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DEPARTMENT OF ECONOMICS

Delegation in Decision-Making: Who gets the power?

Carlos Bowles

Thesis submitted for assessment with a view to obtaining the degree of Doctor of Economics of the European University Institute

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[1] This chapter was published in a slightly different format in Annals of Operation Research (2005) 137, p.21-44

Introduction

Power is a fascinating phenomenon. While this is something different than money, somehow I always had the feeling that Economic sciences could provide useful tools to facilitate its understanding. In what follows, I tried to provide some – necessarily modest – contributions to its analysis, focusing mainly on how the delegation of decision-making often required by the complexity of our societies can make the true distribution of power differ from the apparent one.

In the first chapter ("Discretionary Power in Mission Setting as a Preference for Flexibility") I am trying to understand why public institutions or government agencies very often end up being given some large room of maneuver regarding the definition of the missions that they will carry out on behalf of their constituencies – something we can realize by noticing how many institutions are typically doing things which have nothing to do anymore with the reasons for which they were set up. The reason I am providing is an intuitive one – yet, to my knowledge, new: in an uncertain world, institutions sometimes need to be given some flexibility in their mission setting to be able to adjust to circumstances as they arise. Institutions being run by individuals with private agenda, one should however be mindful that conflict of interests may arise, thereby requiring some balance between a too large and a too restrictive mandate.

The second chapter ("Generating functions for coalitional power indices: an application to the IMF"), a result of a collaboration with José-Maria Alonso-Meijide, is focused on a similar kind of delegation problem, but restricted to cases where the delegation has to comply with some voting scheme, like in a democratic assemble for instance. Here, the power merely lies in the hand of the one who is able to exert an influence over the final outcome. In cooperative game theory, this one is called "the pivotal player", because he/she is able to turn a losing coalition into a winning one. Some tools are already existing to address the issue in some specific cases, like the Electoral College in the United States, or the Security Council at the United Nations. But no tools were easily available to tackle the issue for one of the most important financial institution, namely the International Monetary Fund. The large number of players involved – 184 members states – made it more suitable for the decision process to have a two-step voting scheme, where each of the individual countries has to delegate its power to a group of countries (the so-called "Constituencies", technically referred to as "a priori unions" in the chapter) represented by one of the 25 Executive Directors in the IMF Executive Board. The contribution of the chapter is to provide “ready-to-apply” methods allowing the computation of the so-called “power index” of players, that is: a quantified measure of how much power each of them will have in the decision-making process. Some computer science related results have been needed as intermediary steps, which are presented as an appendix to the chapter.[2]

Drawing on this methodology, the third chapter (“Should European Union represent European States at the IMF?”), also the result of a collaboration, this time with Agnès Benassy-Quéré, investigates what kind of delegation could best serve the power of the European Union within the International Monetary Fund. The issue there is not anymore theoretical, as it has been under intense discussions within and outside the EU over now many years. While the intention behind the creation of a single EU seat at the IMF is to increase the EU influence in this institution, our analysis suggests such a step would rather reduce it in most scenarios looked at. Intuitively, being united here does not help to be stronger, as the current spreading of EU members across many constituencies helps getting leverage in each of them.[3]

[2] The respective contributions of the authors to this paper are as follows: my co-author found the generating function and its proof, while I found the application domain (IMF), wrote the algorithms to compute the indices, provided the proof of their complexity, made the computations of all indices and drafted the paper.

[3] The respective contributions of the authors to this paper are as follows: I computed the IMF quotas under all the different scenario (reintegrating intra-EU trade and financial flows), I identified the power indices approach best fit to the situation and I carried out the computation of the various power indices under the different scenarii, while my co-author acted as the main drafter of the paper presenting the results. Both of us reflected and carried out a set of interviews with Finance Minister Officials in order to understand which methodology could best fit our initial intention to assess the impact that a EU single seat at the IMF.

Chapter 1.
Discretionary power in mission setting as a preference for flexibility

Discretionary Mission Setting as a Preference for Flexibility *

Carlos Bowles†

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Abstract

The mandate of government agencies is often leaving a substantial room of maneuver for them to decide the specific tasks they will carry out on behalf of their constituency, in order to allow them to adjust to new situations as they arise. In order to formalize this intuition, I extend Aghion and Tirole (1997)'s model of formal and real authority to a situation where the principal has state-dependent preferences, thereby leaving scope for exhibiting a preference for Flexibility à la Kreps (1979). I show that the principal can either decide to give a large or a restricted mandate to her agency, depending on the extent to which they share the same interests and the level of the monitoring costs. When the congruence of interests is high and the monitoring costs are low, flexibility is preferred.

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1 Introduction

Public institutions or government agencies are typically given large powers. As it is well known that the interests of those running the agencies may sometimes differ from the interests of those for which the agencies are run, one would expect that the precise tasks that the agencies will carry out are clearly defined in their mandate, to avoid having them doing something different than what they should do. In practice, however, the mandates given to institutions and agencies are very often defined in only general terms, leaving them a lot of room of manoeuver to decide the type of activity in which they will engage or not. In fact, it may even be the case that agencies end up doing something completely different than what was initially foreseen. For instance, the European Central Bank, who was initially given the mandate to carry out monetary policy tasks for the euro area, ended up being engaged into the implementation of fiscal adjustment programs in so-called "mission countries" (Greece, Portugal, Ireland, etc.). The International Monetary Fund, whose initial task was to ensure exchange rate stability, ended up financing members with balance of payment problems and assisting members in poverty reduction programs after the collapse of the Bretton Wood fixed exchange rate system had made its initial task irrelevant looking forward. In a non financial domain, the French Constitutional Court, whose initial task was to make sure that the various branches of the government (legislative, executive and judiciary) would not go beyond the limits assigned to them by the 1958 Constitution, ended up also assessing the compliance of laws regarding fundamental rights, a taks which was not explicitly assigned to them by the Constitutional Committee who designed the Constitution.

Why would society let public institutions decide to do what they want at the risk of having them doing the wrong thing? So far, the one explanation provided by the literature to explain why agencies are given some freedom to decide on which task they will engage is based on incentive to efforts reasons. The bulk of the reasoning is that the principal is in practice better off to let the agent decide on the task (often referred to in the litterature as a project instead of a task) it will engage into as long as there is some convergence of interests between them, for the risk of having the agent's choice overruled by the principal would otherwise reduce the agent's efforts to gather information on the pay-off of possible tasks/projects. In this paper, I am trying to bring another -simple, intuitive and yet, to my knowledge, new - explanation: agencies are given some freedom to decide on which task they will engage into because the future is uncertain and they need to be able to adapt to circumstances as they arise. The change in the way agencies carry out their mandate over time would therefore simply reflect the need to adjust to a changing world. The explanation I propose is therefore based on flexibility needs, similar in essence as those presented by Kreps (1979)

To proceed with the explanation, I am drawing on an extension of the principal-agent model built by Aghion and Tirole (1997). In their paper, a principal (she) and an agent (he) can implement one or zero project. The principal hires the agent to collect information and implement the project. The projects yields different pay-offs to the principal and the agent. The principal can either retain authority and over-rule the agent's choice or let the agent decide on which project to implement, once the information on pay-offs is known. In this setting, a basic trade-off between loss of control and initiative arise. The principal may decide to keep authority over decisions, but that will be at the expense of having the agent making less effort to ascertain the pay-off structure of available projects. The principal may decide to let the agent decide on which project to implement, which will induce the agent to make more effort to gather information, but at the expense of not having the principal's preferred project eventually picked. In this setting, it is however not possible to explain why tasks would need to be adjusted to adapt to a changing world because the world is basically not changing: the preference of the principal over the available projects do not change depending on the state of the world. In order to introduce that explanation, I will therefore, contrary to Aghion and Tirole (1997), assume that the preferences of the principal are state-dependent. Another difference is that I will assume that the principal has the possibility to restrict the set of projects available for the agent to pick (which I refer to as tasks, rather than projects, for the sake of stressing the institutional dimension). I will also proceed with further simplifications of their model to keep as much as possible the focus on the explanation I am bringing. In particular, in order to avoid having a trade-off between delegation and authority retention blurring the focus of the analysis, I will take away the trade-off from the model by assuming that the principal always needs the agent to implement the project.

In essence, my model works as follows. There is one principal (she) and one agent (he). The principal needs the agent to carry out a task (or project), although she does not know ex-ante which task she would like to be carried out because this will depend on how the world will look like tomorrow. One may for instance think about a society delegating monetary policy to a central bank, without specifying whether such a policy should be restrictive or expansive because this will depend on whether the economy is in recession or in expansion, which is itself not known when the mandate is drafted. To keep things simple, I assume that there are only two main possible tasks and two main possible states of the world (for technical reasons, the model also features a third task and a third state of the world, but this can be ignored in this presentation - I will come back to this in the modelling section). In the one state of the world, both the agent and the principal want to implement the same task. In the other state of the world, their preferred task differs, although they still derive some utility from having their non-preferred task implemented (in the above example of monetary policy, this may the bias of the agent towards one specific task may be compared to the so-called restrictive bias of central bankers, which may differ from the preference of the citizens at a given point in time). The possible tasks, the possible states of the world and their associated probabilities of materialisation are known by both the principal and the agent. The state of the world is however not observable without engaging into information gathering, which comes at some cost for the agent. I assume that the principal is not able to observe the state of the world, but is able to detect with some probability when the agent does not pick the principal's preferred task when aware of the state of the world. The probability to be detected is also known by the agent, and is decided upon by the principal taking into account that it will take her some effort. Once the agent knows the state of the world he is in, he decides on the task he will implement, which will also depend on how much room for maneuver he's been initially given by the principal before the state of the world was known. Should he end up picking the task non-preferred by the principal, he will receive an exogenous penalty. To sum up, the situation we describe is as follows: the principal decides the set of tasks among which the agent can chose, the agents gather some information about the state of the world and decides on which task to implement, the principal possibly punish the agent in case she detects some non-desired activity.

With these modelling assumptions, I obtain the following new result that the principal may decide to give some discretion to the agent on which task to implement because this may help her adjust to new circumstances. The discretion left to the agent is therefore not necessarily motivated by the need to induce him to make more efforts to collect information on pay-offs. Another result I obtain - which is this time more counterintuitive - is that the principal may decide to take away the possibility for the agent to implement the principals' preferred task. The reason for this is that excluding this task also means that the principal will be able to save on monitoring costs, while the agent will in turn know that he will not risk any punishment should he not take the decision he prefers. Finally, I also confirm that it may be sometimes interesting for the principal to also allow the implementation of her non-preferred task in order to incentivize the agent to produce some efforts.

2 Relation to the existing litterature

My paper relates to the literature on delegation and incentives. A very relevant paper is that of Armstrong and Vickers (2010), which also present a principal agent model where the principal influences the agent's behaviour by specifying the permitted set of projects among which the agent can chose his preferred one. They find that the principal will exclude some desirable projects from the permitted set to avoid that some projects which are highly valued by the agent but less so by the principal end up being picked (which they do via the characterisation of a so-called 'threshold rule' excluding all projects whose intrinsic utility for the principal is too low). They also find that the more the principal cares about the utility of the agent, the more discretion the agent is given in the sense that a higher fraction of projects are permitted (a similar result to the "ally principle"). In a variant of their benchmark model, they introduce agent's incentive to discover projects similar as in my paper, where by exerting some costly effort the agent finds a project with some probability. They find that the principal may allow some undesirable project for the sake of stimulating agent's efforts.

In my paper, I do also find that it may be desirable to exclude the agent's preferred task from the contract to avoid having the task being picked up at the expense of the principal's preferred task. I also find that the bigger the convergence of interests between the principal and the agent, measured by the opportunity cost resulting from not picking the preferred project, the wider the discretion left to the agent generally. However, I also find that when the probability that the interests will be the same is high enough, it may be even better to restrict the set of permitted tasks in order to save on the monitoring costs needed to ensure that the agents does not pick its preferred task at the expense of the principal should the divergence of interest materialise. Turning back to for incentive to efforts reasons, I also find that the principal allows the agent to possibly take on non-preferred tasks. This only happens, however, when the opportunity cost resulting from not picking the preferred project is not too high, otherwise restricting choices still make sense as one saves monitoring and the agent will generally be better off by carrying the project than doing nothing.

My paper is also related to Bester and Krähmer's paper on Delegation and Incentives (2008). Their paper is close to mine in the sense that they do introduce a project selection stage in which the principal decides to let the agent choose the project he wishes. In their setting, they find that the principal is less likely to let the agent decide which project to pick for incentive to effort reasons, as the efforts of the agent takes place after the project has been determined and the agents always derive a private benefit from the project completion. The principal can therefore always ensure herself at least the same pay-off as under delegation. This differs from my paper where, as in Aghion and Tirole (1997), the principal allows the agent to take on non-preferred tasks for incentive to efforts reason. My paper also differs from theirs as my setting only considers delegation cases, where the agent is the only one who can select the project to implement. While in their setting the agent always end up choosing his preferred project if he has been delegated authority, in my setting the agent may sometimes choose the principal's project to avoid being punished by the principal for having cheated.

Finally, another relevant paper is that of Alonso & Matouschek (2008) on Optimal Delegation, which models a situation where the principal commits to a set of decision from which the agent chooses his preferred one, in a situation where the principal cannot implement contingent transfers. The authors then characterize the optimal delegation set and perform comparative statics on the principal's willingness to delegate and the agent's discretion. They find that the conditions for so-called interval delegation (that is, letting the agent make any decision from a single interval, a la Holmström (1977) to be optimal are satisfied when the agent's preferences are sufficiently aligned. In contrast, my paper shows that the principal may still find it optimal to let the agent have some discretion regarding the project choices even when the probability that their interest diverge is large.

3 The model

3.1 Basic Settings

The model is a stylised extension of Aghion and Tirole's (1997) model on formal and real authority to a situation where utilities are state-contigent.

Players and projects. There is a principal P (she) and an agent A (he). The agent can implement a project $z \in Z = \{z_1, z_2, z_3\}$, while the principal cannot (for instance because some technical competence is needed, which only the agent has). The principal is however able to restrict the choice of projects that the agent will be able to implement to some subset Z_P of 2^Z (say, because she can forbid by law the agent to do things that she clearly does not want him to do). She is also able to inflict a penalty F to the agent in case the agent does not behave as agreed. Monetary transfers are excluded.

Pay-off structure. The implementation of the project brings some pay-off to both the agent and the principal, which depend on the state of the world they are in. The respective pay-off are shown in the table below, where 1 (resp. α,with $0 \leq \alpha \leq 1$) is the utility of the principal (resp. the agent) when the project z_1 is achieved in the state of the world s_1, whose probability is p_1:

	$U^P(z), U^A(z,e)$		
	s_1	s_2	s_3
Prob	p_1	p_2	p_3
z_1	$1, \alpha$	α, α	$-\infty, -\infty$
z_2	$\alpha, 1$	$1, 1$	$-\infty, -\infty$
z_3	$0, 0$	$0, 0$	$0, 0$

Hence, there is one state of the world (s_2) where both the principal and the agent have the same preference over the projects (z_2 being their preferred one), while in another state of the world (s_1) their preference diverge (with z_1 being preferred by the principal while z_2 is preferred by the agent). In a third state of the world (s_3), both the principal and the agent would suffer an infinite disutility should z_1 or z_2 be implemented. One can think of that state of the world as standing for a situation where doing standard things may amount to doing a big mistake - a situation which may not happen very often but may still be worth avoiding. In that situation, then the best thing to do may be to opt for a safe project (z_3), which may not bring any pay-off but does not bring anything negative either. The reason for introducing such a situation in the model is to represent situations where the agent will prefer avoiding taking any chances in case the information he has on which state he is in is not good enough for him to decide. One can for instance think about the case of a judge, who may hesitate between giving a big or a small sanction to punish someone charged with some crime, but finally opt for no sanction at all because the evidence available is too weak to prevail and the doubt eventually benefits to the accused person. For that reason, we assume that the project z_3 is always available for the agent to implement.[1]

Information structure. The distribution of probability of the respective states of the world is common knowledge. The principal and the agent do not know which state of the world they are in but they can obtain the information with some probability e^* (resp. E^*), ranging from at the cost of some disutility $g_A(e^*)$ (resp. $g_P(E^*)$). For $i = A, P$ we also assume that $g_i(.)$ is strictly increasing and strictly convex, with $g_i(0) > 0, g_i'(0) = 0, g_i'(1) = \infty$. The assumption that $g_i(0) > 0$ (which basically means that even ignorance

[1] Having infinitely negative entries in the pay-off table are not strictly needed to generate a situation where the agent opt for the safe action in case of ignorance. It would be enough to have some "sufficiently big " negative pay-off, whose value could be computed depending on the parameters. However, this would bring additional complexity in the model, without brining additional insight.

does not come up for free) reflects the existence of fixed costs in the information technology. One can see these costs as some amount that the principal and the agent would have to pay before entering in the game, in order to make sure that the information technology is available for them to useafter the game has started.

Monitoring technology. The principal is able to inflict a penalty F if she observes that the agent does not implement her preferred project among the permitted subset of actions available to him. The level of penalty is exogenous. We assume that there is some rigidity in the monitoring technology, in the sense that the principal *has* to punish the agent once she receives the information that the agent did not implement her preferred project while he could have. One could think of this mechanism as a pre-commitment device, whereby the principal is ex-post forced to inflict the penalty she announced ex-ante. This could describe a situation where the nature of the monitoring technology is such that the triggering of the penalty is tied with the discovery of the "breach " (for instance in a computer-run monitoring device which would automatically suspend an account in case fraudulent movements are detected). This could also represent a situation where the principal needs to comply with the penalty announced out of credibility reasons non modelled here.

Timing. The principal designs a contract C^* specifying the set of permitted projects for the agent $Z_P^* \in Z_P \equiv \{\{z_1, z_3\}, \{z_2, z_3\}, \{z_1, z_2, z_3\}\}$ and the project to be implemented depending on the state of the world observed by the agent (e.g. a message $m : Z_P \times S \to \{z_1, z_2, z_3\}$ where $S = \{s_1, s_2, s_3\}$). She also decides on her level of effort E^*, which is observable by the agent, and pays the cost $g_P(E)$.The agent accepts or rejects the contract. On acceptance, the agent decides on his effort level e^* and pay the cost $g_A(e^*)$. Nature picks the state of the world. The principal and the agent are informed on the state of the world with probability E^* and e^*. Based on the information available, the agent makes his project choice. The principal observes the agent's project choice. If the principal knows the state of the world and if it turns out that the agent did not implement the principal's preferred project while he could have, the agent suffers the penalty F.

Actions and strategies. The Principal's set of possible actions is defined over $C \times E$ where C stands for the set of possible contracts and $E \equiv [0, 1]$ is the interval of possible effort choices. The Agent 'set of actions is defined over $C\times$ $e \times \{z_1, z_2, z_3\}$ where $e \equiv [0, 1]$ is the interval of possible agent effort choices.The combination of the possible actions for the principal and the agent define the set of all profiles of pure strategies.

Equilibrium concept. We will use subgame perfection as equilibrium concept and solve the game via backward induction.

3.2 Model resolution

We solve the model by starting with the last decision to be made, which is the agent decision regarding the project to pick among the set of permitted projects and his level of effort. We then look into the principal's decision regarding the set of permitted projects and her level of effort, incorporating the impact this will have on agent's decisions.

3.2.1 Agent's program

The agent's behaviour will depend on the set of permitted projects. We will refer to the set of permitted project $\{z_1, z_2, z_3\}$ as the flexibility case, while the set of permitted projects $\{z_1, z_3\}$ and $\{z_2, z_3\}$ will be referred to as inflexibility cases.

The agent will select the effort level e^* which maximizes his utility U^A :

$$e^* = \arg\max U^A(z, e)$$

The utility of the agent depends on the set of permitted projects (as decided by the principal) and on the states of the world. We need to look at each of the three possible set of permitted projects.

Case 1: The set of permitted projects is $\{z_1, z_3\}$. In this case, either the efforts of the agent paid off, meaning that he knows the state of the world and picks z_1, or his efforts did not pay off, meaning that he his not informed about the state of the worlds and picks z_3. His utility function therefore writes:

$$U^A(\{z_1, z_3\}, e) = e(p_1 + p_2)\alpha - g_A(e)$$

and the First Order Condition is:

$$\frac{\partial U^A}{\partial e} = 0 \Rightarrow e^* = g_A'^{-1}((p_1 + p_2)\alpha)$$

Note that e^* is defined as $(p_1+p_2)\alpha \in [0, 1]$ and $g' : [0, 1] \to [0, +\infty]$ meaning that $g_A'^{-1}((p_1+p_2)\alpha) \in [0, 1]$.

Case 2: The set of permitted projects is $\{z_2, z_3\}$. In this case, either the efforts of the agent paid off, meaning that he knows the state of the world and picks z_2, or his efforts did not pay off, meaning that he his not informed about the state of the worlds and is picks z_3. His utility function writes:

$$U^A(\{z_2, z_3\}, e) = e(p_1 + p_2) - g_A(e)$$

and the First Order Condition is:

$$\frac{\partial U^A}{\partial e} = 0 \Rightarrow e^* = g_A'^{-1}((p_1 + p_2))$$

Case 3 ("flexibility "): The set of permitted projects is $\{z_1, z_2, z_3\}$. This case is a bit more complicated to formalize, as the agent may find himself in the state of the world s_1 where he has to chose between picking the principal's preferred project or picking his own preferred project but with the risk of being detected by the principal with some probability E^* and, if this is the case, pay some punishment F. We formalise this choice via a binary variable c taking value over $\{0, 1\}$ where $c = 0$ means that the agent chooses the principal's preferred project while $c = 1$ means that he choses his own preferred project. Under flexibility, the utility function of the agent therefore becomes:

$$U^A(\{z_1, z_2, z_3\}, e) = e[p_1((1-c)\alpha + c(1 - E^*F)) + p_2] - g_A(e)$$

The First Order Condition is :

$$\begin{aligned} \frac{\partial U^A}{\partial e} &= 0 \Rightarrow g'_A(e^*) = p_1((1-c)\alpha + c(1-E^*F)) + p_2 \\ &\Leftrightarrow e^* = g'^{-1}_A(p_1((1-c)\alpha + c(1-E^*F)) + p_2) \end{aligned}$$

the agent will therefore only pick his own preferred project if the benefit he gets for doing so, taking into account the expected punishment he will suffer (e.g. $1 - E^*F$) is above the benefit he would get when picking the principal's preferred project (e.g.α). We therefore have:

$$\begin{aligned} \text{if } \alpha &\geq 1 - E^*F \text{ then we have } c = 0 \text{ and } e^* = g'^{-1}_A(p_1\alpha + p_2) \\ \text{otherwise, if } \alpha &< 1 - E^*F, \text{ then we have } c = 1 \text{ and } e^* = g'^{-1}_A(p_1(1 - E^*F) + p_2) \end{aligned}$$

3.2.2 Principal's Program

The principal designs the contract that maximizes her expected utility taking into account the optimal level of effort of the agent.

She therefore needs to decide on how much flexibility she wants to give to her agent, depending on the expected agent's behaviour, which itself depends on the value of the parameters. To do so, the principal will have to compare the utility she can expect from each of the three possible set of permitted projects. We have to write down the utility of the principal depending on each of the three cases.

Case 1: The set of permitted projects is $\{z_1, z_3\}$. In this situation, the agent is informed of the state of the world with some probability $e^* = g'^{-1}_A((p_1 + p_2)\alpha)$, in which case he always pick project z_1 which for the principal brings a utility of 1 when the state of the world is s_1 or a utility of α when the state of the world is p_2. Using the F.O.C of the agent in case 1, the utility of the principal writes:

$$\begin{aligned} U^P(\{z_1, z_3\}, e^*) &= e^*(p_1 + p_2\alpha) \\ &= g'^{-1}_A((p_1 + p_2)\alpha)(p_1 + p_2\alpha) \end{aligned}$$

Case 2: The set of permitted projects is $\{z_2, z_3\}$. In this situation, the agent is informed of the state of the world with some probability $e^* = g'^{-1}_A((p_1 + p_2))$, in which case the agent always pick project z_2 which for the principal brings a utility of α in state s_1 and 1 in state s_2. Using the F.O.C of the agent in case 2, the utility of the principal writes:

$$
\begin{aligned}
U^P(\{z_2, z_3\}, e^*) &= e^*(p_1\alpha + p_2) \\
&= g_A'^{-1}((p_1 + p_2))(p_1\alpha + p_2)
\end{aligned}
$$

Case 3 ("flexibility "): The set of permitted projects is $\{z_1, z_2, z_3\}$. This case is, again, slightly more complicated to formalize as the principal also needs to decide on its level of monitoring. From the program of the agent, we know that the monitoring only brings incentives for the agent to pick the principal's preferred task when the probability for the agent to be detected is above a certain threeshold $E^* \geq \frac{1-\alpha}{F}$. This costs however $g_P(E^*)$ to the principal. Going above this threeshold does not bring additional incentives for the agent, only more costs for the principal. Going below the threeshold does result into having the agent picking the principal's project, while the principal 's efforts are costly. Overall, the principal should either decide to monitor and set $E^* \geq \frac{1-\alpha}{F}$ or renounce to the monitoring and set $E^* = 0$, depending of the value of the parameters. We need to look at both possibilities in turn.

When the principal decides to engage into monitoring, her expected utility writes down:

$$
\begin{aligned}
U^P(\{z_1, z_2, z_3\}, e^*) &= g_A^{'-1}(p_1\alpha + p_2)(p_1 + p_2) - g_P(E^*) \\
&= g_A^{'-1}(p_1\alpha + p_2)(p_1 + p_2) - g_P(\frac{1-\alpha}{F})
\end{aligned}
$$

When the principal decides not to engage into monitoring, her expected utility writes down:

$$
\begin{aligned}
U^P(\{z_1, z_2, z_3\}, e^*) &= g_A^{'-1}(p_1 + p_2)(p_1\alpha + p_2) - g_P(E^*) \\
&= g_A^{'-1}(p_1 + p_2)(p_1\alpha + p_2) - g_P(0)
\end{aligned}
$$

In that last situation, it becomes clear that allowing the set of permitted projects to be $\{z_1, z_2, z_3\}$ without engaging into monitoring does not bring better utility than restricting the set of permitted projects to $\{z_2, z_3\}$ as the principal will have to pay $g_P(0) > 0$. Allowing $\{z_1, z_2, z_3\}$ therefore only makes sense when the principal engages into monitoring with $E^* = \frac{1-\alpha}{F}$

Comparing the utility derived by the principal in all three cases, it becomes clear that flexibility is chosen by the principal if and only if :

$$
U^P(\{z_1, z_2, z_3\}, e^*) > U^P(\{z_1, z_3\}, e^*)
$$

that is

$$g_A^{'-1}(p_1\alpha+p_2)(p_1+p_2)-g_P(\frac{1-\alpha}{F})>g_A^{\prime-1}((p_1+p_2)\alpha)(p_1+p_2\alpha)$$

and

$$U^P(\{z_1,z_2,z_3\},e^*)>U^P(\{z_2,z_3\},e^*)$$

that is

$$g_A^{'-1}(p_1\alpha+p_2)(p_1+p_2)-g_P(\frac{1-\alpha}{F})>g_A^{\prime-1}(p_1+p_2)(p_1\alpha+p_2)$$

This brings us to the following propositition:

Proposition 1 *Flexibility is strictly preferred by the principal if and only if:*

$$g_A^{'-1}(p_1\alpha+p_2)(p_1+p_2)-g_A^{\prime-1}((p_1+p_2)\alpha)(p_1+p_2\alpha)>g_P(\frac{1-\alpha}{F}) \tag{1}$$

and

$$g_A^{'-1}(p_1\alpha+p_2)(p_1+p_2)-g_A^{\prime-1}(p_1+p_2)(p_1\alpha+p_2)>g_P(\frac{1-\alpha}{F}) \tag{2}$$

3.3 Trade-offs

The table below summarizes the situation from the perspective of the agent's effort and the principal's utility in the three cases:

	Inflexibility 1 (Case 1)
Set of permitted projects	$\{z_1,z_3\}$
Agent's optimal effort level	$e^*=g_A^{\prime-1}((p_1+p_2)\alpha)$
Principal's utility achieved	$U^P(z,e^*)=g_A^{\prime-1}((p_1+p_2)\alpha)(p_1+p_2\alpha)$

	Flexibility (Case 3)
Set of permitted projects	$\{z_1,z_2,z_3\}$
Agent's optimal effort level	$e^*=g_A^{\prime-1}(p_1\alpha+p_2)$
Principal's utility achieved	$U^P(z,e^*)=g_A^{'-1}(p_1\alpha+p_2)(p_1+p_2)-g_P(\frac{1-\alpha}{F})$

	Inflexibility 2 (Case 2)
Set of permitted projects	$\{z_2,z_3\}$
Agent's optimal effort level	$e^*=g_A^{\prime-1}((p_1+p_2))$
Principal's utility achieved	$U^P(z,e^*)=g_A^{\prime-1}((p_1+p_2))(p_1\alpha+p_2)$

The table shows that the agent's efforts are bigger when he knows that he will be able to pick his preferred action in all state of the worlds (Inflexibility 2), compared to a situation where he would only be able to pick his preferred action sometimes (Flexibility) or never (Inflexibility 1) - as $g_A'^{-1}$ is increasing and $p_1 + p_2 \geq p_1\alpha + p_2 \geq (p_1 + p_2)\alpha$. There is therefore for the principal a cost to extend the set of projects permitted to the agent or even to restrict it in a way that the agent would be forced to only pick his non-preferred project.

The cost for the principal of extending the set of permitted projects may however be counterbalanced by the added utility that the principal will obtain from having her preferred project picked in all states of the world, which can be seen by comparing her utility in the case Flexibility against her utility in the case Inflexibility 2, $(p_1 + p_2)$ being above $(p_1\alpha + p_2)$. This situation will however be only interesting under the condition that the monitoring costs $g_P(\frac{1-\alpha}{F})$ are comparatively not too high.

When the monitoring costs are too high, it may prove more profitable to restrict the set of permitted projects. It may however be preferable for the principal to force the agent to always pick her non-preferred project, even though this may result into lowering the agent's efforts, should this be compensated by the utility increase coming from the fact that the action preferred by the principal may be much more often picked. This would happen when p_1 is "sufficiently big " and α is "not too low " in order to make the difference $(p_1 + p_2\alpha) - (p_1\alpha + p_2) = (p_1 - p_2)(1 - \alpha)$ sufficiently positive to compensate the loss induced by the fact that $g_A'^{-1}((p_1 + p_2)\alpha)$ is below $g_A'^{-1}((p_1 + p_2))$.

A simple way to grasp the essence of the trade-offs is to assume that p_3 is sufficiently negligible to be ignored, which helps simplifying notations by setting $p_1 = 1 - p_2$. One can then reformulate the principal's utility as a function of the variable $x = (1 - \alpha)p_1$, as in the table below:

	Principal's utility achieved
Inflexibility 1 (Case 1)	$g_A'^{-1}(\alpha)(x + \alpha)$
Flexibility (Case 3)	$g_A'^{-1}(1 - x) - g_P(\frac{1-\alpha}{F})$
Inflexibility 2 (Case 2)	$g_A'^{-1}(1)(1 - x)$

As can be observed in the table, the functions showing the utility achieved by the principal in the inflexibility cases are both linear in x, with the first one upward sloping and the second one downward sloping.

4 Example with a specific functional form

4.1 Derivation of the specific equations

We take a specific parameterized form for the cost functions $g_i(\cdot)$. For $i = A, P$, and we define :

$$g_i(x) = -\beta \ln(1 - x^2)$$

where $\beta > 0$ is a parameter.

It follows that :

$$g_i'(x) = \beta \frac{2x}{1-x^2}$$

We can check that $g_i'(0) = 0$ and $g_i'(1) = \infty$ and that $g_i(\cdot)$ is increasing and strictly convex.

The reciprocal of $g_i'(x)$ can be obtained by reassembling all terms into a second degree equation and solving in x:

$$y = \beta \frac{2x}{1-x^2}$$

which can be re-expressed as a second order equation:

$$yx^2 + 2\beta x - y = 0$$

and factorized as follows:

$$(x - \frac{\sqrt{(\beta^2 + y^2)} - \beta}{y})(x - \frac{\beta - \sqrt{(\beta^2 + y^2)}}{y}) = 0$$

We therefore have :

$$g_i'^{-1}(x) = \frac{1}{x}(\sqrt{(\beta^2 + x^2)} - \beta)$$

or

$$g_i'^{-1}(x) = -\frac{1}{x}(\sqrt{(\beta^2 + x^2)} - \beta)$$

depending on the definition domain at stake.

The conditions for a preference for flexibility to appear are therefore (assuming the parameters value are such that the correct reciprocal function for g_i' is the first of the two ones above):

$$U^P(\{z_1, z_2, z_3\}, e^*) > U^P(\{z_1, z_3\}, e^*)$$

that is, as derived in equation (1) above:

$$g_A'^{-1}(p_1\alpha + p_2)(p_1 + p_2) - g_A'^{-1}((p_1 + p_2)\alpha)(p_1 + p_2\alpha) - g_P(\frac{1-\alpha}{F}) > 0$$

which is

$$\frac{1}{(p_1\alpha + p_2)} \left[\sqrt{\beta^2 + (p_1\alpha + p_2)^2} - \beta\right](p_1 + p_2)$$

$$-\frac{1}{(p_1+p_2)\alpha}\left[\sqrt{\beta^2+((p_1+p_2)\alpha)^2}-\beta\right](p_1+p_2\alpha)+\beta\ln(1-(\frac{1-\alpha}{F})^2)>0$$

and

$$U^P(\{z_1,z_2,z_3\},e^*)>U^P(\{z_2,z_3\},e^*)$$

that is, as derived in equation (2) above:

$$g_A^{'-1}(p_1\alpha+p_2)(p_1+p_2)-g_A^{'-1}(p_1+p_2)(p_1\alpha+p_2)-g_P(\frac{1-\alpha}{F})>0$$

which is

$$\frac{1}{(p_1\alpha+p_2)}\left[\sqrt{(\beta^2+(p_1\alpha+p_2)^2}-\beta\right](p_1+p_2)$$

$$-\frac{1}{(p_1+p_2)}\left[\sqrt{(\beta^2+(p_1+p_2)^2)}-\beta\right](p_1\alpha+p_2)+\beta\ln(1-(\frac{1-\alpha}{F})^2)>0$$

Assuming p_3 is sufficiently negigible to set $p_2=1-p_1$, it is possible to simplify the equations as follows:

$$\Leftrightarrow\frac{1}{(p_1(\alpha-1)+1)}\left[\sqrt{\beta^2+(p_1(\alpha-1)+1)^2}-\beta\right]$$

$$-\frac{1}{\alpha}\left[\sqrt{\beta^2+(\alpha)^2}-\beta\right](p_1(\alpha-1)+\alpha)+\beta\ln(1-(\frac{1-\alpha}{F})^2)>0$$

$$\Leftrightarrow\frac{1}{(p_1(\alpha-1)+1)}\left[\sqrt{\beta^2+(p_1(\alpha-1)+1)^2}-\beta\right]$$

$$-\left[\sqrt{(\beta^2+1)}-\beta\right](p_1(\alpha-1)+1)+\beta\ln(1-(\frac{1-\alpha}{F})^2)>0$$

4.2 Comparative statics

Figure 1 illustrates the comparative statics at play with given values for α,β and F. In situations where the frequency of convergent interests is low, it is in the interest of the principal to force the agent to only pick his non-preferred task. When the frequency of converging interests increases, "investing " in monitoring becomes more interesting for the sake of capturing the benefit arising from flexibility. However, there is one level of frequency of converging interests above which it is just better for the principal to save on the monitoring costs and increase the agent's efforts by letting the agent only pick the task he prefers. The impact of a change in the parameter α is illustrated in Figure 2. Reducing the α makes flexibility generally less interesting compared to other menus as it increases monitoring costs.

The relation between α and p_1 is depicted in Figure 3,[2] which provides the area where each of the three menu is preferred. The higher the α, the bigger the area where flexibility is preferred. Figure 4 depicts the impact of an increase in the exogenous penalty factor F (from 1 to 1.5), which is to increase the area where flexibility is preferred, because of a reduction in monitoring costs. A similar impact is obtained when changing the parameter β of the functional form taken for the monitoring costs. Figure 5 shows that an increase of β (from 0.8 to 1.2) leads to a reduction of the area for which flexibility is optimal.

5 Conclusion

In this paper, I formalized the intuition that the discretionary power of institutions in mission setting may reflect the preference for flexibility of the principal, in the sense that the principal may prefer to give some room for maneuver to her agent in order to let him adapt to new circumstances. Preference for flexibility framework models a two-step process where a decision-maker uncertain about her future preferences first chooses an opportunity set among the set of all the subset of possible alternatives. In a second stage, once the uncertainty is resolved, the decision-maker finally picks one alternative among the opportunity set. I extended this analysis to an agency theoretic framework by introducing the hazard moral problem analyzed by Aghion and Tirole (1997) into the two-stage decision process of Kreps (1979). The crucial point of my model is that the first stage decision is made by the principal that chooses a subset of allowed tasks among the set of possible tasks, while in the second stage, the decision is made by the agent. I could show that the principal may decide to let some flexibility to the agent when the convergence of interests is relatively high. Using a specific functional form, I could show that the size of the set of parameters for which flexibility is preferred is decreasing with the monitoring costs.

An interesting avenue for future research could be to give some more "economic flesh" to the missions considered in this theoretical model, investigating the different domains of economic policy where agencies end up having some freedom of choice regarding their mandate. For instance, there is currently a substantial discussion regarding whether quantiative easing is or is not within the mandate given to the European Central Bank, with the line of divide often reflecting differences in national preferences among the voters of different EU member states but also difference in priors regarding the need for extraordinary measures in the wake of an unprecedented financial crisis. It could be interesting to try to determine under which conditions a central bank could be given some flexibility in the set of instrument it will use, depending on the probability of appearance of a systemic crisis, also taking into account the risk that overstepping the mandate may sometimes go against the preference of the majority of the voters from which the central bank is deriving its mandate.

[2] Figure 3 and Figure 4 have been obtained using a loop in Matlab investigating for each value of α and p_1 the menu bringing the highest utility for the principal. This lead to the area shown in the figures.

Figure 1

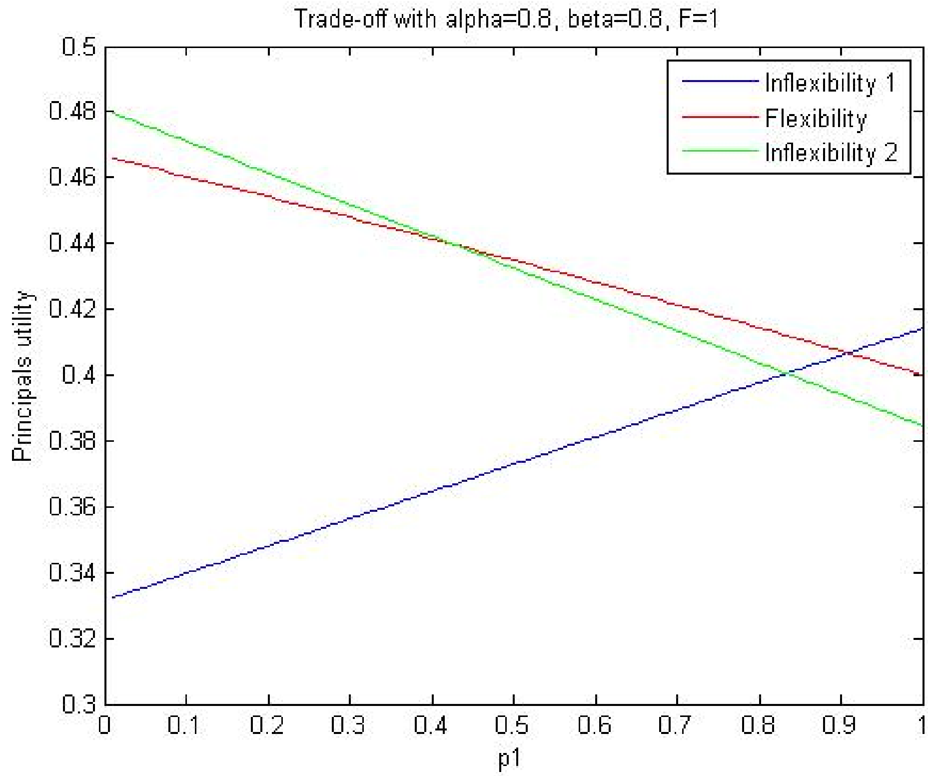


Figure 2

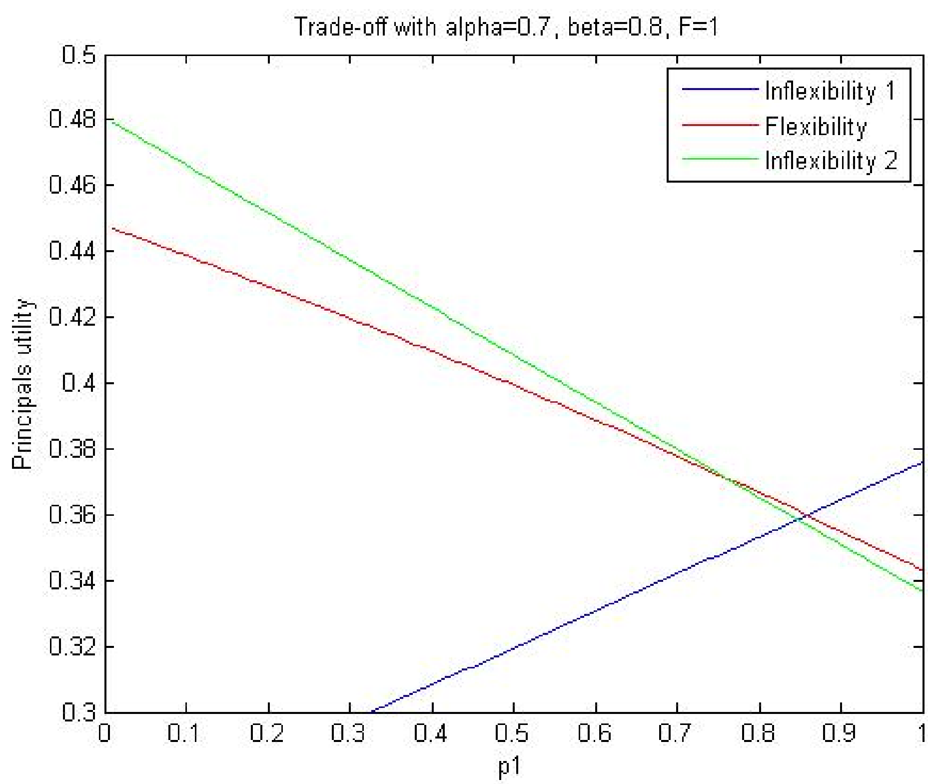


Figure 3 (beta=0.8,F=1)

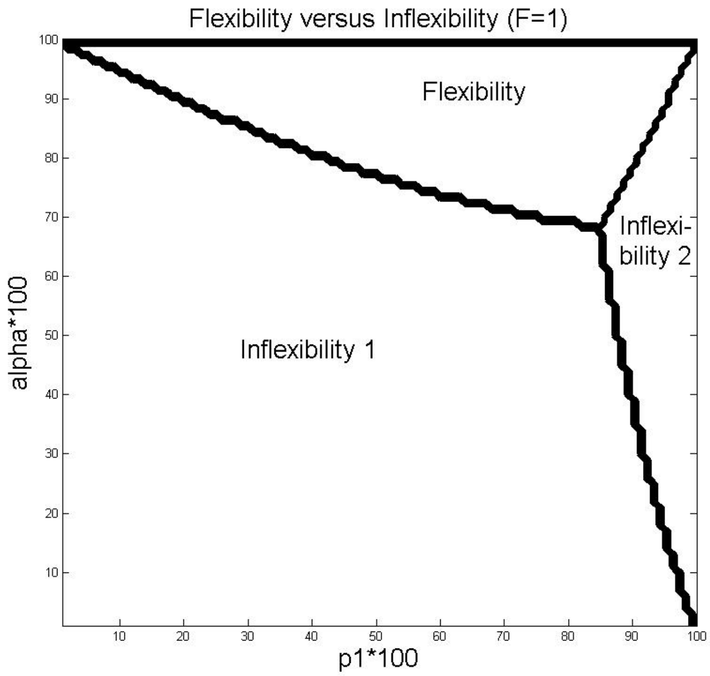


Figure 4 (beta=0.8,F=1.5)

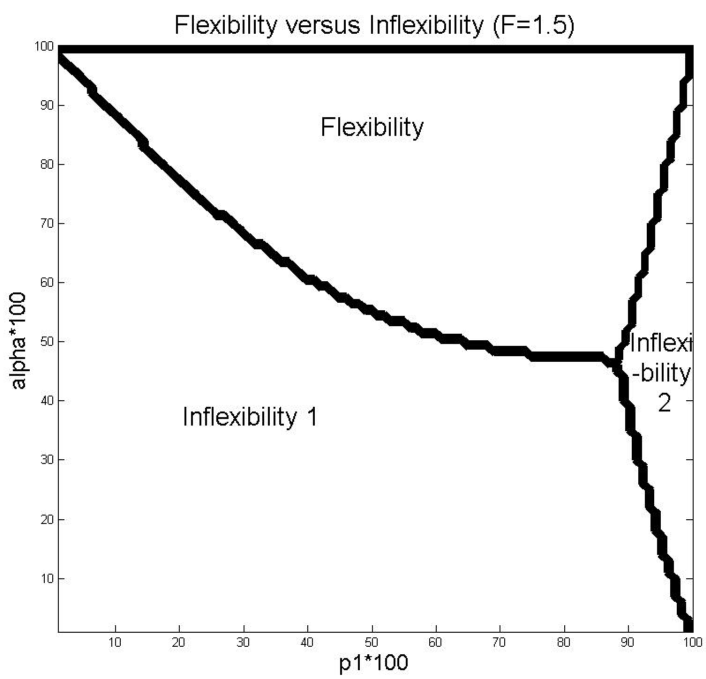
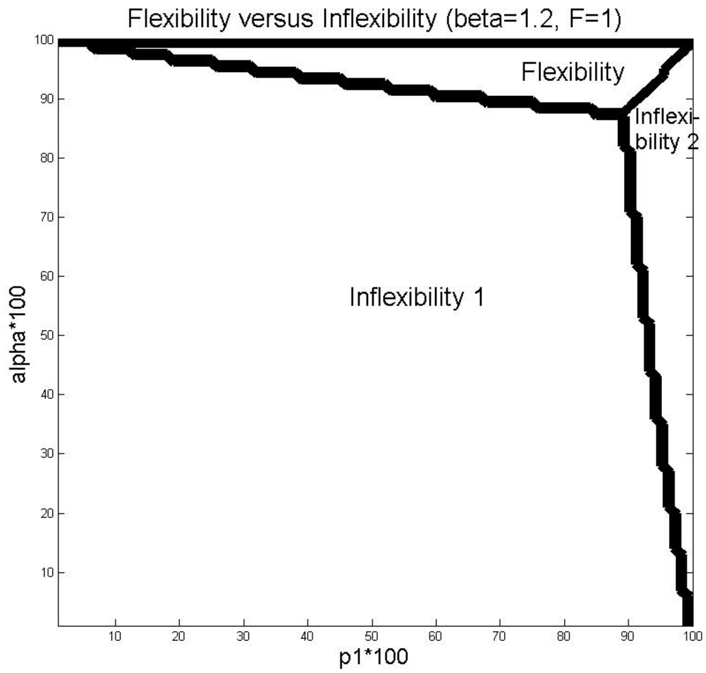


Figure 4 (beta=1.2,F=1)



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Chapter 2.
Generating functions for coalitional power indices: an application to the IMF[4]

(jointly written with José-Alonso María Meijide)

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Generating functions for coalitional power indices: an application to the IMF

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Abstract

This paper provides "ready-to-apply" procedures, based on generating functions, which allow to compute power indices in weighted majority games restricted by an a priori system of unions. We illustrate these methods by an application to the International Monetary Fund. We compare the empirical properties of the coalitional and traditional power indices keeping the game fixed or allowing for variations in its set of parameters.

Keywords: simple game; coalitional value; generating function; IMF

AMS classification: 91A12,91A80.

1 Introduction

How can we assess the relative distribution of power in a Committee when the voting rule requires the players to vote by groups instead of voting separately but let them free to choose the group they will join ?

This situation, though a bit odd, is not unlikely at all. The IMF statutes, for instance, ask the member states to meet in a limited number of groups -the so-called constituencies- before allowing a vote at the Executive Board, its main decision-making committee. The number of players is indeed so large (184 countries) that efficiency of the mechanism design requires to put some constraints on the size of the committee that will decide for the whole (in this case, 24 groups, each of them represented by a single Executive Director). Under this specific set of rules, the vote does not take place with the initial set of players, but with a partition involving only some subsets of it (the representatives of the constituencies). Once the groups are formed, each of them is endowed with the

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sum of the voting rights of its individual components without having the right to split them.

The traditional power indices derived from cooperative game theory, namely the Banzhaf and Shapley-Shubik power indices (Banzhaf, 1965; Shapley and Shubik, 1954), are unable to help us in measuring the distribution of power in this case. Their main strength is to provide a sufficient statistic of the probability that some player might be pivotal during a vote, in the sense that he/she might be able to transform a losing coalition into a winning one. Assuming some a priori uniform distribution under a veil of ignorance is indeed a strong and simplifying advantage. But when there exists some additional information about the coalition structure this assumption becomes too weak.

One way to take care about this additional information, without throwing away solution concepts that proved to be powerful in the past (playing a bit with words), is to apply them to a slightly modified game. Instead of considering a game over the whole set of players, define it over some subsets of the set of players. This can still provide a useful information about the distribution of power, as Leech (2002) did it for the IMF Executive Board in a recent issue of this journal. But what about the distribution of power within such restricted coalitions ? In particular: how should we take into account that some players pertaining to a given group must be rewarded for bringing their voting rights to it ? Here, inference about power needs the use of different instruments.

From a theoretical point of view, the a priori union framework initiated by Owen (1977) turned out to be the relevant one for addressing the aforementioned problem. The main advantage of the method is to correct the analytical expression of the indices, taking into account the modification of the probability space, while keeping track of their fundamental axiomatic properties with respect to both *within* and *between* unions allocation of rewards. We consider three power indices in this context: the Owen value (Owen, 1977), a modification of the Shapley-Shubik power index, the Banzhaf-Owen value (Owen, 1982), a modification of the Banzhaf power index, and the symmetric coalitional Banzhaf index (Alonso-Meijide and Fiestras-Janeiro, 2002) at half-distance between the Shapley-Shubik and Banzhaf indices.

This paper aims at providing some "ready-to-apply" procedures for computing the coalitional power indices. One of the strengths of these procedures is that they allow to get exact values of the indices, including for large games - no need for laborious approximation methods. Furthermore, the time required for the computation is in practice much lower than the one required for the computation of the traditional Banzhaf and Shapley-Shubik indices. This is so because the method, drawn from Cantor's (see Lucas, 1983) and Brams and Affuso (1976) early work, extended since by Fernández et al. (2002) for the Myerson value and by Algaba et al. (2003) for weighted multiple majority games, takes advantage of the properties of formal series called generating functions. Roughly, a generating function is a polynomial that allows to enumerate the set of possible coalitions, while keeping track of their respective weights.

This paper also applies these procedures to the IMF distribution of power. The application of the traditional power indices to situations where there are some a priori unions is shown to lead to a mistaken assessment of the distribution of power in a weighted majority game - justifying *ex post* and not only *ex ante* the interest of these refinements. One related striking interest is that coalitional power indices are sensitive to changes in the coalition structure, while traditional power indices are not.

The paper is organized as follows. In section 2 we recall some preliminary definitions and the notions of generating function and power indices for games with an a priori system of unions. In section 3 we introduce the procedures to compute these power indices by means of generating functions, and provide a simple example of computation "by hand". In section 4 we apply these procedures to the IMF weighted majority game in order to illustrate by a large game the difference in behavior of the three coalitional power indices considered in this paper. Section 5 is dedicated to concluding remarks.

2 Preliminaries

An n-person cooperative game with transferable utility (TU game) is a pair (N, v) where $N = \{1, 2, \ldots, n\}$ is the set of players and v, the characteristic function, is a real valued function on the subsets of N such that $v(\emptyset) = 0$. A subset S of N is called a coalition. A $(0-1)$ game is a TU game in which the function v only takes the values 0 and 1. A $(0-1)$ game is a simple game if it is not identically 0, and obeys the condition of monotonicity $(v(T) \leq v(S)$ whenever $T \subseteq S)$. In simple games, a coalition S is winning if $v(S) = 1$, and losing if $v(S) = 0$. We will denote by W the set of all winning coalitions of a simple game (N, v) and by $SI(N)$ the set of simple games with player set N.

A simple game (N, v) is a weighted majority game if there exists a set of weights $w_1, w_2, ..., w_n$ for players, with $w_i \geq 0, 1 \leq i \leq n$, and a quota $q \in \mathbb{R}^+$ such that $S \in W$ if and only if $w(S) \geq q$, where $w(S) = \sum_{i \in S} w_i$. A weighted majority game is represented by $[q; w_1, w_2, ..., w_n]$.

A power index is a function $f : SI(N) \longrightarrow \mathbb{R}^n$ which assigns to a simple game (N, v) a vector $f(N, v)$, where the real number $f_i(N, v)$ is the power of player i in the game (N, v) according to f.

The most important power indices are the Banzhaf index and the Shapley-Shubik index (hereafter BZ and SH). These indices can be written in this way:[1]

$$f_i(N, v) = \sum_{S \subseteq N \setminus i} p_S^i (v(S \cup i) - v(S)), \text{ for any } i \in N,$$

where $p_S^i = 1/2^{n-1}$ for the BZ index and $p_S^i = s!(n-s-1)!/n!$ for the SH index. These two probability measures are, according to Felsenthal and Machover (1998), the main two underlying the various existing power indices in the literature. The BZ index is based on the combination of players and the SH index

[1] Let S be a finite set. We denote by s the cardinality of S, i.e., $|S| = s$. As it has become common practice, given $i \in S$ we will write for simplicity $S \setminus i$ instead of $S \setminus \{i\}$, and given $i \notin S$ we will write $S \bigcup i$ instead of $S \bigcup \{i\}$.

is based on their permutations. Intuitively, the basic unit of analysis for the BZ index is the coalition, while the SH index focuses on the ranking of individual preferences over a given outcome. In both cases, each basic measurable event - coalition or ranking of preferences - is assumed to have an equal probability.

Taking into account that for a simple game $v(T) = 1$ if $T \in W$ and $v(T) = 0$ otherwise, it holds that $v(S \cup i) - v(S) = 1$ if and only if S is losing and $S \cup i$ is winning. In this case, we say that the pair of coalitions $(S, S \cup i)$ is a swing for player i. Given a simple game (N, v), SH and BZ power indices of a player $i \in N$ depend on the number of swings for player i.

In general, the computation of the previous indices needs a great number of operations. The generating function is one of the procedures to compute them. Generating functions give a method to count the number of elements $c(r)$ of a finite set, where these elements have a configuration that depends on a characteristic r. An application of these functions in the field of simple games allows to recover the number of possible coalitions of a given kind from the set of its coefficients, while the voting power of the coalition can easily be read looking at its set of exponents. Brams and Affuso (1976) provide an example of generating function very easy to understand for the clasical power indices.

The generating function of the numbers $a = \{a_0, a_1, a_2, \ldots\}$ is the formal series $f_a(t) = \sum_j a_j t^j$, and can be finite or infinite. The variable t serves to identify a_j as the coefficient corresponding to t^j in $f_a(t)$.

In some cases, we will employ generating functions of several variables, for example

$$S(x, y, z) = \sum_{k \geq 0} \sum_{j \geq 0} \sum_{l \geq 0} c(k, j, l)\, x^k y^j z^l,$$

where $c(k, j, l)$ are real numbers that depend on k, j and l.

Let us consider a finite set $N = \{1, \ldots, n\}$. We will denote by $P(N)$ the set of all partitions of N. An element $P \in P(N)$ is called a coalition structure or a system of unions of the set N. A simple game with a coalition structure is a triplet (N, v, P), where $(N, v) \in SI(N)$ and $P \in P(N)$. We will denote by $SU(N)$ the family of all simple games with player set N and a coalition structure.

In this case, a power index is a function $f : SU(N) \longrightarrow \mathbb{R}^n$ which assigns a vector $f(N, v, P)$ to a simple game with an a priori system of unions (N, v, P), where the real number $f_i(N, v, P)$ is the power of player i in the game (N, v, P) according to f.

We consider three power indices for $SU(N)$: the Banzhaf-Owen index (hereafter BO), the Symmetric Coalitional Banzhaf index (hereafter SCB) and the Owen index (hereafter OW). These indices can be written in this way:

$$f_i(N, v, P) = \sum_{R \subseteq M \setminus k} \sum_{T \subseteq P_k \setminus i} p^i_{R,T} \left(v(Q \cup T \cup i) - v(Q \cup T) \right), \text{ for any } i \in N,$$

where $M = \{1, \ldots, m\}$, $P = \{P_1, \ldots, P_m\}$, $Q = \cup_{r \in R} P_r$, and $P_k \in P$ is the union such that $i \in P_k$. For the case of the BO index $p^i_{R,T} = \frac{1}{2^{m-1}} \frac{1}{2^{p_k - 1}}$, for the SCB index $p_{R,T}^{i} = \frac{1}{2^{m-1}} \frac{t!(p_k-t-1)!}{p_k!}$ and finally, for the OW index $p_{R,T}^{i} = \frac{r!(m-r-1)!}{m!} \frac{t!(p_k-t-1)!}{p_k!}$ (Owen, 1982; Alonso-Meijide and Fiestras-Janeiro, 2002; Owen, 1977).

According to the probability model underlying the choice of these probability distributions, one can see that all the three coalitional power indices can be derived from a two-level bargaining process. First, unions split the total amount according to the SH or the BZ type of allocation. Then, each union allocates its total reward among its members taking into account the possibility that they might join another union using again the SH or the BZ type of allocation.

3 Using generating functions to compute power indices restricted by a priori unions

This section is dedicated to the derivation of the generating functions of the power indices restricted by a priori unions, that will allow us to compute them easily. We begin first with some definition.

Definition 1 *Given a game $(N, v, P) \in SU(N)$, where $P = \{P_1, P_2, \ldots, P_m\}$, a coalition $S \subseteq N$ is compatible with the a priori system of unions P for a player $i \in P_j$, if $S = \cup_{k \in R \subseteq M \setminus j} P_k \cup T$, with $T \subseteq P_j$.*

If a coalition $S \subseteq N$ is compatible with the a priori system of unions P for a player $i \in P_j$, then S is compatible with P for any player $k \in P_j$. Then we will denote by $C(j, P)$ the set of compatible coalitions with the a priori system of unions for players of the union P_j in a game $(N, v, P) \in SU(N)$.

Definition 2 *Given a game $(N, v, P) \in SU(N)$, a compatible swing with P for a player $i \in P_j$, is a pair of coalitions $(S, S \cup i)$ such that S is losing, $S \in C(j, P)$ and $S \cup i$ is winning.*

We will denote by $\eta_i(N, v, P)$ the number of compatible swings with P for a player $i \in N$, in the game $(N, v, P) \in SU(N)$.

3.1 The Banzhaf-Owen index

Given a game $(N, v, P) \in SU(N)$, where $P = \{P_1, P_2, \ldots, P_m\}$, the BO index of a player $i \in P_j$ is equal to:

$$\Psi_i(N, v, P) = \frac{\eta_i(N, v, P)}{2^{m+p_j-2}}.$$

In the next result, we propose a method to compute $\eta_i(N, v, P)$ in a weighted majority game with an a priori system of unions.

Lemma 1 *Let (N, v, P) be a weighted majority game with an a priori system of unions, with $v = [q; w_1, w_2, \ldots, w_n]$ and $P = \{P_1, P_2, \ldots, P_m\}$. The number of compatible swings with P for a player $i \in P_j$ is:*

$$\eta_i (N, v, P) = \sum_{k=q-w_i}^{q-1} b_k^i,$$

where b_k^i is the number of coalitions $S \in C(j, P)$ such that $i \notin S$ and $w(S) = k$.

Proof. Let us take a weighted majority game with an a priori system of unions $(N, v, P) \in SU(N)$.

The compatible coalitions with P, S, for a player $i \in P_j$, such that $i \notin S$ and, whose weight is between $q - w_i$ and $q - 1$, are losing coalitions. When player i joined this coalition, its weight is greater or equal than q, and then, the coalition becomes a winning one.

If we denote by b_k^i the number of compatible coalitions with P for i, S, such that $i \notin S$ with weight k between $q - w_i$ and $q - 1$, the number of compatible swings with P for i in (N, v, P) is obtained by adding the numbers $\left\{b_k^i\right\}_{k\geq 0}$ for values k between $q - w_i$ and $q - 1$. Thus, $\eta_i (N, v, P) = \sum_{k=q-w_i}^{q-1} b_k^i$. ∎

In the next result, we present the generating function of the numbers $\left\{b_k^i\right\}_{k\geq 0}$.

Theorem 1 *Let (N, v, P) be a weighted majority game with an a priori system of unions, with $v = [q; w_1, w_2, \ldots, w_n]$ and $P = \{P_1, P_2, \ldots, P_m\}$. The generating function of numbers $\left\{b_k^i\right\}_{k\geq 0}$ for a player $i \in P_j$ is given by:*

$$B_i (x) = \prod_{r=1, r\neq j}^{m} \left(1 + x^{w(P_r)}\right) \prod_{l=1, j_l \neq i}^{p_j} \left(1 + x^{w_{j_l}}\right),$$

where $w(P_r) = \sum\limits_{j\in P_r} w_j$ and $P_j = \left\{j_1, j_2, ..., j_{p_j}\right\}$.

Proof. Let us take a game $(N, v, P) \in SU(N)$ with $v = [q; w_1, w_2, \ldots, w_n]$ and a player $i \in P_j$. We consider the function:

$$\left(1 + x^{w(P_1)}\right) \ldots \left(1 + x^{w(P_{j-1})}\right) \left(1 + x^{w(P_{j+1})}\right) \ldots \left(1 + x^{w(P_m)}\right) \left(1 + x^{w_{j_1}}\right) ...$$

$$\ldots \left(1 + x^{w_{j_{p_j}}}\right) = 1 + \sum_{S\in C(j,P)} \prod_{l\in S} x^{w_l} = 1 + \sum_{S\in C(j,P)} x^{\sum_{l\in S} w_l} = 1 + \sum_{S\in C(j,P)} x^{w(S)}.$$

Grouping exponents of the same order and taking $k = w(S)$, the previous function is equal to:

$$\sum_{k=0}^{w(N)} b_k x^k,$$

where $b_0 = 1$ and b_k, for $k > 0$ is the number of compatible coalitions with P with weight k for $i \in P_j$. If we want to compute the numbers $\left\{b_k^i\right\}_{k\geq 0}$, it is sufficient to drop the factor $(1 + x^{w_i})$. ∎

3.2 The Symmetric Coalitional Banzhaf index

Given a game $(N,v,P) \in SU(N)$, where $P = \{P_1, P_2, \ldots, P_m\}$ the SCB index of a player $i \in P_j$, is equal to:

$$\Pi_i(N,v,P) = \sum_{\{S \in C(i,P)/S \notin W \ and \ S \cup i \in W\}} \frac{1}{2^{m-1}} \frac{|S \cap P_j|!\,(p_j - |S \cap P_j| - 1)!}{p_j!} =$$

$$\sum_{l=0}^{p_j-1} \frac{1}{2^{m-1}} \frac{l!\,(p_j - l - 1)!}{p_j!} d_l^i,$$

where d_l^i represents the number of compatible swings with P, $(S, S\cup i)$, for player i, in the game (N,v,P), such that $|S \cap P_j| = l$. For any value of l between 0 and $p_j - 1$,

$$d_l^i = \sum_{k=q-w_i}^{q-1} a_{kl}^i,$$

where a_{kl}^i is the number of compatible swings with P, $(S, S\cup i)$, for player i in (N,v,P) with $w(S) = k$ and $|S \cap P_j| = l$.

Theorem 2 *Let (N,v,P) be a weighted majority game with an a priori system of unions, with $v = [q; w_1, w_2, \ldots, w_n]$ and $P = \{P_1, P_2, \ldots, P_m\}$. The generating function of numbers $\left\{a_{kl}^i\right\}_{k\geq 0, l \geq 0}$, for a player $i \in P_j$ is given by:*

$$S_i(x,z) = \prod_{r=1, r\neq j}^{m} \left(1 + x^{w(P_r)}\right) \prod_{l=1, j_l \neq i}^{p_j} \left(1 + x^{w_{j_l}} z\right).$$

where $w(P_r) = \sum_{j \in P_r} w_j$ and $P_j = \left\{j_1, j_2, ..., j_{p_j}\right\}$.

Proof. Let us take a game $(N,v,P) \in SU(N)$ with $v = [q; w_1, w_2, \ldots, w_n]$ and a player $i \in P_j$. We consider the function:

$$\left(1 + x^{w(P_1)}\right) \ldots \left(1 + x^{w(P_{j-1})}\right) \left(1 + x^{w(P_{j+1})}\right) \ldots \left(1 + x^{w(P_m)}\right) \left(1 + x^{w_{j_1}} z\right) \ldots$$

$$\ldots \left(1 + x^{w_{j_{p_j}}} z\right) = 1 + \sum_{S \in C(i,P)} \prod_{k \in S \setminus P_j} x^{w_k} \prod_{k \in S \cap P_j} x^{w_k} z =$$

$$1 + \sum_{S \in C(j,P)} x^{\sum_{k \in S} w_k} z^{|S \cap P_j|} = 1 + \sum_{S \in C(j,P)} x^{w(S)} z^{|S \cap P_j|}.$$

Grouping exponents of the same order and taking $k = w(S)$, the previous function is equal to:

$$\sum_{k=0}^{w(N)} \sum_{l=0}^{p_j} a_{kl} x^k z^l,$$

where $a_{00} = 1$ and a_{kl} when at least one of the indices k or l is greater than 0, is the number of compatible coalitions with P, S, for $i \in P_j$ with $w(S) = k$ and $|S \cap P_j| = l$. To compute $\left\{a_{kl}^i\right\}_{k \geq 0, l \geq 0}$, it is sufficient to drop the factor $(1 + x^{w_i})$. ■

The previous result gives a method to compute $\left\{a_{kl}^i\right\}_{k \geq 0, l \geq 0}$. Now, it is necessary to obtain the values d_l^i, that can be identified by the coefficients of

$$g_i(z) = \sum_{l=0}^{p_j - 1} d_l^i z^l,$$

and, taking into account that $d_l^i = \sum_{k=q-w_i}^{q-1} a_{kl}^i$, it holds that:

$$g_i(z) = \sum_{l=0}^{p_j-1} d_l^i z^l = \sum_{l=0}^{p_j-1} \left[\sum_{k=q-w_i}^{q-1} a_{kl}^i \right] z^l.$$

Thus,

$$S_i(x, z) = \sum_{l=0}^{p_j-1} \left[\sum_{k=0}^{w(N)-w_i} a_{kl}^i x^k \right] z^l.$$

The coefficients of $g_i(z)$ can be computed, selecting for any exponent of variable z in $S_i(x, z)$, the coefficients of those terms $x^k z^l$ such that k takes values between $q - w_i$ and $q - 1$.

3.3 The Owen index

Given a game $(N, v, P) \in SU(N)$ where $P = \{P_1, P_2, \ldots, P_m\}$, the OW index of a player $i \in P_j$, is equal to:

$$\Phi_i(N, v, P) =$$

$$\sum_{\substack{\{S \in C(i,P)/ \\ S \notin W \text{ and } S \cup i \in W\}}} \frac{|m_j(S)|!\,(m - |m_j(S)| - 1)!}{m!} \frac{|S \cap P_j|!\,(p_j - |S \cap P_j| - 1)!}{p_j!} =$$

$$\sum_{r=0}^{m-1} \sum_{l=0}^{p_j-1} \frac{r!\,(m-r-1)!}{m!} \frac{l!\,(p_j - l - 1)!}{p_j!} d_{rl}^i,$$

where $m_j(S) = \{k \in M \backslash j / P_k \subseteq S\}$ and d_{rl}^i is the number of compatible swings with P, $(S, S \cup i)$, for a player $i \in P_j$ in (N, v, P), such that $|m_j(S)| = r$ and $|S \cap P_j| = l$. Then, for any value of r between 0 and $m - 1$ and, any value of l between 0 and $p_j - 1$,

$$d_{rl}^i = \sum_{k=q-w_i}^{k=q-1} a_{krl}^i,$$

where a_{krl}^i is the number of compatible swings with P, $(S, S \cup i)$, for a player $i \in P_j$, such that $w(S) = k$, $|m_j(S)| = r$ and $|S \cap P_j| = l$.

Theorem 3 *Let (N, v, P) be a weighted majority game with an a priori system of unions, with $v = [q; w_1, w_2, \ldots, w_n]$ and $P = \{P_1, P_2, \ldots, P_m\}$. The generating function of the numbers $\left\{a^i_{krl}\right\}_{k\geq 0, r\geq 0, l\geq 0}$ for a player $i \in N$ is given by:*

$$S_i(x, t, z) = \prod_{r=1, r\neq j}^{m} \left(1 + x^{w(P_r)} t\right) \prod_{l=1, j_l\neq i}^{p_j} \left(1 + x^{w_{j_l}} z\right)$$

where $w(P_r) = \sum\limits_{j\in P_r} w_j$ *and* $P_j = \left\{j_1, j_2, ..., j_{p_j}\right\}$.

Proof. Let us take a game $(N, v, P) \in SU(N)$, with $v = [q; w_1, w_2, \ldots, w_n]$ and a player $i \in P_j$. We consider the function:

$$\left(1 + x^{w(P_1)} t\right) \ldots \left(1 + x^{w(P_{j-1})} t\right) \left(1 + x^{w(P_{j+1})} t\right) \ldots \left(1 + x^{w(P_m)} t\right) \left(1 + x^{w_{j_1}} z\right) \ldots$$

$$\ldots \left(1 + x^{w_{j_{p_j}}} z\right) = 1 + \sum_{S\in C(j,P)} \prod_{r\in m_j(S)} \left(x^{w(P_r)} t\right) \prod_{k\in S\cap P_j} \left(x^{w_k} z\right) =$$

$$1 + \sum_{S\in C(j,P)} x^{\sum_{k\in S} w_k} t^{|m_j(S)|} z^{|S\cap P_j|} = 1 + \sum_{S\in C(j,P)} x^{w(S)} t^{|m_j(S)|} z^{|S\cap P_j|}.$$

Grouping exponents of the same order and taking $k = w(S)$, the previous function is equal to:

$$\sum_{k=0}^{w(N)} \sum_{r=0}^{m-1} \sum_{l=0}^{p_j} a_{krl} x^k t^r z^l$$

where $a_{000} = 1$ and a_{krl}, where at least one of the indices k, r or l is greater than 0, is the number of compatible coalitions with P, S, for a player $i \in P_j$ with $w(S) = k$, $m_j(S) = r$ and $|S \cap P_j| = l$. To compute the numbers $\left\{a^i_{krl}\right\}_{k\geq 0, r\geq 0, l\geq 0}$, it is sufficient to drop the factor $(1 + x^{w_i})$. ■

The previous result gives a method to compute $\left\{a^i_{krl}\right\}_{k\geq 0, r\geq 0, l\geq 0}$. Now, it is necessary to obtain the values d^i_{rl}, that can be identified by the coefficients of

$$g_i(t, z) = \sum_{r=0}^{m-1} \sum_{l=0}^{p_j - 1} d^i_{rl} t^r z^l,$$

and, taking into account that $d^i_{rl} = \sum\limits_{k=q-w_i}^{q-1} a^i_{krl}$, it holds that :

$$g_i(t, z) = \sum_{r=0}^{m-1} \sum_{l=0}^{p_j - 1} d^i_{rl} z^l t^r = \sum_{r=0}^{m-1} \sum_{l=0}^{p_j - 1} \left[\sum_{k=q-w_i}^{q-1} a^i_{krl}\right] t^r z^l.$$

By the previous result:

$$S_i(x, t, z) = \sum_{r=0}^{m-1} \sum_{l=0}^{p_j - 1} \left[\sum_{k=0}^{w(N)-w_i} a^i_{krl} x^k\right] t^r z^l.$$

The coefficients of $g_i(t,z)$ can be computed, selecting for each pair of exponents of variables z and t in $S_i(x,t,z)$, the coefficients of those terms $x^k t^r z^l$ such that k takes values between $q - w_i$ and $q - 1$.[2]

3.4 A simple example

The Parliament of the Balearic Islands, one of the Spain seventeen autonomous communities, is made up of 59 members. Following elections on 13 June, 1999, the Parliament was composed of 28 members of the conservative party PP, 16 members of the socialist party PSOE, 5 members of the socialist regional party PSM-EN, 4 members of EU-EV, a coalition of communist and other left-wing parties, 3 members of the middle-of-the-road regional party UM, and 3 members of PROG, a coalition of progressive parties that presented candidature only on some island. Analyzing this Parliament as a weighted majority game, the quota is 30 and it can be represented as $[30; 28, 16, 5, 4, 3, 3]$.

3.4.1 How does it work ? The simple Banzhaf case

To understand how the generating function works, let's consider first that there is no a priori union and compute the Banzhaf index of player two. Let $BZ(x)$ be the generating function for the game. We have :

$$BZ(x) = (1+x^{28})(1+x^{16})(1+x^5)(1+x^4)(1+x^3)(1+x^3).$$

As we can see, each of the exponents of the variable x corresponds to the voting weights associated to a given player. In order to get the generating function relevant for player two, say $BZ_2(x)$, we drop the factor corresponding to this player, that is $(1+x^{16})$:

$$BZ_2(x) = (1+x^{28})(1+x^5)(1+x^4)(1+x^3)(1+x^3).$$

The next step is to expand the polynomial :

$$\begin{aligned} BZ_2(x) &= (1+x^{28})(1+x^5)(1+x^4)(1+x^3)(1+x^3) \\ &= 2x^3 + x^4 + x^5 + x^6 + 2x^7 + 2x^8 + x^9 + x^{10} + x^{11} \\ &\quad +2x^{12} + x^{15} + x^{28} + 2x^{31} + x^{32} + x^{33} + x^{34} + 2x^{35} \\ &\quad +2x^{36} + x^{37} + x^{38} + x^{39} + 2x^{40} + x^{43} + 1. \end{aligned}$$

[2] The results presented in this section can be extended for (hierarchical) level structures (Winter, 1989) in a straightforward way. A level structure is a finite sequence of partitions $B = (B_1, B_2, ..., B_m)$ such that if $S \subset B_i$, then $S \subset T$ for some $T \in B_{i+1}$. For a two-levels structure, the generating function of the Owen Index might for instance be written as :

$R_i(x,t,z,y) = \prod_{l=1,l\neq k}^{u} \left(1 + x^{w(U_l)} y\right) \prod_{r=1,r\neq j}^{m_l} \left(1 + x^{w(P_{lr})} t\right) \prod_{k=1,j_k\neq i}^{p_j} \left(1 + x^{w_{j_k}} z\right)$

where $U = \{U_1, U_2, ..., U_u\}$ and $U_l = \{P_{l1}, P_{l2}, ..., P_{lm_l}\}$ represents a partition of the set of all the partitions of the set of players. We thank Eyal Winter for having pointed out this possible generalization.

We are now able to understand the "trick" underlying the use of generating functions. The property of the power function, which can transform the multiplication of a given variable into a sum of different exponents, is used to recollect the sum of the weights of a given coalition. For instance, if player four (represented by the monom x^4 because she has 4 voting rights) forms a coalition with player six (represented by the monom x^3), the total weight of the coalition can be read in the exponent of $x^4 \cdot x^3 = x^{4+3} = x^7$. Recollecting all the monoms with the same exponents allows to get the number of coalitions with the same weight. For example, $2x^7$ can be read as: there are two coalitions whose weight is equal to seven (player five with player six and player four with player seven).

We are now able to count the number of coalitions for which player two is pivotal. This will be the case whenever the coalitions formed before her arrival have a weight above 14 and below 29. There are only two of them: one whose total weight is equal to 15 (represented by x^{15}), and another whose total weight is equal to 28 (represented by x^{28}). Given that the total number of possible coalitions without player two is equal to $2^5 = 32$, the (non-normalized) BZ index of player two is 1/16.

3.4.2 How does it work ? The a priori union cases

We may presume the existence of agreements among the left-wing parties. Moreover, from the regionalist opinions of PSM-EN and UM we may assume that both parties will form a union. We consider the following system of unions:

$$P = \{\{PP\}, \{PSOE, EU - EV, PROG\}, \{PSM - EN, UM\}\},$$

where P_1 has 28 seats, P_2 has 23 seats and P_3 has 8 seats.

A1. BO index of player 4. We take the function:

$$B_4(x) = \left(1 + x^{28}\right)\left(1 + x^8\right)\left(1 + x^{16}\right)\left(1 + x^3\right) =$$

$$x^{55} + x^{52} + x^{47} + x^{44} + x^{39} + x^{36} + x^{31} + x^{28} + x^{27} +$$

$$x^{24} + x^{19} + x^{16} + x^{11} + x^8 + x^3 + 1.$$

Then, we obtain $\eta_4(v) = \sum_{k=26}^{29} b_k^4 = 2$. In a similar way, it holds that $\eta_1(v) = \eta_3(v) = \eta_4(v) = \eta_5(v) = \eta_6(v) = 2$.

The BO index is: $\Psi(N, v, P) = (1/2, 1/8, 1/4, 1/8, 1/4, 1/8)$.

A2. SCB index of player 3. We take the function:

$$S_3(x, z) = \left(1 + x^{28}\right)\left(1 + x^{23}\right)\left(1 + x^3 z\right) =$$

$$x^{54}z + x^{31}z + x^{26}z + x^3 z + x^{51} + x^{28} + x^{23} + 1.$$

We choose the coefficients of terms $x^k z^j$ such that k takes values between 25 and 29 ($x^{26}z$ and x^{28}), then

$$g_3(z) = \sum_{j=0}^{1} d_j^3 z^j = z + 1,$$

and the SCB index of player 3 is equal to:

$$\Pi_3(N,v,P) = \frac{1}{2^2}\sum_{j=0}^{1}\frac{j!\,(2-j-1)!}{2!}d_j^3 = \frac{1}{4}\frac{1!0!}{2!}1 + \frac{1}{4}\frac{0!1!}{2!}1 = \frac{1}{4}.$$

The SCB index is: $\Pi(N,v,P) = (1/2, 1/6, 1/4, 1/6, 1/4, 1/6)$.

A3. OW index of player 6. We take the function:

$$S_6(x,t,z) = \left(1+x^{28}t\right)\left(1+x^{8}t\right)\left(1+x^{16}z\right)\left(1+x^{4}z\right) =$$

$$x^{56}tz^2 + x^{48}tz^2 + x^{28}tz^2 + x^{20}z^2 + x^{52}tz + x^{44}tz + x^{40}tz +$$

$$x^{32}tz + x^{24}tz + x^{12}tz + x^{16}z + x^{4}z + x^{36}t + x^{28}t + x^{8}t + 1.$$

We choose the coefficients of terms $x^k t^r z^l$ such that the exponent k takes values between 27 and 29 ($x^{28}tz^2$ and $x^{28}t$), then

$$g_6(t,z) = \sum_{r=0}^{2}\sum_{l=0}^{2} d_{rl}^i t^r z^l = tz^2 + t,$$

and the OW index of player 6 is equal to:

$$\Phi_6(N,v,P) = \sum_{r=0}^{2}\sum_{l=0}^{2}\frac{r!\,(m-r-1)!}{m!}\frac{l!\,(p_j-l-1)!}{p_j!}d_{rl}^i =$$

$$\frac{1!1!}{3!}\frac{0!2!}{3!}1 + \frac{1!1!}{3!}\frac{2!0!}{3!}1 = \frac{1}{9}.$$

The OW index is: $\Phi(N,v,P) = (1/3, 1/9, 1/6, 1/9, 1/6, 1/9)$.

In general, however, games with a large number of players will not allow to get these indices by hand, and the use of a computer is required. This is the procedure that we used in the following application to the IMF. This aims at illustrating the difference in behavior of the coalitional power indices. The procedure has been programmed under the Mathematica system, using a modification of codes written by Tannenbaum (1997) and Bilbao (2000).[3] The results differ from the previous research on the topic presented in this journal (Leech, 2002).

[3] We are happy to make our Mathematica codes available upon request.

4 An application to the IMF

Besides the analysis of parliamentary systems, power indices were widely applied to international organizations, such as the UN Security Council (Shapley and Shubik, 1954), the European Union (Widgren, 1994), the European Central Bank (Brückner, 2000) or the International Monetary Funds (Dreyer and Schotter, 1980; Schmidtchen (2001a), Schmidtchen (2001b) and Leech, 2002). It turns out that the case of IMF is a particularly good example of the limitations that were imposed on this kind of analysis. This is due to the fact that the tools we provide in this paper were missing.

4.1 IMF a priori unions

According to its statutes, decision making at IMF is supposed to be made with respect to a weighted majority rule, using a majority requirement that is varying depending on the importance of the decision to be taken (50%, 70% or 85%). Nevertheless, instead of allowing the whole set of possible coalitions, IMF Article of Agreements imposed the following restrictions on the cooperation structure. First, the 184 members states have to meet by groups of countries, the so-called constituencies, in order to choose an Executive Director who will represent them at the Executive Board. Then, the 24 Executive Directors vote according to a weighted majority rule, each of them casting the total of the votes of the constituency they represent (without having the right to split the voting rights they cast). As a result, the set of feasible coalitions is restricted to happen by a priori blocs of countries. Such a restriction on the set of allowed coalitions obviously comes from the willingness to ease everyday life decision making[4].

This mechanism design calls for the analysis to be carried out for a two-level bargaining process. As stressed by Leech (2002):

> "Given the existence of the constituencies around the election of executive directors, it might be considered appropriate [...] to model the power relationships [...] in terms of a two-stage process: first, members use their weighted votes within their group to elect a director; second, their elected director casts their combined votes as a bloc in the Executive Board. [...] This two-stage approach, however, has not been pursued here since it would require the existence of the constituencies to be fixed independently of the outcome of the first stage, which cannot be assumed under the rules. [...] It is an interesting topic that is not considered here and remains for future works".

Leech did not follow further his reasoning because he was much more thinking about the so-called v-composition of games (see Owen, 1995). Here, such a

[4] And this is indeed the case, as the power of the collectivity to act (as defined in Coleman, 1971) is increased from 8.25393E-06 without this restriction to 7.17461E-04 with it (using a 85% majority requirement).

composition does not work well as it would indeed assign a zero power to countries pertaining to a constituency where some members have more than half of the total of votes within it (Italy or Brazil for instance). In fact, the definition of the constituencies (that is, the identification of the countries that pertain to a given constituency) does not result itself from IMF Articles of Agreements, but from an informal bargaining process among members. It is obviously hard to believe that a member state will accept to give its voting rights to such a dominant country, knowing that the Executive Director of the constituency is going to decide without taking into account its opinion. The former would instead prefer to join another constituency without such a dominant country. The a priori union framework takes into account this implicit power due to the existence of an outside option. It assumes that the allocation of power within a constituency is proportional to the voting rights the country adds to the union he joins, the proportionality factor depending on the index used.[5] As a result, it fits better to the particular information we have on the existing coalition structure.

In what follows, we provide the BO, the SCB and the OW indices using the IMF voting rights as well as the a priori union structure for September 2002.[6] For comparison purposes, we also provide two kind of indices. The first are the traditional BZ and SH indices using the set of weights of the quotient game. This game is defined for 24 players, where each player is an Executive Director endowed with the total of votes of the countries who elected him. The second are the BZ and SH indices using the set of weights given by the voting rights of the 184 members states without any a priori union structure (better said, with the trivial one where each country votes as a single entity). Hence, the first kind of indices analyzes the Executive Board weighted majority game without being able to decompose the voting rights within each constituency, while the second kind of indices analyzes the Governor's Board weighted majority game, the Governor Board being a different committee whose size makes it unable to be involved in everyday life IMF decision-making. The indices are computed for the three majority requirements, and presented in their normalized form (e.g. divided so that the totals sum up to one). It has to be stressed that the generating function method allowed us to compute these indices for a large set of players without having to use any approximation method.

4.2 Comparison of the indices

As can be seen in table 1, the indices differ whenever they are computed with or without an a priori union framework. The differences become very sensitive to the majority requirement. For instance, the Banzhaf-type of power (using the 85% majority requirement) of the actor with the higher weight, namely United States, is roughly divided by two when it is measured without a priori unions.

[5] We have received a confirmation of the legitimacy of the a priori union approach, compared to the v-composition one, from private conversations with anonymous officials that were involved in the IMF decision-making process.

[6] Both can be found on the IMF website : http://www.imf.org.

If the decision lies within the Board of Governor, power is accurately measured by standard power analysis. But when the decision lies within the Executive Board, the coalitional power indices may provide a more relevant measure.

[Insert Table 1 here]

4.2.1 Comparison with the indices without a priori unions

One can also compare the behavior of the indices computed with a priori unions to the behavior of the indices computed without a priori unions but using the set of weights of the quotient game (Table 2). It is quite appealing to see that, in this case, the union-sums of the power indices are roughly equal to the traditional BZ and SH indices computed on the quotient game. In fact, for the SCB and OW case, the differences are only due to roundings made on the set of weights used in the computation[7] but they should be regarded as equal to the BZ and SH (respectively) computed on the quotient game. Hence, the behavior of the two former indices reproduces the feature of the BZ and SH indices, but adds information to the analysis because it is now possible to disaggregate the value at the individual level. For instance, if the SH index roughly yields 4.3 percent of power to the constituency containing Spain and Venezuela, we are now able to say, looking at the OW index, that these countries respectively exert 1.4 and 1.2 percent of the power from an individual point of view.

[Insert Table 2 here]

4.2.2 Modifying the cooperation structure without changing the set of weights

Interestingly, the a priori union framework also allows us to analyze what would be the member states' power in another coalition structure which would not affect the distribution of individual voting rights. Assume for instance that European states were to form a European constituency, a question that is actually under debate and that Benassy-Queré and Bowles (2004) analyze in more details in a separate paper. Would such an a priori union increase or decrease the power of European Union as a whole with respect to, say, the United States ?

[7] IMF voting rights are obtained by giving 250 basic votes plus one voting right for each 100 000 SDR quota a member states has. When expressed in percentage form, nothing prevents the voting weights of member states from being irrational numbers. In order to get integer weights (needed by the use of the generating function method) while keeping a two-digits precision, we multiplied all the weights by 100 in the case of BO and SCB, BZ indices (10 for the OW and SH indices because their computation was more time-consuming). Within the a priori union framework, the rounding is made on an individual basis while it is made on an aggregate basis when the BZ and SH are computed. This, together with the impact of the rounding on the majority requirement, explains entirely the differences. It should be mentioned that the very small differences in voting weights between small countries diminish by rounding.

The BZ and SH indices cannot help us in addressing the question: they will remain unchanged as far as the voting rights will not change. Conversely, the BO, SCB and OW indices change. The striking feature is that this yields to an increased power of the United States under the 85% majority requirement, while the aggregate power of European Union decreases, while the opposite result holds under the 50% majority requirement. On the other hand, small countries or constituencies are likely to benefit from the European consituencies under the 85% majority threshold, indicating that the unification may offer a scope for a better representation of developping countries.

Without detailing the whole set of results, we provide part of them as they also shed light on the behavior of the three former indices when the coalition structure is to be changed (please see Table 3). Indeed, this a priori configuration first illustrates the importance of the symmetry in the quotient game property. While the union-sum of the OW and SCB indices still roughly correspond to the SH and BZ indices computed on the quotient game (whose weights changed), the BO index exhibits huge differences. First, the higher the majority requirement, the higher the difference. Second, this difference seems to introduce a bias between single-player unions and multiple-players unions, which negatively affects multiple players unions when the majority requirement is high.

[Insert Table 3 here]

How should we account for these differences between BO and SCB ? They are obviously due to the combination of three effects. The first one comes from a characteristic feature of the Banzhaf-type measure of power, which becomes less and less efficient when the majority requirement increases. This is roughly because the cardinality of the set of swings is decreasing in the latter. The second effect is due to the way the surplus is allocated within unions. BO still use a Banzhaf-type allocation, while SCB use an efficient one. As a result, the non-normalized SCB index of a given player will always be weakly above the BO one (the equality holding whenever a player is single in her union). Finally, the normalization of the indices introduces a third effect: the weak inequality is transformed into a strict one for multiple-players unions, as long as the a priori union structure is not the trivial one.

4.2.3 Sensitivity of the indices to a variation in the majority requirement

The following graphs illustrate the sensitivity of the indices to a variation of the majority requirement. Using the current set of weights and a priori union structure, we computed the three indices for majority requirements ranging from 50% to 100%.[8] BO and SCB roughly reproduce the behavior of the Banzhaf

[8] These graphs can be compared to the ones provided by Leech (1998) for the approximated BZ and SH indices computed for the quotient game only.

index when the sequence of games tends to the unanimity game, while Owen index reproduces the SH one. The power of the large state measured by BO and SCB quickly decreases, while it remains still high under the OW index as long as the player is still above the veto threshold (85%).

[Insert Graphs 1,2 and 3 here]

5 Concluding Remarks

We believe that the procedures presented in this paper can be helpful in shedding some light on the theoretical work on the comparison and properties of the existing power measures. The analysis of the principles of dominance, transfer and bloc done by Felsenthal and Machover (1995) for several power measures could be extended to the BO, OW and SCB indices using the numerical results based on the generating function approach. Turning now to monotonicity properties, the numerical analysis can show that the three indices may for instance exhibit violations of the local monotonicity criteria proposed by Holler and Napel (2003)[9]. The presentation of a detailed set of numerical results allowing this kind of analysis was beyond the scope of this paper. A large set of such numerical results based on our procedures can be found in Benassy-Queré and Bowles (2004).

Besides this challenging analysis, another interest of the procedures proposed here is that they provide practical tools for decision-makers. Indeed, our approach gives at hand a method to compare the value of constituencies for newcomers and to evaluate the power implications of their choice. For instance, the creation of a European Seat at the International Monetary Fund is currently under discussion among several European administrations. First of all, it is not completely clear whether such a seat may increase or decrease the power of the European Union as a whole. Second, the net impact on the European Union power may hide variations in the relative power of the European countries within the European constituency. Obviously, the results presented here are unrealistic because the United States, whose agreement is needed to change IMF statutes, will never accept to face a European Representative having the highest voting rights among the IMF members. As evidenced in Benassy-Queré and Bowles (2002), the most likely implementation of the European Seat would rank the EU slightly below the US in terms of voting rights[10]. Nevertheless,

[9] We thank Manfred Holler and Stefan Napel for having pointed out these violations. These results are available upon request.

[10] The "trick" allowing this would consist in consolidating the European financial accounts in order to get the voting rights of the new european entity slightly below the United States ones. Indeed, these voting rights are based on economic and financial variables. The extraction of intra-european trade and financial flows in the computation of the voting rights may allow a situation in which the voting rights of the EU as a whole are much lower than the sum of the voting rights of the EU countries.

the application of our procedures to a set of more realistic scenarii can be found in the aforementioned paper. It is worth pointing out that this work has been done upon request of European officials involved in the issue[11].

There is, as always, no free lunch. What makes the interest of our decision-toolkit is also responsible for its limitations. First of all, the simplicity of the cooperative approach has an obvious shortcoming in that it does not allow for an "equilibrium-based" approach. As pointed by an anomymous referee, it is possible that no set of constituencies is in equilibrium in the sense that there may always be countries that can benefit from switching constituencies. For instance, the creation of a European constituency may induce a modification of the perimeter of the other existing constituencies. This may cancel the potential benefits of the EU Seat[12]. The analysis of the existence and unicity of such an equilibrium would require a more complicated non-cooperative approach that, according to our knowledge, has not been yet developped.

In the same vein, our approach adopted a highly stylized view on the real bargaining process. For instance, the cooperative formalization of the game is purely static, while the real decision-making is intrinsically dynamic and definitively more complex. There is for example a temporal discrepancy between the frequency of the constituencies formations (taking place every two years), and the frequency of the Executive Board meetings (once a week). This temporal discrepancy deserves extensive analysis. The constituencies' composition is rather static, i.e., what we called "the outside option" (joining another constituency) is an only very seldom realised option. Once a constituency is formed and its Executive Director elected, it may indeed happen that the former ignores the interests of some member of his/her constituency at some point in time. In this case, the "outside option" cannot be immediately exerted. It will in reality not even be exerted at all if the Executive Director tries to compensate this member later on about a different issue, before the constituencies' renegotiation takes place. Hence the static allocation of rewards given by the indices only captures some kind of (discounted) average of the benefits associated to the participation to a given bloc. Some informal arrangements may emerge to ensure that the representation of all the countries from a given constituency be fair in the long-run. The nordic countries, for instance, have adopted a rotating scheme. But this kind of informal arrangements are not and cannot be modelled by a cooperative approach. We can only hope that they result in the same allocation than the one derived from the static and highly stylized view.

Finally, the three indices presented here have the same drawbacks as the classical power indices based on the use of a uniform distribution. All possible coalitions (restricted by a priori unions) are not equally likely and may depend, like the constituencies' composition, on regional, historical and geostrategic aspects. A modification of these a priori measures taking into account informations like ideological or economical proximities similar to the spatial measures proposed by Shapley (1977) for the classical power indices may be interest-

[11] This request is indeed the initial motivation prevailing the writing of this paper.

[12] Examples of the potential dynamic effects induced by the merging of some players are quoted in Straffin (1994).

ing. Similarly, other issue-specific alliances like in Schmidtchen (2001a) and Schmidtchen (2003) may prove useful in reflecting the real bloc building. As all the decision-toolkits, our approach makes simplifying assumptions providing some insights about complicated problems but is far from exhausting the field. We would even be happy to find out that the research field is now a bit larger than before.

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Table 1. Voting Power at the IMF for selected countries

		Coalitional Power Indices						Classical Power Indices			
	Voting Rights	Banzhaf-Owen		Symmetric Coalitional Banzhaf		Owen		Banzhaf		Shapley-Shubik	
Country	Sept. 2002	q=50	q=85	q=50	q=85	q=50	q=85	q=50	q=85	q=50	q=85
United States	17.1	21.6	6.5	21.6	6.5	20.1	19.7	24.7	3.2	20.6	19.2
Japan	6.1	5.9	5.9	5.9	5.8	6.1	6.2	5.5	3.2	6.2	6.5
Germany	6.0	5.7	5.8	5.7	5.8	6.0	6.1	5.4	3.2	6.1	6.4
France	5.0	4.7	5.3	4.7	5.3	5.0	5.1	4.5	3.2	4.9	5.1
United Kingdom	5.0	4.7	5.3	4.7	5.3	5.0	5.1	4.5	3.2	4.9	5.1
Austria	0.9	0.8	0.9	0.8	0.9	0.9	0.9	0.8	1.4	0.9	0.9
Belarus	0.2	0.2	0.2	0.2	0.2	0.2	0.2	0.2	0.3	0.2	0.2
Belgium	2.1	2.0	2.1	2.0	2.1	2.1	2.1	2.0	2.7	2.1	2.1
Czech Republic	0.4	0.4	0.4	0.4	0.4	0.4	0.4	0.4	0.6	0.4	0.4
Hungary	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.8	0.5	0.5
Kazakhstan	0.2	0.2	0.2	0.2	0.2	0.2	0.2	0.2	0.3	0.2	0.2
Luxembourg	0.1	0.1	0.1	0.1	0.2	0.1	0.1	0.1	0.2	0.1	0.1
Slovak Republic	0.2	0.2	0.2	0.2	0.2	0.2	0.2	0.2	0.3	0.2	0.2
Slovenia	0.1	0.1	0.1	0.1	0.1	0.1	0.1	0.1	0.2	0.1	0.1
Turkey	0.5	0.4	0.5	0.4	0.5	0.5	0.5	0.4	0.7	0.5	0.5
Armenia	0.1	0.0	0.1	0.0	0.1	0.0	0.0	0.0	0.1	0.0	0.0
Bosnia and Herz.	0.1	0.1	0.1	0.1	0.1	0.1	0.1	0.1	0.1	0.1	0.1
Bulgaria	0.3	0.3	0.3	0.3	0.4	0.3	0.3	0.3	0.5	0.3	0.3
Croatia	0.2	0.2	0.2	0.2	0.2	0.2	0.2	0.2	0.3	0.2	0.2
Cyprus	0.1	0.1	0.1	0.1	0.1	0.1	0.1	0.1	0.1	0.1	0.1
Georgia	0.1	0.1	0.1	0.1	0.1	0.1	0.1	0.1	0.1	0.1	0.1
Israel	0.4	0.4	0.5	0.4	0.5	0.4	0.4	0.4	0.7	0.4	0.4
Macedonia, former	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.1	0.0	0.0
Moldova	0.1	0.1	0.1	0.1	0.1	0.1	0.1	0.1	0.1	0.1	0.1
Netherlands	2.4	2.3	2.4	2.3	2.4	2.4	2.4	2.2	2.8	2.4	2.4
Romania	0.5	0.5	0.5	0.5	0.6	0.5	0.5	0.5	0.8	0.5	0.5
Ukraine	0.6	0.6	0.7	0.6	0.7	0.6	0.6	0.6	1.0	0.6	0.6
Costa Rica	0.1	0.1	0.1	0.1	0.1	0.1	0.1	0.1	0.1	0.1	0.1
El Salvador	0.1	0.1	0.1	0.1	0.1	0.1	0.1	0.1	0.1	0.1	0.1
Guatemala	0.1	0.1	0.1	0.1	0.1	0.1	0.1	0.1	0.2	0.1	0.1
Honduras	0.1	0.1	0.1	0.1	0.1	0.1	0.1	0.1	0.1	0.1	0.1
Mexico	1.2	1.1	1.3	1.1	1.4	1.2	1.2	1.1	1.8	1.2	1.2
Nicaragua	0.1	0.1	0.1	0.1	0.1	0.1	0.1	0.1	0.1	0.1	0.1
Spain	1.4	1.4	1.6	1.4	1.6	1.4	1.4	1.3	2.1	1.4	1.4
Venezuela	1.2	1.2	1.4	1.2	1.4	1.2	1.2	1.1	1.8	1.2	1.2
Albania	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
Greece	0.4	0.4	0.5	0.4	0.5	0.4	0.4	0.4	0.6	0.4	0.4
Italy	3.3	3.1	3.7	3.1	3.6	3.3	3.3	3.0	3.1	3.3	3.3
Malta	0.1	0.1	0.1	0.1	0.1	0.1	0.1	0.1	0.1	0.1	0.1
Portugal	0.4	0.4	0.5	0.4	0.5	0.4	0.4	0.4	0.7	0.4	0.4
San Marino	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
Antigua and Bar.	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
Bahamas, The	0.1	0.1	0.1	0.1	0.1	0.1	0.1	0.1	0.1	0.1	0.1
Barbados	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.1	0.0	0.0
Belize	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
Canada	2.9	2.8	3.1	2.8	3.1	2.9	2.9	2.7	3.0	2.9	2.9
Dominica	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
Grenada	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
Ireland	0.4	0.4	0.5	0.4	0.5	0.4	0.4	0.4	0.6	0.4	0.4
Jamaica	0.1	0.1	0.2	0.1	0.2	0.1	0.1	0.1	0.2	0.1	0.1
St. Kitts and Nevis	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
St. Lucia	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
St. Vincent	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
Denmark	0.8	0.7	0.9	0.7	0.9	0.8	0.8	0.7	1.2	0.8	0.8
Estonia	0.0	0.0	0.1	0.0	0.1	0.0	0.0	0.0	0.1	0.0	0.0
Finland	0.6	0.6	0.7	0.6	0.7	0.6	0.6	0.5	0.9	0.6	0.6
Iceland	0.1	0.1	0.1	0.1	0.1	0.1	0.1	0.1	0.1	0.1	0.1
Latvia	0.1	0.1	0.1	0.1	0.1	0.1	0.1	0.1	0.1	0.1	0.1
Lithuania	0.1	0.1	0.1	0.1	0.1	0.1	0.1	0.1	0.1	0.1	0.1
Norway	0.8	0.7	1.0	0.7	1.0	0.8	0.8	0.7	1.2	0.8	0.8
Sweden	1.1	1.1	1.4	1.1	1.4	1.1	1.1	1.0	1.7	1.1	1.1

Nota bene : the difference in shading aims to indicate the participation to the same Constituency

Table 2. Comparison of the indices with the current set of a priori unions

Contituency (*)	Actual voting rights	Totals by union (**)										Computation of the indices on the quotient game			
		Power indices restricted by a priori unions						Power indices non restricted by a priori unions							
		Banzhaf-Owen Index		Symmetric Coalitional Banzhaf Index		Owen Index		Banzhaf Index		Shapley-Shubik Index		Banzhaf Index		Shapley-Shubik Index	
	Sept. 2002	q=50	q=85	q=50	q=85	q=50	q=85	q=50	q=85	q=50	q=85	q=50	q=85	q=50	q=85
United States	17.1	21.6	6.5	21.6	6.5	20.1	19.7	24.7	3.2	20.6	19.2	21.6	6.4	19.8	19.6
Japan	6.1	5.9	5.9	5.9	5.8	6.1	6.2	5.5	3.2	6.2	6.5	5.9	5.8	6.1	6.2
Germany	6.0	5.7	5.8	5.7	5.8	6.0	6.1	5.4	3.2	6.1	6.4	5.7	5.8	6.0	6.1
France	5.0	4.7	5.3	4.7	5.3	5.0	5.1	4.5	3.2	4.9	5.1	4.7	5.3	4.9	4.9
United Kingdom	5.0	4.7	5.3	4.7	5.3	5.0	5.1	4.5	3.2	4.9	5.1	4.7	5.3	4.9	4.9
(Belgium)	5.1	5.2	5.2	5.1	5.5	5.4	5.2	4.9	5.4	5.2	5.2	4.9	5.4	5.1	5.1
(Netherlands)	4.9	4.6	5.1	4.6	5.2	4.8	4.9	4.5	6.8	4.7	4.7	4.6	5.2	4.8	4.8
(Spain)	4.3	4.1	4.7	4.1	4.8	4.2	4.3	3.9	6.4	4.2	4.2	4.1	4.8	4.2	4.3
(Italy)	4.2	4.0	4.8	4.0	4.7	4.1	4.2	3.8	4.6	4.1	4.2	4.0	4.8	4.1	4.2
(Canada)	3.7	3.6	4.1	3.6	4.1	3.5	3.4	3.4	4.3	3.4	3.5	3.5	4.4	3.6	3.6
(Sweden)	3.5	3.3	4.3	3.3	4.3	3.5	3.6	3.2	5.5	3.5	3.5	3.3	4.2	3.4	3.4
(Australia)	3.3	3.2	4.0	3.2	4.0	3.1	3.1	3.1	5.1	3.1	3.1	3.2	4.0	3.2	3.2
Saudi Arabia	3.2	3.1	3.9	3.1	3.9	3.1	3.1	3.0	3.1	3.2	3.2	3.1	3.9	3.1	3.1
(Nigeria)	3.2	3.0	3.8	3.0	3.9	2.9	2.9	2.9	5.1	2.9	2.9	3.1	3.9	3.1	3.1
(Indonesia)	3.1	3.0	3.8	3.0	3.8	2.9	2.9	2.9	5.0	2.9	2.9	3.0	3.8	3.0	3.0
(Kuwait)	2.9	2.8	3.6	2.8	3.6	2.7	2.7	2.7	4.7	2.6	2.6	2.8	3.6	2.8	2.8
China	2.9	2.8	3.7	2.8	3.6	2.8	2.8	2.7	3.0	2.9	2.9	2.8	3.6	2.8	2.8
Russia	2.7	2.6	3.5	2.6	3.4	2.7	2.7	2.5	3.0	2.7	2.7	2.6	3.4	2.6	2.6
(Switzerland)	2.6	2.5	3.3	2.5	3.3	2.3	2.3	2.4	3.9	2.3	2.3	2.5	3.2	2.5	2.5
(Brazil)	2.5	2.3	3.1	2.3	3.1	2.3	2.3	2.3	3.7	2.2	2.2	2.3	3.1	2.4	2.3
(India)	2.4	2.3	3.1	2.3	3.0	2.3	2.3	2.2	3.3	2.3	2.4	2.3	3.0	2.3	2.3
(Iran)	2.4	2.3	3.0	2.3	3.0	2.3	2.3	2.2	3.8	2.3	2.3	2.3	3.0	2.3	2.2
(Argentina)	2.0	1.9	2.6	1.9	2.6	2.0	2.0	1.9	3.2	1.9	1.9	1.9	2.6	1.9	1.9
(Ivory Cost)	1.2	1.1	1.5	1.1	1.5	0.8	0.7	1.1	1.9	0.8	0.8	1.1	1.5	1.1	1.1

(*) The presence of parenthesis indicates that there are several countries in the Constituency. The name of the member state with the higher voting rights within the Constituency is reported between the parenthesis The absence of brackets indicates a single-country Constituency.

(**) The totals reported in the table are the sum of the voting rights/power indices of the Constituency. For single-country Constituencies, this total is equal to the voting rights/power of the country.

Table 3. Modification of the coalition structure: a European Constituency at the IMF

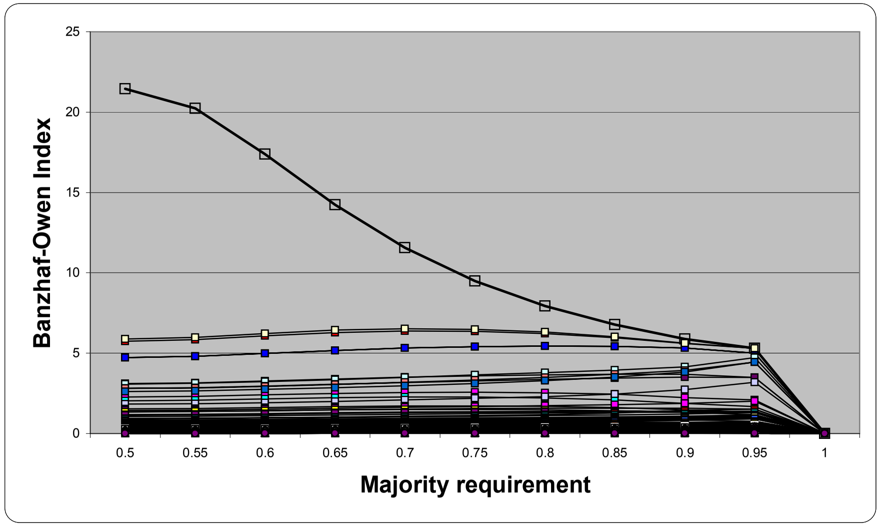
		Power indices restricted by a priori unions : total by unions (**)						Computation on the quotient game			
	Voting rights	Banzhaf-Owen Index		Symmetric Coalitional Banzhaf Index		Owen Index		Banzhaf Index		Shapley-Shubik Index	
Contituency (*)		q=50	q=85	q=50	q=85	q=50	q=85	q=50	q=85	q=50	q=85
United States	17.1	8.1	10.0	8.2	9.1	15.2	25.0	8.2	9.2	15.2	24.7
Japan	6.1	5.8	8.8	5.8	8.1	5.7	6.6	5.8	8.1	5.7	6.6
EU15	29.8	41.9	1.5	41.3	9.1	38.9	25.0	41.3	9.2	38.0	24.7
(Hungary)	2.0	1.9	3.5	1.9	3.2	1.8	2.0	1.9	3.2	1.7	1.8
(Ukraine)	2.5	2.3	4.2	2.4	3.9	2.1	2.3	2.4	3.9	2.1	2.3
(Venezuela)	2.9	2.7	4.9	2.8	4.5	2.6	2.8	2.8	4.5	2.6	2.7
(Canada)	3.3	3.2	5.7	3.2	5.2	2.8	3.0	3.2	5.2	3.0	3.2
(Norway)	1.1	1.1	2.1	1.1	1.9	1.0	1.4	1.1	2.0	1.0	1.2
(Australia)	3.3	3.2	5.6	3.2	5.2	2.9	3.1	3.2	5.2	3.0	3.3
Saudi Arabia	3.2	3.1	5.5	3.1	5.1	2.9	3.1	3.1	5.1	2.9	3.1
(Nigeria)	3.2	3.0	5.3	3.1	5.0	2.7	2.9	3.1	5.1	2.9	3.1
(Indonesia)	3.1	3.0	5.3	3.0	4.9	2.7	2.9	3.0	4.9	2.8	3.0
(Kuwait)	2.9	2.8	5.0	2.8	4.6	2.5	2.7	2.8	4.7	2.7	2.8
China	2.9	2.8	5.1	2.8	4.6	2.6	2.8	2.8	4.6	2.6	2.8
Russia	2.7	2.6	4.7	2.6	4.3	2.5	2.7	2.6	4.3	2.4	2.6
(Uzbekistan)	2.6	2.5	4.5	2.5	4.1	2.1	2.3	2.5	4.1	2.3	2.4
(Brazil)	2.5	2.3	4.3	2.4	3.9	2.1	2.3	2.4	3.9	2.1	2.3
(India)	2.4	2.3	4.2	2.3	3.8	2.1	2.3	2.3	3.8	2.1	2.2
(Iran)	2.4	2.3	4.1	2.3	3.8	2.1	2.3	2.3	3.8	2.1	2.2
(Argentina)	2.0	1.9	3.5	1.9	3.2	1.8	2.0	1.9	3.2	1.7	1.8
(Ivory Cost)	1.2	1.2	2.3	1.2	2.1	0.7	0.8	1.1	2.0	1.0	1.2

(*) The presence of parenthesis indicates that there are several countries in the Constituency. The name of the member state with the higher voting rights within the Constituency is reported between the brackets. The absence of parenthesis indicates a single-country Constituency.

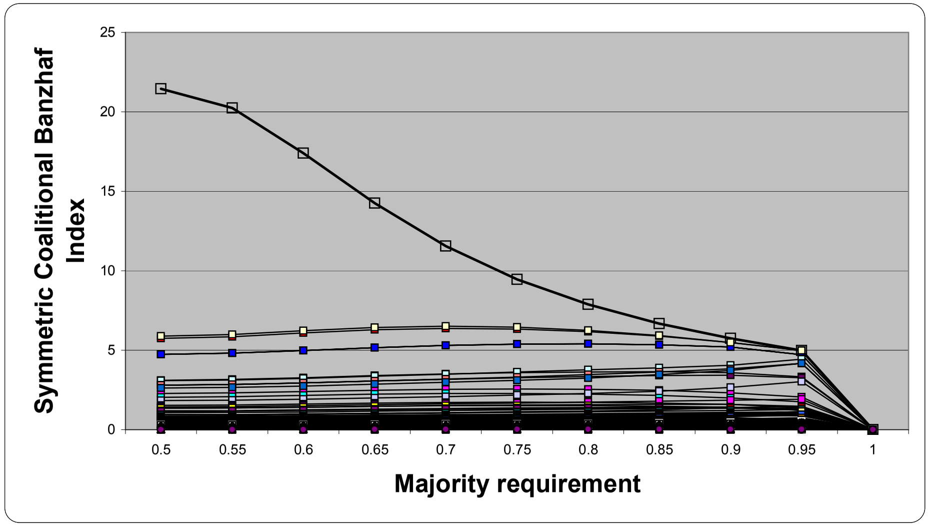
(**) The totals reported in the table are the sum of the voting rights/power indices of the Constituency. For single-country Constituencies, this total is equal to the voting rights/power of the country.

Sensitivity of the power indices to a variation in the quota requirement (*)

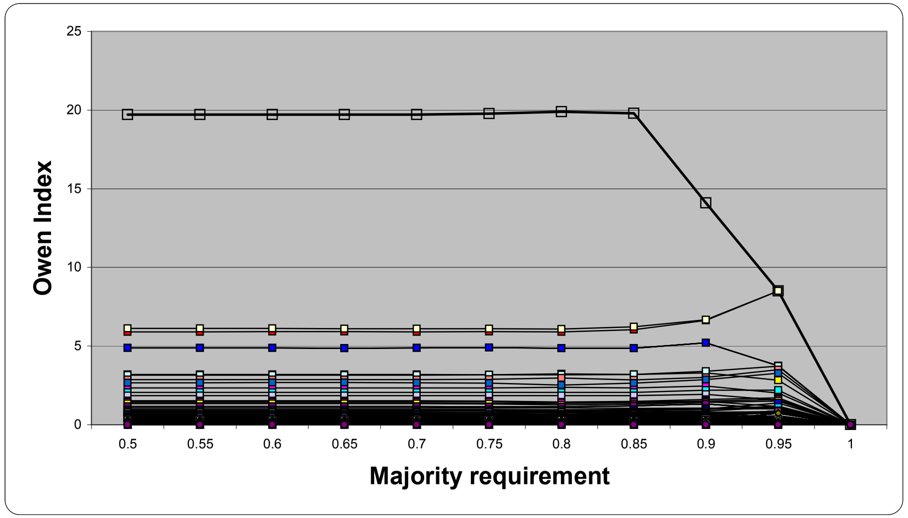
Graph 1. Banzhaf-Owen Index



Graph 2. Symmetric Coalitional Banzhaf Index



Graph 3. Owen Index



(*) The variables drawn on the graphs are the power indices of each of the 184 members states. Each serie represents the evolution of the power index of a given player (vertical axis), depending on the level of the majority requirement (horizontal axis).

1 Appendix : Computational Complexity, Algorithms and Mathematica Code

In this appendix, we provide some results about the complexity of the procedures used, which basically show that the indices can be computed in polynomial time. The following two subsections respectively present the algorithms and an example of programming under the `Mathematica` system.

1.1 Computational Complexity

We first start with a traditional definition in computational complexity theory:

Definition 1 *Let $f : Z_+ \to Z_+$. and $g : Z_+ \to Z_+$. We say that $f(n) = O(g(n))$ if there exist C, n_0 such that $f(n) < Cg(n)$ for each $n > n_0$*

Hence, informally, $f(n) = O(g(n))$ means that f does not grow at a faster rate than g. Computational complexity is defined as the cost $f(n)$, expressed in time, when the input size is n. When $f(n)$ is $\mathcal{O}(n^2)$, the computational complexity is quadratic in the measure of the input size. As recalled by Bilbao (2000), polynomial complexity is often viewed as a desirable property from the computational point of view.

Proposition 1 *Let (N, v, P), with $v = [q; w_1, w_2, \ldots, w_n]$ a weighted majority game with an a priori system of unions. The number C of nonzero coefficients of the polynomial $B^{(j)}(x) = \prod_{r=1,r\neq j}^{m} \left(1 + x^{w(P_r)}\right) \prod_{j_l=1}^{p_j} (1 + x^{w_{j_l}})$, where $P_j = \left\{j_1, j_2, ..., j_{p_j}\right\}$ and $w(P_r) = \sum\limits_{j\in P_r} w_j$ is such that :*

$$m + p_j + 1 \leq C \leq \min(2^{m+p_j}, w(N) + 1)$$

Proof. Assume that $w_{j_1} = w_{j_2} = ... = w_{p_j} = w$ and that $w(P_r) = w$ for each $r \neq j$ The number of nonzero coefficients of the polynomial :

$$(1 + x^w)^{m+p_j}$$

will always be below the number of nonzero coefficients of $\prod_{r=1,r\neq j}^{m} \left(1 + x^{w(P_r)}\right) \prod_{j_l=1}^{p_j} (1 + x^{w_{j_l}})$ for any set of weights.

Notice from before that $\prod_{r=1,r\neq j}^{m} \left(1 + x^{w(P_r)}\right) \prod_{j_l=1}^{p_j} (1 + x^{w_{j_l}}) = \sum_{k=0}^{w(N)} b_k x^k$ is a polynomial whose power is at most $w(N)$. From what we have $C \leq w(N)+1$. Moreover, when the powers of the polynomial are all different, C is equal to the number of subsets of N compatible with P, that is 2^{m+p_j}. ■

Proposition 2 *Let* (N, v, P), *with* $v = [q; w_1, w_2, \ldots, w_n]$ *a weighted majority game with an a priori system of unions. The expansion of the polynomial* $B^{(j)}(x) = \prod_{r=1, r\neq j}^{m} \left(1 + x^{w(P_r)}\right) \prod_{j_l=1}^{p_j} \left(1 + x^{w_{j_l}}\right)$, *where* $P_j = \left\{j_1, j_2, ..., j_{p_j}\right\}$ *and* $w(P_r) = \sum_{j \in P_r} w_j$ *requires a time* $\mathcal{O}(nC)$, *where* C *is the number of nonzero coefficients of the polynomial* $B^{(j)}(x)$.

Proof. The function $B^{(j)}(x)$ can be computed by two successive loops :

$B^{(j)}(x) \leftarrow 1$
for $r \in \{1, ..., m\}$ **with** $r \neq j$ **do**
$B^{(j)}(x) \leftarrow B^{(j)}(x) + B^{(j)}(x)x^{w(P_r)}$
endfor
for $j_l \in \left\{j_1, j_2, ..., j_{p_j}\right\}$ **do**
$B^{(j)}(x) \leftarrow B^{(j)}(x) + B^{(j)}(x)x^{w_{j_l}}$
endfor

The time to compute the line in one loop is $\mathcal{O}(C)$. This line is computed $p_j + m - 1$ times, so that the time to compute this function is $\mathcal{O}((m + p_j)C)$. The worst case happens when $m = n - p_j$, for which the computation time is $\mathcal{O}(nC)$. ∎

Iterating the procedure for the n players yields the following corollary.

Corollary 1 *Let* (N, v, P), *with* $v = [q; w_1, w_2, \ldots, w_n]$ *a weighted majority game with an a priori system of unions. If* C *is the number of nonzero coefficients of* $B^{(j)}(x)$ *where* j *is such that* $p_j \in Max\{p_1, p_2, ..., p_r\} = p$ *the time complexity of the generating algorithm for the Banzhaf-Owen indices is* $\mathcal{O}(n^2C)$.

In the same way, we have the following propositions (the proofs are similar to those of the preceding propositions and corollary and are therefore omitted). Considering first the SCB index, we have:

Proposition 3 *Let* (N, v, P), *with* $v = [q; w_1, w_2, \ldots, w_n]$ *a weighted majority game with an a priori system of unions. The number* C *of nonzero coefficients of the polynomial* $S^{(j)}(x) = \prod_{r=1, r\neq j}^{m} \left(1 + x^{w(P_r)}\right) \prod_{j_l=1}^{p_j} \left(1 + x^{w_{j_l}} z\right)$, *where* $P_j = \left\{j_1, j_2, ..., j_{p_j}\right\}$ *and* $w(P_r) = \sum_{j \in P_r} w_j$ *is such that :*

$$m + p_j + 1 \leq C \leq \min(2^{m+p_j}, p_j w(N) + 1)$$

We also have :

Proposition 4 *Let* (N, v, P), *with* $v = [q; w_1, w_2, \ldots, w_n]$ *a weighted majority game with an a priori system of unions. The expansion of the polynomial* $S^{(j)}(x, z) = \prod_{r=1, r\neq j}^{m} \left(1 + x^{w(P_r)}\right) \prod_{j_l=1}^{p_j} \left(1 + x^{w_{j_l}} z\right)$, *where* $P_j = \left\{j_1, j_2, ..., j_{p_j}\right\}$ *and* $w(P_r) = \sum_{j \in P_r} w_j$ *requires a time* $\mathcal{O}(nC)$, *where* C *is the number of nonzero coefficients of the polynomial* $S^{(j)}(x)$.

From which we can deduce:

Corollary 2 *Let* (N, v, P), *with* $v = [q; w_1, w_2, \ldots, w_n]$ *a weighted majority game with an a priori system of unions. If* C *is the number of nonzero coefficients of* $S^{(j)}(x, z)$ *where* j *is such that* $p_j \in Max\{p_1, p_2, ..., p_r\} = p$ *the time complexity of the generating algorithm for the Symmetric Coalitional Banzhaf indices is* $\mathcal{O}(n^2 C)$.

With regard to the computational complexity of the OW index, the following propositions and corollary hold. The first deals with the complexity bounds:

Proposition 5 *Let* (N, v, P), *with* $v = [q; w_1, w_2, \ldots, w_n]$ *a weighted majority game with an a priori system of unions. The number* C *of nonzero coefficients of the polynomial* $S^{(j)}(x, z, t) = \prod_{r=1, r\neq j}^{m} \left(1 + x^{w(P_r)} t\right) \prod_{j_l=1}^{p_j} \left(1 + x^{w_{j_l}} z\right)$, *where* $P_j = \left\{j_1, j_2, ..., j_{p_j}\right\}$ *and* $w(P_r) = \sum\limits_{j \in P_r} w_j$ *is such that :*

$$m + p_j + 1 \leq C \leq \min(2^{m+p_j}, p_j(m-1)w(N) + 1)$$

The second deals with the complexity order:

Proposition 6 *Let* (N, v, P), *with* $v = [q; w_1, w_2, \ldots, w_n]$ *a weighted majority game with an a priori system of unions. The expansion of the polynomial* $S^{(j)}(x, z, t) = \prod_{r=1, r\neq j}^{m} \left(1 + x^{w(P_r)} t\right) \prod_{j_l=1}^{p_j} \left(1 + x^{w_{j_l}} z\right)$, *where* $P_j = \left\{j_1, j_2, ..., j_{p_j}\right\}$ *and* $w(P_r) = \sum\limits_{j \in P_r} w_j$ *requires a time* $\mathcal{O}(nC)$, *where* C *is the number of nonzero coefficients of the polynomial* $S^{(j)}(x, z, t)$.

From which we can deduce:

Corollary 3 *Let* (N, v, P), *with* $v = [q; w_1, w_2, \ldots, w_n]$ *a weighted majority game with an a priori system of unions. If* C *is the number of nonzero coefficients of* $S^{(j)}(x, z, t)$ *where* j *is such that* $p_j \in Max\{p_1, p_2, ..., p_r\} = p$ *the time complexity of the generating algorithm for the Owen indices* $\mathcal{O}(n^2 C)$.

1.2 Algorithms

In this subsection, we present algorithms for the computation of the three coalitional power indices.

1.2.1 Banzhaf-Owen index

Algorithm *BanzhafOwen* $(\{\{w_1, w_2, ..., w_{p_1}\}, ..., \{w_1, w_2, , ..., w_{p_m}\}\}, q)$

$m \leftarrow \#\{\{w_1, w_2, ..., w_k\}, ..., \{w_l, w_{l+1}, , ..., w_n\}\}$

for $r \in \{1, ..., m\}$ **do**

$w(P_r) \leftarrow \sum\limits_{j \in P_r} w_j$

endfor r

for $r \in \{1, ..., m\}$ **do**

$l \leftarrow \#P_r$

$s_r(x) \leftarrow \prod\limits_{k=1, k \neq r}^{m} (1 + x^{w(P_k)})$

for $i \in \{1, ..., l\}$ **do**

$s_i'(x) \leftarrow \prod\limits_{t=1, t \neq i}^{l} (1 + x^{w_t})$

$B_i(x) \leftarrow s_r(x) \times s_i'(x)$

$$\left\{ B_i(x) = \sum_{k=0}^{w(N \setminus i)} b_k^i x^k \right\}$$

$\eta_i \leftarrow \sum\limits_{k=q-w_i}^{q-1} b_k^i$

$\Psi_i \leftarrow \dfrac{\eta_i}{2^{m+l-2}}$

endfor i

endfor r

output $\{\Psi_1, ..., \Psi_n\}$

1.2.2 Symmetric coalitional Banzhaf index

Algorithm *SCBanzhaf* $(\{\{w_1, w_2, ..., w_{p_1}\}, ..., \{w_1, w_2, , ..., w_{p_m}\}\}, q)$

$m \leftarrow \#\{\{w_1, w_2, ..., w_k\}, ..., \{w_l, w_{l+1}, , ..., w_n\}\}$

for $r \in \{1, ..., m\}$ **do**

$w(P_r) \leftarrow \sum\limits_{j \in P_r} w_j$

endfor r

for $j \in \{1, ..., m\}$ **do**

$p_j \leftarrow \#P_j$

$s_r(x) \leftarrow \prod\limits_{k=1, k \neq r}^{m} (1 + x^{w(P_k)})$

for $i \in \{1, ..., p_j\}$ **do**

$s_i'(x, z) \leftarrow \prod\limits_{t=1, t \neq i}^{p_j} (1 + z x^{w_t})$

$S_i(x, z) \leftarrow s_r(x) \times s_i'(x, z)$

$$\left\{ S_i(x,z) = \sum_{l=0}^{p_j-1} \left[\sum_{k=0}^{w(N)-w_i} a^i_{kl} x^k \right] z^l \right\}$$

$g_i(z) \leftarrow \sum_{l=0}^{p_j-1} d^i_l z^l$

$$\left\{ g_i(z) = \sum_{l=0}^{p_j-1} \left[\sum_{k=q-w_i}^{q-1} a^i_{kl} \right] z^l = \sum_{l=0}^{p_j-1} d^i_l z^l \right\}$$

$\Pi_i \leftarrow \sum_{l=0}^{p_j-1} \frac{1}{2^{m-1}} \frac{l!(p_j-l-1)!}{p_j!} d^i_l$

endfor i

endfor j

output $\{\Pi_1, ..., \Pi_n\}$

1.2.3 Owen index

Algorithm *Owenindex* $(\{\{w_1, w_2, ..., w_{p_1}\}, ..., \{w_1, w_2, , ..., w_{p_m}\}\}, q)$

$m \leftarrow \#\{\{w_1, w_2, ..., w_k\}, ..., \{w_l, w_{l+1}, , ..., w_n\}\}$

for $r \in \{1, ..., m\}$ **do**

$w(P_r) \leftarrow \sum_{j \in P_r} w_j$

endfor r

for $j \in \{1, ..., m\}$ **do**

$p_j \leftarrow \#P_j$

$s_r(x,t) \leftarrow \prod_{k=1, k \neq r}^{m} (1 + x^{w(P_k)} t)$

for $i \in \{1, ..., p_j\}$ **do**

$s'_i(x,z) \leftarrow \prod_{t=1, t \neq i}^{p_j} (1 + z x^{w_t})$

$S_i(x,z,t) \leftarrow s_r(x,t) \times s'_i(x,z)$

$$\left\{ S_i(x,z,t) = \sum_{r=0}^{m-1} \sum_{l=0}^{p_j-1} \left[\sum_{k=0}^{w(N)-w_i} a^i_{klr} x^k \right] t^r z^l \right\}$$

$g_i(z,t) \leftarrow \sum_{r=0}^{m-1} \sum_{l=0}^{p_j-1} d^i_{rl} t^r z^l$

$$\left\{ g_i(z) = \sum_{r=0}^{m-1} \sum_{l=0}^{p_j-1} \left[\sum_{k=q-w_i}^{q-1} a^i_{klr} \right] \times z^l = \sum_{r=0}^{m-1} \sum_{l=0}^{p_j-1} d^i_{rl} z^l t^r \right\}$$

$\Phi_i \leftarrow \sum_{r=0}^{m-1} \sum_{l=0}^{p_j-1} \frac{r!(m-r-1)!}{m!} \frac{l!(p_j-l-1)!}{p_j!} d^i_{rl}$

endfor i

endfor j

output $\{\Phi_1, ..., \Phi_n\}$

1.3 Mathematica Code

The Code is written for the `Mathematica` system and can be runned under the version 3.0[1]. The functions Banzhafowen, Banzhafcs and Owenindex allows to get the non-normalized indices. The syntax requires to enter a set of weights and a majority requirement. The weights of the players pertaining to a given coalition are regrouped into the same subset, and the set of weights used by the functions is defined as the union of the a priori unions subsets. For instance, the set of weights of the Parliament of Balearic Islands case has to be written as **{{28},{16,4,3},{4,3}}** where **{16,4,3}** represents the a priori union composed by player two, four and six.

The three algorithms are similar to those proposed by Tannenbaum (1997) or Bilbao (2000) for the BZ and SH power indices and work as follows. The variable **m** is assigned the number of existing a priori unions and the sum of the weights of each union is stocked into a table called **wp**. Then, two imbricated loops starts. The first one expands the polynomial corresponding to each union. The second one expands the polynomial of each player within the given union. Then, the variable **coefi** stores the set of coefficients of the generating function (using the function **CoefficientList**). Among them, the coefficients corresponding to a pivotal case (between $q-w_i$ and $q-1$) are stored in the variable **monom**. Finally, the variable **index** is assigned the weighted sum of these coefficients, using the probability distribution corresponding to the index under computation.

The example of the Parliament of Balearic Islands is provided together with the code (input in bold, output in normal text).

1.3.1 Banzhaf-Owen index

The Mathematica Code for the Banzhaf-Owen index is:

```
Banzhafowen[weights_List,q_Integer]:=
 Module[{m=Length[weights],wp,p,s,s1,s2,coefi,monom,index},
 wp=Table[Apply[Plus,weights[[r]]],{r,m}];Do[delun=Delete[wp,r];
s1=Times@@(1+x^delun);
Do[delplay=Delete[weights[[r]],i];
 s2=Times @@(1+x^delplay);s=s1*s2;p=Length[weights[[r]]];
 coefi=CoefficientList[s,x];
monom=
 Apply[Plus, coefi[[Range[Max[1,q-Extract[weights[[r]],i]+1],
 Min[q,Length[coefi]]]]]];
 index[Sum[Length[weights[[h]]],{h,r-1}]+i]=monom/2^(p-1),
{i,Length[weights[[r]]]}],{r,m}];
Table[index[i],{i,Sum[Length[weights[[r]]],{r,m}]}]/2^(m-1)]
```

[1] Compatibility with previous versions will require to substitute **wr=weights[[r]]**; ...; **wr[[i]]** for the function **Extract[weights[[r]],i]**

Game={{28},{16,4,3},{4,3}}
{{28},{16,4,3},{4,3}}

Banzhafowen[Game,30]
{1/2, 1/8, 1/8, 1/8, 1/4, 1/4}

1.3.2 Symmetric Coalitional Banzhaf index

The Mathematica Code for the Symmetric Coalitional Banzhaf index is:

```
SCBanzhaf[weights_List,q_Integer]:=
 Module[{m=Length[weights],wp,p,s,s1,s2,coefi,monom,index},
 wp=Table[Apply[Plus,weights[[r]]],{r,m}];
 Do[delun=Delete[wp,r];s1=Times@@(1+x^delun);
 Do[delplay=Delete[weights[[r]],i];
 s2=Times @@(1+z*x^delplay);s=s1*s2;p=Length[weights[[r]]];
 coefi=CoefficientList[s,x];
 monom=Apply[Plus,
 coefi[[Range[Max[1,q-Extract[weights[[r]],i]+1],
 Min[q,Length[coefi]]]]]];
 index[Sum[Length[weights[[h]]],{h,r-1}]+i]=
 Sum[Coefficient[monom,z,l]*l!(p-l-1)!/p!,{l,0,p-1}],{i,
 Length[weights[[r]]]}],{r,m}];
 Table[index[i],{i,Sum[Length[weights[[r]]],{r,m}]}]/2^(m-1)]
```

Game={{28},{16,4,3},{4,3}}
{{28},{16,4,3},{4,3}}

SCBanzhaf[Game,30]
{1/2, 1/6, 1/6, 1/6, 1/4, 1/4}

1.3.3 Owen index

The Mathematica Code for the Owen index is:

```
Owenindex[weights_List,q_Integer]:=
 Module[{m=Length[weights],wp,p,s,s1,s2,coefi,monom,index},
 wp=Table[Apply[Plus,weights[[r]]],{r,m}];
 Do[delun=Delete[wp,r];s1=Times@@(1+t*x^delun);
 Do[delplay=Delete[weights[[r]],i];
 s2=Times @@(1+z*x^delplay);s=s1*s2;p=Length[weights[[r]]];
 coefi=CoefficientList[s,x];
 monom=Apply[Plus,
 coefi[[Range[Max[1,q-Extract[weights[[r]],i]+1],
 Min[q,Length[coefi]]]]]];
```

```
index[Sum[Length[weights[[h]]],{h,r-1}]+i]=
Sum[Coefficient[Sum[Coefficient[monom,z,l]*l!(p-l-1)!/p!,{l,0,p-1}],
 t,k]*k!*(m-k-1)!,{k,0,m-1}],{i,Length[weights[[r]]]}],{r,m}];
 Table[index[i],{i,Sum[Length[weights[[r]]],{r,m}]}]/m!]
```

```
Game={{28},{16,4,3},{4,3}}
{{28},{16,4,3},{4,3}}
```

```
Owenindex[Game,30]
{1/3, 1/9, 1/9, 1/9, 1/6, 1/6}
```

Chapter 3.
Should the European Union Represent European States at the IMF?

(jointly written with Agnès Bénassy-Quéré)

Should the European Union Represent European States at the IMF?

Agnès Bénassy-Quéré[b] and Carlos Bowles[d]

This draft: September 2014

ABSTRACT

This paper investigates whether a single European representation may increase the power of the European Union at the IMF. The two-step decision process of the IMF is formalized as a weighted majority voting game under the a priori union framework. We investigate a set of various scenarios, depending on the geographical coverage of such a single representation (franco-german, euro area, EU15 or EU25) and its juridical setting (European constituency or single seat). According to the various power indices considered, a single European representation is likely to reduce the power of the European Union at the IMF, contradicting other existing results.

JEL Classification:
Key Words: power indices, international monetary fund, governance, quotas.

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1. INTRODUCTION

The question of country representation in international institutions – and especially in the International Monetary Fund – constitutes a major issue in the reform of world governance. The IMF is goverened by two decision-making bodies: the Board of Governors, which brings together 184 representatives from member countries once per year; and the Executive Board, whose 24 Executive Directors take concrete decisions several times a week. Given the large number of Governors and the low frequency of their meetings, the role of the Board of Governors is largely formal. Hence, the Executive Board may be considered de facto as the main decision body of the IMF.

Among the 24 Executive Directors, eight represent each a single country,[1] whereas the remaining 16 are elected by country groups called constituencies. While there is little formal voting at the Executive Board, the relative power of the Directors depends on the voting rights they have been attibuted. The voting rights of each Director corresponds to the voting right of the country or constituency he (she) represents, and the voting is based on qualified majorities (50%, 70% or 85%), depending on the importance of the decision to be made.

One main concern about the governance of the IMF is the low share of voting rights attributed to developing countries. Indeed, the industrial countries hold 60% of the voting rights, the United States alone accounting for 17%. This is as much as the developing Asian countries and Latin America combined, and enough to block all decisions requiring an 85% majority. In fact, the voting rights are quasi proportional to the quotas of member countries at the IMF the latter being more dependent on their capacity to contribute to the Fund than on their needs. Following the Cooper proposal of 2000, there has been an intensive work to figure out a way to rebalance voting rights in favour of developing countries.[2] The basic conclusion is that raising the voting rights of developing countries will hardly be possible without some disconnection between quotas and voting rights. This complicated and political topic was on the agenda of the twelfth general review of quotas, completed by the end of 2003, which did not result into a proposal for a quota change. Ad hoc quota increases for 54 member countries have been decided in 2008, which became effective as of March 2011 and enhance the representation of low income countries. Building on this, the Fourteenth General Quota review took place in 2010. A Plan was adopted in November 2010 "to reflect the increasing

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[1] The countries are China, France, Germany, Japan, Russia, Saudi Arabia, the United Kingdom and the United States.

[2] The Cooper proposal and studies by the IMF on the quota formulas are available at www.imf.org.

importance of emerging market and developing economies",[3] via a doubling of the quota and a proposed increase of 6 ppts of the quota share in favour of the so-called "dynamic emerging market and developing countries (EMDC). The plan, which was supposed to come into force by end 2012, is now pending the ratification of the US. A fifteenth General Quota Review is expected to be completed by January 2015.

A parallel topic is the representation of the European Union in both the Board of Governors and the Executive Board. On the basis of current quotas, the enlarged EU will account for 32% of the quotas, the present euro area alone standing at 23%. However the voting rights are distributed across constituencies where EU and non-EU countries are often mixed up. One topical question is whether grouping EU countries into a single representation, with possibly lower voting rights, could lead to higher voting power for the EU as a whole and for each member country. This is a very important question for the EU, which may feel its influence lower than that of the US at the IMF despite higher cumulated voting rights; it is also important for low developed and emerging countries which could be transferred the voting rights abandoned by the EU.

A popular tool for studying voting power is provided by the cooperative game theory. Power indices are based on the ability of each player to move a losing coalition into a winning one by using its votes. Such approach has been applied to the United Nations Security Council (Shapley-Shubik, 1954), to the European Central Bank (Brückner, 1997) or to the EU Council (Barr and Passarelli, 2003). It has been applied to the IMF Board of Governors by Leech (2002) and by Bini Smaghi (2005). The former shows the relative power of member countries to be very dependent on the qualified majority used (50%, 70% or 85%):

> "The results show that, first (…), the effect of the special 85% supermajority requirement is to equalise voting power to a great extent. Second, the results for ordinary decisions using the 50% majority rule show that power is more unequally distributed than intended" (p. 388).

It follows that the United States, which enjoys the largest voting right, should not ask for supermajority requirements. The intuition behind this result is that, although the United States alone has a veto power with the supermajority requirement, hence a high power to block a decision, this requirement also offers a wide range of possible coalitions to block a decision. Hence, the United States also displays a low power to initiate action with the supermajority requirement. More generally, the 50% requirement produces the most unequal distribution of

[3] IMF website: http://www.imf.org/external/np/exr/facts/quotas.htm

voting powers (depending on the distribution of voting rights), whereas voting powers converge as the majority requirement rises; a unanimity requirement leads to equal voting power for all member countries whatever their voting rights. According to Leech (1998), the same would happen should the EU members vote as a block: they would enjoy a very high voting power (at the expense of the United States) with a 50% majority requirement, but this prominence would be erased for supermajority voting. Similar results are presented in Bini Smaghi (2005).

This striking result, which is derived from the calculation of Banzhaf and Coleman power indices, applies to a single-stage game like that of the Board of Governors. The case of the Executive Board is complicated by the existence of constituencies.[4] The latter reduce the number of players, from 188 (number of member countries) to 24 (number of Executive Directors) and provide some a priori information on the possible coalitions. Most Executive Directors are elected by a group of countries and must take into account the balance of opinions in their own constituency before they proceed to a vote. Hence one could think of the Executive Board as a two-stage game: for each decision, first there is a vote within each constituency, and then a quotient game at the Executive Board. However the constituencies are not fixed forever. Indeed, the Articles allow for a country to join another constituency when Executive Directors are elected. One consequence is that an Executive Director is obliged to account for the opinion of any country in his (her) constituency, since there is a risk that the latter leaves the constituency, reducing the voting rights he (she) is endorsed with. Another consequence is that a two-stage approach cannot be carried out since the outcome of the first stage (the vote within each constituency) may influence the definition of the first stage game itself (the number and nature of the players in each constituency).[5]

In a related paper, Leech (1998) provides a comparison between the Banzhaf index of power (Banzhaf, 1985) and the Shapley-Shubik one (Shapley and Shubik, 1954). The former, a coalition is defined as a combination of any number of players with no specific order, whereas in the latter, each coalition corresponds to a specific ordering of all players, with one pivotal player in each combination. Although only the Shapley-Shubik index has been shown to satisfy desirable properties (for instance, the Banzhaf index must be re-normalised to sum to

[4] As a first approximation, Leech (1998) applies the same, single-stage methodology to the Executive Board as to the Board of Governors. Only the number of players and their voting rights is modified. But then, it is no longer possible to track the voting power of each member of a constituency, except if the voting power can be attributed to the country holding the Executive Directorship.

[5] This point is noted by Leech (2002).

unity, but then it cannot be compared across games with different majority requirements), the Banzhaf index has generally be preferred by political scientists, perhaps due to easier computability. While both indices provide similar results for the 50% majority requirement, the Banzhaf indices of the various countries converge continuously when the majority threshold is raised (from 50% to 100%, see above), whereas the Shapley-Shubik indices converge only for very high threshold (70% or more). However the Banzhaf index displays non-linarities and high sensitiveness to the data. On the whole, it is difficult to assess which index is closer to the reality since no comprehensive image of reality is available.

Alonso-Meijide and Bowles (2005) propose to study the balance of power at the Executive Board with the concept of a priori unions initiated by Owen (1977). In this framework, the power of each country in a given coalition is related to the threat it represents should it leave the coalition: this threat defines the amount of power the country should be rewarded with by the coalition. Alonso-Mejide and Bowles use three power indices: the Owen index, which is based on the Shapley-Shubik index corrected for a priori unions; the Banzhaf-Owen index, which is similar to the Owen index but relies on the Banzhaf index of power (see Alonso-Meijide, 2002 for a derivation of this index), and the so-called Symmetric Coalitional Banzhaf index, where the allocation of power between unions is made according to a Banzhaf-type index, whereas it is made within unions according to a Shapley-type index (see Alonso-Meijide and Fiestra-Janeiro, 2002). The main advantage of the latter index is that it allows for symmetry between unions, ie the aggregate power of two constituencies entitled with equal voting rights is similar. This desirable property is obtained with the Owen index, but not with the Banzhaf-Owen one. Alonso-Meijide and Bowles (2005) provide "ready-to-use" procedures to compute these three indices for large numbers of players. They show the coalition power indices to be very sensitive to the coalition structure, ie, in our case, to the number and composition of constituencies. In particular, they find that aggregating the voting rights of EU countries would lead to a reduction in the voting power of Europe, even without any reduction in EU voting rights. The drop of EU power is more marked for the 85% threshold than for the 50% requirement, which can be related with the equalisation of voting powers when the threshold rises.

The present paper draws on this methodology to study the possible impact of various scenarios concerning the representation of the European Union at the Executive Board. The originality of the paper lies both on the application of a priori union power indices and on the definition of the scenarios for European representation, with a re-calculation of voting rights along the quota formulas in the case of single seat scenarios.

The reform of quota decided in 2010 has not yet came into place, due to the refusal of the US to ratify the reform until now, and it is yet unclear whether those will be adopted or supplemented by a different proposal coming from the Fifteenth quota review. The computations presented in the paper have therefore been done of the basis of the pre-reform situation. In order to keep some comparability with previous studies, we decided to use the quota as they stood in mid-2003. For consistency reasons, the scenario investigated in the paper also refer to the situation existing at that time from the perspective of geographical coverage.[6] While the EU and the Euro Area have experienced changes in their composition since that date, the relative size of the new member states only had a marginal impact on the variables looked at, meaning that the results obtained under these assumptions remain fairly representative of today's situation.

2. EUROPE'S REPRESENTATION AT THE IMF

As already mentioned, voting rights at the IMF rely on quotas, the fixed part of voting rights being negligible for most countries.[7] The countries belonging to the European Union before enlargement account for 30% of the quotas, and the thirteen new members (since May 2004[8]) represent another 2%.[9] Europe thus has a large weight in the decision bodies of the IMF. Nevertheless, its point of view does not always seem to find its way through the twists and turns of the collective decision-making process.[10] A key explanation for this discrepancy is the dissemination of voting rights across various constituencies. As shown in Table 1, most European countries do not have a permanent Director, and some of them belong to constituencies where the Executive Director is not an EU member.

Table 1. Representation of EU25 members in the Executive Board (August 26, 2003)

[6] That is: we will investigate the euro area with its composition in the year 2003, etc.

[7] Each member is endowed with 250 voting rights for every SDR 100,000 quotas. Even for the smallest country in terms of the quota (Palau), the fixed part of the voting right is very small (7.5% of Palau's voting right).

[8] That is: the 10 new members of May 2004, plus Bulgaria and Romania (who joined in 2007) and Croatia (who joined in 2013).

[9] Excluding Bulgaria, Romania and Crotia, which roughly adds another 1%.

[10] See Coeuré and Pisani-Ferry (2001).

Single-state EU constituencies	EU elected executive director and EU countries in the corresponding constituencies.	EU countries in constituencies with non-EU Executive Director
France	Belgium: Austria, Luxembourg, Czech Rep., Hungary, Slovak Rep., Slovenia	Spain (Venezuela)
Germany	Italy: Greece, Portugal, Malta	Ireland (Canada)
United Kingdom	Netherlands: Cyprus	Denmark, Finland, Sweden, Estonia, Latvia, Lithuania (Iceland)*
	Spain	Poland (Switzerland)

* rotating Executive Director.

Source: www.imf.org/external/np/sec/memdir/eds.htm

Several scenarios could allow EU members to co-ordinate their views better within the IMF. The first one would consist in bringing all EU members into one or several constituencies. A single European constituency and a single Executive Director (though preserving the representation of each country on the Board of Governors) would allow a reducing the overall number of constituencies, from 24 (at present), down to 19 (as set out in the Fund's Articles). But the prominent weight of the EU constituency, which would account for 32% of the quotas once the EU enlarges, is unlikely to make this scenario acceptable to third countries.

Here we explore another scenario which consists in merging EU representations into a single seat. In this case, the European Union would have a single representative both on the Board of governors and on the Executive Board. Of course, this raises a number of legal problems, as IMF members must be States, not groups of States[11]. Nevertheless, it would be more acceptable politically than a single constituency since the EU quota would be adjusted downwards, on the ground that the balance of payments of the aggregate member would have to be consolidated (thus eliminating current account payments intra-zone). The position of the EU in the IMF decision-making process would then be decided by the EU Council of Ministers, on the basis of the Nice Treaty.

However, such a scenario would only be possible in line with further political integration in the EU, a prospect that does not gather unanimity. Hence, there is little probability that EU members be represented by a single seat in the near future. Conversely, cooperation on IMF-

[11] See Mahieu, Ooms and Rottier (2003).

competence issues could be engaged on a lower scale within the “enhanced cooperation” framework, which allows a sub-group of EU countries to proceed to further cooperation provided other EU members stay free to join and are not disadvantaged by the enhanced cooperation.

Hence, three scenarios are considered here and compared to the status quo:

- an EU25 single seat;
- an EU15 single seat;
- an Euro area single seat (as of 2003);
- a Franco-German single seat.

The quotas to be attributed to the single European seat in each scenario have been recalculated in Bénassy-Quéré and Bowles (2002) under two polar hypothesis:

(i) the present discrepancy between theoretical quotas (stemming from the five official formulas) and the observed quotas (after political negociation) is kept unchanged in percentage for the European seat as a whole;

(ii) the present discrepancy between theoretical and observed quotas is eliminated.

The results are reported in Table 2.

Table 2. Simulated quotas for a single European seat

As a % of total IMF quotas	Quotas in Sept 2002	Possible scenarios			
		Franco-German seat	Eurozone seat	EU15 seat	EU25 seat
France – Germany	11,2 – 12,2	**9,8 – 10,8**			
Eurozone 12	23,5 – 28,2	22,1 – 27,0	**12,9 – 16,1**		
EU15	30,3 – 36,6	29,2 – 35,6	20,9 – 26,0	**15,6 – 19,8**	
EU25	32,3 – 38,6	31,3 – 37,6	23,2 – 28,3	18,1 – 22,3	**15,9 – 20,0**
United States	17,5 – 16,6	17,7-16,9	19,6 – 19,2	20,7 – 20,7	21,2 – 21,3
Japan	6,3 – 8,3	6,3-8,5	7,0 – 9,7	7,4 – 10,4	7,6 – 10,7
Developing and transition countries	38,6 – 31,9	39,2-32,5	44,2 – 37,5	47,3 – 40,9	(46,0 – 39,6)

Note : The first figure is that of observed quotas (first column) or of a situation in which the spread between the theoretical and observed quotas is retained. The second figure corresponds to the theoretical quotas resulting from a strict application of the Fund's formulas. For the EU25 seat, new Member states have been removed from the « developing and transition countries » group.

Source : Bénassy-Quéré and Bowles (2002).

3. VOTING POWER WITH A PRIORI UNIONS

The notion of power has been extensively investigated in the political sciences litterature, which ended up with several and sometimes contradicting definitions. The analysis implemented here is based on the concept of voting power, which tries to quantify the power of a given player participating in a voting game using a so-called power index. The starting point of the analysis is that the amount of voting rights allocated to a given player is not enough to get a measure of how much he/she will be able to influence the final outcome. This ability, indeed, will also depend on how he/she will be able to engage or even create winning coalitions. One can even imagine situation in which a "small" player is able to federate many players in a strong coalition, which may then be able to overrule the decision of a "big" player. To account for this, the voting power analysis looks at all the possible coalitions that can be formed in a given voting game, distinguishing between the winning ones and the losing ones. A player would be then said to be "pivotal" whenever his participation is able to tranform a losing coalition into a winning one. Overall, the higher the number of coalitions where the player is pivotal, the higher his/her voting power will be.

Formally, the voting power of player i is written:

$$P_i = \sum_{S \subseteq N \setminus i} p_S \left(v(S \cup i) - v(S) \right)$$

where N is the set of players, p_S the probability that the coalition S will form and v a function such that $v(S \cup i) - v(S) = 1$ if and only if S is losing and $S \cup I$ is winning.

Of course, the power thereby defined will depends on the probability distribution chosen.Two classical probability measures are generally used, named after the two "founding fathers" of the voting power analysis. The Banzhaf probability measure assumes that all coalitions are equally probable, independently of their size, that is $p_S = 1/2^{n-1}$(Banzhaf, 1965). The Shapley probability measure, instead, considers all orderings of players as equally probable, that is $p_S = s!(n-s-1)!/n!$, where s and n denotes the cardinality of S and N, respectively (Shapley, 1954). This latter probability measure is made under the assumption that all players can be ranked according to the intensity of their preference over the final outcome (and hence the orderings). For each ordering, the "pivotal" player will be the one without which all the players with an higher desire to "pass the bill" will be precluded to do so, while all the players with a lesser desire will not be needed to make the coalition winning. In other words, the Shapley measure reflects the idea that voting committees are always governed by the center.

Another way to understand the difference of these two probability measures is to see how much probability weight they give to non-winning coalitions. In the Banzhaf framework, both winning and non-winning coalitions can emerge. For this reason the, the sum of the Banzhaf power index of the players does not add up to one, although the results are generaly normalized for convenience. In the Shapley framework, only winning coalitions are considered. In other words, the former framework sees the coalition formation process as not necessarily "efficient", while the latter framework sees it as "efficient". In both cases, the coalition formation is not modelled and only its result (that is the set of possible coalitions) is considered as given. One could always define a probability measure that is more suitable for the specific case under consideration, as some coalitions may be more likely than others because, say cultural or historical reasons may favour the communication between specific groups of players (the conclusion gives some other suggestions that could be followed in further research). One interest of the two measures, however, is that they attempt to measure the voting power implied by the single structure of the voting game, where the game is solely defined by the number of players, their respective voting rights, and the majority threshold.

These two indices cannot be directly applied in the case of the IMF, however. Indeed, according to the IMF rules, the 184 member states have to meet first in groups of countries, the so-called constituencies, in order to elect 25 Executing Directors that will decide over the final set of outcomes. In other words, the coalition formation process is restricted. Alonso-Meijide and Bowles (2005) proposed to use the so-called a priori union framework (Owen, 1977) in order to take this specificity into account. In this framework, the set of players is partitioned into groups of players or “a priori union”, and the set of possible coalition if restricted to be compatible with this coalition structure. In other words, whenever a voting game takes places, players are constrained to vote by group of players.

As for the classical framework, the index will depend on the probability measure used. But the introduction of a two level coalition structure makes it possible to choose a specific probability distribution for both levels, that is within a given union and between the various unions. Four indices can therefore be defined (Owen, 1977, Alonso-Meijide&Fiestras-Janeiro 2002), Amer, Carreras&Jimenez 2002), which are summarized in the table below:

Table 3. Probability measures used in the various coalitional power indices under a priori union

Probability measure used	Banzhaf-Owen (BO)	Symmetric Coalitional Banzhaf (SCB)	Amer-Carreras-Jimenez (ACJ)	Owen (OW)
Between unions	Banzhaf	Banzhaf	Shapley	Shapley
Within unions	Banzhaf	Shapley	Banzhaf	Shapley

One interest of the framework is that a change in the coalition structure can change the power of a given player without any change in the allocation of voting rights may affect its power, just by changing the set of coalitions the players will be allowed to join.

Following our interpretation in terms of efficiency of the coordination process, each index above will provide some information depending on the configuration of the game. For instance, if, for some reason, there is a good coordination at the union level but a bad coordination between the union, then the symmetric coalition banzhaf may be the more suitable index to look at. In practice, the degree of coordination is not that easy to assess and all four indices may be looked at.

In the following section, the various indices are applied to a set a specific scenario presented in section 2. Unfortunately, we were not aware of the existence of the ACJ index when the (quite intensive) numerical computation was started, so that only results for the BO, the SCB and the OW index are presented below.

4. THE RESULTS

We distinguished between the European Constituency and the Single Seat scenario.

a) the European Consituency scenario

- Under the 50% majority threshold, the formation of a European Consituency would likely increase the power of the EU at the IMF (see Table 4-6).[12] The larger the Constituency, the larger the power of the EU as a whole. The voting power of the United States, instead, decreases. The results are robusts to the three different indices used.

- The formation of a European Consituency would decrease the power of the EU for the 85% majority threshold, however, while the one of the US would increase (see Tables 7-9). In this case also, the results seem to be quite robusts to the index used. The decrease of power would be quite drastic according to the SCB and the BO index. This reflects one property of the Banzhaf type allocation, for which the speed of convergence to the unanimity game (100% majority threshold, where all players have the same power) is higher. Intuitively, the higher the majority threshold, the closer we are to the one player one vote situation. Reducing the number of european players do mechanically reduce the power of EU as a whole, part of this lost being allocated to the US.

Tables 4-6. The European Consituency (50% majority threshold)

	Scenario		
	Current	EU12	EU15
Voting power			
FR+GR	11.00	13.17	14.24
EU12	22.88	27.59	29.92
EU15	29.71	34.24	38.91
EU25	31.88	36.33	40.84
US	20.07	17.89	15.24

	Scenario		
	Current	EU12	EU15
Voting power			
FR+GR	10.44	12.26	15.43
EU12	21.88	25.75	31.81
EU15	28.38	32.91	41.30
EU25	30.43	35.11	43.36
US	21.61	13.25	8.20

	Scenario		
	Current	EU12	EU15
Voting power			
FR+GR	10.44	12.12	15.37
EU12	21.88	25.64	32.25
EU15	28.38	32.79	41.86
EU25	30.43	35.02	43.88
US	21.60	13.19	8.14

The numbers presented in the shaded areas are the sum of the power index of the European Consituency and the power indices of the remaining european states.
Source: author's calculations

Table 7-9. The European Consituency (80% majority threshold)

[12] Only the results for the EU12 and the EU15 are presented there, but the results for the EU25 go in the same direction.

	Owen Index 85% Scenario				Symmetric Coalitional Banzhaf Index 85% Scenario				Banzhaf-Owen Index 85% Scenario		
	Current	EU12	EU15		Current	EU12	EU15		Current	EU12	EU15
Voting power				Voting power				Voting power			
FR+GR	11.19	11.22	9.31	FR+GR	11.04	2.89	2.58	FR+GR	11.17	1.00	0.44
EU12	23.22	22.97	19.25	EU12	24.09	8.29	7.09	EU12	24.07	2.84	1.14
EU15	30.19	29.77	25.04	EU15	31.67	17.45	9.15	EU15	31.70	12.55	1.47
EU25	32.35	31.81	27.18	EU25	34.17	20.67	12.60	EU25	34.12	15.99	5.24
US	19.71	22.79	25.04	US	6.46	8.00	9.15	US	6.53	8.47	9.98

The numbers presented in the shaded areas are the sum of the power index of the European Consituency and the power indices of the remaining european states.
Source: author's calculations

b) The Single EU Seat scenario

The particularity of the Single EU Seat scenario is that the voting rights of the EU are lowered by the unification process because of the need to consolidate at the european level the economic variables that enters in the computation of the quota, in particular trade and capital flows. As discussed in Benassy-Quere and Bowles (2002), these theoretical voting rights, however, are just the starting point of a political process after which the final voting rights are decided. Two kinds of results are therefore presented below. The first ones rely on the theoretical voting rights that the EU should have after the unification (column Th. in the various tables, to be compared with the theoretical ones currently prevailing in the column current-th). The second rights rely on the "real" voting rights that the EU would have assuming that the difference between the theoretical voting rights and the current voting rights will be as big as it is now (column Sim., to be compared with the column Current (obs)).

Two main results can be observed (see Table 10 to 15):

- the Single EU Seat is likely to unambiguously decrease the power of the EU power for most of the scenarios (EU12, EU15 and EU25). The results are convergent for the three voting power indices and the two majority requirements (50% and 85%). Conversely, the power of the US would increase. This contradicts both Leech (1998) and Bini Smaghi (2005).;

- the only case where the EU power could increase is the scenario of the franco-german chair;

Table 10-12. The Single EU Seat (50% majority threshold)

Owen Index 50%

	Current (obs)	Current (th.)	Scenario FR+GR Th.	FR+GR Sim.	EU12 Th.	EU12 Sim.	EU15 Th.	EU15 Sim.	EU25 Th.	EU25 Sim.
Voting power										
FR+GR	11.00	12.09	11.32	9.49						
EU12	22.88	27.78	27.43	24.60	17.26	13.28				
EU15	29.71	36.18	35.87	32.60	20.01	15.35	21.85	16.45		
EU25	31.88	38.06	37.94	34.90	22.27	17.88	24.25	18.87	21.79	16.75
US	20.07	23.42	19.26	19.06	21.75	22.34	23.42	23.83	23.84	24.77

The numbers presented in the shaded areas are the sum of the power index of the European Consituency and the power indices of the remaining european states.
Source: author’s calculations

Symmetric Coalitional Banzhaf Index 50%

	Current (obs)	Current (th.)	Scenario FR+GR Th.	FR+GR Sim.	EU12 Th.	EU12 Sim.	EU15 Th.	EU15 Sim.	EU25 Th.	EU25 Sim.
Voting power										
FR+GR	10.44	11.79	10.90	9.06						
EU12	21.88	27.26	26.97	23.94	16.00	11.84				
EU15	28.38	35.43	35.35	31.83	18.79	13.89	19.75	14.01		
EU25	30.43	37.32	37.41	34.09	20.99	16.34	22.22	16.57	19.87	14.28
US	21.61	23.42	19.66	20.00	21.13	23.15	21.48	23.18	22.12	24.11

The numbers presented in the shaded areas are the sum of the power index of the European Consituency and the power indices of the remaining european states.
Source: author’s calculations

Banzhaf Owen Index 50%

	Current (obs)	Current (th.)	Scenario FR+GR Th.	FR+GR Sim.	EU12 Th.	EU12 Sim.	EU15 Th.	EU15 Sim.	EU25 Th.	EU25 Sim.
Voting power										
FR+GR	10.44	11.78	10.89	9.05						
EU12	21.88	27.25	26.98	23.95	15.99	11.46				
EU15	28.38	35.43	35.36	31.83	18.78	13.58	19.74	13.97		
EU25	30.43	37.33	37.43	34.10	20.98	16.15	22.21	16.56	19.86	14.31
US	21.60	23.42	19.65	19.98	21.12	22.41	21.47	23.13	22.11	24.16

The numbers presented in the shaded areas are the sum of the power index of the European Consituency and the power indices of the remaining european states.
Source: author’s calculations

Table 13-15. The Single EU Seat (85% majority threshold)

Owen Index 85%

	Current (obs)	Current (th.)	Scenario FR+GR Th.	FR+GR Sim.	EU12 Th.	EU12 Sim.	EU15 Th.	EU15 Sim.	EU25 Th.	EU25 Sim.
Voting power										
FR+GR	11.19	11.99	12.16	9.89						
EU12	23.22	27.32	28.03	24.96	21.51	15.22				
EU15	30.19	35.56	36.29	33.03	24.01	17.34	23.09	21.65		
EU25	32.35	37.45	38.25	35.28	26.03	19.58	25.31	24.11	23.30	21.92
US	19.71	23.42	20.27	19.79	21.51	20.49	23.09	21.65	23.30	21.92

The numbers presented in the shaded areas are the sum of the power index of the European Consituency and the power indices of the remaining european states.
Source: author’s calculations

Symmetric Coalitional Banzhaf Index 85%

Voting power	Current (obs)	Current (th.)	FR+GR Th.	FR+GR Sim.	EU12 Th.	EU12 Sim.	EU15 Th.	EU15 Sim.	EU25 Th.	EU25 Sim.
			Scenario							
FR+GR	11.04	12.75	8.33	6.95						
EU12	24.09	30.61	27.17	23.34	9.44	7.45				
EU15	31.67	39.41	36.95	32.21	13.30	10.26	10.48	8.19		
EU25	34.17	41.61	39.44	34.87	16.44	13.46	14.18	11.96	11.55	8.73
US	6.46	23.42	8.44	7.09	9.44	7.46	10.48	8.19	11.55	8.73

The numbers presented in the shaded areas are the sum of the power index of the European Consituency and the power indices of the remaining european states.
Source: author's calculations

Banzhaf Owen Index 85%

Voting power	Current (obs)	Current (th.)	FR+GR Th.	FR+GR Sim.	EU12 Th.	EU12 Sim.	EU15 Th.	EU15 Sim.	EU25 Th.	EU25 Sim.
			Scenario							
FR+GR	11.17	12.99	8.40	7.05						
EU12	24.07	30.73	27.04	23.16	9.49	7.21				
EU15	31.70	39.43	36.83	32.14	13.34	10.08	10.52	8.25		
EU25	34.12	41.53	39.26	34.66	16.49	13.28	14.22	12.09	11.60	9.01
US	6.53	23.42	8.51	7.19	9.49	7.21	10.52	8.25	11.60	9.01

The numbers presented in the shaded areas are the sum of the power index of the European Consituency and the power indices of the remaining european states.
Source: author's calculations

5. CONCLUSION

Overall, the results suggest the unification of EU is likely to reduce its power at the IMF, which contradicts previous results. The single scenario which could increase the EU power, namely the European Constituency Scenario, is the one who is less likely to emerge, as it is based on the assumption that the quota would not be renegotiated after the merger.

Two remarks should be made regarding these results, however. First, these computations have been made using "standard" probability measures, which do not take into account the fact countries may tend to collaborate more actively than implied by our assumptions, owing to cultural or historical reasons for instance. One way to take this into account could be to correct these indices, using for instance the correlation structure of voting rights of the various countries at the United Nations for instance, where some more information is available. Second, the decrease of power of the EU following some unification process should not be considered as the single criteria. It would indeed also have some implications regarding the voting power of the other members and may be a way to redistribute some voting powers to developing countries

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