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ENDOGENOUS FLUCTUATIONS IN A TWO-SECTOR OVERLAPPING GENERATIONS ECONOMY

by
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INTRODUCTION.

It is widely known by now that competitive equilibrium models may generate endogenous fluctuations and that these fluctuations are entirely consistent with complete markets and perfect foresight.

Most of the literature on the subject has focused on the overlapping generations (OG) model. In fact, it is generally recognized that this is the only neoclassical model requiring sequential trading (i.e., having a truly dynamic structure) in the absence of market imperfections.

OG economies may have a very complicated dynamics when agents live for several periods and/or one allows for the existence of a multiplicity of goods. In particular, it has been shown (see Kehoe and Levine (1985)) that when the number of consumers in each generation is sufficiently high, the dynamic properties of the OG model are completely "generic" up to very mild restrictions. This is a consequence of the Sonnenschein-Mantel-Debreu Theorem, since the dynamics of equilibrium trajectories of OG models is completely specified in terms of aggregate excess demand functions. Therefore, one may not be surprised that the existence of equilibrium fluctuations is a possible outcome of dynamic disaggregative economies.

The existing literature, however, has shown that complicated dynamic phenomena may exist in OG models with a very simple structure. In other words, one doesn't need to rely on the heterogeneity of agents' prefences and the multiplicity of goods in order to prove the existence of endogenous fluctuations.

From the studies of Benhabib and Day (1982) and Grandmont (1985) we know that a one-good, pure exchange OG model in which identical agents live for two periods may generate periodic equilibrium trajectories and also "chaotic" dynamics around the golden rule (GR) stationary state if the agents' excess demand functions are characterized by a significant income effect[1].

Following these findings, the natural question has arised if the income effect is the only potential cause of endogenous fluctuations in aggregative OG models. In order to investigate this question one has to look at economies whose structure is not completely characterised by agents' preferences. For this reason, some work has studied OG models with production in the setting developed by Diamond (1965).

As it turns out, by introducing productive capital into the OG economy, one can prove the existence of endogenous cycles without assuming a strong income effect. Farmer (1986) has shown that when the GR steady state involves negative outside money, a Hopf bifurcation may occur giving rise to the emergence of invariant circles. The same phenomenon may be observed in the vicinity of a non-monetary stationary state if the agents' labor supply is wage-elastic and there is enough complementarity in the production function (see Reichlin (1986)).

A recent study by Benhabib and Laroque (1986) has confirmed and extended these results in the framework of the Diamond's model with wage-elastic labor supply and outside money. The study is an attempt to characterize different sets of economies in terms of their dynamic behavior by looking at some relevant parameters. Mainly, the elasticity of substitution between factors in the production function (ESP) and the interest rate elasticity of saving (IES). The following statements can be derived:

(a) cycles are either associated with a negative IES or a low ESP. However, an increased ESP will always reduce the scope for cycles.

(b) When the IES is positive, a non-positive outside money is necessary for a low ESP to be associated with endogenous fluctuations.

Thus, the role of a low ESP in generating cycles is not clearly separated from the existence of an evolving debt of the private sector and/or a labor supply response to wage rate fluctuations.

Moreover, there seems to be a tight relation between the type of cycles and the characteristics of the economies. In particular, the existence of chaotic dynamics is only associated with economies where agents' excess demand functions are characterized by a very strong income effect. On the other hand, the existence of trajectories lying on a closed curve around the stationary state has only been proved in economies where private agents' net wealth is negative and evolving over time (Farmer (1986)) and/or labor supply is wage-elastic (Reichlin (1986)).

In the present paper I will slightly modify Diamond's framework in order to evaluate the generality of the above statements. The only departure from the models considered so far concerns the production side. In particular, I will assume the existence of a two-sector technology producing a consumption and a capital good. Moreover, in the attempt to simplify the exposition and confine the analysis to the questions at hand, I will also assume an exogenously fixed labor supply and a zero ESP in both sectors[2]. In this way, the role of complementarity in production is isolated from any other potential cause of endogenous fluctuations. Notice also that a zero ESP rules out any facor intensity reversal between sectors, which can be proved to be a potential cause of dynamic complexities in optimal growth models with infinite lived agents (see Benhabib and Nishimura (1985)).

The following analysis shows that:

(a) periodic and chaotic dynamics are possible with a positive IES;

(b) invariant circles around golden rule stationary states are not necessarily associated with a wage-elastic labor supply or a non positive outside money.

Two parameters of the model are crucial in proving these results: the IES and a purely technological parameter θ whose magnitude depends on the relative factor intensities in the two sectors of production and the rate of depreciation of the capital stock.

The paper is organised in the following way. The next section briefly describes the technology of the model. Section 2 studies the behavior of an autarkic economy, i.e., an economy where no outside money is allowed and private wealth equals the aggregate stock of capital. This economy is shown to possess a wide range of potential dynamic behaviors: from saddle path stability to chaotic dynamics, even for a fixed positive IES. In section 3 agents are allowed to hold outside money beside capital. The possibility of generating invariant circles around a GR steady state involving positive outside money is investigated. This dynamic phenomenon is shown to imply a negative IES.

The model that I am going to analyse, the two-sector OG economy, has been studied for the first time by Gale in 1972. In his formulation the young generation consumes a constant fraction of income. Therefore, anticipations about next period variables do not affect agents' saving decisions. It may be worth emphasizing that this assumption is enough to rule out any endogenous fluctuations in my specification of the model.

I. THE TECHNOLOGY.

The economy that I am going to describe produces a consumption and a capital good, denoted by c and I respectively, using labor and capital in fixed proportionality. Thus, the technology is represented by the following technical coefficients:

a_o, b_o = the labor-output ratios in sectors c and I;

a_1, b_1 = the capital-output ratios in c and I.

Following Diamond (1965), I will assume that the stock of capital K is productive with a lag of one period. Therefore, assuming that the total supply of labor is equal to 1, the equilibrium condition in the factor markets reads:

$$1 \geq a_o c_t + b_o I_t \quad (1a)$$

$$K_t \geq a_1 c_{t+1} + b_1 I_{t+1} \quad (1b)$$

Now let $\delta \in [0,1]$ be the depreciation rate of capital. Then, the latter evolves according to:

$$K_t = (1-\delta)K_{t-1} + I_t \quad (2)$$

Perfect competition in this economy implies that production activity does not allow positive profits to be earned. Assuming that firms take into consideration capital gains and losses in evaluating profits and letting the price of the consumption good be the numeraire, we have:

$$1 + (1-\delta)a_1 q_t \leq a_o w_t + a_1 R_t q_{t-1} \quad (3a)$$

$$q_t [1 + (1-\delta)b_1] \leq b_o w_t + b_1 R_t q_{t-1} \quad (3b)$$

where q is the relative price of the capital good, w is the real wage and R the real interest factor.

An obvious interpretation of (3) is the following. Firms are able to finance their production plan from t to t+1 (i.e., to buy the total stock $q_t K_t$) by issuing a bond whose relative price is $1/R_{t+1}$.

Among the set of equilibria compatible with the present technology, I will only study the ones in which capital, labor and consumption are not free goods. These are associated with prices q and capital stocks K such that:

either: $q \in I_q^1 = [(b_1/a_1),(b_o/a_o)]$,

$K \in I_k^1 = [(b_1/b_o),(a_1/a_o)]$,

or: $q \in I_q^2 = [(b_o/a_o),(b_1/a_1)]$,

$K \in I_k^2 = [(a_1/a_o),(b_1/b_o)]$.

The above are necessary and sufficient conditions for an efficient production plan to imply positive production of c and I and full employment of labor and capital (I am assuming that full employment and diversification of production occurs also when q is tangent to the boundaries of the feasible sets (1a), (1b)). Therefore, they also imply that (3a) and (3b) are satisfied with the equality sign.

Under these conditions we have:

$$K_t = (a_1/\Delta) - (1/\theta)K_{t-1} \quad (4)$$

$$w_t = (\theta/\Delta)[1 + b_1(1-\delta) - a_1 x_t] \quad (5)$$

$$q_t = (\theta/\Delta)[b_o - \Delta x_t] \quad (6)$$

for $\Delta \neq 0$, $\theta \neq 0$, and

$$K_t = a_1/a_o$$

$$w_t = (1/a_1) + (b_1/a_1)(1 - \delta - R_t)$$

$$q_t = b_1/a_1$$

for $\Delta = \theta = 0$, where:

$$\Delta = a_1 b_o - a_o b_1$$

$$\theta = \Delta[a_o - (1-\delta)\Delta]^{-1}$$

$$x_t = R_t q_{t-1}.$$

The parameters Δ and θ play a central role in the dynamics described in (4)-(6). The first may be thought of as a measure of the curvature of the factor price frontier and its sign depends on the relative capital intensity of the two sectors. The latter crucially depends on the magnitude of δ and Δ and affects the dynamics of K and the relation between the interest factor x and the other prices. In order to avoid pathological situations, I will assume:

(A1) $(1-\delta)\Delta \neq a_o$.

At a stationary state (w^*, q^*, R^*), prices are linked by the following relationships:

$$q^* = (b_o/a_o)[1 + \theta(1-\delta)(1 + \theta R^*)]^{-1} \quad (7)$$

$$w^* = (1/b_o)[1 - b_1(R^* - (1-\delta))]q^* \quad (8)$$

Therefore, the steady state equilibrium of the interest rate must be such that:

$$(1/b_1) + (1-\delta) \geq R^* \geq 0 \qquad \text{if } \theta \geq 0$$

$$(1/b_o) + (1-\delta) \geq R^* > -1/\theta \qquad \text{if } \theta < 0.$$

As far as the capital stock is concerned, a stationary state is readily found to be:

$$K^* = a_1(a_o + \delta\Delta)^{-1}$$

In order to insure that the stationary stock of capital is compatible with a full employment equilibrium, I assume:

(A2) $\delta b_1 \leq 1$.

In fact, $K^* \in I_k^i$ $(i = 1,2)$ implies (A2). From this assumption we also have that $K^* \in I_k^1$ if $\Delta > 0$ and $K^* \in I_k^2$ if $\Delta < 0$.

Equations (5) and (6) show that there is a one-to-one relation between x_t, w_t and q_t. Thus, from the dynamics of a sequence $\{x_t;\ t=1,2,\ldots\}$ we can infer the dynamics of the sequence $\{w_t, q_t, R_{t+1};\ t=1,2,\ldots\}$, whereas stationary values of w, q and R correspond to a stationary value of x and viceversa. For this reason, in order to study the evolution of prices, I can focus on x with no loss of information.

Now, $q \in I_q^i$ $(i = 1,2)$ implies:

$x \in I_x^1 = [b_o(1-\delta)/a_o, (1/a_1) + b_1(1-\delta)/a_1]$ if $\Delta \geq 0$,

$x \in I_x^2 = [(1/a_1) + b_1(1-\delta)/a_1, b_o(1-\delta)/a_o]$ if $\Delta \leq 0$.

A consequence of these restrictions on the range of x is that if $\theta < 0$, then $\Delta < 0$. In fact, let $\theta < 0$ and $\Delta > 0$. Then, we have $(\theta/\Delta) < 0$. By (5), in order for wages to be non-negative we have to impose $x \geq [(1/a_1) + b_1(1-\delta)/a_1]$, i.e., $x \in I_x^2$. However this is a contradiction, since $\Delta > 0$ implies $x \in I_x^1$.

II. A WEALTH-CAPITAL ECONOMY.

As it was mentioned in the introduction, the model that I intend to set up is a general equilibrium model based on Diamond's (1965) classical paper. In particular, I will assume that there are two types of identical agents in any t, young and old. Population and preferences are constant over time and the economic activity is started by a generation of old people whose exclusive role is to initiate the production process with a given stock of capital K_o. All consumers live for two periods and supply labor inelastically only when they are young. For simplicity, no bequests are allowed, so that, as in Diamond's original formulation, old people consume all of their wealth and young agents start life with an initial wealth of zero. Finally, I will also assume that young agents are endowed with a given amount E of the consumption good. This assumption, whose importance will be clearer later, essentially widens the set of economies for which there exist equilibrium trajectories with strictly positive private wealth.

In the last section it has been shown that a competitive production activity implies the zero profit condition (3). Therefore, at full employment we have:

$$c_t - w_t = R_t q_{t-1} K_{t-1} - q_t K_t .$$

Letting W_t be the private sector's total wealth in t, one can combine the above equation with the consumers' budget constraint to get:

$$R_t W_{t-1} - W_t = R_t q_{t-1} K_{t-1} - q_t K_t .$$

Therefore, a steady state equilibrium (W^*, K^*, R^*, q^*) satisfies the following condition:

$$(R^* - 1)W^* = (R^* - 1)q^*K^*$$

and we see that it must be one of two types:

wealth-capital (WC): with $W^* = q^*K^*$

golden rule (GR): with $R^* = 1$.

This classification is used by Gale (1972). The term wealth-capital simply refers to the fact that the corresponding equilibria imply the equality between the aggregate stock of capital and total private wealth. On the other hand, in a GR equilibrium this equality will not hold in generic cases[3]. However notice that, as opposed to pure exchange economies, where non-GR stationary states are called autarkic (see Gale (1973)), both types of equilibria imply the existence of intergenerational trade.

In this section I will consider economies (WC economies) in which the only asset available to private agents is productive capital. In this setting all competitive equilibria imply $W_t = q_t K_t$ and R^* will be different from 1 in generic cases. In the next section this assumption will be relaxed by allowing people to hold an alternative asset.

The problem now arises of proving that full employment equilibria with positive wealth are feasible. Formally, we need to show that there are non-negative values of q_t, w_t and K_t satisfying:

$$q_t K_t \leqq w_t + E$$

In what follows I will focus on equilibria with K_t in small neighborhoods of K^*. Thus, consider the constraint:

$$q_t K^* < w_t + E.$$

After some manipulations we get:

$$E > K^*[x_t - (1-\delta b_1)/a_1].$$

Now, it is immediate from the definition of I^i_x that the right hand side of the above inequality is bounded for all the equilibria that I am going to consider. Therefore, for any neighborhood N of K^* we can clearly find a value $E > 0$ at which $qK \leqq w + E$ for any $x \in I^i_x$ $(i = 1,2)$, $K \in N$.

Without formalizing the representative agent's decision problem, I will simply impose the existence of an aggregate saving function $S(w,R)$, derived from the solution of a standard problem of preference maximization. For simplicity, the preference structure is assumed to be such that:

(A3) $S(w_t, R_{t+1})$ is a C^r $(r \geq 1)$ function with $S_w > 0$, $S_R \neq 0$ for all $x \in I_x^i$ $(i = 1,2)$.

Now, denoting (5) and (6) by $w(x)$ and $q(x)$ respectively, we can define a perfect foresight equilibrium for the economy as a sequence $\{x_t, K_{t-1};\ t = 1,2,\ldots\}$ satisfying:

$$q(x_t)K_t = S[w(x_t), x_{t+1}/q(x_t)] \tag{9a}$$

$$K_{t+1} = [(1+\theta)/\theta]K^* - (1/\theta)K_t \tag{9b}$$

$$K_o = \bar{K}$$

with $x_t \in I_x^i$ and $K_t \in I_k^i$ $(i = 1$ or $2)$ for all t.

Notice that the dynamics of capital is totally independent of the dynamics of x. This implies that the existence of a non-stationary perfect foresight full employment equilibrium from any t on, requires a very simple assumption, i.e., the stability of equation (9b). More precisely:

(A4) Either $\theta = 0$ or $|\theta| > 1$.

From (A4) it follows that:

<u>Prop.1.</u> If $K^* > 0$, $\theta \neq 0$ and (A4) holds, then Δ is positive and $\theta > 1$.

<u>Proof.</u> $\theta < -1$ implies $\delta\Delta < -a_o$, which in turn implies the negativity of K^*. If $\theta > 1$, then $\Delta > a_o[1 + (1-\delta)]^{-1} > 0$.

From now on I will only consider parameter values at which $K^* > 0$ and (A4) holds. Then, either the model has a linear factor price frontier, or the consumption sector is more capital intensive.

Assume now that system (9) has at least one stationary equilibrium (x^*,K^*), then the latter satisfies:

$$S[w(x^*),R^*] = q(x^*)K^*.$$

where $R^* = x^*/q(x^*)$.

Now, by (A3) it follows that there exists a neighborhood of x^* where a unique function $x_{t+1} = F(x_t,K_t)$ is implicitly defined by:

$$q(x_t)K_t = S[w(x_t),F(x_t,K_t)/q(x_t)].$$

Differentiating with respect to x at the stationary state we get:

$$F_x(x^*,K^*) = (R^*/e^*)[S_w^*(1+\theta) - \theta(1+e^*)]$$

where e^* is the elasticity of saving with respect to the interest rate and S_w^* the marginal propensity to save out of wage income both evaluated at (x^*,K^*).

The expression for $F_x(x^*,K^*)$ shows that the local dynamic behavior of equilibrium trajectories depends on the magnitude as well as on the sign of a set of parameters $(e^*,S_w^*,\theta,\ldots)$

describing the preferences and the technology of the model. Among these parameters I will single out the influence of θ on the dynamics of (9).

Notice that:

<u>Prop.2.</u> If $F_x(x^*,K^*) \neq 1$ for some θ', then there exists a continuous and differentiable function $x^* = x^*(\theta)$ for θ in a neighborhood of θ'.

<u>Proof.</u> Letting $f(x,\theta) = S[w(x),x/q(x)] - q(x)K^*$, we have:

$f_x = (e/R)q(x)K^*[1 - F_x(x,K^*)]$.

Thus, if the above expression is different from 0 at (x^*,θ'), by the implicit function theorem we get the proposition.

Let θ^o be defined as the parameter value at which the present economy is such that:

$F_x(x^*(\theta^o),K^*(\theta^o)) = -1$.

Under this circumstance, one of the following relations is satisfied:

$e^* = [S_w^*(1+\theta^o) - \theta^o]R^*(\theta^o R^* - 1)^{-1}$ if $R^* \neq 1/\theta^o$

$S_w^*(1+\theta^o) = \theta^o$ if $R^* = 1/\theta^o$

Now, using bifurcation theory, it may be possible to prove the existence of periodic solutions generated by models in which one of the above equalities "almost" hold. By the flip bifurcation theorem, we get the following proposition:

Prop.3. Assume that:

(a) the θ-derivative of $F_x(x^*(\theta),K^*(\theta))$ at θ^o is non zero;

(b) $(1/2)[\partial^2 F(x,K^*(\theta))/\partial x^2]^2 + [\partial^3 F(x,K^*(\theta))/\partial x^3]/3 \neq 0$ at (x^*,θ^o);

then, (9) generates a two-period cycle for θ in a neighborhood of θ^o.

Proof. The Jacobian of (9) at (x^*,K^*) has the following eigenvalues:

$$\lambda_1 = F_x(x^*,K^*),\ \lambda_2 = -1/\theta.$$

Since a solution (x_t,K_t) of (9) through (x^*,K^*) lies on the curve $K = h(x) = K^*$, the latter is an invariant manifold for the dynamical system. When $\lambda_1 = -1$, $h(x) = K^*$ is the center manifold of (9) since $h(x^*) = K^*$ and $h'(x^*) = 0$. Then, by the center manifold theorem, the local behavior of solutions of (9) can be proved by studying a first order equation of the following type:

$$x_{t+1} = F(x_t,K^*)$$

By applying to the above equation the flip bifurcation theorem[4] for one-dimensional systems, we get the proposition.

In generic cases conditions (a) and (b) are clearly verified in the present model. Therefore, "flip" cycles may arise for any $\theta > 1$ belonging to an interval containing θ^o. These cycles are clearly possible even in the absence of an inverse relation between saving and the interest rate. In particular, a two-period cycle in which the interest elasticity of saving is positive may occur provided that:

either: $S_w^* \geq \theta^o/(1+\theta^o)$, $R^* \geq 1/\theta^o$

or: $S_w^* \leq \theta^o/(1+\theta^o)$, $R^* \leq 1/\theta^o$.

It may also be noticed that the parameter θ plays a central role in the dynamics of (9) despite the restriction imposed by the stability assumption (A4).

A closer inspection of the dynamical system shows that F(x,K) is likely to be non monotonic when saving is an increasing function of R. In fact, assume that x rises, then the price of capital and the wage rate will have to decrease. This will depress saving through the income effect but it may also induce expectations of an increased interest rate. In fact, it is possible to prove that the dynamics of (9) may be more complicated than shown by Proposition 3. This will be accomplished by assuming that $K_o = K^*$. Then, (9) becomes a one-dimensional system. In other words, I am going to focus exclusively on the dynamics of prices and disregard non-stationary sequences of the capital stock.

Now, define $G(x) = F(x,K^*)$ and let the saving function S be invertible in I_x^1. If G maps I_x^1 into I_x^1, then, for all x $\in I_x^1$, we have:

$$G'(x) = G(x)\{(S_w(1+\theta) - \theta)/e - \theta/q(x)\}$$

$$G'(x^*) = F_x(x^*,K^*).$$

Clearly, the above expression may change sign in I_x^1. Since q(x) is decreasing in x, even if S_w and e are constant parameters G'(x) changes sign at most once at some point x^m such that:

$$[S_w(1+\theta) - \theta] = e\theta/q(x^m).$$

Now, assume $x^m \in I_x^1$, then G(x) is unimodal on I_x^1 and G'(x) > 0 (< 0) for $x < x^m$ ($> x^m$). Then, we have:

<u>Prop.4.</u> Let $G^i(x)$ be the i-th iterate of x under G. If:

$$x^m \geq G^3(x^m) > G^2(x^m)$$

then, G has a 3-period cycle.

<u>Proof.</u> Since G maps I_x^1 into itself and recalling the definition of x^m, we have:

$$G(x^m),\ G^2(x^m) \in I_x$$

$$G(x^m) \geq G^2(x^m)$$

$$G(x^m) \geq x^m.$$

Now, by the inequality $G^2(x^m) < x^m$, the two intervals $I_a = [G^2(x^m), x^m]$, $I_b = [x^m, G(x^m)]$ are distinct. Moreover, if $G^3(x^m) \leq x^m$, they satisfy the relation:

$$G(I_a) \supset I_b \quad \text{and} \quad G(I_b) \supset (I_a \cup I_b).$$

The relation above implies the existence of a period 3 cycle (see Devaney (1986)).

Three remarks are in order:

(a) by Sarkovskii's theorem (see Devaney (1986) pg.62), period 3 implies the existence of all other periods.

(b) By Li and Yorke's theorem (Li and Yorke (1975)), there exists an uncountable set of aperiodic points whose trajectories wander "randomly" in I_x (chaos)[5].

(c) The square $[G^2(x^m),G(x^m)]^2$ is a "trapping region" for the dynamics induced by G, in the sense that all points lying in this square remain there after any number of iterations of G.

Intuitively, Prop.4 is verified when the map G is steep enough in the sub-intervals in which it is monotonic. The expression for G'(x) shows that different circumstances may account for the occurrence of a three-period cycle.

The nature of the trajectories potentially generated by G can be illustrated by the following example.

Let the utility function of the young representative agent be:

$$U = c_t - (1/2)c_t^2 + c_{t+1}.$$

Then, for any interest rate satisfying $1 \geq R \geq (1-w)$, the saving function is given by:

$$S(w_t, R_{t+1}) = w_t - (1 - R_{t+1}). \qquad (10)$$

Now assume that $\delta = b_o = 1$, $E = b_1 = 0$ and $a_o + a_1 = 1$. In order to satisfy (A4) we have to impose $a_1 > a_o$. With this specification, $w(x) = q(x)$ and prices evolve according to:

$$R_{t+1} = 1 - q_t(1 - K^*). \qquad (11)$$

It is then straightforward to verify that $1 \geq R_t > (1-w_t)$ for all t and for any positive price of the capital stock.

If we now express every variable in terms of x, then equation G assumes the following specification

$$x_{t+1} = \theta(1 - a_1 x_t)x_t \qquad (12)$$

with $x_t \in [0,1/a_1]$ for all t.

It is immediate that the function G(x) implied by (12) maps $[0,1/a_1]$ into itself if $\theta \leq 4$, it is piecewise monotone with a unique maximum at $x^m = 1/2a_1$, it has two fixed points, the origin and $x^* = (1 - \theta)/a_1$, and it is such that $G(1/a_1) = 0$.

It may be worth noticing that, despite the fact that saving is a simple linear function of w and R, equation (12) is highly non-linear.

By making the change of variable $y = a_1 x$, a new function $g(y) = a_1 G(y/a_1)$ mapping [0,1] into itself is defined by:

$$y_{t+1} = \theta(1 - y_t)y_t \tag{13}$$

for $\theta \in (1,4]$. Obviously, the dynamic properties of G are completely invariant under this transformation.

The logistic function (13) has been thouroghly studied in the existing literature on non-linear dynamics (see May (1976) and Collet and Eckmann (1980)). Therefore, I only need to summarize the main results. These are the following:

(a) for $\theta \in (1,3)$, g has an attractive fixed point at $y^* = (1 - \theta)$ and a repelling fixed point at 0. y^* attracts all of the points in [0,1] except for 0, y^* and 1.

(b) As θ passes through 3 there is a periodic point of period 2 attracting all of the points in [0,1] except for 0,y^* and 1.

(c) As θ continues to increase from 3 to 4, there are periodic points of any period and also aperiodic points attracting "almost all" of the points in [0,1].

III. A SAMUELSON ECONOMY.

I will now analyse an economy in which the total value of private agent's wealth may differ from the value of capital. In the tradition initiated by Samuelson's (1958) article, I assume that consumers are allowed to trade in a purely monetary asset, whose (negative or positive) real value is denoted by B. Since B has no intrinsic value, under perfect foresight the no arbitrage condition implies:

$$B_t = R_t B_{t-1}$$

Therefore, a perfect foresight full employment equilibrium for this economy is now defined as a sequence $\{x_t, B_{t-1};\ t = 1,2,\ldots\}$ satisfying:

$$q(x_t)K_t + B_t = S[w(x_t), x_{t+1}/q(x_t)] \quad (14a)$$

$$B_{t+1} = [x_{t+1}/q(x_t)]B_t \quad (14b)$$

$$K_{t+1} = [(1+\theta)/\theta]K^* - (1/\theta)K_t \quad (14c)$$

with $x_t \in I^1_x$, $K_t \in I^1_k$ for all t.

As I mentioned in the last section, system (14) has two types of steady states, WC and GR. Under the present specification, WC is characterized by equilibria $(x^*, K^*, 0)$, where (x^*, K^*) represents the set of steady state solutions of equation (9), and GR is the unique equilibrium $(\bar{x}, K^*, \bar{B})$ given by:

$$\bar{x} = b_o(1+\theta(1-\delta))/a_o(1+\theta), \quad \bar{B} = S[w(\bar{x}), 1] - q(\bar{x})K^*.$$

I am going to look now at the local properties of this dynamical system in the vicinity of WC and GR equilibria to see whether bifurcation phenomena may be generated by the map defined under (14). Before doing this, notice that, since the dynamics of K_t is completely determined by the linear function (14c), we do not need to focus on the complete system. In particular, it is immediately seen that one of the eigenvalues of the Jacobian of (14) evaluated at any stationary state is always equal to $-1/\theta$. Moreover, the curve $K = K^*$ is an invariant manifold of the system. Therefore, by the center manifold theorem, all bifurcation phenomena take place (at most) on a two-dimensional manifold given by the solutions of the following system:

$$q(x_t)K^* + B_t = S[w(x_t), x_{t+1}/q(x_t)] \quad (15a)$$

$$B_{t+1} = [x_{t+1}/q(x_t)]B_t \quad (15b)$$

Thus, the bifurcation phenomena and the asymptotic behavior of (14) can be observed by studying (15) with no loss of generality.

(A3) implies that, in a neighborhood of a stationary state, we can write (15) in the following form:

$$x_{t+1} = H(x_t, z_t) \quad (16a)$$

$$z_{t+1} = x_t \quad (16b)$$

where $H(x,z)$ is implicitly defined by:

$$S[w(x), H/q(x)] - q(x)K^* = \{S[w(z), x/q(z)] - q(z)K^*\}x/q(z)$$

I will denote by (x^*, z^*) and $(\bar{x}, \bar{z})$ the WC and the GR stationary equilibria respectively. Notice that $x^* = z^*$ and $\bar{x} = \bar{z}$.

From the Jacobian of system (16) evaluated at WC and GR, we obtain the following two characteristic polynomials:

$$P_{WC}(\lambda) = \lambda^2 - [h(1)+R^*]\lambda + h(1)R^* = 0$$

$$P_{GR}(\lambda) = \lambda^2 - [h(\sigma)+1+(1-\sigma)/\bar{e}]\lambda + [h(\sigma)-\theta(1-\sigma)/\bar{e}] = 0$$

where:

$$\sigma = q(\bar{x})K^*/S[w(\bar{x}),1],$$

$$h(1) = (R^*/e^*)[S_w^*(1+\theta) - \theta(1+e^*)] = F_x(x^*,K^*),$$

$$h(\sigma) = (\sigma/\bar{e})[\bar{S}_w(1+\theta) - \theta(1+(\bar{e}/\sigma))],$$

$\bar{e}$ is the interest rate elasticity of saving and $\bar{S}_w$ the propensity to save out of wage income, all evaluated at the GR steady state.

Notice that if $B_t = 0$ for some t, then $B_s = 0$ for all $s \geq t$. Moreover, $B_t > (<) \; 0$ implies $B_s > (<) \; 0$ for all $s \geq t$. Therefore, the equilibrium trajectories generated by (15) differ from those of (9) only if they involve non-zero outside money for any $t < +\infty$. This section will only focus on monetary (M) or asymptotically non-monetary (NM) equilibrium trajectories.

<u>Prop.5.</u> NM trajectories near a WC equilibrium are locally monotone.

<u>Proof.</u> The roots of $P_{WC}(\lambda)$ are:

$$\lambda_1 = h(1), \; \lambda_2 = R^*.$$

Therefore, there are NM trajectories near WC if and only if locally WC is either a saddle point or a sink for the dynamical system (15). This implies that all trajectories converging to WC are locally monotone.

As the literature on endogenous cycles has pointed out, M trajectories may be fluctuating in the vicinity of a GR equilibrium. This can be easily shown within the present model. I will consider now the necessary conditions for having a flip and a Hopf bifurcation. The former is associated with the existence of periodic solutions (flip cycles), the latter gives rise to quasi-periodic trajectories lying on a closed curve around the fixed point of the dynamic system (Hopf cycles).

Flip cycles have already been shown to exist in the last section when agents were not allowed to hold money. We have seen also that the existence of these cycles is not necessarily due to a negative interest elasticity of saving. Here we can show that, by allowing the existence of money, this proposition can still be proved.

Notice that if $\bar{B} \neq 0$, the GR equilibrium gives rise to a family of fixed points indexed by the parameter θ and represented by the function:

$$\bar{x}(\theta) = (b_o/a_o)[1+\theta(1-\delta)]/(1+\theta)$$

which is simply the fixed pint of the equation $q = q(x)$.

If, for some value $\bar{\theta}$, there are admissible economies such that one of the roots of $P_{GR}(\lambda) = 0$ is equal to -1, then there is the possibility of a flip bifurcation taking place. In this case the center manifold theorem allows us to reduce the dimension of the problem. More precisely, the local behavior of solutions of (15) can be proved by studying a first order equation of the following type:

$$x_{t+1} = H(x_t, \xi(x_t)) = \Psi(x_t) \qquad (17)$$

where ξ represents the center manifold of (15). In a neighborhood of $(\bar{x}(\bar{\theta}), \bar{z}(\bar{\theta}))$, $\xi(x)$ is a C^2 function satisfying:

$$\xi[H(x,\xi(x))] - x = 0 \qquad (18)$$

$$H(\bar{x}(\bar{\theta}),\xi(\bar{x}(\bar{\theta})) = \bar{x}(\bar{\theta})$$

$$(d/dx)H(x,\xi(x)) = -1 \text{ at } x = \bar{x}(\bar{\theta}). \qquad (19)$$

Carr (1981) and Guckenheimer and Holmes (1983) show how $\xi(x)$ can be found. Consider a function Φ: $\mathbb{R} \to \mathbb{R}$ having a fixed point in x^* and same slope of ξ at x^*. Formally, Φ satisfies:

$$\Phi(x^*) = x^*$$

$$\Phi'(x^*) = -[1 + H_x(\bar{x},\bar{z})]/H_z(\bar{x},\bar{z}).$$

Now, define:

$$N(x) = \Phi[H(x,\Phi(x))] - x.$$

The following theorem establishes that we can approximate $\xi(x)$ as closely as we wish by seeking series solutions of $N(x)$.

Theorem. (Carr(1981)). Let $N(x) = O(|x-x^*|^q)$ as $x \to x^*$ for some $q > 1$. Then:

$$\xi(x) = \Phi(x) + O(|x-x^*|^q) \quad \text{as } x \to x^*.$$

Now consider the behavior of solutions of (15) near the steady state $(\bar{x},\bar{z})$ on the center manifold, that is:

$$\Psi(x) = H[x,\Phi(x)+O(|x-x^*|^q)]$$

with $q > 1$. Then, by the flip bifurcation theorem, we have:

Prop.6.[6] If:

(a) the θ-derivative of $\Psi'(x(\theta))$ is non-zero at $(\bar{x},\bar{\theta})$;

(b) $(1/2)[\partial^2\Psi/\partial x^2]^2 + [\partial^3\Psi/\partial x^3]/3 \neq 0$ at $(\bar{x},\bar{\theta})$;

then, system (15) (and (14)) has a two-period cycle for θ in a neighborhood of $\bar{\theta}$.

By condition (19), an economy in which $\theta = \bar{\theta}$ is such that:

$$\bar{e} = \sigma[(\bar{\theta}+1)/(\bar{\theta}-1)][\bar{S}_w - 1/2] - 1/2. \qquad (20)$$

One may notice from the above equality that the existence of "flip" cycles in this economy is not necessarily linked to any given sign of $\bar{e}$ or $\bar{B}$. In particular, $\bar{e} > 0$ when:

$$\bar{S}_w > (1/2)[1 + (1/\sigma)(\bar{\theta}-1)/(\bar{\theta}+1)].$$

One may be interested in comparing the above inequality with the analogue conditions for the existence of a flip bifurcation in a WC economy. Assuming that $R^* > 1/\theta$ (so that the WC steady state involves a positive interest rate), recall that a WC economy may have a two-period cycle for a given θ near a steady state at which the interest elasticity of saving is positive, provided that:

$$S_w^* > \theta/(1+\theta).$$

Let $\Sigma^* = [\theta/(1+\theta),+\infty]$ be the set of values of the marginal propensity to save out of wage income at which a flip bifurcation may occur in a WC economy having $R^* > 1/\theta$ and $e^* > 0$. With obvious meaning define also the interval $\Sigma' = [m(\sigma),+\infty]$, where:

$$m(\sigma) = (1/2)[1 + (1/\sigma)(\theta-1)/(\theta+1)].$$

Clearly, we have $m(1) = \theta/(\theta+1)$. Moreover, notice that:

$$\theta/(\theta+1) < m(\sigma) \text{ if } \sigma < 1$$

$$\theta/(\theta+1) > m(\sigma) \text{ if } \sigma > 1.$$

Therefore, Σ^* is larger than Σ' when the GR stationary state of the Samuelson economy involves positive outside money and vice versa.

Let us now investigate whether the present economy may generate a Hopf bifurcation. A necessary condition for this to happen is that $P_{GR} = 0$ has a pair of complex conjugate roots (λ, $\bar{\lambda}$) and $|\lambda|$ crosses the value 1 as a parameter of the system is varied (see Guckenheimer and Holmes (1983)). Let θ^* be the value of the parameter vector θ at which $|\lambda| = 1$. Then, $P_{GR}(\lambda) = 0$ has complex eigenvalues at θ^* if:

$$[1 + (1-\sigma)/\bar{e} + h(\sigma)]^2 - 4 < 0$$

The above condition, together with $|\lambda| = 1$, implies:

$$(1-\sigma)(1+\theta^*)/\bar{e} < 0. \qquad (21)$$

Cycles due to $\bar{e} < 0$ and (or) $\bar{B} < 0$ have been shown to exist in recent studies (Benhabib and Day (1982), Grandmont (1985), Farmer (1986)). Farmer (1986) has pointed out that a necessary condition for the emergence of Hopf cycles is $\bar{B} < 0$. Condition (21), however, shows that this need not be the case in the two sector economy considered here. In particular, the present model may generate Hopf cycles even when $B > 0$. However, the occurrence of these cycles implies the failure of the gross substitutability between present and future consumption.

<u>Prop.7.</u>[7] Assume:

(a) $S(w,R)$ is C^r $(r \geq 5)$ in I_x;

(b) λ is not a k^{th} root of unity for $k = 1$-5 at $(\bar{x},\bar{z})$;

(c) the θ-derivative of $[h(\sigma)-\theta(1-\sigma)/e]$ is positive at $(\bar{x},\bar{z},\theta^*)$;

then, by making a smooth change of coordinates in a neighborhood of $(\bar{x},\bar{z},\theta^*)$, we can express system (15) in the form:

$$r_{t+1} = [h(\sigma)-\theta(1-\sigma)/e]r_t + \gamma_1(\theta)r_t^3 + O(5)$$

$$\omega_{t+1} = \omega_t + \alpha + \gamma_2(\theta)r_t^2 + O(5)$$

where (r,ω) are polar coordinates. If, in addition, $\gamma_1(\theta^*) < 0$ at $(\bar{x},\bar{z})$, there exists a closed curve $\Gamma(\theta)$ in the form $r = r(\theta)$ defined for θ in a right neighborhood of θ^*, and invariant under the map generating system (14). Moreover, $\Gamma(\theta)$ is attracting in a neighborhood of $(\bar{x},\bar{z})$ and $\Gamma(\theta) \to (\bar{x},\bar{z})$ as $\theta \to \theta^*$.

Notice that if the θ-derivative of $[h(\sigma)-\theta(1-\sigma)/e]$ is negative at $(\bar{x},\bar{z},\theta^*)$ and $\gamma_1(\theta^*) > 0$, then, after the bifurcation, the invariant circle is repelling while $(\bar{x},\bar{z})$ is attracting.

FOOTNOTES

(1) It may be noticed that the same phenomenon which is responsible for the cycles in pure exchange economies, i.e., the strong income effect, is generally responsible for the indeterminacy of stationary equilibria. This is not surprising because the emergence of cycles in these models are always detected using bifurcation theory, i.e., they arise when, under a perturbation of the economy, some eigenvalue of the associated dynamical system crosses the unit circle at a stationary state. As noted by Woodford (1984), examples characterized by indeterminate Pareto optimal steady states must always involve excess demand functions displaying significant income effects even within more general pure exchange OG models. Therefore, it may be conjectured that the existence of cycles in the vicinity of a GR equilibrium are associated to the income effect even in more disaggregative models.

(2) This same model has been used by Calvo (1978) to provide examples of indeterminate perfect foresight equilibria in OG economies.

(3) As noted by Gale (1972, pg.90), "the inequality of wealth and capital is possible only because our model is 'open-ended'."

(4) See Guckenheimer and Holmes (1983).

(5) As noted by Grandmont (1983), however, Li and Yorke's theorem does not imply that the set of chaotic points is of positive Lebesgue measure.

(6) See Guckenheimer and Holmes (1983).

(7) See Guckenheimer and Holmes (1983).

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