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OUTPUT-INFLATION CYCLES IN AN ECONOMY WITH STAGGERED WAGE SETTING

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Abstract

The paper shows that endogenous deterministic cycles of output and inflation may exist in a nonlinear model with staggered wage setting. The existence of the cycles depends on the presence of a strong effect of inflation on aggregate demand relative to the real balance effect. Bifurcation theory is used to prove that there exists a relation between the amplitude of the cycles and the average contract length. This relation is shown to be negative in a variety of cases.

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INTRODUCTION.

In the past fifteen years, theories of the business cycle have been mostly developed under the assumption of imperfect information. These theories view agents' misperceptions about an economic environment perturbed by exogenous shocks as the essential reason for the observed economic fluctuations. Recent studies on nonlinear economic dynamics, however, have shown that the existence of perfect foresight and a completely stationary environment does not rule out the possibility of having economic fluctuations in standard neoclassical equilibrium models[1]. These findings have interesting implications regarding the consequences of assuming rational expectations and the role of government policy.

The literature on endogenous cycles with forward looking and optimizing agents has been mostly developed in the framework of simple equilibrium growth models (either the overlapping generations or the Cass (1965)-Koopmans (1965) model). These models are not particularly suitable for analysing one of the most relevant issues in macroeconomics, such as the relation between unemployment and price dynamics. The current macroeconomic literature, however, has been greatly influenced by equilibrium models. Intertemporal optimization and rational expectations are commonly assumed even in keynesian-type models embodying market failures and unemployment. The present paper intends to explore the possibility and the consequences of having endogenous cycles in a keynesian model in which agents' anticipations of future variables are recognized.

The model that I am going to study is borrowed from the staggered contracts literature where wages and prices are assumed to be set on the basis of non-synchronos multiperiod contracts. Using a deterministic nonlinear model, I will show that the existence of staggered contracts implies that fluctuations of the output-inflation pair may be permanent and completely endogenous so that a constant rate of growth of money may be ineffective in stabilizing the economy both in the short and in the long run.

The reason for the existence of endogenous cycles is very simple. Let the price setting rule be as in Taylor (1979) and Calvo (1983). Then:

(i) individual contracts last for several periods and they are overlapping with each other;

(ii) agents' objective is to set prices in accordance with their anticipations of future price and output levels.

From (i) and (ii) it follows that output is negatively correlated with the expected change of inflation (the existence of this "higher order inverse Phillips curve" has been emphasized by Calvo (1983)). Assume now that, as in most traditional macromodels, inflation has a positive impact on aggregate demand. Then, when output is increasing, inflation will increase at a decreasing rate. This process will eventually exert a negative effect on aggregate demand and lead to a fall in output. However, as inflation decreases along the cycle, real money balances will increase and have a stimulating effect on aggregate demand. This may be an element working against the cycle as I will show in the following sections.

Most of the analysis will focus on the consequences on the dynamics of the economy of varying some of the parameters of the model. To this purpose, bifurcation theory is shown to be extremely useful in order to detect the appearance of periodic or aperiodic orbits around the stationary state. The parameter on which most of the attention is dedicated is a measure of the average contract length. It is shown that stability and existence of endogenous cycles are highly dependent on the magnitude of this parameter. Whether these dynamic phenomena are more or less likely when the average contract length is increased is ambiguous. The answer to this question can only be found by looking at the relative importance of the effect of inflation, output and real money balances on aggregate demand.

One of the debated questions in the current macroeconomic literature is whether increased price flexibility have a stabilizing effect on output[2]. In order to analyse this problem in a deterministic framework, the methodology used in this paper can be fruitfully applied. In fact, once the existence of endogenous cycles is proved, one can actually evaluate the amplitude of the orbits by applying a standard procedure in the theory of nonlinear dynamical systems. The paper shows that there are robust examples in which a reduction of the average length of the contracts increases the amplitude of the cycle. These results are in contrast with a standard conjecture in the current macroeconomic literature according to which long term contracts and price inflexibility are generally responsible for cyclical fluctuations.

The paper is divided in three sections. The first gives a description of the basic features of the staggered contract model and the way in which price dynamics is modeled. The last two sections analyze the dynamics of the economy using the traditional IS-LM model and shows how the average contract length interacts with the real balance effect and the effect of inflation on aggregate demand to determine the dynamic properties of output and prices.

1. PRICE DYNAMICS AND STAGGERED CONTRACTS.

Modelling the dynamics of prices in Keynesian macromodels has always been a controversial matter. Most of the early literature on the subject views output and price variability as heavily dependent on money illusion and misperceptions of future variables on the part of agents.

In recent years attempts have been made to set up models of "disequilibrium dynamics" which do not rely on erroneous expectations. One of the most interesting developments emphasizes the fact that nominal wages are not continuously revised and that they are set in a non-synchronos way (Phelps (1978), Taylor (1979, 1980), Calvo (1983), Blanchard (1983, 1986)). The term structure of these contracts acts as a constraint for rational agents, whose objective is to set them in accordance with their anticipations of future price and output levels.

This model is consistent with some of the ideas about labor market behavior developed by keynesian economists in recent years. Various considerations, such as costs of changing jobs, employers' efforts to hold down quit rates and minimize setup costs, provide arguments for the existence of long-term contracts in the labor market[3]. The same considerations also point to wage comparisons as a central criterion for employers and workers in setting contracts. In Tobin's (1980) words, agents' "chief concern is the wage rate relative to wages in competing firms or in comparable occupations and situations, and not the availability of cheaper workers at the factory gates"[4].

In the present paper I will basically follow Taylor's (1979, 1980) and Calvo's (1983) formulation of the staggered contracts model and their assumption about the way in which the dynamics of prices affect aggregate output. This formulation differs from most of the traditional keynesian models where business cycle fluctuations essentially follow from real wage movements causing firms to adjust their demand for labor through the marginal productivity schedule. As is well known, these models leave unexplained a major empirical regularity, i.e., the fact that real wages do not move countercyclically during the business cycle (in fact, most economic data show that average labor productivity moves procyclically). On the other hand, in the present model employment fluctuations need not go along with real wage movements. In Taylor's (1979) words, the model deemphasizes "relative price shifts due to asymmetrical rigidities (for example, wages are rigid while prices are flexible) [and focuses on] the general persistence of all prices due to non-synchronous prices (or wages)". For simplicity, labor productivity is assumed to be constant and output fluctuations derive exclusively from demand shifts.

The key element of the mechanism generating the dynamics of the model is the effect of prices on aggregate demand. In a number of papers, Taylor (1979, 1980) has focused on what he seems to consider the most prominent of them, namely, the "Keynes effect". The latter arises when a fall of the price level increases the real money balances and causes the interest rate to fall. A more sophisticated version of the model includes the effect of expected inflation on aggregate demand (see Calvo (1983)) following the argument that anticipations of future price changes affect the real interest rate and, thus, investment decisions. The interaction of these effects with the process of contract formation easily produces serial correlation for output and prices. In what follows it is proved that the two effects may be responsible for the occurrence of a number of interesting dynamic phenomena which have been disregarded by the current literature.

A contract signed at time t specifies a nominal wage from t to t+h. In accordance with Taylor (1979, 1980) I will assume that prices are set by imposing a constant mark-up over the wage rate. Therefore, real wages are constant over time and agents (firms and/or employees) are uniquely concerned about the average wage expected to prevail during the contract period and an indicator of aggregate demand conditions. Letting the integer h be the contract lenghth, under perfect foresight we have:

$$w_t = (1/h)\sum_{s=t}^{t+h} [W_s + \alpha y_s] \qquad (1)$$

where y is the log of the aggregate output deviation from some natural rate value (which will be assumed to be 1 for simplicity), w is the log of the individual wage rate, W the log of the average wage rate and α is a positive coefficient.

Equation (1) has been proposed by Taylor (1979, 1980) to describe the wage setting rule at a macroeconomic level. Working with such specification, however, leads to some analytical difficulties. In particular, when the reduced form of the model is derived, one has to deal with a multidimensional system of difference equations with a very complicated dynamics unless h is specified to be a small integer. Calvo (1983) has proposed a way to simplify the problem without losing the general characteristics of the model[5]. Let the duration of the contract be a random variable with the following density function:

$$f(h) = (1-\delta)\delta^h \text{ for } h = 0,1,2,\ldots, \text{ and } 0 \leq \delta < 1,$$

$$f(h) = 0 \text{ otherwise.}$$

Then, the expected value of h is $\bar{h} = \delta/(1-\delta) > 0$. Notice that the density function does not depend on the date at which the contract has been signed. While this may be considered an unrealistic assumption, it is necessary for the sake of simplicity. With this modification, equation (1) becomes:

$$w_t = (1-\delta)\sum_{s=t}^{+\infty} [W_s + \alpha y_s]\delta^{s-t} \qquad (2)$$

Assume now that "contract setters" are an infinite number of identical agents uniformly distributed over the interval [0,1]. Then, the log of the average wage is given by the following expression:

$$W_t = (1-\delta)\sum_{s=-\infty}^{t} w_s \delta^{t-s} \quad (3)$$

From (2) and (3) we get:

$$w_t - w_{t-1} = (1-\delta)[w_t - W_{t-1} - \alpha y_{t-1}]$$

$$W_t - W_{t-1} = (1-\delta)[w_t - W_{t-1}]. \quad (4)$$

Therefore:

$$w_t - w_{t-1} = (W_t - W_{t-1}) - (1-\delta)\alpha y_{t-1}. \quad (5)$$

Since prices are proportional to nominal wages, we can define the log of the gross rate of inflation as $\Pi_t = W_t - W_{t-1}$. Using this notation, (4) and (5) imply:

$$\Pi_{t+1} = \Pi_t - \beta y_t \quad (6)$$

where $\beta = (1-\delta)\alpha/\bar{h} > 0$, i.e., it is a decreasing function of the expected length of the contract.

Equation (6) is a "second order" inverse Phillips Curve and can be obtained in most versions of the staggered contract model[6]. The output-inflation trade-off is perfectly accelerationist, in the sense that there is no way that output can be raised permanently above (below) its natural level without decelerating (accelerating) rates of inflation.

2. MODEL I.

We are in a position now to complete the model. Let D be the log of aggregate demand. In accordance with the traditional macroeconomic literature, this variable is specified as a function of y and the real interest rate. Thus:

$$D = D(y, r-\pi) \qquad (7)$$

where r denotes the nominal interest rate and it is assumed that $D_1 \geqq 0$, $D_2 \leqq 0$. Letting m be the log of real money balances, portfolio equilibrium in the asset market is given by:

$$r = L(m,y) \qquad (8)$$

whith $L_1 \leqq 0$, $L_2 \geqq 0$. Plugging (8) in (7), we get:

$$D = D[y, L(m,y)-\pi]$$

and thus:

$$D_\pi = -D_2 \geqq 0,\ D_y = D_1 + D_2L_2,\ D_m = D_2L_1 \geqq 0.$$

Assume now that the log of the gross rate of expansion of money supply is a constant parameter μ, then we have the relation:

$$m_{t+1} = m_t + \mu - \pi_{t+1} \qquad (9)$$

If output instantaneously adjusts to equate aggregate demand, equilibrium implies y = D. Then, assuming $0 < D_y < 1$ and solving explicitly for y, we get $y = F(\pi,m)$ where F is such that:

$$F_\pi = D_\pi(1-D_y)^{-1} = (-1/L_1)F_m \geqq 0.$$

The complete dynamical system is the following:

$$\pi_{t+1} = \pi_t - \beta F(\pi_t, m_t) \qquad (10a)$$

$$m_{t+1} = m_t + \mu - \pi_t + \beta F(\pi_t, m_t) \qquad (10b)$$

System (10) has a steady state (π^*, m^*) characterized by $F(\pi^*, m^*) = 0$, $\pi^* = \mu$. Evaluating the linearized version of (10) in the vicinity the steady state, it is verified that the Jacobian of (10) at (π^*, m^*) has real eigenvalues. Moreover, letting λ_2 be the larger eigenvalue, we have $\lambda_2 \geqq 1$, $\lambda_1 \leqq 1$ for any admissible parameter values and $\lambda_1 \leqq -1$ if and only if:

$$\beta F_\pi \geqq 4(2 - \gamma)^{-1}$$

where $\gamma = (-L_1^*) < 2$ and the partial derivatives in the above expression are evaluated at the stationary state.

The dynamic behavior of the model crucially depends on the relative magnitude of the parameters F_π and γ. Any non negative point (F_π, γ) above the curve $\beta F_\pi = 4/(2-\gamma)$ implies $\lambda_1 < -1$ and thus the instability of the stationary state (Π^*, m^*). Conversely, for any parameter value in the region below the same curve or such that $\gamma > 2$, the model exhibits saddle path stability. In the latter case there exists a unique local stable manifold in $\mathbf{R}^2$ where (Π_t, m_t) asymptotically converges to (Π^*, m^*). From this brief analysis of the linearized version of the model we can conclude that the stationary state is a locally determinate perfect foresight equilibrium when:

either $\gamma > 2$

or $0 \leqq \gamma < 2$ and $\beta F_\pi < 4(2 - \gamma)^{-1}$.

In other words, if we restrict our attention to trajectories characterized by an initial condition of the predetermined variable m "very close" to (Π^*, m^*), there is a unique stable perfect foresight equilibrium for any such initial condition.

One may tend to conclude that when the elasticity of the demand for money with respect to the interest rate is sufficiently low (the Keynes effect is high in absolute value) the model possesses the saddle path property for any value of β. On the other hand, a high value of this elasticity (the model approaching some form of liquidity trap) may imply the local instability of the steady state if the price change effect F_π and/or β (which is inversely related with the average length of the contracts) are high enough. In other words, "more sluggish" price dynamics may help to restore stability in an economy in which the effect of nominal variables on aggregate demand tend to be destabilizing. However, trajectories diverging from the steady state are also candidates for representing perfect foresight equilibria of the model. In fact, these trajectories may converge to some limit set not containing the stationary point. In order to see this one has to take into account the nonlinear components of the dynamical system.

I will show now that, for some positive (F_π,γ) with γ smaller than 2 and close to the curve $\beta F_\pi = 4/(2-\gamma)$, system (10) may possess an oscillatory solution converging to a two-period cycle.

Assume that the fixed point of (10) is hyperbolic (i.e., there are no eigenvalues on the unit circle). Then, there are two local invariant manifolds U_1 and U_2 going through the fixed point and here tangent to the eigenspaces associated with λ_1 and λ_2 respectively. Clearly, any trajectory lying on U_2 diverges from the steady state. Therefore, the steady state is either a source ($|\lambda_1| > 1$) or a saddle point ($|\lambda_1| < 1$). In the former case U_1 is unstable, in the latter it is stable.

From the theory of bifurcation we know that some interesting dynamic phenomenon may be generated as one of the eigenvalues crosses the unit circle. In our case we can find the emergence of periodic orbits when λ_1 crosses the value -1. Since the trajectories subject to bifurcation phenomena are the only ones whose stability with respect to the fixed point changes as the bifurcation occurs (i.e., they are on U_1), we can concentrate only on the dynamics of the system restricted to the manifold U_1. As a consequence, the relevant dynamical system on which we are going to be interested has the dimension of the manifold whose stability changes with the bifurcation.

A widespread method in the analysis of macroeconomic models assuming rational expectations is to consider only dynamic trajectories belonging to the stable manifold. Sometimes, restrictions on the parameter values are imposed to guarantee that the dimension of the stable manifold is equal to the number of non-predetermined variables. In the context of a nonlinear model these restrictions may lead to the consideration of only a subset of the possible stable trajectories. By the method used in the following sections the analysis focuses on trajectories belonging to a manifold the stability of which changes as a parameter of the model crosses a critical value.

The procedure goes as follows. In a first step I will apply center manifold theory to reduce the dynamics of (10) to U_1. Then, I will apply the flip bifurcation theorem to prove the existence of the cycle.

a. Center manifold reduction.

From now on I will single out the influence of the parameter β on the dynamics of the model. With more compact notation, system (10) can be written as:

$$X_{t+1} = J(\beta)X_t + f(X_t,\beta)\underline{e} \qquad (11)$$

where J is the Jacobian of (10) evaluated at the steady state and:

$X = [\Pi-\mu, m-m^*]'$

$\underline{e} = [-1,1]'$

$f(X,\beta) = \beta[X'D^2F(\mu,m^*)X](1/2) + O(3)$

$O(3)$ = terms of degree 3 or more.

Since the eigenvalues of J are real, we can find new coordinates for X in $\mathbf{R}^2$ to put the linear part of (11) in diagonal form. In fact, letting $q^1(\beta)$, $q^2(\beta)$ be the eigenvectors of $J(\beta)$ associated with the eigenvalues $\lambda_1(\beta)$ and $\lambda_2(\beta)$ respectively, we have:

$$J(\beta) = Q(\beta)\mathrm{diag}\{\lambda_1(\beta),\lambda_2(\beta)\}Q^{-1}(\beta)$$

where $Q(\beta) = [q^1(\beta),q^2(\beta)]$. Defining X in the new basis as a vector $[x,k]'$, we have $[x,k] = Q^{-1}(\beta)X$.

Now, with a suitable normalization, it is verified that:

$q^i = [\sigma_i(\beta),1]'$ with $i = 1,2$;

where $\sigma_i(\beta) = -\beta F_\pi \gamma/(\lambda_i(\beta)+\beta F_\pi - 1)$ for $i = 1,2$. Notice that:

$\sigma_2(\beta) > 0$ for $\beta F_\pi \gamma \neq 0$, $\sigma_2(\beta) = 0$ for $\beta F_\pi = 0$;

$\sigma_1(\beta) < 0$ for $\beta F_\pi \gamma \neq 0$, $\sigma_1(\beta) = 0$ for $\beta F_\pi = 0$.

Under the same normalization:

$\Pi = \mu + \sigma_1 x + \sigma_2 k$

$m = m^* + x + k$.

Now define $\beta^* = 4/(F_\pi(2-\gamma))$ as the bifurcation point of (10). Recall that $\lambda_1(\beta^*) = -1$ and thus $\lambda_2(\beta^*) = \beta^* F_\pi - 1$. It is implicitly assumed from now on in this section that $0 < \gamma < 2$ and $F_\pi > 0$. Notice that:

$\lambda_1'(\beta^*) = -(F_\pi/4)(2 - \gamma)^2$

so that as β crosses β^* from the left λ_1 crosses the unit circle from inside with non-zero velocity.

In the new basis, system (10) can be written as:

$$x_{t+1} = -x_t + g^1(x_t,k_t,\beta_t) \qquad (12a)$$

$$k_{t+1} = \lambda_2(\beta^*)k_t + g^2(x_t,k_t,\beta_t) \qquad (12b)$$

$$\beta_{t+1} = \beta_t \qquad (12c)$$

where:

$$g^1 = (\lambda_1(\beta_t)+1)x_t - \Sigma_2(\beta_t)f[Q(\beta_t)(x_t,k_t)',\beta_t] + O(3)$$

$$g^2 = (\lambda_2(\beta_t)-\lambda_2(\beta^*))k_t + \Sigma_1(\beta_t)f[Q(\beta_t)(x_t,k_t)',\beta_t] + O(3)$$

$$\Sigma_i(\beta) = (1+\sigma_i(\beta))/(\sigma_1(\beta)-\sigma_2(\beta)) \text{ for } i = 1,2.$$

Now let Φ: $\mathbf{R}^3 \to \mathbf{R}^3$ be the map defining system (12). Then $D\Phi(0,0,\beta^*)$ has two eigenvalues with modulus 1 and one eigenvalue greater than 1. Moreover, g^1 and g^2 vanish together with each of their derivatives at the fixed point. Therefore, if g^1 and g^2 are C^2, there exists a center manifold h:

$R^2 \to R$ with $h(0,\beta^*) = 0$ and $Dh(0,\beta^*) = 0$. In fact, the following theorem can be used in the present case:

Theorem 1 (Carr (1981)).

Let T: $R^{n+m} \to R^{n+m}$ have the following form:

$$T(x,y) = [Ax + f(x,y), By + g(x,y)]$$

where $x \in R^n$, $y \in R^m$, A and B are square matrices such that each eigenvalue of A has modulus 1 and each eigenvalue of B has modulus different from 1, f and g are C^2 and f, g and their first order derivatives are zero at the origin. Then, there exists a center manifold H: $R^n \to R^m$ for T. More precisely, for some $\epsilon > 0$ there exists a C^2 function H: $R^n \to R^m$ with $H(0) = 0$, $DH(0) = 0$ such that $|x| < \epsilon$ and $(x_1, y_1) = T[x, H(x)]$ implies $y_1 = H(x_1)$.

Thus, h solves the equation:

$$h[-x + g^1(x,h(x,\beta),\beta)] = \lambda_2(\beta^*)h(x,\beta) + g^2(x,h(x,\beta),\beta).$$

Carr (1981) shows how h can be approximated by using Taylor series at $x = 0$, $\beta = \beta^*$. Define a function ϕ: $R^2 \to R$ such that $\phi(0,\beta^*) = 0$ and $D\phi(0,\beta^*) = 0$ and consider:

$$N(x,\beta) = \phi[-x + g^1(x,\phi(x,\beta),\beta)] - \lambda_2(\beta^*)\phi(x,\beta) - g^2(x,\phi(x,\beta),\beta).$$

The Taylor series expansion of N about $(0,\beta^*)$ gives:

$$N(x,\beta) = (1/2)(x,\beta-\beta^*)'D^2N(0,\beta^*)(x,\beta-\beta^*) + O(3)$$

since $N(0,\beta^*) = 0$ and $DN(0,\beta^*) = 0$. Now impose the condition $D^2(0,\beta^*) = 0$. The latter implies $\phi_{12} = \phi_{22} = 0$ and:

$$\phi_{11}(0,\beta^*) = a^* = [\Sigma_1(\beta^*)/(2-\beta^*F_\pi)][\sigma_1^2(\beta^*)F_{\pi\pi} + \sigma_1(\beta^*)2F_{\pi m} + F_{mm}].$$

Under the above assumptions we have $N(x,\beta) = O(3)$. Now the following theorem can be stated:

Theorem 2 (Carr (1981)).

If a C^1 function ϕ: $\mathbf{R}^2$ --> $\mathbf{R}$, with $\phi(0,\beta^*) = 0$ and $D\phi(0,\beta^*) = 0$, can be found such that $N(x,\beta) = O(|(x,\beta-\beta^*)|^p)$ for some $p > 1$ as $|(x,\beta-\beta^*)|$ --> 0 then it follows that:

$$h(x,\beta) = \phi(x,\beta) + O(|(x,\beta-\beta^*)|^p) \quad \text{as } |(x,\beta-\beta^*)| \to 0.$$

From theorem 2 we obtain:

$$h(x,\beta) = (a^*/2)x^2 + O(3)$$

and the dynamical system reduced on the center manifold, for x close to 0 and β close to β^*, is a nonlinear map:

$$x_{t+1} = G(x_t,\beta) \qquad (13)$$

with:

$$G(x,\beta) = \lambda_1(\beta)x - (\beta/2)x^2\Sigma_2(\beta)\{F_{\pi\pi}[\sigma_1(\beta)+(\sigma_2(\beta)a^*/2)x]^2$$
$$+ 2F_{\pi m}[\sigma_1(\beta)+(\sigma_2a^*/2)x][1+(a^*/2)x] + F_{mm}[1+(a^*/2)x]^2\} + O(3)$$

and $x = [(\bar{\pi}-\mu)-\sigma_2(\beta)(m-m^*)]/(\sigma_1(\beta)-\sigma_2(\beta))$.

Notice that, if we restrict the equilibria of the present economy to the trajectories generated by system (10) for which it is possible to find a set on which they are bounded, then these equilibria are equal to the stable trajectories generated by system (13), for any predetermined initial condition of x. Thus, there is no possibility of having multiple equilibria in this economy. In the next section, however, we will see that there are admissible parameter values for which equilibria diverge from the steady state and converge to a periodic orbit. Usually, perfect foresight equilibria are only identified with the set of asymptotically stable trajectories with the steady state as the limit point. The use of the center manifold reduction and of bifurcation theory allow us to consider a larger set of perfectly determinate equilibria.

b. Flip bifurcation.

Using bifurcation theory it is now possible to understand the dynamics of (10) when β varies in a neighborhood of the critical value β^*. Since we know that the eigenvalues of the linearized system at (μ,m^*) are real, we can only try to prove the existence of a bifurcation of the flip type. This phenomenon occurs when one eigenvalue crosses -1 as a parameter of the dynamical system is varied. From the previous discussion we know that, in order for $\lambda_1(\beta)$ to cross -1, we must have β going through β^*.

At this point the center manifold reduction operated in the previous section proves to be very useful. In fact, by center manifold theory, all bifurcation phenomena take place on $h(x,\beta)$ and we can restrict our analysis to the dynamic equation (13) with no loss of generality.

Notice that the map G has the following properties:

$G_x(0,\beta^*) = -1$

$G_\beta(0,\beta) = 0$ for all β

$G_{x\beta}(0,\beta^*) = \lambda_1'(\beta^*) = -F_\pi(2-\gamma)^2/4 < 0.$

As shown by the above equalities, the dynamic behavior of equation (13) changes abruptly when β crosses the value β^*. In particular, the stationary point $x = 0$ is stable for $\beta < \beta^*$ and becomes unstable for $\beta > \beta^*$.

Now consider the expression:

$d = (1/2)G_{xx}(0,\beta^*)^2 + (1/3)G_{xxx}.$

Then, we have the following theorem:

Theorem 3 (Guckenheimer and Holmes (1983)).

If $d \neq 0$, there exists a smooth curve of fixed points of $G(x,\beta)$ passing through $(0,\beta^*)$ the stability of which changes at $(0,\beta^*)$. There is also a smooth curve γ passing through $(0,\beta^*)$ so that $\gamma - \{(0,\beta^*)\}$ is a union of hyperbolic period 2 orbits. The curve γ has quadratic tangency with the line $\beta = \beta^*$ at $(0,\beta^*)$. Moreover, if $d > 0$ (< 0) the orbits are stable (unstable) for β in a right (left) neighborhood of β^*.

The situation is illustrated in fig. 1 for the case in which d is positive and in fig. 2 for the case $d < 0$. Consider the former. If $\beta < \beta^*$ the fixed point $x = 0$ is stable. As β crosses β^* from the left there is a bifurcation generating a family of stable fixed points of the second iteration $G(G(x,\beta))$ of the map G.

[figures 1 and 2]

The case $d > 0$ is more interesting since it establishes the existence of a stable orbit around the stationary state, i.e., the existence of permanent endogenous fluctuations. Notice that an increase of the average length of the contracts (a decrease of β) has always a beneficial effect on the economy. In fact:

(i) when $d < 0$, $\beta > \beta^*$ implies that the economy is completely unstable;

(ii) when $d > 0$, $\beta > \beta^*$ implies that the economy will permanently fluctuate and never return to its stationary state. Moreover, the amplitude of the cycle is, at least locally, an increasing function of β (see fig. 1).

An evaluation of the parameter d, whose sign determines the direction of bifurcation and the stability of the orbits of period two, gives the following expression:

$$d = [(4F_{\pi\pi} - 4F_{\pi m} + F_{mm})/F_{\pi}^{2}]\{F_{\pi\pi}(2+F_{\pi}(2-\gamma))$$
$$- F_{\pi m}(2-(F_{\pi}/2\gamma)(2-\gamma)(1+\gamma)) + F_{mm}((1/2)-(F_{\pi}/8\gamma)(2-\gamma)(2+\gamma))\}.$$

The expression for d is very complicated and requires some understanding of the relative magnitude of the nonlinear components of the aggregate demand function. Consider the following application.

One of the most debated issues in traditional keynesian theory is the relation between the nominal interest rate and real money balances. This issue concerns the effectiveness of monetary policy and may be thought of as a discussion about the shape of the LM curve. Some Keynesian theorists would argue that for low values of the nominal interest rate the economy may fall in a liquidity trap where further increases of the real stock of money have little effect on the nominal interest rate. In this respect it may be crucial to assume that the function $r = L(m,y)$ is nonlinear in m[7]. To simplify the matter I will consider the case in which the only nonlinear component of the model is given by the liquidity preference. Thus, let L_2, D_1, D_2 be constants and $L_{11} \neq 0$. Then, it follows that $F_{\pi\pi} = F_{\pi m} = 0$, $F_{mm} = -F_{\pi}L_{11}$.

Under these assumptions we have:

$$d = (L_{11})^{2}[(1/2) - (F_{\pi}/8\gamma)(2-\gamma)(2+\gamma)]$$

Therefore the sign of d does not depend on the sign of L_{11} but only on the magnitude of F_{π} and γ. In particular, the orbits are stable if and only if $F_{\pi} < 4\gamma(4 - \gamma^{2})^{-1}$. Notice the similarity between the latter stability condition (of periodic orbits) and the stability condition of the steady state (μ,m^{*}), i.e., $\beta F_{\pi} < 4(2 - \gamma)^{-1}$. Thus, we can conclude that, a high γ relative to F_{π} is always desirable from the point of view of the stability of the system.

The inverse relation between the rate of change of inflation and output is one of the key elements to understand how cycles may be generated in the present model. This inverse relation, which is represented by equation (6), simply derives from the overlapping structure of the contracts. The other essential element to be considered is the price change effect, i.e., the stimulating effect of inflation on aggregate demand. When the price change effect is strong enough, cycles are clearly possible. In fact, if we are in a stage of increasing inflation, then aggregate demand is rising along with output. However, through equation (6), this economic expansion will soon reduce the speed at which inflation is increasing and eventually produce lower rates of inflation. As a result, aggregate demand will be depressed and this will tend to recreate the conditions for a new increase of the rate of change of inflation. Now consider the effect of real balances on aggregate demand. This element is surely working against the cycle. In fact, when inflation is rising, m is falling and aggregate demand may be depressed despite the stimulating effect of inflation.

A final remark is about the characteristics of the cycles generated by the present model. The existing trajectories of (Π, m) for β close to β^* alternate from one side of (μ, m^*) to the other along the direction of the eigenvector associated to the eigenvalue -1. These are not the type of cycles that one would observe from actual economic data. In fact, inflation rates generally stay above (or below) the trend values for more than one period. This particular feature of the trajectories generated by the present model essentially derives from the low dimensionality of the dynamical system. In the next section I will study a model with a higher dimensionality due to some source of persistence (like costs of immediate adjustment) of lagged output levels along the cycle.

3. MODEL II.

In this section I will modify the model used so far by assuming sluggish output adjustment. It is shown that this modification allows for the existence of cycles of a different kind. These cycles are given by trajectories possibly converging toward closed curves around the steady state. It is shown that neither the stability nor the existence of the orbits depend upon the magnitude of the real balance effect. Moreover, while the effect of price changes and the parameter β have a crucial role in proving the existence of cycles, the way in which this role is exerted is ambiguous.

Assume that output sluggishly adjusts to changes in aggregate demand (inventory holdings may account for any discrepancy between production and sales). This circumstance is captured by the following equation of output dynamics:

$$y_{t+1} = E(\Pi_t, y_t, m_t) \qquad (14)$$

where E is some increasing function of excess demand, i.e., of the difference $D(y, r-\Pi) - y$. Equation (14) adds to equations (6) and (9) to form a new complete dynamical system. A steady state of this system is a fixed point $(\Pi, y, m) = (\mu, 0, m^*)$ such that $E(\mu, 0, m^*) = 0$. Assuming that $E = 0$ if and only if $D = 0$, the stationary value(s) of m in the present model is equal to the one(s) found in the model of the previous section. Now assume E is a C^∞ function and let J be the Jacobian of system (6), (9), (14) evaluated at $(\mu, 0, m^*)$. It is verified that $|J| = E_y + \beta E_\pi$, $Tr(J) = 2 + E_y$. Denoting by λ_i (i = 1,2,3) the eigenvalues of J, we have:

$$\lambda_1 \lambda_2 \lambda_3 = E_y + \beta E_\pi \qquad (15)$$

$$\lambda_1 + \lambda_2 + \lambda_3 = 2 + E_y \qquad (16)$$

$$\lambda_1 \lambda_2 + \lambda_1 \lambda_3 + \lambda_2 \lambda_3 = 1 + E_y + \beta E_\pi \qquad (17)$$

Local determinateness of the steady state in the present economy now implies that two of the eigenvalues are inside the unit circle and one outside. In fact, the only non-predetermined variable of the model is Π. This circumstance is not always verified as one may conclude from a study of the eigenvalues of the system. It is worth noticing that, contrary to the model analysed in the previous section, the direct effect of m on aggregate demand does not affect the eigenvalues of the system. The local dynamic behavior of the latter is only determined by the parameters β, E_y, E_π.

As in the last section I will now consider a situation in which the parameters of the model vary in some interval where there is a change in the dynamic properties of the system.

Notice that if we impose $\lambda_1\lambda_2 = 1$ and $|J| \neq 0$, then $\lambda_1 + \lambda_2 = 1$ and $\beta E_\pi = 1$. In turn, the above conditions imply that λ_1, λ_2 have to be complex conjugate numbers with unitary modulus whereas the remaining eigenvalue is real and such that $\lambda_3 > 1$ (< 1) if $E_y > 0$ (< 0).

From now on I will single out the influence of the parameter β on the behavior of the system and define $\beta^* = 1/E_\pi$. Clearly, for β in a neighborhood of β^*, λ_1 and λ_2 are still complex conjugete numbers, which I denote by $\lambda(\beta) = a(\beta) + ib(\beta)$ and $\bar{\lambda}(\beta) = a(\beta) - ib(\beta)$.

At the critical point β^* we have:

$$a(\beta^*) = 1/2,\ b(\beta^*) = (3)^{1/2}/2,\ \lambda_3(\beta^*) = 1 + E_y.$$

In what follows I will be concerned with the modifications which the dynamics of the model incurs when β varies in an interval whose interior contains β^*.

If λ_1 and λ_2 are complex conjugate numbers in equations (15)-(17), we have:

$$|\lambda(\beta)|^3 - |\lambda(\beta)|^2(1+E_y+\beta E_\pi) + |\lambda(\beta)|(2+E_y)(E_y+\beta E_\pi) - (E_y+\beta E_\pi)^2 = 0$$

which implicitly defines the function $M(\beta) = |\lambda(\beta)|$. By differentiating the above equation at β^* we get:

$$M'(\beta^*) = E_\pi(E_y + 1)[1 + E_y + E_y^2]^{-1}.$$

Since the denominator of this derivative is positive for any value of E_y, then $M'(\beta^*) > 0$ if and only if $E_y > -1$ and viceversa. At any rate, $M(\beta)$ crosses the value 1 with non-zero slope as β crosses β^*.

In order to simplify the analysis it is useful to assume that $\lambda_3(\beta)$ does not cross the unit circle for any interval where β will be allowed to vary. Then, letting I_β be this interval (whose interior contains β^*), I will assume $|\lambda_3(\beta)| \neq 1$ for all β belonging to I_β.

Notice that $|\lambda_3(\beta)| < 1$ implies $-2 < E_y < 0$ and the steady state is locally indeterminate for $M(\beta) < 1$ and is locally unstable for $M(\beta) > 1$. On the contrary, $|\lambda_3(\beta)| > 1$ implies either $E_y > 0$ or $E_y < -2$ and we have a locally determinate steady state if $M(\beta) < 1$ and a locally unstable one if $M(\beta) > 1$. Therefore an increase in the expected length of the contracts has a "local" destabilizing effect when $E_y < -1$ and β is close to β^*. The opposite is true when $E_y > -1$.

I will now proceed to analyze the nonlinear components of the model to check for the existence of equilibrium trajectories bounded away from the stationary state. The procedure is analogous to the one used in the last section. I will first try to find a center manifold reduction and then prove the existence of nonstationary orbits as limit sets for the dynamics of the system.

By the center manifold reduction (see Appendix A), the system represented by equations (6), (9) and (14) can be put in the following form:

$$Y_{t+1} = A(\beta)Y_t + \Psi(Y_t) \qquad (18)$$

where $Y \in \mathbf{R}^2$, Ψ: $\mathbf{R}^2 \rightarrow \mathbf{R}^2$ is a nonlinear function such that $\Psi(0) = D\Psi(0) = 0$ and:

$$A(\beta) = \begin{bmatrix} a(\beta) & -b(\beta) \\ b(\beta) & a(\beta) \end{bmatrix}$$

Now, the theory of bifurcation for two-dimensional systems establishes:

<u>Theorem 4 (Devaney (1986))</u>.

Suppose $\lambda(\beta)$ is not a k^{th} root of unity for k = 1-5. Then, in a neighborhood of the fixed point, with a smooth change of coordinates, (18) assumes the form:

$$\rho_{t+1} = M(\beta)\rho_t - \eta(\beta)\rho_t^3 + O(5) \qquad (19)$$

$$\omega_{t+1} = \omega_t + \theta(\beta) + \zeta(\beta)\rho_t^2 + O(5) \qquad (20)$$

where (ρ,ω) are polar coordinates, $\theta(\beta)$ is such that $\lambda(\beta) = \cos\theta(\beta) + i\sin\theta(\beta)$ and $\eta(\beta)$ and $\zeta(\beta)$ are constants depending on the second and third partial derivatives of $E(\Pi,y,m)$ at the steady state.

The use of the above theorem allows us to reduce the nonlinear terms of the system to two simple components: a cubic term and a remainder containing relatively small fifth or higher powers of ρ. The dynamics is particularly simple if we assume $O(5) = 0$. In this case the map defined in (21) has an invariant circle given by:

$$\rho^* = [(M(\beta) - 1)/\eta(\beta)]^{1/2}$$

provided we have $(M(\beta) - 1)/\eta(\beta) > 0$. Thus, there are two possible cases:

(i) $\eta(\beta) > 0$. Then, if $M(\beta) > 1$ the invariant circle exists and it attracts all neighboring points (see the phase portrait in figure 3). On the other hand, when $M(\beta) < 1$ the invariant circle disappears and the steady state becomes stable. Recall that the case $M'(\beta) < 0$ (> 0) is associated with the case $E_y < 1$ (> 1). Therefore, the invariant circle will appear:

- for any β in a left neighborhood of β^* if $E_y < -1$;
- for any β in a right neighborhood of β^* if $E_y > -1$.

(ii) $\eta(\beta) < 0$. Then, for $M(\beta) < 1$, the origin is attracting and the invariant circle is repelling. As $M(\beta)$ passes through 1 (β passes through β^*), the invariant circle disappears and the origin becomes repelling.

[figure 3]

Most of the conclusions derived from the analysis of system (19), (20) under the assumption $O(5) = 0$ are preserved when the fifth and higher order terms are included. More specifically, we can state the following theorem:

Theorem 5 (Devaney (1986)).

Under the same assumption of theorem 4:

(i) if $M'(\beta^*) > 0$ (< 0) and $\eta(\beta^*) > 0$ (< 0) there exists an invariant attracting (repelling) circle for β in a right neighborhood of β^*;

(ii) if $M'(\beta^*) > 0$ (< 0) and $\eta(\beta^*) < 0$ (> 0) there exists an invariant repelling (attracting) circle for β in a left neighborhood of β^*.

In other words, the invariant attracting circle may exists for β in a right neighborhood of β^* if $E_y > -1$ and for β in a left neighborhood of β^* if $E_y < -1$. Clearly the most interesting case is when the invariant circle is attracting, since it establishes that, even if the stationary state is completely unstable, we still have equilibrium perfect foresight trajectories contained in a bounded set for any t. Notice also that the existence of an attractive invariant circle is not necessarily connected with the indeterminacy of the equilibria. Such indeterminacy only occurs when $|\lambda_3(\beta)| <$ 1, i.e., $-2 < E_y < 0$. In any other case, the dimension of the manifold h where the attracting orbit exists is equal to the number of predetermined variables (y, m). Thus, the stability criterion provides a reason to restrict the dynamics of the model to the manifold h, i.e., to study a two-dimensional difference equation with two predetermined variables as given by (18).

Consider now the simple case in which the cause of the nonlinearity of the system only comes from the liquidity preference. In particular, I will assume, as in the previous section, that L_2, D_1, D_2 are constant and $L_{11} \neq 0$. Moreover, for simplicity any other partial derivative will be assumed to be zero. Now let $E_{mm} = \sigma$ where the sign of σ is equal to sign of L_{11} since E is increasing in m. Then, it is proved in Appendix B that:

$$\eta(\beta^*) = -[(\sigma/2E_\pi)/(1 + E_y - E_y^2)]^2[B_1(E_y)/B_2(E_y)E_y]$$

where $B_1 = (E_y^4 + 6E_y^3 + 12E_y^2 + 11E_y + 6)$ and $B_2 = (E^2 + 3E_y + 3)$. It is immediately verified that $\eta(\beta^*) < 0$ if $E_y > 0$. Moreover:

$$\eta(\beta^*) > 0 \text{ if and only if } B_1(E_y) > 0.$$

The latter inequality is satisfied for all $E_y < 0$ except for a closed interval [a,b] with $a \sim -3$ and $b \sim -2$. Notice also that the sign of $\eta(\beta^*)$ does not depend on the sign of σ. In other words, endogenous cycles would be impossible if the demand for money equation were linear ($\sigma = 0$) but both the existence and the stability of the invariant circles do not depend on the sign of E_m, nor on the sign of σ.

To see how the coefficient η is affected by the average length of the contracts, it is convenient to chose the constant coefficient E_π as the parameter of bifurcation. Since the change of stability of the dynamical system occurs when $\beta E_\pi = 1$, the bifurcation point in this case is readily found to be $E_\pi^* = 1/\beta$. Then, expressing η as a function of the new bifurcation parameter, $\eta(E_\pi^*)$ is increasing in β when the latter is positive. Now, since $\rho^* = [(M(\beta) - 1)/\eta(E_\pi)]^{1/2}$ is an approximate measure of the average ray of the attracting closed curve and $\eta(E_\pi^*) \sim \eta(E_\pi)$, we can conclude that $[(M(\beta) - 1)/\eta(E_\pi^*)]^{1/2} \sim \rho^*$ is an approximate measure of the output-inflation variability along the business cycle. Now, by increasing the average length of the contracts, such a variability is likely to be decreased if $E_y < -1$ (i.e., $M'(\beta^*) < 0$), whereas it is unclear what happens when $E_y > -1$.

The economic interpretation of the cycles in model II is entirely analogous to the interpretation already given in the last section. These cycles are due to the coexistence of two elements in the model: the overlapping structure of the contracts, i.e., the negative relation between output and the rate of change of inflation, and the stimulating effect of inflation on aggregate demand. However, the type of trajectories displayed by the nonstationary equilibria of the present model is completely different from the ones generated by model I. Here we may have quasiperiodic orbits lying on a closed curve around the steady state. This is an interesting result from an economic point of view because it shows that (i) output, inflation and prices can be persistently (i.e., for several periods) above and below their trend values and (ii) the relation between output and inflation depends on which stage of the cycle the economy is going through.

The last statement deserves some more comments. First of all, we can easily determine the direction of the orbits on the clesed curve. In fact, consider equation (6). This relation establishes that when output is above its trend value inflation is falling and vice versa. Then, the direction of the trajectories on the closed curve is clearly clockwise. In fact, assume that output is at its lowest level. Then, inflation is equal to μ but, because of the dynamics implied by (6), its rate of change is positive. Therefore, π will continue to grow and to stimulate output until a peak is reached. From this point the relation between π and y becomes a decreasing relation until output reaches its peak level. An opposite behavior of output and inflation results from the analysis of the cycle when output is falling. Clearly, we cannot derive any monotonic relation between output and inflation like the Phillips curve. However, we can say that π reaches its minimum level only when the economy is experiencing a falling output, whereas the highest level of inflation can only be associated with a recovery from a period of high unemployment.

Figures 4, 5 and 6 show some simulations of the model where the nonlinearity is restricted to the demand for money. All simulations are performed under the assumption $E_m = E_\pi = 1$, $E_y = -0.9$ and an output-inflation cycle is shown for $\beta = 1.01$, $\sigma = -0.1$ in fig. 4 and $\beta = 1.3$, $\sigma = -0.1$ in fig. 5. Comparing figures 4 and 5 one immediately notice that a fall in the average length of the contracts implies a greater variability of the output-inflation pair (greater amplitude of the closed curve). Finally, figure 6 shows the behavior of the real stock of money along the cycle plotted against output. It is worth noticing that m stays permanently above its stationary value.

[figures 4, 5, 6]

CONCLUSION

I have shown that an economy with staggered wage setting may exhibit endogenous fluctuations of the relevant variables: output, inflation and real money balances. These fluctuations seem to be associated with a sufficiently high effect of price changes on aggregate demand. This effect, as commonly stated in textbook macroeconomic models, comes from the negative influence of the real interest rate on private expenditure.

The characteristics of the endogenous cycles are particularly interesting in a model (such as Model II) where output sluggishly adjust in response to excess demand variability. In these cycles output and inflation lie on a closed curve around the stationary state and move clockwise. The behavior of the real money balances along the cycle depends on the shape of the two-dimensional manifold containing the closed curve. With the help of a simulation I have shown that real balances may fluctuate in a region bounded away from the stationary state.

Using some techniques from the theory of bifurcation it has been possible to show that the amplitude of the cycle depends on the average contract length and the Jacobian and the Hessian of the dynamical system. This amplitude can be approximately calculated and used to evaluate the impact of a change in the average length of the contracts on the output-inflation variability. Whereas a general statement about the direction of this impact seems to depend on the other parameters of the model, it has been shown that there are robust examples in which a reduction of the average length of the contracts increases the amplitude of the cycle.

One of the main features of the model presented in this paper is that permanent fluctuations are completely consistent with perfect foresight. This allows us to study the effects of changes in the policies announced by the government in a setting in which people are able to immediately learn and react to these changes. A thourogh investigation of the best policy regimes needed to stabilize the economy is outside the goal of the present paper. It is worth emphasizing, however, that any pre-fixed level of money growth or public expenditure are completely ineffective in stabilizing the economy.

APPENDIX A.

The first step is to bring the system in a form which is best suitable for the center manifold reduction. By taking a Taylor series expansion of system (6), (9), (14), we can write it as follows:

$$X_{t+1} = J(\beta)X_t + f(X_t,\beta)\underline{e} \qquad (A1)$$

where $J(\beta)$ is the Jacobian of the system evaluated at $(\mu,0,m^*)$ and:

$$X = [\Pi-\mu, y, m-m^*]'$$

$$\underline{e} = [0,1,0]'$$

$$f(X',\beta) = (1/2)X'D^2E(\mu,0,m^*)X + O(3).$$

Now consider the following:

<u>Theorem 6 (Hirsch and Smale (1974)).</u>

Let T be a linear operator on R^3 with eigenvalues:

$$\lambda(\beta) = a(\beta) + ib(\beta),\ \bar{\lambda}(\beta),\ \lambda_3(\beta)$$

where $a(\beta)$, $b(\beta)$, $\lambda_3(\beta)$ are real numbers. Then, there is a matrix representation $B(\beta)$ for T such that:

$$B(\beta) = \text{diag}\{A(\beta),\lambda_3(\beta)\}$$

$$A(\beta) = \begin{bmatrix} a(\beta) & -b(\beta) \\ b(\beta) & a(\beta) \end{bmatrix}$$

Under the above theorem, given a matrix representation J for T in the standard basis of $\mathbf{R}^3$, if $J(\beta)$ has two complex eigenvalues there exists an invertible matrix $Q(\beta)$ such that $Q^{-1}(\beta)J(\beta)Q(\beta) = B(\beta)$. Moreover, $Q(\beta)$ is readily found to be$Q(\beta) = [v(\beta),u(\beta),q(\beta)]$ where $[u(\beta) \pm iv(\beta)] \in C^3$ are the complex conjugate eigenvectors of $J(\beta)$ associated to the eigenvalues $\lambda(\beta)$ and $\bar{\lambda}(\beta)$ respectively, and $q \in \mathbf{R}^3$ is the real eigenvector of $J(\beta)$ associated to $\lambda_3(\beta)$.

Now let $Q(\beta)^{-1}X = [Y',j]'$ where $Y \in \mathbf{R}^2$, $j \in \mathbf{R}$. Then system (A1) can be written as:

$$Y_{t+1} = A(\beta^*)Y_t + H(Y_t,j_t,\beta_t) \quad \text{(A2a)}$$

$$j_{t+1} = \lambda_3(\beta^*)j_t + g(Y_t,j_t,\beta_t) \quad \text{(A2b)}$$

$$\beta_{t+1} = \beta_t \quad \text{(A2c)}$$

where:

$$H(Y,j,\beta) = (A(\beta)-A(\beta^*))Y + f[Q(\beta)(Y',j)']\tilde{p}(\beta) + O(3)$$

$$g(Y,j,\beta) = (\lambda_3(\beta)-\lambda_3(\beta^*))j + f[Q(\beta)(Y',j)']p_{32}(\beta) + O(3)$$

$$\tilde{p}(\beta) = (p_{12}(\beta),p_{22}(\beta))'$$

$$p_{ij}(\beta) = \text{generic element of } Q^{-1}(\beta)$$

Defining Φ: $\mathbf{R}^4 \dashrightarrow \mathbf{R}^4$ as the map in (A2), $D\Phi(0,0,0,\beta^*)$ has three eigenvalues with modulus 1. Moreover, H and g vanish together with each of their derivatives at the fixed point. Now, assuming $\lambda_3(\beta^*) \neq 1$, i.e., $E_y \neq 0$, the same method used in section 3 can be applied here to establish the existence, in a neighborhood of the steady state, of a C^2 center manifold h: $\mathbf{R}^3 \dashrightarrow \mathbf{R}$.

Define now the functions ϕ: $\mathbf{R}^3 \dashrightarrow \mathbf{R}$ and N: $\mathbf{R}^3 \dashrightarrow \mathbf{R}$ such that:

$\phi(0,0,\beta^*) = 0$, $D\phi(0,0,\beta^*) = 0$ and:

$$N(Y,\beta) = \phi\{[A(\beta^*)Y + H(Y,\phi(Y,\beta))],\beta\} - \lambda_3(\beta^*)\phi(Y,\beta) - g(Y,\phi(Y,\beta),\beta).$$

It is immediately verified that $N(0,\beta^*)$ and $DN(0,\beta^*)$ are identically zero. Imposing:

$$[Y',\beta-\beta^*]D^2N(0,\beta^*)[Y',\beta-\beta^*]' = 0$$

one finds a matrix $W = D^2\phi(0,\beta^*)$ for which the above equation is satisfied. Then, we can state that, for (Y',β) close enough to $(0,\beta^*)$, it is:

$$\phi(Y,\beta) = \xi(Y,\beta) + O(3), \quad h(Y',\beta) = \xi(Y,\beta) + O(3)$$

where $\xi(Y,\beta) = (1/2)[Y',\beta-\beta^*]W[Y',\beta-\beta^*]'$ and the dynamical system reduced on the center manifold is a nonlinear map:

$$Y_{t+1} = A(\beta)Y_t + f[Q(\beta)(Y,\xi(Y,\beta))']\tilde{p}(\beta) + O(3) \quad (A3)$$

APPENDIX B

Assume that the nonlinearity of the model only comes from the liquidity preference, in particular, $L_{11} \neq 0$ and any other partial derivative of equal or greater order is zero. Then, in (A1) $f(X',\beta) = \sigma(m - m^*)^2$. Letting (x,k,j,β) be the coordinates of the system in normal form (as in (A2)), after some lengthy calculations one can find that the equation of the center manifold of (A2) is $h(x,k,\beta) = \sigma p_{32}(\beta)[d_{11}x^2 + d_{21}xk + d_{31}k^2] + O(3)$, where p_{ij} is the generic element of $Q(\beta^*)^{-1}$ and:

$$d_{11} = -[E_y^2 + (9/4)E_y + (3/2)]/D$$

$$d_{21} = (3)^{1/2}E_y/4D$$

$$d_{31} = -(3/2)((1/2)E_y + 1)/D$$

$$D = E_y(E_y^2 + 3E_y + 3)$$

The system reduced on the center manifold is:

$$x_{t+1} = a(\beta)x_t - b(\beta)k_t + p_{12}(\beta)\sigma\psi(x_t,k_t) + O(4) \quad (B1a)$$

$$k_{t+1} = b(\beta)x_t + a(\beta)k_t + p_{22}(\beta)\sigma\psi(x_t,k_t) + O(4) \quad (B1b)$$

where $\psi(x,k) = [x + \sigma p_{32}(d_{11}x^2 + d_{21}xk + d_{31}k^2)]^2$. For the generic case in which $\lambda(\beta)$ is not a k^{th} root of unity for k = 1-5, (B1) can be transformed into a system like (19), (20).

In order to compute the value of the coefficient $\eta(\beta)$, I will apply the following procedure. Let z = x + ik and define (B1) in complex coordinates. Then, the system will have the following form:

$$z_{t+1} = \lambda(\beta)z_t + P_1(z_t,\bar{z}_t) + P_2(z_t,\bar{z}_t) + O(4)$$

where:

$$P_1(z,\bar{z}) = G_{11}z^2 + G_{12}z\bar{z} + G_{22}\bar{z}^2$$

$$P_2(z,\bar{z}) = G_{111}z^3 + G_{112}z^2\bar{z} + G_{122}z\bar{z}^2 + G_{222}\bar{z}^2.$$

Now we can state the theorem:

<u>Theorem 7 (Wan (1978))</u>.

Under the assumptions of theorem 5, $\eta(\beta^*)$ is given by:

$$\mathrm{Re}\{(1-2\lambda)\bar{\lambda}G_{11}G_{12}(1-\lambda)^{-1}\} + (1/2)|G_{12}| + |G_{22}| - \mathrm{Re}\{\bar{\lambda}G_{112}/2\}$$

where every parameter is evaluated at the bifurcation point β^*.

In the present case we have:

$$\eta(\beta^*) = -[(\sigma/2E_\pi)/(1 + E_y - E_y^2)]^2[B_1(E_y)/B_2(E_y)E_y]$$

where $B_1 = (E_y^4 + 6E_{yy}^3 + 12E_y^2 + 11E_y + 6)$ and $B_2 = (E^2 + 3E_y + 3)$.

FOOTNOTES

1. See the Symposium on Nonlinear Economic Dynamics (1986).

2. See De Long, Bradford and Summers (1984, 1986) and Chadha (1986).

3. The literature on this field is very much developed. See Okun (1981) for original contributions and basic references.

4. Proving the optimality of the specific staggered contracts model, however, is a more difficult question to assess. The proponents of this approach have referred to the literature on implicit contracts (Azariadis and Bailey (1984)) and to models of optimizing firms facing a cost of changing prices (Sheshinski and Weiss (1977). This literature has not been proved yet to provide a complete justification for adopting the assumption of an exogenously fixed contract length and more investigation is needed on this ground.

5. Calvo (1983) uses a continuoum time version of the model that I am going to employ. A discrete time version has been proposed by Chadha (1986).

6. The existence of an inverse relation between the expected rate of change of inflation and aggregate output in the staggered contract model is emphasized by Calvo (1983).

7. Even a log-linear form may not be sufficient to capture the essence of the liquidity trap since this functional form implies the existence of portfolio equilibrium with positive real money balances even for $r = 0$.

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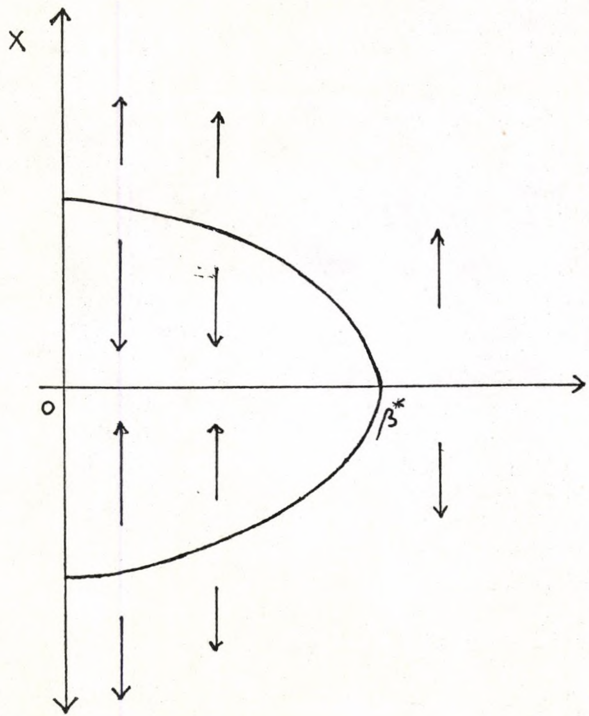

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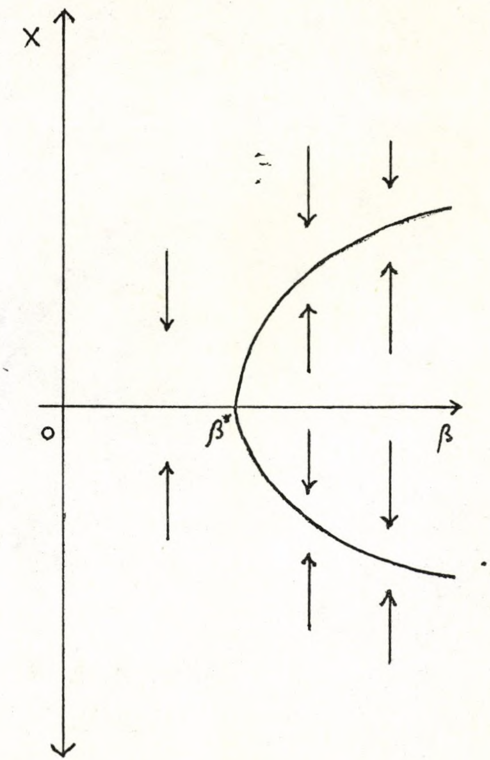

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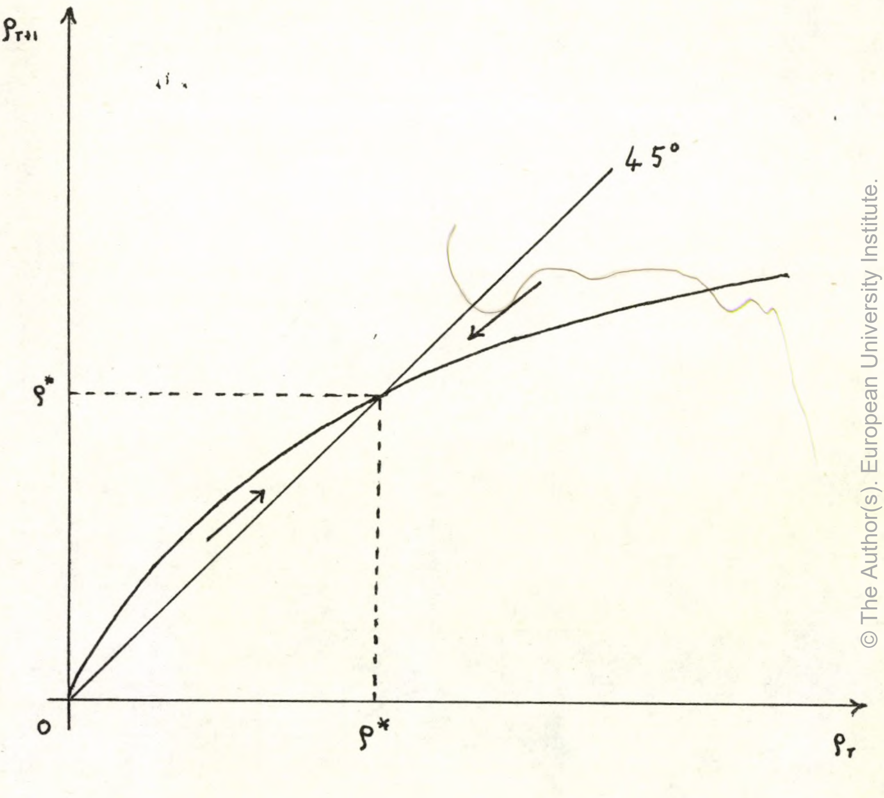

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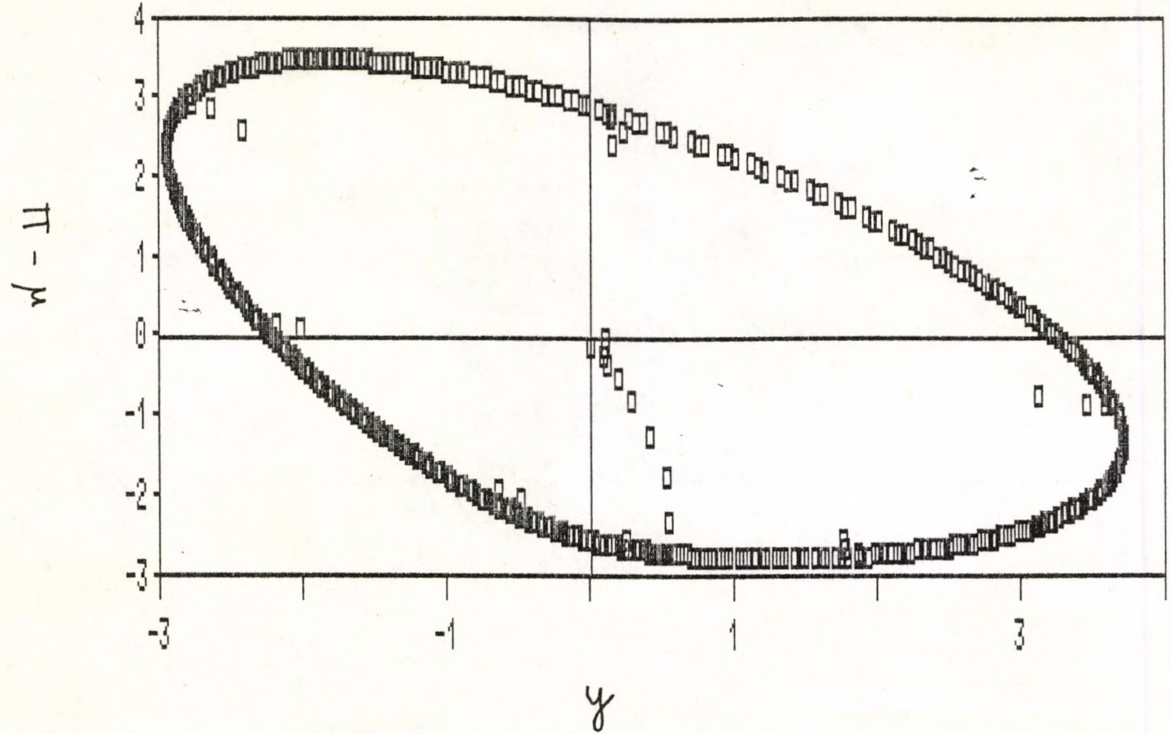

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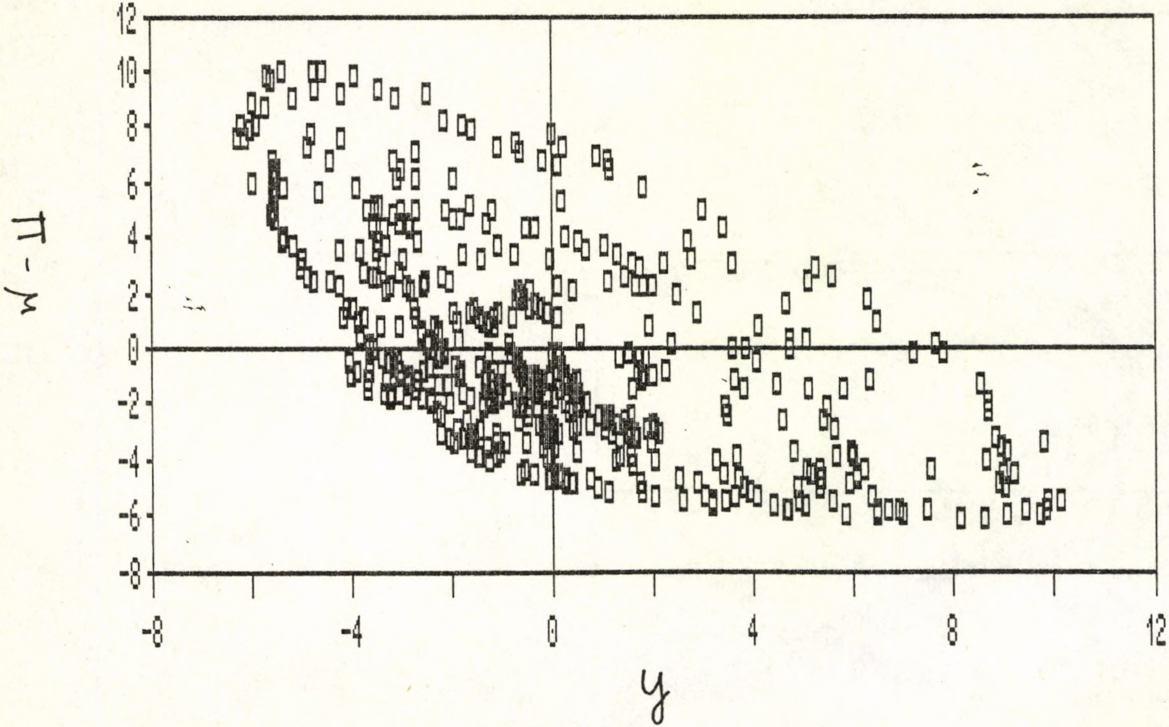

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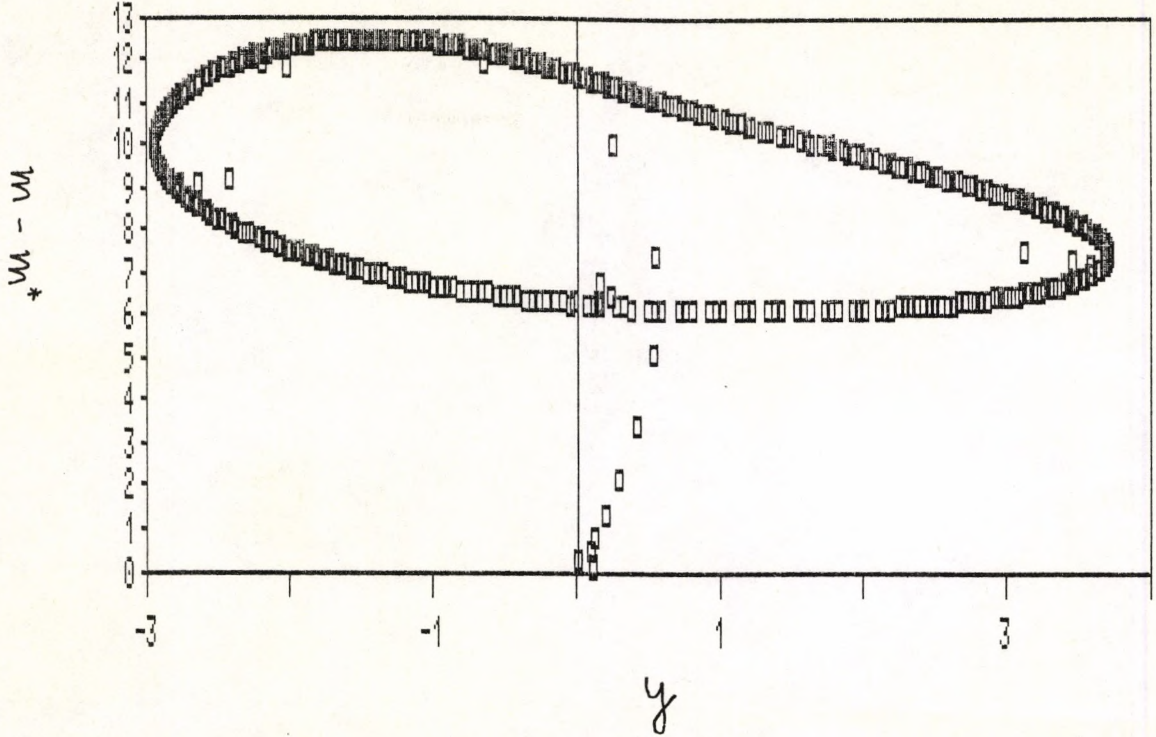

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