



Industrial Organization and Behaviour

Hinnerk Gnutzmann

Thesis submitted for assessment with a view to obtaining the degree
of Doctor of Economics of the European University Institute

Florence, 10 September 2013

European University Institute
Department of Economics

Industrial Organization and Behaviour

Hinnerk Gnutzmann

Thesis submitted for assessment with a view to obtaining the degree of Doctor of Economics of the European University Institute

Examining Board

Prof. Thomas Gehrig, University of Vienna
Prof. Piero Gottardi, Supervisor, European University Institute
Prof. Andrea Mattozzi, European University Institute
Prof. Domenico Menicucci, University of Florence

© Hinnerk Gnutzmann, 2013

No part of this thesis may be copied, reproduced or transmitted without prior permission of the author

To my parents

Acknowledgments

This thesis represents the conclusion of a long, varied, exciting and at times difficult journey. I am immensely grateful to have had the opportunity to undertake it, and in the course relied on the help of many people. To all of them, I am deeply grateful.

Piero Gottardi, my first supervisor, trusted me to undertake this project, critically reviewed each argument I made, and offered help whenever needed; from him, I learned a lot. Domenico Menicucci, my external co-supervisor, read my work in great detail, sent long and invaluable emails with detailed comments, made time for meetings and even attended my presentations at the EUI. My second supervisor, Andrea Mattozzi, was likewise a great help. Their insights, intellectual and moral support, and unbounded generosity made this project possible.

Salvator Ortiguera and Massimo Morelli followed the thesis in its earlier stages. I'm grateful for the TAship under Fernando Vega-Redondo and the research assistance to Jerome Adda. Thanks to Piotr Spiewanowski, not just for his comments on most of the thesis, but his friendship.

I spent the last year at the Università Cattolica del Sacro Cuore in Milan, who generously allowed me to finish my thesis there. Luigi Filippini encouraged my work, advised me and helped in innumerable ways. Thanks for the discussions with Salvatore Piccolo. Carsten Nielsen trusted me to be TA for his course. Finally, Giovanni Ursino shared the office and the ups and downs with me.

Many thanks to the Deutsche Akademische Austauschdienst (DAAD), who supported me with a scholarship not just during the PhD but also before, and the European University Institute for financial support. Thanks to my classmates for the pleasant working environment, and Jessica Spataro, Lucia Vigna, Marcia Gastaldo, Julia Valerio, and Thomas Bourke for their impeccable support.

During the PhD, I enjoyed the hospitality of the OSCE Academy in Bishkek, and the New Economic School in Moscow. Thanks are due to these institutions, and in particular Maxim Ryabkov and Natalia Volchkova. Thanks also to Alexander Phlekanov for discussions.

Life does not start with the PhD program. I'm grateful to Paul Kattuman for encouraging me to start research, and to Tom Karkinsky for long study sessions, may he rest in peace. Peter de Gijssel, Hans Amman, Jurjen Kamphorst, Rob Alessie, Harry Garretsen and my co-authors Killian McCarthy and Brigitte Unger led me through the Research Masters. It was a unique experience, which means very much to me. In Karlsruhe, Christof Weinhardt trusted in me; it was with Marc Adam that I first worked on Penny Auctions.

In Hannover, the *Visionaries* have been my friends for many years. My parents' unwavering support, with heart and hand, has been *the* constant since I was born; I dedicate this thesis to them.

To Arevik, my main discovery in these past years, partner, friend and co-author: without you, I could not have done it. You bore the cross with me, at times for me. To many more years!

Contents

1	Paying Consumers to Stay: Retention Pricing and Market Competition	7
1.1	Introduction	7
1.2	Switching Processes and Retention Pricing	10
1.3	Model	11
1.4	Equilibrium Analysis	13
1.4.1	Mature Market	14
1.4.2	Introductory Pricing	23
1.5	Strategic Effects of Retention	25
1.5.1	Uniform Example	29
1.6	Welfare	29
1.7	Discussion	31
1.7.1	Price Matching	31
1.7.2	Myopic Consumers	33
1.7.3	The Role of Consumer Heterogeneity	34
1.8	Conclusion	35
2	History–Based Price Discrimination and Welfare in a Growing Market	39
2.1	Introduction	39
2.2	Model	42
2.3	Equilibrium Analysis	45
2.3.1	Unregulated Duopoly	45
2.3.2	Duopoly with Regulated Price Discrimination	49
2.3.3	Entrant or Incumbent Monopoly	51
2.3.4	Entry Stage	53
2.4	Welfare and Price Discrimination	54
2.4.1	Price Discrimination in Oligopoly	55
2.4.2	Entry, Price Discrimination and Welfare	57

2.5	Conclusion	60
3	Of Pennies and Prospects: Understanding Behaviour in Penny Auctions	63
3.1	Introduction	63
3.2	Data and Stylised Facts	66
3.3	Expected Utility Model	72
3.4	Prospects and Pennies	77
3.5	Estimation	82
3.6	Results	82
3.6.1	Risk Attitudes	83
3.6.2	Prospect Theory	84
3.7	Understanding Penny Auctions?	86
3.8	Conclusion	88

Abstract

This thesis collects three papers in industrial organization and behaviour, unified in their focus on the “digital economy”. The first two papers study markets for subscription goods, which – by the nature of the contract – allow for rich forms of behaviour-based price discrimination between consumers. The third paper investigates a novel and complex auction mechanism used in online auctions.

Paying Consumers to Stay: Retention Pricing and Market Competition studies the competitive effect of retention discounts. These discounts are given to consumers identified by their current firm as considering a switch to a new provider. In telecommunications markets, firms may obtain such information when consumers initiate a switching process. I model retention as a screening device in a two-period model of switching cost and behaviour-based price discrimination. In the second period, firms offer discounts to switching consumers, but use retention to defend the consumer against competing firms. This leads to fierce price competition, causing *all* prices to fall; first period prices generally rise, but don’t offset the pro-competitive effect. Retention reduces social welfare but raises consumer surplus.

History-Based Price Discrimination and Welfare in a Growing Market studies settings where an incumbent, who has a dominant market share among old consumers, faces potential entry. The incumbent is protected by switching costs in this segment, and can observe the purchase status of all consumers – hence, third-degree price discrimination is feasible. The entrant may induce consumers to reveal their purchase history through suitable offers. Under duopoly, price discrimination leads to lower prices and profits, but also lowers consumers surplus due to brand misallocation and inefficient switching. Thus, banning price discrimination increases welfare. However, regulation may also cause entry into an otherwise monopolised industry; this occurs only when entry is inefficient; thus optimal policy calls for bans on price discrimination unless they cause entry.

Of Pennies and Prospects: Understanding Behaviour in Penny Auctions studies a novelty

type of ascending auctions where bidders pay a fee for each bid they place. The last bidder wins the auction – thus a bid is essentially a bet on being the last bidder, adding a dimension of gambling to the auction. Also, like lotteries and slot machines, these auctions are often, but not always, highly profitable for the seller. The paper then applies prospect theory to penny auctions, obtaining estimates for the probability weighting and value functions that are in line with the literature. Moreover, the model can fit several stylised facts of bidder behaviour that are not explained by the alternative hypothesis of risk-loving bidders.

Chapter 1

Paying Consumers to Stay: Retention Pricing and Market Competition

Abstract. Retention pricing – offering discounts to consumers considering a switch to a competitor – is an established strategy in many markets for short-term contracts. I model retention as a screening device in a two-period model of switching cost and behaviour-based price discrimination. In the second period, firms offer discounts to switching consumers, but use retention to defend the consumer against competing firms. This leads to fierce price competition, causing *all* prices to fall; first period prices generally rise, but don't offset the pro-competitive effect. Retention reduces social welfare but raises consumer surplus

■

Keywords. retention, switching cost, subscription markets, price discrimination ■

1.1 Introduction

In markets with switching costs, firms often want to charge a high price to their existing (“loyal”) consumers, and a low switching price to lure consumers away from the competition. Thus firms “pay consumers to switch”, as in [Chen \(1997\)](#), and such price discrimination may allow for positive profits when consumers are heterogeneous in their switching cost. But so far, the literature has not allowed firms to “pay consumers to stay”, that is, to offer retention discounts to consumers that may be at risk of switching.

Retention pricing is a widespread business practice ([Ofcom, 2010](#)). In the telecommunications industry, consumers trying to port their number to a competing provider often

receive an offer of a reduced monthly fee together with the portability code. When trying to cancel a cable TV subscription, free extra channels may be offered to persuade the consumer to stay. Consumers calling to cancel a credit card may be offered extra air miles if they stay. In the marketing literature, the design of retention offers has been studied since at least [Rosenberg and Czepiel \(1984\)](#).

Common to the examples we have discussed is that consumers need to actively contact their firm to qualify for a retention offer. Doing so requires effort, but arguably less so than switching entirely to a competitor. This suggests that retention offers can be used for *screening* of consumers: a firm sets the retention price to keep some consumers with relatively low switching cost, who would otherwise have moved to a competitor, balanced against the cost of marginal high-cost consumers moving from loyal pricing to retention. This introduces *incentive-compatible*, or *second-degree price discrimination* [Stole \(2007\)](#) into an environment where firms already discriminate based on the observable purchase status, i.e. *third-degree price discrimination*.

How retention affects market competition is thus an important concern, especially in regulated industries, where instruments may be available to constrain such strategies. Do retention offers “dull incentives of providers to compete for rival’s customers” ([Ofcom, 2010](#))? Does retention lead to greater price dispersion, and do consumers with high switching cost lose out? Since retention can only be practiced once a consumer has been initially acquired, are dynamic considerations – the competition for market share – important in the analysis? Who would benefit from a possible ban on retention prices? How does the adoption of retention strategies depend on the industry structure?

In this paper, we develop a two-period price competition model to address these questions. Two or more firms produce a homogeneous good, and compete for market share in the first period. Then, consumers are privately informed about their switching cost; firms only observe the distribution of costs in the population. The model is thus one of vertical differentiation ([Armstrong and Vickers, 2001](#)): if all firms were to offer the same price, all consumers would stay with their current firm. Mindful of this environment, firms set mature market prices for loyal and retained consumers, which are available to their own consumer base, and switching offers, only available to those that purchased from a competitor. Consumers then choose among the offers available to them, incurring a utility cost equal to their type when switching (“switching cost”), or a fraction of said cost when claiming retention (“retention cost”).

Our model is closely related to the literature on behaviour-based price discrimination (BBPD). The addition of the retention offer is the key innovation over [Taylor \(2003\)](#) and

Chen (1997). In this line of literature, price discrimination is based on (observable) purchase status. One is then interested in the differences in market outcomes vis-a-vis a situation of uniform pricing (i.e., firms offer a single price to all consumers). As the survey by Fudenberg et al. (2006) shows, the effect of BBPD in general is complex. In situations where all firms are able to discriminate, one often finds a situation of best-response asymmetry Corts (1998): for a given uniform price by the opponent, firms want to raise price for existing consumers and lower prices for switching consumers. Then, “all out competition” may occur in the mature market, causing both loyal and switching prices to be lower under BBPD than uniform pricing. The effect is, partly or wholly, counteracted when first period competition is allowed, as the reduced value of market share makes competition less fierce. In asymmetric situations, things may be different. Gehrig et al. (2011) considers a model where an incumbent has a dominant position and can practice BBPD, while an entrant must charge a uniform price. However, the entrant is indifferent over a move to uniform pricing by the incumbent, since the potential benefits from reduced competition for loyal consumers are offset by fiercer competition for new consumers. In Chen and Zhang (2009), there is no market growth and hence the possible entrant unambiguously loses when BBPD is allowed. Esteves (2009) develops a model where consumers obtain information from advertising only, and shows that BBPD can in some cases raise industry profit.

In second-degree price discrimination, firms offer a *menu* of price and quantity/quality pairs to consumers (Maskin and Riley (1984) is early study; McAfee (2008) provides a survey and discusses anti-trust implications). Consumers then self-select the bundle they prefer the most, so both participation and incentive compatibility constraints apply. With a continuum of types, this means firms offer a continuum of menus. In contrast, in the retention model, firms do not discriminate on quality but only on the signal of whether a consumer tries to cancel a contract/request a switching code. Thus, even with a continuum of types, the number of menu items is limited to two. As emphasised by Spulber (1989), price discrimination trades off extra-marginal gains (from attracting new consumers) versus infra-marginal losses; but in a competitive setting, the participation constraint faced by each firm is determined by equilibrium, rather than exogenously.

We are aware of one simultaneous paper studying retention, Esteves (2012). In this paper, the product is heterogeneous due to brand preference. Consumers have a fixed switching cost, and crucially, do not claim retention strategically. Hence the dimension of second-degree price discrimination is absent, and in contrast to our model, prices for loyal consumers rise in the mature market. Moreover, due to lack of cost of claiming retention, the Esteves model predicts a social welfare from retention; as in the present paper, retention reduces

firm profits.

The paper proceeds as follows. Section 1.2 presents a brief case study from the telecoms industry, describing the manner in which retention offers are made and the current regulatory concerns. In section 1.3, we present the formal model and conduct an equilibrium analysis in section 1.4. We discuss how the ability of firms to make retention offers affects competition compared to the benchmark of BBPD without retention in section 1.5, and also conduct comparative statics on the cost, α , of claiming retention. Welfare implications are discussed in 1.6. In section 1.7, we provide some extensions to the argument. Finally, section 1.8 concludes.

1.2 Switching Processes and Retention Pricing

The literature typically takes the existence of switching costs as exogenously given and pays scant attention to the process through which consumers switch. Yet this process may matter, because firms may acquire information on their consumers in the process.

Ofcom (2010) provides a useful taxonomy of switching processes. First, especially in unregulated industries, there may be a simple “cease and re–provide” (C&R) regime in place. Consumers determine the order in which they cancel an existing contract and sign a new one. If a new contract is already signed, retention is generally considered infeasible as minimum contract periods etc. already apply with the new provider; however, when the consumer sends a cancellation notice to the existing firm ahead of a new contract being signed – e.g. due to notice periods – retention offers are possible in such a framework.

In regulated industries, the design of the switching process determines whether retention is feasible. Under *Losing–Provider Led* (LPL) processes, consumers contact their existing provider to obtain a code (known as “porting authorization codes”, “migrations authorization code” etc.) which allows the new provider to initiate number portability. However, at the time of the code request, no new contract is yet signed, and the provider at risk of losing the consumer may make a retention offer. In practice, retention offers may be sent automatically through a text message or after a telephone conversation; the retention offers are not publicly advertised, so little systematic data is available about them. One may however find some discussions on the topic in online consumer forums; the broad picture is that retention discounts can be substantial.

With a *Gaining–Provider Led* (GPL) switching regime, the consumer finalises a contract with the new provider before the losing provider is advised that a switch is in process; at the time the e.g. number portability is initiated, there is no more scope for a retention offer

(examples include the “notice of termination” process).

1.3 Model

Set-Up: There are N firms, producing a single homogeneous code good. Marginal cost is constant and normalised to zero. They simultaneously compete in prices to sell the product to a unit mass of consumers, and markets open for two periods: in the first period, firms charge an *introductory price* and consumers, not yet “locked in” to any firm, choose a firm. Then, at the onset of the second period, consumers are privately informed of their switching cost. Firms simultaneously set mature market prices – loyal and retention offers, available to consumers who purchased from them in the first period, and a switching price available to those who did not. Consumers then choose which offer to take, and payoffs are realised. The timing of the model is summarised in figure 2.1

Consumers: Consumers vary in their switching cost, so let the consumer’s type θ denote her switching cost. In the population, the distribution of types, denoted $F(\theta)$, has support on the unit interval $[0, 1]$ and satisfies log-concavity. This assumption essentially assures that extremes of the type space do not occur too frequently and guarantees uniqueness of equilibria¹. Formally,

Assumption 1. *The switching cost has left and right increasing hazard rates*

1. $F(\theta)$ is log-concave
2. $1 - F(\theta)$ is log-concave

The consumer is presented with *offers* at each stage in the game. An offer is a combination of price and switching effort required to secure it. In the introductory stage, consumers are

¹See Bergstrom and Bagnoli (2005) for a review on log-concave probability and its economic applications. Log-concavity does not provide us with sufficient structure to tie down welfare results, so some later results are presented in the context of a uniform distribution

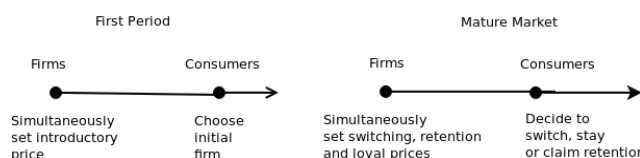


Figure 1.1: Timeline

unattached to firms, so no effort is required to purchase an introductory offer: $(p_i^I, 0)$ is offered by all firms. In the mature market, consumers of firm i may choose the loyal offer, $(p_i^L, 0)$, available as “default” with no effort required; or they may contemplate a switching offer from any other firm, each of which requires unit effort to obtain: $(p_j^S, 1)$, $j \in N - i$. Finally, a retention offer may be available, from firm i only, which requires effort level α to obtain. The θ consumer’s period payoff for a given offer is given by

$$u_\theta(p, e) = \beta - p - \theta e \quad (1.3.1)$$

where $\beta > 1$ is the valuation of the good, sufficiently high to ensure full market coverage.

Firms: There are $N \geq 2$ firms. Due to zero marginal costs, profit maximisation is revenue maximisation. To simplify the notation, denote the firm’s strategy as $\vec{p}_i = (p_i^I, p_i^S, p_i^R, p_i^L) \in R^4$. The mass of consumers choosing firm i in the first period is given by function $\gamma_i(p_i^I, \underline{p}^I)$, where $\underline{p}^I = \min\{p_i^I\}$ over $i \in N$ is the set of cheapest introductory prices. In the second period, the best switching offer available to firm i consumers is given by $\underline{p}_i^S = \min\{p_j^S\}$, for $j \in N - i$; the share of firm i consumers choosing the retention offer is given by $\rho_i(\underline{p}_i^S, p_i^R)$ and share $\lambda_i(p_i^R, p_i^L)$ chooses the loyal offer. The mass of consumers switching from firm j to i is given by the function $\sigma_{ij}(p_i^S, \underline{p}_j^S, p_j^R)$. Of course, due to the extensive form nature of the game, the strategy space is richer than this. But will show that the restrictions made here are equilibrium outcomes and do not affect the results.

The profit of firm i is thus given by

$$\begin{aligned} \pi_i(\vec{p}_i, \vec{p}_{-i}) &= \gamma_i(p_i^I, \underline{p}^I)p_i^I + \gamma_i\left(\rho_i(\underline{p}_i^S, p_i^R)p_i^R + \lambda_i(p_i^R, p_i^L)p_i^L\right) \\ &+ \sum_{j \in N-i} \gamma_j \sigma_{ij}(p_i^S, \underline{p}_j^S, p_j^R)p_i^S \end{aligned} \quad (1.3.2)$$

where the first term denotes profits in the introductory period. The remaining terms concern the mature market: the second contains profits from the existing consumer base, and the third profits from “poaching” consumers from competitor. Let σ_i define the tie-breaking rule:

$$\sigma_{ij}(p_i^S, \underline{p}_j^S, p_j^R) = \frac{1(p_S^i = \min_{j \in N} p_S^j)}{|\min(p_S^j)|} \quad (1.3.3)$$

There is no discounting, but it can be shown that equal discount rates of consumers and firms do not affect the results.

Finally, to resolve payoff ties, we rely on uniform tie breaking:

Assumption 2 (Uniform Tie Breaking). *In case consumers are indifferent between several firms, each firm receives an equal mass of consumers*

The assumption is only for convenience; without tie breaking, there would be multiple equilibria, but each agent receives the same payoff as in the unique equilibrium derived under the UTB assumption.

1.4 Equilibrium Analysis

The structure of the game implies that we have four classes of information sets. First, there is a singleton introductory pricing stage in which each firm sets only a single price. Then, consumers make their first purchase decisions, having observed the introductory prices offered by all firms. Now, the second period starts. Each possible first-period history maps to a singleton mature market pricing information set, in which all firms simultaneously set their loyal, retention and switching prices; however, we will show that it is an equilibrium outcome that the equilibria in this game do not depend on the history. Hence, to unclutter the notation, we will simply write the prices as a 4-vector. Finally, consumers of each firm choose their mature market offer; again, we will demonstrate that the equilibrium actions depend only on the firm purchased from in the first period and mature market prices available to the consumer and denote the choice rule accordingly.

Our solution concept is subgame perfect Nash equilibrium. This leads to the following definition of the equilibrium:

Definition 1 (Equilibrium). *A set of prices $(p_i^{*I}, p_i^{*S}, p_i^{*R}, p_i^{*L})$ for all firms $i \in N$ and consumer choice rules C_1^*, C_2^* such that*

1. *Consumers choose optimally in the mature market*

$$C_2^*(\theta, j, p_i^L, p_i^R, p_{-i}^S) = \underset{(p_i^L, 0), (p_i^R, \alpha), (p_{-i}^S, 1)}{\operatorname{argmax}} (\beta - p_2 - \theta e_2)$$

$$\forall \theta \in \Theta, j \in N, (p_j^L, p_j^R, p_{-j}^S) \in \mathbb{R}^3$$

2. *Firms maximise mature market profits:*

$$(p_i^{*L}, p_i^{*R}, S^{*j}) = \operatorname{argmax} \pi(p_i^I, p_i^L, p_i^R, p_i^S; p_{-i}^I, p_{-i}^{*L}, p_{-i}^{*R}, p_{-i}^{*S}; C_1, C_2^*)$$

$$\forall j \in N, (p_i^I, p_{-i}^I) \in \mathbb{R}^N, C_1 \in \mathcal{C}_1$$

3. Consumers choose optimally in the first period:

$$C_1^*(\theta, p_1^I \dots, p_N^I) = \underset{j \in N}{\operatorname{argmax}} (\beta - p_j^I) + u_2^*(\theta, C_1^*, p_j^I, \vec{p}_2^*) \\ \forall (p_1^I \dots, p_N^I) \in R^N$$

4. Profit Maximisation in the first period:

$$p_i^I = \operatorname{argmax} \pi(p_i^I, p_{-i}^{*I}; \vec{p}_2^*) \forall j \in J$$

1.4.1 Mature Market

Consumer Behaviour: Given the offers available to a consumer, which depends on the firm she bought from in the first period, different types may in principle choose different actions. A small extra complication arises here that consumers can choose between three offers, and thus it may be that one of the offers is strictly worse than the others for all types. In such a case, the “marginal consumer” approach would not be well-defined. To solve the model, first observe that consumer choices in the mature market are threshold strategies, as required for the marginal consumer approach:

Lemma 1.4.1 (Threshold Strategies). *Under the mature market choice rule c_2 , the optimal choice must have consumers playing a threshold strategy.*

Proof. Suppose firm i consumers θ_1 and θ_2 , optimally choose, without loss of generality, the loyal offer; hence they must weakly prefer loyal over retention and the best switching offer available. This implies $\beta - p_L^i \geq \beta - p_R^i - \alpha\theta$ and $\beta - p_L^i \geq \beta - \underline{p}_S^i - \theta$ for $\theta = \{\theta_1, \theta_2\}$. Now consider any convex combination of the types, $\bar{\theta} = \lambda\theta_1 + (1 - \lambda)\theta_2$. Since the payoff to each action is linear in θ , it is immediate to verify that $\bar{\theta}$ will also weakly prefer the loyal offer. The argument for other firms and offers is exactly analogous; hence, there will be convex sets of consumer types choosing the same offer. \square

It turns out that there are two cases: for sufficiently high retention prices, some consumers strictly prefer switching and others strictly prefer the loyal offer over retention. However, as the retention price falls, eventually a positive mass of consumers also chooses the retention offer. Thus, we have to consider two cases in our analysis:

Lemma 1.4.2. *If $p_R^i > \tilde{p}_R^i = (1 - \alpha)p_L^i + \alpha\underline{p}_S^i$, no consumer of firm i chooses retention*

Proof. For a consumer θ to choose retention, it must be weakly preferred to switching, requiring $\theta \geq \frac{p_R^i - p_S^i}{1 - \alpha}$, and loyal, i.e. $\theta \leq \frac{p_L^i - p_R^i}{\alpha}$. When $p_R^i > \tilde{p}_R^i = (1 - \alpha)p_L^i + \alpha p_S^i$, the two conditions yield a contradiction; hence each consumer either weakly prefers switching or loyal over retention. \square

The critical retention price \tilde{p}_R^i , is simply a convex combination of the loyal and switching prices; to see why a retention price slightly below the loyal price may not attract consumers, note that in general the effort cost securing retention is strictly positive. Thus, the retention price has to be sufficiently below the loyal price to attract any consumers.

Combining these two lemmas, we can now derive the marginal consumers in the no-retention and retention cases respectively:

Lemma 1.4.3. *Optimal consumer choice c_2^* is given by*

1. *If $p_R^i > \tilde{p}_R^i$, there is a marginal consumer $\theta_{SL} = L - S$, such that all consumers below θ_{SL} switch and others stay loyal.*
2. *If $p_R^i \leq \tilde{p}_R^i$, there are two marginal consumers. Consumers below $\theta_S = \frac{p_R^i - p_S^i}{1 - \alpha}$ switch; and consumers above $\theta_R = \frac{p_L^i - p_R^i}{\alpha}$ choose the loyal offer; all others choose retention.*

Proof. 1. The previous lemma establishes that no consumer chooses retention for these prices. It remains to find the marginal consumer between loyal and switching. For the marginal consumer $\beta - p_L^i = \beta - p_S^i - \theta$ holds, giving the required condition.

2. From the previous lemma, there are consumer types choosing retention. Consumers with $\beta - p_S^i - \theta = \beta - p_R^i - \alpha\theta$ are marginal between switching and retention; and consumer $\beta - p_R^i - \alpha\theta = \beta - p_L^i$ is marginal between retention and loyal, yielding the desired conditions. \square

Mature Market Pricing: in view of the fact that there are both retention and no-retention cases, it is clear that in principle separate analysis of both regions is required. However, it is a simple observation that once a firm charges a sufficiently high retention price that no consumer would accept it, for all players it becomes payoff-irrelevant just how far above the no-retention constraint this price really is. Secondly, we must have all mature market prices at least equal to zero, for otherwise at least one firm would incur (avoidable) losses. Thirdly, given any of the surviving switching prices, is not in a firm's interest to undercut the rival's switching price with a lower retention or loyal price. Finally, setting loyal and

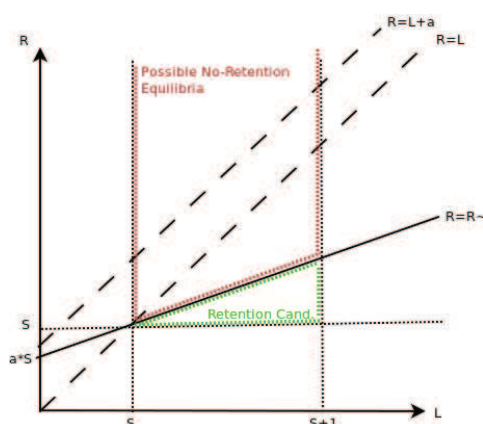


Figure 1.2: A cut of the strategy space

retention prices sufficiently high that all consumers switch to the competition is also not an optimal strategy.

Through this reasoning, illustrated in figure ??, we narrow down the search of equilibria considerably. First, define the subset \mathcal{D} (green in the figure) of the strategy space:

$$\mathcal{D} = \{ \underline{p}_S^i, \tilde{p}_R^i, \underline{p}_L^i \} \text{ such that } \tilde{p}_R^i \leq \tilde{p}_R^i, \underline{p}_R^i, \underline{p}_L^i \leq \underline{p}_S^i + 1, 0 \leq \underline{p}_i^S; \}$$

Lemma 1.4.4. *If there exists a mature market equilibrium outside \mathcal{D} , there exists a payoff-equivalent equilibrium inside \mathcal{D} .*

Proof. First, suppose that $\min\{\underline{p}_i^S, \underline{p}_i^R, \underline{p}_i^L\} < 0$. Then, there must be a firm making positive sales at a negative price; a deviation to raising price would be a profitable deviation. So we must have each of the three prices at least zero.

Now, consider prices such that $\underline{p}_i^R < \underline{p}_i^S$ and $\underline{p}_i^L < \underline{p}_i^S$, the bottom left square in the figure. In this case, a deviation to setting $\underline{p}_i^R = \underline{p}_i^L = \underline{p}_i^S$ will not reduce the mass of consumers (all consumers buy from firm i), but increases revenue per consumer. Hence this case also cannot be an equilibrium. If $\underline{p}_i^L \geq \underline{p}_i^S$ but $\underline{p}_i^R < \underline{p}_i^S$, similarly a deviation to setting $\underline{p}_i^R = \underline{p}_i^S$ increases revenue per consumer but does not reduce market share.

Now consider prices $\underline{p}_i^L > \underline{p}_i^S + 1$. From lemma 1.4.3, the marginal consumer will then be outside the domain of F , so no consumer chooses the loyal offer. Hence a reduction in the loyal price to $\underline{p}_i^L = \underline{p}_i^S + 1$ will not affect the payoff of any firm. Similarly, if the retention price is above the threshold, $\underline{p}_i^R > \tilde{p}_i^R$, indicated by the “no-retention line” in the figure, no consumer chooses retention. Again, a reduction of the retention price to $\underline{p}_i^R = \tilde{p}_i^R$ leaves the

payoffs of all firms unchanged. Hence, if there are equilibria outside \mathcal{D} , they must be payoff equivalent to an equilibrium on the boundary of \mathcal{D} . \square

Hence, we can rewrite the payoff function of a firm i in the marginal consumer form:

$$\begin{aligned} \Pi_i^2(p_R^i, p_L^i; \underline{p}_S^i) &= \underbrace{\gamma_i \{ [1 - F(\theta_R(p_R^i, p_L^i))] p_L^i + [F(\theta_R(p_R^i, p_L^i)) - F(\theta_S(\underline{p}_S^i, p_R^i))] p_R^i \}}_{\Pi_i^{2,RL}} \\ &+ \underbrace{\sum_{j \in N-i} \gamma_j \sigma_{ij}(p_i^S, \underline{p}_j^S, p_j^R) F(\theta_S(p_i^S; p_R^{-i})) p_S^i}_{\Pi_i^{2,S}} \end{aligned}$$

The profit function consists of two parts. The first bracket, weighed by the market share inherited from the first period, denoted γ_i , is the profit made on the own the existing consumer base. Since marginal cost is zero, this simply the sum of revenue in the loyal market segment, where consumers with $\theta \geq \theta_R$ pay p_L^i , and revenue from retained consumers. These are the consumers with lower switching cost than the loyal consumers, $\theta \leq \theta_R$, but for whom a full-scale switch is still too costly ($\theta \geq \theta_S$).

The second part is profit from switching consumers. Due to the uniform tie-breaking rule, if several firms charge the lowest switching price, business from any switching consumers is evenly divided between those firms. The industry revenue from switching consumers depends on the the lowest switching price in the market and the retention prices of each firm, as shown inside the final bracket.

Observe that the two parts of the profit function are additively separable in the switching and loyal/retention price respectively. This feature is shared with the canonical model of [Chen \(1997\)](#): because it is effectively observable who bought from which firm in the first period, the firm's switching price does not compete with the loyal and retention offers for the same consumer. Hence, maximising profit from the existing consumer base and attracting switching consumers are separate concerns. We first consider the problem of pricing the former group:

Best Response Analysis: Own consumer base. To emphasise the relationship with second-degree price discrimination models, note that – on the domain \mathcal{D} – the problem of maximising profit $\Pi_i^{2,RL}$ can be equivalently written as

$$\begin{aligned} \max_{p_i^R, p_i^L} \quad & \gamma_i \{ [1 - F(\theta_R)] p_L^i + [F(\theta_R) - F(\theta_S)] p_R^i \} \\ \text{s.t.} \quad & \beta - p_i^L \geq \beta - p_i^R - \alpha\theta && \text{for } \theta \in [\theta_R, 1] && \text{(IC)} \\ & \beta - p_i^R - \alpha\theta \geq \beta - \underline{p}_i^S - \theta && \text{for } \theta \in [\theta_S, \theta_R] && \text{(PC)} \end{aligned}$$

In the standard BBPD model, prices are set conditional on observables; thus only a participation constraint applies. Retention, on the other hand, is *incentive-compatible* price discrimination, so at the margin, the IC constraint binds. Our model thus combines elements of third-degree and second-degree price discrimination. Note that market share simply scales the profit function but does not affect the first-order conditions; this is again a consequence of the fact that each consumer's purchase status is observable.

The first derivatives are given by:

$$\frac{d\Pi_i^{2,RL}}{dp_L^i} = 1 - F(\theta_R) - \frac{f(\theta_R)}{\alpha} (p_L^i - p_R^i) \quad (1.4.1)$$

$$\frac{d\Pi_i^{2,RL}}{dp_R^i} = F(\theta_R) - F(\theta_S) + \frac{f(\theta_R)}{\alpha} (p_L^i - p_R^i) - \frac{f(\theta_S)}{1-\alpha} p_R^i \quad (1.4.2)$$

The interpretation is standard: when raising the loyal price, $1 - F(\theta_R)$ consumers pay more, the infra-marginal gain; at the margin, there are $f(\theta_R)$ consumers, with elasticity $1/\alpha$, that will opportunistically claim retention. In retention pricing, there is first the infra-marginal gain; second, at the upper margin, consumers move into the loyal segment, creating an extra-marginal gain. Finally, at the lower margin, consumers switch to the competitor, an extra-marginal loss.

When is retention profitable? It turns out that retention is profitable precisely when the switching price exceeds marginal cost:

Lemma 1.4.5. *Only if the switching price exceeds marginal cost, the best response retention price is below the threshold where no consumer accepts retention, i.e. only if $\underline{p}_S^i > 0$, then $p_i^{*R} < \tilde{p}_R^i$*

Proof. Holding \underline{p}_S^i fixed, consider any prices (\tilde{p}_R^i, p_L^i) on the no-retention line. Then, we must have $\theta_R = \theta_S$. Evaluating $\frac{d\Pi_i^{2,RL}}{dp_R^i}$ then yields $f(\theta_R) \frac{\tilde{p}_R^i - (1-\alpha)p_L^i}{\alpha(1-\alpha)} = f(\theta_R) \frac{-\underline{p}_S^i}{(1-\alpha)}$, where the second equality follows from the definition of \tilde{p}_R^i .

Hence, whenever $\underline{p}_S^i > 0$, a deviation to lowering retention price below the no-retention line increases profits. \square

To see the intuition of the lemma, consider the following example for a consumer of firm i . Suppose loyal price is $p_L^i = \frac{1}{2}$, $\alpha = \frac{1}{2}$, and the cheapest switching price $\underline{p}_S^i = 2\epsilon$. Then, by lemma 1.4.2, the lowest price at which no consumer chooses the retention offer is $\tilde{p}_R^i = \frac{1}{2} + \epsilon$. Now consider a small reduction in the retention price. The mass of consumers moving from switching to retention is $2f(\theta)$, equal to the mass moving from loyal to retention. On the first group of consumers, the firm's incremental profit is $\frac{1}{4} + \epsilon$, since

they were previously switchers. The previously loyal consumers however pay less, leading to a loss of $\frac{1}{4} - \epsilon$ per consumer. Thus when $\epsilon > 0$, the gain exceeds the loss; but with zero switching price, the two effects exactly balance. For $\alpha \neq \frac{1}{2}$, the same intuition holds since the critical retention price \tilde{p}_i^R depends on α itself.

Lemma 1.4.6. *Best response prices for the existing consumer base are implicitly defined by*

$$0 = \alpha \frac{1 - F(\theta_R)}{f(\theta_R)} + p_R^i - p_L^i$$

$$0 = \begin{cases} (1 - \alpha) \frac{1 - F(\theta_S)}{f(\theta_S)} - p_R^i & \text{if } \underline{p}_S^i \leq \frac{1 - \alpha}{f(0)} \\ \underline{p}_S^i - p_R^i & \text{otherwise} \end{cases}$$

Proof. As we argued above, the relevant domain of the problem is \mathcal{D} . First, let's consider corners where $F(\theta_R) = 1$, i.e. no consumer purchases the loyal offer. At such points, the first order condition 1.4.1, is negative, hence calling for a lower loyal price. Second, at the corner where $F(\theta_R) = 0$, equation 1.4.1 reduces to $\alpha(1/f(0) - 0) \geq 0$, after substituting out the definition of θ_R . Thus, by appeal to continuity, the optimal loyal price is always interior and hence the first-order condition is sufficient.

Retention pricing: Since the first-order condition for the loyal price is sufficient, we can substitute it into 1.4.2. This yields the simplified first-order condition for p_R^i :

$$\frac{d\Pi_i^{2,RL}}{dp_R^i} = 1 - F(\theta_S) - \frac{f(\theta_S)}{1 - \alpha} p_R^i$$

Observe that, subject to the loyal price being optimally chosen, the retention price depends *only* on the competitor's best switching price.

The first order condition has a discontinuity at $F(\theta_S) = 1$; but as we already showed, $F(\theta_R) < 1$ in any best response, and from the definition of \mathcal{D} , we must have $\theta_S \leq \theta_R$; thus the right-hand discontinuity is outside the domain. It remains to check the case $F(\theta_S) = 0$. The first-order condition is positive at this point if $1 - 0 - f(0) \frac{\underline{p}_S^i}{1 - \alpha} \geq 0$, i.e. $\underline{p}_S^i \leq \frac{1 - \alpha}{f(0)}$. In this case, again by appeal to continuity, the best response retention price will be interior and $\frac{d\Pi_i^{2,RL}}{dp_R^i} = 0$ must hold. Otherwise, the best response is at the corner: $p_R^i = \underline{p}_S^i$. \square

Switching Consumers: The profit function on switching consumers is given by

$$\Pi_i^{2,S} = \sum_{j \in N-i} \gamma_j \sigma_{ij}(p_i^S, \underline{p}_j^S, p_j^R) F(\theta_S(p_S^i; p_R^{-i})) p_S^i$$

Analysing the best response for all possible loyal/retention prices by all firms is unreasonably complex and yields no additional insight. Since our interest is in equilibria of the

pricing game, we instead characterise the switching prices that can occur in equilibrium, that is, subject to (p_S^i, p_R^{-i}) being in the best response by all firms:

Lemma 1.4.7. *Switching prices*

1. When there are more than two firms $N > 2$, the market prices for switching consumers must be zero: $\underline{p}_S^{*i} = 0$ for all $i \in N$.
2. Under duopoly, the equilibrium switching price satisfies

$$0 = (1 - \alpha) \frac{F(\theta_S^*)}{f(\theta_S^*)} - p_S^{*i}$$

for both firms

Proof. 1. Follows from standard Bertrand arguments. Fix firm i and suppose the lowest switching price offered i consumers is positive: $\underline{p}_S^i > 0$. If $|\min(p_S^j)| > 1$, i.e. several firms charge the lowest price, a deviation to lowering p_S^j by one of them discontinuously improves profit. On the other hand, if $|\min(p_S^j)| = 1$, there must be firms that make zero profit from switching consumers. Pick any of these firms and consider a deviation to $p_S^k = \underline{p}_S^i - \epsilon$. This enables firm k to capture the entire market for switchers and improves profit. On the other hand, when $\underline{p}_S^i = 0$, no such deviations exist. Thus in any equilibrium with more than two firms, the switching price must equal marginal cost.

2. In the duopoly case, the payoff from switching consumers becomes $F(\theta_S(p_S^i; p_R^{-i}))p_S^i$. This has the first-order condition

$$\frac{d\Pi_i^{2,S}}{dp_S^i} = F(\theta_S) - f(\theta_S) \frac{p_S^i}{1 - \alpha}$$

We know from the previous lemma that if $p_S^i > \frac{1-\alpha}{f(0)}$, the retention price will be set to match the switching price, leaving $\theta_S = 0$ and zero revenue from switching consumers. A small deviation to lowering the price is thus profitable. Secondly, consider an equilibrium where the switching price is zero. From the previous lemma, the best response retention price in this case is positive, and $\theta_S > 0$; hence, a deviation towards raising the switching price would improve profit from zero to a positive amount. Thus, the best-response switching price should be interior in any equilibrium and the first-order condition again sufficient. □

Equilibrium Analysis: Collecting the earlier results, we are now ready to state the equilibrium of the mature market pricing game:

Proposition 1.4.1 (Duopoly Mature Market Equilibrium). *With $N = 2$, there is an equilibrium such that, for each firm $i \in N$,*

1. *The marginal consumers are given by*

$$0 = \frac{1 - F(\theta_R^*)}{f(\theta_R^*)} - \theta_R^* \quad (1.4.3)$$

$$0 = \frac{1 - 2F(\theta_S^*)}{f(\theta_S^*)} - \theta_S^* \quad (1.4.4)$$

2. *Prices are given by*

$$0 = \alpha \frac{1 - F(\theta_R^*)}{f(\theta_R^*)} + p_R^{*i} - p_L^{*i} \quad (1.4.5)$$

$$0 = (1 - \alpha) \frac{1 - F(\theta_S^*)}{f(\theta_S^*)} - p_R^{*i} \quad (1.4.6)$$

$$0 = (1 - \alpha) \frac{F(\theta_S^*)}{f(\theta_S^*)} - p_S^{*i} \quad (1.4.7)$$

Moreover, the equilibrium is payoff-unique.

Proof. Consumer behaviour was found in 1.4.3 and is only stated for completeness. To derive the equilibrium prices, combine lemma 1.4.6 and the duopoly case of lemma 1.4.7.

For the marginal retained consumer, substitute the definition $\theta_R = (p_L^i - p_R^i)/\alpha$ into equation . Similarly, for the marginal switcher, subtract equation from and substitute for the definition of $\theta_S = (p_R^i - p_S^i)/(1 - \alpha)$. \square

For switching and retention pricing, the equilibrium conditions tell us that the price must equal the ratio of infra-marginal loss (given by $F(x)$ and $1 - F(x)$ respectively) over the extra-marginal gain of consumers, which is $f(x)$. These expressions are scaled by the elasticity, $1 - \alpha$. Thus each firm sets prices to maximises revenue, not taking into account the business stealing externality on the rival. However, when setting loyal prices, the firm takes into account that the marginal consumer would move to the retained segment, which still yields revenue for the firm; thus there is no externality at this margin.

In equilibrium, the marginal consumers are fixed, independent of the effort cost of retention (parameter α). As can be seen from the best responses, say a rise in α increases the elasticity of demand for switching/retained prices; thus, at given prices, a reduction in both

the switching and retention price is called for. Second, for a given price difference, more consumers switch as α rises. By combining the best responses, we see that the two effects exactly cancel and hence changes in the parameter do not influence marginal consumers.

In general, log-concavity gives us relatively little structure regarding the sizes of marginal, retained and loyal segments. Within the general model, we were only able to show a weak upper bound on the amount of switching consumers:

Lemma 1.4.8. *Less than half consumers switch, i.e. $F(\theta_S^*) < \frac{1}{2}$.*

Proof. Suppose not. Then $\frac{1-2F(\theta_S^*)}{f(\theta_S^*)} < 0$, so we must have $\theta_S^* < 0$, by equation 1.4.4. But this contradicts $\theta_S^* \in (0, 1)$ \square

We now turn to the competitive case. In this case, the equilibrium is as follows:

Proposition 1.4.2 (Competitive Mature Market Prices). *With $N > 2$, there is an equilibrium such that, for each firm $i \in N$,*

1. *Consumers with $\theta < \theta_S^* = \theta_R^*$ switch and all others remain loyal; the marginal consumer satisfies*

$$0 = \frac{1 - F(\theta_S^*)}{f(\theta_S^*)} - \theta_S^* \quad (1.4.8)$$

2. *The market prices are given by*

$$0 = \alpha \frac{1 - F(\theta_R^{i*})}{f(\theta_R^{i*})} + p_R^{*i} - p_L^{*i} \quad (1.4.9)$$

$$0 = (1 - \alpha) \frac{1 - F(\theta_S^{i*})}{f(\theta_S^{i*})} - p_R^{*i} \quad (1.4.10)$$

$$0 = \underline{p}_S^{*i} \quad (1.4.11)$$

Moreover, the equilibrium is payoff-unique.

Proof. We follow exactly the steps of proving proposition 1.4.1, but this time combining lemma 1.4.6 with the competitive case in lemma 1.4.7.

Since the switching price must be zero, using equation 1.4.10 provides the condition for the marginal switcher, $(1 - F(\theta_S^*)) / f(\theta_S^*) - \theta_S^* = 0$, after substituting the definition of θ_S^* . But this is exactly the same condition as obtained for θ_R from equation 1.4.9; hence the consumer marginal between switching and retention is also marginal to loyal, and the retained segment is empty, as claimed.

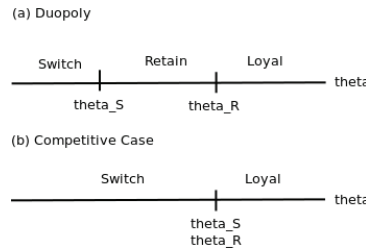


Figure 1.3: Equilibrium Choices

Payoff-uniqueness: First, profit from switching consumers must be zero in any equilibrium as $p_S^i = 0$ for all i . But for retained and loyal consumers, the best response is single-valued, tying down the profit $\Pi_i^{*,RL}$ for all i . \square

Observe that the equilibrium conditions are *symmetric*: that is, for all firms, the loyal, retention and lowest switching price are identical. This feature will be important in analysis of the first period, since it implies that the continuation utility of a consumer does not depend on which firm was chosen in the first period.

Figure ?? illustrates the structure of the mature market equilibrium in the two cases. Under duopoly, the lowest-cost consumers switch, and highest-cost consumers remain loyal; those with intermediate costs claim the retention offer. In the competitive case, the mass of consumers choosing the loyal offer is the same as under duopoly; however, there is no retention in equilibrium, but instead a higher switching rate.

1.4.2 Introductory Pricing

In the first period, no consumer has yet bought from any firm. However, they take into account the continuation utility of being “locked in” to a firm in the second period due to switching cost, in addition to the introductory price. Thus there is a link between the two periods; however, since the second-period prices are symmetric, the continuation utility of being a consumer of firm i or j is identical; thus, given subgame perfection, consumer choice only depends on the introductory price:

Lemma 1.4.9 (First Period Consumer Behaviour). *In the first period, consumers purchases are evenly divided among the firms charging the lowest price:*

$$C_1(i, p_I^i, \dots) = \begin{cases} 0 & \text{if } p_i^I > \min\{p_j^I\} \\ \frac{1}{|\min\{p_j^I\}|} & \text{if } p_i^I = \min\{p_j^I\} \end{cases}$$

Proof. In the first period, consumers are not yet informed of their effort cost. The utility of a consumer buying from firm i is thus given by the period payoff plus the expected payoff of being a firm i consumer in the second period:

$$\{\beta - p_I^i\} + \{\beta - \int_0^{\theta_S} (p_S^{*i} + x)f(x)dx - \int_{\theta_S}^{\theta_R} (p_R^{*i} + \alpha x)f(x)dx - \int_{\theta_R}^1 p_L^i f(x)dx\}$$

From propositions 1.4.2 and 1.4.1, the equilibria in both the duopoly and competitive case are symmetric; thus the continuation utility in the second period does not depend on the firm chosen in period 1. Hence, only the first bracket – first–period payoff – is marginal for the purchase decision. If $p_I^i > \min\{p_I^j\}$, there exists a firm j that offers a lower firm and is strictly preferred by all consumers. On the other hand, if $p_I^i = \min\{p_I^j\}$, the consumer is indifferent between firm i and any of the firms also charging the minimum price. Thus, by uniform tie breaking, consumers split evenly between the firms. \square

It remains only to solve the introductory pricing stage. On the one hand, in the first period only firms charging the lowest price will make positive sales; moreover, they obtain positive profits from their consumer base in the mature market. This suggests that first–period competition should be fierce. On the other hand, there is the “outside option” of forgoing consumers in the first period and enjoying switching profits in the second period only; this serves to restrain first–period competition when switching profits are positive. These considerations thus determine introductory prices:

Proposition 1.4.3. *The first–period pricing equilibrium is characterised by*

1. *First period prices are given by $p_I^I = \Pi_i^{*,S} - \Pi_i^{*,RL}$ for all firms*
2. *Consumers evenly split between firms in the first period*
3. *Intertemporal profits equal $\Pi_i^{*,S}$*

Moreover, the equilibrium is payoff unique.

Proof. The firm’s profit from the first–period perspective is given by

$$\Pi_i = \begin{cases} p_I^i + \Pi_i^{*,RL} & \text{if } p_I^i < \min\{p_I^j\} \\ \frac{1}{|\min\{p_I^j\}|} \{p_I^i + \Pi_i^{*,RL}\} + \left(1 - \frac{1}{|\min\{p_I^j\}|\right) \Pi_i^{*,S} & \text{if } p_I^i = \min\{p_I^j\} \\ \Pi_i^{*,S} & \text{otherwise} \end{cases}$$

reflecting the cases of either attracting all consumers, some consumers or no consumers in the first-period.

Consider first the case with $N \geq 3$ firms. Then, from proposition 1.4.2, $\Pi_i^{*,S} = 0$ for all firms. When $p_I^i = -\Pi_i^{*,RL}$, the firm's equilibrium profit is zero; the profit from deviating to any higher price is $\Pi_i^{*,S} = 0$, a deviation to lowering the introductory price by ϵ captures the entire market, but results in a loss of ϵ , so such a deviation is also not profitable. Hence the claimed strategies are an equilibrium in the competitive case.

Now consider the duopoly case. In a symmetric equilibrium, both firms are charging the lowest price. Hence, for absence of profitable price-raising, we must have $1/2(p_I^i + \Pi_i^{*,RL}) + 1/2\Pi_i^{*,S} = \Pi_i^{*,S}$; solving for the introductory price yields the claimed condition.

Payoff uniqueness: Suppose there is a firm with payoff greater than $\Pi_i^{*,S}$. Clearly, this firm must be making positive sales at price $p_i^I > p_i^{*I}$; for sales to be positive, no firm should charge a price lower than p_i^I . Then either (a) all other firms are charging more than p_i^I . In this case, any of the other firm has a profitable deviation to lowering price $p_i^I - \epsilon$. Or (b), there is another firm charging the same price p_i^I and sharing the market with firm i . In this case, the profitable deviation is for said firm to lower its price by an ϵ . \square

1.5 Strategic Effects of Retention

How do changes in the cost of claiming retention affect market competition? This question is of interest in the case of duopoly, as we already showed that in competitive markets retention does not arise in equilibrium.

First, we establish that the retention model nests the model with “pure” behaviour based price discrimination along the lines of [Chen \(1997\)](#) and [Taylor \(2003\)](#) for the case where parameter $\alpha = 0$:

Proposition 1.5.1. *When $\alpha = 0$, the payoff unique equilibrium of the game is characterised as follows*

1. *In the mature market, consumers with $\theta < \theta^{*S}$ switch and others stay loyal with*

$$0 = \frac{1 - 2F(\theta^{*S})}{f(\theta^{*S})} - \theta^{*S} \quad (1.5.1)$$

2. Mature market prices are given by

$$\frac{1 - F(\theta^{*S})}{f(\theta^{*S})} = p^{*L} = p^{*R} \quad (1.5.2)$$

$$\frac{F(\theta^{*S})}{f(\theta^{*S})} = p^{*S} \quad (1.5.3)$$

3. Consumers evenly split between firms in the first period

4. Introductory prices satisfy

$$p^{*I} = \frac{F(\theta^{*S})}{f(\theta^{*S})} F(\theta^{*S}) - \frac{1 - F(\theta^{*S})}{f(\theta^{*S})} (1 - F(\theta^{*S})) \quad (1.5.4)$$

5. Payoffs of all players coincide with the reduced game where the retention action is deleted

Proof. When $\alpha = 0$, all consumers choose the cheaper of the loyal/retained offers:

$$\theta^R = \begin{cases} 1 & \text{if } p_i^R \geq p_i^L \\ \theta_S & \text{otherwise} \end{cases} \quad (1.5.5)$$

The maximisation problem on the own consumer base then becomes:

$$\max_{p_i^R, p_i^L} \gamma_i (1 - F(\theta^S)) \min\{p_i^R, p_i^L\}$$

(ii) Thus, the payoff only depends on the smaller of the two prices and without loss of generality we can set them equal. Solving the first order-condition then yields the claimed loyal/retention price condition. The pricing for switching consumers is unchanged from lemma 1.4.7.

(i) By subtracting the conditions for optimal switching and loyal price

(iii), (iv) follow directly from proposition 1.4.3

(v) By direct comparison of equilibrium strategies. \square

Thus, since our model is continuous, we can consider comparative statics at $\alpha = 0$ to investigate the effects of introducing retention. Moreover, our comparative statics are monotone; this allows us to evaluate also rather discontinuous policy movements where α “jumps” into the interior.

As α rises, inspection of the pricing equation in proposition 1.4.1 shows us that the demand for switching and retained offers becomes more price elastic, while the demand for

the loyal offer becomes less elastic. Intuitively, as $\alpha \rightarrow 1$, the retention offer becomes a closer substitute for switching and a more distant one for the loyal offer. One may thus think that the switching and retention prices fall. For the loyal price, there are two effects: first, the increasingly inelastic demand calls for a higher price. Second, the falling retention price calls for a lower price. Due to the log-concave type distribution, the latter effect always outweighs the former. Thus, in fact, rising cost of retention leads to *all* mature market prices falling:

Proposition 1.5.2. *In the duopoly case, all mature market prices are decreasing in α . Moreover, $\frac{dp^{*R}}{d\alpha} < \frac{dp^{*S}}{d\alpha} < \frac{dp^{*L}}{d\alpha} < 0$, that is, the retention price falls strongest, followed by the switching price; the effect on the loyal price is weakest, but still unambiguously negative.*

Proof. From proposition 1.4.1, part (1), it is clear that marginal consumer are fixed independent of α . Thus, to find the price effects, we can simply differentiate the implicit functions for equilibrium prices, treating θ_R^* and θ_S^* as constants. This yields

$$\begin{aligned}\frac{dp^{*S}}{d\alpha} &= -\frac{F(\theta_S^*)}{f(\theta_S^*)} \\ \frac{dp^{*R}}{d\alpha} &= -\frac{1 - F(\theta_S^*)}{f(\theta_S^*)} \\ \frac{dp^{*L}}{d\alpha} &= \frac{1 - F(\theta_R^*)}{f(\theta_R^*)} - \frac{1 - F(\theta_S^*)}{f(\theta_S^*)}\end{aligned}$$

From lemma 1.4.8 we have that $0 < F(\theta_S^*) < \frac{1}{2}$. Hence it is immediate that $\frac{dp^{*S}}{d\alpha} < \frac{dp^{*R}}{d\alpha}$; and the price effects are strictly negative. To see that the loyal price is decreasing, note that $\frac{1-F(\theta)}{f(\theta)}$ is a strictly decreasing function, by log-concavity, and combine with $\theta_S^* < \theta_R^*$. \square

Since all prices fall in the mature market, profits must fall as well. In particular, the profits from switching consumers fall. Since the “reservation profit” from switching consumers determines intertemporal profits, these profits are falling:

Proposition 1.5.3. *In the duopoly case, intertemporal profits are strictly decreasing in α .*

Proof. By proposition 1.4.3, intertemporal profits equal switching profits in the mature market. Differentiating with respect to α ,

$$\frac{d\Pi}{d\alpha} = F(\theta^{*S}) \frac{dp^{*S}}{d\alpha} = -\frac{F(\theta^{*S})^2}{f(\theta^{*S})} < 0 \quad (1.5.6)$$

where the second equality is from proposition 1.5.2. \square

The comparative statics of the introductory price pose an interesting challenge. The introductory price is determined by the *difference* between switching and loyal/retained profits in the mature market. Clearly, both expressions are falling, but which effect dominates? It is surprisingly difficult to give a definite answer to this question. To proceed further with the analysis, we introduce the following condition:

Assumption 3 (Well-Behaved Condition). $F(\theta)$ is such that

$$F(\theta^{*R}) \leq \frac{F(\theta^{*S})}{f(\theta^{*S})\theta^{*R}}$$

The “well-behaved condition” effectively places a lower bound on the mass of loyal consumers, required for comparative statics from now on. It is not overly restrictive – for example, the uniform distribution satisfies it; moreover, in numerical simulations, all symmetric Beta distributions satisfy this condition. However, further results remain conjecture at this point.

Under this condition, the first-period price will not rise by more than the loyal price falls:

Proposition 1.5.4. *Under the well-behaved condition, first period price rises in duopoly are bounded by*

$$\frac{dp^{*I}}{d\alpha} \leq -\frac{dp^{*L}}{d\alpha}$$

Proof. The introductory price is given by

$$p_i^{*I} = \Pi_i^{*,S} - \Pi_i^{*,RL} = \int_0^{\theta_S} p_S^* f(x) dx - \int_{\theta_S}^{\theta_R} p_R^* f(x) dx - \int_{\theta_R}^1 p_L^* f(x) dx$$

where the second equality is from substituting equilibrium producer and consumer behaviour from proposition 1.4.1. Differentiating with respect to the parameter,

$$\begin{aligned} \frac{dp_i^{*I}}{d\alpha} &= \int_0^{\theta_S} \frac{dp^{*S}}{d\alpha} f(x) dx - \int_{\theta_S}^{\theta_R} \frac{dp^{*R}}{d\alpha} f(x) dx - \int_{\theta_R}^1 \frac{dp^{*L}}{d\alpha} f(x) dx \\ &= F(\theta_S) \underbrace{\left(\frac{dp^{*S}}{d\alpha} + \frac{dp^{*R}}{d\alpha} \right)}_{-1/f(\theta^{*S})} - F(\theta_R) \underbrace{\left(\frac{dp^{*R}}{d\alpha} - \frac{dp^{*L}}{d\alpha} \right)}_{-(\theta_R)} - \underbrace{\frac{dp^{*L}}{d\alpha}}_{(-)} \\ &= \underbrace{F(\theta_R)\theta_R - \frac{F(\theta_S)}{f(\theta^{*S})}}_{<0} - \frac{dp^{*L}}{d\alpha} \end{aligned}$$

where the first equality applies Leibnitz’ rule, recognising the fact that marginal consumers are fixed. We then solve the integral and finally use the well-behaved condition. \square

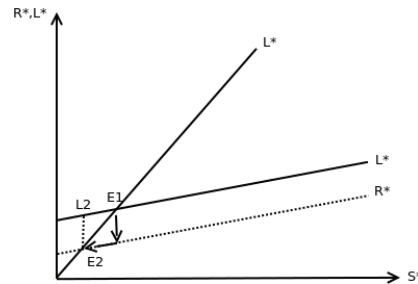


Figure 1.4: Effects of Retention in the Mature Market (Uniform Example)

1.5.1 Uniform Example

For concreteness, consider an example with a uniform distribution on $[0, 1]$. From lemmas 1.4.6 and 1.4.7, we obtain parametric expressions for the best-response functions:

$$p^{*S} = \frac{p^R}{2}$$

$$p^{*R} = \frac{1 - \alpha + p^S}{2}$$

$$p^{*L} = \frac{1 + p^S}{2}$$

Thus, in the uniform case, the best response loyal and switching prices do not depend on the parameter. This facilitates a graphical analysis of the effect of retention pricing. In figure ??, the intersection of lines L^* and S^* , denoted $E1$, is the equilibrium price profile when retention is “banned”, or alternatively the effort to secure retention is zero. Consider now a situation where $\alpha > 0$. Then the immediate effect is a downwards shift in the retention best response function, R^* ; there is no change in the best response loyal and switching functions, a specific feature of the uniform example. The equilibrium with retention is reached at $E2$, and the loyal price is $L2$. Thus, all mature market prices are below the initial level $E1$; this feature, as we have discussed, is more general.

By substitution into proposition 1.5.4, we find that the equilibrium first period price is $p^{*I} = -\frac{(4-\alpha)}{12}$, which is increasing in the parameter. Thus, the effect of more fierce mature market competition is counteracted by weaker first-period competition.

1.6 Welfare

Retention, in an imperfectly competitive setting, certainly harms firm profitability, as we have already discussed; yet the overall welfare effects remain to be addressed. First, observe

that due to the model having inelastic demand, we do not allow for the possibility that price falls can raise welfare by increasing consumption. As [Stole \(2007\)](#) emphasises, one must thus take care to interpret welfare results, bearing in mind that the model underestimates possible gains when there are gains to be had by increasing market coverage. In our model, social welfare must fall when retention is allowed: consumers spend some effort to secure the retention offer, and this effort is dead weight loss. But under the well-behaved condition, consumer surplus rises. In fact, the rise is sufficiently strong that *each* consumer type is better off.

Consumer surplus is defined as the expected utility of a consumer given equilibrium strategies:

$$CS(\alpha) = \beta - p^{*I}(\alpha) + \beta - \int_0^{\theta_S^*} (p_S^*(\alpha) + x)f(x)dx \quad (1.6.1)$$

$$\begin{aligned} & - \int_{\theta_S^*}^{\theta_R^*} (p_R^*(\alpha) + \alpha x)f(x)dx - \int_{\theta_R^*}^1 p_L^*(\alpha)f(x)dx \\ & = 2\beta - \Pi^{*S}(\alpha) - \Phi(\alpha) \end{aligned} \quad (1.6.2)$$

where $\Pi^{*S}(\alpha)$ is producer surplus, as discussed already, and $\Phi(\alpha)$ is the dead weight loss of switching effort:

$$\Phi(\alpha) = \int_0^{\theta_S^*} xf(x)dx + \int_{\theta_S^*}^{\theta_R^*} \alpha xf(x)dx$$

Hence, indeed, social welfare is affected by retention only through switching effort, and falls as the parameter rises:

$$W(\alpha) = CS(\alpha) + \Pi^{*S}(\alpha) = 2\beta - \Phi(\alpha)$$

Proposition 1.6.1. *Under the well-behaved condition, consumer surplus is strictly increasing in α ; moreover, as α rises, each type is weakly better off*

Proof. .

1. The derivative of consumer surplus is

$$\begin{aligned} \frac{dCS}{d\alpha} &= -\frac{dp^{*I}}{d\alpha} - F(\theta^{*S})\frac{dp^{*S}}{d\alpha} - [F(\theta^{*R}) - F(\theta^{*S})]\frac{dp^{*S}}{d\alpha} - (1 - F(\theta^{*R}))\frac{dp^{*L}}{d\alpha} \\ &\geq \frac{p^{*L}}{d\alpha} - \frac{p^{*L}}{d\alpha} = 0 \end{aligned}$$

where the second equality follows from proposition 1.5.2 and the third from proposition 1.5.4.

2. First, for switching consumers, we have

$$\frac{du^*}{d\alpha}|_S = -\frac{dp^{*I}}{d\alpha} - \frac{dp^{*S}}{d\alpha} > \frac{dp^{*L}}{d\alpha} - \frac{p^{*S}}{d\alpha} > 0$$

for loyal consumers, the result is immediate. It remains to check for retained consumers:

$$\begin{aligned} \frac{du^*}{d\alpha}|_R &= -\frac{dp^{*I}}{d\alpha} - \frac{dp^{*R}}{d\alpha} - \theta \\ &\geq \left(\frac{1 - F(\theta_R^*)}{f(\theta_R^*)} - \frac{1 - F(\theta_S^*)}{f(\theta_S^*)} \right) + \frac{1 - F(\theta_S^*)}{f(\theta_S^*)} - \theta \\ &\geq 0 \end{aligned}$$

where the last inequality follows because the highest-cost retained consumer is at θ_R^* . □

The consumer gains from retention are not evenly distributed. Indeed, switching consumers benefit the most from retention; retained consumers enjoy the largest price fall, but also bear the increased cost of effort as α rises. Finally, loyal consumers enjoy a mild intertemporal price fall, all under the well-behaved condition.

In conclusion, the model provides little support on consumer welfare grounds for banning retention activity.

1.7 Discussion

1.7.1 Price Matching

In price discrimination, firms choose prices as a best reply to opponent's prices. On the other hand, in models of price matching guarantees (such as Salop (1986)), firms can commit ex ante to (part of) their pricing strategy. This is with the intent to change the pricing incentives of the opponent, and has been shown to be anti-competitive in some static models: because a lower price is inevitably matched, a firm cannot gain market share by reducing its price. Thus price matching may sustain collusion². The British regulator suggested in Ofcom (2010) that a similar intuition may apply to retention offers. In this section, we briefly explore the implications of price matching in the context of our model.

²Although there remains a theoretical and empirical debate (Arbatskaya et al., 2004) on this

Suppose before the start of the second period, each firm has the option to commit to setting the retention price equal to the switching price. In subgames where the firm is committed, the price matching logic applies: for any switching price, the competitor will not make positive sales; so any switching price can be supported in equilibrium. The loyal price is a best response to whatever switching price is chosen in equilibrium. In subgames where the firm chooses not to commit, the mature market as analysed before unfolds.

In such an extended model, we can have equilibria where retention is price matching. However, these equilibria are *more* desirable from both a consumer and social welfare perspective. This arises because, under price matching, the continuation profit Π^{*S} is zero; hence first competition is sufficiently fierce so that consumers can secure the entire surplus.

Thus, allowing for price matching even strengthens our results regarding the pro-competitive effects of retention, but for the opposite reason (fierce first-period competition) as opposed to the price discrimination model.

Example 1.7.1 (Equilibrium with Price Matching). *In the subgame where the firm chooses price matching, the profit on switching consumers, $\theta_R = 0$ for all p , precisely due to the guarantee. Hence, any switching price is a best response. For existing consumers, the problem becomes*

$$\begin{aligned} \max [1 - F(\theta_R)]p_L^i + [F(\theta_R) - F(\theta_S)]p_R^i \\ = [1 - F(\theta_R)]p_L^i + [F(\theta_R) - 0]p_S^i \end{aligned}$$

which yields the same best response expression $p_{L^*}^i(p_S^i) = \alpha \frac{1-F(\theta_R)}{f(\theta_R)} + p_S^i$ using the steps as in lemma 1.4.6.

Hence we may have an equilibrium where, for example, $p_{L^*}^i = \beta, p_{S^*}^i = p_{L^*}^i{}^{-1}(\beta)$ under price matching. This assures the monopoly profit to the seller, which is better than archived under price discrimination. Hence, the firm will choose price matching. But then the first-period price equals $-\Pi^{*2,RL}$, since switching profits are zero. So intertemporal prices are zero, consumers extract all the surplus, and full efficiency is achieved since there is no switching. Thus, allowing for price matching does not weaken our results about retention being pro-competitive.

On the other hand, there may be equilibria where in the price matching continuation, switching prices are low – say zero. Then, the firm would not choose to commit, and the equilibrium outcomes correspond to the model without price matching.

1.7.2 Myopic Consumers

Throughout the model, we required consumers to have a high degree of rationality; in particular, consumers are well-informed and correctly anticipate that retention offers will be made. In the first period, consumers anticipate that the “bargain” prices in the first period are followed by “rip-offs” in the mature market, to use the terminology of the Farrell survey. Hence it is important to ask how the conclusions would be affected if consumers fell short of this rather strict benchmark.

We now introduce a rather strong form of myopia into the model. Let us assume that consumers in the mature market do not anticipate that retention offers will be made; thus, a consumer first decides whether to switch, based on considerations of loyal and switching prices. The marginal consumer is thus between switching and loyal, and has type $\theta_{SL} = p_L^i - p_S^i$. Consumers with lower type then switch and are “surprised” to receive a retention offer. The offer will be accepted if $\beta - p_R^i - \alpha\theta > \beta - \underline{p}_S^i - \theta$; hence there is a marginal consumer $\theta_{SR} = \frac{p_R^i - \underline{p}_S^i}{1 - \alpha}$. In the introductory period, consumers base their purchase decisions only on the prices offered that period and do not take into account the continuation utility of being locked-in in the second period.

This leads to a revised payoff function for the firm’s own consumer base:

$$\begin{aligned} \max_{p_i^R, p_i^L} \quad & \gamma_i \{ [1 - F(\theta_{SL})] p_L^i + F(\theta_{SL}) F(\theta_{SR} | \theta < \theta_{SL}) p_R^i \} \\ \text{s.t.} \quad & \beta - p_i^L \geq \beta - \underline{p}_S^i - \theta && \text{for } \theta \in [\theta_R, 1] && \text{(PC)} \\ & \beta - p_i^R - \alpha\theta \geq \beta - \underline{p}_S^i - \theta && \text{for } \theta \in [\theta_S, \theta_R] && \text{(PC)} \end{aligned}$$

The associated best response functions are

$$\begin{aligned} p_i^{*L} &= \frac{1 - F(\theta_{SL})}{f(\theta_{SL})} \\ p_i^{*R} &= (1 - \alpha) \frac{1 - F(\theta_{SR})}{f(\theta_{SR})} \end{aligned}$$

By comparison with lemma 1.4.6, it is apparent that there is an upwards shift in the loyal best response function and no change in the retention best response function. However, for

switching consumers, we have

$$\begin{aligned} \max_{p_i^S} & F(\theta_{SL})F(\theta_{SR}|\theta < \theta_{SL})p_i^S \\ \text{s.t.} & \beta - p_i^L \geq \beta - \underline{p}_i^S - \theta \quad \text{for } \theta \in [\theta_R, 1] \quad (\text{PC}) \\ & \beta - p_i^R - \alpha\theta \geq \beta - \underline{p}_i^S - \theta \quad \text{for } \theta \in [\theta_S, \theta_R] \quad (\text{PC}) \end{aligned}$$

which again, yields an unchanged best-response function vis-a-vis the rational case:

$$p_i^{*S} = \alpha \frac{F(\theta_{SR})}{f(\theta_{SR})}$$

In the first period, myopia obviously does not play a role since the continuation utilities of buying are equal between firms (see discussion in [Farrell and Klemperer \(2007\)](#)), so the introductory price is still determined by the difference between switching and loyal/retention profit. Hence we can conclude that myopia has only mild impacts on our results: The mature market switching and retention prices are unaffected; the loyal price is generally at a higher level than under forward-looking consumers, but still decreasing in α (due to strategic complementarity with the switching price, which is decreasing).

1.7.3 The Role of Consumer Heterogeneity

The core model of the paper focuses on the case where effort costs follow some distribution on the unit interval. While it is clear that heterogeneity is crucial – if the distribution were degenerate at a point, the model would simply have switching prices at marginal cost, loyal prices and the switching cost and introductory prices set to set intertemporal profits to zero, with no basis for retention pricing – it remains interesting to ask which *form* of heterogeneity really affects prices.

To investigate this question, we follow [Bouckaert et al. \(2010\)](#) and consider a continuous uniform distribution on $[\underline{e}, \bar{e}]$, where $0 < 2\underline{e} < \bar{e}$. The latter inequality assures that the distribution has sufficient dispersion for an equilibrium with price discrimination to exist.

First, substitute the expressions of uniform density into intertemporal profits of the duopoly case:

$$\Pi^{*S} = (1 - \alpha) \frac{(\bar{e} - 2\underline{e})^2}{9} \quad (1.7.1)$$

For the case of $\alpha = 0$, the expression coincides with [Bouckaert et al. \(2010\)](#). The fundamental insight here is that a *mean-preserving spread*, where \bar{e} is increased by the same amount as \underline{e}

decreases raises profits; the intuition being that more heterogeneity restrains the incentives of either firm to cut prices. Less intuitively, a raise in the mean effort cost, but keeping dispersion constant, *reduces* profits: at the margin, it comes more attractive to fight for switching consumers, and thus there is price pressure.

As retention is introduced – so α is non-zero – the fall in industry profits is larger the greater the heterogeneity in effort cost. Moreover, it is easily seen that the higher firm profits are in the absence of retention, the greater would be the associated profit falls when retention is introduced. Finally, directly relevant for our purposes is the interaction of consumer heterogeneity and the impact of retention. The relevant expression being:

$$\frac{dp_i^R}{d\alpha} = -\frac{2b - a}{3}$$

Hence, for given α , more consumer heterogeneity leads to a steeper drop in prices; however, the functional form of the heterogeneity is different from the case of profits analysed above.

1.8 Conclusion

This paper studied the impact of retention pricing on market competition, using an intertemporal model of price-discrimination. To do so, we combined existing models of behaviour-based price discrimination (BBPD) with second-degree price discrimination. When setting the retention price, firms thus face two considerations: at the lower margin, some consumers switch to the competition, resulting in a revenue loss (participation constraint); at the upper margin, some consumers that previously purchased the retention offer switch to become loyal consumers, paying more (incentive-compatibility constraint).

The model showed that retention pricing is a phenomenon of imperfectly competitive markets: in a competitive setting, firms do not gain from introducing such offers since, around the equilibrium price, the gains from “saving” consumers exactly balance the loss in revenue from loyal consumers that opportunistically claim the retention offer (but, in the counter-factual, would not have switched).

However, in an imperfectly setting, retention prices are an equilibrium phenomenon. We showed that, although the mass of consumers claiming retention is relatively small, once retention prices are introduced, all prices in the mature market fall. This occurs because once a firm introduces retention prices, the competitor responds through a lower switching price; the resulting price fall puts pressure on the loyal price as well, and – given our assumption of a log-concave distribution – is sufficient to cause all mature market prices to fall.

The model predicts a fall in firm profitability when retention is introduced. Moreover, retention reduces social welfare due to the increased dead-weight loss from claiming retention. However, as we discussed, the social welfare results have limitations. Finally, under an added regularity condition, it can be shown that retention strictly improves consumer welfare: each consumer is better off when retention is introduced, because the mature market gains are sufficiently large relative to the adverse effect from rising introductory prices.

HERE

Bibliography

- Arbatskaya, M., Hviid, M., Shaffer, G., 2004. On the incidence and variety of low-price guarantees. *Journal of Law and Economics* 47, 307–589.
- Armstrong, M., Vickers, J., 2001. Competitive price discrimination. *RAND Journal of Economics*, 579–605.
- Bergstrom, T., Bagnoli, M., 2005. Log-concave probability and its applications. *Economic theory* 26, 445–469.
- Bouckaert, J., Degryse, H., Provoost, T., 2010. Enhancing market power by reducing switching costs. *Economics Letters* 109 (2), 131–133.
- Chen, Y., 1997. Paying customers to switch. *Journal of Economics & Management Strategy* 6 (4), 877–897.
- Chen, Y., Zhang, Z., 2009. Dynamic targeted pricing with strategic consumers. *International Journal of Industrial Organization* 27 (1), 43–50.
- Corts, K., 1998. Third-degree price discrimination in oligopoly: all-out competition and strategic commitment. *The RAND Journal of Economics*, 306–323.
- Esteves, R., 2009. Customer poaching and advertising. *The Journal of Industrial Economics* 57 (1), 112–146.
- Esteves, R.-B., 2012. Customer poaching with retention strategies. mimeo, Universidade do Minho, December 2012.
- Farrell, J., Klemperer, P., 2007. Coordination and lock-in: Competition with switching costs and network effects. *Handbook of industrial organization* 3, 1967–2072.
- Fudenberg, D., Villas-Boas, J., Hendershott, T., 2006. Behavior-based price discrimination and customer recognition. *Handbook on economics and information systems* 1, 377–436.

- Gehrig, T., Shy, O., Stenbacka, R., 2011. History-based price discrimination and entry in markets with switching costs: a welfare analysis. *European Economic Review* 55 (5), 732–739.
- Maskin, E., Riley, J., 1984. Monopoly with incomplete information. *The RAND Journal of Economics* 15 (2), 171–196.
- McAfee, R., 2008. Price discrimination. prepared for ABA's Issues in Competition and Policy.
- Ofcom, 2010. Strategic Review of Consumer Switching. Ofcom.
- Rosenberg, L., Czepiel, J., 1984. A marketing approach for customer retention. *Journal of Consumer Marketing* 1 (2), 45–51.
- Salop, S., 1986. 9 practices that (credibly) facilitate oligopoly co-ordination. *New developments in the analysis of market structure*, 265.
- Spulber, D., 1989. Product variety and competitive discounts. *Journal of Economic Theory* 48 (2), 510–525.
- Stole, L., 2007. Price discrimination and competition. *Handbook of industrial organization* 3, 2221–2299.
- Taylor, C., 2003. Supplier surfing: Competition and consumer behavior in subscription markets. *The RAND Journal of Economics* 34 (2), 223–246.

Chapter 2

History–Based Price Discrimination and Welfare in a Growing Market

Abstract. An incumbent faces an entrant in a market for a horizontally differentiated product. The incumbent’s advantage is twofold: the existing consumer base is protected by switching cost, and its purchase history is observed by the incumbent; third–degree price discrimination on purchase status is thus feasible for the incumbent. The entrant may induce consumers to reveal their purchase history through suitable offers. Under duopoly, price discrimination leads to lower prices and profits, but also lowers consumers surplus due to brand misallocation and inefficient switching. Thus, banning price discrimination increases welfare. However, regulation may also cause entry into an otherwise monopolised industry; this occurs only when entry is inefficient; thus optimal policy calls for bans on price discrimination unless they cause entry. ■

2.1 Introduction

History–based price discrimination – charging consumers differently depending on what their purchase history – is a growing business practice. When an established incumbent uses such strategies to compete against a small entrant, the issue becomes contentious. Does price discrimination exclude an efficient potential entrant? How strong is the incumbent’s strategic advantage from having existing consumer records, while the entrant starts without such data? Does price discrimination sharpen or blunt the forces of price competition? Do consumers benefit, and what is the impact on social welfare?

These concerns have surfaced in a number of well–known antitrust cases ([Geradin and](#)

Petit (2005), Motta (2004, ch. 7.4)). In the textbook AKZO case, the firm offered discounts to consumers of a smaller competitor; the European Commission found this practice abusive due to its potential for excluding the competitor from the market. The Swedish Competition Authority (unsuccessfully) sued TeliaSonera for offering discounts to consumers that switched to entrant Bredbandsbolaget while keeping prices high for others, alleging intent to exclude.

History-based price discrimination (HBPD) is often studied in the context of poaching offers (Fudenberg et al., 2006), discounts to consumers who previously purchased from a competitor. But an incumbent with a dominant position has few or no potential switchers. In such settings, the incumbent's HBPD may involve a high price for existing consumers, but the (possibly) discounted price would be targeted at new consumers joining the market, a theme developed in Gehrig et al. (2011). Reflecting this, in my model there is a share of "old" consumers that have previously purchased from the incumbent – so the incumbent inherits a 100% market share in this segment – and face a switching cost when moving to the entrant. "New" consumers join the market and do not face any switching costs. In both segments, consumers' brand preferences vis-a-vis entrant or incumbent are distributed on the Hotelling line. The incumbent can observe the identities of old consumers from a corporate database, and thus charge different prices to old and new consumers.

The entrant may have an incentive to offer poaching discounts to the incumbent's old consumers. While not directly observing a consumer's purchase history, it is conceivable that she may offer discounts that induce old consumers to reveal their status, i.e. second-degree price discrimination. Allowing for this possibility is a central departure from the model of Gehrig et al. (2011), who a priori impose uniform pricing on the entrant. As the analysis will show, the entrant generally has an incentive charge old consumers a lower price than new consumers; hence the consumers' incentive compatibility constraint is not binding and such discrimination is feasible. Moreover, the entrant's ability to discriminate has important consequences for equilibrium entry and welfare results.

Since competition law generally allows restraints only on large firms – "dominant undertakings" in the terminology of Article 102 of the *Treaty on the Functioning of the European Union* – possible interventions regarding price discrimination can be targeted only at the incumbent; the entrant will remain unconstrained in her ability to discriminate¹. This is captured in the model through a constraint, set by a competition authority, on the maximum price difference between old and new segments that the incumbent is allowed to impose. The case of uniform pricing, typically considered in the literature, is a special case of this. The

¹See (Geradin and Petit, 2005, section 2.2.) for a detailed discussion of the scope of article 102(c), which applies to price discrimination

policy is determined at the onset of the game; then the entrant decides whether to join the market and the incumbent whether to remain active or exit. Finally, duopoly or incumbent monopoly is played out.

In this setting, price discrimination has important welfare consequences. Given that both firms have entered, tighter regulation forces the incumbent to trade off capturing rents on old consumers with the goal of attracting new consumers. In equilibrium, this leads to higher prices and greater entrant's market penetration among new consumers, with opposite results for old consumers². Hence old consumers benefit from regulation while new ones are hurt; in the aggregate, consumer surplus *rises*. From a social welfare perspective, the reduction in switching rates brings a welfare gain that outweighs the costs of distorting brand choices among new consumers. When regulation is marginal to entry, one must trade off the reduction in transportation cost under the best-regulated duopoly regime against the cost of the entrant's fixed cost and switching cost. But for regulation to affect entry, fixed costs of the entrant must be high – sufficiently high that entry is inefficient. Thus, optimal policy calls for *bans on incumbent price discrimination, unless they cause entry*. Thus, the potential of price discrimination to consolidate an industry – discussed e.g. in [Spector \(2005\)](#) – should not be unambiguously seen as negative.

In [Gehrig et al. \(2011\)](#), entry considerations do not arise because the entrant's profit is invariant to the incumbent's regulation. This relies on the assumption of uniform pricing. But my model shows that the entrant's incentive to price discriminate *grows* when the incumbent is regulated. But this opens the possibility of exclusionary price discrimination, which should not be neglected. [Chen \(2008\)](#) argues that history-based price discrimination increases consumer surplus as long as it does not cause exit. His model is based on a homogenous product where lower prices unambiguously increase welfare, and because the consumer surplus standard disregards the social costs of excess entry³. In contrast, in the present paper, history based price discrimination reduces consumer surplus under duopoly: the welfare effects differ between old and new segment, and the losses of the latter – due to more effective exploitation of rent under discrimination – more than outweigh the gains of new consumers from fierce competition in that segment. This emphasises the importance of paying close attention to the specifics of the market structure and welfare standard when

²This is contrast to models of best-response asymmetry [Corts \(1998\)](#), in which one firm's strong market is another firm's weak market, and vice versa. In these settings, price discrimination may lead to “all out competition”, i.e. lower prices overall ([Bester and Petrakis \(1996\)](#); [Thisse and Vives \(1988\)](#); [Chen \(1997\)](#) and indeed the retention paper in chapter 1 are examples of such models).

³See [Farrell and Katz \(2006\)](#) for an interesting discussion of welfare standards applied to anti-trust, arguing that both standards have limitations and anti-trust is not strictly welfarist

assessing price discrimination. Indeed, [Gehrig et al. \(2012\)](#) develop a model with a single population of consumers on the Hotelling line, where the initial asymmetry market shares is interior. This re-introduces elements of best response asymmetry, and consumers benefit from price discrimination unless switching cost is high or initial dominance is weak.

[Bouckaert et al. \(2013\)](#) consider regulation price discrimination by dominant firms. As [Armstrong and Vickers \(1993\)](#), they consider an incumbent with an exogenously sheltered and a potentially competitive segment subject to potential entry. In a two-period model, they consider constraints that ban price increases for sheltered consumers or rival's consumers respectively. Their results depend strongly on the form of the constraint, and they argue that bans price cuts for rival consumers maximise consumer surplus.

Price discrimination and entry are especially salient issues in network industries [Haucap \(2003\)](#). In [Karlinger and Motta \(2012\)](#), a more efficient entrant considers joining the market. Each firm's technology has constant returns to scale, but the size of the network exerts a positive externality on the consumers' valuation of the product by particular firm. In particular, the firm's network must have a certain minimum size for consumers to have a positive valuation for the product. There are no switching costs in their model, so entry is always efficient. In this setting, they show that price discrimination by the incumbent reduces the range of efficient entry equilibria, due to a miscoordination effect as in [Segal and Whinston \(2000\)](#). Like the present paper, they assume an inherited asymmetry in market share, and entry is less likely under price discrimination. However, the welfare results differ since their model precludes the possibility of inefficient entry.

The paper proceeds by setting up the model in section 2.2. Section 2.3 then solves for the equilibrium under the different possible market structures and regulation regimes. Then, section 2.4 conducts the welfare analysis and finally section 2.5 concludes.

2.2 Model

Set-Up. The framework is a two-stage model of entry and price competition. In the first stage, both the incumbent firm A and potential entrant firm B decide whether to be active (A) in the market or exit (E); the resulting market structure may thus involve a duopoly (both firms enter), incumbent monopoly (no entry) or entrant monopoly (incumbent exit). In the second stage, new consumers join the market; in addition, old consumers – who previously purchased from the incumbent – remain in the market. The latter incur a switching cost if they move to the entrant; in both segments, consumers vary in their brand preference for either A or B 's product. Hotelling price competition then unfolds, given the market



Figure 2.1: Timeline

structure in place. The timing of the model is summarised in figure 2.1.

Consumers. At the onset of the pricing stage, there are two kinds of consumers. First, a mass θ of “new” consumers joins the market. These consumers vary in their brand preference x for either firm, which is assumed to be distributed uniformly on the unit interval, $x \sim U[0, 1]$. Facing prices p_{NEW}^A and p_{NEW}^B by incumbent and entrant respectively, their utility function is given by

$$u_{NEW}(x; p_{NEW}^A, p_{NEW}^B) = \begin{cases} \beta - p_{NEW}^A - tx & \text{if buying from A} \\ \beta - p_{NEW}^B - t(1-x) & \text{if buying from B} \\ 0 & \text{otherwise} \end{cases} \quad (2.2.1)$$

Second, the incumbent A has an inherited consumer base (“old consumers”); these consumers purchased from the incumbent in an earlier, unmodelled period and amount to a mass $1 - \theta$. Like new consumers, these consumers have uniformly distributed brand preference; however, in case an old consumers switches to B , she incurs a (real) switching cost s . Old consumers may choose between the incumbent’s product, at price p_{OLD}^A , and the entrant’s offer intended for old consumers, p_{OLD}^B . This price is available to old consumers that voluntarily choose to reveal that they previously purchased from the incumbent. Finally, old consumers can masquerade as new consumers and obtain the new consumer offer from B , priced at p_{NEW}^B . This gives the utility function

$$u_{OLD}(\bullet) = \begin{cases} \beta - p_{OLD}^A - tx & \text{if staying with A} \\ \beta - p_{OLD}^B - t(1-x) - s & \text{switch to B, reveal status} \\ \beta - p_{NEW}^B - t(1-x) - s & \text{switch to B, masquerade} \\ 0 & \text{otherwise} \end{cases} \quad (2.2.2)$$

Denote by \tilde{x}_{NEW} and \tilde{x}_{OLD} respectively the marginal consumer type indifferent between the two firms.

To give focus for the welfare analysis, we now impose the assumption of full market coverage, so that all consumers purchase from one of the firms:

Assumption 4 (Full Market Coverage). *The utility of the good, β , is sufficiently high to ensure full market coverage under all market structures*

This concentrates our attention on the issues of distortion of consumers between firms, and the trade-off between product variety and duplication of fixed costs.

Firms. Firm $i \in \{A, B\}$ incurs a constant marginal cost c_i for each unit of the good produced. In addition, a fixed cost F_i is sunk when the firms decides to enter in the first stage. We allow the fixed cost to vary between firms, but impose the assumption of common marginal costs⁴:

Assumption 5. *The two firms have equal marginal costs, i.e. $c = c_A = c_B$*

In the second period, there are two asymmetries between entrant and incumbent. First, the incumbent is protected by switching cost among old consumers (old consumers pay a utility cost when moving to the entrant; see below for details). Second, the incumbent can recognise old consumers (perhaps because there is a contract in place) from new ones. This allows the incumbent to offer different prices p_{OLD}^A and p_{NEW}^B for the two consumer segments, engaging in price discrimination on observables. The entrant cannot directly distinguish consumers, but may come to know this information if the consumers chooses to reveal it (e.g. by showing a receipt from the incumbent). Thus the entrant engages in second-degree price discrimination, and offers p_{OLD}^B and p_{NEW}^A respectively. Assuming $p_{OLD}^B < p_{NEW}^A$ – so the entrant’s price discrimination is incentive compatible – the profit functions of the two firms, after fixed costs have been sunk, can be written as

$$\begin{aligned} \pi^A(\vec{p}) &= (1 - \theta)\tilde{x}_{OLD}(p_{OLD}^A, p_{OLD}^B)(p_{OLD}^A - c) \\ &\quad + \theta\tilde{x}_{NEW}(p_{NEW}^A, p_{NEW}^B)(p_{NEW}^A - c) \end{aligned} \tag{2.2.3}$$

$$\begin{aligned} \pi^B(\vec{p}) &= (1 - \theta)(1 - \tilde{x}_{OLD}(p_{OLD}^A, p_{OLD}^B))(p_{OLD}^B - c) \\ &\quad + \theta(1 - \tilde{x}_{NEW}(p_{NEW}^A, p_{NEW}^B))(p_{NEW}^B - c) \end{aligned} \tag{2.2.4}$$

where for notational simplicity, $\vec{p} = (p_{OLD}^A, p_{NEW}^A, p_{OLD}^B, p_{NEW}^B)$, the vector of second-period prices.

⁴This assumption is for convenience and without loss of generality

Limits to Price Discrimination. Behaviour based price–discrimination means charging different prices according to purchase status. In line with common competition law practice, we consider the possibility that the degree of price discrimination by the incumbent may be constrained but do not allow for constraints to the entrant’s pricing strategy. In particular, a competition authority may limit the incumbent to a maximum price difference q between the two market segments:

$$|p_{OLD}^A - p_{NEW}^A| \leq q \quad (2.2.5)$$

Thus $q = 0$ would correspond to a complete ban of price discrimination, while intermediate values impose some restraint on the pricing strategy of the incumbent without banning BBPD entirely.

For the purpose of the present analysis, we take q to be a parameter and simulate the competitive and welfare consequences of possible restrictions on price discrimination; hence the competition authority is not a strategic player in the model.

Solution Concept. Due to the sequential nature of the game, our solution concept is subgame perfect Nash equilibrium (SPNE). Equilibrium proceeds by analysing market outcomes in each of the possible types of subgames (competition, incumbent monopoly, entrant monopoly), before turning to the market entry stage.

2.3 Equilibrium Analysis

This section develops the subgame perfect equilibria of the model. First, I analyse price competition under duopoly with and without limits to price discrimination and provide relevant comparative statics results. Then I analyse monopoly pricing when only the incumbent or entrant is active in the market. Finally, subsection 2.3.4 turns to the first stage in which firm entry/exit is determined.

2.3.1 Unregulated Duopoly

Consumer Behaviour: Given that both firms are active in the market, old consumers have a choice between p_{OLD}^A, p_{OLD}^B and p_{NEW}^B if they choose not to reveal their purchase status to the entrant. Old consumers switching to the entrant reveal their purchase status if $p_{NEW}^B \geq p_{OLD}^B$ ⁵; otherwise, old consumers switching to B masquerade as new and pay p_{NEW}^B . Thus

⁵Precisely, the assumption is that old consumers can prove their status – e.g. by showing a receipt issued by firm A in the first period – while new consumers cannot produce a falsified receipt

the marginal consumers are given by:

$$\tilde{x}_{OLD} = \begin{cases} \frac{p_{OLD}^B - p_{OLD}^A + t + s}{2t} & \text{if } p_{OLD}^B \leq p_{NEW}^B \\ \frac{p_{OLD}^B - p_{NEW}^A + t + s}{2t} & \text{otherwise} \end{cases} \quad (2.3.1)$$

$$\tilde{x}_{NEW} = \frac{p_{NEW}^B - p_{NEW}^A + t}{2t} \quad (2.3.2)$$

When switching costs are very high, the entrant may not be able effectively compete for the old segment; in this case, old consumers would be a “sheltered segment”. As will become clear shortly, the segment of old consumers has positive switching to the entrant only if condition C1 holds:

$$s \leq 3t \quad (C1)$$

We shall assume that condition C1 is satisfied. Hence, given prices (p_{OLD}^A, p_{NEW}^A) and (p_{OLD}^B, p_{NEW}^B) , equation 2.3.1 gives the mass \tilde{x}_{OLD} of consumers purchasing from firm A in the old consumer segment. By the full coverage assumption, the mass of consumers purchasing from B is simply $1 - \tilde{x}_{OLD}$; analogous expressions hold for the case of new consumers.

Price Setting: Consider first the incumbent firm A . Solving the first-order condition of equation 2.2.3 with respect to p_{OLD}^A and p_{NEW}^A respectively yields

$$p_{OLD}^{*A}(\vec{p}^B) = \begin{cases} \frac{p_{OLD}^B + t + s + c}{2} & \text{if } p_{OLD}^B \leq p_{NEW}^B \\ \frac{p_{NEW}^B + t + s + c}{2} & \text{otherwise} \end{cases} \quad (2.3.3)$$

$$p_{NEW}^{*A}(\vec{p}^B) = \frac{p_{NEW}^B + t + c}{2} \quad (2.3.4)$$

Due to the concavity the model, it can be verified directly that the second-order conditions hold. Second, given assumption C1, the equilibrium is guaranteed to be interior.

Now turn to the entrant B . The problem is 2.2.4, subject to the incentive-compatibility constraint of consumers $p_{OLD}^B \leq p_{NEW}^B$. Writing in Kuhn-Tucker form yields

$$\begin{aligned} \mathcal{L} = & (1 - \theta)(1 - \tilde{x}_{OLD})(p_{OLD}^A, p_{OLD}^B) (p_{OLD}^B - c) \\ & + \theta(1 - \tilde{x}_{NEW})(p_{NEW}^A, p_{NEW}^B) (p_{NEW}^B - c) - \lambda(p_{OLD}^B - p_{NEW}^B) \end{aligned} \quad (2.3.5)$$

Consider first the case where the incentive constraint does not bind ($\lambda = 0$). Then, the solution of the problem is given by

$$p_{OLD}^{*B}(\vec{p}^A) = \frac{p_{OLD}^A + t - s + c}{2} \quad (2.3.6)$$

$$p_{NEW}^{*B}(\vec{p}^A) = \frac{p_{NEW}^A + t + c}{2} \quad (2.3.7)$$

which is feasible as long as $p_{OLD}^A \leq p_{NEW}^A + s$.

Now consider the case where the constraint binds ($\lambda \neq 0$). Then, we must have as solution

$$p_{OLD}^{*B}(p^A) = p_{NEW}^{*B}(p^A) = \frac{(1 - \theta)p_{OLD}^A + \theta p_{NEW}^A - t + s - c}{2} \quad (2.3.8)$$

Value of Consumer Information: The first important observation concerns the extent to which entrant is disadvantaged by the lack of her ability to observe consumers' purchase status. As the best response analysis showed, for some price profiles by the incumbent, this constraint is binding and may adversely impact profits. This occurs when the incumbent's price offered to old consumers exceeds the price offered to new consumers by more than s . Yet, the following proposition shows that the incumbent will not set such prices in equilibrium:

Proposition 2.3.1. *The incentive compatibility constraint of the entrant is not binding on pricing strategies in any equilibrium*

Proof. Consider an equilibrium strategy profile in which the incentive constraint on B is binding. This implies $p_{OLD}^B = p_{NEW}^B$. Second, equilibrium requires that (p_{OLD}^A, p_{NEW}^A) are in the response, i.e. satisfying equations 2.3.3 and 2.3.4. Combining these two yields $p_{OLD}^A = p_{NEW}^A + \frac{s}{2}$, which must hold in the equilibrium.

However, in this case the entrant B has an unconstrained best response $p_{OLD}^B < p_{NEW}^B$, which is also feasible (since $p_{OLD}^A \leq p_{NEW}^A + s$). Hence, the strategy profile with binding incentive compatibility constraint cannot constitute an equilibrium. \square

Thus, the entrant is able to extract the information on purchase status from consumers through second-degree price discrimination; the equilibrium strategy and profits thus coincide with an alternative model where the purchase status is publicly observable. In contrast, Gehrig et al. (2011) do not allow the entrant to use second-degree price discrimination, directly imposing uniform pricing as a constraint on the entrant's strategy.

Any equilibrium thus involves price discrimination by *both* firms. It remains to derive the explicitly the market outcomes:

Proposition 2.3.2. *Under duopoly and no constraint on price discrimination is imposed, the unique equilibrium has*

1. *Old consumers with $\hat{x}_{OLD} \leq \frac{1}{2} + \frac{s}{6t}$ and new consumers with $\hat{x}_{NEW} \leq \frac{1}{2}$ purchase from firm A; otherwise, they purchase from B*

2. Firm A sets prices $p_{OLD}^A = t + c + \frac{s}{3}$ and $p_{NEW}^A = t + c$; Firm B charges $p_{OLD}^B = t + c - \frac{s}{3}$ and $p_{NEW}^B = t + c$

3. Equilibrium profits are given by

$$\begin{aligned}\pi_{DUO}^{*A} &= \frac{s^2(1-\theta)}{18t} + \frac{s(1-\theta)}{3} + \frac{t}{2} \\ \pi_{DUO}^{*B} &= \frac{s^2(1-\theta)}{18t} - \frac{s(1-\theta)}{3} + \frac{t}{2}\end{aligned}$$

Proof. By proposition 2.3.1, the equilibrium must involve price discrimination by the entrant. Taking the appropriate sections of the best response functions equation (2.3.3-2.3.7) yields a 4x4 linear system with full rank. Solving this system yields as the unique solution the prices claimed in (ii). Substituting these prices into equations (2.3.1-2.3.2) yields equilibrium marginal consumers found in (ii). Finally, part (iii) contains the profit functions (2.2.3-2.2.4) evaluated at equilibrium.

To assure that assumption 4 is satisfied, the utility of each consumer has to exceed the reservation level of zero. Since the marginal old consumer has the lowest utility in equilibrium, evaluating equation 2.2.2 at the equilibrium yields the constraint on the parameters required to satisfy the assumption:

$$\beta \geq \frac{3t + s}{2} + c \tag{P1}$$

□

Under unconstrained price discrimination, entrant and incumbent effectively compete separately in the old and new consumer markets. In the new consumer market, there is no rent to be earned from switching costs; given that marginal costs are equal, entrant and incumbent divide this market evenly. For old consumers, the incumbent charges a premium price to recover rent on the switching cost; the entrant “pays to switch” (Chen, 1997) and offers a discount vis-a-vis the price new consumers pay. Furthermore, the incumbent remains dominant in the market for old consumers – having more than 50% market share – as in Gehrig et al. (2011) and Bouckaert et al. (2013).

The higher switching costs are, the greater the profit of the incumbent and the lower profits of the entrant⁶. Greater transportation costs t improve the profits of both firms. Finally note that the marginal cost does not affect profits in equilibrium, because there is perfect cost pass-through to consumer prices.

⁶Monotonicity of entrant’s profit with respect to switching cost follows from C1

2.3.2 Duopoly with Regulated Price Discrimination

We now allow for limits to price discrimination imposed by the competition authority on the incumbent's extent of price discrimination.

Proposition 2.3.3. *Under duopoly with a binding constraint on price discrimination, $q \in [0, \frac{s}{3})$, the unique equilibrium has*

1. *Old consumers with $\tilde{x}_{OLD} \leq \frac{s\theta+6t+2s-3q\theta}{12t}$ and new consumers with $\tilde{x}_{NEW} \leq \frac{s\theta+6t-s+3q(1-\theta)}{12t}$ purchase from firm A, and all others from firm B*
2. *Firm A sets prices $p_{OLD}^A = \frac{s(1-\theta)}{3} + q\theta + t + c$, $p_{NEW}^A = \frac{s(1-\theta)}{3} - q(1-\theta) + t + c$. Firm B's prices are $p_{OLD}^B = \frac{(3q-s)\theta}{6} - \frac{s}{3} + t + c$ and $p_{NEW}^B = -q\frac{1-\theta}{2} + \frac{s(1-\theta)}{6} + t + c$*
3. *Equilibrium profits are given by*

$$\begin{aligned}\pi_{DUO}^{*A} &= \frac{s^2(1-\theta)}{18t} + \frac{s(1-\theta)}{3} + \frac{t}{2} - \frac{q(q-s)(1-\theta)\theta}{4t} \\ \pi_{DUO}^{*B} &= \frac{s^2(1-\theta)}{18t} - \frac{s(1-\theta)}{3} + \frac{t}{2} + \frac{(s-q)^2\theta(1-\theta)}{8t}\end{aligned}$$

Proof. Applying inequality constraint (equation 2.2.5) to the incumbent's problem, the incumbent's problem in Kuhn-Tucker form becomes:

$$\begin{aligned}\mathcal{L} &= (1-\theta)\tilde{x}_{OLD}(p_{OLD}^A, p_{OLD}^B)(p_{OLD}^A - c) \\ &+ \theta\tilde{x}_{NEW}(p_{NEW}^A, p_{NEW}^B)(p_{NEW}^A - c) - \lambda(p_{OLD}^A - p_{NEW}^A - q)\end{aligned}\quad (2.3.9)$$

Suppose first the constraint does not bind ($\lambda = 0$). Then, the solutions of the problem are given by equations (2.3.3-2.3.4). Combining with the best responses of the entrant (2.3.6-2.3.7) we thus have the same equilibrium as in proposition 2.3.2. By examining the incumbent's price difference $p_{OLD}^{*A} - p_{NEW}^{*A} = \frac{s}{3}$, this equilibrium is feasible for $q \geq \frac{s}{3}$.

When the competition authority's constraint is binding ($\lambda \neq 0$), the first-order condition yields:

$$p_{OLD}^A(p_{NEW}^B) = \frac{((1-\theta)p_{OLD}^B + \theta p_{NEW}^B) + (2q-s)\theta + t + s + c}{2} \quad (2.3.10)$$

$$p_{NEW}^A(p_{NEW}^B) = \frac{((1-\theta)p_{OLD}^B + \theta p_{NEW}^B) + (2q-s)\theta + t + s + c}{2} - q \quad (2.3.11)$$

From proposition 2.3.1, it is immediate that the entrant is able to engage in price discrimination a fortiori if the incumbent is constrained (as the price difference shrinks).

Combining equations (2.3.10-2.3.11) and (2.3.6-2.3.7) again yields a full-rank 4x4 system, and which can be solved to yield the unique equilibrium prices claimed in part (ii) of the proposition. Parts (i) and (iii) are analogous to proposition 2.3.2. \square

When a binding constraint on price discrimination is introduced, holding the entrant's prices constant, the incumbent lowers the prices for old consumers and increases them for new consumers. Observe that this constraint on the incumbent's price discrimination actually enhances the entrant's price discrimination – the price difference B charges new and old consumers actually increases.

Collecting systematically the comparative statics results, and noting that a *fall* in q means stricter regulation, we have:

Proposition 2.3.4 (Effects of Limiting Price Discrimination). *As price discrimination on the incumbent is constrained ($dq < 0$),*

1. *the incumbent's market share among old consumers rises ($d\tilde{x}_{OLD}/dq < 0$) and falls among new consumers ($d\tilde{x}_{NEW}/dq > 0$); across segments, market shares are unchanged ($d(\theta\tilde{x}_{NEW} + (1 - \theta)\tilde{x}_{OLD})/dq = 0$)*
2. *both incumbent and entrant reduce prices for old consumers $dp_{OLD}^i/dq > 0$ and raise prices for new consumers ($dp_{NEW}^j < 0$)*
3. *entrant profit rises, $d\pi^{*B}/dq < 0$ while the incumbent's profit falls $d\pi^{*A}/dq > 0$*

Proof. Using proposition 2.3.3 directly,

1. For old consumers, $d\tilde{x}_{OLD}/dq = -\frac{\theta}{4t} < 0$; while for new consumers $d\tilde{x}_{NEW}/dq = \frac{1-\theta}{4t} < 0$.

Taking $d((1 - \theta)\tilde{x}_{OLD} + \theta\tilde{x}_{NEW})/dq = -\frac{(1-\theta)\theta}{4t} + \frac{(1-\theta)\theta}{4t} = 0$, as claimed.

2. For the incumbent, $dp_{OLD}^{*A}/dq = \theta > 0$, $dp_{NEW}^{*A}/dq = -(1 - \theta) < 0$; while for the entrant, $dp_{OLD}^{*B}/dq = \frac{\theta}{2} > 0$, $dp_{NEW}^{*B}/dq = -\frac{(1-\theta)}{2} < 0$,

3. For the incumbent and entrant respectively:

$$\frac{d\pi_{DUO}^{*A}}{dq} = \frac{(s - 2q)(1 - \theta)\theta}{4t} > 0 \quad (2.3.12)$$

$$\frac{d\pi_{DUO}^{*B}}{dq} = -\frac{(s - q)(1 - \theta)\theta}{4t} < 0 \quad (2.3.13)$$

Noting that the proposition indicates $q \in [0, s/3)$, the expressions are definitely signed.

□

Regulation of price discrimination affects the old and new consumer segments differently. For old consumers, the incumbent's price is depressed; since the entrant's price does not fall one-for-one, this results in a falling price difference $p_{OLD}^A - p_{OLD}^B$, and thus shifts the marginal consumer towards the incumbent. Thus, regulation strengthens the market share of the incumbent in the old segment. In contrast, the incumbent's price for new consumers rises; again, the entrant partially follows suit, hence gaining market share among new consumers. Regulation does not affect the overall market shares of the two firms.

As one may expect, the regulation harms profitability of the incumbent. The entrant gains. This is as in [Bouckaert et al. \(2013\)](#) and [Chen \(2008\)](#), but not as in [Gehrig et al. \(2011\)](#) where the entrant is compelled to uniform pricing.

2.3.3 Entrant or Incumbent Monopoly

Now, I solve for the market outcomes when either incumbent or entrant is in a monopoly position:

Incumbent Monopoly. Consider first the case that only the incumbent is active in the market. In this case, the marginal consumer is indifferent between purchasing from firm A and abstaining from the market. Solving for the marginal type using equations 2.2.2 and 2.2.1 respectively,

$$\tilde{x}_{OLD} = \frac{\beta - p_{OLD}^A}{t} \quad \tilde{x}_{NEW} = \frac{\beta - p_{NEW}^A}{t} \quad (2.3.14)$$

Two points are noteworthy about these expressions. First, it is clear that there is no competitor to switch to under incumbent monopoly; thus it is also intuitive that uniform pricing will arise in this setting. Second, the full market coverage assumption ties down the prices directly in the monopoly case; it remains to find boundaries on the parameters for which full market coverage can arise in equilibrium.

Proposition 2.3.5. *Under incumbent monopoly,*

1. All old and new consumers purchase from firm A , $\tilde{x}_{OLD} = \tilde{x}_{NEW} = 1$
2. The incumbent charges prices $p_{OLD}^A = p_{NEW}^A = \beta - t$
3. Incumbent profit equals $\pi_{MON}^{*A} = \beta - t - c$

Proof.

1. This is an immediate consequence of assumption 4, i.e. that the market is fully covered.

2. To find monopoly prices, note the incumbent's problem is given by

$$\begin{aligned} \max_{\{p_{OLD}^A, p_{NEW}^A\}} \pi^A = & (1 - \theta)\tilde{x}_{OLD}(p_{OLD}^A) (p_{OLD}^A - c) \\ & + \theta\tilde{x}_{NEW}(p_{NEW}^A) (p_{NEW}^A - c) \end{aligned} \quad (2.3.15)$$

The first-order condition for either price p_{OLD}^A is given by

$$\frac{d\pi^A}{dp_{OLD}^A} = \begin{cases} 0 & \text{if } p_{OLD}^A \geq \beta \\ \frac{1-\theta}{t} (\beta + c - 2p_{OLD}^A) & \text{if } \beta - t \leq p_{OLD}^A \leq \beta \\ -1 & \text{if } p_{OLD}^A < \beta - t \end{cases}$$

To satisfy the full market coverage assumption, we can immediately observe from equation 2.3.14 that $p_{OLD}^A = p_{NEW}^A = \beta - t$. For this to be a solution of the firm's pricing problem, we must have

$$\beta \geq 2t + c$$

3. Follows directly by substituting the equilibrium prices and marginal consumers into the payoff function. □

Entrant Monopoly. Under entrant monopoly, all old consumers face switching cost. This yields a modified marginal consumer condition in this segment. For new consumers, the expression remains unchanged from the previous section:

$$\tilde{x}_{OLD} = \frac{\beta + t + s - p_{OLD}^B}{t} \quad \tilde{x}_{NEW} = \frac{\beta + t - p_{NEW}^B}{t} \quad (2.3.16)$$

Given the symmetry of the model, the appropriate condition to satisfy the full coverage assumption is that $\tilde{x}_j = 0$ in both segments. Solving the pricing problem then yields

Proposition 2.3.6. *Under entrant monopoly,*

1. All old and new consumers purchase from firm B, $\tilde{x}_{OLD} = \tilde{x}_{NEW} = 0$
2. The incumbent charges prices $p_{OLD}^B = \beta - t - s$ and $p_{NEW}^B = \beta - t$ to old and new consumers respectively

3. The entrant's profit equals $\pi_{MON}^{*B} = \beta - t - c - s(1 - \theta)$

Proof.

1. Follows directly from market coverage, as before

2. The entrant's problem is given by

$$\begin{aligned} \max_{\{p_{OLD}^B, p_{NEW}^B\}} \pi^B = & (1 - \theta)(1 - \tilde{x}_{OLD})(p_{OLD}^B)(p_{OLD}^B - c) \\ & + \theta(1 - \tilde{x}_{NEW})(p_{NEW}^B)(p_{NEW}^B - c) \end{aligned} \quad (2.3.17)$$

The first-order conditions are given by

$$\begin{aligned} \frac{d\pi^B}{dp_{OLD}^B} &= \begin{cases} 0 & \text{if } \beta - s \leq p_{OLD}^B \\ \frac{1-\theta}{t}(\beta - s + c - 2p_{OLD}^B) & \text{if } \beta - t - s \leq p_{OLD}^B \leq \beta - s \\ -1 & \text{otherwise} \end{cases} \\ \frac{d\pi^B}{dp_{NEW}^B} &= \begin{cases} 0 & \text{if } \beta \leq p_{OLD}^B \\ \frac{\theta}{t}(\beta - c - 2p_{NEW}^B) & \text{if } \beta - t \leq p_{NEW}^B \leq \beta \\ -1 & \text{otherwise} \end{cases} \end{aligned}$$

To satisfy the full market coverage assumption, we can immediately observe from equation 2.3.14 that $p_{OLD}^B = \beta - t - s$ and $p_{NEW}^B = \beta - t$. For this to be a solution of the firm's pricing problem, we must have

$$\beta \geq 2t + c + s \quad \beta \geq 2t + c$$

for the FOC of old and new consumers respectively. Since the first constraint is tighter, the full coverage assumption implies $\beta \geq 2t + c + s$.

3. By substitution of equilibrium prices into the payoff function. □

2.3.4 Entry Stage

The model admits for the equilibrium market structures of duopoly, monopoly by entrant (exit) or monopoly by the incumbent (no entry). Depending on the parameters – fixed cost of entry, switching cost, regulation of price discrimination – different outcomes are possible. We derive the monopoly equilibrium, then first establish an ordering of the profits, and finally find for which parameter cases a particular market structure is an equilibrium.

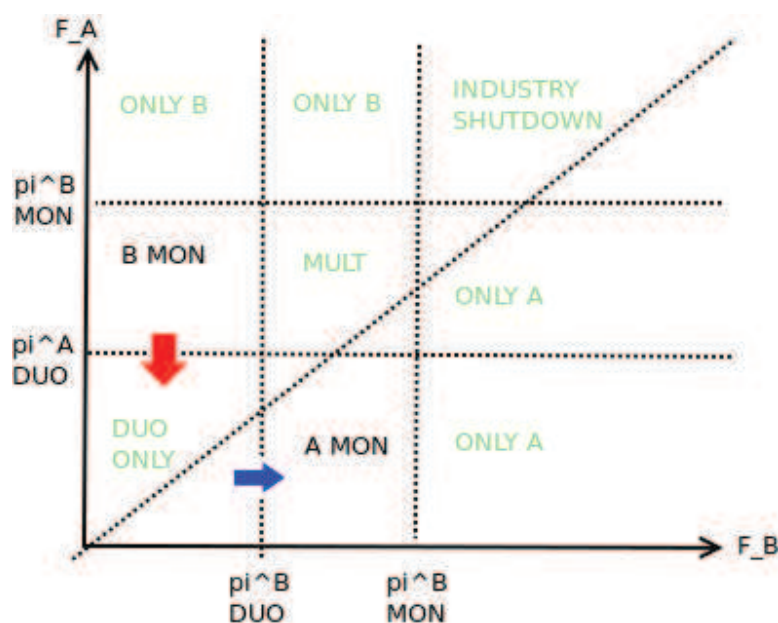


Figure 2.2: Constellation of Equilibrium Market Structures

Proposition 2.3.7. *Given the parameters, the equilibrium market structure is given by*

1. $F_A < \pi_{DUO}^{*A}, F_B < \pi_{DUO}^{*B}$: unique equilibrium where both firms enter
2. $F_A < \pi_{DUO}^{*A}, \pi_{DUO}^{*B} < F_B < \pi_{MON}^{*B}$: incumbent monopoly is the unique equilibrium
3. $\pi_{DUO}^{*A} < F_A < \pi_{MON}^{*A}, F_B < \pi_{DUO}^{*B}$: entrant monopoly is the unique equilibrium
4. $\pi_{DUO}^{*A} < F_A < \pi_{MON}^{*A}, \pi_{DUO}^{*B} < F_B < \pi_{MON}^{*B}$: either entrant or incumbent monopoly are equilibria.

Proof. The entry stage is a 2x2 game. First, consider the case that $F_i < \pi_{DUO}^{*i}$. In this case, it is a dominant strategy for firm i to enter the market. Second, if $\pi_{DUO}^{*i} < F_i < \pi_{MON}^{*i}$, firm i 's best response is to enter if the rival does not, and otherwise abstain from the market.

The claimed market structures in the proposition then follow directly from combining the cases just discussed. \square

Figure 2.2 summarises graphically the possible market structures.

2.4 Welfare and Price Discrimination

Consumer Surplus:

$$\begin{aligned}
CS &= \beta + \theta \left(\int_0^{\tilde{x}_{NEW}} -tx - p_{NEW}^A dx + \int_{\tilde{x}_{NEW}}^1 -t(1-x) - p_{NEW}^B dx \right) \\
&+ (1-\theta) \left(\int_0^{\tilde{x}_{OLD}} -tx - p_{OLD}^A dx + \int_{\tilde{x}_{OLD}}^1 -t(1-x) - p_{OLD}^B - s dx \right)
\end{aligned} \tag{2.4.1}$$

Profits were already derived before. Social welfare is given by

$$\begin{aligned}
W &= \beta + \theta \left(\int_0^{\tilde{x}_{NEW}} -tx dx + \int_{\tilde{x}_{NEW}}^1 -t(1-x) dx \right) \\
&+ (1-\theta) \left(\int_0^{\tilde{x}_{OLD}} -tx dx + \int_{\tilde{x}_{OLD}}^1 -t(1-x) - s dx \right)
\end{aligned} \tag{2.4.2}$$

The expression makes clear the nature of the welfare discussion. Given the full coverage assumption, the sources of inefficiency in the model are (a) misallocation of consumers away from the preferred brand and (b) switching costs.

2.4.1 Price Discrimination in Oligopoly

We now derive the welfare-maximising policy on price discrimination (i.e. q), given that a duopoly is in place. It turns out this policy is to ban price discrimination:

Proposition 2.4.1. *Given duopoly, the welfare-maximising policy calls for a complete ban of behaviour-based price discrimination by the incumbent. This policy maximises both consumer surplus and producer surplus.*

Proof. Substituting into equations (2.4.1–2.4.2) the equilibrium marginal consumers found in proposition 2.3.3, we find after taking derivatives with respect to q

$$\frac{dCS}{dq} = -\frac{(s-q)(1-\theta)\theta}{8t} \tag{2.4.3}$$

$$\frac{dPS}{dq} = -\frac{q(1-\theta)\theta}{4t} \tag{2.4.4}$$

$$\frac{dW}{dq} = -\frac{(s+q)(1-\theta)\theta}{8t} \tag{2.4.5}$$

Since $q \in [0, s/3]$, the first expression is unambiguously negative; similarly, the derivative of welfare with respect to q is unambiguously negative. Thus $q = 0$ maximises both social and producer surplus as claimed. \square

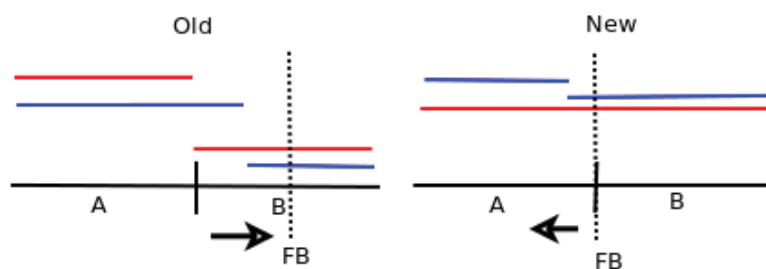


Figure 2.3: Welfare Effects of Price Discrimination

Price discrimination is thus very damaging to welfare in the present model, and leads to a “double loss” for both consumers and producers. Producer surplus falls because the incumbent loses more than the entrant gains, as discussed above and in line with other models.

Figure 2.3 illustrates the effects of price discrimination on the structure of the equilibrium. The first-best allocation divides consumers evenly between the two firms in the new segment ($\tilde{x}_{NEW}^{FB} = \frac{1}{2}$), as one would expect given the uniformly distributed brand preference and equality of marginal costs. When price discrimination is not regulated, this allocation is also implemented by the market through equal prices for old consumers offered by firms A and B (the red line indicates pre-regulation prices). As regulation is imposed, the incumbent’s price for new consumers rises faster than the entrant’s price; the new price structure is indicated in blue, and entails a gain in market share for the entrant among new consumers.

Among old consumers, the first best has a marginal consumer $\tilde{x}_{OLD}^{FB} = \frac{1}{2} + \frac{s}{2t}$ of firm A . But as indicated in proposition 2.3.2, under unfettered price discrimination the market share of A is, in fact, below this value. Thus, the model exhibits excessive switching due to the incumbent harvesting rent from switching costs (prices offered before regulation are again indicated in red, with A charging more than B). As price discrimination is restricted, the incumbent is forced to lower prices for old consumers; since the entrant’s price cut is smaller, the marginal consumer shifts towards A , which tends to improve efficiency.

Social welfare rises when price discrimination is curtailed. Regulation improves welfare in the old segment by moving the marginal consumer closer to the first. This gain is strong, because the distortion – due to inefficient switching – is strong absent regulation; the loss of welfare in the new segment is smaller, establishing the welfare result. Consumer surplus falls in the new segment, which is hurt by higher prices. But old consumers benefit both from a price fall and the reduction in switching effort. Again, the gain to old consumers is sufficiently strong to drive the overall result.

Under the welfare-maximising policy – which, by the proposition is a ban on price discrimination, the maximum welfare that can be attained under duopoly is:

$$W_3 = \beta - c - \frac{t}{4} - \frac{7(s\theta)^2 + 13s^2\theta}{144t} - \frac{s(1-\theta)}{2} + \frac{5s^2}{36t} - F_b - F_a \quad (2.4.6)$$

2.4.2 Entry, Price Discrimination and Welfare

This section establishes the central results of the paper relating to welfare, price discrimination and entry/exit. The first proposition focuses on the case where the entry of firm B is potentially affected by price discrimination; it shows that, if price discrimination is marginal to entry, this must imply that B 's fixed cost is sufficiently high to make entry inefficient. Thus, the normative result, building on the analysis of preceding section, states that price discrimination should be banned if and only if it does not cause entry. I then move to the case where the incumbent may exit; here, the analysis is somewhat more intricate because – in case of exit – all old consumers bear the switching cost to firm B .

Incumbent monopoly. All consumers get the good, so under incumbent monopoly we have the following welfare expression:

$$\begin{aligned} W_1 &= \int_0^1 (\beta - tx) dx - c - F_A \\ &= \beta - \frac{t}{2} - c - F_A \end{aligned} \quad (2.4.7)$$

Now, the crucial result:

Proposition 2.4.2. *Given that the incumbent does not exit, banning price discrimination by the incumbent maximises social welfare if and only if it does not cause entry*

Proof. To ensure that the incumbent remains in the market, we must have $F_A \leq \pi_{DUO}^{*A}$. Then, two cases can be distinguished:

1. Price discrimination not marginal to entry.

By proposition 2.3.7, if $F_b \leq \pi_{DUO}^{*B} | q = s/3$, for any policy q , the unique equilibrium has both firms active in the market. By appeal to proposition 2.4.1, under duopoly, it is welfare maximising to ban price discrimination of the incumbent ($q = 0$).

2. Price discrimination is marginal to entry

When $\pi_{DUO}^{*B}|q = s/3 \leq F_b \leq \pi_{DUO}^{*B}|q = 0$, there is an equilibrium with incumbent monopoly if price discrimination is unrestrained, and an equilibrium with duopoly if price discrimination is restricted. With entry, we have the gain in product variety (lower transportation cost), but need to sink the fixed cost of firm B and create switching costs.

Computing the welfare gain of duopoly:

$$W_3 - W_1 = \frac{t}{4} - \frac{7(s\theta)^2 + 13s^2\theta}{144t} - \frac{s(1-\theta)}{2} + \frac{5s^2}{36t} - F_b \quad (2.4.8)$$

$$\leq \frac{t}{4} - F_b \quad (2.4.9)$$

$$\leq \frac{t}{4} - \left(\frac{t}{2} + \frac{s^2(1-\theta)}{18t} - \frac{s(1-\theta)}{3}\right) < 0 \quad (2.4.10)$$

where the first inequality is the upper bound on welfare gains, if switching costs are disregarded. The second inequality substitutes the lower bound on fixed cost of the entrant. Combining, we obtain that welfare is higher under incumbent's monopoly than duopoly for the claimed range of entrant's fixed cost using condition C1.

□

When price discrimination is not marginal to entry, the welfare results obtained previously apply: excessive switching among old consumers is more costly from a welfare perspective than the distortion of market shares among new consumers. Moreover, both consumers and producers benefit from a ban on price discrimination in this case, as discussed above. Matters are slightly more complex when price discrimination affects the entry decision of firm B . Since by construction the incumbent's activity in the market is not at stake, from a welfare perspective her fixed costs are considered sunk. The entry of the incumbent then promotes welfare through a reduction in transportation costs, but creates costs through inefficient switching and the fixed cost of entry that must be spent. In order for price discrimination to affect entry, however, it must be that the entrant's fixed costs are sufficiently high that even disregarding the switching cost component of welfare, entry is inefficient. Thus, the result obtains.

Entrant monopoly. Again, all consumers get the good, but now all old consumers need to

switch:

$$\begin{aligned} W_2 &= \theta \int_0^1 \beta - t(1-x) dx + (1-\theta) \int_0^1 \beta - t(1-x) - s dx - c - F_B \quad (2.4.11) \\ &= \beta - \frac{t}{2} - (1-\theta)s - c - F_B \end{aligned}$$

Proposition 2.4.3. *Given that the entrant joins the market, banning price discrimination by the incumbent maximises social welfare if it does not cause exit or the switching cost is below transportation cost ($s < t$)*

Proof. To ensure entry by firm B , we must have $F_B \leq \pi_{DUO}^*{}^A$ for all possible policies q . Analogously to the previous proposition, two cases need to be distinguished:

1. Price discrimination not marginal to exit

If $F_A \leq \pi_{DUO}^*{}^A|q = 0$, the incumbent will stay in the market for any policy q . From proposition 2.4.1, the welfare maximising policy is to ban price discrimination.

2. Price discrimination is marginal to exit.

When $\pi_{DUO}^*{}^A|q = 0 \leq F_a \leq \pi_{DUO}^*{}^A|q = s/3$, policy may bring about both equilibria in which the industry is a duopoly (when regulation of PD is lax) or the entrant becomes a monopolist while the incumbent exits (for tight regulation of PD).

$$W_3 - W_2 = \frac{t}{4} - \frac{7(s\theta)^2 + 13s^2\theta}{144t} + \frac{s(1-\theta)}{2} + \frac{5s^2}{36t} - F_a \quad (2.4.12)$$

$$\leq \frac{t}{4} + s(1-\theta) - F_a \quad (2.4.13)$$

$$= -\frac{t}{4} - \frac{s^2(1-\theta)}{18t} + \frac{2s(1-\theta)}{3} \quad (2.4.14)$$

When $s < t$, the expression is guaranteed to be negative, so entrant monopoly and hence regulation of price discrimination maximises social welfare.

□

A potential exit by the incumbent is damaging to welfare because *all* old consumers must incur the switching cost to move to the entrant. This naturally weakens the welfare rationale for entrant monopoly. To guarantee that social welfare rises in this case, a further condition on the parameters is required, i.e. the transportation cost should be below the switching cost.

2.5 Conclusion

This paper studied a model of history-based price discrimination in the context of a growing market, where old and new consumers may be treated differently. The key contribution of the present study was to allow the potential entrant to engage in price discrimination, so long as the consumers' incentive compatibility constraint is met. This constraint arises because the entrant does not directly observe who previously purchased from the incumbent; however, as the model shows, this presents no strategic disadvantage – in any equilibrium, the incentive constraint is non-binding, and *both* firms engage in price discrimination.

Price discrimination in this setting has very adverse welfare consequences. In particular, the market of old consumers is heavily distorted through excessive consumer switching. While the market for new consumers attains an efficient allocation under price discrimination, the welfare costs in the old segment are sufficiently strong to assure that price discrimination hurts consumer and producer surplus. Thus, given that a duopoly market structure is in place, banning price discrimination is the optimal policy in this framework.

Regulation may also affect entry and exit decisions, requiring a careful welfare analysis. When an initially monopolised industry becomes subject to competition, one must trade off – from a social welfare perspective – the gains arising due to reduced transportation cost against the added fixed cost expended by the entrant. Since the entrant partially “steals” rent from the incumbent, her incentive to enter the industry is generally inefficiently strong. In fact, I show that price discrimination is only marginal to entry when entry would be inefficient. Therefore, price discrimination bans that encourage entry are not welfare maximising in this model, although they may benefit consumers. Conversely, when the incumbent is under threat of exit, a ban on price discrimination may consolidate the industry into an entrant monopoly. A sufficient condition for this to be optimal is that switching costs are not larger than transportation costs.

Bibliography

- Armstrong, M., Vickers, J., 1993. Price discrimination, competition and regulation. *The Journal of Industrial Economics*, 335–359.
- Bester, H., Petrakis, E., 1996. Coupons and oligopolistic price discrimination. *International Journal of Industrial Organization* 14 (2), 227–242.
- Bouckaert, J., Degryse, H., Dijk, T., 2013. Bertrand competition with an asymmetric no-discrimination constraint. *The Journal of Industrial Economics* 61 (1), 62–83.
- Chen, Y., 1997. Paying customers to switch. *Journal of Economics & Management Strategy* 6 (4), 877–897.
- Chen, Y., 2008. Dynamic price discrimination with asymmetric firms. *The Journal of Industrial Economics* 56 (4), 729–751.
- Corts, K., 1998. Third-degree price discrimination in oligopoly: all-out competition and strategic commitment. *The RAND Journal of Economics*, 306–323.
- Farrell, J., Katz, M. L., 2006. The economics of welfare standards in antitrust.
- Fudenberg, D., Villas-Boas, J., Hendershott, T., 2006. Behavior-based price discrimination and customer recognition. *Handbook on economics and information systems* 1, 377–436.
- Gehrig, T., Shy, O., Stenbacka, R., 2011. History-based price discrimination and entry in markets with switching costs: a welfare analysis. *European Economic Review* 55 (5), 732–739.
- Gehrig, T., Shy, O., Stenbacka, R., 2012. A welfare evaluation of history-based price discrimination. *Journal of Industry, Competition and Trade* 12 (4), 373–393.
- Geradin, D., Petit, N., 2005. Price discrimination under ec competition law. The pros and cons of price discrimination. Swedish Competition Authority, Stockholm, 21–64.

- Haucap, J., 2003. Selective price cuts and uniform pricing rules in network industries. *Journal of Industry, Competition and Trade* 3, 269–291.
- Karlinger, L., Motta, M., 2012. Exclusionary pricing when scale matters. *The Journal of Industrial Economics* 60 (1), 75–103.
- Motta, M., 2004. *Competition policy: theory and practice*. Cambridge University Press.
- Segal, I. R., Whinston, M. D., 2000. Naked exclusion: comment. *The American Economic Review* 90 (1), 296–309.
- Spector, D., 2005. The strategic uses of price discrimination. The pros and cons of price discrimination. Swedish Competition Authority, Stockholm, 187–211.
- Thisse, J.-F., Vives, X., 1988. On the strategic choice of spatial price policy. *The American Economic Review*, 122–137.

Chapter 3

Of Pennies and Prospects: Understanding Behaviour in Penny Auctions

Abstract. Penny auctions are ascending auctions where bidders pay a fee for each bid they place. The last bidder wins the auction – thus a bid is essentially a bet on being the last bidder, adding a dimension of gambling to the auction. Also, like lotteries and slot machines, these auctions are often, but not always, highly profitable for the seller. The paper explains these facts using prospect theory, obtaining estimates for the probability weighting and value functions that are in line with the literature. In contrast to the competing hypothesis of risk-loving bidders, the model can explain why auctions for low-value objects are unprofitable.

■

3.1 Introduction

Penny auctions are a novelty online auction format, where each bidder pays a fixed fee to place a bid. This fee raises the current price in the auction by a fixed amount (the price increment). The last agent to place a bid wins the good, as in a standard auction – such as those conducted on eBay. But, crucially, participants pay a fee for each bid they place, even if they do not go on to win the auction. Thus, the auction can be seen as a hybrid with a lottery: placing a bid essentially amounts to placing a bet that no other participants will place bids in the next round, in the hope of winning the object at a fraction of its retail price. These auctions are empirically very interesting, because – in the words of [Thaler \(2009\)](#) –

participants “pay a price for the thrill of the hunt”: on average, the sellers revenue, from winners and losers combined, exceeds the value of goods sold by nearly 50%!

Swoopo, a now-defunct German company, first introduced these auctions to the market. Using a rich dataset of game auctions conducted on its United Kingdom website between 2009 and 2010, containing over 25000 auctions, I develop some stylised facts on penny auctions. As [Augenblick \(2012\)](#), [Platt et al. \(2013\)](#) and [Hinnosaar \(2010\)](#) have noted, penny auctions are indeed, on average, very profitable for the seller. However, this aggregate masks considerable heterogeneity; in particular, auctions for low-priced products yielded less revenue than the value of the good on average. Furthermore, I develop some facts on the role of the price increment and (monetary) returns to bidding during different rounds of auctions, using both variations between auctions for different objects and within auctions for the same object. This shows that a version of the long-shot bias [Ottaviani and Sørensen \(2008\)](#) familiar from many betting markets is present in penny auctions.

The central contribution of the paper is to explain the stylised facts of penny auction behaviour using prospect theory ([Kahneman and Tversky, 1979](#); [Tversky and Kahneman, 1992](#)). Perhaps the leading alternative theory to expected utility ([Starmer, 2000](#)), prospect theory incorporates regularities of behaviour found in laboratory studies¹ to provide a *descriptive* account of human attitudes to risk. The theory is able to explain high revenue in penny auctions through probability weighting: humans overweight low-probability events, such as winning in a lottery. Thus, humans may be willing to participate in a penny auction even if the odds in a given equilibrium are very low. However, diminishing marginal sensitivity – akin to a diminishing marginal utility of wealth – and loss aversion work against excessive bidding in the auction. Using the functional forms of [Tversky and Kahneman \(1992\)](#), I show that prospect theory may imply a “cut off” so that auctions for items below the cut-off retail value are unprofitable. Moreover, I estimate the prospect theory model using maximum likelihood. I show that the parameters are qualitatively and quantitatively in line with results from experimental and observational studies.

It appears that the main alternative hypothesis to prospect theory in the penny auction setting is that agents have expected utility functions, but are risk-loving. [Platt et al. \(2013\)](#) advance this hypothesis in the context of penny auctions, and estimate their model on a dataset similar to one used in the present paper. I similarly estimate their model on my dataset, and compare the predicts qualitatively and quantitatively to the prospect theory model. Since the risk-loving and prospect theory models are not nested, model selection is not just a statistical exercises. My approach is thus to compare the predictions of both

¹But not only: see [Trepel et al. \(2005\)](#) for an interesting neuroeconomic study

models against the stylised facts and assess their out of sample performance.

A number of recent and closely related papers investigate penny auctions. [Hinnosaar \(2010\)](#) developed the theory of penny auctions in a risk-neutral setting, which I discuss below as a benchmark. [Augenblick \(2012\)](#) suggests that a naive sunk cost fallacy on the side of penny auction participants can explain the high revenue; this theory similarly has the limitation that it predicts excess revenue for *all* auctions. [Wang and Xu \(2013\)](#) find that the overwhelming number of players lose money, as one would expect, but also that they leave the platform quickly. This gives some support to behavioural explanations, as agents have an incentive to leave when their misperceptions become exposed. [Byers et al. \(2010\)](#) discuss “asymmetries” in penny auctions, and show how – for example – misperceptions of the number of players can lead to excess revenue.

Winner-pays online auctions, such as eBay are by now well studied (see [Ockenfels et al. \(2006\)](#) as survey). Closely related to the present paper is [Malmendier and Szeidl \(2008\)](#), who develop a model of behavioural bidders in the eBay context. This study also seeks to explain excess revenue, which amounts to 8% in the estimates of [Malmendier and Lee \(2011\)](#), i.e. less than one-sixth of the average found in penny auctions.

Finally, the paper is related to other prospect theory using observational data ([Camerer, 2004](#)). [Gurevich et al. \(2009\)](#) finds evidence for probability weighting and diminishing marginal sensitivity in data on financial options. [Jullien and Salanié \(2000\)](#) estimate a prospect theory model for bets in horse-races; relatedly, [Snowberg and Wolfers \(2010\)](#) investigate the hypothesis of probability misperception, also in the horse race context. Compared to these studies, we find evidence of stronger probability weighting biases. This may well arise because, unlike in the markets studied by the previous papers, penny auctions do not provide a rational agent with an opportunity to “bet against” the market other than by abstaining from the auction.

The paper proceeds as follows. Section 3.2 presents the dataset and develops the crucial stylised facts for theory to address. Section 3.3 then presents the expected utility model of penny auctions, considering the risk-neutral and risk-loving cases. Then, section 3.4 develops the alternative model, which is based on prospect theory. Estimation is discussed in section 3.5. Section 3.6 presents the estimation results, followed by 3.7, which compares predictions of the models to the stylised facts. Section 3.8 concludes.

Variable	Unit	Description
retail_price	GBP	Retail price of the product, as quoted by Swoopo
imputed_revenue	GBP	Revenue, calculated from final price
margin	–	Ratio of revenue to retail price
winner_savings_pcnt	%	Winning bidder's savings relative to retail price
flag_endprice	Dummy	1 if auction was in “final price off” format
flag_onecent	Dummy	1 if price increment was GBP 0.01
flag_5cents	Dummy	1 if price increment was GBP 0.05
flag_20cents	Dummy	1 if price increment was GBP 0.20

Figure 3.1: Variables

3.2 Data and Stylised Facts

The present study is based on a dataset of more than 25000 auctions. All of these auctions were conducted on the British website of Swoopo <http://www.swoopo.co.uk> (earlier known as [telebid.co.uk](http://www.telebid.co.uk)). The dataset was collection using an automated “spider” program, which downloaded the data from the completed auction page of the web site, and spans auctions running from May 2008 through December 2009. In constructing the dataset, we selected auctions for products that were sold at least 500 times during the sample period; this yields a rich panel in which we observe not only variation between different items, but also considerable variation within auctions for the same product – on average, each product in the sample was sold 1370 times. Figure 3.1 describes the variables of the dataset, and figure 3.2 provides descriptive statistics of the sample.

High Revenue: The most striking observation from figure 3.2 is that, on average, Swoopo makes a revenue more than GBP 300 per auction for products worth a mean retail price of only GBP 175.15. Thus at least on the surface, it appears that penny auctions are in practice a profitable mechanism for the seller: the average margin (defined as ratio of revenue over retail price) is 1.47, indicating that in a typical penny auction, the seller's revenue is nearly 50% higher than the retail price of the object. However, penny auctions are also risky for the seller: the coefficient of variation, defined as the ratio of standard deviation to means, in revenue is 2.02, compared to 1.03 for the retail price. The median auction yields an excess revenue of 23%.

The high expected revenue has attracted considerable research interest, and thus receives pride of place as the first stylised fact:

Variable	Mean	Median	N.Obs.	Std.Dev
retail_price	175.1543	949	25919	181.44
imputed_revenue	308.0433	140.5000	25919	624.52
margin	1.4747	1.2390	25919	1.22
imputed_bids	614.7159	279.0000	25919	1249.04
winner_savings_pcnt	55.6737	61.0000	25919	29.65
flag_endprice	0.0452	0.0000	25919	0.21
flag_onecent	0.0813	0.0000	25919	0.27
flag_2cents	0.0136	0.0000	25919	0.12
flag_5cents	0.0070	0.0000	25919	0.08
flag_20cents	0.0004	0.0000	25919	0.02

Figure 3.2: Descriptive Statistics

Stylised Fact 1. *On average, penny auction revenue is approximately 50% higher than the value of the object being sold on average*

After taking into account bidding fees, the winning bidder “saves” approximately 55% of the value of the good. Since average revenue is close to 150% of the value, this means that losing bidders account for the lion’s share of revenue: combined, the losers pay more than 100% of the value of the good. Because the bidding fee is small compared to the value of the good, auctions must receive many bids on average. This is clearly the case: the typical auctions receives more than 614 bids! These observations clearly show the lottery-like nature of the penny auction. In contrast, standard auctions, such as First or Second Price, typically yield a small surplus for the winning bidder and zero for all others.

Products being sold: In contrast to more established online auction platforms, Swoopo did not sell used or rare items. The sample contains 25 products. Except for a GBP 40 cash reward, all products are consumer electronics that are readily available from online retailers or local stores. Furthermore, all products were sold by Swoopo themselves, rather than through third-party sellers. While studies of eBay, for example, are complicated by the fact that bidding behaviour depends not only on the type of product but also the reputation rating of a seller², these issues do not arise in the present dataset.

Regression Analysis: Given the large number of observations, is instructive to set up a regression model to understand key features of the dataset. Since the key outcome of interest

²See, e.g., Resnick et al. (2006) for an interesting controlled experiment

Model	(1)	(2)
(Intercept)	-0.11** (0.04)	-0.13** (0.04)
log(retail_price)	0.33*** (0.01)	0.33*** (0.01)
flag_20cents	0.30 (0.37)	
flag_5cents	0.15† (0.09)	
flag_onecent	0.15*** (0.03)	0.15*** (0.03)
flag_endprice	0.81*** (0.04)	0.81*** (0.04)
N	25919	25919
R^2	0.08	0.08

Standard errors in parentheses

† significant at $p < .10$; * $p < .05$; ** $p < .01$; *** $p < .001$

Figure 3.3: Regression Results

is the profitability – or lack thereof – of a penny auction, the following model is natural:

$$margin_i = f(retail_price_i, X_i) + \epsilon_i \quad (3.2.1)$$

where i denotes the index of the observation, $margin$ and $retail_price$ are as defined above, and X_i is a vector of auction characteristics. Given the sample at hand, this means that X_i contains dummy variables indicating the price increment used in a given auction. Based on statistical fit, the following functional form was chosen:

$$\begin{aligned} margin_i = & \beta_0 + \beta_1 \log(retail_price_i) + \beta_2 flag_20cents_i \\ & + \beta_3 flag_5cents_i + \beta_4 flag_onecent_i \\ & + \beta_5 flag_endprice + \epsilon_i \end{aligned} \quad (3.2.2)$$

The model was estimated by OLS; results are reported in figure 3.3. Model (1) presents results for the full model; after “testing down” to drop insignificant variables, the preferred

equation (2) results. At with an R^2 of 8%, for both models, it is clear that considerable variation remains unexplained by the model; this is as one would expect, since theoretical considerations point to considerable randomness, given observables, in the outcomes of each auction. More importantly, the coefficients are very precisely estimated; in model (2), all coefficients are significant at the 1% level.

Variation by Retail Price: At the high-price end of the product spectrum, Swoopo sold semi-professional cameras; for example, more than 1100 auctions are recorded for the Nikon D90 camera, with retail price quoted at GBP 791.49, and yielding an average revenue of GBP 1995.60. In the mid-range, one finds game consoles like the *Nintendo DS* and *Playstation 3*, with retail prices between GBP 99.99 and GBP 299.23. These auctions also tended to be rather profitable, with revenues around 50% higher than the values of the object.

Auctions for lower-priced products tended to fail to show results in the auction, with average revenues often falling short of the retail value. For example, in 648 auctions for a *Kingston Micro SD* reader with retail price GBP 19.99, revenue on average reached only GBP 5.69. The *Freecom Databar 16GB* achieved average revenue of GBP 24.34, barely exceeding the retail price of GBP 23.08 in 633 auctions.

The low, indeed negative, profit margin for low-value goods is confirmed by regression analysis. For a 10p auction, for example, the model predicts a margin less than one for products with a retail price less than GBP 28.89. Generally, a one-percent increase in retail price is associated with a 0.33 increase in the profit margin; thus, as expected from the prior discussion, high-value goods are indeed predicted to have high profit margin, not just high absolute excess revenue over the retail price. Figure 3.4 plots the predicted margin, as compared to the “zero-profit” case of a unit margin for the case of a 10p auction.

Moreover, perhaps surprisingly, this fact appears to have been overlooked in the literature on penny auctions so far. Thus it is an important challenge for theory to explain why low-priced items incur losses on average, when – under the same auction format – high-priced items generally do well:

Stylised Fact 2. *The profit margin of a penny auction is increasing in the retail value of the product. While high-value products are often also highly profitable, auctions for low-price objects tend to incur losses on average.*

Variation by Price Increment: In all auctions, the bid fee was GBP 0.50. However, there is variation in the price increment – in the “default” auction, each bid raises the price by GBP 0.10; this format was used in 85.25% of auctions. In only 11 auctions (0.04%) of the sample

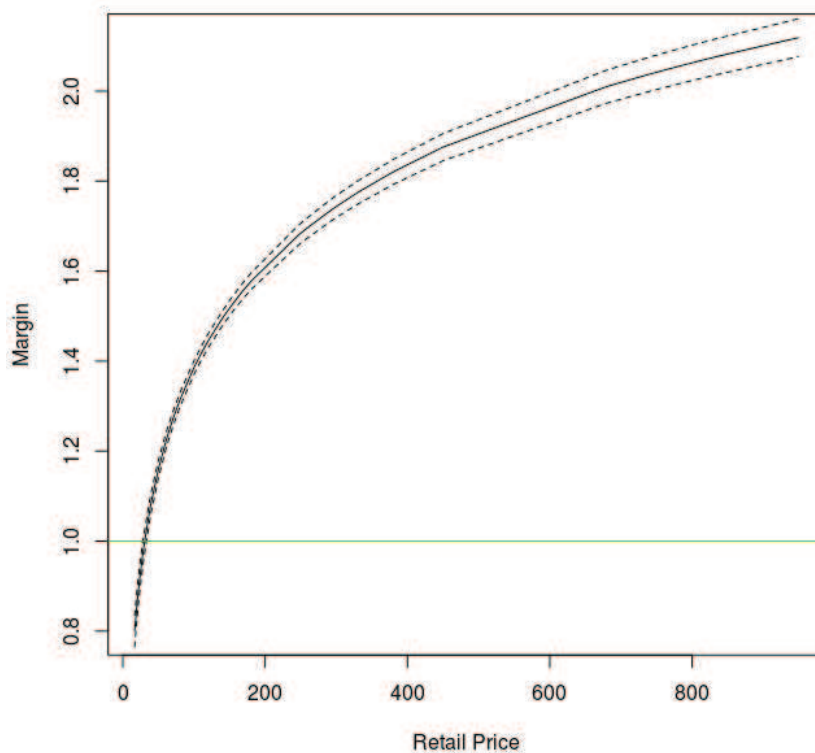


Figure 3.4: Retail Price and Margin: 10p auctions

was a higher increment of GBP 20 used, while 0.7% of auctions were conducted with a GBP 0.05 increment and 0.01% with a GBP 0.02 increment. 8% of auctions were conducted with a single-penny price increment. In the remaining 4.5% of auctions, the auction was conducted with a zero increment: in these auctions, the winning bidder received the “final price off”, as these auctions were called on Swoopo.

In some cases, we observe variation in the price increment for different auctions of the same product. For example, the Nikon D90 was sold 351 times with a GBP 0.10 increment, yielding average revenue of GBP 1859.62, and 625 times with a one penny increment. In the latter format, average revenue – for the same product – amounted to GBP 2167.53! While exploratory analysis suggests that lower price increments are associated with improved profitability of the auction, other things held equal, it is clear that the relationship is subject to noise: for example, GBP 0.05 auctions for the D90 were less profitable than GBP 0.10 ones, although the sample size is relatively small here (181 observations).

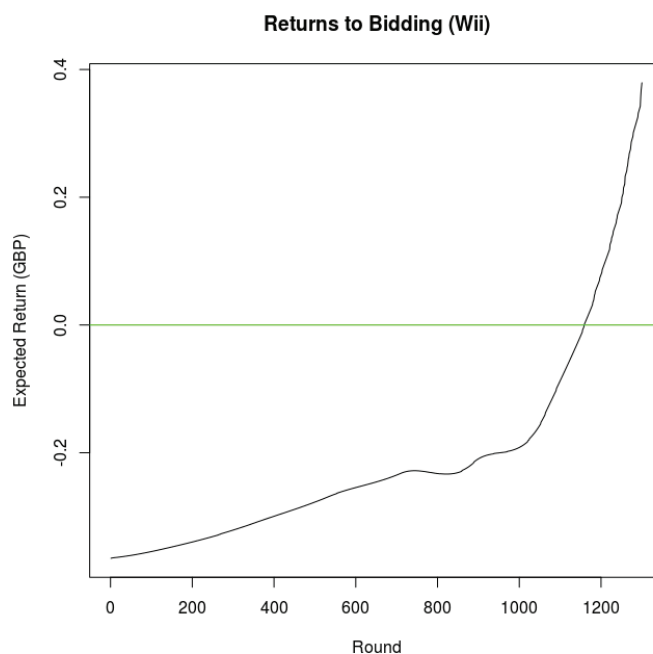


Figure 3.5: Expected Returns to Bidding (Wii Auctions, 10p)

Regression analysis shows that final price off auctions are especially profitable, associated with a 0.81 increase in margin over the 10p auction benchmark. Similarly, one-cent auctions are more profitable than the standard, although the margin is lower: they are associated with a 0.15 higher margin. For 5p and 20p auctions, the differences to the default penny auction are not significant at the 5% level, presumably because we observe so few auctions in this configuration. Hence,

Stylised Fact 3. *Given the retail value of the object, lower price increments are associated with higher profit margins in the auction*

Variation between Rounds: Since the dataset is *wide*, i.e. contains many observations for auctions of the same product, is important to consider variation within auctions for the same objects. Given that an auction reached round i , one may compute the empirical probability of the auction ending in this round (in the terminology of Gurevich et al. (2009), this is the “physical probability”). Taking into account the current price in round i and the value of the object, this allows for a straightforward computation of the expected (monetary) return to a bid.

Figure 3.5 plots the expected returns to bidding in Nintendo Wii auctions with a 10p price increment; the picture is similar for other auction. It is clear that in early rounds, the expected returns to bidding are very strongly negative: in fact, in round 1, the expected return is GBP -0.40 on a bidding stake of GBP 0.50. As the auction progresses, the expected return to bidding rises; in late rounds, the expected return to bidding even becomes positive. For example, in say, round 1200, when the price has reached GBP 120.00 for a console worth GBP 180.00, the probability of winning is sufficiently high that a bid yields a positive profit of GBP 0.20 in expectation. Of course, relatively few auctions reach such late rounds, implying that in practice the arbitrage opportunity for a risk-neutral bidder may be limited:

Stylised Fact 4. *The expected return to bidding in a penny auction increases with the round; in early rounds, the return is negative, and in late rounds turns positive.*

A positive correlation between winning probability and expected return has been observed in horse race betting markets since Griffith (1949), and is known as the “long-shot bias”. As in our data, in Snowberg and Wolfers (2010) and Jullien and Salanié (2000), returns for long-shot bets are sharply negative but bets on high-probability outcomes (here: placing a bid late in a penny auction) yield positive return. This suggests that penny auctions should be added to the already long list of speculative markets exhibiting a long shot bias³.

3.3 Expected Utility Model

This section presents the canonical expected utility model of penny auction bidding, following very closely Platt et al. (2013), Augenblick (2012) and Hinnosaar (2010).

Auction Rules: There is a single good commonly valued at V by all potential bidders, of which there are N , and the non-strategic seller. The auction proceeds through a series of rounds, $i = 0, 1, 2, \dots$. In round zero, the starting price is also equal to zero – so there are no reserve prices – and rises by an exogenously fixed increment $\beta > 0$ each round⁴. Thus, in round i , the current price is simply βi . From round 1 onwards, there is a *current highest bidder*, who is randomly selected from all agents that placed a bid in the previous round and pays a bidding fee f for her bid. The current highest bidder is, by assumption, not allowed

³For example, Woodland and Woodland (1994) present evidence of the long shot bias for baseball betting, and Hodges et al. (2003) for futures options

⁴Although some auctions with $\beta = 0$ are observed, in this case the auction reduces to a War of Attrition. Since this case is well known, it is omitted from the present analysis

to place a bid. Any other agents that placed bids and were not chosen to become current highest bidder also do not pay the fee. If in round $i > 0$ no bid is submitted, the good is awarded to the current highest bidder who pays the current price βi . If no bid is submitted in round zero, the auction ends immediately and the seller keeps the good.

Strategies: We are looking for equilibria in Markovian mixed strategies. In each round i , the $j = 1, \dots, N - 1$ current non-highest bidders place a bid with probability p_i^j . As we show below, without loss of generality we can confine attention further to the set of symmetric equilibria. Thus the strategy space of the game can be written as the infinite sequence, $\{p_i\}_0^\infty$.

Hazard Rates: Let $h(i)$ be the *hazard rate* of the auction in round i , i.e. the probability of the auction ending in round i , conditional on having reached this round. Since this quantity will be essential for empirical analysis, observe that by the rules of the game the hazard rate is related to individual bidding probabilities as follows:

$$h(i) = \begin{cases} (1 - p_i)^{N-1} & \text{if } i \geq 1 \\ (1 - p_i)^N & \text{if } i = 0 \end{cases} \quad (3.3.1)$$

Furthermore let $f(i)$ denote the probability of the auction ending in round i (ending probability). This is a direct function of the hazard rate:

$$f(i) = h(i) \prod_{k=0}^{i-1} (1 - h(k)) \quad (3.3.2)$$

Seller's Revenue: Suppose the auction ends after round i . Then, the current price is βi , and i bids have been placed. This yields the following expression for expected revenue:

$$E(R) = \sum_{i=0}^{\infty} (f + \beta)i f(i) \quad (3.3.3)$$

Payoffs: Players have an expected utility function $u(x)$ according to which outcomes are evaluated. We will variously assume this function to be either risk-neutral or risk-loving. Furthermore, let $V_N(i)$ denote the continuation utility of a player reaching round i as non-highest bidder, and $V_H(i)$ the continuation utility of being highest bidder.

Figure 3.6 illustrates the rules of the game for a penny auction with 3 players. Suppose the game is currently in round $i \geq 1$, and player 1 is the current highest bidder. Players

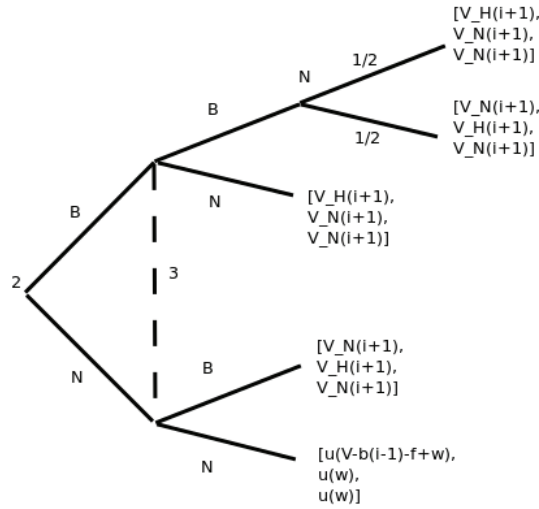


Figure 3.6: Stage Game with N=3

2 and 3 simultaneously decide whether to place a bid. In case neither submits a bid, the auction ends. Player 1 wins the good at price $\beta(i - 1)$, and players 2 and 3 are left with their reservation utility. In case only one of players 2 and 3 places a bid, this player becomes the the next highest bidder, receiving continuation utility $V_H(i + 1)$ upon entry to the next round. The other players enter round $i + 1$ as non-highest bidders, with continuation utility $V_N(i + 1)$. Finally, in case both 2 and 3 bid, nature randomly selects one bid to be accepted, the other bid being discarded.

The Last Round: Since the price of the auction increases by a constant increment each round, at some point the current price of the auction will exceed the retail price. In fact, this occurs in round

$$\tilde{i} = \frac{v - f}{\beta} \tag{3.3.4}$$

It is clear that no agent will bid at such prices:

Proposition 3.3.1. *For all rounds $i \geq \tilde{i}$, in any subgame perfect equilibrium $p_i = 0$*

Proof. Consider an agent placing a bid that is accepted in round \tilde{i} . This yields a utility of at most $u(w + V - \beta(\tilde{i} + 1) - f) < u(w)$, in the event of the agent winning the auction outright with this bid in round $\tilde{i} + 1$. But even in this case the player has a better response to refrain from bidding; thus $p_i = 0$ in any subgame perfect equilibrium. \square

Risk-Neutrality: Attention now turns to determining the equilibrium strategies for the remaining rounds in the case of risk-neutral expected utility. We have

Proposition 3.3.2. *Given $u(w) = w$, there exists a subgame perfect equilibrium such that bidding probabilities are given by*

$$p_i = \begin{cases} 1 & \text{if } i = 0 \\ 1 - \sqrt[N-1]{\frac{f}{v-\beta i}} & \text{if } 0 < i < \tilde{i} \\ 0 & \text{if } i \geq \tilde{i} \end{cases} \quad (3.3.5)$$

implying the hazard function

$$h(i) = \begin{cases} 0 & \text{if } i = 0 \\ \frac{f}{v-\beta i} & \text{if } 0 < i < \tilde{i} \\ 1 & \text{if } i \geq \tilde{i} \end{cases} \quad (3.3.6)$$

Proof. First, the equilibrium strategies for the case $i \geq \tilde{i}$ follow from proposition 3.3.1.

Now, turn to rounds $i < \tilde{i}$. We will assume that $V_N(i) = V_H(i) = 0$ in all these rounds, and show that it holds under the claimed equilibrium strategies. First, consider the continuation utility of being the highest bidder in round i . With probability $h(i)$ this leads to winning the auction, and otherwise to being non-highest bidder in round $i + 1$. This gives,

$$V_H(i) = h(i)(v - \beta i - f) + (1 - h(i))(V_N(i + 1) - f) \quad (3.3.7)$$

$$= \frac{f}{v - \beta i}(v - \beta i) - f = 0 \quad (3.3.8)$$

For the non-highest bidder, with probability $h(i)$ the auction ends in the current round, leaving the bidder with her wealth, normalised to zero in this section for readability. Otherwise, the auction continues and she may either become non-highest bidder, or conditional on having an accepted bid, highest bidder. This yields

$$V_N(i) = h(i)0 + (1 - h(i))(\zeta(i)V_H(i + 1) + (1 - \zeta(i))V_N(i + 1)) \quad (3.3.9)$$

$$= 0 \quad (3.3.10)$$

where the function $\zeta(i)$ reflects the probability of a submitted bid being accepted, i.e. any tie-breaking in round i . Thus, zero continuation utility are consistent with the equilibrium strategies. Given this, the player's utility of either bidding or not bidding is zero in each round. Thus, all mixtures are in the best response; hence, the claimed strategies constitute an equilibrium. \square

The intuition of the equilibrium is as follows: placing a bid in round i is essentially a gamble that no agent will overbid in round $i + 1$. The hazard rates in the proposition ensure that this gamble has a zero expected value, just like staying out of the auction. Hence, agents will be indifferent over any mixture of bidding/not bidding, and we are free to choose the winning probabilities in round $i + 1$ to make agents indifferent about bidding in round i .

As [Platt et al. \(2013, Proposition 2\)](#) show, other equilibria can arise in the risk-neutral setting. However, these equilibria require the auction ending either in round $i = 0$ or $i = 1$; intuitively, if in any round i a player strictly prefers to bid, then no player wants to bid in round $i - 1$, hence unravelling the equilibrium. Since such outcomes are not observed empirically, we focus attention on the non-degenerate equilibrium (and its versions under risk-aversion and prospect theory) in the subsequent analysis.

Since the model has no free parameters to estimate, the predictions of the model can be derived directly:

Proposition 3.3.3. *Under the equilibrium in proposition 3.3.4,*

1. *The expected return to a bid is zero in round $i < \tilde{i}$*
2. *Seller's expected revenue equals the retail price, $E(R) = v$*

Proof. 1. Follows immediately from the proof of proposition 3.3.4

2. See [Platt et al. \(2013, section 2.2\)](#)

□

In other words, the model fails to match any of the stylised facts. Expected revenue equals the retail value of the good, in contradiction of stylised fact 1. The price increment has no impact on revenue, and the expected return to bidding is zero in each round, not increasing as per stylised fact 4. Furthermore, profits are always – in contradiction of stylised fact 2, there is no dependence on the value of the good.

This prompted [Platt et al. \(2013\)](#) to extend the Penny Auction model for risk loving preferences, in the tradition of [Weitzman \(1965\)](#). That is, let the player now have CARA utility functions of the form

$$u(w) = \frac{1 - e^{-\alpha w}}{\alpha} \tag{3.3.11}$$

This leads to the revised equilibrium:

Proposition 3.3.4. *With CARA utility, there exists an equilibrium such that hazard rates are given by*

$$h(i) = \begin{cases} 0 & \text{if } i = 0 \\ \frac{e^{\alpha f} - 1}{e^{\alpha f} - e^{\alpha(f + \beta i - v)}} & \text{if } 0 < i < \tilde{i} \\ 1 & \text{if } i \geq \tilde{i} \end{cases} \quad (3.3.12)$$

supported by bidding probabilities

$$p_i = \begin{cases} 1 & \text{if } i = 0 \\ 1 - N^{-1} \sqrt{h(i)} & \text{if } 0 < i < \tilde{i} \\ 0 & \text{if } i \geq \tilde{i} \end{cases} \quad (3.3.13)$$

Proof. The steps are directly analogous to proposition 3.3.4. See Platt et al. (2013, section 2.3) for further details \square

With risk-loving preferences, agents will accept unfair some gambles. By continuity, the hazard rates that make bidders indifferent in the risk-loving case, must be *lower* than under risk-neutrality. Thus, the expected monetary return to bids will be negative. Hence the duration of the auction $f(i)$ first order stochastically dominates the duration under risk-neutrality; by the expression for expected revenue, this means that the seller must make higher expected revenue than the value of the good. Thus, the risk-loving model is in principle able to explain stylised fact 1.

However, further properties of the model under risk-loving preferences are difficult to obtain Platt et al. (2013). The authors conduct some simulations to show that lower price increments increase revenue, and profitability rises with the value of the good. We investigate below the performance of the risk-loving model empirically.

3.4 Prospects and Pennies

Prospect Theory, developed in Kahneman and Tversky (1979) and refined in Tversky and Kahneman (1992), seeks to be a *descriptive* theory of human behaviour under risk⁵. The key

⁵See Wakker (2010, ch. 9) for an exposition of the theory as applied in this section. See Barberis (2012a) for an application to slot-machine gambling

ingredients of the theory are as follows: first, lotteries are evaluated relative to a *reference point* rather than in terms of their absolute consequences, as in expected utility theory. Second, the decision-maker may not consider all possible outcomes of the lottery, but only an especially salient subset (“narrow framing”, see Barberis (2012b)). In principle, this frame may be manipulated to extract surplus from an agent. Third, the *value function* according to which outcomes are evaluated implies risk-aversion for gains relative to the reference point, and conversely risk-loving behaviour for losses relative to the reference. Relatedly, losses “loom larger than gains”: the disutility of a loss $-x$ exceeds in absolute the value the value gain x (loss aversion). Finally, outcomes of the lottery are not weighed by their objective probabilities, but subjective *decision weights* that are obtained from a probability weighting function.

The section proceeds by developing the prospect theory model of penny auction behaviour first for the case where the current bid is the reference point, and sketches some implications of the theory.

A decision-maker contemplates placing a bid in round $i < I$ of the auction, and entertains in her mind the possible outcomes of winning and not winning the auction with this bid, where the former outcomes occurs with objective probability μ_i . Conditional on having the bid accepted, the lottery is represented as the *prospect*

$$(V - \beta i - f, \mu_i; -f, 1 - \mu_i)$$

which is a *mixed prospect*, i.e. in case of winning, yields to a gain, and otherwise a loss.

The prospect is evaluated as

$$PT_i(\mu_i) = w^+(\mu_i)v(V - \beta i - f) + w^-(1 - \mu_i)v(-f) \quad (3.4.1)$$

where $v(x)$ is the value function, which is concave for $x > 0$ and convex for $x < 0$, with $v(x) > -v(-x)$ to capture loss aversion. We will be following the functional form used in Tversky and Kahneman (1992) in the analysis:

$$v(x) = \begin{cases} x^\alpha & \text{if } x > 0 \\ 0 & \text{if } x = 0 \\ -\lambda(-x)^\alpha & \text{if } x < 0 \end{cases} \quad (3.4.2)$$

where $\alpha \in (0, 1]$, and $\lambda \geq 1$. Secondly, the probability weighting functions $w^+(p)$ and $w^-(p)$ capture the notion of “diminishing marginal sensitivity”: for small probabilities, they

induce a disproportionately high decision weight ($w(p) > p$ when p small), while large probabilities are under-weighted. This idea is captured using the one-parameter functional form developed in [Lattimore et al. \(1992\)](#):

$$w(p) = \frac{p^\gamma}{p^\gamma + (1-p)^\gamma} \quad (3.4.3)$$

which nests probability weighting according to objective probabilities for the case of $\gamma = 1$.

For tractability, we impose the assumption of *reflection*, meaning that $w^+(p) = w^-(p)$, i.e. gains and losses are subject to the same probability weighting.

The alternative to the prospect of bidding is for players to remain at the reference point with probability one, which has value $v(0) = 0$. As under expected utility, the principle is to make bidders indifferent over their participation in each round on the equilibrium path; this is achieved through the determination of an appropriate sequence of hazard rates. Given the indifference of the players, we are free to set their probability of bidding so that the required hazard rates are implemented.

This leads to the following definition equilibrium:

Proposition 3.4.1. *An equilibrium under prospect theory is a sequence of hazard rates solving*

$$\left(\frac{p}{1-p}\right)^\gamma = \lambda \left(\frac{f}{v - \beta i - f}\right)^\alpha \quad (3.4.4)$$

for rounds $i = 1, \dots, \tilde{i} - 1$. Additionally, we have $p(0) = 1$ and $p(i) = 0$ for $i \geq \tilde{i}$. The equilibrium can be supported through a sequence of symmetric mixed strategies.

Proof. As in the expected utility case, for a behavioural agent to be indifferent over bidding and not bidding, we must have

$$PT_i(\mu_i^*) = 0 \text{ for } i = 0, \dots, I$$

Since $w(1) = 1$, the last round in which it is possible to find an interior winning probability coincides with the expression derived in proposition [3.3.1](#).

Now, consider interior rounds. Solving directly for the indifference condition yields:

$$\left(\frac{p}{1-p}\right)^\gamma = \lambda \left(\frac{f}{v - \beta i - f}\right)^\alpha \quad (3.4.5)$$

for all $i = 0, \dots, \tilde{i} - 1$.

Since players are indifferent between the prospects, as before, we are free to assign bidding probabilities to individual players that support the required hazard rates.

□

Observe that the risk-neutral equilibrium derived above is nested as a special case of the prospect theory model when there is no loss aversion and the exponents of value and probability weighting function are equal:

Lemma 3.4.1. *Let $\lambda = 1$ and $\alpha = \gamma$. Then, the hazard rates of the prospect theory model coincide with the risk-neutral benchmark*

Proof. The result follows from direct substitution. We have

$$\frac{p}{1-p} = \left(\frac{f}{V - \beta i - f} \right) \quad (3.4.6)$$

$$p = \frac{f}{V - \beta i} \quad (3.4.7)$$

□

Interest now turns to the question if, through suitable parameters, the prospect theory model is able to explain the stylised facts of penny auction behaviour that are observed empirically. First, consider the case without loss aversion:

Lemma 3.4.2. *Let $\lambda = 1$. If $\gamma < \alpha$, rounds with $V - \beta i < 2f$ have a positive expected return to bidding; and rounds with $V - \beta i > 2f$ have a negative expected return to bidding*

Proof. The implicit function for the ending probability in this case is

$$\left(\frac{p}{1-p} \right)^{\frac{\gamma}{\alpha}} = \left(\frac{f}{v - \beta i - f} \right) \quad (3.4.8)$$

By the results of the previous section, the risk-neutral ending probabilities yield a zero expected return. The task is thus to compare the solutions of the present equation to those of 3.4.6; whenever the p solving the current equation is below the solution of 3.4.6 (p_{RN} , equilibrium probability of winning is lower than under risk-neutrality, and so is the expected (monetary) return to a bid.

First, let $V - \beta i = 2f$. In this case, $p = p_{RN}$, and the expected return to a bid under PT coincides with the expected return to a bid under RN. Second, let $v - \beta i < 2f$. In this case, for $\gamma/\alpha < 1$, $p > p_{RN}$ by simple algebra. Hence, the expected return to bidding is positive in the case that the utility of winning is low. Finally, if $v - \beta i > 2f$, the expected return to bidding is negative as $p < p_{RN}$.

□

Thus, under prospect theory, there exists a critical round \hat{i} where the expected return to bidding changes sign. In rounds before \hat{i} , the potential gains from winning are high. Equilibrium requires a low probability of winning, which is in the region where the agent overweights probabilities. Hence, the expected return to bidding is negative. In late rounds, the gain from winning is small, and equilibrium requires a probability of winning. But high probabilities are under-weighted by our behavioural agents, and thus the monetary expected return to bidding is positive. Qualitatively, we can thus capture stylised fact 4. Quantitatively, the model without loss aversion is less successful: for example, in the case of a fixed-price auction with bidding fee GBP 0.50, we would have positive excess revenue whenever the value of the object exceeds GBP 1.00. But this cut-off is far lower than discussed in stylised fact 3. Thus, the model predicts excess revenue, but in too many cases compared to what we observe in the data.

Now, consider the loss aversion parameter λ . In general, loss aversion makes the prospect of bidding less attractive for the agent. Thus, agents will require higher probabilities of winning to be indifferent about participating in the auction; this results in lower revenue for the seller. To find the critical round where expected revenue changes sign, we compute

$$\left(\frac{p}{1-p}\right)^{\frac{\gamma}{\alpha}} \lambda^{\frac{-1}{\alpha}} = \left(\frac{p}{1-p}\right) = \left(\frac{f}{v - \beta i - f}\right) \quad (3.4.9)$$

After some algebra, this yields

$$v - \beta i = f \left(1 + \lambda^{\frac{1}{\alpha-\gamma}}\right) \quad (3.4.10)$$

Which is increasing in λ , as expected.

To fix ideas, consider again a numerical example with a final-price off auction. Let $\alpha = 1$, so the agent is risk-neutral, and fix $\lambda = 2.25$ in line with [Tversky and Kahneman \(1992\)](#). Then, given the bidding fee of GBP 0.50, a probability weighting parameter $\gamma = 0.80$ predicts the break-even value of the good $v = 29.33$, which is chosen to match stylised fact 2 quantitatively. Applying the same parameters to obtain the expected revenue for an auction of the Nikon D90, with retail price GBP 791.49, yields a revenue prediction GBP 1809.33. This compares to an observed value of GBP 1995.60 in the sample (stylised fact 1). Moreover, consider the Nintendo Wii example from stylised fact 4. Solving equation [3.4.10](#) for the critical round where expected return to bidding is positive yields $\tilde{i} = 1506.67$. This is higher than the empirical value (1180), but at least an approximation. Thus, even with “manual calibration”, it appears that the model can match several stylised facts of the dataset.

3.5 Estimation

The theories of bidder behaviour, both under expected utility and prospect theory, generate predictions regarding the *hazard rate*, i.e. the probability that an auction will end in a given round, depending on the parameters. This suggests a maximum likelihood approach to model estimation. Thus, let X denote the random variable that a penny auction will end in round x . For this to occur, the auction there must have been no hazard in any previous round and hazard in round x , thus the probability of the event is given by

$$f(x; \theta) = h(x; \theta) \prod_{j=1}^{x-1} (1 - h(j; \theta)) \quad (3.5.1)$$

The hazard rate is implied by the equilibrium strategies:

$$h(i) = \prod_{j=0}^{i-1} (1 - \sigma_j) \quad (3.5.2)$$

In a sample of observations $o = 1, \dots, O$, where o_j denotes the duration of auction j in the sample, the log likelihood function is then given by

$$llf(\theta) = \sum_{o=1}^O \log(f(o_i; \theta)) \quad (3.5.3)$$

Since closed form expressions are not available for $f(x; \theta)$, optimization must be carried out numerically. This yields estimates $\hat{\theta}$ for each of the models considered. For inference, we rely on the standard property that

$$\sqrt{N}(\hat{\theta} - \theta) \rightarrow N(0, \mathcal{I}(\theta)^{-1}) \quad (3.5.4)$$

in sufficiently large samples, where $\mathcal{I}(\theta)$ is the Fisher information matrix. Thus, the estimator has a standard sampling distribution and usual significance tests can be carried out. These procedures we implemented in GNU R.

3.6 Results

This section applies the methods of section 3.5 to the data. For estimation, a sub-sample was used. First, the *Nintendo Wii* was included since it is the most frequently sold product in the

dataset (N=7258 observations), and observations are available for both the 10 cent and one-cent price increment format. Second, the *Playstation 3* was included, on the grounds of being a close substitute product and similarly a large number of auctions (N=1775) being included in the sample. The use of a sub-sample was necessitated by computational constraints.

We proceed by developing the results for the risk-loving case (subsection 3.6.1). Then, subsection 3.6.2 turns to prospect theory.

3.6.1 Risk Attitudes

Estimates for the CARA parameter α are provided in figure 3.7. For both products, the coefficients are precisely estimated, as indicated by significance at the 1% level. In the case of the Nintendo Wii, the estimated risk-love parameter -0.012 is very mild; for the Playstation 3, an even lower value of -0.006 is obtained.

	(1)	(2)
Sample	Wii	PS3
α	-0.012^{***}	-0.006^{***}
	(0.0001)	(0.0002)
$-2\log L$	104645.3	27236.35
N	7258	1775

Figure 3.7: Estimate of Risk-Attitudes Model

In both cases, the negative values of α indicate risk-loving preferences, and are thus qualitatively consistent with the theory developed in section 3.3. However, especially in the case of the Playstation 3, the bidders' taste for risk is rather mild: at a risk-love of only -0.006 , preferences are very close to risk-neutrality (which obtains as $\alpha \rightarrow 0$). Secondly, the instability of parameters between the products may give rise to some concern.

These estimates agree with Platt et al. (2013), which is to be expected given the similarity of the dataset. Secondly, their estimates are also rather unstable between auctions, ranging from from -0.0001 to -0.09 for the risk-love parameter α . Jullien and Salanié (2000) estimate $\alpha = -0.055$ from the horse betting dataset⁶. In comparison, the estimated deviation from risk-neutrality seems to be minor.

⁶Since they do not observe the stake size (a in their notation) the parameter α is not identified. The reported estimate is for a unit stake

3.6.2 Prospect Theory

Turning to the prospect theory model, figure 3.8 provides the results. For the Wii dataset, all coefficients are precisely estimated and significant at the 1% level. Unfortunately, the smaller Playstation 3 dataset yields less precise estimates, with only one parameter significant at the 10% level. This may be attributed to the fact that the more complex model, with fewer degrees of freedom compared to the risk-loving case, makes rather strong demands on sample size for estimation. Regardless, the estimated values of the parameters are close for both sub-samples.

	(1)	(2)
Sample	Wii	PS3
α	0.6765*** (0.2650)	0.6639* (0.4874)
γ	0.4912*** (0.1926)	0.4334* (0.3172)
λ	1.83639*** (0.4487)	2.8946 (2.3107)
-2logL	104917	27270.57
N	7258	1775

Figure 3.8: Estimates of Prospect Theory Model

Qualitative Fit: The estimated parameters are consistent with the theory developed in section 3.4:

1. $\alpha < 1$: Value function has diminishing marginal sensitivity over gains. As required, they exhibit risk-aversion over gains risk-love over losses.
2. $\gamma < 1$: Low probabilities are overweighted and large probabilities are underweighted
3. $\lambda > 1$: Losses loom larger than gains, so the preferences exhibit loss-aversion

Quantitatively, the estimates for risk-aversion ($\alpha = 0.67$ and 0.66 respectively) indicate substantial diminishing marginal sensitivity. In [Tversky and Kahneman \(1992\)](#), the corresponding parameter was estimated at 0.88 for the median experimental subject. In contrast, the degree of loss aversion appears to be somewhat weaker than in experimental studies.

In the Wii dataset, $\lambda = 1.83$ is estimated, as opposed to $\lambda = 2.25$ from [Tversky and Kahneman \(1992\)](#). As [Barberis \(2012a\)](#) emphasises, the degree of loss aversion may depend on the context of decision problem; it is reasonable to believe that (self-selected) participants in the penny auction, who are inclined to gamble, have lower than median loss aversion.

Probability Weighting: Probability weighting is the fundamental ingredient of the prospect theory model to explain excess revenue (where it arises). At an estimate of $\gamma = 0.49$ (Wii dataset; $\gamma = 0.43$ for PS3), a strong degree of probability weighting is indicated in the sample. For example, a physical probability of 1% would have a decision weight close to 10% ($w(0.01) = 0.09$).

Self-selection into the penny auction may explain why probability weighting is so strong in the dataset. The majority of observations contain rounds where the physical probability of winning is low and returns are high; only agents with strong probability weighting would participate in such a game. Thus, the estimates appear natural to the penny auction setting.

Figure 3.9 compares the obtained probability weighting function to the experimental study of [Camerer and Ho \(1994\)](#), and the observational study of [Gurevich et al. \(2009\)](#). In the Camerer and Ho's study, participants did not self-select, and their estimates of probability weighting should thus represent the bias of a typical subject. Gurevich et al., in contrast, used data from financial markets; in these markets, it is possible to "bet against" any behavioural biases. As one would expect, the probability weighting function is close to linearity in their setting. Contrasting with this, in the penny auction, a rational bidder would in most rounds have as best response only to abstain from the auction – and hence not creating a market incentive for probability weighting to disappear.

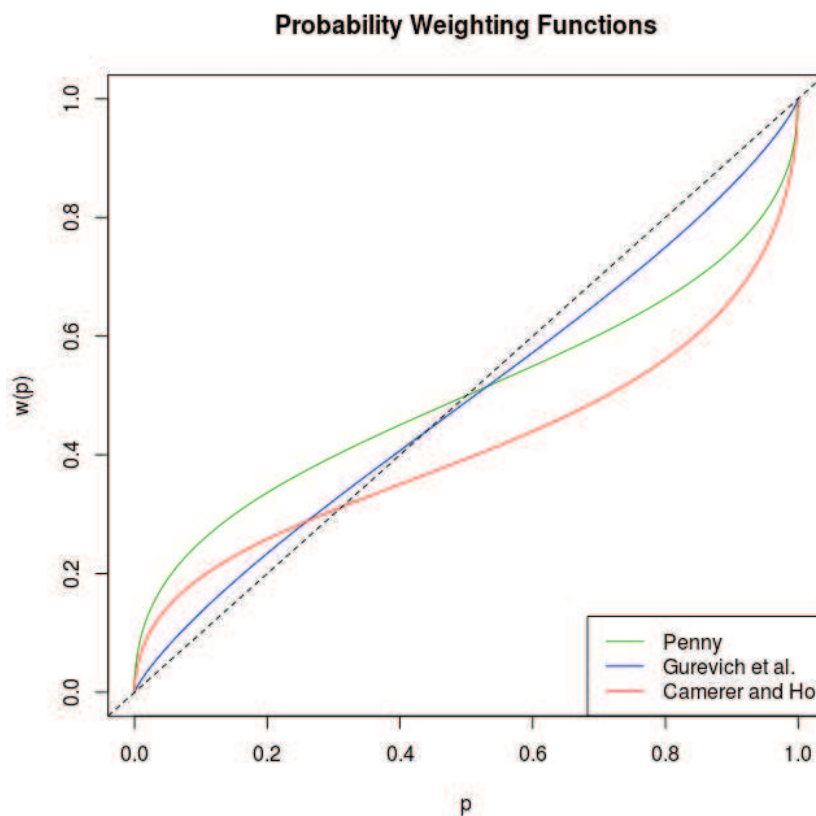


Figure 3.9: Comparison of Probability Weighting Functions

3.7 Understanding Penny Auctions?

Excess Revenue: Figure 3.10 shows the predicted revenue implied by the estimates of the risk-loving and prospect theory model for the products in the sample. Qualitatively, both models capture the fact that expected revenue exceeds the retail value of the object. This is to be expected from the theoretical discussion, as the risk-loving model is guaranteed to produce revenue greater than retail value; in the case of prospect theory, the sample contains high-priced objects, and these are expected to yield positive profits.

Quantitatively, the expected revenue predictions of the two models are similar for the case of 10p auctions, and tend to somewhat over-predict revenue. For 10p auctions, the Wii's observed expected revenue was 297.2, while the prospect theory (risk attitudes) model predicts 346.68 (358.68); thus, both models over-predict, but the prospect theory model is closer to the observed values. This pattern also holds for the PlayStation auctions. Thus,

we conclude that both models can explain stylised fact 1.

Product	Price Increment	Observed	Risk Attitudes	CPT
Nintendo Wii	0.10	297.27	354.68	346.57
Nintendo Wii	0.01	423.38	538.40	453.00
PlayStation 3	0.10	455.23	554.44	547.67

Figure 3.10: Predicted and Actual Revenue

Price Increment: For Wii auctions, the sub-sample also contains auctions with a GBP 0.01 price increment. These yielded higher revenue than 10p auctions for the same object, consistent with stylised fact 3. As figure 3.7, both models qualitatively match this increased revenue. The prospect theory model comes close to matching expected revenue quantitatively, too: it predicts GBP 453.00 revenue, compared to an observed value of GBP 423.38. In contrast, the risk-loving model substantially over-predicts revenue (GBP 554.44). In conclusion, the effect of the price increment is better matched by the prospect theory model.

Unprofitable Auctions: Auctions for low-priced objects tend to be unprofitable, as highlighted in stylised fact 3. The risk attitudes model is not able to match this fact either qualitatively or quantitatively.

For the prospect theory model, equation 3.4.10 provides a threshold such that all auctions with retail price below that value are unprofitable. Applying the equation to the estimates of 3.8, yields a prediction that auctions will retail value below GBP 14.86 are unprofitable. In contrast, the threshold estimated from the regression model reported in figure 3.3 imply a threshold of GBP 28.89. Clearly, the prospect theory model qualitatively matches the stylised fact. Quantitatively, the model's implied threshold is too low, pointing to possible functional form issues. However, given that the estimation was carried out for products with a retail value almost an order of magnitude away from the threshold, the out of sample performance is reasonable. At any rate, prospect theory delivers a good qualitative fit of stylised fact 3.

Expected Return to Bidding: As the auction progresses into later rounds, the expected return to placing bid rises as per stylised fact 4. Figure 3.11 compares the actual expected return to the values implied by both models.

For Wii auctions, the risk-loving model does very well in early rounds of bidding, match-

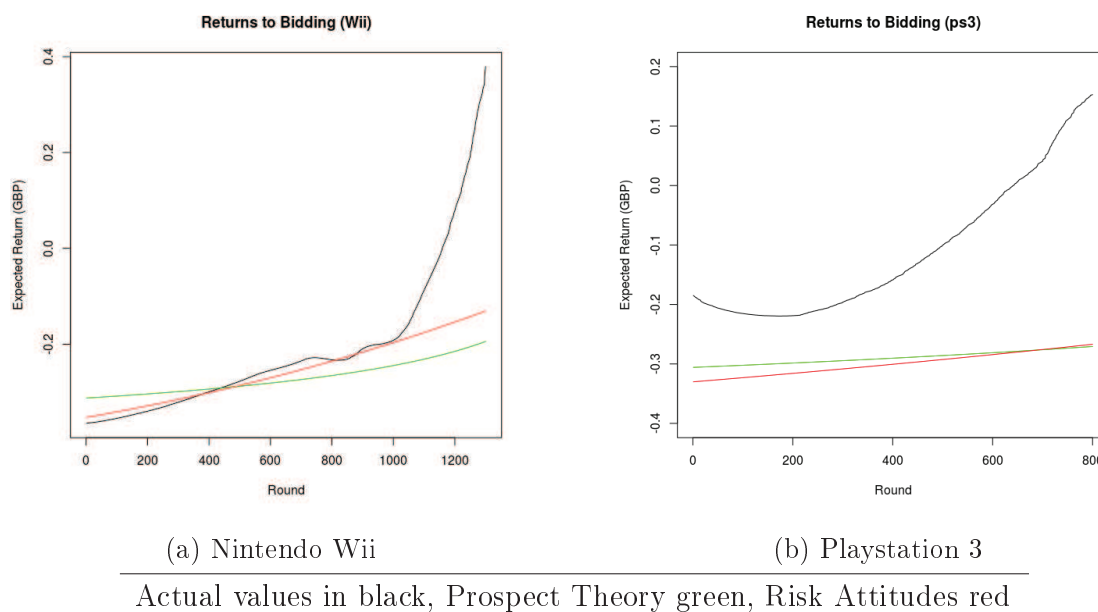


Figure 3.11: Actual and Predicted Expected Returns to Bidding

ing qualitatively and quantitatively the data. In late rounds, the observed return rises quickly above the predicted value, and the model is unable to explain even qualitatively the positive return enjoyed by bidders in late rounds. In contrast, the prospect theory model catches less of the variation between rounds, over-predicting returns in early rounds and under-predicting by a greater margin late in the game. Both models provide a poor fit of within-product variation for PlayStation (3) auctions, as is evident from part (b) of the figure. Returns are under-predicted consistently, in line with the over-predicted revenue discussed above; understanding why both models fail for this product is an interesting question for future research.

3.8 Conclusion

Penny auctions may well be among the largest economic field experiments ever conducted, and as such have yielded a rich dataset to investigate human behaviour under risk. This shows that penny auctions are often profitable for the seller, especially if high-value items are sold; but losses may loom for low-value items. Lower price increments are associated with greater profitability, and the expected return to bidding rises with the auction rounds. These facts have their analogues in other betting markets, such as for horse races.

We then applied prospect theory to understand the features in the data. The fitted model implies that agents are loss averse and have a diminishing marginal sensitivity over gains, as found in other studies, laboratory and observational. However, bidders appear to strongly overweight low probabilities – and underweight high probabilities – which explains why these auctions can be profitable. The theory matches the stylised fact qualitatively that auctions for low-value objects are often unprofitable. Thus, one may see penny auctions as a mechanism to extract a small amount of surplus from many behavioural bidders.

The alternative hypothesis is risk-loving behaviour. The risk-loving model can also explain high revenue, but quantitatively over-predicts revenue and suffers from parameter instability. Moreover, it was not able to explain why auctions for low-priced products are often loss making even in qualitative terms.

In conclusion, prospect theory provides a very appealing framework to understand behaviour in penny auctions. It offers very good qualitative, and often quantitative fit, with the stylised facts identified. In future research, it would be very interesting to consider alternative functional forms or non-parametric estimation to further refine our understanding of penny auctions.

Bibliography

- Augenblick, N., 2012. Consumer and producer behavior in the market for penny auctions: A theoretical and empirical analysis. Unpublished manuscript. Available at www.stanford.edu/~ned789.
- Barberis, N., 2012a. A model of casino gambling. *Management Science* 58 (1), 35–51.
- Barberis, N. C., 2012b. Thirty years of prospect theory in economics: A review and assessment. Tech. rep., National Bureau of Economic Research.
- Byers, J. W., Mitzenmacher, M., Zervas, G., 2010. Information asymmetries in pay-per-bid auctions. In: *Proceedings of the 11th ACM conference on Electronic commerce*. ACM, pp. 1–12.
- Camerer, C. F., 2004. Prospect theory in the wild: Evidence from the field. Colin F. Camerer, George Loewenstein, and Matthew. Rabin, eds., *Advances in Behavioral Economics*, 148–161.
- Camerer, C. F., Ho, T.-H., 1994. Violations of the betweenness axiom and nonlinearity in probability. *Journal of risk and uncertainty* 8 (2), 167–196.
- Griffith, R. M., 1949. Odds adjustments by american horse-race bettors. *The American Journal of Psychology* 62 (2), 290–294.
- Gurevich, G., Kliger, D., Levy, O., 2009. Decision-making under uncertainty-a field study of cumulative prospect theory. *Journal of Banking & Finance* 33 (7), 1221–1229.
- Hinnosaar, T., 2010. Penny auctions are unpredictable.
- Hodges, S., Tompkins, R., Ziemba, W., 2003. The favorite/long-shot bias in s&p 500 and ftse 100 index futures options: the return to bets and the cost of insurance. In: *EFA 2003 Annual Conference Paper*. No. 135.

- Jullien, B., Salanié, B., 2000. Estimating preferences under risk: The case of racetrack bettors. *Journal of Political Economy* 108 (3), 503–530.
- Kahneman, D., Tversky, A., 1979. Prospect theory: An analysis of decision under risk. *Econometrica: Journal of the Econometric Society*, 263–291.
- Lattimore, P. K., Baker, J. R., Witte, A. D., 1992. The influence of probability on risky choice: A parametric examination. *Journal of economic behavior & organization* 17 (3), 377–400.
- Malmendier, U., Lee, Y. H., 2011. The bidder's curse. *The American Economic Review* 101 (2), 749–787.
- Malmendier, U., Szeidl, A., 2008. Fishing for fools. Tech. rep., Berkeley University mimeo.
- Ockenfels, A., Reiley, D., Sadrieh, A., 2006. Online auctions. Tech. rep., National Bureau of Economic Research.
- Ottaviani, M., Sørensen, P. N., 2008. The favorite-longshot bias: an overview of the main explanations. *Handbook of Sports and Lottery Markets* (eds. Hausch, DB and Ziemba, WT), North-Holland/Elsevier, 83–102.
- Platt, B. C., Price, J., Tappen, H., 2013. The role of risk preferences in pay-to-bid auctions. *Management Science* (in print).
- Resnick, P., Zeckhauser, R., Swanson, J., Lockwood, K., 2006. The value of reputation on ebay: A controlled experiment. *Experimental Economics* 9 (2), 79–101.
- Snowberg, E., Wolfers, J., 2010. Explaining the favorite-longshot bias: Is it risk-love or misperceptions? Tech. rep., National Bureau of Economic Research.
- Starmer, C., 2000. Developments in non-expected utility theory: The hunt for a descriptive theory of choice under risk. *Journal of economic literature* 38 (2), 332–382.
- Thaler, R., 2009. Paying a price for the thrill of the hunt.
URL <http://www.nytimes.com/2009/11/15/business/economy/15view.html>
- Trepel, C., Fox, C. R., Poldrack, R. A., 2005. Prospect theory on the brain? toward a cognitive neuroscience of decision under risk. *Cognitive Brain Research* 23 (1), 34–50.

- Tversky, A., Kahneman, D., 1992. Advances in prospect theory: Cumulative representation of uncertainty. *Journal of Risk and uncertainty* 5 (4), 297–323.
- Wakker, P. P., 2010. *Prospect theory: For risk and ambiguity*. Cambridge University Press Cambridge.
- Wang, Z., Xu, M., 2013. Selling a dollar for more than a dollar? evidence from online penny auctions. Tech. rep., Resources for the Future.
- Weitzman, M., 1965. Utility analysis and group behavior: An empirical study. *The Journal of Political Economy* 73 (1), 18–26.
- Woodland, L. M., Woodland, B. M., 1994. Market efficiency and the favorite-longshot bias: The baseball betting market. *The Journal of Finance* 49 (1), 269–279.

