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Abstract
This article evaluates the effects of patent rights on the timing of innovation. As in Shleifer (1986), firms in different sectors receive cost-saving ideas exogenously and sequentially, from which they can make temporary monopoly profits. In the presence of sectoral demand externalities, firms might opt to postpone the implementation of their ideas so that they innovate together with firms from other sectors. I show that a prolongation of patent rights limits the appeal of this possibility, and, for ideas which are not too radical, it can lead to a welfare improvement.

Keywords
Patent rights, timing of innovation, implementation cycles with capital, temporary monopolies, demand externalities, multiple equilibria

JEL Classification: D43, E32, O3

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1 Introduction

A veil of mystery often surrounds the release of new products. Especially technology firms are particularly secretive, not just about the products they develop, but also about the timing of their releases. While the former suggests that an innovating firm's prospective profits in the presence of potential imitators are short-lived, the latter suggests that timing has an impact on them. Through the lens of this last remark, this article revisits a classic question: are longer-lasting patent rights desirable?

To provide a clear-cut answer, this article takes a novel approach. First, it completely abstracts from how inventions and ideas are generated only to focus on *when* inventions and ideas are implemented. Second, it lets the timing of innovation be sensitive to the demand externalities present across the different sectors of an economy. This article, then, evaluates the effects of patent rights on the timing of innovation in an economy with demand externalities across its sectors.

A fine starting point in this attempt is the work by Shleifer (1986), which introduces the "implementation cycles" concept. The concept of implementation cycles is simple: when innovating firms across different sectors share beliefs about the timing of a boom, they might find it beneficial to coordinate the implementation of their inventions so that the anticipated boom becomes true, even though inventions might have been developed at different points in time.

I assume that innovation comes in the form of cost-reducing ideas and I let innovation take place in the two capital sectors an economy comprises. Within each capital sector, there are a number of Bertrand-competing capital makers, which use foregone consumption (investment) to produce the capital good they specialize in. In the even-numbered periods, a cost-reducing idea reaches randomly, exogenously, and costlessly a capital maker in sector 1, and in the odd-numbered ones, a capital maker in sector 2. As in Shleifer (1986), capital makers can make monopoly profits out of an idea, but they can do so only temporarily: once a new idea is implemented, the innovating capital maker's competitors can costlessly reverse-engineer it, driving sector profits to zero—until a new idea arrives in the sector.

It is precisely the presence of imperfect competition that invites demand externalities between the two capital sectors. Furthermore, since ideas on improved technologies reach a sector every two periods, capital makers might implement their inventions and ideas as soon as they receive them, but they might also decide to implement them in the following period. As Matsuyama (1995) notes, it is precisely this intertemporal decision that capital makers face in combination with the presence of intratemporal demand externalities that can invite (welfare-rankable) multiple cyclical equilibria. For expectations accordingly formed, it may be the case, then, that capital makers implement their ideas as soon as they receive them, which implies that ideas are put into practice at the perfectly smooth rate of their arrival ("immediate implementation equilibrium"); or capital makers may instead coordinate the implementation of their ideas and innovate together, in which case implementation cycles with capital are generated ("synchronized implementation equilibrium").

I perform the following policy experiment: initially, I let capital makers make monopoly profits out of an idea for only one period. This is the period following the implementation period of a new idea, and that, in turn, is because the ideas concern capital technology, and it takes one period to build capital. Subsequently, I extend the patent horizon to two periods. I find that in the former case an immediate implementation equilibrium is always sustainable, and, for plausible parameter values, so can a synchronized implementation one. In the latter case, an immediate implementation equilibrium is always sustainable too. However, a synchronized implementation equilibrium cannot be sustained, rendering, therefore, implementation cycles impossible and incompatible with prolonged patent rights. The model is tailored so that it delivers this last stylized result, which reflects the more general intuitive one that the longer a firm's profit horizon for an idea is, the earlier the firm will implement it, and in the limit case that a firm has an infinitely-lasting monopoly over an idea, it will do nothing but implement it immediately.

To answer whether prolonged patent rights are desirable, I compare the representative agent's lifetime utility in the immediate implementation equilibrium prevailing for prolonged (two-period) patent rights to the agent's lifetime utility in the two types of equilibria which can prevail when patent rights last one period. When comparing the immediate implementation equilibria, the answer is negative: as the patent horizon becomes longer, capital makers can profit from their ideas and inventions for longer, and therefore ideas diffuse more slowly to the implementing capital makers' competitors. Consequently, the economy reaches a certain consumption, and therefore welfare, level more slowly. Nevertheless, the answer can be positive when comparing the immediate implementation equilibrium prevailing with prolonged patent rights to the synchronized implementation equilibrium prevailing when patent rights last one period. This is because on the one hand a prolongation of patent rights implies that some ideas (here, the ones in sector 2) diffuse to the economy later as they are appropriated for longer, but on the other hand it implies that some other ideas (here, the ones in sector 1) are implemented earlier. It turns out that for ideas which are not too radical the "early-implementation" effect dominates the "late-diffusion" one, and thus a prolongation of patent rights leads to a welfare improvement, which is this article's central result.

Relation to the literature. The literature on patent protection and intellectual property rights is vast and ever-growing. It dates back at least to Schumpeter (1942) and Arrow (1962), while recent contributions include, among others, Boldrin and Levine (2002, 2008a,b) and Henry and Ponce (2011) (one may also check Chapter 12 in Acemoglu (2009), Holmes and Schmitz (2010), Rockett (2010) and the references therein). The vast majority of works, however, are concerned with how patent rights affect the incentives to innovators, and thus the number of inventions and ideas generated. The present article completely abstracts from how ideas are generated and whether innovators have sufficient incentives to invent, and focuses instead on how patent rights affect the timing of the implementation of ideas and inventions, i.e. on how patent rights affect the timing of innovation.

There are a few papers which also explore the effects of patent rights on the timing of innovation, or, what is equivalent in these setups, the timing firms select to file for a patent and, as a result, disclose their innovation. They do so, though, from a completely different angle, and thus their approach should be viewed as complementary. For instance, Hopenhayn and Squintani (2014) explore the effects of patent rights on the timing of innovation within a model of sequential innovation which allows for preemptive entry by an R&D firm's competitors. As a result, innovators might file for a patent too early if their fear of preemption by their competitors dominates, or too late if their failure to take into account the spillovers their innovation has on future research dominates. Scotchmer and Green (1990) also employ a sequential R&D model coupled with the assumption that sequential innovations displace previous ones, and find that firms file for patents too late. Also within frameworks that allow for patent races, Horstmann et al. (1985) show that firms might opt for secrecy for fear of imitators, while Matutes et al. (1996) show that firms might patent ideas which set the basis for future applications too late (one may also check Hall et al. (2014) for an excellent overview of the empirical and theoretical work on the "patents vs. secrecy" decision). Here, I completely abstract from such considerations: capital makers do not develop ideas, but they receive them exogenously and costlessly. Equally importantly, capital makers can appropriate ideas, either immediately or with delay, without the fear of preemption or imitation, and without (the implementation of their ideas itself) affecting the generation or value of future ideas at all.

Finally, as already discussed, the model employed here builds on that in Shleifer (1986),[1] who introduced the "implementation cycles" idea, but completely abstracted from any patent rights considerations. I frequently refer to how the two models relate to each other in the rest of this article. However, it is essential to stress early on that the model employed here does allow for capital—and, nevertheless, implementation cycles can be sustained, and lets innovation affect the investment technology. Section 6 discusses these points in detail.

The rest of the article is organized as follows. Section 2 presents the model, Section 3 provides the equilibrium definition, and Section 4 pins down the equilibria for one-period (Section 4.1) and two-period patent rights (Section 4.2). Section 5 discusses welfare and presents this article's central result (Proposition 3). Section 6 makes additional remarks and concludes.

[1] With expectations arbitrarily centered around one of the possible multiple equilibria, this article naturally relates to the "sunspots" literature too, which, among other articles, includes Azariadis (1981), Cass and Shell (1983), and Grandmont (1985). Benhabib and Farmer (1999) offer an overview of this literature, while an earlier complementary survey emphasizing endogenous cycles is by Boldrin and Woodford (1990). Certainly, this article also shares features with the endogenous growth literature, prominent contributions to which include Romer (1990), Grossman and Helpman (1991), and Aghion and Howitt (1992), the literature on strategic delay (for instance, Chamley and Gale (1994)) as well as Matsuyama (1999) and Jovanovic (2009).

2 Environment

The economy is populated by an infinitely-lived representative agent (Section 2.1). The agent consumes a single storable final good produced by a representative final-good firm (Section 2.2), to which he supplies his labor inelastically. In addition to labor, the final-good firm uses two storable capital goods as inputs for its production, and it uses them in an additively-separable way. For each capital good, there is a respective sector which comprises at least two capital-good firms (henceforth, "capital makers"). Capital makers are managed by respective capital-good producers (Section 2.3), and Bertrand-compete for the production of the capital good they specialize in. It takes precisely one period to produce a capital good, and foregone consumption ("investment") is the only input used in its production. Capital makers become the recipients of exogenously-arriving, cost-saving ideas, and once they implement them they can temporarily make monopoly profits, until their ideas get imitated by their competitors (Section 2.3.1).

There is no uncertainty: agents and capital-good producers have perfect foresight. In addition, capital markets are perfect, which allows agents and capital makers to perfectly borrow against their future incomes. Time is discrete with an infinite horizon, and commences in period 0.

2.1 Representative agent

The preferences of the representative agent are given by

$$\sum_{t=0}^{\infty} \beta^t \log x_t \,, \tag{1}$$

where x_t denotes consumption of the final good, and $\beta \in (0,1)$ parametrizes the agent's time preference.[2]

The representative agent is endowed with one unit of time which he supplies inelastically, owns all firms in the economy, and, with capital markets being perfect, he can freely borrow against his perfectly-foreseen future income. The assumption of perfect capital markets allows

[2]An earlier version of this article allowed for variable labor supply without this having any noteworthy impact on the results that follow.

me then to use the agent's intertemporal budget constraint, which is given by

$$\sum_{t=0}^{\infty} m_t \, x_t \;\leq\; \sum_{t=0}^{\infty} m_t \, [w_t \,+\, \Pi_t^f \,+\, \sum_{i=1}^{2} \Pi_{t,i}\,], \tag{2}$$

where $m_0 \;=\; 1\,,\; m_1 \;=\; \frac{1}{R_0}\,$, and $m_t \;=\; \frac{1}{R_0 \ldots R_{t-1}}$ for $t > 1\,$. R_t denotes the real gross interest rate paid in period $t+1\,$, and w_t denotes the real wage paid by the final-good firm, both expressed in units of the final good. Π_t^f denotes the profits that accrue to the agent from the final-good firm in period t, and $\Pi_{t,i}$ denotes the profits that accrue to the agent from all the capital makers in sector $i\,$, where $i \;=\; 1, 2\,$, in period $t\,$.

The agent chooses $\{x_t\}_{t=0}^{\infty}\,$, where x_t takes only non-negative values, to maximize his lifetime utility given by (1), subject to his intertemporal budget constraint given by (2). The solution to his problem leads to the following, usual, intertemporal relations:

$$\frac{x_{t+1}}{x_t} \;=\; \beta \, R_t \tag{3}$$

$$\frac{x_{t+1}}{x_{t-1}} \;=\; \beta^2 \, R_{t-1} \, R_t \,. \tag{4}$$

2.2 Final-good firm

The final-good firm is competitive in both the final-good and the input markets. Its technology is given by

$$F\,(n_t\,,\, k_{t,1}\,,\, k_{t,2}) \;=\; n_t^{\alpha}\,(k_{t,1}^{1-\alpha} \,+\, k_{t,2}^{1-\alpha}), \tag{5}$$

where n_t denotes labor and $k_{t,1}\,,\; k_{t,2}$ denote capital of types 1 and $2\,$, respectively, rented in period $t\,$; $\alpha \in (0, 1)$ parametrizes both the labor share and the elasticity of substitution between the two types of capital. As in Romer (1990), the marginal products of the two types of capital are conditionally on labor independent of each other. To make the analysis tractable, I further assume that capital of both types fully depreciates within a period.

The final-good firm chooses non-negative quantities $\{n_t\,,\, k_{t,1}\,,\, k_{t,2}\,,\, y_t\}$ each period to maximize its period profits given by

$$\Pi_t^f \;=\; y_t \,-\, w_t \, n_t \,-\, q_{t,1} \, k_{t,1} \,-\, q_{t,2} \, k_{t,2}\,,$$

where $y_t \le F(n_t, k_{t,1}, k_{t,2})$, and $q_{t,i}$ denotes the real rental price of capital of type i, where $i = 1, 2$, in period t.

The firm's maximization problem yields the following inverse demand functions:

$$w_t = \alpha\, n_t^{\alpha-1} (k_{t,1}^{1-\alpha} + k_{t,2}^{1-\alpha}) \tag{6}$$

$$q_{t,i} = (1-\alpha)\, n_t^{\alpha}\, k_{t,i}^{-\alpha} \ , \text{ for i} = 1, 2 . \tag{7}$$

Observe that both the labor and the capital demand functions are elastic, with respective elasticities $\epsilon_w = -\frac{1}{1-\alpha}$ and $\epsilon_q = -\frac{1}{\alpha}$, and that, by implication of constant returns to scale, profits are equal to zero each period.

2.3 Capital makers

For each type of capital good,[3] there are a number of capital makers, which Bertrand-compete for its production with no capacity constraints. With no loss of generality, I index capital makers of both capital goods with j, where $j = 1, 2, \ldots, J$ and $J \ge 2$.

Capital makers operate a constant-returns-to-scale technology. The technology of capital maker j manufacturing capital of type i is given by

$$k_{t,i,j} = \psi_{t-1,i,j}\, i_{t-1,i,j} \ , \tag{8}$$

where $k_{t,i,j}$ denotes the quantity of capital type i produced by capital maker j in period t and $i_{t-1,i,j}$ denotes capital maker j's corresponding investment in period $t-1$, which, as usual, comes in the form of foregone consumption.

Aggregate investment, i_t, in period t is then equal to

$$i_t = \sum_{i=1}^{2} \sum_{j=1}^{J} i_{t,i,j} . \tag{9}$$

[3] This is only an abstraction. What we, in fact, need is two baskets of capital goods, with each basket containing a sufficiently high number of capital goods so that each capital maker is small enough within the "basket" it is contained in. The target implication of this assumption is that capital makers take aggregate outcomes as given, which implies that any strategic interaction among capital makers within and across baskets is ruled out. For simplicity, let each basket contain a continuum of capital goods of unit measure. That is $k_{t,i} = \int_{i' \in [0,1]} k_{t,i,i'}\, di'$, where $k_{t,i,i'}$ denotes capital good i' in basket i in t. Assuming a symmetric treatment of capital goods within a composite capital good allows me, henceforth, to mention capital good of type i and actually refer to the representative capital good i' in basket i.

2.3.1 Pattern of ideas

As in Shleifer (1986), I make two assumptions on the arrival of ideas.

Assumption 1. Ideas on improved technologies arrive exogenously, and they reach sectors sequentially, at a perfectly smooth rate. With no loss of generality, an idea reaches a capital maker in sector 1 randomly in the even periods, and a capital maker in sector 2 in the odd ones. Ideas lower the marginal production cost of capital of type i by μ:

$$\frac{\psi^*_{t+1,i}}{\psi^*_{t-1,i}} = \mu \,,$$

where $\psi^*_{\cdot,i}$ denotes the *state-of-the-art* technology, or equivalently the highest inverse marginal cost of capital, prevailing among capital makers in sector i, and μ parametrizes the innovation rate, which I assume to be greater than one.

Since time commences in period 0, an even period, I impose that $\psi^*_{0,1} = 1$ and $\psi^*_{0,2} = \mu^{\frac{1}{2}}$ in order to remove the first-mover advantage of sector 1, and ensure thereby symmetric treatment of the capital-good sectors. In this way, the lead in the technology race might alternate between the two sectors, however their relative technology distance remains fixed and equal to $\mu^{\frac{1}{2}}$. Equivalently, it is as if time had instead commenced at $-\infty$.

The assumption that ideas arrive periodically could be partially relaxed. What really matters is that the probability that an idea reaches a sector before the time indicated is sufficiently low. The fact that ideas reach sectors at a perfectly smooth rate renders cycles harder to generate and crystallizes the forces underpinning them. Were ideas to arrive in a cyclical fashion, their cyclical implementation would then come as little surprise.

Assumption 2. A capital maker can make monopoly profits out of an idea, but only temporarily. Eventually, its competitors can copy the idea costlessly, driving prices down to marginal costs and profits to zero. In Section 4.1, I let capital makers appropriate an idea for only one period, just as in Shleifer (1986). In Section 4.2, I let them do so for two periods.

The fact that the firms' monopoly profits are temporary is an assumption in line with the Schumpeterian growth literature, originating in Aghion and Howitt (1992), which focuses on quality improvements ("process innovations"), and comes in contrast with Romer (1990), where firms' rights over an idea last forever. Of course, unlike the whole endogenous growth literature, this article abstracts entirely from how ideas are generated.

2.3.2 Profits

As it takes one period to build capital, the profits of capital maker j producing capital good i are given by

$$\Pi_{t,i,j} \;=\; q_{t,i,j}\, k_{t,i,j} \;-\; R_{t-1}\, i_{t-1,i,j}\,.$$

Capital maker j of capital good i chooses non-negative quantities of $\{k_{t,i,j}\,,\, i_{t-1,i,j}\}$ each period to maximize its profits subject to its technology, given by (8). Revenue is realized one period after investment is made, and I allow capital makers to be able to perfectly borrow against it.

I distinguish between two cases: in the first case, all capital makers in sector i operate the same technology, in which case a firm, say j^*, is randomly selected to produce capital of type i. In the second case, firm j^* has a technological advantage over its competitors, which allows it to make temporary monopoly profits.

Perfect competition. Let $\psi_{\cdot,\cdot,-j^*}$ denote the inverse marginal cost of the competitors of capital maker j^*. When $\psi_{t,i,j^*} \;=\; \psi_{t,i,-j^*}$, there is perfect competition in sector i, which, given the capital makers' linear technology, implies zero profits for firm j^*.

Firm j^* in sector i supplies capital good i at a price given by

$$q^*_{t,i,j^*} \;=\; \frac{R_{t-1}}{\psi_{t-1,i,-j^*}}\,. \tag{10}$$

Inverse demand for capital from the final-good firm (7) pins down the competitive quantity:

$$k^*_{t,i,j^*} \;=\; \left[\frac{(1-\alpha)\,\psi_{t-1,i,-j^*}}{R_{t-1}}\right]^{\frac{1}{\alpha}} n_t\,. \tag{11}$$

Monopoly in the presence of a competitive fringe. Capital maker j^* chooses k_{t,i,j^*} to maximize its profits, given by

$$\Pi_{t,i,j^*} \;=\; q_{t,i,j^*}\, k_{t,i,j^*} \;-\; R_{t-1}\,\frac{k_{t,i,j^*}}{\mu\,\psi_{t-1,i,-j^*}}\,, \tag{12}$$

subject to the inverse demand for capital, given by (7). Since the demand for capital is elastic, the solution is well-defined:

$$q^m_{t,i,j^*} \;=\; \frac{R_{t-1}}{(1-\alpha)\,\mu\,\psi_{t-1,i,-j^*}} \;\text{ and }\; k^m_{t,i,j^*} \;=\; \left[\frac{(1-\alpha)^2\,\mu\,\psi_{t-1,i,-j^*}}{R_{t-1}}\right]^{\frac{1}{\alpha}} n_t\,.$$

Henceforth, I restrict attention to "limit pricing," which occurs when $q^m_{t,i,j^*} \;\geq\; q^*_{t,i,j^*}$. Since capital maker j^* cannot sell at a price greater than the inverse marginal cost of its competitors, the limit price, q^*_{t,i,j^*}, ensues. Requiring $q^m_{t,i,j^*} \;\geq\; q^*_{t,i,j^*}$ implies the following constraint on the values that parameters μ and α can take:

$$\mu\,(1-\alpha) \;\leq\; 1\,. \tag{LPC}$$

The lower the innovation rate, μ, and the less elastic the demand for capital is, the more easily the limit-pricing Condition (LPC) is satisfied. For a certain level of the elasticity of demand for capital, Condition (LPC) imposes an upper bound on the innovation rate, and vice versa.

A monopolist then sells the quantity corresponding to its competitors' technology level given by (11), at the limit price given by (10), and makes profits because of its lower, by μ relative to its competitors, marginal cost of producing capital. Combining (12) with (10) and (11) implies that a monopolist's profits are given by

$$\Pi_{t,i,j^*} \;=\; (1-\alpha)^{\frac{1}{\alpha}}\,\psi^{\frac{1}{\alpha}-1}_{t-1,i,-j^*}\left(\frac{\mu-1}{\mu}\right)\frac{n_t}{R^{\frac{1}{\alpha}-1}_{t-1}}\,. \tag{13}$$

By virtue of the elastic demand for capital, profits depend negatively on the interest rate paid in the period they are made, and positively on the technology level of a monopolist's competitors.

2.3.3 The implementation decision

Suppose capital maker j^* in sector i receives an idea in period $t-1$, and needs to decide whether to implement it immediately or in the following period. Capital maker j^* will implement its idea immediately instead of in the following period as long as its present-discounted period t profits exceed its present-discounted period $t+1$ profits. Letting superscripts denote the date an idea arrives and subscripts the date profits are made, it must then be that

$$\frac{\Pi^{t-1}_{t,i,j^*}}{R_{t-1}} \geq \frac{\Pi^{t-1}_{t+1,i,j^*}}{R_{t-1}\, R_t}\,.$$

By (13), this boils down to

$$\left(\frac{R_t}{R_{t-1}}\right)^{\frac{1}{\alpha}-1} \geq \frac{1}{R_t}\,. \tag{IPC}$$

We can see from (IPC) that the lower the interest rate paid one period after an idea arrives relative to that paid two periods afterwards, the more costly it is for a capital maker to wait, and the more desirable the immediate implementation of the idea becomes.

In the complementary case, in which

$$\left(\frac{R_t}{R_{t-1}}\right)^{\frac{1}{\alpha}-1} < \frac{1}{R_t}\,, \tag{SPC}$$

capital maker j^* opts to postpone implementation to the following period.

Note that a monopolist's implementation decision is independent of its competitors' technology level, while an assumption implicit in the above is that no capital maker can affect the real interest rate with its actions alone.

3 Balanced Growth Path Equilibria

I define a perfect-foresight equilibrium as follows:

Definition 1 (Equilibrium)**.** *A no-storage, perfect-foresight equilibrium consists of a set of interest rates* $\{R_t\}_{t=0}^{\infty}$ *and prices* $\{w_t\,,\,(q_{t,i})_{i=1}^2\}_{t=0}^{\infty}$, *an allocation* $\{x_t\}_{t=0}^{\infty}$ *for the representative agent, an allocation* $\{n_t\,,\,(k^d_{t,i})_{i=1}^2\,,\,y_t\}_{t=0}^{\infty}$ *for the final-good firm, and an allocation* $\{(k^s_{t,i,j^*}\,,\,i_{t-1,i,j*})_{t=0}^{\infty}\}_{i=1,2}$ *for the technology-leading capital makers in the two capital-good sectors, such that:*

1. *(Optimality) The allocations of the agent, the final-good firm, and the technology-leading capital makers solve their respective problems, laid out in Section 2, at the stated prices. The transversality condition (TVC) must also be met.*

2. *(Market clearing)* $k^d_{t,i} = k^s_{t,i,j^*}$ *for all* i, $n_t = 1$, *and* $y_t = x_t + i_t$, *for all* t.

3. *(Consistency) For expectations arbitrarily centered around an equilibrium ("sunspots"), capital makers must find it optimal to implement their ideas when conjectured.*

4. *(No storage) No storage takes place.*

Let me start with the no-storage requirement in Definition 1 . Assuming a one-to-one storage technology, the no-storage condition is as follows:

Condition 1 (No storage)**.** $R_t > 1$ *for all* t *rules out storage in equilibrium.*

Proof. See Appendix A.1 . □

I will discuss two types of equilibria: an acyclical, immediate implementation equilibrium, and a cyclical, synchronized implementation one. In the former, capital makers implement their ideas as soon as they receive them. In the latter, capital makers in the two sectors coordinate the implementation of their ideas, which requires at least one of them to wait before implementing its idea.

For each type of equilibrium, I will proceed as follows. I will first ensure that the optimality and market clearing conditions are met (requirements (1) and (2) in Definition 1), assuming that the TVC and the no-storage condition are met too. I will next specify the conditions under which the conjectured timing of the implementation of the ideas is optimal for the capital makers receiving them (req. (3) in Definition 1). Finally, I will confirm that both the TVC (second part of req. (1) in Definition 1) and the no-storage condition (req. (4) in Definition 1) are met.

I will restrict attention to balanced growth path (BGP) equilibria, which involve cycles of constant periodicity. Along the balanced growth path, consumption, output and investment all grow at the same long-run rate since, eventually, all ideas are implemented. However, since an idea reaches a sector every other period, the growth rate may differ between even and odd periods.

Henceforth, I let τ denote an odd period. Since time commences in an even period, discussing only period τ, an odd period, and the even periods immediately before and after it is enough to pin down the balanced growth path equilibria. Along a balanced growth path equilibrium, consumption must satisfy the following stationarity conditions:

$$\frac{x_{\tau+1}}{x_{\tau}} = v \tag{14}$$

$$\frac{x_{\tau+1}}{x_{\tau-1}} = \lambda\,. \tag{15}$$

Combining (14) and (15) with the agent's optimality conditions, given by (3) and (4), yields the following pattern for the real interest rate:

$$R_t = \begin{cases} \frac{v}{\beta} & \text{if } t \text{ odd} \\ \frac{\lambda}{v\,\beta} & \text{if } t \text{ even} \end{cases}\,. \tag{16}$$

and leads to the following remark, which I will use in the remainder of the paper:

Remark 1. $R_t\,R_{t+1} = \frac{\lambda}{\beta^2}\,.$

4 Characterization of equilibria

I will first characterize the equilibria in which capital makers can appropriate an idea only for one period (Section 4.1). I will subsequently characterize the equilibria in which they can do so for two periods (Section 4.2).

4.1 Benchmark Equilibria with One-Period Patent Rights

Let capital makers be able to appropriate an idea for only one period. I will first characterize the immediate implementation equilibrium (Section 4.1.1), and I will subsequently turn to the synchronized implementation one (Section 4.1.2).

4.1.1 Immediate implementation equilibrium

In the immediate implementation equilibrium, capital makers implement their ideas immediately, and they expect each other to do so. This means that sectors alternate in innovating, with capital makers in sector 1 innovating in the even periods and capital makers in sector 2 in the odd ones.

Period $\tau - 1$. Capital maker j^* in sector 1 receives an idea which it immediately implements. Since I assume that Condition (LPC) holds throughout, limit pricing ensues. Hence, capital maker j^* sets the same price as its competitors would, which is given by (10), and produces the same quantity as its competitors would, which is given by (11). Let the technology level of j^*'s competitors, $\psi_{\tau-1,1,-j^*}$, be denoted by ψ, where ψ can take any positive value. The technology level of j^* is greater by μ relative to that of its competitors, i.e. $\psi_{\tau-1,1,j^*} = \mu\,\psi$. Given the assumption on the initial conditions, the technology level in capital sector 2 is $\psi_{\tau-1,2} = \mu^{\frac{1}{2}}\,\psi$, where the notation $\psi_{t,i}$ will henceforth denote the technology level of all capital makers in sector i in period t, which is implicitly the same. This will be the case when no new idea is implemented in a sector.

Capital in period τ is then given by

$$k_{\tau,1} = \left[\frac{(1-\alpha)\,\psi}{R_{\tau-1}}\right]^{\frac{1}{\alpha}} \tag{17}$$

$$k_{\tau,2} = \left[\frac{(1-\alpha)\,\mu^{\frac{1}{2}}\,\psi}{R_{\tau-1}}\right]^{\frac{1}{\alpha}}, \tag{18}$$

where I have used the fact that $n_t = 1$ for all t.

By (8) and (9), investment is given by

$$i_{\tau-1} = \frac{k_{\tau,1}}{\mu\,\psi} + \frac{k_{\tau,2}}{\mu^{\frac{1}{2}}\,\psi}. \tag{19}$$

Combining (19) with (17) and (18) together implies that investment in period $\tau - 1$ is equal to

$$i_{\tau-1} = \left[\mu^{-1} + \mu^{\frac{1}{2}(\frac{1}{\alpha}-1)}\right]\,\psi^{\frac{1}{\alpha}-1}\left(\frac{1-\alpha}{R_{\tau-1}}\right)^{\frac{1}{\alpha}}. \tag{20}$$

As expected, investment depends negatively on the interest rate to be paid in the following period, which is a consequence of the elastic demand for capital.

Likewise, since $\psi_{\tau-2,1,-j^*} = \psi$ and $\psi_{\tau-2,2,-j^*} = \mu^{-\frac{1}{2}}\,\psi$,

$$k_{\tau-1,1} = \left[\frac{(1-\alpha)\,\psi}{R_{\tau-2}}\right]^{\frac{1}{\alpha}} \tag{21}$$

$$k_{\tau-1,2} = \left[\frac{(1-\alpha)\,\mu^{-\frac{1}{2}}\,\psi}{R_{\tau-2}}\right]^{\frac{1}{\alpha}} . \tag{22}$$

Combining (5) with (21) and (22) yields

$$y_{\tau-1} = \left[1+\mu^{-\frac{1}{2}(\frac{1}{\alpha}-1)}\right]\,\psi^{\frac{1}{\alpha}-1}\left(\frac{1-\alpha}{R_{\tau-2}}\right)^{\frac{1}{\alpha}-1} . \tag{23}$$

Output depends negatively on the interest rate currently paid: the higher the interest rate currently paid, the lower the investment in capital in the period before the current one has been, and thus the lower current production is.

Market clearing in the final-good market (simply subtract (20) from (23)) implies that consumption is equal to

$$x_{\tau-1} = \psi^{\frac{1}{\alpha}-1}\,(1-\alpha)^{\frac{1}{\alpha}}\left[\frac{1+\mu^{-\frac{1}{2}(\frac{1}{\alpha}-1)}}{(1-\alpha)\,R_{\tau-2}^{\frac{1}{\alpha}-1}} - \frac{\mu^{-1}+\mu^{\frac{1}{2}(\frac{1}{\alpha}-1)}}{R_{\tau-1}^{\frac{1}{\alpha}}}\right] . \tag{24}$$

I follow exactly the same process for periods τ and $\tau+1$, with the derivations collected in Appendix B.1 . As we will need it below, I only report consumption in period $\tau+1$, which is equal to

$$x_{\tau+1} = \psi^{\frac{1}{\alpha}-1}\,(1-\alpha)^{\frac{1}{\alpha}}\left[\frac{\mu^{\frac{1}{\alpha}-1}+\mu^{\frac{1}{2}(\frac{1}{\alpha}-1)}}{(1-\alpha)\,R_{\tau}^{\frac{1}{\alpha}-1}} - \frac{\mu^{\frac{1}{\alpha}-2}+\mu^{\frac{3}{2}(\frac{1}{\alpha}-1)}}{R_{\tau+1}^{\frac{1}{\alpha}}}\right] . \tag{25}$$

So far, I have made sure that the first-order conditions and market clearing, which are (part of) requirements (1)-(2) in Definition 1, are met. I will now exploit stationarity to check in turn for consistency, no storage and the transversality condition.

Starting with consistency, for expectations accordingly formed, an immediate implementation equilibrium is sustainable as long as each capital maker which receives a new idea finds it optimal to implement it immediately. As in Shleifer (1986), I will argue in two steps, which I respectively label "profit condition 1 (IPC1)" and "profit condition 2 (IPC2)." I will first specify the two profit conditions, and I will afterwards argue why they are necessary and sufficient for consistency.

Profit condition 1 . With the flow of new ideas reaching the economy remaining constant, it has to be the case in an immediate implementation equilibrium that all aggregate variables grow at a constant rate too. This, of course, implies that the real interest rate remains constant over time, i.e. it must be that $R_{\tau-1} = R_{\tau}$, where $R_{\tau-1}$ refers to the real interest rate prevailing in even periods, and R_{τ} refers to the real interest rate prevailing in the odd ones. Since the two are equal to each other in an immediate implementation equilibrium, I will henceforth simply use R to denote the, constant across periods, real interest rate.

A capital maker prefers to implement an idea immediately to doing so in the following period if and only if Condition (IPC) is satisfied. Since the real interest rate is constant across periods, Condition (IPC) simplifies to

$$R \geq 1 \,. \tag{IPC1}$$

Profit condition 2 . No capital maker with a new idea has an incentive to wait for two periods before implementing it, irrespectively of the fact that a new idea will arrive in its sector, rendering obsolete the one it currently possesses, as long as

$$\frac{\Pi_{t,i}^{t-1}}{R_{t-1}} \geq \frac{\Pi_{t+2,i}^{t-1}}{R_{t-1}\, R_t\, R_{t+1}} \,.$$

Since, in an immediate implementation equilibrium, both the real interest rate and profits are constant across time, and, at the same time, capital makers are too small to affect them with their implementation decisions alone, the above condition also boils down to

$$R \geq 1 \,. \tag{IPC2}$$

Taken together, Conditions (IPC1) and (IPC2) imply that no capital maker postpones innovation to any period after the next even one in the case of capital makers in sector 1, or the next odd one in the case of capital makers in sector 2. To see why, note that, by (IPC2), capital makers possessing a new idea, for instance, in sector 1 do not wait until the next even period, whereas by (IPC1) they do not wait until the odd period following this, and so forth, and likewise for capital makers in sector 2. Conditions (IPC1) and (IPC2) *combined* then imply that capital makers in both sectors implement their new ideas as soon as they receive them.

Since the real interest rate is constant, it follows from (16) that $v = \lambda^{\frac{1}{2}}$, whereas (15), (24) and (25) together imply that $\lambda = \mu^{\frac{1}{\alpha}-1}$. The period growth rate of consumption, output and investment is then equal to $\mu^{\frac{1}{2}(\frac{1}{\alpha}-1)}$, and by (16) the real interest rate along the balanced growth path is equal to

$$R = \mu^{\frac{1}{2}(\frac{1}{\alpha}-1)} / \beta ,$$

which increases in the innovation rate, μ, and falls in the subjective discount factor, β.

Since $\mu > 1$, $\alpha \in (0,1)$ and $\beta \in (0,1)$, we have that $R > 1$, which implies that the profit conditions, (IPC1) and (IPC2), and the no-storage Condition 1 are, as desired, always satisfied. In Appendix A.3, I show that the same is true for the transversality condition. The following corollary then ensues:

Corollary 1 (Steady growth)**.** *When patent rights last one period, an immediate implementation (steady-growth) equilibrium is always sustainable for expectations accordingly formed.*

4.1.2 Synchronized implementation equilibrium

In a synchronized implementation equilibrium—of the type I focus on, capital makers in one of the two sectors find it optimal to save their ideas and implement them in the following period, together with the capital makers in the other sector receiving ideas then. With no loss of generality, in the synchronized implementation equilibrium I focus on, I let capital makers in sector 1, which receive ideas in even periods, postpone the implementation of their ideas to the following odd period so that they implement together with the capital makers in sector 2, which receive ideas then.

Period $\tau - 1$. An idea reaches a capital maker in sector 1, which chooses not to implement it, but instead to store it and implement it in period τ. Of course, an unimplemented idea is an idea that has effectively never arrived. Thus, in each sector, capital makers set the same price, given by (10), and produce the same quantity of capital, given by (11), at the same marginal cost as their competitors would. Period τ's capital is, therefore, produced with technologies $\psi_{\tau-1,1,-j^*} = \psi$ and $\psi_{\tau-1,2} = \mu^{\frac{1}{2}} \psi$, and is given by

$$k_{\tau,1} = \left[\frac{(1-\alpha)\,\psi}{R_{\tau-1}}\right]^{\frac{1}{\alpha}} \tag{26}$$

$$k_{\tau,2} = \left[\frac{(1-\alpha)\,\mu^{\frac{1}{2}}\,\psi}{R_{\tau-1}}\right]^{\frac{1}{\alpha}} . \tag{27}$$

Investment in period $\tau - 1$ is given by

$$i_{\tau-1} = \frac{k_{\tau,1}}{\psi} + \frac{k_{\tau,2}}{\mu^{\frac{1}{2}}\,\psi}, \tag{28}$$

which, after using (26) and (27), becomes equal to

$$i_{\tau-1} = \left[1 + \mu^{\frac{1}{2}(\frac{1}{\alpha}-1)}\right] \psi^{\frac{1}{\alpha}-1} \left(\frac{1-\alpha}{R_{\tau-1}}\right)^{\frac{1}{\alpha}} . \tag{29}$$

Since in period $\tau - 2$ the technologies of the competitors of the implementing capital makers were given by $\psi_{t-2,1,-j^*} = \mu^{-1}\psi$ in the case of sector 1, and $\psi_{t-2,2,-j^*} = \mu^{-\frac{1}{2}}\psi$ in the case of sector 2, capital in period $\tau - 1$ is given by

$$k_{\tau-1,1} = \left[\frac{(1-\alpha)\,\mu^{-1}\,\psi}{R_{\tau-2}}\right]^{\frac{1}{\alpha}}$$

$$k_{\tau-1,2} = \left[\frac{(1-\alpha)\,\mu^{-\frac{1}{2}}\,\psi}{R_{\tau-2}}\right]^{\frac{1}{\alpha}} .$$

Output in period $\tau - 1$ is then equal to

$$y_{\tau-1} = \left[\mu^{-(\frac{1}{\alpha}-1)} + \mu^{-\frac{1}{2}(\frac{1}{\alpha}-1)}\right] \psi^{\frac{1}{\alpha}-1} \left(\frac{1-\alpha}{R_{\tau-2}}\right)^{\frac{1}{\alpha}-1} . \tag{30}$$

Finally, market clearing in the final-good market implies that consumption in period $\tau-1$ is equal to

$$x_{\tau-1} = \left[1 + \mu^{\frac{1}{2}(\frac{1}{\alpha}-1)}\right] \psi^{\frac{1}{\alpha}-1} (1-\alpha)^{\frac{1}{\alpha}} \left[\frac{1}{(1-\alpha)\,\mu^{\frac{1}{\alpha}-1}\,R_{\tau-2}^{\frac{1}{\alpha}-1}} - \frac{1}{R_{\tau-1}^{\frac{1}{\alpha}}}\right]. \tag{31}$$

Period τ. In period τ, both sectors implement their new ideas. The technology across capital makers in the two sectors is $\psi_{\tau,1,-j^*} = \psi < \psi_{\tau,1,j^*} = \mu\,\psi$ and $\psi_{\tau,2,-j^*} = \mu^{\frac{1}{2}}\,\psi < \psi_{\tau,2,j^*} = \mu^{\frac{3}{2}}\,\psi$.

Therefore, capital in period $\tau + 1$ is given by

$$k_{\tau+1,1} = \left[\frac{(1-\alpha)\,\psi}{R_\tau}\right]^{\frac{1}{\alpha}} \tag{32}$$

$$k_{\tau+1,2} = \left[\frac{(1-\alpha)\,\mu^{\frac{1}{2}}\,\psi}{R_\tau}\right]^{\frac{1}{\alpha}}. \tag{33}$$

Investment in period τ is given by

$$i_\tau = \frac{k_{\tau+1,1}}{\mu\,\psi} + \frac{k_{\tau+1,2}}{\mu^{\frac{3}{2}}\,\psi},$$

which, after using (32) and (33), becomes equal to

$$i_\tau = \left[\frac{1+\mu^{\frac{1}{2}(\frac{1}{\alpha}-1)}}{\mu}\right]\,\psi^{\frac{1}{\alpha}-1}\,\left(\frac{1-\alpha}{R_\tau}\right)^{\frac{1}{\alpha}}. \tag{34}$$

Combining the production function (5) with (26) and (27) yields output in period τ:

$$y_\tau = \left[1+\mu^{\frac{1}{2}(\frac{1}{\alpha}-1)}\right]\,\psi^{\frac{1}{\alpha}-1}\,\left(\frac{1-\alpha}{R_{\tau-1}}\right)^{\frac{1}{\alpha}-1}. \tag{35}$$

Finally, market clearing in the final-good market implies that

$$x_\tau = \left[1+\mu^{\frac{1}{2}(\frac{1}{\alpha}-1)}\right]\,\psi^{\frac{1}{\alpha}-1}\,(1-\alpha)^{\frac{1}{\alpha}}\,\left[\frac{1}{(1-\alpha)\,R_{\tau-1}^{\frac{1}{\alpha}-1}} - \frac{1}{\mu\,R_\tau^{\frac{1}{\alpha}}}\right]. \tag{36}$$

Period $\tau + 1$. I proceed in the same way as in the case of the previous even period, $\tau - 1$, bearing in mind that technologies in the two sectors now are $\psi_{\tau+1,1,-j^*} = \mu\,\psi$ and

$\psi_{\tau+1,2} = \mu^{\frac{3}{2}}\,\psi$. Confirm then that

$$i_{\tau+1} = \mu^{\frac{1}{\alpha}-1}\left[1+\mu^{\frac{1}{2}(\frac{1}{\alpha}-1)}\right]\psi^{\frac{1}{\alpha}-1}\left(\frac{1-\alpha}{R_{\tau+1}}\right)^{\frac{1}{\alpha}}$$

$$y_{\tau+1} = \left[1+\mu^{\frac{1}{2}(\frac{1}{\alpha}-1)}\right]\psi^{\frac{1}{\alpha}-1}\left(\frac{1-\alpha}{R_{\tau}}\right)^{\frac{1}{\alpha}-1}$$

$$x_{\tau+1} = \left[1+\mu^{\frac{1}{2}(\frac{1}{\alpha}-1)}\right]\psi^{\frac{1}{\alpha}-1}(1-\alpha)^{\frac{1}{\alpha}}\left[\frac{1}{(1-\alpha)\,R_{\tau}^{\frac{1}{\alpha}-1}} - \frac{\mu^{\frac{1}{\alpha}-1}}{R_{\tau+1}^{\frac{1}{\alpha}}}\right]. \tag{37}$$

A two-period implementation cycle of the type that I focus on requires, first, capital makers in sector 1 receiving ideas in the even periods to find it optimal to wait exactly one period before implementing them, and, second, capital makers in sector 2 receiving ideas in the odd periods to find it optimal to implement them immediately. To see when or whether, both requirements are met, I will take the same two steps as in the case of the immediate implementation equilibrium.

Profit condition 1 . A capital maker in sector 1 prefers to implement its idea in the following period to doing so immediately as long as (SPC) holds, that is when

$$R_{\tau-1}^{\frac{1}{\alpha}-1} > R_{\tau}^{\frac{1}{\alpha}}\,. \tag{SPC}$$

As I show in Appendix A.2 , (SPC) boils down to

$$\frac{\mu\,[1\,+\,\beta\,(1-\alpha)]}{\mu\,+\,\beta\,(1-\alpha)} > \frac{1}{\beta}\,. \tag{SPC1}$$

To get a sense of the synchronized implementation condition (SPC1), let $\alpha = 2/3$, in which case (LPC) calls for $\mu \leq 3$, and $\beta = 0.97$. Condition (SPC1), then, requires approximately that $\mu > 1.14$. Condition (SPC1) is more easily satisfied as β increases. This is because a higher subjective discount factor, β, implies that capital makers are more patient, hence more likely to wait. To this end, note that in the limit case where $\beta \to 1$, (SPC1) is always met. A higher innovation rate, μ, implies higher profits, and, in the presence of demand externalities, this is translated into a greater incentive for capital makers to coordinate as long as β is high enough and α low enough. Parameter α parametrizes both the capital share, which is equal to $1-\alpha$, and the elasticity of substitution between the two capital types, which is equal to $1/\alpha$. Greater values of the former and lower of the latter imply that implementation cycles are more easily sustained. As α falls, both increase. It turns out, however, that for high enough values of β and μ the former effect dominates, in which case (SPC1) is more easily satisfied as α falls.

Profit condition 2 . No capital maker must find it optimal to postpone implementation past the two-period cycle. As before, and as Shleifer (1986) suggests, for this to be the case it suffices to show that no capital maker has an incentive to wait until the next implementation period, which here is period $\tau+2$.

Consider, for instance, a capital maker in sector 1 . Given the stationary structure of the economy, if the capital maker in question prefers to implement in period τ rather than in period $\tau-1$, i.e. when Condition (SPC) holds, then it also prefers to postpone implementation from the next even period, which, in this example, is period $\tau+1$, to the next odd period, which is $\tau+2$. In other words, Condition (SPC) implies that capital makers in sector 1 prefer to implement in odd periods. Showing, then, that a capital maker in sector 1 opts to implement in period τ rather than in period $\tau+2$, and, by the same token, in any future odd period, is what we need to complete the consistency requirement of Definition 1 . The following condition, then, must be satisfied in the case of a capital maker in sector 1 :

$$\frac{\Pi_{\tau+1,1}^{\tau-1}}{R_{\tau-1}\,R_{\tau}} \geq \frac{\Pi_{\tau+3,1}^{\tau-1}}{R_{\tau-1}\,R_{\tau}\,R_{\tau+1}\,R_{\tau+2}}\,. \tag{38}$$

Since by stationarity $R_{\tau-1} = R_{\tau+1}$ and $R_{\tau} = R_{\tau+2}$, (38) simplifies to

$$R_{\tau}\,R_{\tau+1} \geq 1\,. \tag{SPC2}$$

Condition (SPC2) implies that profits are discounted at an average positive net real interest rate, and we can think of this as a weak version of the transversality condition. Further, note that the argument which underpins Condition (SPC2) is independent of the fact that a new idea will reach sector 1 in period $\tau+1$, rendering obsolete the idea received in $\tau-1$.

A similar logic and the same as in the case of the immediate implementation equilibrium applies to capital makers in sector 2 . In particular, Conditions (SPC) and (SPC2) together imply that a capital maker in sector 2 finds it optimal to implement immediately.[4]

Combining equations (15) and (16) with equations (31) and (37) implies that

$$\lambda \,=\, \mu^{\frac{1}{\alpha}-1}\,. \tag{39}$$

Since μ is greater than one, λ is greater than one too. This fact combined with Remark 1 and (SPC2), which draws on Remark 1, leads to the following lemma:

Lemma 1. *Condition (SPC2) is always met.*

Given (39), what remains to be pinned down in order to characterize the equilibrium interest rates, given by (16), is v . This is given by

$$v \,=\, \left[\mu^{(\frac{1}{\alpha}-1)^2-1}\,\,\frac{\mu\,+\,\beta\,(1-\alpha)}{1\,+\,\beta\,(1-\alpha)}\right]^{\frac{\alpha}{2-\alpha}}\,, \tag{40}$$

with derivations leading to (40), collected in Appendix A.2 . Observe that v increases in the innovation rate, μ , for α not higher than $2/3$, whereas it falls in the subjective discount factor, β . Numerical simulations reveal, further, that v is not lower than one for values of α not greater than $2/3$. However, v can generally take values lower than one (for instance, when $\mu\,=\,3$, $\beta\,=\,0.99$ and $\alpha\,=\,0.8$). Since v can be lower than one, so can be the real interest rate prevailing in the odd periods, R_τ , in which case the no-storage Condition 1 needs to be binding. Claim 1 below follows directly from eq. (40):

Claim 1. $v \,<\, \mu^{\frac{1}{2}(\frac{1}{\alpha}-1)}$

Proof. See Appendix A.2 . □

[4]To see this, note that in the case of sector 2 profit condition 2 is exactly like (SPC2). Turning to profit condition 1, a capital maker in sector 2 implements immediately when

$$\frac{\Pi^{\tau}_{\tau+1,i}}{R_\tau} \,\geq\, \frac{\Pi^{\tau}_{\tau+2,i}}{R_\tau\,R_{\tau+1}}\,.$$

Substituting for profits, given by (13) , implies that this condition boils down to

$$R_{\tau+1}\left(\frac{R_{\tau+1}}{R_\tau}\right)^{\frac{1}{\alpha}-1} \,\geq\, 1\,,$$

which holds when (SPC) and (SPC2) hold together.

Claim 1 , eq. (39) and the fact that $\beta \in (0, 1)$ imply that the real interest rate in the even periods, $R_{\tau-1}$, (see (16)) is always greater than one, hence, in that case, the no-storage Condition 1 is met. Last, I show in Appendix A.3 that the transversality condition is always met.[5] Proposition 1 then ensues:

Proposition 1. *When patent rights last one period, a synchronized implementation equilibrium can be sustained as long as Condition (SPC1) and the no-storage Condition 1 hold.*

Proposition 1 is essential to this paper. This is because, in sharp contrast with the conjecture made in Shleifer (1986, page 1183), implementation cycles with capital can be generated, and they can be for plausible values of the parameters. Importantly, this happens in the presence of storable commodities, and in the absence of borrowing constraints and investment irreversibilities, which are points Section 6 discusses in detail.

What makes implementation cycles possible is the presence of demand externalities between the two capital-good sectors. The following remark, which follows directly from Claim 1 , bears testimony to their presence:

Remark 2. $R_\tau < R < R_{\tau-1}$.

Since $R_\tau < R$, where R_τ denotes the real interest rate prevailing in the implementation periods in the synchronized implementation equilibrium, and R denotes the real interest rate prevailing in the immediate implementation equilibrium, the capital makers which implement their ideas immediately in both equilibria make greater profits (with profits given by (13)) when they implement their ideas together with a capital maker from the other sector, which is the case in the synchronized implementation equilibrium, rather than when they implement their ideas alone, which is the case in the immediate implementation one. This is true because profits in a certain sector are eventually translated into greater demand for the product of the other sector, leading to greater profits for an innovating capital maker in that sector, which in turn raises demand and profits for the innovating sector in question, and so forth.

[5] This is only because preferences are logarithmic in consumption. For general CRRA preferences, the transversality condition is more easily satisfied as the intertemporal elasticity of substitution falls.

4.2 Equilibria with Two-Period Patent Rights

Let capital makers make monopoly profits out of an idea for two periods instead of one, and leave all the other assumptions unchanged. I take the same steps as in Section 4.1 : I first characterize the immediate implementation equilibrium (Section 4.2.1), and I subsequently turn to the synchronized implementation one (Section 4.2.2).

4.2.1 Immediate implementation equilibrium

As in Section 4.1.1, I characterize the aggregate variables for period $\tau - 1$, relegating the analysis for periods τ and $\tau+1$ to Appendix B.2 . I subsequently specify the profit conditions, and go on to show that an immediate implementation equilibrium when patent rights last two periods always exists.

Period $\tau-1$. A capital maker in sector 1 receives an idea, which it implements immediately, whereas the capital maker in sector 2 which implemented its idea in the previous period, $\tau - 2$, enters the second and last period of its monopoly. The technology levels of the capital makers in the two sectors are $\psi_{\tau-1,1,-j^*} = \psi < \psi_{\tau-1,1,j^*} = \mu\,\psi$ and $\psi_{\tau-1,2,-j^*} = \mu^{-\frac{1}{2}}\,\psi < \psi_{\tau-1,2,j^*} = \mu^{\frac{1}{2}}\,\psi$.

Capital in period τ is then given by

$$k_{\tau,1} \;=\; \left[\frac{(1-\alpha)\,\psi}{R_{\tau-1}}\right]^{\frac{1}{\alpha}} \tag{41}$$

$$k_{\tau,2} \;=\; \left[\frac{(1-\alpha)\,\mu^{-\frac{1}{2}}\,\psi}{R_{\tau-1}}\right]^{\frac{1}{\alpha}} . \tag{42}$$

Given the technology levels of the implementing capital makers, indexed by j^* as usual, investment is given by

$$i_{\tau-1} \;=\; \frac{k_{\tau,1}}{\mu\,\psi} \;+\; \frac{k_{\tau,2}}{\mu^{\frac{1}{2}}\,\psi} . \tag{43}$$

Plugging (41) and (42) into (43) implies that investment in period $\tau - 1$ is equal to

$$i_{\tau-1} \;=\; \left[\mu^{-1} \;+\; \mu^{-\frac{1}{2}\left(\frac{1}{\alpha}+1\right)}\right]\,\psi^{\frac{1}{\alpha}-1}\,\left(\frac{1-\alpha}{R_{\tau-1}}\right)^{\frac{1}{\alpha}} . \tag{44}$$

Along the same lines, since $\psi_{\tau-2,1,-j^*} = \mu^{-1}\,\psi$ and $\psi_{\tau-2,2,-j^*} = \mu^{-\frac{1}{2}}\,\psi$, capital in period $\tau - 1$ is given by

$$k_{\tau-1,1} = \left[\frac{(1-\alpha)\,\mu^{-1}\,\psi}{R_{\tau-2}}\right]^{\frac{1}{\alpha}}$$

$$k_{\tau-1,2} = \left[\frac{(1-\alpha)\,\mu^{-\frac{1}{2}}\,\psi}{R_{\tau-2}}\right]^{\frac{1}{\alpha}} .$$

I take familiar steps to find output and consumption, which are given respectively by

$$y_{\tau-1} = \left[\mu^{-\left(\frac{1}{\alpha}-1\right)} + \mu^{-\frac{1}{2}\left(\frac{1}{\alpha}-1\right)}\right]\,\psi^{\frac{1}{\alpha}-1}\,\left(\frac{1-\alpha}{R_{\tau-2}}\right)^{\frac{1}{\alpha}-1} \tag{45}$$

$$x_{\tau-1} = \psi^{\frac{1}{\alpha}-1}\,(1-\alpha)^{\frac{1}{\alpha}}\left[\frac{\mu^{-\left(\frac{1}{\alpha}-1\right)} + \mu^{-\frac{1}{2}\left(\frac{1}{\alpha}-1\right)}}{(1-\alpha)\,R_{\tau-2}^{\frac{1}{\alpha}-1}} - \frac{\mu^{-1} + \mu^{-\frac{1}{2}\left(\frac{1}{\alpha}+1\right)}}{R_{\tau-1}^{\frac{1}{\alpha}}}\right] . \tag{46}$$

Turning to the profit conditions, as each capital maker can make profits from an idea for two periods, profit condition 1 in the case of a capital maker receiving a patent in, say, period $t-1$ is

$$\frac{\Pi_t}{R_{t-1}} + \frac{\Pi_{t+1}}{R_{t-1}\,R_t} \geq \frac{\Pi_{t+1}}{R_{t-1}\,R_t} + \frac{\Pi_{t+2}}{R_{t-1}\,R_t\,R_{t+1}} .$$

Profits in the above expression are expressed in period $t-1$ terms. The LHS refers to discounted profits made when an idea is implemented immediately, which is period $t-1$ in our example, whereas the RHS refers to discounted profits made when an idea is implemented in the following period, which is period t in our example.

Likewise, profit condition 2 is

$$\frac{\Pi_t}{R_{t-1}} + \frac{\Pi_{t+1}}{R_{t-1}\,R_t} \geq \frac{\Pi_{t+2}}{R_{t-1}\,R_t\,R_{t+1}} + \frac{\Pi_{t+3}}{R_{t-1}\,R_t\,R_{t+1}\,R_{t+2}} .$$

In the steady-growth equilibrium, investment, output and consumption all grow at the same, constant over time, rate equal to $\mu^{\frac{1}{2}\left(\frac{1}{\alpha}-1\right)}$, and the time-invariant interest rate is equal to $R = \mu^{\frac{1}{2}\left(\frac{1}{\alpha}-1\right)}/\beta$. As a result, period profits remain constant too, which in turn implies that both profit conditions boil down to $R \geq 1$.

With the profit, no-storage, and transversality conditions always satisfied, Corollary 2 ensues:

Corollary 2. *An immediate implementation equilibrium can always be sustained when patent rights last two periods.*

4.2.2 Synchronized implementation equilibrium

In the synchronized implementation equilibrium of the type I considered in Section 4.1.2, a capital maker in sector 1 postpones the implementation of its idea to the following odd period so that it can implement it simultaneously with a capital maker from sector 2. This time, I directly start with the profit conditions, and I subsequently show that a synchronized implementation equilibrium is never sustainable when rights over an idea last two periods.

A capital maker in sector 1 which receives an idea in, say, period $\tau - 1$ prefers to implement it in the following period, period τ, to doing so immediately if

$$\frac{\Pi_{\tau+1}}{R_{\tau-1}\, R_{\tau}} + \frac{\Pi_{\tau+2}}{R_{\tau-1}\, R_{\tau}\, R_{\tau+1}} > \frac{\Pi_{\tau}}{R_{\tau-1}} + \frac{\Pi_{\tau+1}}{R_{\tau-1}\, R_{\tau}} . \tag{47}$$

The first term on the LHS and the second term on the RHS cancel out so that (47) becomes

$$\frac{\Pi_{\tau+2}}{R_{\tau-1}\, R_{\tau}\, R_{\tau+1}} > \frac{\Pi_{\tau}}{R_{\tau-1}} . \tag{48}$$

We can see from (48) that postponing implementation to the following period when the monopoly horizon is two periods is equivalent to postponing implementation to two periods afterwards when the monopoly horizon is one period.

Since, in a stationary equilibrium, interest rates remain constant, controlling for the period being odd or even, it follows that $\Pi_{\tau} = \Pi_{\tau+2}$. This implies that (48) becomes

$$R_{\tau}\, R_{\tau+1} < 1 . \tag{SPC1'}$$

Turning to profit condition 2, this requires

$$\frac{\Pi_{\tau+1}}{R_{\tau-1}\, R_{\tau}} + \frac{\Pi_{\tau+2}}{R_{\tau-1}\, R_{\tau}\, R_{\tau+1}} \geq \frac{\Pi_{\tau+3}}{R_{\tau-1}\, R_{\tau}\, R_{\tau+1}\, R_{\tau+2}} + \frac{\Pi_{\tau+4}}{R_{\tau-1}\, R_{\tau}\, R_{\tau+1}\, R_{\tau+2}\, R_{\tau+3}} . \tag{49}$$

By stationarity, one can confirm that (49) simplifies to

$$\left(1 - \frac{1}{R_{\tau+1}\, R_{\tau+2}}\right) \left(\Pi_{\tau+1} + \frac{\Pi_{\tau+2}}{R_{\tau+1}}\right) \geq 0 .$$

However, in equilibrium the gross real interest rate cannot be less than one as this would violate the no-storage condition 1, let alone negative as this would imply that profits are negative, with the non-negativity of profits ensured by the fact that capital makers would rather not implement an idea if it were to lead to negative profits. Taking these claims and stationarity into account implies that the last expression further simplifies to

$$R_\tau \, R_{\tau+1} \, \geq \, 1 \,, \tag{SPC2'}$$

which is the same as (SPC2) in Section 4.1.2 .

It is now clear that only the empty set satisfies the two profit conditions, (SPC1') and (SPC2'), and this is what Proposition 2 reflects:

Proposition 2. *A synchronized implementation equilibrium can never be sustained when patent rights last two periods.*

All is now set to address the main question of this article: can a prolongation of patent rights be desirable?

5 Welfare

A first remark is that, due to the periodic presence of monopolies in the capital-good markets, all equilibria are suboptimal.[6] Importantly, though, the equilibria can be welfare-ranked.

The remaining steps to answer this article's question are the following ones: in Section 5.1, I specify the representative agent's lifetime utility for both the two types of equilibria; in Section 5.2 , I welfare-rank the equilibria which can prevail when patent rights last one period (henceforth, "one-period-patent equilibria"), and I conclude in Section 5.3 with the welfare comparison between the one-period-patent equilibria and the equilibrium prevailing

[6]Since the implementation of a patent improves the technology of an implementing firm's competitors, one would suggest that externalities constitute an additional source of inefficiency. My argument why this is not true is the same as the one in Shleifer (1986) (in particular, see Shleifer (1986, page 1178) and fn. 10 there). In principle, this potential problem could be corrected for by allowing for decentralized markets on a one-to-one basis between the firm possessing an idea and one of its competitors, with the former setting the price in exchange for sharing the rights to its idea with the latter (we could equivalently think in terms of bilateral contracts of a "take-it-or-leave-it" type). The presence of constant returns to scale in the capital-good's technology implies that competitors, which behave symmetrically, will demand zero at any positive price, since they will afterwards Bertrand-compete at least with the firm owning the patent, which, in turn, is not willing to propose a zero price. Hence, markets would clear at positive and small enough prices, so that demand and supply are both equal to zero in each period. This proves that externalities do not constitute an additional source of inefficiency.

when patent rights last two periods (henceforth, "two-period-patent equilibrium"). Out of this last welfare comparison emerges this article's central result (Proposition 3).

5.1 Equilibrium lifetime utilities

The representative agent's lifetime utility is given by (1).

In the immediate implementation equilibrium, the agent's lifetime utility, which I denote by U_i, is equal to

$$U_i = \log x_0^i + \beta \left(\log \mu^{\frac{1}{2}(\frac{1}{\alpha}-1)} x_0^i \right) + \beta^2 \left(\log \mu^{\frac{1}{\alpha}-1} x_0^i \right) + \dots ,$$

where I have used x_0^i to denote consumption in period 0 in the immediate implementation equilibrium, and I have taken into account that consumption grows in each period by $\mu^{\frac{1}{2}(\frac{1}{\alpha}-1)}$. It is easy to confirm that the above expression boils down to

$$U_i = \frac{1}{1-\beta} \left[\frac{\beta}{2(1-\beta)} \log \mu^{\frac{1}{\alpha}-1} + \log x_0^i \right]. \tag{50}$$

To distinguish between the immediate implementation equilibrium when patent rights last one period and the immediate implementation equilibrium when patent rights last two periods, I will use U_i to denote lifetime utility in the former and $\tilde{U}_i$ to denote lifetime utility in the latter, and likewise x_0^i and $\tilde{x}_0^i$ to distinguish between initial consumption in the two equilibria. The agent's lifetime utility in the two-period-patent, immediate implementation equilibrium is then given by

$$\tilde{U}_i = \frac{1}{1-\beta} \left[\frac{\beta}{2(1-\beta)} \log \mu^{\frac{1}{\alpha}-1} + \log \tilde{x}_0^i \right]. \tag{51}$$

In the synchronized implementation equilibrium of the type I considered in Section 4.1.2, the lifetime utility of the agent, which I denote as U_s, is given by

$$U_s = \log x_0^s + \beta \left(\log \frac{\mu^{\frac{1}{\alpha}-1}}{v} x_0^s \right) + \beta^2 \left(\log \mu^{\frac{1}{\alpha}-1} x_0^s \right) + \dots ,$$

where v is given by (40). In this expression, I have taken into account that consumption grows as (14) and (15) prescribe, and I have used x_0^s to denote consumption in period 0 of the synchronized implementation equilibrium. This last expression simplifies to

$$U_s = \frac{1}{1-\beta} \left(\frac{\beta}{1-\beta^2} \log \mu^{\frac{1}{\alpha}-1} - \frac{\beta}{1+\beta} \log v + \log x_0^s \right). \tag{52}$$

To make welfare comparisons, simply subtract (52) from (50) to get

$$U_i - U_s = \frac{1}{1-\beta}\left[\frac{\beta}{1+\beta}\left(\log v - \log \mu^{\frac{1}{2}(\frac{1}{\alpha}-1)}\right) + \log x_0^i - \log x_0^s\right], \tag{53}$$

and switch notation to $\tilde{U}_i$ and $\tilde{x}_0^i$ when referring to the two-period-patent, immediate implementation equilibrium.

5.2 Welfare comparison between the one-period-patent equilibria

For all the considered parametrizations, welfare is greater in the immediate implementation equilibrium than in the synchronized implementation one (assuming that the parameters satisfy (SPC1) and Condition 1 so that the latter exists as well as Condition (LPC)). To show why, I will draw a distinction between the welfare difference attributed to the different initial consumption levels in the two equilibria (or equivalently, the consumption levels at the beginning of an implementation cycle) captured by the third and the fourth terms in eq. (53) and the welfare difference attributed to the different consumption growth rates, captured by the first two terms.

The latter, by Claim 1, takes a negative value. To see why, first note that the growth rate every two periods is the same in the two equilibria because all ideas are eventually implemented. It has to be the case then that the welfare difference attributed to the different consumption growth rates in the two equilibria is entirely attributable to the growth rate difference *within* an implementation cycle, which is precisely what the first two terms in eq. (53) capture. Claim 1, then, attests that consumption grows faster in the synchronized implementation equilibrium. Maintaining the assumption that an implementation cycle starts in an even period, and controlling for the differences in the initial consumption levels, which I deal with next, this means that consumption in odd periods is higher in the synchronized implementation equilibrium than in the immediate implementation one. This is because in a synchronized implementation equilibrium both sectors implement their ideas in odd periods, which causes investment to drop by more compared to when, as in the immediate implementation equilibrium, only one sector innovates, leaving thereby a greater amount of output available for consumption (confirm this by juxtaposing (34) and (68), and (35) and (69)).

Turning to the initial consumption levels, for all the parametrizations considered, initial consumption is higher in the immediate implementation equilibrium. More precisely, it turns out that in the immediate implementation equilibrium initial output is greater and, for high enough values of α, initial investment lower relative to their respective levels in the synchronized implementation equilibrium. Investment is lower in the immediate implementation equilibrium (compare (20) with (29)) because one capital maker (from sector 1) implements an idea as opposed to none doing so in the synchronized implementation one, whereas output in the immediate implementation equilibrium is greater (compare (23) with (30)) because ideas in sector 1 are implemented and thus diffuse to the economy faster (one period ahead), which leads more quickly to a greater level of capital of type 1, and thereby to a greater level of output.

It turns out that the welfare difference attributable to the different initial consumption levels drives this welfare comparison, hence welfare is higher in the immediate implementation equilibrium. That said, it is useful to add that the welfare difference between the two equilibria is increasing in the innovation rate μ and the discount factor β.[7]

5.2.1 Initial levels of investment

The analysis so far has completely abstracted from the initial amount of capital needed in each equilibrium, where by "initial amount of capital" one could equivalently think of the amount of capital that would have been installed at time 0 if time had commenced at $-\infty$. This is what I will now take care of.

Since there are two types of capital, it is more convenient instead to pin down the level of initial investment required by each equilibrium, as in this way comparisons will be made in output terms. To pin down the level of initial investment, I set $\psi = 1$ and divide the RHS

[7] I have not explicitly explored welfare in the case of a synchronized implementation equilibrium in which implementation booms take place in the even periods. Such an equilibrium would be symmetric to the synchronized implementation equilibrium considered here, and, since time here is assumed to commence in an even period, it is safe to expect that the welfare results discussed would be overturned.

of both (68) and (34) by $\mu^{\frac{1}{\alpha}-1}$. This yields respectively

$$i^i_{-1} \,=\, \left[1 \,+\, \mu^{-\frac{1}{2}(\frac{1}{\alpha}+1)}\right] \left(\frac{1-\alpha}{R}\right)^{\frac{1}{\alpha}} \tag{54}$$

$$i^s_{-1} \,=\, \left[\mu^{-\frac{1}{\alpha}} \,+\, \mu^{-\frac{1}{2}(\frac{1}{\alpha}+1)}\right] \left(\frac{1-\alpha}{R_\tau}\right)^{\frac{1}{\alpha}}, \tag{55}$$

where it should be recalled that $R \,=\, \mu^{\frac{1}{2}(\frac{1}{\alpha}-1)} \,/\, \beta$ and $R_\tau \,=\, v \,/\, \beta$ with v given by (40).

For all the parametrizations considered, initial investment is greater in the immediate implementation equilibrium. As a result, the two equilibria can only be welfare-ranked conditional on their initial investment difference.

5.3 Welfare comparison between equilibria with different patent horizons

In this section, I compare the agent's lifetime utility in the two-period-patent immediate implementation equilibrium given by (51) with the agent's lifetime utility in the one-period-patent equilibria. First, with the agent's lifetime utility in the one-period-patent immediate implementation equilibrium given by (50) (Section 5.3.1), and subsequently with the agent's lifetime utility in the synchronized implementation equilibrium given by (52) (Section 5.3.2). Out of this last comparison follows this article's central result (Proposition 3).

5.3.1 Welfare comparison between the immediate implementation equilibria

The difference in lifetime utilities is given by

$$U_i \,-\, \tilde{U}_i \,=\, \frac{1}{1-\beta}\,\left(\log x^i_0 \,-\, \log \tilde{x}^i_0\right). \tag{56}$$

I show in Appendix A.4 that $x^i_0 \,>\, \tilde{x}^i_0$, that is initial consumption is always greater in the one-period-patent equilibrium, which should come as no surprise, especially since the sequence of interest rates is the same in the two equilibria: ideas might be implemented at the same time in both equilibria, but in the two-period-patent equilibrium capital makers can profit from them for one additional period, which implies that ideas become available to the implementing capital makers' competitors and hence to the economy with a one-period delay. As a result, the economy reaches any given consumption level more slowly.

5.3.2 Welfare comparison between the two-period-patent immediate implementation equilibrium and the one-period-patent synchronized implementation one

The difference in lifetime utilities is given by

$$\tilde{U}_i - U_s = \frac{1}{1-\beta}\left[\frac{\beta}{1+\beta}\left(\log v - \log \mu^{\frac{1}{2}\left(\frac{1}{\alpha}-1\right)}\right) + \log \tilde{x}_0^i - \log x_0^s\right]. \tag{57}$$

I show below that what characterizes the welfare comparison between the two equilibria is the trade-off between the earlier implementation of some (sector-1) ideas in the two-period-patent immediate implementation equilibrium and the earlier diffusion of some other ones (the sector-2 ones) in the one-period-patent synchronized implementation equilibrium. It turns out that, for agents patient enough, there is no dominant effect, and in particular for sufficiently low values of the innovation rate, μ, welfare is greater in the two-period-patent equilibrium (see also Figure 1).

I take the same steps as in Section 5.2. First, as in Section 5.2 by Claim 1, the difference between the consumption growth terms takes a positive value, which favors the synchronized implementation equilibrium. Unlike in Section 5.2, this is the case here because odd/implementation-period output in the synchronized implementation equilibrium is greater than in the two-period-patent immediate implementation equilibrium (juxtapose (35) with (75)), which in turn is because ideas in sector 2 diffuse faster in the synchronized implementation one-period-patent equilibrium. Odd/implementation-period investment, however, and after controlling for interest rate differences, is the same in the two equilibria (juxtapose (34) with (74)).

While consumption growth favours the (one-period-patent) synchronized implementation equilibrium, the difference in the initial consumption levels, which are given by (31) and (46), favors the two-period-patent immediate implementation equilibrium, and this is what I shed light on below. To do so, I will again need to decompose initial consumption into initial output and initial investment.

Starting with the initial output levels, we can see from (30) and (45), after letting $\psi = 1$, that

$$\Delta y \equiv \tilde{y}_0^i - y_0^s = \left[\mu^{-\left(\frac{1}{\alpha}-1\right)} + \mu^{-\frac{1}{2}\left(\frac{1}{\alpha}-1\right)}\right](1-\alpha)^{\frac{1}{\alpha}-1}\left(\frac{1}{R^{\frac{1}{\alpha}-1}} - \frac{1}{R_\tau^{\frac{1}{\alpha}-1}}\right). \tag{58}$$

By Remark 2, $R_\tau < R$, which implies that the last term is negative, hence initial output in the (one-period-patent) synchronized implementation equilibrium is greater than in the two-period-patent immediate implementation one. This outcome should have been expected because ideas on capital of type 2 might be implemented simultaneously in the two equilibria, but in the case of the two-period-patent equilibrium, they diffuse to all capital makers in sector 2 one period later, which is a point I elaborate on below. Furthermore, for high enough values of α the initial output difference grows in the innovation rate μ.

Turning to initial investment levels, we can see from (29) and (44) that

$$\Delta i \equiv \tilde{i}_0^i - i_0^s = \left[\mu^{-1} + \mu^{-\frac{1}{2}\left(\frac{1}{\alpha}+1\right)}\right](1-\alpha)^{\frac{1}{\alpha}}\left[\frac{1}{R^{\frac{1}{\alpha}}} - \frac{\mu^{\frac{1}{2}\left(\frac{1}{\alpha}+1\right)}}{R_{\tau-1}^{\frac{1}{\alpha}}}\right]. \tag{59}$$

For all the parametrizations considered, the last term is negative, and for high enough values of α the initial investment difference grows in μ.

It follows, then, from the above that both initial output and initial investment are greater in the synchronized implementation one-period-patent equilibrium relative to the two-period-patent immediate implementation one. Starting with the initial output differences, or equivalently differences in the level of output at the beginning of an implementation cycle, they are entirely attributable to differences in the interest rates in the two equilibria, which favor the synchronized implementation one-period-patent equilibrium, rather than to directly technology/implementation-related reasons. This merely reflects the fact that in the odd periods, which precede the beginning of an implementation cycle, and which are when the following (even) period's capital is built, the technology level of the implementing capital makers' competitors is the same in both equilibria.

On the other hand, the initial investment differences can be accounted for not only by general equilibrium effects but, crucially, by technology/implementation-related differences too. To see this, note first that ideas in sector 2 diffuse to the economy faster in the (one-period-patent) synchronized implementation equilibrium, and, second, ideas in sector 1 are implemented earlier in the two-period-patent, immediate implementation equilibrium, with their implementation crucially taking place in the even periods. Both remarks imply that initial investment is lower in the two-period-patent immediate implementation, equilibrium.

In order to analyze these two remarks, it needs to be clear how the *appropriation* and how the *diffusion* of an idea affect investment. When a capital maker appropriates an idea on some type of capital, then, abstracting from interest-rate considerations, it produces the same amount of capital as in the period immediately before the idea had first been implemented, but at a lower cost. In other words, when an idea on some type of capital is appropriated, investment in this type of capital is lower than in the period immediately preceding the idea's implementation period. However, when an idea passes on to all capital makers in a sector, perfect competition prevails in that sector and therefore investment in the type of capital the idea relates to can do nothing but increase—and increase the following period's output too.

To see the first remark, simply juxtapose (27) and (42), and, controlling for the interest rate differences, confirm that more capital of type 2 needs to be installed in the synchronized implementation equilibrium in the next odd period—which follows the (even) period marking the start of a new cycle. This is because even though sector-2 ideas are implemented simultaneously in the two equilibria capital makers appropriate them for one extra period in the two-period-patent one, and as a result they become available to the implementing capital makers' competitors with a one-period delay—which, of course, as discussed above, has an adverse affect on the following period's output. To see the second remark, simply recall that sector-1 ideas are implemented immediately in the two-period-patent equilibrium, whereas they are with a one-period delay in the synchronized implementation, one-period-patent one (confirm this by juxtaposing (28) and (43)). Controlling for differences in the interest rates, investment is then lower in the two-period-patent immediate implementation equilibrium because of the combined benefits (solely from an investment point of view) of the prolonged appropriation, or, equivalently, belated diffusion, of one idea (first remark) and the earlier implementation of another one (second remark).

Since "technology" reasons favour the two-period-patent immediate implementation equilibrium, then, as one would expect, the interests rates—in the case of the synchronized implementation one-period-patent equilibrium—would adjust so as to smooth out the prevailing initial consumption differences between the two equilibria. The different interest rates do not overturn the above-discussed implementation/technology-caused implications for initial investment, but, as already discussed, the interest rates do move initial output in the opposite direction, which favors the one-period-patent synchronized implementation equilibrium. Is there a dominant effect on initial consumption? For all the parametrizations considered, which, of course, need to satisfy (SPC1) and Condition 1 so that a synchronized implementation equilibrium can be sustained, as well as Condition (LPC), the answer is that the "investment effect" dominates, hence initial consumption is always higher in the case of the two-period-patent immediate implementation equilibrium. It is crucial, though, that for plausible parametrizations which include values of β close to 1 and values of α around 2/3 there is a threshold value of μ past which a greater innovation rate decreases the initial consumption difference.

Will the faster diffusion of (sector-2) ideas in the one-period-patent synchronized implementation equilibrium, or the earlier implementation of (sector-1) ideas in the two-period-patent, immediate implementation equilibrium dominate? For sufficiently low values of μ (and not too impatient agents), the two-period-patent immediate implementation equilibrium is welfare-superior to the one-period-patent synchronized implementation one—assuming, of course, that the parameters are such that the latter is sustainable. For instance, when $\alpha = 2/3$ and $\beta = 0.97$ the threshold value in the welfare comparison is $\mu^{**} \simeq 2.24$ (see also Figure 1), which suggests that for empirically plausible parametrizations welfare in the two-period-patent immediate implementation equilibrium exceeds welfare in the one-period-patent synchronized implementation equilibrium.

Proposition 3 states the conclusion of the above analysis:

Proposition 3. *For a sufficiently low innovation rate, a prolongation of patent rights can be welfare-improving.*

5.3.3 Initial level of investment

As in Section 5.2.1 , to pin down the initial level of investment in the two-period-patent, immediate implementation equilibrium, I set $\psi = 1$ and divide the RHS of (74) by $\mu^{\frac{1}{\alpha}-1}$, which yields

$$\tilde{i}^i_{-1} = \left[\mu^{-\frac{1}{\alpha}} + \mu^{-\frac{1}{2}(\frac{1}{\alpha}+1)}\right] \left(\frac{1-\alpha}{R}\right)^{\frac{1}{\alpha}} . \tag{60}$$

After comparing (60) with (54) and (55), it is easy to confirm that initial investment in the two-period-patent equilibrium is lower than in both one-period-patent equilibria. We can therefore conclude that the welfare improvement to which a prolongation of patent rights can lead is unconditional on, and therefore robust to, differences in the initial level of investment.

The next, and final, section of this article elaborates on Proposition 1 which appears in Section 4.1.2 , and concludes by suggesting directions for future work.

6 Discussion

The importance of Proposition 1 should by now be clear. It states that implementation cycles with capital are possible, and they are so in the presence of storable commodities, and in the absence of borrowing constraints and investment irreversibilities. What might be less clear, though, is how and why implementation cycles with capital are possible, especially since this comes in sharp contrast with the conjecture made in Shleifer (1986, page 1183). Following Shleifer's line of thought, one would expect that in the prospect of future profits the representative agent would like to reduce current savings and thereby future capital stock in order to smooth out consumption across periods. This would be accompanied by a (real) interest rate increase, necessary to limit the agent's borrowing in the period before the wealth expansion. Both effects would imply that the capital makers' present discounted profits in an implementation boom would fall, which could eventually eliminate their incentives to postpone the implementation of their ideas until then.

This intuition does not apply here. This is because, in sharp contrast with Shleifer (1986), in which innovations are sector-neutral, i.e. they enhance total factor productivity (TFP), innovations here are investment-specific, as in Greenwood et al. (1997, 2000). This difference in modelling technological change is important. Unlike changes in TFP, investment-specific technological changes introduce a one-period discrepancy between the date capital makers invest and the date they receive their revenue. As a result of this time discrepancy, a coordinated implementation of ideas implies a concurrent considerable fall in savings/investment—in fact, investment can even undershoot—due to the reduced cost of producing capital, and a considerable increase in the wealth of agents in the following period. The former implies that consumption grows considerably—by Claim 1, above trend—in an implementation boom, even after taking into account the effects an implementation boom has on output. The latter implies that the consumption boom takes place *before* the wealth boom. This not only eliminates the need to smooth consumption away from the wealth boom to the period before it, but, on the contrary, it implies that the interest rate linking the implementation period to the one when revenue is realized and wealth expands actually falls (Remark 2). As a result, capital makers discount future profits less rather than more. The fall in the real interest rate, in turn, brings about two interrelated effects. First, it pushes investment in the implementation period upwards—without overturning, though, the implementation effect discussed above as Claim 1 is a general equilibrium result—smoothing out, thereby, consumption, but in the opposite direction from the conjectured one. Second, it leads to an increase in the capital stock in the period in which revenue is realized relative to its level when implementation takes place. To see this, compare equations (26)-(27) with (32)-(33) using Remark 2 . With the demand for capital being elastic, this implies that the profits that capital makers make following an implementation boom grow rather than fall relative to the profits they would have made had they implemented alone. Taking everything into account, present discounted profits increase, which makes implementation cycles with capital possible, leading to Proposition 1 .[8]

Those said, the full capital depreciation which I have assumed throughout effectively rules out disinvestment, and it could therefore be argued that I have imposed investment irreversibilities, which according to Shleifer (1986, page 1183) could render implementation

[8]Francois and Lloyd-Ellis (2008) also generate implementation cycles with capital which can further be sustained as the unique equilibrium outcome. Their model is quite different from the one here, and I will illustrate two key differences. First, a central assumption the authors make is that ideas arrive after firms have incurred an endogenous search cost. An implication of this assumption is that ideas reach all sectors simultaneously, which comes in sharp contrast with their perfectly smooth rate of arrival in Shleifer (1986) and here: if ideas arrive periodically, they can more easily be implemented in cycles too. A second key difference is that in Francois and Lloyd-Ellis (2008) ideas are not on the technology of capital, but, instead, they are on the technology of intermediate goods, which only uses labor as input. Consequently, the wealth and the consumption booms happen simultaneously. Here, ideas affect the investment technology, which, as explained in the main text, naturally introduces a one-period discrepancy between the consumption and the wealth booms.

cycles with capital possible. My interpretation, though, of Shleifer's argument is that what he, in fact, expects to rule implementation cycles with capital out is too volatile investment. By allowing here for full capital depreciation, I maximize the investment volatility since new capital needs to be installed every period, and it is precisely the excessive investment volatility that renders implementation cycles with capital possible here.

To conclude, this paper has employed a macro model to answer a classic micro question, namely whether prolonged patent rights are desirable. The exogenous generation of ideas has allowed it to abstract entirely from the effects of patent rights on the generation of patents, and to instead focus on the effects of patent rights on the implementation of ideas, or equivalently their period of secrecy. It has shown that a prolongation of patent rights leads to a faster implementation of ideas, which, importantly, can increase welfare in the case of not too radical ideas.

The, arguably, highly stylized model employed certainly has its limitations. From a macro perspective, consumption seems too volatile, whereas, from a more theoretical perspective, the analysis has been silent about transitional dynamics. Addressing these issues could be a direction for future work.

A Appendix

A.1 Proof of Condition 1 (No storage) :

There are two types of storable commodities in the economy, the capital goods and the final good. In both cases, I assume that the storage technology is one-to-one. I discuss in detail below no storage for the case of capital goods, and one can easily show that the no-storage condition is the same in the case of the final good.

Assuming that capital depreciates only if used, suppose that capital maker j produces an additional unit of capital good i in period $t-1$, and, instead of selling it to the final-good firm in period t, it instead stores and sells it in period $t+1$. The cost of producing it as of $t-1$ is, say, $\frac{1}{\psi}$, whereas the revenue generated out of it as of $t-1$ is $\frac{q_{t+1}}{R_t\, R_{t-1}}$, where q is the competitive price offered by the final-good firm, given by (10). No storage takes place if

$$\frac{q_{t+1}}{R_t\, R_{t-1}} < \frac{1}{\psi}\,.$$

Since by period $t{+}1$ the idea will have diffused to the capital maker in question's competitors, the competitive price will prevail. That is, $q_{t+1} = \frac{R_t}{\psi}$, and the above condition boils down to

$$R_{t-1} > 1\,. \tag{61}$$

If, instead, the capital maker in question considers selling an additional unit of capital in period $t+2$, and ignoring the possibility that a new idea will further lower the capital-good price, the no-storage condition becomes

$$R_t\, R_{t-1} > 1\,.$$

It follows then that (61) suffices to rule out storage in this case too, and, by the same token, in all cases involving periods after period $t+2$. Positive net interest rates, therefore, rule out storage in equilibrium.

In the above arguments, I have assumed that a capital maker sells at least an infinitesimally small quantity of the capital good it specializes in in period t, which implies that, in the case that it possesses and implements a new idea, that idea becomes publicly available in t, so that its competitors copy it, and the competitive price prevails in $t+1$. It is profit conditions 1 and 2 in the main text that deal with the possibility that a capital maker possesses a new idea, which it may consider not implementing immediately. In other words, the profit conditions and the no-storage condition act in a somewhat complementary way. The former ensure that a capital maker implements an idea, and, since it maximizes profits, meets the whole demand for the type of capital it specializes in when it is conjectured to do so, whereas the latter rules out the possibility that a capital maker produces an additional amount of capital, which it stores in order to sell in the future.

A.2 Derivations in Section 4.1.2 :

Profit condition (SPC1) . By stationarity, $\frac{x_{\tau+1}}{x_\tau} = v$ and $R_{\tau-1} = R_{\tau+1}$. Using (36) and (37) for x_τ and $x_{\tau+1}$ respectively yields

$$\frac{1}{(1-\alpha)\,R_\tau^{\frac{1}{\alpha}-1}} - \frac{\mu^{\frac{1}{\alpha}-1}}{R_{\tau-1}^{\frac{1}{\alpha}}} = v\left[\frac{1}{(1-\alpha)R_{\tau-1}^{\frac{1}{\alpha}-1}} - \frac{1}{\mu R_\tau^{\frac{1}{\alpha}}}\right]. \tag{62}$$

Rearranging terms in (62) yields

$$\frac{1}{1-\alpha} + \frac{v}{\mu R_\tau} = \left(\frac{R_\tau}{R_{\tau-1}}\right)^{\frac{1}{\alpha}-1}\left(\frac{\mu^{\frac{1}{\alpha}-1}}{R_{\tau-1}} + \frac{v}{1-\alpha}\right). \tag{63}$$

Combine (63) with (16), taking into account that $\lambda = \mu^{\frac{1}{\alpha}-1}$, to get

$$R_{\tau-1}^{\frac{1}{\alpha}-1} = R_\tau^{\frac{1}{\alpha}}\left\{\frac{\beta\,\mu\,[1+\beta(1-\alpha)]}{\mu+\beta(1-\alpha)}\right\}. \tag{64}$$

Profit condition (SPC1) requires

$$R_{\tau-1}^{\frac{1}{\alpha}-1} > R_\tau^{\frac{1}{\alpha}}.$$

Using, then, (64) leads directly to profit condition (SPC1) .

Derivation of (40) . Substituting for the interest rates given by (16) into (64) using (39), and solving for v leads to (40).

Proof of Claim 1 . Since the fraction term in (40) divided by μ is lower than one, to prove that $v < \mu^{\frac{1}{2}(\frac{1}{\alpha}-1)}$, it suffices to show that

$$\mu^{(\frac{1}{\alpha}-1)^2\,\frac{\alpha}{2-\alpha}} < \mu^{\frac{1}{2}(\frac{1}{\alpha}-1)}.$$

It is easy to confirm that this is always true.

A.3 Transversality condition :

The transversality condition (TVC) requires the agent's present-discounted lifetime wealth to converge. I explore this here in the case of the synchronized implementation equilibrium, and one could follow the same process for the immediate implementation equilibrium and for different patent horizons, with the final conclusion always being that the TVC always holds.

The present-discounted lifetime wealth of the agent is the agent's present-discounted stream of profits, which I will denote by $D\Pi$, and which is generated by his ownership of the capital makers. Present discounted profits grow by $\mu^{\frac{1}{\alpha}-1}$ every two periods, and they are equal to

$$D\Pi \;=\; \frac{\Pi_{2,1} \,+\, \Pi_{2,2}}{R_0\,R_1}\left[1 \,+\, \frac{\mu^{\frac{1}{\alpha}-1}}{R_0\,R_1} \,+\, \left(\frac{\mu^{\frac{1}{\alpha}-1}}{R_0\,R_1}\right)^2 \,+\, \ldots\right].$$

Since, by Remark 1 and (39), $R_0\,R_1 \;=\; \mu^{\frac{1}{\alpha}-1} / \beta^2$, the above expression converges, and the TVC is always satisfied.

A.4 Proof in Section 5.3.1 :

Subtracting (46) from (24) having set $\psi \;=\; 1$ in both implies that

$$\begin{aligned} x_0^i \,-\, \tilde{x}_0^i \;&=\; (1-\alpha)^{\frac{1}{\alpha}-1}\, R^{-\frac{1}{\alpha}} \left(R\left[1 \,-\, \mu^{-\left(\frac{1}{\alpha}-1\right)}\right] \,-\, (1-\alpha)\,\mu^{\frac{1}{2}\left(\frac{1}{\alpha}-1\right)}\left(1 \,-\, \mu^{-\frac{1}{\alpha}}\right)\right) \\ &>\; (1-\alpha)^{\frac{1}{\alpha}-1}\, R^{-\frac{1}{\alpha}}\, \mu^{\frac{1}{2}\left(\frac{1}{\alpha}-1\right)} \left[1 \,-\, \mu^{-\left(\frac{1}{\alpha}-1\right)} \,-\, (1-\alpha)\left(1 \,-\, \mu^{-\frac{1}{\alpha}}\right)\right] \\ &=\; (1-\alpha)^{\frac{1}{\alpha}-1}\, R^{-\frac{1}{\alpha}}\, \mu^{\frac{1}{2}\left(\frac{1}{\alpha}-1\right)} \left[\alpha \,-\, \mu^{-\left(\frac{1}{\alpha}-1\right)} \,+\, (1-\alpha)\,\mu^{-\frac{1}{\alpha}}\right]. \end{aligned}$$

In the second line, I use the fact that $\beta \in (0,1)$, having taken into account that $\mu \;>\; 1$ and $\alpha \in (0,1)$, which imply that $1 \,-\, \mu^{-\left(\frac{1}{\alpha}-1\right)} \;>\; 0$.

The next and final step is to show that

$$\alpha \,-\, \mu^{-\left(\frac{1}{\alpha}-1\right)} \,+\, (1-\alpha)\,\mu^{-\frac{1}{\alpha}} \;\geq\; 0\,,$$

or, equivalently, that

$$\alpha\,\mu^{\frac{1}{\alpha}} \,+\, 1-\alpha \;\geq\; \mu\,. \tag{65}$$

With $\mu \;>\; 1$ and $\alpha \in (0,1)$, one can confirm that eq. (65) is always true. Hence, $x_0^i \;>\; \tilde{x}_0^i$ as desired.

B Omitted derivations

B.1 Omitted derivations in Section 4.1.1:

Period τ**.** A capital maker in sector 2 receives and implements an idea immediately, whereas no new idea arrives in sector 1. Technology in the two sectors in period τ then is $\psi_{\tau,1} = \mu\,\psi$ and $\psi_{\tau,2,-j^*} = \mu^{\frac{1}{2}}\,\psi < \psi_{\tau,2,j^*} = \mu^{\frac{3}{2}}\,\psi$.

Capital in period $\tau+1$ is then given by

$$k_{\tau+1,1} = \left[\frac{(1-\alpha)\,\mu\,\psi}{R_\tau}\right]^{\frac{1}{\alpha}} \tag{66}$$

$$k_{\tau+1,2} = \left[\frac{(1-\alpha)\,\mu^{\frac{1}{2}}\,\psi}{R_\tau}\right]^{\frac{1}{\alpha}}. \tag{67}$$

By (8) and (9), investment is

$$i_\tau = \frac{k_{\tau+1,1}}{\mu\,\psi} + \frac{k_{\tau+1,2}}{\mu^{\frac{3}{2}}\,\psi}.$$

Combined with (66) and (67), investment in period τ is then equal to

$$i_\tau = \left(\mu^{\frac{1}{\alpha}-1} + \mu^{\frac{1}{2\alpha}-\frac{3}{2}}\right)\,\psi^{\frac{1}{\alpha}-1}\left(\frac{1-\alpha}{R_\tau}\right)^{\frac{1}{\alpha}}. \tag{68}$$

Substituting (17) and (18) in the production function given by (5) implies that output in period τ is equal to

$$y_\tau = \left[1 + \mu^{\frac{1}{2}(\frac{1}{\alpha}-1)}\right]\,\psi^{\frac{1}{\alpha}-1}\left(\frac{1-\alpha}{R_{\tau-1}}\right)^{\frac{1}{\alpha}-1}. \tag{69}$$

Finally, market clearing in the final-good market implies that consumption in period τ is given by

$$x_\tau = \psi^{\frac{1}{\alpha}-1}\,(1-\alpha)^{\frac{1}{\alpha}}\left[\frac{1+\mu^{\frac{1}{2}(\frac{1}{\alpha}-1)}}{(1-\alpha)\,R_{\tau-1}^{\frac{1}{\alpha}-1}} - \frac{\mu^{\frac{1}{\alpha}-1} + \mu^{\frac{1}{2\alpha}-\frac{3}{2}}}{R_\tau^{\frac{1}{\alpha}}}\right].$$

Period $\tau+1$**.** A capital maker in sector 1 receives and implements a new idea, whereas no new idea arrives in sector 2. The technology in the two sectors then is $\psi_{\tau+1,1,-j^*} = \mu\,\psi < \psi_{\tau+1,1,j^*} = \mu^2\,\psi$ and $\psi_{\tau+1,2} = \mu^{\frac{3}{2}}\,\psi$.

Capital in period $\tau + 2$ is then given by

$$k_{\tau+2,1} = \left[\frac{(1-\alpha)\,\mu\,\psi}{R_{\tau+1}}\right]^{\frac{1}{\alpha}} \tag{70}$$

$$k_{\tau+2,2} = \left[\frac{(1-\alpha)\,\mu^{\frac{3}{2}}\,\psi}{R_{\tau+1}}\right]^{\frac{1}{\alpha}} . \tag{71}$$

Proceeding in the same way as before, investment in period $\tau + 1$ is given by

$$i_{\tau+1} = \frac{k_{\tau+2,1}}{\mu^2\,\psi} + \frac{k_{\tau+2,2}}{\mu^{\frac{3}{2}}\,\psi} ,$$

which combined with (70) and (71) yields

$$i_{\tau+1} = \left[\mu^{\frac{1}{\alpha}-2} + \mu^{\frac{3}{2}(\frac{1}{\alpha}-1)}\right]\,\psi^{\frac{1}{\alpha}-1}\left(\frac{1-\alpha}{R_{\tau+1}}\right)^{\frac{1}{\alpha}} .$$

Substituting (66) and (67) into (5) implies that output in period $\tau + 1$ is equal to

$$y_{\tau+1} = \left[\mu^{\frac{1}{\alpha}-1} + \mu^{\frac{1}{2}(\frac{1}{\alpha}-1)}\right]\,\psi^{\frac{1}{\alpha}-1}\left(\frac{1-\alpha}{R_{\tau}}\right)^{\frac{1}{\alpha}-1} .$$

Finally, market clearing in the final-good market implies that consumption in period $\tau+1$ is given by eq. (25) in the main text.

B.2 Omitted derivations in Section 4.2.1:

Period τ. A capital maker in sector 2 receives an idea, which it implements immediately. In period τ, the technology levels of the capital makers in sector 1 remain as in the preceding period, period $\tau - 1$, that is, $\psi_{\tau,1,-j^*} = \psi < \psi_{\tau,1,j^*} = \mu\,\psi$, whereas in sector 2 they become $\psi_{\tau,2,-j^*} = \mu^{\frac{1}{2}}\,\psi < \psi_{\tau,2,j^*} = \mu^{\frac{3}{2}}\,\psi$. Capital in the following period is then given by

$$k_{\tau+1,1} = \left[\frac{(1-\alpha)\,\psi}{R_{\tau}}\right]^{\frac{1}{\alpha}} \tag{72}$$

$$k_{\tau+1,2} = \left[\frac{(1-\alpha)\,\mu^{\frac{1}{2}}\,\psi}{R_{\tau}}\right]^{\frac{1}{\alpha}} . \tag{73}$$

Investment in τ is then given by

$$i_\tau \;=\; \frac{k_{\tau,1}}{\mu\,\psi} \;+\; \frac{k_{\tau,2}}{\mu^{\frac{3}{2}}\,\psi}\,,$$

which combined with (72) and (73) yields

$$i_\tau \;=\; \left(\mu^{-1} + \mu^{\frac{1}{2\alpha}-\frac{3}{2}}\right)\,\psi^{\frac{1}{\alpha}-1}\,\left(\frac{1-\alpha}{R_\tau}\right)^{\frac{1}{\alpha}}\,. \tag{74}$$

Output and consumption are respectively given by

$$y_\tau \;=\; \left[1 + \mu^{-\frac{1}{2}(\frac{1}{\alpha}-1)}\right]\,\psi^{\frac{1}{\alpha}-1}\,\left(\frac{1-\alpha}{R_{\tau-1}}\right)^{\frac{1}{\alpha}-1} \tag{75}$$

$$x_\tau \;=\; \psi^{\frac{1}{\alpha}-1}\,(1-\alpha)^{\frac{1}{\alpha}}\,\left[\frac{1+\mu^{-\frac{1}{2}(\frac{1}{\alpha}-1)}}{(1-\alpha)\,R_{\tau-1}^{\frac{1}{\alpha}-1}} \;-\; \frac{\mu^{-1}\;+\;\mu^{\frac{1}{2\alpha}-\frac{3}{2}}}{R_\tau^{\frac{1}{\alpha}}}\right].$$

Period $\tau+1$ **.** The technology levels of the capital makers in the two sectors are $\psi_{\tau+1,1,-j^*} = \mu\,\psi \;<\; \psi_{\tau+1,1,j^*} \;=\; \mu^2\,\psi$ and $\psi_{\tau+1,2,-j^*} \;=\; \mu^{\frac{1}{2}}\,\psi \;<\; \psi_{\tau+1,2,j^*} \;=\; \mu^{\frac{3}{2}}\,\psi\,.$

Investment, output, and consumption are then respectively given by

$$i_{\tau+1} \;=\; \frac{k_{\tau,1}}{\mu^2\,\psi} \;+\; \frac{k_{\tau,2}}{\mu^{\frac{3}{2}}\,\psi} \;=\; \left[\mu^{\frac{1}{\alpha}-2} + \mu^{\frac{1}{2\alpha}-\frac{3}{2}}\right]\,\psi^{\frac{1}{\alpha}-1}\,\left(\frac{1-\alpha}{R_{\tau+1}}\right)^{\frac{1}{\alpha}}$$

$$y_{\tau+1} \;=\; \left[1 + \mu^{\frac{1}{2}(\frac{1}{\alpha}-1)}\right]\,\psi^{\frac{1}{\alpha}-1}\,\left(\frac{1-\alpha}{R_\tau}\right)^{\frac{1}{\alpha}-1}$$

$$x_{\tau+1} \;=\; \psi^{\frac{1}{\alpha}-1}\,(1-\alpha)^{\frac{1}{\alpha}}\,\left[\frac{1+\mu^{\frac{1}{2}(\frac{1}{\alpha}-1)}}{(1-\alpha)\,R_\tau^{\frac{1}{\alpha}-1}} \;-\; \frac{\mu^{\frac{1}{\alpha}-2}+\mu^{\frac{1}{2\alpha}-\frac{3}{2}}}{R_{\tau+1}^{\frac{1}{\alpha}}}\right].$$

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Figure 1: Welfare comparison in Section 5.3.2