Three Essays on Frictional Labour Markets

David Pothier

Thesis submitted for assessment with a view to obtaining the degree of Doctor of Economics of the European University Institute

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European University Institute
Department of Economics

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Abstract

This thesis contributes to the understanding of how socio-economic factors affect the functioning of modern labour markets. It belongs to the strand of academic literature that departs from the standard Walrasian model of the labour market, and considers matching and information frictions to be important determinants of observed labour market phenomena. Within this general framework, this thesis analyses how different forms of agent heterogeneity - socio-demographic identity, productivity, and wealth - affect wage rates and the level of employment in competitive labour markets.

The first chapter studies how occupational segregation - the sorting of workers across occupations based on their demographic characteristics - affects the allocation of talent in the labour market. When job vacancy information is transmitted via workers' group-biased social contacts, occupational segregation is found to be a robust equilibrium outcome. The chapter shows that while occupational segregation implies benefits in terms of the job-finding probability of individual workers, it may also engender significant allocative inefficiencies when workers differ in terms of their productivity across occupations.

The second chapter examines how heterogeneous workers and firms sort across formal (market-based) and informal (network-based) recruitment channels. When worker and firm productivity are unobservable the two recruitment channels effectively compete in terms of their screening capability. Matching frictions are shown to generate a sorting externality that leads to a multiplicity of equilibrium outcomes, depending on the skill-bias within social networks and the productivity dispersion among workers and firms.

The third chapter, co-authored with Damien Puy, examines to what extent variations in wages and employment over the business-cycle can explain the counter-cyclical properties of the income distribution. We show that demand composition effects are an important channel through which aggregate supply shocks are propagated through the economy, and that these have important distributional consequences. In particular, we find income inequality (as measured by the Gini coefficient) to be counter-cyclical. Consistent with empirical evidence, this is shown to be largely due to changes in the level of employment and to a lesser degree to variations in relative factor prices.

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Preface

Neoclassical economics traditionally considers the labour market to function as a pure spot market populated by non-strategic price-taking agents. According to this framework, labour hours are traded at a competitive price (the wage rate), and the level of employment is determined by the equalisation of labour supplied by households and labour demanded by firms. However, delving back into the history of economic thought, one quickly realises that the Classics had a much more nuanced view of how the market for labour operates. Indeed, economic thinkers ranging back from Smith all the way up to Marshall argued that one cannot properly analyse the labour market without taking into account key factors such as the social relations of production, long-term contractual arrangements, worker and manager incentives, in addition to non-market institutions like trade unions and socioeconomic networks [36].

In reality, labour markets are marred by informational asymmetries and coordination failures. The inability of standard neoclassical theory to address these issues has led to a significant revamping of the discipline, and most economists have, by and large, come to the conclusion that the Walrasian framework is ill suited to study labour market phenomena. As a result, numerous theoretical models have been developed in order to explicitly model the structural "frictions" that commonly characterise labour markets. These can be broadly grouped into one of two categories: (i) *matching frictions* that inhibit the unrestricted pairing of unemployed workers and job vacancies [28], [29]; and (ii) *informational frictions* that arise due to the presence of asymmetric information between workers and firms [41], [72], [73]. Further understanding how these two types of frictions interact, and how they affect the functioning of labour markets constitutes, in the broadest of terms, the focus of this thesis.

This thesis is organised into three chapters. The first studies how occupational segregation affects the allocation of talent in a competitive labour market. The second focuses on how heterogenous workers and firms sort across competing recruitment channels. The third chapter examines, among other issues, to what extent variations in labour income over the business cycle can explain the counter-cyclical properties of the income distribution. While these three research questions address fundamentally different issues, each one identifies a distinct channel through which matching and/or informational frictions affect the workings of the labour market, and each provides a novel theoretical framework

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with which to interpret a number of understudied empirical facts.

The first chapter, entitled *Occupational Segregation and the (Mis)allocation of Talent*, examines how homophilic job search networks, via their effects on occupational segregation, affect the allocation of talent in a market economy. To this end, the chapter develops a model of occupational choice where heterogenous workers must rely on their social contacts to acquire job vacancy information. Workers' heterogeneity is assumed to affect both: (i) the cost they incur in order to specialise in a particular occupation, and (ii) their ability on the job. The chapter begins by showing that occupational segregation can arise in equilibrium due to the strategic complementarities engendered by the homophilic properties of workers' social contacts. The purpose of the ensuing analysis is then to determine whether this equilibrium occupational segregation is actually efficient. The model shows that the widespread use of homophilic job search networks generates both positive externalities (due to improved labour market matching), and negative externalities (due to a poor allocation of talent). Which of these two effects dominate depends on the properties of the information transmission mechanism, as well as the correlation between workers' skills and their ability on the job. A key normative conclusion of the chapter is that the degree of occupational segregation in competitive labour markets is generally not efficient. Using CPS data, we also find suggestive empirical evidence supporting the claim that average wages are negatively correlated with the degree of occupational segregation within occupations.

The second chapter, *Competing Recruitment Channels*, studies a matching model of the labour market where workers and firms must choose whether to match through a profit-maximising employment agency or through an informal referral network. The productivity of workers and firms is assumed to be private information, and the two recruitment channels effectively compete in terms of their screening technologies. The employment agency has access to a proprietary technology which allows it to screen job applicants, and charges firms wanting to use its services a subscription fee. Referral hiring, on the other hand, is costless, and its screening capability is determined by the degree of skill homophily in workers' social networks. The main objective of the chapter is to understand how the recruitment method chosen by workers and firms varies as a function of their productivity. Due to a classic sorting externality, the model supports a multiplicity of equilibrium outcomes. Under certain equilibrium configurations, highly productive firms and workers seek to match through the labour market intermediary rather than through the referral

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network. Interestingly, while this result goes against the conclusions of the existing theoretical literature studying job referral networks, it is consistent with recent empirical evidence.

The third chapter, titled *Unemployment, Inequality and the Business Cycle* and co-authored with Damien Puy, is primarily focused on the counter-cyclical properties of the income distribution, and understanding to what extent these are driven by variations in labour income. Contrary to the first two chapters, in which workers were assumed to differ in terms of their productivity, this chapter focuses on the effects of wealth inequality. In particular, the chapter develops a multi-sector general equilibrium model with labour market frictions in which agents differ in terms of their ownership shares of the aggregate capital stock. Consumers are assumed to have non-homothetic preferences, and only begin to consume non-essential goods after satiating their demand for more basic goods. Moreover, production technologies are such that the factor share of capital is greater in sectors producing more basic goods. We first show that changes in the composition of aggregate demand (and its implications for relative factor rewards) is an important channel through which supply shocks are propagated through the economy. Second, we show that the *ex ante* distribution of capital ownership affects the magnitude of these demand-driven effects. Lastly, we study the *ex post* distributional consequences of aggregate productivity shocks. Consistent with empirical evidence, we find that inequality rises during recessions because higher unemployment and lower wages worsen the relative position of low-income groups.

The remainder of this thesis is organised as follows. The three models discussed above are presented in Chapters 1-3. In order to facilitate the reading of each chapter, proofs have not been included in the main text but have been relegated to an appendix at the end of each chapter. All bibliographic material is to be found at the end of this document.

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Chapter 1

Occupational Segregation and the (Mis)allocation of Talent

1.1 Introduction

Occupational choice often not only depends on idiosyncratic characteristics (e.g innate ability), but also on the occupations chosen by family, friends, and peers. Indeed, numerous studies in the sociology literature indicate that sociocultural identity is an important factor explaining peoples' occupational choice decisions [5]. This has led sociologists to coin the term 'occupational segregation' in order to refer to the sorting of individuals across occupations based on their social, religious, ethnic and/or gender identity. Such segregation is known to be an important cause of wage and employment disparities between social groups [4], [47]. However, surprisingly little is known about how it affects the allocation of talent (the matching of skills to tasks) in the labour market. This paper studies the allocative implications of occupational segregation in a competitive market environment.

Few formal theories have been proposed addressing the presence and persistence of occupational segregation. 'Demand-side' theories based on discriminatory preferences or statistical discrimination often fail to explain why we should observe workers segregating across occupations [33], [52], [53], [68]. Another line of research emphasising 'supply-side' effects, first suggested by Arrow [7], argues that occupational segregation can arise due to the widespread use of referral networks in labour markets.[1] In a recent paper, Buhai and van der Leij [16] formalise Arrow's original intuition and explicitly model the mechanisms by which network effects in job search can lead to occupational segregation.[2] They show

[1]Empirical work dating back to Granovetter [39] suggests that between 30% and 50% of all jobs are found using such informal social networks.

[2]The theoretical framework developed by Buhai and van der Leij [16] is closely related to the social interactions literature [30], [55], [71]. More specifically, their model is an adapted version of the one developed by Benabou [11] to study how residential segregation affects workers' human capital accumulation decisions.

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that occupational segregation can be supported in equilibrium whenever individuals are disproportionately likely to form social ties with other individuals belonging to the same social group - a phenomenon often referred to as "homophilic inbreeding."[3]

In reality, occupational choice is not driven only by social considerations but also by inherent differences among workers regarding their ability to perform tasks in different occupations. Allowing for such heterogeneity implies that the way workers sort across occupations has important allocative consequences. To study these effects, we propose a model of occupational choice in which heterogeneous workers must rely on their social contacts to acquire job vacancy information. Workers are assumed to differ in terms of some publicly unobservable skill characteristic that determines both the cost of specialising in different occupations and their productivity if employed by a firm. We show that in such an environment occupational segregation may engender significant allocative inefficiencies. While Buhai and van der Leij [16] find that occupational segregation is desirable from a welfare perspective because it increases the aggregate level of employment, allowing for (unobservable) worker heterogeneity dramatically changes the normative implications of occupational segregation. In particular, a constrained efficient allocation in this environment trades-off an increase in employment from more efficient job search when workers segregate across occupations, with the associated decrease in productivity implied by a misallocation of talent. In a decentralised economy, a worker ignores how his occupational choice decision affects both the job finding probability of other workers and the allocation of talent across occupations. Instead, he chooses an occupation by comparing the expected wage with the associated idiosyncratic specialisation cost, taking as given the occupational choice decision of other workers in the economy.

The presence of network effects in occupational choice implies that workers' best response functions are highly non-linear. Because of this, we are unable to provide a full analytical characterisation of the set of equilibria. Nonetheless, we can still answer some interesting questions such as: (i) when can occupational segregation arise in equilibrium, and (ii) under what conditions is occupational segregation actually efficient? In particular, we identify conditions (most notably restrictions on the variable parameterising the degree of homophilic inbreeding) under which occupational segregation ca be supported in equilibrium. We find that these conditions are generally not the same as those need for

[3]There exists a wealth of empirical evidence demonstrating that such homophilic inbreeding is a widespread social phenomena. See for example the landmark study by McPherson, et al [59].

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occupational segregation to be efficient. This divergence between equilibrium and efficient outcomes is caused by a search externality that leads workers to segregate too little, and a pecuniary externality that leads workers to segregate too much in equilibrium. Which of these two effects dominate is shown to depend on the efficiency of the job search technology and the variability of workers' abilities across occupations.

Additionally, the model yields some interesting and novel testable predictions about the relationship between the degree of occupational segregation and wages: *ceteris paribus*, increases in the degree of occupational segregation should lead to a decrease in average wages due to a misallocation of talent in the labour market. In the last section of this paper, we provide some suggestive empirical evidence supporting this theoretical prediction. In particular, we obtained CPS data on median weekly wages, the total number of employed workers, and the demographic composition of occupations in the US between 2003 and 2010. Using standard panel data techniques, we find evidence of a negative correlation between the degree of racial/ethnic segregation within occupations and median weekly wages in those occupations. Moreover, we argue that this negative correlation is not driven by wage differences between workers belonging to different racial/ethnic groups.

A key building block of this paper is the assumption that workers' reliance on homphilic social networks to access job vacancy information introduces a degree of strategic complementarity in the occupational choice decision they face before entering the labour market. When workers differ in terms of the cost of specialising in different occupations, they face a trade-off when choosing an occupation. On one hand, workers prefer to choose an occupation that is popular among individuals in their social network as this increases the probability that they find a job.[4] On the other hand, workers would like to choose an occupation in which they are relatively more able, as this minimises the specialisation cost they must incur before entering the labour market. We show that workers' reliance on social contacts in job search generates a positive externality: by choosing to specialise in a given occupation, an individual increases the probability that other individuals in his social network choosing the same occupation are successfully employed. If in addition to affecting his cost of specialisation, a worker's skill-type also affects his productivity on

[4]This effect is consistent with recent empirical evidence showing that the use of social contacts in job search increases the probability of employment. For example, using a panel of local authority-level data from England between 1993 and 2003, Patacchini and Zenou [67] find that increases in ethnic population density (meant to proxy for social networks and the transmission of job vacancy information) increases the ethnic employment rate. See also Munshi [64] and Topa [74].

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the job, workers' reliance on social contacts in job search generates a negative externality: when choosing an occupation, a worker does not internalise how his occupational choice decision affects the allocation of talent, and thereby aggregate labour productivity.

In general, this paper contributes to the literature studying the interaction between informal social networks and competitive labour markets. Most of the existing research on this topic has emphasised the beneficial effects of workers' reliance on social contacts in job search. For example, Montgomery [63] argues that homophilic social networks can be exploited by profit-maximising firms to costlessly screen job applicants, while Kugler [50] argues that referals lower monitoring costs and allow firms to pay lower efficiency wages (thereby reducing distortions generated by the presence of asymmetric information in the labour market). An exception is a closely related paper by Bentolila, et al [12] that studies the implications of social contacts for occupational mismatch. As in this paper, they find that social contacts imply both benefits (in terms of job-finding probability) and costs (in terms of labour productivity). Their paper, however, focuses solely on the wage and employment effects of the network-based recruitment channel. In particular, they assume an exogenous correlation between workers' skill-types and the skill-type of their social contacts. The distortions that arise due to occupational mismatch in their model thus fundamentally depend on this exogenous parametre, and they do not analyze the (endogenous) sorting of workers across occupations based on their group-identity nor its implications for occupation segregation.

The remainder of this paper is organised as follows. Section 2 describes the primitives of the model. Section 3 gives a definition of the equilibrium concept and the relevant welfare benchmark. Equilibrium and welfare analysis is presented in Section 4. Empirical results are found in Section 5. Section 6 concludes.

1.2 The Model

1.2.1 Workers

Consider an economy populated by a continuum of risk-neutral workers. Let N denote the set of workers, with (Lebesgue) measure normalised to two. Workers are *ex ante* heterogenous and differ in terms of their idiosyncratic skill-type. We denote the type space by $\Theta = [0, 1]$. Moreover, workers are equally divided into two social groups: reds (R)

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and greens (G). For simplicity, we assume skill-types to be uniformly distributed in both groups, so that $\theta \sim U[0,1]$ for all $X \in \{R, G\}$.[5] This implies that no one group is *ex ante* predisposed to any particular occupation. Consistent with Loury's [54] axiom of *anti-essentialism*, any occupational segregation that arises in equilibrium will therefore be due to strategic considerations among the workers, rather than some presupposed productivity difference between individuals belonging to different social groups. An individual worker's social 'colour' in this context thus only serves as a social marker, and is otherwise completely payoff irrelevant.

There exists a binary set of occupations for workers to choose from, denoted by $\Phi = \{A, B\}$. Before entering the labour market, workers must choose to specialise in one of these two occupations. Importantly, we assume that a worker cannot be hired by a firm unless he has specialised in an occupation.[6] A worker's skill-type determines the idiosyncratic cost he must incur in order to specialise in one of the two occupations. Let $c_\phi(\theta) \in \mathbb{R}_+$ denote the cost incurred by a type θ worker choosing to specialise in occupation ϕ.

Assumption 1: The cost functions $c_\phi(\theta)$ for $\phi \in \{A, B\}$ satisfy the following conditions

1. Symmetry: $c_A(1-\theta) = c_B(\theta)$
2. Monotonicity: $c'_A(\theta) > 0$ and $c'_B(\theta) < 0$
3. Weak Concavity: $c''_\phi(\theta) \leq 0$ and $c'''_\phi(\theta) \leq 0 \ \forall \phi \in \{A, B\}$

The monotonicity assumption implies that workers located on the left hand side (near 0) of the unit interval find it relatively easier to specialise in occupation A, while workers located on the right hand side (near 1) of the unit interval find it relatively easier to specialise in occupation B. The symmetry assumption implies that, absent any network effects, workers located at the midpoint of the unit interval will be indifferent between specialising in occupation A or occupation B.

A worker's skill-type also determines his productivity on the job. This productivity is negatively correlated with workers' specialisation costs; e.g workers that find it relatively

[5] The uniform assumption is made to simplify the derivations. The qualitative nature of the results would be unchanged had we instead assumed a single-peaked symmetric distribution function.

[6] The decision should therefore be viewed as an investment in some observable and publicly recognised certificate needed for employment within a particularly industry (e.g a law or architecture degree).

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costly to specialise in a given occupation are also relatively less able at performing tasks if employed in that occupation. Formally, let $z_\phi(\theta) \in \mathbb{R}_+$ denote the productivity of a type θ worker employed in occupation ϕ.

Assumption 2: The productivity functions $z_\phi(\theta)$ for $\phi \in \{A, B\}$ satisfy the following conditions

1. Symmetry: $z_A(1-\theta) = z_B(\theta)$
2. Monotonicity: $z'_A(\theta) < 0$ and $z'_B(\theta) > 0$
3. Weak Concavity: $z''_\phi(\theta) \leq 0$ and $z'''_\phi(\theta) \leq 0 \ \forall \phi \in \{A, B\}$

1.2.2 Social Network

After having chosen an occupation, workers enter a competitive labour market. All matches on the labour market necessarily take place via workers' social contacts. Network-mediated search is subject to frictions insofar as workers are more likely to receive a job offer when they have more friends specialised in the same occupation. In fact, we assume social ties to workers specialised in a different occupation provide no job vacancy information whatsoever.[7]

We model workers' social network as an Erdos-Renyi random graph formed as the result of a binomial link formation process. This stochastic process is subject to an inbreeding bias so that workers are disproportionately likely to form ties with other workers belonging to the same social group. Let $\alpha \in (1/2, 1)$ denote the conditional probability that a randomly chosen worker is 'linked' to another worker belonging to the same social group. This parametre effectively measures the degree of homophilic inbreeding in workers' social network. Importantly, we assume that workers make their specialisation decisions *before* the stochastic network is realised. This is consistent with the interpretation whereby an individual's social network is constituted of "weak" or "instrumental" ties.[8] Consequently,

[7]Intuitively, one should interpret the two occupations as being very different in terms of the skills they require. Building on the example provided in footnote 7, job vacancies for architects are unlikely to be of interest to someone holding a law degree.

[8]Granovetter [39], among others, has shown that job vacancy information is more likely to be obtained from social connections made at university or in the labour market (and thus after workers have made some fixed investment in a career path), rather than from family or kinship ties.

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workers' occupational choice decisions are not conditioned on the realised structure of their social networks. Let η_ϕ^X denote the (expected) measure of a worker's neighbours specialised in occupation ϕ when a worker belongs to group X. The probability that a worker receives a job offer is then denoted by $q(\eta_\phi^X) \in (0,1)$.

Assumption 3: The job search function is linear such that $q(\eta_\phi^X) = \gamma\eta_\phi^X$, where $\gamma > 0$.

This exogenous job search technology should be interpreted as a reduced-form representation of a dynamic job search process. The dynamics of such network-mediated job search have been studied in detail by Calvo-Armengol [18], and Calvo-Armengol and Jackson [19], [20] (among others). As we are only concerned with the overall network externality generated by network-mediated job search, we choose to model the job search process in this way in order to simplify the analysis. Note that due to the structure of the job search technology, a positive mass of workers remain unemployed in equilibrium. The linearity assumption is equivalent to saying that there are constant returns-to-scale to job search (i.e there are no congestion effects).

1.2.3 Firms

Production is organised in two stages. Firms in two intermediate sectors $\phi \in \{A, B\}$ employ workers specialised in each occupation and produce two input goods, which are then combined in the final goods sector to produce the consumption good. Importantly, both intermediate goods are assumed to be essential and complements in the production of the final good. We normalise the price of the consumption good to unity, and denote the price of the intermediate goods by $p_\phi \in \mathbb{R}_+$. The *effective* labour supply of workers in each intermediate good sector - i.e the mass of workers specialised in each occupation weighted by the respective employment rates - is denoted by $l_\phi \in \mathbb{R}_+$. The effective labour supply in efficiency units - i.e the effective labour supply weighted by workers' productivity - is denoted by $\tilde{l}_\phi \in \mathbb{R}_+$. Firms in the intermediate goods sector have access to a linear production technology denoted by $y_\phi = \tilde{l}_\phi$, while firms in the final goods sector have access to a symmetric Cobb-Douglas technology denoted by $f(y_A, y_B) = \sqrt{y_A, y_B}$.[9]

[9] The Cobb-Douglas assumption allows us to solve for equilibrium allocations in closed-form, but results would be qualitatively similar given any symmetric production function satisfying the standard Inada conditions. The symmetry assumption could be relaxed in order to study how occupational segregation

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1.2.4 Timing

The timing of the model can be summarised as follows:

- **Stage 1:** Workers choose to specialise in occupation $\phi \in \{A, B\}$ and incur the cost $c_\phi \in \mathbb{R}_+$.
- **Stage 2:** Workers randomly and non-strategically form a network of social connections.
- **Stage 3:** Workers enter the labour market and are hired with probability $q(\eta_\phi^X) \in (0, 1)$. Conditional on being hired in occupation $\phi \in \{A, B\}$, workers receive the wage $w_\phi \in \mathbb{R}_+$.

1.3 Equilibrium and Welfare: Definitions

1.3.1 Equilibrium

Let $\sigma^X(\theta) \in [0, 1]$ denote the probability with which a type θ worker belonging to group X chooses occupation A, and define $\sigma^X = \int_0^1 \sigma^X(\theta)d\theta$ to be the total measure of workers in group X choosing occupation A. The payoff function of a type θ worker belonging to group X and choosing to specialise in occupation ϕ is given by

$$U_\phi^X\left(\sigma^X, \sigma^{X'}; \theta\right) = q\left(\eta_\phi^X\left(\sigma^X, \sigma^{X'}\right)\right) w_\phi - c_\phi(\theta) \tag{1.1}$$

The first term of the right hand side of this equation equals a worker's expected wage when choosing occupation ϕ, while the second term equals the cost he must incur in order to specialise in this occupation. The objective function of the firms in intermediate good sector ϕ is given by

$$\Pi_\phi = p_\phi y_\phi - w_\phi l_\phi, \quad \forall \phi \in \{A, B\} \tag{1.2}$$

while the objective function of the (representative) firm in the final goods sector is given by

$$\Pi = f(y_A, y_B) - p_A y_A - p_B y_B \tag{1.3}$$

affects wage inequality.

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Definition 1: An equilibrium is defined as a specialisation strategy $\sigma^X(\theta)$ for all $X \in \{R, G\}$ and $\theta \in [0,1]$; a labour demand schedule $(l_\phi)_{\phi \in \{A,B\}}$; an intermediate goods demand schedule $(y_\phi)_{\phi \in \{A,B\}}$; wages $(w_\phi)_{\phi \in \{A,B\}}$; and prices $(p_\phi)_{\phi \in \{A,B\}}$ such that

1. Each worker chooses a specialisation strategy $\sigma^X(\theta)$ to maximise his utility, taking wages and the occupational choice decision of other workers as given.
2. Firms in each intermediary good sector $\phi \in \{A, B\}$ choose labour demands l_ϕ to maximise their profits, taking wages, prices and the occupational choice decision of workers as given.
3. Firms in the final good sector choose an input demand schedule $(y_\phi)_{\phi \in \{A,B\}}$ to maximise their profits, taking prices as given.
4. The labour and goods markets clear.

Utility maximisation implies that workers' specialisation decisions must satisfy

$$\begin{aligned} \sigma^X(\theta) = 0 \quad &\text{if} \quad \Delta_\phi U^X(\theta) < 0 \\ \sigma^X(\theta) \in [0,1] \quad &\text{if} \quad \Delta_\phi U^X(\theta) = 0 \\ \sigma^X(\theta) = 1 \quad &\text{if} \quad \Delta_\phi U^X(\theta) > 0 \end{aligned} \tag{1.4}$$

Definition 2: An equilibrium in *threshold strategies* is an equilibrium such that the specialisation strategy of workers $\sigma^X(\theta)$ satisfies the following condition

$$\exists \hat{\theta}^X \in [0,1] \quad \forall X \in \{R, G\}: \quad \sigma^X(\theta) = 1 \quad \text{if} \quad \theta < \hat{\theta}^X \quad \text{and} \quad \sigma^X(\theta) = 0 \quad \text{if} \quad \theta > \hat{\theta}^X$$

The properties of the cost functions - in particular, the monotonicity and continuity assumptions - imply that restricting attention to equilibria in threshold strategies is without loss of generality.

Lemma 1: All equilibria are necessarily in threshold strategies.

Proof: See Appendix B.

Rewriting the optimality condition of workers in terms of threshold strategies, we have

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that any (interior) equilibrium threshold profile $(\hat{\theta}^R, \hat{\theta}^G) \in (0,1)^2$ must satisfy the following indifference conditions

$$q(\eta_A^X) w_A(\hat{\theta}^X, \hat{\theta}^{X'}) - q(\eta_B^X) w_B(\hat{\theta}^X, \hat{\theta}^{X'}) = c_A(\hat{\theta}^X) - c_B(\hat{\theta}^X), \quad \forall X \in \{R, G\} \tag{1.5}$$

For corner solutions, so that $(\hat{\theta}^R, \hat{\theta}^G) \in \{(1,0),(0,1)\}$, these conditions may hold as inequalities. Using Lemma 1, we can derive explicit expressions for the (expected) measure of a worker's neighbours specialised in each occupation as follows

$$\eta_A^X = \alpha \hat{\theta}^X + (1-\alpha)\hat{\theta}^{X'} \tag{1.6}$$

$$\eta_B^X = \alpha(1-\hat{\theta}^X) + (1-\alpha)(1-\hat{\theta}^{X'}) \tag{1.7}$$

The effective labour supply of workers in each occupation is thus equal to

$$l_A = \sum_{X \in \{R,G\}} q(\eta_A^X)\hat{\theta}^X \tag{1.8}$$

$$l_B = \sum_{X \in \{R,G\}} q(\eta_B^X)(1-\hat{\theta}^X) \tag{1.9}$$

and the effective labour supply in efficiency units is given by

$$\tilde{l}_A = \sum_{X \in \{R,G\}} q(\eta_A^X) \int_0^{\hat{\theta}^X} z_A(\theta) d\theta \tag{1.10}$$

$$\tilde{l}_B = \sum_{X \in \{R,G\}} q(\eta_B^X) \int_{\hat{\theta}^X}^1 z_B(\theta) d\theta \tag{1.11}$$

Free entry in the intermediate goods sectors implies that labour demand schedules must satisfy

$$w_\phi = p_\phi \left(\frac{\tilde{l}_\phi}{l_\phi} \right), \quad \forall \phi \in \{A, B\} \tag{1.12}$$

while profit maximisation in the final goods sector implies that input demands are such that

$$p_\phi = \frac{\partial f(y_A, y_B)}{\partial y_\phi}, \quad \forall \phi \in \{A, B\} \tag{1.13}$$

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The following result is rather intuitive if one interprets workers' occupational choice decisions as a classic coordination game: given that no worker chooses to segregate, the individual gains to choosing an occupation other than the one in which a worker holds a skill advantage are zero.

Lemma 2: The non-segregated threshold profile $(\hat{\theta}^R, \hat{\theta}^G) = (1/2, 1/2)$ can always be supported as an equilibrium.

Proof: See Appendix B.

1.3.2 Social Welfare

The welfare criterion we use is based on the notion of utilitarian efficiency, rather than the more common notion of Pareto efficiency. The utilitarian welfare benchmark is a compelling one, especially if one is interested in comparing different outcomes from an *ex ante* perspective [43], [44], [76], [77]. Given that workers located at the extrema of the type space have a payoff advantage relative to those located in the middle of the type space, it makes sense to consider *ex ante* rankings of different potential equilibrium outcomes. Aggregate utilitarian welfare is given by

$$W = \sum_{X \in \{R,G\}} \int_0^1 \left(\sigma^X U_A^X(\theta) + (1 - \sigma^X) U_B^X(\theta)\right) d\theta$$

Rewriting this welfare function in terms of threshold profiles, we obtain

$$W(\hat{\theta}^R, \hat{\theta}^G) = \sum_{X \in \{R,G\}} \left(\int_0^{\hat{\theta}^X} U_A^X(\theta) d\theta + \int_{\hat{\theta}^X}^1 U_B^X(\theta) d\theta \right) \tag{1.14}$$

Definition 3: A threshold profile $(\hat{\theta}^R, \hat{\theta}^G) \in [0,1]^2$ is constrained efficient if it maximises aggregate utilitarian welfare (1.14) subject to the technological constraints (1.8)-(1.9).

The task of a social planner seeking to maximise aggregate utilitarian welfare consists of specifying an allocation rule $\sigma_i^X(\theta) : \Theta \rightarrow \Phi$ for all $X \in \{R, G\}$ and $i \in N$ that deterministically allocates each worker of type $\theta \in [0, 1]$ to a specific occupation $\phi \in \{A, B\}$.

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The definition of constrained efficiency implies that a social planner faces the same technological constraints as the competitive market, and is required to match workers with vacancies using the network-mediated job search technology. Note that the constrained efficient allocation need not coincide with the first-best social optimum. Indeed, if the social planner could frictionlessly match workers with vacancies, the non-segregated threshold profile would always be the unique efficient allocation.

Using the definition of the payoff function as given by condition (1.1), together with the labour supply conditions (1.8)-(1.9) and the assumption that the production functions exhibit constant returns-to-scale (so that factor incomes exhaust total output), we can rewrite condition (1.14) as

$$W(\hat{\theta}^R, \hat{\theta}^G) = f(y_A, y_B) - C(\hat{\theta}^R, \hat{\theta}^G) \tag{1.15}$$

where

$$C(\hat{\theta}^R, \hat{\theta}^G) = \sum_{X \in \{R,G\}} \left(\int_0^{\hat{\theta}^X} c_A(\theta) d\theta + \int_{\hat{\theta}^X}^1 c_B(\theta) d\theta \right) \tag{1.16}$$

The social planner thus seeks to maximise total output net of the aggregate specialisation costs. The salient trade-off which characterises the efficient allocation can be summarised as follows. On the one hand, the social planner would like to segregate workers belonging to different groups across occupations, as this increases the efficiency of the job search technology and thereby maximises aggregate employment. On the other hand, he must balance this against the increase in specialisation costs and the decrease in average labour productivity that arise when workers segregate. To gain a better intuition of this underlying trade-off, consider the two benchmark cases without inbreeding bias and with homogenous skill-types, respectively. Absent any inbreeding bias, the social planner would have each worker choose the occupation in which he is most able, as this minimises total costs and allocative inefficiencies, while leaving total output unchanged. If workers were homogenous but the social network exhibited a positive inbreeding bias, the social planner would instead have workers completely segregate across occupations, as this maximises aggregate employment while engendering no misallocation effects. In general, the symmetry properties imposed on the cost, productivity and production functions imply that the efficient allocation must also be symmetric. This leads us to the following result.

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Lemma 3: The welfare maximising threshold profile $(\hat{\theta}^R, \hat{\theta}^G)$ is necessarily symmetric such that $\hat{\theta}^R = 1 - \hat{\theta}^G$.

Proof: See Appendix B.

1.4 Equilibrium and Welfare: Analysis

The objective of this section is to characterise the properties of the equilibrium and efficient allocations. Formally speaking, occupational segregation is defined as any deviation from the non-segregated threshold profile $(\hat{\theta}^R, \hat{\theta}^G) = (1/2, 1/2)$, whereby each worker chooses the occupation in which he is relatively more able. Complete occupational segregation is defined as one of the two corner solution threshold profiles: i.e $(\hat{\theta}^R, \hat{\theta}^G) \in \{(1,0),(0,1)\}$. Any intermediate threshold profile is referred to as partial occupational segregation. We are particularly interested in characterising the conditions under which complete occupational segregation can be supported in equilibrium, and comparing these to the conditions needed for complete occupational segregation to be efficient. More specifically, we show that there exist cut-off values of the inbreeding bias parameter, $\underline{\alpha}^{EQ} \in (1/2, 1)$ and $\underline{\alpha}^{SW} \in (1/2, 1)$, above which complete occupational segregation can be supported in equilibrium and is efficient, respectively. Interestingly, these two cut-off values generally do not coincide. We conclude this section with a detailed examination of the factors responsible for the divergence between the equilibrium and efficient allocations.

1.4.1 An Example

We begin by considering an example using specific functional form assumptions. Although these assumptions are restrictive, they allow us to solve for the allocations in closed-form, and thereby provide a more robust intuition of the underlying workings of the model. In particular, we assume the cost and productivity functions are linear. Formally,

$$c_A(\theta) = k\theta \quad \text{and} \quad c_B(\theta) = k(1-\theta), \quad k > 0$$

and

$$z_A(\theta) = h(1-\theta) \quad \text{and} \quad z_B(\theta) = h\theta, \quad h > 0$$

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In Appendix A, we solve a similar example assuming that workers' productivity on the job is independent of their skill-type. This allows us to isolate the effect of the externalities generated by the network-mediated job search technology. The results we obtain in this case confirm the main findings of Buhai and van der Leij [16]. In particular, we find that occupational segregation can be supported in equilibrium for sufficiently high values of the inbreeding bias. Intuitively, this follows from the fact that as the degree of homophilic inbreeding in workers' social networks increases, the strategic complementarities implied by the network-mediated job search technology become more pronounced. Moreover, we find that whenever workers segregate in equilibrium, it is also efficient for them to do so. In fact, the network-mediated job search technology generates a positive externality which leads workers to segregate "too little" in equilibrium: i.e for intermediate values of the inbreeding bias, workers choose not to segregate even though it would be efficient for them to do so. Extending the model to allow for worker heterogeneity, however, we find that this efficiency result is very specific to the case where workers are assumed to be homogeneous, and does not in general extend to the case where the productivity of workers varies across occupations.

Result 1: When workers' productivity depends linearly on their skill-type, complete occupational segregation can be supported in equilibrium for sufficiently high values of the inbreeding bias. Moreover, if complete occupational segregation can be supported in equilibrium then it is also stable, while the non-segregated threshold profile is not.

Proof: See Appendix B.

The intuition behind this result is the same as in the homogeneous productivity case alluded to above. Qualitatively speaking, allowing for heterogeneous worker productivity does not affect the strategic complementarities workers face when making their occupational choice decisions. However, while the conditions needed for occupational segregation to be supported in equilibrium do not significantly change, the welfare-properties of such segregation are radically different when workers differ in terms of their underlying productivity.

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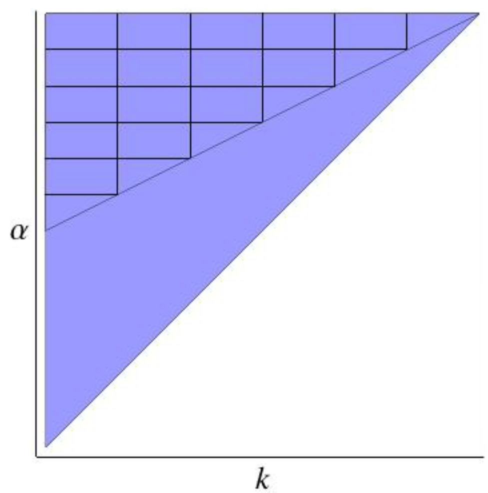

Figure 1.1: Complete Occupational Segregation (heterogeneous productivity) - equilibrium (shaded) and efficient (checkered).

Result 2: When workers' productivity depends linearly on their skill-type, complete occupational segregation can be supported in equilibrium even though it is inefficient.

Proof: See Appendix B.

A graphical depiction of this result is presented in Figure 1.1. The figure clearly shows the set of parametre values $(\alpha, k) \in (1/2, 1) \times \mathbb{R}_{++}$ under which complete occupational segregation is efficient to be a strict subset of those needed for complete occupational segregation to be supported in equilibrium. Why do the welfare properties of the competitive equilibrium differ so dramatically compared to homogeneous productivity case? Heuristically speaking, the difference arises because heterogeneous worker productivity, coupled with the fact that individual worker productivity cannot be observed by firms, generates a negative pecuniary externality. In this particular case, this (negative) externality dominates the positive externality generated by the job search technology. The following section delves into this issue in more detail.

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1.4.2 General Results

In this section, we do away with the specific functional forms for the cost and productivity functions. This allows us to gain a more general understanding of the direction and magnitude of the externalities driving the divergence between the equilibrium and efficient outcomes. Moreover, it allows us to precisely identify the (general) conditions under which the degree of occupational segregation in the competitive market exceeds that implied by the welfare maximising allocation.

Building on Lemma 3, we restrict attention to symmetric allocations such that $\hat{\theta}^R = 1 - \hat{\theta}^G$. This implies that labour supply and wage rates will be equal across occupations in equilibrium. Focusing on interior solutions, the indifference condition (1.5) becomes

$$w(\hat{\theta})\left(q(\eta_A) - q(\eta_B)\right) - \left(c_A(\hat{\theta}) - c_B(\hat{\theta})\right) = 0 \tag{1.17}$$

Lemma 4: There exists a unique segregated equilibrium.

Proof: See Appendix B.

Proposition 1: Wages are strictly decreasing in the degree of occupational segregation.

Proof: See Appendix B.

Proposition 1 constitutes a key testable implication of the model. It implies that increases in the degree of occupational segregation are associated with lower equilibrium wages for *all* workers, irrespective of their group identity. It comes about because occupational segregation leads to a poor matching of skills to tasks in the labour market (relative to the non-segregated case). This misallocation of talent reduces the average productivity of labour across occupations, and this in turn has a depressing effect on the wages offered by profit-maximising firms. The magnitude of this effect depends on the variability of workers' productivity across occupations. In the limiting case with homogeneous productivity, so that $z_A(\theta) = z_B(\theta)$ for all $\theta \in [0, 1]$, there would be no productivity loss from misallocating talent, thereby implying that changes in the degree of occupational segregation would have no effect on equilibrium wages.

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We now turn to study the normative properties of the segregated equilibrium identified in Proposition 1. By symmetry, we can rewrite the social welfare function (1.15) as follows

$$W = \left(q(\eta_A) \int_0^{\hat{\theta}} z_A(\theta) d\theta + q(\eta_B) \int_{\hat{\theta}}^1 z_B(\theta) d\theta \right) - 2 \left(\int_0^{\hat{\theta}} c_A(\theta) d\theta + \int_{\hat{\theta}}^1 c_B(\theta) d\theta \right) \quad (1.18)$$

Lemma 5: The social planner's problem has a unique solution.

Proof: See Appendix B.

Proposition 2: The first-order condition characterising the efficient allocation is given by

$$w(\hat{\theta}) \left(q(\eta_A) - q(\eta_B) \right) - \left(c_A(\hat{\theta}) - c_B(\hat{\theta}) \right) + \mathcal{E}^S + \mathcal{E}^P = 0 \quad (1.19)$$

where

$$\mathcal{E}^S = w(\hat{\theta}) q'(\cdot) \eta_A'(\hat{\theta})(2\hat{\theta} - 1) > 0 \quad (1.20)$$

and

$$\mathcal{E}^P = w'(\hat{\theta}) l(\hat{\theta}) < 0 \quad (1.21)$$

Proof: See Appendix B.

Corollary 1: Complete occupational segregation is inefficient if the following inequality holds

$$q'(\cdot) < \frac{(1-\alpha) z_A(0) - \alpha z_A(1)}{(2\alpha - 1) \int_0^1 z_A(\theta) d\theta}$$

Proof: See Appendix B.

Discussion The uniqueness result established in Lemma 5 implies that the first-order condition of the social planner's problem fully characterises the (interior) efficient allocation. Comparing conditions (1.17) and (1.19), we find that the equilibrium and welfare-optimal allocations differ by two additive terms: $\mathcal{E}^S$ and $\mathcal{E}^P$. Condition (1.20) measures the positive search externality generated by the job search technology. The key term of condition (1.20) is $q'(\cdot)\eta_A'(\hat{\theta})$, which measures the effect of changes in the threshold $\hat{\theta}$ on

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the job-finding probability of workers' neighbours specialised in occupation ϕ. This is the source of the search externality generated by the job search technology: in the competitive market, a worker does not take into account how his occupational choice decision affects the job-finding probability of other workers in his social network choosing the same occupation. Unsurprisingly, the magnitude of this externality is found to be strictly increasing in the value of the inbreeding bias parametre $\alpha \in (1/2, 1)$ and the efficiency of the job search technology, as measured by the slope of the job search function $q(\cdot)$.

Condition (1.21) measures the negative pecuniary externality that arises when workers' ability on the job varies as a function of their skill-type. The key term of condition (1.21) is $w'(\hat{\theta})$, which measures the effect of changes in the threshold $\hat{\theta}$ on wages. This is the source of the pecuniary externality: when choosing an occupation, a worker does not internalise how his occupational choice decision affect the allocation of talent in the labour market, and thereby equilibrium wages. It depends critically on two assumptions. The first is that workers' skill-types cannot be observed by firms. If this were not the case, firms could offer a menu of wages which varies as a function of workers' productivity. As is well known, in such a complete markets setting the pecuniary externalities generated by workers' occupational choice decisions would not imply an efficiency loss, as wages would adjust so that workers receive their marginal product regardless of which occupation they choose [42]. The second is that workers' occupational choice decisions are strategic complements due to the presence of network effects in job search. If these were absent, and workers' occupational choice decisions were non-strategic, then the competitive equilibrium would be efficient. Even though workers' skill-types would still be unobservable and markets incomplete, specialisation costs - which by assumption are negatively correlated with workers' ability on the job - would drive workers to choose the occupation in which they are relatively more able. Hence, both the search externalities and pecuniary externalities fundamentally depend on the degree of homophily in workers' social networks.

Under what conditions should we expect the equilibrium level of occupational segregation to exceed or fall short of the constrained efficient level? For any allocation $\hat{\theta} \in (1/2, 1)$, if the LHS of condition (1.17) is less than the LHS of condition (1.19), the competitive market supplies "too little" occupational segregation. Similarly, if the LHS of condition (1.17) exceeds the LHS of condition (1.19), then the competitive market supplies "too much" occupational segregation. It follows that the relative magnitudes of the externality terms $\mathcal{E}^S$ and $\mathcal{E}^P$ determine how the degree of occupational segregation in equilibrium compares to the welfare maximising allocation. Building on the example discussed above, Corollary 1 gives the general condition under which complete occupational segregation is inefficient. Heuristically speaking, the condition states that if workers' ability on the job is very sensitive to their skill-type and the job search technology is relatively inefficient (implying that $q'(\cdot)$ is small in magnitude), then complete occupational segregation can arise in equilibrium even though it is inefficient.

These normative results mirror closely the conclusions reached by Bentolila et al [12], but it is important to underline in what ways they differ. They propose a search-and-matching model of the labour market a la Pissarides [69] in which workers can rely on their social contacts to find a job. As in this paper, they find that reliance on social contacts in job search imply both benefits (in terms of job-finding probability) and costs (in terms of labour productivity). In equilibrium, the level of occupational mismatch exceeds the socially optimal level because workers do not internalise the adverse effect that a reduction in aggregate labour productivity has on vacancy creation. This inefficiency is closely related to the pecuniary externality identified by Acemoglu [1]: i.e undirected search and *ex post* wage bargaining imply incomplete wage contracts which distort investment incentives. As pointed out by Acemoglu [1], the consequences of these pecuniary externalities are conceptually similiar to technology-driven increasing returns-to-scale. That being said, they fundamentally depend on the fact that the bargaining protocol results in there being no direct mapping from workers' productivity to the wage they receive in equilibrium. For example, the inefficiencies would disappear in an environment with *ex ante* wage posting and directed search. This limitation does not apply to the externalities identified in the present paper as wages are set competitively in a Walrasian market. More importantly perhaps, the inefficiency result of Bentolila et al [12] fundamentally depends on the correlation between a workers' skill-type and the skill-type of his social contacts, which they treat as exogenous. Consequently, they cannot address the endogenous sorting of workers across occupations based on their group-identity, nor its implications for occupational segregation. In addition, their model remains silent about the conditions under which the positive search externality generated by network effects in job search dominates, or is dominated by, the negative pecuniary externality implied by a misallocation of talent in the labour market.

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1.5 Empirical Extension

A key positive implication of the model analysed above (see Proposition 1) is that, *ceteris paribus*, increases in the degree of occupational segregation should lead to a decrease in wages for *all* workers, irrespective of their gender or race/ethnicity. Most, if not all, of the existing theoretical and empirical work studying occupational segregation has focused on its distributional implications for wage inequality *between* social groups [6], [49].[10] However, if the positive results of the model studied above are valid, occupational segregation should also imply significant allocative inefficiencies, and these should in turn have a negative effect on equilibrium wages. This section provides some suggestive empirical evidence in support of this (predicted) negative relationship between occupational segregation and wages.

1.5.1 Data

The data we use was obtained from the Current Population Survey (CPS). The CPS is a monthly survey of households conducted by the Bureau of Census for the US Bureau of Labor Statistics (BLS). It provides a comprehensive body of data on the labour force, employment, earnings, and other demographic and labour force characteristics. We extracted national-level data on employment and median weekly earnings of full-time wage and salary workers by detailed occupation, decomposed by race and Hispanic or Latino ethnicity. Using this raw data, we constructed a panel of 263 occupations over the 2003-2010 period.[11] In particular, we collected data on median weekly wages, the total number of employed workers, and the proportion of blacks, asians and hispanics employed in each occupation. Table 1.1 provides a description of the key variables and summary statistics are presented in Table 1.2.

Using this data, we constructed a measure of occupational segregation (denoted by $Seg_{j,t}$) based on Duncan's [31] Index of Dissimilarity. This index measures the proportion

[10] In addition to Buhai and van der Leij [16], recent work by Bowles, et al [15] and Kim [46] also examines the conditions under which inequality can emerge and persist between *ex ante* identical social groups in a competitive market environment.

[11] The classification system for occupations is derived from the Standard Occupational Classification (SOC). Broad classifications are aggregated into minor groups, which are in turn aggregated into major groups. We disregard major and minor groups so as to restrict attention to the most disaggregated level of occupational classifications.

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Table 1.1: Description of Variables

Variable	**Description**
$Wage_{j,t}$	Median weekly earnings (in US dollars) of full-time wage and salary workers employed in occupation j in year t.
$Emp_{j,t}$	Total number (in thousands) of full-time wage and salary workers in occupation j in year t.
$bProp_{j,t}$	Proportion (in %) of full-time wage and salary Black or African American workers employed in occupation j in year t.
$aProp_{j,t}$	Proportion (in %) of full-time wage and salary Asian workers employed in occupation j in year t.
$hProp_{j,t}$	Proportion (in %) of full-time wage and salary Hispanic or Latino workers employed in occupation j in year t.
$rProp_{j,t}$	Proportion (in %) of full-time wage and salary workers not belonging to black, asian, or hispanic race and ethnic groups in occupation j in year t.
$Seg_{j,t}$	Measure of segregation in occupation j in year t.

of workers that would need to change jobs in order for the demographic composition of workers employed in each occupation to reflect the demographic composition of the population at large. To construct the index, we obtained data from the 2010 US Census on the proportion of the population belonging to black, asian or hispanic race and ethnic groups, and calculated the proportion of the US population *not* belonging to either of these race and ethnic groups.[12,13,14] We also calculated the proportion of workers for each

[12]The Census indicates that 12.6% of the population was black, 4.8% of the population was asian, and 16.4% of the population was hispanic. The proportion of the population belonging to the excluded demographic groups was thus equal to 66.2%.

[13]The majority of the excluded demographic group consists of "white" workers (72.4% of the US population in 2010) or belonging to "some other race" (6.2% of the US population in 2010). Note, however, that the US Census divides race and ethnicity into two different categories, and Hispanics and Latinos often classify themselves as "white" or belonging to "some other race." Consequently, the excluded demographic group in the sample consists of all "white" or "other race" workers whose ethnicity is neither Hispanic nor Latino.

[14]As a robustness test, we also ran the regression model using sample means rather than population

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Table 1.2: Summary Statistics

Variable	Mean	Std. Dev.	N
Wage ($)	764	301.47	2018
Emp (x1000)	348	434.99	2080
bProp (%)	10.5	6.03	2062
aProp (%)	4.8	4.56	2062
hProp (%)	13.2	9.84	2062
rProp (%)	71.5	12.76	2062
Seg (%)	14.2	7.09	2062

occupation j in each year t *not* belonging to black, asian or hispanic race and ethnic groups (denoted by $rProp_{j,t}$). An index of occupational segregation was then constructed for each race, ethnic and residual group as the deviation of the occupation-specific proportion from the respective proportion in the population. Formally,

$$xSeg_{j,t} = |(\text{ \% of } x \text{ in the population}) - xProp_{j,t}|, \quad \forall x \in \{b, a, h, r\}$$

The composite index of occupational segregation was then calculated as follows

$$Seg_{j,t} = \frac{1}{2} \sum_{x \in \{b,a,h,r\}} xSeg_{j,t}$$

As an illustration, consider the occupation of Chief Executives in 2010. The data indicates that the proportion of black, asian and hispanic workers employed as Chief Executives in 2010 was equal to 2.8%, 3.2% and 4.8%, respectively. Hence, by construction, the proportion of Chief Executives belonging to the excluded group was equal to 89.2%. The deviation from the respective proportion of black, asian and hispanics in the population was equal to 9.8%, 1.6% and 11.6%. The deviation for the excluded group was equal to 23%. Thus, the total proportion of workers that would need to change jobs in order for the demographic composition of Chief Executives to reflect the demographic composition of the US population in 2010 was equal to 23%.[15]

means. None of the results changed in qualitative terms.

[15]The minimum value of the $Seg_{j,t}$ variable in the sample equals 0.4% (Transportation attendants in 2010), and the maximum value equals 44.9% (Textile and garment workers in 2007). The (within-group) coefficient of variation of the $Seg_{j,t}$ variable is equal to 7%.

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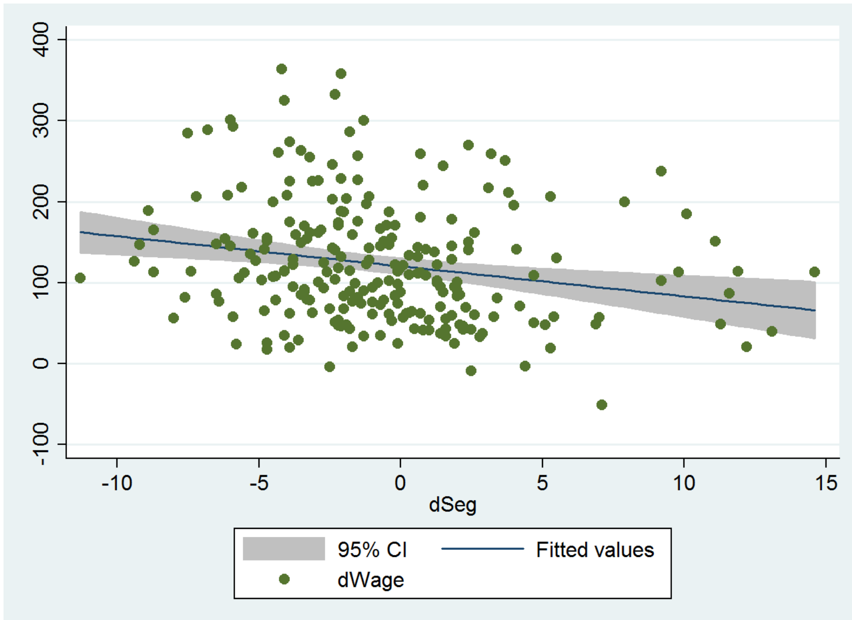

Figure 1.2: Scatter plot of the change in segregation on the change in median weekly wages.

Figure 1.2 consists of a 'naive' scatter plot of the change in the segregation index on the change in median weekly wages in each of the 263 occupations in the sample between 2003 and 2010. The figure indicates that occupations that were subject to the greatest increase in occupational segregation witnessed the most modest increase in wages over the 2003-2010 period. This suggests a negative correlation between the degree of occupational segregation within occupations and median weekly wages. Below we present the results of a simple panel regression in order to control for potential spurious time-varying factors driving this (suggested) negative correlation.

1.5.2 Econometric Specification

The econometric specification we adopt attempts to measure how changes in occupational segregation over time contribute to the observed variations in median weekly earnings within occupations. Of course, many factors aside from the degree of occupational segregation affect equilibrium wages. First and foremost among these are differences in total factor productivity across occupations. In order to control for these productivity differ-

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ences, we run a simple panel regression controlling for occupation-specific fixed effects. Year-specific fixed effects are also included in order to control from time trends in the data. The baseline regression model we consider is thus given by

$$Wage_{j,t} = \beta_0 + \beta_1 Seg_{j,t} + \beta_2 Emp_{j,t} + O_j + Y_t + \epsilon_{j,t} \tag{1.22}$$

where O_j denotes a vector of occupation dummies, and Y_t denotes a vector of year dummies. We also control for the possible wage effects of changes in the level of total employment by including the variable $Emp_{j,t}$ in the regression model. The null-hypothesis we wish to test is $H_0 : \beta_1 < 0$, implying a negative correlation between the degree of segregation within occupations and median weekly wages in those occupations.

1.5.3 Results

The results of the baseline regression model are presented in Table 1.3. We are particularly interested in the third column of Table 1.3 (i.e the specification that controls for both occupation-specific and year-specific fixed effects), even though the results are qualitatively similar across all three specifications. The estimated coefficient implies that a one standard deviation increase in the degree of occupational segregation is associated with a $10 decrease in median weekly wages. Although small in magnitude, the estimated coefficient is found to be strongly significant. Changes in the level of total employment within occupations are found to have no significant effect on wages. The results thus seem to confirm the negative correlation between occupational segregation and median weekly wages suggested graphically by Figure 1.2.

Two key assumptions of the baseline regression model are that: (i) labour productivity is identically distributed across demographic groups, and (ii) there is no wage discrimination between workers based on their race or ethnicity. If either of these two assumptions are violated in the data, then it is likely that the measured effect of occupational segregation on wages is biasedly estimated. To fix ideas, consider what would happen if workers from a specific minority group are paid lower wages on average than workers from other groups, due for example to either differences in human capital or discrimination in the labour market. In this case, increases in the proportion of workers belonging to this minority group within a given occupation will drive down wages in that occupation. If the minority workers are also over-represented in this occupation (compared to their pro-

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Table 1.3: Baseline Regression Model

Variable	**Coefficient** (Std. Err.)	**Coefficient** (Std. Err.)	**Coefficient** (Std. Err.)
Seg	**-2.156****	**-2.766****	**-1.399****
	(0.602)	(0.672)	(0.397)
Emp	0.009	0.019	-0.004
	(0.032)	(0.041)	(0.027)
Fixed Effects	No	Yes	Yes
Time Effects	No	No	Yes
N	2008	2008	2008
R^2(within)	0.015	0.015	0.529

*Note: * denotes significance at 5% level and ** at 1% level.*

portion in the population), then the regression model estimated above will consider the observed fall in wages to be driven by increases in the degree of occupational segregation. In Appendix C, we present results of a modified regression model that attempts to control for this potential "composition" bias. We argue that the negative correlation between occupational segregation and median weekly wages presented in Table 1.3 is not driven by wage differences between workers belonging to different racial/ethnic groups.

1.6 Conclusion

This paper studies how occupational segregation affects the allocation of talent in a competitive labour market. A model of occupational choice was proposed in which heterogenous workers must rely on their social contacts to acquire job vacancy information. In this environment, network effects in job search lead to occupational segregation arising in equilibrium. We showed that the equilibrium level of occupational segregation does not generally coincide with the constrained efficient level. Inefficiencies arise because workers do not internalise how their individual occupational choice decisions affect either: (i) the job finding probability of other individuals belonging to their social network, or (ii) the average productivity of labour across occupations. More specifically, we found that the

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divergence between the equilibrium and optimal allocations arises due to the presence of a (positive) search externality and a (negative) pecuniary externality. Which of these two effects dominates depends on the properties of the job search and ability functions: whenever the job search technology is relatively inefficient and the ability of workers on the job is very sensitive to their skill-type, the level of occupational segregation supplied by the competitive market will exceed the welfare-maximising level.

This normative analysis was complemented with an empirical investigation of one of the model's key positive implications: *ceteris paribus*, increases in the degree of occupational segregation should lead to a decrease in observed equilibrium wages due to the lowering of average labour productivity engendered by a misallocation of talent in the labour market. Using CPS data, we found evidence of a negative correlation between the degree of segregation within occupations and median weekly wages in the United States over the 2003-2010 period. Notwithstanding the inherent limitations of the dataset, we argued that this negative correlation is not driven by wage differences between workers belonging to different racial/ethnic groups.

As a clarifying remark, this paper does not wish to argue that gender-based or race-based discrimination is not an important cause of observed occupational segregation. Rather, the model developed here seeks to highlight another channel through which occupational segregation can arise in competitive labour markets. To this extent, it serves to underline the view that even in societies in which the level of overt discrimination is on the wane (due to either broad societal changes or more specific political causes), there is reason to believe that occupational segregation will persist as a salient feature of labour markets. What is more, the conclusions presented above indicate that in economic environments in which (i) social contacts play an important role in allocating workers to vacancies, and (ii) workers' productivity is private information, occupational segregation may lead to an inefficient allocation of talent in the labour market. This suggests that some form of policy intervention in labour markets may be justified, even though a detailed discussion of the specific features of such policy intervention lies beyond the scope of this paper.[16]

[16]Marimon and Zilibotti [56] examine how unemployment insurance affects the allocation of talent and the level of unemployment in a search-and-matching model of the labour market. See also Acemoglu and Shimer [2], [3].

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1.7 Appendix A: Homogenous Abilities

Result A1: When worker ability is homogenous, complete occupational segregation can be supported in equilibrium for sufficiently high values of the inbreeding bias.

Proof: Normalising the productivity of labour to unity, we have that $z_A(\theta) = z_B(\theta) = 1$ $\forall \theta \in \Theta$. The Cobb-Douglas production technology implies that wages are equal to

$$w_A = p_A = \frac{1}{2}\sqrt{\frac{y_B}{y_A}} \quad \text{and} \quad w_B = p_B = \frac{1}{2}\sqrt{\frac{y_A}{y_B}}$$

Utility maximisation implies that the threshold skill-type is implicitly determined by the following conditions

$$\hat{\theta}^X = \frac{1 + k^{-1}\Delta_\phi E[w]^X}{2}, \quad \forall X \in \{R, G\}$$

Notice that

$$\lim_{k\to\infty} \hat{\theta}^X(k) = \frac{1}{2}, \quad \forall X \in \{R, G\}$$

implying that as specialisation costs become arbitrarily large, the unique equilibrium threshold profile is $(\hat{\theta}^R, \hat{\theta}^G) = (1/2, 1/2)$. The difference in expected wages across occupation is given by

$$\Delta_\phi E[w]^X = \left(\alpha\hat{\theta}^X + (1-\alpha)\hat{\theta}^{X'}\right)\frac{1}{2}\sqrt{\frac{y_B}{y_A}} - \left(\alpha(1-\hat{\theta}^X) + (1-\alpha)(1-\hat{\theta}^{X'})\right)\frac{1}{2}\sqrt{\frac{y_A}{y_B}}$$

The linearity of the job search function, together with the assumption that skill-types are uniformly distributed, imply that we can write the labour supply (and thus the input demand) functions as

$$y_A = \sum_{X\in\{R,G\}} \left(\alpha\hat{\theta}^X + (1-\alpha)\hat{\theta}^{X'}\right)\hat{\theta}^X$$
$$y_B = \sum_{X\in\{R,G\}} \left(\alpha(1-\hat{\theta}^X) + (1-\alpha)(1-\hat{\theta}^{X'})\right)(1-\hat{\theta}^X)$$

These conditions together define a threshold equilibrium for this economy. Recall that complete occupational segregation is defined as a threshold profile such that $(\hat{\theta}^R, \hat{\theta}^G) \in$

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$\{(1,0),(0,1)\}$. Since the problem is symmetric, it suffices to show that one of these threshold profiles constitutes an equilibrium. We focus attention on the case where $(\hat{\theta}^R, \hat{\theta}^G) = (1,0)$, and proceed to prove the claim by construction. In this case, the labour supply of workers across occupations is equal and given by $l_A = l_B = \alpha$. Wages across occupations are also equal and given by $w_A = w_B = 1/2$. It follows that the difference in expected wages for workers belonging to group $X \in \{R, G\}$ equals

$$\Delta E[w]^R = \alpha - \frac{1}{2} \quad \text{and} \quad \Delta E[w]^G = \frac{1}{2} - \alpha$$

Plugging these equations into the optimality condition given above, we find that for the equilibrium to exist the following two inequalities must be satisfied

$$\hat{\theta}^R = \frac{1 + k^{-1}\left(\alpha - \frac{1}{2}\right)}{2} \geq 1$$
$$\hat{\theta}^G = \frac{1 + k^{-1}\left(\frac{1}{2} - \alpha\right)}{2} \leq 0$$

Solving this system of inequalities, we find that complete occupational segregation can be supported in equilibrium whenever $\alpha \geq \underline{\alpha}^{EQ} = \frac{1}{2} + k$, for $k < 1/2$. $\square$

Result A2: When worker ability is homogenous, if complete occupational segregation can be supported in equilibrium then it is also efficient.

Proof: With linear cost functions, the welfare function (1.15) becomes

$$W = y_A^{1/2} y_B^{1/2} - k \sum_{X \in \{R,G\}} \left((\hat{\theta}^X)(\hat{\theta}^X - 1) + \frac{1}{2} \right)$$

From Lemma 3, we know that we can restrict attention to symmetric allocations such that $\hat{\theta}^R = 1 - \hat{\theta}^G$. The social welfare function can then be rewritten as follows

$$W(\hat{\theta}; \alpha, k) = \alpha(1 - 2\hat{\theta})^2 - 2\hat{\theta}(\hat{\theta} - 1) - 2k \left(\hat{\theta}(\hat{\theta} - 1) + \frac{1}{2} \right)$$

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Twice differentiating the social welfare function with respect to $\hat{\theta}$ we obtain

$$\frac{d^2W(\hat{\theta};\alpha,k)}{d\hat{\theta}^2} = 8\alpha - 4k - 4 \lesseqgtr 0$$

It follows that the social welfare function is either globally concave or globally convex, depending on the values of α and k. Global convexity of the social welfare function implies that complete occupational segregation is efficient. Thus, the cut-off value of the inbreeding bias that determines whether or not complete occupational segregation is efficient is equal to $\alpha \geq \alpha^{SW} = \frac{1}{2}(1+k)$ for $k < 1$. It follows immediately that $\underline{\alpha}^{EQ} > \underline{\alpha}^{SW}$. $\square$

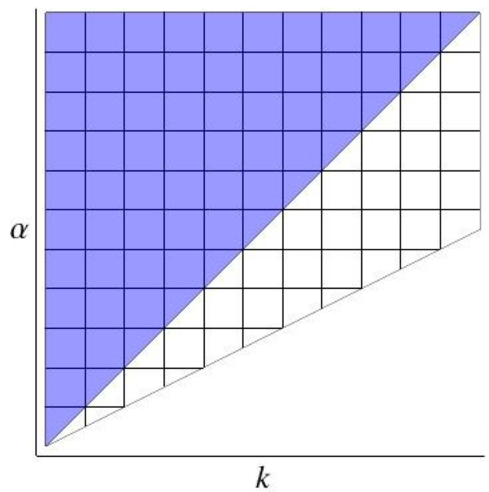

Figure 1.3: Complete Occupational Segregation (homogenous ability) - equilibrium (shaded) and efficient (checkered).

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A graphical depiction of this last result is presented in Figure 1.3. The figure shows that the set of parametre values $(\alpha, k) \in (1/2, 1) \times \mathbb{R}_{++}$ under which complete occupational segregation can be supported in equilibrium is a strict subset of those needed for complete occupational segregation to be efficient.

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1.8 Appendix B: Proofs

1.8.1 Proof of Lemma 1

Since the problem is symmetric for workers of both groups, it suffices to show that the claim holds for workers of one group. Hence, without loss of generality, we restrict attention to workers belonging to group $X = R$. For notational simplicity, we denote the difference in expected wages across occupations by $\Delta_\phi E[w]^X = q(\eta_A^X)w_A - q(\eta_B^X)w_B$. Consider first the case of a candidate equilibrium strategy profile $(\sigma^R(\theta), \sigma^G(\theta))_{\theta\in\Theta}$ such that $U_A^R(\theta) > U_B^R(\theta)$ for some $\theta \in \Theta$ and $U_A^R(\theta') < U_B^R(\theta')$ for some $\theta' \in \Theta$. Then, utility maximisation implies that we must have $\Delta_\phi c_\phi(\theta) < \Delta_\phi E[w]^R < \Delta_\phi c_\phi(\theta')$. Since $c_A(\theta)$ is increasing in θ and $c_B(\theta)$ is decreasing in θ, this implies that $\theta' > \theta$. Moreover, by continuity and monotonicity of the functions $c_A(\theta)$ and $c_B(\theta)$ it must be that $\exists!\bar{\theta} \in \Theta$ such that $U_A^R(\bar{\theta}) = U_B^R(\bar{\theta})$. Since $U_A^R(\theta)$ is decreasing in θ while $U_B^R(\theta)$ is increasing in θ, it must be that $U_A^R(\theta) < U_B^R(\theta)\ \forall\theta > \bar{\theta}$ while $U_A^R(\theta) > U_B^R(\theta)\ \forall\theta < \bar{\theta}$. This implies that in equilibrium we must have $\sigma^R(\theta) = 1\ \forall\theta < \bar{\theta}$ and $\sigma^R(\theta) = 0\ \forall\theta > \bar{\theta}$. But this is just the definition of a threshold strategy. The argument easily extends to the case of candidate equilibrium strategy profiles where either $U_A^R(\theta) > U_B^R(\theta)\ \forall\theta \in \Theta$, so that $\sigma^R(\theta) = 1\ \forall\theta \in \Theta$, or $U_A^R(\theta) < U_B^R(\theta)\ \forall\theta \in \Theta$, so that $\sigma^R(\theta) = 0\ \forall\theta \in \Theta$. $\square$

1.8.2 Proof of Lemma 2

We prove the claim by construction. We begin by showing that this candidate threshold profile implies that labour supply is constant and equal across occupations. Specifically, $l_A(1/2, 1/2) = l_B(1/2, 1/2) = q(1/2)$. Moreover, the symmetry of the productivity functions implies that the labour supply in efficiency units (and thus wages and prices) will also be constant across occupations. It follows that the difference in expected wages $\Delta_\phi E[w]^X$ will be the same across groups and equal to zero, so that $\Delta_\phi E[w]^X(1/2, 1/2) = \Delta E[w]^{X'}(1/2, 1/2) = 0$. By the symmetry of the cost functions, we have that $c_A(1/2) = c_B(1/2)$. As this implies that the indifference condition $\Delta_\phi E[w]^X = \Delta c_\phi(\hat{\theta}^X)$ is satisfied, it confirms the claim that the threshold profile $\hat{\theta}^X = 1/2$ for all $X \in \{R, G\}$ constitutes an equilibrium. $\square$

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1.8.3 Proof of Lemma 3

The claim follows from the symmetry assumptions imposed on the cost, productivity and production functions. Since the social welfare function is additively separable, we prove the claim by demonstrating that the production function $f(\cdot)$ reaches its maximum value and the aggregate cost function $C(\cdot)$ reaches its minimum value on the line defined by $\hat{\theta}^R = 1-\hat{\theta}^G$. Without loss of generality, we can write $\hat{\theta}^R = x + \hat{\theta}^G$, where $x \in [-1, 1]$. Substituting this condition into the cost functions yields $C(\hat{\theta}^R, \hat{\theta}^G) = C(\hat{\theta}^G; x)$. Differentiating this function with respect to $\hat{\theta}^G$ and using the symmetry assumption $c_A(\theta) = c_B(1-\theta)$ we obtain

$$\left(c_A(\hat{\theta}^G) - c_A(1-\hat{\theta}^G)\right) + \left(c_A(x+\hat{\theta}^G) - c_A(1-x-\hat{\theta}^G)\right) = 0$$

It is easily verified that this condition holds if $\hat{\theta}^G = (1-x)/2$. To verify that this is indeed a unique minimum, notice that

$$\frac{dC^2(\hat{\theta}^G; x)}{d\hat{\theta}^{G2}} = c_A'(\hat{\theta}^G) + c_A'(1-\hat{\theta}^G) + c_A'(x+\hat{\theta}^G) + c_A'(1-x-\hat{\theta}^G) > 0$$

This inequality follows from the assumption that the function $c_A(\cdot)$ is monotonically increasing, implying that the cost function $C(\hat{\theta}^G; x)$ is globally convex. Turning now to the production function $f(\cdot)$, notice that the symmetry assumption implies that the maximum must satisfy the following condition

$$\frac{\partial f(y_A, y_B)}{\partial y_A} = \frac{\partial f(y_A, y_B)}{\partial y_B} \Rightarrow y_A = y_B \Rightarrow \tilde{l}_A = \tilde{l}_B$$

Using conditions (1.10)-(1.11), together with $\hat{\theta}^R = x + \hat{\theta}^G$, this last equality implies

$$\begin{aligned} &q(\hat{\theta}^G + \alpha x) \int_0^{\hat{\theta}^G + x} z_A(\theta) d\theta + q(\hat{\theta}^G + (1-\alpha)x) \int_0^{\hat{\theta}^G} z_B(\theta) d\theta = \\ &q(1-\hat{\theta}^G - \alpha x) \int_{\hat{\theta}^G + x}^1 z_A(\theta) d\theta + q(1-\hat{\theta}^G - (1-\alpha)x) \int_{\hat{\theta}^G}^1 z_B(\theta) d\theta \end{aligned}$$

It can be verified that this condition is again satisfied if and only if $\hat{\theta}^G = (1-x)/2$. The final step of the proof simply requires to notice that since $x \in [-1, 1]$, the locus of points defined by the condition $\hat{\theta}^G = (1-x)/2$ is nothing more than the line $\hat{\theta}^R = 1 - \hat{\theta}^G$. □

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1.8.4 Proof of Result 1

The proof follows closely the proof of Result A1. Again, without loss of generality, we focus on the case where $(\hat{\theta}^R, \hat{\theta}^G) = (1, 0)$. To simplify notation, we set $h = 1$. As in the case with homogeneous productivity, complete occupational segregation implies that the labour supply of workers across occupations is equal so that $l_A = l_B = \alpha$. Given the linear productivity functions, the labour supply in efficiency units equals $\tilde{l}_A = \tilde{l}_B = \alpha/2$ and prices equal $p_A = p_B = 1/2$. This implies that wages across occupations are equal to $w_A = w_B = 1/4$. The difference in expected wages for workers belonging to group $X \in \{R, G\}$ is thus given by

$$\Delta E[w]^R = \frac{\alpha}{2} - \frac{1}{4} \quad \text{and} \quad \Delta E[w]^G = \frac{1}{4} - \frac{\alpha}{2}$$

Plugging these equations into the optimality conditions outlined in the proof of Result A1 and solving the resulting system of inequalities, we find that the cut-off value of the inbreeding bias above which complete occupational segregation can be supported in equilibrium to be given by $\alpha \geq \underline{\alpha}^{EQ} = \frac{1}{2} + 2k$, for $k < 1/4$. Following [11], [16], [60] the stability concept we use is based on a standard myopic tatonnement adjustment process. That is, we consider stable equilibria to be the stationary points of the following system of (ordinary) differential equations

$$\dot{\hat{\theta}}^X = \Delta_\phi E[w]^X(\hat{\theta}^X, \hat{\theta}^{X'}) - \Big(c_A(\hat{\theta}^X) - c_B(\hat{\theta}^X)\Big), \quad \forall X \in \{R, G\}$$

This leads us to the following definition

Definition A1: An equilibrium threshold profile $(\hat{\theta}^{R*}, \hat{\theta}^{G*})$ is stable if and only if

$$\left.\frac{\partial \Delta_\phi U^X(\hat{\theta}^X, \hat{\theta}^{X'})}{\partial \hat{\theta}^X}\right|_{(\theta^X, \theta^{X'}) = (\theta^{R*}, \theta^{G*})} < 0, \quad \forall X \in \{R, G\}$$

and

$$\det(J(\hat{\theta}^{R*}, \hat{\theta}^{G*})) \equiv \begin{vmatrix} \frac{\partial \Delta_\phi U^R(\hat{\theta}^{R*}, \hat{\theta}^{G*})}{\partial \hat{\theta}^R} & \frac{\partial \Delta_\phi U^R(\hat{\theta}^{R*}, \hat{\theta}^{G*})}{\partial \hat{\theta}^G} \\ \frac{\partial \Delta_\phi U^G(\hat{\theta}^{R*}, \hat{\theta}^{G*})}{\partial \hat{\theta}^R} & \frac{\partial \Delta_\phi U^G(\hat{\theta}^{R*}, \hat{\theta}^{G*})}{\partial \hat{\theta}^G} \end{vmatrix} > 0$$

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Begin by considering the threshold profiles $(\hat{\theta}^{R*}, \hat{\theta}^{G*}) = \{(1,0),(0,1)\}$. Calculating the entries of the (symmetric) Jacobian matrix, we obtain

$$\frac{\partial \Delta_\phi U^R(\hat{\theta}^{R*}, \hat{\theta}^{G*})}{\partial \hat{\theta}^R} = \frac{\partial \Delta_\phi U^G(\hat{\theta}^{R*}, \hat{\theta}^{G*})}{\partial \hat{\theta}^G} = \frac{\alpha - 8k\alpha - 1}{4\alpha}$$
$$\frac{\partial \Delta_\phi U^R(\hat{\theta}^{R*}, \hat{\theta}^{G*})}{\partial \hat{\theta}^G} = \frac{\partial \Delta_\phi U^G(\hat{\theta}^{R*}, \hat{\theta}^{G*})}{\partial \hat{\theta}^R} = \frac{\alpha - 1}{4\alpha}$$

The determinant of the Jacobian matrix is thus equal to

$$\det(J(\hat{\theta}^{R*}, \hat{\theta}^{G*})) = 4k^2 - \left(1 - \frac{1}{\alpha}\right) k$$

Applying the conditions outlined in Definition A1, we find that complete occupational segregation is stable if and only if

$$\alpha \geq \underline{\alpha}^{EQ} = \frac{1}{2} + 2k$$

which is precisely the condition needed for occupational segregation to be supported in equilibrium. Performing the same exercise for the non-segregated threshold profile $(\hat{\theta}^{R*}, \hat{\theta}^{G*}) = (1/2, 1/2)$, we have that the entries of the Jacobian matrix are given by

$$\frac{\partial \Delta_\phi U^R(\hat{\theta}^{R*}, \hat{\theta}^{G*})}{\partial \hat{\theta}^R} = \frac{\partial \Delta_\phi U^G(\hat{\theta}^{R*}, \hat{\theta}^{G*})}{\partial \hat{\theta}^G} = \frac{3\alpha - 8k - 3}{4}$$
$$\frac{\partial \Delta_\phi U^R(\hat{\theta}^{R*}, \hat{\theta}^{G*})}{\partial \hat{\theta}^G} = \frac{\partial \Delta_\phi U^G(\hat{\theta}^{R*}, \hat{\theta}^{G*})}{\partial \hat{\theta}^R} = -\frac{3\alpha}{4}$$

The determinant of the Jacobian matrix in this case is equal to

$$\det(J(\hat{\theta}^{R*}, \hat{\theta}^{G*})) = \frac{(3 + 8k)(3 + 8k - 6\alpha)}{16}$$

It is easily verified that no values of α satisfy the conditions set out in Definition A1 when $\alpha \geq \underline{\alpha}^{EQ}$ (so that complete occupational segregation is also an equilibrium). □

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1.8.5 Proof of Result 2

Again from Lemma 3, we can restrict attention to symmetric allocations such that $\hat{\theta}^R = 1 - \hat{\theta}^G$. The social welfare function is then given by

$$W = \frac{1}{2}\left(\alpha(1-2\hat{\theta})^2 - 3\hat{\theta}(\hat{\theta}-1)\right) - 2k\left(\hat{\theta}(\hat{\theta}-1) + \frac{1}{2}\right)$$

As in the proof of Result A2, we proceed by twice differentiating the social welfare function with respect to $\hat{\theta}$, which yields

$$\frac{d^2W(\hat{\theta};\alpha,k)}{d\hat{\theta}^2} = 4\alpha - 4k - 3 \lesseqgtr 0$$

Again, it follows that the social welfare function is either globally concave or globally convex depending, on the values of α and k. Global convexity of the social welfare function implies that complete occupational segregation is efficient. In this case, the cut-off value of the inbreeding bias that determines whether or not complete occupational segregation is efficient is equal to $\alpha \geq \alpha^{SW} = \frac{3}{4} + k$ for $k < 1/4$. It is immediate to verify that $\underline{\alpha}^{SW} > \underline{\alpha}^{EQ}$. $\square$

1.8.6 Proof of Lemma 4

In order to guarantee the existence of an interior equilibrium, we must impose some restrictions on the parametre space.

Assumption B1: The inbreeding bias parametre is such that

$$\frac{1}{2} + \frac{c_A'(1/2)}{2q'(\cdot)\int_0^{1/2} z_A(\theta)d\theta} < \alpha < \frac{1}{2} + \frac{c_A(1) - c_A(0)}{\int_0^1 z_A(\theta)d\theta}$$

Given the symmetry restriction, the indifference condition reads

$$I(\hat{\theta}) \equiv w(\theta)q\left((2\alpha-1)(2\hat{\theta}-1)\right) - \left(c_A(\hat{\theta}) - c_B(\hat{\theta})\right) = 0$$

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where

$$w(\hat{\theta}) \equiv p_\phi\left(\frac{\tilde{l}_\phi}{l_\phi}\right) = \frac{1}{2}\frac{q(\eta_A)\int_0^{\hat{\theta}} z_A(\theta)d\theta + q(\eta_B)\int_{\hat{\theta}}^1 z_B(\theta)d\theta}{q(\eta_A)\hat{\theta} + q(\eta_B)(1-\hat{\theta})}, \quad \forall\phi \in \{A, B\}$$

Notice that since the non-segregated threshold profile is always an equilibrium, we have that $I(1/2) = 0$. By Assumption 4, we also have $I(1) < 0$. Differentiating the indifference condition with respect to $\hat{\theta}$, we obtain

$$I'(\hat{\theta}) = w'(\hat{\theta})q\left((2\alpha-1)(2\hat{\theta}-1)\right) + 2w(\hat{\theta})q'(\cdot)(2\alpha-1) - \left(c'_A(\hat{\theta}) - c'_B(\hat{\theta})\right)$$

Again, from Assumption 4 we have that $I'(1/2) > 0$. To prove the claim, it thus suffices to show that the function $I(\hat{\theta})$ is globally concave. Differentiating again with respect to $\hat{\theta}$ yields

$$I''(\hat{\theta}) = w''(\hat{\theta})q\left((2\alpha-1)(2\hat{\theta}-1)\right) + 4w'(\hat{\theta})q'(\cdot)(2\alpha-1) - \left(c''_A(\hat{\theta}) - c''_B(\hat{\theta})\right) < 0$$

where the inequality follows from the properties of the wage function $w(\hat{\theta})$ (see Proposition 1) and Assumption 1. □

1.8.7 Proof of Proposition 1

Given the structure of the wage function $w(\hat{\theta})$, we have that

$$w'(\hat{\theta}) < 0 \Longleftrightarrow \tilde{l}'(\hat{\theta})l(\hat{\theta}) < l'(\hat{\theta})\tilde{l}(\hat{\theta})$$

where

$$l'(\hat{\theta}) = (q(\eta_A) - q(\eta_B)) + q'(\cdot)(2\alpha-1)(2\hat{\theta}-1)$$

and

$$\tilde{l}'(\hat{\theta}) = (q(\eta_A)z_A(\hat{\theta}) - q(\eta_B)z_B(\hat{\theta})) + q'(\cdot)(2\alpha-1)\left(\int_0^{\hat{\theta}} z_A(\theta)d\theta - \int_{\hat{\theta}}^1 z_B(\theta)d\theta\right)$$

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For notational simplicity, we write $l'(\hat{\theta})\tilde{l}(\hat{\theta}) = A_1(\hat{\theta}) + B_1(\hat{\theta})$ where

$$A_1(\hat{\theta}) = q(\eta_A)^2 \int_0^{\hat{\theta}} z_A(\theta)d\theta - q(\eta_B)^2 \int_{\hat{\theta}}^1 z_B(\theta)d\theta - q(\eta_A)q(\eta_B)\left(\int_0^{\hat{\theta}} z_A(\theta)d\theta - \int_{\hat{\theta}}^1 z_B(\theta)d\theta\right)$$

and

$$B_1(\hat{\theta}) = q'(\cdot)(2\alpha - 1)(\hat{\theta} - (1-\hat{\theta}))\left(q(\eta_A)\int_0^{\hat{\theta}} z_A(\theta)d\theta + q(\eta_B)\int_{\hat{\theta}}^1 z_B(\theta)d\theta\right)$$

Similarly, we write $\tilde{l}'(\hat{\theta})l(\hat{\theta}) = A_2(\hat{\theta}) + B_2(\hat{\theta})$ where

$$A_2(\hat{\theta}) = q(\eta_A)^2 z_A(\hat{\theta})\hat{\theta} - q(\eta_B)^2 z_B(\hat{\theta})(1-\hat{\theta}) - q(\eta_A)q(\eta_B)\left(z_B(\hat{\theta})\hat{\theta} - z_A(\hat{\theta})(1-\hat{\theta})\right)$$

and

$$B_2(\hat{\theta}) = q'(\cdot)(2\alpha - 1)\left(q(\eta_A)\hat{\theta} + q(\eta_B)(1-\hat{\theta})\right)\left(\int_0^{\hat{\theta}} z_A(\theta)d\theta - \int_{\hat{\theta}}^1 z_B(\theta)d\theta\right)$$

We begin by demonstrating that $B_1(\hat{\theta}) > B_2(\hat{\theta})$. After simplifying, this condition implies

$$\hat{\theta}\int_{\hat{\theta}}^1 z_B(\theta)d\theta > (1-\hat{\theta})\int_0^{\hat{\theta}} z_A(\theta)d\theta$$

Rearranging and using the fact that the productivity functions are symmetric yields

$$\left(\frac{2\hat{\theta}-1}{1-\hat{\theta}}\right)\frac{z_A(1-\hat{\theta})}{z_A(1-\hat{\theta})} > \frac{\int_{1-\hat{\theta}}^{\hat{\theta}} z_A(\theta)d\theta}{\int_0^{1-\hat{\theta}} z_A(\theta)d\theta}, \quad \forall\hat{\theta} \in (1/2, 1)$$

where the inequality follows from the fact that $z_A(\theta)$ is monotonically decreasing. We next show that $A_1(\hat{\theta}) > A_2(\hat{\theta})$. Since $z_A(\cdot)$ is monotonically decreasing, we must have

$$\left(\int_0^{\hat{\theta}} z_A(\theta)d\theta - z_A(\hat{\theta})\hat{\theta}\right) - \left(\int_0^{1-\hat{\theta}} z_A(\theta)d\theta - z_A(1-\hat{\theta})(1-\hat{\theta})\right) > 0$$

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and

$$\int_{1-\hat{\theta}}^{\hat{\theta}} z_A(\theta)d\theta < z_A(1-\hat{\theta})\hat{\theta} - z_A(\hat{\theta})(1-\hat{\theta})$$

This completes the proof. □

1.8.8 Proof of Lemma 5

To guarantee that the efficient allocation is interior, we must impose some restrictions on the parametre set.

Assumption B2: The inbreeding bias parametre is such that

$$\frac{1}{2} + \frac{c_A'(1/2)}{2q'(\cdot)z_A(1/2)} - \frac{z_A'(1/2)}{8q'(\cdot)z_A(1/2)} < \alpha < \frac{1}{2} + \frac{c_A(1) - c_A(0)}{\int_0^1 z_A(\theta)d\theta} + \frac{(1-\alpha)z_A(0) - \alpha z_A(1)}{2q'(\cdot)\int_0^1 z_A(\theta)d\theta}$$

Differentiating the social welfare function (1.18) with respect to $\hat{\theta}$ yields the following first-order condition

$$E(\hat{\theta}) \equiv \left(q(\eta_A)z_A(\hat{\theta}) - q(\eta_B)z_B(\hat{\theta})\right) + q'(\cdot)(2\alpha-1)\left(\int_0^{\hat{\theta}} z_A(\theta)d\theta - \int_{\hat{\theta}}^1 z_B(\theta)d\theta\right) - 2\left(c_A(\hat{\theta}) - c_B(\hat{\theta})\right)$$

Begin by noticing that $E(1/2) = 0$. By Assumption 5, we have that $E(1) < 0$. Differentiating the first-order condition with respect to $\hat{\theta}$, we have that

$$E'(\hat{\theta}) = \left(q(\eta_A)z_A'(\hat{\theta}) - q(\eta_B)z_B'(\hat{\theta})\right) + 2q'(\cdot)(2\alpha-1)\left(z_A(\hat{\theta}) + z_B(\hat{\theta})\right) - 2\left(c_A'(\hat{\theta}) - c_B'(\hat{\theta})\right)$$

Again, from Assumption 5, we have that $E'(1/2) > 0$. To prove the claim, it thus suffices to show that the function $E(\hat{\theta})$ is globally concave. Differentiating again with respect to $\hat{\theta}$ yields

$$E''(\hat{\theta}) = \left(q(\eta_A)z_A''(\hat{\theta}) - q(\eta_B)z_B''(\hat{\theta})\right) + 3q'(\cdot)(2\alpha-1)\left(z_A'(\hat{\theta}) + z_B'(\hat{\theta})\right) - 2\left(c_A''(\hat{\theta}) - c_B''(\hat{\theta})\right) < 0$$

where the inequality follows from Assumptions 1 and 2. □

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1.8.9 Proof of Proposition 2

From the proof of Proposition 1, we have that

$$w'(\hat{\theta}) = l(\hat{\theta})^{-1}\left(\frac{E(\hat{\theta})}{2} + (c_A(\hat{\theta}) - c_B(\hat{\theta})) - w(\hat{\theta})\left(q(\eta_A) - q(\eta_B)\right) - w(\hat{\theta})q'(\cdot)(2\alpha - 1)(2\hat{\theta} - 1)\right)$$

Solving this equation for $E(\hat{\theta})$ and simplifying yields

$$w(\hat{\theta})(q(\eta_A) - q(\eta_B)) - (c_A(\hat{\theta}) - c_B(\hat{\theta})) + w(\hat{\theta})q'(\cdot)(2\alpha - 1)(2\hat{\theta} - 1) + w'(\hat{\theta})l(\hat{\theta}) = 0$$

where $l(\hat{\theta}) = q(\eta_A)\hat{\theta} + q(\eta_B)(1 - \hat{\theta}) = (2\alpha - 1)(2\hat{\theta} - 1)$. □

1.8.10 Proof of Corollary 1

From the proof of Lemma 4, we know that complete occupational segregation can be supported in equilibrium whenever

$$\alpha > \underline{\alpha}^{EQ} = \frac{1}{2} + \frac{c_A(1) - c_A(0)}{\int_0^1 z_A(\theta)d\theta}$$

Similarly, from the proof of Lemma 5, we know that complete occupational segregation is efficient whenever

$$\alpha > \underline{\alpha}^{SW} = \frac{1}{2} + \frac{c_A(1) - c_A(0)}{\int_0^1 z_A(\theta)d\theta} + \frac{(1-\alpha)z_A(0) - \alpha z_A(1)}{2q'(\cdot)\int_0^1 z_A(\theta)d\theta}$$

Hence, we have that complete occupational segregation can be supported in equilibrium even though it is inefficient whenever $\underline{\alpha}^{SW} > \underline{\alpha}^{EQ}$. Rearranging this inequality, we obtain the desired condition. □

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1.9 Appendix C: Empirical Appendix

To allow for possible race or ethnic wage differentials (relative to the excluded demographic group), we control for the proportion of black, asian and hispanic workers employed in occupation j in year t. The modified regression model is given by

$$Wage_{j,t} = \beta_0 + \beta_1 Seg_{j,t} + \beta_2 Emp_{j,t} + \sum_{x \in \{b,a,h\}} \beta_x xProp_{j,t} + O_j + Y_t + \epsilon_{j,t}$$

The results of this modified regression model are presented in Table 1.4 below. Even after controlling for the proportion of workers belonging to the different demographic groups, we still find occupational segregation to have a negative and significant effect on median weekly wages.

Table 1.4: Modified Regression Model (Full Sample)

Variable	**Coefficient** (Std. Err.)	**Coefficient** (Std. Err.)	**Coefficient** (Std. Err.)
Seg	**-2.229****	**-3.092****	**-1.508****
	(0.672)	(0.628)	(0.399)
Emp	0.007	0.020	-0.004
	(0.028)	(0.038)	(0.026)
bProp	**-3.145****	-0.793	-0.610
	(0.752)	(0.796)	(0.538)
aProp	**4.950****	**3.836****	0.621
	(1.387)	(0.960)	(0.586)
hProp	-0.288	**3.287****	**-1.217****
	(0.569)	(0.651)	(0.431)
Fixed Effects	No	Yes	Yes
Time Effects	No	No	Yes
N	2008	2008	2008
R^2(within)	0.014	0.046	0.532

*Note: * denotes significance at 5% level and ** at 1% level.*

The first column results suggest that black workers are employed in occupations that pay

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Table 1.5: Modified Regression Model (Restricted Sample)

Variable	Coefficient (Std. Err.)	Coefficient (Std. Err.)	Coefficient (Std. Err.)
Seg	**-4.125****	**–3.709****	**-2.03****
	(1.384)	(1.167)	(0.631)
Emp	0.023	0.062	-0.009
	(0.032)	(0.057)	(0.024)
bProp	**-4.458****	-0.654	-0.853
	(1.082)	(1.010)	(0.702)
aProp	**6.468****	**4.556****	0.723
	(1.594)	(1.079)	(0.669)
hProp	-2.032	**4.175****	**-2.721****
	(1.805)	(1.711)	(1.043)
Fixed Effects	No	Yes	Yes
Time Effects	No	No	Yes
N	1569	1569	1569
R^2(within)	0.027	0.060	0.559

*Note: * denotes significance at 5% level and ** at 1% level.*

lower wages, while asians are employed in occupations that pay higher wages (relative to the excluded demographic group). This is consistent with existing empirical evidence.[17] The second column results suggest that asians and hispanics were employed in occupations that witnessed higher average wage growth over the 2003-2010 period (again, relative to the excluded demographic group). Importantly, once we control for both occupation-specific and year-specific fixed effects, we find the proportion of black and asian workers to have no effect on wages. This suggests that (on average) the wage gap within occupations between blacks and asians on one hand, and the excluded demographic group on the other, is negligible.

However, an increase in the proportion of hispanic employees is found to have a negative and significant effect on wages, even after controlling for occupation-specific and year-specific fixed effects. This suggests that hispanics employed in a given occupation are paid

[17] See for example the *Statistical Abstract of the United States: 2012* published by the US Census Bureau.

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lower wages than workers belonging to the excluded demographic group. Consequently, there is a risk that "composition effects" are biasing the estimated effect of occupational segregation on wages. As long as the hispanic wage gap is constant across occupations and across time, including the proportion of hispanics employees in the regression model should suffice to control for the bias implied by such composition effects. However, if the hispanic wage gap is correlated with the measure of occupational segregation, then the estimated effect of occupational segregation on wages will be biased even in the modified regression model. For example, the estimated coefficient will be downwardly biased if the wage gap between hispanics and the excluded demographic group increases when the degree of hispanic segregation increases. Ideally, one could control for this using data on median weekly wages decomposed by race and ethnic groups. Unfortunately, the aggregate data obtained from the CPS does not provide this information. Nonetheless, we argue that we can control for this potential bias by restricting the sample. In particular, we run the modified regression model anew after dropping all observations in which the proportion of hispanic employees exceeds 16.4% (the proportion of hispanics in the population at large). By construction, this implies that none of the increase in the segregation index in the restricted sample will be caused by increases in hispanic segregation. Restricting the sample in this way should thus in principle control for the potential downward bias caused by a positive correlation between the hispanic wage gap and the measure of occupational segregation.

Summary statistics for the restricted sample (not included) are similar to those for the full sample. The regression results for the restricted sample are presented in Table 1.5 above. The coefficient on the segregation index is still negative and significant at the 1% level. This suggests that wage differences between hispanics and members of the excluded demographic group are not driving the negative correlation between occupational segregation and median weekly wages.

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Chapter 2

Competing Recruitment Channels

While economists have a lot to say about how a firm can incentivise an employee, they have far less to say about how the firm should go about finding the right employee in the first place... Little work has been done to understand how different firms approach the hiring problem and what determines firm-level heterogeneity in hiring strategies. Oyer and Schaefer (2011) [66]

2.1 Introduction

Empirical work in the sociology literature dating back to Granovetter [39] suggests that between 30% and 50% of all jobs are found through friends and/or relatives. At the same time, surveys of the job-finding methods used by workers indicate that intermediaries play an active role in modern labour markets [9],[40].[1] Notwithstanding Montgomery's [63] seminal model of social networks and labour market outcomes, we still have a very sparse understanding of how formal (market-based) and informal (network-based) recruitment channels interact.[2] More generally, as suggested by the above quotation, there is a noticeable lack of theoretical work examining the reasons why we observe so much cross-sectional heterogeneity in the recruitment methods chosen by workers and firms.

The existing literature studying competing recruitment platforms focuses almost exclusively on the interaction between a referral network and an anonymous competitive market. Consequently, there is much to be learned from extending these models to allow for a greater variety of recruitment methods. This paper proposes a model of competing recruitment channels in which heterogeneously productive workers and firms must choose to match either through a profit-maximising labour market intermediary, or through an informal referral network. We are particularly interested in understanding how the recruitment method chosen by workers and firms varies as a function of their productivity.

[1] These range from public employment agencies which usually operate on a not-for-profit basis, to private for-profit specialised intermediaries (colloquially referred to as 'head hunters').

[2] The issue of competition between formal and informal exchange mechanisms is also addressed by Kranton [48] in her influential article on reciprocal exchange.

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Contrary to existing models of job referral networks, we consider a labour market subject to matching frictions. As mentioned above, existing research restricts attention to economic environments in which firms must choose to recruit new workers either by referral or on a Walrasian labour market subject to adverse selection frictions, and focuses on how referral networks are used by firms to minimise the distortions caused by asymmetric information about workers' productivity. By construction, these models cannot address distortions on the extensive margin caused by matching frictions on the labour market. Contrary to these models, this paper allows for workers to remain unemployed and vacancies to remain unfilled in equilibrium. As a result, an interesting trade-off arises in equilibrium between the probability of matching and match quality on different recruitment platforms. The presence of matching frictions also implies that, under certain equilibrium configurations, highly productive firms and workers seek to match through the labour market intermediary rather than through the referral network. This goes against the results of the existing literature, which find that highly productive workers and firms are disproportionately likely to use referrals [22],[63]. Moreover, it is consistent with recent empirical evidence (e.g Dustmann, et al [32]) which finds that less productive firms use informal hiring channels more frequently than more productive firms.

The remainder of this paper is organised as follows. Section 2 provides a brief overview of the relevant theoretical literature. Section 3 describes the primitives of the model and the structure of the game. Section 4 solves the benchmark case where the two recruitment channels operate independently one from the other. Section 5 analyses the equilibrium properties of the full model with competing recruitment channels. Section 6 concludes.

2.2 Literature

This paper draws from two main branches of the economics literature. The first consists of formal models of job referral networks, beginning with Montgomery's [63] seminal contribution. The second consists of new developments in the study of matching markets, especially recent models of competing matchmakers in two-sided markets.

Job Referral Networks Montgomery [63] was the first to propose a formal model of job referral networks. He studied the interaction between an informal referral network and an anonymous competitive market subject to adverse selection frictions. Montgomery

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argued that a ‘homophilic’ bias underlying the formation of social ties, whereby individuals of a certain skill are disproportionately likely to form links with other similarly skilled individuals, effectively allows firms hiring by referral to costlessly screen job applicants. He found that workers sharing many links with other high-skilled workers will tend to receive higher wages in equilibrium, thus prompting him to declare that his model gave a theoretical justification to the popular claim that “it’s not *what* you know but *who* you know” that determines peoples’ labour market outcomes.

More recent papers by Casella and Hanaki [21], [22] extend Montgomery’s original model to allow for more general information channels. In particular, they allow workers who enter the anonymous labour market to invest in a costly certification technology that can be used to signal their publicly unobservable productivity. Notwithstanding the introduction of this more precise signaling technology, the authors find that the referral network is remarkably resilient, even when the signaling technology is perfectly separating. Heuristically speaking, the reason why the referral network is so robust stems from the fact that the signaling technology must acheive two contradictory objectives: in order to be informative, the signaling technology must be costly; but if the signaling technology is too costly, the network can easily undercut it. Moreover, since information transmission on the referral network is only bilaterally observable (i.e by the worker and the firm to which he is linked), firms can extract some positive surplus when recruiting workers by referral. This surplus is fully appropriated by workers on the anonymous labour market since workers’ certification decisions are publicly observable and there is free entry of firms.

Competing Matchmakers The employment agency in our model effectively plays the role of a matchmaker in a two-sided matching market. Although the study of two-sided matching markets has a long history in the economics literature [70], our approach is largely inspired by the model of a price-discriminating matchmaker developed by Damiano and Hao, Li [27]. They study the problem of a monopoly matchmaker that uses a schedule of subscription fees to sort different types of agents participating in a two-sided matching market. They extended the analysis to the case of competitive matchmakers in Damiano and Hao, Li [26], where they consider how price competition in a duopolistic market affects the equilibrium outcome. This latter paper identifies an important ‘sorting externality:’ the expected quality pool of participants in a given submarket affects players’ decisions of which matchmaker’s platform to join. Contrary to standard Bertrand price competition,

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the participation decisions of agents in these matching environments are not completely determined by prices, as they also depend on the participation decisions of agents on the other side of the market. A closely related article by Bloch and Ryder [14] studies equilibria in a marriage market game with search frictions in which men and women of heterogenous abilities must choose whether to engage in decentralised costly search, or to subscribe to the services of a profit-maximising intermediary.

Although Damiano and Hao, Li [27],[26] establish a number of important results, their models are not directly applicable to the study of employment agencies. The reason for this is that they postulate that the matchmakers can charge subscription fees to both sides of the market. By law, however, employment agencies are almost always prohibited from charging workers subscription fees. Another recent paper by Tregouet [75] develops a model of competition among matchmakers when one side of the market is exempted from payment. We model employment agencies in a way that closely resembles the approach developed by Tregouet.

2.3 The Model

2.3.1 Workers and Firms

Consider an economy populated by a continuum of risk-neutral workers and firms. Let I denote the set of workers and let J denote the set of firms, both of (Lebesgue) measure equal to two. Workers and firms are *ex ante* heterogeneous and differ in terms of a publicly unobservable characteristic, referred to as the worker's or firm's type. Formally, worker types are denoted by θ with binary support $\Theta = \{\theta_H, \theta_L\}$, where $\theta_L = 1$. Firm types are denoted by ω with binary support $\Omega = \{\omega_H, \omega_L\}$. Without loss of generality, we restrict attention to the symmetric case where $\Theta = \Omega$ and assume there to be an equal measure of high- and low-types of both workers and firms. The following assumption is imposed on the type space in order to reduce the number of possible equilibrium outcomes which need to be considered.

Assumption 1 (Sufficient Productivity Dispersion): $\theta_H > 2$ and $\omega_H > 2$.

Firms have one vacancy which they seek to fill. If filled, a match between a type θ

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worker and type ω firm produces a match value $y(\theta, \omega) = \theta\omega$ to both the worker and the firm.[3] The payoff to unemployed workers and unmatched firms is normalised to zero. We assume there to be no decentralised matching market on which workers and firms can meet. Rather, all matches take place either through a profit-maximising employment agency or through a referral network.

2.3.2 Employment Agency

The employment agency is characterised by an exogenous matching rule and a proprietary screening technology, and charges firms subscription fees in order to maximise its profits. Subscription fees are denoted by $p \in \mathbb{R}_{++}$. The set of workers that choose to join the agency are partitioned according to a screening technology which provides the agency with an informative but noisy signal of workers' true type. Let $\sigma : \Theta \to \Theta$ denote the signal observed by the agency. The signal is assumed to satisfy the following 'informativeness' criterion

$$Prob(\sigma = \theta_H | \theta = \theta_H) = Prob(\sigma = \theta_L | \theta = \theta_L) = \phi,$$
$$Prob(\sigma = \theta_H | \theta = \theta_L) = Prob(\sigma = \theta_L | \theta = \theta_H) = 1 - \phi$$

where $\phi \in [1/2, 1)$ measures the precision of the signal.

Obtaining an informative signal is costly. Specifically, in order to obtain a signal of quality ϕ, the agency must incur a cost $c : [1/2, 1) \to \mathbb{R}_+$, where $c(\cdot)$ is a $\mathcal{C}^2$ function such that $c'(\phi) > 0$ and $c''(\phi) > 0$ for all $\phi \in [1/2, 1)$, $c'(1/2) = 0$ and $\lim_{\phi \to \infty} c'(\phi) = \infty$. Importantly, we assume the employment agency commits to use a matching rule whereby firms paying the subscription fee are (randomly) matched with registered workers obtaining the high-signal only. That is, the employment agency commits to leave registered workers obtaining the low-signal unmatched with probability one. This implies that by modifying the signal quality, not only does the employment agency modify the expected type of workers that will be matched with firms, but also directly affects the matching probability

[3] Although certainly the exception rather than the rule, the assumption that utility between firms and workers is nontransferable is not new to economics. Motivating the use of the NTU assumption in labour market models, Burdett and Wright [17] argue that if a "worker is enthusiastic about a job (say, because of its location), but the employer is not so keen on the worker (say, because of his personality),... there is a limit as to what the former can do to convince the latter to hire him. For example, if the worker offers to work for a reduced wage, we might restrict this wage to be above some lower bound determined outside of the match, like a legislated minimum wage or an industrywide union-negotiated wage. In the simplest case wages and all other terms of the relationship are fixed in advance and there is nothing for the pair to negotiate after they meet."

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of workers.

2.3.3 Social Network

Instead of matching through the employment agency, workers and firms can also match through a referral network. We model the referral network as a bipartite Erdos-Renyi graph. More specifically, we assume that workers can form one 'link' to a random firm. This should be interpreted as a worker forming a link to a current employee within a firm. As in Montgomery [63], we assume this link formation process to be subject to an inbreeding bias: workers are more likely to form a link to a firm of the same type. Formally, we assume the probability that a type θ_t worker forms a link to a type ω_t firm is equal to $\alpha \in (1/2, 1)$ for all $t \in \{H, L\}$. Even though workers and firms do not observe the type of the firm or worker to which they are linked, the magnitude of the bias parametre α is common knowledge. If a firm is linked to several workers and decides to recruit by referral, we assume the firm makes a job offer to a random worker.

2.3.4 Timing

The economy described above can be modeled as an extensive-form game. The timing of the game proceeds as follows:

- **Stage 1:** The employment agency sets a subscription fee and chooses a publicly observable signal quality $(p, \phi) \in \mathbb{R}_{++} \times [1/2, 1)$.
- **Stage 2:** Workers choose whether to form a link to a firm, to (costlessly) register with the employment agency or to remain unmatched. Firms decide whether to post a referral offer, to pay the subscription fee to the employment agency or to remain unmatched.
- **Stage 3:** Workers receiving a referral offer choose whether to accept or reject the offer. The employment agency matches registered firms with workers obtaining the high-signal.

Importantly, we assume the two matching markets to be exclusive: workers and firms cannot simultaneously participate in both the agency-mediated matching market and the referral network. Given this, it is always a dominant strategy for workers receiving a referral offer to accept the offer.

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2.3.5 Matching Sets

For ease of exposition, it will be useful to define the following subsets of workers and firms. Let F denote the set of firms paying the subscription fee, W denote the set of workers registered with the employment agency, H denote the set of firms posting a referral offer, and G denote the set of workers forming a link to a firm. Furthermore, let $F_j \subseteq F$ denote the subset of firms paying the subscription fee that are of type $j \in \{H, L\}$, $W_i \subseteq W$ denote the subset of registered workers that are of type $i \in \{H, L\}$, $\tilde{W}_s \subseteq W$ denote the subset of registered workers obtaining the signal θ_s for $s \in \{H, L\}$, $H_j \subseteq H$ denote the subset of firms posting referral offers that are of type $j \in \{H, L\}$, and $G_i \subseteq G$ denote the subset of linked workers that are of type $i \in \{H, L\}$. We denote the Lebesgue measure of these subsets by the corresponding lower-case letters, e.g $f_j = \lambda(F_i)$. Vectors are denoted in bold, e.g $\mathbf{f} = (f_H, f_L)$.

2.3.6 Payoffs

This section explicitly derives the payoff functions of worker and firms depending on which recruitment channel they decide to use. We also specify the objective function of the employment agency.

Workers The payoff of a type $i \in \{H, L\}$ worker seeking a job through the employment agency will depend on the distribution of firm-types joining the employment agency. Given the participation decision of high- and low-type firms, define the expected type of firms subscribing to the agency as follows

$$\tilde{\omega} = \begin{cases} 0 & \text{if} \quad f_H = f_L = 0 \\ \frac{f_H \omega_H + f_L \omega_L}{f_H + f_L} & \text{otherwise} \end{cases}$$

The expected payoff function of a worker then equals

$$U_i^A = \underbrace{Prob(\sigma = \theta_H | \theta_i) \min\left\{1, \frac{f}{\tilde{w}_H}\right\}}_{\text{matching probability}} \tilde{\omega}\theta_i \tag{2.1}$$

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Notice that the matching probability of workers registered with the agency varies as a function of the signal quality. This is because we have assumed the employment agency commits to match firms only with high-signal workers.

Let $s_{ij} \in [0,1]$ denote the probability that a type $i \in \{H, L\}$ worker is hired through the referral network by a type $j \in \{H, L\}$ firm. The expected payoff function of a type $i \in \{H, L\}$ worker participating in the referral network is then given by

$$U_i^R = (s_{ii}\omega_i + s_{ij}\omega_j)\,\theta_i \tag{2.2}$$

Firms The payoff of a type $j \in \{H, L\}$ firm recruiting through the employment agency will depend on the type of worker it expects to be matched with if it pays the subscription fee. Define

$$\tilde{\theta}(\phi) = \begin{cases} 0 & \text{if} \quad w_H = w_L = 0 \\ \frac{\phi w_H \theta_H + (1-\phi) w_L \theta_L}{\phi w_H + (1-\phi) w_L} & \text{otherwise} \end{cases}$$

This corresponds to the expected type of a worker recruited through the employment agency, which varies as a function of the signal quality. The expected payoff function of a firm recruiting through the agency equals

$$V_j^A = \underbrace{\min\left\{1, \frac{\tilde{w}_H}{f}\right\}}_{\text{matching probability}} \tilde{\theta}(\phi)\omega_j - p \tag{2.3}$$

The matching probability implies that whenever the measure of firms paying the subscription fee is greater than the measure of workers obtaining a high-signal, firms rationally expect to remain unmatched with a strictly positive probability.

Let $r_{ij} \in [0,1]$ denote the probability that a type $j \in \{H, L\}$ firm hires a type $i \in \{H, L\}$ worker through the referral network. The expected payoff function of a type $j \in \{H, L\}$ firm recruiting through the referral network is then given by

$$V_j^R = (r_{Hj}\theta_H + r_{Lj}\theta_L)\,\omega_j \tag{2.4}$$

The probability vectors $\mathbf{r} \in [0,1]^4$ and $\mathbf{s} \in [0,1]^4$ are explicitly derived in Appendix A.

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Employment Agency The profit function of the employment agency is given by

$$\Pi = f(p, \phi)p - c(\phi) \tag{2.5}$$

where $f : \mathbb{R}_{++} \times [1/2, 1) \to [0, 2]$ denotes the 'demand' for the employment agency's services; i.e the measure of firms paying the subscription fee. The employment agency sets the subscription fee p and the signal quality ϕ in order to maximise its profits.

2.4 Separate Recruitment Channels

In this section, we consider the two benchmark cases in which only one of the two recruitment channels is active. This is intended to provide some intuition regarding the underlying incentive problem faced by the employment agency.

2.4.1 The Employment Agency without Referrals

Consider first the case where the referral network is inactive so that all workers choose to register with the employment agency and no firm posts a referral offer. Under these conditions, the participation constraints facing the employment agency are given by

$$\min\left\{1, \frac{1}{f}\right\}\tilde{\theta}(\phi)\omega_j - p \geq 0, \quad \forall j \in \{H, L\} \tag{2.6}$$

The participation constraints simply state that the payoff to firms joining the employment agency must be weakly greater than their exogenously specified outside option. Note that because all workers join the agency, the measure of workers obtaining the high-signal equals $\phi + (1 - \phi) = 1$ regardless of the signal quality chosen by the agency. The employment agency seeks to maximise (2.5) subject to (2.6). Since the agency is restricted to posting a unique subscription fee, it must choose whether to set a low price and supply the whole market or whether to set a high price and serve only high-type firms. If the employment agency were to serve both types of firms, the subscription fee would be set in order to bind the low-type firms' participation constraint

$$p_L(\phi) = \frac{1}{2}\left(\phi\theta_H + (1 - \phi)\theta_L\right)\omega_L \tag{2.7}$$

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Note that this price function also satisfies the high-type firms' participation constraint, implying that high-type firms will also subscribe to the agency's services at this price for any $\phi \in [1/2, 1)$. Maximising (2.5) subject to the price function (2.7), we find that the profit-maximising screening level in this case equals

$$\phi^* : c'(\phi^*) = \Delta\theta\omega_L \tag{2.8}$$

If, instead, the employment agency were to choose to exclude low-type firms from the market, the subscription fee would be set in order to bind the high-type firms' participation constraint

$$p_H(\phi) = (\phi\theta_H + (1-\phi)\theta_L)\omega_H \tag{2.9}$$

Note that this price function is incentive compatible: i.e, no low-type firm will have an incentive to misreport its type and subscribe to the employment agency at this price for any $\phi \in [1/2, 1)$. Given this alternative price function, the profit-maximising screening level equals

$$\phi^{**} : c'(\phi^{**}) = \Delta\theta\omega_H \tag{2.10}$$

Since the function $c(\cdot)$ is convex, this implies that $\phi^{**} > \phi^*$: i.e, the profit-maximising signal quality is greater when the employment agency excludes low-type firms from the market. Also, notice that since $c(1/2) = 0$, there always exists a $\phi \in [1/2, 1)$ such that the profit-maximising fee-signal pair (p, ϕ) satisfies the employment agency's participation constraint. The difference in profits implied by the two pricing schemes is given by

$$\Pi_H(\phi) - \Pi_L(\phi) = (\phi\theta_H + (1-\phi)\theta_L)\,\Delta\omega > 0, \quad \forall\phi \in [1/2, 1)$$

It follows that the employment agency will opt to serve only high-type firms. The reason why excluding low-type firms from the matching market turns out to be profitable for the employment agency follows directly from Assumption 1. The trade-off facing the employment agency is as follows. On the one hand, since the agency is restricted to setting a unique price, if it decides to serve the whole market it must leave substantial information rents to high-type firms. On the other hand, if the agency restricts participation to high-type firms it necessarily reduces the number of firms paying the subscription fee, thereby decreasing its revenue for any given price. When deciding whether to serve the whole

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market or to restrict participation to high-type firms, the agency therefore faces a trade-off between lost revenue on the intensive versus extensive margin. When the productivity difference between high- and low-types is large enough, the loss on the intensive margin dominates, implying that the employment agency will opt to restrict participation and only serve high-type firms. Low-type firms remain unmatched and receive their reservation payoff. High-type firms enter the agency-mediated matching market but also obtain a payoff equal to their reservation value since the employment agency extracts all the surplus. The expected payoff of high-type workers equals

$$U_H^A = \phi^{**}\omega_H\theta_H$$

where ϕ^{**} solves condition (2.10). Similarly, the expected payoff of low-type workers is equal to

$$U_L^A = (1-\phi^{**})\omega_H\theta_L$$

2.4.2 Referrals without the Employment Agency

Consider now the case where all workers form links to firms and all firms post referral offers. The expected utility of a firm recruiting through the referral network is given by

$$V_j^R = (1-e^{-1})\left(\alpha\theta_j + (1-\alpha)\theta_i\right)\omega_j, \quad \forall j \in \{H, L\} \tag{2.11}$$

The term $(1-e^{-1})$ equals the probability that a firm is linked to a worker (of any type), given that all workers join the referral network. The expected utility of workers participating in the referral network equals

$$U_i^R = (1-e^{-1})\left(\alpha\omega_i + (1-\alpha)\omega_j\right)\theta_i, \quad \forall i \in \{H, L\} \tag{2.12}$$

The term $(1-e^{-1})$ in this case also equals the probability that a worker receives a referral offer conditional on being linked to a firm, given that all workers of both types join the referral network. Firms (of both types) are unambiguously better off using the referral network compared to the employment agency's monopoly contract (recall that the agency extracts all surplus from participating firms). On the other hand, whether high- and low-type workers obtain a higher expected utility from the employment agency's monopoly

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contract or the referral network depends on the informativeness of the agency's screening technology relative to the value of the inbreeding bias. When the screening technology is very precise, low-type workers will unconditionally prefer the referral network, as they will face a very low matching probability on the agency-mediated matching market. The opposite holds true for high-type workers, who benefit (in terms of matching probability) when the employment agency uses a more precise screening technology.

2.5 Competing Recruitment Channels

We now turn to the more challenging task of analysing the interaction between the referral network and the employment agency. The equilibria of the game can be thought of in terms of competition between intermediaries in a matching market. The 'intermediaries' in question consist of firms' current employees, who effectively take on the role of informal intermediaries on the referral network, and the profit-maximising employment agency. The former have access to an exogenous screening technology parameterised by the inbreeding bias underlying the stochastic link formation process, while the latter has access to an 'endogenous' screening technology whose precision varies as a function of the agency's subscription fee. The referral decision of existing employees is non-strategic and referral hiring is costless for firms. Contrary to this, the employment agency seeks to maximise its profits and sets its subscription fee and signal quality accordingly.

2.5.1 Participation Subgame

We begin by analysing the participation subgame in Stage 2; i.e, the subgame in which workers and firms simultaneously decide which recruitment channel to use, given the subscription fee p and signal quality ϕ announced by the employment agency in Stage 1. Importantly, we restrict attention to symmetric pure-strategy equilibria in which all workers and firms of the same type choose the same strategy profile. Following Daminao and Li, Hao [27], we refer to the outcome of the Stage 2 subgame as a matching market structure (MMS). Since we are only interested in studying symmetric pure-strategy equilibria, we only need to consider six different possible matching market structures. Two of these consist of pooling MMSs, where workers and firms of both types all use the same recruitment channel (either the referral network or the agency-mediated matching market). Another

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two consist of positive assortative MMSs, where workers and firms use both recruitment channels and sort by type. The remaining two consist of negative assortative MMSs, where both recruitment channels are active in equilibrium but workers and firms of the same type choose different recruitment channels.

Let $\Gamma(p, \phi; \alpha)$ denote the strategic-form subgame induced by a given subscription fee p, signal quality ϕ and network bias α. Moreover, let $I_i^x \subseteq I$ denote the subset of type $i \in \{H, L\}$ workers choosing recruitment channel $x \in \{A, R, \varnothing\}$ and let $J_j^x \subseteq J$ denote the subset of type $j \in \{H, L\}$ firms choosing recruitment channel $x \in \{A, R, \varnothing\}$.

Definition 1: A symmetric equilibrium in pure strategies of $\Gamma(p, \phi; \alpha)$ is defined as a distribution of agents $\mathbf{g} \in \{0, 1\}^2$, $\mathbf{w} \in \{0, 1\}^2$, $\mathbf{h} \in \{0, 1\}^2$ and $\mathbf{f} \in \{0, 1\}^2$ such that for any participation decisions $x \in \{A, R, \varnothing\}$ and $x' \neq x$

- $\lambda(I_i^x) = 1$ iff $U_i^x(\mathbf{g}, \mathbf{w}, \mathbf{h}, \mathbf{f}; p, \phi, \alpha) \geq U_i^{x'}(\mathbf{g}, \mathbf{w}, \mathbf{h}, \mathbf{f}; p, \phi, \alpha), \quad \forall i \in \{H, L\}$
- $\lambda(J_j^x) = 1$ iff $V_j^x(\mathbf{g}, \mathbf{w}, \mathbf{h}, \mathbf{f}; p, \phi, \alpha) \geq V_j^{x'}(\mathbf{g}, \mathbf{w}, \mathbf{h}, \mathbf{f}; p, \phi, \alpha), \quad \forall j \in \{H, L\}$

where the functions $U_i^x(\cdot)$ and $V_j^x(\cdot)$ for $x \in \{A, R\}$ are given by conditions (2.1)-(2.4) above.

The definition of equilibrium requires workers and firms of both types to optimally choose a recruitment channel, given their beliefs about the participation decision of the other players and the type of these players. Moreover, when considering deviations from equilibrium, players treat the distribution of agents across recruitment channels as a parametre; that is, workers and firms of both types ignore the infinitesimally small effect their individual participation decision has on the aggregate matching probabilities and type distributions.

The following claims show that the set of equilibrium MMSs is generically not unique. We begin by demonstrating that a negative assortative MMS can never be supported as an equilibrium. We then move on to characterise the subset of fee-signal pairs under which the remaining pooling and positive assortative MMSs can be supported as equilibria.

Claim 1: No equilibrium of the participation subgame results in a negative assortative matching market structure.

Proof: See Appendix B.

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The claim follows from the fact that there always exists a subset of players who have an incentive to deviate in any negative assortative MMS. Broadly speaking, in the case where low-type workers match with high-type firms through the agency, and high-type workers match with low-type firms on the referral network, high-type workers will always have an incentive to deviate and join the employment agency. In the case where high-type workers match with low-type firms through the agency, and low-type workers match with high-type firms on the referral network, there does not exist a price $p \in \mathbb{R}_{++}$ such that the incentive compatibility constraints of both high-type and low-type firms are satisfied.

Cliam 2: A pooling matching market structure with all workers and firms (of both types) joining the referral network (called $P1$) can always be supported as an equilibrium of the participation subgame.

Proof: Omitted.

The claim follows immediately from Definition 1. Intuitively, it comes about due to the coordination-game structure of the participation subgame. More specifically, if no worker registers with the employment agency, firms will have no incentive to pay the subscription fee. All firms will post referral offers since the probability of being matched will be strictly greater than zero. Given this, no worker will find it profitable to deviate.

Lemma 1: A pooling matching market structure with no workers or firms (of both types) joining the referral network (called $P2$) can be supported as an equilibrium of the participation subgame providing the fee-signal pair satisfies the following condition

$$p(\phi) \notin \Bigg(\underbrace{1/2(\phi\hat{\theta} + (1-\phi))}_{PC_L}, \underbrace{(\phi\hat{\theta} + (1-\hat{\theta}))}_{IC_L}\Bigg), \quad \forall \phi \in [1/2, 1) \tag{2.13}$$

Proof: See Appendix B.

The restriction comes from the incentive compatibility constraint of low-type firms. More specifically, a subscription fee that belongs to the interval specified in condition (2.13) fails

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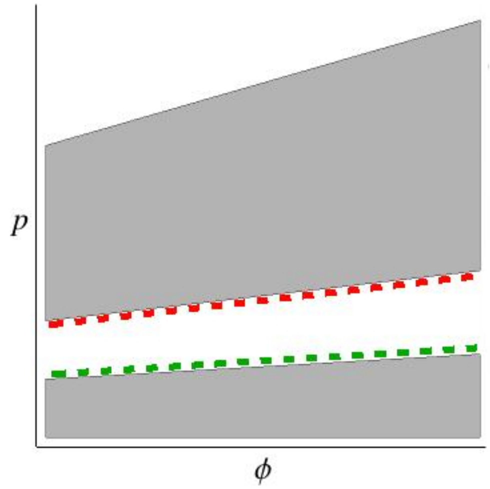

Figure 2.1: Equilibrium Set of the MMS $P2$ generated for parametre values $\hat{\theta} = 2.5$ and $\alpha = 0.9$; green: PC_L (firm), red: IC_L (firm).

to satisfy the participation constraint of low-type firms, implying that only high-type firms join the employment agency. This, however, implies a discontinuous jump in matching probabilities. Recall from Definition 1 that the participation decision of individual agents is assumed to have a negligible effect on the aggregate matching probabilities. Individual low-type firms will thus find it profitable to unilaterally deviate and join the employment agency whenever the subscription fee lies in this interval, thereby breaking any candidate equilibrium. Figure 2.1 provides a graphical representation of the price-signal pairs satisfying condition (2.13): the green line represents the participation constraint of low-type firms, while the red line represents the incentive-compatibility constraint of low-type firms.

Lemma 2: A positive assortative matching market structure with high-types joining the employment agency and low-types joining the referral network (called $S1$) can be supported as an equilibrium of the participation subgame for fee-signal pairs belonging to the set

$$\mathcal{C}^{S1} = \left\{(p,\phi) : p \in \left[e^{-\alpha}\left(1 - e^{\alpha} + e^{\alpha}\phi\hat{\theta}\right), \frac{e\phi\hat{\theta}^2 + e^{\alpha}\hat{\theta} - e\hat{\theta}}{e}\right], \phi \in \left[\frac{e^{-\alpha}(1 - e^{\alpha} + e^{\alpha}\hat{\theta})}{\hat{\theta}}, 1\right)\right\}$$

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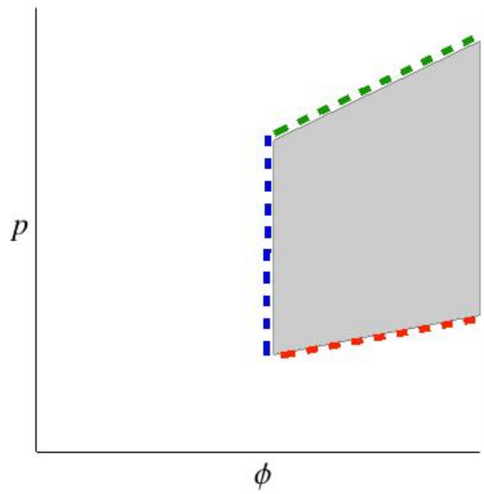

Figure 2.2: Equilibrium Set of the MMS $S1$ generated for parametre values $\hat{\theta} = 2.5$ and $\alpha = 0.9$; red: IC_L (firm), green: IC_H (firm), blue: IC_L (worker).

Proof: See Appendix B.

We provide here a a heuristic sketch of the logic underlying the formal proof. Consider first the incentives of workers. Participation in both recruitment channels is costless. Consequently, if the two channels are completely separating, then *ceteris paribus* all workers will prefer to participate in the recruitment channel chosen by high-type firms. However, if the signal quality is sufficiently high so that $\phi \geq \underline{\phi}$ the probability that a low-type worker obtains a high signal from the employment agency will be relatively low. If this probability is sufficiently low, then low-type workers will not want to deviate from the referral network, even though it is common knowledge that only low-type firms participate in the network. This incentive-compatibility constraint of low-type workers is represented graphically by the blue line in Figure 2.2. The incentives of high-type workers are more straightforward: given that their probability of matching (with a high-type firm) is high if they remain in the agency-mediated matching market, they will not want to deviate and join the referral network.

We now turn to the incentives of firms. Clearly, high-type firms will only participate in the agency-mediated matching market if the gain (in terms of match quality) outweighs

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the cost (of the subscription fee). For subscription fees that are low enough so that $p \leq \bar{p}$ high-type firms will be willing to subscribe to the agency's services. The incentive-compatibility constraint of high-type firms is represented graphically by the green line in Figure 2.2. The subscription fee cannot be too low, however, in order to avoid low-type firms from imitating high-type firms and subscribing to the employment agency as well. That is, there must also exist a lower bound for the subscription fee $p \geq \underline{p}$ such that low-type firms prefer to stay in the referral network and match with low-type workers. The incentive-compatibility constraint of low-type firms is represented by the red line in Figure 2.2. The existence of a non-empty price interval $p \in [\underline{p}, \bar{p}]$ is guaranteed by the assumed functional form of the match value function. Specifically, the supermodularity of the match value function implies that firms' return function exhibits increasing differences. This implies that high-type firms have a higher willingness-to-pay for high-type workers than low-type firms. Graphically, it implies that the slope of the incentive-compatibility constraint of high-type firms is steeper than for low-type firms.

Lemma 3: For $\alpha > \underline{\alpha}$ and $\hat{\theta} < \bar{\theta}(\alpha)$, a positive assortative matching market structure with high-types joining the referral network and low-types joining the employment agency (called $S2$) can be supported as an equilibrium of the participation subgame for fee-signal pairs belonging to the set

$$\mathcal{C}^{S2} = \left\{ (p, \phi) : p \in \left(0, \frac{e - e\hat{\theta} + e^{\alpha}\hat{\theta} - \phi e}{e} \right], \quad \phi \in \left(\frac{1}{2}, \frac{e - e\hat{\theta} + e^{\alpha}\hat{\theta}}{e} \right) \right\}$$

Proof: See Appendix B.

Once again, we provide a heuristic summary of the formal proof. The Lemma holds for sufficiently large values of the inbreeding bias and relatively small differences in productivity between high- and low-types. For sufficiently low levels of the signal quality $\phi \leq \bar{\phi}$ workers will have no incentive to deviate. The large inbreeding bias assures that high-type workers will match with high-type firms with a high probability. If they were to deviate, they would match with low-type firms with a smaller probability. Even though low-type workers prefer to match with high-type firms, for sufficiently small values of ϕ and sufficiently large values of α the probability of matching through the employment agency

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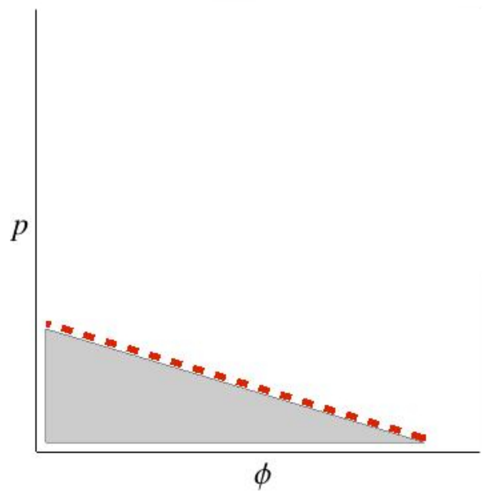

Figure 2.3: Equilibrium Set of the MMS $S2$ generated for parametre values $\hat{\theta} = 2.5$ and $\alpha = 0.9$; red: IC_L (firm).

will be so much larger than the probability of matching through the referral network that they will not want to deviate.

If the inbreeding bias is large enough, high-type firms will have no incentive to deviate and join the employment agency. The high inbreeding bias guarantees that the probability of matching in the referral network is high. If they were to deviate and join the agency, not only would they match with low- rather than high-type workers but they would also need to pay the subscription fee. Most of the structure of the equilibrium is therefore given by the low-type firms' incentive compatibility constraint. Note that in equilibrium low-type firms pay the subscription fee and match with low-type workers. If they were to deviate, they would save on the subscription fee and match with high-type workers. Consequently, in order to satisfy the incentive constraint we need to impose conditions on the signal quality, the inbreeding bias, the subscription fee and the productivity difference between high-type and low-type workers such that the expected payoff to low-type firms from staying in the agency-mediated matching market exceeds their deviation payoff. The specific conditions are given in Appendix B. Figure 2.3 provides a graphical representation of the set $S2$, where the red line depicts the incentive-compatibility constraint of low-type firms. It may seem counter-intuitive that the incentive-compatibility constraint of low-types firms is *decreasing* in the signal quality ϕ. The reason for this comes from the fact

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that only low-type workers join the employment agency. Given that the agency commits only to match registered firms with workers receiving a high signal, decreasing the signal quality improves the matching probability of firms registered with the employment agency.

2.5.2 Profit-Maximising Matching Contracts

Having identified the set of equilibria in the participation subgame for given values of the fee-signal pair (p, ϕ), we now turn to the problem of characterising the set of profit-maximising matching contracts posted by the employment agency. The analysis of the previous section indicates that the participation subgame supports multiple equilibria. Rather than imposing refinements directly on the set of equilibria, we solve the agency's profit-maximisation problem for each possible equilibrium selected by workers and firms in the ensuing subgame.

The multiplicity of equilibria in the participation subgame requires that we specify the off-equilibrium-path beliefs held by the employment agency. For simplicity, we assume that any deviation on the part of the employment agency from a candidate equilibrium matching contract has workers and firms coordinating on the pooling MMS where all workers and firms participate in the referral network. Recall that this MMS always exists for all possible parametre values, and has the employment agency obtaining zero revenue for sure. Let $\mathcal{G}(\alpha)$ denote the extensive form game given the network inbreeding bias α, and let $\beta(p, \phi) \in [0, 1]$ denote the agency's belief that no firm will pay the subscription fee when it posts the fee-signal pair (p, ϕ).

Definition 2: An agency-optimal equilibrium of the extensive game $\mathcal{G}(\alpha)$ is defined as a fee-signal pair $(p, \phi) \in \mathbb{R}_{++} \times (1/2, 1]$, a belief function $\beta : \mathbb{R}_{++} \times (1/2, 1] \to [0, 1]$, and a distribution of agents such that:

- $(p^*, \phi^*) \in \arg\max_{p,\phi} \Pi = fp - c(\phi)$
- $\beta(p, \phi) = 1$ for all $(p, \phi) \neq (p^*, \phi^*)$
- $\lambda(I_i^x) = 1$ iff $U_i^x(\mathbf{g}, \mathbf{w}, \mathbf{h}, \mathbf{f}; p^*, \phi^*, \alpha) \geq U_i^{x'}(\mathbf{g}, \mathbf{w}, \mathbf{h}, \mathbf{f}; p^*, \phi^*, \alpha), \forall x' \neq x, i \in \{H, L\}$
- $\lambda(J_j^x) = 1$ iff $V_j^x(\mathbf{g}, \mathbf{w}, \mathbf{h}, \mathbf{f}; p^*, \phi^*, \alpha) \geq V_j^{x'}(\mathbf{g}, \mathbf{w}, \mathbf{h}, \mathbf{f}; p^*, \phi^*, \alpha), \forall x' \neq x, j \in \{H, L\}$

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Pooling MMS Consider first the case in which workers and firms coordinate on the pooling MMS where only the referral network is active. Given this, the contract posted by the employment agency is inconsequential: whatever fee-signal pair it posts no worker nor firm will join the agency and it will always make zero revenue.

Proposition 1: If workers and firms coordinate on the pooling matching market structure $P1$ given any fee-signal pair $(p, \phi) \in \mathbb{R}_{++} \times [1/2, 1)$, then the matching contracts take on the following general form

$$(p, \phi) : \left\{p \in \mathbb{R}_{++}, \quad \phi^{*P1} = 1/2\right\}$$

Proof: Omitted.

The result follows immediately from the fact that since no firm registers with the agency, the agency makes zero revenue whatever subscription fee it sets. Consequently, the agency will seek to minimise its costs, which is accomplished by minimising the signal quality. Even though there exists a continuum of equilibria, the outcome of the game is the same regardless of the specific equilibrium strategy chosen by the employment agency: the agency always makes zero profits, and the payoffs accruing to workers and firms are given by conditions (2.11)-(2.12) derived above.

Now consider the case where workers and firms coordinate on the pooling MMS where only the employment agency is active (when it exists). Given the analysis presented in Section 4, we know that the profit-maximising pricing policy in this case for any given $\phi \in [1/2, 1)$ is to set the subscription fee equal to the participation constraint of high-type firms, leaving low-type firms unmatched. Since this pricing policy also satisfies the incentive-compatibility constraint of low-type firms, and thereby condition (2.13), it also constitutes an equilibrium matching contract. This leads us to the following result.

Proposition 2: If the conditions of Lemma 1 are satisfied, and workers and firms coordinate on the pooling matching market structure $P2$, then the matching contract is unique and given by the solution to conditions (2.9)-(2.10).

Proof: Omitted.

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Firms' and workers' payoffs in this case coincide with those derived in Section 4. The profits of the employment agency are given by the solution to the following system of equations

$$\Pi^{*P2}(\phi) = \underbrace{\Big(\phi\hat{\theta} + (1-\phi)\Big)\hat{\theta}}_{p_H(\phi)} - c(\phi), \quad \phi^{*P2} : c'(\phi) = (\hat{\theta}-1)\hat{\theta}$$

Separating MMS We now solve for the equilibrium matching contracts in the case where workers and firms coordinate on one of the two positive assortative MMSs. We first examine the case where the fee-signal pair $(p, \phi) \in \mathcal{C}^{S1}$. Recall that only high-type firms participate in the agency-mediated matching market in this case. Consequently, the employment agency will seek to set its price function in order to bind the participation constraint of high-type firms. Formally, the participation constraint of high-type firms is given by

$$p(\phi) \leq \phi\hat{\theta}^2$$

The maximisation problem of the employment agency therefore consists of the following

$$\begin{aligned} &\max_{p,\phi} p - c(\phi) \\ &\text{s.t} \quad (p, \phi) \in \mathcal{C}^{S1}, \quad p(\phi) \leq \phi\hat{\theta}^2 \end{aligned}$$

Notice that the upper bound of the price interval in $\mathcal{C}^{S1}$ lies below the participation constraint of high-type firms for all values of $\phi \in [1/2, 1)$. This implies that the participation constraint will never bind, and that high-type firms will enjoy positive rents in equilibrium. The reasons for this follows from the fact that the high-type firms always have the possibility to leave the agency-mediated market and join the referral network, thereby guaranteeing them a strictly positive payoff. The interesting question is which fee-signal pair the employment agency will choose within the set $\mathcal{C}^{S1}$? Clearly, the answer to this question will depend on the properties of the cost function $c(\cdot)$. What we know for certain is that, whatever the functional form of the cost function $c(\cdot)$, the profit-maximising fee-signal pair must lie on the upper boundary of the trapezoid formed by the set $\mathcal{C}^{S1}$. In other words, the incentive-compatibility constraint of high-type firms must bind in

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equilibrium. Note, however, that the existence of such an equilibrium matching contract is not guaranteed. The reason for this is that, together with the incentive-compatibility constraint of high-type firms, the equilibrium contract must also satisfy the participation constraint of the employment agency

$$\Pi(\phi) = p(\phi) - c(\phi) \geq 0$$

Consequently, some restrictions have to be imposed on the set of admissible cost functions $c(\cdot)$ in order to guarantee the existence of the equilibrium.

Proposition 3: If the conditions of Lemma 2 are satisfied, and workers and firms coordinate on the positive assotative matching market structure $S1$, then the matching contract (if it exists) satisfies the conditions

$$p^{S1*}(\phi) = \phi\hat{\theta}^2 - (1 - e^{\alpha-1})\hat{\theta}, \quad \phi^{S1*} : c'(\phi) = \hat{\theta}^2$$

Proof: See Appendix B.

It is interesting to note that the equilibrium signal quality in this case is greater than the signal quality provided by the employment agency when there is no interaction between the two recruitment channels (see Section 4). High-type firms obtain a strictly positive payoff in equilibrium, which varies as a function of the inbreeding bias. Specifically, the surplus accruing to high-type firms equals $V_H^A(\alpha) = (1 - e^{\alpha-1})\hat{\theta}$, which can easily be shown to be a decreasing function of α. The reason for this comes from the fact that only low-type workers join the referral network, implying that as α tends to one the matching probability of high-type firms deviating to the referral network tends to zero. Consequently, the outside option of high-type firms also tends to the zero, and the incentive-compatibility constraint converges to the the participation constraint.

Finally, we solve for the equilibrium matching contract in the case where firms and workers coordinate on the positive assortative matching market structure $S2$. Recall that, contrary to the two pooling MMSs and matching market structure $S1$, the matching market structure $S2$ only exists for a subset of the parametre space. Formally, this subset

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is equal to[4]

$$\mathcal{R} = \left\{ (\hat{\theta}, \alpha) : \hat{\theta} \in \left(2, \frac{e}{2(e - e^{\alpha})}\right) \wedge \alpha \in (1 - 2\ln(2) + \ln(3), 1) \right\}$$

Given these parametric restrictions, we consider the case where the employment agency posts matching contracts $(p, \phi) \in \mathcal{C}^{S2}$ and where firms and players subsequently coordinate on the positive assortative MMS where low-types join the employment agency and high-types participate in the referral network. Again, the employment agency will seek to set its price function so as to bind the participation constraint of the firms that register to use its services (in this case the low-type firms). The participation constraint is given by

$$p(\phi) \leq (1 - \phi)$$

The maximisation problem of the employment agency can thus be written as follows

$$\begin{aligned} &\max_{p,\phi} p - c(\phi) \\ &\text{s.t} \quad (p, \phi) \in \mathcal{C}^{S2}, \quad p(\phi) \leq (1 - \phi) \end{aligned}$$

Notice that the sequence of subscription fees belonging to the upper boundary of $\mathcal{C}^{S2}$ is decreasing in ϕ. This implies that the employment agency will post a matching contract that specifies the minimal possible signal quality, thereby minimising its costs and maximising the price it can obtain for its services.

Proposition 4: If the conditions of Lemma 3 are satisfied, and workers and firms coordinate on the positive assortative matching market structure $S2$, then there exists a unique matching contract given by

$$p^{S2*} = \frac{1}{2} - (1 - e^{\alpha - 1})\hat{\theta}, \quad \phi^{S2*} = \frac{1}{2}$$

Proof: See Appendix B.

As before, firms using the services of the employment agency (in this case the low-type

[4]See Appendix B for details on how this subset is derived.

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firms) will enjoy positive rents in equilibrium. In fact, the payoff accruing to low-type firms is the same as that accruing to high-type firms before: i.e $V_L^A(\alpha) = (1 - e^{\alpha-1})\hat{\theta}$. For the same reasons as outlined above, the rent accruing to firms participating in the agency-mediated matching market is strictly decreasing in the value of the inbreeding bias.

2.6 Conclusion

This paper studied how worker and firms participating in a frictional labour market sort among competing formal and informal recruitment channels. We developed a model in which workers of heterogenous skill and firms of heterogenous quality much choose between a profit-maximising intermediary and an informal referral network in order to successfully match on the labour market. The model supports a multiplicity of equilibrium outcomes, which depend on the degree of inbreeding bias in the referral network and the productivity dispersion among workers and firms. Moreover, equilibrium sorting patterns critically depend on the probability of matching and the match quality across different recruitment channels. Contrary to the existing theoretical literature, and consistent with recent empirical evidence, we find that there exist equilibrium outcomes in which high-skill workers and high-quality firms hire the services of the profit-maximsing labour market intermediary, while low-skill workers and low-quality firms seek to match through the informal referral network. More generally, this paper contributes to the literature studying the use of social networks in job search. Until now, this literature had focused exclusively on the information-provision role played by referrals, without considering their effect on the matching rate between unemployed workers and job vacancies when the labour market is subject to frictions.

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2.7 Appendix A: General Network Structure

What is the vector $\mathbf{r} = (r_{ij})_{i\in\{H,L\},j\in\{H,L\}} \in [0,1]^4$ equal to? To calculate $\mathbf{r}$, we first need to solve for the probability that a firm has at least one link to a worker. Consider first the finite version of the economy described above, where there are $N \in \mathbb{N}_1$ workers and firms of each type. Let $\gamma_i^N \in \{0, \frac{1}{N}, \frac{2}{N}, ...1\}$ denote the fraction of type $i \in \{H, L\}$ workers who form a link to a firm. Given the structure of the link formation process described above, the probability that a firm is linked to a worker of the same type is α/N, and the probability that a firm is linked to a worker of a different type is $(1-\alpha)/N$. Consequently, the probability that a type $j \in \{H, L\}$ firm has m links to workers of the same type is given by the binomial distribution

$$\binom{\gamma_i^N N}{m}\left(\frac{\alpha}{N}\right)^m\left(1-\frac{\alpha}{N}\right)^{\gamma_i^N N-m}$$

In the limit this can be approximated by a Poisson distribution

$$\frac{(\alpha g_i)^m}{m!}e^{-\alpha g_i}$$

The probability that a high-type firm has no connection to a worker equals

$$\left(1-\frac{\alpha}{N}\right)^{\gamma_H^N N}\left(1-\frac{1-\alpha}{N}\right)^{\gamma_L^N N}$$

Similarly, the probability that a low-type firm has no connection to a worker equals

$$\left(1-\frac{\alpha}{N}\right)^{\gamma_L^N N}\left(1-\frac{1-\alpha}{N}\right)^{\gamma_H^N N}$$

In the limit, the probability that a high-type firm has at least one connection to a worker is thus given by

$$\xi_H = \lim_{N\to\infty}\left(1-\left(1-\frac{\alpha}{N}\right)^{\gamma_H^N N}\left(1-\frac{1-\alpha}{N}\right)^{\gamma_L^N N}\right) = 1-e^{-(\alpha g_H+(1-\alpha)g_L)}$$

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The probability that a low-type firm has at least one connection to a worker equals

$$\xi_L = 1 - e^{-(\alpha g_L + (1-\alpha) g_H)}$$

Given this, we now turn to calculating the probability that a type $j \in \{H, L\}$ firm with at least one link randomly selects a worker of the same type. Conditional on having at least one link, the probability that a high-type firm has k additional links equals

$$\begin{aligned}\zeta_H(k) &= \sum_{l=0}^{k} \left(\frac{(\alpha g_H)^{k-l}}{(k-l)!} e^{-\alpha g_H} \frac{((1-\alpha) g_L)^l}{l!} e^{-(1-\alpha) g_L} \right) \\ &= \frac{(\alpha g_H + (1-\alpha) g_L)^k}{k!} e^{-(\alpha g_H + (1-\alpha) g_L)}\end{aligned}$$

Similarly, for low-type firms we have

$$\zeta_L(k) = \frac{(\alpha g_L + (1-\alpha) g_H)^k}{k!} e^{-(\alpha g_L + (1-\alpha) g_H)}$$

Consequently, conditional on having at least one link, the probability that a high-type firm with k additional links hires a worker of the same type is

$$\frac{1}{\zeta_H(k)} \sum_{l=0}^{k} \left(\left(\frac{k-l}{k} \right) \frac{(\alpha g_H)^{k-l}}{(k-l)!} e^{-\alpha g_H} \frac{((1-\alpha) g_L)^l}{l!} e^{-(1-\alpha) g_L} \right) = \frac{\alpha g_H}{\alpha g_H + (1-\alpha) g_L}$$

The unconditional probability is therefore equal to

$$r_{HH}(\mathbf{g}) = \frac{\alpha g_H}{\alpha g_H + (1-\alpha) g_L} \xi_H$$

Symmetrically, for low-type firms

$$r_{LL}(\mathbf{g}) = \frac{\alpha g_L}{\alpha g_L + (1-\alpha) g_H} \xi_L$$

The probability that a high-type firm hires a worker of a different type is

$$r_{LH}(\mathbf{g}) = \left(1 - \frac{\alpha g_H}{\alpha g_H + (1-\alpha) g_L} \right) \xi_H$$

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Symmetrically, for low-type firms

$$r_{HL}(\mathbf{g}) = \left(1 - \frac{\alpha g_L}{\alpha g_L + (1-\alpha) g_H}\right) \xi_L$$

We now turn to calculating the vector $\mathbf{s} = (s_{ij})_{i \in \{H,L\}, j \in \{H,L\}} \in [0,1]^4$. The probability that a type $i \in \{H, L\}$ worker is linked to a firm of the same type who chooses to hire by referral equals αh_i. Similarly, the probability that a type $i \in \{H, L\}$ worker is linked to a firm of a different type who chooses to hire by referral equals $(1-\alpha)h_j$. To calculate the vector $\mathbf{s}$ we need to solve for the probability that a worker who is linked to a high type firm will receive a referral offer conditional on that firm choosing to hire though the referral network

$$\rho_H = 1 - \sum_{k=1}^{\infty} \frac{k}{k+1} \zeta_j(k) = 1 - \frac{e^{-(\alpha g_H + (1-\alpha) g_L)} + (\alpha g_H + (1-\alpha) g_L) - 1}{\alpha g_H + (1-\alpha) g_L}$$

Similarly, for a worker linked to a low type firm

$$\rho_L = 1 - \frac{e^{-(\alpha g_L + (1-\alpha) g_H)} + (\alpha g_L + (1-\alpha) g_H) - 1}{\alpha g_L + (1-\alpha) g_H}$$

Consequently, the probability that a type $i \in \{H, L\}$ worker is hired through the referral network by a firm of the same type is given by

$$s_{ii}(\mathbf{g}, \mathbf{h}) = \alpha h_i \rho_i \quad \forall i \in \{H, L\}$$

and the probability that a type $i \in \{H, L\}$ worker is hired through the referral network by a firm of a different type $j \neq i$ equals

$$s_{ij}(\mathbf{g}, \mathbf{h}) = (1-\alpha) h_j \rho_j \quad \forall i, j \in \{H, L\}, j \neq i$$

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2.8 Appendix B: Proofs

2.8.1 Proof of Claim 1

We prove the claim by contradiction.[5] Consider first the mixing matching market structure where low-type workers match with high-type firms through the agency, and high-type workers match with low-type firms on the referral network. The incentive compatibility constraint such that high-type workers participating in the referral network have no incentive to deviate and register with the employment agency is

$$\underbrace{\left(1 - e^{-(1-\alpha)}\right)}_{s_{HL}} \hat{\theta} \geq \phi\hat{\theta}^2$$

Using the restrictions $\hat{\theta} > 2$ and $\phi \geq 1/2$, this condition implies

$$\left(1 - e^{-(1-\alpha)}\right) > 1 \Rightarrow \text{Contradiction}$$

We now turn to the case where high-type workers match with low-type firms in the agency-mediated matching market while low-type workers and high-type firms participate in the referral network. The firms' incentive compatibility constraints are given by

$$\phi\hat{\theta} - p \geq \underbrace{(1 - e^{-\alpha})}_{r_{LL}} \quad \text{and} \quad \underbrace{\left(1 - e^{-(1-\alpha)}\right)}_{r_{HL}} \hat{\theta} \geq \phi\hat{\theta}^2 - p$$

Solving for p and rearranging these constraints yields the inequality

$$\left(\phi\hat{\theta} - (1 - e^{-\alpha})\right) \geq p \geq \left(\phi\hat{\theta} - \left(1 - e^{-(1-\alpha)}\right)\right)\hat{\theta}$$

Again, using the restriction $\hat{\theta} > 2$ this condition implies

$$(1 - e^{-\alpha}) < \left(1 - e^{-(1-\alpha)}\right) \Rightarrow \alpha < 1/2 \Rightarrow \text{Contradiction}$$

[5] All proofs in Appendix B impose the parametric restrictions on the type space described in Section 3.1: i.e $\theta_H = \hat{\theta} > 2$, $\theta_L = 1$ and $\Theta = \Omega$.

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2.8.2 Proof of Lemma 1

For any $\phi \in [1/2, 1)$, all prices $p \leq p_L(\phi)$ necessarily satisfy the participation constraint of all high- and low-type firms. All firms will therefore optimally choose to participate in the agency-mediated matching market. Given this, no worker will have an incentive to deviate and join the referral network. Similarly, for all $p \geq p_H(\phi)$, neither high- nor low-type firms' participation constraints will be satisfied, and no firm will pay the subscription fee. One has to be a bit more careful when considering prices belonging to the interval $p(\phi) \in (p_L(\phi), p_H(\phi))$: i.e prices which satisfy the participation constraint of high-type firms but do not satisfy the participation constraint of low-type firms. Any equilibrium in this case would have high-type firms paying the subscription fee and low-type firms opting to remain unmatched. Consider two candidate equilibria: one where $p = p_L(\phi) - \epsilon$ such that all firms join the agency, and one where $p' = p_L(\phi) + \epsilon$ such that only high-type firms join the agency. Notice that the matching probability of firms joining the agency jumps discontinuously from 1/2 to 1 when going from p to p'. As argued above, there cannot be an equilibrium with both types of firms joining the employment agency when $p > p_L(\phi)$. However, an individual low-type firm will have an incentive to unilaterally deviate and join the employment agency when the subscription fee equals p', since this unilateral deviation will have a negligible affect on the firm's matching probability. Consequently, any equilibrium where $p_L(\phi) \leq p \leq p_H(\phi)$ must satisfy the incentive compatibility constraint of low-type firms: formally, this requires $p > (\phi\hat{\theta} + (1 - \phi))$, which equals low-type firms' deviation payoff when $p \in (p_L(\phi), p_H(\phi))$. $\square$

2.8.3 Proof of Lemma 2

We prove the claim by construction. We begin by calculating the agents' payoffs at the candidate equilibrium. We then identify the deviation payoff for each type of each agent and proceed to show that the area defined by the set of corresponding incentive compatibility constraints is non-empty. The value of ϕ is implicitly pinned down by the incentive compatibility constraint of low-type workers

$$(1 - e^{-\alpha}) \geq (1 - \phi)\hat{\theta} \quad \Rightarrow \quad \phi \geq 1 - \frac{(1 - e^{-\alpha})}{\hat{\theta}}$$

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The price interval is determined using the incentive compatibility constraints of high- and low-type firms

$$\left(\phi\hat{\theta} - \left(1 - e^{-(1-\alpha)}\right)\right)\hat{\theta} \geq p \geq \left(\phi\hat{\theta} - (1 - e^{-\alpha})\right)$$

These two linear inequalities define a polyhedron that can be represented in terms of nested inequalities. This polyhedron effectively defines the parametre subspace which supports the separating matching market structure in equilibrium. This separating matching market structure always exists for any value of $\hat{\theta} > 2$ and $\alpha \in (1/2, 1)$. The set of fee-signal pairs (p, ϕ) that support the separating matching market structure is given by

$$\mathcal{C}^{S1} = \left\{(p,\phi) : p \in \left[e^{-\alpha}(1 - e^{\alpha} + e^{\alpha}\phi\hat{\theta}), \frac{e\phi\hat{\theta}^2 + e^{\alpha}\hat{\theta} - e\hat{\theta}}{e}\right], \phi \in \left[\frac{e^{-\alpha}(1 - e^{\alpha} + e^{\alpha}\hat{\theta})}{\hat{\theta}}, 1\right)\right\}$$

2.8.4 Proof of Lemma 3

We again prove the claim by construction. The incentive constraint of low-type firms is

$$(1 - \phi) - p \geq \left(1 - e^{-(1-\alpha)}\right)\hat{\theta}$$

For given values of $\hat{\theta}$, ϕ and α this constraint pins down the upper threshold value for the subscription fee p. The incentive constraint of high-type workers is

$$(1 - e^{-\alpha})\hat{\theta}^2 \geq \phi\hat{\theta}$$

This pins down the upper threshold value for ϕ for given values of $\hat{\theta}$ and α. The incentive constraint of high-type firms is

$$(1 - e^{-\alpha})\hat{\theta}^2 \geq (1 - \phi)\hat{\theta} - p$$

These conditions pin down the upper threshold values for $\hat{\theta}$ for given values of p, ϕ and α. Finally, these three constraints together pin down the lower threshold value for α. Solving, we find that for the subspace of the parametre space defined by

$$\mathcal{R} = \left\{(\hat{\theta},\alpha) : \hat{\theta} \in \left(2, \frac{e}{2(e - e^{\alpha})}\right) \wedge \alpha \in (1 - 2\ln(2) + \ln(3), 1)\right\}$$

there exists a separating matching market structure such that high-types join the referral network and low-types join the employment agency, providing the subscription fee p and signal quality ϕ belong to the following set

$$\mathcal{C}^{S2} = \left\{ (p, \phi) : p \in \left(0, \frac{e - e\hat{\theta} + e^{\alpha}\hat{\theta} - \phi e}{e}\right], \quad \phi \in \left(\frac{1}{2}, \frac{e - e\hat{\theta} + e^{\alpha}\hat{\theta}}{e}\right) \right\}$$

2.8.5 Proof of Proposition 3

To see that the participation constraint of high-type firms never binds, just notice that

$$\phi\hat{\theta}^2 - (1 - e^{\alpha-1})\hat{\theta} < \phi\hat{\theta}^2$$

since $\alpha \in (1/2, 1)$, thereby implying that $e^{\alpha-1} < 1$. Next, we show that any profit-maximising matching contract must lie on the upper boundary of the set $\mathcal{C}^{S1}$, defined as

$$\left\{ p(\phi) = \phi\hat{\theta}^2 - (1 - e^{\alpha-1})\hat{\theta}, \phi \in \left[\frac{e^{-\alpha}(1 - e^{\alpha} + e^{\alpha}\hat{\theta})}{\hat{\theta}}, 1\right) \right\}$$

Assume, to the contrary, that the profit-maximising matching contract (p^*, ϕ^*) lies within the set $\mathcal{C}^{S1}$. Then, we can always find an alternative fee-signal pair (p', ϕ') such that $\Pi(p', \phi') > \Pi(p^*, \phi^*)$. For example, consider the case where $p' > p^*$ and $\phi' = \phi^*$. Since (p^*, ϕ^*) is not on the upper boundary of $\mathcal{C}^{S1}$, this alternative fee-signal pair exists and yields strictly larger profits. Plugging in the boundary condition into the objective function of the employment agency yields

$$\Pi^{S1}(\phi) = \phi\hat{\theta}^2 - (1 - e^{\alpha-1})\hat{\theta} - c(\phi)$$

Maximising with respect to ϕ yields the profit-maximising contract. An equilibrium matching contract will only exist if there exists a fee-signal pair $(p, \phi) \in \mathcal{C}^{S1}$ such that the employment agency makes non-negative profits. Formally, it must be that: $\exists (p, \phi) \in \mathcal{C}^{S1} : \Pi^{S1}(\phi) \geq 0$. This last condition implicitly puts restrictions on the set of admissible cost functions $c(\cdot)$. $\square$

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2.8.6 Proof of Proposition 4

The proof as to why the equilibrium matching contract must lie on the upper boundary of the set $\mathcal{C}^{S2}$ is the same as the one provided in the proof of Proposition 3. This implies that the price function is given by

$$p(\phi) = (1-\phi) - (1-e^{\alpha-1})\hat{\theta}$$

Plugging this into the object function of the employment agency yields

$$\Pi^{S2}(\phi) = (1-\phi) - (1-e^{\alpha-1})\hat{\theta} - c(\phi)$$

Notice that this function is monotonically decreasing in ϕ. Consequently, the employment agency will set $\phi^{S2*} = 1/2$. Given this, the equilibrium subscription fee is equal to

$$p^{S2*} = \frac{1}{2} - (1-e^{\alpha-1})\hat{\theta}$$

It is easily verified that this equilibrium matching contract satisfies the participation constraint of low-type firms since $(1-e^{\alpha-1})\hat{\theta} > 0$. Also, notice that since $\phi^{S2*} = 1/2$, the participation constraint of the employment agency is always satisfied. □

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Chapter 3

Unemployment, Inequality and the Business Cycle

with Damien Puy

3.1 Introduction

There is a longstanding tradition in the economics literature studying the link between inequality and macroeconomic performance. Indeed, for David Ricardo, understanding how economic output is divided among the various classes in society constituted the "principal problem of Political Economy." Within the neoclassical representative-agent paradigm, however, distributional issues were long considered to be of second-order importance and thought to have negligible effects on aggregate production and economic performance. This neglect by mainstream economics has been partly addressed by several recent contributions, focusing on issues as varied as human capital accumulation [38], international trade [57], and economic development [10]. That being said, surprisingly little work has been done studying how inequality affects short-run macroeconomic performance. While the counter-cyclical properties of the Gini coefficient have been widely documented [61], the underlying economic mechanisms responsible for this empirical regularity remain largely unexplored. Moreover, virtually nothing is known about whether and how the *ex ante* level of inequality affects the propagation of shocks at business cycle frequencies; i.e whether relatively more or less unequal economies react differently to similar macroeconomic shocks. This paper proposes a theoretical framework addressing these two issues.

This paper argues that the cyclical properties of the income distribution are driven by heterogenous sectoral responses to business-cycle frequency shocks. Using US industry-level data from the Bureau of Economic Analysis over the period 1977-2010, a recent empirical study by Jin and Li [45] found that labour-intensive sectors expand disproportionately more than capital-intensive sectors during booms, implying that the share of production, investment and employment in capital-intensive sectors significantly drops

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during economic expansions. The reverse tends to happen during recessions. In our opinion, understanding the distributional consequences of such heterogenous sectoral response to aggregate shocks is key in order to thoroughly understand the counter-cyclical properties of income inequality. In particular, we argue that the novel empirical facts documented by Jin and Li [45] are driven by a recomposition of aggregate demand over the business cycle. Preliminary evidence points to the validity of such a demand-driven mechanism. For example, examining household consumption, Charles and Stephens [24] find significant changes in the composition of consumption bundles over the business cycle, with the largest changes occurring in low-income households. Such demand composition effects have been shown to play an important role in explaining long-run structural change [58],[34]. To our knowledge, ours is the first paper to examine the short-run macroeconomic consequences of such demand composition effects. The theoretical framework we develop addresses the following three research questions:

1. What are the general equilibrium effects of aggregate productivity shocks when agents' consumption bundles vary as a function of their income and sectors differ in terms of their relative labour- and capital-intensity?

2. Does the *ex ante* distribution of capital ownership affect the propagation of these aggregate productivity shocks?

3. What are the *ex post* distributional consequences of such productivity shocks?

To address these questions, we develop a multi-sector general equilibrium model with labour market frictions. Three key assumptions drive the main mechanism of the model. First, consumers have non-homothetic preferences. This implies that aggregate consumption shares vary with aggregate income, and that aggregate productivity shocks affect the allocation of capital across sectors. More specifically, building on the hierarchic preferences developed by Matsuyama [58], we assume that consumers only begin to consume non-essential goods after satiating their demand for more basic goods. Second, production technologies are such that the factor share of capital is greater in sectors producing more 'basic' goods. Consequently, labour-intensive sectors are particularly sensitive to aggregate fluctuations and experience greater volatility in output and employment. Third, the ownership of capital is unequally distributed among the population. We show that the

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magnitude of the demand composition effect, and the implied capital reallocation across sectors, depends on the initial distribution of capital ownership.

In order to gain some intuition about the underlying mechanics of the model, consider the effect of a Hicks-neutral productivity shock in an economic environment like the one described above. Clearly, the first *direct effect* will be a drop in aggregate output due to lower productivity. As consumers have non-homothetic preferences, this drop in income will engender a recomposition of demand away from non-essential (labour-intensive) goods towards basic (capital-intensive) goods. This *demand composition effect* in turn will translate into a further fall in labour demand, over and above that generated by the aggregate productivity shock itself. In a frictional labour market, this *factor demand effect* will result in an increase in the equilibrium unemployment rate and drop in the wage rate, thereby worsening the relative position of lower quantiles. Finally, the initial distribution of capital ownership, insofar as it determines the magnitude of the change in the composition of demand, will play an important role in determining the aggregate effects of the shock on output and employment. The rest of this paper provides a formal treatment of this argument.

The theoretical results we obtain go a long way in rationalising the key empirical facts mentioned above. First, we find the sectoral propagation of aggregate shocks to be driven by the reallocation of capital investment across sectors. This is consistent with the novel empirical work of Jin and Li [45], who find that the share of investment in capital-intensive sectors increases by about 5% during recessions. Second, we show that this capital reallocation effect depends in interesting ways on the initial distribution of capital ownership in the economy. In particular, any redistribution of wealth which increases the recomposition of aggregate demand over the cycle is found to amplify the effects of aggregate shocks. Third, the cyclical properties of the income distribution are driven by the capital reallocation effect, and its implications for relative factor rewards. Consistent with the data, we find that inequality rises during recessions because higher unemployment and lower wages worsen the relative position of low-income groups [61]. Moreover, we find that the dynamics of the income distribution are essentially independent from the concentration of wealth. This is consistent with the findings of Castaneda et al [23], which as yet had received no clear theoretical justification.

The rest of this paper is organised follows. Section 2 provides a brief overview of the relevant empirical and theoretical literature. Section 3 solves for the equilibrium and ex-

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	Correlation with Output	Volatility
1st Quintile (0-20%)	0.53	1.07
2nd Quintile (20-40%)	0.49	0.48
3rd Quintile (40-60%)	0.31	0.26
4th Quintile (60-80%)	-0.29	0.17
Next 15% (80-95%)	-0.64	0.36
Top 5% (95%-100%)	0.00	0.74

Table 3.1: Cyclical Behaviour of Income Share by Quintile for US 1948-1986. *(Source: Castaneda et al [23])*

amines its comparative static properties in the case where the labour market is Walrasian. Section 4 introduces labour market frictions, and studies the effects of unemployment. Concluding remarks are provided in Section 5.

3.2 Empirical Motivation

In this section, we discuss the existing empirical evidence supporting the theoretical model described above. The key empirical claims of the model can be briefly summarised as follows: (i) the Gini coefficient for income is counter-cyclical, increasing during recessions and diminishing during booms; (ii) the counter-cyclical properties of income inequality are driven by the excess volatility of labour-intensive sectors because higher unemployment and lower wages worsen the relative position of the poor; and (iii) demand composition effects imply that capital reallocation over the business cycle favours labour-intensive sectors. To what extent are these claims supported by empirical evidence? Below, we provide a cursory overview of existing empirical work suggesting that all three are largely confirmed by the data.

Claim 1: Counter-Cyclical Gini Coefficient. Table 3.1 is taken from Castaneda et al [23], and documents the cyclical properties of income shares decomposed by quintile for the US between 1948 and 1986. The data is taken from the Current Population Surveys (CPS) compiled by the Bureau of the Census for the US Bureau of Labour Statistics (BLS). The correlations presented in Table 3.1 clearly show that the income share earned by the lowest quintile is both the most volatile and the most pro-cyclical. Moreover, the pro-cyclicality

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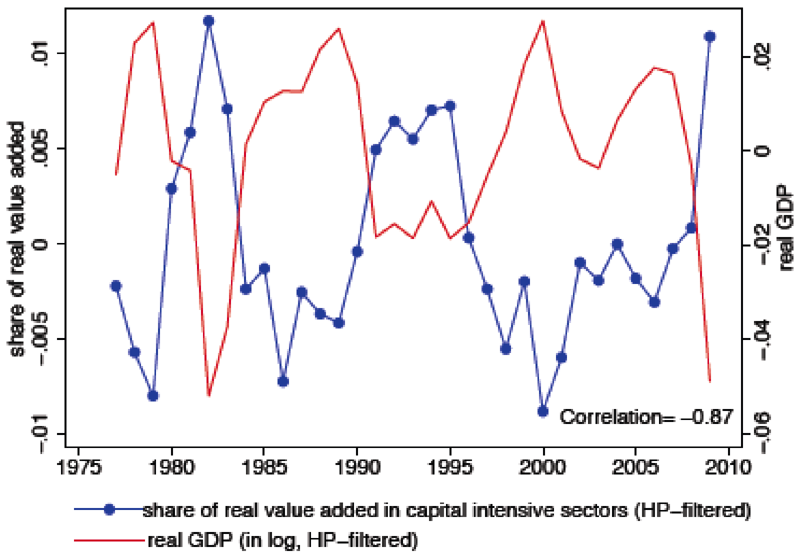

Figure 3.1: Correlation between the share of value added in capital-intensive sectors and GDP for US 1977-2009. (*Source: Jin and Li [45]*)

of the income shares is monotonically decreasing up to the 5th percentile. These results indicate that income dispersion in the US over the post-war years was strongly counter-cyclical, thereby supporting the claim that income inequality (as measured by the Gini coefficient) is pro-cyclical.

Claim 2: Pro-Cyclicality of Labour-Intensive Sectors. Figures 3.1 and 3.2 are taken from Jin and Li [45] and document the compositional change in output and employment over the business cycle.[1] The figures were generated with data taken from the US Bureau of Economic Analysis (BEA) Industry Economic Accounts for private sectors at the NAICS 2-4 digit level from 1977 to 2009. The two figures clearly show the counter-cyclicality of (detrended) output and employment shares of capital-intensive sectors. In particular, the correlation of the share of value added in capital-intensive sectors with output is -0.87, while the correlation of the share of employment in capital-intensive sec-

[1]Capital shares at the industry level are constructed as follows: $(\text{capital share}) = 1 - \frac{(\text{compensation of employees})}{(\text{value-added})-(\text{taxes less subsidies})}$. Capital-intensive sectors are then defined as all sectors where the capital share is greater than the median.

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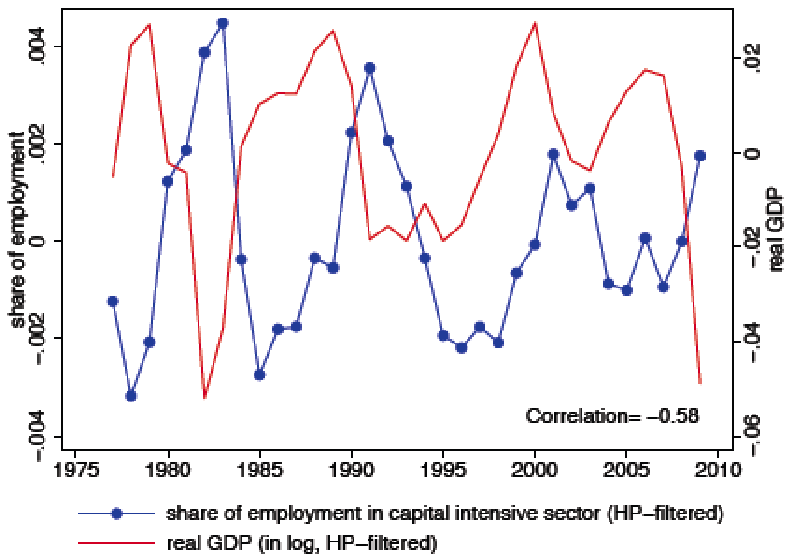

Figure 3.2: Correlation between the share of employment in capital-intensive sectors and GDP for US 1977-2009. *(Source: Jin and Li [45])*

tors with output is −0.58. These results indicate that output and employment shares in labour-intensive sectors are strongly pro-cyclical.

Claim 3: Counter-Cyclical Investment in Capital-Intensive Sectors Figure 3.3 is again taken from Jin and Li [45]. It documents the (detrended) share of investment in capital-intensive sectors in total investment over the 1977 to 2009 period. Their results indicate that the correlation between the share of investment in capital-intensive sectors and output is -0.70. The magnitude of this investment reallocation is deemed to be significant: the share of investment in capital-intensive sectors increases by about 5% during recessions. This evidence supports the capital reallocation effect identified in the theoretical model below as an important channel via which aggregate shocks are propagated through the economy.

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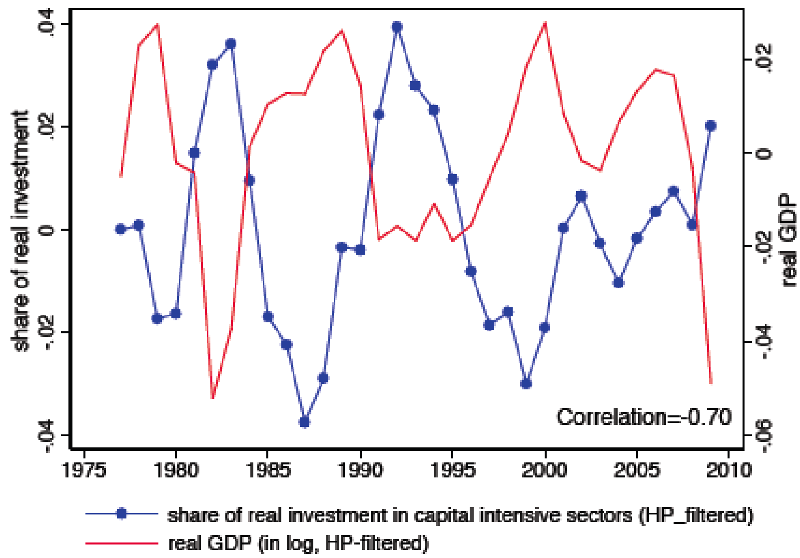

Figure 3.3: Correlation between the share of investment in capital-intensive sectors and GDP for US 1977-2009. (*Source: Jin and Li [45]*)

3.3 Literature

This paper builds on the nascent theoretical literature studying how non-homothetic consumer preferences interact with income distribution effects to explain the sectoral distribution of output and employment.[2] This literature has predominately focused on long-run macroeconomic performance, in particular issues related to structural change, growth and the process of industrialisation. Murphy et al [65] show how aggregate demand spillovers and imperfectly competitive product markets can rationalise Rosenstein-Rodan's 'big-push' theory of industrialisation. In their model, the distribution of income affects the process of development via changes in the composition of demand. This idea is further developed by Matsuyama [58], who studies how demand composition and income distribution effects interact to explain the rise of 'mass consumption' societies. As in this paper, a key assumption of Matsuyama's model is that consumer preferences are hierarchic, so that as households' income increases, they expand the range of consumer goods they purchase rather than purchasing greater quantities of the same goods. Among other things, this

[2]See Bertola [13] for a survey of this literature.

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implies that the market size for each consumption good does not depend only on the level of aggregate income, but also on the distribution of income across households. A similar mechanism is studied by Foellmi and Zweimuller [34].

As mentioned above, while a considerable amount of work has been done studying how the distribution of income and demand composition effects interact to explain long-run structural change, much less is known about their implications for short-run macroeconomic performance. An important exception is the recent paper by Foellmi and Zweimuller [35], who study how inequality affects the level of aggregate employment in an economy in which consumers have non-homothetic preferences and product markets are monopolistically competitive.[3] In this sense, Foellmi and Zweimuller [35] build on earlier work by Gali [37] studying the effect of changing demand elasticities on monopolistically competitive firms' optimal output. A key result of Foellmi and Zweimuller's analysis is that inequality can lead to higher unemployment due to changes in the level of aggregate labour demand. This mirrors closely a result of the present paper. However, the underlying mechanisms driving this common result are markedly different. First, while unemployment arises in Foellmi and Zweimuller's model due to imperfect competition in the product market, we explicitly model unemployment as the result of matching frictions in the labour market. Second, Foellmi and Zweimuller consider a model with only labour as a factor of production, and focus on labour income inequality (measured in terms of heterogenous labour endowments). This paper, instead, considers a model with both capital and labour as factors of production, and focuses on capital income inequality (measured in terms of heterogenous ownership shares of the capital stock). Importantly, the introduction of an additional factor of production endogenises the income distribution through changes in relative factor prices. This, in turn, allows us to explicitly address the counter-cyclical properties of the income distribution, an issue which is markedly absent from Foellmi and Zweimuller's analysis.

This paper is also related to the literature studying the cyclical properties of the income distribution. In particular, Castaneda et al [23] build a model to explore to what extent unemployment spells and cyclically moving factor shares can account for the counter-cylical properties of Gini coefficient. Their results indicate that: (i) cyclically moving factor shares play a small role in explaining the counter-cyclicality of income inequality;

[3] Another (later) paper by Chiang [25] also studies the effects of inequality on unemployment in a very similar model.

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and (ii) that the cyclical properties of the income distribution are essentially independent from the wealth distribution. Their results suggest that capital income fluctuations play a small role in accounting for the counter-cyclicality of the Gini coefficient. Interestingly, the model we develop below confirms this result. In fact, we find that the cyclical properties of the income distribution are primarily driven by changes in wages and the unemployment rate. Not only are our results consistent with existing empirical studies [61], but they also go a long way in providing a (qualitative) answer to the puzzles raised by Castaneda et al [23].

Finally, at a broader level, this paper is also related to the recent work studying the possible links between inequality and macro-financial fragility. This issue has very much come to the fore due to recent empirical evidence pointing to an intriguing correlation in macroeconomic data between inequality and economic crises [8]. For example, Kumhof and Ranciere [51] have suggested that increases in inequality may lead to increased financial instability through an increase in household debt leverage. The mechanism developed in this paper proposes a different, yet complementary, channel through which the *ex ante* level of inequality can affect the propagation of macroeconomic shocks.

3.4 The Model

3.4.1 Environment

Preferences Consider an economy populated by a continuum of risk-neutral agents, indexed by $i \in [0,1]$. We denote the set of agents by $N = [0,1]$ and its Lebesgue measure by $\mathcal{N} = 1$. The production-side of the economy consists of two sectors, one producing basic goods and the other producing secondary goods. We index sectors by $s \in \{1,2\}$, and refer to sector 1 as the basic good sector and sector 2 as the secondary good sector. Consumers are assumed to have identical non-homothetic preferences, implying that the bundle of goods they demand will depend on their income. This non-homotheticity is captured by the hierarchic structure of consumer preferences. Formally, we assume consumer preferences can be represented by the following utility function

$$u(c_1, c_2) = \begin{cases} c_1 & \text{if} \quad c_1 \leq \bar{c}_1 \\ \bar{c}_1 + c_2 & \text{if} \quad c_1 = \bar{c}_1 \end{cases}$$

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where $\bar{c}_1 > 0$ denotes the satiation point for consumers' demand of the basic good. The structure of preferences implies that agents will only consume the basic good until they reach the satiation point. After this point, agents will continue to consume this fixed amount of the basic good, and spend all additional income on the consumption of the secondary good.

Technology Production takes place using two factors of production: capital (K) and labour (L). Importantly, we assume that the two factors of production are used in different intensities in the two sectors. In particular, we assume the secondary good sector to be relatively labour intensive, while the basic good sector to be relatively capital intensive. To simplify the analysis, we assume that production of the basic good requires only capital as an input, while the production of the secondary good combines both factors of production using a Cobb-Douglas production technology. Formally, the production technology in the basic good sector is given by

$$Y_1(K_1) = AK_1$$

while the production technology in the secondary good sector is given by

$$Y_2(K_2, L_2) = AK_2^{\alpha} L_2^{1-\alpha}$$

where $\alpha \in (0, 1)$ and $A > 0$ denotes a Hicks-neutral productivity parametre.[4] We assume the capital stock to be in fixed supply so that $K_1 + K_2 = \bar{K}$, where $\bar{K} > 0$ denotes the aggregate capital stock.

Endowments All agents are endowed with one unit of labour, but differ in terms of their ownership of the aggregate capital stock $\bar{K}$. Formally, we assume that each agent is characterised by a publicly observable type $\theta_i \in \Theta = [0, 1]$, where $\theta_i \in [0, 1]$ denotes an agent's ownership share of the capital stock such that $\int_{i \in N} \theta_i di = 1$. Ownership shares are assumed to be continuously distributed in the population according to the cumulative distribution function $G : \Theta \to [0, 1]$, with associated probability density function $g : \Theta \to$

[4] The assumption that labour does not enter the production of basic goods is without loss of generality in the sense that all results would hold even in the case of a Cobb-Douglas production function

$$Y_1(K_1, L_1) = K_1^{\phi} L_1^{1-\phi}$$

providing that $\phi > \alpha$.

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$[0,1]$. By inverting the cumulative distribution function we obtain the quantile function $Q : \mathcal{N} \to [0,1]$ and associated quantile density function $q : \mathcal{N} \to [0,1]$, where $Q(\cdot) \equiv G^{-1}(\cdot)$ and $q(\cdot) \equiv Q'(\cdot)$. Without loss of generality, we order agents by their ownership shares such that the index of agent i also denotes the Lebesgue measure of the set $[0,i]$. This implies that we can write $\theta_i = q(i)$, where by definition we must have $\int_0^1 q(i)di = Q(1) = 1$. In order to measure the degree of wealth inequality, we define a scaling parametre $\beta > 0$ which determines the statistical dispersion of the probability distribution $G(\theta;\beta)$. As β gets large, the distribution of shares becomes increasingly unequal; as β goes to zero, the distribution of shares becomes increasingly uniform. This leads us to the following two assumptions.

Assumption 1: The distribution function $G : \Theta \to [0,1]$ is such that the quantile density function is continuously differentiable and monotonically increasing such that $q'(\cdot) > 0$.

Assumption 2: The distribution function $G : \Theta \to [0,1]$ is such that the quantile function is monotonically decreasing in β such that $Q_\beta(\cdot;\beta) < 0$.

3.4.2 Optimality Conditions

Utility Maximisation Taking the price of the secondary good as the numeraire so that $p_2 = 1$, we can write the budget constraint of agent i as follows

$$p_1 c_{i,1} + c_{i,2} \leq I_i \equiv wl_i + \theta_i r\bar{K}, \quad \forall i \in N$$

where $w > 0$ and $r > 0$ denote the wage and interest rate, respectively. Note the since agents incur no disunity from labour, we will have $l_i = 1$ $\forall i \in N$ in equilibrium. It follows that the aggregate labour supply will be constant an equal to $\bar{L} \equiv \int_{i\in N} l_i di = 1$. The utility maximisation problem of consumers is simply given by

$$\max_{c_{i,1},c_{i,2}} u(c_{i,1}, c_{i,2}) \quad : \quad p_1 c_{i,1} + c_{i,2} \leq w + \theta_i r\bar{K}$$

Given the hierarchic structure of consumer preferences, utility maximisation implies that agent i consumes a positive quantity of the secondary good if and only if the following

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condition is satisfied

$$\frac{w + \theta_i r\bar{K}}{p_1} > \bar{c}_1 \tag{3.1}$$

Since an agent's income is strictly increasing in the value of his ownership share θ_i, it follows that any equilibrium must have a threshold structure: i.e only agents with an ownership share greater than some (endogenous) threshold $\theta_i > \hat{\theta} \in [0,1]$ will consume a positive quantity of the secondary good. From the above condition, we can derive an expression for the threshold ownership share as follows

$$\hat{\theta} = \frac{p_1\bar{c}_1 - w}{r\bar{K}} \tag{3.2}$$

We denote by $\hat{\imath} \in [0,1]$ the marginal agent such that $\theta_{\hat{\imath}} = \hat{\theta}$.

Profit Maximisation The objective function of a (representative) firm in sector $s \in \{1,2\}$ is given by

$$\max_{K_s, L_s} \Pi_s = p_s Y_s(K_s, L_s) - wL_s - rK_s$$

Given the production technologies outlined above, profit maximisation in the secondary good sector implies that the equilibrium interest rate must satisify

$$r(K_2, L_2) = \frac{\partial Y_2}{\partial K_2} = \alpha A \left(\frac{L_2}{K_2}\right)^{1-\alpha} \tag{3.3}$$

while the equilibrium wage rate will be such that

$$w(K_2, L_2) = \frac{\partial Y_2}{\partial L_2} = (1-\alpha) A \left(\frac{K_2}{L_2}\right)^{\alpha} \tag{3.4}$$

In what follows, it will be useful to have an expression for the wage-interest rate ratio, given by

$$\rho(K_2; \alpha) \equiv \frac{w}{r} = \frac{1-\alpha}{\alpha} K_2 \tag{3.5}$$

Importantly, notice that this ratio is strictly increasing in K_2. Finally, free-entry in the basic good sector pins down the price of the basic good as a function of the interest rate

$$p_1(r; A) = \frac{r}{A} \tag{3.6}$$

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Definition 1: A Walrasian equilibrium consists of relative prices (r^*, w^*, p_1^*, p_2^*) and quantities $(c_1^*, c_2^*, K_1^*, K_2^*, L_2^*)$ such that

- All agents $i \in N$ choose consumption bundles $(c_{i,1}, c_{i,2})$ in order to maximise their utility subject to their budget constraints, taking prices as given.
- Firms in both sectors $s \in \{1, 2\}$ choose factor inputs (K_1, K_2, L_2) in order to maximise their profits, taking prices as given.
- Labour, capital and goods markets clear.

3.5 Equilibrium Analysis

In what follows, we restrict attention to interior equilibria such that $\hat{\theta} \in (0, 1)$. This requires us to impose some parametric restrictions so that agents are neither too rich (so that not all agents consume both goods) nor too poor (so that some agent consumes both goods). Formally, the condition guaranteeing that the equilibrium is interior is given by

$$\frac{w + \theta_0 r\bar{K}}{p_1} < \bar{c}_1 < \frac{w + \theta_1 r\bar{K}}{p_1}$$

Using the price equations (3.3)-(3.6) derived above, and recalling that $\theta_i = q(i)$, we are led to the following assumption.

Assumption 3: The distribution of ownership shares is such that

$$(1 - \alpha(1 - q(0)))A\bar{K} < \bar{c}_1 < q(1)A\bar{K}$$

Market clearing in the basic good sector implies

$$\int_0^{\hat{i}} \left(\frac{w + q(i)r\bar{K}}{p_1} \right) di + (1 - \hat{i})\bar{c}_1 = AK_1 \tag{3.7}$$

where $\hat{i} \in (0, 1)$ denotes the measure of constrained agents: i.e agents too poor to demand a positive quantity of the secondary good. Recall from condition (3.1) that constrained agents will spend all their income on the basic good, while unconstrained agents will

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demand a constant quantity of the basic good equal to $\bar{c}_1$. Using the free-entry condition $p_1 = r/A$ and the feasibility constraint $K_1 + K_2 = \bar{K}$, the market clearing condition simplifies to

$$A\left(\hat{\imath}\rho(K_2) + Q(\hat{\imath})\bar{K}\right) + (1-\hat{\imath})\bar{c}_1 = A(\bar{K} - K_2) \tag{3.8}$$

where $\rho(K_2) > 0$ denotes the wage-interest rate ratio. Finally, using the price equations (3.3)-(3.6) and the fact that $\hat{\theta} = q(\hat{\imath})$, we can rewrite the threshold condition (3.2) as follows

$$q(\hat{\imath}) = \frac{\bar{c}_1}{A\bar{K}} - \frac{\rho(K_2)}{\bar{K}} \tag{3.9}$$

These last two conditions define a system of two non-linear equations in two unknowns: the measure of constrained agents $\hat{\imath} \in (0,1)$ and the capital supplied to the secondary good sector $K_2 \in \mathbb{R}_{++}$. Its solution fully characterizes the equilibrium prices and quantities for this economy.

Proposition 1: If Assumptions 1 and 3 are satisfied, there exists a unique (interior) Walrasian equilibrium.

Proof: See Appendix B.

3.5.1 Capital Reallocation Effect

Building on this existence result, we are particularly interested in understanding how equilibrium prices and quantities - especially the equilibrium allocation of capital across sectors - varies as a function of the productivity parametre A. Moreover, we would like to know whether dispersion in capital ownership affects the way in which the economy reacts to such productivity shocks.

Corollary 1: Following a positive/negative Hicks-neutral productivity shock, capital is reallocated from the basic/secondary good sector to the secondary/basic good sector.

Proof: See Appendix B.

What is the mechanism driving the reallocation of capital across sectors? For illustrative purposes, consider the case of a negative Hicks-neutral shock. The productivity shock

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obviously has as an immediate consequence a reduction of income for all agents. However, the non-homothetic preferences of consumers results in this productivity shock also engendering a recomposition of demand away from secondary goods and towards basic goods. In other words, a greater share of aggregate income is now spent on the basic good. Because of this *demand composition effect*, a greater share of capital (which is in fixed supply) is reallocated from the luxury goods sector to the basic goods sector.

Since capital and labour are complements in production of the luxury good, the reallocation of capital has as a consequence a lowering of the marginal product of labour in the luxury goods sector. As labour is inelastically supplied, this results in a lowering of the wage rate, and thereby a further decrease in the income of workers over and above the magnitude of the initial productivity shock. Below, we study these effects on factor prices in more detail.

3.5.2 Factor Prices

Given this capital reallocation effect following a Hicks-neutral productivity shock, we now investigate the effects on the wage and interest rate. Differentiating the wage condition (3.4) yields

$$\frac{dw^*}{dA} = \underbrace{(1-\alpha)K_2^{*\alpha}}_{\text{direct effect}} + \underbrace{\alpha(1-\alpha)AK_2^{*\alpha-1}\frac{dK_2^*}{dA}}_{\text{reallocation effect}} > 0$$

The first term of this derivative corresponds to the direct effect of a productivity shock on the marginal product of labour, for a given supply of capital to the secondary good sector. The second term corresponds to the indirect effect of a productivity shock on the marginal product of labour engendered by the reallocation of capital to or from the secondary good sector (recall that capital and labour are complements in production). Turning now to the interest rate, by differentiating condition (3.3) we obtain

$$\frac{dr^*}{dA} = \underbrace{\alpha K_2^{*\alpha-1}}_{\text{direct effect}} - \underbrace{\alpha(1-\alpha)AK_2^{*\alpha-1}\frac{\frac{dK_2^*}{dA}}{K_2^*}}_{\text{scarcity effect}} \lesseqgtr 0$$

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The change in the interest rate following a Hick-neutral productivity shock again consists of a direct (productivity) component and an indirect (reallocation) component. Why is the interest rate, contrary to the wage rate, not always increasing in A? The reason lies in the fact that even though capital becomes more/less productive following a positive/negative productivity shock, it also becomes relatively less/more scarce (i.e the demand for the capital-intensive good increases/decreases in relative terms). This (negative) scarcity effect counterbalances the (positive) productivity effect. It can be shown that for sufficiently small values of $\bar{K}$, the negative (scarcity) effect can in fact dominate the positive (productivity) effect, so that the interest rate will be decreasing in A. However, regardless of whether the interest rate increases or decreases, the wage-interest rate ratio will always be increasing in the productivity parametre A. This is the *factor demand effect*. Formally, we have

$$\frac{d\rho(K_2)}{dA} = \rho'(K_2)\frac{dK_2}{dA} > 0$$

where, using the factor price equations (3.3)-(3.4), we have

$$\rho'(K_2) = \frac{1-\alpha}{\alpha} > 0$$

3.5.3 Inequality and Reallocation

***Ex Ante* Inequality** We now examine how changes in the distribution of capital ownership affect the degree of capital reallocation across sectors following a Hicks-neutral productivity shock. More specifically, we are interested in knowing how the capital reallocation effect identified and discussed above varies as a function of the scaling parametre $\beta > 0$ (where greater values of β imply a more unequal distribution of wealth).

Corollary 2: If Assumption 2 is satisfied and $\frac{\partial q(\hat{i})}{\partial \beta} \geq 0$, the magnitude of the capital reallocation effect is increasing in the scaling parametre β.

Proof: See Appendix B.

This result implies that the way in which changes in *ex ante* inequality affect the propagation of aggregate shocks will depend on the way in which wealth is reshuffled among the agents. Generally speaking, an increase in inequality will amplify the capital reallocation

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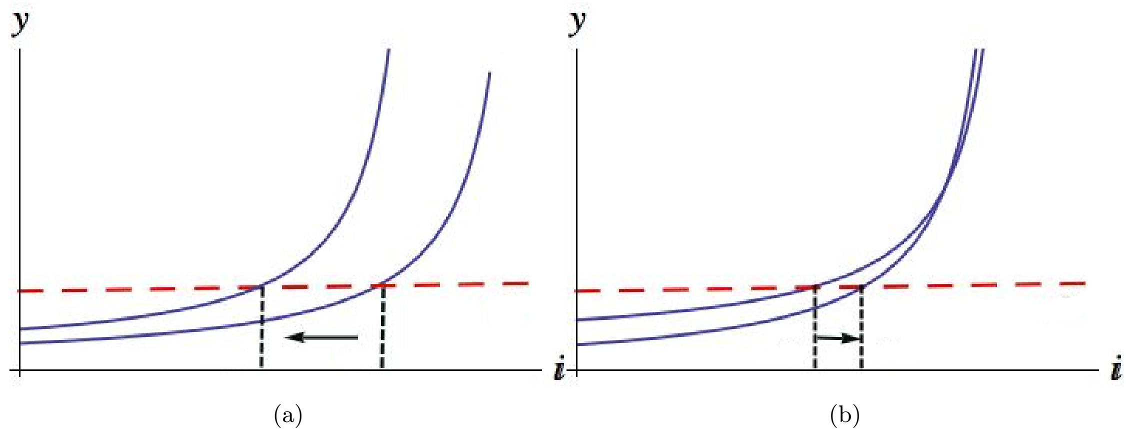

Figure 3.4: Wealth Redistribution Effects. Panel (a) Redistribution of wealth such that the mass of constrained agents decreases. Panel (b) Redistribution of wealth such that the mass of constrained agents increases.

effect if the reallocation of wealth results in the measure of constrained agents falling. More specifically, the sufficient condition provided in Corollary 2 tells us that an increase in wealth inequality will amplify the capital reallocation effect if it increases the marginal agent's share of capital. It is worth emphasising that this is only a sufficient condition. In particular, the amplification effect would persist even if the marginal agent's share of capital decreases, provided that wages rise enough following the redistribution so as to offset the drop in capital income.

Figure 3.4 illustrates graphically two ways in which wealth can be redistributed among the agents, for a given level of productivity. The dashed red line represents the satiation point for agents' demand of the basic good. Agents' whose income lies above this threshold consume a positive quantity of both basic and secondary goods, while agents' whose income lies below this threshold consume only the basic good. Panel (a) represents a case where wealth is redistributed such that the marginal agent gains, and the mass of constrained agents falls. Panel (b), on the other hand, represents a case where the marginal agent loses, and the mass of constrained agents rises. Consider now what happens following a positive Hicks-neutral productivity shock in each case: since the capital reallocation effect is decreasing in the measure of constrained agents, it follows that the redistribution considered in Panel (a) will amplify the capital reallocation effect, while the redistribution

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considered in Panel (b) will dampen the capital reallocation effect.

***Ex Post* Inequality** We now turn to the task of examining how the *ex post* distribution of income changes following a Hicks-neutral shock to aggregate productivity. To this end, we begin by deriving the equilibrium distribution of income. Using the budget constraint of agents, it follows that individual income is given by

$$y_i \equiv I_i = w^* + \theta_i r^* \bar{K}$$

Recall that since $\theta_i \sim G(\theta)$, we must have

$$y_i \sim H(y) \equiv G\left(\frac{y - w^*}{r^* \bar{K}}\right)$$

where $y \in [y_l, y_h]$ with $y_l = w^*$ and $y_h = w^* + r^* \bar{K}$. We use the Gini coefficient to measure the degree of *ex post* income inequality.

Definition 2: Given a piecewise differentiable distribution function $H(y) : [y_l, y_h] \rightarrow [0, 1]$ with associated density function $h(y) : [y_l, y_h] \rightarrow [0, 1]$, the Gini coefficient Γ is defined as

$$\Gamma = \frac{\int_{y_l}^{y_h} H(y)(1 - H(y))dy}{\int_{y_l}^{y_h} yh(y)dy}$$

Using this definition, we are lead to the following result.

Proposition 2: If Assumption 1 is satisfied, the Gini coefficient is decreasing in the productivity parametre A.

Proof: See Appendix B.

The counter-cyclicality of the Gini coefficient is a direct consequence of changes in the wage-interest rate ratio. Note that because the Gini coefficient is scale invariant, changes in the wage and interest rate *pari passu* do not affect the degree of income inequality.[5]

[5]NB: Scaling or multiplying all incomes by the same factor does not change the value of the Gini coefficient.

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However, since the wage-interest rate ratio is increasing in A, the factor by which an agent's income changes following a productivity shock is decreasing in the level of his *ex ante* wealth. This can be seen formally by noticing that the relative change in agents' income after a shock varies as a function of agents' capital ownership position

$$\frac{d\rho(K)}{dA} > 0 \quad \Leftrightarrow \quad \frac{d}{d\theta_i}\left(\frac{y_i + \frac{dy_i}{dA}}{y_i}\right) < 0$$

Alternatively, a simple way to interpret the cyclical dynamics of the income distribution is to notice that labour income is uniformly distributed across the population, while capital income is not. Therefore, whenever the wage increases/decreases relatively more than than the interest rate, the share of aggregate income that is uniformly distributed increases/decreases relative to the share that is unequally distributed.

3.6 Extension: Labour Market Frictions

As pointed out in the introduction, a large part of the variation in income inequality over the business cycle appears to be due to changes in labour income, and more specifically variation in the unemployment rate [61]. Unfortunately, the Walrasian economy analysed above cannot account for changes in the level of employment. In particular, demand composition effects had no implications for aggregate output because factors of production were always used to full capacity. Although productivity shocks induced changes in relative prices, there was no variation along the extensive margin. To address this shortcoming, we extend the model to account for labour market frictions so that some agents remain unemployed in equilibrium. When frictions are introduced, changes in the composition of demand (insofar as they change the matching rate on the labour market) directly affect the level of employment, and thus the level of aggregate output.

3.6.1 Labour Market

We model frictions as in the competitive search literature pioneered by Moen [62]. In a competitive search equilibrium, firms post wage offers. Workers observe all wage offers and apply to at most one job vacancy. Firms that receive at least one application hire one worker, pay the announced wage, and produce. Workers who are not hired re-

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main unemployed, while unfilled jobs remain vacant. In this section, we solve for the (partial) equilibrium in the labour market, treating the matching frictions as an exogenous technological constraint. Interested readers are referred to Appendix A in which the micro-foundations of the matching frictions are derived in full.

Environment Contrary to the Walrasian economy studied above, the secondary good is no longer produced by a representative firm using a Cobb-Douglas production technology. Instead, we assume the secondary good sector to consist of a continuum of homogenous firms, each employing at most one worker. Let $F \subset \mathbb{R}_+$ denote the set of active firms in the secondary goods sector, and denote its Lebesgue measure by $\mathcal{F} \in \mathbb{R}_+$. Each firm needs at least $\kappa > 0$ units of capital to produce, which it rents on a competitive credit market at the interest rate $r > 0$. For simplicity, we assume $\kappa = 1$. Production takes place using a constant returns-to-scale technology: i.e we assume each firm employing a worker produces $A > 0$ unit of the secondary good. Aggregate capital demanded by the secondary goods sector is thus given by

$$K_2 = \int_0^{\mathcal{F}} dj = \mathcal{F} \tag{3.10}$$

Matching frictions imply that not every active firm succeeds in hiring a worker, and hence not every active firm produces output in equilibrium. We assume the probability that firm j successfully hires a worker to be given by[6]

$$\mu(\mathcal{F}) = (1 - e^{-\frac{1}{\mathcal{F}}}) \tag{3.11}$$

Note that this probability is strictly decreasing in the measure of firms active in the secondary goods sector. Each firm posts a wage $w_j \geq 0$ in order to maximise its expected profits. Since firms are homogenous, we will have $w_j = w$ in equilibrium. The expected profits of firms is thus given by

$$\mathbb{E}[\pi] = \mu(\mathcal{F})(A - w) - r \tag{3.12}$$

[6]See Appendix A for a detailed explanation of how this matching function is derived.

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Free-entry of firms into the secondary good sector implies that expected profits must be equal to zero in equilibrium.

Partial Equilibrium We show in Appendix A that the equilibrium wage posted by firms is equal to

$$w(\mathcal{F}; A) = \frac{A}{\mathcal{F}(e^{\frac{1}{\mathcal{F}}} - 1)} \tag{3.13}$$

The free-entry condition pins downs the equilibrium measure of active firms as a function of the interest rate. Substituting the equilibrium wage into the objective function of firms (3.12) and solving for r yields an implicit condition pinning down the capital demanded by the secondary good sector. Formally, we obtain

$$r = A\left(1 - \left(1 + \frac{1}{\mathcal{F}(r; A)}\right)e^{-\frac{1}{\mathcal{F}(r;A)}}\right) \tag{3.14}$$

We refer interested readers to Lemma A1 in Appendix A for a formal proof of the existence and uniqueness of the partial equilibrium in the labour market. The level of employment in this economy is equal to the measure of active firms successfully hiring a worker. Formally, this is given by

$$L_2(\mathcal{F}) = \mu(\mathcal{F})\mathcal{F} \tag{3.15}$$

This leads us to the following result.

Lemma 1: The level of employment is strictly increasing in the quantity of capital allocated to the secondary good sector.

Proof: See Appendix B.

3.6.2 Equilibrium

In this section, we return to the general equilibrium model and introduce the matching frictions outlined above. Contrary to the Walrasian economy, agents now differ both in terms of their initial ownership of the aggregate capital stock and their employment status (i.e whether they are employed or unemployed). Importantly, an individual agents' employment status is independent of his capital ownership position. Given this, the market

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clearing condition (3.7) in the basic good sector becomes

$$(1-L_2)\left(\int_0^{\hat{i}^U} \frac{q(i)r\bar{K}}{p_1}di + (1-\hat{i}^U)\bar{c}_1\right) + L_2\left(\int_0^{\hat{i}^E}\left(\frac{w+q(i)r\bar{K}}{p_1}\right)di + (1-\hat{i}^E)\bar{c}_1\right) = AK_1$$

where L_2 denotes the employment rate as defined by condition (3.15), $\hat{i}^U \in (0,1)$ denotes the marginal unemployed agent, and $\hat{i}^E \in [0,1)$ denotes the marginal employed agent. Note that, as before, we restrict attention to interior equilibria, implying that some (but not all) unemployed agents consume a positive quantity of the secondary good. Since unemployed agents receive no wage income, Assumption 3 simplifies to the following condition.

Assumption 4: The distribution of ownership shares is such that

$$q(0)A\bar{K} < \bar{c}_1 < q(1)A\bar{K}$$

Using the threshold condition (3.2), we can derive explicit expressions for the marginal unemployed and employed agent. Formally, these threshold conditions are given by

$$\hat{i}^U = q^{-1}\left(\frac{\bar{c}_1}{A\bar{K}}\right) \quad \text{and} \quad \hat{i}^E = \max\left\{0, q^{-1}\left(\frac{\bar{c}_1}{A\bar{K}} - \frac{\rho(K_2)}{\bar{K}}\right)\right\}$$

Notice that the condition pinning down the measure of constrained unemployed agents does not depend on the allocation of capital across sectors. Hence, even though the measure of unemployed agents varies as a function of the quantity of capital allocated to the secondary good sector, the quantity of basic good demanded by each unemployed agent will be constant. Intuitively, this is because unemployed agents by definition do not earn a wage, and are thus unaffected by changes in the wage-interest rate ratio.

Using the free-entry condition $p_1 = r/A$ and the feasibility condition $K_1 + K_2 = \bar{K}$, we can rewrite the market clearing condition as follows

$$A(\bar{K}-K_2) = (1-L_2)\left((1-\hat{i}^U)\bar{c}_1 + Q(\hat{i}^U)A\bar{K}\right) + L_2\left((1-\hat{i}^E)\bar{c}_1 + Q(\hat{i}^E)A\bar{K} + \hat{i}^E A\rho(K_2)\right)$$

As before, these conditions constitute a system of two non-linear equations in two unknowns: the capital supplied to the secondary good sector $K_2 \in \mathbb{R}_{++}$ and the measure of constrained employed agents $\hat{i}^E \in (0,1)$. This leads us to the following existence result.

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Proposition 3: If Assumptions 1 and 4 are satisfied, there exists a unique (interior) competitive equilibrium in the model with frictions.

Proof: See Appendix B.

Given this, we now show that the capital reallocation effect remains when frictions are introduced. Moreover, from Lemma 1, this implies that the level of employment varies as a function of aggregate productivity.

Corollary 3: Following a positive/negative Hicks-neutral productivity shock, capital is reallocated from the basic/secondary good sector to the secondary/basic good sector. Moreover, the level of employment is increasing in the productivity parametre A.

Proof: See Appendix B.

Broadly speaking, this result stems from the fact that productivity shocks, insofar as they change the composition of demand due to the non-homotheticity of consumer preferences, change the measure of firms active in the secondary good sector. As total employment is proportional to the measure of active firms in the secondary good sector, productivity shocks will directly affect the level of equilibrium employment. Contrary to the Walrasian case in which productivity shocks only affected relative prices, the model with frictions is also able to capture variation along the extensive margin. This, in turn, implies that changes in the *ex post* distribution of income will now be determined both by changes in the wage-interest rate ratio and by changes in the level of employment.

3.6.3 Inequality and Unemployment

Analytical Characterisation In this section, we examine how unemployment affects *ex post* inequality in this economy. To do this, we must once again derive the distribution of income in equilibrium. Deriving the income distribution in the model with frictions is somewhat more involved than in the Walrasian case, since the set of agents is now partitioned into employed and unemployed workers. However, the task is simplified by the fact that an individual agent's employment status is independent of his wealth. Partitioning

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agents based on their employment status, we have that

$$y_i^E = w^* + \theta_i r^* \bar{K} \quad \text{and} \quad y_i^U = \theta_i r^* \bar{K}$$

where y_i^E and y_i^U denotes the income of employed and unemployed agents, respectively. Again, since $\theta_i \sim G(\theta)$ we have that

$$y_i^E \sim G\left(\frac{y^E - w}{r\bar{K}}\right) \quad \text{and} \quad y_i^U \sim G\left(\frac{y^U}{r\bar{K}}\right)$$

where $y^E \in [w^*, w^* + r^*\bar{K}]$ and $y^U \in [0, r^*\bar{K}]$. It follows that the distribution of income is given by the following piecewise continuous function

$$y_i \sim H(y) \equiv \mathbf{1}_{y \leq w} G\left(\frac{y}{r\bar{K}}\right)(1 - L_2) +$$
$$\mathbf{1}_{w<y<r\bar{K}}\left(G\left(\frac{y}{r\bar{K}}\right)(1 - L_2) + G\left(\frac{y-w}{r\bar{K}}\right)L_2\right) + \mathbf{1}_{y \geq r\bar{K}}\left((1 - L_2) + G\left(\frac{y-w}{r\bar{K}}\right)L_2\right)$$

Although well defined, deriving an analytical expression for the Gini coefficient using this income distribution function is quite tedious. Consequently, we turn to some simple numerical simulations in order to analyse how the distribution of income is affected by aggregate productivity shocks.

Numerical Simulation In what follows, we assume that ownership shares are exponentially distributed across the population. The cumulative distribution function of the truncated exponential distribution is given by

$$\theta_i \sim G(\theta) = \frac{e^{\beta} - e^{\beta(1-\theta)}}{e^{\beta} - 1}$$

Table 3.2 provides numerical estimates of the Gini coefficient of income, for both the Walrasian economy and the economy with labour market frictions, for different values of the productivity parametre A and scaling parametre β.

The numerical results illustrate how the introduction of labour market frictions affect the cyclical properties of the income distribution. Note that the qualitative properties of the two models are largely identical. In particular, the introduction of labour market frictions does not change the counter-cyclical properties of the Gini coefficient. That

	$\beta=3$	$\beta=5$	$\beta=10$		$\beta=3$	$\beta=5$	$\beta=10$
$A=3.7$	0.357	0.357	0.356	$A=3.7$	0.328	0.271	0.162
$A=3.8$	0.355	0.356	0.355	$A=3.8$	0.326	0.269	0.161
$A=3.9$	0.353	0.355	0.354	$A=3.9$	0.323	0.268	0.160
$A=4$	0.351	0.354	0.353	$A=4$	0.321	0.266	0.160

Table 3.2: Numerical Simulations of Gini coefficients: Frictions (left) and Walrasian (right).

being said, equilibrium unemployment implies that the Gini coefficient in the model with frictions always exceeds the Gini coefficient in the Walrasian economy. Moreover, while the dispersion of *ex post* income in the Walrasian case is highly sensitive to changes in the dispersion of capital ownership, the Gini coefficient for the economy with frictions is remarkably stable. The reason for this is that the presence of unemployment increases the measure of constrained agents in equilibrium. Since the capital reallocation effect is decreasing in $\hat{\imath}$ (the measure of constrained agents), this implies that the distributional consequences of shocks will be less pronounced in economies featuring a high level of unemployment. Thus, the introduction of unemployment affects both the value and the cyclical dynamics of the Gini coefficient.

As a footnote, it might seem counterintuitive that the dispersion in *ex post* income is *decreasing* in β (which measures the dispersion in *ex ante* wealth). The reason for this comes from a specific feature of the truncated exponential distribution. Recall that, broadly speaking, the dispersion in *ex post* income will be decreasing in the quantity of capital supplied to the secondary good sector, due to the implied increase in the wage-interest rate ratio. Solving the threshold condition (3.9) for the capital supplied to the secondary good sector, we obtain

$$K_2 = \frac{\alpha}{1-\alpha}\left(\frac{\bar{c}_1}{A} - \bar{K}q(\hat{\imath};\beta)\right)$$

Differentiating this condition with respect to β yields

$$\frac{dK_2}{d\beta} = -\frac{\alpha}{1-\alpha}\bar{K}\left(q'(\hat{\imath})\frac{di}{d\beta} + \frac{\partial q}{\partial \beta}\right)$$

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In the case of the truncated exponential distribution, we have that

$$-\frac{\partial q}{\partial \beta} > q'(\hat{i})\frac{d\hat{i}}{d\beta} > 0$$

Thus, as the dispersion in *ex ante* wealth increases, the equilibrium measure of constrained agents also increases. This, in itself, should imply that the capital allocated to the secondary good sector is decreasing in β, while the dispersion in *ex post* income should be increasing in β. However, in this case, this effect is counteracted by the fact that the wealth of the marginal agent is strictly decreasing in β; or, equivalently, the cumulative wealth of the unconstrained agents is strictly increasing in β. In fact, this second effect, which implies the capital allocated to the secondary good sector should be increasing in β, is so strong that it dominates the increasing measure of constrained agents. Consequently, in aggregate, we have that as the *ex ante* dispersion in wealth increases, the capital supplied to the secondary good sector also increases, thereby reducing the dispersion in *ex post* income. It is important to emphasise that this is not a general property, but rather a consequence of the specific functional form assumed for $G(\theta)$.

3.7 Conclusion

This paper studied how inequalities affect short-run macroeconomic performance. To this end, we developed a multi-sector general equilibrium model, and studied to what extent changes in the composition of demand can explain the sectoral variation of output and employment over the business cycle. Three key features drive the underlying mechanism of the model: (i) consumers have non-homothetic preferences, implying that aggregate consumption shares vary with aggregate income; (ii) production technologies are such that the factor share of capital is greater in sectors producing more 'basic goods;' and (iii) the ownership of the aggregate capital stock is unequally distributed among the population. The model shows that demand composition effects have important implications for changes in relative factor rewards, and are thus an important channel through which supply shocks are propagated through the economy. In particular, we found that the reallocation of capital investment across sectors is a key factor driving the heterogenous sectoral response to aggregate shocks. This is consistent with new empirical evidence showing that the share of investment in capital-intensive sectors increases during recessions.

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We then used this theoretical framework to study how the *ex ante* level of inequality affects the propagation of aggregate shocks. We found the magnitude of the aforementioned capital reallocation effect to depend in interesting ways on the initial distribution of capital ownership. In particular, any redistribution of wealth which increases the recomposition of aggregate demand over the cycle was found to amplify the effects of productivity shocks. We also examined the *ex post* distributional consequences of short-run macroeconomic fluctuations. In order to capture variations in the level of unemployment (which are known empirically to be an important driver of changes in inequality over the business cycle), we introduced labour market frictions into the baseline model. The model predicts that the Gini coefficient for income is counter-cyclical. Moreover, this result was shown to be primarily driven by changes in wages and the level of unemployment, confirming existing empirical work studying the cyclical properties of the income distribution.

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3.8 Appendix A: Competitive Search Equilibrium

This section provides formal derivations of the partial equilibrium in the labour market.

Environment Firms post wage announcements $w_j \geq 0$. After having observed the distribution of wage announcements, each worker chooses a (symmetric) application strategy, denoted by $\sigma_j \in [0,1]$ for all $j \in F$ such that $\int_0^{\mathcal{F}} \sigma_j dj = 1$. The workers' application strategies induce an expected queue length at each firm, denoted by $\lambda_j \geq 0$. This corresponds to the expected number of job applicants at a firm posting wage w_j. Given the assumption that application strategies are symmetric and independent across workers, the actual number of applicants at a firm posting wage w_j is a Poisson random variable with mean λ_j. Each firm posting wage w_j receives $z \in \{0, 1, 2, ...\}$ applicants with probability $\frac{\lambda_j^z e^{-\lambda_j}}{z!}$. It follows that the probability that a worker applying to a firm posting a wage w_j is hired is equal to

$$\nu(\lambda_j) = \lim_{\bar{z} \to \infty} \sum_{z=0}^{\bar{z}} \frac{1}{(z+1)} \frac{\lambda_j^z e^{-\lambda_j}}{z!} = \frac{1 - e^{-\lambda_j}}{\lambda_j}$$

It follows that we must have

$$\mu(\lambda_j) \equiv \lambda_j v(\lambda_j) = (1 - e^{-\lambda_j})$$

The queue lengths are determined such that each worker obtains an expected utility of at least $V > 0$ from applying to any active firm. Since a worker facing a queue of length λ_j is hired with probability $\nu(\lambda_j)$, this implies the following indifference condition must hold in equilibrium

$$\nu(\lambda_j) w_j = V$$

Labour market clearing requires that the total number of workers searching for a job must equal the aggregate labour supply. Formally, this implies

$$\int_0^{\mathcal{F}} \lambda_j dj = 1$$

Definition A1: A competitive search equilibrium is defined as a tuple $\langle w, \lambda, V, \mathcal{F} \rangle$ such that

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- Firms choose wages w to maximize expected profits, taking as given workers' expected utility V and queue lengths λ.
- Each worker applies to exactly one firm thereby inducing queue lengths λ, taking the profile of wages w as given.
- Queue lengths λ and the measure of firms entering the market $\mathcal{F}$ are such that the labour market clears.

Equilibrium We now solve for the partial equilibrium in the labour market, taking the interest rate $r > 0$ as exogenous. Begin by substituting the indifference condition into the objective function of firms as given by condition (3.12)

$$\mathbb{E}[\pi_j] = A(1 - e^{-\lambda_j}) - \lambda_j V - r$$

Differentiating this equation with respect to λ_j yields the first-order condition

$$\lambda_j = \log\left(\frac{A}{V}\right), \quad \forall j \in \mathcal{F}$$

Since the RHS of this condition does not depend on j, it must be that the equilibrium queue lengths (and thus the equilibrium wage announcements) are the same for all active firms. Given this, the labour market clearing condition implies

$$\lambda(\mathcal{F}) = \frac{1}{\mathcal{F}}$$

Combining the last two equations allows us to solve for the equilibrium expected utility of workers

$$V(\mathcal{F}; A) = Ae^{-\frac{1}{\mathcal{F}}}$$

Plugging this condition into the indifference condition of workers, we can solve for the wage posted by firms in equilibrium

$$w(\mathcal{F}; A) = \frac{A}{\mathcal{F}(e^{\frac{1}{\mathcal{F}}} - 1)}$$

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Substituting this into the expected profit condition (3.12) and simplifying, we obtain

$$\mathbb{E}[\pi] = A\left(1 - \left(1 + \frac{1}{\mathcal{F}}\right)e^{-\frac{1}{\mathcal{F}}}\right) - r$$

Free-entry of firms into the secondary good sector implies expected profits are equal to zero in equilibrium. This pins down the equilibrium measure of active firms as a function of the interest rate. Setting the last condition equal to zero and solving for r yields

$$r = A\left(1 - \left(1 + \frac{1}{\mathcal{F}(r;A)}\right)e^{-\frac{1}{\mathcal{F}(r;A)}}\right)$$

This leads us to the following existence result.

Lemma A1: Given any interest rate $r \in (0, A]$, there exists a unique competitive search equilibrium. Moreover, the equilibrium measure of active firms $\mathcal{F}^*$ is decreasing in r and increasing in A.

Proof: Begin by noticing that expected gross revenue of a firm is a continuous and monotonically decreasing function of the measure of active firms, beginning at A when $\mathcal{F} = 0$ and converging to 0 as $\mathcal{F} \to \infty$. Formally,

$$\frac{d}{d\mathcal{F}}\left(A\left(1 - \left(1 + \frac{1}{\mathcal{F}}\right)e^{-\frac{1}{\mathcal{F}}}\right)\right) = -\frac{e^{-\frac{1}{\mathcal{F}}}}{\mathcal{F}^3} < 0$$

It follows that given any (exogenous) interest rate $r \in (0, A]$, there exists a unique and finite equilibrium measure of active firms $\mathcal{F}^*(r; A)$. $\square$

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3.9 Appendix B: Proofs

3.9.1 Proof of Proposition 1.

Recall that the LHS of condition (3.8) corresponds to the aggregate demand for the basic good, while the RHS equals the aggregate supply of the basic good. It is easy to verify that the RHS is monotonically decreasing in K_2 from $[A\bar{K}, 0]$ on the interval $K_2 \in [0, \bar{K}]$. Differentiating the LHS with respect to K_2, we obtain

$$\frac{d\hat{i}}{dK_2} \underbrace{\left(A\left(\rho(K_2) + q(\hat{i})\bar{K}\right) - \bar{c}_1\right)}_{=0} + A\left(\frac{1-\alpha}{\alpha}\right)\hat{i} > 0$$

where the inequality follows from Assumption 3, since it implies that $\hat{i} \in (0,1)$ is such that

$$\frac{w + q(\hat{i})r\bar{K}}{p_1} = \bar{c}_1$$

It follows that aggregate demand for the basic good is monotonically increasing in K_2. Evaluating the LHS of the market clearing condition (3.8) at $K_2 = 0$, we have that

$$Q(\hat{i})A\bar{K} + (1-\hat{i})\bar{c}_1 < A\bar{K}$$

since $\rho(0) = 0$. Rearranging, we obtain

$$(1 - Q(\hat{i}))A\bar{K} > (1-\hat{i})\bar{c}_1$$

where the inequality follows from Assumption 1 since $Q(i) < i$ for all $i \in (0,1)$, and Assumption 3 since $A\bar{K} > \bar{c}_1$. It follows that there exists a unique equilibrium. □

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3.9.2 Proof of Corollary 1.

Rewriting the market clearing condition (3.8) and differentiating with respect to A yields

$$\frac{dK_2^*}{dA} = (1-\hat{i}^*)\frac{\bar{c}_1}{A^2} - \hat{i}^*\rho'(K_2^*)\frac{dK_2^*}{dA} + \frac{d\hat{i}^*}{dA}\underbrace{\left(\frac{\bar{c}_1}{A} - \rho(K_2^*) - q(\hat{i}^*)\bar{K}\right)}_{=0}$$

Solving for dK_2^*/dA, we obtain

$$\frac{dK_2^*}{dA} = \omega(\hat{i}^*;\alpha)\frac{\bar{c}_1}{A^2} > 0$$

where

$$\omega(\hat{i};\alpha) = \frac{\alpha(1-\hat{i})}{\alpha + (1-\alpha)\hat{i}} \in (0,1)$$

This completes the proof. □

3.9.3 Proof of Corollary 2.

Differentiating the comparative static condition above with respect to β, we obtain

$$\frac{dK_2^{*2}}{dAd\beta} = \underbrace{\frac{\partial\omega(\hat{i}^*;\alpha)}{\partial\hat{i}}}_{(-)}\frac{\bar{c}_1}{A^2}\frac{d\hat{i}^*}{d\beta}$$

It follows that the capital reallocation effect will be increasing in β if and only if $\frac{d\hat{i}^*}{d\beta} < 0$. Solving the threshold condition (3.9) for K_2, we obtain

$$K_2 = \frac{\alpha}{1-\alpha}\left(\frac{\bar{c}_1}{A} - \bar{K}q(\hat{i})\right)$$

Substituting this into the market clearing condition (3.8) and rearranging yields

$$\left(\frac{\alpha}{1-\alpha}\right)q(\hat{i}) - Q(\hat{i}) = \frac{1}{1-\alpha}\frac{\bar{c}_1}{A\bar{K}} - 1$$

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This equation implicitly pins downs the equilibrium measure of constrained agents. Taking the derivative with respect to β, we find

$$\frac{d\hat{i}^*}{d\beta} = \frac{1}{q'(\hat{i}^*)}\left(\frac{\partial Q(\hat{i}^*)/\partial\beta}{\left(\frac{\alpha}{1-\alpha}+\hat{i}^*\right)} - \frac{\partial q(\hat{i}^*)}{\partial\beta}\right)$$

Given Assumption 2, it follows that a sufficient condition for the capital reallocation effect to be increasing in β is

$$\frac{\partial q(\hat{i}^*)}{\partial\beta} > 0$$

This completes the proof. □

3.9.4 Proof of Proposition 2.

We begin by rewriting the Gini coefficient in terms of the quantile function. Formally,

$$\Gamma = 1 - 2\int_0^1 L(x)dx$$

where

$$L(x) = \frac{\int_0^x H^{-1}(p)dp}{\int_0^1 H^{-1}(p)dp}$$

is the Lorenz curve and $H^{-1}(p) = w^* + Q(p)r^*\bar{K}$ is the income quantile function. It follows that the Gini coefficient will be decreasing in A if and only if the Lorenz curve is increasing in A. Formally,

$$\frac{d}{dA}\frac{\int_0^x w^* + Q(p)r^*\bar{K}dp}{\int_0^1 w^* + Q(p)r^*\bar{K}dp} > 0$$

Multiplying and dividing by r, we have

$$\frac{d}{dA}\frac{\int_0^x \rho^*(A) + Q(p)\bar{K}dp}{\int_0^1 \rho^*(A) + Q(p)\bar{K}dp} > 0$$

which implies

$$\left(\int_0^x \frac{d}{dA}\rho^*(A)dp\right)\int_0^1 H^{-1}(p)dp - \left(\int_0^1 \frac{d}{dA}\rho^*(A)dp\right)\int_0^x H^{-1}(p)dp > 0$$

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Simplifying, we obtain

$$\left(x\int_0^1 H^{-1}(p)dp - \int_0^x H^{-1}(p)dp\right)\frac{d}{dA}\rho^*(A) > 0$$

From Assumption 1, we must have

$$x > L(x) = \frac{\int_0^x H^{-1}(p)dp}{\int_0^1 H^{-1}(p)dp}$$

Since $\rho^*(A)$ is always increasing in A, this completes the proof. □

3.9.5 Proof of Lemma 1.

The result is obtained by differentiating the employment condition (3.15) with respect to the measure of active firms $\mathcal{F}$. It is easy to verify that $\frac{dL_2}{d\mathcal{F}} > 0$. Since the capital demand condition (3.10) implies that $K_2 = \mathcal{F}$, the result follows immediately. □

3.9.6 Proof of Proposition 3.

Differentiating the RHS of the marketed clearing condition with respect to K_2 yields

$$\underbrace{\frac{dL_2}{dK_2}}_{(+)}\left(D_1^E(K_2) - \bar{D}_1^U\right) + L_2\left(\underbrace{\frac{d\hat{i}^E}{dK_2}\left(A\rho(K_2) + q(\hat{i}^E)A\bar{K} - \bar{c}_1\right)}_{=0} + \hat{i}^E A\underbrace{\rho'(K_2)}_{(+)}\right)$$

where $D_1^E(K_2)$ and $\bar{D}_1^U$ denotes the quantity of basic good demanded by employed and unemployed agents, respectively. Notice that, contrary to the Walrasian case, all employed agents can be unconstrained in equilibrium. That is, we can have

$$\frac{w + q(\hat{i}^E)r\bar{K}}{p_1} > \bar{c}_1$$

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implying that $\hat{i}^E = 0$. Notice that by definition in such a case we will have $d\hat{i}^E/dK_2 = 0$. Using the capital demand, wage and free-entry conditions (3.10), (3.13)-(3.14) we have

$$\rho'(K_2) = \rho(K_2)^2 \left(2 - \left(1 - \frac{1}{K_2}\right) e^{\frac{1}{K_2}} - \left(1 + \frac{1}{K_2} + \frac{1}{{K_2}^2}\right) e^{-\frac{1}{K_2}}\right) > 0$$

It follows that the aggregate demand of basic good will be monotonically increasing in $\mathcal{F}$ if and only if employed agents demand strictly more basic good than unemployed agents. Formally,

$$D_1^E(K_2) - \bar{D}_1^U = (\hat{i}^U - \hat{i}^E)\bar{c}_1 + \hat{i}^E A\rho(K_2) - Q(\hat{i}^U - \hat{i}^E)A\bar{K} > 0$$

Dividing by $A\bar{K}$ and noticing that $q(\hat{i}^U) = \bar{c}_1/A\bar{K}$, we obtain

$$(\hat{i}^U - \hat{i}^E)q(\hat{i}^U) + \hat{i}^E \frac{\rho(K_2)}{\bar{K}} - Q(\hat{i}^U - \hat{i}^E) > 0$$

where the inequality follows from Assumption 1 as long as $\hat{i}^U \neq \hat{i}^E$. From Assumption 4, we have that $\hat{i}^U > \hat{i}^E$ since $\hat{i}^U > 0$ and $\hat{i}^E < 1$. Evaluating aggregate demand for the basic good at $K_2 = 0$, and noticing that $L_2 = 0$ when $K_2 = 0$, we must have

$$Q(\hat{i}^U)A\bar{K} + (1 - \hat{i}^U)\bar{c}_1 < A\bar{K}$$

Rearranging, we obtain

$$(1 - Q(\hat{i}^U))A\bar{K} > (1 - \hat{i}^U)\bar{c}_1$$

which is always the case as long as $\hat{i}^U < 1$ since $Q(\hat{i}^U) < \hat{i}^U$ and $A\bar{K} > \bar{c}_1$ by assumption. Finally, since aggregate supply of the basic good is monotonically decreasing in K_2 starting at $A\bar{K}$ when $K_2 = 0$, it follows that there exists a unique competitive equilibrium. □

3.9.7 Proof of Corollary 3.

Rearranging the market clearing condition, we obtain

$$K_2^* = \bar{K} - (1 - L_2)\left((1 - \hat{i}^U)\frac{\bar{c}_1}{A} + Q(\hat{i}^U)\bar{K}\right) - L_2\left((1 - \hat{i}^{E*})\frac{\bar{c}_1}{A} + Q(\hat{i}^{E*})\bar{K} + \hat{i}^{E*}\rho(K_2^*)\right)$$

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Differentiating this condition with respect to A yields

$$\frac{dK_2^*}{dA} = \frac{(1-L_2)(1-\hat{i}^U)\bar{c}_1 + L_2(1-\hat{i}^{E*})\bar{c}_1}{A^2} -$$
$$(D_1^E(K_2^*) - \bar{D}_2^U)\frac{\partial L_2}{\partial K_2}\frac{dK_2^*}{dA} - L_2\left(\hat{i}^{E*}\rho'(K_2^*)\frac{dK_2^*}{dA} + \underbrace{\frac{d\hat{i}^{E*}}{dA}\left(\rho(K_2^*) + q(\hat{i}^{E*})\bar{K} - \frac{\bar{c}_1}{A}\right)}_{=0}\right)$$

where again we have that whenever $\hat{i}^E = 0$ we will have $d\hat{i}^E/dA = 0$. Rearranging yields the following comparative static condition

$$\frac{dK_2^*}{dA} = \frac{(1-L_2)(1-\hat{i}^U)\bar{c}_1 + L_2(1-\hat{i}^{E*})\bar{c}_1}{A^2}\left(1 + \hat{i}^{E*}\rho'(K_2^*)L_2 + (D_1^E(K_2^*) - D_2^U)\frac{dL_2}{dK_2}\right)^{-1} > 0$$

This completes the proof. □

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