ pays one unit of account in state $s$ and zero otherwise. Security $s$ can be thought of as state-$s$ money, or state-$s$ dollars, dollars accepted in state $s$ and only in state $s$. In order to avoid punishment, consumer $h$ must meet the commodity-market budget.
constraint:

\[ (4.2.ii) \sum_{j=1}^{j=l} b_{h}^{j}(s) \leq \sum_{j=1}^{j=l} q_{h}^{j}(s) + \frac{\sum_{k=1}^{k=n} b_{k}^{j}(s)}{\sum_{k=1}^{k=n} q_{k}^{j}(s)} + \frac{\sum_{k=1}^{k=n} q_{k}^{m}(s)}{\sum_{k=1}^{k=n} b_{k}^{m}(s)} \]

for \( s = 1, \ldots, r \).

The consumption, \( \tilde{x}_{h}^{i}(s) \), of consumer \( h \) is given by

\[
\tilde{x}_{h}^{i}(s) = \omega_{h}^{i} - q_{h}^{i}(s) + b_{h}^{i}(s) - \frac{\sum_{k=1}^{k=n} q_{k}^{i}(s)}{\sum_{k=1}^{k=n} b_{k}^{i}(s)}
\]

(4.3) if the Budget Constraints (4.2.i)-(4.2.ii) hold,

\[
\tilde{x}_{h}^{i}(s) = 0 \text{ otherwise,}
\]

for \( i = 1, \ldots, i \) and \( s = 1, \ldots, r \).

The System of Equations (4.3) is consistent with the following auditing-punishment procedure: Trade takes place on the securities market, and if Constraint (4.2.i) is violated, consumer \( h \) is punished on the spot market no matter which state of nature occurs, i.e., \( x_{h}^{i}(s) = 0 \) for \( s = 1, \ldots, r \). Then the referee audits the consumers' spot market plans. If it is the case that in some state of nature \( s \), consumer \( h \) violates Constraint (4.2.ii), then he is punished on the spot market no matter which state of nature occurs, i.e., \( x_{h}^{i}(s') = 0 \) for \( s' = 1, \ldots, r \).

Generalizing Peck-Shell (1985b), we have constructed from the Market Game
T, the probability device P, and the signalling devices I_h, h = 1,...,n, the Securities Game  . The game T can be thought of as the noncompetitive analogue of the Arrow (1974) securities model. For the special case of symmetric information, our model can also be thought of as the noncompetitive analogue of the particular Cass-Shell (1983) sunspot model in which there are no restrictions on market participation. The Securities Game T is completely specified. The strategy sets are  (h = 1,...,n); cf. Equation (4.1). The outcomes (x_h(1),...,x_h(r)) (h = 1,...,n) are given by Equation (4.3), and the payoffs are the expected utilities v_h (h = 1,...,n) at probabilities {π(s)}_s=1^{sr}. We adopt the standard definition of Nash Equilibrium.

4.4. Definition. A (pure-strategy) Nash Equilibrium to the Securities Game T is a  ∈  with the property

\[ v_h(\tilde{x}_h(\sigma)) = \max_{\sigma' \in S_h} \{ v_h(\tilde{x}_h(\sigma|\sigma')) \} \]

for h = 1,...,n. The corresponding NE allocation is  = (\tilde{x}_1(\sigma),...,\tilde{x}_h(\sigma),...,\tilde{x}_n(\sigma)) ∈ \mathbb{R}_+^{nr}.

We next establish that the game T is “individually rational” for each of the n consumers.

4.5. Lemma. Let  be consumer h’s best response to the strategies  in the Market Game T, i.e.,

\[ v_h(\tilde{x}_h(\sigma)) = \max_{\sigma' \in S_h} \{ v_h(\tilde{x}_h(\sigma|\sigma')) \} . \]

Then, we have

\[ v_h(\overset{\sim}{x}_h(\sigma)) \geq v_h(\overset{\sim}{\omega}_h) = v_h(\omega_h, \ldots, \omega_h) \]

\[ = \sum_{s=1}^{r} \pi(s)u_h(\omega_h) = u_h(\omega_h). \]

**Proof:** If, in response to the strategies, \( \hat{\sigma}_h \), of the others, consumer \( h \) plays the trivial strategy (which is trivially measurable with respect to \( I_h \)) given by

\[ h_h(s) = 0, q_h(s) = 0, b_h(s) = 0, \text{ and } q_h(s) = 0, \]

for \( i = 1, \ldots, \ell \) and \( s = 1, \ldots, r \), then from Equation (4.3), we have

\[ x_h^i(s) = \omega_h^i \text{ for } i = 1, \ldots, \ell \text{ and } s = 1, \ldots, r. \]

Hence, we have

\[ v_h(\overset{\sim}{x}_h(\sigma)) \geq v_h(\overset{\sim}{\omega}_h) = v_h(\omega_h, \ldots, \omega_h) \]

\[ = \sum_{s=1}^{r} \pi(s)u_h(\omega_h) = u_h(\omega_h). \]

The inequality above reflects consumer \( h \)'s ability to "defend his endowments". The last equality above is a consequence of \( \sum_{s=1}^{r} \pi(s) = 1. \)

We next show that \( \hat{\Gamma} \) has a trivial NE. Later we show that there is also a nontrivial NE for \( \hat{\Gamma} \).

**4.6. Lemma.** Let \( \hat{\sigma} \) be the vector in \( \hat{S} \) with each component zero. Then \( \hat{\sigma} \)

is a NE strategy for \( \hat{\Gamma} \). The corresponding NE allocation is \( \hat{x} = \)
\[(\tilde{x}_1, \ldots, \tilde{x}_h, \ldots, \tilde{x}_n) = \tilde{\omega} = (\tilde{\omega}_1, \ldots, \tilde{\omega}_h, \ldots, \tilde{\omega}_n) \in \mathbb{R}^{trn}.\]

**Proof:** Obvious. \(\square\)

### 4.7. Definition
A NE strategy is said to be **interior** if it entails each market (including the markets for securities) being open, i.e., we have
\[
\sum_{k=1}^{n} b^j_k(s) > 0 \text{ for } j = 1, \ldots, k \text{ and } s = 1, \ldots, r, \text{ and } \sum_{k=1}^{n} b^m_k(s) > 0 \text{ for } \text{all } s = 1, \ldots, r, \text{ and } k = n.
\]

### 4.8. Definition
We say that **sunspots do not matter** if in the allocation of consumption goods, we have

\[(4.9) \quad x_h(s) = x_h(s')\]

for \(h = 1, \ldots, n\) and \(s, s' = 1, \ldots, r\). Otherwise, **sunspots matter**. A NE to \(\hat{T}\) in which Condition (4.9) is satisfied (resp; not satisfied) is called a **Nonsunspot NE** (resp. **Sunspot NE**) to \(\hat{T}\).

It is easy to display an interior Nonsunspot NE to \(\hat{T}\) if \(I_h\) is independent of \(h\), i.e., signals are perfectly correlated. This is done in the next proposition.

### 4.10. Proposition
Let \(I_h\) be the finest partition of \(P\) for \(h = 1, \ldots, n\) and let there be at least two states of nature, i.e., \(r \geq 2\). Then the Securities Game \(\hat{T}\) has an interior Nonsunspot NE.

**Proof:** Let \(\sigma = \{(b^*_k, q_k)\}_{k=1}^{k=n} \in S\) be an interior NE of the certainty game \(T\).
(analyzed in Section 2). We know that there is such a strategy \( \sigma \). We now construct \( \hat{\sigma} \in \hat{S} \) measurable with respect to the \( I_h \) to be an interior Nonsunspot NE to \( \Gamma \):

\[
b^i_h(s) = b^i \quad \text{for } i = 1, \ldots, l; s = 1, \ldots, r; h = 1, \ldots, n;
\]

\[
q^i_h(s) = q^i \quad \text{for } i = 1, \ldots, l; s = 1, \ldots, r; h = 1, \ldots, n;
\]

(4.11)

\[
b^m_h(s) = \pi(s) \quad \text{for } s = 1, \ldots, r; h = 1, \ldots, n;
\]

\[
q^m_h(s) = 1 \quad \text{for } s = 1, \ldots, r; h = 1, \ldots, n.
\]

No income is being transferred between states if \( \hat{\sigma} \) defined by (4.11) is the strategy vector for \( \hat{\Gamma} \). Hence the Constraint (4.2.1) holds with equality. Then, Constraint (4.2.11) holds with equality since \( \sigma \) is an interior NE on \( \Gamma \).

Since all markets are open in the securities game \( \hat{\Gamma} \) for the strategy \( \hat{\sigma} \) described in Equations (4.11), the first-order conditions for utility maximization under binding Constraints (4.2) are necessary and sufficient for optimality. These conditions are

\[
(4.12) \quad \frac{\lambda^i_h(s)}{\lambda^i_h(s')} = \frac{\pi(s)}{\pi(s')} \frac{3u_h(x^i_h(s))/\partial x^i_h(s)}{3u_h(x^i_h(s'))/\partial x^i_h(s')} \left[ \sum_{k \neq h} b^i_k(s) \left[ \frac{\sum_{k=1}^{n} q^i_k(s)}{\sum_{k=1}^{n} b^i_k(s)} \right]^2 \right] 2 \left[ \sum_{k \neq h} b^j_k(s') \left[ \frac{\sum_{k=1}^{n} y^j_k(s')}{\sum_{k=1}^{n} b^j_k(s')} \right]^2 \right] 2 \left[ \sum_{k=1}^{n} y^j_k(s') \right]
\]

and
\[
\frac{\lambda_h(s)}{\lambda_h(s')} = \frac{\sum_{k \neq h} q^m_k(s)}{\sum_{k \neq h} b^m_k(s)} \left( \frac{\sum_{k=1}^n b^m_k(s)}{\sum_{k=1}^n q^m_k(s)} \right)^2 = \frac{\sum_{k \neq h} b^m_k(s')}{\sum_{k \neq h} b^m_k(s')} \left( \frac{\sum_{k=1}^n q^m_k(s')}{{\sum_{k=1}^n b^m_k(s')}^2} \right)^2
\]

for \( h = 1, \ldots, n \); \( i, j = 1, \ldots, \ell \); and \( s, s' = 1, \ldots, r \); and \( \lambda_h(s) \) and \( \lambda_h(s') \) are (respectively) the Kuhn-Tucker-Lagrange multipliers associated with Constraint (4.2.ii) for states \( s \) and \( s' \).

Substitute the data from (4.11) into the right-hand side of Equation (4.13), which is consistent if

\[
\frac{\lambda_h(s)}{\lambda_h(s')} = \frac{\pi(s)}{\pi(s')}
\]

If we substitute from Equation (4.14), Equation (4.12) must hold because of First-order Condition (2.10).

Thus, \( \hat{\sigma} = \hat{\sigma} \) defined by Equations (4.11) is an interior NE for \( \hat{\Gamma} \).

Since we have \( x_h(s) = x_h(s') \) for \( s, s' = 1, \ldots, r \) and \( h = 1, \ldots, n \), \( \hat{\sigma} \) is also a Nonsunspot NE for the Securities Game \( \hat{\Gamma} \).

4.15. Remark. A careful reading of the proof of Proposition (4.10) shows that for every NE strategy \( \sigma \in \hat{S} \) (with corresponding allocation \( x(\sigma) \in K_{++}^n \)) for the Market Game \( \Gamma \) there is an "equivalent" NE strategy \( \hat{\sigma} \in \hat{S} \) for the Securities Game \( \hat{\Gamma} \) (with corresponding allocation \( \hat{x}(\hat{\sigma}) \in K_{nr}^n \)). The strategies \( \sigma \) and \( \hat{\sigma} \) are equivalent in the sense

\[
x_h(s; \hat{\sigma}) = x_h(\sigma)
\]

for \( s = 1, \ldots, r \) and \( h = 1, \ldots, n \). Thus \( \hat{x}(\hat{\sigma}) \) is a Nonsunspot NE for \( \hat{\Gamma} \).

Essentially, the NE to \( \Gamma \) reappear as the Nonsunspot NE to \( \hat{\Gamma} \).
If endowments are Pareto optimal, then there are no Sunspot NE to \( \hat{\Gamma} \). If the probability mechanism \( P \) along with the signalling devices \( I^1_h \) are nondegenerate, then we have for endowments not Pareto-optimal there must exist Sunspot NE to \( \hat{\Gamma} \). These ideas are formalized in the following proposition.

**4.17 Proposition.** (1) Let the endowment vector \( \omega \in \mathbb{R}_{++}^n \) in the Market Game \( \Gamma \) be Pareto-optimal. Then there is no Sunspot NE to the corresponding Securities Game \( \hat{\Gamma} \). (2) Let the endowment vector \( \omega \in \mathbb{R}_{++}^n \) in the Market Game \( \Gamma \) be not Pareto-optimal. Let there be a common coarsening of the \( I^1 \), which contains (at least) two elements. Then there is a Sunspot NE \( \sigma \in \hat{S} \) to the corresponding Securities Game \( \hat{\Gamma} \).

**Proof:** (1) Assume that \( \omega \) is Pareto-optimal in \( \Gamma \). Clearly, then \( \tilde{\omega} \) is also Pareto-optimal in \( \hat{\Gamma} \). Assume that \( \tilde{x} = (\tilde{x}_1, \ldots, \tilde{x}_h, \ldots, \tilde{x}_n) \) is a Sunspot NE allocation in \( \hat{\Gamma} \). By Lemma (4.5), we have

\[
(4.18) \quad v_h(\tilde{x}_h) \geq v_h(\omega_h, \ldots, \omega_h) = u_h(\omega_h).
\]

Because of the strict concavity of \( u_h \), a Sunspot allocation cannot be Pareto-optimal. We have a contradiction. If \( \tilde{\omega} \) is Pareto-optimal in \( \hat{\Gamma} \), there are no Sunspot NE for \( \hat{\Gamma} \). Indeed, the only NE allocation is \( \tilde{x} = \tilde{\omega} = (\tilde{\omega}_1, \ldots, \tilde{\omega}_n) \in \mathbb{R}_{++}^n \).

(2). Assume that \( \omega \) is not Pareto-optimal. Then there are at least two NE strategies for \( \Gamma \), \( \sigma' = 0 \in S \) (with the corresponding allocation \( x' = ((x_1)', \ldots, (x_h)', \ldots, (x_n)') = \omega \in \mathbb{R}_{++}^n \)), and an interior NE strategy \( \sigma'' = \{(q^h_h)_{h=1}^{h=n}, (b^h_h)_{h=1}^{h=n}\} \in \mathbb{R}_{++}^n \) (with the corresponding allocation \( x'' = ((x_1)'', \ldots, (x_h)'', \ldots, (x_n)'') \in \mathbb{R}_{++}^n \)). Cf. Lemma (2.5) and Proposition (2.12). From Proposition (2.9), we know that \( x' \) and \( x'' \) are not equal.
By hypothesis, we can partition the states of nature \( P \) into two subsets, \( A \) and \( B \) which are each elements of a common coarsening of the \( I_h \). We have \( A \cup B = P, A \cap B = 0, A \neq \emptyset, \) and \( B \neq \emptyset \). We construct the Sunspot NE \( \tilde{\sigma} \) for the Securities Game \( \hat{\Gamma} \) from these two NE of the Market Game \( \Gamma \) as follows:

\[
\begin{align*}
\hat{b}_h^i(s) &= 0, \\
\hat{q}_h^i(s) &= 0, \\
(4.19.A) & \\
\tilde{b}_h^m(s) &= 0, \\
\tilde{q}_h^m(s) &= 0,
\end{align*}
\]

for \( s \in A, h = 1, \ldots, n, \) and \( i = 1, \ldots, \ell \); and

\[
\begin{align*}
\hat{b}_h^i(s) &= \hat{b}_h^i\left(\tilde{b}_h^i\right), \\
\hat{q}_h^i(s) &= \hat{q}_h^i\left(\tilde{q}_h^i\right), \\
(4.19.B) & \\
\tilde{b}_h^m(s) &= 0, \\
\tilde{q}_h^m(s) &= 0,
\end{align*}
\]

for \( s \in B, h = 1, \ldots, n, \) and \( i = 1, \ldots, \ell \), where \( \hat{b}_h^i \) and \( \hat{q}_h^i \) are bids and offers in the interior NE strategy \( \hat{\sigma} \) for the Market Game \( \Gamma \). The strategy \( \hat{\sigma} \in \hat{S} \) is clearly a Sunspot NE for the Securities Game \( \hat{\Gamma} \).

The proof of Proposition (4.17) is complete. \( \Box \)

The Sunspot NE allocation \( \tilde{x} \) constructed in Proposition (4.17) is a lottery over (certainty) NE from the underlying Market Game \( \Gamma \). As such, the
allocation \( \tilde{x} \) is also a Correlated Equilibrium allocation to the Market Game \( \Gamma \). Next we show that a Correlated Equilibrium to \( \Gamma \) is always a NE to \( \hat{\Gamma} \).

4.20. Proposition. Let \( \tilde{x} \in \mathbb{R}^{2n}_{++} \) be a Correlated Equilibrium allocation for the Market Game \( \Gamma \), where the probability-signalling mechanism is given by \( P, \{\pi(s)\}_{s=1}^{s}, \{\lambda_h\}_{h=1}^{h} \). Then \( \tilde{x} \in \mathbb{R}^{2n}_{++} \) is also an NE allocation to the Securities Game \( \hat{\Gamma} \) generated by \( \Gamma, P, \{\pi(s)\}_{s=1}^{s} \) and \( \{\lambda_h\}_{h=1}^{h} \).

Proof: Let \( \sigma = (\tilde{b}, \tilde{q}) \in \mathbb{R}^{2n}_{+} \) be the Correlated Equilibrium to \( \Gamma \) corresponding to \( \tilde{x} \). Define \( \hat{\sigma} \in \mathbb{R}^{2(n+1)}_{+} = (\hat{b}, \hat{q}) \) by \( \hat{b} = (\tilde{b}, 0, \ldots, 0) \) and \( \hat{q} = (\tilde{q}, 0, \ldots, 0) \). Then \( \hat{\sigma} \in \hat{S} \) is clearly a NE to \( \hat{\Gamma} \) which supports the NE allocation \( \tilde{x} \).

5. Examples.

This section is devoted to numerical examples. We begin by computing some pure-strategy NE to the Market Game \( \Gamma \). It is easy to find other correlated equilibria to \( \Gamma \) (and hence pure-strategy NE to \( \hat{\Gamma} \)) which are simple randomizations over the pure-strategy NE to \( \Gamma \). The more interesting examples involve either asymmetric information which generates the imperfectly correlated equilibria for \( \Gamma \) (and corresponding Sunspot NE to \( \hat{\Gamma} \)), or transfer of income across states which generates Sunspot NE to \( \hat{\Gamma} \) which are not correlated equilibria to \( \Gamma \).

First, we present examples of NE in a Market Game \( \Gamma \). These computed solutions will be used repeatedly in the sequel.

5.1. Example. Let there be two consumers \((h = 1,2)\) and two commodities \((i = 1,2)\), so that \(n = 2\) and \(\ell = 2\). The following data about consumer preferences and endowments complete the description of the Market Game \( \Gamma \):
\[ u_h(x_h^1, x_h^2) = \log x_h^1 + \log x_h^2 \quad \text{for} \quad h = 1, 2 \]

(5.2)

\[ \omega_1 = (\omega_1^1, \omega_1^2) = (80, 20); \quad \omega_2 = (\omega_2^1, \omega_2^2) = (20, 80). \]

Solution 1 to Example (5.1): The example exhibits a skew-symmetry between the two consumers. Hence, if each of the consumers offers 100% of his endowment for sale, we have the skew-symmetric interior NE to \( \Gamma \) displayed below. This is a thick-market solution. Trading is substantial, but since this game is neither cooperative nor perfectly competitive, the allocation of consumption goods is still far from Pareto-optimal.

<table>
<thead>
<tr>
<th></th>
<th>Commodity 1</th>
<th>Commodity 2</th>
</tr>
</thead>
<tbody>
<tr>
<td>( b_1 )</td>
<td>.3333</td>
<td>.1667</td>
</tr>
<tr>
<td>( q_1 )</td>
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<tr>
<td>( x_1 )</td>
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<td>33.3333</td>
</tr>
<tr>
<td>( b_2 )</td>
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<td>.3333</td>
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<tr>
<td>( q_2 )</td>
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<td>80.0000</td>
</tr>
<tr>
<td>( x_2 )</td>
<td>33.3333</td>
<td>66.6667</td>
</tr>
</tbody>
</table>

Solution 1 to the 2 x 2 Game \( \Gamma \) of Example (5.1): Each Consumer Offers All of his Endowments
Solution 2 to Example (5.1): Here markets are thinner than before. Each
consumer offers for sale only 25% of his endowments. Trading is substantially
less than in the first example. Lack of consumer confidence is self-justi-
lying. Skew-symmetry is preserved. Each consumer is worse off in Solution 2
than in Solution 1. The NE is interior.

<table>
<thead>
<tr>
<th></th>
<th>Commodity 1</th>
<th>Commodity 2</th>
</tr>
</thead>
<tbody>
<tr>
<td>b₁</td>
<td>.2892</td>
<td>.2108</td>
</tr>
<tr>
<td>q₁</td>
<td>20.0000</td>
<td>5.0000</td>
</tr>
<tr>
<td>x₁</td>
<td>74.4603</td>
<td>25.5397</td>
</tr>
<tr>
<td>b₂</td>
<td>.2108</td>
<td>.2892</td>
</tr>
<tr>
<td>q₂</td>
<td>5.0000</td>
<td>20.0000</td>
</tr>
<tr>
<td>x₂</td>
<td>25.5397</td>
<td>74.4603</td>
</tr>
</tbody>
</table>

Solution 2 to the 2 x 2 Game Γ of Example (5.1): Each Consumer Offers 25% of his Endowments
Solution 3 to Example (5.1): The NE is interior, but the skew-symmetry is broken. Consumer 1 offers 100% of his endowments, but Consumer 2 offers only 25%. These strategies are self-justifying. Markets are thin relative to those in Solution 1. Each consumer is worse off than in Solution 1. Consumer 2 is worse off here than he is in Solution 2, while Consumer 1 is better off here than he is in Solution 2. Indeed, in moving from Solution 2 to Solution 3, Consumer 1 gives up .2367 units of Commodity 1, the marginal utility of which is relatively low, in exchange for 4.2573 units of Commodity 2, the marginal utility of which is relatively high.

\[
\begin{array}{c|c|c}
 & \text{Commodity 1} & \text{Commodity 2} \\
\hline
b_1 & .6836 & .1618 \\
q_1 & 80.0000 & 20.0000 \\
x_1 & 74.2236 & 29.7970 \\
\hline
b_2 & .0992 & .0554 \\
q_2 & 5.0000 & 20.0000 \\
x_2 & 25.7764 & 70.2029 \\
\end{array}
\]

Solution 3 to the 2 x 2 Game \( \Gamma \) of Example (5.1): Consumer 1 Offers All of his Endowments; Consumer 2 Offers 25% of his Endowments

The following is an example of a Securities Game \( \Gamma \) which is based on the Market Game \( \Gamma \) described by Equations (5.2) (cf. Example (5.1)).

5.3. Example. Let \( \Gamma \) be described by the Data (5.2). Let there be two states \( s = \alpha, \beta \) (i.e., \( r = 2 \)) and assume that the extrinsic random variable \( s \) obeys the probability law \( \pi(\alpha) = \pi(\beta) = 1/2 \). Let \( I_h(h = 1, 2) \)
be the finest partition of $P = \{a, \beta\}$. Hence, signals are perfectly correlated. Let $\hat{\Gamma}$ be the corresponding securities game.

Next, we compute three Sunspot NE for $\hat{\Gamma}$. The allocation of resources varies across states of nature as market thickness varies. These solutions establish:

$$\begin{align*}
(\text{i}) & \text{ Some Sunspot NE to } \hat{\Gamma} \text{ are interior; others are not.} \\
(\text{ii}) & \text{ Some interior Sunspot NE to } \hat{\Gamma} \text{ involve nonzero net trades on the securities market; others do not.} \\
(\text{iii}) & \text{ Some interior Sunspot NE allocations to } \hat{\Gamma} \text{ are also Correlated Equilibrium allocations to } \hat{\Gamma}; \text{ others are not.} 
\end{align*}$$

Solution 1 to Example (5.3): State $\alpha$ is the "good state", which corresponds to the interior NE for $\Gamma$ given in Solution 1 to (5.1). State $\beta$ is the "bad state", which corresponds to the interior NE for $\Gamma$ given in Solution 2 to (5.1). "Confidence" drops in moving from $\alpha$ to $\beta$; all offers are reduced by 75%, creating thin markets. Each consumer is worse off in $\beta$ than in $\alpha$. The price of the $\alpha$-security in terms of the $\beta$-security is

$$\frac{(155.8731 + 155.8731)}{(100 + 100)} = 3.53 \frac{(44.1269 + 44.1269)}{(100 + 100)} = * *.$$

Despite their relative poverty in state $\beta$, each consumer is (just) willing to give up 3.53 state-$\beta$ dollars in exchange for a single state-$\alpha$ dollar. "Needs" are greater in $\beta$ than in $\alpha$, but "opportunities" in $\alpha$ are very much greater than in $\beta$, which is reflected in the exchange rate between $\alpha$-dollars and $\beta$-dollars.

Because of the various symmetries, it turns out that in equilibrium Consumer 1 and Consumer 2 have the same relative utility weights for state-$\alpha$.
income versus state-β income. There are then no social gains to be made from transferring income across the states of nature. Hence, we have an interior Sunspot NE to \( \hat{\tau} \) in which net securities trades are zero. Therefore, this interior Sunspot NE allocation can be taken as a lottery over interior NE for the (certainty) Market Game \( \Gamma \); (cf. Solutions 1 and 2 to Example (5.1)). Furthermore, this interior Sunspot NE allocation (to \( \hat{\tau} \)) is also a Correlated Equilibrium allocation for the Market Game \( \Gamma \).

<table>
<thead>
<tr>
<th></th>
<th><strong>State α</strong></th>
<th><strong>State β</strong></th>
</tr>
</thead>
<tbody>
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<td></td>
<td>Comm. 1</td>
<td>Comm. 2</td>
</tr>
<tr>
<td>( b_1 )</td>
<td>.3333</td>
<td>.1667</td>
</tr>
<tr>
<td>( q_1 )</td>
<td>80.0000</td>
<td>20.0000</td>
</tr>
<tr>
<td>( x_1 )</td>
<td>66.6667</td>
<td>33.3333</td>
</tr>
<tr>
<td>( b_2 )</td>
<td>.1667</td>
<td>.3333</td>
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<tr>
<td>( q_2 )</td>
<td>20.0000</td>
<td>80.0000</td>
</tr>
<tr>
<td>( x_2 )</td>
<td>33.3333</td>
<td>66.6667</td>
</tr>
</tbody>
</table>

Solution 1 to the Game \( \hat{\Gamma} \) defined in Example (5.3): Markets are Thick in State α and Thin in State β. Net Securities Purchases are Zero.
Solution 2 to Example: State $\alpha$ is the good state. Each consumer offers 100% of his endowment in state $\alpha$; the commodity markets in state $\alpha$ are thick. Consumer 1 offers 100% of his endowment in state $\beta$, the bad state, but Consumer 2 offers only 25% of his endowment in state $\beta$. Both consumers are worse off in $\beta$ than in $\alpha$, although the impact on Consumer 2 turns out, in this particular case, to be the more dramatic. The securities markets are closed. Hence, this solution can be taken as a lottery over interior NE Solutions (1) and (3) (to Example (5.1)) in the Market Game $T$. This solution is a Sunspot NE which is not interior. The corresponding Sunspot NE allocation is also a Correlated Equilibrium allocation to the Market Game $T$.

<table>
<thead>
<tr>
<th></th>
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<th>State $\beta$</th>
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<td>Comm. 2</td>
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<td>$b_1^*$</td>
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<td>$q_1$</td>
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<td>$x_1^*$</td>
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<tr>
<td>$q_2$</td>
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<td>80.0000</td>
</tr>
<tr>
<td>$x_2^*$</td>
<td>33.3333</td>
<td>66.6667</td>
</tr>
</tbody>
</table>

Solution 2 to the Game $\tilde{T}$ defined in Example (5.3): Consumer 2 Reduces his Offers to 25% in State $\beta$. The Securities Markets are Closed.
Solution 3 to Example (5.3): Solution 3 is very much like Solution 2 with one very important difference: In Solution 3, the securities markets are open and net securities purchases are nonzero. The price of the α-security in terms of the β-security is

\[
\frac{(116.3231 + 116.3031)}{(100 + 100)} / \frac{(83.6768 + 83.6968)}{(100 + 100)} = 1.39
\]

Consumer 1's purchases of the α-security (or better, α-money) are

\[200 \left( \frac{116.3211}{116.3211 + 116.3031} \right) = 100.0077 \text{ units}\]

His net purchases of the α-money are hence .0077 units. Consumer 1's bids for commodities in state α sum to .5087 (= .3376 + .1710) state-α dollars, of which .0077 state-α dollars (amounting to 1.5% of the total) are financed by his purchases of α-money in the securities market.

Consumer 1 transfers income into state α, while consumer 2 transfers income into state β. Consumer 1 seeks commodity 2 in state β, but Consumer 2 offers little of this commodity. Hence, Consumer 1 parts with commodity 1 in state β in exchange for state-β money, a substantial portion of which he then exchanges for state-α money. The state-α money is used to finance his purchases of commodities in state α.

Compare Solutions (2) and (3) (Example (5.3)). With open security markets (Solution 3), Consumer 1 increases his consumption of both commodities in (the good state) α and reduces his consumption of both commodities in (the bad state) β.
Solution 3 to the Game $\hat{\Gamma}$ defined in Example (5.3): Consumer 2 reduces his offers to $25\%$ in State $\beta$. The Securities Markets are Open, and Net Purchases of Securities are Nonzero.

Solution 3 is especially noteworthy. It is an interior Sunspot NE solution to $\hat{\Gamma}$ with open and active securities markets. Hence, this solution cannot be considered to be a lottery over NE solutions to $\Gamma$. In the next lemma, we establish that the Sunspot NE allocation displayed in Solution 3 is not a Correlated Equilibrium allocation to the corresponding Market Game $\Gamma^\ast$.

In what follows, we use Solution 3 to Example (5.3) to establish that for some Market Games $\Gamma$, the set of Correlated Equilibrium allocations to $\Gamma$ is a proper subset of the set of pure strategy NE allocations to the corresponding Securities Game $\hat{\Gamma}$. The basic idea is that the Securities Game allows income to be transferred across states, but the self-enforcing nature of Correlated Equilibrium in $\Gamma$ excludes income transfer across states.

5.4. Lemma. It is not always the case that a pure-strategy NE allocation for the Securities Game $\hat{\Gamma}$ is a Correlated Equilibrium allocation for the corresponding Market Game $\Gamma$. 

<table>
<thead>
<tr>
<th></th>
<th>State $\alpha$</th>
<th>State $\beta$</th>
</tr>
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<tbody>
<tr>
<td></td>
<td>Comm. 1</td>
<td>Comm. 2</td>
</tr>
<tr>
<td>$b_1$</td>
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<td>80.0000</td>
</tr>
<tr>
<td>$x_2$</td>
<td>32.4796</td>
<td>65.8019</td>
</tr>
</tbody>
</table>
Proof: We shall be considering the Sunspot NE allocation to Example (5.3) which is presented as Solution 3. We need to show that \( \tilde{x} = ((x_1^1(\alpha), x_1^2(\alpha), x_1^1(\beta), x_1^2(\beta)), (x_2^1(\alpha), x_2^2(\alpha), x_2^1(\beta), x_2^2(\beta)) = ((67.5204, 34.1981, 73.3364, 29.0214), (32.4796, 65.8019, 26.6636, 70.9786)) \) is not a Correlated Equilibrium allocation to the Market Game \( \Gamma \) defined in Example (5.1).

Assume the contrary, i.e., that \( \tilde{x} \) is a Correlated Equilibrium allocation to \( \Gamma \). If neither player could distinguish between state \( \alpha \) and state \( \beta \), the measurability assumption would imply that the Correlated Equilibrium allocation would be independent of the state of nature. This is not the case for the allocation \( \tilde{x} \). If one player could not recognize the difference between states, the other player's best response would lead to an allocation independent of states. Hence, we have shown that each of the two players can see sunspots: neither is blind to solar activity.

Given the strategy of consumer \( k \), we have from Conditions (2.1)-(2.2) that the frontier of consumer \( h \)'s budget set in \( (x_1^1(s), x_1^2(s)) \)-space is given by the intersection of the equation defined by

\[
\frac{b_1^1(s)(\omega_h - x_1^1(h(s)))}{\omega_h + q_k^1(s) - x_1^1(h(s))} + \frac{b_2^1(s)(\omega_h - x_2^1(s))}{\omega_h + q_k^2(s) + x_1^2(h(s))} = 0
\]

and the positive orthant, where \( h \neq k \). But if Mr. \( h \) is optimizing, it is also the case that his budget frontier is tangent to his indifference curve at the consumption point, which yields

\[
\frac{x_1^2(s)}{x_1^1(s)} = \frac{q_k^1(s)}{q_k^2(s)} \left[ \frac{2 - x_1^2(s)}{\omega_h - x_1^1(s)} \right]^2
\]

for \( s = \alpha, \beta \) as a consequence of the utility-function specification in (5.2). Let \( h = 2, k = 1, \) and \( s = \alpha \). First-order Conditions (5.6) then
From the definition of the (strategy) set $S_1$, we have $q_1^1(\alpha) \leq 80 = \omega_1^1$ and $q_1^2(\alpha) \leq 20 = \omega_1^2$. Substituting $q_1^2(\alpha) \leq 20$ into (5.7) yields

$$q_1^1(\alpha) \geq \frac{20(12.4796)}{20(4.384) - 7.9732} > 80,$$

which is a contradiction. The allocation $\tilde{x}$ is not a Correlated Equilibrium allocation for $\Gamma$.

In Lemma (5.4), we compute a nontrivial Sunspot NE to $\Gamma$ which cannot be interpreted as Correlated Equilibrium to $\Gamma$. The driving force of the example is income transfer across states of nature. Information in this example (and the other computed examples) is symmetric across consumers, i.e., the information partitions $I_h$ are independent of $h$. In the next example, we construct (but do not compute) a nontrivial Correlated Equilibrium. Since income cannot be transferred across states in a Correlated Equilibrium, the example must be driven by asymmetric information, i.e., the information partitions $I_h$ are not independent of $h$.

5.8. Example (Nontrivial Correlated Equilibrium):

We construct a Correlated Equilibrium to the Market Game $\Gamma$ in which the allocation is neither a NE nor a simple randomization over NE. The example is driven by asymmetric information.
Consider first the Market Game $\Gamma(n)$ based on $n$ consumers, $n \geq 2$, and $\ell$ commodities, $\ell \geq 2$. We know from Proposition (2.12) that there is an interior (pure-strategy) NE $\sigma(n) = \{(b_h(n), q_h(n))\}_{h=1}^{h=n} \in S$ with the corresponding allocation $x(n) = \{(x_h(n))\}_{h=1}^{h=n} \in \mathbb{R}^n_+$. Let $\sigma'(n) = \{b'_h(n), q'_h(n)\}_{h=1}^{h=n}$ be the strategy defined by

$$b'_h(n) = Mb_h(n)$$

and

$$q'_h(n) = q_h(n),$$

where $M \in \mathbb{R}_+$ is a positive scalar. We know from Equations (2.2), that the allocation $x(n)$ is homogenous of degree zero in the bids $b(n)$. Hence, the NE $\sigma'(n)$ has the same allocation as the NE $\sigma(n)$, i.e., $x'(n) = x(n)$.

Introduce two states ($r = 2$) of nature, $s = \alpha, \beta$. Assume that each of the $n$ consumers can distinguish between $\alpha$ and $\beta$, i.e., we have that $I_h$ is the finest partition of $P$ for $h = 1, \ldots, n$. Define the trivial Correlated Equilibrium for $\Gamma(n)$, $\tilde{\sigma}(n) = (\sigma(n;\alpha), \sigma(n;\beta)) \in \mathbb{R}^4n_+$, by

$$\sigma(n;\alpha) = \sigma(n)$$

and

$$\sigma(n;\beta) = \sigma'(n).$$

The Correlated Equilibrium allocation, $\tilde{x}(n) \in \mathbb{R}^{2n}_+$, is defined by $\tilde{x}(n) = (x(n;\alpha), x(n;\beta)) = (x(n), x'(n)) = (x(n), x(n))$.

We have constructed a very trivial Correlated Equilibrium allocation. Bids in state $\beta$ are all $M$ times those in state $\alpha$. The general price level in state $\beta$ is $M$ times the general price level in state $\alpha$. 

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Consumption is unaffected by sunspots (although consumers must be alert to price level changes caused by sunspots).

Now, introduce a new consumer, Mr. (n+1) and create from \( \Gamma(n) \) the new Market Game \( \Gamma(n+1) \). The preferences and endowments of Mr. (n+1) are chosen so that his best response in \( \Gamma(n+1) \) of strategies of the others, \( \sigma(n) \), would involve some nonzero net trades. That is, if \( \sigma_{-(n+1)}(n+1) = \sigma(n) \), then (n+1)'s best response implies \( x_{n+1}(n+1) \neq \omega_{n+1} \).

Assume that Mr. (n+1) cannot recognize the difference between state \( a \) and state \( b \): \( I_{n+1} \) is the coarsest partition of \( P \). We claim that, if the scalar \( M \) is sufficiently large and the probability \( \pi(a) > 0 \) is sufficiently small, his best response to the correlated strategies of the others, \( \tilde{\sigma}_{-(n+1)}(n+1) \), defined by

\[
\tilde{\sigma}_{-(n+1)}(n+1) = \tilde{\sigma}(n) = (\sigma(n), \sigma'(n)),
\]

is to make zero net trades. Hence, we claim that the Correlated Equilibrium allocation for \( \Gamma(n+1) \), \( \tilde{x}(n+1) \in \mathbb{R}^{2t(n+1)} \) is defined by

\[
\tilde{x}(n+1) = (x(n), \omega_{n+1}, x(n), \omega_{n+1}).
\]

The Correlated Equilibrium allocation, \( \tilde{x}(n+1) \), does not depend on sunspots but it is not based on a NE to \( \Gamma(n+1) \); \( (x(n), \omega_{n+1}) \) is not a NE to the Market Game \( \Gamma(n+1) \).

Before we prove our claim, we provide some heuristics. If the first \( n \) players were playing the pure strategy \( \sigma(n) \) (or even \( \sigma'(n) \)), Mr. (n+1) would respond with nonzero net trades. Neither \( \sigma(n) \) or \( \sigma'(n) \) can form the basis for a NE strategy to \( \Gamma(n+1) \). Because Mr. (n+1) cannot distinguish
between \(\alpha\) and \(\beta\), his randomized strategy must be degenerate in that it is constant across states. Because of the extreme bankruptcy rule, he will not accept any positive probability of bankruptcy. His bids are bounded from above in order to avoid bankruptcy originating from state \(\alpha\). State \(\beta\) is the inflationary state. If consumer \((n+1)\) makes positive bids, they would have to be small to avoid state-\(\alpha\) bankruptcy. This means that he would make substantial offers with meager state-\(\beta\) bids (relative to the state-\(\beta\) bids of others). This would lead to a loss in utility on his state-\(\beta\) trades which cannot be offset by the gains from his state-\(\alpha\) trades if the probability \(\pi(\alpha)\) is sufficiently small and the inflation rate \(M\) is sufficiently large.

5.9. Proof of Claim:

Mr. \((n+1)\) maximizes

\[
\pi(\alpha) \sum_{n+1} \left( \ldots, w_{n+1}^{i} - q_{n+1}^{i} + \sum_{k=1}^{n+1} b_{n+1}^{i} \frac{q_{k}^{i}}{q_{n+1}^{i}}, \ldots \right)
\]

\[
+ \pi(\beta) \sum_{n+1} \left( \ldots, w_{n+1}^{i} - q_{n+1}^{i} + \sum_{k=1}^{n} b_{n+1}^{i} \frac{q_{k}^{i}}{q_{n+1}^{i}}, \ldots \right)
\]

subject to

\[
\sum_{j=1}^{n+1} b_{n+1}^{j} \leq \sum_{j=1}^{n+1} b_{n+1}^{j} \left( \frac{q_{n+1}^{j}}{q_{n+1}^{k}} + \sum_{k=1}^{n+1} b_{n+1}^{k} \sum_{k=1}^{n+1} \frac{q_{k}^{j}}{q_{n+1}^{k}} \right)
\]

and

\[
\sum_{j=1}^{n+1} b_{n+1}^{j} \leq \sum_{j=1}^{n+1} b_{n+1}^{j} \left\{ \frac{q_{n+1}^{j}}{q_{n+1}^{k}} \left[ b_{n+1}^{j} + M \sum_{k=1}^{n+1} b_{n+1}^{k} \right] \right\}.
\]
Suppose there is some $i$ for which $b_{i}^{n+1} > 0$. Then the first-order condition

\[
\pi(a)\left(\frac{\partial u_{n+1}}{\partial x_{n+1}(a)}\right) - \lambda_{n+1}(a) \left[\begin{array}{c}
\sum_{k=1}^{n} q_{k}^{j} \\
\sum_{k=1}^{n+1} b_{k}^{j}
\end{array}\right] 2^n + \pi(\beta)\left(\frac{\partial u_{n+1}}{\partial x_{n+1}(\beta)}\right) b_{i}^{n+1} + M \sum_{k=1}^{n} b_{k}^{j} 2^n = 0
\]

(5.13)

must be satisfied, if $\lambda_{n+1}(a)$ is the Lagrangean multiplier associated with Constraint (5.11). From (5.13) we have that for every scalar $\varepsilon > 0$ there is a pair $(\pi(a), M)$ such that for $\pi(a) < \varepsilon(a)$ and $M > M$, we have $\lambda_{n+1}(a) < \varepsilon$.

Next consider the expression

\[
\pi(a)\left(\frac{\partial u_{n+1}}{\partial x_{n+1}(a)}\right) + \pi(\beta)\left(\frac{\partial u_{n+1}}{\partial x_{n+1}(\beta)}\right) b_{i}^{n+1} + M \sum_{k=1}^{n} b_{k}^{j} 2^n
\]

(5.14)

\[
- \lambda_{n+1}(a) \left[\begin{array}{c}
\sum_{k=1}^{n+1} q_{k}^{j} \\
\sum_{k=1}^{n+1} b_{k}^{j}
\end{array}\right] 2^n
\]

The parameters $\pi(a) > 0$ and $M > 0$ can be chosen so that Expression (5.14) is positive for $j = 1, \ldots, \ell$, which would imply that all offers from Mr. $(n+1)$ are zero, $q_{n+1}^{j} = 0$ for $j = 1, \ldots, \ell$. This contradicts the assumption that there is a positive bid since from (5.11) (or (5.12)) we have...
\[ b_{n+1}^j = 0 \text{ for } j = 1, \ldots, \ell. \]

By choosing \( M > 0 \) sufficiently large and \( \pi(\alpha) > 0 \) sufficiently small, we have shown that Mr. \((n+1)\)'s best response is zero bids, zero offers, and hence zero trade. □

Since we have \( \bar{\sigma}_{n+1} = 0 \), consumers 1, \ldots, n will be content with strategies \( \{\sigma_n, \sigma'_n\}_{h=1}^{h=n} \). Hence, we have constructed a Correlated Equilibrium to \( \Gamma(n+1) \) driven by asymmetric information.

5.15. Conjecture. The Correlated Equilibrium was not easy to construct. Our example relies heavily on our specification of the bankruptcy rule. We conjecture that correlated equilibria driven by asymmetric information are not "generic" for such a severe bankruptcy penalty. If we ease the extent of the bankruptcy penalty (making it a smooth function of the degree of default) then more, interesting correlated equilibria could appear. In fact, the Correlated Equilibrium exhibited in the Aumann-Peck-Shell (1985) notes was relatively easy to construct, because the example allows only real trades in such a way as to obviate the bankruptcy problem.
6. References


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R. Selten (1975), Re-examination of the perfectness concept for equilibrium points in extensive games, Int. J. Game Theory, 4, 25-55.


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<td>François Duchene</td>
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<td>85/156</td>
<td>Political and Economic Fluctuations in the Socialist System</td>
<td>Domenico Mario Nuti</td>
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<tr>
<td>85/157</td>
<td>On the Determination of Macroeconomic Policies with Robust Outcome</td>
<td>Christophe Deissenberg</td>
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<td>85/161</td>
<td>A Critique of Orwell's Oligarchic Collectivism as an Economic System</td>
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<td>85/162</td>
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