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Understanding Competitiveness

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European University Institute  
**Max Weber Programme**

## **Understanding Competitiveness**

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**Abstract**

Using firm level data, we analyze the factors that drive the evolution of the aggregate Unit Labor Costs – the main European competitiveness indicator – in France, Germany, Italy and Spain. The evolution of the aggregate Unit Labor Cost is not driven by the evolution of the firm level Unit Labor Costs, but rather by an important factor for the competitiveness of a country: the reallocation of resources among the firms of the economy. Using the methodology of Hsieh and Klenow (2009), we show the importance of an efficient allocation of resources for productivity gains.

**Keywords**

Unit labour costs, competitiveness, misallocation, European Union

**JEL Codes:** F02, F15, J30, O47, O57

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# 1 Introduction

The latest world crisis and the increase of debt in Europe have reopened in the last few years a debate forgotten in the good times, the competitiveness of an economy. Currently the relevant measure of competitiveness in the European Union is the evolution of unit labor costs. The unit labor cost is a macroeconomic aggregate that measures the labor cost per unit of product and is calculated as the ratio of total labor costs to real output. A rise in labor costs higher than the rise in labor productivity may be a threat to an economy's cost competitiveness if other costs are not adjusted in compensation.

The use of aggregate price-cost based indicators, like the unit labor costs, may not be informative enough to determine the competitiveness of a country. For example, Spain's aggregate unit labor cost has grown faster than in the other European countries in the last decade. Then, we should see a decrease in the world's export shares reflecting the decrease in the ability to sell their products. However, the exports shares have decreased less than those of the other European countries. This "Spanish paradox" is explained by the different relative weight of firms in the unit labor costs and the economy's total exports. Firms that export are usually the largest and most productive of the economy (Clerides et al. (1998) and Bernard and Bradford Jensen (1999)), and they account for the main share of firms that export. However, for the aggregate unit labor cost all the firms in the economy are taken into account, not just the exporters. Recent literature in industrial organization and international trade (di Giovanni and Levchenko (2009) and Bernard et al. (2011)) has provided abundant empirical evidence supporting the idea that the evolution of macroeconomic aggregates is determined closely by the decisions and characteristics of the firms in the economy, and in particular by the behavior and productivity of a subgroup of them: the most productive ones. Then, an adequate competitiveness measure should be able to take into account the role of firms and their heterogeneity.

In this paper, we analyze the ability of the aggregate unit labor costs evolution to capture adequately the firm heterogeneity of a country. We calculate, using firm level data, a weighted change of the aggregate unit labor costs between 2002 and 2007 for four European countries: France, Germany, Italy and Spain. The components of the weighted average are then decomposed according to a Laspeyres decomposition into three main elements: the first captures changes in firm level unit labor costs, keeping the initial domestic market shares of firms constant; the second quantifies the reallocation of market shares within the domestic economy, keeping the initial unit labor costs constant; and the

third measures the interaction between the first two. If the aggregate ULC was a measure that captured adequately the heterogeneity existent at the firm level ULC, its evolution should be driven by the evolution of the firm level ULC. Then we should observe the within component to be the most relevant in the explanation of the aggregate ULC evolution.

The results reveal that the evolution of the firm level unit labor cost does not explain the evolution of the aggregate unit labor costs, rather it is the resource reallocation and the interaction effect that explain around 90% of the changes in ULCs for all the countries in the sample. Furthermore, Germany is the country that presents a greater reallocation of resources in the period 2002 to 2007. In comparison with Germany, the lower resource reallocation led to competitiveness losses of around 4.3% in the case of France, 6.4% in Italy and 8% in Spain.

Motivated by the significant role of the reallocation of resources to explain the evolution of the aggregate ULC, we apply the methodology of [Hsieh and Klenow \(2009\)](#) to explain how much of the differences in productivity in Europe is due to an inefficient allocation of resources. As a result of distortions that affect production, firms produce different amounts than what would be dictated by their productivity. In order to determine the gains from an efficient allocation of resources, we calculate the hypothetical “efficient” output in each country — the output if these distortions did not exist — and compare it with actual output levels.

An efficient allocation of resources would boost aggregate manufacturing TFP in 2008 by 22.7% in France, 27.9% in Germany, 43.5% in Italy and 28.2% in Spain. More interestingly, we observe that over the period of 2002 to 2008, the “misallocation” of resources decreases in Germany, remains fairly constant in France and increases in Italy and Spain. This is actually consistent with the higher reallocation of resources present in the evolution of Germany’s aggregate unit labor costs, which is followed by France, Italy and Spain.

Our empirical analysis of the unit labor costs as a competitiveness measure reveals the need to open the “black boxes” that the macroeconomic indicators often are, by using firm level data to understand clearly what are the driving factors behind their evolution. While the evolution of the aggregate unit labor cost does not reflect adequately the evolution of the firm level unit labor costs, and therefore does not capture the firm heterogeneity present in an economy, it highlights the importance of the reallocation of resources between firms in an economy. Our results suggest that an efficient reallocation of resources leads to productivity gains of at least 20% in all countries. Attending to the definition of [Porter \(1990\)](#), the competitiveness of a nation is the productivity with which a nation utilizes its



human, capital and natural resources. Therefore, our results indicate that the evolution of the ULC is driven by an important factor for the competitiveness of a country.

This paper contributes to the competitiveness literature by showing that the evolution of the aggregate unit labor costs is driven by the reallocation of resources in the economy, and by quantifying potential gains through an efficient reallocation of resources. Our paper relates to two strands in the literature. First, the literature that studies the effectiveness of aggregate macroeconomic indicators and their effectiveness to be used as policy indicators (Boone et al. (2007) and Felipe and Kumar (2011)). Boone et al. (2007) claim that the use of the price cost margin as a competitiveness measure may be potentially misleading since it tends to misrepresent the development of competition over time in markets with few firms and high concentration. And Felipe and Kumar (2011) analyze if the reduction of unit labor costs through a significant reduction in nominal wages is the best policy to exit the current crisis for some countries of the eurozone. Their analysis reveals that the aggregate unit labor costs reflects the distribution of income between wages and profits, and that the unit capital costs have also increased in the last decade. Therefore, a large reduction in nominal wages simply will not solve the problem. Second, our paper is related to the literature that studies the efficient allocation of resources. In particular, we follow the methodology of Hsieh and Klenow (2009) who use micro data on manufacturing establishments to quantify the potential extent of resource misallocation in China and India versus the United States.

The rest of the paper is organized as follows. In Section 3.2, we describe the firm level data used throughout the exercise. In Section 3.3, we discuss the traditional indicators of competitiveness and their limitations, particularly regarding their inability to account for the role of firms and their heterogeneity. In Section 3.4, we analyze if the aggregate evolution of the unit labor costs captures adequately the evolution of the same variable for the individual firms. In Section 3.5, we explain how much of the differences in productivity and output due to an inefficient allocation of resources. Section 3.6 concludes.

## 2 Data

We analyze balance sheet data from the AMADEUS dataset, managed by Bureau van Dijk, which has been integrated with the EFIGE survey, a representative sample<sup>1</sup> at the country level for the manufacturing industry of several European economies.

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<sup>1</sup> Altomonte and Aquilante (2012) provide more information on the construction of the dataset and a comprehensive set of validation measures.

The analysis is centered on France, Germany, Italy and Spain.<sup>2</sup> While for the analysis of the ULC only the cost of employees and the turnover of the firm are needed, the study of the impact of an efficient reallocation of resources requires data both from the balance sheet and the survey which we specify in detail later.

For each surveyed firm, nine years of usable balance sheet information has been retrieved, from 2001 to 2009. France, Italy and Spain are the countries with best quality in the balance sheet data, with a coverage<sup>3</sup> of 88.6%, 86.86% and 90.56% respectively. For Germany, the coverage is irregular. For the period of 2004-2008, there is a fairly good coverage of 70% to 80% of the firms, however for the years 2001-2003 and 2009 it drops to levels between 30-45% on average.

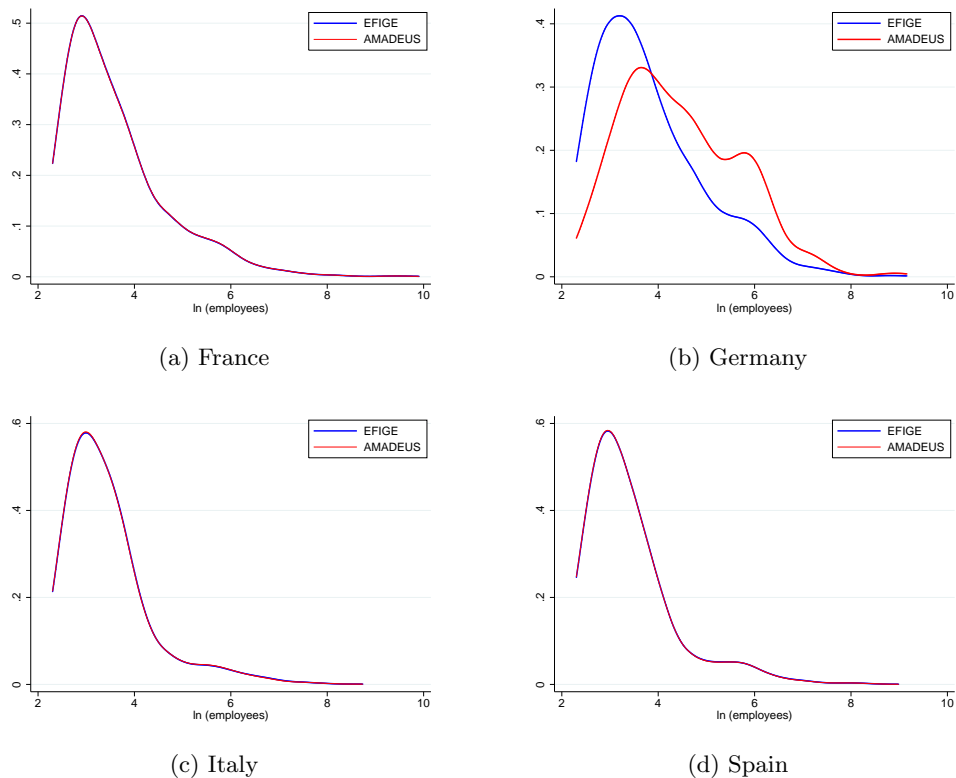


Figure 1: Distribution of Plant Size

<sup>2</sup>In the EFIGE dataset there is also information about three more European countries: Austria, Hungary and United Kingdom. Due to the poor quality of the balanced data for these countries, they have not been included in the analysis.

<sup>3</sup>The reference variable for the coverage is the turnover of the firm.

In [Figure 1](#), we present the distribution of firms by employment size for all the surveyed firms in EFIGE and the sample covered by the AMADEUS database. For all the countries with the exception of Germany, the firm size distribution of the subsection of firms present in AMADEUS matches almost perfectly the firm size distribution of the surveyed firms in EFIGE. Within the subsection of firms present in the AMADEUS dataset for Germany, the number of small firms is slightly under-represented while the number of medium firms is slightly over-represented with respect to the distribution of all the surveyed firms in EFIGE. Hence, we should be cautious in the interpretation of results for Germany and make sure is that they are not biased by this fact.

### 3 Limitations of the Traditional Competitiveness Indicators

[Porter \(1990\)](#) defines the competitiveness of a nation as the productivity with which a nation utilizes its human, capital and natural resources. The OECD considers the ability of a country to sell its products in the international markets while [Krugman \(1994\)](#) refers to competitiveness as a poetic way of speaking about productivity, and warns about the danger of obsessing about the competitiveness of a country. Most of these definitions of competitiveness allude to the relative position of a country in international trade. This position, in principle, depends on price and cost factors because if they have a negative evolution in relation with those from others economies, the ability to sell products at home and abroad is damaged. This argument, combined with the easy availability of data, makes price-cost competitiveness indicators especially attractive for the analysis of a country's economic situation. This is why the classical macroeconomic textbooks relate the competitiveness of nations to the comparison of their relative prices.

Currently the price-cost indicator of reference to measure competitiveness in the European Union is the unit labor cost (ULC), which measures the labor cost by unit of product and is calculated as the ratio of total labor costs to real output.<sup>4</sup> A rise in an economy's ULC represents an increased reward for labor's contribution to output. However, a rise in labor costs higher than the rise in labor productivity may be a threat to an economy's cost competitiveness, if other costs are not adjusted in compensation.

A simple comparison of the evolution of prices and costs between two countries may

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<sup>4</sup>An assumption implicit in the use of cost based indicators is that in the short run the capital is fixed, and therefore the cost of capital should not differ between similar countries. This assumption can be a limitation of the cost-competitiveness measures, see [Felipe and Kumar \(2011\)](#) for further details.

not be informative enough to determine the competitiveness of a country, and therefore, the ULC may be a measure of competitiveness with a very limited prediction power. If an increase in the ULC index indicates a loss in competitiveness of the country, then we should see a decrease in a country's export shares whenever aggregate ULC goes up. Figure 2 shows the so called *Spanish competitiveness paradox*, an example that a loss in competitiveness does not imply necessarily a loss in the world's export shares. Figure 2a shows the evolution of the ULC for Spain and the main developed economies, while in Figure 2b shows the evolution of these countries worlds' export share during the 2000's. The Spanish ULC has grown faster than in the main developed countries, but on the other hand, its export shares have decreased less than those of other countries, the only exception being Germany.

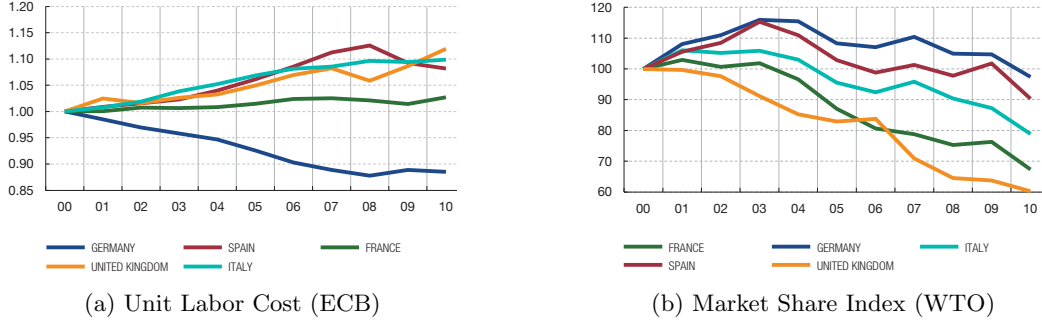


Figure 2: Competitiveness Indicators Vis-à-Vis the Euro Area

Antràs et al. (2010) show that large Spanish firms experienced both lower ULC growth and higher export growth than other countries, yet this differential is not reflected in aggregate price indicators due to aggregation and dispersion bias (Altomonte et al. (2012)). In the calculation of the ULC all the firms are taken into account while to calculate the economy's total exports, only the exporters are taken into account. Firms that export are usually the largest and most productive of the economy (Clerides et al. (1998) and Bernard and Bradford Jensen (1999)). The different relative weight in the aggregate ULC and in the economy's total export, helps therefore to explain the *Spanish paradox*.

An adequate competitiveness measure should be able to capture the role of firms and their heterogeneity. Several questions arise then. First, why is heterogeneity so important? Second, why should a competitiveness measure take into account the heterogeneity within the firms of an economy? And third, how adequately do traditional competitiveness

measures capture the heterogeneity?

To understand the importance of the heterogeneity between firms, the concept of productivity is essential since it allows high wages and high capital returns in an economy (See Porter (2005)). Recent literature in industrial organization and international trade has provided abundant empirical evidence supporting the idea that the evolution of macroeconomic aggregates is determined closely by the decisions and characteristics of the firms in the economy, and in particular by the behavior and productivity of a subgroup of them: the most productive ones. This is evident in the case of exporting firms. Exporter firms from a sector or a country are a minority and, in general, they are those that behave better in terms of productivity, size and innovation. The higher performance is present before these firms become exporters (see Clerides et al. (1998) and Bernard et al. (2011)).

Table 1 and Table 2 illustrate why a competitiveness measure should take into account this heterogeneity. Table 1 shows the export probability (extensive margin) of a firm in relation to its size for each of the countries in the database of EFIGE, while Table 2 reports the percentage of production that each firm exports (intensive margin). It is observed that for two similar sized firms from different countries, the probability of exporting and the export proportion are roughly similar. For example, among firms with 50 to 249 employees in France and Spain, the probability of exporting is 75.4% and 76.2% respectively, less than a 1 percentage point difference. Furthermore, the difference in the export intensity of these firms is only 0.3 percentage points.

Employees	Austria	France	Germany	Hungary	Italy	Spain	UK
10 – 19	69.8	44.7	45.7	58.0	65.4	51.2	54.9
20 – 49	63.8	59.1	65.4	64.7	73.3	63.5	62.8
50 – 249	88.6	75.4	78.2	79.3	86.6	76.2	76.8
Over 249	90.8	87.6	84.0	97.4	92.6	88.0	90.7
Aggregate	72.6	57.9	63.4	67.3	72.2	61.1	61.0

Table 1: Extensive margin of exports (%), by country and company size.

Employees	Austria	France	Germany	Hungary	Italy	Spain	UK
10 – 19	26.2	23.0	25.9	30.2	30.4	21.4	26.2
20 – 49	33.3	27.0	28.1	43.6	34.2	24.5	27.8
50 – 249	55.9	33.0	33.9	53.2	42.2	33.3	33.2
Over 249	64.7	41.2	37.8	66.6	52.6	40.6	34.2
Aggregate	40.4	28.5	30.0	44.8	34.6	25.9	29.1

Table 2: Intensive margin of exports (%), by country and company size.

In the aggregate, the differences between France and Spain in the export probability and the export intensity are higher. These differences in the exports aggregated by size, sector or country do not come from differences in two similar firms from different countries, they are due to differences in the allocation of resources between the sectors of the economy and differences in the firm size distribution within sectors.

Barba-Navaretti et al. (2011) estimate that if Spain had the industrial structure and firm size distribution of Germany, the exports of Spain would increase 25%. The differences in the aggregates were due to differences in the allocation of resources between the sectors of the economy and differences in the firm size distribution of the firms within sectors. That is, within a sector there can be as much firm heterogeneity as there can be between firms in different sectors.

To address how adequately traditional competitiveness measures capture firm heterogeneity, in the next section we study whether the firm level ULC evolution drives the aggregate ULC evolution or whether it is driven by other factors.

## 4 ULC Decomposition

In this section we analyze how adequately the evolution of the Unit Labor Cost captures the firm heterogeneity present in a country. We decompose the evolution of the ULCs of four European countries given the firm level information in EFIGE. The exercise analyzes if the aggregate evolution of the ULC between years 2002 and 2007 captures adequately the evolution of the same variable for the individual firms.<sup>5</sup>

<sup>5</sup>Unfortunately, the coverage of Amadeus for Germany does not let us use the whole sample from 2001 to 2009.

For that purpose, we calculate at firm level a weighted change of the ULC as:

$$ULC_{t+1} - ULC_t = \sum_{i \in I_{t+1}} ms_{i,t+1} ulc_{i,t+1} - \sum_{i \in I_t} ms_{i,t} ulc_{i,t}$$

where  $ulc_{i,t}$  is the ULC of a given firm  $i$  at time  $t$  and  $ms_{i,t}$  is its market share at that time. The components of the weighted average are decomposed as follows, according to a Laspeyres decomposition.<sup>6</sup>

$$\begin{aligned} ULC_{t+1} - ULC_t &= \sum_{i \in I_{t+1}} ms_{i,t+1} ulc_{i,t+1} - \sum_{i \in I_t} ms_{i,t} ulc_{i,t} \\ &= \underbrace{\sum_{i \in I} ms_{i,t} (ulc_{i,t+1} - ulc_{i,t})}_{\text{Within}} + \underbrace{\sum_{i \in I} ulc_{i,t} (ms_{i,t+1} - ms_{i,t})}_{\text{Reallocation}} \\ &+ \underbrace{\sum_{i \in I} (ms_{i,t+1} - ms_{i,t}) (ulc_{i,t+1} - ulc_{i,t})}_{\text{Interaction}} \\ &+ \underbrace{\sum_{i \in I_{t+1} \setminus I} ms_{t+1} ulc_{t+1} - \sum_{i \in I_t \setminus I} ms_{i,t} ulc_{i,t}}_{\text{Entry-Exit}} \end{aligned}$$

The first element, the *within* component, is the change attributable to the evolution of the firms' ULC given their market share: a positive sign would imply a relevant loss in competitiveness at the firm level. The second element, the *reallocation* component, accounts for the redistribution of market shares among the firms, holding the ULC constant: a negative sign implies a reallocation of market shares towards firms with initial lower ULC. The third element, the *interaction* component, gives information about the underlying dynamics: a negative sign would show that ULCs and market shares are moving in different directions, either because their activity is expanding thanks to a reduction in ULC or because the importance of their sector is decreasing after an increase in the ULC. The fourth element, the *entry and exit* component is indicative of the market dynamics that follow the removal of barriers fostering entry, and the exogenous shocks that can oblige some firms to exit. As we already discussed in Section 3.2, the EFIGE survey is not designed to keep track of entry and exit of firms, therefore this element is simply a residual

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<sup>6</sup>Note that the latter decomposition is also discussed by Boone et al. (2007), as the starting point of the indicator of competition, and by (Altomonte et al., 2010).

of the calculation, and will be ignored in the discussion.

If the aggregate ULC was a measure that captured adequately the heterogeneity existent at the firm level ULC, its evolution should be driven by the evolution of the firm level ULC. Then we should observe the *within* component to be the most relevant in the explanation of the aggregate ULC evolution.

Table 3 shows the result of the decomposition of the change in aggregate ULC in manufacturing between years 2002 and 2007 annualized. First, on average, for the period considered, the real ULCs have decreased in all countries indicating an improvement in the cost competitiveness of the countries — which is supported as well by results using the EU-KLEMS database. Second, the weight of the change in competitiveness within firms is small, particularly in Italy and Spain, where it is 0.17% and  $-0.21\%$  respectively. Third, the interaction effect has the desired sign, negative. Unfortunately we can not infer if it is due to the activity of firms expanding thanks to a reduction in ULC or because the importance of their sector is decreasing after an increase in the ULC. Fourth, the reallocation of resources is the component that explains most of the evolution of the ULC for all the countries in the sample. The relative intensity differs between countries: the largest reallocation of resources occurs in Germany, followed by France, then Italy and Spain. Not only is the the reallocation of resources in France and Germany larger, but it is also the most important factor in the explanation of the evolution of the aggregate ULC. In Italy and Spain, the interaction effect has a similar weight as the reallocation of resources effect in the explanation of the evolution of the aggregate ULC.

	Total	Within	Reallocation	Interaction	Entry-Exit
France	-2.62	-1.19	-1.87	-0.61	1.06
Germany	-3.25	-1.55	-2.69	-0.43	1.42
Italy	-1.38	0.17	-1.35	-1.42	1.22
Spain	-2.06	-0.21	-1.19	-1.27	0.61

Table 3: Changes in the ULCs of each country (annualized rate), 2002-2007

Table 4 shows the relative accumulated evolution of the ULC of each country with respect to the evolution of Germany for the period 2002 to 2007. A positive number indicates the possible gain associated with each effect if these countries had had the evolution



of Germany. The change in competitiveness within firms was particularly small in Italy and Spain, which implies losses of competitiveness with respect to Germany of 8.75% in Italy and 7% in Spain. More importantly, the smaller reallocation of resources with respect to Germany between 2002 and 2007 implies losses of competitiveness around 4.3% in France, 6.4% in Italy and 8% in Spain.

	Total	Within	Reallocation	Interaction
France	5.22	1.86	4.27	-0.91
Italy	10.37	8.75	6.39	-4.77
Spain	10.82	7.00	7.95	-4.14

Table 4: Changes in the ULCs of each country relative to Germany, 2002-2007

Even though the exercise has limitations since we are only looking at manufacturing firms, recent empirical research with sectoral data shows that the reallocation of resources within the sector is key to understand the evolution of aggregate ULC. Given the importance of the reallocation of resources to explain the evolution of the ULC, in the next section we focus on understanding what would be the productivity gains in each of these countries if there were no misallocation, that is, if all the resources were allocated efficiently.

## 5 Resources' Misallocation: Source of Country Differences in Productivity

The ability to reallocate resources within the firms of the economy has a very significant role in the explanation of the evolution of the aggregate ULC. In this section we apply the methodology of [Hsieh and Klenow \(2009\)](#) to explain the impact of an efficient allocation of resources in the productivity and output of France, Germany, Italy and Spain.

### 5.1 [Hsieh and Klenow \(2009\)](#) Methodology

[Hsieh and Klenow \(2009\)](#) propose an empirical framework to investigate if large differences in output per worker across countries (or sectors) are due to the fact that there is “misallocation” across plants, firms and sectors. The empirical framework proposed, while based on specific parametric assumptions on preferences and production technology, enables a

clean representation of the potential impact of “misallocation” on sectoral or aggregate productivity.

Consider an economy consisting of  $S$  sectors and aggregate output is defined as:

$$Y = \prod_{s=1}^S Y_s^{\theta_s} \text{ where } \sum_{s=1}^S \theta_s = 1. \quad (1)$$

Let  $P = \prod_{s=1}^S \left(\frac{P_s}{\theta_s}\right)^{\theta_s}$  represent the price of the final good, where  $P_s$  refers to the price of industry output  $Y_s$ . Then, cost minimization implies

$$P_s Y_s = \theta_s P Y. \quad (2)$$

Industry output  $Y_s$  is itself a C.E.S. aggregate of  $M_s$  differentiated products:

$$Y_s = \left( \sum_{i=1}^{M_s} Y_{si}^{\frac{\sigma-1}{\sigma}} \right)^{\frac{\sigma}{\sigma-1}},$$

and each firm in sector  $s$  has a Cobb-Douglas production function that depends on firm TFP, capital and labor<sup>7</sup>:

$$Y_{si} = A_{si} K_{si}^{\alpha_s} L_{si}^{1-\alpha_s}.$$

Hsieh and Klenow (2009) assume that there are firm specific distortions affecting total production and capital which are modelled as taxes. They denote distortions that increase the marginal products of capital and labor by the same proportion as an output distortion  $\tau_Y$ , and denote distortions that raise the marginal product of capital relative to labor as the capital distortion  $\tau_K$ . As a result, firms produce different amounts than what would be dictated by their productivity and also may have different capital-labor ratios.<sup>8</sup>

Combining the aggregate demand for capital and labor in a sector, the expression for the price of aggregate industry output and Equation 2, aggregate output can then be expressed as a function of  $K_s$ ,  $L_s$ , and industry TFP:

$$Y = \prod_{s=1}^S (TFP_s \cdot K_s^{\alpha_s} \cdot L_s^{1-\alpha_s})^{\theta_s}. \quad (3)$$

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<sup>7</sup>Note that capital and labor shares may differ across industries but not across firms within an industry.

<sup>8</sup>See the Appendix for a full derivation of the firm’s maximization problem.

To determine the formula for industry productivity  $TFP_s$  it has to be noted that when industry deflators are used, differences in plant specific prices show up in the customary measure of plant TFP. Foster et al. (2008) stress the distinction between “physical productivity” (TFPQ) and “revenue productivity” (TFPR).

$$TFPQ_{si} \triangleq A_{si} = \frac{Y_{si}}{K_{si}^{\alpha_s} L_{si}^{1-\alpha_s}},$$

$$TFPR_{si} \triangleq P_{si} A_{si} = \frac{P_{si} Y_{si}}{K_{si}^{\alpha_s} L_{si}^{1-\alpha_s}}.$$

If there are no firm specific distortions and all firms within a sector have the same markup (assumed by this framework but obviously not true in general), TFPR will be equalized across firms. In the absence of distortions, more labor and capital should be allocated to plants with higher TFPQ to the point where their higher output results in a lower price and the exact same TFPR as smaller plants. TFPR is proportional to a geometric average of the plant’s marginal revenue products of labor and capital:

$$TFPR_{si} \propto \frac{(1 + \tau_{K_{si}})^{\alpha_s}}{1 - \tau_{Y_{si}}}. \quad (4)$$

High plant TFPR is a sign that the plant faces barriers that raise the plant’s marginal products of labor and capital, rendering the plant smaller than optimal. In general, variation of TFPR within a sector will be a measure of misallocation.

Then, the relevant measure of sectoral TFP can be written as:<sup>9</sup>

$$TFP_s = \left( \sum_{i=1}^{M_s} \left( TFPQ_{si} \cdot \frac{\overline{TFPR}_s}{TFPR_{si}} \right)^{\sigma-1} \right)^{\frac{1}{\sigma-1}}, \quad (5)$$

where  $\overline{TFPR}_s$  is the geometric average of the average marginal revenue product of capital and labor in sector  $s$ . Intuitively, the extent of misallocation is worse when there is greater dispersion of marginal products.

To see this more clearly, consider a special case where  $TFPQ_{si}$  and  $TFPR_{si}$  are jointly

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<sup>9</sup>See the Appendix for the full derivation of Equation 5.

lognormally distributed, then the expression in Equation 5 implies:

$$\log TFP_s = \frac{1}{\sigma - 1} \log \left( \sum_{i=1}^{M_s} A_{si}^{\sigma-1} \right) - \frac{\sigma}{2} \text{var}(\log TFP_{R_{si}}),$$

so that the negative effect of distortions can be summarized by the variance of log TFPR.

## 5.2 Gains of an Efficient Allocation of Resources in Europe

In order to determine the gains from an efficient allocation of resources, we calculate “efficient” output in each country so we can compare it with actual output levels. If there are no firm specific distortions, TFPR will be equalized across firms within a sector. Then, industry TFP would be  $\overline{A}_s = \left( \sum_{i=1}^{M_s} A_{si}^{\sigma-1} \right)^{\frac{1}{\sigma-1}}$ . For each industry, we calculate the ratio of actual TFP (Equation 5) to this efficient level of TFP, and then aggregate this ratio across sectors using the Cobb-Douglas aggregator (Equation 1):

$$\frac{Y}{Y_{efficient}} = \prod_{s=1}^S \left[ \sum_{i=1}^{M_s} \left( \frac{A_{si} \overline{TFPR}_s}{\overline{A}_s \overline{TFPR}_{si}} \right)^{\sigma-1} \right]^{\frac{\theta_s}{\sigma-1}} \quad (6)$$

To calculate the effects of resource misallocation, we need to estimate key parameters (industry output shares, industry capital shares, and the firm-specific distortions) from the data.

The data for France, Germany, Italy and Spain are drawn from the joint EFIGE-Amadeus dataset. We use are the plant’s industry (four-digit level), age (based on reported birth year), wage payments, value-added, export revenues, and capital stock. For labor input we use the plant’s wage bill<sup>10</sup> rather than its employment to measure  $L_{si}$ . As a later robustness check, we measure  $L_{si}$  as employment. We define capital stock as the book value of fixed capital net of depreciation.

We set the rental price of capital (excluding distortions) to  $R = 0.10$ , we have in mind a 5% real interest rate and a 5% depreciation rate.<sup>11</sup> We set the elasticity of substitution between plant value added to  $\sigma = 3$ , which ranges within the estimates of the substitutabil-

<sup>10</sup>The Amadeus data only report wage payments; the information on non-wage compensation is not reported.

<sup>11</sup>The actual cost of capital faced by plant  $i$  in industry  $s$  is denoted  $(1 + \tau_{K_{si}})R$ , so it differs from 10% if  $\tau_{K_{si}} \geq 0$ . Because our hypothetical reforms collapse  $\tau_{K_{si}}$  to its average in each industry, if  $R$  is set incorrectly, it will affect the average capital distortion but not the experiment itself.

ity of competing manufactures in the trade and industrial organization literature (Broda and Weinstein (2006)). Later, we entertain the higher value of 5 and a lower value of 2 for  $\sigma$  as a robustness check. We set the elasticity of output with respect to capital in each industry ( $\alpha_s$ ) to be 1 minus the labor share in the corresponding industry in Germany in 2008. We adopt the German shares as the benchmark.

On the basis of the other parameters and the plant data, we infer the distortions and productivity for each plant in each country-year as follows:

$$1 + \tau_{K_{si}} = \frac{\alpha_s}{1 - \alpha_s} \frac{wL_{si}}{RK_{si}} \quad (7)$$

$$1 - \tau_{Y_{si}} = \frac{\sigma}{\sigma - 1} \frac{wL_{si}}{(1 - \alpha_s)P_{si}Y_{si}} \quad (8)$$

$$A_{si} = \frac{(P_{si}Y_{si})^{\frac{\sigma}{\sigma-1}}}{K_{si}^{\alpha_s} L_{si}^{1-\alpha_s}} \quad (9)$$

Before calculating the gains from our hypothetical liberalization, we trim the 1% tails of  $\log(TFPR_{si}/\overline{TFPR_s})$  and  $\log(A_{si}/\overline{A_s})$  across industries to make the results robust to outliers. We then recalculate  $wL_s$ ,  $K_s$ ,  $P_s Y_s$ ,  $\overline{TFPR_s}$  and  $\overline{A_s}$ .

Table 5 provides percent TFP gains in each country from fully equalizing TFPR across plants in each industry for the years 2002 to 2008, where the entries are  $100(Y_{efficient}/Y - 1)$ . As we discussed in Section 3.2, a major shortcoming of the unification of the EFIGE and AMADEUS dataset is that the coverage of Amadeus for the firms surveyed is not 100%. In this exercise, for the years 2002 to 2008, for France, Italy and Spain there is a coverage of 80% to 90% of the firms, whereas for Germany it is considerably lower. Particularly, for the years 2002 and 2003 there is information for less than 50% of the firms, and for the years 2004 to 2008 it ranges between 50% and 70%. Hence, in Table 5 we do not report hypothetical gains from an efficient allocation of resources for Germany for the years 2002 and 2003, and the variation in these gains is calculated for the years 2008-2004 instead of 2008-2002.

Removing all barriers, by this calculation, would boost aggregate manufacturing TFP in 2008 by 22.7% in France, 27.9% in Germany, 43.5% in Italy and 28.2% in Spain. More interestingly, we observe that between the years 2002 to 2008, the gains from efficient allocation decrease in Germany (-8.50%), increase in Italy and Spain (6.93% and 6.97%), and are constant in France (-0.82%). This reveals that within this period, in Italy and Spain the “misallocation” of resources within the sector has increased while in France

it remains constant and in Germany it decreases. An increase in the “misallocation” of resources in Italy and Spain, reveals an increase in the distortions or barriers to production present in these countries which is consistent with their smaller ability to reallocate market shares towards firms with initially smaller ULC as reported in Table 3. At the same time, the decrease in the “misallocation” of resources in Germany is also reflected by the greater ability of reallocating market shares to firms with an initially lower ULC. The results of the decomposition in the evolution of ULC and an hypothetical efficient allocation of resources are complementary to each other.

Year	France	Germany	Italy	Spain
2002	23.55		36.41	21.23
2003	19.29		30.46	21.68
2004	22.07	36.41	32.75	23.30
2005	22.43	31.90	30.46	24.66
2006	23.88	32.30	32.97	24.70
2007	20.95	33.25	34.54	28.71
2008	22.74	27.92	43.34	28.20
$\Delta_{2008-2002}$	-0.82	-8.50	6.93	6.97

Table 5: TFP Gains from Equalizing TFPR within Industries

Figure 3 plots the “efficient” versus actual size distribution of plants in year 2008, where size is measured as plant value added. In all the countries except Germany, the hypothetical efficient distribution is more dispersed than the actual one. In particular, in all countries, there should be fewer mid-sized plants and more small and large plants. The popular belief is that there are less large firms than there should be due to distortions in the economy, but not that there are less small firms than there should be like the flattening of these distributions is predicting. Hsieh and Klenow (2009) find similar predictions for their analysis of China, India and the United States, which suggest that the shape of the efficient plant size distribution is robust across countries. In Germany, the efficient distribution is more dispersed as well, but we observe a shift to the right in the distribution rather than a flattening as it happens in the other countries. The reason behind the different behavior in Germany lies probably in the bias in the size distribution of the German firms present

in the AMADEUS dataset as we have explained in Section 3.2. The small firms in terms of employment are very under-represented in the subsection of German firms present in the AMADEUS side of the data (see Figure 1), hence the explanation to why there is no flattening in the efficient distribution and the exercise predicts that a large group of the medium sized firms in terms of output should decrease their size.

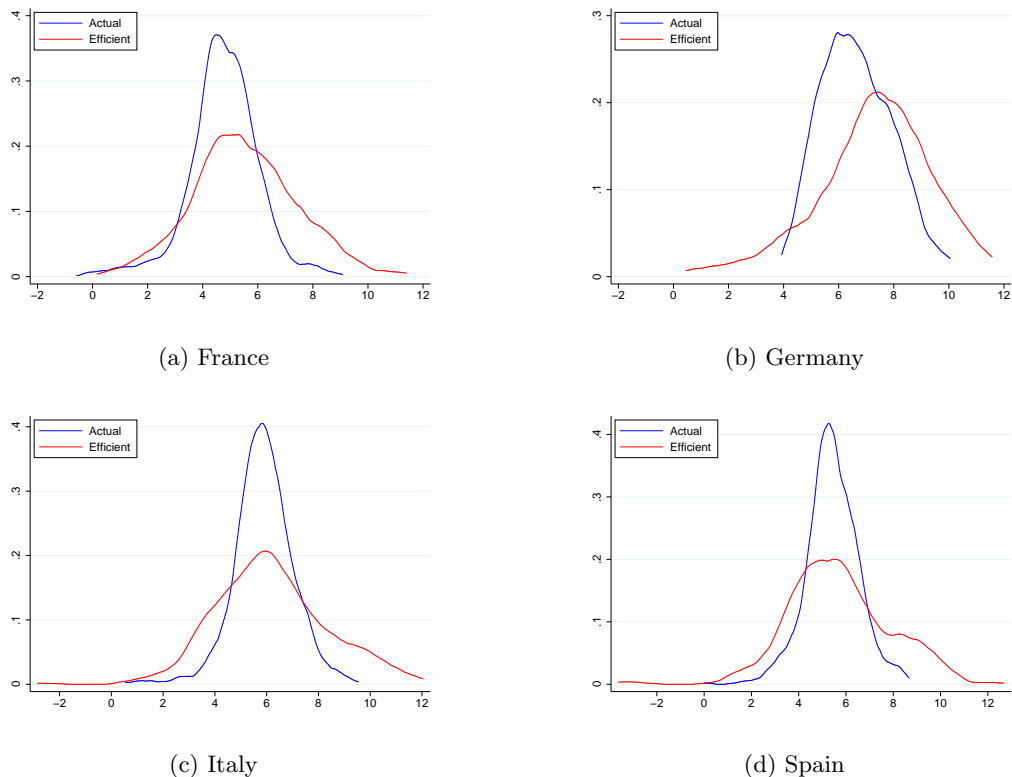


Figure 3: Distribution of Plant Size

Table 6 shows how the size of initially big vs. small plants would change if TFPR were equalized in each country. The entries are unweighted shares of plants. The rows are actual plant size quartiles, and the columns are bins of efficient plant size relative to actual size: 0% – 50% — the plant should shrink by a half or more, 50% – 100% — the plant should shrink by less than half, 100% – 200% — the plant should increase but not double in size, > 200% — the plant should at least double in size.

		[0% – 50%]	[50% – 100%]	[100% – 200%]	> 200%
France	1st quartile	3.84	2.25	8.70	10.29
	2nd quartile	11.97	0.47	0.47	12.07
	3rd quartile	8.04	14.87	1.50	0.56
	Top quartile	1.22	7.39	14.31	2.06
Germany	1st quartile	1.75	2.62	10.92	10.04
	2nd quartile	10.48	2.62	0.0	12.23
	3rd quartile	10.48	14.41	0.0	0.0
	Top quartile	2.18	5.68	14.41	2.62
Italy	1st quartile	2.44	0.57	5.61	16.41
	2nd quartile	14.13	3.49	0.16	7.23
	3rd quartile	7.31	13.81	3.57	0.32
	Top quartile	1.14	7.15	15.68	0.97
Spain	1st quartile	2.91	0.97	9.06	12.08
	2nd quartile	12.84	0.65	0.76	10.79
	3rd quartile	8.20	16.07	0.54	0.22
	Top quartile	1.08	7.34	14.67	1.83

Table 6: Actual size vs. Efficient size (Percent of Plants)

In all countries, firms with initial smaller size should increase. Particularly for Italy and Spain, not only is there a large number of firms that should increase their size but also that should at least double in size. In all countries, firms with initial size in the 2nd quartile should either shrink by half or at least double in size. This indicates that there is a large number of small medium sized firms that should not be there. In all countries, firms with initial size in the 3rd quartile should shrink. This is particularly relevant for Germany. Finally, firms with initial size in the top quartile should not shrink as much and actually should increase their size, but not double it. That is, large firms should be larger in all countries, whereas medium productivity firms should shrink and there are some small firms that should increase their size given their real productivities.



### 5.3 Robustness Tests

We now provide a number of robustness checks to our baseline [Table 5](#) calculations of hypothetical efficiency gains. We have measured plant labor input using its wage bill. The logic is that wages per worker adjust for plant differences in hours worked per worker and worker skills. However, wages could also reflect rent sharing between the plant and its workers. If so, we might be interpreting differences in TFPR across plants because the most profitable plants have to pay higher wages. We therefore recalculate the gains from equalizing TFPR in France, Germany, Italy and Spain using simply employment as our measure of plant labor input. The gains from an efficient allocation remain almost unchanged for all countries with the exception of Germany — 21.18% for France, 35.44% for Germany, 42.56% for Italy and 27.58% for Spain in 2008. The intuition behind the smaller gains for Germany when we use the wage bill rather than the employees is that wage differences may be limiting the TFPR differences.

We have assumed an elasticity of substitution within industries ( $\sigma$ ) of 3. However the literature on business cycles puts it at 2 while the literature more close to international trade puts it at 5. Our estimates are sensitive to this parameter, with an increase between 10% and 20% in the gains from efficient allocation if  $\sigma = 5$ , and a decrease of 5% to 10% if  $\sigma = 2$ . The intuition behind these results, is that when the elasticity of substitution within industries is larger, then TFPR gaps are closed more slowly in response to reallocation of inputs from low to high TFPR plants, enabling bigger gains from equalizing TFPR gains.

Given the dispersion in the size of the firms within the sectors and between countries,<sup>12</sup> a last valid concern might be that the trimming of the productivity measures is large. Firms with extreme productivity values have a high relative weight (following a trend more similar to a Pareto distribution than a Normal distribution), which means that the behavior of the sector aggregates are strongly influenced by the behavior of the largest firms ([di Giovanni and Levchenko \(2009\)](#), [Altomonte et al. \(2010\)](#) and [Altomonte et al. \(2011\)](#)). Hence, less trimming (or no trimming at all) in the right tail of the distribution, implies a higher dispersion in the data observed, and we expect larger gains from an hypothetical efficient allocation of resources. To analyze the robustness of the calculations to the dispersion in firm size, we trim only 0.5% of the right tail of  $\log(TFPR_{si}/TFPR_s)$  before calculating the hypothetical gains. While the results prove to be sensitive to this trimming, and as

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<sup>12</sup>In Italy and Spain there is a smaller number of large firms than in Germany and France. See [Crespo \(2012\)](#) and [Rubini et al. \(2012\)](#).

expected there is an increase in the gains from an efficient allocation, this increase is similar across countries (around 5%) — 26.86% in France, 33.97% in Germany, 49.33% in Italy and 35.46% in Spain. Between 2002 and 2008, the predicted gains from an efficient allocation decrease in 3.64% in France, decrease in 9.20% in Germany, increase in 9.07% in Italy and increase in 10.56%. While the variations are slightly larger, the ranking is unchanged and therefore the conclusions of our exercise are consistent.

## 6 Conclusions

In this paper, we have analyzed the ability of the change in the aggregate unit labor cost to capture the change in the competitiveness of a country.

Using firm level data, we calculate a weighted change of the aggregate unit labor costs between 2002 and 2007 for four European countries: France, Germany, Italy and Spain. The components of the weighted average are then decomposed according to a Laspeyres decomposition into three main elements: the first captures changes in firm level unit labor costs, keeping the initial domestic market shares of firms constant; the second quantifies the reallocation of market shares within the domestic economy, keeping the initial unit labor costs constant; and the third measures the interaction between the first two. The results reveal that the evolution of the firm level unit labor cost does not explain the evolution of the aggregate unit labor costs, rather it is the resource reallocation that drives the evolution of the aggregate unit labor costs.

Motivated by the significant role of the reallocation of resources to explain the evolution of the aggregate ULC, we apply the methodology of [Hsieh and Klenow \(2009\)](#) to analyze the extent to which aggregate productivity differences between these four European countries relate to inefficient resource reallocation. As a result of distortions that affect production, firms produce different amounts than what would be dictated by their productivity. An efficient allocation of resources would boost aggregate manufacturing TFP in 2008 by 22.7% in France, 27.9% in Germany, 43.5% in Italy and 28.2% in Spain.

The empirical analysis of the unit labor costs as a competitiveness measure reveals the need to use microeconomic data to understand the driving factors behind the evolution of macroeconomic aggregates. And the decomposition of the aggregate indicator shows that there are relevant differences among countries which in the aggregate cannot be observed due to the noisiness of the measure.

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## Appendix

The production function for each differentiated product is given by a Cobb Douglas function of firm TFP, capital and labor:

$$Y_{si} = A_{si} K_{si}^{\alpha_s} L_{si}^{1-\alpha_s}.$$

Since there are distortions affecting the production of firms, the profits of a firm are given by:

$$\pi_{si} = (1 - \tau_{Y_{si}}) P_{si} Y_{si} - w L_{si} - (1 + \tau_{K_{si}}) R K_{si}.$$

Profit maximization yields the standard optimal price and capital-labor ratio:

$$\begin{aligned} P_{si} &= \frac{\sigma}{\sigma - 1} \left( \frac{R}{\alpha_s} \right)^{\alpha_s} \left( \frac{w}{1 - \alpha_s} \right)^{1-\alpha_s} \frac{(1 + \tau_{K_{si}})^{\alpha_s}}{A_{si} (1 - \tau_{Y_{si}})}, \\ \frac{K_{si}}{L_{si}} &= \frac{\alpha_s}{1 - \alpha_s} \cdot \frac{w}{R(1 + \tau_{K_{si}})}. \end{aligned}$$

The marginal revenue product of labor is proportional to revenue per worker:

$$MRPL_{si} = P_{si} \frac{\partial Y_{si}}{\partial L_{si}} = (1 - \alpha_s) \frac{P_{si} Y_{si}}{L_{si}} = \left( \frac{\sigma}{\sigma - 1} \right) \frac{w}{1 - \tau_{Y_{si}}}.$$

The marginal revenue product of capital is proportional to the revenue-capital ratio:

$$MRPK_{si} = P_{si} \frac{\partial Y_{si}}{\partial K_{si}} = \alpha_s \frac{P_{si} Y_{si}}{K_{si}} = \left( \frac{\sigma}{\sigma - 1} \right) \frac{R(1 + \tau_{K_{si}})}{1 - \tau_{Y_{si}}}.$$

To derive  $K_s$  and  $L_s$ , first we derive the aggregate demand for capital and labor in a sector by aggregating the firm level demands for the two factor inputs. Then, we combine the aggregate demand for the factor inputs in each sector with the allocation of total expenditure across sectors.

$$\begin{aligned} L_s &\equiv \sum_{i=1}^{M_s} L_{si} = L \frac{(1 - \alpha_s) \theta_s / \overline{MRPL_s}}{\sum_{s'=1}^S (1 - \alpha_{s'}) \theta_{s'} / \overline{MRPL_{s'}}}, \\ K_s &\equiv \sum_{i=1}^{M_s} K_{si} = K \frac{\alpha_s \theta_s / \overline{MRPK_s}}{\sum_{s'=1}^S \alpha_{s'} \theta_{s'} / \overline{MRPK_{s'}}}, \end{aligned}$$

where

$$\begin{aligned}\overline{MRPL}_s &\triangleq \frac{w}{\sum_{i=1}^{M_s} (1 - \tau_{Y_{si}}) \frac{P_{si} Y_{si}}{P_s Y_s}} \\ \overline{MRPK}_s &\triangleq \frac{R}{\sum_{i=1}^{M_s} \frac{1 - \tau_{Y_{si}}}{1 + \tau_{K_{si}}} \frac{P_{si} Y_{si}}{P_s Y_s}}\end{aligned}$$

The  $TFPR_{si}$  is defined as follows:

$$\begin{aligned}TFPR_{si} &= \frac{\sigma}{\sigma - 1} \left( \frac{\overline{MRPK}_{si}}{\alpha_s} \right)^{\alpha_s} \left( \frac{\overline{MRPL}_{si}}{1 - \alpha_s} \right)^{1 - \alpha_s} \\ &= \frac{\sigma}{\sigma - 1} \left( \frac{R}{\alpha_s} \right)^{\alpha_s} \left( \frac{w}{1 - \alpha_s} \right)^{1 - \alpha_s} \frac{(1 + \tau_{K_{si}})^{\alpha_s}}{1 - \tau_{Y_{si}}}.\end{aligned}$$

Then,

$$\begin{aligned}\overline{TFPR}_s &\triangleq \frac{\sigma}{\sigma - 1} \left[ \frac{R}{\alpha_s \left( \sum_{i=1}^{M_s} \frac{1 - \tau_{Y_{si}}}{1 + \tau_{K_{si}}} \frac{P_{si} Y_{si}}{P_s Y_s} \right)} \right]^{\alpha_s} \left[ \frac{w}{\left( (1 - \alpha_s) \sum_{i=1}^{M_s} (1 - \tau_{Y_{si}}) \frac{P_{si} Y_{si}}{P_s Y_s} \right)} \right]^{1 - \alpha_s} \\ &= \frac{\sigma}{\sigma - 1} \left( \frac{\overline{MRPK}_s}{\alpha_s} \right)^{\alpha_s} \left( \frac{\overline{MRPL}_s}{1 - \alpha_s} \right)^{1 - \alpha_s}.\end{aligned}$$

Using these expressions, we can derive [Equation 5](#):

$$\begin{aligned}TFP_s &\triangleq \frac{Y_s}{K_s^{\alpha_s} L_s^{1 - \alpha_s}} = \frac{\left[ \sum_i^{M_s} (A_{si} K_{si}^{\alpha_s} L_{si}^{1 - \alpha_s})^{\frac{\sigma - 1}{\sigma}} \right]^{\frac{\sigma}{\sigma - 1}}}{\left( \sum_i^{M_s} L_{si} \right)^{1 - \alpha_s} \left( \sum_i^{M_s} K_{si} \right)^{\alpha_s}} \\ &= \frac{\left[ \sum_i^{M_s} \left( A_{si} \frac{1 - \tau_{Y_{si}}}{(1 + \tau_{K_{si}})^{\alpha_s}} \right)^{\sigma - 1} \right]^{\frac{1}{\sigma - 1}}}{\left( \sum_i^{M_s} \frac{1 - \tau_{Y_{si}}}{1 + \tau_{K_{si}}} \frac{P_{si} Y_{si}}{P_s Y_s} \right)^{\alpha_s} \left( \sum_i^{M_s} (1 - \tau_{Y_{si}}) \frac{P_{si} Y_{si}}{P_s Y_s} \right)^{1 - \alpha_s}} \\ &= \left[ \sum_i^{M_s} \left( A_{si} \frac{\overline{TFPR}_s}{\overline{TFPR}_{si}} \right)^{\sigma - 1} \right]^{\frac{1}{\sigma - 1}}.\end{aligned}$$