European University Institute

ROBERT SCHUMAN CENTRE FOR ADVANCED STUDIES

EUI WORKING PAPERS

RSCAS 2013/50
Robert Schuman Centre for Advanced Studies
Loyola de Palacio Programme on Energy Policy

Statistical description of the error on wind power forecasts via a Lévy α-stable distribution

Kenneth Bruninx, Erik Delarue and William D'haeseleer

European University Institute
Robert Schuman Centre for Advanced Studies
Loyola de Palacio Programme on Energy Policy

Statistical description of the error on wind power forecasts via a Lévy α-stable distribution

Kenneth Bruninx, Erik Delarue and William D'haeseleer

EUI Working Paper **RSCAS** 2013/50

ISSN 1028-3625

Printed in Italy, July 2013
European University Institute
Badia Fiesolana
I – 50014 San Domenico di Fiesole (FI)
Italy
www.eui.eu/RSCAS/Publications/
www.eui.eu
cadmus.eui.eu

Robert Schuman Centre for Advanced Studies

The Robert Schuman Centre for Advanced Studies (RSCAS), created in 1992 and directed by Stefano Bartolini since September 2006, aims to develop inter-disciplinary and comparative research and to promote work on the major issues facing the process of integration and European society.

The Centre is home to a large post-doctoral programme and hosts major research programmes and projects, and a range of working groups and *ad hoc* initiatives. The research agenda is organised around a set of core themes and is continuously evolving, reflecting the changing agenda of European integration and the expanding membership of the European Union.

Details of the research of the Centre can be found on:
http://www.eui.eu/RSCAS/Research/

Research publications take the form of Working Papers, Policy Papers, Distinguished Lectures and books. Most of these are also available on the RSCAS website:
http://www.eui.eu/RSCAS/Publications/

The EUI and the RSCAS are not responsible for the opinion expressed by the author(s).

Loyola de Palacio Energy Policy Chair

The Loyola de Palacio Energy Policy Chair was created in October 2008 at the RSCAS in honour of Loyola de Palacio, former Vice President of the European Commission and Commissioner for Energy and Transportation in the Prodi Commission. It promotes research in the area of energy policy. It is funded by contributions from donors. Professor Jean-Michel Glachant is the holder of the Chair.

The Chair focuses on the fields of energy economics, law, regulation, as well as geo-politics. It addresses topics such as the *achievement of the EU internal energy market; sustainable energy systems and the environment; energy security of supply; the EU model of energy regulation; the EU energy competition policy; the EU policy towards carbon free energy systems in 2050.*

The series of working papers aims at disseminating the work of academics on the above-mentioned energy policy issues.

For further information
Loyola de Palacio Energy Policy Chair
Nicole Ahner (scientific coordinator)
Email contact: Nicole.Ahner@eui.eu
Robert Schuman Centre for Advanced Studies
European University Institute
Via delle Fontanelle, 19
I-50016 San Domenico di Fiesole (FI)
Fax: +39055 4685755
http://www.loyola-de-palacio-chair.eu

Statistical description of the error on wind power forecasts via a Lévy α-stable distribution

Kenneth Bruninx, Erik Delarue and William D'haeseleer

Abstract

As the share of wind power in the electricity system rises, the limited predictability of wind power generation becomes increasingly critical for operating a reliable electricity system. In most operational & economic models, the wind power forecast error (WPFE) is often assumed to have a Gaussian or so-called β-distribution. However, these distributions are not suited to fully describe the skewed and heavy-tailed character of WPFE data. In this paper, the Lévy α-stable distribution is proposed as an improved description of the WPFE. Based on 6 years of historical wind power data, three forecast scenarios with forecast horizons ranging from 1 to 24 hours are simulated via a persistence approach. The Lévy α-stable distribution models the WPFE better than the Gaussian or so-called β-distribution, especially for short term forecasts. In a case study, an analysis of historical WPFE data showed improvements over the Gaussian and β-distribution between 137 and 567% in terms of cumulative squared residuals. The method presented allows to quantify the probability of a certain error, given a certain wind power forecast. This new statistical description of the WPFE can hold important information for short term economic & operational (reliability) studies in the field of wind power.

Index Terms

Error analysis, Lévy α-stable distribution, Statistical analysis, Stable process, Wind power forecasting, Wind power generation

K. Bruninx, E. Delarue and W. D'haeseleer are with the University of Leuven (KU Leuven) Energy Institute, TME Branch (Energy Conversion), Celestijnenlaan 300A post box 2421, B-3001 Leuven, Belgium.

Corresponding author: William D'haeseleer, tel: +3216/322510 , fax: +3216/322985, e-mail: william.dhaeseleer@mech.kuleuven.be.

I. INTRODUCTION

As the penetration of intermittent renewables in the grid rises, new operational and regulatory challenges emerge. One of the most mature renewable energy technologies is wind power. In this case, the variability and unpredictability of the wind power generation are two critical aspects in operating a reliable electricity system that need to be addressed. In this paper, the focus is on the statistical, distribution-based description of the error on wind power forecasts.

Various forecasting methods are being used and even more are under development, ranging from basic persistence methods to complex statistical and physical models based on weather predictions [1]. None of these forecasting methods can generate a perfect wind power forecast (WPF). The error on a WPF has various sources: errors on the wind prediction, local effects due to the terrain, non-uniformity of the wind in a wind park, non-linearity in the dynamics of wind turbines, unplanned outages etc. [1]–[4]. Detailed knowledge of this error is indispensable when one wants to study or operate an electricity system with a considerable amount of wind power.

The operational impact of the uncertainty on WPFs in the planning of electricity generation has been studied extensively [5]–[13]. Makarov et al. [5] study the load-following capabilities of conventional power plants when faced with erroneous WPF and load forecasts in California. The wind power forecast error (WPFE) is modelled as a truncated normal distribution and fitted to measured wind power data. Doherty and O'Malley [6] present a probabilistic method to quantify the demand for reserves in power systems with significant wind capacity. The WPFE is modelled as a Gaussian stochastic variable with a zero mean and a certain standard deviation. Bouffard and Galiana [7] formulate a short-term electricity market-clearing problem in which the demand for reserves can be (partly) covered by wind power. The WPFE is modelled as Gaussian stochastic variable with a zero mean and non-zero variance. Bouffard and Galiana justify this assumption via a Central Limit Theorem (CLT) argument. Methaprayoon et al. [8] incorporate WPF uncertainty in an unit commitment (UC) model via confidence intervals, based on the assumption of a normal distribution of the WPFE. Ummels et al. [9] investigate the benefits of an optimal UC for a thermal generation unit. To characterize the WPFE, a Gaussian distribution is assumed. Delarue et al. [10], [11] investigate the effect of WPFEs on the economic dispatch problem, given a day-ahead unit commitment. A random Gaussian distributed forecast error (zero mean, predefined standard deviation) is imposed on the wind power forecasts. Bludszuweit et al. [12], [13] employ the β-distribution to investigate the impact of WPFEs on the needed power ratings of energy storage systems.

Similarly, the economic impact of WPFEs has been studied [14]–[16]. De Vos et al. [14] reviewed the support system to reduce imbalance costs for offshore wind power generators. Empirical statistical models are used to analyse the imbalance settlements. Bathurst et al. [15] assess the effect of imbalance prices and uncertain wind generation on the market behaviour of a generator. They propose a method to determine the optimum level of contract energy to be sold on the advance markets based on Markov probabilities for a wind farm. Fabbri et al. [16] quantify the cost due to WPFEs. In their analysis, they employ a β-distribution to describe the WPFE.

A key element in all these studies is the probability density function (pdf) that is assumed for the wind power forecast error (WPFE). As the shape of the WPFE pdf is dependent on the forecast horizon and method, a proper definition for this pdf is hard to find. Up to today, mainly the Gaussian distribution [3], [17] and the so-called β-distribution (see Section IV-D) [12], [16], [18], [19] are employed to describe the WPFE. The analysis presented by Bludszuweit et al. [12] shows that the Gaussian distribution cannot describe the heavy-tailed character of the WPFE, a statement confirmed by Hodge et al. [20], [21]. Bludszuweit et al. note that in some cases, the β-distribution is not sufficiently heavy-tailed to model the leptokurtosis[1] of the WPFE data, leading to an underestimation of the frequency of the largest errors.

In this paper, the Lévy α-stable distribution is proposed as an alternative to describe the WPFE. In an approach similar to that of Bludszuweit et al. [12], the method presented permits the quantification of the probability of a certain error ϵ given a certain prediction p^*. The improved performance of the Lévy α-stable distribution compared to that of the Gaussian and β-distribution is demonstrated.

Throughout the analysis, a system perspective (aggregated wind farms in for example the control zone of a TSO or the portfolio of a GenCo) will be maintained, albeit the analysis can be done for a single wind farm or turbine as well. The focus

[1]The kurtosis is a measure for the peakedness or tail weight of a distribution. It is the fourth standardized moment. Excess kurtosis is the fouth standardized moment minus the kurtosis of the Guassian distribution (3). Leptokurtic distributions are heavy-tailed compared to the Gaussian distribution (kurtosis > 3, excess kurtosis > 0).

is on short term forecasts (forecast horizon up to 24 hours). The effect of the temporal and spatial resolution of the data has not been taken into account explicitly[2].

First, the literature on the statistical description of the WPFE is reviewed. In the following section, the Lévy α-stable distribution is introduced. Third, the proposed methodology is presented. Fourth, the feasibility of the stable distribution as a description of the WPFE is demonstrated. Last, a conclusion is formulated and some possible improvements are outlined.

II. LITERATURE STUDY: STATISTICAL DESCRIPTION OF THE WPFE

The WPFE is often described by means of standard statistical error measures: the normalized mean absolute error (NMAE), the normalized root mean square error (NRMSE), the bias and the normalized standard deviation (NSTDEV) [1], [23]. To compare the performance of different WPF techniques, skill scores (relative improvement compared to a reference method) or the coefficients of determination R^2 are used [1]. For a more detailed description of the WPFE, a number of authors have considered a distribution-oriented approach, based on the joint distribution[3] of the forecasts p^* and the WPFE ϵ.

Up to today, mainly the Gaussian distribution (see above and [3], [17]) and the β-distribution [12], [16], [18], [19] are employed to describe the WPFE. Other distributions have been examined as well, such as the Cauchy distribution [20], the hyperbolic distribution [21], the γ-distribution [25] and the Weibull distribution [26]. However, WPFE data exhibits heavy tails (leptokurtosis) and skewness – shape characteristics that are not well captured by Gaussian or β-distributions. A number of approaches have been proposed to circumvent these shortcomings.

In [3], [17] Lange explores the idea to transform the well-behaved Gaussian distributions of the wind speed prediction errors into non-Gaussian distributions of the WPFE via Taylor expansions of the power curve of the wind farm. However, this analysis only takes into account the effect of the power curve on the level of the wind turbine (park), neglecting all other effects.

Pinson [2], [27]–[30] focusses on the description of the WPFE from an operational vantage point. The WPFE is characterized via a non-parametric method. These characteristics are then employed to estimate and evaluate prediction intervals. Pinson demonstrates the non-Gaussian character of the WPFE extensively. These results are further employed in the evaluation and design of ensemble prediction methods. However, one should treat the conclusions in this work with caution as the wind farms under consideration are relatively small (max. 21 MW) and therefore may not represent all effects present in present-day, large (aggregations of) wind farms.

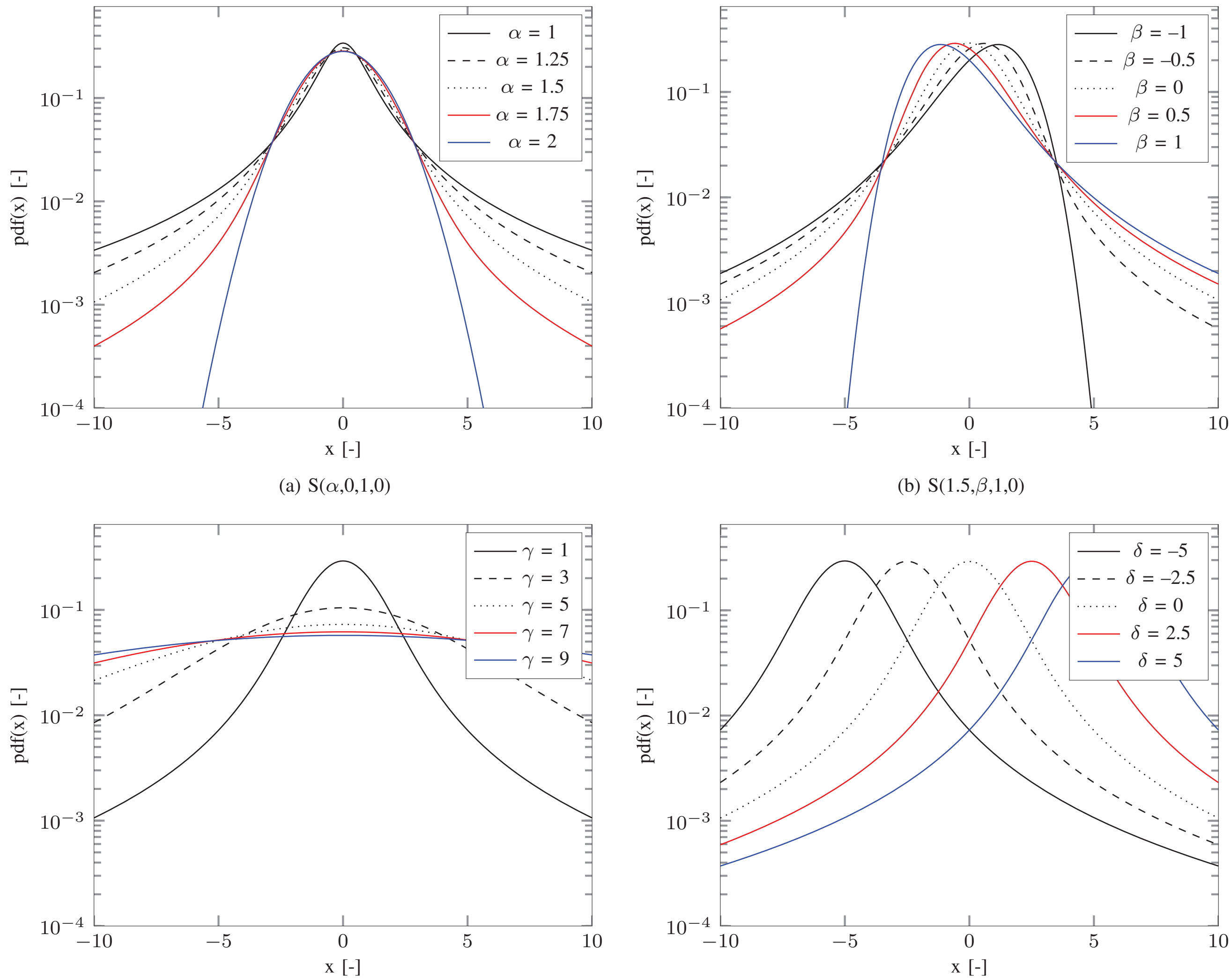
Bludszuweit et al. [12] proposes a β-distribution to describe the pdf to describe the WPFE - based on the approach by Luig et al. [18], [19] and Fabbri et al. [16]. The analysis presented by Bludszuweit et al. shows that the Gaussian distribution cannot describe the WPFE when the WPFE is heavy-tailed (e.g. when the forecast horizon is shorter). Bludszuweit et al. use a persistence forecast method to generate wind power time series based on measured wind power production, allowing them to study the effect of different forecast horizons. Bludszuweit et al. note that in some cases, the β-distribution is not sufficiently heavy-tailed to model the kurtosis of the WPFE data, leading to an underestimation of the frequency of the largest errors.

III. THE LÉVY α-STABLE DISTRIBUTION

In this section, the stable distributions and two important properties (stability or invariance under addition and the Generalized Central Limit Theorem) are introduced. The goal is to give a brief overview for practical use, not to provide a full overview or proof of the properties of the stable distributions. For this, the reader is referred to the specialized literature, such as (amongst others) Nolan [31]–[33], Zolotarev [34], [35] and Smaorodnitsky and Taqqu [36]. For the implementation of the stable distribution probability density function in MATLAB®, see Nolan [31].

[2]In general, the WPFE declines as the number of turbines (or the geographical dispersion) under consideration rises [3], [22] and the forecast horizon is shorter [2]. The former effect is due to the spatial correlations between wind power generation and WPF. This phenomenon is called spatial smoothing and vanishes on a length scale of about 750 km [22]. These effects could be taken into account by varying the temporal and/or geographical scope of the data.

[3]Some authors study the WPFE independently of the forecast. Examples can be found in Hodge et al. [20] and Tewari et al. [24]. In this paper, only joint distributions are considered. These allow for an more accurate description of the WPFE, as the behaviour of the error is highly dependant on the forecast. For example, when one forecasts 0 p.u., the error is limited to positive values. On the other hand, if one forecasts 1 p.u. of wind power, only negative values are possible. This can not be captured in one distribution.



(a) S(α,0,1,0) (b) S(1.5,β,1,0)

(c) S(1.5,0,γ,0) (d) S(1.5,0,1,δ)

Fig. 1. The effect of the four parameters in the Lévy α-stable distribution. In each figure, one parameter is varied, while the others are kept constant. From left to right, top to bottom: the effect of α, β, γ and δ in the stable distribution.

A. *The Lévy α-stable distribution and its parameters*

The pdf and cumulative probability density function (cdf) of a Lévy α-stable distribution cannot be expressed in analytical form. The characteristic function[4] $\phi(u)$ of a random stable variable X can be parametrized and is most often written as in Samorodnitsky and Taqqu [33], [36] :

$$\phi(u) = E\left[\exp(iuX)\right] = \tag{1}$$
$$\begin{cases} \exp\left(-\gamma^{\alpha}|u|^{\alpha}[1 - i\beta - \tan(\frac{\pi\alpha}{2}) \cdot \mathrm{sign}(u)] + i\delta u\right) & \alpha \neq 1 \\ \exp\left(-\gamma|u|[1 + i\beta - \frac{2}{\pi} \cdot \mathrm{sign}(u) \cdot \ln|u|] + i\delta u\right) & \alpha = 1 \end{cases}$$

The parameters of this family of distributions $\mathrm{S}(\alpha, \beta, \gamma, \delta)$ are:

- α index of stability ($0 < \alpha \leq 2$)
- β a skewness parameter ($-1 \leq \beta \leq 1$)
- γ a scale parameter ($\gamma > 0$)
- δ a location parameter ($\delta \in \Re$)

[4]For a random variable X with cumulative distribution function F(x), the characteristic function $\phi(u)$ is defined as $\phi(u) = E\left[\exp(iuX)\right] = \int_{-\infty}^{\infty} \exp(iux) dF(x)$ with $u \in \Re$ and $i = \sqrt{-1}$.

The effect of the various parameters is visualized in Fig. 1[5]. The index of stability α determines the total probability contained in the tails, thus the kurtosis, of the distribution. The probability in the tails is inversely proportional to α. A positive skewness parameter β yields a distribution skewed to the right. The degree of skewness is larger as β rises. Similar reasoning applies to negative β-values. The third parameter γ defines the scale of the distribution and is linked to the variance σ^2 for $\alpha = 2$. The location parameter δ coincides with the mean of the distribution for $\alpha \geq 1$. For $\alpha < 1$, the mean of the distribution is not defined and δ will be some other parameter which describes the location of the distribution[6].

The Gaussian ($\mathrm{N}(\mu, \sigma^2) \rightarrow \mathrm{S}(2, \beta, 2^{-0.5} \cdot \sigma, \mu)$), the Cauchy (scale γ and location δ: $\mathrm{C}(\delta, \gamma) \rightarrow \mathrm{S}(1, 0, \gamma, \delta)$) and Lévy distribution (scale γ and location δ: $\mathrm{L}(\delta, \gamma) \rightarrow \mathrm{S}(0.5, 1, \gamma, \delta)$) are all stable distributions that can be described via the parametrization above. Only in these cases, the probability density function can be expressed analytically.

B. Parameter estimation

This lack of closed form density functions complicates statistical inference for stable distributions, such as parameter estimation. Multiple methods have been developed. In general, one can distinguish the following methods [37]:

- **Sample Quantile Methods:** These methods are considered to be the fastest, but as well the least accurate methods. The best known method is that of McCulloch [38], a robust approach for $\alpha \geq 0.6$. This approach has been used in this paper, see section IV.
- **Sample Characteristic Function Methods:** Given n independent and identically distributed (iid) random samples, the sample characteristic function $\phi^*(u)$ is defined as $\frac{1}{n}\sum_{i=1}^{n}\exp(itx_i)$ and used as an approximation of the characteristic function $\phi(u)$. Based on that approximative characteristic function, the parameters can be estimated.
- **Maximum Likelihood Method** In this group of techniques, the log-likelihood function is maximized for a set of parameters, given an iid random sample. These techniques provide the highest accuracy, but sometimes suffer from robustness issues and are slower.

Other methods such as Monte Carlo-based schemes or indirect inference-methods and estimators of the tail index α (for example the Hill-estimator [39]), are not discussed here.

C. Stability or invariance under addition

Stability or invariance under addition can be defined as follows (see Nolan [33]):

Definition 1. *Non-degenerate X is **stable** if and only if for all $n > 1$, there exist constants $c_\mathrm{n} > 0$ and $d_\mathrm{n} \in \Re$ such that*

$$\sum_{\mathrm{i}=1}^{\mathrm{n}} X_\mathrm{i} = c_\mathrm{n} X + d_\mathrm{n} \tag{2}$$

*where X_i are n iid copies of X. X is **strictly stable** if and only if $d_\mathrm{n} = 0$ for all n.*

It can furthermore be shown that the only possible choice for the scaling constant is $c_\mathrm{n} = n^{\frac{1}{\alpha}}$ with some $\alpha \in (0, 2]$. This can be translated in a parametrization of the characteristic function $\phi(u)$ for the stable distributions. It can be shown that this characteristic function has the same form as that of the Lévy α-stable distribution family (see Nolan [33]). Therefore, by definition (Eq. (1)), a Lévy α-stable distribution is any distribution that is stable. Based on this stability property, any linear weighted sum of iid stable distributions results in a stable distribution. With the same characteristic exponent α but different

[5]The Lévy α-stable distributions is non-zero for (the *support* of the distribution) [33]

$$\text{support } \mathrm{S}(\alpha, \beta, \gamma, \delta) = \begin{cases} [\delta, \infty) & \alpha \leq 1 \quad \& \quad \beta = 1 \\ (-\infty, \delta] & \alpha \leq 1 \quad \& \quad \beta = -1 \\ [-\infty, \infty] & \text{otherwise} \end{cases}$$

However, in this paper, the domain of x is limited to $-10 \leq x \leq 10$ ($-1 \leq \epsilon \leq 1$ for WPFE). The pdf is normalized such that the integral of the pdf equals 1 on the supported domain.

[6]This can be generalized as follows: the p^{th} moment of a stable random variable is finite if and only if $p \leq \alpha$ [37]. Thus, for $\alpha < 1$, the first moment (mean) is not finite. For $1 \leq \alpha < 2$ the mean is finite, but the second moment (variance) is infinite. The variance is finite if and only if $\alpha = 2$.

location, scale and skewness parameters (δ_{i}, γ_{i}, β_{i}) and weights p_{i}, this yields [40][7]:

$$\begin{aligned}&\sum_{\mathrm{i}=1}^{\mathrm{n}} p_{\mathrm{i}} \cdot S(\alpha, \beta_{\mathrm{i}}, \gamma_{\mathrm{i}}, \delta_{\mathrm{i}}) \\ &= S(\alpha, \frac{\sum_{\mathrm{i}=1}^{\mathrm{n}} \gamma_{\mathrm{i}} |p_{\mathrm{i}}|^{\alpha} \beta_{\mathrm{i}}}{\sum_{\mathrm{i}=1}^{\mathrm{n}} \gamma_{\mathrm{i}} |p_{\mathrm{i}}|^{\alpha}}, \sum_{\mathrm{i}=1}^{\mathrm{n}} \gamma_{\mathrm{i}} |p_{\mathrm{i}}|^{\alpha}, \sum_{\mathrm{i}=1}^{\mathrm{n}} p_{\mathrm{i}} \delta_{\mathrm{i}})\end{aligned} \tag{3}$$

D. Generalized Central Limit Theorem (GCLT): stable distributions as limiting distributions

The classical central limit theorem states that the normalized sum of iid terms with a finite variance converges to a normal distribution. More precise (see [41]):

Theorem 1. *Let X_{i} be n iid random variables with a finite variance $\sigma^2 = E\left[X_{\mathrm{i}} - E(X)\right]^2$. The* ***Central Limit Theorem*** *then states that*

$$\lim_{\mathrm{n}\to\infty} n^{-0.5} \sigma^{-1} \sum_{\mathrm{i}=1}^{\mathrm{n}} \left(X_{\mathrm{i}} - E\left[X\right]\right) \tag{4}$$

is a reduced Gaussian variable.

This result is the basis for the presumed occurrence of the Gaussian law in many practical applications, such as the description of the WPFE. One argues that the sum of a large number of iid variables – such as the various sources of the WPFE – from a finite-variance distribution will be (asymptotically) normally distributed. However, this theoretical result has been contradicted by empirical findings in many fields. Often, this is caused by infinite-variance distributed variables. If however the assumption of a finite variance is dropped in the CLT, one can show via the Pareto-Doeblin-Gnedenko conditions[8] (power law behavior) and stability arguments (section III-C) that the limit (4) exits in a real sense and that this is the stable law (see section III-C and [40], [41]). Therefore, if the variance of a large set of random variables tends to infinity, a GCLT-argument may be employed to justify a Lévy α-stable distribution as the description of the sum of these variables. This is summarized in the Generalized Central Limit Theorem (GCLT):

Theorem 2. *Let X_{i} be n iid random variables with a variance σ^2. The* ***Generalized Central Limit Theorem*** *then states that Eq. (4) is a reduced variable that satisfies the stability equation. If X has a finite variance, the limiting distribution is Gaussian (CLT). If X does not have a finite variance, the limit is stable non-Gaussian if and only if the conditions of Pareto-Doeblin-Gnedenko are satisfied for some stability index $\alpha \in (0, 2]$.*

E. Applications of stable distributions

Nolan [31] reports three reasons to employ a stable distribution. First, when there are solid (theoretical) reasons to expect a non-Gaussian stable model: for example the reflection off a rotating mirror, yielding a Cauchy distribution. The second reason is the Generalized Central Limit Theorem. Many observed quantities are the sum of many small terms, thus could be described via a stable distribution. The third reason according to Nolan is purely empirical: many data sets show heavy tails and skewness, properties that are well described by a stable distribution. The hypothesis of stability is however impossible to prove [31]. No standard, widely-accepted tests for assessing stability is available (see [31], [37] and references herein).

Stable, α-stable, stable Paretian or Lévy stable laws were first introduced by Lévy [42]. Despite of the lack of closed expressions for the pdf and the standard stability tests, they have been proposed to model several physical and economic processes [33][9]. For example: the price of a stock [37], [40], [41], [43], multi-variate financial portfolios [44], [45], loss distribution models [46], network traffic [47], wave power [48], engineering [48]–[50] and other fields [31], [33], [37], [43], [51]–[53]. For a comprehensive overview of applications of stable distributions, see Nolan [33].

[7] Eq. (3) only holds for the parametrization of Eq. (1). Other parametrizations will yield other expressions for the β, γ and δ-parameters. Furthermore, if α = 1, the resulting δ-parameter has to be calculated as $\sum_{\mathrm{i}=1}^{n} p_{\mathrm{i}}\delta_{\mathrm{i}} - \frac{2}{\pi}\sum_{\mathrm{i}=1}^{\mathrm{n}} \beta_{\mathrm{i}} p_{\mathrm{i}} \gamma_{\mathrm{i}} \ln|p_{\mathrm{i}}|$ (see Nolan [33]).

[8] See Mandelbrot [41]. In the description of the WPFE, it is assumed that these requirements are met.

[9] The first author to propose the stable distribution to describe heavy-tailed financial data was Mandelbrot [40], [41]. After initial support in the first years of his publication, Mandelbrots stable distribution hypothesis was questioned. In the mid 1990s the stable distribution has made a dramatic comeback in economics [37].

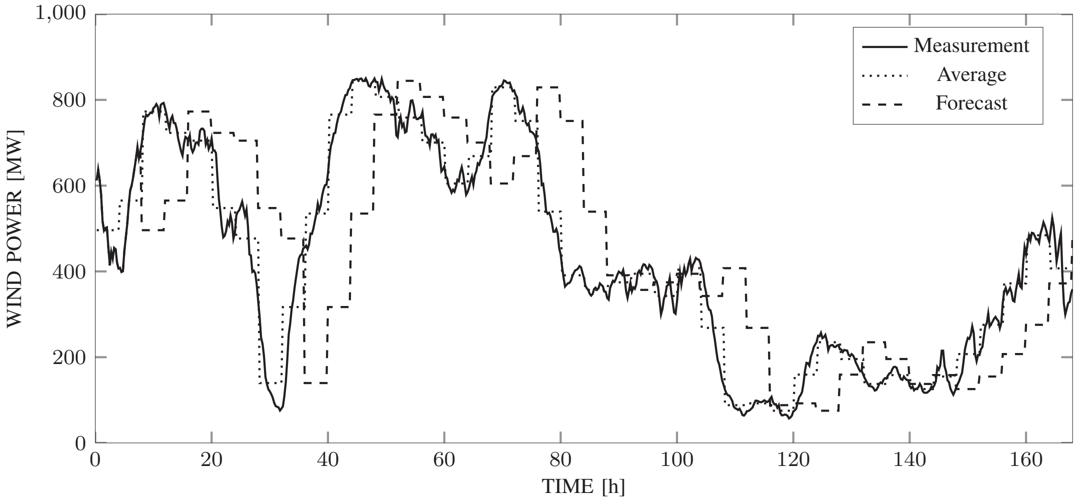


Fig. 2. In the persistence method, the wind power prediction over an interval T is computed as the average of an interval of the same length. The time difference between the start of the forecast interval and the interval used to calculate the average is the prediction horizon k. T and k have been chosen equal to 4 hours in the figure.

F. Stable distributions as a WPFE description

The rationale developed in this paper stems from the similarities that can be found between WPFE data and some financial data, such as the price of a stock. Many of the concepts developed in finance and WPFE analysis rest upon the assumption that asset returns follow a normal distribution. The strongest statistical argument for this choice is the Central Limit Theorem. However, the empirical observations exhibit fat tails (financial data: [40], [41], [43], [52], [53], WPFE: [12], [20], [21]) – a characteristic that cannot be described through a normal distribution. Although various distributions could be used as heavy-tailed alternatives for the Gaussian distribution, the GCLT is invoked to motivate the use of stable distributions. Similar reasoning applies to the WPFE description. The WPFE is the result of various sources of uncertainty and exhibits fat tails. Therefore, under the GCLT, the stable distribution is here proposed to describe the WPFE.

IV. Methodology

First, the persistence method is introduced. This method will be used to generate various forecast scenarios, allowing to test the performance of the stable distribution as a WPFE description. Second, the methodology used to derive the parameters of the Gaussian, stable and β-distribution is discussed.

A. Generating forecast scenarios via the persistence method

The persistence method is a very simple method to generate forecasts based on historical data. It is often used as a benchmark to compare various forecasting methods [1]. Here it is employed to generate forecast-scenarios of various quality from a given wind power time series. The persistence method estimates the wind power generation on a certain interval $[t+k, t+k+T]$ as the average of a previous interval $[t-T, t]$ with the same length. This can be expressed for each time step a forecast is made t as [12]:

$$\forall j \in [t+k, t+k+T]: \;\; p^*(t+k+j|t) = \frac{1}{T}\sum_{\mathrm{j}=0}^{\mathrm{n}} p(t - j\Delta t) \tag{5}$$

with

- $p^*(t+k+j|t)$: the wind power forecast for time step $t+k+j$ made at time t,
- k: the prediction horizon,
- T: the prediction interval length,
- $p(t - i\Delta t)$: the historical wind power values for time t and the n previous time steps within T ($T = n\Delta t$).

This method is illustrated in Fig. 2. For a more detailed description, we refer to Monteiro et al. [1] and Madsen et al. [23].

B. Fitting the distributions to the WPFE data

As the behavior of the WPFE is highly dependent on the forecast, we focus on the joint distribution of the WPFE ϵ and the forecast p^*, further denoted by $f(\epsilon, p^*)$. This joint distribution can be studied indirectly by focusing on the conditional (here: the observations given the forecasts $f(\epsilon|p^*)$) and the marginal (here: the observations $f(p^*)$) distributions. The distribution of the WPFE can hereby be approximated as $f(\epsilon, p^*) \approx f(p^*) \cdot f(\epsilon|p^*)$. This approach is known as the Murphy-Winkler verification framework [54].

The methodology employed in this paper consists of three main steps. First, the conditional WPFE is calculated. In a second step, the parameters of the various distributions are obtained. Last, the unconditional WPFE pdf $f_{\mathrm{tot}}(\epsilon)$ is assembled.

C. Step 1: calculating the conditional error

To obtain an empirical distribution for the WPFE, a similar methodology as in [12] is employed. First, the measured and forecasted wind power time series are sorted into power classes or bins (index i, set I) with a certain width ω according to their forecasted value p^*. Then, the WPFE is calculated on each time step (index j, set J). The normalized conditional prediction error $\epsilon_{\mathrm{i,j}}$ is defined as the difference between the measured and the forecasted wind power:

$$\epsilon_{\mathrm{i,j}} = p_{\mathrm{j}} - p^*_{\mathrm{j}} \quad \Leftrightarrow \quad ll_{\mathrm{i}} \leq p^*_{\mathrm{j}} < ul_{\mathrm{i}} \tag{6}$$

In this equation, p^*_{j} is the wind power forecast that belongs to power bin i (lower limit ll_{i} and upper limit ul_{i}) on time step j and similarly, p_{j} is the measured wind power. With this WPFE time series, the histogram of the error $\epsilon_{\mathrm{i,j}}$ can be obtained for each forecast bin i. The resolution of this histogram (width of the intervals) is chosen equal tot that of the WPF.

D. Step 2: obtaining the distributions parameters

The Gaussian, β and Lévy α-stable distributions are used to describe the WPFE. The parameters of all distributions are optimized for each power bin via a least squares fitting method, applied to the pdf[10]. The starting values depend on the distribution:

a) The Gaussian distribution $N(\mu_{\mathrm{i}}, \sigma_{\mathrm{i}})$: The mean μ_{i} and the standard deviation σ_{i} are calculated directly from the data sample.

b) The β-distribution $B(a, b, p_{\mathrm{i}}, q_{\mathrm{i}})$: The β-distribution according to Johnson [55] is characterized by four parameters. This distribution yields non-zero values on the interval $[a, b]$, here fixed on $[-1, 1]$. The parameters p_{i} and q_{i} are restricted to positive values. The mean μ_{i} and the variance σ^2_{i} of the data are related to the p_{i} and q_{i}-shape parameters of the β-distribution via the method-of-moments [55]:

$$p_{\mathrm{i}} = \frac{(1-\mu_{\mathrm{i}}) \cdot \mu^2_{\mathrm{i}}}{\sigma^2_{\mathrm{i}}} - \mu_{\mathrm{i}} \qquad\qquad q_{\mathrm{i}} = \frac{1-\mu_{\mathrm{i}}}{\mu_{\mathrm{i}}} \cdot p_{\mathrm{i}} \tag{7}$$

c) The Lévy α-stable distribution: The parameters of the stable distribution are estimated via the quantile approach by McCulloch [38], the equivalent Gaussian and Cauchy distribution. In the latter case, γ_{i} has been chosen arbitrary equal to one while the location parameter δ_{i} has been set to the position of the maximum in the empirical histogram.

E. Step 3: assembling the probability density function

To obtain an unconditional pdf for the WPFE, the pdf of each bin $f_{\mathrm{i}}(\epsilon_{\mathrm{i}})$ are combined via the empirical probability that a wind power forecast will be an element of that bin $f_{\mathrm{emp}}(p^*_{\mathrm{i}})$:

$$f_{\mathrm{tot}}(\epsilon|a^1, \ldots, a^{\mathrm{k}}) = \sum_{\mathrm{i}=1}^{\mathrm{m}} f_{\mathrm{emp}}(p^*_{\mathrm{i}}) \cdot f_{\mathrm{i}}(\epsilon_{\mathrm{i}}|a^1_{\mathrm{i}}, \ldots, a^{\mathrm{k}}_{\mathrm{i}}) \tag{8}$$

where m is the number of bins defined, k is the number of parameters in the chosen distribution $f_{\mathrm{i}}(\epsilon_{\mathrm{i}})$ and $f_{\mathrm{emp}}(p^*_{\mathrm{i}})$ the empirical distribution of the forecasts over the power bins. In case of the stable distribution, this resulting distribution $f_{\mathrm{tot}}(\epsilon|a^1, \ldots, a^{\mathrm{k}})$

[10]Empirical research has shown that this yields the lowest error in the pdf and the cdf-fit.

is stable as well under the stability property (see paragraph III-C). If the stability index α is equal for all power bins (or if this is imposed on the optimization), Eq. (3) can be used to describe the unconditional WPFE via one stable distribution.

V. Results and discussion

In this section, the performance of the stable distribution as a WPFE description is compared the performance of the Gaussian & β-distribution. In two steps, we will demonstrate the higher quality of the WPFE description based on the stable distribution. In the first step, wind power forecasts are simulated via the persistence method, based on a historical wind power generation time series (section V-A). Second, we take a closer look at the performance of the stable distribution as a WPFE description in a case study, where we analyse a historical WPFE time series (section V-B).

Wind power data for 2006-2012 was taken from 50Hertz [56][11]. Based on these time series, we constructed the historical, normalized wind power time series (production and forecast errors).

A. Performance of the Lévy α-stable distribution under persistence forecasts

Three forecast scenarios are generated from the historical wind power generation time series from 50Hertz [56], similar to the approach in Bludszuweit et al. [12]. In the best case scenario ($k = -T$), the forecast is generated as the average of the measured values on that interval. A scenario with an intermediate forecast performance is generated via the persistence method without a forecast horizon ($k = 0$). In the third scenario ($k = T$), the forecast horizon increases linearly with the forecast interval. The forecast interval ranges from 1 to 24 hours. The WPFE increases as the forecast interval is longer and exhibits characteristics in line with values reported in literature. The performance of the Lévy α-stable distribution in terms of squared residuals[12] is summarized in Fig. 3. As Bludszuweit et al. [12] do not perform a least-squares fitting, the residuals prior to optimization are shown as well.

In the $k = -T$ *scenario*, the normalized root mean square error (NRMSE) rises from about 1% to 10% with increasing forecast intervals T. Fig. 3a shows that the stable distribution outperforms the other distributions in terms of residuals of the pdf-fit. On average, the Gaussian distribution yields residuals that are 1.62 times higher compared to the stable distribution. In case of the β-distribution, this factor rises to 9.4. In absolute values, the residuals are highest for short forecast intervals and decrease rapidly with increasing forecast intervals. This decrease is the fastest and the strongest for the stable distribution. This explains the trend of the increasing relative difference in performance between the distributions as the prediction interval is longer. This can be observed for all forecast scenarios before optimization. In the cdf, the Gaussian and stable distributions are characterized by very low residuals compared to the β-distribution (Fig. 3c). After optimization, the residuals of the Gaussian and β-fit again decrease considerably slower than those of the stable distributions – although faster than prior to optimization –, yielding the high relative differences for forecast intervals between 3 and 6 hours (Fig. 3b). The poor performance of the stable distribution for low forecast intervals is caused by high residuals in the first forecast power bin. The stable distribution tries to capture the extreme kurtosis (values over 40 are observed) of the data in this power bin. For the pdf, this yields good results, but leads to poor performance in the cdf-fit (Fig. 3d). For all other forecast intervals, the residuals of the cdf are situated in the range 0.15 and 0.5. The residuals of the Gaussian and β-distribution for these forecast intervals are considerably higher than those of the stable fit.

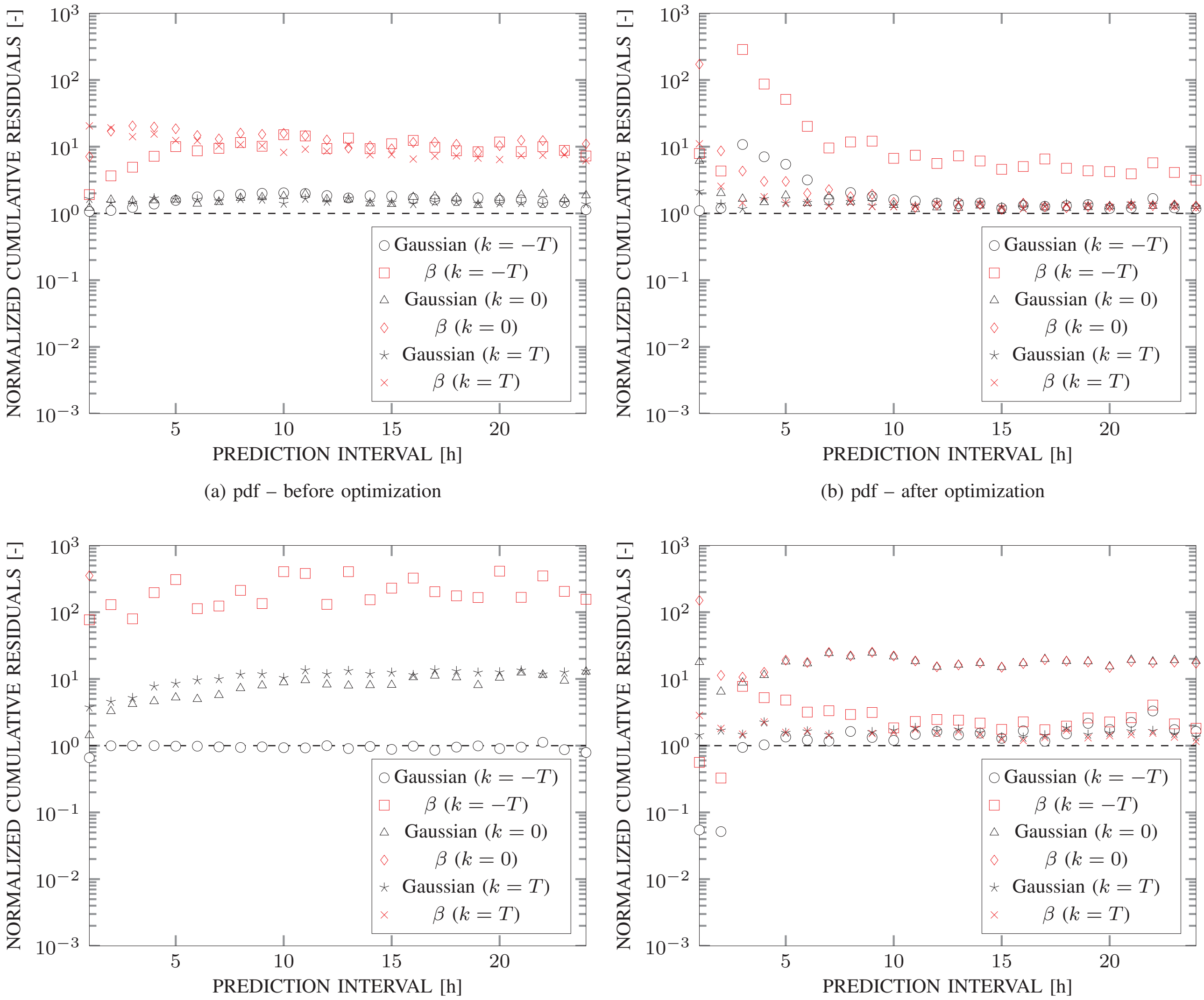
When the WPFE data is generated with a *zero prediction horizon*, this yields NRSME values between 2.5% and 21.5%. Before optimization, the stable distribution outperforms the Gaussian and β-distribution by a factor up to 20 (Fig. 3a). The

[11]The German TSO 50Hertz publishes forecasted wind power levels and actual production levels with a temporal resolution of 15 minutes online as of 2006 [56]. The time horizon of the forecasts used is 24 hours. For the period 2006-2008, only hourly forecast values are reported. The wind power capacity installed over time is published on the same website and is used to normalize measurements and forecasts. This large data set consists of over 230 000 data points. The current WPF is based on energy and meteo services: EuroWind, Fraunhofer IWES, meteomedia and WEPROG. The WPF is published at 6 p.m. for the following day (8 a.m to 8 a.m). No details are given on the wind power forecasting model employed, nor on the methodology of the measurements.

[12]The residuals visualized here correspond to the sum of the residuals over all power bins:

$$res_{\mathrm{cum}} = \sum_{\mathrm{i}=1}^{\mathrm{n}} \sum_{\mathrm{k}=1}^{\mathrm{m}} \left(f_{\mathrm{emp}}(\epsilon_{\mathrm{k}}|p_{\mathrm{i}}^{*}) - f_{\mathrm{dist}}(\epsilon_{\mathrm{k}}|p_{\mathrm{i}}^{*}) \right)^{2},$$

with n the number of forecast power bins, m the number of error power bins, $f_{\mathrm{emp}}(\epsilon_{\mathrm{k}}|p_{\mathrm{i}}^{*})$ the empirical probability of errors ϵ_{k} belonging to error power bin k and $f_{\mathrm{dist}}(\epsilon_{\mathrm{k}}|p_{\mathrm{i}}^{*}$ the probability of that error calculated via a fitted distribution. In Fig. 3, the cumulative residuals of the Gaussian & β-distribution have been normalized with respect to those of the stable distribution.



(a) pdf – before optimization

(b) pdf – after optimization

(c) cdf – before optimization

(d) cdf – after optimization

Fig. 3. The Lévy α-stable distribution outperforms the Gaussian & β-distribution in terms of cumulative residuals over a wide range of forecast scenarios. Top left: the residuals of the pdf-fit of the Gaussian and β-distribution, relative to the residuals of the stable distribution before least-squares fit. Top right: same figure, after least-squares fit. Bottom left: resulting residuals of the cdf of the Gaussian and β-distribution, relative to the residuals of the stable distribution before least-squares fit. Bottom right: same figure, after least-squares fit. The normalized cumulative residuals of the cdf for the β-distribution before optimization in the cases $k = 0$ and $k = T$ are not displayed on Fig. 3c as they are higher than 10^3.

performance of the Gaussian and β-distribution is similar to their performance in the $k = -T$ scenario. In terms of the cdf (Fig. 3c), the stable distribution outperforms the Gaussian distribution by a factor 8 on average. For the β-distribution, this factor exceeds 10^3 (not visible on Fig. 3c), caused by poor performance of the method-of-moments to calculate the distribution parameters. After optimization, the residuals of the pdf-fit (Fig. 3b) of the stable and Gaussian distribution rise as the forecast interval T increases. Simultaneously, the performance of the Gaussian and β-distribution converge. Once more, the β-distribution yields high residuals for short forecast horizons. Looking at the cdf (Fig. 3d), the performance of the Gaussian and β-distribution is comparable over the whole range of forecast intervals T considered. For all distributions, the residuals rise with the forecast interval T. The stable distribution outperforms the Gaussian and β-distribution by a factor 17.5 and 23.2 on average respectively.

NRSME values between 5% and 28% are observed in the $k = T$ *scenario*. Before optimization, the stable distribution yields the lowest residuals in the pdf and cdf-fit. The residuals of the β-fit in the pdf (Fig. 3a) are high for short forecast interval and sharply decrease as T increases. The residuals of the stable and Gaussian distribution are in a narrow interval over the T-range considered. The cdf-fit (Fig. 3c) yields high residuals for the β-distribution (not in visualised range on Fig. 3c). The

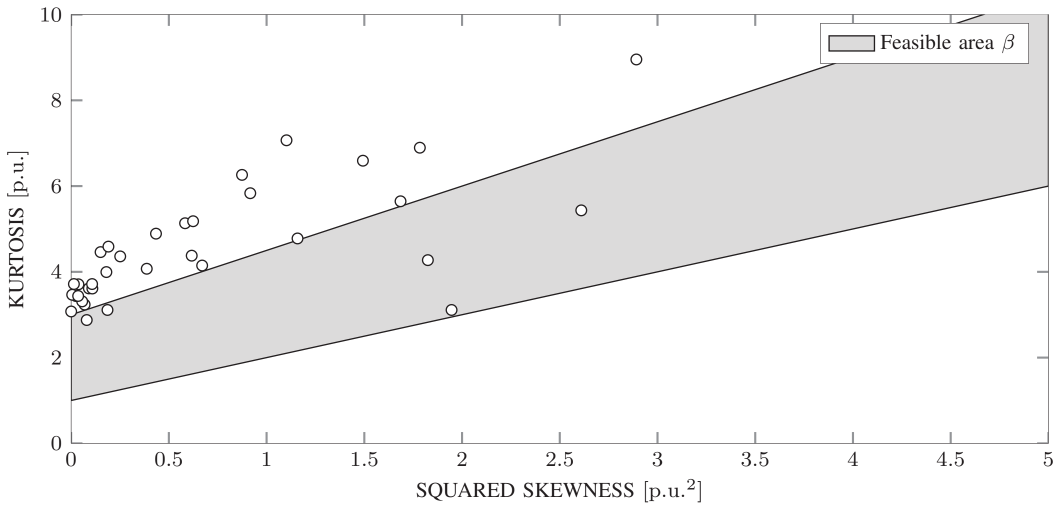


Fig. 4. The majority of the observed values of (squared) skewness and kurtosis in the WPFE data are situated well out of the feasible area of the Gaussian (kurtosis 3, skewness 0) and β-distribution (grey area). The data of power bin 1 and 37 have been omitted due to their extreme kurtosis values.

stable and Gaussian distributions result in residuals of the same order of magnitude. These residuals increase modestly as the forecast interval is longer. After optimization, the residuals of the pdf-fit (Fig. 3b) again increase with the forecast interval T. The performance of the Gaussian and β-distribution is similar. The stable distribution yields the lowest residuals for all T-values in the pdf (Fig. 3b) and the cdf-fit (Fig. 3d).

B. A closer look at the performance of the stable distribution: a case study

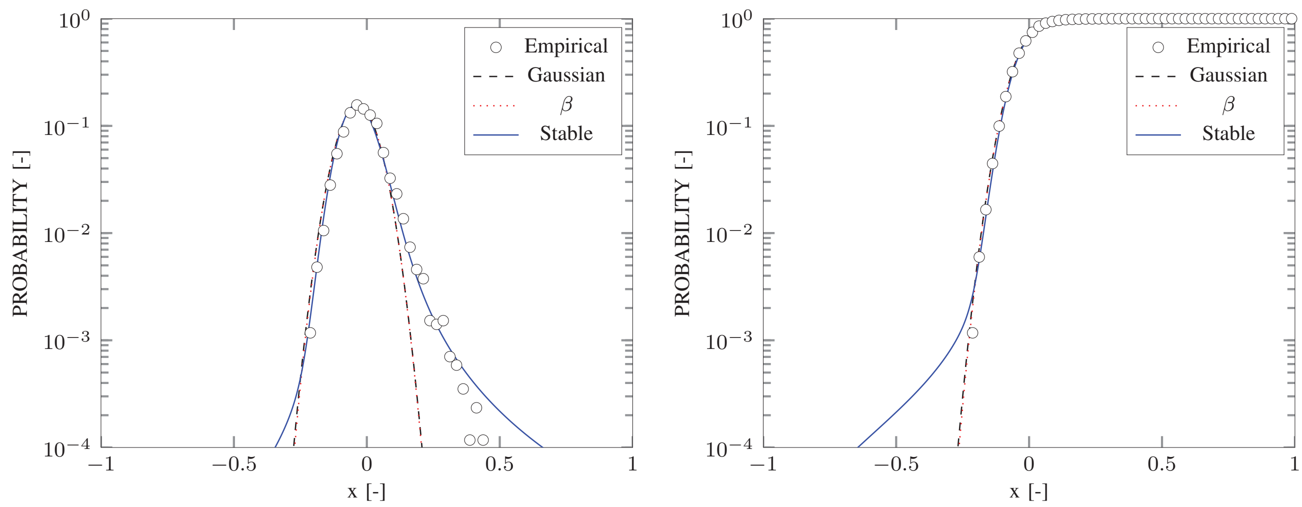
Based on the wind power data (forecasts & measurements) for 2006-2012 from 50Hertz [56], we constructed the historical, normalized WPFE time series. This section describes the analysis of that data set in detail.

1) Statistical analysis of the WPFE data: The NRMSE varies between 0.018 and 0.19 p.u. The high values for the NRMSE in certain power bins should be interpreted with caution, as these power bins contain few data points – a remark that holds for the remainder of this case study. The bias varies from -0.18 to 0 p.u. over the power bins. The average bias amounts to -0.027 p.u. The average standard deviation equals 0.078 p.u. and varies between 0.02 and 0.107 p.u. The skewness, as a measure for the asymmetry of the data, exhibits positive values in the lower power bins and negative values in the higher power bins, in line with our expectations. The skewness varies between -4.7 and 5.1. The kurtosis, as a measure of the peakedness of the distribution, varies here between 2.876 and 55.6. If the extreme kurtosis values of the first and the last power bins are excluded, a maximum kurtosis of 10.2 is observed. These values are in line with the literature [12], [20], [21] and confirm the reported leptokurtic character of WPFE data.

Note that the non-zero bias implies that the zero-bias assumption in, amongst others, [5]–[7], [9] does not hold. Furthermore, looking at Fig. 4, it is clear that the β-distribution does not allow to capture all the asymmetry and peakedness of the WPFE data. The vast majority of the power bins are however characterized by kurtosis values above the feasible range of the β-family. This is directly visible in the behavior of the tail index of the stable distribution. The estimated[13] α is situated between 1.19 and 2 with an average of 1.35.

2) Performance of the stable distribution: The stable distribution is compared against the performance of the normal and the β-distribution. An example of the fitted distributions is displayed in Fig. 5 for the 10^{th} power bin. The data is clearly asymmetric (skewness: 0.79) and leptokurtic (kurtosis: 5.18). The β and Gaussian fit do not allow for this behavior, while the stable distribution does (Fig. 5).

[13]These values are obtained via the percentile approach by McCulloch [53]. As shown by Weron [57], large datasets ($O(10^6)$ samples) are needed to adequately estimate the tail index, especially when the actual tail index is close to 2. Some estimators, such as the log-log regression method or the Hill estimator, may lead to gross overestimations of the tail index. The McCulloch approach [53] is more robust and will not yield values greater than 2. Results should however still be regarded as estimates and interpreted with caution.



(a) Histogram and fitted distributions (pdf)

(b) Cumulative histogram and fitted distributions (cdf)

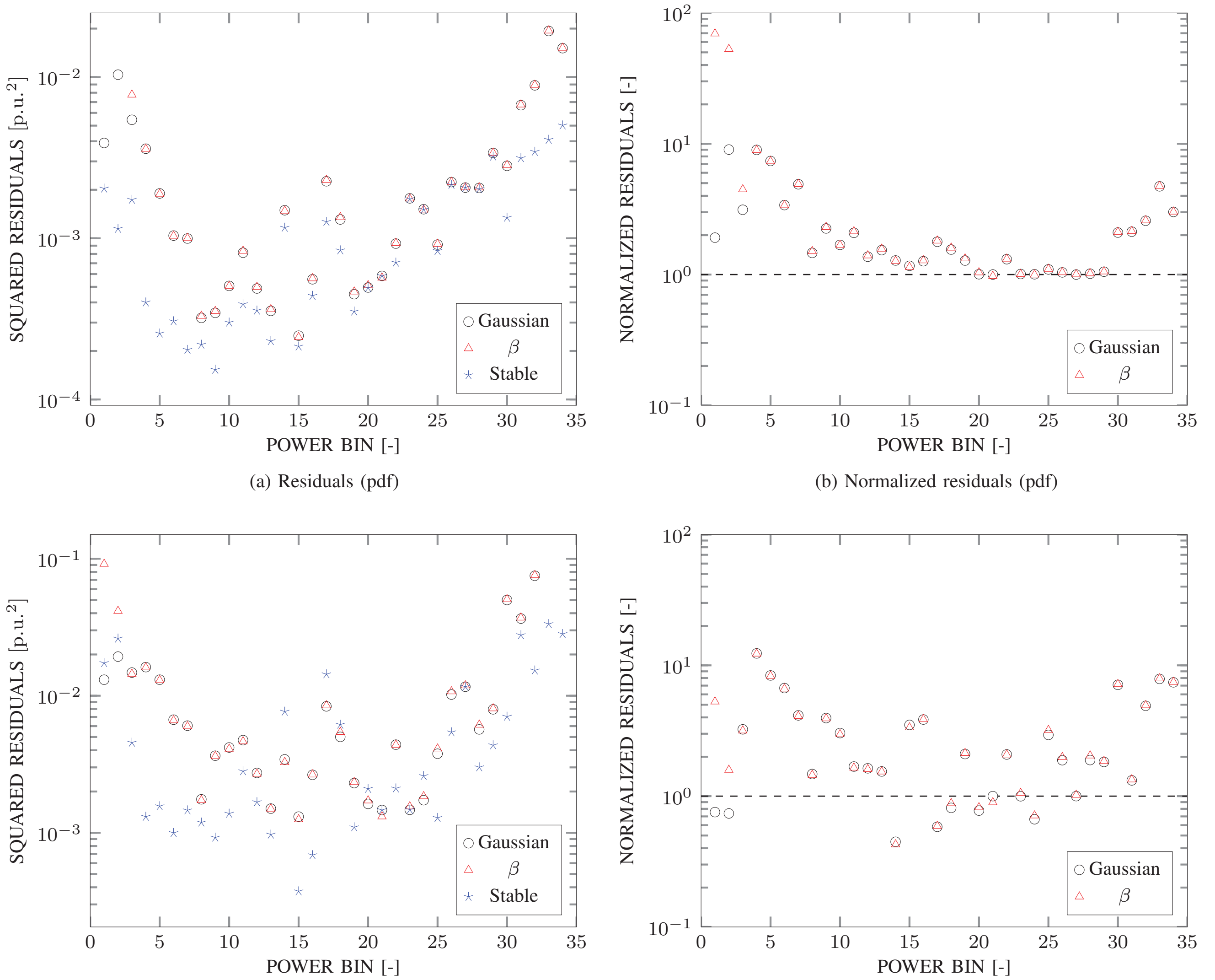
Fig. 5. The histogram, the fitted pdf and cdf for the 10^{th} power bin. The β– and Gaussian distribution exhibit a comparable performance. The stable distribution allows for the best fit. Note the asymmetry of the data: as the 10^{th} power bin contains forecasts between 0.225 and 0.25 p.u., the error must be in the interval [-0.25,0.775].

The residuals of the least squares fit in each power bin are shown and compared in Fig. 6. In Fig. 6, a general U-shape in the residuals can be observed. In the lower power bins, residuals are high and decrease with the power bin number. For the higher power bin numbers, a trend of rising residuals with the power bin number can be observed. As the predicted wind power is closer to the limits of the physical range of the WPFE ($\epsilon \in [-1, 1]$), the empirical histogram exhibits more asymmetry. This is more difficult to model through any of the distributions, yielding higher residuals. This asymmetry is visible as well in Fig. 5.

Looking at the pdf, the stable distribution outperforms the other distributions in all power bins except power bin 21 and 24. In these power bins, the β-distribution yields a 2.56% and 0.18% improvement over the stable distribution. The performance of the β- and the Gaussian distribution is similar, except for the first two power bins. As the rise of the residuals in the lower and higher power bins (the U-shape) is less pronounced for the stable distribution, the highest performance improvements are observed in these power bins. Residuals of the stable distribution are up to 70 times lower compared to the β-distribution. When comparing the overall residuals (the sum of all residuals over the power bins), the stable distribution results in an improvement in performance of 137% compared to the Gaussian distribution and 567% compared to the β-distribution.

The story is somewhat different for the cdf. The stable distribution is here out performed by the Gaussian and β-distribution in 6 to 8 power bins. However, the differences between the different distributions in performance are rather small. The Gaussian and β-distribution again show a similar performance, except for the first two power bins. In terms of the cumulative squared residuals, the stable distribution still outperforms all other distributions considered. Improvements of 240% (compared to the Gaussian distribution) and 284% (compared to the β-distribution) are observed.

3) The Lévy α-stable distribution: The parameters, obtained via the least-squares fit, are listed in table I. The optimized tail index α varies between 1.05 and 2, with an average value of 1.6716 (heavy tailed data). However, no clear trend can be distinguished in the tail index α. The trend of decreasing skewness with rising power bin number observed in the data can also be found in the values of the optimized skewness parameter β. The location parameter δ exhibits the same behavior as the bias of the WPFE data: low power bin numbers show a positive bias, while higher power bin numbers are characterized by a negative bias. The scale parameter (γ) is situated between 0.0027 - 0.0718 and has an average value of 0.0484. The data in table I shows that the stable distribution is a normal distribution (tail index $\alpha \to 2$) in power bin 21 and 27. The kurtosis of the data in these power bins equals 3.72 and 3.11 respectively. The skewness is limited to -0.12 and -0.43. These values should be compared with the skewness (0) and kurtosis (3) of the normal distribution. The standard deviation is 0.11 and 0.0935, similar to the range standard deviation of the other power bins and is related to the scale parameter γ. The bias equals -0.0271 and -0.0111 and is to be compared with the location parameter δ.



(a) Residuals (pdf)

(b) Normalized residuals (pdf)

(c) Residuals (cdf)

(d) Normalized residuals (cdf)

Fig. 6. In terms of squared residuals, the stable distribution outperforms the Gaussian & β-distribution. The values in the figures on the right are normalized against the corresponding values for the Lévy α-stable distribution. Note that no fit is generated for power bins 35 to 40 as they contain too few samples to adequately represent the distribution in these power bins.

VI. Conclusion & Future Research

Wind power is one of the largest sources of renewable electricity. This form of electricity production is however characterized by a limited predictability. A correct description of the wind power forecast error (WPFE) holds important information for short term economical & operational (reliability) studies in the field of wind power. The focus of this paper therefore is the statistical, distribution-based description of the WPFE. To this goal, the relevant literature on statistical distributions used to model the WPFE has been reviewed.

From the literature and own calculations, it has been shown that WPFE data exhibits heavy tails. In analogy with models proposed in the financial sector, the Lévy α–stable distribution is proposed as an alternative description of the WPFE. These distributions allow to model the skewness and kurtosis observed in the WPFE data – in contrast to the Gaussian and β-distributions currently proposed in the literature. Furthermore, in the GCLT the stable distributions are postulated as the only possible non-trivial limit distribution of normalized sums of independent and identically distributed random variables, such as the various sources of the WPFE.

The performance of the Lévy α–stable distribution is compared to that of the Gaussian and the β-distribution in numerous WPF scenarios generated via a persistence approach. The effect of the prediction interval and horizon has been investigated. The

TABLE I
THE PARAMETERS OF THE LÉVY α-STABLE DISTRIBUTION OBTAINED VIA THE LEAST SQUARES FIT.

Power bin	Lower limit	Upper limit	Lévy α-stable distribution			
			α	β	γ	δ
$[-]$	$[p.u.]$	$[p.u.]$	$[-]$	$[-]$	$[-]$	$[-]$
1	0	0,025	1,490956	-0,17701	0,002745	-0,01264
2	0,025	0,05	1,200699	1	0,013463	0,018465
3	0,05	0,075	1,728345	1	0,019416	-0,01628
4	0,075	0,1	1,641842	0,999999	0,025312	-0,00796
5	0,1	0,125	1,648228	0,928762	0,030159	-0,00997
6	0,125	0,15	1,759023	0,98355	0,033693	-0,01261
7	0,15	0,175	1,63697	0,530829	0,035516	-0,01664
8	0,175	0,2	1,805913	-0,0238	0,039359	-0,02792
9	0,2	0,225	1,713584	0,433488	0,042915	-0,02031
10	0,225	0,25	1,773031	0,820393	0,044469	-0,02246
11	0,25	0,275	1,630213	-0,01097	0,0457	-0,03425
12	0,275	0,3	1,778905	-0,02646	0,050355	-0,03861
13	0,3	0,325	1,826589	0,495014	0,052551	-0,03011
14	0,325	0,35	1,703064	0,780857	0,057736	-0,01966
15	0,35	0,375	1,927614	0,982638	0,061633	-0,02776
16	0,375	0,4	1,855012	0,561979	0,060008	-0,03092
17	0,4	0,425	1,307637	0,158414	0,061526	-0,02031
18	0,425	0,45	1,518471	-0,04812	0,063369	-0,03109
19	0,45	0,475	1,781856	0,244511	0,067507	-0,02817
20	0,475	0,5	1,853991	0,569429	0,068698	-0,02717
21	0,5	0,525	2	0	0,071804	-0,04196
22	0,525	0,55	1,709676	0,398764	0,068326	-0,04196
23	0,55	0,575	1,97504	0,916131	0,063813	-0,03455
24	0,575	0,6	1,825685	0,289011	0,064353	-0,02617
25	0,6	0,625	1,834317	-0,14999	0,05863	-0,03763
26	0,625	0,65	1,848721	-0,20268	0,058917	-0,03294
27	0,65	0,675	2	0	0,063434	-0,01222
28	0,675	0,7	1,907413	-0,52005	0,063278	-0,0392
29	0,7	0,725	1,890048	-0,99998	0,058302	-0,0356
30	0,725	0,75	1,461485	-1	0,047861	-0,05625
31	0,75	0,775	1,107461	-0,2942	0,042111	-0,08605
32	0,775	0,8	1,369544	-0,87744	0,042385	-0,05988
33	0,8	0,825	1,050504	-0,79263	0,035065	-0,36706
34	0,825	0,85	1,271564	-1	0,030773	-0,09872

analysis has shown that the Lévy α–stable distribution outperforms the Gaussian and β-distribution. Partly this improvement can be explained by the higher number of parameters used in the stable distribution. However, the improvement cannot be attributed to this fact alone. The results indicate that the data originates from a stable process and thus can best be described by a stable distribution. The improved performance of the approach presented in this paper has been demonstrated in detail in a case study of the historical WPFE on the wind farms in the 50Hertz grid.

The improved representation of the WPFE allows researchers and industry to develop improved operational models that reflect the heavy-tailed nature of the WPFE. This can further facilitate the integration of wind power in the power system through advanced operational techniques, such as dynamic reserve levels and stochastic simulations.

This research may be further strengthened on the following fields. First, the starting values for the optimization of the parameters of the stable distribution affect the resulting parameters and thus performance considerably and may require further attention. Second, the objective function (min. squared residuals) utilized in this paper has merely been chosen to demonstrate the superiority of the stable distributions in capturing the shape of the WPFE. If one aims to characterize specific shape characteristics of the WPFE (e.g. the tails or peaks), this criterion might not be suitable. Third, the effect of the width of the power bins has not been examined explicitly. This power bin width is a compromise between the level of detail needed to describe the distribution adequately within a power bin – pushing for a small power bin width – and the amount of data in each power bin, allowing a statistical analysis, yielding a lower limit for the power bin width. Other power bin widths may lead to lower residuals.

REFERENCES

[1] C. Monteiro, R. Bessa, V. Miranda, A. Botterud, J. Wang, G. Conzelmann, Decision and Information Sciences, and INESC Porto, "Wind Power Forecasting : State-of-the-Art 2009," tech. rep., Argonne National Laboratory, United States, 2009.

[2] P. Pinson, *Estimation of the uncertainty in wind power forecasting*. PhD thesis, Ecole des Mines de Paris, 2006.

[3] M. Lange, *Analysis of the Uncertainty of Wind Power Predictions*. PhD thesis, Carl von Ossietzky Universität Oldenburg, 2003.

[4] C. Moehrlen, *Uncertainty in wind energy forecasting Uncertainty in Wind Energy Forecasting*. PhD thesis, University College Cork, 2004.

[5] Y. Makarov, C. Loutan, and P. de Mello, "Operational Impacts of Wind Generation on California Power Systems," *IEEE Transactions on Power Systems*, vol. 24, pp. 1039–1050, May 2009.

[6] R. Doherty and M. O'Malley, "A New Approach to Quantify Reserve Demand in Systems With Significant Installed Wind Capacity," *IEEE Transactions on Power Systems*, vol. 20, pp. 587–595, May 2005.

[7] F. Bouffard and F. D. Galiana, "Stochastic security for operations planning with significant wind power generation," in *2008 IEEE Power and Energy Society General Meeting - Conversion and Delivery of Electrical Energy in the 21st Century*, pp. 1–11, IEEE, July 2008.

[8] K. Methaprayoon, C. Yingvivatanapong, W.-J. Lee, and J. R. Liao, "An Integration of ANN Wind Power Estimation Into Unit Commitment Considering the Forecasting Uncertainty," *IEEE Transactions on Industry Applications*, vol. 43, no. 6, pp. 1441–1448, 2007.

[9] B. C. Ummels, M. Gibescu, E. Pelgrum, W. L. Kling, and A. J. Brand, "Impacts of Wind Power on Thermal Generation Unit Commitment and Dispatch," *IEEE Transactions on Energy Conversion*, vol. 22, pp. 44–51, Mar. 2007.

[10] E. Delarue and W. D'haeseleer, "Adaptive mixed-integer programming unit commitment strategy for determining the value of forecasting," *Applied Energy*, vol. 85, pp. 171–181, Apr. 2008.

[11] E. D. Delarue, P. J. Luickx, and W. D'haeseleer, "The actual effect of wind power on overall electricity generation costs and CO2 emissions," *Energy Conversion and Management*, vol. 50, pp. 1450–1456, June 2009.

[12] H. Bludszuweit, J. Dominguez-Navarro, and A. Llombart, "Statistical Analysis of Wind Power Forecast Error," *IEEE Transactions on Power Systems*, vol. 23, pp. 983–991, Aug. 2008.

[13] H. Bludszuweit and J. Dominguez-Navarro, "A Probabilistic Method for Energy Storage Sizing Based on Wind Power Forecast Uncertainty," *IEEE Transactions on Power Systems*, vol. 26, no. 3, pp. 1651–1658, 2011.

[14] K. De Vos, J. Driesen, and R. Belmans, "Assessment of imbalance settlement exemptions for offshore wind power generation in Belgium," *Energy Policy*, vol. 39, pp. 1486–1494, Mar. 2011.

[15] G. Bathurst, J. Weatherill, and G. Strbac, "Trading wind generation in short term energy markets," *IEEE Transactions on Power Systems*, vol. 17, pp. 782–789, Aug. 2002.

[16] A. Fabbri, T. Gomez San Roman, J. Rivier Abbad, and V. Mendez Quezada, "Assessment of the Cost Associated With Wind Generation Prediction Errors in a Liberalized Electricity Market," *IEEE Transactions on Power Systems*, vol. 20, pp. 1440–1446, Aug. 2005.

[17] M. Lange, "On the Uncertainty of Wind Power Predictions – Analysis of the Forecast Accuracy and Statistical Distribution of Errors," *Journal of Solar Energy Engineering*, vol. 127, no. 2, p. 177, 2005.

[18] A. Luig, S. Bofinger, and H. Beyer, "Analysis of confidence intervals for the prediction of regional wind power output," in *European Wind Energy Conference '01*, (Copenhagen, Denmark), pp. 725–728, Dept. of Electrical Engineering, University of Applied Sciences Magdeburg-Stendal, 2001.

[19] S. Bofinger, A. Luig, and H. Beyer, "Qualification of wind power forecasts," in *2002 Global Wind Power Conference (GWPC'02)*, vol. 1996, (Paris, France), Dept. of Electrical Engineering, University of Applied Sciences Magdeburg-Stendal, 2002.

[20] B.-M. Hodge and M. Milligan, "Wind power forecasting error distributions over multiple timescales," in *Power & Energy Society General Meeting*, no. March, (Detroit, Michigan, USA), 2011.

[21] B.-M. Hodge, D. Lew, M. Milligan, and H. Holttinen, "Wind Power Forecasting Error Distributions: An International Comparison," in *Transmission Networks for Offshore Wind Power Plants Conference*, no. September 2012, (Lisbon, Portugal), 2012.

[22] U. Focken, M. Lange, K. Mönnich, H.-P. Waldl, H. G. Beyer, and A. Luig, "Short-term prediction of the aggregated power output of wind farmsa statistical analysis of the reduction of the prediction error by spatial smoothing effects," *Journal of Wind Engineering and Industrial Aerodynamics*, vol. 90, pp. 231–246, Mar. 2002.

[23] H. Madsen, R. P. Plads, T. S. Nielsen, and P. Pinson, "A Protocol for Standardizing the Performance Evaluation of Short-Term Wind Power Prediction Models," *Wind Energy*, vol. 29, no. 6, pp. 475–489, 2004.

[24] S. Tewari, C. J. Geyer, and N. Mohan, "A Statistical Model for Wind Power Forecast Error and its Application to the Estimation of Penalties in Liberalized Markets," *IEEE Transactions on Power Systems*, vol. 26, pp. 2031–2039, Nov. 2011.

[25] N. Menemenlis, M. Huneault, and A. Robitaille, "Computation of Dynamic Operating Balancing Reserve for Wind Power Integration for the Time-Horizon 1-48 Hours," *IEEE Transactions on Sustainable Energy*, vol. 3, pp. 692–702, Oct. 2012.

[26] K. Dietrich, J. M. Latorre, L. Olmos, A. Ramos, and J. P. Ignacio, "Stochastic Unit Commitment Considering Uncertain Wind Production in an Isolated System," in *4th Conference on Energy Economics and Technology*, (Dresden, Germany), pp. 1–6, 2009.

[27] P. Pinson, H. A. Nielsen, and H. Madsen, "Methods for the Estimation of the Uncertainty of Wind Power Forecasts," tech. rep., 2007.

[28] P. Pinson, H. Nielsen, H. Madsen, and G. Kariniotakis, "Skill forecasting from ensemble predictions of wind power," *Applied Energy*, vol. 86, pp. 1326–1334, July 2009.

[29] P. Pinson, H. Madsen, H. A. Nielsen, G. Papaefthymiou, and B. Klöckl, "From probabilistic forecasts to statistical scenarios of short-term wind power production," *Wind Energy*, vol. 12, pp. 51–62, Jan. 2009.

[30] P. Pinson, T. Ranchin, and G. Kariniotakis, "Short-term Wind Power Prediction for Offshore Wind Farms - Evaluation of Fuzzy-Neural Network Based Models," in *Global Wind Power Conference (GWPC'10)*, pp. 1–9, 2010.

[31] J. P. Nolan, "Fitting Data and Assessing Goodness-of- fitt with Stable Distributions." Online: http://academic2.american.edu/~jpnolan/stable/DataAnalysis.pdf, 1999.

[32] J. P. Nolan, "An overview of multivariate stable distributions." Online: http://academic2.american.edu/~jpnolan/stable/overview.pdf, 2008.

[33] J. P. Nolan, *Stable Distributions - Models for Heavy Tailed Data*. Boston: Birkhauser, 2013. In progress, Chapter 1 online at http://academic2.american.edu/~jpnolan.

[34] V. Zolotarev, *One-Dimensional Stable Distributions*. Translations of Mathematical Monographs, American Mathematical Society, 1986.

[35] V. V. Uchaikin and V. M. Zolotarev, *CHANCE and STABILITY - Stable Distributions and their Applications*. VSP, 1999.

[36] G. Samorodnitsky and M. Taqqu, *Stable Non-Gaussian Random Processes: Stochastic Models With Infinite Variance*. Stochastic Modeling, Chapman & Hall, 1994.

[37] S. Borak, A. Misiorek, and R. Weron, "Models for Heavy-tailed Asset Returns." 2010.

[38] J. McCulloch, "Simple consistent estimators of stable distribution parameters," *Communications in Statistics, Simulation and Computation*, vol. 15, pp. 1109–1136, April 1986.

[39] B. M. Hill, "A Simple General Approach to Inference About the Tail of a Distribution," *The Annals of Statistics*, vol. 3, pp. 1163–1174, Sept. 1975.

[40] E. F. Fama, "The behavior of Stock-Market Prices," *The Journal of Business*, vol. 38, no. 1, pp. 34–105, 1965.

[41] B. Mandelbrot, "The Variation of Certain Speculative Prices," *The Journal of Business*, vol. 36, no. 4, pp. 349–419, 1963.

[42] P. Lévy, *Calcul des probabilités*. PCMI collection, Gauthier-Villars, 1925.

[43] S. Borak, W. K. Härdle, and R. Weron, "Stable distributions - SFB 649 discussion paper No. 2005.008." Online: http://hdl.handle.net/10419/25027, 2005.

[44] T. Kozubowski, "Geometric stable laws: Estimation and applications," *Mathematical and Computer Modelling*, vol. 29, pp. 241–253, May 1999.

[45] T. Kozubowski and A. Panorska, "Multivariate geometric stable distributions in financial applications," *Mathematical and Computer Modelling*, vol. 29, pp. 83–92, May 1999.

[46] K. Dutta and J. Perry, "A Tale of Tails : An Empirical Analysis of Loss Distribution Models for Estimating Operational Risk Capital," Tech. Rep. 212, Federeal Reserve Bank of Boston, 2007.

[47] R. J. Adler, R. Feldman, and M. Taqqu, *A practical guide to heavy tails: statistical techniques and applications*. Birkhäuser, 1998.

[48] R. Pierce, "Application of the positive alpha-stable distribution," in *Proceedings of the IEEE Signal Processing Workshop on Higher-Order Statistics*, pp. 420–424, IEEE Comput. Soc, 1997.

[49] C. Nikias and M. Shao, *Signal processing with alpha-stable distributions and applications*. Adaptive and learning systems for signal processing, communications, and control, Wiley, 1995.

[50] C.-Y. Yang, K.-C. Hsu, and K.-C. Chen, "The use of the Levy-stable distribution for geophysical data analysis," *Hydrogeology Journal*, vol. 17, pp. 1265–1273, Dec. 2008.

[51] J. P. Nolan, "Modeling financial data with stable distributions." Online: http://academic2.american.edu/~jpnolan/stable/StableFinance23Mar2005.pdf, 2005.

[52] J.-P. Bouchard and M. Potters, *Theory of financial risks - From statistical physics to risk management*. Cambridge University Press, 2000.

[53] J. H. McCulloch, "Financial applications of stable distributions," in *Statistical Methods in Finance* (G. Maddala and C. Rao, eds.), vol. 14 of *Handbook of Statistics*, pp. 393 – 425, Elsevier, 1996.

[54] A. H. Murphy and R. L. Winkler, "A General Framework for Forecast Verification," *Monthly Weather Review*, vol. 115, pp. 1330–1338, July 1987.

[55] N. L. Johnson and S. Kotz, *Continuous univariate distributions - 2*. John Wiley & Sons, Ltd, 1970.

[56] 50Hertz, "50hertz." Online: www.50hertz.com.

[57] R. Weron, "Levy-stable distributions revisited: tail index > 2 does not exclude the Levy-stable regime," *International Journal of Modern Physics*, vol. 12, no. 2, pp. 209–223, 2001.

Author contacts:

William D'haeseleer (Corresponding author)

TME Branch (Energy Conversion)

Celestijnenlaan 300A

post box 2421

B-3001 Leuven

Belgium

Email: william.dhaeseleer@mech.kuleuven.be