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ON RATIONALIZING EXPECTATIONS

by

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"When we first begin to believe anything, what we believe is not a single proposition, it is a whole system of propositions . . .
Bit by bit there forms a system of what is believed, and in that system some things stand unshakeably fast and some more or less liable to shift. What stands fast does so, not because it is intrinsically obvious or convincing; it is rather held fast by what lies around it."

Ludwig Wittgenstein in On Certainty (§141, 144, New York, 1969).

Summary

The problem of maintaining the consistency of state estimates with a decision maker's expectations is considered. The discrepancy between the estimate and its expectation defines a jump in the state estimate. The jump makes the estimate consistent with its expectation. A method is described for generating corresponding covariance matrices for the Kalman filter in order to make the filter consistent with such a jump. It is shown that the method is able to alter the estimates in the precise direction indicated by the expectations. The corresponding alteration to the Kalman gain is also discussed. Convergence becomes evident if expectations become, through learning, increasingly accurate estimates of the corresponding state vector.

1. Introduction: The Problem*

It is important, for optimal decision making, that the economic consequences of policy measures be predicted accurately and detected quickly. Conventional approaches to policy, prediction and detection, have been subject to a sustained series of theoretical and empirical objections by the New Classical Macroeconomists.(1) In particular, the crucial issue of the policy invariance of parameters defining the structure of an econometric model (used in the formulation and evaluation of policy) has led to a resurgence of interest in recursive schemes of estimation. Indeed, it is a sign of the times, perhaps, that a classic in applied mathematics: "Prediction and Regulation by Linear Least-Square Methods" (Whittle (1963), (1983)), is now republished with the encouragement of a leading exponent of the New Classical Paradigm, viz: T.J. Sargent.

Lucas and Sargent, in their influential essay on "After Keynesian Macroeconomics", point out, for example:

> ". . . it has been only a matter of analytical convenience and not of necessity that equilibrium models have used the assumption of stochastically stationary shocks and the assumption that agents have already learned the probability distributions they face. Both of these assumptions can be abandoned. . . . In fact, within the framework of quadratic objective functions, in which the 'separation principle' applies, one can apply *the Kalman filtering formula* to derive optimum linear decision rules with time-dependent coefficients. In this framework, the Kalman filter permits a neat application of Bayesian learning to updating optimal forecasting rules from period to period *as new information becomes available*."
>
> (Lucas and Sargent (1979), p. 315 in Lucas and Sargent (1981). Italics added.)

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*Various versions of this paper have been presented at seminars in the Universities of Lund and Bologna and at the European University in Florence. Critical--but constructive--comments by participants at these seminars, as well as useful comments by Professors Wolfgang Gebauer and Michael Kuczynski, have helped in the clarification of many obscure points. The usual disclaimer applies.

We are left with the impression that the magic wand of a Kalman filter restores learning processes in an interesting way in linear rational expectations models. It is this over-simplification with which we are exclusively concerned in our development.

The updating that Lucas and Sargent refer to is via the innovations term in the recursive scheme in Kalman filtering and is of an almost mechanical Bayesian variety. Initial conditions completely determine the evolution of the conditional densities which are used for prediction and control. 'As new information becomes available', it is not only the mechanical updating of an estimate on the basis of the description between expected and realized values, weighted by the Kalman gain, that is crucial: the Kalman gain itself must be re-examined in terms of the new information that becomes available. This latter point has been totally neglected in the applied New Classical literature (and, indeed, in all applications of Kalman filtering techniques in economics)--somewhat understandably, in the initial flush of enthusiasm surrounding the exciting possibilities made feasible by recursive techniques. There is, obviously, a deeper reason for this neglect--at least by the New Classical Economists. That, of course, is partly due to the modelling philosophy adopted by the New Classicals: there is only one true model of the economy and a fortiori all agents view the world through the commonality of this one true model. There seems, therefore, no scope for the individual agents to have a set of beliefs of their own, and thus to have the possibility of constructing (at least in their minds--and hence for intelligent behaviour) models of the economy in which they are the actors. All learning is pseudo-learning. Beliefs, conditioning the learning processes, are constrained by the postulate of the existence of this one true model of the economy.

The prototype for this setting is, of course, Lucas' (1975) essay. In that pioneering work, Lucas uses, as state variables, money, capital and an economy-wide average "belief". Townsend (1983) tries to relax the assumption of an economy-wide average "belief" within the framework of a partial equilibrium model of investment by trying to formalize, using recursive techniques, the problem of 'forecasting the forecast of others'. Thus, ostensibly, agents are allowed to operate with disparate "beliefs". In a more recent work[(2)] concerned with 'modelling of the movements of average opinion', Townsend attempts to integrate the problem of expectations of others' expectations with the standard approach of monistic beliefs in the New Classical literature. However, in both cases his attempts fail to lead to an interesting breakthrough precisely because the Kalman gain remains invariant. Thus, even as Phelps and Frydman point out:

> "The crucial assumption made by Townsend is that although the vector of prior means differs across agents, the covariance matrix . . . of beliefs is the same."
>
> (Phelps and Frydman, 1983, p. 21.)

Clearly, the libertarian philosophy underlying New Classical Economics would benefit from attempts to remove this restriction on agents' beliefs. We outline a method to relax this restriction in the next section.

In the subsequent discussion we interpret the state of the economy in a general manner. The state may be taken to be as a vector of ordinary economic variables or a vector of model parameters, considered as time varying variables by the decision maker (economic agent). We do not consider combinations of variables and parameters that lead to non-linearities and thence to the inapplicability of certainty equivalence (but not necessarily the separation principle) in optimal decisions. The application of the Kalman filter to linear econometric models with time varying parameters are discussed by Athans (1976) and Rustem and Velupillai (1978).

Before concluding this introductory section a minor doctrine-historical digression may not be out of place.

The connection between recent developments in Macroeconomic methodology--particularly in its New Classical variants--and the impressive contributions by Wicksell's followers in the Stockholm of the '30s is an unwritten (as yet) chapter of intellectual history. Of the latter it was perhaps Erik Lindahl who was most uncompromisingly theoretical and, as such, most instrumental in developing new concepts and original frameworks.(3) Long before Hayek, and almost simultaneously with Keynes and Knight, Lindahl had been grappling with the problem of incorporating intelligent expectation formation and its revision by individual economic agents in a monetary macroeconomic setting (cf. Lindahl (1924)). The crucial role played by expectation and its revision in the formulation of monetary policy was almost the unifying thread in all his macroeconomic writings.(4) It was, therefore, not surprising that when he came to write, almost 30 years later, his masterly evaluation of 'Keynes' Economic System' (cf. Lindahl (1954)), the role of expectations and its revision in <u>equilibrium</u> formulation was stressed. Although it was a particular technical deficiency in certain New Classical models that suggested the questions we have tried to answer in this paper, it was, in fact, a re-reading of Lindahl (1954) that gave us the hint towards a reasonable solution. It may, therefore, be of some use to quote at least a few of the relevant sections from Lindahl (op. cit.):

> "It also seems reasonable to postulate an interdependence between the variables entering an economic system in the case concerning the determination of the conditions for CORRECTLY ANTICIPATED processes. <u>These conditions are that the individuals have such expectations of the future that they act in ways which are necessary for their expectations to be fulfilled</u>. It follows that the interdependence between present and future magnitudes is conditioned in this case by the fact that the latter, via correct anticipations, influence the former. If we also choose to describe such developments as equilibrium processes, this implies that we widen the concept of equilibrium to include also economic systems describing changes over time where the changes which take place from period to period do not cause any interruption in, but, on the contrary, are an expression of, the continuing adjustment of the variables to each other."

(Lindahl (op. cit.), p. 27, second set of italics added.)

He then goes on to point out that:

> "In the construction of these equilibrium processes it is not necessary to presume perfect foresight in the sense that all economic subjects anticipate the actual development with full certainty. It is sufficient to presume that the economic subjects which have a determining influence in their respective markets have anticipated the actual development as the MOST PROBABLE of various possibilities, in which case both their planning and their actions to a certain extent are influenced by the presence of uncertainty (the risk factor). Further, *we need not presume that the anticipations should be correct for a long time ahead*. But the earlier course of development must have been such as to have generated fairly correct anticipations for the relevant period. *The development must, consequently, be so stable that, disregarding exogenous factors, it does not cause any great surprise in the near future*."
>
> (Lindahl (op. cit.), p. 28, second set of italics added. But cf. also f.n. 11, p. 28 in Loc. cit.)

The method discussed in the next section observes the discrepancy between the decision maker's expectations and the corresponding state estimate. This discrepancy defines a jump in the state estimate in order to make the filter consistent with such a jump. It is shown that the method is able to alter the estimates in the exact direction indicated by the expectations.

2. A Method for Updating the Covariance Matrices of the Kalman Filter

In this section a method is discussed for updating the covariance matrices of the Kalman Filter. The method is designed to translate the decision maker's (or private agent's) expectations of future states of the economic system to an update of the covariance matrices. It is shown below that these updates move the optimal state estimates of the Kalman filter in the direction indicated by the policy maker's expectations. The method is of an iterative nature in that the decision maker is allowed to revise his expectations until a state estimate consistent with his requirements is obtained.

Let the system be described by the linear vector difference equation

$$x_{k+1} = F_k x_k + G w_{k+1}, \quad k = 0, 1, \ldots \tag{1}$$

where x_k is the n-vector state at time k, F_k is an n x n nonsingular matrix, G is n x r and $\{w_k, k = 1, \ldots\}$ is an r-vector white Gaussian sequence, $w_k \sim N(0, Q_k)$. Discrete, linear observations of the system are taken at time instants, k, using

$$y_k = H_k x_k + v_k \tag{2}$$

where y_k is the m-dimensional observation vector, H_k is an m x n matrix and $\{v_k, k = 1, 2, \ldots\}$ is an m-vector, white Gaussian sequence, $v_k \sim N(0, R_k)$, $R_k > 0$. x_0, $\{w_k\}$, $\{v_k\}$ are assumed to be independent.

The well-known optimal filter for the discrete system (1), (2) consists of equations of evolution for the state estimate $\hat{x}$ and its covariance matrix P (see e.g. Jazwinski, 1970). We define

$$\hat{x}_{k|k} \triangleq E\{x_k | Y_k\} \tag{3}$$

where $Y_k \triangleq \{y_1, \ldots, y_k\}$. In view of (1), we have

$$\hat{x}_{k+1|k} = E\{x_{k+1} | Y_k\}$$

$$= F_k \hat{x}_{k|k} \tag{4}$$

The covariance matrix at observations, corresponding to (3), is defined as

$$P_{k|k} \triangleq E\{(x_k - \hat{x}_{k|k})(x_k - \hat{x}_{k|k})^T | Y_k\}$$
$$= E\{(x_k - \hat{x}_{k|k})(x_k - \hat{x}_{k|k})^T\} \tag{5}$$

with P_o as an initial condition of the filter. The covariance matrix, between observations, corresponding to (4), can be written in view of (1) as

$$P_{k+1|k} = E\{(x_{k+1} - \hat{x}_{k+1|k})(x_{k+1} - \hat{x}_{k+1|k})^T | Y_k\}$$
$$= F_k P_{k|k} F_k^T + G_k Q_k G_k^T \tag{6}$$

The Kalman filter equations are (4), (6) and

$$\hat{x}_{k|k} = \hat{x}_{k|k-1} + K_k(y_k - H_k \hat{x}_{k|k-1}) \tag{7}$$

$$P_{k|k} = P_{k|k-1} - K_k H_k P_{k|k-1} \tag{8}$$

where

$$K_k = P_{k|k-1} H_k^T (H_k P_{k|k-1} H_k^T + R_k)^{-1} \tag{9}$$

is the Kalman gain.

Suppose the decision maker computes $\hat{x}_{k|k}$ and does not find this estimate consistent with his expectations based upon his own perceptions of the performance of the economic system. The following scheme provides a way in which $\hat{x}_{k|k}$, and subsequently the Kalman filter can be made consistent with these expectations. This leads to an "ultimate" Bayesian framework in which the decision maker can influence the estimate and its covariance matrix, generated by the Kalman filter.

The method confronts the decision maker with the state estimate $\hat{x}_{k|k}$. If this is inconsistent with the decision maker's expectations, then he is asked to specify the estimate that would be consistent. Let this be $\hat{x}^e_{k|k}$ where the superscript e signifies that this is the value consistent with his <u>expectations</u> of the state estimate. We define the correction vector

$$\delta \triangleq \hat{x}^e_{k|k} - \hat{x}_{k|k} \tag{10}$$

which represents a jump in the state estimate $\hat{x}_{k|k}$ to make it consistent with the decision maker's expectations. Let $P_{k|k}$ be nonsingular and $\bar{\delta}$ be the solution of

$$P_{k|k} \bar{\delta} = \delta \tag{11}$$

Using δ, we compute a rank-one update of $P_{k|k-1}$

$$\bar{P}_{k|k-1} = P_{k|k-1} + \alpha P_{k|k-1} \bar{\delta} \bar{\delta}^T P_{k|k-1} \tag{12}$$

and replace $P_{k|k-1}$ in the Kalman filter by $\bar{P}_{k|k-1}$, where α is a scalar, chosen to reflect the emphasis of the update in (12). The effect of α is also illustrated in the following result:

<u>Proposition</u> (13)

Let $\bar{K}_k = \bar{P}_{k|k-1} H_k^T (H_k \bar{P}_{k|k-1} H_k^T + R_k)^{-1}$ be the Kalman gain corresponding to (12). Then

$$\bar{K}_k = (I + \beta(\alpha,\bar{\delta}) P_{k|k} \bar{\delta} \bar{\delta}^T) K_k \tag{14}$$

where

$$\beta(\alpha, \bar{\delta}) \triangleq \alpha/(1 + \alpha \bar{\delta}^T K_k H_k P_{k|k-1} \bar{\delta}) \tag{15}$$

Let $\bar{P}_{k|k} = \bar{P}_{k|k-1} - \bar{K}_k H_k \bar{P}_{k|k-1}$ be the covariance matrix (5) corresponding to (12), then

$$\bar{P}_{k|k} = P_{k|k} + \beta(\alpha, \bar{\delta}) P_{k|k} \bar{\delta} \bar{\delta}^T P_{k|k} \tag{16}$$

Let $\bar{\hat{x}}_{k|k}$ be the estimate corresponding to (12), then

$$\bar{\hat{x}}_{k|k} = \hat{x}_{k|k} + \mu(\alpha, \bar{\delta}) \delta \tag{17}$$

where

$$\mu(\alpha, \bar{\delta}) \triangleq \beta(\alpha, \bar{\delta}) (\bar{\delta}^T K_k (y_k - H_k \hat{x}_{k|k-1})) \tag{18}$$

and $\mu(\alpha, \bar{\delta}) \geq 0$ if

$$\bar{\delta}^T K_k (y_k - H_k \hat{x}_{k|k-1}) \geq 0 \tag{19}$$

and $\alpha \geq 0$. If (19) is not satisfied, then $\mu(\alpha, \bar{\delta}) \geq 0$ for $0 > \alpha > -1/(\bar{\delta}^T K_k H_k P_{k|k-1} \bar{\delta})$.

Proof

To evaluate $\bar{K}_k$, we use $\bar{P}_{k|k-1}$ given by (12) and evaluate the resulting expression using the Sherman-Morrison formula (Householder, 1964). We have

$$\bar{K}_k = (P_{k|k-1} + \alpha P_{k|k-1} \bar{\delta} \bar{\delta}^T P_{k|k-1}) H_k^T (H_k (P_{k|k-1} + \alpha P_{k|k-1} \bar{\delta} \bar{\delta}^T P_{k|k-1}) H_k^T + R_k)^{-1}$$

$$= K_k + \beta(\alpha, \bar{\delta}) P_{k|k-1} \bar{\delta} \bar{\delta}^T P_{k|k-1} H_k^T (H_k P_{k|k-1} H_k^T + R_k)^{-1}$$

$$-\beta(\alpha, \bar{\delta}) P_{k|k-1} H_k^T (H_k P_{k|k-1} H_k^T + R_k)^{-1} H_k P_{k|k-1} \bar{\delta} \bar{\delta}^T P_{k|k-1} H_k^T (H_k P_{k|k-1} H_k^T + R_k)^{-1}$$

$$= K_k + \beta(\alpha, \bar{\delta}) (P_{k|k-1} - P_{k|k-1} H_k^T K_k^T) \bar{\delta} \bar{\delta}^T K_k$$

which, in view of (8), can be written as (14). The covariance matrix $\bar{P}_{k|k}$ can be evaluated using

$$\bar{P}_{k|k} = (P_{k|k-1} + \alpha P_{k|k-1} \bar{\delta} \bar{\delta}^T P_{k|k-1}) - \bar{K}_k H_k (P_{k|k-1} + \alpha P_{k|k-1} \bar{\delta} \bar{\delta}^T P_{k|k-1})$$

$$= (P_{k|k-1} - K_k H_k P_{k|k-1}) + \alpha (P_{k|k-1} - K_k H_k P_{k|k-1}) \bar{\delta} \bar{\delta}^T P_{k|k-1}$$

$$-\beta(\alpha, \bar{\delta}) (P_{k|k-1} - K_k H_k P_{k|k-1}) \bar{\delta} \bar{\delta}^T (K_k H_k P_{k|k-1} + \alpha (\bar{\delta}^T K_k H_k P_{k|k-1} \bar{\delta}) P_{k|k-1})$$

Using (8), this can be written as

$$\bar{P}_{k|k} = P_{k|k} + \alpha P_{k|k} \bar{\delta} \bar{\delta}^T P_{k|k-1}$$

$$-\beta(\alpha, \bar{\delta}) P_{k|k} \bar{\delta} \bar{\delta}^T (-P_{k|k} + (1 + \alpha (\bar{\delta}^T K_k H_k P_{k|k-1} \bar{\delta}) P_{k|k-1}))$$

which, in view of (15), yields (16). The state estimate $\bar{\hat{x}}_{k|k}$ is obtained from

$$\bar{\hat{x}}_{k|k} = \hat{x}_{k|k-1} + \bar{K}_k (y_k - H_k \hat{x}_{k|k-1})$$

and using (14) we obtain

$$\bar{\hat{x}}_{k|k} = \hat{x}_{k|k-1} + \mu(\alpha, \bar{\delta}) P_{k|k} \bar{\delta}$$

which, using (11) yields the result (17). $\mu(\alpha, \bar{\delta}) \geq 0$ follows from (19) and the positive (semi)-definiteness of $K_k H P_{k|k-1}$.

<u>Remark</u> (20)

$\bar{P}_{k|k-1}$ and $\bar{P}_{k|k}$ are positive (semi-)definite if $P_{k|k-1}$ and $P_{k|k}$ are positive (semi-)definite. This can be confirmed by inspection of (12) and (16) respectively for $\alpha > 0$ or for sufficiently small $|\alpha|$, $\alpha < 0$.

<u>Remark</u> (21)

Expression (16) shows that $P_{k|k}$ need not be recalculated when $P_{k|k-1}$ is altered by (12). The rank-one modification in (16) yields the required covariance matrix.

<u>Remark</u> (22)

If $P_{k|k-1}$ is positive definite or, otherwise, if a $\bar{\delta}$ can be specified such that $\bar{\delta}^T P_{k|k-1} \bar{\delta} \neq 0$, then one choice for α is given by

$\alpha = 1/(\bar{\delta}^T P_{k|k-1}\bar{\delta})$ which, in view of (15) yields $\beta(\alpha, \bar{\delta}) = 1/(\bar{\delta}^T (I + K_k H_k) P_{k|k-1}\bar{\delta}))$. An example of negative α is

$$\alpha = -1/(\bar{\delta}^T P_{k|k-1} \bar{\delta}) \tag{23}$$

provided $\bar{P}_{k|k-1}$ remains positive semi-definite. In view of (8), (15) this yields

$$\beta(\alpha, \bar{\delta}) = -1/(\bar{\delta}^T P_{k|k}\bar{\delta}) \tag{24}$$

Note that, for this choice, $\bar{P}_{k|k-1}\bar{\delta} = \bar{P}_{k|k}\bar{\delta} = 0$. An alternative choice for α is

$$\alpha = 1/(\bar{\delta}^T K_k (y_k - H_k \hat{x}_{k|k}) - \bar{\delta}^T K_k H_k P_{k|k-1} \bar{\delta})$$

which ensures $\mu(\alpha, \bar{\delta}) = 1$. This can only be achieved with $\alpha > 0$ when

$$\bar{\delta}^T K_k (y_k - H_k \hat{x}_{k|k-1}) > \bar{\delta}^T K_k H P_{k|k-1} \bar{\delta}$$

It can be observed from (12) and (16) that $\bar{P}_{k|k-1}$ and $P_{k|k-1}$ can be automatically ensured to be positive (semi-)definite for $\alpha > 0$. Nevertheless, negative values of α may still be acceptable provided these covariance matrices remain positive (semi-)definite.

Remark (25)

The non-negativity condition (19) on $\bar{\delta}$ can clearly be relaxed. This condition implies that the chosen $\bar{\delta}$ should be at an acute angle with the innovations term $K_k(y_k - H_k \hat{x}_{k|k-1})$ in (7). This, in turn, implies that the decision maker's expectations of the state of the economy should not entirely contradict the direction, $\hat{x}_{k|k} - \hat{x}_{k|k-1}$ which the filter, based on the latest available information, has chosen in order to correct the previous estimate $\hat{x}_{k|k-1}$. This restriction is, nevertheless, only necessary to ensure the positive (semi-)definiteness of $\bar{P}_{k|k-1}$, $\bar{P}_{k|k}$ with $\alpha \geq 0$. Therefore, it can be relaxed, if necessary, to allow for general $\bar{\delta}$ provided α takes negative values such that $\mu(\alpha, \bar{\delta}) \geq 0$ whenever $\bar{\delta}^T K_k(y_k - H_k \hat{x}_{k|k-1}) \leq 0$. It should be noted that in this case, as in Proposition (13), $|\alpha|$ can be chosen small enough to maintain the positive (semi-)definiteness of the covariance matrices, for $\alpha \in (-1/(\bar{\delta}^T K_k H_k P_{k|k-1} \bar{\delta}), 0)$.

Remark (26)

The significance of (17) is that, provided $\mu(\alpha, \bar{\delta}) \geq 0$, the estimate $\bar{\hat{x}}_{k|k}$ is given by a correction of $\hat{x}_{k|k}$ in the precise direction δ, specified by the decision maker. This is clearly a desirable property in that it allows the estimate $\hat{x}_{k|k}$ to be corrected consistently with both the decision maker's expectations and the filter itself.

Remark (27)

The new Kalman gain, given by (14), is affected by $\bar{\delta}$, specified by the decision maker. It is also a function of $P_{k|k}$, as opposed to K_k given by (9) which is a function of $P_{k|k-1}$ but not $P_{k|k}$.

At any given time k, the updating process (12) can be repeated iteratively, if necessary, resulting in several evaluations of (17) in which $\bar{\hat{x}}_{k|k}$ is relabelled $\hat{x}_{k|k}$ after each evaluation. The practical importance of maintaining the positive (semi-)definiteness of $\bar{P}_{k|k}$, $P_{k|k-1}$ (relabelled $P_{k|k}$, $\bar{P}_{k|k-1}$ after each evaluation) becomes evident if such iterations are undertaken.

3. Forecasts Consistent with Expectations

Suppose, given the information at time k, Y_k, the forecasted state estimate $\hat{x}_{k+1|k}$, given by (4), is found to be inconsistent with the decision maker's expectations. Let $\hat{x}^e_{k+1|k}$ be the state expected by the decision maker and let

$$\delta' \triangleq \hat{x}^e_{k+1|k} - \hat{x}_{k+1|k} \qquad (27)$$

be the correction vector, representing a jump in the state estimate $\hat{x}_{k+1|k}$ to make it consistent with the decision maker's expectations. In order to make the filter consistent with these expectations, a corresponding jump in $\hat{x}_{k|k}$ has to be determined. Let this jump be δ as in (10), then by (4)

$$\hat{x}^e_{k+1|k} = \hat{x}_{k+1|k} + \delta' = F_k(\hat{x}_{k|k} + \delta)$$

and, as F_k is nonsingular, $\delta = F_k^{-1}\delta'$. Using (11) and (12), we obtain from (17)

$$\bar{\hat{x}}_{k|k} = \hat{x}_{k|k} + \mu(\alpha, \bar{\delta})\delta$$

$$\bar{\hat{x}}_{k+1|k} = F_k(\hat{x}_{k|k} + \mu(\alpha, \bar{\delta})\delta)$$

$$= \hat{x}_{k+1|k} + \mu(\alpha, \bar{\delta})\delta'$$

Thus, the old forecast state estimate $\hat{x}_{k+1|k}$ is corrected with a jump in the specified direction δ', to yield the new forecast $\bar{\hat{x}}_{k+1|k}$. Hence, the method in Section 2 can be applied to adjust state forecasts according to the decision maker's expectations.

4. The Rationalization of Expected Values of y_k with the Filter

Suppose, at time k, the decision maker's expected value of y_k, denoted by y_k^e, is different from the observed value. We set

$$\delta \triangleq y_k^e - y_k \tag{28}$$

as the corresponding jump in the observed values. In economics, observed data is often subsequently revised. This indicates that the decision maker might well be not entirely convinced with the observed y_k. Thus, the decision maker might crediby have some expected value, different from the one observed. A related reason for such an expected value may be that, at or just after the observation at k, events have occurred or are expected to occur, that will rapidly alter the observed value y_k towards the expected value y_k^e. The decision maker can thus no longer use the estimate $\hat{x}_{k|k}$ (7), but has to revise this estimate to $\bar{\hat{x}}_{k|k}$ in order to make it consistent with y_k^e.

To obtain the estimate $\bar{\hat{x}}_{k|k}$, consistent with y_k^e and with the filter, we use the following rank one formula to update R_k:

$$\bar{R}_k = R_k + \alpha\, \delta\delta^T \tag{29}$$

where α is a scalar.

Proposition (30)

Let $\bar{K}_k = P_{k|k-1}H_k^T(H_kP_{k|k-1}H_k^T + R_k)^{-1}$, then

$$\bar{K}_k = K_k(I - \bar{\beta}(\alpha, \delta)\ \delta\delta^T(H_kP_{k|k-1}H_k^T + R_k)^{-1}) \tag{31}$$

where

$$\bar{\beta}(\alpha, \delta) \triangleq \alpha/(1 + \alpha\delta^T(H_kP_{k|k-1}H_k^T + R_k)^{-1}\delta) \tag{32}$$

Let $\bar{P}_{k|k} = P_{k|k-1} - \bar{K}_kH_kP_{k|k-1}$, then

$$\bar{P}_{k|k} = P_{k|k} + \bar{\beta}(\alpha, \delta)\ K_k\delta\delta^TK_k^T \tag{33}$$

and for $\bar{\hat{x}}_{k|k} = \hat{x}_{k|k-1} + \bar{K}_k (y_k - H_k \hat{x}_{k|k-1})$

$$\bar{\hat{x}}_{k|k} = \hat{x}_{k|k} + \bar{\mu}(\alpha, \delta) K_k \delta \tag{34}$$

where

$$\bar{\mu}(\alpha, \delta) \triangleq \bar{\beta}(\alpha, \delta) \delta^T (H_k P_{k|k-1} H_k^T + R_k)^{-1} (H_k \hat{x}_{k|k-1} - y_k)$$

and

$$\bar{\mu}(\alpha, \delta) \geq 0 \tag{35}$$

if $\alpha \geq 0$ and

$$\delta^T (H_k P_{k|k-1} H_k^T + R_k)^{-1} (H_k \hat{x}_{k|k-1} - y_k) \geq 0 \tag{36}$$

If (36) is not satisfied, then (35) is still satisfied for $0 > \alpha > -1/\delta^T (H_k P_{k|k-1} H_k^T + R_k)^{-1} \delta$

<u>Proof</u>

We have

$$\begin{aligned}\bar{K}_k &= P_{k|k-1} H_k^T (H_k P_{k|k-1} H_k^T + R_k - \alpha\delta\delta^T)^{-1} \\ &= P_{k|k-1} H_k^T (H_k P_{k|k-1} H_k^T + R_k)^{-1} (I - \bar{\beta}(\alpha,\delta) \delta\delta^T (H_k P_{k|k-1} H_k^T + R_k)^{-1})\end{aligned}$$

which yields (31). $\bar{P}_{k|k}$ in (33) and $\bar{\hat{x}}_{k|k}$ in (34) can simply be derived using (31). Inequality (35) follows from the positive definiteness of $(H_k P_{k|k-1} H_k^T + R_k)$.

The matrix $\bar{P}_{k|k}$ has, as in Section 2, to be ensured to remain positive (semi-)definite. This can be done for positive α or for negative α with $|\alpha|$ sufficiently small.

Condition (35) on δ, as condition (19), can clearly be relaxed by using negative α with $|\alpha|$ sufficiently small.

Substituting (7) into (34) yields

$$\bar{\hat{x}}_{k|k} = \hat{x}_{k|k-1} + K_k (y_k - \bar{\mu}(\alpha, \delta) \delta - H_k \hat{x}_{k|k-1})$$

which illustrates the effect of (28) in that the innovations term now uses $y_k + \bar{\mu}(\alpha, \delta)\delta$ instead of y_k. Thus, the observation y_k is shifted by a jump proportional to that specified by the decision maker's expectations.

5. The Rationalization of Expected Future Observed Values with the Filter

If the forecast value of y_{k+1}, given y_k, is

$$\hat{y}_{k+1|k} = H_k \hat{x}_{k+1|k} = H_k F_k \hat{x}_{k|k}$$

and if this value differs from the value expected by the decision maker, denoted by y^e_{k+1}, it is possible to generate a state estimate $\hat{x}_{k+1|k}$ which is consistent with expectation y^e_{k+1}. The existence of such y^e_{k+1} assumes some anticipation of future events. This, in turn, invalidates well-known assumptions of nonanticipativity which imply that the decision maker (controller) is not clairvoyant (see e.g. Wonham, 1968). Let

$$\delta_y \triangleq y^e_{k+1} - \hat{y}_{k+1|k}$$

The corresponding jump in the state $\hat{x}_{k+1|k}$ is δ'. This jump is the solution of

$$\hat{y}_{k+1|k} + \delta_y = H_k (\hat{x}_{k+1|k} + \delta')$$

and is given by

$$\delta' = H^{\dagger}_k \delta_y \qquad (37)$$

with $H_k H^{\dagger}_k \delta_y = \delta_y$. We then use this value as δ' in Section 3 to yield

$$\bar{y}_{k+1|k} = H_k \bar{x}_{k+1|k} = H_k (\hat{x}_{k+1|k} + \mu(\alpha, \bar{\delta})\ \delta')$$

where, as in Section 3, $\delta = F^{-1}_k \delta'$ and $\mu(\alpha, \bar{\delta})$ and $\bar{\delta}$ are given by (17) and (11). From (37) we have

$$\bar{y}_{k+1|k} = \hat{y}_{k+1|k} + \mu(\alpha, \bar{\delta})\ \delta_y$$

Thus, the state forecast $\hat{x}_{k+1|k}$ is now amended to $\bar{x}_{k+1|k}$ in the Kalman filter so that the observation forecast $\bar{\hat{y}}_{k+1|k}$ is shifted in the precise direction indicated by the decision maker's expectations.

Concluding Notes

The method in Section 2 also indicates the existence of a convergence problem: does the state estimate generated by the filter converge to the true state estimate? The convergence of the Kalman filter being a separate issue, the answer seems to be affirmative if there is a learning process in which the decision maker's expectations become, through learning, increasingly accurate estimates of the corresponding state vector (i.e. $\delta = \hat{x}^e_{k|k} - \hat{x}_{k|k} = 0$ in (10) or $\delta = \hat{x}^e_{k+1|k} - \hat{x}_{k+1|k} = 0$ in (27)) or if the expected and actual observations converge (i.e. $\delta = y^e_k - y_k = 0$ in (30)).

It would be unrealistic to postulate identical adjustment parameters α and δ over all agents. Surely Lucas is right when he asserts that:

> "The assumption of rational expectations is appealing /in the context of yielding important insights into the workings of market economies/ not only because by applying the principle of optimization to _the acquisition of information_ it maintains an internally consistent framework, but _also because it limits to some extent the range of possible outcomes_."

(Lucas, 1984, p. 1653. Italics added.)

But then 'acquisition of information' is pre-eminently a sequential process and by making explicit the nature of the modifications necessary to relax the assumption of an invariant Kalman Gain we have, it is hoped, cast some new light on the constraints imposed on agents in New Classical Models.

If we are to consider seriously the problem of modelling expectations and therefore that of 'forecasting the forecasts of others', it is clear that the interaction between agents must not be trivialized. The extent to which the ethical, aesthetic and economic components of value determine the strictly economic behaviour of agents is of crucial importance in imputing intelligent and useful values for the adjustment parameters α and δ. It will no longer be purely a question of imperfect information in the limited sense of confusing local and global information structures. It is, instead, a question of incomplete perception of the ethical, aesthetic and economic components of the individual agent's value system.

In the review of Grandmont from which we have quoted from Lucas above (Lucas, op. cit.) he went on to cast some doubt on the former's approach to modelling expectations:

> "Grandmont's analysis in contrast /to RE/ demonstrates that *if expectations are purely subjective then anything can happen and probably will*. . . ."
>
> (Lucas, op. cit., p. 1653. Italics added.)

However, the moral of the story we have developed seems to be that Libertarian modelling propped up by unbridled utilitarianism is as undesirable --if not unpractical--as unconstrained subjectivism. Rawls, faced with the same dilemma, appealed to intuitionism to get tempered utilitarianism as a workable alternative. This seems to us to be a most promising avenue to explore, especially in the variants introduced by Harsanyi and his followers to incorporate, in addition to strictly economic components of value, also ethical and other considerations (cf. Harsanyi, 1977, §3.6 in conjunction with ch. 4 of the same book).[(5)] To some extent Harsanyi's tracing procedure, within a framework in which agents' preference functions incorporate also ethical considerations, is a sequential updating procedure to which the method we have developed can be applied (cf. Harsanyi (1979)).

There remains, finally, the almost unanswerable question whether the stochastic calculus and linear rational expectation models are the best that modern 'theoretical technology' can offer us for macroeconomic modelling strategies. And if not, what of the concepts that are a direct outcome of such tools? As followers of a method popularized by Mitchell and the NBER, and as believers in the positivistic philosophy endorsed by Friedman, there may be some consistency in New Classical over-indulgence with the elementary tools of the stochastic calculus. But 'theoretical technology' offers, for those who seek, also other and more exotic tools. The theory of dynamical systems, in particular, and the vast vistas opened by Mandelbrot's (1983) explorations in the "Fractal Geometry of Nature" are two convenient examples. The problem of the 'Business Cycle' is, inherently, a dynamic problem. The Lucasians have studied it with the exclusive use of the tools of stochastic calculus. The concepts in their theoretical formulation have thus been developed within the limits imposed by such tools. An approach to the problem of the dynamics of 'Business Cycles' with other tools, as mentioned above, may prove even more fruitful in interpreting the role of the individual agent in his social setting--and hence that much more useful in the prediction and regulation of socio-economic events of an aggregate nature.

Footnotes

(1) By New Classical Macroeconomists or Lucasians we denote the class of economic model builders whose assumptions are predicated by, at least, the following postulates for the agents and values that characterize the economy:

(a) Rational Expectations

(b) Equilibrium Values (i.e., Auction Market Clearing) and

(c) Imperfect Information.

It is only the technical aspects of (a) (and, to some extent (c)), that are of concern to us for the subject matter of this paper. In fact, however, the assumption of "Auction Market Clearing" is more about centralization and knowledge than about rationality. There is, therefore, a curious inconsistency in the triple pillars that characterize the New Classicals and to concentrate, as we do here, on (a) and (c) alone is not unjustified, at least from a theoretical and methodological point of view.

(2) We have not had access to it except in the form of the summary in the introductory essay by Phelps and Frydman in the volume which contains Townsend's recent essay.

(3) New Classical Macroeconomists (Lucas in particular) have, indeed, made attempts to link their work with the developments in monetary and business cycle theory in the 20s and 30s. In particular the theoretical work of Hayek and Knight and the NBER statistical methodology have been important sources of inspiration for the 'Rational Macroeconomists' (cf. also Fitoussi-Velupillai (1984)). However, the work of the Swedes seems to have entirely escaped their notice.

(4) This qualification ("macroeconomic writings") is mainly to distinguish it from his equally important contribution to Public Finance and the Theory of Taxation.

(5) We would perhaps endorse an even further step 'beyond utilitarianism' and go all the way to formalization in terms of deontic logic. That, on the other hand, implies considerable imprecision in outcomes and, hence, incompatible with the simpler variants of positivism underlying standard statistical methodology. It is, perhaps, the price we must pay for recognizing the essentially 'soft science' nature of economics.

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