Uncovered Interest Rate Parity and
the Expectations Hypothesis of the Term Structure:
Empirical Results for the U.S. and Europe

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Uncovered Interest Rate Parity and the Expectations Hypothesis of the Term Structure: Empirical Results for the U.S. and Europe*

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Abstract

A system of U.S. and euro area short- and long-term interest rates is analyzed. According to the expectations hypothesis of the term structure the interest rate spreads should be stationary and according to the uncovered interest rate parity the difference between the U.S. and euro area long-term interest rates should also be stationary. If all four interest rates are integrated of order one, one would expect to find three linearly independent cointegration relations in the system of four interest rate series. Combining German and European Monetary Union data to obtain the euro area interest rate series we find indeed the theoretically expected three cointegration relations, in contrast to previous studies based on different data sets.

Keywords: Expectations hypothesis of the term structure, uncovered interest rate parity, unit roots, cointegration analysis

JEL classification: C32

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1 Introduction

According to the expectations hypothesis of the term structure (EHT) the nominal long-term interest rate ($R_t$) is a weighted average of expected short-term interest rates ($r_t$) plus a term premium. Assuming that the two interest rates are integrated of order one ($I(1)$) and the term premium is constant, Campbell & Shiller (1987) show that the EHT implies a stationary spread $R_t - r_t$, in short, $R_t - r_t \sim I(0)$. Furthermore, considering bonds in two different countries denominated in different currencies, the difference between their nominal interest rates should be determined by the expected relative change in the associated exchange rate, according to the uncovered interest rate parity hypothesis (UIP). Denoting the foreign long-term interest rate by $R_t^*$ and assuming that exchange rate changes are generated by a stationary process, UIP implies that $R_t - R_t^* \sim I(0)$ (Wolters (2003)). Thus, considering two countries with $I(1)$ short- and long-term nominal interest rates, there should be three linearly independent cointegration relations within four interest rate series, that is, the two spreads and the difference between the long-term interest rates of different countries should be stationary.

Wolters (2003) points out the importance of these relations for monetary policy and investigates whether the relations can be found for U.S. and euro area interest rates. Based on monthly data from 1994 to 2001 he finds no evidence of EHT and UIP to hold jointly and concludes that this result may be due to the insufficient sample size. Regarding the European interest rate series used one might add that they are artificially constructed by the European Central Bank (ECB) for a period where substantial convergence processes were going on in Europe because many countries were adjusting their economic systems in preparation for the euro. Therefore Wolters’ finding may indeed be a data problem.

In the following we will therefore use a different approach and combine German data for the period 1985 to 1998 with euro area data from 1999 to 2004. This approach was successfully used by Brüggemann & Lütkepohl (2004) in a related context. They argue that at the time when the Maastricht criteria were announced, Germany satisfied them almost and, hence, no major adjustments and related convergence processes were necessary for Germany. Thus, it makes sense to regard Germany as the anchor country to which the others converged. Such a view is also supported by the finding of Kirchgässner & Wolters (1993) and others that German monetary policy had a dominant position within Europe before the euro was introduced. Consequently, combining German interest rates before the European Monetary Union (EMU) started with euro area interest rates from 1999 onwards may reflect more properly the current situation in the EMU.

Admittedly, a number of studies have considered EHT and UIP jointly for Germany and the U.S. with quite mixed results and generally more evidence against than in favor of both hypotheses holding jointly (see Wolters (2003) for a brief summary of some of the related literature). Hence, one may not expect to find evidence for the two hypotheses to hold by combining German with EMU euro area data. However, although in our view strong arguments can be put forward for starting the sample in the middle of the 1980s, different sampling periods were considered by other authors. In the next
section we will provide more information on our data and sample period. In fact, in contrast to other studies, we find some evidence of both EHT and UIP to hold for the U.S. and the euro area.

The structure of the following study is as follows. In the next section we discuss the data used and in Section 3 the empirical study is presented. Conclusions are drawn in the final section.

2 The Data

We use monthly data for four interest rate series for the period 1985M1-2004M12. The short-term interest rates, denoted by $r^U_t$ and $r^E_t$ for the U.S. and Europe, respectively, are three-month money market rates and the long-term rates, denoted by $R^U_t$ and $R^E_t$ for the U.S. and Europe, respectively, are 10-year bond rates. The European interest rate series are German interest rates until the end of 1998 and the corresponding euro area rates afterwards. Details on the data used in the analysis and their sources are provided in Appendix A.

The year 1985 was chosen as the sample beginning because it was argued by other authors that the monetary transmission mechanism in Europe and in particular in Germany has changed in the first half of the 1980s due to increased monetary integration (see, e.g., Juselius (1998)). To avoid distortions in our results due to possible structural changes in the generation mechanism of the German interest rates we decided to use data from 1985 onwards only. The end of the sample was determined by the data availability at the time when the study was conducted. In total we have 20 years of monthly data and, hence, an overall sample size of $T = 240$.

The four interest rate series are plotted in Figure 1. Our empirical analysis will focus on the properties of the interest rate spreads $(R^U_t - R^E_t$ and $R^E_t - R^E_t)$ which are depicted in Figure 2 and the parities $(R^U_t - R^E_t$ and $r^U_t - r^E_t)$ shown in Figure 3. In the next section we will in particular analyze the orders of integration of these series.

3 Empirical Analysis

As we have argued in the introduction, if the four interest rates are all $I(1)$, EHT and UIP imply that the spreads as well as the bond rate differential $R^U_t - R^E_t$ are $I(0)$. In this case also the across-country difference between the short-term interest rates, $r^U_t - r^E_t$, must be stationary. Hence, we will start our empirical analysis by checking the orders of integration of the interest rate series and then continue by testing for unit roots in the spreads and parity series.

3.1 Unit Root Tests

Because the order of integration is crucial for our conclusions we present detailed results of augmented Dickey-Fuller (ADF) unit root tests for our series in Table 1. Because deterministic linear time trends are implausible in interest rate series we follow Wolters (2003) and include constants in the test regressions for the level interest rate series. Hence, no deterministic term appears in the tests.
For the first differences. Also, because the lag length is known to have an impact on the results of unit root tests, we have performed tests with different lag lengths. Note that in Table 1 the numbers of lagged differences suggested by different lag selection criteria and the corresponding values of the ADF statistics are given. The maximum lag length considered in the lag length selection was 12. However, we also increased the maximum lag to 24 and obtained similar results. Clearly, there is strong evidence for all interest rate series to be $I(1)$.

Based on this outcome it is reasonable to proceed with unit root tests for the spreads and parity series. The results are also shown in Table 1. Using the lag lengths suggested by the AIC, a unit root can be rejected in both spread series at least at the 10% level. We have double checked these results by performing KPSS tests (see Kwiatkowski, Phillips, Schmidt & Shin (1992)) which check the stationarity null hypothesis. The results are not shown in the table and we just mention that they overall confirm the stationarity result. Of course, not being able to reject the stationarity null hypothesis may be due to lack of power. However, together with the ADF test results, the evidence against a unit root in the spread series is quite strong.

The situation is somewhat different for the two parity series. Including a constant term in the test regression, the ADF test can reject a unit root in $r_t^{US} - R_t^{EU}$ at least at the 10% level, depending on the lag length chosen (see Table 1). On the other hand, a unit root cannot not be rejected in $r_t^{US} - r_t^{EU}$ at the 10% level. Of course, this result is inconsistent with the previous findings because

$$r_t^{US} - r_t^{EU} = (r_t^{US} - R_t^{US}) + (R_t^{US} - R_t^{EU}) + (R_t^{EU} - r_t^{EU})$$

and is thus a sum of three $I(0)$ series which must be stationary as well. Hence, the result based on a test with a constant term may be due to lack of power. In fact, in this case, it is not clear, why a constant should be included in the test regression because, based on the UIP, one might expect the parities to have zero mean and also the intercept terms were not significant in the ADF regressions for $R_t^{US} - R_t^{EU}$ and $r_t^{US} - r_t^{EU}$. Therefore we also give results for ADF tests without a constant term for both parity series in Table 1. For the long-term rates these tests clearly reject a unit root at the 1% level for different lag lengths whereas for the short-term rates nonstationarity is rejected at the 5% or 10% level depending on the number of lagged differences used in the tests. Thus, overall there is considerable evidence in favor of UIP.

In summary, the unit root tests provide strong evidence for both EHT and UIP to hold jointly for the U.S. and Europe. These results are based on univariate analyses only and it is also of interest to consider a multivariate model for the four interest rate series to study their relation in more detail. A related multivariate analysis is also a good robustness check for the previous results. Therefore a cointegration analysis within a vector error correction model (VECM) framework is presented next.

### 3.2 System Analysis

The cointegration analysis starts by investigating the cointegration rank of the system of four interest rates. Some results of Johansen (1995) trace tests for the cointegration rank with a constant restricted to the cointegration relations are presented in Table 2. Excluding a linear deterministic trend term is
in line with the unit root analysis. Based on that analysis one would expect to find cointegration rank \( r = 3 \) in a four dimensional VECM for \( y_t = (R_{US}^t, r_{US}^t, R_{EU}^t, r_{EU}^t)' \). Given that the tests are known to have low power in higher dimensional systems, the results in Table 2 provide some evidence in favor of the expected three cointegration relations. For a lag order of three\(^1\), as suggested by AIC when a maximum lag order of 12 is considered, all cointegration ranks less than \( r = 3 \) are rejected at least at the 10% level. The situation is not quite so clear if the lag order selected by the HQ and SC criteria is considered. Even then the test values for null hypotheses with \( r < 3 \) are quite close to the 10% critical values of the tests. Given that Lütkepohl & Saikkonen (1999) found the tests to perform slightly better when the lag order is chosen by AIC than by SC, the test results provide overall evidence in favor of a cointegration rank of \( r = 3 \).

Based on the VECM with three lagged differences and a cointegration rank of \( r = 3 \) we have also conducted a number of further tests to support the results from the unit root analysis. In particular, we have tested the spread and parity restrictions for the three cointegration vectors. Specifically we have tested

\[
H_0: \beta' = \begin{bmatrix}
1 & -1 & 0 & 0 \\
0 & 0 & 1 & -1 \\
1 & 0 & -1 & 0
\end{bmatrix},
\]

where \( \beta \) denotes the cointegration matrix. Under standard assumptions, the corresponding LR statistic has an asymptotic \( \chi^2(3) \) distribution. For our null hypothesis we obtained a test value of 5.20 which corresponds to a \( p \)-value of 0.16 and thus \( H_0 \) is not rejected at conventional significance levels.

Our results imply that a single stochastic trend is driving the long-term development of the interest rates and hence the monetary conditions in the U.S. and Europe. Clearly, it is of interest to know to what extent the central banks in the two currency regions have an impact on this stochastic trend. To explore this question, we have used a VECM with three lagged differences and cointegration rank three in performing a structural forecast error variance decomposition (SFEVD). Given that there are three cointegration relations, there may be three shocks which have transitory effects only and at least one shock must have permanent effects. Using the setup and terminology of Breitung, Brüggemann & Lütkepohl (2004), we choose a B-model with restrictions

\[
\Xi_B = \begin{bmatrix}
* & 0 & 0 & 0 \\
* & 0 & 0 & 0 \\
* & 0 & 0 & 0 \\
* & 0 & 0 & 0
\end{bmatrix} \quad \text{and} \quad B = \begin{bmatrix}
* & * & 0 & 0 \\
* & * & * & * \\
* & * & 0 & 0 \\
* & * & * & *
\end{bmatrix}, \quad (3.1)
\]

where \( \Xi_B \) denotes the matrix of long-run effects and \( B \) is the matrix of instantaneous effects of the structural shocks. In these matrices the asterisks denote unrestricted elements, that is, the first shock is the only one which is allowed to have permanent effects. The identification of the transitory shocks is to some extent arbitrary. Specifying zero restrictions for \( B \) as in (3.1) means that the second transitory shock has no instantaneous impact on the U.S. long-term rate and the third transitory shock (the

\(^1\)That is, three lagged differences of all variables are included which corresponds to a VAR order of four for the levels variables.
last shock in the system) has no instantaneous impact on any of the two long-term rates. Notice that we have not imposed any restrictions on the cointegration relations or loading coefficients in the structural analysis. In particular, we have not imposed the spread and parity restrictions. Such restrictions, although not rejected by the data, will condition the results of a structural analysis in an undesirable way which we try to avoid at this stage.

The resulting SFEVD is shown in Figure 4. The permanent shock is seen to determine to a large extent the U.S. short-term interest rate. More precisely, for short horizons around 80% or more of the forecast error variance of $r_{US}^t$ are accounted for by the permanent shock and for longer horizons the importance increases even further to more than 90%. Hence, we regard this shock as a U.S. monetary policy shock. It has a substantial impact on all other interest rates and in particular also on the European short- and long-term rates. Thus, viewing the permanent shock as a U.S. monetary policy shock, one may conclude that the European interest rates are to some extent influenced by U.S. policy. It is also interesting to see that the last two transitory shocks determine to some extent the European interest rates but contribute very little to the U.S. rates. Thus, they may be regarded as European monetary policy shocks. Only one of the transitory shocks has a sizable impact on both U.S. and European interest rates. Hence, based on this analysis it appears that European monetary policy pays more attention to the U.S. policy than vice versa. On the other hand, it is also clear that European interest rates are determined to a substantial extent by other factors than U.S. monetary conditions. Hence, there is no evidence that the ECB simply follows U.S. monetary policy.

There are several possible concerns one could raise against such an analysis and interpretation of our results. First of all, our VECM with three lagged differences of all variables is not a fully satisfactory representation of the data generation process. In particular, there may be some remaining residual autocorrelation. Therefore we have repeated the analysis with a model with twelve lagged differences and the SFEVD results were qualitatively the same. Another criticism could be that the identifying restrictions may determine our results to some extent. Although this cannot be denied, it may be worth noting that in our setup the permanent shock is fully identified by the assumption that the cointegration rank is three and there are correspondingly three transitory shocks. As we have seen in the unit root and cointegration analysis, there is considerable evidence in favor of three cointegration relations in the four-dimensional system. Thus, our conclusion to view the permanent shock as a U.S. monetary policy shock is supported by the previous findings from the unit root and cointegration analysis. It is independent of the way the transitory shocks are identified and, of course, it does not depend on where the shock is placed in the vector of structural innovations. For example, it may be placed alternatively last in the vector of structural innovations. In that case the last column of $\Xi B$ remains unrestricted and all other columns are restricted to zero.

Our conclusions regarding the interpretation of the transitory shocks is on less firm grounds. Changing the identifying restrictions does indeed change the SFEVDs somewhat. However, we have checked a number of other just-identifying restrictions for the transitory shocks and always found that two of them have little or no impact on the U.S. interest rates while a third one contributes to the forecast error variances of U.S. and European rates. Thus, even in this respect the results are qualitatively
robust with respect to changes in the identifying restrictions.

4 Conclusions

In this note we have analyzed the relation between short-term and long-term interest rates in the U.S. and the euro area based on monthly data from 1985-2004. The euro area data are constructed by combining German data for the years 1985-1998 with EMU data for 1999-2004. We have argued that constructing the European data in this way avoids potential problems due to the adjustment processes in Europe in the run-up period to the EMU. These adjustment processes may be responsible for different findings by other authors.

We find evidence in favor of both the EHT and the UIP. More precisely, we find that all four interest rate series considered are $I(1)$ and the U.S. spread as well as the European spread are $I(0)$. Moreover, the differences between the U.S. and European long-term rates is $I(0)$. Our evidence is based on univariate unit root tests as well as on a VECM analysis. A more detailed analysis of the relations between the variables based on SFEVDs provides support for the hypothesis that European monetary policy responds to changes in U.S. interest rates whereas there is little evidence for a reverse causal link from European policy to the U.S..

References


### A Variables and Data Sources

Monthly data for the period 1985M1-2004M12 are used. Euro area interest rate series correspond to German interest rates for the period 1985M1-1998M12 and to euro area interest rates for the period 1999M1-2004M12. Monthly values are averages over all business days. The data are taken from sources listed below:


Figure 1: Interest rate time series analyzed.

Figure 2: Interest rate spreads.

Figure 3: Interest rate parities.
Figure 4: Forecast error variance decompositions.
Table 1: Augmented Dickey Fuller Unit Root Tests. Sample: 1985M1-2004M12.

<table>
<thead>
<tr>
<th>Variable</th>
<th>Lagged differences</th>
<th>Deterministic term</th>
<th>Test statistic</th>
<th>Critical values</th>
</tr>
</thead>
<tbody>
<tr>
<td>RUS</td>
<td>AIC, HQ, SC: 3</td>
<td>c</td>
<td>−2.43</td>
<td>−2.57 −2.86 −3.43</td>
</tr>
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<td>rUS</td>
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<td>c</td>
<td>−1.68</td>
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<td></td>
<td>HQ, SC: 1</td>
<td>c</td>
<td>−1.56</td>
<td></td>
</tr>
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<td>AIC: 9</td>
<td>c</td>
<td>−1.48</td>
<td></td>
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<td>−1.19</td>
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<td>rEU</td>
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<td>c</td>
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<td></td>
<td>HQ: 10</td>
<td>c</td>
<td>−2.10</td>
<td></td>
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<tr>
<td></td>
<td>SC: 1</td>
<td>c</td>
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<tr>
<td>ΔRUS</td>
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<td></td>
<td>−7.74</td>
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<td></td>
<td>HQ, SC: 0</td>
<td></td>
<td>−11.6</td>
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<tr>
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<td>HQ: 9</td>
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<td>−2.65</td>
<td></td>
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<tr>
<td></td>
<td>SC: 0</td>
<td></td>
<td>−11.0</td>
<td></td>
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<td>−3.02</td>
<td>−2.57 −2.86 −3.43</td>
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<td>−1.62 −1.94 −2.56</td>
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<tr>
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<tr>
<td></td>
<td>SC: 4</td>
<td></td>
<td>−1.85</td>
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</table>

Note: c denotes a constant. The number of lagged differences has been determined using information criteria with a maximum lag order of $p_{max} = 12$. All critical values are obtained from Table 20.1 in Davidson & MacKinnon (1993). Computations are performed with JMulTi, Version 3.2 (see Lütkepohl & Krätzig (2004)).
Table 2: Cointegration Tests for $y_t = (\Delta RUS_t, \Delta rUS_t, \Delta EU_t, \Delta rEU_t)'$. Sample: 1985M1-2004M12.

<table>
<thead>
<tr>
<th>lagged differences</th>
<th>$H_0$</th>
<th>test value</th>
<th>10%</th>
<th>5%</th>
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</thead>
<tbody>
<tr>
<td>AIC: 3</td>
<td>$r = 0$</td>
<td>54.51</td>
<td>50.50</td>
<td>53.94</td>
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<td></td>
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<td>33.52</td>
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<td></td>
<td>$r = 3$</td>
<td>5.94</td>
<td>7.60</td>
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<td>$r = 0$</td>
<td>50.07</td>
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<td>5.79</td>
<td>7.60</td>
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</table>

*Note:* Results are for Johansen (1995) trace tests with a constant restricted to the cointegration relations. The number of lagged differences has been determined using information criteria with a maximum lag order of $p_{max} = 12$. Critical values are computed from the response surface given in Trenkler (2004). Computations are performed with JMulTi, Version 3.2 (see Lütkepohl & Krätzig (2004)).