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SEGMENTED TRENDS AND NONSTATIONARY TIME SERIES

by

Peter **RAPPOPORT*** and Lucrezia **REICHLIN****

* Federal Reserve Bank of New York and New York University

** European University Institute

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ABSTRACT

This paper explores an alternative method of detrending nonstationary time series, conforming to the notion that economic series undergo infrequent changes in their trend rates of growth. The "segmented trend" models examined are intermediate between trend stationary and difference stationary models examined by Nelson and Plosser, in the flexibility they permit in the trend. The paper shows that it is easy to confuse these models with difference stationary models, especially when the alternative specification is trend stationary Examination of the long historical data series for the United States shows that segmented trends perform favourably against difference stationary models. The implications of these results for modelling of macroeconomic time series are also discussed.

*The opinions expressed in this paper are those of the authors, and do not represent the opinions of the Federal Reserve Bank of New York.

Until recently, the standard approach to removing non-stationarity from an economic time series was to regress the series on time, and treat the residuals as the cyclical component. Obviously, the effect of this approach was to minimize the portion of the variance of growth rates attributed to permanent or trend movements, and to maximize the explanatory power of the cyclical components. It fitted well with "neoclassical synthesis" Keynesian models, in which the stable long run equilibrium path evolves as the result of population growth and the steady advance of technological progress, and temporary shocks are propagated into cycles by sluggish adjustment of wages and prices.

Influential papers by Nelson and Kang (1981) and Nelson and Plosser (1982) disputed the appropriateness of this method. Nelson and Kang demonstrated that regressions of a random walk on time produce residuals with marked cyclical characteristics. However, these are purely an artifact of the erroneous method used to remove the non-stationary component. Using tests developed by Dickey and Fuller (1979, 1981), Nelson and Plosser found that all the nonstationary series they examined were "difference-stationary" (DS), that is, they required differencing.[1/] The important characteristic of a DS process is that the trend rate of growth varies over time. As a result, some of the apparent cyclical fluctuations that result from treating series as stationary movement around linear trends (which Nelson and Plosser call "trend-stationary" (TS)) belong to the trend, or permanent component. Indeed, Nelson and Plosser deduced that innovations to the permanent component dominate temporary innovations in explaining the variance of output changes. They interpreted their results as supporting equilibrium business cycle models, in which markets clear instantaneously, and variability comes from permanent shocks to technology.[2/]

This paper examines an alternative view of nonstationarity that combines elements of both TS and DS models. TS and DS models can be characterized by the flexibility they permit in the trend or permanent components. In a TS model, the trend rate of growth never changes, while in a DS model it is permitted to change (at least) every period, typically according to a normal distribution. In between these two extremes, lie models in which the trend line changes slope less frequently than the data are sampled. We call these models "segmented trends" following Gallant and Fuller (1973).

Segmented trend models are of interest for two reasons. First, they can be used to model economic processes that are subject to infrequent "regime changes" or "structural changes".[3/] The sort of events that typically are regarded as precipitating such changes are dramatic technological innovations, such as the completion of canal and railway networks in the 19th century; changes in labour productivity, such as that which occurred in the mid-seventies; or an alteration of the behaviour of the government, such as its increased participation in the economy following the Great Depression and World War II. During any given regime, ecomonic series thus behave as TS processes. However, over long stretches of time, this characterization will not be correct, as the trend growth rate changes. Indeed, if data are sampled sufficiently <u>infrequently</u>, the process can resemble a DS process, since adjacent observations do not come from the same regime. In view of the association of equilibrium business cycles with DS processes, this characterization is reminiscent of the view that equilibrium models describe long-term but not short-term fluctuations in economic time series.

Viewing segmented trends as models that provide more than one observation per regime also motivates consideration of other aspects of nonstationarity. Thus, there is no reason why the variance and serial correlation properties of cyclical movements should not also change as the regime changes.[4/] Indeed, long-term historical data have been examined from this perspective by several authors (c.f. for example De Long and Summers (1986)) as possible evidence of the changing effects of the government on the economy.

A second reason for drawing attention to segmented trends is that they can be easily confused with DS processes. We argue below that the residuals from fitting a TS model to a segmented trend can display the same characterestics as those that result from applying the same procedure to a series of the DS class. This parallels the results of Nelson and Kang (1981), discussed above. In addition, if the true process is a segmented trend, and one chooses between a DS model and a TS model, there is a tendency for the Dickey-Fuller tests to favour the DS model. A similar result emerges when a DS model is compared to a segmented trend with less segments than the true process. The implication is that the DS specification is a "default" model: it will appear to fit the data best if competing models are not adequately parameterized.

As mentioned above, segmented trends offer an alternative perspective on the decomposition of variance into trend and cycle. Obviously, a greater portion of the variance of first differences is attributed to changes in trend, than in a TS model, but potentially less than in a DS model. It also is particularly important to distinguish among TS, DS and segmented trend processes for the purpose of forecasting. Both DS and TS models provide biased estimates of the out-of-sample trend of a segmented model. This contrasts with the case of forecasting a TS process using a DS model, for which long horizon forecasts will not be biased.

To assess the relevance of segmented trend models, we reconsider the historical series examined by Nelson and Plosser, by estimating a segmented trend model with a single break in 1940. This date is chosen in the light of the belief of many economists that the changed role of the government in the economy around this time altered the performance of macroeconomic aggregates. In contrast to Nelson and Plosser's finding that all non-stationary series are DS, we find that, after suitable corrections for nonstationarity in variances, the series are divided into two distinct types. The "price" series are all of the DS class, while the quantity series (output, employment, etc.) all reject the DS model decisively in favour of the segmented trend model.

The plan of the paper is as follows. Section 1 briefly reviews the existing methods for discriminating between TS and DS models, introduces the notion of a segmented trend, and shows the pitfalls of comparing DS and TS models using segmented trend data. Section 2 amends Dickey and Fuller's procedure for the purposes of testing a DS model against a segmented trend alternative, and the next section applies these tests to the data used by Nelson and Plosser. An interpretation of the results of the paper in the light of earlier findings is contained in Section 4, and Section 5 summarizes the paper.

1. Models of Non-stationarity

In this section, we consider the relationship of segmented trends to TS and DS models. We show that, according to criteria that are common in the literature, segmented trends fall into neither class. Instead, it is useful to regard segmented trends as the middle ground of an ordered class of models, of which TS and DS models represent the extremes. The number of parameters increases as we move from the TS model to models with more and more segments until (loosely), one reaches the DS class. This motivates the results we present below, to the effect that, when the true process is a segmented trend, a DS model will asymptotically be favoured over a TS model (or a lower-order segmented trend) by standard regression procedures. The message is thus that care must be taken in specifying alternatives to DS models.

(a) Trend-stationary, Difference-Stationary and Segmented Trend Models

Segmented trends are non-stationary processes in which the slope of the trend changes intermittently. The simplest example of such a process is [5/]

$$(1)\quad y_t^T = \begin{cases} a_1+b_1 t & t \leq t^* \\ a_2+b_2 t & t^* \leq t \end{cases}$$

$$a_1 \neq a_2,\ b_1 \neq b_2,$$

Here y^T represents the trend part of y, the logarithm of the original data. y_t and y_t^T are related by the familiar decomposition:

$$(2)\quad y_t = y_t^T + y_t^C,$$

where y_t^C represents the (stationary) cyclical component of y_t. According to this model, the nonstationary component of e^y grows along trend at rate b_1 up to t*, after which time it grows at the trend rate of b_2. Thus, y follow a (different) TS process before and after t*.

To motivate the need for examining segmented trends, consider the comparison of TS and DS models used by Dickey, Bell and Miller (1986):

$$(3) \quad : \quad \phi(L)\,(1-\rho L)(y_t-a-bt) = \Theta(L)u_t, \quad \rho<1 \quad \text{(TS)}$$

$$(4) \quad : \quad \phi(L)\,(1-L)(y_t-a-bt) = \Theta(L)u_t, \qquad \text{(DS)}$$

Here, u_t is stationary and serially uncorrelated, and the polynomials $\phi(.)$ and $\Theta(.)$ have no unit roots. These equations say that, if y_t is a DS process, its deviation from a linear trend will still have to be first-differenced in order to be rendered stationary, while it will only require ρ-differencing ($\rho<1$) if the process is TS.

Neither of the transformations of y in (3) and (4) will make a segmented trend stationary. Consider, for example, the two-segment model, (1). The quantity

$$(5) \quad (1-\rho L)\,(y_t^T - a - bt) \qquad \rho < 1$$

which appears in (3), depends on t over at least one of the segments $t\leq t^*$ or $t^*\leq t$ for any a and b. So the segmented trend process described by (1) does not fit model (3) for stationary u_t. Of course, the first difference of (1), which would be produced by the DS model (4), is not stationary either. However, the nature of the resulting non-stationarity in

$$(6) \quad (1-L)\,(y_t^T - a - bt) = \begin{cases} b_1-b, & t \leq t^* \\ b_2-b, & t^* \leq t \end{cases}$$

is different in an important respect from that in (5). The sample variance of (5) explodes as the sample size tends to infinity,[6/] whereas the sample variance of (6) is constant for different sample sizes.

The autocorrelation functions of the transformations of y^T in equations (5) and (6) are actually quite similar.[7/] However, one typically observes y, not y^T. When y contains the segmented trend (1), the autocorrelations of $(1-\rho L)(y_t-a-bt)$ will be similar to those of expression (5), since the explosive term in (5) will typically dominate in the autocovariance and the variance. However, it is possible that the first difference of the cyclical term in $(1-L)(y_t-a-bt)$ can dominate (6) in the autocorrelation function, giving it a damped appearance. In this case, comparison of the ρ-differenced and first-differenced deviations of y_t from a linear trend will again suggest that the linear trend is not adequate, but that the DS model is.

Equation (6) also illustrates the dangers of using a DS model to remove non-stationarity from a segmented trend process. Typically, one would estimate the first difference model with an unchanging intercept, b, (i.e. the levels process is assumed to have constant drift). However, this procedure attributes a portion of the change from b_1 to b_2 to the innovation in the process. This may bias the assessment of the cyclical volatility of the series. Similarly, extrapolation of y at the rate of drift b will lead to biased forecasts.

This discussion of a model with one change in the trend carries over to the case where there are more dates such as t*, when the trend rate of growth changes. It is instructive to consider what happens when the number of turning points in the model is equal to the sample size. The graph the realisations is given by

(7) $y^T_\tau = a_t+b_t\tau, \; t-1 \leq \tau \leq t$

which simply joins adjacent observations by straight lines. As in the case of

(3), the limits on τ in (9) imply that the trend lines for each period join, i.e. $a_{t-1} + b_{t-1}t = a_t + b_t t$. Consequently

$$(8) \quad y_t^T - y_{t-1}^T = b_t.$$

Equation (8) obviously holds for any arbitrary series of observations. However if we impose the additional conditions that b_t is a stationary invertible stochastic process, then y^T is a DS process.[8/]

It therefore appears useful to classify nonstationary models according to the number of changes that occur in the trend parameters. DS and TS models may be regarded as particular extremes of this class. One can consider each member of the class of segmented trend models as realizations of an "arrival time" distribution, with the DS model being the particular limiting case where the frequency of occurences is always greater than that frequency of observation. For example, the interval between changes in trend could follow as an exponential distribution, in which case, the number of changes in the trend over any historical period would follow a Poisson process. This process would thus determine whether or not $b_t = b_{t-1}$. If b_t and b_{t-1} are to differ, i.e. there is an arrival at t, the new value, b_t, can be drawn from any distribution.[9/] Hence, b_t can be modelled as a mixture of the Poisson process with some other distribution that scales b_t. As is well known, a Poisson process tends to a normal distribution as its mean (the expected number of arrivals per period) tends to infinity. This corresponds to the case of the DS model.

This framework entails a particular interpretation of the use of different models of non-stationarity. When one specifies a non-DS segmented trend model (along with a date of the change point), as will be done in this paper, one performs inference conditional on a particular realization of the process generating b_t. On the other hand, when one uses a DS model, one estimates the parameters of the distribution of b_t. This is reminiscent of the dichotomy between fixed and random effects models in the literature on cross-section time-series models. It is also implicit in the current treatment of TS and DS models: the trend component of the TS model is typically regarded as deterministic while that of the DS model is regarded as stochastic.

(b) Inferences based on comparisons of TS and DS model when the true process is a segmented trend

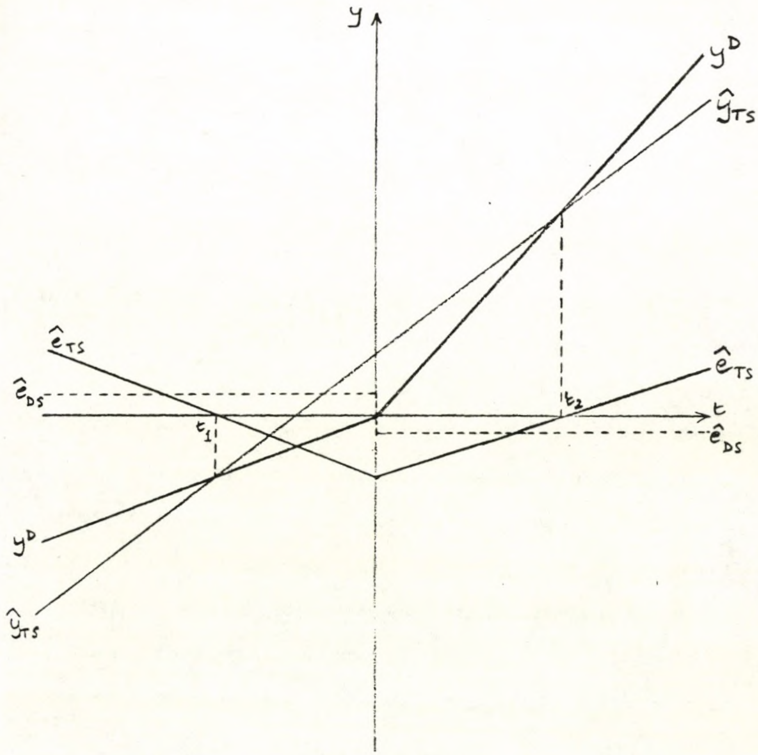
If the true process is a segmented trend, but one fits DS and TS models to the data, which will be favoured by traditional regression procedures? The answer is provided by the following result:

Proposition

> If the true model contains K+1 segments, then any fitted model involving K or less segments will, asymptotically yield a larger sum of squared residuals than the DS model.

The proof of this proposition is contained in the appendix. The intuitive idea is quite simple, and is illustrated here for the case K = 1. Thus equation (1) describes the true model, to which we consider fitting alternately a DS and a TS (linear trend) model.[10/] The situation is indicated in Figure 1 on which are illustrated the actual data (y^T), the fitted part of the TS model ($\hat{y}_{TS}$) and the residuals from the TS and DS models ($\hat{e}_{TS}$ and $\hat{e}_{DS}$ respectively).[11/] Let T_1 and T_2 be the numbers of observations in each segment. Consider the behaviour of the sums of squared residuals as T_1 and T_2 tend to infinity, while T_1/T_2 remains fixed. The residuals for the DS model remain bounded while those for the TS model grow linearly with T_1 and

FIGURE 1



T_2. Hence, the sum of squared residuals grows at the rate T^3 for the TS model, but only at the rate T for the DS model.

The sample size at which the DS sum of squared residuals is overtaken by the TS sum of squared residuals depends on the autocovariance properties of the random components of the process, and the difference between the trend rates of growth, b_i. In the two segment case, with $T_1=T_2$, it can be shown that the critical condition is:

$$\frac{T_1^4+2T_1^3-10T_1^2-11T_1-3}{12(2T_1+1)^2} > \frac{\sigma^2-2\gamma_1}{(b_2-b_1)^2}$$

where σ^2 and γ_1 are the variance and first autocovariance of the random movements around the linear trends of the two regimes. To get an idea of the practical significance of this condition, consider the case of the real GNP data used below. When the sample of 62 observations is split in the middle, $(\hat{b}_2-\hat{b}_1)^2 = .000144$. If $\gamma_1=0$, the variance of the first difference of real GNP is an estimate of $2\sigma^2$, and yields $\sigma^2 = .001825$. Hence, the right hand side of the above condition is 12.67, which is exceeded by the left hand side when T_1 25, or when the total sample exceeds 50. Hence, even without assuming the "cyclical component" to be serially correlated, comparing TS and DS models of this sample of real GNP data will tend to favour the DS model when the true model is a segmented trend. If the movements around trend are positively serially correlated, as one would expect, the critical size of T_1 declines. If $\gamma_1 = 0.5\sigma^2$, then the critical value of T_1 is 3, that is, the DS model will dominate in samples of more than six observations. (The first autocorrelation of real GNP exceeds 0.5 in both subsamples.)

The principal tests for discriminating between TS and DS models have been formulated by Dickey and Fuller (1979, 1981) who suggest estimating the equation

(9) $y_t = \alpha + \gamma t + \rho y_{t-1} + e_t$

and testing the hypothesis:[12/]

(10) $H_0 \; : \; \rho = 1$

If this null hypothesis is accepted, then the series is taken to be DS. A straightforward corollary of the above proposition cautions that this procedure will tend to favour the DS model over the TS alternative when the time process is a segmented trend.

Corollary

> If y follows a segmented trend, then the OLS estimator of ρ in equation (9) tends to 1 in probability.

The implication is that underparameterization of a segmented trend will, asymptotically and in samples of typical size, lead the data to evidence a closer fit to the DS model than to underparameterized segmented models. It should also be noted that these results are based on the assumption that the points at which the trends change are chosen optimally (with respect to the sum of squared residuals criterion). Fixing the change points arbitrarily in advance of estimation biases the results even more strongly against underparameterized segmented trend models.[13/]

Another criterion that has been suggested for discriminating between TS and DS models can also lead one to favour a DS interpretation instead of an underparameterized segmented trend. Nelson and Kang (1981), building on earlier work by Chan, Hayya and Ord (1977), show that a symptom that a DS process has been inappropriately detrended by use of a TS model is the pronounced damped cyclical shape of the autocorrelation function of the residuals. (Figure 2). Nelson and Kang show that the shape of the autocorrelation function depends only on the sample size. Thus, for example, the first place at which the autocorrelation function crosses the horizontal axis is always at a lag that is approximately 15% of the sample size.

One must be careful in interpreting residual autocorrelations from TS models that resemble Figure 2 as the sign that a DS model is appropriate. Examination of Figure 1 reveals that the residual autocorrelations from applying a TS models to a segmented trend process with a single break also follow a distinctive pattern. The residuals in the figure are positive up to time t_1 and after t_2, and negative between t_1 and t_2. Thus, autocovariances at short and long lags will be positive, while autocovariances at intermediate lags will be negative. It can be shown that the autocorrelation function of these models follows a third order polynomial in the ratio of the autocorrelation lag to the sample size. (A proof is available on request). The behavior of this autocorrelation function for the same size sample as Nelson and Kang used is plotted in Figure 3.[14/] It is evident that this function would be easy to confuse with Figure 2. Hence, while such cyclical patterns in the residual autocorrelations from a TS model are a sign that all is not well, they do not show that a DS model is relevant.

These results suggest that extreme care must be taken in interpreting the inadequacy of a TS model as justification for using a DS model. In the sequel, these models are also compared with trend models containing more than one segment.

2. Testing DS Processes Against Segmented Trends.

In this section, we adapt Dickey and Fuller's testing procedure to take account of the case where the alternative to the unit root (DS) model is that y_t follows a segmented trend. Our starting point is an augmented version of equations (3) and (4):

FIGURE 2

Theoretical Autocorrelations of Residuals from Detrended Random Walk (Nelson and Kang (1981))

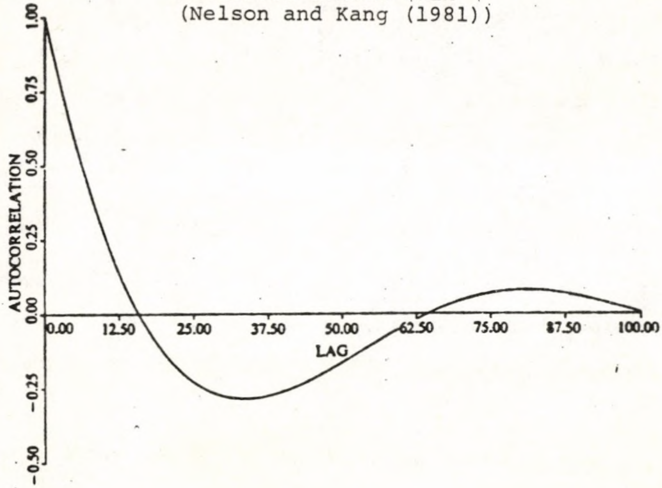
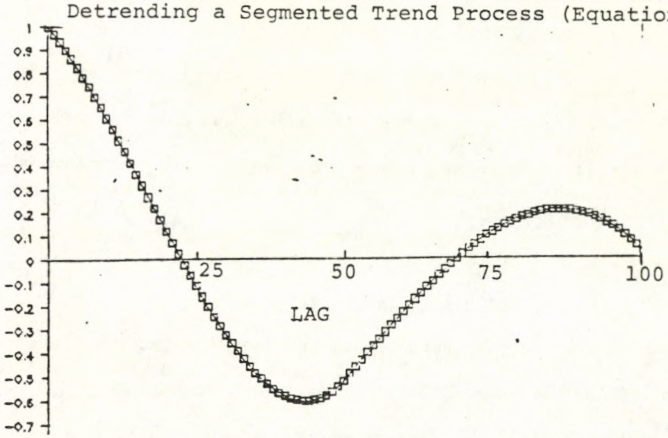


FIGURE 3

Theoretical Autocorrelations of Residuals from Detrending a Segmented Trend Process (Equation (3))



$$(11) \quad \phi(L)(1-\rho L)(y_t - a_1 - b_1 t - \sum_{i=2}^{K} (b_i - b_{i-1})(t - t^*_{i-1})d_{i-1,t}) = \theta(L)u_t \quad (\rho<1)$$

$$(12) \quad \phi(L)(1-L)(y_t - a_1 - b_1 t - \sum_{i=2}^{K} (b_i - b_{i-1})(t - t^*_{i-1})d_{i-1,t}) = \theta(L)u_t$$

In these equations, t^*_{i-1} is the date of the switch from the (i-1)th to the ith segment, and $d_{i-1,t}$ is a dummy variable, equal to zero prior to t^*_{i-1} and one from t^*_{i-1} onwards.[15] The interpretation of these equations is analogous to that given for (3) and (4) above. Equation (12) says that the deviation of y_t from a segmented trend still needs to be first-differenced in order to yield a stationary process. In contrast, equation (11) implies that the deviations of y_t from the segmented trend are already stationary.

To develop an equation suitable for testing the DS hypothesis against segmented trend models, we solve (11) for y_t. To avoid unnecessary algebra, we focus on the two segment model (K=2). Equation (11) yields the following analogue to equation (1):[16]

$$(13) \quad y_t = \alpha_1 + \alpha_2 d_{1t} + \gamma_1 t + \gamma_2 d_{1t} t + \rho y_{t-1} + e_t.$$

Under the condition $\rho=1$ equation (13) reduces to

$$(14) \quad y_t = b_1 + (b_2 - b_1)d_{1t} + y_{t-1} + e_t$$

This equation differs from a pure DS model, in that it permits the drift term to change at t^*_1. Such a model is perhaps more in the spirit of a segmented trend model than a DS model, in that the first difference of a process satisfying (14) is not stationary. Thus, even if the hypothesis $\rho=1$ is not rejected, it does not follow that the DS model of section 1 is adequate, for which it is also required that $\alpha_2=0$.

As in the Dickey-Fuller procedure, the computed t-statistic for $\hat{\rho}$ in equation (13) does not have a standard distribution. Moreover, the critical values vary with K, the number of segments in the estimated model. In order to arrive at the appropriate critical values, we performed a number of Monte-Carlo experiments, in which tests of ρ=1 were carried out on realizations of a driftless random walk, using samples of 100 observations.

The results are shown in Table 1, the first row of which gives the critical values for the t-statistics of $\hat{\rho}$ in equation (1) (from Fuller (1976)). Rows (b) and (c) show the results for two segment models with t* = 50, 25, respectively, and row (d) contains the critical values for a 3-segment model with change points at t=25 and t=75. These critical values are appreciably higher than those for Dickey and Fuller's. Evidently, the more (spurious) regimes are included in the model, the larger in absolute size are the values of the t-ratio consistent with the null.[17/] In the two segment model, the critical values do not appear to depend much on the position of the change in regime. Whether this is so for higher-order segmented trend models is not clear, but as K increases, the number of permutations to be examined becomes unwieldy. As a consequence, we shall only base our statistical inferences on two-regime models, using other approaches to account for the effects of additional regimes.

One difficulty in using equation (13) is that estimation is performed conditional on a choice of the date t*. Typically, this choice will not occur independently of the realization (data series) on which the equation is to be run. There is a danger that "ex post" choice of a change point biases the tests away from the DS model. A safeguard against this problem is to compute critical values for the

Table 1

Critical Values* of t-statistic for H_0: $\rho=1$ (T=100)

Model	Probability of larger value 0.01	0.05	0.10
a) 1 Segment	4.04	3.45	3.15
b) 2 Segments, t*=50	4.81	4.23	3.95
c) 2 Segments, t*=25	4.73	4.08	3.74
d) 3 Segments, $t_1^*=25$, $t_2^*=75$	5.45	4.76	4.43
e) 2 Segments, t* at extreme	4.66	4.08	3.74
f) 3 Segments t* at max and min	5.43	4.68	4.37

Sources: a) Fuller (1976, p 373). b) - f): Monte Carlo simulations of t-statistic for $\rho=1$ in regression models of the form of equation (13). The same 5000 samples of 100 observations were used in each case.

* The data in the Table are the negative of the actual values. Where $\hat{\rho} < 1$, a rejection at the given significance level occurs if the calculated t-statistic is larger in absolute value than the number in the Table.

case where t* is chosen after viewing the series. On way to approach this is to allow the computer to set the break point at the date of the extreme value of the realization (i.e. at its maximum or minimum, whichever turns out to be farthest from the initial value of zero).[18/] The results of the Monte-Carlo simulation for this method of selecting t* are shown in row (e). Evidently, they do not differ dramatically from rows (b) and (c). Of course, the number of break points (as well as their position) may also be chosen after the data have been examined. To understand the consequences of this strategy, we reran the simulation with break points at the maximum and minimum values of each realization (row (e)). The critical values are uniformly lower than those in row (d). Hence, no apparent bias towards rejection of the DS model is created by selecting change points at extremes, after viewing the data.

3. Empirical Results

This section of the paper applies the ideas developed above to the data examined by Nelson and Plosser (1982). They assembled a number of long historical series, in some cases stretching from the mid-nineteenth century to 1970.[19/] The series include wages, prices, output and financial data, and so constitute a good cross-section of the types of data encountered by macroeconomists. Nelson and Plosser used the Dickey-Fuller testing procedure to discriminate between TS and DS models of these series, and found the DS model to be favoured in all but one case (the unemployment rate) where the series appeared to be stationary. Here, we shall consider the effects of pitting the DS model against members of the segmented trend class, using the testing procedure outlined in the last section.

Before we can proceed, we need to pick the date dividing the two historical segments, which we set at 1940. We base our choice on the popular belief that it was around this time that the government started to take a more active role in the running of the economy, both in terms of its fiscal behaviour, and as the consequence of the introduction of certain regulatory activities, particularly in the financial area. One can quarrel both with the belief in the validity of this story and in the timing we have chosen, and we do not intend to provide a historical defense of these assumptions here. However, in view of the Monte-Carlo results described above, if the structural change argument is false, then this should not affect the tests of the unit root hypothesis, as long as the correct critical value is chosen. Further, if the structural change approach is correct, but the date of the switch is not 1940, then this should only serve to bias the results in favour of the DS model.

A second issue concerns the specification of the non-trend components in the model. As is usual, in order to ensure valid inference, these must be specified in such a way as to cause the resulting errors in the model to have a scalar covariance matrix. Nelson and Plosser include sufficient lagged differences of the dependent variable ($y_{t-1}-y_{t-2}$, $y_{t-2}-y_{t-3}\ldots$, etc.) to reduce the sample residual autocorrelations to those of a white noise series. This approach implicitly assumes that the distribution of the non-trend components is unchanged over the entire sample period. However, a number of authors have suggested that both the amplitude of the observed business cycle and the size of the random shocks hitting the economy have changed since the Great Depression. (C.f. for example, Delong and Summers (1986), Romer (1986)).[20/] These considerations suggest that the estimated model be amended to account for non-stationarity in the errors as well.

In order to assess the relevance of changing "cyclical" characteristics, we estimated the model

$$(15)\quad y_t = \alpha + \gamma t + \rho y_{t-1} + \delta_1(y_{t-1}-y_{t-2}) + \delta_2(y_{t-2}-y_{t-3}) + e_t$$

for each series over four different periods. One is the entire span of each data series, another two run from the beginning date to 1929, and 1939 respectively, and the last runs from 1940 to 1970. Table 2 reports the estimates of δ_1, δ_2 and σ_e (the standard error of the regression). Each series shows a distinct variation over time in at least one of these parameters.[21] Almost every series exhibits a markedly higher standard error in the earlier periods than in the later one. (A particular exception is the consumer price index.) This corresponds to the observation of Delong and Summers that the variance of the shocks to the economy has diminished over time. Variation over time in δ_1 and δ_2 is less common, but is nevertheless quite pronounced for half the series. Similarly, there is a tendency for series to exhibit (absolutely) larger values of δ_1 in the later period. Finally, except for the price series, estimates of these parameters for the pre-1940 era depend critically on whether the 1930's are included in the sample. This observation will figure in the subsequent interpretation of the results.

In view of the results displayed in Table 2, it is advisable to take account of changing error structure in the estimated model. Thus, for each series, we estimated a model of the form:

$$\begin{aligned}(16)\quad y_t = {} & \alpha_1 + \gamma_1 t + \rho y_{t-1} \\ & + \delta_{11}(y_{t-1}-y_{t-2}) + \delta_{21}(y_{t-2}-y_{t-3}) \\ & + d_{1t}\cdot[\alpha_2 + \gamma_2 t + \delta_{12}(y_{t-1}-y_{t-2}) + \delta_{22}(y_{t-2}-y_{t-3})] + e_t\end{aligned}$$

In this equation, both the variance and serial correlation pattern of the non-trend component are allowed to vary, depending on which of the two segments obtains. Thus e_t is an independently distributed random variable, with mean zero and variance σ_i^2 during regime i.

In models where it is assumed that the error variance changes, equation (16) was estimated in two stages by weighted least squares. Equation (15) was first estimated for each regime, and the standard error of the regression was retained. This was used to scale the observations of the regime in a second stage regression of the form of (16), which produced the t-statistic for testing the unit root hypothesis (ρ=1)[22/].

The results of these tests for several specifications are displayed in Table 3. The first column of the Table contains the results for equation (15), which compares DS and TS models. Only one of the series (industrial production) has a test statistic close to the 5% critical value of 3.45. Equation (15) is of the same form as that run by Nelson and Plosser, except that they estimate a longer lag distribution in the first differences of some series to eliminate serial correlation. This produces lower test statistics, particularly for industrial production. Model (ii) allows the trend to change at 1940. Although the test statistics are almost all higher than those for model (i), only one series, common stock prices, rejects the DS model. This particular result is somewhat surprising in view of the voluminous literature on efficient markets. As we shall discuss below, it appears to be the consequence of including the 1930's in the data sample.

Table 3

t – statistics for Tests of ρ=1a/

	Including 1930's					Excluding 1930's			
	(i)	(ii)	(iii)	(iv)	(v)	(vi)	(vii)	(viii)	(ix)
No. of Segments	1	2b/	1	2	4c/	1	2	1	2
Non-stationary Variance Correction			X	X	X			X	X
Nominal Wage	2.39	2.79	3.21	3.11	3.36	3.33	3.74	3.54*	3.24
Real Wage	2.57	3.21	2.63	4.06	4.76	3.86*	4.02	3.44	4.22
CPI	2.00	2.68	2.26	2.37	2.65	2.12	2.43	2.53	2.37
GNP Deflator	2.57	2.79	3.49*	2.95	2.98	3.46*	3.59	3.85*	2.93
Common Stock Prices	2.26	4.38*	2.10	4.28*	4.79	1.73	2.79	1.76	2.86
Velocity	1.34	3.35	1.30	3.26	3.41	1.34	2.53	1.30	2.47
Money	2.98	2.98	4.02*	4.15	5.52	2.86	4.30*	3.36	5.10**
Real GNP	3.02	3.39	4.11**	4.40*	6.62	4.64**	4.49*	4.98**	5.48**
Nominal GNP	2.28	3.15	3.46	3.69	4.54	3.90*	4.01	3.92*	4.28*
Real GNP per head	3.03	3.19	4.07**	4.01	6.32	4.87**	4.68*	5.42**	5.27**
Ind. Production	3.63*	2.68	3.81*	4.06	6.60	3.48*	4.97**	3.48*	5.59**
Employment	3.13	3.19	3.52*	3.74	5.32	2.79	4.47*	3.00	4.75*
Unemployment	2.98	2.95	3.62*	3.49	4.80	3.95*	4.36*	4.25**	4.54*

a/ Statistics displayed are the negative of those estimated.

b/ Models with two segments have first observation of second segment at 1940.

c/ The four segment models have a change in the trend at 1929, 1933, and 1940.

d/ First segment error characteristics are estimated using data up to 1939 for models (1) – (5), and up to 1929 for models (6) – (9).

* Significant at 5% level. ** Significant at 1% level. Critical values for the two-segment models are given in Table 1, Row (b).

Model (iii) augments Model (i) by allowing the serial correlation and variance characteristics of the non-trend component to differ before and after 1940. This model evidences a substantial rise in the t-values. Five of the series now reject the DS model at the 5% level, and three others are close (employment, nominal GNP and the GNP deflator). From these results, it appears superficially that it is not necessary to invoke segmented trends in order to dispute empirically the finding that standard macroeconomic series follow a DS process. All that seems necessary is to use the appropriate correction for the non-trend component. Extra weight is added to this view by the results for model (iv), in which the segmented trend of model (ii) is combined with the nonstationary variance corrections of model (iii). The test statistics for model (iv) are not appreciably higher than for model (iii).

A closer look at the characteristics of the data suggests, however, that the variance correction used in these models is not quite adequate. Table 2 documented that for many series there are substantial changes in the persistence, measured by δ_1 and δ_2, and the residual error variance, according as the 1930's are included in the sample or not. Thus, for example, the standard error of industrial production rises by 30% when the 1930's are included, while those of real GNP and employment rise by about 16%. Several of these series fall dramatically from 1929 to 1933, and rise quickly from 1933 to 1940. An obvious question is, do these movements represent changes in the cycle or the trend rate of growth? Was the tailspin that followed the Great Crash merely an extremely pronounced cyclical downturn, or a secular movement that was only arrested by the New Deal? There does not seem to be any direct way of addressing these questions. However, the evidence on the behaviour of the data during the 1930's does seem to raise the necessity of dealing with the period separately.

Sufficient degrees of freedom are not available to permit estimation of a model of the non-trend component with coefficients that vary during the 1930's. An alternative approach is to estimate a model with a trend that changes in view of the developments of the 1930's. This is done in model (v), where the trend is allowed to change slope in 1929, 1933, and 1940. Here, the t-statistics for the hypothesis $\rho=1$ jump dramatically, with the exception of the price series.[23/] This rise in the test statistics is consistent with the demonstration of the last section that an underparameterized segmented trend model (i.e. models (ii) and (iv)) will be dominated in large samples by the DS model. However, the relevant critical values for model (v) are not known, although one would imagine then to be higher than those exhibited in Table 1 for the three regime model. Hence, the implications of the results for model (v) are difficult to evaluate.

Another approach to dealing with the 1930's is simply to drop these observations from the sample. While this involves a loss of information, it does aid inference by restricting the sample to relatively homogeneous data. Moreover, if the DS model is correct, its performance should not be affected by this omission. Models (vi) - (ix) replicate models (i) - (iv) respectively. Model (vi) shows rejections of the DS hypothesis for five series, compared to only one when the 1930's are included. In contrast to model (ii), the two segment model (vii) shows a quite dramatic rise in the test statistics over the one-segment model (vi). Here, an interesting pattern emerges for the first time: the "quantity" data (output, employment, money) appear to follow segmented trend models, while the price series (nominal wages, CPI and stock prices in particular) seem to conform more to the DS process. The only exception to this is the real wage.

Given the nonstationary nature of the non-trend components documented in Table 2, models (vi) and (vii), which do not correct for nonstationary autocorrelations and variance, are not a reliable basis for inference. Models (viii) and (ix) replicate models (vi) and (vii), allowing for the usual changes in the parameters of the non-trend processes. The results confirm the qualitative findings of models (vi) and (vii). In contrast to the models including the 1930's data inclusion of the segmented trend term (vii) makes as great a contribution as the variance correction (viii) to the rejections of the DS model. These results suggest that an important role of the variance correction (or the inclusion of more segmented trend lines as in (v)), is to account for the 1930's. Once this period is removed from the sample, the contribution of the variance correction to the test statistics diminishes. Finally, model (ix) shows the effect of both a segmented trend and the variance correction on the data excluding the 1930's. The results are qualitatively similar to model (vii), except more pronounced. Thus, with the exception of the real wage, all the price series are well described by a DS model. In particular, the anomalous behavior of common stock prices in models (ii) and (iv) is no longer present. In contrast all the "quantity" series significantly reject the DS model in favour of a segmented trend.

In Section 1, we motivated the need to consider segmented trends by showing how a DS model would be favoured over an underparameterized trend model. The reason for this is that the differences of a segmented trend make a smaller contribution to the residual sum of squares that do its (squared) deviations from a linear trend. Hence, the estimate of ρ is pushed towards unity. If this is the explanation for the superiority of the segmented trend models over DS models of quantity data, then there should be observable consequences when subsamples are examined. In comparisons of DS and TS models (equation (15)), subsample estimates of ρ should be smaller than full sample estimates, and subsample tests of the DS model against the TS model should be more likely to reject the DS model.

The relevant data are provided in Table 4, which shows the results of estimating equation (15) over a variety of subsamples. For the quantity data, the first thing to note is that $\hat{\rho}$ is uniformly larger over the whole sample than it is in the pre-1929 and post-1940 samples. Second, when the 1930's are added to the earlier sample, there is a dramatic rise in the estimated value of ρ. Both these results conform to the framework of Section 1: segmented trend data will tend to favour a DS model when the alternative is an underparameterized segmented trend (in this case, a TS model). When segmented trend data are split up into subperiods containing only one segment, the estimated value of ρ is much lower. Third, the post-1940 quantity series reject the DS model in favour of the TS alternative, as the two-segment model would predict. In contrast, the t-statistics are insignificant for the data prior to 1929. However, this appears to be more a reflection of the low precision of the estimates of ρ than of their being close to unity. In several cases, the hypothesis ρ=0 cannot be rejected at conventional significance levels.

The price data evidence markedly different patterns. There is little variation in $\hat{\rho}$ among sample periods, and a similar uniformity is displayed by the t-statistics. These results are exactly what one would expect from DS processes: no matter how the sample is dissected, the estimate of ρ should stay close to unity.

Table 4

Estimates of ρ and Dickey-Fuller Tests, Selected Intervals[a/]

	Entire Sample		Beginning-1929		Beginning-1939		1940-70	
	$\hat{\rho}$	t	$\hat{\rho}$	t	$\hat{\rho}$	t	$\hat{\rho}$	t
Nominal Wage	.93	2.39	.76	2.47	.92	1.53	.88	2.36
Real Wage	.88	2.58	.60	2.13	.66	2.51	.61	3.61*
CPI	.97	1.94	.95	1.89	.97	2.36	.91	1.56
GNP Deflator	.91	2.58	.81	2.60	.91	1.70	.91	1.81
Common Stock Prices	.90	2.26	.83	1.62	.60	4.21**	.57	2.73
Velocity	.95	1.37	.90	1.32	.83	2.31	.68	2.61
Money	.92	3.04	.73	2.77	.97	1.62	.85	4.28*
Real GNP	.82	3.03	.41	2.34	.64	2.70	.50	4.90**
Nominal GNP	.90	2.33	.60	2.05	.81	2.26	.72	3.77*
Real GNP per head	.80	3.05	.33	2.21	.56	2.86	.54	4.81**
Ind. Production	.80	3.57*	.64	3.25	.86	2.12	.45	4.72**
Employment	.84	3.15	.73	2.32	.88	1.78	.61	4.24*
Unemployment	.75	2.98	.42	3.16	.77	1.98	.62	3.40

a/ Estimates are from equation (15). The t-statistics are for the hypothesis ρ=1. The critical values of the t-statistics vary according to the sample size (see Table 2), and are taken from Fuller (1976, p. 373). For the period 1940-1970, we used a 5% value of 3.6 and a 1% value of 4.38, which are relevant for a sample of size 25.

In summary, this section has tested a large number of specifications of the trend component in widely used macroeconomics data series. The results show that Nelson and Plosser's finding that all series followed a DS model is not robust to more appropriate error corrections. Examination of the data suggests that the 1930's be treated differently from other periods, but the only method that permits statistical inference to be carried out involves dropping these observations. When the 1930's are excluded from the sample, a new uniformity emerges from the results: the quantity series obey segmented trend models, whereas the price series follow DS processes.

3. Economic Interpretations

The results of the last section run counter to a growing body of literature that finds the DS model to be adequate in describing the non-stationarity in macroeconomic time-series, and draws conclusions from this finding for the validity of theoretical models of economic fluctuations. In this section, we discuss the relationship of our results to this discussion.

Nelson and Plosser (1982) use their findings to develop inferences concerning the relative contributions of the cyclical (y_t^C) and stochastic trend (y_t^T) components of output (equation (3)). Using the empirical findings that output is a DS process and that its first differences follow a MA (1) process, they infer that innovations to y_t^T contribute more to the variance of $y_t - y_{t-1}$, than do innovations to y_t^C. As a result of the DS specification, innovations to y_t^T have a permanent effect on y_t, whereas those affecting y_t^C only have a transitory effect. Since "monetary disturbances have no permanent effects" (p. 159) (aside from the Tobin-Mundell effect), they conclude "that real (non-monetary) disturbances are likely to be a much more important source of output fluctuations that monetary disturbances" (ibid).[24/] Nelson and Plosser note that "real business cycle" models, in which only real shocks to technology and preferences are present, are capable of generating the kind of fluctuations observed in empirical series. A characteristic of such models is that the business-cycle-like behaviour of aggregates is generated from models that nevertheless are in competitive equilibrium at each point in time (C.f. for example, Kydland and Prescott (1982), Long and Plosser (1983)).

The finding that non-stationarity is represented by a segmented trend means that the first difference of output can be broken down into three components: the change in the trend, and the innovations to y^T and y^C. The latter two components both have transitory effects in this model, and so their separate contributions cannot be identified. Thus, it is not possible to measure the relative contributions of trend and cycle innovations to variation in output. However, a segmented trend model does potentially attribute smaller fluctuations to the non-deterministic trend component than a TS model.[25/]

While there has been a tendency to identify the observed DS characteristics of macroeconomic time series with equilibrium business cycles (C.f. for example, Gagnon (1986)), there are other models which involve the persistence of shocks to real variables. Campbell and Mankiw (1986) suggest that such persistence is consistent with Keynesian models of "secular stagnation" (Diamond (1984), Weitzman (1982)) that exhibit locally stable multiple equilibria. As they put it: "if the economy gets stuck in a 'bad' equilibrium, there is no force driving the economy back to a Pareto-dominating equilibrium". (Campbell and Mankiw (1986, p. 21)). Thus, the shocks that push the economy from one equilibrium to another will exhibit persistence in time-series analysis of historical data.

Neither the equilibrium business cycle models nor the Keynesian models described above are inconsistent with the finding that real variables follow a segmented trend. These models simply point to the persistent, as opposed to decaying, effects of shocks. But they are completely agnostic as to the frequency of these shocks. In interpreting the models, economists have implicitly assumed that the permanent shocks occur with a frequency greater than that at which the data are observed. However, hard information has yet to be produced that the permanent shocks do not occur every thirty years, rather than every thirty days. If the permanent shocks are separated by long stretches of time, one could conceivably observe non-stationarity of the segmented trend, rather than the DS variety, yet the sources of these shocks could be precisely the phenomena treated by the above models. For example, the greater role played by the government in the economy after the Great Depression could be treated as a shock to (the government's) preferences, or as a change in the Keynesian equilibrium around which the economy oscillates. Similarly, the relevant technology shocks could be of the order of the "transportation revolutions" of the 19th century (rather than imperceptible changes occuring every period). One is accustomed to referring to these large shocks as "structural changes", and to regard them as outside the purview of economic models, but this is more the result of their infrequent occurence than their intrinsic nature. Of course, the implication of this perspective is that the phenomena examined by real business cycle models and multiple equilibrium models do not explain the high-frequency components of observed data.

The empirical results of the last section turned up a distinction between quantity series, which tended to follow segmented trends, and price series, which were better fit by DS processes. One is accustomed to associate long term movements in prices with those in the money stock. While prices follow DS processes, the money stock appears to follow a segmented trend, so the transmission from money to prices is not likely to be a simple one. Walsh (1987) discusses a model developed by Goodfriend, in which prices can be DS irrespective of the behavior of the money stock. However, this result appears to hinge on the assumption that the error in the money demand equation is itself difference stationary.

An alternative is the possibility that differences between price and quantity series result from the methods of compiling these two types of data. Prices, such as the CPI, are roughly) estimates made at a point in time, while quantity date represent flows over intervals of time. All other things being equal, quantities should be less erratic than prices, although this does not explain why the latter exhibit unit roots and the former do not. However, the GNP deflation is the quotient of nominal actual GNP series which are themselves flows, and so the GNP deflator is also a flow series. However, it displays a unit root. Similarly, the real wage is the ratio of two point observations, and yet it comes close to following a segmented trend. Hence, it appears that the explanation should be sought in the economic distinctions between prices and quantities. One is tempted to try to explain the unit root in prices by appeal to efficient markets. But then substantial information or contract lags have to be invoked to explain the serial correlation in inflation rates.Evidently, this is a subject for further research.

A final question that deserves brief mention concerns the use of predictions from models of non-stationary data. Several authors (C.f. for example, Campbell and Mankiw (1986)) mention their discomfort with the implication of TS models, that the forecast error variance is independent of the forecast horizon. Since they feel more uncertain about the distant than the near future, the DS model appears more satisfactory, as its forecast error variance is linear in the forecast horizon. The precision of forecasts from segmented trend models is closer to that of the DS model than the TS model. Forecasts from a segmented trend model only have constant variance as the forecast horizon grows, conditional on the current segment obtaining in the future. Should there be a move to a new trend rate of growth, at some time in the future, the variance of the errors of forecasts based on the current trend grow with the sample size. Of course, it may not be possible to arrive at very precise estimates of the probability of a change in the trend rate of growth, but this only contributes to the expected forecast error variance. Hence, while there is a qualitative difference between the expected characteristics of DS and TS forecast errors, the characteristics of DS and segmented trend forecast errors are similar.

4. Summary

This paper has examined a class of models of non-stationarity which includes as extreme cases the TS and DS models popularized by Nelson and Plosser (1982). Members of this class of models display trends that change infrequently, and are named "segmented trend" models. Comparing TS and DS models using segmented trend data can be misleading, since several standard indicators of performance can lead one to believe that a DS model is adequate. When a DS model is tested against a segmented trend model using the data examined by Nelson and Plosser, the price series still exhibit DS characteristics, but the quantity series convincingly reject the DS model in favour of the segmented trend.

We note that while segmented trend models are favoured over DS models, they share characteristics that set both apart from TS models. Thus, a segmented trend does not attribute all deviations from a linear trend to the "cyclical" component of the series. Similarly, forecasts from segmented trend models exhibit growing uncertainty as the forecast horizon increases, since it is not known when the current "segment" will give way to a new trend rate of growth.

Segmented trend models capture the idea that the economy experiences "epochs" that are of long duration relative to the frequency with which data are observed. It is possible that the shocks dealt with by equilibrium business cycle models and Keynesian models of multiple equilibria are more relevant to the description of these low frequency epochs, than they are to the high-frequency movements to which they have so far been addressed.

FOOTNOTES

1/ Similar results were found, for example, by Nelson and Plosser (1982), Mankiw and Shapiro (1985) and Stock and Watson (1986).

2/ Quah (1986) and McCallum (1986) dispute whether such inferences can be made.

3/ Thus, for example, Gordon (1982), defines natural real GNP from 1890 and 1980 by linear interpolation between actual GNP observations that are from five to twenty-one years apart.

4/ Changing error properties are also consistent with a DS model, and may be implemented using varying parameter of ARCH models. The difference is that these methods impose a certain homogeneity on the way in which the statistical properties of the errors change, whereas focusing on regimes of relatively long duration allows changes in statistical properties to be estimated.

5/ The fact that both regimes include t* implies that the segments join at t*. C.f Gallant and Fuller (1973) for an elegant general formulation of such models.

6/ In order to make the asymptotic approximation useful in the consideration of segmented trends, it is necessary to preserve the segmented nature of the data or model under consideration as the sample size grows. Hence, if T_i is the number of observations in the ith segment when the total sample is T, we consider what happens as T_i tends to infinity maintaining the ratio T_i/T constant.

7/ As the sample size gets large, they tend to unity at all lags. Similarly, all partial autocorrelation beyond the first are zero in both cases.

8/ This is similar to the interpretation given by Harvey and Todd (1983, p. 300), who describe their more flexible unobserved components model as "a local approximation to a linear trend".

9/ A process with a similar flavour is Quah's (1986) "clinging model", in which there is positive probability each period that the series will return to some fixed value.

10/ We abstract here from the non-deterministic part of the series. Since this is assumed in any case to be stationary, its omission does not affect the argument.

11/ The "fitted part" of the DS model is simply a translation of the actual data.

12/ If $\phi(L)$ has no unit roots, equation (3) may be premultiplied by $\phi(L)$ to yield an equation identical to (9), where $\alpha=a(1-\rho) + \rho b$, $\gamma = b(1-\rho)$ and $e_t=\phi^{-1}(L)\theta(L)u_t$. Thus, under the null hypothesis (2), $\gamma=0$, although this restriction is not tested directly. Nelson and Plosser point out that, if the first difference of y_t is stationary (as it is for the series they examine) then it is inconsistent that $\rho=1$ and $\gamma\neq0$, since the latter condition implies that the first difference of y_t is non-stationary.

13/ Even if the correct number of change points is used, locating them wrongly can bias the results against the segmented trend model. However, one presumably selects these change points using some criterion such as the belief that a particular historical event is responsible for a change. It seems plausible t suppose that the error in locating the event stays fixed as the sample size becomes large. Hence, such errors disappear asymptotically.

14/ Both of these autocorrelation functions tend to zero as the lag of the autocorrelation approaches the sample size, as a result of the formula used by Nelson and Kang. This is an estimate of $((T-K)/T)\ (\gamma_K/\gamma_o)$ where γ_K is the autocovariance at lag K. Hence, as K approaches T, the first term in the ratio tends to zero.

15/ The sum in these expressions is understood to be zero if K=1.

16/ In this equation $\alpha = a_1\ (1-\rho) + \rho b_1$, $\alpha_2 = (b_2-b_1)\ (\rho-(1-\rho)t^*)$, $\Upsilon = b_1\ (1-\rho)$, $\Upsilon_2 = (b_2-b_1)\ (1-\rho)$, and e_t is as before.

17/ One would expect the critical values to climb in this way as the number of segments increases, in view of the characterization given above of the DS process as the limiting case of a segmented trend.

18/ If one wanted to maximize the chance that a segmented trend best fit the data, one would choose a change point at t* such that the absolute difference between the "slopes" $(y_{t^*}-y_1)/(t^*-1)$ and $(y_T-y_{t^*})/(T-t^*)$ is maximized. This is achieved when y_{t^*} is an extremum.

19/ For a description of these series, see Nelson and Plosser (1982, p. 146). All series used are the natural logarithms of the published data.

20/ Delong and Summers and others have interpreted these findings in terms of the greater role of the government after the Depression, discussed above. In contrast, Romer's explanation is that output and price indices were sparser in their coverage and gave greater weight to more volatile products and commodities in the early period.

21/ This change in the error structure is also noted by Harvey (1985), who breaks the sample at 1947.

22/ Fuller (1976, pp. 373-377) shows that autocorrelation of e_t (in equation (9), for example) can be accounted for by including lagged differences of the dependent variable, without altering the asympototic distribution of $\hat{\rho}$. Since his argument relies only on the boundedness of the moments of e_t, it extends simply to the current cast as well, justifying the use of the critical values in Table 1.

23/ These are the series that do not evidence large change in δ_1, δ_2 and σ_e when the 1930's are excluded from the early sample (See Table 2).

24/ McCallum (1986) has criticized Nelson and Plosser's reasoning on the grounds that it is difficult to distinguish whether ρ is equal to or slightly less than unity. In the latter case, he shows that

Nelson and Plosser's decomposition would attribute the larger part of the variance to the trend component, even though it belongs to the cycle.

25/ In fact, these do not appear to be of any practical significance. Excluding the 1930's, the contribution of the change in drift of real GNP to the variance of its first difference is 1.13%.

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APPENDIX

Proof of Proposition of Section 1

> If the true model contains K+1 segments, then any fitted segmented trend model involving K or less segments will, asymptotically, yield a larger sum of squared residuals than the DS model fitted to the same data.

The proof proceeds by induction on K. The sample, T, is assumed to grow asymptotically such that the proportion of observations in each actual segment, T_i/T (i=1,..., K+1) is assumed constant as T, the total sample size, tends to infinity.

(i). K=1

The two actual segments are given by

$$y_t = \begin{cases} a_1 + b_1 t + \varepsilon_t & t \leq 0 \\ a_2 + b_2 t + \varepsilon_t & t \geq 0 \end{cases}$$

Here, ε_t is a mean-zero stationary process. Without loss of generality, let $a_1=a_2=0$, so that the two lines join at the origin (see Diagram 1). The OLS line is $\hat{y}_t=\hat{a}_T+\hat{b}_T t$. The dates $\tau_1(T)$ and $\tau_2(T)$ are the points of intersection of $\hat{y}_t(T)$ with regimes 1 and 2 respectively.

The OLS residuals can be expressed as:

$$l_t = y_t-\hat{y}_t(T) = \begin{cases} (b_1-\hat{b}_T)(t-\tau_1(T)) + \varepsilon_t & t < 0 \\ (b_2-\hat{b}_T)(t-\tau_2(T)) + \varepsilon_t & t \geq 0 \end{cases}$$

Define $\Delta_T = \max_{i=1,2} (b_i-\hat{b}_T)^2$. Since $b_1 \neq b_2$, Δ_T is strictly positive and exceeds $((b_2-b_1)/2)^2$ for all T. Let a '*' denote the regime i such that $(b_i-\hat{b}_T)^2 = \Delta_T$. (This regime will in general change with T). Thus, for example, T_* is T_1 or T_2 depending on whether, at stage T, $(b_1-\hat{b}_T)^2$ or $(b_2-\hat{b}_T)^2$ is the larger. Define:

$$S_{*T} = \sum_{0}^{T^*} l_t^2 = \Delta_T O(T_*^3) + O_p(T^{3/2})$$

Since $T_1 \leq T_* \leq T_2$, and T_1/T and T_2/T are constant,

$$S_{*T} = O_p(T^3).$$

Finally, as S_{*T} is the contribution to the sum of squared residuals of only one regime, the total sum of squared residuals, SSTS, is of the same order as S_{*T}. That is,

$$\underline{SSTS = O_p(T^3)}$$

In contrast, SSDS, the sum of squares from fitting a DS model (with drift) is:

$$SSDS < \Sigma(y_t-y_{t-1})^2 \leq Tb^2 + \Sigma(\varepsilon_t-\varepsilon_{t-1})^2 + O_p(T)$$

where $b^2 = \max(b_1^2, b_2^2)$. Hence

$$\underline{SSDS = O_p(T)}$$

<u>(ii) $K > 1$</u>

Now assume the result holds for fitting k segments to k+1 actual segments, where $k < K$. In this part of the proof, the stationary random error term ε_t is omitted for simplicity. This does not materially alter the results, since the terms in ε_t are again $O_p(T^{3/2})$, we shall show that the SSR from the under-parameterized segmented trend model is again $O(T^3)$, while the first difference model yields a SSR that is O(T).

It will be useful to define the following terms. Subsequently, dependence on T will be suppressed where no ambiguity arises. The index 'i' denotes <u>actual</u> segments and runs from 1 to K+1; the index 'j' denotes <u>fitted</u> segment and runs from 1 to K.

A_i, $F_j(T)$ subsets of $\{1,\ldots, T\}$ occupied by ith actual and jth fitted segments, respectively

$\Gamma(T) = \{ C_{ij}(T)$ s.t. $C_{ij}(T) = A_i(T) \cap F_j(T)$ and $C_{ij}(T)$ is non-empty$\}$.

$\tau_{ij}(T)$: point in $C_{ij}(T)$ s.t. $(y_t - \hat{y}_t(T))^2$ is minimized for all $t \in C_{ij}(T)$

S(X) : Number of data points in $X = \{1,\ldots,T\}$.

An illustration, for the case K = 2, is given overleaf, in Figure 4.

The sum of squared residuals from fitting the segmented tend model can then be written as:

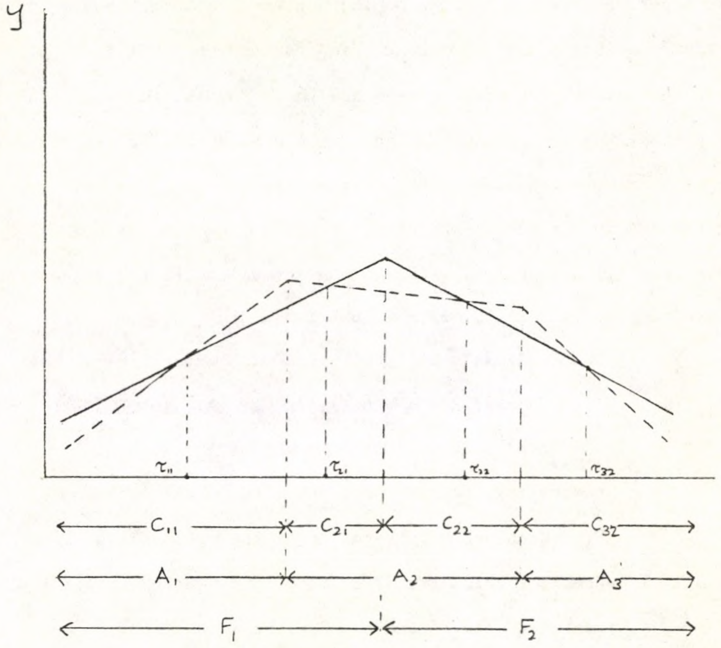
$$SSST = \sum_{C_{ij}\in\Gamma} \sum_{t\in C_{ij}} [\hat{b}_j - b_i]^2 [t-\tau_{ij}]^2$$

Define $$\Delta_T = \max_{\substack{i,j \\ \text{s.t. } C_{ij}\in\Gamma}} [\hat{b}_j - b_i]^2$$

Since these are less fitted than actual segments, the fitted segmented trend cannot equal the actual segmented trend at all points for any sample size. Hence

$$\lim_{T\to\infty} \Delta_T = \Delta > 0$$

FIGURE 4



Let a '*' denote the i-j combination such that for $C_{ij} \varepsilon \Gamma$, $\lim[\hat{b}_j - b_i]^2 = \Delta$.

Then SST is eventually bounded from below by

$$\Delta \sum_{t \varepsilon C_*} (t - \tau_*(T))^2 \qquad \text{(A1)}$$

The order of this sum depends on the number of observations in C_* as T grows. Since $C_* \subset A_i$ for some i and for every i, $S(A_i) = O(T)$, it follows that $S(C_*) \leq O(T)$. There are three possible cases, depending on whether the end points of C* coincide with actual or fitted regimes. In each case, we shall show that either the SSR in C_* is $O(T^3)$, or the SSR in the complement of C_* is $O(T^3)$. We use the notation 'A_*' and 'F_*' to refer to the actual and fitted segments containing C_*, and i_* to refer to the index of A_*.

a) Both endpoints of C_* coincide with those of A_*. Then $C_* = A_*$ and $S(C_*) = O(T)$, and so (A1) is $O(T^3)$.

b) One endpoint coincides with one of A_* (without loss of generality, say the left one), the other with one of F_*. Consider omitting C_* from the optimisation problem, which is now divided into two: there are i_*-1 regimes to be fitted to the left of C_* and $K+1 - (i_*-1)$ to the right (there are $K+1 - i_*$ to the right of A_*, plus $A_* - C_*$).
To accomplish this, K fitted regimes are available. If these are divided among the left and right actual regimes, then at least one will have less fitted than actual regimes. But both i_*-1 and $K+1 - (i_*-1)$ are less than K+1 for i_* 1, and so the earlier stages of the induction for $k < K$ show that the SSR in either of these regions will be

$O(T^3)$. For $i_*=1$, the left endpoint belongs both to A_1 and F_1, so the right must belong to F_1. If $S(C_*)$ ($=S(C_{11})$) is $<O(T)$ then $S(A_1-C_*)$ is $O(T)$, and omitting C_* leaves K+1 actual regimes to be fitted by K-1 segments. The SSR from this exercise is greater than that from fitting K-1 segments to K regimes, which yields an SSR of $O(T^3)$, by the K-1th stage of the induction.

c) Both endpoints coincide with those of F_*. Then omitting C* leaves at least a total of K+1 actual regimes to be fitted by K-1 segments, and earlier stages of the induction shows again that SSR is $O(T^3)$.

The SSR from the DS model is again bounded from above by $\sum_{i=1}^{K+1} T_i b^2$

where $b^2 = \max_i \; b_i^2$, and so is $O(T)$.

Hence, SSST is $O(T^3)$ while SSDS is $O(T)$, and so, asymptotically the DS model yields a lower SSR than an underparameterized segmented trend.

Proof of Corollary

> If y follows a segmented trend, then plim $\hat{\rho}=1$, where $\hat{\rho}$ is the OLS estimator of ρ in equation (1).

Let $Z_T = [i_T, t_T]$, i.e. a Tx2 matrix, one column of which contains '1's, and the other the integers from 1 through T. Then

$$\hat{\rho} = (y'_{-1}M_z y_{-1})^{-1}(y'_{-1}M_z y)$$

where y_{-1} contains the once lagged values of y, $M_z = I-Z(Z'Z)^{-1}Z'$, and the time subscript has been suppressed for simplicity.

Hence

$$\hat{\rho}-1 = (y'_{-1}M_Z y_{-1})^{-1}(y'_{-1}M_Z(B+(1-L)\varepsilon)),$$

where B is a Tx1 vector of slopes of trends, b_i, and ε is a Tx1 error vector. The first term in parentheses on the left is the SSR from regressing y on a single trend, and so is $O_p(T^3)$ by the above theorem. The largest term in the parentheses on the right is $O_p(T^2)$. Consequently, $\hat{\rho}-1$ is $O_p(T^{-1})$, and so its probability limit is zero.

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