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Three Essays on Liquidity Risk

Grégory Claeys

Thesis submitted for assessment with a view to obtaining the degree of
Doctor of Economics of the European University Institute

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Department of Economics

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Abstract

Liquidity risk is inherent to the very nature of the banking activity which is to transform short term liabilities into long term assets. That is why liquidity crises are in one way or another implied in most financial crisis episodes. This thesis contributes to the understanding of how liquidity risk and liquidity crises in the banking and financial sector affect the allocation of resources and the functioning of the economy. It also discusses what could be the best institutional arrangements to share liquidity risk across agents and the best economic policies to avoid liquidity crises. It consists of three chapters focusing on diverse aspects of this topic. The first chapter, co-authored with Katerina-Chara Papioti, provides a new way to measure liquidity risk in the financial sector using the bidding behavior of banks in the bond auctions conducted by central banks. The second chapter examines risk-sharing between agents prone to liquidity shocks obtained through generational and intergenerational coalitions and asset trading in overlapping generation economies. Various institutional arrangements including financial intermediaries, stock markets and government interventions are studied in order to compare their risk sharing performance and optimality. The third chapter examines the international dimension of the liquidity issue and studies theoretically what combination of exchange rate regime and central bank policy is less vulnerable to a combined currency and banking crisis focusing on the sudden stop of capital flows as an underlying source of instability.

à Chantal Claeys (1951-2013)

Acknowledgments

This section is dedicated to all the people who supported me throughout my Ph.D.

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Finally, I am dedicating this Ph.D thesis to Aurélie Balcon who supported me before and during my PhD and who will hopefully continue to do so in the decades to come (even though my more-complex-than-average career choices asked her to go against her natural aversion to moving and traveling often in order to follow me from Paris to Florence, to Barcelona, to Santiago de Chile and finally to Brussels). I am also dedicating this thesis to my mother, Chantal Claeys, who always supported me and helped me in fulfilling my dreams (however random they seemed sometimes) and taught me to always aim for the best.

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Preface

Liquidity risk is inherent to the very nature of the banking activity which is to transform short term liabilities into long term assets. That is why liquidity crises are in one way or another implied in most financial crisis episodes. This thesis contributes to the understanding of how liquidity risk and liquidity crises in the banking and financial sector affect the allocation of resources and the functioning of the economy, but also discusses what could be the best institutional arrangements to share liquidity risk across agents and the best central bank policies to try to avoid liquidity crises. More precisely it consists of three chapters focusing on liquidity from three different but complementary perspectives: how to measure liquidity risk in the financial sector? what is the best institutional arrangement to share liquidity risk across agents of the same and different generations? and what are the best central bank policies and exchange rate regimes to avoid an international liquidity crisis?

The first chapter, co-authored with Katerina-Chara Papioti, provides a tool for central banks to measure liquidity risk in their financial sector using the bidding behavior of banks in bond auctions. First, we build up a model combining the auction literature and the financial economics literature to understand precisely the effect of the liquidity risk affecting banks on their bidding strategies in those auctions. We develop a benchmark version of the model with no insurance against the liquidity shock, and another with a lender of last resort to see how the behavior of the banks is affected by this policy. Second, we use these theoretical results and a unique dataset collected at Central Bank of Chile containing all the details of its open market operation auctions (where it sells bonds to drain money from the banking sector) between 2002 and 2012 to estimate the distribution of the liquidity risk across Chilean banks and its changes over time. The evolution of the estimated distribution seems to capture well the main episodes of liquidity stress of the last decade in the Chilean banking sector. This measuring tool could be used by other central banks conducting similar open market operations and in need of evaluating in real time the evolution of the liquidity risk affecting their financial sector.

The second chapter examines from a theoretical perspective risk-sharing obtained through coalitions of agents and asset trading in overlapping generation economies prone to non-verifiable idiosyncratic liquidity shocks. We attempt to synthesize the extant and extend the overlapping generation version of the Diamond and Dybvig (1983) model in order to compare various institutional arrangements including financial intermediaries, stock markets and government interventions. We show that generational financial intermediaries are able to achieve – whether they are combined with a stock market or not – an allocation with perfect insurance and full investment as long as a government can implement intergenerational transfers between financial institutions. Incidentally, we also show that, in this framework, financial intermediaries and markets can not coexist unless there is a government transfer scheme ensuring that the first best allocation is achieved. Finally, we show that there exists a feasible path leading to the steady state optimal allocation when the economy has a starting date in which the first generation has only its endowment to start the intergenerational risk-sharing mechanism. Unlike the previous authors examining this question we have chosen not to restrict ourselves to solutions ensuring a constant level of expected utility to all members of all generations but to look for a finite path to the optimal steady state allocation taking into account the participation constraints of the generations living during this transition.

The third chapter contributes to the debate between flexible and fixed exchange rate in light of the recent events. Indeed, the purpose of this chapter is to determine what combination of exchange rate regime and central bank policy is less vulnerable to a combined currency and banking meltdown. Chang and Velasco (2000) showed that flexible exchange rates dominate all other monetary arrangements (currency board, fixed rate, etc.) in the sense that it is the only regime that is not vulnerable to either a banking or a currency crisis, provided that the central bank stands ready to act as a Lender of Last Resort. Nevertheless, they did not take into account the sudden stop phenomenon as an underlying source of instability to the financial system. We intend to amend their model and results by giving a prevailing position to foreign investors and to the sudden stop phenomenon in accordance with what was observed during some of the most recent financial crises. In this setup, flexible rates still dominate all other combinations of exchange rate regime and central bank policy under some strict condition regarding the size of foreign borrowing. However, if the financial sector is too dependent on short-term debt in foreign currency, then the flexible rate regime is no panacea anymore as it can not ensure that a twin crisis will be avoided in the case of a sudden stop. The main result is that a flexible exchange rate supported by a lender of last resort policy of the central bank dominate all other combinations of exchange rate regime and central bank policy under some strict condition regarding the size of foreign borrowing. Indeed, if the financial sector is too dependent on short-term debt in foreign currency, then the flexible rate regime is no panacea as it can not ensure that a twin crisis will be avoided in the case of a sudden stop in capital flows.

Chapter 1

Bond Auctions and Financial Sector Liquidity Risk

1.1 Introduction

Liquidity risk is inherent to the very nature of the banking activity which is to transform short term liabilities into long term assets. Traditionally, this was done by collecting deposits in order to make loans for long term projects. However, in the two decades preceding the 2008 financial crisis, this risk has been pretty much neglected by banks and regulators alike. Banks around the world have been relying more and more on short term wholesale funding such as the asset-backed commercial paper market, the repo market and the overnight interbank market. The counterpart of this rapid growth in wholesale funding was a parallel decrease of historically more stable retail deposits in the funding of the banks[1]. A good – although a bit extreme – example of this trend is the trajectory of the British bank Northern Rock in the years leading to the financial crisis, with its ratio of deposits to total liabilities declining from 62.7% in 1997 to only 22.4% in 2006 (Bank of England, 2007b).

However, during the financial turmoil that started in 2007 and worsened especially after Lehman Brothers' failure in September 2008 – and described at length in Brunnermeier (2009) – those crucial sources of short term funding for financial institutions all vanished

[1]Deposits are indeed considered a more stable source of funding for banks since the implementation of deposit insurance in many countries to avoid the phenomenon of bank runs that was prevalent during financial crises until the 1930s (for instance the US FDIC was created in 1933)

at the same time. Consequently, banks relying on this kind of funding had difficulties to meet their obligations and some of them actually failed, meanwhile others like Northern Rock were nationalized to avoid bankruptcy, highlighting the fact that liquidity risk had not disappeared.

Since then, there has been a renewed interest in this issue of liquidity risk in the financial sector. That is why the main purpose of this paper[2] is to build a tool for central banks and regulators to measure this risk in order to be able to assess the fragility of their financial sector. Note that the measuring tool described in this paper has been constructed especially for the Central Bank of Chile (BCCh) but that it could be used by any other central bank conducting similar open market operations.

Before explaining in detail how we intend to proceed, let's give first a careful definition of liquidity risk because the word liquidity has often been used to describe different concepts, and we want to be clear about what we intend to understand and measure in this paper. We define liquidity as the ability of an agent to settle its obligations with immediacy. It is clearly a binary concept: an agent is liquid or illiquid. On the contrary, liquidity risk is a continuous concept and we define it as the probability to become unable to settle its obligations over a specific horizon. This is what we want to measure in this paper. Finally, it is also interesting to distinguish, as in Brunnermeier and Pedersen (2009), funding liquidity: the ease to access funding (which is agent specific), from market liquidity: the ease to sell an asset (which is asset specific), because in the paper we want to focus exclusively on funding liquidity risk.

There exists already various measures of liquidity risk in the literature. Some of them are based on data from the balance-sheets of banks like the Liquidity Coverage Ratio and the Net Stable Funding Ratio introduced by Basel III after the subprime crisis (BIS, 2013), other are market-based like the very simple LIBOR-OIS spread or more complex like the composite financial market liquidity indicator introduced by the Bank of England (2007a). However, we consider that those measures of liquidity risk are imperfect either because they depend heavily on stress scenarios and are very sensitive to expert categorizations of assets and liabilities (in the case of balance-sheet based indices) or because they do not allow to disentangle liquidity risk from other risks like solvency risk (in the case of

[2]This paper was written in collaboration with Katerina-Chara Papioti. We are grateful to Elena Carletti, Massimo Morelli and Andrea Mattozzi for their support and advice, to Rodrigo Cifuentes and Juan-Francisco Martinez from the BCCh for their help and the access they gave us to the data of the Open Market Operations, and to the participants of the various conferences and seminars where we have presented this paper for their remarks. All remaining errors are ours.

market-based indices). We want to build a new tool to measure liquidity risk that will not be subject to those flaws. We think that, if it is difficult for central banks to assess the liquidity risk of a specific bank or of the whole banking sector using data from balance-sheets or financial markets, we can still assume two things: first, banks are aware of their own liquidity risk and, second, they may be less willing to give up their cash if this risk is high, to avoid bankruptcy. That is why the basic idea behind this paper is to try to extract this information directly from the banks using their bidding behavior in the bond auctions of the BCCh.

Why do we think that the banks' bids in those auctions reveal their liquidity risk? First, it is important to note that the BCCh sells bonds to the Chilean commercial banks in order to drain money from the banking sector to control the quantity of money in circulation in the economy and meet its inflation target. It is purely a technical open market operation by the central bank and not at all a way to finance itself or to finance the Chilean government. Therefore, those bonds issued by the central bank bear absolutely no solvency risk whatsoever: the BCCh can always print pesos to reimburse the banks when the bonds mature. That is why, in our opinion, the trade-off faced by a bank on whether to hold cash or bonds depends only on the liquidity risk of the bank between the moment the bond is bought and the moment it matures.

In this respect, we think that Figure 1.1 is quite informative. This chart depicts the evolution of two different variables from 2002 to 2012. On one side, the red curve represents the famous LIBOR-OIS spread[3], which we consider an imperfect measure of liquidity risk but a good first approximation for the liquidity stress in international financial markets. On the other side, the blue curve represents the difference between the average interest rate asked by Chilean commercial banks to buy the short-term bonds of the BCCh and the deposit rate at the BCCh where banks can leave their reserves and have access to them whenever they want. At first glance, it appears that there is a clear correlation between periods of international liquidity stress and an increase in the premium asked by banks on those bonds, especially during the 2007-08 financial turmoil. This seems to confirm our initial intuition that banks ask for a higher liquidity premium on the BCCh bonds to give up their cash if they anticipate a higher probability to face a liquidity problem.

[3]The LIBOR-OIS spread is defined as the difference between the 3-month LIBOR rate at which banks borrow unsecured funds from each other in the London wholesale money market and the Overnight Indexed Swap which is roughly equivalent to an overnight rate rolled-over every day for 3 months.

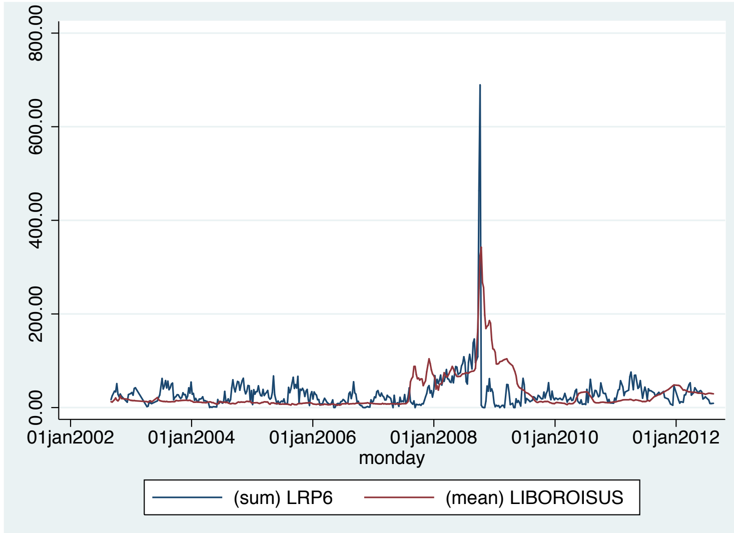

Figure 1.1: Spread between rate bids for ST Bonds and deposit rate vs LIBOR-OIS spread (in basis points)

In practice, how can we use those auctions to measure liquidity risk in the financial sector? The first step is to build a simple model to understand, and quantify, how the possibility of liquidity shock at the bank level affects their bidding strategies in a bond auction similar to the one conducted by the BCCh. In terms of methodology, the idea is to combine the theoretical research in multi-unit auctions initiated by Wilson (1979) with the literature on liquidity crises initiated by Diamond and Dybvig (1983). By itself, this would fill a hole in the theoretical literature, as most of the articles discussing liquidity crises assume exogenous (deterministic or stochastic) asset returns. On the contrary, the auction framework will allow us to endogenize the interest rate and understand fully how it is determined. The model will therefore contribute to both the financial literature and the auction theory by showing how the possibility of a liquidity shock affects bidders' strategies and the auction outcome. To address these questions we develop a 3-period model. In the first period, there is an auction where sell bonds that mature in the third period. Before the auction takes place, banks discover their own private liquidity risk and the distribution of this risk across bidders. In the second period, an idiosyncratic liquidity shock materializes for some banks which makes those holding bonds illiquid because its cash is invested in the bonds. This allows us to obtain theoretical bidding strategies as a function of the probability to have a liquidity shock. Intuitively, a bank with a higher probability to get the liquidity shock should be inclined to ask for a higher rate to insure itself against the shock if it is awarded some bonds in the auction. We develop two versions of the model, one with no insurance against the liquidity shock and another one with a Central Bank acting as a lender of last resort to see how the bidding behavior of the banks is affected by this policy.

The second step is to use the theoretical bidding strategies of the banks from the model and the dataset containing all the details of the bond auctions conducted by the BCCh between 2002 and 2012 to estimate the distribution of liquidity risk across Chilean banks and its changes over time. To perform those estimations we build on the literature on structural econometrics in auctions reviewed extensively for instance in Paarsch and Hong (2006). Besides, once the parameters of the distribution are estimated for each period, this method also allows us to retrieve for each bank participating in the auction its probability to be hit by a liquidity shock during the duration of the bond just by inverting its bid thanks to the theoretical bidding function obtained in the model. In the end, this helps us create an interesting measuring tool for Central Banks conducting similar open market operations and in need of evaluating in real time the evolution of the liquidity risk affecting their financial sector in general or each of its banks more precisely.

The paper is organized as follows. Section 2 explains how open market operations in general and bond auctions in particular are conducted at the BCCh. In this section we also describe our unique dataset collected with the help of the BCCh containing all the open market operations details for a whole decade. Section 3 presents the theoretical model of the auction in two different settings. The estimation of the distribution of liquidity risk in the Chilean financial sector across time is discussed in section 4. Section 5 concludes.

1.2 Open Market Operations at the Central Bank of Chile

1.2.1 Liquidity Management Framework at the BCCh

Like many central banks around the World, the BCCh has adopted inflation targeting framework for its monetary policy. It started with a partial inflation targeting framework in 1990 and moved to a full adoption, in combination with a flexible foreign exchange regime, in September 1999. To meet its inflation target, the central bank uses the overnight nominal interest rate as its main policy instrument. It sets a notional level for the monetary policy rate (MPR), and then adjusts the quantity of money in the market to bring the overnight interbank interest rate around that level. It also offers permanent overnight deposit (FPD) and lending facilities (FPL) to Chilean commercial banks with a view to keeping the interbank lending rate close to the MPR. Like the Fed, the ECB or the Bank of England, the BCCh can inject money via repo operations. However, unlike those central banks, this is not the main tool for liquidity management[4] in Chile. Indeed, because of important capital inflows in the past and the constitution of large foreign exchange reserves in the 1990s before the adoption of a fully flexible foreign exchange rate regime, the BCCh controls the quantity of money in the economy not by injecting money every week in the financial sector but by draining money from it. Thus, the adjustment operations (like the repo operations) employed to inject money in the financial sector if necessary are used more sporadically and mainly for fine-tuning.

The main operations of the BCCh are therefore the regular structural Open Market Operations performed weekly or sometimes even bi-weekly to drain money from the banking sector. Those operations take the form of auctions where the BCCh issues different types of bonds and sell them to the commercial banks. These bonds are short-term notes (PDBC) due in 30 to 360 days, nominal bonds with maturities of 2, 5 and 10 years (BCP2, BCP5 and BCP10 respectively) and inflation-indexed bonds with maturities of 5 and 10 years (BCU5 and BCU10).

Those various bonds can be purchased by agents authorized to participate in the primary market. In practice, in 2012, the participants to these auctions included twenty-three banks, four pension fund administrators, the unemployment fund administrator, three insurance companies and four stock brokers.

Among the bonds sold by the BCCh, PDBC (the short term notes) are the most heavily used to manage and regulate the quantity of money in circulation in the financial system within a given month or from one month to the next. The auction schedule for these notes is announced monthly, when the Bank operations calendar for the month is made public. The program planning takes into account the liquidity demand forecast, maturing issues from previous periods, strategies for complying with reserve requirements and seasonal factors affecting liquidity in the period.

The reader interested in more details in the liquidity management by the BCCh should

[4] Note that in this subsection, the term liquidity is used not to define the ability for an agent to settle its obligation as in the introduction or the rest of the article, but to define the management by the central bank of the quantity of money in circulation in various markets or more generally in the economy.

consult the document published by the central bank (2012) on this topic.

1.2.2 Bond Auction Format at the BCCh

To sell short term bonds (which are the one that will be of interest in the last section of this paper), the BCCh carries out mainly multi-unit uniform auctions. It is a multi-unit auction because a fixed number of identical units of a homogenous commodity (bonds) are sold, and it is a uniform auction because all the winners of the auction receive the same interest rate on the bonds they buy, regardless of their actual bid.

In this type of auction, the BCCh reveals first the volume of bonds it wants to sell to the bidders. Then, each participant submits to the BCCh the minimum rate at which they would accept to buy a given volume of PDBC. In the award procedure, the BCCh ranks the bidders' offers by interest rate, ranking the bids from the smallest to the highest rates and awarding bonds to the smallest until the quantity being auctioned is totally allotted. The results are announced at the close of the auction and all the banks awarded with bonds get the cut off rate (the highest rate at which bonds have been awarded). The BCCh retains the option to award a different amount than scheduled, which in the case of bonds is $+/-$ 20% of the amount auctioned, or to declare unilaterally the auction as deserted because the rates asked by the bank are too high. We have not noted any occurrence of such a decision in our dataset but possibility has been taken into account, in our opinion in order for the BCCh to cancel the auction in the case it suspects a collusion of the banks to obtain a higher rate.

1.2.3 Dataset of Open Market Operations (2002-2012)

Our dataset contains all the details of the open market operations of the BCCh from September 2002 to august 2012. In particular, it consists of all the bidding information, in every bond auctions conducted by the BCCh during that period. The information includes the total volume of bonds allotted by the central bank in each auction, the marginal (or cut-off) rate and more importantly the bidders' identities and the rates asked by each bidder. This dataset is not publicly available and usually only the information on the total volume allotted and marginal rates are available on the BCCh web site.

Year	PDBC30	PDBC90	PDBC180	PDBC360	BCP2	BCP5	BCP10
2005	335	348	0	0	153	221	235
2006	569	560	0	0	57	86	37
2007	427	406	9	13	86	90	12
2008	534	315	133	133	217	213	113
2009	574	550	316	81	137	73	0
2010	751	279	112	127	228	261	0
2011	668	446	96	6	89	216	176
2012	500	175	35	0	123	178	150
Total	4392	3095	701	360	1090	1338	723

Table 1.1: Descriptive statistics: Number of bond auctions per year

Table 1.1 and table 1.2 summarize respectively the descriptive statistics on the number of auctions performed by the BCCh and on the marginal rates for all types of bonds sold from 2005 to 2012.

Year	PDBC30	PDBC90	PDBC180	PDBC360	BCP2	BCP5	BCP10
2005	3.83	4.19	4.04	4.32	5.05	5.95	6.63
	(0.45)	(0.43)	(0.74)	(0.67)	(0.74)	(0.56)	(0.38)
2006	4.93	5.09	5.25	5.43	5.99	6.47	6.85
	(0.26)	(0.16)	(0.11)	(0.12)	(0.23)	(0.33)	(0.39)
2007	5.33	5.41	5.47	5.48	5.77	6.10	6.35
	(0.34)	(0.39)	(0.44)	(0.46)	(0.49)	(0.42)	(0.36)
2008	7.10	7.18	7.20	7.05	7.07	6.96	6.95
	(0.86)	(0.83)	(0.83)	(0.77)	(0.73)	(0.64)	(0.56)
2009	1.96	1.69	1.57	1.86	2.91	4.64	5.35
	(2.22)	(1.89)	(1.56)	(1.19)	(0.76)	(0.56)	(0.58)
2010	1.53	1.76	2.09	2.72	3.72	5.11	5.86
	(1.01)	(1.11)	(1.17)	(1.03)	(0.63)	(0.19)	(0.22)
2011	4.76	4.78	4.82	4.88	5.19	5.56	5.83
	(0.77)	(0.60)	(0.50)	(0.50)	(0.61)	(0.57)	(0.54)
2012	5.00	4.93	4.87	4.80	4.96	5.21	5.45
	(0.07)	(0.07)	(0.13)	(0.23)	(0.31)	(0.28)	(0.23)

Table 1.2: Descriptive statistics: Marginal rates (standard deviations in parentheses)

1.3 Model of a Bond Auction with Liquidity Risk

The idea of the model is to replicate as much as possible the auction performed by the Central Bank of Chile to sell its bonds to the banking sector.

1.3.1 The Basic Setup

There are 3 dates, $t \in \{0, 1, 2\}$. There is an institution[5] wanting to raise an amount of money Q by selling a number Q of 2-period bonds at price one in an auction in $t = 0$. The money is then paid back with interest (determined in the auction) to the bond buyers in $t = 2$.

There are N potential buyers of the bonds (i.e. banks) maximizing profits. At the beginning of $t = 0$, each bank $i \in [1, N]$ gathers one unit of cash at interest rate r_0 normalized to 0. After each bank has gathered its funds, the liquidity risk of their funding is privately revealed to each of them. This risk is represented by p_i the probability for bank i to be subject to an idiosyncratic liquidity shock in $t = 1$. This p_i is a random variable independently drawn by each bank from a common distribution F defined on the interval $[0, 1]$, with positive and continuous pdf f.

The auction in t=0

The auction taking place in $t = 0$ is a uniform multi-unit auction. First, the bond seller announces the number Q of bonds it wants to sell. Second, the banks place their bids. A bid consists of the minimum net interest rate r_i at which the bank is willing to accept to buy a bond. The bank proposing the smallest rate gets a bond, then the bank proposing the second smallest rate get the bond, then the third, etc. until the total volume Q allotted by the bond seller is reached. The cut-off rate r_s is the highest rate at which the supply of bonds Q is exhausted. A uniform auction is defined by the fact that all winning bidders receive the market clearing cut off interest rate r_s in $t = 2$. To say it differently, bank i is awarded 1 bond if it bids $r_i \leqslant r_s$ and 0 otherwise.

Finally, the banks which are not awarded any bonds in the auction invest their cash in a risk free technology yielding an interest rate r_L per period. This risk free investment can be thought as the overnight deposit facility of the central bank where the money can

[5]This institution can be interpreted as a central bank draining liquidity temporarily from the banking sector (as it will be the case in the application in the next section).

be withdrawn by the bank at any time (the rate r_L would be fixed by the central bank) or it can simply be thought as cash with 0 interest rate. Without loss of generality and in order to simplify the model's notation, we just assume $r_L = 0$

The liquidity shock in t=1

In $t = 1$, if the idiosyncratic liquidity shock does not materialize for bank i, the bank's creditors do not withdraw the cash from the bank in the intermediate period and wait until $t = 2$ to take it back. If the idiosyncratic liquidity shock materializes, the bank i has to give back the cash to its creditors immediately in $t = 1$. However, in this case, if the bank has invested in a bond, it has no cash left to finance the withdrawal.

If there is no way to raise some cash in $t = 1$ we assume that the bank goes bankrupt and makes a loss equivalent to the money it can not reimburse to its creditors. In a second step, we also explore what changes if there exists a lender of last resort policy where the bank can borrow from the central bank lending facility at a fixed high rate r_H if it becomes illiquid in $t = 1$.

Collection of profits in t=2

For the banks which have not suffered the liquidity shock in the intermediate period, in $t = 2$, the banks collect their profits r_s from the maturing bonds if they have some and reimburse their creditors.

1.3.2 Bidding Strategy of the Banks in the Bond Auction

Let's say that the seller wants to sell $Q = k + 1$ bonds in the auction. It means that among the N potential buyers, $k + 1$ bidders are served: one is the cut-off bidder who has proposed r_s in the auction and k bidders have proposed rates smaller than r_s. We assume that $N \geq 2$ and that $k \in [0, N - 2]$ so that there is always at least one bidder that gets nothing in the auction, otherwise all banks would have an incentive to bid a rate equal to infinity which would make the problem trivial and uninteresting.

The strategies available to the players are the rates they bid. Since we focus on symmetric equilibria, we suppose that all banks use the same bidding strategy β which is a function of their probability to get an idiosyncratic shock in $t = 1$. Let's determine this function $\beta(p_i)$.

The maximization problem of representative bank i, bidding r_i when other banks bid according to the symmetric increasing function $\beta(p_i)$, can be written as:

$$\begin{aligned}\max_{r_i} \quad & Pr(r_i < \beta(Y_k)).[(1-p_i)E(\beta(Y_k)/\beta(Y_k) > r_i) - p_i] \\ & +Pr(\beta(Y_k) < r_i < \beta(Y_{k+1})).[(1-p_i)r_i - p_i] \\ & +Pr(r_1 > \beta(Y_{k+1})).0\end{aligned} \tag{1.1}$$

where Y_k is the k^{th} order statistic attached to the random variables p_j sorted into ascending order (i.e the k^{th} highest among $N-1$ random variables), for $j \in [0, N]$ and $j \neq i$ (because for bank i, p_i is perfectly known and not random at that point).

The objective function maximized in equation (1.1) is easy to understand. The first line represents the expected profits if the bid of bank i is smaller than the cut off rate (i.e. the k^{th} highest rate proposed, bank i excluded) and is therefore awarded a bond in the auction: in this case, either the shock does not materialize (with probability $1-p_i$) and the bank makes a profit equal to the expected cut off rate given that this rate is higher than its bid, or the shock materializes (with probability p_i) and it makes a loss equal to 1. Similarly, the second line represents the expected profits if the bid of bank i is the cut off rate: in this case, either the shock does not materialize and the bank makes a profit equal to its bid, or the shock does materializes and it makes a loss equal to 1 again. The third line is the expected profits if bank i is not awarded any bond in the auction. It is equal to 0 as the bank does not make any investment and is indifferent between giving back the money to its creditors in $t = 1$ or 2.

Since the bidding function β is continuous and increasing in p_i, the maximization problem can be rewritten as:

$$\begin{aligned}\max_{r_i}(1 - Pr(Y_k < \beta^{-1}(r_i))).[(1-p_i)E(\beta(Y_k)/Y_k > \beta^{-1}(r_i)) - p_i] \\ + Pr(Y_k < \beta^{-1}(r_i)) < Y_{k+1}).[(1-p_i)r_i - p_i]\end{aligned}$$

where all the probabilities and conditional expectations are easily computable. Taking first order condition with respect to r_i and imposing symmetry, such that $r_i = \beta(p_i)$, we obtain after some manipulations the following differential equation:

$$\beta'(p_i) + \beta(p_i)\frac{(N-k-1)f(p_i)}{1-F(p_i)} = \frac{(N-k-1)f(p_i)}{1-F(p_i)}\frac{p_i}{1-p_i} \tag{1.2}$$

Equation (1.2) is among the few differential equations that have closed-form solutions. Following the method proposed by Boyce and DiPrima (1977), we can obtain the following equilibrium-bidding function:

$$\beta(p_i) = \frac{-1}{(F(p_i)-1)^{N-k-1}} \int_{p_i}^{1} (F(p)-1)^{N-k-1} \frac{p}{1-p} f(p) \frac{N-k-1}{1-F(p)} dp$$

which is indeed increasing in p_i and such that $\lim_{p_i \to 1} \beta(p_i) = +\infty$.

With p_i distributed uniformly on $[0,1]$ the strategy is depicted in figure 1.2.

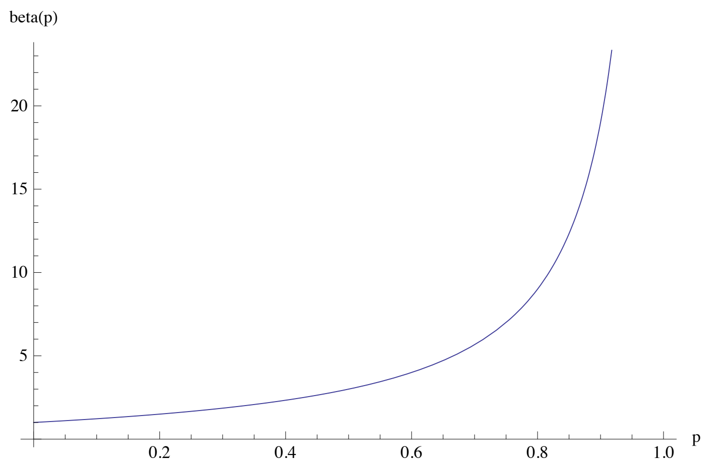

Figure 1.2: Bidding strategy with a uniform distribution, $N = 8$ and $Q = 6$

The intuition behind this result is quite simple. Because there is no other way for the banks to insure themselves against the shock when they hold a bond, the only way for them to compensate this risk is by asking a higher rate on those bonds. The fact that $\lim_{p_i \to 1} \beta(p_i) = +\infty$ is understandable as the bank with $p_i = 1$ is sure to have the shock in $t = 1$ so it does not really want to buy the bond because it is sure to make a loss if it holds it.

An interesting policy implication of the model for the moment is that we have shown that an increase of the liquidity risk in the financial sector (for instance with a shift of its distribution towards a more riskier one) can have a huge impact on the rates asked by banks in a bond auction even if there isn't any change in the fundamentals of those bonds or any solvency problem whatsoever.

1.3.3 Introduction of a Lender of Last Resort in the Framework

Since the concept was introduced and developed by Thornton (1802) and Bagehot (1873) in the 19th century, most central banks have been acting as lenders of last resort in order to save illiquid but solvent banks from bankruptcy by allowing them to borrow from the discount window whenever it seems necessary for the stability of the financial sector. Given the importance taken by this central bank function, we thought it could be interesting to introduce it in our model in order to see its impact on the strategies of the banks in the auction.

In our model, a lender of last resort policy can be represented by the fact that a bank holding a bond subject to a liquidity shock in $t = 1$ can now borrow from the discount window of the central bank to reimburse its creditors and avoid bankruptcy. In this case, in $t = 2$, the bank collects its profits from the bond but it also has to repay its loan with an interest rate r_H fixed by the central bank.

Therefore, the maximization problem of bank i (bidding r_i^L when other banks bid according to the symmetric increasing function $\beta_L(p_i)$)can be rewritten as:

$$\begin{aligned}\max_{r_i^L} \quad & Pr(r_i^L < \beta_L(Y_k)).[(1-p_i)E(\beta_L(Y_k)/\beta_L(Y_k) > r_i^L) + p_i(E(\beta_L(Y_k)/\beta_L(Y_k) > r_i^L) - r_H)] \\ & +Pr(\beta_L(Y_k) < r_i < \beta_L(Y_{k+1})).[(1-p_i)r_i^L + p_i(r_i^L - r_H)] \\ & +Pr(r_i^L > \beta(Y_{k+1})).0\end{aligned} \tag{1.3}$$

where again Y_k is the k^{th} order statistic attached to the random variables p_j sorted into ascending order for $j \in [0, N-1]$ and $j \neq i$.

The main difference between objective functions (1.1) and (1.3) lays in in the fact that, if bank i suffers a liquidity shock, it can avoid bankruptcy by borrowing from the central bank which modifies its expected profits as it can keep the bond until maturation. As before, with a bidding function β increasing in p_i, this becomes:

$$\begin{aligned}\max_{r_i}&(1 - Pr(Y_k < \beta_L^{-1}(r_i^L))).[E(\beta_L(Y_k)/\beta_L(Y_k) > r_i^L) - p_i r_H] \\ &+ Pr(Y_k < \beta_L^{-1}(r_i^L)) < Y_{k+1}).[r_i^L - p_i r_H]\end{aligned}$$

where all the probabilities and conditional expectations are easily computable. Taking first order condition and imposing symmetry, such that $r_i^L = \beta_L(p_i)$, we obtain after some manipulations the following differential equation:

$$\beta_L'(p_i) + \beta_L(p_i)\frac{(N-k-1)f(p_i)}{F(p_i)-1} = p_i r_H \frac{(N-k-1)f(p_i)}{F(p_i)-1}$$

Using the same method as before to solve the differential equation, we obtain the following equilibrium-bid function:

$$\beta_L(p_i) = p_i r_H + \frac{r_H}{(1-F(p_i))^{N-k-1}} \int_{p_i}^{1} (1-F(p))^{N-k-1} dp$$

which is increasing in p_i and such that $\lim_{p_i \to 1} \beta_L(p_i) = r_H$.

With p_i distributed uniformly the strategy is depicted in figure 1.3.

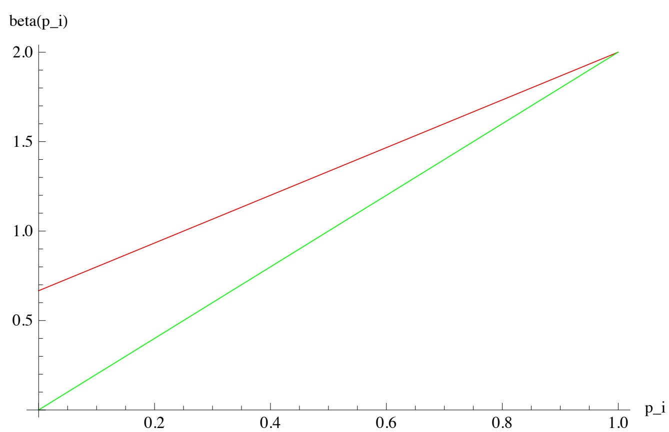

Figure 1.3: Bidding strategy (in red), $p_i.r_H$(in green), $N = 8$ and $Q = 6$, $r_H = 2$

As it can be seen in figure 1.3, unlike the previous case, the rates are now bounded by the rate r_H charged by the central bank for its loan in $t = 1$. This makes sense because even if it is costly to use the lending facility, the lender of last resort provides an ex post insurance to the banks buying bonds. For instance, the bank with probability one to get the shock is now indifferent between getting the bond in the auction at the rate r_H and not getting the bond because in any case its profit is going to be 0.

In fact, this is an interesting side result in terms of policy implication, as it shows that the introduction of a lender of last resort willing to lend cash to banks suffering a liquidity shock can have a huge impact on the strategies of the bank and on the result of a bond auction by bounding the rate with its lending rate.

However, although the LoLR extension of the model is theoretically interesting, it does not fit the data discussed in the next section, since rates bid during the crisis exceeded the lending facility rate (the FPL rate or r_H in the model) asked that the Central Bank of Chile would use when playing its role as LoLR. It is therefore not used in the following empirical part. One possible explanation is the fact that in reality banks are not completely sure whether the central bank will intervene as a LoLR and lend them money in case of illiquidity. A possible extension left for future research would be to introduce a non zero probability of the central bank not acting as a LoLR in the model. This probability perceived by the banks could also vary through time and could be interesting to estimate.

1.4 Structural Analysis of the Bond Auctions of the BCCh

The objective of this section is to use the theoretical bidding strategies of the banks from the previous section in order to put some structure on the dataset from BCCh bond auctions by estimating the parameters of the distribution of the liquidity shock probabilities across the Chilean Banks and its changes over time.

1.4.1 Choice of the Distribution Form

In order to be able to perform this estimation, we need to assume that the distribution of the liquidity shock probabilities across banks takes a particular distribution form. We have chosen the Kumaraswamy distribution (introduced by the author of the same name in 1980) for various reasons. First, as explained in Jones (2009), it is defined on $[0, 1]$ which perfectly suits our needs because we are interested in the distribution of a probability. Second, it is particularly straightforward with only two parameters a and b and has a very simple PDF: $f(x; a, b) = abx^{a-1}(1 - x^a)^{b-1}$ and CDF: $F(x; a, b) = 1 - (1 - x^a)^{b-1}$. Nonetheless, this density function is very flexible and can take various type of shapes (unimodal, uniantimodal, increasing, decreasing, monotone or even constant depending on the values of the parameters[6]) as it can be seen in figure 1.4. That is why we believe that this distribution form does not impose too much restriction on the data.

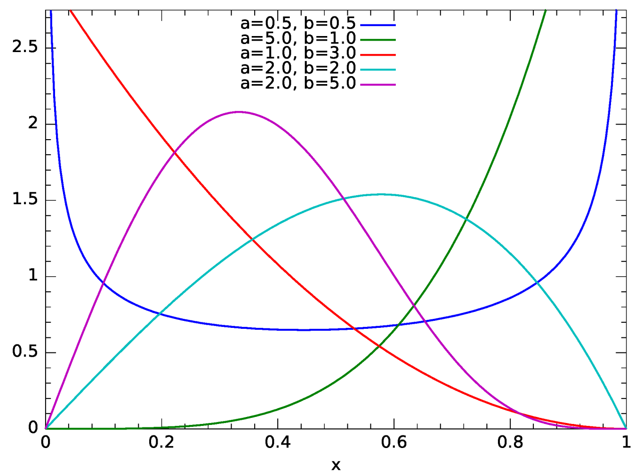

Figure 1.4: The Kumaraswamy probability density function for different parameters

1.4.2 Data Selection

In terms of data, in order to estimate the liquidity risk distribution, we decided to restrict ourselves to the observed bids from the 30-day bond (PDBC) auctions from 2002 to 2012 and to put aside the other auctions of the BCCh. The main reason behind this choice is simply the shorter maturity of those bonds. We believe that it makes those bonds a better substitute for cash or reserves at the central bank and that the main difference would come from the expectation of a liquidity shock in the following month which would suit perfectly our model. Another reason to use this auction is that we believe that these short term bonds are more likely to be kept in the balance sheet of the banks (as it is assumed in our model) until they mature than the long term bonds and not resold in a secondary market. Finally, given that they are really short-term, we can also assume that there is no inflation premium asked as it would be the case with longer maturity bonds.

[6]In that sense the Kumaraswamy distribution is quite similar to the Beta distribution but with a simple explicit formula for its distribution function not involving any special functions

More practical reasons include the fact that the 30-day PDBC auctions are the most frequent open market operations of the BCCh as it can bee seen in table 1.1 (with 4392 auctions performed between 2005 and 2012) and also the auctions with the largest number of participants (for more details, see the summary statistics of the PDBC auctions from 2002 to 2012 in table 1.3).

Numbers of periods-months	118
Average Number of Bidders / period	48
Average Number of Winning Bidders / period	29
Average Rate Bid	3.62
Average Cut Off Rate	3.61
Average Standard deviation of bids / period	0.25

Table 1.3: Summary Statistics for 30-day PDBC Auctions

We have also decided to group auctions of the same month together to get enough data per period of estimation. We know that this is a strong assumption because it means that the liquidity risk distribution varies every month but is a constant in a given month. However, within the model framework it could simply be interpreted as banks drawing a new liquidity risk from the new distribution at the beginning of each month.

Finally, instead of using the rate bids alone as it was done in the model, we prefer to use in this empirical section the difference between the rate bids and the deposit rate at the central bank at the time of the auction (remember that for the sake of simplicity we assumed $r_L = 0$ in the model). Indeed, we think that the liquidity premium should be observed between those two variables and not between the rate bids and simply cash earning 0% interest rate, because this spread takes into account the variations in the monetary policy rate (the deposit facility follows exactly the monetary policy rate with only minus 0.25 percentage point) inducing some volatility in the bids not related to the evolution of the liquidity risk.

1.4.3 Estimation of the Liquidity Risk Distribution

In this section we perform a structural estimation of the parameters of the distribution for each period (each month) of our sample using our theoretical results from the previous section. We use the maximum likelihood method to estimate the parameter vector $\theta = (a, b)$ according to equation 1.4.

$$\max_{a,b} \prod_{i=1}^{N} Pr(p_i = \beta^{-1}(b_i, a, b)) \tag{1.4}$$

Using the data previously described, we can estimate the parameters of the distribution for each period of our sample. Figure 1.5 presents the estimated density functions at various interesting points in time during the studied decade.

As can be seen in Figure 1.5, at the beginning of 2004 the density function has a decreasing form and is definitely skewed to the right before becoming more and more symmetric in 2005. Then, making a big jump in time, we can see that during the worst months of the financial turmoil in september and october 2008 after the bankruptcy of Lehman Brothers, the density function definitely moved to the right revealing what appears to be a huge increase in liquidity risk in the Chilean financial sector.

In response to this risk and to the deterioration of international financial markets, the BCCh decided to increase the quantity of money in the domestic financial system at the end of 2008 and 2009. They used part of their international reserves and some swap agreements with the Fed to lend some U.S. dollars to the Chilean commercial banks that needed some. Aside from that, during six month, the BCCh put in place regular repo operations aimed at injecting pesos every week. To encourage the participation in those repo operations they expanded the eligible collaterals. At the beginning of 2009, the BCCh modified its debt schedule by suspending the issue of five-year peso bonds (BCP5), five-year and ten-year UF bonds (BCU5 and BCU10) in the primary market in order to reduce the usual money draining resulting from these open market operations[7]. All these measures seem to have worked well and resulted in an enormous decrease in the liquidity risk of the financial sector, as can be seen in the density function for May 2009. All in all, it seems that the evolution of the estimated distribution capture well the main episodes of liquidity stress of the last decade in the Chilean banking sector.

[7]This gives us another reason not to use the auctions for those bonds to do our estimations: these changes decided by the BCCh made those auctions unusable to monitor the liquidity risk when they became an instrument of the central bank to reduce this liquidity risk.

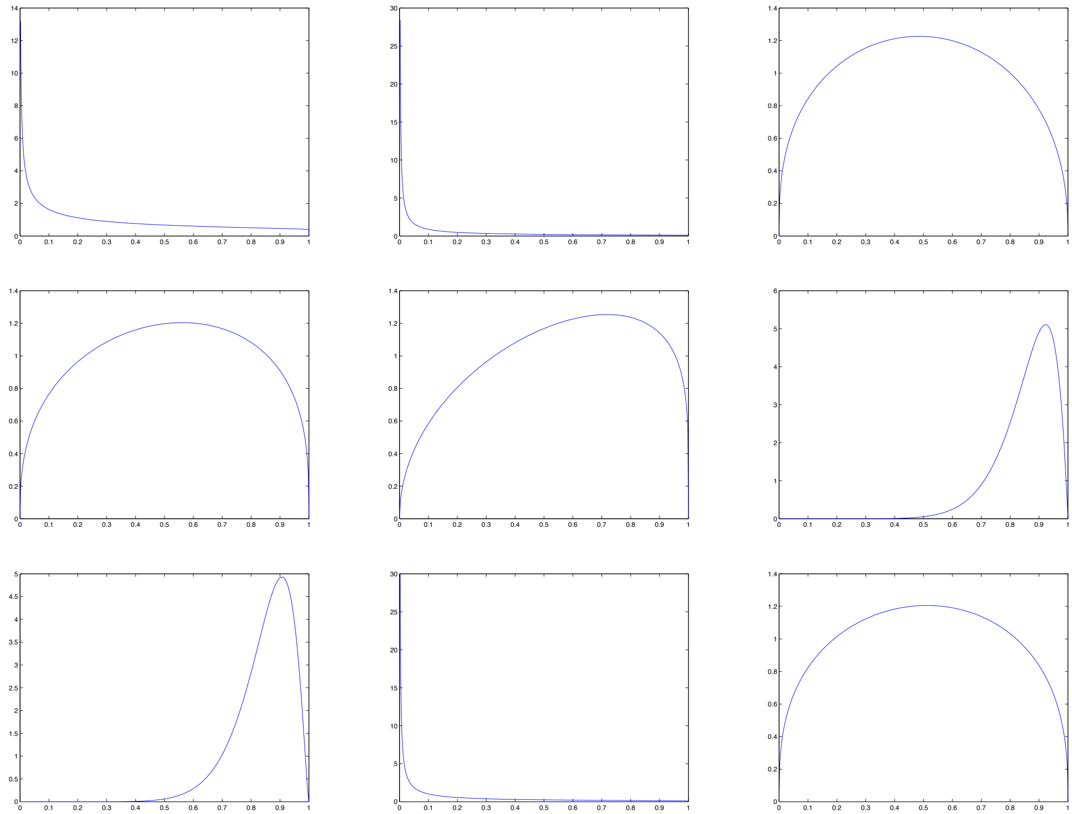

Figure 1.5: From left to right and top to bottom, estimated PDF $f(p_i)$ for January, May and August 2004, January and November 2005, September and October 2008, May 2009, August 2010.

Besides, the estimation of the parameters of the PDF of the liquidity risk across banks for each period allows us to infer the probability to be hit by a liquidity shock during the duration of the bond for all the banks participating in the auction just by inverting their bids thanks to our theoretical bidding function obtained in the previous section. So, not only the method proposed in this paper provides to the central bank a way to monitor the liquidity risk of its financial sector as a whole but also to monitor each of the Chilean commercial banks participating to its bond auctions[8].

[8]For confidentiality reasons concerning the Chilean banks, it is impossible to show individuals results for any particular banks in the present paper. Given the small number of banks and the structure of the banking sector in Chile, it would also be difficult to show them with hidden identities without making them easily recognizable.

1.5 Conclusions

In the end, the paper proposes a ready-to-use tool for all central banks conducting similar open market operations to the BCCh and in need of evaluating in real time the evolution of the liquidity risk affecting their financial sector in general or their commercial banks individually. To do that, we modeled the bond auction conducted by the BCCh and used the theoretical results obtained to estimate the parameters of the distribution of liquidity risk in the Chilean banking sector for each month in the period going from 2002 to 2012.

In addition, this paper also contributes to the financial literature and to the auction theory by proposing a simple model of bidding behavior in a multi-unit auction where bidders are affected by liquidity risk. The model carefully explores the link between liquidity risk in the financial sector and bond rates. Although the model is designed in a very simple way because it was constructed mainly to be used in the structural estimation, it also has some interesting policy implications. The comparison of our two different settings is particularly enlightening as it shows that the introduction of a central bank acting as a lender of last resort willing to lend cash to banks suffering a liquidity shock can have a huge impact on the bidding strategies of the banks by caping their rate with the lending rate it proposes at the discount window.

Finally, we think that it could be interesting to extend the analysis of the model presented in this paper and to develop it further. A possible extension could be for instance to replace the lender of last resort in the second period by a careful modelization of a secondary market in which the banks hit by the liquidity shock would be able to sell the bond to the banks that were not awarded bonds in the auction of the initial period. The type of insurance provided by this secondary market would depend on the market liquidity and therefore on which banks have been awarded bonds in the auction and which banks will get the shock. Given that there is a finite number of bidders, this feature would result in aggregate uncertainty in the model. This could lead to a result analog to the cash-in-the-market concept developed in Allen and Gale (1994) where financial institutions must sell assets to obtain liquidity, and because the supply and demand of liquidity are inelastic in the short-run, a small degree of aggregate uncertainty could cause large fluctuations in asset prices. In this case, the rates asked by investors in the auction may be distorted by their own liquidity risk but also by the aggregate uncertainty coming from the secondary market. In any case, these features will have for sure some interesting implications on the bidding strategies of the banks. We leave this possible extension for future research.

Appendix

In this appendix we prove that the increasing symmetric bidding function in the LoLR case is a Nash Equilibrium. The proof for the no insurance case is analogous. In order for the bidding function obtained (necessary condition) to be a Nash Equilibrium, the second order (sufficient) condition needs to be satisfied. The maximization problem of bank i is:

$$\begin{aligned} max_{r_i}\Pi = \quad & \textstyle\int_{\beta^{-1}(r_i)}^{1}\beta(p)\frac{(N-1)!}{(k-1)!(N-1-k)!}f(p)F(p)^{k-1}(1-F(p)^{N-1-k})dp \\ & -[1-\textstyle\sum_{j=k}^{N-1}\binom{N-1}{j}(F(\beta^{-1}(r_i)))^{j}(1-F(\beta^{-1}(r_i)))^{N-1-j}](p_i r_h) \\ & +\tfrac{(N-1)!}{(N-1-k)!k!}F(\beta^{-1}(r_i))^{k}(1-F(\beta^{-1}(r_i)))^{N-1-k}(r_i-p_i r_h) \end{aligned}$$

The derivative of the profit with respect to r_i is given by:

$$\begin{aligned} \Pi_{r_i} = \quad & (r_i-p_i r_h)\tfrac{f(\beta^{-1}(r_i))}{\beta'(\beta^{-1}(r_i))}\tfrac{(N-1)!}{(N-2-k)!k!}F(\beta^{-1}(r_i))^{k}(1-F(\beta^{-1}(r_i)))^{N-2-k} \\ & +\tfrac{(N-1)!}{(N-1-k)!k!}F(\beta^{-1}(r_i))^{k}(1-F(\beta^{-1}(r_i)))^{N-1-k} \end{aligned} \tag{1.5}$$

At the symmetric solution $p_i = \beta^{-1}(r_i)$. Then when maximizing profits the first order condition when imposing symmetry yields:

$$\begin{aligned} \frac{(N-1)!}{(N-1-k)!k!}F(\beta^{-1}(r_i))^{k}(1-F(\beta^{-1}(r_i)))^{N-1-k} = \\ (\beta^{-1}(r_i)r_h-r_i)\frac{f(\beta^{-1}(r_i))}{\beta'(\beta^{-1}(r_i))}\frac{(N-1)!}{(N-2-k)!k!}F(\beta^{-1}(r_i))^{k}(1-F(\beta^{-1}(r_i)))^{N-2-k} \end{aligned} \tag{1.6}$$

By substituting (1.6)into (1.5) we get:

$$\Pi_{r_i} = \frac{f(\beta^{-1}(r_i))}{\beta'(\beta^{-1}(r_i))}\frac{(N-1)!}{(N-2-k)!k!}F(\beta^{-1}(r_i))^{k}(1-F(\beta^{-1}(r_i)))^{N-2-k}(\beta^{-1}(r_i)r_h-p_i r_h)$$

Suppose all other banks are bidding according to the symmetric bidding function $\beta(p)$. Also suppose that bank i decides not to play according to the symmetric equilibrium, but instead chose some $r_i \neq \beta(p_i)$ such that $r_i = \beta(q)$ with $q \neq p_i$. Then for any q' such that $q' > p_i$ we have $\Pi_{r_i} > 0$, and accordingly for any q'' such that $q'' < p_i$ we have $\Pi_{r_i} < 0$. We have $\Pi_{r_i} = 0$ only for $q = p_i$ i.e. when bank i bids according to the symmetric bidding function. Thus, the symmetric bidding function $\beta(p_i)$ is indeed an equilibrium.

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Chapter 2

Risk-sharing through Financial Intermediaries and Financial Markets: an Intergenerational Perspective

2.1 Introduction

This paper[1] contributes to the long-standing theoretical debate on the relative merits of financial intermediaries and financial markets in promoting investment and providing insurance against some risks. More precisely, its main purpose is to examine risk-sharing obtained through contractual coalitions of agents and asset trading in overlapping generation (OLG) economies prone to non-verifiable idiosyncratic liquidity shocks.

In an economy where (1) agents are facing stochastic consumption needs in terms of timing, (2) investment is taking a long time to mature (i.e. there is some lag in the production technology) and is illiquid in the meantime, and (3) insurance markets are incomplete because agents' claims concerning their time preferences are not verifiable, institutional arrangements such as financial intermediaries and markets improve welfare of

[1] I am grateful to Elena Carletti, Russell Cooper, the participants of the "Cooper & Students" Discussion Group at EUI and the participants of the CREI International Workshop at Pompeu Fabra (in particular Jaume Ventura and Alberto Martin) for their useful comments and suggestions. All remaining errors are mine.

agents by providing them liquidity.

Starting with Diamond and Dybvig (1983), numerous 3-period models have been studying the role of financial intermediaries and financial markets in providing generational insurance to agents. However, adding an overlapping generation (OLG) structure to this economy allows for the possibility to share liquidity risk across generations. Despite its apparent importance (in retirement systems, pension funds and in the financial sector in general), intergenerational insurance has received little attention in this literature. The combination of the Diamond and Dybvig framework and of overlapping generation structure provides us with a unified framework to study all possible configurations of the financial sector providing both generational and intergenerational risk-sharing and to compare the welfare properties of those configurations. We set out the basic framework of the so called Diamond and Dybvig overlapping generation model in section 2.2.

The previous articles adopting this particular theoretical setup – Qi (1994), Fulghieri and Rovelli (1998), Bhattacharya and Padilla (1996) – suggest that in steady state, and omitting consideration of the very first generation problem, financial markets achieve allocations that are inferior to infinite intergenerational financial intermediaries, except if a government is able to implement a transfer scheme with the same informational requirements as those imposed on financial intermediaries. The main results of those papers are carefully summarized in section 2.3. However, Penalva and Van Bommel (2009) suggest that if those infinite financial intermediaries are governed by their living depositors, their risk-sharing capacity is severely limited by the temptation of those depositors to liquidate the institution's assets.

Given this relevant criticism of infinite financial intermediaries, one objective of this paper is try to find alternative ways to decentralize the optimal allocation, in particular with generational financial intermediaries. This is what it is done in section 2.4. Our paper sheds light on the fact that generational financial intermediaries, that are not subject to Penalva and van Bommel's governance critique, can achieve (whether they are combined with a stock market or not) the first best steady state allocation as long as a government can implement transfers between financial intermediaries.

In addition to this problem, the literature adopting this particular framework, except for Qi (1994), has been silent on the transitional dynamics of this type of economy and the analysis is confined to steady states in which inheritance of a stationary level of investment is taken as given. We believe this is a really restrictive assumption, especially concerning the first generation that only has its endowment to start the intergenerational risk-sharing mechanism – whether the mechanism chosen is a financial intermediary or a financial market. That is why in section 2.5 we analyze the dynamics of this model and show that there exists a feasible finite path leading to the steady state optimal allocation that takes the outside options of agents into account.

2.2 The Setup of the Diamond & Dybvig OLG model

The basic setup is similar to the setup of the classic Diamond and Dybvig (1983) model except that there are infinitely many overlapping generations of agents. Specifically, consider a sequential economy with a single consumption good. At each time $t \in \mathbb{Z}$, the set of integers, a new generation of agents is born. Each generation is identical and its size is constant over time, with a continuum of measure 1 of agents that live three periods.

Agents are ex ante identical but individually observe a private preference type during the second period of their life. The preferences of the agents from generation t are represented by $u(C_{t+1,E})$ if he is an early consumer (i.e. he must absolutely consume in $t+1$), which happens ex ante with probability λ, and by $u(C_{t+2,L})$ if he is a late consumer (i.e. he must absolutely consume in $t+2$), which happens ex ante with probability $(1-\lambda)$; where $\lambda \in (0,1)$, where $C_{t+1,E}$ denotes generation t early type agent's consumption at time $t+1$ and $C_{t+2,L}$ denotes generation t late type agent's consumption at time $t+2$, and where u is an increasing, strictly concave, and twice-continuously differentiable von Neumann-Morgenstern utility function satisfying Inada conditions: $u'(0) = \infty$ and $u'(\infty) = 0$. Ex ante the expected utility of a representative agent of generation t is hence represented by:

$$\lambda u(C_{t+1,E}) + (1-\lambda)u(C_{t+2,L}) \tag{2.1}$$

An agent's type is only revealed to him privately in the intermediate period of life. There exists no technology to verify the type of an agent. As a consequence, Arrow-Debreu contracts are not available in this economy because if this type of contracts were offered, agents would have an incentive to make non-verifiable claims of being of a particular type in order to receive payments.

Each agent is endowed with y units of the consumption good at the beginning of his life and none after. Two infinitely divisible investment technologies are publicly available. The first one is a one-period storage technology whose per-unit payoff is 1 after one period. The second is a two-period production (or long-term) technology whose per-unit payoff is

$R > 1$ after two periods. If the long-term technology is interrupted after one period, its per-unit payoff is only $Q \in [0, 1)$, i.e. the long-term technology is partially illiquid.

2.3 Main Results from the Literature

In this section, we carefully analyze the most interesting institutional settings of the literature that has adopted the Diamond and Dybvig overlapping generation setup. This will allow us to understand fully some results that will be used in the following sections but also the main critiques addressed to them.[2] As we will see the authors of these papers focus mainly on steady state allocations and omit consideration of the very first generation.

2.3.1 Individual Autarky

As a benchmark let's consider the case in which new-borns invest individually their endowment in the two technologies. Let K and S denote the respective investments in the long-term technology and in the storage technology, the problem of an agent that is in autarky (i.e the agent is not able to form any type of coalition with other agents or to trade with them in any way) is therefore:

$$\max_{K,S,C_E,C_L} \lambda u(C_E) + (1-\lambda)u(C_L) \tag{2.2}$$

such that:

$$y = K + S \tag{2.3}$$

$$C_E = S + QK \tag{2.4}$$

$$C_L = S + RK \tag{2.5}$$

Equation (2.3) is the budget constraint of new-borns as they invest their endowment in storage and in the long-term technology. Equation (2.4) is the feasibility constraint of agents who turn out to be early consumers: they consume what they have stored and liquidate the long term investment after one period. If they are late consumers, they consume what they have stored and the return of the long term technology, as it is stated in equation (2.5) after two periods.

[2] We also homogenize here the assumptions and the notations across the various environments in order to adopt precisely those presented in section 2.2.

The chosen allocation is such that $\lambda(1-Q)u'(y-K+QK) = (1-\lambda)(R-1)u'(y-K+RK)$. Agents can not insure themselves against the idiosyncratic liquidity shock: inefficient liquidation takes place in the middle period if agents turn out to be early consumers and inefficient storage is used if they are late consumers.

2.3.2 Social Planner: the Optimal Allocation

Let's now look for the optimal allocation. The social planner maximizes the expected utility of a representative agent given an aggregate resource constraint:

$$\max_{K,S,C_E,C_L,L,S} \lambda u(C_E) + (1-\lambda)u(C_L) \tag{2.6}$$

such that:

$$\lambda C_E + (1-\lambda)C_L + K + S = y + R(K-L) + QL + S \tag{2.7}$$

$$C_E, C_L, K, S, L \geq 0, L \leq K, K \leq y \tag{2.8}$$

As can be seen in equation (2.7), at each date the social planner gives consumption to early type middle-aged agents λC_E and to late type old agents $(1-\lambda)C_L$, invests in the long term technology K and possibly in storage S. To do that, it has the following resources: the endowment of the new generation y, the return of what was invested in the long term technology two periods ago minus what was possibly liquidated one period ago $K-L$, what was stored last period S, and finally the proceeds of the liquidation of some part of what was invested in the long-term technology one-period ago QL.

The optimal allocation is therefore: $K^{SP} = y$, $L^{SP} = S^{SP} = 0$ and $C_E^{SP} = C_L^{SP} = Ry$. In steady state, the Social Planner invests the whole endowment of the new-borns in the long-term technology and uses the return of the long-term technology to give consumption equally to early and late consumers. Liquidation and storage are not necessary. Thus, full investment of the endowment in the productive technology and perfect insurance from the liquidity shock are possible thanks to intergenerational insurance. For a detailed proof, see Fulghieri and Rovelli (1998).

2.3.3 Generational Financial Intermediary

If agents are able to form a coalition only with the other agents from the same generation, this case is equivalent to the classical Diamond and Dybvig (1983) case and the problem of this coalition is defined by:

$$\max_{K,S,L,C_E,C_L} \lambda u(C_E) + (1-\lambda)u(C_L) \tag{2.9}$$

such that:

$$y = K + S \tag{2.10}$$

$$\lambda C_E = S + QL \tag{2.11}$$

$$(1-\lambda)C_L = R(K-L) \tag{2.12}$$

Equation (2.10) is the budget constraint for the newly-created intermediary: it uses the endowment to invest in the long-term technology and in storage. Equation (2.11) states that it uses storage and possibly liquidation to give some consumption to its early type depositors. In its last period, it uses the return on the long-term technology (minus what has possibly been liquidated) to give consumption to its late type depositors, as shown in equation (2.12).

The chosen allocation is such that: $y < C_E^{DD} < C_L^{DD} < Ry$. More precisely, if the utility function is of the CRRA form with parameter $\theta > 1$, the consumption stream is given by $C_E^{DD} = \frac{y}{\lambda+(1-\lambda)R^{\frac{1-\theta}{\theta}}}$ and $C_L^{DD} = \frac{R^{\frac{1}{\theta}}y}{\lambda+(1-\lambda)R^{\frac{1-\theta}{\theta}}}$. Forming a generational financial intermediary allows agents to smooth consumption because agents of the same generation insure each others through the coalition (thanks to the law of large numbers) thus avoiding premature liquidation of the long term investment. This constitutes an improvement over the autarkic allocation as long as agents are risk averse. However, it can not provide as much insurance as the Social Planner because the planner uses insurance between generations. Because $C_E < C_L$, this allocation is incentive compatible in the sense that late consumers will not have an incentive to withdraw early. Note however that this allocation is not immune to side trades as pointed out by Jacklin (1987)[3], and also that this contract

[3] Jacklin's critique is basically the following: if agents can perform side trades, they have an incentive to stay outside of the financial intermediary and to self-invest in the long-term asset. If they turn out to be late consumers they get the full Ry and if they are early consumers they can trade their claim on the long-term investment with a late consumer participating to the financial intermediary that will claim to be an early type in order to withdraw after one period, and both agents will benefit from this side trade.

can only be offered to new-born agents otherwise it would not be incentive compatible anymore as late types could withdraw early (i.e. after one period) in order to reinvest in the newly created generational financial intermediary.

2.3.4 Intergenerational Financial Intermediary

The problem of an infinite financial intermediary is quite similar to the one of the social planner. It maximizes the expected utility of the representative depositor – equation (2.6) – and it is constrained by the same aggregate resource constraint, given by equation (2.7). The only difference is that the contract offered to the depositors has to be incentive compatible in order to avoid that late consumers have an incentive to withdraw when they are middle-age and store the good until they are old.

If the generation to which agents belong is publicly known or if there exists a way to implement a no-redepositing rule so that the demand deposit contract is only offered to new-borns, an infinite intergenerational financial intermediary can implement the socially optimal allocation: $K^{IB} = y$, $L^{IB} = S^{IB} = 0$ and $C_E^{IB} = C_L^{IB} = Ry$. Note that this allocation is incentive compatible (as $C_E^{IB} \leq C_L^{IB}$) and, contrary to the previous one, is immune to Jacklin's critique, as there is no incentive to stay outside of the intermediary because it allows all its depositors to obtain the optimal consumption. For detailed proofs, see Qi (1994) and Fulghieri and Rovelli (1998).

2.3.5 Anonymous Intergenerational Financial Intermediary

However, if generations are not publicly known and agents are fully anonymous, the optimization problem of the infinite financial intermediary is subject to another incentive compatibility constraint in order to avoid withdrawal and redeposit by late consumers. It is such that:

$$C_E = r.y \ \ and \ \ C_L = r^2.y \tag{2.13}$$

where r is defined as the one-period return offered by the intermediary. In this case the optimal allocation is given by $K^{AIB} = y$, $L^{AIB} = S^{AIB} = 0$, $C_E^{AIB} = \frac{\sqrt{\lambda+4(1-\lambda)R}-\lambda}{2(1-\lambda)}y$ and $C_L^{AIB} = \left(\frac{\sqrt{\lambda+4(1-\lambda)R}-\lambda}{2(1-\lambda)}\right)^2 y$ so $C_E^{AIB} < Ry < C_L^{AIB}$. Note that, even if the investment in the long term technology is optimal, the expected utility is less than the one offered in

However, in that case, no agent will have an incentive to enter the coalition which therefore will not be created in the first place.

the the previous case because the anonymous infinite financial intermediary can not offer as much as insurance as before because of the new incentive compatibility constraint.

It is also interesting to note that, as it has been suggested by Bhattacharya and Padilla (1996), this allocation can not be obtained when an interbank market is open because it would invite competing coalitions to invest in each other instead of in the production technology (because $Ry < C_L^{AIB}$), so that, eventually, the whole system of intermediation would collapse. This critique parallels the one made by Jacklin (1987) in a 3-period environment. To avoid this, the interbank market would have to be ruled out. For the detailed proofs of those results, see Qi (1994), Fulghieri and Rovelli (1998) and Bhattacharya and Padilla (1996).

2.3.6 Intergenerational Stock Market

Let's now investigate what happen when liquidity insurance is not provided by financial intermediaries but through what can be called a stock market where agents can individually buy and sell shares of the long-term technology when the production in its intermediate stage (i.e. after one period). Let K_t^j, I_t^j and S_t^t denote the respective investments in the long term technology, in shares of the intermediate production and in the storage technology, where the subscript t denotes the period in which the investment is made and the superscript $j = \{n, i\}$ denotes the age of the agent: n if he is a new-born and i if he is its the intermediate period. The generation t agent's problem is therefore:

$$\max_{K_t^n, I_t^n, S_t^n, K_{t+1}^i, I_{t+1}^i, S_{t+1}^i, C_{t+1,E}, C_{t+2,L}} \lambda u(C_{t+1,E}) + (1-\lambda) u(C_{t+2,L}) \tag{2.14}$$

such that

$$y = K_t^n + S_t^n + p_t I_t^n \tag{2.15}$$

$$C_{t+1,E} = RI_t^n + p_{t+1} K_t^n + S_t^n \tag{2.16}$$

$$C_{t+2,L} = RI_{t+1}^i + p_{t+2} K_{t+1}^i + S_{t+1}^i \tag{2.17}$$

$$with \quad RI_t^n + p_{t+1} K_t^n + S_t^n = p_{t+1} I_{t+1}^i + K_{t+1}^i + S_{t+1}^i \tag{2.18}$$

The market clearing condition for shares in t is :

$$I_t^n + (1-\lambda) I_t^i = K_{t-1}^n + (1-\lambda) K_{t-1}^i \tag{2.19}$$

and for goods:

$$\lambda C_{t-1,E} + (1-\lambda)C_{t-2,L} + K_t^n + (1-\lambda)K_t^i = y + R\left(K_{t-2}^n + (1-\lambda)K_{t-2}^i\right) \tag{2.20}$$

The no-arbitrage conditions are:

$$p_{t+1} = \frac{R}{p_t} \tag{2.21}$$

and

$$p_{t+1}p_{t+2} = R \tag{2.22}$$

Equation (2.15) is the budget constraint for the new-born: he uses his endowment to invest in the long term technology, in shares and in storage. In the second period of his life his wealth is constituted by the dividends from the shares bought when young, the proceeds of the sale of the shares of the intermediate production and what he has stored in the previous period. Equation (2.16) states that if the agent turns out to be an early consumer, he will consume all his wealth when he is middle-aged. However, if he turns out to be a late consumer, he uses his wealth to invest in shares, in production and in storage in the intermediate period, as stated in equation (2.18), and consume the proceeds of those when old, as can be seen in equation (2.17).

Concerning the market clearing conditions, equation (2.19) states that the demand of shares (LHS) by young and late middle-age agents has to be equal to the supply of shares (RHS) by middle age agents (both early and late) and by old agents, whereas equation (2.20) just states that in each period the quantity of goods consumed by early and late agents and invested in production by young and late middle-age agents has to be equal to the endowment, plus the return from production (omitting storage). Finally, no arbitrage conditions (2.21) and (2.22) are there to ensure that investment in shares and in production have the same one-period return so agent have an incentive to hold both assets (otherwise there would be no production or no shares to sell).

An equilibrium in this economy is a sequence of prices $\{p_t\}_{t\in\mathbb{Z}}$ and investment decisions $\{K_t^n, I_t^n, S_t^n, K_t^i, I_t^i, S_t^i\}_{t\in\mathbb{Z}}$ such that all agents are maximizing expected utility and markets clear. There exists an infinity of equilibria with the following properties:

1. $p_t p_{t+1} = R$ with $p_t \in [1, R]$
2. $C_{t,E} = p_t y$ and $C_{t,L} = Ry$

3. if $p_t > 1$ then $S_t^n = S_t^i = 0$ and $K_t = K_t^n + (1-\lambda)K_t^i = y - y\lambda\frac{R-p_t}{R-1}$

Among those, two possible equilibria are worth mentioning:

- the stationary one where $p = \sqrt{R}$, $C_E = \sqrt{R}y$ and $C_L = Ry$.
- the two-periodic one where for odd t, $p_t = 1$ and $C_{t,E} = C_{t,L} = Ry$ and for even t, $p_t = R$, $C_{t,E} = y$ and $C_{t,L} = R$y.

The steady state allocation is suboptimal because even if the presence of a stock market ensure that there is neither premature liquidation of the long term technology nor storage, this results in underinvestment because part of the endowment is diverted onto the stock market. Thus, the stock market can not decentralize the optimal allocation. The proof can be found in various form in Fulghieri and Rovelli (1998), Bhattacharya and Padilla (1996) and Bhattacharya, Fulghieri, and Rovelli (1998).

Finally, note that if it is not possible to close the interbank market in the case analyzed in section 2.3.5, the anonymous intergenerational financial intermediary would deliver the same steady state allocation as the stock market.

2.3.7 Intergenerational Stock Market with Government Transfers

However, if there exists a government collecting lump-sum age-dependent taxes and subsiding some agents, the stock market can attain the steady state optimal allocation, as shown in Bhattacharya and Padilla (1996). Steady state optimality can therefore not only be obtained through the intergenerational financial intermediary described in section 2.3.4 but also through a market combined with a government intervention as long as the age (i.e. the generation) of agents is public information.

Indeed, in this case, the agent problem is quite similar to the previous case (and the following equations could be explained in the same way), except for the lump-sum subsidies of the government to the new born s_t and to the middle-age agents z_t financed by the lump-sump tax of the old x_t :

$$\max_{K_t^n, I_t^n, S_t^n, K_{t+1}^i, I_{t+1}^i, S_{t+1}^i, C_{t+1,E}, C_{t+2,L}} \lambda u(C_{t+1,E}) + (1-\lambda)u(C_{t+2,L}) \tag{2.23}$$

such that

$$y + s_t = K_t^n + S_t^n + p_t I_t^n \tag{2.24}$$

$$C_{t+1,E} = RI_t^n + p_{t+1}K_t^n + S_t^n + z_t \tag{2.25}$$

$$C_{t+2,L} = RI_{t+1}^i + p_{t+2}K_{t+1}^i + S_{t+1}^i - x_t \tag{2.26}$$

$$with \quad RI_t^n + p_{t+1}K_t^n + S_t^n = p_{t+1}I_{t+1}^i + K_{t+1}^i + S_{t+1}^i \tag{2.27}$$

The market clearing conditions in t are:

$$I_t^n + (1-\lambda)I_t^i = K_{t-1}^n + (1-\lambda)K_{t-1}^i \tag{2.28}$$

and

$$\lambda C_{t-1,E} + (1-\lambda)C_{t-2,L} + K_t^n + (1-\lambda)K_t^i = y + R\left(K_{t-2}^n + (1-\lambda)K_{t-2}^i\right) \tag{2.29}$$

The no-arbitrage conditions are:

$$p_{t+1} = \frac{R}{p_t} \;\; and \;\; p_{t+1}p_{t+2} = R \tag{2.30}$$

The government budget constraint is:

$$s_t + z_t = x_t \tag{2.31}$$

The optimal steady state allocation $K = K^i + K^n = y$, $L = S = 0$, $C_E = C_L = Ry$ is a solution to this optimization problem if the government implements a transfer scheme with lump-sum taxes to the young $s = \sqrt{R}y - (1-\lambda)Ry$ and to the middle-age agents $z = \sqrt{R}((1-\lambda)Ry - y)$ and subsidies to the old agents $x = R(\sqrt{R}y - y)$.

2.3.8 Welfare Comparisons

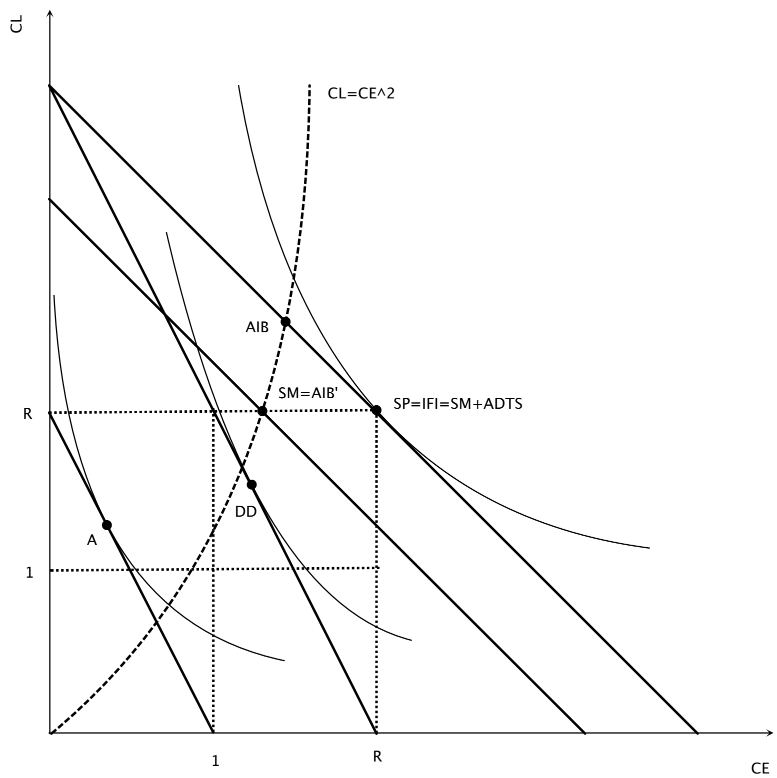

Figure 2.1: Comparison of steady state allocations (with $y = 1$)

Figure 2.1 summarizes the steady state consumption allocations that are available under the different institutional settings studied throughout this section. They are represented as follows: autarky A, Diamond and Dybvig generational financial intermediary DD, stock market SM, social planner SP, infinite intergenerational financial intermediary (for new-born only agents) IFI, anonymous intergenerational financial intermediary with a closed interbank market AIB, anonymous intergenerational financial intermediary with an open interbank market AIB', and finally stock market with an age-dependent transfer scheme $SM+ADTS$. The dashed curve represents the no roll-over constraint and the thin convex curves are utility indifference curves.

2.4 Alternative Institutional Arrangements

What have we learned so far? The main lesson from this extensive literature review is that in steady state – and in the case where the age of the agents is public information – infinite financial intermediaries are able to achieve the optimal allocation, whereas financial markets are able to do so only if a government implements a transfer scheme with the same informational requirements as those imposed on financial intermediaries.

However, as suggested by Penalva and Van Bommel (2009), if those infinite financial intermediaries are governed by their living depositors, those depositors have an incentive to liquidate the institution's assets to increase their own consumption at the expense of the following generations. Indeed, suppose that there is a general meeting of the living depositors between date $t-1$ and t to decide what to do with the infinite intermediary. If this is the case, only the late consumers of the generation born in $t-2$ and all the depositors from generation $t-1$ will attend to this meeting (as the early agents from generation $t-2$ have already withdrawn and the generation t agents are not yet born). Because in steady state the infinite intermediary has invested y in $t-2$ and in $t-1$ in the long-term technology, it will obtain a return of Ry in t and in $t+1$. However, at that point in time, it owes Ry in t (λRy to the early generation $t-1$ depositors and $(1-\lambda)Ry$ to the late generation $t-2$ depositors) and only $(1-\lambda)Ry$ in $t+1$ (to the late generation $t-1$ depositors). It is therefore over-funded because the new generation t has not yet deposited its endowment, so the current depositors should decide to share the assets of the intermediary in order to increase their utility.

Given this limitation of the infinite financial intermediaries, let's find alternative ways to decentralize the optimal allocation with generational financial intermediaries as those described in section 2.3.3 in order to see what interesting role can financial intermediaries play in this environment.

2.4.1 Generational Financial Intermediary with Lump-Sum Age-Dependent Transfers

An interesting and unexplored institutional arrangement is the one combining generational intermediaries and a government intervention. Instead of implementing transfers between agents as in section 2.3.7, the transfer could take place between intermediaries of different "age" (taking advantage of the fact that there exist overlapping financial institutions) in order to help them deliver the optimal consumption stream to their depositors. Indeed, in this case, the generational financial intermediary problem is simply:

$$\max_{K_t, S_t^n, S_{t+1}^i, L_{t+1}, C_{t+1,E}, C_{t+2,L}} \lambda u(C_{t+1,E}) + (1-\lambda) u(C_{t+2,L}) \tag{2.32}$$

such that:

$$y + s_t = K_t + S_t^n \tag{2.33}$$

$$\lambda C_{t+1,E} = S_t^n + QL_{t+1} + z_{t+1} - S_{t+1}^i \tag{2.34}$$

$$(1-\lambda) C_{t+2,L} = S_{t+1}^i + R(K_t - L_{t+1}) - x_{t+2} \tag{2.35}$$

whereas the government budget constraint is:

$$s_t + z_t = x_t \tag{2.36}$$

Equations (2.33) to (2.35) can be interpreted exactly in the same way as equations (2.10) to (2.12), except for the slightly different notation concerning storage (S_t^j with the superscript $j = \{i, n\}$ denoting the age of the intermediary: n if it is in its first period and i if it is its the intermediate period) and more importantly for the lump-sum subsidies of the government to the newly-created intermediaries s_t and to the middle-aged intermediaries z_t financed by the lump-sump tax of the old intermediaries x_t.

Proposition 1. *In steady state, a financial intermediary can implement the first best allocation* $K_t = y$, $L_{t+1} = S_t^n = S_{t+1}^i = 0$, $C_{t,E} = C_{t,L} = Ry$ $\forall t \in \mathbb{Z}$ *if a government implements a transfer scheme with lump-sum taxes and subsidies depending on the age of the financial intermediaries. More precisely the government will tax old financial intermediaries to subsidize intermediate financial intermediaries.*

Proof. It is immediate. Assume the optimal allocation $\forall t \in \mathbb{Z}$ $K_t = y$, $L_t = S_t^i = S_t^n = 0$ and $C_{t,E} = C_{t,L} = Ry$. Given this allocation, equation (2.33) implies that $s_t = 0$. Equation (2.34) yields: $z_t = \lambda Ry$ whereas equation (2.35) yields $x_t = \lambda Ry$. For such values of s_t, z_t and x_t the government budget constraint, equation (2.36), is satisfied. □

Hence, we have shown in a very simple but never explored institutional arrangement that there exists a governmental intervention that ensures the optimality of the decentralized equilibrium with a generational financial intermediary.[4]

[4]This result has the flavor of the Shell (1971) result where the social planner can improve total welfare

What about all the critiques that came up while we were examining the institutional arrangements from previous papers? Do they apply to our new setup? First, this allocation is incentive compatible in the sense that late type will not withdraw to store the good themselves because $C_{t+2,L} \geq C_{t+1,E}$. Second, it is immune to Jacklin's critique as there is no incentive to stay outside of the financial intermediary to perform side trades. Third, it is also immune to Bhattacharya and Padilla (1996)'s critique as an interbank market can be open (in the sense that financial intermediaries can invest in each other) without destroying the equilibrium. Fourth, it's immune to Penalva and Van Bommel (2009)'s critique about the governance of an infinite financial intermediary. Indeed, contrary to the infinite financial intermediary controlled by its living depositors, this setup allows us to go back to Wallace (1988)'s cash dispenser interpretation of the Diamond and Dybvig generational financial intermediary where all the decisions are taken by the coalition of new born depositors in the initial period and then the financial intermediary just act as a cash dispenser in the following ones.

Nevertheless, it is true that the intergenerational government transfer scheme exposed here – as the one proposed in Bhattacharya and Padilla (1996) – could be criticized because it could be cancelled by a majority-vote of the living citizens who would like to increase their own consumption at the expense of the following generations. However, this criticism could be dismissed by assuming that we face a benevolent (although non-democratic) government that cares for future generations.

Finally, if a no-redepositing condition is not in place, the financial intermediary is not immune to interest rate arbitrage by its late depositors in the sense that they will have an incentive to withdraw in the intermediate period of their life in order to redeposit in a newly-formed financial intermediary. Thus the financial intermediaries need to know the age of the depositors. Moreover, the government needs also to know the "age" of the financial intermediaries as its transfer scheme to implement the optimal allocation is age-dependent.

2.4.2 Generational Financial Intermediary with Proportional Transfers

In the case in which the government is unable to verify the age of financial intermediaries, it can implement another type of transfer scheme where subsidies are proportional to the

by organizing some transfers from young to old agents to take into account the initial old, even if here the transfer goes in the opposite direction.

investment in the long-term technology (and where taxes are proportional to the revenues coming from the long-term technology. In this case, the generational financial intermediary's problem is similar to the one explained in the previous section except that s_t denotes the subsidy rate unit of investment in the productive technology and x_t denotes the tax rate per unit of return of the long-term technology:

$$\max_{K_t, S_t^n, S_{t+1}^i, L_{t+1}, C_{t+1,E}, C_{t+2,L}} \lambda u(C_{t+1,E}) + (1-\lambda)u(C_{t+2,L}) \tag{2.37}$$

such that:

$$y + s_t K_t = K_t + S_t^n \tag{2.38}$$

$$\lambda C_{t+1,E} = S_t^n + Q L_{t+1} - S_{t+1}^i \tag{2.39}$$

$$(1-\lambda)C_{t+2,L} = S_{t+1}^i + (1 - x_{t+2})R(K_t - L_{t+1}) \tag{2.40}$$

and the government budget constraint is:

$$s_t K_t = x_t R(K_{t-2} - L_{t-1}) \tag{2.41}$$

Proposition 2. *In steady state, a financial intermediary can implement the steady state first best allocation $K = y$, $L = S^i = 0$, $C_E = C_L = Ry$ if a government implements a transfer scheme with proportional taxes on dividends and subsidies on investment.*

Proof. It is immediate. Assume that in steady state the optimal allocation $\forall t \in \mathbb{Z}$ $K_t = y$, $L_{t+1} = S_{t+1}^i = 0$, and $C_{t+1,E} = C_{t+2,L} = Ry$. Given this allocation, equation (2.39) implies that $S_t^n = \lambda R y$. Equation (2.38) yields: $s_t = \lambda R$ whereas equation (2.40) yields $x_{t+2} = \lambda$. For such values of s_t and x_t the government budget constraint, equation (2.41), is satisfied. □

We have shown that there exists another governmental intervention that ensures the optimality of the decentralized equilibrium with a generational financial intermediary. Yet, in this particular institutional arrangement the government does not even need to know the "age" of the financial intermediaries as its transfer scheme is not age-dependent anymore.

As the previous one, this allocation is incentive compatible ($C_L \geq C_E$), immune to all the critiques previously formulated by (Jacklin, 1987), Bhattacharya and Padilla (1996) and Penalva and Van Bommel (2009). But, as before, the financial intermediaries is not immune to interest rate arbitrage by its late depositors if a no-redepositing condition is not in place, so the financial intermediaries need to know the age of the depositors. However, this solution could be interesting to implement if the government is unable to verify the age of the financial intermediaries.

2.4.3 Generational Financial Intermediary Combined With a Stock Market

What would happen if generational intermediaries are combined with a stock market? This would allow financial intermediaries to trade shares of the production by the long-term technology in its intermediate state in the same way as individual agents in the paper by Fulghieri and Rovelli (1998) described in section 2.3.6. Let's see if this institutional arrangement can attain the first best allocation.

The generational financial intermediary's problem becomes:

$$\max_{K_t^n, I_t^n, S_t^n, K_{t+1}^i, I_{t+1}^i, S_{t+1}^i, C_{t+1,E}, C_{t+2,L}} \lambda u(C_{t+1,E}) + (1-\lambda) u(C_{t+2,L}) \tag{2.42}$$

such that

$$y = K_t^n + S_t^n + p_t I_t^n \tag{2.43}$$

$$\lambda C_{t+1,E} = R I_t^n + p_{t+1} K_t^n + S_t^n - p_{t+1} I_{t+1}^i - K_{t+1}^i - S_{t+1}^i \tag{2.44}$$

$$(1-\lambda) C_{t+2,L} = R I_{t+1}^i + p_{t+2} K_{t+1}^i + S_{t+1}^i \tag{2.45}$$

The market clearing conditions in t are:

$$I_t^n + I_t^i = K_{t-1}^n + K_{t-1}^i \tag{2.46}$$

and

$$\lambda C_{t-1,E} + (1-\lambda) C_{t-2,L} + K_t^n + K_t^i = y + R\left(K_{t-2}^n + K_{t-2}^i\right) \tag{2.47}$$

The no-arbitrage conditions are:

$$p_{t+1} = \frac{R}{p_t} \text{ and } p_{t+1} p_{t+2} = R \tag{2.48}$$

Similarly to section 2.3.6, let K_t^j, I_t^j and S_t^t denote the respective investments in the long-term technology, in shares and in the storage technology where the subscript t denotes the period in which the investment is made and the superscript $j = \{n, i\}$ denotes the "age" of the financial intermediary: n if it is a newly-created one and i if it is in its intermediate age.

Equation (2.43) is the budget constraint for the newly created intermediary: it uses the endowment to invest in the long-term technology, in shares and in storage. In the second period, its wealth is constituted by the dividends from the shares bought in its first period, the proceeds of the sale of the shares of the intermediate production and what was stored in the previous period. Equation (2.44) states that it uses this wealth to give some consumption to its early type depositors and also to invest in the long term technology, in shares and in storage. In its last period, it uses its wealth to give consumption to its late type depositors.

Concerning the market clearing conditions, equation (2.46) states that the demand of shares by new and intermediate intermediaries has to be equal to the supply of shares by intermediate and old intermediaries, whereas equation (2.47) just states that in each period the quantity of goods consumed and invested in production has to be equal to the endowment, plus the return from production (omitting storage). Finally, no arbitrage conditions in (2.48) are there to ensure that investment in shares and in production have the same one-period return so that intermediaries have an incentive to hold both assets (otherwise there would be no production or no shares to sell).

Prima facie, the steady state efficient allocation appears to be feasible here, because it respects the budget constraints of the financial intermediary and the market clearing conditions. For example, assuming $K = K^i = y$, $L = S = K^n = 0$ and $C_E = C_L = Ry$ we obtain a positive price $p = \frac{\sqrt{R(\lambda^2 R+4)}-\lambda R}{2}$, $I^n = \frac{y}{p}$ and $I^i = \frac{y}{p}(\frac{R}{p} - \lambda R - 1)$ as long as $\lambda \in [0, 1]$ and $R \geq \frac{1}{1-\lambda}$ (to avoid $I^i < 0$). However, this can not be an equilibrium because the no-arbitrage condition is not respected as $p < \sqrt{R}$ for $\lambda > 0$. If this were the case, the financial intermediary would only invest in the long-term technology in the intermediate period, so there would be only financial intermediary in their initial period buying shares, which would lead, given (2.46), to a contradiction.

And if we just assume $p = \sqrt{R}$, $C_E = C_L = Ry$ and $L = S = 0$ without imposing any restriction on K^n, K^i, I^n and I^i, we can not find any solution to the system of equations, implying that it is not possible to achieve in this environment the optimal allocation where all depositors receive an equal amount of goods and where there is neither liquidation, nor storage.

This negative result is nonetheless interesting. If the efficient allocation can not be obtained in this environment combining a financial intermediary and a stock market, then no other less efficient equilibrium can exist at all in this setup because of Jacklin's critique. The presence of the stock market in the economy allows agents to do side trade (unless you forbid agents to use the market individually), and because the financial intermediary is not able to provide the optimal allocation, agents will have an incentive to stay outside of the bank and trade with insiders if they are early consumers, but because everybody has an incentive to do this, intermediation would disappear. Thus, it appears that generational financial intermediaries and stock markets can not coexist in this setup.

2.4.4 Generational Financial Intermediary Combined With a Stock Market and Lump-Sum Age-Dependent Government Transfers

Finally, let's try to see what happen if we combine a generational financial intermediary, a stock market and a transfer scheme by the government to see if in this configuration they can coexist. In this case, the generational financial intermediary problem is similar to the previous one except for the transfers:

$$\max_{K_t^n, I_t^n, S_t^n, K_{t+1}^i, I_{t+1}^i, S_{t+1}^i, C_{t+1,E}, C_{t+2,L}} \lambda u(C_{t+1,E}) + (1-\lambda)u(C_{t+2,L}) \tag{2.49}$$

such that

$$y + s_t = K_t^n + S_t^n + p_t I_t^n \tag{2.50}$$

$$\lambda C_{t+1,E} = RI_t^n + p_{t+1}K_t^n + S_t^n - p_{t+1}I_{t+1}^i - K_{t+1}^i - S_{t+1}^i + z_{t+1} \tag{2.51}$$

$$(1-\lambda)C_{t+2,L} = RI_{t+1}^i + p_{t+2}K_{t+1}^i + S_{t+1}^i - x_{t+2} \tag{2.52}$$

The market clearing conditions in t are:

$$I_t^n + I_t^i = K_{t-1}^n + K_{t-1}^i \tag{2.53}$$

and

$$\lambda C_{t-1,E} + (1-\lambda)C_{t-2,L} + K_t^n + K_t^i = y + R\left(K_{t-2}^n + K_{t-2}^i\right) \tag{2.54}$$

The no-arbitrage conditions are:

$$p_{t+1} = \frac{R}{p_t} \text{ } and \text{ } p_{t+1}p_{t+2} = R \tag{2.55}$$

The government budget constraint is:

$$s_t + z_t = x_t \tag{2.56}$$

Proposition 3. *In steady state, a financial intermediary combined with a stock market can implement the first best allocation $K = K^i + K^n = y$, $L = S = 0$, $C_E = C_L = Ry$ if and only if a government implements a transfer scheme with lump-sum taxes and subsidies depending on the age of the financial intermediaries.*

Proof. Assume that $p = \sqrt{R}$, therefore respecting the two equations in (2.55), $K = K^i + K^n = y$, $L = S = 0$, $C_E = C_L = Ry$ and also without loss of a generality that $K^n = y$ and so that $K^i = 0$. Given this allocation, equation (2.33) implies that $I^n = \frac{s}{\sqrt{R}}$ and equation (2.53) that $I^i = y - \frac{s}{\sqrt{R}}$. Equation (2.34) yields: $z = \lambda Ry - s(1 + \sqrt{R})$ whereas equation (2.35) yields $x = R(\lambda y - \frac{s}{\sqrt{R}})$. Equations (2.54) and (2.56) do not bring more information, therefore the subsidy to the new financial intermediaries is indeterminate: $s \in \mathbb{R}$. However, for I^n and I^i to be positive, we need to have $s \in [0, \sqrt{R}y]$. □

Moreover, as you can see in figure 2.2, if s is chosen by the government such that:

- $s \in [0, \frac{\lambda Ry}{1+\sqrt{R}}]$, the government will tax old financial intermediaries to subsidize new and intermediate financial intermediaries.
- $s \in [\frac{\lambda Ry}{1+\sqrt{R}}, \lambda\sqrt{R}y]$, the government will tax intermediate and old financial intermediaries to subsidize new financial intermediaries.
- $s \in [\lambda\sqrt{R}y, \sqrt{R}y]$, the government will tax intermediate financial intermediaries to subsidize new and old financial intermediaries.

Note that of course $K^n = y$ is only one of the many possible solutions and that if for example the financial intermediary had chosen $K^i = y$ and $K^n = 0$, x and z would be defined the same way but we would have $I^n = \frac{y+s}{\sqrt{R}}$ and $I^i = y - \frac{y+s}{\sqrt{R}}$ so s would have to be chosen in the interval $[-y, \sqrt{R}y - y]$, which would lead to different cases as to when the financial intermediary is taxed and when it is subsidized depending on the value of the parameters.

Yet, we have shown that there exists a set of governmental interventions that ensures the optimality of the decentralized equilibrium in which coexist a generational financial intermediary and an equity market. This allocation obtained through this institutional arrangement shares the qualities and flaws as the allocation obtained in section 2.4.1: it is incentive compatible, immune to Jacklin's, Bhattacharya and Padilla (1996)'s and Penalva and Van Bommel (2009)'s critiques, but financial intermediaries need to know the age of depositors and the government needs to know when the financial intermediaries were formed.

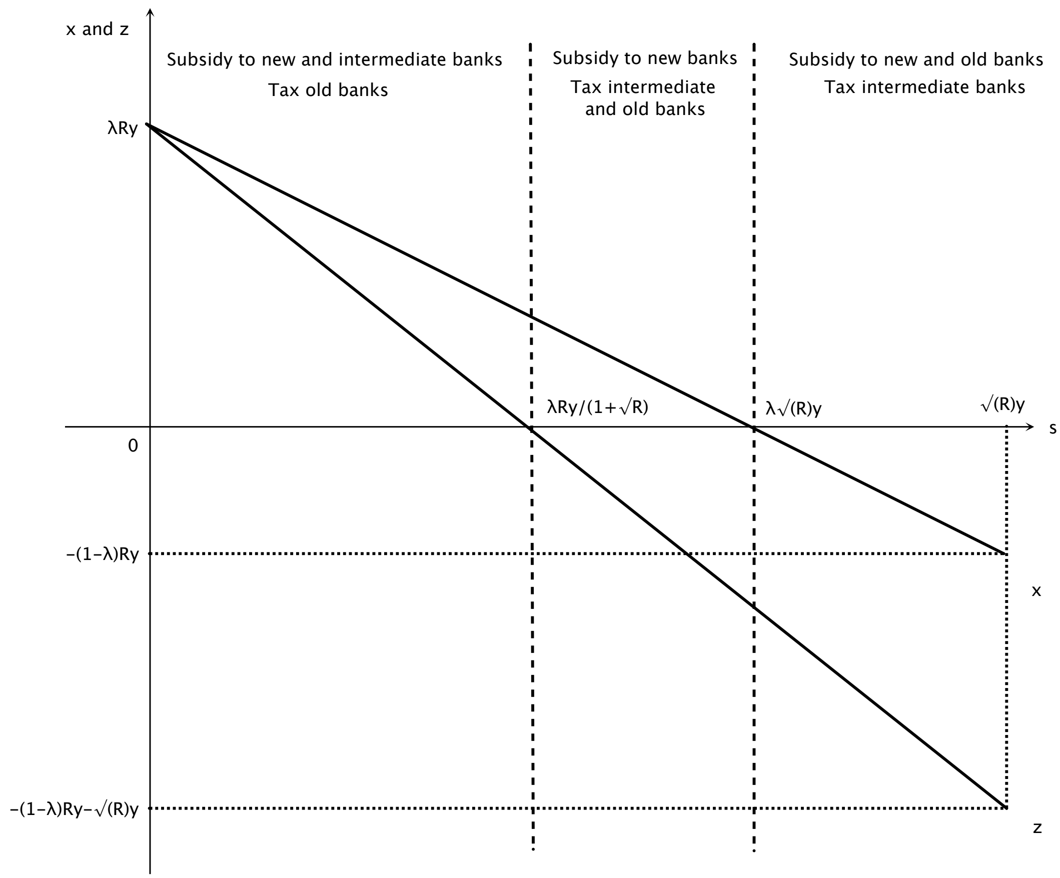

Figure 2.2: Government transfers to achieve the optimal allocation (for the case in which $K^n = y$)

2.5 Transition to the Optimal Steady State Allocation: Forming an Intergenerational Financial Intermediary

In all the previously mentioned papers and the alternative institutional arrangements proposed in section 2.4, the analysis has been confined to steady states and does not consider the problem of the dynamics of the economy from the very first generation.

Qi (1994) is the only one to examine this issue, but he makes the extreme choice of looking only for a solution ensuring a constant level of expected utility to all members of all generations. In this case, the infinite financial intermediary is formed without prior assets and liabilities and the only resources it has at the beginning are the deposit of the endowment of the first generation (born in $t = 0$). Given these initial conditions, the intergenerational financial intermediary is subject to tighter budget constraints than in the case assuming directly that it is in steady state (for example in section 2.3.4) and it has to rely on new deposits to pay off some of the deposit withdrawals. Although the allocation achieved is better than the one obtained through a generational financial intermediary by investing a greater proportion of the deposits in the more productive long-term technology, it prevents the financial intermediary to reach the most efficient allocation where all the deposits are invested in the illiquid technology and where all agents are perfectly insured and receive $C_E = C_L = Ry$. Indeed, the allocation is characterized by cyclical long-term investment and storage and by a consumption stream given by $C_E = \frac{2R}{R+1}y$ and $C_L = Ry$ for all generations.

However, suppose that instead of limiting the analysis to steady state payoffs that offer all current and future depositors the same ex-ante utility as in Qi (1994), we have a infinite financial intermediary that wants to maximize the long-run average of the expected utilities of the different generations as in Allen and Gale (1997):

$$\lim_{T\to+\infty} T^{-1} \sum_{t=0}^{T} \lambda u(C_{t+1,E}) + (1-\lambda)u(C_{t+2,L}) \tag{2.57}$$

In this case, given the initial conditions[5], the following constraints must be satisfied:

$$K_0 + S_0 \leq y \tag{2.58}$$

[5] We maintain Qi's assumption that there is no prior assets and liabilities and that the only resources the infinite intermediary has at the beginning are the deposit of the endowment of the first generation.

$$\lambda C_{1,E} + K_1 + S_1 \leq y + S_0 \tag{2.59}$$

$$\lambda C_{t,E} + (1-\lambda)C_{t,L} + S_t + K_t \leq y + RK_{t-2} + S_{t-1} \quad \forall t \geq 2 \tag{2.60}$$

$$C_{t,E}, C_{t,L}, K_t, S_t \geq 0 \;\; and \;\; K_t \leq y \tag{2.61}$$

$$C_{t+1,E} \leq C_{t+2,L} \tag{2.62}$$

Equation (2.58) is the budget constraint in $t = 0$: the infinite intermediary receives the endowment of the first generation as deposits and uses it to invest in the long-term technology and in storage. Equation (2.59) is the constraint for $t = 1$: the intermediary uses the endowment from the second generation and the storage from the previous period to repay the depositors from the first generation that are early consumers and, again, to invest in the long-term technology and storage. As can be seen in equation (2.60), from $t = 2$ on, in addition to the endowment of the new-born generation and to the storage from the previous period, the intermediary starts receiving the return from the investment in the long-term technology started two periods before. It uses these sources to invest and to repay depositors born two periods before who turn out to be of the late type and those born in the previous period who are early consumers. In addition to that, (2.61) ensures that consumption and investments are positive and that the problem is properly bounded. Finally, the incentive compatibility constraint (2.62) ensures that late depositors do not have an incentive to withdraw early.

2.5.1 Forming a Generational Coalition as an Outside Option

Note that, in all these budget constraints, the endowments of the new-born generation appear every period in the resources of the intermediary. However, for this to be the case, the intermediary has to attract the agents by offering them better payoffs than what they could achieve by acting on their own or by depositing their endowment in another structure. Consequently, the intergenerational financial institution has to offer them more than a generational intermediary (à la Diamond and Dybvig), which means that it also has to satisfy the following participation constraint:

$$\lambda u(C_{t+1,E}) + (1-\lambda)u(C_{t+2}) \geq \lambda u(C^{DD}_{t+1,E}) + (1-\lambda)u(C^{DD}_{t+2}) \tag{2.63}$$

One way to look at that problem is to try to find out a finite way to reach the optimal steady state allocation ($K_t = y$, $S_t = 0$, $C_{t,E} = C_{t,L} = Ry$ $\forall t > \tau$ where τ is the period in which the steady state is reached) while giving the early generations on the transition path at least their outside option period by period[6].

If the utility function is of the CRRA type with a risk aversion parameter θ, there exists a transition path[7] to the steady state such that:

$$K^T_{2k} = y \quad \forall k \in \mathbb{Z}^+ \tag{2.64}$$

$$\begin{aligned} K^T_{2k+1} &= R^k(y - \lambda C^{DD}_E) + \sum_{i=0}^{k-1} R^i(y + Ry - 2\lambda C^{DD}_E - 2(1-\lambda)C^{DD}_L) \\ &= \frac{\lambda(R - R^{k+1} + R^{\frac{1}{\theta}}) - R^{\frac{1}{\theta}}}{\lambda(R^{\frac{1}{\theta}} - R) - R^{\frac{1}{\theta}}} y \quad \forall k \in \mathbb{Z}^+ \end{aligned} \tag{2.65}$$

$$S^T_{2k+1} = S_0 = 0 \quad \forall k \in \mathbb{Z}^+ \tag{2.66}$$

$$S^T_{2k} = Ry - \lambda C^{DD}_E - (1-\lambda)C^{DD}_L \quad \forall k \in \mathbb{Z}^{++} \tag{2.67}$$

$$C^T_E \geq C^{DD}_E = \frac{y}{\lambda + (1-\lambda)R^{\frac{1-\theta}{\theta}}} \tag{2.68}$$

$$C^T_L \geq C^{DD}_L = \frac{R^{\frac{1}{\theta}} y}{\lambda + (1-\lambda)R^{\frac{1-\theta}{\theta}}} \tag{2.69}$$

until period $t = \tau$ where the condition $K^T_{2k+1} = y$ can be satisfied. When this is the case, the economy has reached the steady state and from the next period on, the allocation will be:

$$\forall t > \tau, \;\; K_t = y, \;\; S_t = 0, \;\; C_{t,E} = C_{t,L} = Ry \tag{2.70}$$

Note that the length of the transition and the exact value of C^T_E and C^T_L depend on the value of R:

- if $R = 2$: the condition $K_{2k+1} = y$ is fulfilled in $t = 3$ and $C^T_{1,E} = C^T_{2,E} = C^T_{3,E} = C^{DD}_E$ and $C^T_{2,L} = C^T_{3,L} = C^{DD}_L$
- if $R > 2$: the condition $K_{2k+1} = y$ is fulfilled in $t = 3$ but because there is more resources than necessary (i.e. $\frac{\lambda(R-R^{k+1}+R^{\frac{1}{\theta}})-R^{\frac{1}{\theta}}}{\lambda(R^{\frac{1}{\theta}}-R)-R^{\frac{1}{\theta}}} y - y > 0$) in $t = 3$ which means

[6] Of course, giving them their outside option period by period is not necessary but it simplifies the problem heavily and it makes it tractable.

[7] During the transition, the variables are denoted by the superscript T.

there would be over-saving and/or over-investing in the previous periods, the financial intermediary can therefore give to the early generations more than their reservation utility so $C^T_{1,E} = C^T_{2,E} = C^T_{3,E} > C^{DD}_E$ and $C^T_{2,L} = C^T_{3,L} > C^{DD}_L$ (as long as $C^T_{1,E} < \frac{y}{\lambda}$ to ensure that $K_1 > 0$ and as long as $\lambda \frac{R+1}{2} C^T_{t,E} + (1-\lambda) C^T_{t,L} < Ry$ to ensure that $K^T_{2(k+1)+1} > K^T_{2k+1}$ so that the economy is not trapped in the cyclical allocation proposed by Qi)

- if $1 < R < 2$: the length of the transition is such that if $2^{\frac{1}{j}} \leq R < 2^{\frac{1}{j-1}}$ for $j \in \mathbb{Z}^+ - \{0, 1\}$ the condition is fulfilled in $t = \tau = 2j + 1$. As in the two previous cases when the first inequality is binding (i.e when $R = 2^{\frac{1}{j}}$) we will have: $\forall t \leq \tau$, $C^T_{t,E} = C^{DD}_E$ and $C^T_{t,L} = C^{DD}_L$, but when it is not binding (i.e. when $2^{\frac{1}{j}} < R < 2^{\frac{1}{j-1}}$) we will have: $C^T_{t,E} > C^{DD}_E$ and $C^T_{t,L} > C^{DD}_L$. It is also interesting to note that if $R \to 1$ – or equivalently if $\lim_{j \to +\infty} 2^{\frac{1}{j}}$ – the transition tends to infinity.

Figure 2.5.1 illustrates the transition for the case in which $R = 2^{\frac{1}{10}} = 1.07177$, we can see that in this case it takes 21 periods to reach the optimal full investment in the long-term technology and that during the transition it jumps from full investment in even periods to growing levels – as defined by equation (2.65) – in odd periods. In the meantime, consumption levels of the generations living during the transition are defined by their outside option and storage is cyclical. But when the steady state is reached, there is full investment, no storage, and consumption is at its optimal level.

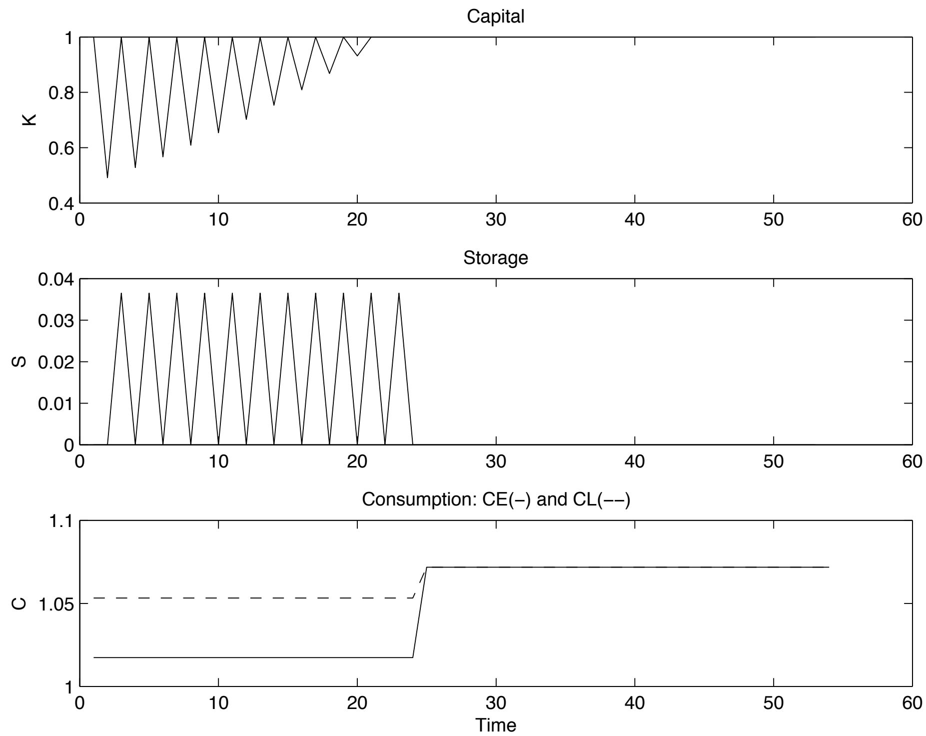

Figure 2.3: Transition to the optimal steady state ($R = 1.07177 = 2^{\frac{1}{10}}$, $\lambda = 0.5$, $\theta = 2$, $y = 1$)

2.5.2 Opening a Stock Market as an Outside Option

Another interesting case is the one where the outside option for the agents is not to form a generational coalition à la Diamond and Dybvig but to open a stock market where they can trade shares of the production by the long-term technology in the interim period. As it is shown in Bhattacharya, Fulghieri, and Rovelli (1998) – for the environment described in section 2.3.6 – if the stock market starts in $t = 0$ then the only equilibrium consistent with the initial conditions is the periodic one. Consequently, the outside option for an agent tempted to open a stock market is the consumption stream given by $C_E^{SM} = y$ and $C_L^{SM} = Ry$, and the participation constraint to attract the agent in the intergenerational financial intermediary is then:

$$\lambda u(C_{t+1,E}) + (1-\lambda)u(C_{t+2}) \geq \lambda u(y) + (1-\lambda)u(Ry) \tag{2.71}$$

In this case, with the same approach as in the previous section, it is possible to show that there exists a feasible transition path to the steady state such that:

$$K^T_{2k} = y \quad \forall k \in \mathbb{Z}^+ \tag{2.72}$$

$$\begin{aligned} K^T_{2k+1} &= R^k(y - \lambda C^{SM}_E) + \sum_{i=0}^{k-1} R^i(y + Ry - 2\lambda C^{SM}_E - 2(1-\lambda)C^{SM}_L) \\ &= y + \lambda(R^k - 2)y \quad \forall k \in \mathbb{Z}^+ \end{aligned} \tag{2.73}$$

$$S^T_{2k+1} = S_0 = 0 \quad \forall k \in \mathbb{Z}^+ \tag{2.74}$$

$$S^T_{2k} = Ry - \lambda C^{SM}_E - (1-\lambda)C^{SM}_L \quad \forall k \in \mathbb{Z}^{++} \tag{2.75}$$

$$C^T_E \geq C^{SM}_E = y \tag{2.76}$$

$$C^T_L \geq C^{DD}_L = Ry \tag{2.77}$$

until period $t = \tau$ where the condition $K^T_{2k+1} = y$ can be satisfied. When this is the case, the economy has reached the optimal steady state and from the next period on, the allocation will be:

$$\forall t > \tau, \;\; K_t = y, \;\; S_t = 0, \;\; C_{t,E} = C_{t,L} = Ry \tag{2.78}$$

Note that, again, the length of the transition and the exact value of C^T_E and C^T_L depend on the value of R:

- if $R = 2$: the condition $K_{2k+1} = y$ is fulfilled in t=3 and $C^T_{1,E} = C^T_{2,E} = C^T_{3,E} = C^{SM}_E$ and $C^T_{2,L} = C^T_{3,L} = C^{SM}_L$
- if $R > 2$: the condition $K_{2k+1} = y$ is fulfilled in t=3 but because there is more resources than necessary (i.e. $\lambda(R^k - 2)y > 0$) in $t = 3$ which means there would be over-saving and/or over-investing in the previous periods, the financial intermediary can therefore give more then their reservation utility to the early generations so $C^T_{1,E} = C^T_{2,E} = C^T_{3,E} > C^{SM}_E$ and $C^T_{2,L} = C^T_{3,L} > C^{SM}_L$ (as long as $C^T_{1,E} < \frac{y}{\lambda}$ to ensure that $K_1 > 0$ and as long as $\lambda\frac{R+1}{2}C^T_{t,E} + (1-\lambda)C^T_{t,L} < Ry$ to ensure that

$K^T_{2(k+1)+1} > K^T_{2k+1}$ so that the economy is not trapped in the cyclical allocation proposed by Qi)

- if $1 < R < 2$: the length of the transition is such that if $2^{\frac{1}{j}} \leq R < 2^{\frac{1}{j-1}}$ for $j \in \mathbb{Z}^+ - \{0,1\}$ the condition is fulfilled in $t = \tau = 2j+1$. As before when the first inequality is binding: $\forall t \leq \tau$, $C^T_{t,E} = C^{SM}_E$ and $C^T_{t,L} = C^{SM}_L$ but when it is not: $C^T_{t,E} > C^{SM}_E$ and $C^T_{t,L} > C^{SM}_L$. It is also interesting to note that if $R \to 1$ – or equivalently if $\lim_{j \to +\infty} 2^{\frac{1}{j}}$ – the transition tends to infinity.

However, if both outside options are possible (i.e. agents can either form a generational intermediary or open a stock market in addition to the possibility to deposit in the infinite intermediary), the only one that is relevant is the generational coalition as long as the agent is risk averse because if this is the case: $\lambda u(C^{DD}_{t+1,E}) + (1-\lambda)u(C^{DD}_{t+2}) \geq \lambda u(C^{SM}_{t+1,E}) + (1-\lambda)u(C^{SM}_{t+2})$

2.5.3 Forming an Intergenerational Coalition with Steady State Payoffs as an Outside Option

What happen if the agents are allowed to form another intergenerational coalition with steady state payoffs that offer all current and future depositors the same ex-ante utility? As noted before, Qi (1994) shows that in the case with steady state payoffs the allocation is characterized by cyclical investment in the long-term technology and in storage forever and by a consumption stream given by $C_E = \frac{2R}{R+1}y$ and $C_L = Ry$ for all generations. So, if this is the outside option in our problem, we will have $K^T_{2(k+1)+1} = K^T_{2k+1}$ $\forall k \in \mathbb{Z}^+$ and thus the investment in the long-term technology will not converge to the optimal and the economy will be stuck in the infinite cycle described by Qi.

Therefore, if forming an intergenerational coalition with steady state payoffs is a possibility for the first generation of agents, it means that the transition to the optimal steady state never takes place because the infinite financial intermediary does not have the possibility to give agents their outside option ($C_E = \frac{2R}{R+1}y$ and $C_L = Ry$) and at the same time to invest a bit more every odd period, even if reaching the steady state optimal allocation would be better in terms of total welfare.

2.5.4 Alternative Decentralization Mechanisms for the Transition

This transition to the optimal steady state described in Section 2.5.1 could also be decentralized in other ways than with the intergenerational financial intermediary. Indeed, it is possible to implement this solution by combining generational intermediaries and a government transfer scheme as in section 2.4.1.

In the case of a lump-sum age-dependent transfer scheme, generational financial intermediaries solve the same problem as the one described by equations (2.32) to (2.35), whereas the government subject to the constraint (2.36) can easily implement the solution described by equations (2.72) to (2.78) by taxing and subsidizing the coexisting generational financial intermediaries. For example[8], for $R = 2$, the transfer scheme defined by: $s_1 = -\lambda C_E^{DD}$ and $s_t = 0$ $\forall t > 1$; $x_1 = 0$, $x_2 = Ry - (1-\lambda)C_L^{DD}$ (so $S_2^i = Ry - (1-\lambda)C_L^{DD} - \lambda C_E^{DD}$), $x_3 = \lambda C_E^{DD}$ and $x_t = \lambda Ry$ $\forall t > 3$; $z_1 = \lambda C_E^{DD}$, $z_2 = Ry - (1-\lambda)C_L^{DD}$, $z_3 = \lambda C_E^{DD}$ and $z_t = \lambda Ry$ $\forall t > 3$ ensures that the optimal steady state allocation is reached after 3 periods and that the generations of the transition receive their outside option as in Section 2.5.1.

Note, however, that the proportional transfer scheme described in section 2.4.2 is not a feasible way to decentralize the transition as there is no returns to tax before $t = 2$. Therefore, because no transfer is possible in $t = 1$, the financial intermediary of the generation 0 has no incentive to invest all its deposits in the long-term technology. Instead, it invests as in the Diamond and Dybvig case (see section 2.3.3), which means that if the government wants to give the agents of this generation their outside option – i.e something that gives them as much utility as the generational financial intermediary – it can not tax the returns on long-term investment in $t = 2$ (otherwise they would get less utility then with their outside option), so the intergenerational transfer scheme can never start.

2.6 Conclusions

In this paper we have synthesized the extant and extended the Diamond and Dybvig overlapping generation framework in order to compare various institutional arrangements including financial intermediaries, stock markets and government interventions. The previous literature shows that in steady state, and omitting consideration of the very first

[8] In the cases where $R \neq 2$, a similar transfer scheme achieving the proposed solution is always feasible but it becomes less tractable during the transition to the steady state as it depends on the parameter values.

generation problem, financial markets achieve allocations that are inferior to infinite intergenerational financial intermediaries, except if a government is able to implement a transfer scheme with the same informational requirements as those imposed on financial intermediaries.

The first contribution of this paper is to provide alternative ways to decentralize the steady state optimal allocation taking into account the criticism addressed to infinite financial intermediaries by Penalva and Van Bommel (2009). We show that generational financial intermediaries can achieve – whether they are combined with a stock market or not – the first best steady state allocation as long as a government can implement transfers between financial intermediaries – whether those transfers are lump-sum or proportional. The main flaw of these solutions is of course that intergenerational government transfers could also be criticized on the basis that they could be cancelled by a majority-vote of the living citizens who would like to increase their own consumption at the expense of the following generations. However, as we said before, we can ignore this problem if we assume the existence of a benevolent government that cares for future generations. Incidentally, we also show that financial intermediaries and markets can not coexist unless there is a transfer scheme ensuring that the steady state optimal allocation is achieved.

The second contribution is to show that there exists a feasible path leading to the optimal allocation when the economy has a starting date in which the first generation has only its endowment to start the intergenerational risk-sharing mechanism. Unlike previous authors examining this question we have chosen not to restrict ourselves to solutions ensuring a constant level of expected utility to all members of all generations but to look for a finite path to the optimal steady state allocation taking into account the participation constraints of the generations living during this transition. Those participation constraints depend on the possible outside options of the agents. On the one hand, if they can form a generational coalition or open a stock market as outside options there will be no problem to convince them to participate to the infinite intermediary during the transition. On the other hand, if they can form an infinite intermediary with steady state payoffs as the one proposed by Qi (1994) then, of course, the first generation does not have an incentive to participate to the transition, so the transition to the optimal steady state never takes place, even if reaching the steady state optimal allocation would be better in terms of total welfare. Finally, we show that this transition can not only be obtained through an infinite financial intermediary but also through a sequence of generational financial intermediary combined with intergenerational lump-sum transfers between these financial institutions.

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Chapter 3

Twin Crisis, Sudden Stop and the Exchange Rate Regime

3.1 Introduction

This paper[1] contributes to the debate between flexible and fixed exchange rate in light of the recent events. From the 1970s to the 1990s, most emerging countries had pegged exchange-rate arrangements.[2] At the time, this type of monetary framework appeared desirable to solve their credibility problems by imposing some external discipline on monetary and fiscal policy. However, after the East-Asian crisis of 1997-98 and the crises in Russia and Argentina, the focus changed rapidly and arguments in favor of flexible exchange rates regained the advantage over the ones favorable to pegs both in policy and academic circles.

In this context, Chang and Velasco's paper (2000c)[3] on exchange rate regime choice and

[1] I am grateful to Elena Carletti, Russell Cooper and the participants of the "Cooper & Students" Discussion Group and the Micro Working Group of the EUI for their useful comments. All remaining errors are mine.

[2] According to the IMF, in the mid-1970s, 85% of developing countries had pegs in various forms: currency boards, fixed rates with small fluctuation bands, crawling pegs, etc.

[3] Chang and Velasco's paper belongs to the so called third generation of currency crisis models developed at the end of the 1990s that included banking crisis in their formalization. Other notable contributions are Chang and Velasco (1999), (2001), Allen and Gale (2000) and Gale and Vives (2002). Before that, the literature on bank runs and on currency crisis mainly developed separately. On the banking front, there has been a huge literature on the micro foundations of bank runs following Diamond and Dybvig (1983). On the currency front, a first generation of models of currency crisis developed following Krugman (1979) to explain the crises of the 1970s. According to these models, crises were mainly the result of fiscal imbalances due to unsustainable government policies. A second generation, put forward by Obstfeld (1994, 1996), was designed to explain currency crises by the fact that central banks were abandoning pegs

financial fragility was influential in terms of policy as it was considered a strong theoretical case for flexible exchange rates. Indeed, the goal of the two authors was to determine what combination of exchange rate regime – currency board, fixed or flexible exchange rate – and central bank policy – acting as Lender of Last Resort (LoLR) or not – is less vulnerable to financial fragility. This financial fragility manifesting itself either through a banking crisis (represented in the model by a bank run provoking the failure of the entire consolidated banking sector) or through a currency crisis (represented by a balance-of-payment crisis occurring when the central bank runs out of foreign exchange reserves).

In order to do that, they develop an open economy version of the Diamond and Dybvig model (1983) in which banking and currency crises are self-fulfilling phenomena linked to multiple equilibria. In this framework, they show that (1) a currency board is not vulnerable to a balance-of-payment crisis but is vulnerable to a bank run, (2) a fixed exchange rate without a LoLR is not prone to currency crises but can be subject to bank runs, and (3) a fixed exchange rate with a LoLR is not subject to bank runs but only to balance-of-payment crises. But their most influential result is that the flexible exchange rate regime dominates all these regimes because it leads to a unique equilibrium that implements the optimal allocation, provided that liabilities in the form of deposits are denominated in domestic currency and the central bank stands ready to act as a LoLR. The intuition behind these results is the following: a crisis is a possible equilibrium only if each depositor expects that others will run and exhaust the country's foreign-exchange reserves. In a crisis, depositors withdraw pesos from the banks to exchange them for dollars at the central bank, while the central bank is printing pesos to help the commercial banks if it acts as a LoLR. With fixed rates, this provokes a self-fulfilling crisis by causing the central bank to run out of dollars. By contrast, with flexible rates, the central bank is no longer obligated to use all its reserves to avoid an attack. Instead, the running depositors are punished by a devaluation, while those who do not run know that there will still be dollars available for withdrawal at a later date. So, with flexible rates, neither early withdrawals nor devaluations occur in equilibrium.

Given this kind of appealing theoretical result and the bad performances of the currency-board countries at that time – notably Argentina and Hong-Kong – some countries decided to abandon their exchange rate-pegs and to let their currency float. Iceland was one of

because of strong recessions and raising unemployment, which suited well the ERM 1992 crisis. However, both types of models ignored the role of the banking system and were useless to understand the Asian crisis, so a third generation of models emerged at that time.

those countries which changed their monetary policy framework at the turn of the century. In March 2001, the central bank ceased to use the exchange rate of the króna as a target and nominal anchor of monetary policy and adopted an inflation targeting framework with a market-determined exchange rate[4].

However, contrary to what was suggested by Chang and Velasco's results, a flexible exchange rate did not prevent the country from experiencing a banking meltdown and a dramatic currency crisis a few years later. Indeed, in October 2008, the three most important commercial banks of Iceland (representing about 85% of the banking assets of the country[5]) – Glitnir, Landsbanki and Kaupthing – all collapsed in a few days, meanwhile the Icelandic currency was experiencing a complete free fall. The simultaneous occurrence of those two events makes it a perfect example of a typical twin crisis combining a systemic banking crisis (i.e. the bankruptcy of a large number of financial institutions) and a dramatic currency crisis (i.e. a strong loss of value of the domestic currency and a huge loss of central bank foreign exchange reserves)[6]. To understand the roots of this event, it is interesting to remind here that in the years preceding the crisis, the Icelandic banking sector experienced an incredibly quick expansion thanks to foreign financing, which allowed it to boost its assets from 100 to almost 900% of GDP in only four years. Indeed, the internationally integrated Icelandic banks used mainly short term debt in foreign currencies from the international wholesale capital market to finance themselves, leaving the Icelandic banking sector totally exposed to a sudden stop, i.e. a sharp reversal of capital flows. That is exactly what happened in the beginning of October 2008, when some international investors did not roll over the short term debt to the Icelandic banks, the banks tried to find other sources of funding but were not able to do so in the context of the global financial crisis that followed Lehman Brothers bankruptcy. They turned next to the central bank of Iceland (ICB) for funding but the ICB could not provide liquidity in currencies other than the Icelandic króna, leaving no other choice to the Icelandic authorities than to declare the banks bankrupt and to take over the bank's operations. Following the collapse of its financial system, the Icelandic government was forced to negotiate a $2.1 billion loan

[4] See Icelandic Central Bank (2001) for details.

[5] All the numbers concerning the Icelandic crisis are from Buiter and Sibert (2008).

[6] The 2008 Icelandic twin crisis is a good example of a coupled bank-currency run. Kaminsky and Reinhart (1999) – the seminal empirical paper in the literature on twin crises – document well the numerous occurrences of those since the 1970s and find that banking troubles help predict currency crises: in a sample of 20 countries, they identify 76 currency crises and 26 banking crises during the period 1970-95. Among those, 23 banking crises took place since 1980, 18 of which happened simultaneously with a currency crisis.

from the International Monetary Fund in order to be able to stabilize the foreign exchange market by supporting the appreciation of the Icelandic króna and to restructure its banking system.

The Icelandic example highlights the main shortcomings of Chang and Velasco's model. First, for a paper that tries to explain the link between banking and currency crises, it never exhibits a twin crisis. There is either one type of crisis or the other but never both at the same time. Basically, when a bank run occurs the central bank can either decide to let the banking sector fail or to save it but in this case a balance-of-payment crisis happens as the central bank runs out of foreign exchange reserves. Second, and more importantly, although empirical papers find that sudden stops[7] in capital flows are one of the main reasons explaining twin crises in internationally opened financial systems, the model does not consider the possibility of foreign creditors withdrawing their funds from the country.

The purpose of our paper is therefore to amend Chang and Velasco's model by giving a prevailing position to foreign investors and to the sudden stop phenomenon (i.e. to allow foreign investors to be at the origin of the crisis when they stop lending and claim what they have lent before) in accordance with what we observed during the Icelandic crisis of 2008[8].

This allows us to qualify the results of Chang and Velasco and to get more insights about the financial mechanisms at work during a combined currency and banking meltdown. More precisely, the model intends to determine which combination of exchange rate regime and central bank policy is less prone to a twin crisis focusing on the sudden stop phenomenon as an underlying source of instability to the financial system. First, we show that having a central bank that is acting as a LoLR in case of a sudden stop is not compatible with having a fixed exchanged rate regime. Contrary to Chang and Velasco, acting as a LoLR or not are not equivalent in the sense that the central bank that acts as LoLR fails without saving the banking sector, therefore provoking a twin crisis instead of a plain banking crisis. Second, our main finding is that the flexible exchange rate regime coupled with a central bank acting as a "Printer of Last Resort" still dominates all other combinations of exchange rate regime and central bank policy under some strict condition

[7]The expression sudden stop was first suggested, and the phenomenon highlighted, in Dornbusch, Goldfajn, Valdés, Edwards, and Bruno (1995).

[8]To be exhaustive, sudden stops are also considered a prominent feature of the financial crises of Argentina (in 1982–83 and 1994–95), Chile (in 1981–83 and 1990–91), Ecuador (in 1995–96), Hungary (in 1995–96), Indonesia (in 1996–97), Malaysia (in 1993–94), Mexico (in 1981–83 and 1993–95), Philippines (in 1996–97), Venezuela (in 1992–94), Korea (in 1996–97), Thailand (in 1996–97) and Turkey (in 1993–94)

regarding the size of foreign borrowing relative to some parameters of the domestic economy. However, if this condition is not fulfilled, then the flexible rate regime is no panacea: the good equilibrium is not unique anymore and a bad equilibrium with a twin crisis becomes a possibility again contrary to Chang and Velasco's main result. This could have some important policy implications because if a country really wants to avoid a crisis, it could try to limit bank borrowing from abroad, but this would come at some cost in terms of efficiency.

The paper is organized as follows. Section 3.2 presents the general environment of the model. Section 3.3 shows the characterization of the optimal allocation. The decentralization of the solution through a banking system with demand deposit contracts and different central bank procedures under fixed exchange rate is discussed in section 3.5. The major insight of this paper is to give a significant role to foreign investors, that is what is done in Section 3.5.3. Section 3.6 presents the case with flexible exchange rate and the possibility for the central bank to act as a "Printer of Last Resort" only when some conditions are met. Section 3.7 concludes.

3.2 The Basic Framework

The models builds on Chang and Velasco (2000c) and uses their structure unless otherwise noted. There are 3 dates, $t \in \{0, 1, 2\}$ and 2 currencies: a foreign currency or \$ and a local currency or Peso that can be created costlessly by the domestic central bank. The exchange rate between the two is denoted by ε_t for $t \in \{0, 1, 2\}$ (i.e. ε_t is the quantity of pesos obtained per dollar). We focus here on a small open economy, i.e. an economy that is small enough not to influence the price of the only good of the model – the international consumption good – whose price will be fixed and normalized to 1\$.

The economy is populated by a continuum of *ex ante* identical agents of measure one – the domestic agents – that have an initial endowment of $e > 0$ units of the good (or e\$ which is the same here) and that can be of two types. Their type is revealed to them at date 1 and is a private information after that: either they are early type with probability λ and they get utility from consuming the international good in $t = 1$, or they are late type with probability $1 - \lambda$ and they get utility from consuming in $t = 2$ but also from holding pesos between date 1 and date 2.

The expected utility of the representative domestic agent can be represented by:

$$\lambda u(x) + (1-\lambda)u(y + \chi(\frac{M}{\varepsilon_2})) \tag{3.1}$$

where u is a smooth, strictly increasing, strictly concave and twice differentiable function that respects Inada conditions, x is the consumption of the early consumers and y of the late ones, M is the quantity of pesos held by late consumers between dates 1 and 2, χ is a concave function such that $\chi(0) = 0$, $\chi'(0) = \infty$ and $\chi'(\bar{m}) = 0$ for some $\bar{m} > 0$. So $\bar{m}$ is the satiation level of pesos. Note that domestic agents derive utility from the quantity of pesos deflated by the exchange rate in $t = 2$ because those pesos will be used at date 2 to purchase the international good.

There are two different assets. On the one hand, agents can decide to invest in a short-term asset that yields zero interest rate and that can be liquidated at any time without any cost. This can be thought as a risk free international capital market where 1\$ (or 1 unit of good) invested in one period brings you 1\$ in the next one. On the other hand, there is a long-term asset that yields $R > 1$ \$ at date 2 for each \$ invested at date 0. As in the Diamond and Dybvig article, the return of the long term asset is deterministic[9]. If this asset has to be liquidated in $t = 1$, it only yields $r < 1$ \$.

The domestic agents are also able to borrow from foreign investors up to an exogenous credit ceiling of f\$ at zero interest rate. They can borrow from abroad in $t = 0$ and 1, as long as the total borrowing does not exceed f\$. However, because we want to study the sudden stop phenomenon, we assume, that all the foreign debt is short-term, contrary to Chang and Velasco[10].

Finally, there exists a central bank that performs three different tasks in this economy: (1) it exchanges \$ for pesos and vice-versa at a fixed or at a flexible exchange rate depending on the chosen regime; (2) it gives credit to banks; and (3) it can act as lender of last resort (LoLR)[11] or not depending on the chosen central bank policy towards banks.

[9]To come back to the Icelandic example, the deterministic nature of the return on the long-term asset could reflect the belief that the banking crisis was not a solvency crisis linked to non-performing loans, but that it was merely the result of a non-resilient highly leveraged banking model vulnerable to self-fulfilling crises. This is consistent with the widespread idea that the investments realized in Iceland in the years preceding the meltdown were mainly sound projects (e.g. 25% of the loans were made to enhance the profitable aluminum production in Iceland).

[10]This can be justified by the fact that banks in emerging countries use mainly short-term debt because it is cheaper and/or because the foreign investors consider it less risky.

[11]The precise definition of acting as a LoLR will be provided in section 3.5.3 when we study the case in which there is one.

3.3 The Social Planner's Solution

The problem for the social planner (which takes charge of the central bank policy and therefore can decide to set the exchange rate ε_2 to 1 because it entirely determined by the supply of pesos by the central bank) is to choose $\{x, y, d_0, d_1, k, M\}$ – where x is the consumption of the early consumers and y of the late ones, M is the quantity of pesos given to late consumers between $t = 1$ and $t = 2$, k denotes the investment in the long-term project, d_0 the borrowing from abroad at date 0 (rolled over at date 1 until date 2) and d_1 the new borrowing from abroad contracted at date 1 (repaid in 2) – in order to maximize the representative agent expected utility (given that no run is anticipated):

$$\lambda u(x) + (1 - \lambda)u(y + \chi(M)) \tag{3.2}$$

subject to:

$$k \leq d_0 + e \tag{3.3}$$

$$\lambda x + d_0 \leq d_0 + d_1 \tag{3.4}$$

$$(1 - \lambda)y \leq Rk - d_0 - d_1 \tag{3.5}$$

$$d_0 + d_1 \leq f \tag{3.6}$$

$$x \leq \chi(M) + y \tag{3.7}$$

$$x, y, M, k, d_0, d_1 \geq 0 \tag{3.8}$$

Equation (3.3) is the period 0 constraint on investment: the planner uses the endowment and long-term borrowing to invest in the long-term asset. Equation (3.4) is the feasibility constraint at date 1: the planner borrows short-term to repay the early consumers – note that the possibility to borrow abroad in 1 allows the planner to avoid investing in the short-term asset at date 0. Moreover, the planner has to roll over the foreign debt contracted in period 0 as we assume that all the foreign debt is short-term. This is why d_0 appears both in the inflows (RHS of 3.4) and in the outflows (LHS) of the bank. Equation (3.5) is the feasibility constraint at date 2: the planner uses the return of the long-term asset minus what it has borrowed from foreign investors in 0 and 1 to repay the late consumers. Equation (3.6) states that the total amount of foreign borrowing at date 0 and 1 cannot exceed the exogenous credit ceiling. Equation (3.7) is the incentive compatibility constraint ensuring that the late type depositors do not have any incentive to lie about their type to withdraw earlier. Finally, (3.8) contains the non-negativity constraints. The solution of this problem will be denoted with tildes:

Proposition 4. *The optimal allocation* $\left\{\tilde{x}, \tilde{y}, \tilde{d}_0, \tilde{d}_1, \tilde{k}, \tilde{M}\right\}$ *satisfies:*

$$\tilde{k} = \tilde{d}_0 + e \tag{3.9}$$

$$\lambda\tilde{x} = \tilde{d}_1 \tag{3.10}$$

$$(1-\lambda)\tilde{y} = R\tilde{k} - \tilde{d}_0 - \tilde{d}_1 \tag{3.11}$$

$$\tilde{d}_0 + \tilde{d}_1 = f \tag{3.12}$$

and the following first order conditions:

$$u'(\tilde{x}) = Ru'(\tilde{y} + \chi(\tilde{M})) \tag{3.13}$$

and

$$\chi'(\tilde{M}) = \chi'(\bar{m}) = 0 \tag{3.14}$$

Given that providing pesos to the late depositors is made at no cost, the optimal level of pesos provided to them by the planner is the satiation level, as (3.14) shows. Note also that, since $R > 1$ and u is increasing and concave, (3.13) guarantees that the incentive constraint (3.7) does not bind.

3.4 The Decentralized Economy

The following sections examine one by one the different ways to decentralize this allocation for all combinations of exchange rate regime (fixed or flexible) and central bank policy towards banks (LoLR or not).

In order to study the twin crisis phenomenon, the best way to implement this solution in a decentralized setup is to resort to a financial intermediary offering demand deposit contracts to the agents as in Diamond and Dybvig (1983). On top of that, the bank and the central bank need to agree on a procedure about how dollars and pesos are exchanged and also on a lending-borrowing relationship in which the central bank gives credit in pesos at 0% interest rate to the bank to finance the withdrawal of pesos by late consumers at date 1 (the one they like to keep between $t = 1$ and 2)[12]. However, these pesos should not be converted into dollars until date 2. Therefore, even if we assume that types are private information, withdrawals should be observable by the central bank, which implies that the currency exchange at the central bank only takes place at the end of the periods after the bank is closed and withdrawals are over.

Figure 3.1 provides a schematic representation of the decentralized economy with all the different institutions and the way they interact.

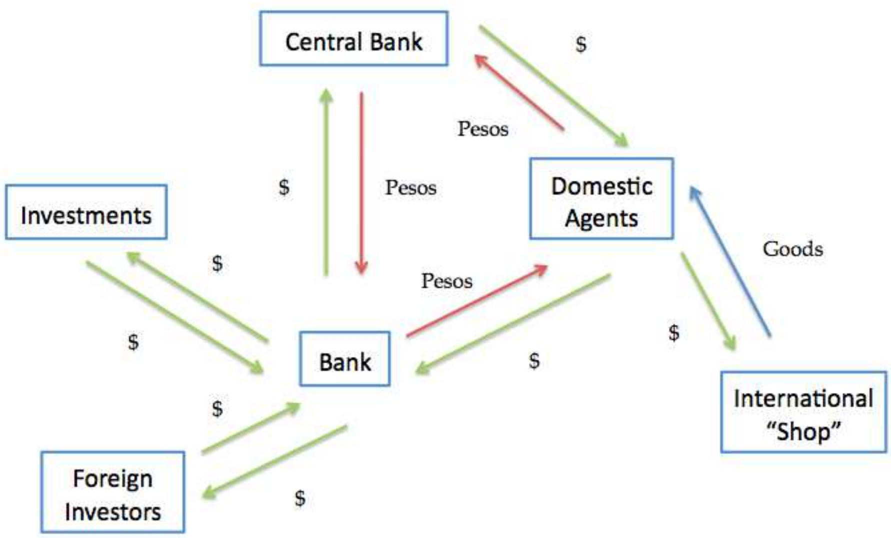

Figure 3.1: Schematic representation of the decentralized economy

To make it simple, agents deposit their endowment in dollars at the bank. The bank invests it, as well as what it borrows from foreign investors, in short and long-term assets in dollars. When the depositors need to withdraw funds in pesos[13], the bank uses the return of its investments (or liquidates them if necessary) to get some dollars that it will exchange

[12] This loan of pesos allows the bank to reach the optimal allocation and distinguishes the fixed and flexible regimes from a currency board regime in which the central bank can only create pesos if it receives dollars in exchange.

[13] We have to assume that agents can only withdraw pesos and not dollars from the bank to give a role to the local currency.

at the central bank against pesos that it will give to its depositors. The depositors then go to the central bank to exchange the withdrawn pesos for dollars that they can finally use to buy some goods.

3.5 Decentralization through a Fixed Exchange Rate Regime

In this section, we try to see if we can implement the optimal allocation in a decentralized fashion in a fixed exchange rate regime. By fixed rate, we mean that the central bank exchanges the dollars for pesos and vice-versa at rate $\varepsilon_1 = \varepsilon_2 = 1$ as long as its foreign exchange reserves allow it to do so. By foreign exchange reserves, we mean the quantity of foreign currency held by the central bank after the bank came to exchange dollars against pesos.

3.5.1 The Bank's Problem

We focus on the bank's optimization problem assuming that the commercial bank and its depositors agree on a demand deposit contract. As in Diamond and Dybvig (1983), we assume that free entry and competition in the banking sector will force the bank to maximize the depositors expected utility[14]:

$$\lambda u(x) + (1-\lambda)u(y + \chi(M)) \tag{3.15}$$

subject to:

$$k \leq d_0 + e \tag{3.16}$$

$$(1-\lambda)M \leq h \tag{3.17}$$

$$\lambda x + d_0 \leq d_0 + d_1 \tag{3.18}$$

$$(1-\lambda)y \leq Rk - h - d_0 - d_1 + (1-\lambda)M \tag{3.19}$$

$$d_0 + d_1 \leq f \tag{3.20}$$

$$x \leq \chi(M) + y \tag{3.21}$$

$$x, y, M, k, l, h, d_0, d_1 \geq 0 \tag{3.22}$$

[14]The bank can also be seen as a coalition of agents pooling their resources together in order to share the liquidity risk and maximize their welfare, as explained in section 2.3.3

The demand deposit contract stipulates that in $t = 0$, the depositors have to surrender to the bank their endowment and their capacity to borrow from abroad. The bank borrows d_0\$ from abroad and invests it in the long-term asset with the endowment of its depositors, as (3.16) shows. In exchange, depositors can withdraw either x pesos in $t = 1$ if they claim to be early type, as stated in equation (3.18)[15], or M pesos in $t = 1$ (financed by the bank by borrowing h pesos from the central bank), as stated in equation (3.17), and $y - M$ pesos in $t = 2$ if they claim to be late type.[16]

The contract and the bank optimal strategy (that does not take run into account) will therefore be determined by:

$$\tilde{h} = (1 - \lambda)\tilde{M} \tag{3.23}$$

because the central bank provides pesos for the late depositors at zero interest in 1 reimbursed in 2, and by conditions (3.9) to (3.14) as in Proposition 4.

3.5.2 The Good Equilibrium

Given this demand deposit contract,

Proposition 5. *There exists a good equilibrium in which there is no sudden stop and in which the domestic depositors do not lie about their types. The outcome is the optimal allocation and neither the commercial bank nor the central bank fails.*

Proof. Suppose that all agents report their types honestly and that foreign investors continue lending to the bank:

In $t = 0$, the banks borrows $\tilde{d_0}$\$ from abroad and invests those with the e\$ in the long-term asset.

In $t = 1$, the foreign investors do not claim the initial debt of $\tilde{d_0}$\$ and roll over their lending to the bank. The early consumers come to the bank to withdraw $\tilde{x}$ pesos, so the bank borrows a total of $\tilde{d_1} = \lambda\tilde{x}$ \$ from abroad, goes to the central bank, exchanges those dollars for pesos at the rate of 1 and gives those $\lambda\tilde{x}$ pesos to the early consumers. The central bank has therefore $\lambda\tilde{x}$ \$ in reserves. The early agents go to the central bank to get dollars against their pesos, the central bank has enough dollars to exchange those at the rate of 1, so they can buy the international goods with their dollars. The late

[15] As in the planner's problem, the bank also has to roll over its initial debt in $t = 1$.

[16] For simplification purposes that will be evident later, we will say that the late depositors give back their M pesos to the bank in $t = 2$ and then get y pesos from it in the same period which is equivalent to get M pesos in $t = 1$ and $y - M$ pesos in $t = 2$.

consumers come to withdraw $\tilde{M}$ pesos that are financed by a central bank loan of pesos to the commercial bank at 0% interest rate.

In $t = 2$, the late consumers give back their pesos to the bank, those are used to reimburse the central bank loan. The bank gets $R\tilde{k}\$$ from the long-term asset. It uses part of it to reimburse the loans from abroad and goes with the rest ($R\tilde{k} - f$ \$) to the central bank to get pesos at the rate of 1, and gives those to late consumers. They go to the central bank for dollars and get them at the rate of 1 so they can buy goods with them.

Those behaviors are all optimal: early consumers have no incentive to lie. If they lie they get $\tilde{M}$ pesos in 1 that they must carry to date 2 when pesos will be of no use for them. Late consumers, if they lie, get $\tilde{x}$ pesos which they must convert in dollars in 1. They will buy goods in 2 using those dollars, but because of equation (3.21) they have no incentive to lie either because they would get less utility acting this way. Concerning the foreign investors, they are risk neutral investors so in this case they are indifferent between roll over lending or not, so there exists an equilibrium where they do not stop lending.

Hence there exists an equilibrium where the optimal allocation is implemented. □

3.5.3 The Possibility of a Sudden Stop

In the last section of Chang and Velasco (2000c), the authors use a sketch of this framework to see if the banking sector is more fragile with or without borrowing from abroad in a fixed exchange rate regime. However, because they assume that the bank is always committed to repay the foreign investors (i.e. it will stop liquidating the long-term asset when it reaches the amount necessary to repay foreign investors in the case of a domestic depositor run), the foreign investors never have an incentive to stop lending so the model can not exhibit a sudden stop provoking a twin crisis (i.e. a simultaneous banking and balance-of-payment crisis). That is why in this section we will focus on sudden stops from foreign investors as a source of instability.

Indeed, because we assume that the bank only uses short term borrowing and that it is not committed to repay the foreign investors (i.e. it will liquidate all the long-term asset in $t = 1$ if necessary), a sudden stop can happen in our new environment. By sudden stop, we mean that in $t = 1$, the foreign investors decide not roll over the initial debt $\tilde{d}_0\$$ lent in $t = 0$, but also that they do not lend the supplementary $\tilde{d}_1\$$ used for the withdrawal of early consumers. We can distinguish two different cases depending on the chosen central bank policy.

3.5.3.1 The central bank does not act as LoLR

By not acting as LoLR, we mean that the central bank does not lend pesos to the bank if the bank does not have enough pesos to repay all the domestic depositors claiming to be early type.[17] In this case, in addition to the good equilibrium described in Proposition 5, there exists another equilibrium:

Proposition 6. *If*

$$\tilde{x} + \tilde{d}_0 > r\tilde{k} \tag{3.24}$$

there exists a bad equilibrium in which a sudden stop and a domestic depositor run occur and in which the bank fails but not the central bank. A sudden stop is a necessary condition for the bank collapse if $\tilde{x} \leq rl^d$*, where* $l^d = \frac{R\tilde{k} - \tilde{d}_0}{R}$ *is the maximum amount of long-term asset liquidated in* $t = 1$ *allowing the bank to be able to repay the initial debt to the foreign investors at date 2.*

Proof. Note first that we only consider the case where $\tilde{x} \leq rl^d$ because we are interested in the case where the instability of the system finds its origins in the borrowing from abroad. Suppose that we would have $\tilde{x} > rl^d$, a domestic run would provoke a bank collapse, whereas in our case the self-fulfilling sudden stop is necessary for the bank to collapse. To say it differently, in the case where $\tilde{x} \leq rl^d$, if the foreign investors do not panic, the bank will not collapse (even if there is a depositor run), that is why this case is the more interesting when studying the sudden stop phenomenon.

Suppose that in $t = 1$ all foreign investors claim their initial debt $\tilde{d}_0$ and all domestic depositors claim to be early type and want to withdraw $\tilde{x}$ pesos. As we assume sequential service, they all queue in the same line in random order at the bank. Condition (3.24) implies that the bank is forced to liquidate all its investment in the long-term assets to meet their demand. Indeed, the LHS of equation 3.18 (the outflows for the bank) becomes $\tilde{x} + \tilde{d}_0$ whereas the RHS becomes rl – where $l \leq k$ is the liquidation of the long-term asset – as this is the only possible inflow for the bank because of the sudden stop. Therefore bank gets $r\tilde{k}\$$: Part of it $(\frac{\tilde{d}_0}{\tilde{d}_0 + \tilde{x}} r\tilde{k}\$)$ is used to reimburse $\tilde{d}_0\$$ to the first $\frac{r\tilde{k}}{\tilde{d}_0 + \tilde{x}}$ foreign investors and part of it $(\frac{\tilde{x}}{\tilde{d}_0 + \tilde{x}} r\tilde{k}\$)$ is exchanged for pesos at the rate of 1 at the central bank to give

[17] The fact that the central bank is not acting as LoLR does not prevent her from lending pesos to the bank in $t = 1$ to finance the withdrawal of $\tilde{M}$ pesos by those claiming to be late depositors.

$\tilde{x}$ Pesos to the first $\frac{r\tilde{k}}{\tilde{d}_0+\tilde{x}}$ domestic depositors in the line. Because it can not repay everyone (domestic and foreign) the bank fails, but there are just enough dollars at the central bank when the domestic agents come to exchange their pesos for dollars at the rate of 1 so there is no balance-of-payment crisis.

In this case, claiming to be early type at date 1 is now optimal for all depositors: because the bank fails at date 1, no withdrawal is possible at date 2. Furthermore, because the central bank will have no dollar left in 2, the $\tilde{M}$ pesos withdrawn in 1 would be worthless. Concerning the foreign investors, because the bank closes in 1, it is optimal for them to claim the early debt and not to lend the additional amount in 1, otherwise they would make a loss. □

3.5.3.2 The central bank acts as LoLR

By acting as a LoLR, we mean that the central bank accepts to lend to the bank as much pesos as necessary in $t = 1$ to repay all the domestic depositors claiming to be early type. In this case, again, in addition to the good equilibrium described in Proposition 5, there is another equilibrium:

Proposition 7. *If $\tilde{x} + \tilde{d}_0 > r\tilde{k}$ there exists a bad equilibrium in which a twin crisis occurs. A sudden stop and a domestic depositor run provoke the failure of the bank and the central bank runs out of foreign currency before it can satisfy all the demand. If $\tilde{x} \leq rl^d$, a sudden stop is a necessary condition for this twin crisis to happen.*

Proof. Note first that we focus again on the case where $\tilde{x} \leq rl^d$ for the reason explained as in Proposition 6.

Suppose, as in the previous section, that in $t = 1$ all foreign investors claim their initial debt and all domestic depositors claim to be early type and want to withdraw $\tilde{x}$ pesos. As we assume sequential service, they all queue in the same line in random order at the bank. Condition (3.24) implies that the bank must liquidate all the long-term asset. Indeed, the LHS of equation (3.18) – the outflows of the bank – becomes $\tilde{x} + \tilde{d}_0$ whereas the RHS becomes this time $rl + b$ – where b is the pesos borrowed from the LoLR – because the bank liquidates the long-term asset but it has also access to the LoLR if the liquidation is not enough to repay the domestic depositors. It gets $r\tilde{k}\$$, part of it $(\frac{\tilde{d}_0}{\tilde{d}_0+\tilde{x}}r\tilde{k}\$)$ is used to reimburse $\tilde{d}_0\$$ to the first $\frac{r\tilde{k}}{\tilde{d}_0+\tilde{x}}$ foreign investors and part of it $(\frac{\tilde{x}}{\tilde{d}_0+\tilde{x}}r\tilde{k}\$)$ is exchanged for pesos at the rate of 1 at the central bank to give $\tilde{x}$ Pesos to the first $\frac{r\tilde{k}}{\tilde{d}_0+\tilde{x}}$ depositors.

The central bank acts as a LoLR and lends the missing pesos ($b = \tilde{x} - \frac{\tilde{x}}{\tilde{d}_0+\tilde{x}} r\tilde{k}$ pesos) to the bank so it can repay all the domestic depositors in pesos, but because it can not repay all the foreign investors the bank fails anyway. However, this time there are not enough dollars at the central bank to exchange at the rate of 1 so for the first time there is a balance-of-payment crisis at the same time as the banking crisis. Therefore, a twin crisis occurs in this environment. As in the previous section, it is easy to check that everyone behave optimally acting this way. □

In the end, these results tell us that having a central bank that is acting as a LoLR in case of a sudden stop is not compatible with having a fixed exchanged rate regime. So, contrary to Chang and Velasco, acting as a LoLR or not acting as LoLR are not equivalent in the sense that if the central bank acts as LoLR fails without saving the banking sector anyway, whereas if it does not act as a LoLR only the bank will fail but not the central bank.

3.6 Decentralization through a Flexible Rate Regime

In this section, we try to see if we can implement the optimal allocation in a decentralized fashion in a flexible exchange rate regime. By flexible rates, we mean that the central bank is not committed anymore to exchange dollars for pesos at the rate of 1. Moreover, from now on, the central bank has a policy of acting not exactly as a LoLR but more as a "Printer of Last Resort" (PoLR) because it will determine the exchange rate by printing pesos in order to save the bank. Our goal in this section is to see if the combination of a flexible exchange rate regime and PoLR policy dominates the previous combinations of Section 3.5 in the sense that it leads to a unique good equilibrium as in Chang and Velasco's paper.

In addition, we continue to assume that at date 1 the central bank gives credit to the bank to finance the withdrawal of pesos by late depositors, that the bank is not committed to repay all foreign investors first and that it only uses short-term borrowing, as in the previous section.

Finally, we continue to assume sequential service when domestic depositors come to the bank to withdraw pesos. However, in a flexible exchange rate regime, when all the agents have finished withdrawing pesos from the bank, we assume that the rate at which pesos are exchanged for dollars is determined by the quantity of dollars that the central bank has in reserve compared to the total quantity of pesos agents want to exchange.

Therefore, a possible devaluation mechanism to save the bank is the following:

- in $t = 1$, when the bank comes to the central bank to exchange dollars for pesos to give to the depositors claiming to be early type, the central bank, which is able to print as many pesos as it wants, delivers the quantity of pesos that is needed by the bank in exchange for "some" dollars (we will clarify this notion when we will define more carefully the good equilibrium). Then when those claiming to be early type come to the central bank (all at the same time), it exchanges their pesos for dollars using what it has in reserves at the rate $\varepsilon_1 = \frac{D_1}{S_1}$ with D_1 denoting the quantity of Pesos that depositors want to exchange at date 1 (i.e. "demand" for dollars), and S_1 denoting the foreign currency reserves of the central bank ("supply" of dollars).

- in $t = 2$, same thing with $\varepsilon_2 = \frac{D_2}{S_2}$ with D_2 denoting the quantity of Pesos that depositors want to exchange at date 2, and S_2 the foreign currency reserves.

Given this central bank procedure let's check if there are still multiple equilibria in this setup:

Proposition 8. *There exists a good equilibrium in which there is no sudden stop and domestic agents do not lie about their types. The outcome is the social optimum, and the exchange rate is 1 at all times.*

Proof. Suppose that all domestic agents report their types honestly and that the foreign investors continue to lend to the bank at date 1.

In $t = 1$, the early consumers withdraw from the bank a total of $\lambda\tilde{x}$ Pesos financed by dollars borrowed abroad and exchanged by the bank at the central bank at the rate of 1. After their withdrawal, the consumers go to the central bank to exchange their pesos for dollars. Therefore the exchange rate establishes itself at $\varepsilon_1 = 1$ because the quantity of pesos to exchange equals the quantity of dollars held at the central bank. The late consumers' withdrawal of $(1 - \lambda)\tilde{M}$ Pesos is financed by a loan at the central bank

In $t = 2$, the late consumers give their pesos back to the bank to reimburse the central bank. The bank get $R\tilde{k}$ \$ from the long-term asset, it reimburses the foreign investors with $(\tilde{d_0} + \tilde{d_1})$ \$ and exchanges the rest at the the central bank to get $R\tilde{k} - (\tilde{d_0} + \tilde{d_1})$ Pesos. The late depositors will then go to the central bank for dollars and get those at the rate of $\varepsilon_2 = 1$. Again, everyone behave optimally acting this way. The outcome of this equilibrium is therefore the same as the good equilibrium in a fixed exchanged rate regime. □

Proposition 9. *A bad equilibrium does not exist if* $\tilde{d}_0 < r\tilde{k}$

Proof. Let's check if it is optimal for late consumers to lie about their type in this framework.

In $t = 1$, suppose that foreign investors take their money back and suppose also that a fraction of domestic agents $\lambda^r > \lambda$ claims to be early type. The bank liquidates part of the long-term asset to do two things: First, to reimburse the foreign debt it contracted at date 0 (in the end $\frac{\tilde{d}_0}{r}$ of the long-term asset will be liquidated for this reason) and second to finance the withdrawal of those claiming to be early depositors (because it can not finance it by borrowing from abroad because of the sudden stop). But the central bank stands ready to exchange whatever amount of dollars against the needed amount of pesos, so the bank should only liquidate a really small portion of the long-term asset for the domestic depositors. Every time an agent claiming to be early type comes to withdraw, the bank liquidates $\frac{\epsilon}{r}\tilde{x}$ units of the long-term asset to get $\epsilon\tilde{x}\$$, then it goes to the central bank which accepts to exchange those dollars for $\tilde{x}$ Pesos. The bank gives those pesos to the agent claiming to be early type. When the withdrawal is over, those agents go to the central bank and exchange a total amount of $\lambda^r\tilde{x}$ pesos for dollars at the rate $\varepsilon_1 = \frac{\lambda^r\tilde{x}}{\lambda^r\epsilon\tilde{x}} = \frac{1}{\epsilon}$ so each of them gets: $\epsilon\tilde{x}\$$. In contrast, the remaining $(1 - \lambda^r)$ depositors claiming to be late consumers will get from the bank $\tilde{M}$ pesos financed by a loan from the central bank.

In $t = 2$, the bank uses the $(1 - \lambda^r)\tilde{M}$ Pesos from the late depositors to reimburse its debt at the central bank. It gets $R(\tilde{k} - \frac{\tilde{d}_0}{r} - \lambda^r\frac{\epsilon}{r}\tilde{x})\$$ from the long-term asset, it goes to the central bank to exchange it for pesos. The central bank does it at the rate of 1 and gives the bank $R(\tilde{k} - \frac{\tilde{d}_0}{r} - \lambda^r\frac{\epsilon}{r}\tilde{x})$ Pesos. The bank will then distribute those pesos to the remaining $(1 - \lambda^r)$ depositors who will go to the central bank to get dollars. Given the reserves of the central bank, the exchange rate will be $\varepsilon_2 = 1$.

In the end, the late truth tellers will get $\chi(\tilde{M}) + R(\tilde{k} - \frac{\tilde{d}_0}{r} - \lambda^r\frac{\epsilon}{r}\tilde{x}).\frac{1}{1-\lambda^r}$ compared to $\epsilon\tilde{x}$ for the late liars. And we have:

$$\epsilon\tilde{x} < \tilde{x} < \chi(\tilde{M}) + \frac{1}{1-\lambda}R(\tilde{k} - \tilde{d}_0 - \lambda\tilde{x}) < \chi(\tilde{M}) + \frac{1}{1-\lambda^r}R(\tilde{k} - \frac{\tilde{d}_0}{r} - \lambda^r\frac{\epsilon}{r}\tilde{x}) \qquad (3.25)$$

The first inequality in (3.25) is obvious, the second one is the incentive compatibility constraint and the third one hold as long as ϵ is chosen such that:

$$\epsilon < \frac{r}{\lambda^r \tilde{x}}\left[\tilde{k} - \frac{\tilde{d}_0}{r} - \frac{1-\lambda^r}{1-\lambda}(\tilde{k} - \tilde{d}_0 - \lambda\tilde{x})\right] \tag{3.26}$$

but because we have sequential service, the bank does not know in advance λ^r so it should choose ϵ such that (3.26) hold even if everybody run – i.e. if $\lambda^r = 1$ – that is:

$$\epsilon < \frac{r\tilde{k} - \tilde{d}_0}{\tilde{x}} \tag{3.27}$$

In this case, it is not optimal for late depositors to run in $t = 1$. So if $\tilde{d}_0 < r\tilde{k}$, the foreign investors will not have any incentive to take back their funds in $t = 1$ because a sudden stop will not trigger a run by domestic depositors, so a sudden stop combined with a domestic run is not an equilibrium if the bank acts this way and the central bank designs the right devaluation policy in advance. □

To sum up, with this specific central bank procedure, the result of Chang and Velasco (2000c) about flexible rates without foreign investors seems to hold in the presence of foreign investors in the special case where $\tilde{d}_0 < r\tilde{k}$ because by making a domestic run not optimal, it makes a foreign investors run not optimal either. However, this is true only if the sudden stop is motivated by the fear of a domestic depositor run, but in the case where $\tilde{d}_0 \geq r\tilde{k}$, another equilibrium can emerge:

Proposition 10. *If*

$$\tilde{d}_0 \geq r\tilde{k} \tag{3.28}$$

there exists a bad equilibrium in which a twin crisis occurs.

Proof. In this case, the central bank will be powerless if a sudden stop happens. Indeed if $\tilde{d}_0$, the debt contracted at date 0, is too high compared to the liquidation value $r\tilde{k}$ then the central bank can not act as a Printer of Last Resort as in the previous case. Suppose that foreign investors stop lending and claim the initial debt in 1 because they fear that the other foreign investors will do so. Condition (3.28) implies that the liquidation value of the long-term assets will not be enough to repay them all and the bank will fail. So whatever the central bank tries to do to avoid the domestic depositors to withdraw it will not work this time as there will be nothing left for them at date 2 because of the sudden stop at date 1. It is then optimal for domestic depositors to run in 1 even if the central bank threatens them with a devaluation. □

To understand more precisely what type of crisis can happen, suppose that in $t = 1$ all foreign investors claim their initial debt and all domestic depositors claim to be early type and want to withdraw $\tilde{x}$ pesos. As we assume sequential service, they all queue in the same line in random order at the bank. Condition (3.28) implies that in the end the bank will liquidate all the long-term asset and get $r\tilde{k}\$$. If the bank tries to act as in the previous case and the central bank tries to act as a PoLR, $\frac{\tilde{d_0}}{\tilde{d_0}+\epsilon\tilde{x}}r\tilde{k}\$$ are used to reimburse the first $\frac{r\tilde{k}}{\tilde{d_0}+\epsilon\tilde{x}}$ foreign investors and $\frac{\epsilon\tilde{x}}{\tilde{d_0}+\epsilon\tilde{x}}r\tilde{k}\$$ are exchanged for $\frac{r\tilde{k}}{\tilde{d_0}+\epsilon\tilde{x}}\tilde{x}$ Pesos at the central bank to give $\tilde{x}$ Pesos to the first $\frac{r\tilde{k}}{\tilde{d_0}+\epsilon\tilde{x}}$ depositors. When the withdrawal is over, those agents go to the central bank to exchange their pesos against the dollars in reserve.The exchange rate will therefore establish itself at $\varepsilon_1 = \frac{1}{\epsilon}$, so each of them gets: $\epsilon\tilde{x}\$$. If strictly speaking there is no balance-of-payment crisis because the central bank does not run out of reserves before the demand is exhausted, there is nonetheless a currency crisis in the form of a strong devaluation of the currency that impacts heavily the purchasing power and hence the consumption of the domestic agents (they will get $\epsilon\tilde{x}$ units of good instead of $\tilde{x}$ units). Contrarily to the previous case, this devaluation is permanent because there is no long-term assets left and no dollar reserves so pesos would be worthless at date 2. And because the bank is not able to repay all the foreign investors and all the depositors, it fails, so a banking crisis and a currency crisis occur simultaneously in this environment: we obtain a different form of twin crisis than the one found in section (3.5.3.2), but in fact much more similar to the situation observed during the 2008 Iceland financial crisis.

Therefore, with foreign investors, a flexible exchange rate regime does not guarantee the uniqueness of the equilibrium in the case where $\tilde{d_0} \geq r\tilde{k}$ or equivalently (in terms of the parameters of the economy) where $\tilde{d_0} \geq \frac{r}{1-r}e$ (from equation 3.9). One of the possible policy implications suggested by this last result is that it could be interesting for a country that would like to avoid absolutely a crisis to limit bank borrowing from abroad below a certain threshold ($\tilde{d_0} < \frac{r}{1-r}e$ if we take the model literally). In this case, the central bank would be able to act as a Printer of Last Resort and therefore to avoid the bad equilibrium in the case of a flexible exchange rate. However, there would be a trade-off between being able to reach the optimal allocation and being sure not to fall into a bad equilibrium. The structure of the model does not allow us to answer which solution is better in terms of welfare because crises are self-fulfilling equilibria with no probability attached so it is not possible to evaluate the expected cost or benefits in terms of welfare that such a policy could have.

3.7 Conclusions

We have developed a simple model – an extension of the influential Chang and Velasco (2000c) article – that is not only able to exhibit twin crises under certain circumstances but also to amend Chang and Velasco's results by giving a prevailing position to foreign investors and to the sudden stop phenomenon.

First, it tells us explicitly that having a central bank that is acting as a LoLR in case of a sudden stop is not compatible with having a fixed exchanged rate regime. So, contrary to Chang and Velasco, acting as a LoLR and not are acting as LoLR are not equivalent in the sense that the central bank acting as LoLR can fail without saving the banking sector anyway.

Second, we show that a flexible exchange rate regime combined to a Printer of Last Resort policy dominates all other combination of exchange rate regime and central bank policy only under a strict condition concerning the size of the foreign debt. However, if this condition is not met, a flexible exchange rate is not sufficient to ensure the uniqueness of the equilibrium and therefore to avoid crises for sure. In terms of policy implications, this means that adopting a particular exchange rate regime and designing a particular central bank policy are not enough: small open economies with an internationally leveraged financial sector will have to find other ways to deal with financial fragility and they may have to adopt strict macro-prudential regulation if they want to avoid crises at any cost.

Besides that, we recognize that some of the assumptions made in this paper and mostly inherited from Chang and Velasco are particularly strong. First, the endowment of domestic agents and the possible investments are all labelled in dollars, so the existence of pesos in this model is artificial and rests essentially on the convenient money-in-the-utility-function assumption. Second, the model does not explain why the bank borrows short term. This could be optimal because it is cheaper as it is the case in Chang and Velasco (2000a), which is itself based on the contribution by Cooper and Ross (1998). Third, the model does not explain why the bank can only borrow in foreign currency: it could be optimal for foreign investors because they do not want to take the exchange rate risk. Finally, the credit ceiling is exogenous, even if it is not really important here, it is an extreme assumption. Nevertheless, we believe that maintaining the same assumptions as in the original paper is useful as it allows us to compare directly our results with those obtained by Chang and Velasco.

However, further research could be undertaken by modifying some of these assumptions in order to match another important stylized fact of twin crises: the currency mismatch that is often observed in bank's balance-sheets prior to twin crises. For example, this kind of mismatch could be obtained in the model by denominating the investment in domestic currency and the debt in foreign currency. Indeed, Kaminsky and Reinhart (1999) observed that in some countries, a vicious circle played an important role in the unwinding of the twin crises as the significative difference in the currency denomination of assets and liabilities gave birth to a currency risk that materialized when a quick and sharp depreciation of the domestic currency linked to some minor banking problems led to an explosion in the domestic currency value of the dollar debt that provoked the banking failure, so it could be interesting to study and model this mechanism to understand more deeply the twin crisis phenomenon.

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