European University Institute

DEPARTMENT OF ECONOMICS

Three Essays in Collective Decision Making

Niall Hughes

Thesis submitted for assessment with a view to obtaining the degree of Doctor of Economics of the European University Institute

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**Department of Economics**


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# Abstract

This thesis is a collection of three essays on voting as a means of collective decision-making. The first chapter builds a model of how voters should optimally behave in a legislative election with three parties under plurality rule. I show that, in contrast to single district elections, properties such as polarisation and misaligned voting can be mitigated in legislative elections. The second chapter studies a model of committee decision making where members have career concerns and a principal can choose the level of transparency (how much of the committees decision he can observe). We show that increased transparency leads to a breakdown in information aggregation, but that this may actually increase the principal's payoff. The theoretical model is then tested in a laboratory experiment. The final chapter introduces a model of legislative bargaining where three parties in the legislature bargain over the formation of government by choosing a policy and a distribution of government perks. I show that when individual politicians are responsible for the policies they implement - that is, those outside of government are not held accountable by voters for the implemented governments policies, while each individual politician in the ruling coalition is - then a given seat distribution can result in almost any two party coalition.

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# Summary

Conventional models of single district plurality elections show that with three parties anything can happen - extreme policies can win regardless of voter preferences. In Chapter 1, I show that when there are multiple district elections for a legislature we get back to a world where the median voter matters: an extreme policy will generally only come about if it is preferred by the median voter in a majority of districts, while the existence of a centrist party can lead to moderate outcomes even if the party itself wins few seats. Furthermore, I show that while single district elections always have *misaligned voting* i.e. some voters do not vote for their preferred choice, equilibria of the legislative election exist with no misaligned voting in any district. Finally, I show that when parties are impatient, a fixed rule on how legislative bargaining occurs will lead to more coalition governments, while uncertainty will favour single party governments.

In Chapter 2, (joint work with Sebastian Fehrler) we show theoretical and experimental results on the role of transparency in committee decision making and deliberation. We present a model in which committee members have career concerns and unanimity is needed to change the status quo. Transparency leads to a break down of information aggregation, causing more incorrect group decisions. However, if the cost of wrongly changing the status quo is high enough, the principal will be better off in expectation under transparency than under secrecy - he is helped by the failure of information aggregation. We test the model in a laboratory experiment with two member committees playing under three levels of transparency. We observe strong effects of transparency on committee error rates and information aggregation that are largely consistent with the

model's predictions. On the individual level, we observe strong effects on deliberative behaviour which go in the predicted direction but are less pronounced than in theory.

In existing legislative bargaining models, the precise division of seats between parties has no bearing on some of: which coalition forms, which policy is adopted, how perks are divided - or even all three. These models also predict that the proposer should reap significantly larger rewards than the other players. Such predictions are, however, at odds with longstanding empirics: government portfolios are generally allocated in proportion to seat share, and there is no proposer advantage. In Chapter 3, I show that when each member of a party faces the electoral consequences of being in government then seat shares matter a great deal: (1) For a given ranking of parties, changing their respective seat shares can bring about almost any coalition; (2) the implemented policy is a function of the coalition parties seat shares; (3) an increase in one coalition party's seats will move the policy towards their preferred point, but may increase or decrease their share of government perks. Furthermore, I show that (4) there can be equilibria in which the largest party is not government, and (5) in many cases the larger coalition member will have all of his rent extracted by the junior member.

# Contents

# Chapter 1

# Voting in Legislative Elections under Plurality Rule

Conventional models of single district plurality elections show that with three parties anything can happen - extreme policies can win regardless of voter preferences. I show that when there are multiple district elections for a legislature we get back to a world where the median voter matters: an extreme policy will generally only come about if it is preferred by the median voter in a majority of districts, while the existence of a centrist party can lead to moderate outcomes even if the party itself wins few seats. Furthermore, I show that while single district elections always have *misaligned voting* i.e. some voters do not vote for their preferred choice, equilibria of the legislative election exist with no misaligned voting in any district. Finally, I show that when parties are impatient, a fixed rule on how legislative bargaining occurs will lead to more coalition governments, while uncertainty will favour single party governments.

## 1.1 Introduction

Plurality rule (a.k.a. first-past-the-post) is used to elect legislatures in the U.S., U.K., Canada, India, Pakistan, Malaysia as well as a host of other former British colonies - yet we know very little about how it performs in such settings. The literature on single district elections shows that plurality rule performs well when there are only two candidates but poorly when there are more.[1] Indeed, plurality has recently been deemed the worst voting rule by a panel of voting theorists.[2] However, the objectives of voters are different in single district and legislative elections. In a legislative election, many districts hold simultaneous plurality elections and the winner of each district takes a seat in a legislature. Once all seats are filled, the elected politicians bargain over the formation of government and implement policy. If voters only care about which policy is implemented in the legislature, they will cast their ballots to influence the outcome of the legislative bargaining stage. A voter's preferred candidate will therefore depend on the results in other districts. Meanwhile, in a single district election, a voter's preference ordering over candidates is fixed, as only the local result matters. These different objectives are at the heart of this paper. I show that when three parties compete for legislative seats and voters care about national policy, several negative properties of plurality rule are mitigated.

While there has been some key work on voting strategies in legislative elections under proportional representation (PR), notably Austen-Smith and Banks (1988) and Baron and Diermeier (2001), there has been scant attention paid to the question of how voters should act when three parties compete in a legislative election under plurality rule. Studies of plurality rule have either focused on two-party legislative competition or else on three-party single district elections, in which voters only care about the result in that district. In the former case, as voters face a choice of two parties, they have no strategic decision to make - they simply vote for their favourite. However, for almost all countries using plurality rule, with the notable exception of the U.S., politics is not a two-party

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[1]See for example Myerson (2000) and Myerson (2002).
[2]See Laslier (2012)

game: the U.K. has the Conservatives, Labour and the Liberal Democrats; Canada has the Conservatives, Liberals, and New Democrats; India has Congress, BJP and many smaller parties.[3] With a choice of three candidates, voters must consider how others will vote when deciding on their own ballot choice.

In a single plurality election, only one candidate can win. Therefore, when faced with a choice of three options, voters who prefer the candidate expected to come third have an incentive to abandon him and instead vote for their second favourite, so that in equilibrium only two candidates receive votes. These are the only serious candidates. This effect, known as Duverger's law[4], was first stated by Henry Droop in 1869:

> "Each elector has practically only a choice between two candidates or sets of candidates. As success depends upon obtaining a majority of the aggregate votes of all the electors, an election is usually reduced to a contest between the two most popular candidates or sets of candidates. Even if other candidates go to the poll, the electors usually find out that their votes will be thrown away, unless given in favour of one or other of the parties between whom the election really lies." (Droop cited in Riker (1982), p. 756)

A vast literature has pointed out two negative implications of Duverger's law in single district elections.[5] First, *"anything goes"*: the equilibrium is completely driven by voters' beliefs, so *any* of the three candidates could be abandoned, leaving the other two to share the vote. This means that, regardless of voter preferences, there can always be polarisation - where a race between the two extreme choices results in an implemented

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[3]Other countries with plurality rule and multiple parties represented in the legislature include: Bangladesh, Botswana, Kenya, Liberia, Malawi, Malaysia, Mongolia, Pakistan, Trinidad & Tobago, Tanzania, Uganda, and Zambia.

[4]The law takes it's name from French sociologist Maurice Duverger who popularised the idea in his book *Political Parties*. While Riker (1982) argues that Duverger's law should be interpreted as the tendency of plurality rule to bring about a two-party system, most scholars use the term to describe the local effect: in any one district only two candidates will receive votes. I also use the local interpretation.

[5]See Palfrey (1989), Myerson and Weber (1993), Cox (1997), Fey (1997), Myerson (2002), Myatt (2007).

policy far away from the centre. Second, when each of the three choices is preferred by some voter, there will always be *misaligned voting.* That is, some people will vote for an option which is not their most preferred. Misaligned voting undermines the legitimacy of the elected candidate: one candidate may win a majority simply to "keep out" a more despised opponent, so the winner's policies may actually be preferred by relatively few voters.

In this paper, I compare my model of legislative elections with "national" voters, who care only about government policy, to the two other types of plurality elections typically studied in the literature: (1) single district elections such as presidential elections, and (2) legislative elections with "local" voters i.e. where voters only care about the winner in their own district. I show that the two negative properties of plurality elections - polarisation and misaligned voting - while always present in the traditional cases, need not hold in my setting.

The intuition for the polarisation result is as follows. For any party to win a majority of seats it must be that they are preferred to some alternative by a majority of voters in a majority of districts. In a presidential election or a legislative election with local voters, it can always be that voters focus on races between the left and right candidates, ignoring the centrist candidate. In such cases we would witness non-centrist policies, for any distribution of voter preferences. Instead, in a legislative election with national voters, the alternative to a left majority will generally not be a right majority but rather a moderate coalition government. Therefore, for an extreme policy to come about, it must be that the median voter in the median district prefers this policy to the moderate coalition policy.

The misaligned voting result stems from the fact that voters condition their ballots on a wider set of events in my setting. In a standard plurality election, voters condition their vote on the likelihood of being pivotal in their district. However, in a legislative election, national voters will condition their ballot choice on their vote being pivotal <u>and</u> their district being decisive in determining the government policy. In many cases a district will be decisive between two policies, even though there are three candidates.

For example, a district might be decisive in either granting a majority of seats to a non-centrist party, say the left party, or bringing about a coalition by electing one of the other parties. Under many bargaining rules this coalition policy will be the same regardless of which of the weaker parties is elected. So, voters only face a choice between two policies: that of the left party and that of the coalition. When voters have a choice over two policies there can be no misaligned voting - everyone must be voting for their preferred option of the two.

I examine the workings of my model under several legislative bargaining settings - varying the scope of bargaining, the patience of politicians, and the bargaining protocol. Government formation processes *do* vary across countries. In some countries potential coalition partners may bargain jointly over policy and perks, while in others perks may be insufficient to overcome ideological differences. The patience of politicians will also differ across countries depending on aspects such as how quickly successive rounds of bargaining occur, and how likely politicians are to be re-elected.[6] A further feature of government formation which has been studied extensively is the protocol for selecting a formateur (a.k.a. proposer). The two standard cases are *random recognition* - where a party's probability of being the formateur in each round of bargaining is equal to its seat share - and *fixed order* - where the largest party makes the first offer, then the second largest, and so on. Diermeier and Merlo (2004) analyse 313 government formations in Western European over the period 1945–1997 and find the data favours random recognition rule. On the other hand, Bandyopadhyay et al. (2011) note that a fixed order of bargaining is constitutionally enshrined in Greece and Bulgaria, and is a strong norm in the U.K. and India, where elections are held under plurality rule.

In my benchmark model, parties in the legislature bargain only over policy and they do not discount the future. Here, if no party holds a majority of seats, the median party's policy will be implemented. Two clear predictions emerge from this benchmark model. First, when the median party wins at least one seat, polarisation is mitigated:

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[6]See Austen-Smith and Banks (2005) and Banks and Duggan (2006) for a discussion of discount rates in legislative bargaining.

the policy of the left or right party can only be implemented if a majority of voters in a majority of districts prefer it to the policy of the median party. Second, if either the left or right party is a serious candidate in less than half of the districts, there can be no misaligned voting. These results change somewhat under different bargaining rules, but their flavour remains the same. When parties bargain over perks of office as well as policy, the polarisation result is strengthened - it is even more difficult to have extreme outcomes - while the misaligned voting result is weakened - it can only be ruled out if a non-centrist party is serious in less than a quarter of districts.

Finally, when politicians are impatient, I show that if a country uses a fixed order of recognition there will be a higher incidence of coalition governments than if it uses a random recognition rule, all else equal. The reason is that a fixed order rule gives a significant advantage to the largest party and also makes it easier for voters to predict which government policy will be implemented after the election. As the difference in policy between, say, a left majority government and a coalition led by the left is quite small, voter preferences must be very much skewed in favour of the left party in order for it to win a majority. With a random recognition rule, however, risk averse voters will prefer the certainty of a non-centrist single-party government to the lottery over policies which coalition bargaining would induce.

This paper contributes primarily to the theoretical literature on strategic voting in legislative elections. The bulk of this works has been on PR. Austen-Smith and Banks (1988) find that, with a minimum share of votes required to enter the legislature, the median party will receive just enough votes to ensure representation, with the remainder of its supporters choosing to vote for either the left or right party. Baron and Diermeier (2001) show that, with two dimensions of policy, either minimal-winning, surplus, or consensus governments can form depending on the status quo.On plurality legislative elections Morelli (2004) and Bandyopadhyay et al. (2011) show that if parties can make pre-electoral pacts, and candidate entry is endogenous, then voters will not need to act strategically. My paper nonetheless focuses on strategic voting because in the main countries of interest, the U.K. and Canada, there are generally no pre-electoral pacts and

the three main parties compete in almost every district, so strategic candidacy is not present.[7]

The paper proceeds as follows. In the next section I introduce the benchmark model and define an equilibrium. In section 1.3, I solve the model and show conditions which must hold in equilibrium. section 1.4 compares the level of polarisation and misaligned voting in the benchmark model with that of the other plurality elections typically studied in the literature. section 1.5 adds perks of office to the bargaining stage of the model, while section 1.6 shows how the benchmark results change when parties discount the future. Finally, section 1.7 discusses the assumptions of the model and concludes.

## 1.2 Model

**Parties** There are three parties; $l, m$, and $r$, contesting simultaneous elections in $D$ districts, where $D$ is an odd number. The winner of each of the $D$ elections is decided by plurality rule: whichever party receives the most votes in district $d \in D$ is deemed elected and takes a seat in the legislature. The outcome from all districts gives a distribution of seats in the legislature, $S \equiv (s_l, s_m, s_r)$, with party $c \in \{l, m, r\}$ having $s_c$ seats and $\sum_c s_c = D$. Party $c$ has a preferred platform $a_c$ in the unidimensional policy space $\mathbb{X} = [-1, 1]$ on which it must compete in every district. A party cannot announce a different platform to gain votes; voters know that a party will always implement its preferred platform if it gains a legislative majority. Once all the seats in the legislature have been filled, the parties bargain over the formation of government and implement a policy $z$. As such, a party cannot commit to implement its platform as the policy outcome $z$ depends on bargaining. Party $c$ has the payoff $W_c = b_c - (z - a_c)^2$, linear in its share of government benefits $b_c$, and negative quadratic in the distance between its platform $a_c$ and the implemented policy $z$. A feasible allocation of benefits is $b = (b_l, b_m, b_r)$ where each $b_c$ is non-negative and $\sum_c b_c \leq B$.

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[7]In the 2010 U.K. General Election, the three main parties contested 631 out of 650 districts (None of them contest seats in Northern Ireland), while in the 2011 Canadian Federal Election the three major parties contested 307 of the 308 seats.

The benchmark model I use is that of Baron (1991), where $B = 0$ so that bargaining is over policy alone.[8] In section 1.5, I consider the more complicated case of $B > 0$ due to Austen-Smith and Banks (1988). If a party has a majority of the seats in the legislature it can form a unitary government and will implement its preferred policy. If no party wins an outright majority we enter a stage of legislative bargaining. I consider two bargaining protocols: one in which the order of bargaining is random and one in which it is fixed. Under the former rule, one party is randomly selected as formateur, where the probability of each party being chosen is equal to its seat share in the legislature. The formateur proposes a policy in $[-1, 1]$, which is implemented if a majority of the legislature support it; if not, a new formateur is selected, under the same random recognition rule, and the process repeats itself until agreement is reached. Under the fixed order rule, the party with the largest number of seats proposes a policy in $[-1, 1]$, which is implemented if a majority of the legislature support it; if not, the second largest party proposes a policy. If this second policy does not gain majority support, the smallest party proposes a policy, and if still there is no agreement, a new round of bargaining begins with the largest party again first to move. I assume for now that parties are perfectly patient, $\delta = 1$, but this is relaxed in section 1.6. A party's strategy specifies which policy to propose if formateur, and which policies to accept or reject otherwise.

**Voters**  Individuals are purely policy-motivated with quadratic preferences on $\mathbb{X}$. As such, a voter does not care who wins his district *per se*, nor does he care which parties form government; all that matters is the final policy, $z$, decided in the legislature. A voter's type, $t \in T \subset \mathbb{X}$, is simply his position on the policy line; his utility is $u_t(z) = -(z - t)^2$.

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[8]A large literature has grown from legislative bargaining model of Baron and Ferejohn (1989), in which legislators bargain over the division of a dollar. See Baron (1991), Banks and Duggan (2000), Baron and Diermeier (2001), Jackson and Moselle (2002), Eraslan et al. (2003), Kalandrakis (2004), Banks and Duggan (2006), and many others. Morelli (1999) introduces a different approach to legislative bargaining whereby potential coalition partners make demands to an endogenously chosen formateur. In contrast with the Baron and Ferejohn (1989) setup, the formateur does not capture a disproportionate share of the payoffs.

I assume $T$ is sufficiently rich that for any tuple of distinct policies, $\{a_l, a_m, a_r\}$, there is at least one voter type who prefers one of the three over the other two. Furthermore, I assume for simplicity that there is no type which is exactly indifferent between two platforms. Let $V \equiv \{v_l, v_m, v_r\}$ be the set of feasible actions an individual can take, with $v_c$ indicating a vote for party $c$. Voting is costless; thus, there will be no abstention.

Following Myerson (2000, 2002), the number of voters in each district $d$ is not fully known but rather is a random variable $n_d$, which follows a Poisson distribution and has mean $n$. The probability that there are exactly $k$ voters in a district is $Pr[n_d = k] = \frac{e^{-n}n^k}{k!}$. section 2.6 summarises several properties of the Poisson model. The use of Poisson games in large election models is now commonplace as it simplifies the calculation of probabilities while still producing the same predictions as models with fixed but large populations.[9]

Each district has a distribution of types from which its voters are drawn, $f_d$, which has full support over $[-1, 1]$. The probability of drawing a type $t$ is $f_d(t)$. The *actual* population of voters in $d$ consists of $n_d$ independent draws from $f_d$. A voter knows his own type, the distribution from which he was drawn, and the distribution functions of the other districts, $\mathbf{f} \equiv \{f_1, \dots, f_d, \dots, f_D\}$, but he does not know the actual distribution of voters that is drawn in any district.

A voter's strategy is a mapping $\sigma : T \to \Delta(V)$ where $\sigma_{t,d}(v_c)$ is the probability that a type $t$ voter in district $d$ casts ballot $v_c$. The usual constraints apply: $\sigma_{t,d}(v_c) \geq 0, \forall c$ and $\sum_c \sigma_{t,d}(v_c) = 1, \forall t$. In a Poisson game, all voters of the same type in the same district will follow the same strategy (see Myerson (1998)). Given the various $\sigma_{t,d}$'s, the expected vote share of party $c$ in the district is

$$\tau_d(c) = \sum_{t \in T} f_d(t)\sigma_{t,d}(v_c) \tag{1.1}$$

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[9]Krishna and Morgan (2011) use a Poisson model to show that in large elections, voluntary voting dominates compulsory voting when voting is costly and voters have preferences over ideology and candidate quality. Bouton and Castanheira (2012) use a Poisson model to show that when a divided majority need to aggregate information as well as coordinate their voting behaviour, approval voting serves to bring about the first-best outcome in a large election. Furthermore, Bouton (2013) uses a Poisson model to analyse the properties of runoff elections.

which can also be interpreted as the probability of a randomly selected voter playing $v_c$. The expected distribution of party vote shares in $d$ is $\tau_d \equiv (\tau_d(l), \tau_d(m), \tau_d(r))$. The realised profile of votes is $x_d \equiv (x_d(l), x_d(m), x_d(r))$, but this is uncertain *ex ante*. As the population of voters is made up of $n_d$ independent draws from $f_d$, where $E(n_d) = n$, the expected number of ballots for candidate $c$ is $E(x_d(c)|\sigma_d) = n\tau_d(c)$. In the extremely unlikely event that nobody votes, I assume that party $m$ wins the seat.[10] Let $\boldsymbol{\sigma} \equiv \{\sigma_1, \ldots, \sigma_d, \ldots, \sigma_D\}$ denote the profile of voter strategies across districts and let $\boldsymbol{\sigma_{-d}}$ be that profile with $\sigma_d$ omitted. Let $\boldsymbol{\tau} \equiv \{\tau_1, \ldots, \tau_d, \ldots, \tau_D\}$ denote the profile of expected party vote share distributions and let $\boldsymbol{\tau_{-d}}$ be that profile with $\tau_d$ omitted. Thus, we have $\boldsymbol{\tau}(\boldsymbol{\sigma}, \mathbf{f})$.

At this point, I could define an equilibrium of the game; however, it is more convenient to define equilibrium in terms of pivotality and decisiveness, so I first introduce these additional concepts below.

**Pivotality, Decisiveness and Payoffs**  A single vote is *pivotal* if it makes or breaks a tie for first place in the district. A district is *decisive* if the policy outcome $z$ depends on which candidate that district elects. When deciding on his strategy, a voter need only consider cases in which his vote affects the policy outcome. Therefore, he will condition his vote choice on being *pivotal* in his district <u>and</u> on the district being *decisive*. The ability to do so is key, as if a voter cannot condition on some event where his vote matters then he does not know how he should vote.

Let $piv_d(c, c')$ denote when, in district $d$, a vote for party $c'$ is pivotal against $c$. This occurs when $x_d(c) = x_d(c') \geq x_d(c'')$ – so that an extra vote for $c'$ means it wins the seat – or when $x_d(c) = x_d(c') + 1 \geq x_d(c'')$ – so that an extra vote for $c'$ forces a tie. In the event of a tie, a coin toss determines the winner.

Let $\lambda_d^i(z^i)$ denote the event in which district $d$ is decisive between policies $z_l^i, z_m^i$, and $z_r^i$: these are the policy outcomes of the bargaining stage when the decisive district elects party $l, m$, or $r$ respectively. Note that these policies need not correspond to

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[10]The probability of zero turnout in a district is $e^{-n}$.

the announced platforms of the parties - typically coalition bargaining will lead to compromised policies. It is useful to classify decisive events into three categories. Let $\lambda(3)$ be a decisive event where all three policies $z_l^i$, $z_m^i$, and $z_r^i$ are different points on the policy line; let $\lambda(2)$ be a case where two of the three policies are identical.[11] Furthermore, let $\lambda(2')$ be an event where there are three different policies but one of them is the preferred choice of no voter. For this to be the case, the universally disliked policy must be a lottery over two or more policies. Two decisive events $\lambda^i$ and $\lambda^j$ are *distinct* if $z^i \neq z^j$. Let $\Lambda$ be the set of distinct decisive events; this set consists of $I$ elements, where $\lambda_d^i$ is the $i$-th most likely decisive event for district $d$. As we will see, the number and type of decisive events in the set $\Lambda$ depends on the legislative bargaining rule used.

Let $G_{t,d}(v_c|n\boldsymbol{\tau})$ denote the expected gain for a voter of type $t$ in district $d$ of voting for party $c$, given the strategies of all other players in the game – this includes players in his own district as well as those in the other $D-1$ districts. The expected gain of voting $v_l$ is given by

$$G_{t,d}(v_l|n\boldsymbol{\tau}) = \sum_{i=1}^{I} Pr[\lambda_d^i]\Big( Pr[piv_d(m,l)]\big(u_t(z_l^i)-u_t(z_m^i)\big) + Pr[piv_d(r,l)]\big(u_t(z_l^i)-u_t(z_r^i)\big)\Big)$$
$$(1.2)$$

with the gain of voting $v_m$ and $v_r$ similarly defined. The probability of being pivotal between two candidates, $Pr[piv_d(c,c')]$, depends on the strategies and distribution of player types in that district, summarised by $\tau_d$, while the probability of district $d$ being decisive depends on the strategies and distributions of player types in the other $D-1$ districts, $\boldsymbol{\tau_{-d}}$. The best response correspondence of a type $t$ in district $d$ to a strategy profile and distribution of types given by $\boldsymbol{\tau}$ is

$$BR_{t,d}(n\boldsymbol{\tau}) \equiv \operatorname*{argmax}_{\sigma_{t,d}} \sum_{v_c \in V} \sigma_{t,d}(v_c) G_{t,d}(v_c|n\boldsymbol{\tau}) \tag{1.3}$$

**Timing** The sequence of play is as follows:

1. In each district, nature draws a population of $n_d$ voters from $f_d$.

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[11]Obviously, $\lambda(1)$ events cannot exist, as if electing any of the three parties gives the same policy, it is not a decisive event.

2. Voters observe platforms $\{a_l, a_m, a_r\}$ and cast their vote for one of the three parties. Whichever party wins a plurality in a district takes that seat in the legislature.

3. A government is formed according to a specified bargaining process and a policy outcome, $z$, is chosen.

**Equilibrium Concept**   The equilibrium of this game consists of a *voting equilibrium* at stage 2 and a *bargaining equilibrium* at stage 3. In a *bargaining equilibrium*, each party's strategy is a best response to the strategies of the other two parties. I restrict attention to stationary bargaining equilibria, as is standard in such games.[12]

The solution concept for the voting game at stage 2 is strictly perfect equilibrium (Okada (1981)).[13]   A strategy profile $\boldsymbol{\sigma}^*$ is a strictly perfect equilibrium if and only if $\exists \epsilon > 0$ such that $\forall \tilde{\tau}_d \in \Delta V : |\tilde{\tau}_d - \tau_d(\boldsymbol{\sigma}^*, \mathbf{f})| < \epsilon$ then $\sigma_{t,d}^* \in BR_{t,d}(n\tilde{\boldsymbol{\tau}})$ for all $(t,d) \in T \times D$. That is, the equilibrium must be robust to epsilon changes in the strategies of players. Bouton and Gratton (2012) argue that restricting attention to such equilibria in multi-candidate Poisson games is appropriate because it rules out unstable and undesirable equilibria identified by Fey (1997). If, instead, Bayesian Nash equilibrium is used there may be knife-edge equilibria in which voters expect two or more candidates to get exactly the same number of votes. Bouton and Gratton (2012) also note that requiring strict perfection is equivalent to robustness to heterogenous beliefs about the distribution of preferences, $\mathbf{f}$. As I am interested in the properties of large national elections, I analyse the limiting properties of such equilibria as $n \to \infty$.

## 1.3   Equilibrium

I solve for the equilibrium of the game by backward induction. The bargaining equilibrium at stage 3 follows from Baron (1991). Of greater interest to us is the voting equilibrium

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[12]An equilibrium is stationary if it is a subgame perfect equilibrium and each party's strategy is the same at the beginning of each bargaining period, regardless of the history of play.

[13]The original formulation of strictly perfect equilibrium was for games with a finite number of players; Bouton and Gratton (2012) extend this to Poisson games.

at stage 2. While there are multiple voting equilibria for any distribution of voter types, I show that every equilibrium has only two candidates receiving votes in each district and I present several properties which must hold in any equilibrium.

**Stage 3: Bargaining Equilibrium**   When no party has a majority of seats and $\delta = 1$, in any stationary bargaining equilibrium $z = a_m$ is proposed and eventually passed with probability one.[14] This is regardless of whether the protocol is fixed order or random. To see this, note that if any other policy is proposed, a majority of legislators will find it worthwhile to wait until $a_m$ is offered (which will occur when party $m$ is eventually chosen as formateur). The equilibrium policy outcome of the legislative bargaining stage is then

$$
z = \begin{cases}
a_l & \text{if } s_l > \frac{D-1}{2} \\
a_r & \text{if } s_r > \frac{D-1}{2} \\
a_m & \text{otherwise}
\end{cases}
\tag{1.4}
$$

Every feasible seat distribution is mapped into a policy outcome, so, at stage 2, voters can fully anticipate which policy will arise from a given seat distribution. The set of distinct decisive events is given by

$$
\Lambda = \{\lambda(a_l, a_m, a_m), \lambda(a_m, a_m, a_r), \lambda(a_l, a_m, a_r)\}
\tag{1.5}
$$

**Stage 2: Voting Equilibrium**   Voting games where players have three choices typically have multiple equilibria; this game is no exception. However, I show that every voting equilibrium involves only two candidates getting votes in each district.

**Proposition 1.1.** *For any majoritarian legislative bargaining rule and any distribution of voter preferences,* $\mathbf{f} \equiv \{f_1, \ldots, f_d, \ldots, f_D\}$*, there are multiple equilibria; in every equilibrium districts are duvergerian.*

*Proof.* See section 2.6. □

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[14]For a proof see Jackson and Moselle (2002).

It is perhaps unsurprising that there are multiple equilibria and districts are always duvergerian, given the findings of the extensive literature on single district plurality elections. The logic as to why races are duvergerian is similar to the single district case; voters condition their ballot choice on being pivotal and decisive, and as $n \to \infty$, the most likely pivotal and decisive event is infinitely more likely to occur than any other pivotal and decisive event.[15] Thus, voters need only consider the most likely event in which their district is decisive and their own vote is pivotal. This greatly simplifies the decision process of voters and means they need only consider the two frontrunners in their district. While we cannot pin down which equilibrium will be played, the following properties will always hold.

1. In each district only two candidates receive votes; call these *serious candidates*.

2. If $\tau_d(c) > \tau_d(c') > 0$, candidate $c$ is the expected winner and his probability of winning goes to one as $n \to \infty$. Let $d_c$ denote such a district.

3. The expected seat distribution is $E(S) = E(s_l, s_m, s_r) = (\#d_l, \#d_m, \#d_r)$.

4. A district with $c$ and $c'$ as serious candidates will condition on the most likely decisive event $\lambda^i \in \Lambda$ such that $z^i(c) \neq z^i(c')$.

The fourth property says: if a district's most likely decisive event, $\lambda_d^1$, is of the type $\lambda(3)$ or $\lambda(2')$, then voters must be conditioning on this event; if $\lambda_d^1$ is of type $\lambda(2)$, voters will be conditioning on it only if they are not indifferent between the two serious candidates.

## 1.4 Analysis of Benchmark Model

This section presents the main results of the paper. I compare my model of national voters, who care only about policy $z$, with two other types of plurality elections: (1) single district elections such as presidential elections, and (2) legislative elections with

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[15]This is due to an application of the Magnitude Theorem (Myerson, 2000) shown in the appendix.

local voters i.e. where voters only care about the winner in their own district. I contrast the degree of polarisation and misaligned voting possible in each. The proofs of results in the rest of the paper rely on graphical arguments, hence, a slight detour is needed to explain this approach.

Recall, from Equation 1.5, that there are three distinct decisive events a district may condition on:

$$\Lambda = \{\lambda(a_l, a_m, a_m), \lambda(a_m, a_m, a_r), \lambda(a_l, a_m, a_r)\}$$

The first two are $\lambda(2)$ events while the final one is a $\lambda(3)$ event. A $\lambda(a_l, a_m, a_m)$ event occurs when a district is decisive in determining whether $l$ wins a majority of seats and implements $z = a_l$, or it falls just short, allowing a coalition to implement $z = a_m$. Here, voter types $t < \frac{a_l + a_m}{2} \equiv a_{lm}$ will vote $v_l$ while those of type $t > a_{lm}$ will coordinate on either $v_m$ or $v_r$, as they are indifferent between the two policies offered. Similarly, in a $\lambda(a_m, a_m, a_r)$ event, a district can secure party $r$ a majority of seats or not; those with $t < a_{mr}$ will vote either $v_l$ or $v_m$, while the rest will choose $v_r$. Finally, when a district finds itself at a point $\lambda(a_l, a_m, a_r)$, it is conditioning on $l$ and $r$ winning half the seats each before $d$'s result is included: $S_{-d} = (\frac{D-1}{2}, 0, \frac{D-1}{2})$. Electing either $l$ or $r$ would give them a majority, while electing $m$ would bring about a coalition. Therefore, depending on the result in $d$, any of the three policies $a_l, a_m$ or $a_r$ could be implemented. These three distinct decisive events are represented in Figure 1.1. The simplex represents the decision problem of voters in district $d$, holding fixed the strategies of those in the other $D - 1$ districts.[16] Each point corresponds to an expected distribution of $D - 1$ seats among the three parties: the bottom left point corresponds to $E(S_{-d}) = (D - 1, 0, 0)$; the bottom right, $E(S_{-d}) = (0, 0, D - 1)$; the apex is $E(S_{-d}) = (0, D - 1, 0)$.

Each district will condition on one of the distinct events $\lambda \in \Lambda$ when voting, and this must be consistent with the equilibrium properties given in the previous section. All $d_l$ have $E(S_{-d_l}) = E(s_l - 1, s_m, s_r)$, all $d_m$ have $E(S_{-d_m}) = E(s_l, s_m - 1, s_r)$, and all $d_r$ have $E(S_{-d_r}) = E(s_l, s_m, s_r - 1)$. In Figure 1.1 this corresponds to the various $E(S_{-d})$

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[16]While the simplex represents the case of $D = 25$, the same would hold for any odd $D$. To avoid the case where two parties could share the seats equally, I ignore the case where $D$ is even.
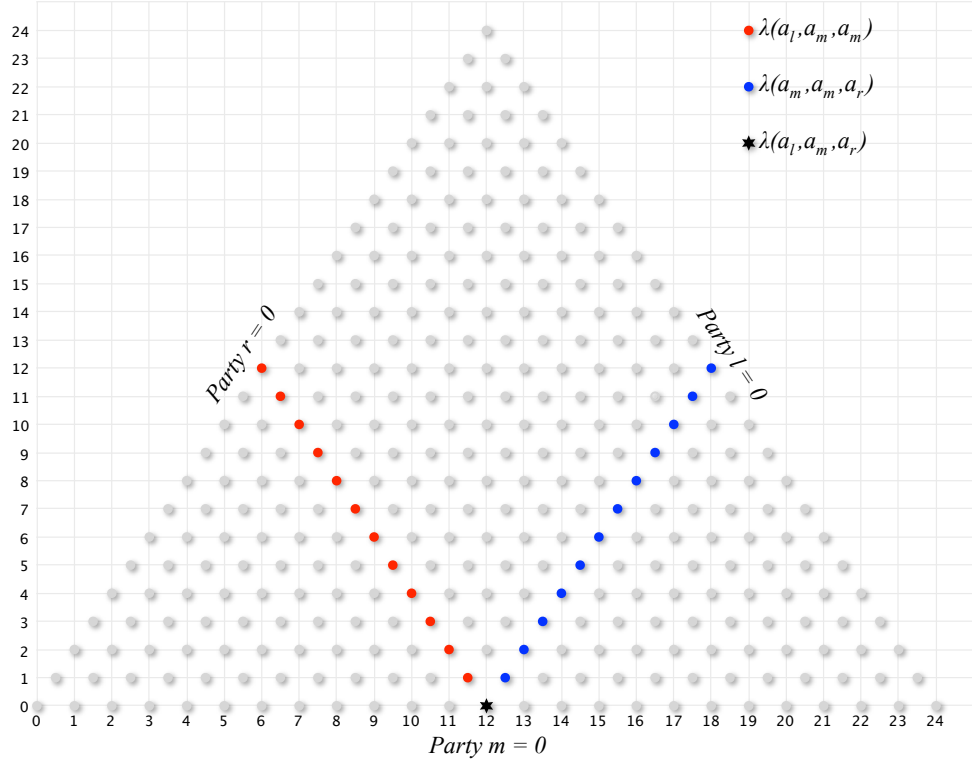
Figure 1.1: Cases in which $d$ is decisive when $D = 25$

forming an inverted triangle: $E(S_{-d_r})$ is one point west of $E(S_{-d_l})$, while $E(S_{-d_m})$ is one point south-west of $E(S_{-d_l})$.

### 1.4.1  Polarisation

Much attention in the U.S. has focused on how a system with two polarised parties has led to policies which are far away from the median voter's preferred point.[17] The same problem may also arise if there are three parties but one of them is not considered a serious challenger. An open question is the degree to which policy outcomes reflect the preferences of voters in a legislative election with three parties. Let $E(t_d)$ be the expected position of the median voter in district $d$, and label the districts so that $E(t_1) < \ldots < E(t_{\frac{D+1}{2}}) < \ldots < E(t_D)$. Then, $E(t_{\frac{D+1}{2}})$ is the expected median voter in

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[17]See McCarty et al. (2008).

the median district. The following proposition shows that, while polarisation of outcomes can always occur in the plurality settings usually considered in the literature, an extreme policy can only be implemented in my benchmark model if it is preferred by the median voter in the median district.

**Proposition 1.2.** *For any distribution of voter preferences,*

1. *In a presidential election or a legislative election with local voters, equilibria always exist where either $z = a_l$ or $z = a_r$.*

2. *In a legislative election with national voters, $\delta = 1$, $E(s_m) > 0$ and where bargaining occurs over policy, $z = a_l$ can only be implemented if the median voter in the median district prefers policy $a_l$ to policy $a_m$; that is, if $E(t_{\frac{D+1}{2}}) < a_{lm}$. Similarly, $z = a_r$ can only be implemented if $E(t_{\frac{D+1}{2}}) > a_{mr}$.*

*Proof. Case 1:* In a presidential election, any two of the three candidates may become focal. If $l$ and $r$ are the serious candidates, then we must have either $z = a_l$ or $z = a_r$. Whichever of the two wins, depends on the location of the median voter.

*Case 2:* In a legislative election with local voters, each district will focus on a race between any two of the three candidates. There must be at least $\frac{D+1}{2}$ districts with either $E(t) < a_{lr}$ or $E(t) > a_{lr}$. If these districts have $l$ and $r$ as serious candidates, the outcome will be either $z = a_l$ or $z = a_r$.

*Case 3:* In a legislative election with national voters and $E(s_m) > 0$, for the policy to be $z = a_l$ it must be that $E(S_{-d})$ is in the bottom left triangle of Figure 1.1 for all districts. Any $d_l$ district must be conditioning on either a $\lambda(a_l, a_m, a_m)$ event or a $\lambda(a_l, a_m, a_r)$ event. However, given $E(s_m) > 0$, the probability of a $\lambda(a_l, a_m, a_m)$ event is strictly greater than the probability of a $\lambda(a_l, a_m, a_r)$ event, making the former is infinitely more likely. Therefore, these $d_l$ districts must be conditioning on $\lambda(a_l, a_m, a_m)$ and as $l$ is the expected winner, must have $E(t) < a_{lm}$. It follows that for $z = a_l$ to be implemented, the median of median voters must be $E(t_{\frac{D+1}{2}}) < a_{lm}$. The proof for $z = a_r$ is analogous. □

The proposition gives a novel insight into multiparty legislative elections under plurality. In the U.K., until recently, a vote for the Liberal Democrats (Lib Dems) has typically been considered a "wasted vote".[18] The popular belief was that the Lib Dems were not a legitimate contender for government and so, even if they took a number of seats in parliament, they would not influence policy. As a result, centrist voters instead voted for either the Conservatives or Labour. My model shows that electing a Lib Dem candidate is far from a waste. Electing just one member of the median party to the legislature will be enough to bring about that party's preferred policy unless voter preferences favour one of the non-centrist parties very much. Indeed, the result suggests that concerns about the average voter not being adequately represented in the U.K. or Canada are misplaced. If the Conservatives win a majority in parliament it must be that a majority of voters in a majority of districts prefer their policy to that of the centrist Lib Dems/Liberals. On the other hand, a coalition implementing $z = a_m$ can come about for any distribution of voter preferences.[19] Supporters of the Lib Dems in the U.K. and the Liberals in Canada are therefore hugely advantaged by the current electoral system; it is the supporters of the non-centrist parties who are disadvantaged.

### 1.4.2 Misaligned Voting

All voters are strategic: a voter chooses his ballot to maximise his expected utility; which ballot this is depends on how the others vote. In any given situation, an individual may cast the same ballot he would if his vote unilaterally decided the district, or the strategies played by the others in the district may mean his best response is to vote for a less desirable option. Following Kawai and Watanabe (2013), I call the latter misaligned voting.

**Definition 1.1.** *A voter casts a **misaligned vote** if, conditioning on the strategies of*

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[18]See "What Future for the Liberal Democrats" by Lord Ashcroft, 2010.

[19]For any **f**, there are always equilibria where $z = a_m$ is the expected outcome. If support is strong for party $r$ then an equilibrium in which districts focus on a $\lambda(a_l, a_m, a_m)$ decisive event will give $z = a_m$. Likewise, if $l$ is popular then a focus on $\lambda(a_m, a_m, a_r)$ will give $z = a_m$.

*voters in other districts, he would prefer a different candidate to win his district than the one he votes for.*

If a voter casts a misaligned vote, he is essentially giving up on his preferred candidate due to the electoral mechanism. In the two types of plurality elections usually considered in the literature, there will always be misaligned voting: In a presidential election there is only one district - all citizens vote in the same plurality election - so there is no conditioning on other districts. With candidates $l, m$ and $r$, one of them will receive no votes, for the usual reason of voters conditioning on the most likely pivotal event. Whichever candidate is least likely to be pivotal will be abandoned by his supporters, leading to a two-party race. Therefore, either all types with $t < a_{lm}$, all types with $t > a_{mr}$, or those in the interval in between will cast a misaligned vote. In a legislative election with local voters, individual's payoffs depend only on the result in their own district, so there is no conditioning on other districts. Each district is akin to its own presidential plurality election with three choices - so once again there must be misaligned voting. There may be more or less misaligned voting in a presidential election than in a legislative election with local voters, but there will always be a significant level of misaligned voting in each. In contrast, proposition 1.3 below shows that there are many equilibria of the legislative election with national voters in which there is no misaligned voting.

**Proposition 1.3.** *For any distribution of voter preferences,*

1. *In a presidential election or in a legislative election with local voters, every equilibrium has misaligned voting.*

2. *Under a legislative election with national voters, bargaining over policy and $\delta = 1$, there always exist equilibria with no misaligned voting in any district. These occur when party $l$ or $r$ receive votes in fewer than $\frac{D-1}{2}$ districts.*

*Proof.* By proposition 1.1, only two candidates will receive votes in each district. With $D$ districts there will be $2D$ serious candidates. If party $r$'s candidates are serious in less

than $\frac{D+1}{2}$ districts, party $r$ can never win a majority of seats. If party $r$'s candidates are serious in less than $\frac{D-1}{2}$ districts, the decisive event in which an extra seat for party $r$ gives them a majority can never come about. Therefore, in any equilibrium where party $r$ is serious in less than $\frac{D-1}{2}$ districts, the only decisive event voters can condition on is $\lambda(a_l, a_m, a_m)$. As this is the only decisive event, in each district, voters with $t < a_{lm}$ will vote $v_l$ while those with $t > a_{lm}$ will vote for whichever of $m$ or $r$ is expected to receive votes. As long as less than $\frac{D-1}{2}$ of these districts coordinate on $r$, there will be no misaligned voting. An analogous result holds when party $l$ is serious in less than $\frac{D-1}{2}$ districts. $\hfill\square$

The crux of the proposition is that when one of the non-centrist parties is a serious candidate in less than half the districts, only one distinct decisive event exists. This event is a choice over two policies; with only two policies on the table there is no strategic choice to make - voters simply vote for their preferred option of the two. So, there can be no misaligned voting. A voter with $t > a_{lm}$ facing a $\lambda(a_l, a_m, a_m)$ decisive event is indifferent between electing $m$ or $r$; he will vote for whichever of the two the other voters are coordinating on.

The proposition gives us a clear prediction on when there will and will not be misaligned voting with three parties competing in a legislative election. It shows that the conventional wisdom - no misaligned with two candidates, always misaligned with three - is wide of the mark; whether there is misaligned voting or not depends on the strength of the non-centrist parties. The proposition also has implications for the study of third-party entry into a two-party system. Suppose, as is plausible, that a newly formed party cannot become focal in many districts - maybe because they have limited resources, or because voters do not yet consider them a serious alternative. Either way, an entering third-party is likely to be weaker than the two established parties. Proposition 1.3 tells us that if a third party enters on the flanks of the two established parties, then there will be no misaligned voting and no effect on the policy outcome as long as this party is serious in less than half the districts. On the other hand, if a third party enters at a policy point in between the two established parties, this can shake up the political landscape. First of

all there will necessarily be misaligned voting, and second of all the policy outcome could be any of $a_l$, $a_m$ or $a_r$ depending on which equilibrium voters focus on. Success for the new party in just one district can radically change the policy outcome. The implication is that parties in a two-party system should be less concerned about the entry of fringe parties and more concerned about potential centrist parties stealing the middle ground.

## 1.5 Legislative Bargaining over Policy and Perks

While the model of bargaining over policy in the previous section is tractable, it lacks one of the key features of the government formation process: parties often bargain over perks of office such as ministerial positions as well as over policy. Here, as parties can trade off losses in the policy dimension for gains in the perks dimension, and vice versa, a larger set of policy outcomes are possible. This section will show that, nonetheless, the results of the benchmark model extend broadly to the case of bargaining over policy and perks.

The following legislative bargain model with $B > 0$ is due to Austen-Smith and Banks (1988). As usual, if a party wins an overall majority it will implement its preferred policy and keep all of $B$. Otherwise, the parties enter into a stage of bargaining over government formation. The party winning the most seats of the three begins the process by offering a policy outcome $y^1 \in \mathbb{X}$ and a distribution of a fixed amount of transferable private benefits across the parties, $b^1 = (b_l^1, b_m^1, b_r^1) \in [0, B]^3$. It is assumed that $B$ is large enough so that any possible governments can form, i.e. $l$ can offer enough benefits to party $r$ so as to overcome their ideological differences. If the first proposal is rejected, the party with the second largest number of seats gets to propose $(y^2, b^2)$. If this is rejected, the smallest party proposes $(y^3, b^3)$. If no agreement has been reached after the third period, a caretaker government implements $(y^0, b^0)$, which gives zero utility to all parties. At its turn to make a proposal, party $c$ solves

$$\max_{b_{c'}, y} B - b_{c'} - (y - a_c)^2 \tag{1.6}$$
$$\text{subject to} \quad b_{c'} - (y - a_{c'})^2 \geq W_{c'}$$

where $W_{c'}$ is the continuation value of party $c'$ and $W_{c''} + (y - a_{c''})^2 > W_{c'} + (y - a_{c'})^2$,

| Seat Share | $3l_r < l_l$ | $2l_r < l_l \leq 3l_r$ | $l_r < l_l \leq 2l_r$ | $l_l = l_r$ | $l_l < l_r \leq 2l_l$ | $2l_l < l_r \leq 3l_l$ | $3l_l < l_r$ |
|---|---|---|---|---|---|---|---|
| $s_l > (D-1)/2$ | $a_l$ | $a_l$ | $a_l$ | $a_l$ | $a_l$ | $a_l$ | $a_l$ |
| $(D+1)/2 > s_l > s_r > s_m$ | $a_{lm}$ | $a_{lm}$ | $a_{lm}$ | $a_{lm}$ | $a_{lm}$ | $2a_m - a_{lr}$ | $a_l$ |
| $(D+1)/2 > s_l > s_m > s_r$ | $a_{lr}$ | $a_{lr}$ | $a_{lr}$ | $a_m$ | $a_m$ | $a_m$ | $a_m$ |
| $s_m > s_l, s_r$ | $a_m$ | $a_m$ | $a_m$ | $a_m$ | $a_m$ | $a_m$ | $a_m$ |
| $(D+1)/2 > s_r > s_m > s_l$ | $a_m$ | $a_m$ | $a_m$ | $a_m$ | $a_{lr}$ | $a_{lr}$ | $a_{lr}$ |
| $(D+1)/2 > s_r > s_l > s_m$ | $a_r$ | $2a_m - a_{lr}$ | $a_{mr}$ | $a_{mr}$ | $a_{mr}$ | $a_{mr}$ | $a_{mr}$ |
| $s_r > (D-1)/2$ | $a_r$ | $a_r$ | $a_r$ | $a_r$ | $a_r$ | $a_r$ | $a_r$ |

Table 1.1: Policy outcomes in Austen-Smith and Banks (1988) for any seat distribution and distance between parties.

so that the formateur makes the offer to whichever party is cheaper. Solving the game by backward induction, Austen-Smith and Banks (1988) show that a coalition government will always be made up of the largest party and the smallest party. They solve for the equilibrium policy outcome, for any possible distance between $a_l, a_m$ and $a_r$.

Table 1.1 shows the policy outcome for each seat distribution and distance between parties, where $l_l \equiv a_m - a_l$ and $l_r \equiv a_r - a_m$. I assume if two parties have exactly the same number of seats, a coin is tossed before the bargaining game begins to decide the order of play. So, if $s_l = s_r > s_m$, then with probability one-half, the game will play out as when $s_l > s_r > s_m$ and otherwise as $s_r > s_l > s_m$.

The set of possible policy outcomes depends on the number of seats in the legislature, and on the distance between party policies. The simplex in Figure 1.2 shows what the policy will be, for any seat distribution, when there are 25 districts and $l_l < l_r \leq 2l_l$. Notice that there are far more policy possibilities than in the case of $B = 0$. Figure 1.3 shows the various decisive cases from the perspective of a single district; it is the analogue of Figure 1.1. While there are many more decisive cases than when $B = 0$, they can be grouped into the three categories defined previously: $\lambda(2), \lambda(2')$ and $\lambda(3)$ events.

The following proposition shows that, even when parties can bargain over perks as well as policy, a non-centrist party can only win a majority if the median voter in the median district prefers its policy to that of the centrist party.
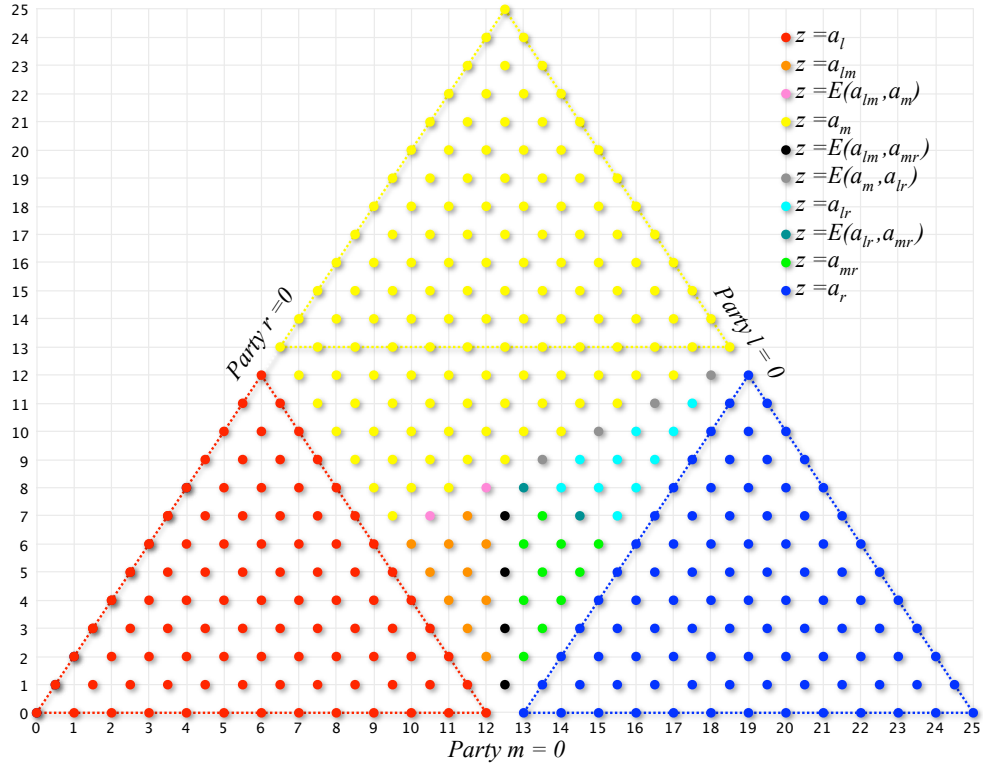
Figure 1.2: Policy outcomes under ASB bargaining, with $D = 25$ and $l_l < l_r \leq 2l_l$
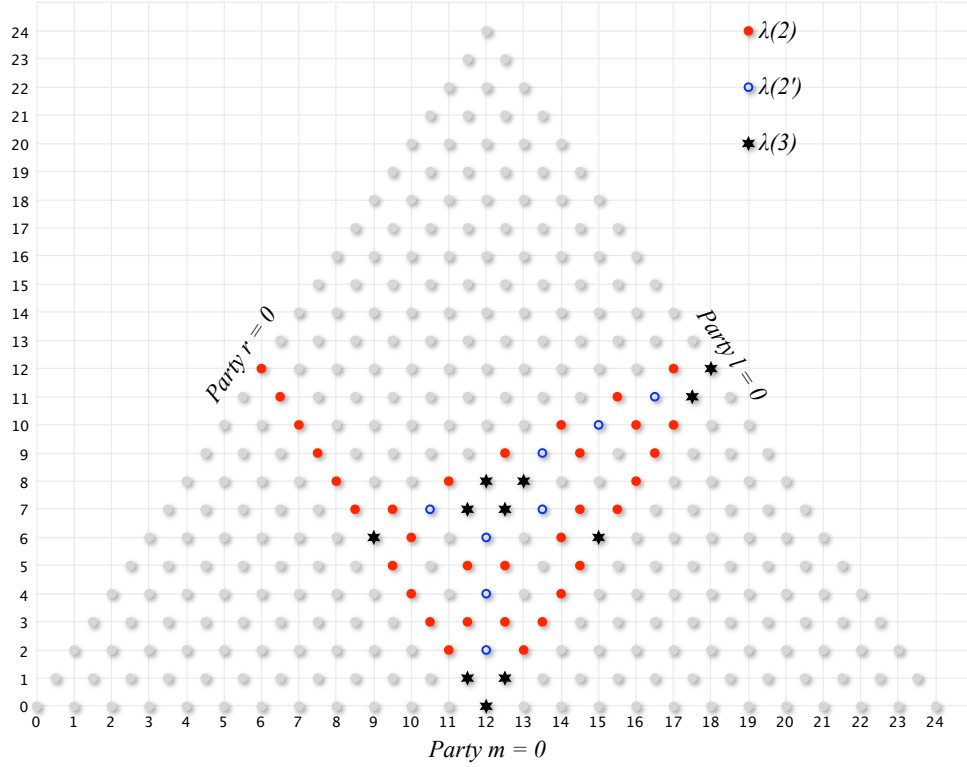


Figure 1.3: Decisive events under ASB bargaining, with $D = 25$ and $l_l < l_r \leq 2l_l$

**Proposition 1.4.** *In a legislative election with a fixed order of bargaining over policy and perks of office and $E(s_m) > 1$, $z = a_l$ can only be implemented if $E(t_{\frac{D+1}{2}}) < a_{lm}$. Similarly, $z = a_r$ can only be implemented if $E(t_{\frac{D+1}{2}}) > a_{mr}$.*

*Proof.* Suppose we have $l_l < l_r \leq 2l_l$, as in Figure 1.3. If the policy outcome is $z = a_l$ and the median party is expected to win more than one seat, each $d_l$ district must be conditioning on one of the following events: $\lambda(a_l, a_m, a_m), \lambda(a_l, a_{lm}, a_{lm})$, or $\lambda(a_l, a_m, a_{lm})$. If a district is conditioning on a race between policies $a_l$ and $a_m$, then $a_l$ will win if $E(t) < a_{lm}$. If it is conditioning on a race between policies $a_l$ and $a_{lm}$, then $a_l$ will win only if $E(t) < \frac{a_l + a_{lm}}{2}$, a stricter condition. Therefore the minimum requirement for a district to elect $l$ is $E(t) < a_{lm}$. For $l$ to win a majority of seats, it must be that this condition is met in at least $\frac{D+1}{2}$ districts. Still with $l_l < l_r \leq 2l_l$, note from that the leftmost policy which could be implemented when $s_r = \frac{D-1}{2}$ is $z = a_m$ (this occurs when $S = (0, \frac{D+1}{2}, \frac{D-1}{2})$). Therefore, in order for the policy $z = a_r$ to never come about, we must have $E(t_{\frac{D+1}{2}}) > a_{mr}$. From Table 1.1, we can see that these bounds of $a_{lm}$ and $a_{mr}$ will apply no matter what the difference between the three party platforms. $\square$

This reaffirms the result of proposition 1.2, that moderate coalitions will be the norm in legislative elections unless the population is heavily biased in favour of one of the non-centrist parties. Moreover, bargaining over perks as well as policy can lead to even less extreme policies than the benchmark case. This can be seen from Figure 1.2: starting from a point $E(S)$ where $E(s_l) > \frac{D-1}{2}$, $E(s_m) > 1$ and $\frac{D-1}{4} < E(s_r) < \frac{D-1}{2}$, the most likely decisive event for each district must be $\lambda(a_l, a_{lm}, a_{lm})$. Therefore, such a party $l$ majority could only come about if $E(t_{\frac{D+1}{2}}) < \frac{a_l + a_{lm}}{2}$ - an even stricter requirement than that of the benchmark case. This result is noteworthy as in U.K. and Canadian elections party seat shares tend to be in line with this case: one of the non-centrist parties wins a majority, the other wins more than a quarter of the seats, while the centrist party wins much less than a quarter.

On the other dimension of interest bargaining over policy and perks does not perform as well; the restrictions required to completely rule out misaligned voting are more severe

than in the benchmark model. However, as proposition 1.6 will show, there are many equilibria in which a large subset of districts have no misaligned voting.

**Proposition 1.5.** *In a legislative election with a fixed order of bargaining over policy and perks of office, there always exist equilibria with no misaligned voting in any district.*

1. *When $a_l$ and $a_r$ are equidistant from the centrist policy, $l_l = l_r$, there is no misaligned voting if either party l or party r receive votes in fewer than $\frac{D-1}{4}$ districts.*

2. *When $a_l$ is closer than $a_r$ to the centrist policy, $l_l < l_r$, there is no misaligned voting if party r receive votes in fewer than $\frac{D-1}{4}$ districts.*

3. *When $a_l$ is further than $a_r$ to the centrist policy, $l_l > l_r$, there is no misaligned voting if party l receive votes in fewer than $\frac{D-1}{4}$ districts.*

*Proof.* See section 2.6. □

The intuition is the same as in proposition 1.3: when a non-centrist party is not a serious candidate in enough districts, there is no hope of it influencing the order of recognition in the legislative bargaining stage. The threshold for relevance is lower than in the benchmark case because under this bargaining protocol the order of parties matters for the policy outcome. From Figure 1.3 we see that once it is possible for party $r$ to win $\frac{D-1}{4}$ districts, two distinct decisive events exist: $\lambda(a_l, a_m, a_m)$ and $\lambda(a_l, a_m, a_{lm})$. No matter which of these two events a district focuses on, and which two candidates are serious, some voters in the district will always be casting misaligned votes.

When party $l$ or $r$ have serious candidates in more than $\frac{D-1}{4}$ districts we cannot rule out misaligned voting. However, there are equilibria in which there is no misaligned voting in a subset of districts. The following proposition holds for all bargaining rules.

**Proposition 1.6.** *There will be no misaligned voting in a district if either*

1. *The most likely decisive event $\lambda_d^1$ is a $\lambda(2')$ event where candidates c and c' are serious <u>and</u> $z^1(c'')$ is preferred by no voter.*

2. *The most likely decisive event $\lambda_d^1$ is a $\lambda(2)$ event where candidates $c$ and $c'$ are serious, $z^1(c) = z^1(c'')$, <u>and</u> all those voting $v_c$ must have $u_t(z^i(c)) > u_t(z^i(c''))$ in the next most likely decisive event $\lambda^i \in \Lambda$ such that $z^i(c) \neq z^i(c'')$.*

*Proof.* See section 2.6. □

The proposition is best understood by way of example. Take a $\lambda(2')$ event, for example, $S_{-d} = (\frac{D-3}{2}, 2, \frac{D-3}{2})$. Electing $l$ will give $s_l > s_r > s_m$ resulting in $z = a_{lm}$, electing $r$ instead will give $s_r > s_l > s_m$ and bring about $z = a_{mr}$, while electing $m$ will lead to a tie for first place between $l$ and $r$. A coin toss will decide which of the two policies comes about, but *ex ante* voters' expectation is $E(a_{lm}, a_{mr})$. As voters have concave utility functions, every voter strictly prefers either $a_{lm}$ or $a_{mr}$ to the lottery over the pair. If this decisive event is the most likely (i.e. infinitely more likely than all others) and the district focuses on a race between $l$ and $r$, nobody in the district is casting a misaligned vote.

To see the second part of the proposition, suppose the most likely decisive event is $S_{-d} = (\frac{D-3}{2}, 3, \frac{D-5}{2})$. Here, electing $l$ or $m$ gives $a_{lm}$ while electing $r$ brings about a coin toss and an expected policy $E(a_{lm}, a_{mr})$. Suppose further that the second most likely decisive event is $S_{-d} = (\frac{D-5}{2}, 3, \frac{D-3}{2})$, where electing $m$ or $r$ gives policy $a_{mr}$ while electing $l$ gives $E(a_{lm}, a_{mr})$. In the most likely event, all voters below a certain threshold will be indifferent between electing $l$ and electing $m$. However, in the second most likely decisive event, all of these voters would prefer to elect $l$ than $m$. Given that each decisive event is infinitely more likely to occur than a less likely decisive event, these voters need only consider the top two decisive events. Any voter who is indifferent between $l$ and $m$ in the most likely decisive event strictly prefers $l$ in the second most likely. So, if the district focuses on a race between $l$ and $r$ there will be no misaligned voting.

Proposition 1.6 is quite useful, as it holds for any bargaining rule. It will allow me to say that in the next section, even though we cannot get results such as proposition 1.3 and proposition 1.5, we do not return to the single plurality election case of "always misaligned voting". Instead, there are again many equilibria in which a subset of districts

have no misaligned voting.

## 1.6 Impatient Parties

In this section, I examine how the results of the benchmark model change when $\delta < 1$, so that parties are no longer perfectly patient. It is likely that the discount rates of politicians vary across countries depending on things such as constitutional constraints of bargaining, the status quo, and the propensity of politicians to be reelected.[20] In the benchmark model it didn't matter whether the bargaining protocol was random or had a fixed order; a coalition would always implement $z = a_m$. However, once parties discount the future, we get vasty different results depending on which bargaining protocol is used. The scope for policy polarisation and misaligned voting not only depends on how the formateur is selected but also on the location of the status quo policy, $Q$. I assume the status quo is neither too extreme, $Q \in (a_l, a_r)$, nor too central $Q \neq a_m$.[21] In each period where no agreement is reached, the status quo policy remains and enters party's payoff functions. All parties discount the future at the same rate of $\delta \in (0, 1)$. Therefore, if a proposal $y$ is passed in period $t$, the payoff of party $c$ is

$$W_c = -(1 - \delta^{t-1})(Q - a_c)^2 - \delta^{t-1}(y - a_c)^2 \tag{1.7}$$

For ease of analysis I assume, without loss of generality, that $a_m = 0$.[22] Banks and Duggan (2000) show that all stationary equilibria are no-delay equilibria and are in pure strategies when the policy space is one-dimensional and $\delta < 1$.

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[20]After the 2010 Belgian elections, legislative bargaining lasted for a record-breaking 541 days, suggesting high values of $\delta$. Conversely, after the 2010 U.K. elections, a coalition government was formed within five days.

[21]If $Q = a_m$ the result is the same as the benchmark case of $\delta = 1$.

[22]Taking any original positions $(a_l, a_m \neq 0, a_r)$, we can always alter $\mathbf{f}$ so that the preferences of all voter types are the same when $(a'_l, a'_m = 0, a'_r)$.

### 1.6.1    Fixed Order Bargaining

The order of recognition is fixed and follows the ranking of parties' seat shares. In section 2.6, I derive the policy outcomes for any ordering of parties; these are presented in Table 1.2 below. From the table we see that the further party $m$ moves down the ranking of seat shares, the further the policy moves away from $a_m$. Figure 1.4 shows the various policy outcomes for any seat distribution in the legislature. Figure 1.5 shows the frequency of the three categories of decisive events.

| Seat Share | Policy |
|---|---|
| $s_l > (D-1)/2$ | $a_l$ |
| $(D+1)/2 > s_l > s_r > s_m$ | $-\sqrt{(1-\delta^2)Q^2}$ |
| $(D+1)/2 > s_l > s_m > s_r$ | $-\sqrt{(1-\delta)Q^2}$ |
| $s_m > s_l, s_r$ | $a_m = 0$ |
| $(D+1)/2 > s_r > s_m > s_l$ | $\sqrt{(1-\delta)Q^2}$ |
| $(D+1)/2 > s_r > s_l > s_m$ | $\sqrt{(1-\delta^2)Q^2}$ |
| $s_r > (D-1)/2$ | $a_r$ |

Table 1.2: Policy outcomes with fixed order bargaining over policy and $\delta < 1$.

The proposition below shows that when the bargaining protocol is fixed, parties discount the future, and the status quo is not exactly $a_m$, it is even more difficult for a non-centrist party to win a majority of seats and implement its preferred policy than is the case in the benchmark model.

**Proposition 1.7.** *In a legislative election with a fixed order of bargaining over policy, $\delta < 1$ and $E(s_m), E(s_r) > 1$; $z = a_l$ can only be implemented if $E(t_{\frac{D+1}{2}}) < \frac{a_l - \sqrt{(1-\delta)Q^2}}{2} < a_{lm}$. Similarly, when $E(s_l), E(s_m) > 1$; $z = a_r$ can only be implemented if $E(t_{\frac{D+1}{2}}) > \frac{a_r + \sqrt{(1-\delta)Q^2}}{2} > a_{mr}$.*

*Proof.* See section 2.6. □

As the difference in policy between, say, an $l$ majority government and a coalition led by party $l$ is quite small, the majority government can only come about if the electorate
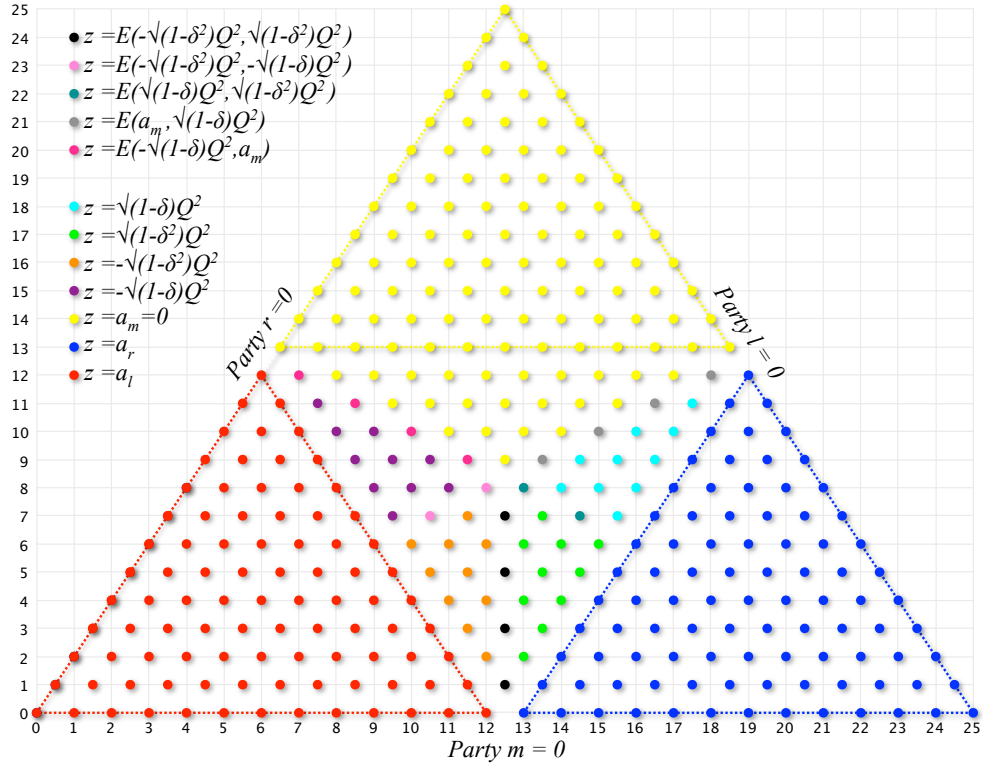
Figure 1.4: Policy outcomes under fixed order bargaining, with $D = 25$ and $\delta < 1$
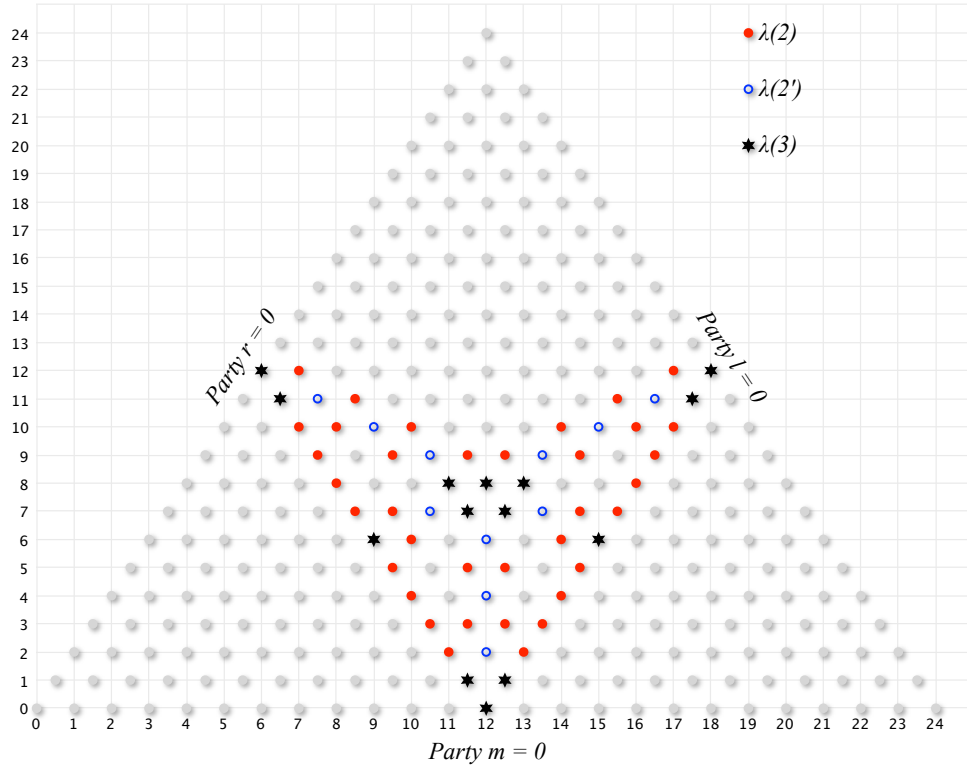


Figure 1.5: Decisive events under fixed order bargaining, with $D = 25$ and $\delta < 1$

is sufficiently biased in favour of policy $a_l$ - even more so than in the benchmark case.[23] The reason is that in the benchmark case every coalition implements $z = a_m$, while with discounting and a fixed order protocol, the largest party has a significant advantage in coalition negotiations and so can use this to get an alternative policy passed. Voters anticipate the power that the leading party $l$ will have in coalition formation and so will only vote to bring about a party $l$ majority if they prefer it to the $l$ led coalition. What the proposition also shows is that the further the status quo policy, $Q$, is from the median party policy, $a_m$, the more likely we are to have coalition governments, all else equal; a more distant status quo gives the formateur even more bargaining power over the median party.

Figure 1.5 shows the frequency of the three types of decisive event for this bargaining rule. While it is quite similar to Figure 1.3, the difference is that now there is no condition we can impose so as to ensure there is no misaligned voting. The corner decisive events of Figure 1.5 are $\lambda(3)$ events, so at least one of them can always be conditioned on. If a district is conditioning on a $\lambda(3)$ event there must be misaligned voting in that district. On the other hand, proposition 1.6 also holds here - so there are equilibria with misaligned voting in only a subset of districts. The following proposition summaries the state of misaligned voting under this bargaining rule.

**Proposition 1.8.** *In a legislative election with a fixed order of bargaining over policy and $\delta < 1$, there always exist equilibria with misaligned voting. However, equilibria exist with no misaligned voting in a subset of districts.*

### 1.6.2 Random Recognition Bargaining

In each period one party is randomly selected as formateur, where the probability of each party being chosen is equal to its seat share in the legislature, $\frac{s_c}{D}$. Party payoffs are

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[23]It is worth mentioning that without the restriction to $E(s_r) > 1$ in the proposition, the threshold becomes $E(t_{\frac{D+1}{2}}) < a_{lm}$ as in the benchmark case. This is because some districts may then condition on $\lambda = (\frac{D-1}{2}, \frac{D-1}{2}, 0)$, and have $l$ and $m$ as serious candidates. In such a case $l$ will win the district only if $E(t) < a_{lm}$.

again given by Equation 1.7. As usual if a party has a majority of seats it will implement its preferred policy. Following Banks and Duggan (2006), when no party has a majority I look for an equilibrium of the form $y_l = a_m - \Delta$, $y_m = a_m$, $y_r = a_m + \Delta$. Cho and Duggan (2003) show that this stationary equilibrium is unique. As bargaining is only over policy, any minimum winning coalition will include party $m$. When there was no discounting this meant party $m$ could always achieve $z = a_m$. Now, however, the presence of discounting allows parties $l$ and $r$ to offer policies further away from $a_m$, which party $m$ will nonetheless support. The median party will be indifferent between accepting and rejecting an offer $y$ when

$$W_m(y) = -(\Delta)^2 = -(1-\delta)(Q)^2 - \frac{\delta(D - s_m)}{D}(\Delta)^2 \qquad (1.8)$$

which, when rearranged gives

$$\Delta = \pm\sqrt{\frac{(1-\delta)Q^2}{1 - \delta\frac{D-s_m}{D}}} \qquad (1.9)$$

Table 1.3 shows the equilibrium offer each party will make when chosen as formateur. Notice that the policies offered by $l$ and $r$ depend on the seat share of party $m$. The more seats party $m$ has, the closer these offers get to zero.

| Formateur | Policy |
|:---:|:---:|
| $y_l$ | $-\sqrt{\frac{1-\delta}{1 - \delta\frac{D-s_m}{D}}Q^2}$ |
| $y_m$ | $a_m = 0$ |
| $y_r$ | $\sqrt{\frac{1-\delta}{1 - \delta\frac{D-s_m}{D}}Q^2}$ |

Table 1.3: Policy proposals with random order bargaining over policy, $\delta < 1$.

For a seat distribution such that no party has a majority, the expected policy outcome from bargaining is

$$E(z) = -\frac{s_l}{D}\left(\sqrt{\frac{1-\delta}{1 - \delta\frac{D-s_m}{D}}Q^2}\right) + \frac{s_m}{D}(0) + \frac{s_r}{D}\left(\sqrt{\frac{1-\delta}{1 - \delta\frac{D-s_m}{D}}Q^2}\right) \qquad (1.10)$$

An extra seat for any of the three parties will increase their respective probabilities of being the formateur and so affect the expected policy outcome. Thus, every district

always faces a choice between three distinct (expected) policies. We also see that as $s_m$ increases, the expected policy moves closer and closer to zero. This occurs for two reasons; firstly because there is a higher probability of party $m$ being the formateur, and secondly because $s_m$ enters the policy offers of $l$ and $r$; as $s_m$ increases the absolute value of these policies shrink.

The proposition below shows that when the bargaining protocol is random, parties discount the future, and the status quo is not exactly $a_m$, it is easier for a non-centrist party to win a majority of seats and implement its preferred policy than is the case in the benchmark model.

**Proposition 1.9.** *For any distribution of voter preferences, in a legislative election with a random order of bargaining over policy, $\delta < 1$ and $E(s_m) > 0$; $z = a_l$ can only be implemented if $E(t_{\frac{D+1}{2}}) < z_l^*$, where $z_l^* > a_{lm}$. Similarly, $z = a_r$ can only be implemented if $E(t_{\frac{D+1}{2}}) > z_r^*$, where $z_r^* < a_{mr}$.*

*Proof.* See section 2.6. □

The proposition implies that we should witness more majority governments than coalition governments when the bargaining protocol is random. The reason is that with a random recognition rule voters face vast uncertainty if they choose to elect a coalition. The implemented policy will vary greatly depending on which party is randomly chosen as formateur. As voters are risk averse, they find the certainty of policy provided by a majority government appealing. The median voter in the median district need not prefer the policy of a non-centrist party to that of party $m$ in order for the former to win a majority of seats. Along with the previous propositions on polarisation, this proposition shows that no matter which of the bargaining rules is used, there is less scope for polarisation in legislative elections with national voters than there is in presidential elections or legislative elections with local voters.

Figure 1.6 below shows the various decisive cases when the random recognition rule is used. Almost all points are $\lambda(3)$ events, and as we know, if a district is conditioning on
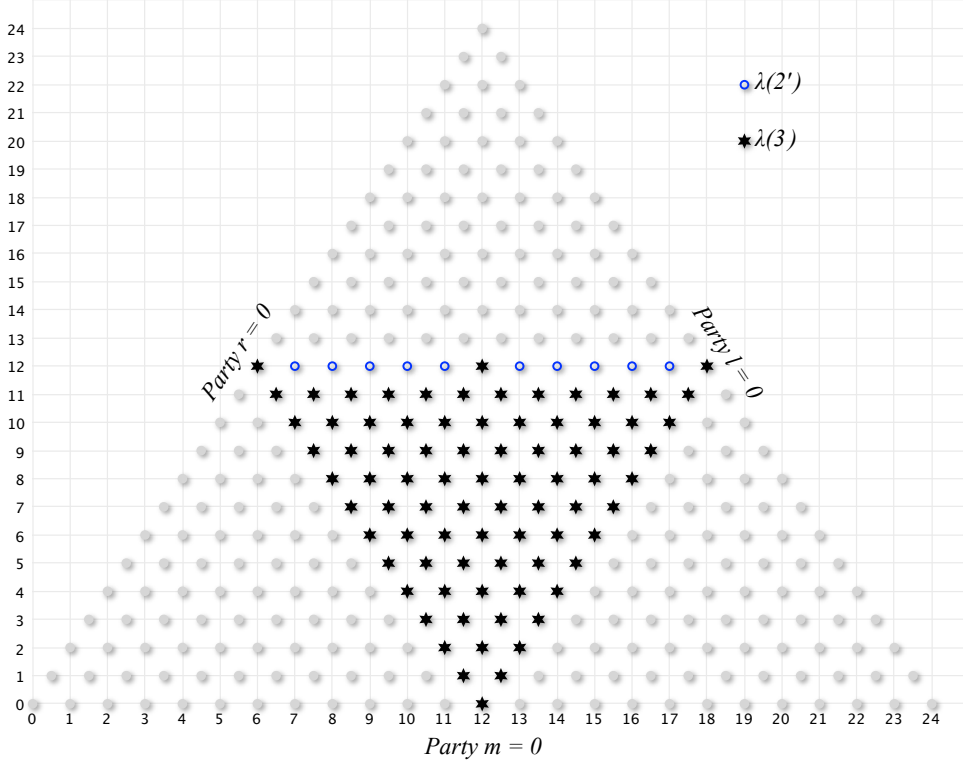
Figure 1.6: Decisive events under random recognition bargaining, with $D = 25$ and $\delta < 1$

such an event it must have misaligned voting.[24] There are however, a selection of $\lambda(2')$ events when an extra seat for party $m$ gives them a majority. If $E(s_m) > \frac{D-1}{2}$, then there are many equilibria in which there is misaligned voting in only a subset of districts.

**Proposition 1.10.** *For any distribution of voter preferences, in a legislative election with a random order of bargaining over policy and $\delta < 1$, there are no equilibria without misaligned voting. However, equilibria exist with only misaligned voting in a subset of districts.*

*Proof.* See section 2.6. □

---

[24]The picture changes somewhat depending on the values of $Q, D$ and $\delta$: for certain values, decisive events where $s_m$ is small may be $\lambda(2')$ events or may even be events where all voters would like to elect $m$. However this does not alter proposition 1.10. Figure 1.6 shows the case of $D = 25$, $\delta = 0.99$ and $|Q| < 0.33$.

At any decisive event, voters may be conditioning on, they face a choice between three distinct (expected) policies. In almost all cases this means that there will be misaligned voting in the districts. However, as I show in the proof, and as can be seen from Figure 1.6, when $m$ is the largest party then the expected policy which comes about by electing the smallest party in the legislature is actually not preferred by any voter type. So, in this case, if a district focuses on the two national frontrunners as the two serious candidates in their district, there will be no misaligned voting. In fact, there may only be misaligned voting in one district. If party $m$ has a majority of seats, party $l$ has one seat, and $r$ has the rest, then as long as all $d_m$ and $d_r$ districts have $m$ and $r$ as serious candidates, only the one $d_l$ district will have misaligned voting. This gives a fresh insight into the idea of "wasted votes": if party $l$ or $r$ is expected to be the smallest of the parties in the legislature, and party $m$ is expected to have a majority, then the least popular national party should optimally be abandoned by voters. Any district which actually elects the weakest national party does so due to a coordination failure; a majority there would instead prefer to elect one of the other two parties. Notice however, that for this to be the case, the median party must be expected to win an overall majority. So, while the idea of a wasted vote does carry some weight, it clearly does not apply to the case of the Liberal Democrats.

## 1.7   Discussion

In this paper, I introduced and analysed a model of three-party competition in legislative elections under plurality rule. I showed that two negative aspects of plurality rule - polarisation and misaligned voting - are significantly reduced when the rule is used to elect a legislature. The degree to which these phenomena are reduced depends on the institutional setup - specifically, on how legislative bargaining occurs.

In the benchmark model, parties are perfectly patient and bargain only over policy. Two clear results emerged from this model. First, while an extreme policy can always come about in a presidential election or legislative election with local voters, in order for

a non-centrist policy to be implemented by national voters it needs broad support in the electorate; specifically, the median voter in the median district must prefer the extreme policy to the median party's policy. Second, while standard plurality elections with three distinct choices always have misaligned voting, in my benchmark model this is the case if the non-centrist parties are serious candidates in more than half the districts - otherwise there is no misaligned voting in any district.

The results of the benchmark model largely hold up under the other bargaining rules considered: the non-centrist parties cannot win for any voter preferences (unlike in standard plurality elections), and there are always equilibria in which there is no misaligned voting (at least in a subset of districts). Moreover, if parties are impatient we gain an additional insight: with a fixed order of formateur recognition we should see more coalitions while when the order is random we should see more single-party governments, all else equal.

In the remainder of this section I discuss the robustness of my modelling assumptions. First, if utility functions are concave rather than specifically quadratic, the benchmark model is unchanged. When bargaining also involves perks, the same is true as long as there are enough perks to allow a coalition between the left and right parties to form.[25] Second, if parties bargain by making demands rather than offers, as in Morelli (1999), the results will be the same as in the benchmark model.[26] Third, if instead of a Poisson model I assumed a fixed population size drawn from a multinomial distribution, the results of my model would still go through.[27]

A key assumption is that voters only care about the policy implemented in the legislature. If they also have preferences over who wins their local district, the results

---

[25]If the perks are not large enough or parties don't value perks enough, a coalition will always involve the median party and we return to the simpler bargaining over policy case.

[26]Whenever there is no clear majority, the head of state selects party $m$ as the first mover, so the coalition policy will be $z = a_m$.

[27]Myerson (2000) shows that the magnitude of an event with a multinomial distribution is a simple transformation of its magnitude with a Poisson distribution. This transformation preserves the ordering of events and so this means that the ordering of sets of pivotal and decisive events in my model would remain unchanged, and therefore so would the equilibria.

of the model no longer hold: the probability of being pivotal locally would outweigh any possible utility gain at the national level so that voters will only consider the local dimension. However, in Westminster systems, a Member of Parliament has no power to implement policy at a local level; he merely serves as an agent of his constituents: bringing up local issues in parliament, helping constituents with housing authority claims, etc. So, if voters *do* have preferences over their local winner, it should only be on a common-value, valence dimension. If this were indeed the case, party policies would be irrelevant for how voters cast their ballots.

Arguably, the assumption of perfect information is unrealistic in a real world election. The asymptotic elements of the model means voters can perfectly rank the probabilities of certain events. In real life we are never that confident: polls may not be accurate, or more often, polls may not exist at the district level. In the future, I would like to extend the model to include greater aggregate uncertainty over voters' preferences.

Finally, the model does not address how party policies are formed nor how the number of competing parties comes about. Endogenising the entry decisions and policy choices of parties would provide a richer model with which to compare different parliamentary systems, and would tackle the question of whether plurality rule really leads to a two-party system, as is often claimed. This is something I wish to address in future research.

## 1.8   Appendix

### 1.8.1   Poisson Properties

The number of voters in a district is a Poisson random variable $n_d$ with mean $n$. The probability of having exactly $k$ voters is $Pr[n_d = k] = \frac{e^{-n}n^k}{k!}$. Poisson Voting games exhibit some useful properties. By *environmental equivalence*, from the perspective of player in the game, the number of other players is also a Poisson random variable $n_d$ with mean $n$. By the *decomposition property*, the number of voters of type $t$ is Poisson distributed with mean $nf_d(t)$, and is independent of the number of other players types. For simplicity here I drop the district subscript. The probability of a vote profile

$x = (x(l), x(m), x(r))$ given voter strategies is

$$Pr[x|n\tau] = \prod_{c \in \{l,m,r\}} \frac{e^{-n\tau(c)}(n\tau(c))^{x(c)}}{x(c)!} \tag{1.11}$$

**Magnitude Theorem** Let an event $E$ be a subset of all possible vote profiles. The magnitude theorem (Myerson (2000)) states that for a large population of size $n$, the magnitude of an event, $\mu(E)$, is:

$$\mu(E) \equiv \lim_{n \to \infty} \frac{\log(Pr[E])}{n} = \max_{x \in E} \sum_{c \in \{l,m,r\}} \tau_d(c)\psi\left(\frac{x_d(c)}{n\tau_d(c)}\right) \tag{1.12}$$

where $\psi(\theta) = \theta(1 - log(\theta)) - 1$. That is, as $n \to \infty$, the magnitude of an event $E$ is simply the magnitude of the most likely vote profile $x \in E$. The magnitude $\mu(E) \in [-1, 0]$ represents the speed at which the probability of the event goes to zero as $n \to \infty$; the more negative its magnitude, the faster that event's probability converges to zero.

**Corollary to the Magnitude Theorem** If two events E and E' have $\mu(E) < \mu(E')$, then their probability ratio converges to zero as $n \to \infty$.

$$\mu(E) < \mu(E') \Rightarrow \lim_{n \to \infty} \frac{Pr[E]}{Pr[E']} = 0 \tag{1.13}$$

It is possible that two distinct events have the same magnitude. In this case, we must use the offset theorem to compare their relative probabilities.

**Offset Theorem** Take two distinct events, $E \neq E'$ with the same magnitude, then

$$\mu(E) = \mu(E') \Rightarrow \lim_{n \to \infty} \frac{Pr[E]}{Pr[E']} = \phi \quad 0 < \phi < \infty \tag{1.14}$$

Suppose we have $\tau(c_1) > \tau(c_2) > \tau(c_3)$, so that the subscript denotes a party's expected ranking in terms of vote share. Maximising Equation 1.12 subject to the appropriate constraints we get

$$
\begin{aligned}
\mu(piv(i,j)) &= \mu(piv(j,i)) \quad \forall i, j \in C \\
\mu(c_1\text{-win}) &= 0 \\
\mu(c_2\text{-win}) = \mu(piv(c_1, c_2)) &= 2\sqrt{\tau(c_1)\tau(c_2)} - \tau(c_3) \\
\mu(c_3\text{-win}) = \mu(piv(c_2, c_3)) = \mu(piv(c_1, c_3)) &= 3\sqrt[3]{\tau(c_1)\tau(c_2)\tau(c_3)} - 1 \quad \text{if } \tau(c_1)\tau(c_3) < \tau(c_2)^2 \\
\mu(c_3\text{-win}) = \mu(piv(c_1, c_3)) &= 2\sqrt{\tau(c_1)\tau(c_3)} - \tau(c_2) \quad \text{if } \tau(c_1)\tau(c_3) > \tau(c_2)^2
\end{aligned}
\tag{1.15}
$$

With a magnitude of zero, by the corollary, the probability of candidate $c_1$ winning goes to 1 as $n$ gets large. Also, as the magnitude of a pivotal event between $c_1$ and $c_2$ is greater than all other pivotal events, a pivotal event between $c_1$ and $c_2$ is infinitely more likely than a pivotal event between any other pair as $n$ gets large.

### 1.8.2 Proof of Proposition 1

I first show that voters will only condition on the most likely event in which their vote is pivotal and decisive. Then I show that all strictly perfect equilibria involve only two vote getters in each district. Finally, I show that there are always multiple equilibria.

Let $\boldsymbol{\tau}$ be a candidate for equilibrium. It summaries the expected vote shares in each district given the strategies of all player types. A voter of type $t$ in district $d$, decides how to vote by comparing his expected gain from each ballot, given by Equation 1.2. In order to exactly compute his expected gain for each ballot, the voter would need to work out the probability of each combination of being both pivotal and decisive, $Pr[piv_d(c, c')]Pr[\lambda_d^i]$ for all $c, c' \in C$ and $i \in I$. The probability of a particular profile of votes across all districts is

$$Pr[x|n\boldsymbol{\tau}] = \prod_{\substack{d \in D \\ c \in \{l,m,r\}}} \left( \frac{e^{-n\tau_d(c)}(n\tau_d(c))^{x_d(c)}}{x_d(c)!} \right) \tag{1.16}$$

After some manipulation, taking the log of both sides, and taking the limit as $n \to \infty$ we get the magnitude of this profile of votes

$$\mu(x) \equiv \lim_{n \to \infty} \frac{log(Pr[x|n\boldsymbol{\tau}])}{n} = \lim_{n \to \infty} \sum_{c \in \{l,m,r\}} \sum_{d \in D} \tau_d(c)\psi\left( \frac{x_d(c)}{n\tau_d(c)} \right) \tag{1.17}$$

Notice that the magnitude of a particular profile of votes across districts, is simply the sum of the magnitudes in each district. So while $\mu(x_d) \in (-1, 0)$ in a single district, when considering the profile of votes in all districts, $x$, we have $\mu(x) \in (-D, 0)$. Just as the magnitude theorem can be used to find the most likely events in a given district, the linear nature of Equation 1.17 means the exact same approach can be used for events which involve multiple districts. Given $\boldsymbol{\tau}$ and a constraint, such as a particular seat

distribution, we can easily find the maximum sum of magnitudes for which the constraint holds.

Once the magnitude of an event $piv_d(c_1, c_2)\lambda_d^i$ is greater than all other magnitudes, it is infinitely more likely to occur, so all voters in $d$ will vote for one of the two top candidates $c_1$ and $c_2$. As we can see from Equation 1.15, this magnitude must be greater than either $piv_d(c_1, c_3)\lambda_d^i$ or $piv_d(c_2, c_3)\lambda_d^i$.

I've shown that voters condition on the most likely event in which their vote is pivotal and decisive. I will now show that in all strictly perfect equilibria, the most likely pivotal and decisive event is unique i.e. races are duvergerian. If the decisive event with the largest magnitude $\lambda_d^1$ is a $\lambda(3)$ or $\lambda(2')$ event then it is immediate that the most likely pivotal and decisive event is $piv_d(c_1, c_2)\lambda_d^1$. However, if $\lambda_d^1$ is a $\lambda(2)$ event, then voters will instead focus on a less likely decisive event if they are indifferent between the two serious candidates in the $\lambda_d^1$ case. Nevertheless, the outcome will once again be duvergerian. Non-Duvergerian outcomes can only occur when the probability of two events with different serious candidates are exactly equal. For the probability of two events to be equal, their magnitudes must be equal, so the following condition must hold

$$\mu(piv_d(c, c'')\lambda_d^i) = \mu(piv_d(c, c')\lambda_d^i) \tag{1.18}$$

for some combination of candidates $c, c', c'' \in C$. Now, from Equation 1.15 we know there is only one case in which two different pivotal events have the same magnitude. That is when $\tau(c_1)\tau(c_3) < \tau(c_2)^2$ so that $\mu(piv(c_2, c_3)) = \mu(piv(c_1, c_3)) = 3\sqrt[3]{\tau(c_1)\tau(c_2)\tau(c_3)} - 1$. This occurs when the least popular candidate has much less support than the other two. For Equation 1.18 to hold and be the largest magnitude it must be that $\lambda_d^1$ is of type $\lambda(2)$. The condition then becomes

$$\mu(piv_d(c_1, c_3)\lambda_d^1) = \mu(piv_d(c_2, c_3)\lambda_d^1) \tag{1.19}$$

If this condition holds, then it is resistant to perturbations, as any change of $\epsilon$ will have the same effect on both sides; so we have a candidate for a strictly perfect equilibrium which is non-duvergerian. However, while these magnitudes are the same, we can use the offset

theorem to see that their probabilities are not. Specifically $piv_d(c_1, c_3) \neq piv_d(c_2, c_3)$. We know from the offset theorem that

$$\frac{Pr[piv_d(c_1, c_3)]}{Pr[piv_d(c_2, c_3)]} = \phi > 0 \tag{1.20}$$

The magnitudes of these events are the same because the most likely event in which they occur is when all three candidates get exactly the same number of votes. However, the events consist of more than just this event. In an event $piv_d(c_1, c_3)$ , candidate $c_2$ must have the same or fewer votes than the others. Similarly, in an event $piv_d(c_2, c_3)$ , candidate $c_1$ must have the same or fewer votes than the others. By the decomposition property, for any given number of votes $c_3$ has, $c_1$ is always more likely to have more votes than $c_2$. Therefore, it must be that $\phi > 1$. Returning to the decision of a voter facing

$$\mu(piv_d(c_1, c_3)\lambda_d^1) = \mu(piv_d(c_2, c_3)\lambda_d^1)$$

and given that $\lambda_d^1$ is a $\lambda(2)$ event, voters are indifferent between having $c_1$ or $c_2$ elected. As I have just shown, $Pr[piv_d(c_1, c_3)] > Pr[piv_d(c_2, c_3)]$, therefore all voters should focus on this event, which will mean the previous second placed candidates loses all her support to the leading candidate, thus ensuring a duvergerian equilibrium. Therefore, all strictly perfect equilibria involve duvergerian races in every district.

A simple example proves the existence of multiple pure strategy equilibria for any bargaining rule where a majority is needed to implement a policy $z$. For any $\mathbf{f}$, suppose the right party is never serious in any district. All races will be between $l$ and $m$. We will have $E(S_{-d_l}) = (k-1, D-k, 0)$ and $E(S_{-d_m}) = (k, D-k-1, 0)$ for some $k \in (0, D)$. As party $r$ receives no votes, every district must be conditioning on the same decisive event $\lambda = (\frac{D-1}{2}, \frac{D-1}{2}, 0)$. When conditioning on this $\lambda$, voters will face a choice between $a_l$, $a_m$, and a third policy which would come about if $r$ wins a seat. All the districts may focus on races between $l$ and $m$, and so it is indeed an equilibrium. Similarly, it is possible that all districts ignore the left party, and so an equilibrium will have every district conditioning on $\lambda = (0, \frac{D-1}{2}, \frac{D-1}{2})$, or that all districts ignore the middle party and all condition on $\lambda = (\frac{D-1}{2}, 0, \frac{D-1}{2})$. These three equilibria always exist for any majoritarian

bargaining rule.

### 1.8.3 Proof of Proposition 5

*Case 1:* From Table 1.1 we see that when $l_l = l_r$, the policy outcome will be $a_m$ if no party has a majority and $m$ is not that smallest party. Given that there are $D$ districts and all equilibria are duvergerian, there will be $2D$ serious candidates. Any one party can be serious in at most $D$ districts. Suppose party $r$ is a serious candidate in less than $\frac{D-1}{4}$ districts; then it can win at most that many seats. Meanwhile, $l$ and $m$ must each be serious candidates in more than $\frac{D+1}{4}$ districts. If party $m$ wins less than $\frac{D-1}{4}$ districts and party $r$ comes second, it must be that party $l$ has an overall majority - thus there will be no decisive events. Similarly if party $l$ wins less than $\frac{D-1}{4}$ districts and party $r$ comes second, it must be that party $m$ will have an overall majority; again no decisive events. Conditional on $r$ winning less than $\frac{D-1}{4}$ districts, the only decisive event is $\lambda(a_l, a_m, a_m)$, when $l$ wins $\frac{D-1}{2}$ seats and $m$ is the second largest party. Given that only this distinct decisive event exists, all voters must be conditioning on it. As electing $m$ or $r$ in this decisive event brings about the same policy, voters are indifferent between the two. In each district, those with $t < a_{lm}$ will vote $v_l$ while those with $t > a_{lm}$ will coordinate on either $v_m$ or $v_r$. As long as the latter group coordinate on $v_r$ in less than $\frac{D-1}{4}$ districts, there is no misaligned voting for any voter in any district. The case of $l$ being a serious candidate in less than $\frac{D-1}{4}$ districts is analogous.

*Case 2:* From Table 1.1 we see that when $l_l < l_r$, if $\frac{D+1}{2} > s_l > s_m > s_r$, then the policy outcome will be $a_m$. As shown above, when $r$ is serious in less than $\frac{D-1}{4}$ districts, the only distinct decisive case is $\lambda(a_l, a_m, a_m)$. All districts will condition on this event and, as before, there is no misaligned voting as long as less than $\frac{D-1}{4}$ districts coordinate on a race between $l$ and $r$.

*Case 3:* From Table 1.1 we see that when $l_r < l_l$, if $\frac{D+1}{2} > s_r > s_m > s_l$, then the policy outcome will be $a_m$. When $l$ is serious in less than $\frac{D-1}{4}$ districts, the only distinct decisive event is $\lambda(a_m, a_m, a_r)$. All districts will condition on this event and there is no misaligned voting as long as less than $\frac{D-1}{4}$ districts coordinate on a race between $l$ and

$r$.                                                                                       □

### 1.8.4   Proof of Proposition 6

*Case 1:* Recall that in all $\lambda(2')$ events under this bargaining rule, one of the outcomes is a lottery over the other two and is thus preferred by no voter. The frequency of these events can be seen in Figure 1.3. If $\lambda_d^1$ is a $\lambda(2')$ event, it must be infinitely more likely to occur than any other decisive event, and given that voters are not indifferent between any of the options, whichever option a voter prefers in this case will also be his preferred over all possible decisive events. Every voter prefers one of the two "pure" policies to the lottery over them, so if acting unilaterally would never choose the lottery candidate. Therefore, as long as the lottery candidate is not one of the serious candidates, there is no misaligned voting in that district.

*Case 2:* Let the most likely decisive event $\lambda_d^1$ be a $\lambda(2)$ event where candidates $c$ and $c'$ are serious. If voters are conditioning on $\lambda_d^1$ it must be that $z^1(c) = z^1(c'')$ or $z^1(c') = z^1(c'')$; Here, I take it to be the former. Without loss of generality let $z^1(c) < z^1(c')$. Any voter type with $t > \frac{z^1(c)+z^1(c')}{2}$ will vote $v_{c'}$, while any voter type will vote $v_c$. The former group cannot be casting misaligned votes as they have $u_t(z^1(c')) > u_t(z^1(c))$, and decisive event $\lambda_d^1$ is infinitely more likely than all others. Next, we need to consider whether any of the voters choosing $v_c$ might be misaligned. All of these voters have $u_t(z^1(c)) = u_t(z^1(c'')) > u_t(z^1(c'))$, so that they want to beat $c'$ but are indifferent between $c$ and $c''$ in this most likely decisive event. If one of these voters could unilaterally decide which candidate coordination takes place on, he would decide by looking at the most likely pivotal event in which $z(c) \neq z(c'')$, call this event $\lambda^i$. If $u_t(z^i(c)) > u_t(z^i(c''))$ then voter type $t$ would prefer coordination to take place on candidate $c$, while if $u_t(z^i(c)) < u_t(z^i(c''))$ she'd want coordination on $c''$. Therefore, if there exists no type such that $u_t(z^i(c)) < u_t(z^i(c''))$ and $u_t(z^1(c)) = u_t(z^1(c'')) > u_t(z^1(c'))$ when $c$ and $c'$ are the serious candidates, then there is no misaligned voting in the district.                    □

### 1.8.5 Bargaining Equilibrium for Fixed Order Protocol and $\delta < 1$

As equilibria are stationary we need only consider two orderings: $l > r > m > l > r > \dots$ and $r > l > m > r > l > \dots$. I will derive the equilibrium offers for the case of $l > r > m$, the other is almost identical. I solve the game by backward induction. At stage 3, party $m$ will make an offer $y_m$ which maximises its payoff subject to the proposal being accepted by either party $l$ or $r$.

At stage 2, party $r$ will either make an offer $y_r(m)$ to attract party $m$, or an offer $y_r(l)$ to attract party $l$. For these proposals to be accepted by $m$ and $l$ respectively requires

$$-y_r(m)^2 \geq -(1-\delta)Q^2 - \delta y_m^2$$

$$-(a_l - y_r(l))^2 \geq -(1-\delta)(a_l - Q)^2 - \delta(a_l - y_m)^2$$

If $y_r(m)$ is chosen then the first inequality will bind and we have $y_r(m) = \sqrt{(1-\delta)Q^2 + \delta y_m^2}$. We can now compare the payoff of party $l$ when $y_r(m)$ and $y_r(l)$ are implemented.

$$-(a_l - \sqrt{(1-\delta)Q^2 + \delta y_m^2})^2 = -a_l^2 - (1-\delta)Q^2 - \delta y_m^2 + 2a_l\sqrt{(1-\delta Q^2 + \delta y_m^2)}$$

$$-(a_l - y_r(l))^2 = -a_l^2 - (1-\delta)Q^2 - \delta y_m^2 + (1-\delta)2a_lQ + \delta 2a_l y_m$$

Party $l$ prefers policy $y_r(l)$ when

$$(1-\delta)2a_lQ + \delta 2a_l y_m > 2a_l\sqrt{(1-\delta Q^2 + \delta y_m^2)}$$

$$2a_l((1-\delta)Q + \delta y_m) > 2a_l\sqrt{(1-\delta Q^2 + \delta y_m^2)}$$

$$(1-\delta)Q + \delta y_m < \sqrt{(1-\delta Q^2 + \delta y_m^2)}$$

the final inequality always holds. As party $l$ gets a higher payoff from $y_r(l)$ than $y_r(m)$, the former must be closer to $a_l$ on the policy line, and therefore further away from $a_r$. Clearly then, party $r$ maximises its utility by choosing $y_r = \sqrt{(1-\delta)Q^2 + \delta y_m^2}$.

At stage 1, party $l$ will either make an offer $y_l(m)$ to attract party $m$, or an offer $y_l(r)$ to attract party $r$. For these proposals to be accepted by $m$ and $r$ respectively requires

$$-y_l(m)^2 \geq -(1-\delta)Q^2 - \delta(-\sqrt{(1-\delta)Q^2 + \delta y_m^2})^2$$
$$-(a_r - y_l(r))^2 \geq -(1-\delta)(a_r - Q)^2 - \delta(a_r - \sqrt{(1-\delta)Q^2 + \delta y_m^2})^2$$

If $y_l(m)$ is chosen then the first inequality will bind and we have $y_l(m) = -\sqrt{(1-\delta^2)Q^2 + \delta^2 y_m^2}$. We can now compare the payoff of party $r$ when $y_l(m)$ and $y_l(r)$ are implemented.

$$-(a_r + \sqrt{(1-\delta^2)Q^2 + \delta^2 y_m^2})^2 = -a_r^2 - (1-\delta^2)Q^2 - \delta^2 y_m^2 - 2a_r\sqrt{(1-\delta^2 Q^2 + \delta^2 y_m^2)}$$
$$-(a_r - y_l(r))^2 = -(1-\delta)(a_r^2 + Q^2 - 2a_r Q) - \delta a_r^2 - \delta(1-\delta)Q^2 - \delta^2 y_m^2 + \delta 2a_r\sqrt{(1-\delta)Q^2 + \delta y_m^2}$$

Party $r$ prefers policy $y_l(r)$ when

$$(1-\delta)2a_r Q + \delta 2a_r\sqrt{(1-\delta)Q^2 + \delta y_m} > -2a_r\sqrt{(1-\delta^2 Q^2 + \delta^2 y_m^2)}$$
$$(1-\delta)Q + \delta\sqrt{(1-\delta)Q^2 + \delta y_m} > -\sqrt{(1-\delta^2 Q^2 + \delta^2 y_m^2)}$$

the final inequality always holds. As party $r$ gets a higher payoff from $y_l(r)$ than $y_l(m)$, the former must be closer to $a_r$ on the policy line, and therefore further away from $a_l$. Clearly then, party $l$ maximises its utility by choosing $y_l = -\sqrt{(1-\delta^2)Q^2 + \delta^2 y_m^2}$.

Now, we can return to stage 3 to show that $y_m = 0$. By stationarity, if $y_m$ is rejected at stage 3, then in stage 4 $y_l = -\sqrt{(1-\delta^2)Q^2 + \delta^2 y_m^2}$ will be proposed and accepted. Parties $l$ and $r$ will accept proposal $y_m$ if

$$-(a_l - y_m)^2 \geq -(1-\delta)(a_l - Q)^2 - \delta(a_l + \sqrt{(1-\delta^2)Q^2 + \delta^2 y_m^2})^2$$
$$-(a_r - y_m)^2 \geq -(1-\delta)(a_r - Q)^2 - \delta(a_r + \sqrt{(1-\delta^2)Q^2 + \delta^2 y_m^2})^2$$

Party $m$'s payoff is maximised when $y_m = 0$ (because $a_m = 0$), so we want to check whether this is an implementable proposal. Letting $y_m = 0$ and rearranging, the two

inequalities above become

$$0 \leq (1 - \delta^3)Q^2 + 2a_l[\delta\sqrt{(1 - \delta^2)Q} - (1 - \delta)Q]$$
$$0 \leq (1 - \delta^3)Q^2 + 2a_r[\delta\sqrt{(1 - \delta^2)Q} - (1 - \delta)Q]$$

The term in square brackets may be positive or negative. If it is positive then, party $r$ will accept $y_m = 0$, if the term is negative then party $l$ will accept $y_m = 0$. Whenever $\delta > 0.543689$ then the term is positive. Given that we mostly care about values of $\delta$ close to one, we can say that it is generally party $r$ who accepts $m$'s offer.

Given $y_m = 0$, we can now characterise the accepted policy proposals (and therefore policy outcomes) for the fixed order protocol when $l > r > m > l > r > \ldots$.

$$
\begin{aligned}
y_l &= -\sqrt{(1 - \delta^2)Q^2} \\
y_r &= \sqrt{(1 - \delta)Q^2} \\
y_m &= 0
\end{aligned}
$$

Instead when $r > l > m > r > l > \ldots$, the same process gives:

$$
\begin{aligned}
y_r &= \sqrt{(1 - \delta^2)Q^2} \\
y_l &= -\sqrt{(1 - \delta)Q^2} \\
y_m &= 0
\end{aligned}
$$

### 1.8.6 Proof of Proposition 7

For $z = a_l$ to be implemented it must be that $E(s_l) > \frac{D-1}{2}$. Given the restriction that $E(s_m), E(s_r) > 1$, the set of distinct decisive events which $d_l$ districts can be conditioning

on is reduced to

$$\Lambda = \{\lambda(a_l, -\sqrt{(1-\delta^2)Q^2}, -\sqrt{(1-\delta^2)Q^2}),$$
$$\lambda(a_l, -\sqrt{(1-\delta^2)Q^2}, -\sqrt{(1-\delta)Q^2}),$$
$$\lambda(a_l, -\sqrt{(1-\delta)Q^2}, -\sqrt{(1-\delta)Q^2})\}$$

Any race between $a_l$ and $-\sqrt{(1-\delta)Q^2}$, where the former is the expected winner, must have $E(t) < \frac{a_l - \sqrt{(1-\delta)Q^2}}{2}$. Any race between $a_l$ and $-\sqrt{(1-\delta^2)Q^2}$, where the former is the expected winner, must have $E(t) < \frac{a_l - \sqrt{(1-\delta^2)Q^2}}{2}$, a stricter condition. Therefore in order to for a party $l$ majority to come about when $E(s_m), E(s_r) > 1$ it must be at least that $E(t_{\frac{D+1}{2}}) < \frac{a_l - \sqrt{(1-\delta)Q^2}}{2}$. Notice that since $-\sqrt{(1-\delta)Q^2} < a_m$, then $\frac{a_l - \sqrt{(1-\delta)Q^2}}{2} < a_{lm}$. Similarly, for party $r$ to win a majority when $E(s_m), E(s_l) > 1$ it must be that $E(t_{\frac{D+1}{2}}) > \frac{a_r + \sqrt{(1-\delta)Q^2}}{2} > a_{mr}$. $\square$

### 1.8.7 Proof of Proposition 9

Suppose party $l$ is expected to win a majority. Then $E(S)$ must be in the bottom left section of Figure 1.6. Each of the decisive events are distinct as by increasing a party's seat share by one, it alters the expected policy outcome $E(z)$. Each district must be conditioning on a decisive event where $s_l = \frac{D-1}{2}$. Given this, the policy choice of the district depends on the number of seats $m$ and $r$ have. A larger number of $r$ seats implies a smaller number of $m$ seats. The larger number of $r$ seats means a higher likelihood of a policy to the right of $a_m$, and on top of this a smaller number of $m$ seats means the policies offered by parties $l$ and $r$ are further from $a_m$. Therefore, given $s_m > 0$, the furthest expected policy from $a_l$ must be at the point $\lambda = (\frac{D-1}{2}, 1, \frac{D-3}{2})$. At this point, electing party $l$ gives them a majority and brings about $z = a_l$, while electing party $r$ leads to a coalition with an ex ante expected policy of

$$E(z) = -\frac{D-1}{2D}\left(\sqrt{\frac{1-\delta}{1-\delta\frac{D-1}{D}}Q^2}\right) + \frac{2}{2D}(0) + \frac{D-1}{2D}\left(\sqrt{\frac{1-\delta}{1-\delta\frac{D-1}{D}}Q^2}\right)$$

A voter will prefer the former if

$$-(a_l - t)^2 > -\frac{D-1}{2D}\left(-\sqrt{\frac{1-\delta}{1-\delta\frac{D-1}{D}}Q^2} - t\right)^2 - \frac{2}{2D}\left(-t\right)^2 - \frac{D-1}{2D}\left(\sqrt{\frac{1-\delta}{1-\delta\frac{D-1}{D}}Q^2} - t\right)^2$$

rearranging this we get that a voter prefers $a_l$ if

$$t < \frac{a_l}{2} - \frac{D-1}{2Da_l}\left(\frac{1-\delta}{1-\delta(\frac{D-1}{D})}Q^2\right)$$

As $a_l < 0$, the right hand side is greater than $\frac{a_l}{2}$, which is the cutoff point in the benchmark case (recalling that $a_{lm} = \frac{a_l}{2}$ when $a_m = 0$). The cutoff for a party $l$ majority is thus given by

$$E(t_{\frac{D+1}{2}}) < z_l^* \equiv \frac{a_l}{2} - \frac{D-1}{2Da_l}\left(\frac{1-\delta}{1-\delta(\frac{D-1}{D})}Q^2\right) > a_{lm}$$

while the cutoff for a party $r$ majority is given by

$$E(t_{\frac{D+1}{2}}) > z_r^* \equiv \frac{a_r}{2} - \frac{D-1}{2Da_r}\left(\frac{1-\delta}{1-\delta(\frac{D-1}{D})}Q^2\right) < a_{mr}$$

$\square$

### 1.8.8   Proof of Proposition 10

I first show that when $s_m < \frac{D-1}{2}$, there will always be misaligned voting in some, if not all, districts. Then I show that when $s_m > \frac{D-1}{2}$ there are equilibria with no misaligned voting in a subset of districts. Finally, I show that in any equilibrium there must be misaligned voting in at least some districts.

*Case 1:* When $s_m < \frac{D-1}{2}$ there will always be misaligned voting in equilibrium.

*Case 1a:* When one of the non-centrist parties is expected to have a majority, there will be misaligned voting.

I examine the case where $l$ is expected to win a majority; the other case is identical. Voters must be conditioning on a decisive event where $s_l = \frac{D-1}{2}$; the expected utility of

electing the three different candidates is

$$
\begin{aligned}
u_t(l) &= -(a_l - t)^2 \\
u_t(m) &= -\frac{D-1}{2D}\left(-\sqrt{\frac{1-\delta}{1-\delta\frac{D-(s_m+1)}{D}}}Q^2 - t\right)^2 - \frac{s_m+1}{D}(t)^2 - \frac{D-1-2s_m}{2D}\left(\sqrt{\frac{1-\delta}{1-\delta\frac{D-(s_m+1)}{D}}}Q^2\right. \\
u_t(r) &= -\frac{D-1}{2D}\left(-\sqrt{\frac{1-\delta}{1-\delta\frac{D-s_m}{D}}}Q^2 - t\right)^2 - \frac{s_m}{D}(t)^2 - \frac{D+1-2s_m}{2D}\left(\sqrt{\frac{1-\delta}{1-\delta\frac{D-s_m}{D}}}Q^2 - t\right)^2
\end{aligned}
$$

It suffices to consider the most extreme types $t = -1$, $t = 0$ and $t = 1$. Subbing these values in we see that a type $t = -1$ will always want to elect $l$ and a type $t = 0$ will always want to elect $m$. For any case $E(s_l) > \frac{D-1}{2}$, $E(s_r) > 0$, there must therefore be misaligned voting as in the districts where $r$ is expected to win, the other voters coordinate on either $l$ or $m$. The supporters of that candidate which is not serious must be casting misaligned votes.

*Case 1b:* For any expected seat distribution where no party has a majority, the expected utility of electing the three different candidates is

$$
\begin{aligned}
u_t(l) &= -\frac{s_l+1}{D}\left(-\sqrt{\frac{1-\delta}{1-\delta\frac{D-s_m}{D}}}Q^2 - t\right)^2 - \frac{s_m}{D}(t)^2 - \frac{s_r}{D}\left(\sqrt{\frac{1-\delta}{1-\delta\frac{D-s_m}{D}}}Q^2 - t\right)^2 \\
u_t(m) &= -\frac{s_l}{D}\left(-\sqrt{\frac{1-\delta}{1-\delta\frac{D-(s_m+1)}{D}}}Q^2 - t\right)^2 - \frac{s_m+1}{D}(t)^2 - \frac{s_r}{D}\left(\sqrt{\frac{1-\delta}{1-\delta\frac{D-(s_m+1)}{D}}}Q^2 - t\right)^2 \\
u_t(r) &= -\frac{s_l}{D}\left(-\sqrt{\frac{1-\delta}{1-\delta\frac{D-s_m}{D}}}Q^2 - t\right)^2 - \frac{s_m}{D}(t)^2 - \frac{s_r+1}{D}\left(\sqrt{\frac{1-\delta}{1-\delta\frac{D-s_m}{D}}}Q^2 - t\right)^2
\end{aligned}
$$

Where I abuse notation slightly to let $s_c$ to be the expected number of seats of party $c$ before district $d$ votes, so that $s_l + s_m + s_r = D - 1$. By subbing in $t = 0$, we see that this type will always want $m$ elected. Whether a type $t = 1$ wants to elect $m$ or $r$ depends on parameters (similarly $t = -1$). Specifically, some algebra shows that a type $t = 1$ prefers to elect $r$ over $m$ when

$$
s_r - s_l > \frac{\sqrt{(1-\delta)DQ^2}D - 2(D - \delta D + \delta s_m + \delta)\sqrt{D - \delta D + \delta s_m}}{2[\sqrt{D - \delta D + \delta s_m + \delta} - \sqrt{D - \delta D + \delta s_m}]}
$$

A type $t = -1$ will prefer to elect $l$ than $m$ if

$$s_l - s_r > \frac{\sqrt{(1-\delta)DQ^2D - 2(D - \delta D + \delta s_m + \delta)\sqrt{D - \delta D + \delta s_m}}}{2[\sqrt{D - \delta D + \delta s_m + \delta} - \sqrt{D - \delta D + \delta s_m}]}$$

These inequalities will generally hold (they may not hold if $Q$ and $D$ are sufficiently large, and $s_m$ is sufficiently small). If these conditions hold, the three types have three different preferred candidates, so will be misaligned voting. However, even if they do not hold, there cannot be an equilibrium without misaligned voting, for the following reason: a $d_r$ district conditions on $r$ having less seats than a $d_l$ district conditions on (Graphically, $E(D_{-d_r})$ is one point to the left of $E(D_{-d_l})$). So, if at point $E(D_{-d_r})$ a $t = 1$ voter prefers $r$ to $m$, then for sure the same type at $E(D_{-d_l})$ would also prefer $r$ to $m$. Hence, at at least one of the three $E(S_{-d})$ points which make up an equilibrium there will be voters who prefer each of the three parties. Therefore, there will be misaligned voting.

*Case 2:* When $s_m > \frac{D-1}{2}$ a subset of districts may have no misaligned voting, but there will be misaligned voting in at least one district.

If $s_m > \frac{D-1}{2}$, all districts must be conditioning on decisive events where $s_l = \frac{D-1}{2}$. In such cases the expected utility of a type $t$ voter is

$$
\begin{aligned}
u_t(l) &= -\frac{s_l + 1}{D}\left(-\sqrt{\frac{1-\delta}{1 - \delta\frac{D+1}{2D}}Q^2} - t\right)^2 - \frac{D-1}{2D}(t)^2 - \frac{D - 1 - 2s_l}{2D}\left(\sqrt{\frac{1-\delta}{1 - \delta\frac{D+1}{2D}}Q^2} - t\right)^2 \\
u_t(m) &= -(t)^2 \\
u_t(r) &= -\frac{s_l}{D}\left(-\sqrt{\frac{1-\delta}{1 - \delta\frac{D+1}{2D}}Q^2} - t\right)^2 - \frac{D-1}{2D}(t)^2 - \frac{D + 1 - 2s_l}{2D}\left(\sqrt{\frac{1-\delta}{1 - \delta\frac{D-s_m}{D}}Q^2} - t\right)^2
\end{aligned}
$$

Note that for $t = 0$ we have $u_t(m) > u_t(l) = u_t(r)$. Any voter type with $t < 0$ has $u_t(l) > u_t(r)$, while any voter with $t > 0$ has $u_t(l) < u_t(r)$. However, it could be that some of these types prefer $u_t(m)$ to either of the other two. In order to check this I

calculate the derivative of each of the expected utilities with respect to $t$.

$$
\begin{aligned}
\frac{d[u(l)]}{dt} &= -2t + \frac{D-3-4s_l}{D}\left(\sqrt{\frac{1-\delta}{1-\delta^{\frac{D+1}{2D}}}}Q^2\right) \\
\frac{d[u(m)]}{dt} &= -2t \\
\frac{d[u(r)]}{dt} &= -2t + \frac{D+1-4s_l}{D}\left(\sqrt{\frac{1-\delta}{1-\delta^{\frac{D+1}{2D}}}}Q^2\right)
\end{aligned}
$$

When $s_l < \frac{D-3}{4}$ then for any $t < 0$ we have $\frac{d[u(m)]}{dt} < \frac{d[u(l)]}{dt} < \frac{d[u(r)]}{dt}$. Combined, with the fact that we have $u_t(m) > u_t(l) = u_t(r)$ for $t = 0$, this means that for $s_l < \frac{D-3}{4}$ there is no type with $u_t(l) > u_t(m), u_t(r)$. When $s_l > \frac{D+1}{4}$ then for any $t > 0$ we have $\frac{d[u(m)]}{dt} < \frac{d[u(r)]}{dt} < \frac{d[u(l)]}{dt}$. Combined, with the fact that we have $u_t(m) > u_t(l) = u_t(r)$ for $t = 0$, this means that for $s_l > \frac{D+1}{4}$ there is no type with $u_t(r) > u_t(m), u_t(l)$.

What this means is that, conditional on $s_m = \frac{D-1}{2}$, if $s_l < \frac{D-3}{4}$ then a district in which $m$ and $r$ are the serious candidates will have no misaligned voting; and if $s_l > \frac{D+1}{4}$ then a district in which $m$ and $l$ are the serious candidates will have no misaligned voting.

However, each equilibrium has misaligned voting in a least one district. Recall that $E(S_{d_l}) = (s_l - 1, s_m, s_r), E(S_{d_m}) = (s_l, s_m - 1, s_r)$ and $E(S_{d_r}) = (s_l, s_m, s_r - 1)$. As all the relevant decisive events occur at $s_m = \frac{D-1}{2}$, $d_l$ and $d_r$ districts will have the same "route" to being decisive. That is, in any equilibrium if $d_l$ districts are conditioning on $(k, \frac{D-1}{2}, \frac{D-1}{2} - k)$, then $d_r$ districts must be conditioning on $(k + 1, \frac{D-1}{2}, \frac{D-1}{2} - (k + 1))$.

When $0 < s_l < \frac{D-3}{4}$ then all $d_m$ and $d_r$ districts are conditioning on $\lambda(2')$ events. In any of these districts if the serious candidates are $m$ and $r$, there is no misaligned voting. However, we know that $d_l$ districts must either be conditioning on a $\lambda(2')$ event or else a $\lambda(3)$ event (if it conditions on $S_{-d} = (0, \frac{D-1}{2}, \frac{D-1}{2})$). Whichever one of these is the case, there will always be misaligned voting in these $d_l$ districts. Indeed, if it conditions on $S_{-d} = (0, \frac{D-1}{2}, \frac{D-1}{2})$, and the other districts are all races between $m$ and $r$, it must be that there is only misaligned voting in this single $d_l$ district. Examining the $\frac{D+1}{4} < s_l < \frac{D-1}{2}$ case gives the same insight for the mirror case; there'll be no misaligned voting in $d_l$ or $d_m$ districts if they focus on races between $l$ and $m$, but there will always

be misaligned voting in the $d_r$ districts. $\qquad\square$

# Chapter 2

# How Transparency Kills Information Aggregation (And Why That May Be Good)

*Joint work with Sebastian Fehrler, University of Zurich.*

We show theoretical and experimental results on the role of transparency in committee decision making and deliberation. We present a model in which committee members have career concerns and unanimity is needed to change the status quo. Transparency leads to a break down of information aggregation, causing more incorrect group decisions. However, if the cost of wrongly changing the status quo is high enough, the principal will be better off in expectation under transparency than under secrecy - he is helped by the failure of information aggregation. We test the model in a laboratory experiment with two member committees playing under three levels of transparency. We observe strong effects of transparency on committee error rates and information aggregation that are largely consistent with the model's predictions. On the individual level, we observe strong effects on deliberative behaviour which go in the predicted direction but are less pronounced than in theory.

**Keywords:** Group Decision Making, Transparency, Career Concerns, Experiments.

**JEL Classification Number:** D71, D72.

## 2.1 Introduction

"I've searched all the parks in all the cities and found no statues of committees."

(G.K. Chesterson)

The quote above illustrates nicely how we are often willing to delegate decision making to committees, yet praise and recognition remain firmly attached to the individual. Moreover, promotions in organisations are usually not granted to groups but to individuals. Many committees are therefore composed of members who seek recognition from the principal in order to advance in their careers. Obvious examples are corporate committees deciding about business strategies, campaign committees of political parties and even search committees in academia.

Recent research has shown that career concerns of a single agent can have negative consequences for a principal (Holmström (1999); Prat (2005); Fox and Van Weelden (2012)). While the principal would like the agent's action to maximise her expected utility, the agent behaves so as to make himself appear smart to the principal. These two objectives don't necessarily align. Further research shows that career concerns might also be of great importance for committee decision making (Levy (2007); Visser and Swank (2007, 2012); Meade and Stasavage (2008); Gersbach and Hahn (2008)). All of these papers study career concerns under different levels of transparency and suggest that the degree of transparency under which a committee deliberates and decides plays an important role.

We construct a new model to investigate the effects of transparency on deliberation and decision making in committees and test its predictions in the laboratory. In a departure from previous models, our's allows for largely unstructured communication. This is a key element in decision making of real committees and has been shown to matter in a recent experimental study (Goeree and Yariv (2011)) when career concerns are not present. As in most career concerns models, there are two types of committee members (high and low). They, first, receive more or less informative signals about the true state of the world, then have the opportunity to deliberate before finally voting for or against

changing the status quo in favour of an alternative option. A committee member's utility depends on the principal's belief about the probability that he is a high type.

We study three different transparency regimes - *secrecy*, where votes and communication are secret, *mild transparency*, where individual votes are made public, and *full transparency*, where both communication and individual votes are public. While each regime has more than one equilibrium, we argue below that some are more plausible than others. Under secrecy the most plausible equilibrium predicts that committee members pool information and vote unanimously for the state with the higher posterior probability. Under mild transparency better informed committee members do not share their information, in order to distinguish themselves from less well-informed members, and vote according to their signal. Finally, under full transparency all members are predicted to truthfully share their information but still vote according to their own private signal in the final vote. This happens because switching would reveal a low quality signal and thereby indicate a low type.

Committee members succeed in aggregating information under the secrecy regime but fail to do so under both forms of transparency. This results in fewer wrong decisions overall under secrecy. However, we show that when the cost of a mistake in one state of the world far outweighs that in the other state, then transparency serves to improve the principal's utility *vis-a-vis* secrecy. Here transparency aids the principal *because* it causes information aggregation to break down. Depending on the level of transparency, committee members either don't pool their information or pool it but disregard it in the final vote. This makes it very difficult for them to come to a unanimous decision. This bias towards the status quo translates into many mistakes in one state of the world but hardly any in the other.

To test these predictions, we run a laboratory experiment with groups consisting of two committee members and one principal. To the best of our knowledge this is the first experiment on committee voting with career concerns. As predicted, we find that players largely tell the truth regarding their signal under both secrecy and full transparency. However, subjects lie to a large extent about their type under full transparency and

information aggregation is much worse in the final vote. As a result, aggregate error rates are very different under secrecy and full transparency. Secrecy gives rise to much lower error rates when changing the status quo is the right thing to do, but higher error rates if sticking to the status quo is the right choice. This leads to higher payoffs for the principals under transparency than under secrecy, as predicted by the model. Aggregate level results regarding error rates under mild transparency are similar to the results under full transparency. However, they result from quite different individual level behaviour. Mild transparency leads to a significant level of deception regarding the signal, which harms information aggregation, whereas full transparency induces low quality members to stick with their signal even when they think they are wrong. Our results thus show that transparency can have very strong effects, and that it can work through quite different channels, which depend on what part of the decision process is made transparent.

We also observe a number of deviations from our predictions regarding individual level behaviour. Interestingly, deliberation regarding the signal under mild transparency and regarding the type under full transparency is more truthful and informative than predicted, even though truthful subjects pay a price for it in terms of lower reputation. This suggests that some subjects face psychological costs of lying, and confirms results from previous experimental studies that found "too much" truth-telling in deliberation (Cai and Wang (2006); Goeree and Yariv (2011)). However, while overly truthful communication survives under different voting rules in Goeree and Yariv's (2011) study, we find that the level of truthfulness varies greatly between our treatments, making the level of transparency appear the more important element of institutional design.

In the next section we present a review of related literature. We then set up and solve the model in full generality. We proceed with the experimental design and theoretical predictions for the chosen parameter values before discussing the aggregate and individual level results. We conclude with a discussion of the main results and future research questions.

## 2.2 Related Literature

Two recent papers deal with the issue of transparency in single principal - single agent settings. Prat (2005) shows in a simple setup that it may be better for the principal to observe only the consequences of the agent's choice rather than observing the choice directly. If actions are observed, the agent will have an incentive to disregard his private signal and simply act as a high type is expected to. This leads to too many wrong decisions and makes it more difficult to separate high and low types. Fox and Van Weelden (2012) show that these results are overturned when the prior on one state of the world is sufficiently strong, and the cost of mistakes are asymmetric across states.

Another string of papers have addressed the issue of transparency in the context of committee voting. Levy (2007) examines how the decisions of careerist experts in a committee of three are affected by the degree of voting information that is revealed. In her model, secrecy (where only the group decision is seen) leads members to pander to the *ex ante* most likely state, while transparency (publishing individual voting records) will make reforms more likely to be accepted. Visser and Swank (2007, 2012) present a model in which committee members will vote in the best interest of the principal when communication is public and individual votes are observed. The fact that committee members are made worse off in terms of reputation spurs them into arranging non-transparent pre-meetings. These results rest on the assumption that the state is not revealed to the boss and that players do not know their own types - this drives the committee to present a "united front" regardless of whether it is the right choice or not. Meade and Stasavage (2008) study committee deliberations in a monetary policy setting, in which a known expert speaks first, followed by two experts of unknown quality. They show that if deliberations are kept private there is a greater likelihood that committee members will dissent (i.e. reveal their signals truthfully) than would be the case if deliberations were public. Gersbach and Hahn (2008) study a two period game in which members of a monetary policy committee must choose an interest rate in each period but each individual may be dismissed after the first period. They find that

when communication and voting are public, low types try to masquerade as high types, resulting in many wrong decisions. This behaviour allows low types to be weeded out by the second period, but the overall effect of transparency on welfare remains negative.

At this point we would like to highlight the differences between these models and ours, to show why we constructed it, instead of taking one of the existing models to the laboratory. There are a number of key differences: (i) in contrast to Levy (2007) we allow for communication because there are few real world committees without deliberation and we expect this feature to matter; furthermore, the other studies that do allow for deliberation put a rigid structure on it, while we allow for free-form communication after an initial straw poll; (ii) in our model committee members know their own type, which we believe to be a reasonable assumption for in many committees, while this is not the case in Visser and Swank (2007, 2012); (iii) we allow agents to change their mind after deliberation and vote for a different alternative than the one they had favoured initially; and (iv) we allow for asymmetric costs of mistakes in the different states of the world, neither of these were allowed for in Gersbach and Hahn (2008); finally,(v) Gersbach and Hahn (2008) compare secrecy with full transparency - where both communication and individual votes are public - while we allow for the intermediate case of mild transparency, where communication is private but votes are not. This form of mild transparency would be consistent with situations where committee members hold secret pre-meetings.

Empirical literature on the role of transparency is quite scarce, as observational data is seldom available and, to the best of our knowledge, no experimental study has been conducted prior to this one.[1] A rare empirical study of career concerned committees is offered by Meade and Stasavage (2008) who examine the Federal Open Market Committee meeting transcripts before and after a change in transparency. They find that the style of deliberations changed after 1993, when transcripts became public. Consistent with their theoretical predictions, they find that openly expressed dissent is lower under transparency.

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[1]There have been some experimental papers on the standard Holmström (1999) career concerns model; notably Irlenbusch and Sliwka (2006) and Koch et al. (2009).

A number of important experimental papers have studied other elements of committee decision making, such as the role of the voting rule or the role of deliberation (most of them in a common value set-up and none of them with career concerned committee members). Guarnaschelli et al. (2000) show that while committees come to very different outcomes under different voting rules without deliberation, if a straw poll is allowed players largely aggregate information successfully - over 90% of reports in the straw poll are truthful. Goeree and Yariv (2011) extend this analysis by allowing free-form communication and some heterogeneity of preferences. They show that communication is overwhelmingly truthful - even when players would benefit from lying. Furthermore, the ability to communicate leads committees to the same outcomes regardless of the voting rule used. They conclude that the voting rule might be a less important element of institutional design than deliberation protocols. Our experiment demonstrates that the theoretically predicted outcome differences under different levels of transparency appear very robust to deviations from equilibrium deliberation.

## 2.3 The Model

A committee of $n$ members must make decision $D \in \{B(lue), R(ed)\}$ on behalf of their boss (the principal). There are two states of the world $S \in \{B, R\}$ where the *ex ante* probability of each state is $Pr(S = B) = p$, $Pr(S = R) = 1 - p$. The utility of the boss in each state is

$$U_{boss}(D = B|S = B) = x$$
$$U_{boss}(D = R|S = B) = U_{boss}(D = B|S = R) = 0$$
$$U_{boss}(D = R|S = R) = 1 - x$$

with $x \in [0.5, 1)$. That is, the boss gets higher utility from a correct group decision when the state is $B$. The group decision is made by unanimity rule, whereby option $R$ is implemented only if all $n$ members vote for it, otherwise the status quo $B$ is upheld.

Committee members are not perfectly informed of the state of the world. Instead, each member gets an informative signal about the true state of the world, where the

level of informativeness depends on his type. Each member $i \in n$ is either a high or low type, $t_i \in \{H, L\}$, where $Pr(t_i = H) = q$ and $Pr(t_i = L) = 1 - q$. They each receive an informative signal $s_{t_i} \in \{B, R\}$, where the accuracy of the signals are given by $Pr(s_H = S) = \sigma_H$ and $Pr(s_L = S) = \sigma_L$ respectively. High types receive more informative signals than low types, $0.5 < \sigma_L < \sigma_H \leq 1$. We assume throughout that $p, 1 - p < \sigma_L$, so that each members signal is informative.

Types and signals are private information. However, players can communicate with each other once nature has chosen the number of $H$ and $L$ types and everyone has received their signals. First, in a simple straw poll, each member simultaneously announces a message $m_i \in \{B, R, \emptyset\}$, i.e. he can announce that his signal was in favour of $B$ or $R$ or can remain silent. Next, the committee enters a round of free-form communication where voters can elaborate on their straw poll announcement, reveal their type, discuss which voting strategy is the best for the group etc. Finally, after both rounds of communication, each voter casts a secret ballot $v_i \in \{B, R\}$. Once the committee has made its decision, the true state of the world is revealed and the utility of the boss is realised.

Committee members' primary concern is not making the correct decision for the boss but rather to advance their own individual careers or reputations. The payoff of a committee member is simply the boss' posterior belief that he is of type $H$, given by $\hat{q} \in [0, 1]$. Before the game starts, the boss' prior for each player is $Pr(t_i = H) = q$. She updates this belief as best she can, where this ability depends on how much of the decision making process she observes, i.e. on the level of transparency. We compare three different transparency regimes: (1) *secrecy*, where the boss only observes the group decision $D$, (2) *mild transparency*, where the boss also sees how each individual votes, and (3) *full transparency*, where the boss witnesses each player's message $m_i$, free-form communication, and final vote $v_i$.

In what follows, we characterise the set of symmetric perfect Bayesian equilibria under the three transparency regimes. As committee members' payoffs depend on the boss' beliefs, there will be many equilibria in each of these settings. We restrict attention to *responsive* equilibria, where each committee member's strategy depends

at least on his own signal, and potentially on other players' messages.[2] We ignore "mirror" equilibria, in which $H$ types vote against what they believe to be the true state. Finally, our equilibria fall into two categories: *Truthful* equilibria, where players believe communication is truthful, and *Non-Truthful* equilibria, where all communication is believed to be babbling.

### 2.3.1 Equilibrium

It is useful to first consider how the boss would act if she observed all of the signals directly and made the decision herself. With no career concerns or informational asymmetries to contend with, she would simply update her beliefs using Bayes' rule and maximise her expected utility, choosing option $R$ if the posterior $\hat{Pr}(R) > x$, and otherwise $B$. In a committee of size $n$ let $h$ be the realised number of $H$ types, let $j$ be the number of $H$ types with the correct signal, and let $k$ be the number of $L$ types with the correct signal. Thus, there will be $2j - h$ net high type signals and $2k - n + h$ net low type signals in favour of the true state. Note that each of these numbers may be positive or negative, and while $h$ is observable, $j$ and $k$ are not. Denoting $\#s_H^R \in \{(h-2j), (2j-h)\}$ as the net number of $H$ type signals observed in favour of $R$, and $\#s_L^R \in \{(n-h-2k), (2k-n+h)\}$ as the net number of $L$ type signals for $R$, the boss will decide in favour of $R$ when

$$\left(\frac{\sigma_H}{1-\sigma_H}\right)^{\#s_H^R} \left(\frac{\sigma_L}{1-\sigma_L}\right)^{\#s_L^R} > \frac{px}{(1-p)(1-x)} \tag{2.1}$$

However, her decision may be the incorrect one *ex post*. This will happen when the evidence pointing in favour of one option is driven by incorrect rather than correct signals. We can define $k(S)^*$ as the threshold number of correct $L$ type signals below which a wrong decision is taken,

$$k(B)^* = 0.5\left[n - h + \frac{(h - 2j)\log\left(\frac{\sigma_H}{1-\sigma_H}\right) - log\left(\frac{px}{(1-p)(1-x)}\right)}{\log\left(\frac{\sigma_L}{1-\sigma_L}\right)}\right] \tag{2.2}$$

$$k(R)^* = 0.5\left[n - h + \frac{(h - 2j)\log\left(\frac{\sigma_H}{1-\sigma_H}\right) + log\left(\frac{px}{(1-p)(1-x)}\right)}{\log\left(\frac{\sigma_L}{1-\sigma_L}\right)}\right] \tag{2.3}$$

---

[2]This rules out equilibria in which voters ignore all information, vote randomly, and all gain $\hat{q} = q$.

As these will be real numbers, it is useful to define $\lfloor k(S)^* \rfloor$ as the integer directly before $k(S)^*$, and $I(k(S)^*)$ as an indicator function which equals one if $k(S)^* > 0$ and otherwise equals zero. The probability of the boss making a mistake under the optimal decision rule is then

$$Pr(D \neq S) = \sum_{h=0}^{n} \binom{n}{h} q^h (1-q)^{n-h} \sum_{j=0}^{h} \binom{h}{j} \sigma_H^j (1-\sigma_H)^{h-j} \sum_{k=0}^{\lfloor k(S)^* \rfloor} \binom{n-h}{k} I(k(S)^*) \sigma_L^k (1-\sigma_L)^{n-h-k}$$

(2.4)

The decision rule leads to more mistakes in state $R$ than in state $B$, but because $x > 0.5$, this is optimal.

**(1) Secrecy**   Here the boss can only see the group decision, and so must hold the same posterior $\hat{q}$ for each individual. For this reason committee members have a common interest in making the correct group decision.

**Proposition 2.1.** *There always exist truthful equilibria in which each member honestly reveals his signal and type, and the group implements whichever decision has a weighted majority of signals. The probability of a wrong decision in each state is given by (2.4) with thresholds (2.2) and (2.3) where $x = 0.5$ in both cases. The posterior beliefs of the boss after correct and incorrect group decisions are given by:*

$$\hat{q}_{sec^T}(D \neq S) = \frac{\sum_{h=0}^{n} \frac{h}{n} Pr_{sec^T}(D \neq S|h)}{Pr(D \neq S)}, \qquad \hat{q}_{sec^T}(D = S) = \frac{\sum_{h=0}^{n} \frac{h}{n} \left(1 - Pr_{sec^T}(D \neq S|h)\right)}{1 - Pr_{sec^T}(D \neq S)}$$

Notice that the probability of a mistake is identical to (2.4) where $x = 0.5$. This means that, while the boss would like to choose $B$ or $R$ to maximise her expected utility, the committee members instead choose the project which is most likely to align with the state - regardless of how much the boss values it. The formal voting rule plays no role here: all players truthfully reveal their information and then as a group decide to vote unanimously for whichever alternative has a higher posterior probability. When the posterior probability of both states are equal, the committee will decide to all vote for $R$ with probability 0.5 and will choose any other vote with probability 0.5. Any other

strategy would mean these indecisive committees would be more likely to choose one project over the other. However, this cannot be an equilibrium as the boss would realise this and give a lower evaluation to whichever project is favoured by these committees.

It is intuitive that committee members with a common interest in choosing the correct state will reveal their information and aggregate it optimally. Coughlan (2000) shows that communication between players with a common interest can lead to full aggregation, while Guarnaschelli et al. (2000) and Goeree and Yariv (2011) show that play corresponds overwhelming to truthful equilibria in the lab.

**(2) Mild Transparency** Under this regime, the boss cannot observe any communication, but she does observe the individual votes of committee members as well as the final group decision. Here there can be strong incentives for $H$ types to separate from $L$ types.

**Proposition 2.2.** *Both truthful and non-truthful equilibria exist under mild transparency. In any truthful equilibrium the probability of a mistake and the posterior beliefs $\hat{q}$ are the same as those in the truthful equilibria under secrecy. In any non-truthful equilibrium all committee members vote according to their signal and we have*

$$
\begin{array}{rcl}
\hat{q}_{mild^{NT}}(v_i = S) & = & \dfrac{q\sigma_H}{q\sigma_H + (1-q)\sigma_L} \\[3mm]
\hat{q}_{mild^{NT}}(v_i \neq S) & = & \dfrac{q(1-\sigma_H)}{q(1-\sigma_H) + (1-q)(1-\sigma_L)} \\[3mm]
Pr_{mild^{NT}}(D = R|S = B) & = & (1 - q\sigma_H - (1-q)\sigma_L)^n \\[2mm]
Pr_{mild^{NT}}(D = B|S = R) & = & 1 - (q\sigma_H - (1-q)\sigma_L)^n
\end{array}
$$

The non-truthful equilibria can come about simply because nobody believes communication to be truthful, or it can be that $L$ types try to communicate their information but $H$ types counteract this by engaging in signal jamming.Note that in the non-truthful equilibria the probability of a mistake in state $B$ is much smaller than the corresponding probability in state $R$. This occurs because the failure of information aggregation makes it very difficult to get a unanimous decision in favour of $R$.

Truthful equilibria must be supported by very specific beliefs such that no player has an incentive to deviate. One such set of beliefs is where the boss believes every player to be of low type unless the vote is unanimous.

**(3) Full Transparency**  Under this regime, the boss sees all stages of communication that occur and observes each individual's vote.

**Proposition 2.3.** *Both truthful and non-truthful equilibria exist under full transparency. In any truthful and any non-truthful equilibrium the probability of a mistake and the posterior beliefs $\hat{q}$ are the same as those in the non-truthful equilibria under mild transparency. In the truthful and non-truthful equilibria all committee members vote according to their signal. The truthful equilibrium is sustained by out-of-equilibrium beliefs that any members who switches from his initial announcement in the final vote is of type L.*[3]

The non-truthful equilibrium requires that the principal does not take the communication stage into account in his evaluation. However, this equilibrium disappears once committee members believe there is a small probability that messages that correspond to the true state of the world positively influence the principal's evaluation. Therefore, we believe that the truthful equilibrium in which no switching occurs is the most plausible.

While the aggregate outcomes of the truthful and non-truthful equilibria under the transparency regimes coincide, individual behaviour in the deliberation stage is very different.

## 2.3.2   The Optimal Level of Transparency

In this section we assume that the non-truthful equilibrium is played under mild transparency, the truthful under secrecy and either the truthful or the non-truthful under full transparency. As she will be indifferent between the two transparency regimes, the question which regime the boss would favour *ex ante* then reduces to whether she prefers secrecy to transparency. As transparency and secrecy lead to different error probabilities

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[3]We discuss different assumptions about out-of-equilibrium beliefs in Appendix 1.

in the different states of the world, the boss' preference depends on the payoffs $x$ and $(1 - x)$ she derives from correct decisions in either state. We can state the following result:

**Proposition 2.4.** *There always exists an $x^*$ such that if $x > x^*$ the principal is better of under transparency than under secrecy.*

*Proof.* The principal will be better off under transparency than under secrecy when

$$px[Pr_{tran}(D = R|B)] + (1 - p)(1 - x)[Pr_{tran}(D = B|R)] <$$
$$px[Pr_{sec}(D = R|B)] + (1 - p)(1 - x)[Pr_{sec}(D = B|R)]$$

which can be rearranged to

$$Pr_{tran}(D = R|B) - Pr_{sec}(D = R|B) < \frac{(1 - p)(1 - x)}{px}[Pr_{sec}(D = B|R) - Pr_{tran}(D = B|R)]$$

If $x$ is infinitesimally close to 1, this condition becomes

$$Pr_{tran}(D = R|B) - Pr_{sec}(D = R|B) < -\epsilon \tag{2.5}$$

Under transparency where players stick to their signals, a mistake will only occur in state $B$ when all players receive incorrect signals in favour of $R$. This event is a subset of the error events which occur under secrecy in state $B$, which is why both sides of (2.5) are negative. $\square$

## 2.4 Experiment

To test the main predictions of the model, we ran a laboratory experiment with two member committees, where each member was a high type with probability $q = 0.25$ and the two states were *ex ante* equally likely ($p = 0.5$). High types received perfectly informative signals ($\sigma_H = 1$) and low types got low accuracy signals ($\sigma_L = 0.55$). The experiment consisted of three treatments, one for each level of transparency.

### 2.4.1   Equilibrium Predictions

*Ex ante* equilibrium error rates are reported in Table 2.1 for the truthful equilibrium under secrecy, the non-truthful under mild transparency and the truthful or the non-truthful under full transparency. As discussed in the previous section, we find these to be the most plausible equilibria. If, instead, subjects under mild transparency played according to the truthful equilibrium, the *ex ante* error rates would be the same as under secrecy.

Table 2.1: *Ex ante* Equilibrium Error Rates (in %)

|                    | $S = B$ | $S = R$ | overall |
|--------------------|---------|---------|---------|
| Secrecy            | 25.3    | 25.3    | 25.3    |
| Mild Transparency  | 11.4    | 56.1    | 33.8    |
| Full Transparency  | 11.4    | 56.1    | 33.8    |

Note: In the experiment $S = B$ ($S = R$) corresponds to
the case of the blue (red) jar (see below).

Table 2.2 reports the principal's predicted evaluation of player types after observing the true state of the world, the group decision and, in case of transparency, the individual decisions. In the same table we also report the *ex ante* expected evaluations for $H$ and $L$ type voters. Type $L$ voters are predicted to do best under secrecy and $H$ types voters under transparency. The reason is that evaluators are expected to be better able to tell them apart under transparency.

Voters should succeed in aggregating information under secrecy but fail completely under the two transparency regimes. Under secrecy all voters are expected to truthfully announce their signal in the straw poll and, if needed, their type in free-form communication. Under mild transparency we expect communication to be non-truthful, particularly from $H$ types - that is we expect the announcement of signals and types to hold no useful information. Finally, under full transparency we predict that all player types truthfully announce their signals in the straw poll. However, in free-form communication there will be no new information, as each player has an incentive to announce himself as a $H$ type.

Table 2.2: Principal's Equilibrium Beliefs About Members' Types $Pr(t_i = H)$ in %

| | Eq. evaluation after correct and wrong decisions | | Eq. expected *ex ante* evaluation for voters | |
|---|---|---|---|---|
| | dec. corr. | dec. wr. | type $H$ | type $L$ |
| Secrecy (group decision) | 33.5 | 0 | 33.5 | 22.2 |
| Mild Transparency (individual decision) | 37.7 | 0 | 37.7 | 20.8 |
| Full Transparency (individual decision) | 37.7 | 0 | 37.7 | 20.8 |

Note: In equilibrium under secrecy the evaluator only takes the group decision into account for her evaluation, under mild transparency he exclusively looks at individual decisions, and under full transparency at messages and individual decisions (which are the same).

So, under mild transparency information aggregation fails because $H$ types withhold information, while mistakes arise under full transparency because $L$ types prefer to stick to their signal announcement, rather than revealing themselves as a $L$ type by switching to the more likely option.

### 2.4.2 Experimental Design

We ran six sessions, two for each transparency regime (see Table 2.3). Each session consisted of 20 rounds with random matching of subjects into groups. In the first round of the experiment, subjects were randomly assigned to matching groups of nine people.[4] In every period, new groups of three were randomly formed within these matching groups. This was done to avoid the emergence of reciprocal behaviour and at the same time provide independent matching groups. In each group and round, one member was randomly assigned the role of the principal (called the "observer" in the instructions) and the other two were assigned the role of committee members (called "voters" in the instructions). With probability $q = 0.25$ a voter was of type $H$ ("well-informed voter"), with probability $1 - q = 0.75$ of type $L$ ("informed voter"). The task of the voters was to vote on the true colour of a randomly selected jar. The blue jar ($S = B$) contained 11

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[4]In one session we had only 15 subjects and therefore only one matching group.

blue and 9 red balls, the red jar ($S = R$) contained 11 red and 9 blue balls. Jars were chosen with equal probability. Type $H$ voters received a ball with the true colour of the jar (the perfectly informative signal), type $L$ voters received a ball that was drawn from the selected jar (the 55% accuracy signal).

Table 2.3: Experimental Sessions

|                   | N sessions | N matching groups | N subjects |
|-------------------|:----------:|:-----------------:|:----------:|
| Secrecy           | 2          | 5                 | 45         |
| Mild Transparency | 2          | 5                 | 45         |
| Full Transparency | 2          | 4                 | 42         |

Note: All sessions were run at the DeSciL Lab at the ETH Zurich in May 2013 with students from the ETH and the University of Zurich.

On the first screen, the observer learned that she is an observer, while voters were informed about their type and the colour of their ball. Each voter then had to simultaneously send a message {red, blue, not specified} to the other voter in their group. On the next screen, voters saw the message from their partner and had the opportunity to chat with him for 90 seconds.[5] In the full transparency treatment, the principal could see the voters' messages and follow the chat on her second screen. In the other two treatments the principal could not. On the third screen, the voters could review the communication and then make their final decision by voting for red or blue. Votes were then aggregated to the group decision (red required two votes, blue only one). After the voting stage, the principal received information on her next screen that depended on the treatment. In the secrecy treatment, she could only see the group decision and the true colour of the jar. In the mild transparency treatment, she could, in addition, see the individual votes of the voters. In the full transparency treatment, she could also review the whole communication (messages and chat) between the voters. On this screen, the

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[5]The timeout was not strictly enforced. When the time was up a message appeared on the screen asking them to finish their sentence and proceed. Most subjects did so immediately and the few others were kindly asked to proceed after 120 seconds by an experimenter.

principal had to indicate her belief about the probability that the voters are of type $H$, by entering this probability in per cent. In the secrecy treatment, in which both voters are indistinguishable to the principal, she had to evaluate one randomly chosen voter from her group, in the other two treatments she had to evaluate both voters in her group. On the final screen of each round, subjects received feedback information regarding the types of the voters, the group decision, the true colour of the jar and their pay-offs.

Subjects earned points in each round. The points of a voter in one round was twice the probability that he was a high type, as entered in percentage points by the principal, e.g., if the principal entered 30% the voter's payoff was 60 points. The principal's payoff was 3 points for a correct group decision if the true state of the jar was blue and 1 point for a correct group decision if the true state was red, reflecting the greater importance of a low error rate in the blue state ($x = 0.75$). In addition, the principal earned a number of points between 0 and 100 for accurate evaluation of the voters' types. In each treatment, the evaluation of one of the two voters was randomly selected and a principal $j$'s earnings were determined by the following quadratic scoring rule:

$$
\text{Points for accuracy} =
\begin{cases}
100 - \frac{1}{100}(100 - Pr_j(t_i = H))^2 & \text{if voter } i \text{ is of type } H \text{ and} \\
100 - \frac{1}{100}(Pr_j(t_i = H))^2 & \text{if voter } i \text{ is of type } L
\end{cases}
\tag{2.6}
$$

where $Pr_j(t_i = H)$ denotes the probability that voter $i$ is of type $H$, as entered by principal $j$, in per cent. This rule makes it optimal for (risk neutral) subjects to truthfully enter their beliefs (see, e.g., Nyarko and Schotter 2002) and subjects were directly told so in the instructions.

Four rounds were randomly chosen at the end of the session and the points earned in these rounds converted to Swiss Francs at a rate of 1 point= CHF 0.15 (at the time of the experiment CHF 1 was roughly worth USD 1.04). Subjects spend about 2 hours in the lab and earned on average CHF 47 in addition to their show-up fee of CHF 10. Earnings per hour are comparable to an hourly wage for student jobs in Zurich.

### 2.4.3   Experimental Results

**Aggregate Behaviour**

Table 2.4 summarises the observed error rates and, for comparison, the *ex post* equilibrium predictions, i.e. the predictions after realisation of the true state of the world (colour of the jar), the types, and the signals. It also contains a column with the hypothetical full information aggregation benchmark case, which assumes perfect information revelation, voting for the state with the higher posterior probability and mixing of committees between voting for $(B)$lue and $(R)$ed if the posterior probabilities are equal.  This coincides with the equilibrium predictions under secrecy and optimal behaviour of a committee that minimises the total error rate.

Table 2.4: Observed, *Ex post* Equilibrium, and Full Information Aggregation Error Rates by State and in Total

| true colour | $S = B$ | | | $S = R$ | | | total | | |
|---|---|---|---|---|---|---|---|---|---|
| | Observed | Eq. | f.i.a. | Observed | Eq. | f.i.a. | Observed | Eq. | f.i.a. |
| Secrecy | 28.3 (5.5) | 27.2 | 27.2 | 25.8 (1.7) | 24.8 | 24.8 | 27 (3.1) | 26 | 26 |
| Mild Transp | 15.5 (1.2) | 9.5 | 24 | 46.1 (3.6) | 57.9 | 25.3 | 30.8 (1.8) | 34 | 24.7 |
| Full Transp | 15.8 (.1) | 8.2 | 21.9 | 50.7 (6.4) | 61.9 | 26.9 | 33.3 (3.5) | 33.9 | 24.3 |

Note: Eq. = likely equilibrium, f.i.a. = full information aggregation. Equilibrium error rates are *ex post* error rates. Standard errors of the observed error rates (in parentheses) are obtained with the delta method, adjusting for clustering in matching groups.

Equilibrium predictions are very accurate for the secrecy treatment.  In the two transparency treatments, committees make more mistakes than equilibrium predicts in state $B$ ($\chi^2$-tests, $p < 0.01$) but do better in state $R$ ($\chi^2$-tests, $p < 0.01$ for mild transparency and $p < 0.1$ for full transparency).  As predicted, both forms of transparency fare much worse than the information aggregation benchmark in state $R$, but better in state $B$.  Consequently, the transparent committees performed significantly and substantially worse than secretive committees in state $R$ ($\chi^2$-tests, $p < 0.01$) but better

in state $B$ ($\chi^2$-tests, $p < 0.05$). Even though the total error rate was higher under transparency than under secrecy, principals earned slightly more points in the transparency treatments because state $R$ is more valuable to them ($x = 0.75$). However, this difference was not statistically significant.

The most likely source for the differences between the treatments are (a) differences in information aggregation and (b) coordination behaviour of groups consisting only of low types. Information aggregation helps if there is one type $H$ voter and one type $L$ voter in a group, they have conflicting signals and the true state is red. In this scenario all groups in the secrecy treatment aggregated information successfully (see Table 2.5). While the error rates are substantially and significantly higher ($\chi^2$-tests, $p < 0.05$) in the transparency treatments, they are much lower than their predicted value of 100%. In the groups with two low types under transparency, not much coordination on voting for red takes place, resulting in very high error rates if the true state is red (close to the predicted 75%) and much fewer errors if it is blue (close to the predicted 25%). This contrasts with the secrecy treatment where the equilibrium prediction is an error rate of 50% for each state, as these committees are predicted to coordinate on voting unanimously for red with 50% probability. Indeed, we observe rates closer to 50% under secrecy. As a consequence, error rates are much lower under secrecy if the true state is red and substantially higher if it is blue, which is exactly what principals dislike.

Surprisingly, not all groups with two signals in the same direction vote for that alternative. As a consequence, the error rates of these groups are not 100% if their signals go in the wrong direction (but range from 82.4% in the mild transparency treatment to 95.6% under secrecy, with 93.3% under full transparency) but also not 0% if they go in the right direction (and range from 4.7% under secrecy to 13.6% under mild transparency, with 7.4% under full transparency, instead).

**Individual Behaviour**

We start our analysis of individual behaviour with the deliberation stage. In the straw poll, we see that under secrecy there is almost completely truthful revelation of signals by

Table 2.5: Information Aggregation: Error Rates in Groups with Conflicting Signals

| true color | $\{H, L\}$ Group $S = R$ | $\{L, L\}$ Group $S = R$ | $S = B$ |
|---|---|---|---|
| Secrecy | 0 | 54.3 (11.0) | 38.7 (15.3) |
| Mild Transp | 22.6 (5.5) | 75.7 (4.4) | 27.9 (6.8) |
| Full Transp | 44.8 (8.3) | 83.3 (10.2) | 27.5 (8.9) |

Note: Standard errors of the observed error rates (in parentheses) are obtained with the delta method, adjusting for clustering in matching groups.

both high and low types, consistent with equilibrium predictions (Table 2.6). Under full transparency, high types are also almost always truthful and reveal their signal. However, there are 8.3% low types that lie about their signal and another 10.5% who stay silent, which goes against our predictions. While lying does not make much sense it is also not very costly as $L$ type signals are not very informative. Staying silent might be motivated by the hope to learn more about the true state and then vote accordingly without being punished for it. However, we will see that this does not work out.

Under mild transparency, 19.2% of the high types lie about their signal and another 4.8% stay silent, while the low types are almost always truthful and reveal their signal. The degree of lying from high types is significantly higher than in the other treatments ($\chi^2$-tests, $p < 0.01$). It is not high enough though to make the straw poll completely uninformative. Low types could update their beliefs and make fewer mistakes than by sticking to their own signal.

Next, we turn to communication in the chat. Similar to Goeree and Yariv (2011) we observe that communication can roughly be divided into two phases - a first phase, where information is shared, and a second with a discussion on how the committee members should vote. 59.1% of the voters announce a type in the chat (38.2% under secrecy, 49.5% under full transparency and 89% under mild transparency). While announced types

Table 2.6: (Non-)Truthful messages from different types

|  | type | lying | silent |
|---|---|---|---|
| Secrecy | $H$ type | 0 | 0 |
|  | $L$ type | .7 (.3) | 1.8 (1.2) |
| Mild Transp | $H$ type | 19.2 (6.6) | 4.8 (1.9) |
|  | $L$ type | 3.5 (1.1) | 3.3 (1.3) |
| Full Transp | $H$ type | 1.4 (0.9) | 0.7 (0.7) |
|  | $L$ type | 8.3 (2.1) | 10.5 (2.8) |

Note: Percentage of non-truthful messages (lying) and "not specified" messages (silent). Standard errors of the observed error rates (in parentheses) are obtained with the delta method, adjusting for clustering in matching groups.

are always truthful under secrecy, 51.35% (9.4%) of the claims to be a high type are lies under full (mild) transparency. Claims to be of low type are almost always truthful (96.2% under mild transparency, 98.2 under full transparency and 100% under secrecy). Despite the frequent lies in the full transparency treatment, the announcements are still partially informative and should thus be taken into account in the evaluations by the principals.

Next, we turn to information aggregation again and study how many voters vote against their signal in the final vote after receiving a low quality blue signal and seeing the other voter send the red message. The numbers are surprisingly high for the transparency treatments where voting according to signal is predicted (Table 2.7). Low types always switch under secrecy when it matters most, i.e., when the other group member is a high type, which is facilitated by the truthfulness of announcements of types in the chat. As announcements of types are also truthful most of the time under mild transparency, low types also switch very often under this regime when the other group member is a high

type.

Table 2.7: Information Aggregation: Percentage of Low Types Voting against their blue Signal when other Voter Reports Red

|  | overall | oth. voter $H$ | oth. voter $L$ |
|---|---|---|---|
| Secrecy | 56.2 (6.5) | 100 | 42.6 (11.6) |
| Mild Transp | 45.9 (2.9) | 80.0 (5.1) | 33.3 (3.7) |
| Full Transp | 41.9 (6.0) | 60.0 (5.7) | 33.3 (7.2) |

Note: Standard errors of the observed error rates (in parentheses) are obtained with the delta method, adjusting for clustering in matching groups.

Next, we study how the principals react. A first look at their evaluations shows that they are well able to distinguish between low types and high types (Table 2.8). Low types do best under secrecy and high types under full transparency. Average evaluations are too high in all treatments and even significantly positive after wrong decisions ($t$-tests, $p < 0.01$), suggesting that evaluators do not fully take the relatively low prior into account or that they also care about the pay-offs to the voters. However, the incentives to make a correct group decision under secrecy and a correct individual decision under transparency are about as strong as in our theoretical predictions and always at least 33.5 percentage points.

Under full transparency, evaluators are very well able to distinguish high and low types even if the individual decision is correct. It seems that the principals learn about the types by observing the deliberation. Regressing the evaluations on the pieces of information that the evaluator sees when evaluating, shows that making the wrong group decision has a big effect on the evaluation under secrecy but no effect under transparency, where she also sees the individual votes (Table 2.9).

Under both forms of transparency, individual mistakes have the biggest influence on the evaluation. Under full transparency, the evaluation is also negatively influenced by switching between the message and the voting stage, and if the voter communicates her

Table 2.8: Evaluations

|  |  | evaluation | | |
|  |  | avg. | dec. corr. | dec. wr. |
|---|---|---|---|---|
| Secrecy | $H$ type | 53.5 (3.4) | 53.5 (3.4) |  |
| (group decision) | $L$ type | 37.4 (1.5) | 55.0 (3.3) | 6.9 (1.9) |
|  | overall | 41.6 (1.5) | 54.4 (3.2) | 6.9 (1.9) |
| Mild Transp | $H$ type | 47.7 (1.2) | 48.3 (1.1) |  |
| (individual decision) | $L$ type | 31.9 (2.3) | 46.4 (2.6) | 11.8 (1.6) |
|  | overall | 35.7 (1.9) | 47.1 (1.9) | 11.7 (1.6) |
| Full Transp | $H$ type | 60.8 (3.2) | 62.8 (3.8) |  |
| (individual decision) | $L$ type | 29.4 (2.9) | 40.2 (2.2) | 14.6 (4.0) |
|  | overall | 37.2 (3.0) | 48.2 (1.7) | 14.7 (4.0) |

Note: Standard errors (in parentheses) are adjusted for clustering in matching groups.

type to be $L$ in the chat. It is positively influenced if the voter communicates her type to be $H$. The principals respond to the informational content regarding types in the chat as claiming to be of type $H$ increases your evaluation while claiming type $L$ lowers it. Staying silent in the straw poll has the same negative effect on evaluations as switching which means that the 10.5% low types who stay silent do not get away with it.

On average, low types who switch between the straw poll and the final vote or stay silent in the straw poll earn 23.6 points less (get an evaluation that is 11.8 percentage points lower) than low types who do not ($t$-test, $p < 0.05$). This result also holds when attention is restricted to the case where the other voter has a different signal.

So, principals react to all the information available to them in the right direction, even if the evaluation levels that they choose are too high, on average, to maximise their pay-offs.

Table 2.9: Evaluation Criteria

| | M1 (Secrecy) | | M2 (Mild Tr.) | | M3 (Full Tr.) | |
|---|---|---|---|---|---|---|
| | dependent variable: evaluation ($Pr(t_i = H)$) | | | | | |
| Group decision wrong | -47.6*** | (4.2) | -0.9 | (3.3) | 0.2 | (3.2) |
| Individual decision wrong | | | -34.8*** | (2.1) | -28.7** | (5.4) |
| Staying silent in straw poll | | | | | -12.0** | (2.2) |
| Switch between $m$ and $v$ | | | | | -10.5* | (4.1) |
| Claimed to be type $H$ in chat | | | | | 16.4*** | (2.1) |
| Claimed to be type $L$ in chat | | | | | -17.4* | (6.5) |
| constant | 54.5*** | (3.4) | 47.1*** | (1.9) | 44.3*** | (2.3) |
| $N$ | 600 | | 600 | | 560 | |
| $N\_clust$ | 5 | | 5 | | 4 | |
| $R^2$ | 0.49 | | 0.31 | | 0.28 | |

Note: * p<0.1, ** p<0.05, *** p<0.01. Standard errors (in parentheses) are adjusted for clusters in matching groups.

## 2.5 Conclusion

We show theoretically that transparency can have a strong effect on group decision making. While transparency has a positive effect on the number of correct group decisions in one state of the world it has a negative effect in the other. The question of whether to choose a transparent or secretive committee thus depends on the relative importance the principal attaches to correct decisions in either state of the world. Moreover, our theoretical results suggest that the failure to aggregate information can work through quite different channels, depending on the type of transparency. If the individual voting records are made public, it works through the incentive not to share information in deliberation. If deliberation is also made public, it works through the incentive to stick to one's announcements even if the posterior probability of the other state is higher after deliberation.

Our experiment is, to our best knowledge, the first experimental study of the effects of career concerns on decision making in groups and our findings mainly confirm our theoretical results. While Goeree and Yariv (2011) find that free-form deliberation eliminates differences in outcomes under different voting rules, our theoretical predictions regarding aggregate outcomes under different levels of transparency largely hold with free-form communication. While these results match the theoretical predictions remarkably well, we do see a considerable level of deviations on the individual level, especially from committee members. As in Goeree and Yariv (2011) and Cai and Wang (2006) players are too truthful in deliberation. Still their behaviour corresponds closely enough to our predictions - less truthfulness under mild transparency and less updating and coordination on changing the status quo under both transparency regimes - to lead to drastic differences between secrecy and transparency on the aggregate level.

We thus show that career concerns play an important role and that the way they play out depends crucially on the level of transparency the committee operates under. This suggests that transparency is a highly important element of institutional design and setting the level wrong might have considerable negative consequences for a principal. An interesting extension of our work might be to consider what happens when the level of transparency is set by the committee members rather than by the principal, and whether leaving such a choice to a committee is a good idea.

## 2.6 Appendix

### 2.6.1 Out-of-Equilibrium Beliefs

In proposition 2.3, we assume the out-of-equilibrium belief that a boss holds is that a committee member who switches between the deliberation message $m$ and the final vote $v$ is of type $L$. This belief is intuitive as under many straw poll results type $H$ voters who find themselves in the minority would still hold the posterior belief that the true state is indicated by their signal, while type $L$ voters' posterior beliefs can be changed by a majority of opposing messages more easily. So, while high types have no reason to switch after many straw poll results, low types would have an incentive to switch, e.g. if they mistakenly decide to help the principal instead of improving their reputation. This argument could be made in a more rigorous and formal way by assuming an $\epsilon$ probability that committee members care about the principal's pay-off rather than about their reputation. Type $H$ committee members of this type would still not want to switch but Type $L$ committee members would want to switch. This would make switching an non-zero probability event in equilibrium where switchers are then low types with certainty.

However, if the number of majority messages in deliberation is high enough, high types who find themselves in the minority will also change their belief about which state is more likely. In this case a different equilibrium is equally plausible. In this equilibrium, neither high nor low types in the minority can change their reputation after a straw poll with sufficiently many majority messages. These members can then switch in the final vote to help the boss. The results of Proposition 2.3 would change as follows.

Let $\theta$ denote the number of majority signals in the straw poll and $\theta^*$ the threshold of majority signals above which high and low types who supported $B$ in deliberation and find themselves in the minority would want to switch:

$$\theta^* = \frac{n-1}{2} + \frac{log\left(\frac{\sigma_H p}{(1-\sigma_H)(1-p)}\right)}{log\left(\frac{q(\sigma_H - \sigma_L)+\sigma_L}{1-q(\sigma_H - \sigma_L)-\sigma_L}\right)} \tag{2.7}$$

**Proposition 2.5.** *Both truthful and non-truthful equilibria exist under full transparency. In any non-truthful equilibrium the probability of a mistake and the posterior beliefs $\hat{q}$ are the same as those in the non-truthful equilibria under mild transparency. In any truthful equilibrium (i) if $\theta < \theta^*$ then the probability of a mistake and posterior beliefs will be as in the non-truthful equilibrium, (ii) if $\theta > \theta^*$ then posterior beliefs are given by*

$$\hat{q}_{full^{NT}}(i \in maj, m_i = v_i = S) = \binom{n}{\theta} \frac{q\sigma_H (q\sigma_H + (1-q)\sigma_L)^\theta (1 - q\sigma_H - (1-q)\sigma_L)^{n-\theta}}{q\sigma_H + (1-q)\sigma_L}$$

$$\hat{q}_{full^{NT}}(i \in maj, m_i = v_i \neq S) = \binom{n}{\theta} \frac{q(1-\sigma_H)(q\sigma_H + (1-q)\sigma_L)^{n-\theta}(1 - q\sigma_H - (1-q)\sigma_L)^\theta}{q(1-\sigma_H) + (1-q)(1-\sigma_L)}$$

$$\hat{q}_{full^{NT}}(i \in min, m_i \neq S) = \binom{n}{\theta} \frac{q(1-\sigma_H)(q\sigma_H + (1-q)\sigma_L)^\theta (1 - q\sigma_H - (1-q)\sigma_L)^{n-\theta}}{q(1-\sigma_H) + (1-q)(1-\sigma_L)}$$

$$\hat{q}_{full^{NT}}(i \in min, m_i = S) = \binom{n}{\theta} \frac{q\sigma_H (q\sigma_H + (1-q)\sigma_L)^{n-\theta}(1 - q\sigma_H - (1-q)\sigma_L)^\theta}{q\sigma_H + (1-q)\sigma_L}$$

This equilibrium is sustained by out-of-equilibrium beliefs that any member who switches from his initial announcement in the final vote is of type $L$ if $\theta < \theta^*$. The resulting error probabilities are:

$$Pr(D = R|S = B) = \sum_{\theta=\theta^*}^{n} \binom{n}{\theta} (1 - q\sigma_H - (1-q)\sigma_L)^\theta (q\sigma_H + (1-q)\sigma_L)^{n-\theta}$$

$$Pr(D = B|S = R) = 1 - \sum_{\theta=\theta^*}^{n} \binom{n}{\theta} (q\sigma_H + (1-q)\sigma_L)^\theta (1 - q\sigma_H - (1-q)\sigma_L)^{n-\theta}$$

Mistakes will happen in state $B$ when $\theta \geq \theta^*$ voters receive the wrong signal. Mistakes will happen in state $R$ whenever less than $\theta^*$ voters get $R$ signals.

Note that Proposition 2.4 still holds, just for a different threshold of $x$ in this case. For $\sigma_H = 1$, the parameter choice in the experiment, $\theta < \theta^*$ always holds.

### 2.6.2 Instructions for Experiment

Instructions for the full transparency treatment (translation; original in German). Instructions for the other treatments where very similar and are therefore omitted here.

# Overview

Welcome to this experiment. We kindly ask you not to communicate with other participants during the experiment and to switch of your phones and other mobile devices.

At the end of the experiment you will be paid out in cash for your participation in today's session. The amount of your pay-off depends in parts on your decisions, on the decisions of other participants and on chance. For this reason it is important that you read the instructions carefully and understand them before the start of the experiment.

In this experiment all interactions between participants are via the computers that you are sitting in front of. You will interact anonymously and your decisions will only be stored together with your random ID number. Neither your name, nor names of other participants will be made public, not today and not in future written analyses.

Today's session consists of several rounds. **At the end, 4 rounds will be randomly selected and paid out. The rounds that are not chosen will not be paid out.** Your pay-off results from the points that you earn in the selected rounds, converted to Swiss Francs, plus your show-up fee of CHF 10. The conversion of points to Swiss Francs happens as follows: Every point is worth 15 cents, which means that

**20 points = CHF 3.00.**

Every participant will be paid out in private at the payment counter, so that no other participant can see how much you have earned.

# Experiment

This experiment consists of 20 procedurally identical rounds. In each round a group decision has to be made, that can be correct or wrong.

Two members in each group of three make the group decision (henceforth we will call them the voters). There are well and less well informed voters and the task of the third group member is to observe the decision process of the other two members and then to indicate the probability with which he thinks that the other group members are well or less well informed (henceforth, we will call this member the observer).

The higher the evaluation of the observer with respect to the level of information of a voter is, the higher is the pay-off to that voter in the round. The more accurate the evaluation of the observer with respect to the level of information of the voters is, the higher is his or her pay-off in the round. In addition, the observer receives a pay-off for correct group decisions.

## The Group

In the **first** round you will be assigned a meta-group of 9 members. In the beginning of **every** round you will be randomly assigned to a **new group** which consists of randomly selected members of your meta-group. Every group has three members: 2 voters nd 1 observer.

Whether you will be assigned the role of a voter or an observer, is randomly determined each round. The voters receive, again randomly, the labels "voter 1" and "voter 2".

All interactions in a round take place within your group of three.

## The Voters

There are two types of voters, well informed (type G) and (less well) informed (type I) voters. Of which type the group members are, is again determined randomly. With probability ¼ (or 25%) a voter receives good information which means he is of type G; with probability ¾ (or 75%) he receives less good information which means he is of type I.

Because the assignment of types to the voters is independent of the assignment to other voters, there can be two voters of type G, two voters of type I, or one of each type in a group.

The voters learn their type on the first screen of a round but not the type of the other voter in their group. The observer learns that he is an observer on the first screen but not the types of the voters in his group.

Later, after observing the behavior of the voters, it will be the task of the observer to estimate the probabilities that voter 1 and voter 2 are of type G.

# The Jar

There are two jars: one red jar and one blue jar. The red jar contains 11 red and 9 blue balls, the blue jar 11 blue and 9 red balls. Each round one jar will be randomly selected.

The task of the voters is to vote on the color of the jar. Each jar has an equal probability of being selected, that is it will be selected with 50% probability.

# The Ball
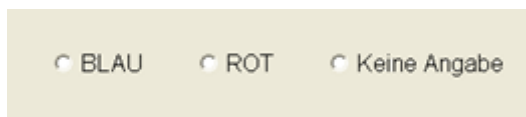
The well informed voters (type G) receive a ball with the actual color of the jar, that is they are directly informed about the color of the jar.

The informed voters (type I) receive a randomly drawn ball from the selected jar. They are not told the color of the jar. If there are two type I voters in a group, each of them receives a ball from the jar. Every ball in the jar has the same selection probability for the type I voters, that is for each voter of type I a ball is drawn from a jar containing 20 balls (11 with the color of the jar, 9 with the other color).

The voters learn the color of their ball on the first screen. Every voter only sees the color of his ball, not the color of the other voter's ball.

# Communication

After learning their type and the color of their ball, the voters can communicate the color of their ball to the other voter in their group. They can also communicate the color that their ball did not have or stay silent. The communication is made through the following entry mask.

○ BLAU     ○ ROT     ○ Keine Angabe

On the following screen the voters learn the message of the other voter in their group and have the option to chat with him. The chat happens via the following entry mask.

Sie können nun mit dem anderen Wähler in ihrer Gruppe chatten. Im Chat bekommen Sie die ID "Wähler 1" und der andere Wähler die ID "Wähler 2". Jede Texteingabe muss mit der Enter-Taste bestätigt werden.

Wähler 2: Hallo
Wähler 1: Chat Test

You can enter arbitrary text messages into the blue entry field. Pay attention to confirm every entry by pressing the enter button to make it visible for the other voter. It will then appear in the grey field above.

The observer cannot participate in the communication but sees the messages of the two voters regarding the color of their ball as well as the chat.

# Group Decision

After the communication stage the voter make their decision in a group vote.

**So, if you are a voter, you have to vote either fro blue or for red.**

Once both voters have made their decision, the votes for blue and red are added up and the group decision results from the following rule:

- **If the color RED receives 2 votes, the group decision is RED**
- **If the color BLUE receives 1 or 2 votes, the group decision is BLUE**

That is for a group decision for blue only one vote is necessary while a group decision for red requires two votes.

# Evaluation of the Observer

After the voters have cast their vote and the group decision is determined, the evaluator learns the group decision as well as the decisions of the individual voters in his group.

Moreover, he learns the true color of the jar, that is, whether the group decision and the individual decisions were correct or wrong.

On the same screen the observer can review the entire communication between the voters in his group once again.

**If you are an observer, you now have to enter for each of the two voters the probability with which you believe that this voter is of type G.**

To do so you enter a number between 0 and 100 wich expresses your evaluation in percentage points. The entry mask looks as follows.



The complete screen of the observer looks as follows (example screen).



# Pay-off in each Round

If you are a **voter** your pay-off is determined by the evaluation of the observer. If the observer believes that you are of type G with **X%** probability, you receive a pay-off of **2*X** points in

this round. This means that your pay-off directly depends on the probability with which the observer believes you are a **well-informed voter (type G).**

If the observer has entered the probability 25%, for example, your pay-off is 50 points, if he has entered 50%, it is 100 points.

If you are an **observer** you receive a pay-off for correct group decisions and a pay-off for the accuracy of your evaluations of the types of the voters.

- If the group decision is RED and the jar is indeed RED, you as an observer receive **1 point.**
- If the group decision is BLUE and the jar is indeed BLUE, you as an observer receive **3 points.**
- If the group decision is wrong, you receive **0 points**, independently of the true color of the jar.

For your evaluation regarding the types of the voters you receive a pay-off between 0 and 100 points. It will be randomly determined whether you will be paid out for the evaluation of voter 1 or voter 2.

If you have evaluated both voters correctly with certainty (that is with 0 or 100%) (if you entered the probability 0 for both voters, for example, and both are indeed not of type G but of type I), you receive 100 points. If you are completely wrong (if both are of type G in the example) you receive 0 points.

The formula that determines your pay-off is a little complicated.

Put simple the formula assures **that it is best for you (gives you the highes expected pay-off) if you truthfully indicate the probability with which you believe that a voter is indeed of type G.** Every other evaluation lowers your expected pay-off.

If you believe, for example, that voter 1 in your group is of type G with 30% probability and voter 2 with 60% probability, it is best fort o enter exactly these values.

In case you want to in more detail how your payoff is determined: for the evaluation of the randomly selected voter you receive:

$100 - \frac{1}{100}\big(100 - prob(voter\ is\ of\ type\ G)\big)^2$, if this voter is of type G and

$100 - \frac{1}{100}\big(prob(voter\ is\ of\ type\ G)\big)^2$ , if this voter is of type I,

where $\big(prob(voter\ is\ of\ type\ G)\big)$ is your indication of probability in percentage points that that voter is of type G. The resulting number is rounded up to a whole number and give,s together with your pay-off in case of a correct group decision, your pay-off in the round.

Remember: At the end of the experiment 4 rounds are randomly selected, the point incomes converted to Swiss Francs and paid out in private. The not selected rounds will not be paid out.

# Questions?

Take your time to read the instructions carefully. If you have any questions, raise your hand. An experimental administrator will then come to your seat.

# Chapter 3

# Legislative Bargaining with Accountability

In existing legislative bargaining models, the precise division of seats between parties has no bearing on some of: which coalition forms, which policy is adopted, how perks are divided - or even all three. These models also predict that the proposer should reap significantly larger rewards than the other players. Such predictions are, however, at odds with longstanding empirics: government portfolios are generally allocated in proportion to seat share, and there is no proposer advantage. In this paper, I show that when each member of a party faces the electoral consequences of being in government then seat shares matter a great deal: (1) For a given ranking of parties, changing their respective seat shares can bring about almost any coalition; (2) the implemented policy is a function of the coalition parties seat shares; (3) an increase in one coalition party's seats will move the policy towards their preferred point, but may increase or decrease their share of government perks. Furthermore, I show that (4) there can be equilibria in which the largest party is not government, and (5) in many cases the larger coalition member will have all of its rent extracted by the junior member.

## 3.1    Introduction

One of the central tenets of a parliamentary democracy is that the people can choose their preferred government by the politicians they elect to the parliament. This is only partly true - if no party wins a majority of the seats then the government that forms and the policies they implement will depend on the outcome of bargaining between the parties. Indeed, post-election coalition bargaining is the norm in much of Europe. An understanding of how legislative bargaining works is important for a number of reasons. If citizens can anticipate how bargaining will unfold, they can use this to make a more informed decision on how to vote: whether they should act strategically, and if so, how. What's more it can inform us on the relationship between the ideological distance of parties, their seat shares, and the final balance of power: does bargaining result in policies preferred by the centrist party? Does the largest party reap an unfair advantage in bargaining?

The formation of coalition governments has by now been extensively studied and typically is modelled as a non-cooperative bargaining game between parties. The main difference between models in this class is (a) what is being bargained over, and (b) how the proposer is chosen. Most papers following Baron and Ferejohn (1989) (henceforth BF) model coalition bargaining as a 'divide the dollar' game, in which parties negotiate over the distribution of a fixed amount of government perks or ministries. A smaller set of papers such as Baron (1991) model the game as bargaining over a government policy to be implemented. Then are are a number of papers which combine both elements (Austen-Smith and Banks (1988); Baron and Diermeier (2001); Jackson and Moselle (2002); Morelli (1999)). The other dimension on which these models differ is their treatment of proposer selection. Most models follow BF in using *random recognition*; that is where, at each stage of bargaining a party is randomly chosen to be the proposer or 'formateur', with each party's probability of selection being equal to its seat share. A less common setup is *fixed order recognition* used in Austen-Smith and Banks (1988) (henceforth ASB) where the largest party is the first to try to form a government and if

this fails the second largest takes its turn, and so on.

For all their variety, these models generate predictions at odds with real world government formation. The both BF and ASB models predict a strong formateur advantage. Under BF the formateur should take two thirds of the dollar while the coalition partner takes the rest - regardless of how many seats the formateur or his partner has. In the ASB setup the largest party is strongly advantaged due to the fixed order protocol - it will implement a policy close to its preferred and reap a large share of the government perks. This is independent of exactly how many seats this party has (assuming it does not have a majority). In both BF and ASB models the first chosen formateur should form a government and extract a large proportion of the surplus. The empirical literature, however has found little by way of a formateur advantage (Warwick and Druckman (2006); Laver et al. (2010)). A related conundrum is that of Gamson's Law, which states that a party's share of cabinet seats is proportional to it's share of seats in the parliament. Gamson's Law is one of the strongest empirical regularities in political science. Warwick and Druckman (2006) show that the relationship between cabinet seats and parliamentary seat remains regardless of whether cabinet posts are weighted by their importance, or we take account of parties real bargaining weights. Neither the BF nor the ASB model can explain Gamson's law. Under BF, the formateur will take the same two thirds share of cabinet seats regardless of seat shares - the only thing which changes is is the probability with which the offer is made to each of the other two parties (**?**). In a model with both policy and perks it is difficult to assess Gamson's Law. However, for a given ranking of parties the ASB model predicts the same outcome in terms of coalition partners, policy and division of perks, regardless of the precise number of seats each party holds. Finally, as players use mixed strategies in divide the dollar games, 'non-connected' coalitions between a left and right party will form in equilibrium. Such non-connected coalitions are also a feature of the ASB model, where the fixed order protocol means coalitions always form between the largest and smallest party. Empirically, however, non-connected coalitions are in fact very rare - parties overwhelmingly tend to form coalitions with other ideologically similar parties.

In this paper I build a modified version of the ASB model. That is, I have three parties bargaining over the ideological location of government policy and over the distribution of a fixed amount of ministries or perks. I use a fixed order of recognition as in the original but I introduce two innovations to the benchmark model. Firstly, I include the number of seats a party has into its utility function. Standard models of bargaining over policy assume that each party's utility depends on the distance between it's preferred policy and the implemented policy. The idea is that the further away the implemented policy is from what the party promised during the election campaign, the more their supporters will dislike this, and the more difficulty they're going to have keeping those votes in the next election. These models treat all parties equally, so that a party would suffer the same loss from a policy regardless of whether the party has ten or one hundred politicians elected.[1] In reality, each politician must go back to his own constituency and convince voters to re-elect him. It stands to reason then that each individual in a party gets a disutility from implementing a policy far away from the party's announced platform. Therefore, in my model, each party's utility from policy is the sum of all of its elected members utilities. So, all else equal, a larger party will find it more costly to implement policies away from its announced policy than a party with few seats will. The second change I make to the ASB model is in terms of which parties are held responsible by voters. If we interpret a party's disutility from policy as a punishment from voters for breaking electoral promises, then it does not seem very reasonable that all parties are blamed for the policies chosen by a majority. Specifically, any parties outside of the governing coalition should not face any repercussions from voters as they had no say in implementing policy and did not 'let down' their voters. We can combine both of these novel assumption to say: If a party is not in the government coalition its utility loss from policy is zero, and if a party is in government its utility loss from policy is the sum of losses of all its elected members.

This new model shows that seat shares matter in determining which coalition forms,

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[1] Jackson and Moselle (2002) have some applications where each party cares about policy and perks to different extents, however this is not connected to party seat shares

which policy is implemented and how perks are divided. For any given seat distribution and size of perks these three are perfectly pinned down. However, varying the seat shares will alter policy and perks shares, and may even change which coalition forms. I show that the implemented policy is a weighed average of the two coalition partner's preferred policies, with the weights given by seat shares. This occurs because the formateur maximises his party's utility by minimising the coalitions joint utility loss from policy. For the same reason, an increase in one coalition party's seat share will push the final policy in their direction but may increase or decrease that party's share of perks. This contrasts sharply with standard bargaining models in which only a party's ability to block coalitions matters rather than its seat share. Furthermore, I find that for a given party ranking almost any coalition can form depending on how seats are distributed. This contrasts with Austen-Smith and Banks (1988), where the largest and smallest parties always form a government. In my model, if the left party is largest and right party is smallest we can have a coalition between left and right, left and middle, middle and right, or even a stalemate where a caretaker government is appointed. In fact, non-connected coalitions between left and right may not occur at all if the the size of parliament or the distribution of seats makes it infeasible. Finally, I show there are numerous equilibria in which there is no formateur advantage - rather there is a disadvantage. If perks are not large enough to allow the formateur to form a coalition with either party then the one party he can join with will extract all of the formateur's surplus. In these equilibria being lower down the pecking order gives a party better bargaining position. The paper proceeds as follows: the next section introduces the model, section 3 presents the main results of the model and then section 4 gives some numerical examples to show the range of possible outcomes for a single seat ordering. Section 5 concludes.

## 3.2 Model

The game begins after a legislative election has taken place. There are $D$ number of seats shared among three parties; $l, m$, and $r$, where $D$ is an odd number. The distribution

of seats in the legislature is $S \equiv (s_l, s_m, s_r)$, with party $c \in \{l, m, r\}$ having $s_c$ seats and $\sum_c s_c = D$. No party has a majority of seats. The parties must bargain over the formation of government and implement a policy $z$.

Party $c$ has an announced policy $a_c$ in the unidimensional policy space $\mathbb{X} = [-1, 1]$ on which it contested the election. Without loss of generality let $a_l < a_m = 0 < a_r$. If party $c$ is part of the government coalition, its payoff is $W_c = b_c - s_c(z - a_c)^2$. The first term is its share of government benefits $b_c$, such as ministerial positions. The second term represents the loss a party incurs from implementing policies different from its announced policy $a_c$. This loss is weighted by the number of seats the party has. As each elected member of the party is held accountable by his constituents, a party with one seat will face a much smaller policy loss for implementing other policies than a party with many seats will. If party $c$ is not part of the government coalition its payoff is zero; its members gain no perks of office, but likewise are not held responsible by voters, as they did not approve the policy $z$. A feasible allocation of benefits is $b = (b_l, b_m, b_r)$ where each $b_c$ is non-negative and $\sum_c b_c \leq B$. The size of the pie $B$ will determine which coalitions are feasible.

The party with the most seats of the three begins the bargaining process by offering a policy outcome $y^1 \in \mathbb{X}$ and a distribution of a fixed benefits across the parties, $b^1 = (b_l^1, b_m^1, b_r^1) \in [0, B]^3$. If the first proposal is rejected, the party with the second largest number of seats gets to propose $(y^2, b^2)$. If this is rejected, the smallest party proposes $(y^3, b^3)$. If no agreement has been reached after the third period, a caretaker government is put in place and all parties receive zero utility.

At its turn to make a proposal, party $c$ solves

$$\max_{b_{c'}, y} B - b_{c'} - s_c(y - a_c)^2 \tag{3.1}$$
$$\text{subject to} \quad b_{c'} - s_{c'}(y - a_{c'})^2 \geq W_{c'}$$

where $W_{c'}$ is the continuation value of party $c'$ and $W_{c''} + s_{c''}(y - a_{c''})^2 > W_{c'} + s_{c'}(y - a_{c'})^2$, so that the formateur makes the offer to whichever party is cheaper. The proposer may potentially find it in his interest to make a proposal surely to be rejected if this maximises

his payoff. The solution concept is subgame perfect Nash equilibrium. As the game and number of strategies are finite an equilibrium always exists.

## 3.3 Results

In this section I show the importance of the seat share distribution in determining which coalition forms, which policy is implemented, and how perks are shared between coalition partners.

**Proposition 3.1.** *Each coalition will consist of only two parties, where the implemented policy, z, will be the one which minimises the joint policy loss of the coalition parties. A coalition of parties $c$ and $c'$ implements $z_{c,c'} = \frac{s_c a_c + s_{c'} a_{c'}}{s_c + s_{c'}}$.*

*Proof.* First, note that the worst each party can do is have a utility of zero. The outside option of not joining a coalition or allowing a caretaker government ensures a utility of zero. It is easy to see that each coalition will consist of only two parties. If a third party was in the coalition, the formateur could deviate by excluding one of the partners, and then divide the excluded party's benefits up between the remaining coalition members. When solving equation 3.1, each player's continuation value is at least zero, so therefore the formateur must offer some benefits to a potential coalition partner in order to offset the policy loss. Maximising with respect to $y$, the formateur will offer $\frac{s_c a_c + s_{c'} a_{c'}}{s_c + s_{c'}}$ and whichever amount $b_{c'}$ is necessary to guarantee the other player accepts. □

The final policy is a function of coalition partners' seat shares. This is a departure from the standard fixed order protocol of ASB where only the ranking of parties and their ideological distance matters in determining the final policy. The reason for the difference is twofold. Firstly, in my model, any party which does not form part of the government coalition will have a utility of zero. This means the best a formateur can do is to offer a potential coalition partner the policy which would minimise the joint loss function of both parties. In ASB and other papers, even a party not in the coalition faces a utility loss from an implemented policy. This can lead to negative continuation

values and this in turn allows a formateur to propose policies which do not minimise the joint loss function. Secondly, in my model each individual member of the party faces a utility loss from being in government. This means that the policy which minimises the joint loss function depends on the number of seats each of the parties has as well their ideological distance, rather than just the latter in ASB.

For each coalition, the joint loss of the parties from implementing the optimal policy is given below.

$$L_{l,m} = s_l(a_l - \frac{s_l a_l}{s_l + s_m})^2 + s_m(\frac{s_l a_l}{s_l + s_m})^2 \tag{3.2}$$

$$L_{r,m} = s_r(a_r - \frac{s_r a_r}{s_r + s_m})^2 + s_m(\frac{s_r a_r}{s_r + s_m})^2 \tag{3.3}$$

$$L_{l,r} = s_l(a_l - \frac{s_l a_l + s_r a_r}{s_l + s_r})^2 + s_r(a_r - \frac{s_l a_l + s_r a_r}{s_l + s_r})^2 \tag{3.4}$$

Let $L(s_1, s_2) \in \{L_{l,m}, L_{r,m}, L_{l,r}\}$ denote the cost of forming a coalition between the largest and second largest party. We can now describe the equilibria of the game for any ordering of parties and any size of $B$.

**Proposition 3.2.** *For a given party seat ordering $s_1 > s_2 > s_3$, the equilibrium depends on the size of $B$.*

  *i.* *If $B > L(s_1, s_2), L(s_1, s_3)$ then a coalition $(1, 3)$ forms. Benefits are $b_1 = B - b_3, b_2 = 0, b_3 = s_3(z_{1,3} - a_3)^2$.*

  *ii.*  *(a) If $L(s_1, s_2) > B > L(s_1, s_3)$ and $L(s_2, s_3) > L(s_1, s_3)$ then a coalition $(1, 3)$ forms. Benefits are $b_1 = s_1(z_{1,3} - a_1)^2, b_2 = 0, b_3 = B - b_1$.*

     *(b) If $L(s_1, s_2) > B > L(s_2, s_3)$ and $L(s_1, s_3) > L(s_2, s_3)$ then a coalition $(2, 3)$ forms. Benefits are $b_1 = 0, b_2 = s_2(z_{2,3} - a_2)^2, b_3 = B - b_2$.*

     *(c) If $L(s_1, s_3) > B > L(s_1, s_2), L(s_2, s_3)$ then a coalition $(1, 2)$ forms. Benefits are $b_1 = s_1(z_{1,2} - a_1)^2, b_2 = B - b_1, b_3 = 0$.*

  *iii.* *If $L(s_1, s_2), L(s_1, s_3), L(s_2, s_3) > B$ then no coalition forms, and a caretaker government in installed.*

*Proof.* See Appendix.         □

**Corollary 3.1.** *When $s_l, s_r > s_m$ an $(l, r)$ coalition can never form.*

The proposition and corollary tell us that with a fixed order protocol almost any coalition is possible; which one forms depends on the size of perks relative to the cost of putting together each coalition. Unlike in ASB, the size of the legislature $D$ and the distribution of seats $S$ determines whether or not $B$ is large enough to allow a certain coalition. Another feature which conflicts with the standard model is that delay is possible (in *ii.(b)* a coalition forms between the second and third largest parties), so the largest party may not be part of the coalition at all. What's more, in many equilibria the larger of the two coalition partners has all of its surplus extracted by the junior coalition member. We see this in all the *(ii)* equilibria, and this contrasts sharply with standard legislative bargaining results showing a large bonus for the formateur. Finally, the proposition shows that non-connected coalitions need not occur in fixed order protocols where negotiations occur over perks and policy. Indeed, non-connected coalitions can only occur if party $m$ is the second largest and the perks of government are big enough to allow the largest party to form a coalition with either of the other two. As the number of seats in the legislature increases, it becomes increasingly difficult for a coalition to form between $l$ and $r$, for a given size of $B$. The next proposition looks at how policy and the distribution of perks within a given coalition changes as parties each gain or lose a seat.

**Proposition 3.3.** *In a given coalition $(c, c')$ where $s_c > s_{c'}$, increasing the seat share of party $c$ will move the policy towards $a_c$, while an increase in $s_{c'}$ will move the policy towards $a_{c'}$. In an equilibrium type $(i)$, a decrease in $s_{c''}$ causes $b_{c'}$ to increase. An increase in $s_{c'}$ at the expense of $s_c$ causes $b_{c'}$ to increase if $s_c > 2s_{c'}$ and decrease otherwise. Under equilibria types $(ii)$, an increase in $s_c$ or a decrease in $s_{c'}$ causes $b_c$ to decrease.*

*Proof.* See Appendix. □

The proposition says that increasing the seats of a party in coalition will move the government policy towards that party's preferred point but may increase or decrease

their share of government perks. While Gamson's Law states that a party's share of perks or ministries should be proportional to their seat share, the division of perks here is more nuanced. Under ($i$) equilibria an increase in the size of the coalition will increase the perks of the junior partner, while under ($ii$) equilibria the fewer seats a coalition partner has, the larger it's share of the perks. This occurs because the junior partner essentially holds the senior coalition member hostage for all of the surplus, and the lower its seat share, the more sense it makes to take utility in terms of perks rather than policy.

## 3.4   Numerical Examples

In this section I vary the seat distributions as well as the policy positions of parties to give a sense of how these parameters effect the types of coalitions that form, and the distribution of benefits in those coalitions.

**Example 1**   Let $D = 101$, and $s_l > s_m > s_r$ where $S = \{45, 30, 26\}$, and $\{a_l, a_m, a_r\} = \{-1, 0, 1\}$. Here we have $L_{l,r} > L_{l,m} > L_{r,m}$, and four different outcomes are possible.

  i. If $B < 13.93$ a caretaker government will form.

 ii. If $13.93 < B < 18$ then an $(m, r)$ coalition will form in the second round. We'll have $z = \frac{26}{56} \approx 0.46$ , $b = \{0, 6.47, B - 6.47\}$ and $w = \{0, 0, B - 13.93\}$. If $s_m$ increases or $s_r$ decreases then $b_m$ gets smaller.

iii. If $18 < B < 65.9$ an $(l, m)$ coalition will form in the first round. We'll have $z = -0.6$ , $b = \{7.2, B - 7.2, 0\}$ and $w = \{0, B - 18, 0\}$. If $s_l$ increases or $s_m$ decreases then $b_l$ gets smaller.

 iv. If $B > 65.9$ an $(l, r)$ coalition will form in the first round. We'll have $z = -\frac{19}{71} \approx -0.268$ , $b = \{B - 41.8, 0, 41.8\}$ and $w = \{B - 65.9, 0, 0\}$. If $s_l$ decreases or $s_m$ increases then $b_r$ gets smaller.

In this case, for a given seat distribution and party positions, very different policies are implemented depending on the size of government perks $B$. As the size of perks

increase, the policy moves from 0.46 to $-0.6$ and then finally to $-0.268$. Another point worth noting is that the largest party has utility of zero in all equilibria where $B < 65.9$. The intuition for this is that it is very costly to make an $(l, r)$ coalition, so party $l$ has no real alternative to an $(l, m)$ coalition. Party $m$ uses this fact to extract all the rents from party $l$. In fact, when $13.93 < B < 18$ then no coalition featuring party $l$ can generate surplus, so in this case $l$ is excluded from government even though it is the largest party. In terms of comparative statics, in the equilibria with $B < 65.9$ the larger of the two coalition partners will gain more perks by losing seats or having the coalition partner win more seats. When $B > 65.9$ an increase in the overall size of the coalition or an increase in the number of $l$ seats at the expense of $r$ seats will increase the perks of party $r$.

**Example 2**   Let $D = 101$, $S = \{49, 48, 4\}$, and $\{a_l, a_m, a_r\} = \{-1, 0, 1\}$. Here we have $L_{l,m} > L_{l,r} > L_{r,m}$, and three different outcomes are possible.

 i. If $B < 3.7$ a caretaker government will form.

 ii. If $3.7 < B < 24.2$ then an $(m, r)$ coalition will form in the second round. We'll have $z = \frac{1}{13} \approx 0.077$ , $b = \{0, 0.28, B - 0.28\}$ and $w = \{0, 0, B - 3.7\}$. If $s_m$ increases or $s_r$ decreases then $b_m$ gets smaller.

 iii. If $B > 24.2$ an $(l, r)$ coalition will form in the first round. We'll have $z = -\frac{45}{53} \approx -0.85$ , $b = \{B - 13.7, 0, 13.7\}$ and $w = \{B - 14.8, 0, 0\}$. If $s_l$ increases or $s_m$ increases then $b_r$ gets smaller.

In the example above the policy points remain the same but the distribution of seats is much closer to a bipartisan legislature, with party $r$ having only a few seats. There are some key differences with respect to the first example. First, there is one less equilibrium. Second, fewer perks are needed to facilitate the various coalition agreements; in previous example an $(l, r)$ coalition would only occur if $B > 65.9$ whereas now it will occur if $B > 24.2$. Third, when an $(l, r)$ coalition forms, party $l$ receives a bonus of 9.4 over and above what it would find acceptable. This occurs because an $(l, m)$ coalition must be feasible for an $(l, r)$ coalition to form. This requires a larger $B$ than usual, which means

the formateur $l$ takes the excess $L(l, m) - L(l, r)$. Finally, when $B > 24.2$ an increase in the number of $l$ seats at the expense of $r$ seats will decrease the perks of party $r$.

**Example 3**  Let $D = 101$, $S = \{45, 30, 26\}$, and $\{a_l, a_m, a_r\} = \{-0.5, 0, 1\}$. Here we have $L_{l,r} > L_{r,m} > L_{l,m}$, and three different outcomes are possible.

 i If $B < 4.5$ a caretaker government will form.

 ii If $4.5 < B < 37$ then an $l, m)$ coalition will form in the first round. We'll have $z = -0.3$ , $b = \{1.8, B - 1.8, 0\}$ and $w = \{0, B - 4.5, 0\}$. If $s_l$ increases or $s_m$ decreases then $b_l$ gets smaller.

 iii If $B > 37$ an $(l, r)$ coalition will form in the first round. We'll have $z = 0.05$, $b = \{B - 23.46, 0, 23.46\}$ and $w = \{B - 37, 0, 0\}$. If $s_l$ decreases or $s_m$ increases then $b_r$ gets smaller.

In this case the seat distribution is the same as the first example but now $a_l$ is twice as close to $a_m$. This eliminates the equilibria with an $(m, r)$ coalition, as this is now relatively less desirable. For a large range of perks an $(l, m)$ coalition forms with party $l$ only having perks of 1.8. As in the other two cases, once $B$ is large enough an $(l, r)$ coalition will form and party $l$ will have at least one third of the perks.

## 3.5   Discussion

By introducing two new assumptions to the Austen-Smith and Banks (1988) model I show that seat shares matter a great deal. When the public hold individual politicians accountable for supporting a government policy then, for a given ranking of parties sizes, we can have wildly different coalition, policies, and perk distributions. The fact that seat shares 'matter' is more in line with the empirical findings on coalition formation than previous legislative bargaining models. While my model does not produce perk shares proportional to seat shares *ala* Gamson's law, this is due to that fact that parties bargain

over both perks and policy. Here, it is policy rather than perks which is determined according to how many seats members of the coalition bring to the table.

The assumption that individuals are censured by their constituents for supporting bad policies is intuitive under systems where local districts elect individuals to the legislature. This is the case with plurality rule in the U.K., Canada, and India, with instant runoff voting in Australia and with single transferable voting in Ireland. Instead under proportional representation systems found in much of Europe, where voters vote for party lists, this local element is somewhat missing. An implication of this is that it should be more difficult to form coalition governments in countries where individual politicians are held accountable.

While I hope this paper has added to our understanding of government coalition bargaining, there is still quite a gap between the theory and the empirics. Laver et al. (2010) criticise the unrealistic structure of bargaining models, particularly random recognition rules where one player is randomly given monopoly rights to make proposals. They claim that in reality coalition bargaining is a much more unstructured process, where several deals may be on the table at the same time between different proto-coalitions. Pre-election pacts are another feature of elections across many countries, yet standard models have little to say here (although Carroll and Cox (2007) build a model with pre-electoral pacts which then give results consistent with Gamson's law).

Yet even more troubling is the question of whether these formal models can really be tested with real world data. Laver et al. (2010) note that empirical work done by Warwick and Druckman (2006) and Snyder Jr et al. (2005) relies on a coding of formateur status which is done ex post - essentially a party is coded as being the formateur if it takes the prime minister position. Given that these government negotiations typically occur behind closed doors and involve unverifiable rumours, it is very difficult to say which party was the formateur at a given stage unless the constitution specifies an explicit protocol.

## 3.6   Appendix

### 3.6.1   Lemmas used for Proof of Proposition 2

**Lemma 3.1.** *If $s_l > s_r > s_m$ then $L_{l,r} > L_{l,m}$. If $s_r > s_l > s_m$ then $L_{l,r} > L_{m,r}$.*

*Proof.* Notice that $\frac{s_l a_l}{s_l + s_m}$ is the point which minimises the joint loss function $s_l(a_l - z)^2 + s_m(z)^2$. So then it must be that $L_{l,m} < s_l(a_l - \frac{s_l a_l + s_r a_r}{s_l + s_r})^2 + s_m(\frac{s_l a_l + s_r a_r}{s_l + s_r})^2 \equiv L^*$. To see that $L_{l,r} > L^*$, we need to show $s_r(a_r - \frac{s_l a_l + s_r a_r}{s_l + s_r})^2 > s_m(\frac{s_l a_l + s_r a_r}{s_l + s_r})^2$. We know that $a_r > 0$ and that $s_l > s_r > s_m$, therefore we must have $L_{l,r} > L^* > L_{l,m}$. Similarly it holds that if $s_r > s_l > s_m$ then $L_{l,r} > L_{m,r}$.  □

**Lemma 3.2.** *If $L_{l,m} > L_{l,r}$ then $L_{l,r} > L_{r,m}$. If $L_{r,m} > L_{l,r}$ then $L_{l,r} > L_{l,m}$.*

*Proof.* Notice that $\frac{s_l a_l}{s_l + s_m}$ is the point which minimises the joint loss function $s_l(a_l - z)^2 + s_m(z)^2$, and that $\frac{s_r a_r}{s_r + s_m}$ is the point which minimises the joint loss function $s_r(a_r - z)^2 + s_m(z)^2$. The point $\frac{s_l a_l + s_r a_r}{s_l + s_r}$ which minimises $s_l(a_l - z)^2 + s_r(a_r - z)^2$ is either greater or less than zero. Suppose it is greater than zero; then it must be that $s_l(a_l - \frac{s_l a_l + s_r a_r}{s_l + s_r})^2 > L_{l,m}$, and so therefore $L_{l,r} > L_{l,m}$. If instead $s_l(a_l - z)^2 + s_r(a_r - z)^2$ is less than zero; then it must be that $s_r(a_r - \frac{s_l a_l + s_r a_r}{s_l + s_r})^2 > L_{r,m}$, and so therefore $L_{l,r} > L_{r,m}$.  □

### 3.6.2   Proof of Proposition 2

I show the proof for the case of $s_l > s_m > s_r$. The other cases are analogous. We solve the game by backwards inductions.

At stage 3 party $r$ can propose an allocation $(y, b_l, b_m, b_r)$. Every party's outside option is a utility of zero if a caretaker government is implemented. As such party $r$ will propose a coalition with party $m$ if $B > L_{m,r}$ and $L_{l,r} > L_{m,r}$, will instead propose a coalition with $l$ if $B > L_{l,r}$ and $L_{m,r} > L_{l,r}$, while if $L_{m,r}, L_{l,r} > B$ party $r$ will make a proposal surely to be rejected by both other parties.

**Let $L_{m,r}, L_{l,r} > B$.** We know that party $r$ makes an offer sure to be rejected in stage 3. At stage 2 party $m$ can form a coalition with either of the other two parties, or make an offer surely to be rejected. As $L_{m,r} > B$, a coalition between $m$ and $r$ will lead to negative utility, so it cannot occur.

- If $L_{m,l} > B$ then $m$ will make an offer to be rejected, and because $L_{m,r}, L_{l,r}, L_{m,l} > B$ no coalition can create positive utility, thus at stage 1 party $l$ also prefers not to form a government.

- If instead $B > L_{m,l}$ then an $(m, l)$ coalition will form at stage 2. At stage 1 party $l$ must choose between the two coalition partners and the outside option. Party $l$ will never choose a coalition with party $r$ as $B < L_{l,r}$ but is indifferent between forming a coalition with party $m$ or making a proposal to be rejected. Whichever of these party $l$ chooses, the outcome is a $(l, m)$ coalition where party $l$ has utility zero with $b_l = s_l(a_l - \frac{s_l a_l}{s_l + s_m})$ while party $m$ has utility $B - L_{l,m}$ with perks of the amount $B - b_l$.

**Let $B > L_{m,r}$ and $L_{l,r} > L_{m,r}$.** At stage 3 party $r$ will prefer to form a coalition with party $m$.

- If $B > L_{l,m}$ then $m$ will form a coalition with party $l$ at stage 2. Then at stage 1, if $B > L_{l,r}$ party $l$ will form a coalition with party $r$ where party $l$ gets utility $B - L_{l,r}$ and benefits $B - b_r$, while party $r$ gets utility zero and benefits $b_r = s_r(a_r - \frac{s_l a_l + s_r a_r}{s_l + s_r})$. If instead $B < L_{l,r}$ then at stage 1 party $l$ would rather form a coalition with party $m$ giving herself utility of zero and $b_l = s_l(a_l - \frac{s_l a_l}{s_l + s_m})$ while party $m$ has utility $B - L_{l,m}$ and perks of the amount $B - b_l$.

- If $B < L_{l,m}$ then $m$ will form a coalition with party $r$ at stage 2 as it is indifferent between this and having a proposal rejected. At stage 1 party $l$ will make a proposal sure to be rejected as a proposal to party $m$ would give a negative utility $B - L_{l,m}$ as would a proposal to party $r$ (which would yield $L_{r,m} - L_{l,m}$).

**Let** $B > L_{l,r}$ **and** $L_{m,r} > L_{l,r}$. At stage 3 party $r$ will form a coalition will party $l$. At stage two if $m$ were to form a coalition with party $r$, the latter's utility would be negative and so will never be chosen.

- If $B > L_{l,m}$ then party $m$ will form a coalition with party $l$ in stage 2. In stage 1 party $l$ will have a utility of zero from a coalition with $m$ or a utility of $B - L_{l,r}$ from a coalition with $r$. As this is positive, the $(l, r)$ coalition forms with party $l$ having utility $B - L_{l,r}$ and benefits $B - b_r$ while party $r$ gets utility zero and benefits $b_r = s_r(a_r - \frac{s_l a_l + s_r a_r}{s_l + s_r})$.

- If $B < L_{l,m}$ then party $m$ makes an offer which will be rejected in stage 2. At stage 1 party $l$ forms a coalition with party $r$. The utility of party $l$ is zero and its benefits are $b_l = s_l(a_l - \frac{s_l a_l + s_r a_r}{s_l + s_r})$, while the utility of party $r$ is $B - L_{l,r}$ with benefits of $B - b_l$.

### 3.6.3   Proof of Proposition 3

For any $z = \frac{s_c a_c + s_{c'} a_{c'}}{s_c + s_{c'}}$ we see that $\frac{dz}{dc} = \frac{s_{c'}(a_c - a_{c'})}{(s_c + s_{c'})^2}$ which is negative if $a_c < a_{c'}$ and positive otherwise. So, any increase in a party's number of seats moves to policy closer to it's announced point (contingent on remaining in the coalition).

In an equilibrium type $(i)$, we have $s_c > s_{c''} > s_{c'}$ and coalition $(c, c')$ forms. We know that $z_{c,c'} = \frac{s_c a_c + s_{c'} a_{c'}}{s_c + s_{c'}}$ and that party $c$ will offer enough perks to $c'$ to give zero utility. So then $b_{c'} = s_{c'}(a_{c'} - \frac{s_c a_c + s_{c'} a_{c'}}{s_c + s_{c'}})^2$. Differentiating with respect to $c$ or $c'$ tells us the effect of an increase of one seat at the expense of the party not in the coalition, $s_{c''}$.

$$\frac{\delta b_{c'}}{\delta s_c} = \frac{2 s_c s_{c'}^2 (a_c - a_{c'})^2}{(s_c + s_{c'})^3} \tag{3.5}$$

$$\frac{\delta b_{c'}}{\delta s_{c'}} = \frac{s_c^2 (a_c - a_{c'})^2 (s_c - s_{c'})}{(s_c + s_{c'})^3} \tag{3.6}$$

Both terms are positive. To investigate the effect of an increase in $s_{c'}$ at the expense of $s_c$ we rewrite the share of perks as $\hat{b_{c'}} = s_{c'}(a_{c'} - \frac{(D - s_{c'} - s_{c''})a_c + s_{c'} a_{c'}}{D - s_{c''}})^2$.

$$\frac{\delta \hat{b_{c'}}}{\delta s_{c'}} = \frac{(s_c - s_{c'})^2 (D - s_{c''} - 3 s_{c'})(D - s_{c''} - s_{c'})}{(D - s_{c''})^2} \tag{3.7}$$

The result is positive if $s_c > 2s_{c'}$ and otherwise negative.

In equilibria of type $(ii)$ where $s_c > s_{c'}$ and a coalition $(c, c')$ forms, the smaller party offers enough perks to ensure the larger party a utility of zero. We have $b_c = s_c(a_c - \frac{s_c a_c + s_{c'} a_{c'}}{s_c + s_{c'}})^2$. Differentiating with respect to $c$ or $c'$ tells us the effect of an increase of one seat at the expense of the party not in the coalition $s_{c''}$.

$$\frac{\delta b_c}{\delta s_c} = \frac{s_{c'}^2 (a_{c'} - a_c)^2 (s_{c'} - s_c)}{(s_c + s_{c'})^3} \tag{3.8}$$

$$\frac{\delta b_c}{\delta s_{c'}} = \frac{2 s_{c'} s_c^2 (a_{c'} - a_c)^2}{(s_c + s_{c'})^3} \tag{3.9}$$

The former is negative because $s_{c'} < s_c$, while the latter is positive. To investigate the effect of an increase in $s_{c'}$ at the expense of $s_c$ we again rewrite the share of perks as $\hat{b}_c = s_c(a_c - \frac{(D - s_c - s_{c''})a_{c'} + s_c a_c}{D - s_{c''}})^2$

$$\frac{\delta \hat{b}_{c'}}{\delta s_{c'}} = \frac{(s_{c'} - s_c)^2 (D - s_{c''} - 3s_c)(D - s_{c''} - s_c)}{(D - s_{c''})^2} \tag{3.10}$$

The result is negative because $s_{c'} < 2s_c$. This means that in these equilibria, any increase in $s_{c'}$ or any decrease in $s_c$ will lead to an increase in $b_c$.

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