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Abstract

The gains from openness to trade and multinational production (MP) depend largely on country size. A large country may attract more foreign firms by closing itself to trade, while a small country may attract a larger amount of MP if trade costs with its neighbors are low because it can be used as an export platform. I develop a model to study these effects, where firms face non-convex decisions of whether to serve a foreign country by exporting from the home country, exporting from a third country, or producing in the foreign country. I calibrate the model separately for South America and Europe. I find that the gains from openness in Europe are double those in South America, and that the distribution of these gains varies less with size in South America. I also find that MP is more important in explaining the gains from openness in large countries, but the export platform mechanism is more important in small countries. Finally, I find that trade and MP have important implications for the size distribution of firms.

Keywords

Trade, Multinational Production, Bridge Multinational Production, South America, Europe.

JEL Codes:F12, F15, F23

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1 Introduction

The gains from openness to trade and multinational production (MP) depend largely on country size. A large country may attract more foreign firms by closing itself to trade, while a small country may attract a larger amount of MP if trade costs with its neighbors are low because it can be used as an export platform. I develop a quantitative theory to assess how trade barriers and country size interact to determine the location of multinational firms, and its effects on GDP, GNP, and firm size distribution.

Trade barriers affect the location decision of multinational firms in two ways. First, trade barriers change the relative cost of exporting compared to producing in the consumption location. A firm may decide to become multinational if it is cheaper to serve a market by MP rather than by exporting. Second, trade barriers change the relative cost of exporting from two different locations. Firms may use a country as an export platform to serve a set of neighbor countries.[1] The importance of these two channels depends critically on country size. For a small country it is difficult to attract multinational production to overcome trade barriers since its domestic market is small. Then, for a small country the way of attracting multinational firms is by offering the possibility of serving other countries, i.e. to be used as an export platform. On the other hand, large countries, as they have large markets, can attract MP even with high trade costs (they may attract even a larger amount of MP with trade barriers than without).

I quantitatively compare the performance of large and small countries in two different regions: South America, with high trade barriers; and Europe, with low trade barriers. In order to do so, I use a heterogeneous firm model of trade with monopolistic competition, asymmetric countries, and allowing for MP and bridge multinational production (BMP). I perform two separate calibrations, one for each region. In the calibration for South America, I include Argentina, Brazil, Chile and Uruguay. In the calibration for Europe, I include four members of the European Union (France, Italy, the Netherlands and the United Kingdom). In both cases I include a fifth country which stands for the rest of the world. To calibrate the model I use data on bilateral trade flows, bilateral FDI flows, firm composition (domestic and foreign, exporters and non-exporters), GDP per capita, manufacturing trade deficit, and labor force size.

To assess the gains from openness I compare the real GDP in the calibrated model economies with the real GDP in autarky. I find that the gains from openness in Europe double those in South America (10.5% versus 5.3% of real GDP), indicating that, as a region, South America is closed, benefitting little from trade and MP. Then, I perform three experiments to disentangle the contribution of the different channels through which

[1] I call this mechanism bridge multinational production following Ramondo and Rodríguez-Clare (2013).

MP affects the gains from openness: (i) by producing and selling in the domestic country (MP itself); (ii) by using the domestic country as an export platform (BMP); and (iii) both effects together. To asses the contribution of MP itself (without considering the BMP channel), I compare the losses of going to autarky in a world with MP and no BMP versus a world without MP (this implies no BMP neither). I find that MP itself plays a bigger role in large countries. In the Netherlands the losses of going to autarky in a world without BMP are 20% larger than in a world without MP, while in Italy they are 84% larger. This means that for the Netherlands most of the gains from MP come from BMP while in Italy most of the gains come from MP itself (not from BMP). To assess the role played by BMP, in the second experiment I compare the losses of going to autarky in the calibrated model economies with the losses in a world in which BMP is not allowed. I find that BMP is more important for small countries. For example, in the Netherlands the losses of going to autarky are 20% higher in the baseline economy than in the world without BMP, while in Italy they are only 10% higher. Finally, to assess the contribution of MP (including both mechanisms), I compare the losses of going to autarky in the calibrated model economies with the losses of going to autarky in a world without MP. I find that MP as a whole is more important for large countries. The losses of going to autarky in Italy are 100% larger than in a world without MP, while in the Netherlands they are only 44% larger. It is worth mentioning that in South America, since economies are more closed (reducing the possibility of exploiting BMP) and foreign firms are much less efficient (reducing the gains from MP itself)[2], the role played by MP and BMP in explaining the gains from openness in large and small countries is changed. Uruguay (the small country) benefits more from BMP than Brazil, but MP itself is equally important in both countries.

My second set of findings is that the differences between what a small and what a big country lose when going to autarky are very different among regions. In South America, losses are more homogeneously distributed (vary less with size) than in Europe. The difference between what Brazil (the largest country) loses in real manufacturing GDP and what Uruguay (the smallest country) loses is of 8.5 percentage points, while in Europe this difference is of 14.7 percentage points. The higher heterogeneity in Europe comes from the fact that Europe is more open than South America, which allows a small country in Europe, like the Netherlands, to take more advantage from trade and MP than a small country in South America, like Uruguay.

Next, I study what would be the gains in South America of improving the degree

[2]Even though I do not explicitly model this aspect, the low efficiency of multinationals (high γ in the model) may be due to institutions, labor market policies, input quality, etc.

of openness. South America is a much more closed region and there could be large gains from openness. To do this, I decrease the variable trade cost in all countries in the calibration for South America setting them to the average level in Europe.[3] I find that all countries benefit from this reduction, but the smallest country, Uruguay, is the one that benefits the most with an increase in manufacturing real GDP of 30%. If, in addition, the efficiency of multinational firms operating in these countries increases 20%, the gains would increase from 30% to 50%. The gains for Uruguay would be even larger if the improvement in efficiency takes place only in Uruguay but not in the other South American countries. This is because in this case Uruguay would face less competition to attract multinational firms. However BMP is crucial to attain the gains from better efficiency. If I do not allow for BMP, the additional gains Uruguay would get by improving efficiency (on top of the ones obtained by reducing trade costs) are close to zero.

Finally, I study how openness affects the size distribution of businesses across countries and regions. In the calibrated model economies, the size distribution of firms changes across countries of different size in the same region, and also across countries of the same size in different regions. I find that openness increases the proportion of large firms (with more than 250 employees) more in small than in large countries, and also that this effect is larger in the open than in the closed regions. In the baseline economy, the Netherlands has 4.2% of large firms while Uruguay has 1.1%, Italy has 1.7% and Brazil has 0.8%. Therefore, internationalization of firms has an important effect on the size distribution of firms. This is, I believe, an important contribution to the misallocation literature on business size distribution. Previous papers have studied the effects of size dependent policies (Guner et al. (2008), Restuccia and Rogerson (2008), García-Santana and Pijoan-Mas (2012)), capital market imperfections (Erosa (2001), Amaral and Quintin (2010), Buera et al. (2011), Greenwood et al. (2010)) and trade (Melitz (2003), Piguillem and Rubini (2012)) on firm size distribution. My paper contributes to this literature by addressing the effect that trade and MP have on the size of businesses.

Previous studies have analyzed the interaction of trade and MP but these studies did not allow for BMP.[4] Recently, some papers have incorporated BMP in their models. Ekholm et al. (2007), developed a model of trade with three countries to study the role

[3]I understand that in the trade barriers I use in the model, I include features like geography or language which vary a lot between regions and are probably not subject to reductions. Still using Europe as the best scenario that South America can reach is a very informative exercise on the size of the gains that could be obtained.

[4]See Helpman (1984), Horstmann and Markusen (1992), Markusen and Venables (2000), Irarrazabal et al. (2013). Helpman et al. (2004) do not include BMP in the main text, but they developed it in the appendix.

of the export platform, however they do not allow for firm heterogeneity. Ramondo and Rodríguez-Clare (2013) use a ricardian model of trade to address the gains from openness (trade and MP). However, they cannot address the effects of country size on the location of multinational firms since they assume perfect competition and as a result they do not model fixed costs of MP. Arkolakis et al. (2013) model trade and MP with monopolistic competition. However, they do not include fixed costs of setting up foreign firms. Fixed costs are important to study the role that the size of a country plays in determining the location of multinationals, which is the goal of my paper. With fixed costs there are increasing returns in production, which makes the size of a market an important variable to make a location decision. The closest paper to mine is Tintelnot (2012). He includes fixed cost of producing and performing MP and studies the gains from openness (trade and MP) in a monopolistic competition set-up. The focus in my theory is on how BMP shapes the impact of country size and geography (the distribution of trade costs across different countries) on the determination of output and trade across countries. In particular, I use my theory to quantitatively assess and compare the geography of trade and multinational production barriers in South America versus Europe. Finally, I also assess the effects of trade, MP and BMP on the distribution of firm sizes.

2 Facts

In this section I document some facts on the relation between trade, FDI and country size for South America and Europe. I use data from the World Bank to measure trade and FDI flows, and data from the United Nations to measure FDI stock.[5] In order to study the relation between trade, FDI and country size I run the following regression:

$$\begin{aligned} y_{it} &= \beta_0 + \beta_1 * Population_{it} + \beta_2 * Mercosur_i + \beta_3 * Mercosur_i * Population_{it} \\ &+ \beta_4 * European\ Union_i + \beta_5 * European\ Union_i * Population_{it} + \sum_t \gamma_t * year_t \end{aligned} \quad (1)$$

where y_{it} is the outcome of interest (either $Trade/GDP$ or FDI/GDP); $year_t$ are year fixed effects; Mercosur is a dummy variable which takes value 1 if the country belongs to the MERCOSUR;[6] Population is the natural logarithm of total population (when using from the WDI) or the labor force (in the case of UNCTAD data); Europe is also a dummy variable which takes the value 1 if the country entered in the European Union

[5]The data is from the United Nations Conference on Trade and Development (UNCTAD) Statistics years 1995-2013, and The World Bank Development Indicators from 1990-2013.

[6]The four countries that originally signed the MERCOSUR agreement in 1991 are Argentina, Brazil, Paraguay and Uruguay.

before 2000.[7] Finally, I include the interaction of the two regional dummies and the population variable.

Table 1 presents the results of running equation 1 on two different samples using $Trade/GDP$ as outcome variable. Column 1 presents the results using the whole set of countries in the WDI sample, while column 2 excludes countries with less than 100 thousand inhabitants.[8] It is a well-known fact in the trade literature that small countries benefit the most from trade. As a result it is expected to observe that small countries are more open than large countries (which should be reflected in negative β_1, β_3 and β_5). As expected β_1 has a negative sign and it is significant. Looking at the coefficients for MERCOSUR and Europe fixed effects (β_2 and β_4 respectively) we observe that both are positive, which means that countries in these regions have a higher $Trade/GDP$ than the rest of the world. However, the coefficient of $Mercosur$ is smaller than the one of $EuropeanUnion$ and it is not statistically significant. The coefficients of the interaction terms have the expected negative signs, which means that small countries in these regions have larger $Trade/GDP$. However, the coefficients for MERCOSUR are again smaller compared to those of Europe and not statistically significant. In summary, as expected $Trade/GDP$ has a negative relation with country size. While being part of the European Union has a positive and significant effect on $Trade/GDP$, being part of the MERCOSUR has a positive but not significant effect. Finally, the estimated coefficient of the interaction between region and country size suggests that small countries in Europe are more open and can benefit more from trade than small countries in MERCOSUR.

[7]The countries I include in Europe are Austria, Belgium, Denmark, Finland, France, Germany, Ireland, Italy, Luxembourg, Netherlands, Portugal, Spain, Sweden and the United Kingdom.

[8]I exclude small countries as a robustness check.

Table 1: Trade and Country Size

	(1)	(2)
Population	-4.153***	-5.786***
	(0.332)	(0.397)
Mercosur	21.352	-4.985
	(49.016)	(49.104)
Mercosur*Population	-3.013	-1.395
	(2.918)	(2.924)
European Union	114.595***	87.724***
	(32.991)	(33.159)
European Union*Population	-6.884***	-5.235***
	(2.000)	(2.011)
Adj. R-squared	0.059	0.075
N	4519	4246

The dependent variable is Trade/GDP. The sample includes all countries in the WDI sample. All regressions include year fixed effects. *** p<0.01, ** p<0.05, * p<0.1.

Table 2 presents the results of running equation 1 using as outcome variable net FDI inflows/GDP (columns 1 and 2), or using as outcome variable FDI stock/GDP (column 3 and 4). Again, for each outcome variable I estimate the equation for two samples, one using all countries, and one excluding countries with less than 100 thousand inhabitants.[9] The expected signs of the parameters of interest of equation 1 are in line with those discussed previously using $Trade/GDP$ as outcome variable: negative for $Population$, positive for the region fixed effects and negative for the interactions of region and population. The results show that, as expected, the coefficient for $Population$ is negative and statistically significant. In the case of regions fixed effects the result for $Mercosur$ differs from the one expected, it is negative and slightly significant (for FDI stock/GDP) or not significant (for net FDI inflows/GDP). For Europe, the results are the expected ones, positive and significant. This suggests that while being part of the European Union may increase the ability of countries to attract foreign firms, being part of MERCOSUR may not. For the estimated coefficients of the interaction term we find similar results, it is negative and significant for Europe, while it is positive and not significant for MERCOSUR. This could indicate that small countries in MERCOSUR cannot benefit from FDI as much as small countries in Europe do.

[9]For the UNCTAD dataset I exclude countries with less than 100 thousand workers.

Table 2: FDI and Country Size

	(1)	(2)	(3)	(4)
Population	-0.809***	-0.743***	-5.653***	-6.363***
	(0.095)	(0.092)	(0.639)	(0.730)
Mercosur	-14.015	-12.938	-77.117*	-83.389*
	(13.379)	(11.072)	(44.024)	(44.423)
Mercosur*Population	0.771	0.705	7.191	7.901
	(0.796)	(0.659)	(4.775)	(4.820)
European Union	72.985***	74.154***	138.011***	131.739***
	(9.372)	(7.776)	(26.493)	(26.843)
European Union*Population	-4.257***	-4.328***	-13.747***	-13.037***
	(0.567)	(0.470)	(3.004)	(3.044)
Adj. R-squared	0.051	0.066	0.037	0.039
N	4283	4061	3430	3267

The dependent variable is Net FDI inflow/GDP for the first two columns and FDI stock/GDP for the last two columns. The sample includes all countries in the WDI and UNCTAD sample. All regressions include year fixed effects. *** p<0.01, ** p<0.05, * p<0.1.

3 Model

The model builds on Melitz (2003) but adds the possibility of multinational production and bridge multinational production. There is a set of countries with different sizes. In each country there is a representative consumer. In the world economy there are two types of goods: a homogeneous good and a differentiated good, both of them tradable. Each differentiated good is produced by a firm with a given productivity. Differentiated goods have three sub-indices: the first one indicates where the good is consumed, the second one where the good is produced and the last one to which country the firm that produced the good belongs. For example, $q_{ijk}(\omega)$ is the quantity of good ω consumed in country i and produced in country j by a firm from country k.

3.1 Countries

The world economy consists of $i = 1, ..., N$ countries; two sectors: a homogeneous good sector (sector 0) and a differentiated good sector (sector 1); one factor of production, labor; and a continuum of goods indexed by $\omega \in \Omega$. All goods in the economy are tradable. Each country has a population of L_i individuals who supply labor inelastically. Let w_i be the wage in country i in terms of the homogeneous good. I set the price of the homogeneous good, P_0, to be the numeraire. In each country there is a large mass of potential firms producing.

3.2 Consumers

In each country there is a representative consumer with Cobb-Douglas preferences:

$$U_i = q_{i,0}^{\mu_0} q_{i,1}^{(1-\mu_0)} \quad , \tag{2}$$

where μ_0 is the share of the homogeneous good in total consumption and q_1 is a Dixit-Stiglitz aggregator[10] :

$$q_{i,1} = \left(\int q_i(\omega)^{\frac{(\sigma-1)}{\sigma}} d\omega \right)^{\frac{\sigma}{\sigma-1}} \quad ,$$

where $\sigma > 1$ is the elasticity of substitution between varieties and $q_{i,1}$ are all the varieties consumed in country i.

The above utility function implies that the representative consumer will spend μ_0 share of his income on the homogeneous good and $1 - \mu_0$ in differentiated goods. Then

[10] Where $\rho = \frac{\sigma-1}{\sigma}$. I will use σ or ρ in my definitions depending on which is the most convenient.

the demand functions are:

$$q_{i0} = \frac{\mu_0 E_i}{P_i^0} \ ,$$
$$q_{i1} = \frac{(1-\mu_0) E_i}{P_i^0} \ , \tag{3}$$

where P_i^0 is the aggregate price index in country i including the homogeneous good sector and E_i is the aggregate expenditure in country i.

Define the expenditure in the differentiated good sector as $(1-\mu_0) * E = E^1$, where E is total expenditure. Then, the demand for variety ω is given by:

$$q_{jki}(\omega) = \frac{E_j^1}{P_j} \left(\frac{p_{jki}(\omega)}{P_j} \right)^{-\sigma} \ , \tag{4}$$

where E_j^1 is the aggregate expenditure of country j in differentiated goods and P_j is the aggregate price in the differentiated good sector in country j.[11] The demand of good $q_{jki}(\omega)$ is increasing in total expenditure and the aggregate price of the country where the good is consumed (E_j^1 and P_j), and decreasing in the price of the good.

3.3 Homogeneous good

Each country has an exogenous endowment z_i of the homogeneous good. This good is traded without any cost. This implies that the price of this good will be equalized among countries. We will denote the price of the homogeneous good as p_0. Each country will be an exporter or importer of this good depending on whether the domestic supply of the good is bigger or smaller than the domestic demand of the good.

Without the homogenous good, the model would require trade imbalances to be compensated by capital account imbalances to get a balanced current account. This would imply that a country having a trade deficit would have a capital account surplus. Capital account surplus in this model means that profits from domestic firms producing abroad are larger than profits from foreign firms producing in the domestic country. Introducing the homogenous good sector allows the model to have countries with both trade deficit in the differentiated good and also capital account deficits, something that is present in the Latin-American countries I am considering.

[11] I will give a formula for P_j later since I still need to define some concepts used in the definition of P_j.

3.4 Differentiated good sector

3.4.1 Production

To produce the differentiated good the only input used is labor.[12, 13] Firms pay a fixed entry cost κ_i^e to make the labor productivity draw ϕ, denominated in labor units (then what a firm pays is $w_i * \kappa_i^e$, where w_i is the wage in country i). I assume that productivities are drawn from a Pareto distribution. After observing the productivity, firms decide whether to produce or not. If they decide to produce there are four activities that they can perform.

Selling domestically. Firms have to pay a fixed cost of operation κ_i^d, also denominated in labor units, to produce domestically. In addition to this fixed cost, firms have to pay the variable cost of production. The variable cost of selling domestically $q_{iii}(\omega)$ units of the good is:

$$c_{iii}(\omega) = \frac{w_i}{\phi} q_{iii}(\omega) \ .$$

Exporting from the domestic country. To export firms have to pay a fixed cost (independent of the selling destination) and an iceberg type cost which is partner specific. Firms producing in country i and exporting to country j pay a fixed cost $w_i * \kappa_i^x$ and an iceberg cost τ_{ji} per unit sold i.e. they have to send $\tau_{ji} \geq 1$ units of the good for one unit to arrive at its destination. The variable cost of exporting $q_{jii}(\omega)$ units of the good is:

$$c_{jii}(\omega) = \frac{\tau_{ji} * w_i}{\phi} q_{jii}(\omega) \ .$$

Producing and selling abroad (MP). When a firm produces abroad its productivity is shifted by a parameter γ. The new productivity of a firm from country i producing in country k is $\hat{\phi} = \frac{\phi}{\gamma_{ki}}$. In addition, a firm from country i producing in country k has to pay a fixed cost $w_k * \kappa_k^{MP}$ which is independent from the source country (all foreign firms producing in country k pay the same fixed cost). κ_k^{MP} includes the domestic cost of producing in country k and an extra cost i.e. $\kappa_k^{MP} \geq \kappa_k^d$. The variable cost of producing abroad and selling $q_{kki}(\omega)$ units in that country is:

$$c_{kki}(\omega) = \frac{\gamma_{ki} * w_k}{\phi} q_{kki}(\omega) \ .$$

[12] As a result, this paper analyzes horizontal FDI in the spirit of Markusen (1995). See Barba Navaretti and Venables (2004) for a review of the literature on FDI (both vertical and horizontal FDI).

[13] The work of Irarrazabal et al. (2013) model trade with vertical FDI. When assuming that foreign firms use imported intermediate goods as input, we are introducing complementarity between trade and MP. With horizontal FDI and allowing for BMP, MP and trade can be substitutes or complements. Ramondo and Rodríguez-Clare (2013) on the other hand model both vertical and horizontal FDI.

Note that the wage that is paid is that of the country where the firm is producing, in this case country k.

Producing abroad and exporting (BMP). Finally, a firm that is producing abroad has the option to export to a third country. In this case, the firm will have to pay an extra fixed cost of exporting $w_k * \kappa_k^x$. The variable cost for a firm from country i producing abroad (in country k) and exporting $q_{jki}(\omega)$ units of the good to country j is

$$c_{jki}(\omega) = \frac{\tau_{jk}\gamma_{ki}w_k}{\phi} q_{jki}(\omega) \ .$$

Maximizing variable profits of a firm from country i for a given activity,

$$\max_{p(\omega)} \pi = p(\omega)q(\omega) - c(\omega) \ , \tag{5}$$

where $q(\omega)$ was defined in equation (4), we get the price of a variety, given by:

$$p_{jki}(\omega) = \frac{w_k \gamma_{ki} \tau_{jk}}{\rho\phi} \ . \tag{6}$$

As each firm produces a different variety we can substitute, without loss of generality, ω by ϕ. Using expression (4) and (6) we obtain the revenue associated with each activity.

$$\begin{aligned}
\text{Selling Domestically} \Rightarrow r_{iii}(\phi) &= E_i^1 P_i^{\sigma-1} \left(\frac{\rho\phi}{w_i}\right)^{\sigma-1} \\
\text{Exporting from the home country} \Rightarrow r_{kii}(\phi) &= E_k^1 P_k^{\sigma-1} \left(\frac{\rho\phi}{w_i \tau_{ki}}\right)^{\sigma-1} \\
\text{Doing MP in country } k \Rightarrow r_{kki}(\phi) &= E_k^1 P_k^{\sigma-1} \left(\frac{\rho\phi}{w_k \gamma_{ki}}\right)^{\sigma-1} \\
\text{Doing BMP in } k \text{ to sell in } j \Rightarrow r_{jki}(\phi) &= E_j^1 P_j^{\sigma-1} \left(\frac{\rho\phi}{w_k \gamma_{ki} \tau_{jk}}\right)^{\sigma-1}
\end{aligned} \tag{7}$$

The next step is to find which firms are going to perform each activity. A firm will perform an activity as long as the activity is profitable. Let's start with firms selling only domestically. A firm will sell domestically if

$$\pi_{iii}(\phi) = \frac{E_i^1 P_i^{\sigma-1}}{\sigma} \left(\frac{\rho\phi}{w_i}\right)^{\sigma-1} - \kappa_i^d w_i \geq 0 \tag{8}$$

As profits are increasing with productivities, there will be one productivity (the cut-off productivity) for which profits will be equal to zero. I will denote the domestic productivity cut-off as ϕ_{iii}^*. All firms with productivities higher than ϕ_{iii}^* will sell domestically. Now, firms can also export or produce abroad. Before continuing, let me assume the following:

Assumption 1: *A variety is defined by the country of origin of the firm and the country where the good is produced.*
Assumption 2: *Any firm from country i performing an activity has to pay the domestic cost of producing in i.*

With assumption 1 a firm from Uruguay producing in Uruguay and exporting to Brazil is going to sell a different variety than the same firm producing in Brazil and selling in Brazil. The fact that varieties are determined also by the production location simplifies the solution of the problem allowing me to treat each activity as independent activities.[14,15] In the absence of assumption 1 a firm from Uruguay will have to choose how to serve the Brazilian market (either by producing in Uruguay and exporting, by doing MP in Brazil or by doing MP in a third country and exporting to Brazil) since the variety sold is the same independently from the production location. Then, without assumption 1 there will be more competition between countries for attracting MP, which would increase the importance of the efficiency of multinationals operating in the domestic country (parameter γ). Also, without assumption 1 BMP becomes a more important factor for attracting MP. In the quantitative section I discuss in more detail the role of assumption 1.

Assumption 2 ensures that there will be no firms exporting or doing MP and not selling in the domestic country.

The profit for a firm from country i exporting to country k is given by:

$$\pi_{kii}(\phi) = \frac{E_k^1 P_k^{\sigma-1}}{\sigma} \left(\frac{\rho\phi}{w_i \tau_{ki}} \right)^{\sigma-1} - \kappa_i^x w_i \tag{9}$$

Setting this equation equal to zero, we can find the cut-off productivity (ϕ^*_{kii}) for a firm from country i exporting to country k. To fix ideas, let us keep aside the possibility of MP. Then, we have two possibilities for defining the exporting cut-offs:

Case 1: If all the exporting cut-offs are higher than the domestic cut-off in country i, that is if $\phi^*_{iii} < \phi^*_{kii}$ $\forall k$, then the domestic and the exporting cut-offs are well calculated. Firms with productivities $\phi^*_{iii} < \phi < \phi^*_{kii}$ only sell in the domestic market, while firms with productivities $\phi > \phi^*_{kii}$ sell domestically and export.

Case 2: If at least one exporting cut-off ϕ^*_{kii} is lower than the domestic cut-off ϕ^*_{iii}, then we have to re-calculate cut-offs. Denote K_i^x the set of countries k for which the

[14]Using assumption 1, I can extend the results from Melitz (2003) considering multinational production and BMP just as an additional activity that simplifies the problem.

[15]It can be that in the end activities are not fully independent, but I can compute costs and profits for each activity as if they were fully independent.

exporting cut-off (from country i to country k) is lower than the domestic cut-off. For countries $k \in K_i^x$ the exporting cut-off is equal to the domestic cut-off ($\phi_{iii}^* = \phi_{kii}^*$). The marginal firm entering in the domestic market (with productivity ϕ_{iii}^*) makes negative profits selling in the domestic market but these negative profits are compensated by the positive profits obtained by exporting to countries $k \in K_i^x$. Then, the productivity cut-off defined in the marginal entrant (ϕ_{iii}^*) solves the following equation:

$$\pi_{iii}(\phi_{iii}^*) + \sum_{k \in K^x} \pi_{kii}(\phi_{iii}^*) = 0. \tag{10}$$

Now let us consider the possibility for MP. Allowing for MP brings new cases for the way the domestic cut-off is defined. The profit for a firm from country i producing and selling in country k (performing MP in country k) is given by:

$$\pi_{kki}(\phi) = \frac{E_k^1 P_k^{\sigma-1}}{\sigma} \left(\frac{\rho\phi}{w_k \gamma_{ki}} \right)^{\sigma-1} - \kappa_k^{MP} w_k \tag{11}$$

To fix ideas, let us ignore the possibility of exporting. We want to focus on how MP affects the calculation of the domestic cut-off. There are two cases again to consider:

Case 3: If all the MP cut-offs are higher than the domestic cut-off in country i, that is if $\phi_{iii}^* < \phi_{kki}^*$ $\forall k$, then the domestic and the MP cut-offs are well calculated. Firms with productivities $\phi_{iii}^* < \phi < \phi_{kki}^*$ only sell in the domestic market, while firms with productivities $\phi > \phi_{kki}^*$ sell domestically and perform MP.

Case 4: If at least one MP cut-off (ϕ_{kki}^*) is lower than the domestic cut-off, then we need to follow similar steps as in case 2. Denote by K_{ki}^{MP} the set of countries (k) for which the MP cut-off in country i (ϕ_{kki}^*) is lower than the domestic cut-off in country i (ϕ_{iii}^*). For countries $k \in K_{ki}^{MP}$ the MP cut-off is equal to the domestic cut-off $\phi_{kki}^* = \phi_{iii}^*$. The marginal firm entering into the domestic market (with productivity ϕ_{iii}^*) makes negative profits selling in the domestic market but these negative profits are compensated by the positive profits obtained by performing MP in countries $k \in K_{ki}^{MP}$. Then, the productivity of the marginal entrant in country i solves the following equation:

$$\pi_{iii}(\phi_{iii}^*) + \sum_{k \in K_{ki}^{MP}} \pi_{kki}(\phi_{iii}^*) = 0 \tag{12}$$

If we assume that firms can export and do MP, the procedure is the same. The only difference is that if we have exporting cut-offs and MP cut-offs that are below the domestic cut-off, then the productivity of the marginal entrant in country i solves the following equation:

$$\pi_{iii}(\phi^*_{iii}) + \sum_{k \in K^x} \pi_{kii}(\phi^*_{iii}) + \sum_{k \in K^{MP}_{ki}} \pi_{kki}(\phi^*_{iii}) = 0 \tag{13}$$

Finally, a firm may want to use a third country as an export platform (BMP). The profit for a firm from country i, producing in country k and selling in country j is given by:

$$\pi_{jki}(\phi) = \frac{E^1_j P^{\sigma-1}_j}{\sigma} \left(\frac{\rho\phi}{w_k \gamma_{ki} \tau_{jk}} \right)^{\sigma-1} - \kappa^x_k w_k \tag{14}$$

Setting the above equation to zero, we can find the BMP cut-off productivity (ϕ^*_{jki}) for a firm from country i producing in country k and selling in country j. As in the previous cases we have two cases:

Case 5 If all the BMP cut-off productivities for firms from country i producing in country k (ϕ^*_{jki} $\forall j$) are above the MP cut-off productivity for firms from country i producing in country k (ϕ^*_{kki}), then the BMP cut-offs are well calculated. Firms with productivities $\phi^*_{kki} < \phi < \phi^*_{jki}$ sell domestically and produce and sell in country k, while firms with productivities $\phi > \phi^*_{jki}$ sell domestically, produce and sell in country k and also do BMP from country k to country j.

Case 6 If at least one BMP cut-off for firms from country i producing in country k (ϕ^*_{jki} $\forall j$) is below the MP cut-off productivity for firms from country i producing in country k (ϕ^*_{kki}), then we have to re-calculate the MP cut-off ϕ^*_{kki}. Define J^{BMP}_{ki} the set of countries for which the BMP cut-off (ϕ^*_{jki}) is lower than the MP cut-off (ϕ^*_{kki}). Then the cut-off productivity for the marginal firm from country i performing MP in country k and BMP to country j solves:

$$\pi_{kki}(\phi^*_{kki}) + \sum_{j \in J^{BMP}_{ki}} \pi_{jki}(\phi^*_{kki}) = 0 \tag{15}$$

As firms performing BMP have to pay the fixed cost of producing abroad (κ^{MP}) also, there will be no firm performing BMP and not MP, which implies that the equilibrium BMP cut-off is not going to be below the MP cut-off. After re-calculating the MP cut-off we have to check if the new MP cut-off is larger than the domestic cut-off. If it is larger, then the MP cut-off is well calculated, otherwise we need to re-calculate the domestic cut-off which will be the one that solves:

$$\pi_{iii}(\phi^*_{iii}) + \sum_{k \in K^{MP}_{ki}} \pi_{kki}(\phi^*_{iii}) + \sum_{k \neq i} \sum_{j \in J^{BMP}_{ki}} \pi_{jki}(\phi^*_{iii}) = 0 \ , \tag{16}$$

In Appendix 3 I present the algorithm to calculate the cut-offs.

Profits

In summary, if $\phi^*_{iii} < \phi^*_{kii}$, $\phi^*_{iii} < \phi^*_{kki}$ and $\phi^*_{kki} < \phi^*_{jki}$ all the cut-offs are the ones that come from equating the profit from each activity to zero, and so the marginal firm performing each activity makes zero profit. Otherwise the marginal firm entering into the domestic market can be making negative profits in the domestic market and compensate these negative profits with positive profits in other activities, like exporting or MP or both. Then, the profit made by a firm from country i is given by: to define profits I need to use an indicator that allows me to know if an activity is operative or not

$$\pi_i(\phi) = \pi_{iii}(\phi) + \sum_{k\neq i} \pi_{kii}(\phi) I^x_{kii} + \sum_{k\neq i} \pi_{kki}(\phi) I^{MP}_{kki} + \sum_{k\neq i}\sum_{j\neq k} \pi_{jki}(\phi) I^{BMP}_{jki} \ , \tag{17}$$

where I^x_{kii} is an indicator function that takes the value 1 if $\phi > \phi^*_{kii}$ and 0 otherwise, I^{MP}_{kki} is an indicator function that takes the value 1 if $\phi > \phi^*_{kki}$ and 0 otherwise, and finally I^{BMP}_{jki} is an indicator function that takes the value 1 if $\phi > \phi^*_{jki}$ and 0.[16] Note that for a firm with productivity ϕ it can be possible that the profit for some activities is negative. For example, it can happen that for this firm the profit of opening a plant in country k and selling to country k ($\pi_{kki(\phi}$), but the profit of producing in country k and selling to country j are positive and more than compensates the negative profit. Finally, as profits from every activity are increasing in ϕ (since $\sigma - 1 > 0$), more productive firms make higher profits, and so if the productivity is high enough a firm performs all the activities.

3.4.2 Productivity distribution

Productivities are drawn from a Pareto distribution with scale parameter ϕ^m_i and shape parameter α_i.[17] Lets define the density function as $g_i(\phi) = \alpha_i \frac{(\phi^m_i)^{\alpha_i}}{\phi^{\alpha_i+1}}$. As only firms with productivities above ϕ^*_{iii} will produce in country i, then the equilibrium distribution of productivities of domestic firms is:

$$\mu_i(\phi) = \frac{g_i(\phi)}{1 - G(\phi^*_{iii})} \text{ if } \phi \geq \phi^*_{iii} \ , \tag{18}$$

[16] In the calibrated model economies there are no exporting or MP cut-offs lower than the domestic cut-off. However, there are some BMP cut-offs smaller than the MP cut-offs.

[17] In a Pareto distribution the scale parameter indicates the minimum value that the random variable can take.

and 0 otherwise. The conditional probability of performing each of the other activities is:

$$\begin{aligned}
\text{Exporting to country } k \Rightarrow \theta_{kii} &= \frac{1-G(\phi^*_{kii})}{1-G(\phi^*_{iii})} \\
\text{Doing FDI in country } k \Rightarrow \theta_{kki} &= \frac{1-G(\phi^*_{kki})}{1-G(\phi^*_{iii})} \\
\text{Doing BMP in } k \text{ to sell in } j \Rightarrow \theta_{jki} &= \frac{1-G(\phi^*_{jki})}{1-G(\phi^*_{iii})}
\end{aligned}$$

The average productivity for each activity can be calculated as:

$$\tilde{\phi}_{jki} = \left[\int_{\phi^*_{jki}}^{\infty} \phi^{\sigma-1} \mu_i(\phi) d\phi \right]^{1/(\sigma-1)} \tag{19}$$

for all i, j and k. Notice that $\tilde{\phi}_{jki}$ only depends on the cut-off productivity.
Following Melitz (2003), we can consider that for each activity there is a representative firm with productivity $\tilde{\phi}_{jki}$. The average productivity $\tilde{\phi}_{jki}$ summarizes all the information concerning each activity. This is convenient because now aggregate variables for each activity can be expressed in terms of $\tilde{\phi}_{jki}$. One difference with respect to the case of Melitz (2003) is that in his case it is possible to calculate an average productivity for the whole economy that depends only on domestic firms. In this paper, the average productivity of a country will be given by the domestic firms producing domestically and also by foreign firms producing domestically. Then, aggregate variables for the whole economy will depend not only on the domestic mass of firms but also on the mass of firms from the rest of the countries.

Evaluating revenues at the average productivity level and making the ratio of this revenue with a revenue evaluated at any other productivity level we find that:

$$\frac{r(\tilde{\phi}_{iii})}{r_{iii}(\phi)} = \frac{E_i^1 P_i^{\sigma-1} \left(\frac{\rho \tilde{\phi}_{iii}}{w_i}\right)^{\sigma-1}}{E_i^1 P_i^{\sigma-1} \left(\frac{\rho \phi}{w_i}\right)^{\sigma-1}} \Rightarrow r(\tilde{\phi_{iii}}) = \left(\frac{\tilde{\phi}_{iii}}{\phi}\right)^{\sigma-1} r_{iii}(\phi) \tag{20}$$

We can get the previous relation for each activity: exporting, doing MP and doing BMP.

$$\begin{aligned}
\text{Exporting to country } k \Rightarrow r(\tilde{\phi}_{kii}) &= \left(\frac{\tilde{\phi}_{kii}}{\phi}\right)^{\sigma-1} r_{kii}(\phi) \\
\text{Doing MP in country } k \Rightarrow r(\tilde{\phi}_{kki}) &= \left(\frac{\tilde{\phi}_{kki}}{\phi}\right)^{\sigma-1} r_{kki}(\phi) \\
\text{Doing BMP in } k \text{ to sell in } j \Rightarrow r(\tilde{\phi}_{jki}) &= \left(\frac{\tilde{\phi}_{jki}}{\phi}\right)^{\sigma-1} r_{jki}(\phi)
\end{aligned}$$

3.4.3 Sales distribution

Sales for a given activity are given by $r_{jki} = E_j^1 \left(\frac{P_j \rho \phi}{w_k \gamma_{ki} \tau_{jk}}\right)^{\sigma-1}$, where E_j^1 is aggregate expenditure in differentiated goods in country j. Given that productivities are drawn from a Pareto distribution it is possible to obtain the distribution of sales for each activity analytically. I will present the result for domestic firms selling domestically, but the expression is analog for the other activities.

$$
\begin{aligned}
prob(r_{iii}(\phi) > y) &= prob\left(E_i^1 \left(\frac{P_i \rho \phi}{w_i}\right)^{\sigma-1} > y\right) \\
&= prob\left(\phi > \left(\frac{y}{E_i^1}\right)^{\frac{w_i}{1-\sigma}} \frac{w_i}{P_i \rho}\right) .
\end{aligned}
$$

As ϕ is distributed Pareto we can calculate this probability to be

$$
prob(r_{iii}(\phi) > y) = \left(\frac{\phi_i^m}{\left(\frac{y}{E_i^1}\right)^{\frac{1}{1-\sigma}} \frac{w_i}{P_i \rho}}\right)^{\alpha} ,
$$

where $\phi_{m,i}$ is the scale parameter (the minimum value that ϕ can take) of the Pareto distribution. We can write the above expression as:

$$
\begin{aligned}
prob(r_{iii}(\phi) > y) &= \left(\frac{(E_i^1)^{1/(\sigma-1)}(P_i \rho \phi_i^m / w_i)}{y^{1/(\sigma-1)}}\right)^{\alpha} \\
prob(r_{iii}(\phi) > y) &= \left(\frac{E_i^1 (P_i \rho \phi_i^m / w_i)^{(\sigma-1)}}{y}\right)^{\alpha/(\sigma-1)} \\
prob(r_{iii}(\phi) > y) &= \left(\frac{r_i^m}{y}\right)^{\alpha/(\sigma-1)}
\end{aligned}
$$

where $r_i^m(\phi_i^m) = E_i^1 (P_i \rho \phi_i^m)^{\sigma-1}$ is the revenue of a firm from country i with productivity equal to $\phi_{m,i}$ producing and selling domestically. Then $r_{iii}(\phi)$ is distributed Pareto with scale parameter $r_{m,i}$ and shape parameter $\alpha/(\sigma-1)$. This would be the distribution of sales if all the firms were producing. But, as we stated previously, there will be some firms (the ones with productivity between $\phi_{m,i}$ and ϕ_{iii}^*) which are not going to produce. Then, the true distribution of sales will be a truncation of the previous distribution. The Pareto distribution has the property that if it is truncated, the remaining distribution is still Pareto with the same shape parameter. Then sales ($r_{iii}(\phi)$) are distributed Pareto with scale parameter $r_{iii}(\phi^*)$ and shape parameter $\alpha/(\sigma-1)$, where $r_{iii}(\phi^*)$ are the sales of a firm with the cut-off productivity.

For the rest of activities we can operate in a similar way and we obtain:

$$\text{Exporting firms} \Rightarrow prob(r_{kii} > y) = \left(\frac{E_k^1 \left(\frac{P_k \rho \phi_i^m}{w_k \tau_{ki}} \right)^{(\sigma-1)}}{y} \right)^{\alpha/(\sigma-1)}$$

$$\text{Doing FDI in country } k \Rightarrow prob(r_{kki} > y) = \left(\frac{E_k^1 \left(\frac{P_k \rho \phi_i^m}{w_k \gamma_{ki}} \right)^{(\sigma-1)}}{y} \right)^{\alpha/(\sigma-1)}$$

$$\text{Doing BMP in } k \text{ to sell in } j \Rightarrow prob(r_{jki} > y) = \left(\frac{E_j^1 \left(\frac{P_j \rho \phi_i^m}{w_k \tau_{jk} \gamma_{ki}} \right)^{(\sigma-1)}}{y} \right)^{\alpha/(\sigma-1)}$$

where the numerator of each equation is the sales for each activity that correspond to the minimum productivity level. As in the case of domestic sales, the equilibrium distribution of sales for each activity is going to be Pareto with shape parameter $\alpha/(\sigma-1)$ and scale parameter $r(\phi_{jki}^*)$, where $r(\phi_{jki}^*)$ is sales of a firm with the cut-off productivity level for a firm from country i producing in country k and selling to country j.

3.4.4 Average Profits

Replacing (19) in the profit equations we can calculate average profits in terms of average productivities. In the case that each individual activity makes zero profit at the cut-off level, we can calculate average profit for each activity as:

$$\text{Selling Domestically} \Rightarrow \bar{\pi}_{iii} = \kappa_i^d w_i \left[\left(\frac{\tilde{\phi}_{iii}}{\phi_{iii}^*} \right)^{\sigma-1} - 1 \right]$$

$$\text{Exporting from the home country} \Rightarrow \bar{\pi}_{kii} = \kappa_i^x w_i \left[\left(\frac{\tilde{\phi}_{kii}}{\phi_{kii}^*} \right)^{\sigma-1} - 1 \right]$$

$$\text{Doing MP in country } k \Rightarrow \bar{\pi}_{kki} = \kappa_k^{MP} w_k \left[\left(\frac{\tilde{\phi}_{kki}}{\phi_{kki}^*} \right)^{\sigma-1} - 1 \right]$$

$$\text{Doing BMP in } k \text{ to sell in } j \Rightarrow \bar{\pi}_{jki} = \kappa_k^x w_k \left[\left(\frac{\tilde{\phi}_{jki}}{\phi_{jki}^*} \right)^{\sigma-1} - 1 \right]$$

If the profit at the cut-off level is not zero, then the average profit for that activity is obtained using equation (20). We can calculate the average profit of a firm from country i as:

$$\bar{\pi}_i = \bar{\pi}_{iii} + \sum_{k \neq i} \theta_{kii} \bar{\pi}_{kii} + \sum_{k \neq i} \theta_{kki} \bar{\pi}_{kki} + \sum_{k \neq j} \sum_{k \neq i} \theta_{jki} \bar{\pi}_{jki} \ . \qquad (21)$$

Notice that profits are a function of aggregate expenditures E_i^1. Aggregate expenditure is determined, among other factors, by the population size. Hence, the profitability of a foreign firm depends on the selling country size. Given two countries with similar variable and fixed trade costs, a multinational plant will prefer to get installed in the bigger country. As a result, a small country will attract less investment than a bigger one. For example, assume that the country where the good is going to be consumed is Uruguay, and a firm from Japan is considering the different possibilities of serving Uruguay. If the fixed export cost in Japan is high, then it could be better to produce the good directly in Uruguay. This will be the case if the fixed cost to open a subsidiary in Uruguay is not very high and also the productivity loss for producing abroad ($\gamma_{Uru,Jap}$) is low. Now, imagine that Japan is also considering to sell to Argentina, and that the productivity loss of producing in Argentina for a Japanese firm is the same as in Uruguay $\gamma_{Arg,Jap} = \gamma_{Uru,Jap}$. Then, as Argentina is bigger, $E^1_{Arg} > E^1_{Uru}$. If aggregate prices, wages, and fixed costs are not very different, the Japanese firm will prefer to produce in Argentina to producing in Uruguay. In other words, the productivity required by a Japanese firm to start producing in Argentina is lower (ceteris paribus) than that required to produce in Uruguay. This implies that more firms get located in Argentina. Size, then, is crucial to attract foreign investment.

3.4.5 Mass of Firms

Define M_i^e to be the total mass of firms making a productivity draw in country i, and M_i as the mass of firms finally operating. By definition, the total mass of firms operating should be equal to the total mass of firms making a productivity draw times the probability of successful entry, which is $\theta_{iii} M_i^e = M_i$.

In the case of an open economy without FDI we can obtain M_i in the same way as in Melitz (2003). $M_i = R_i/\bar{r}_i$, where $R_i = w_i L_i$ denotes aggregate revenue and aggregate expenditure, and $\bar{r}_i$ denotes average revenue. In Melitz (2003), aggregate revenue and total payment to labor are equal because total profits (Π) are equal to the payment to labor used in making the productivity draw ($\kappa_i^e w_i$) in equilibrium and only domestic firms produce in country i.

When foreign firms are allowed to produce in the domestic country $R_i \neq w_i L_i$. The equality does not hold because foreign firms send their profits abroad, and domestic firms producing abroad bring their profits home, making total expenditure in the country also a function of profits of domestic firms abroad. However, it is still true that $w_i L_i^e = \Pi_i$ (where L_i^e is labor used in entering) [18], but the determination for labor used in production

[18] This is obtained using the equation for total payment to labor used in entering and the free entry condition,

(L_i^p) is different. Now the total payment to labor in country i is equal to revenue minus profits of firms producing in i, which can include foreign firms. In equations, $w_i L_i^p = \hat{R}_i - \hat{\Pi}_i$ where $\hat{R}_i$ and $\hat{\Pi}_i$ are revenues and profits of firms producing in country i (domestic or foreign).

The total mass of firms performing each of the other activities is obtained by multiplying the mass of firms operating, M_i, by the conditional probability of performing the activity $M_{jki} = \theta_{jki} M_i$.

3.4.6 Aggregation

We define aggregate price and GDP in country i as:

$$\begin{aligned} P_i &= \Bigg[\int_{\phi^*_{iii}} (p_{iii}(\phi))^{1-\sigma} M_i \mu_i(\phi) d\phi + \sum_{k \neq i} \int_{\phi^*_{ikk}} (p_{ikk}(\phi))^{1-\sigma} M_k \mu_k(\phi) d\phi \\ &+ \sum_{k \neq i} \int_{\phi^*_{iik}} (p_{iik}(\phi))^{1-\sigma} M_k \mu_k(\phi) d\phi + \sum_{k \neq j} \sum_{k \neq i} \int_{\phi^*_{ikj}} (p_{ikj}(\phi))^{1-\sigma} M_j \mu_j(\phi) d\phi \Bigg]^{\frac{1}{1-\sigma}} , \end{aligned} \tag{22}$$

$$\begin{aligned} GDP_i &= \int_{\phi^*_{iii}} r_{iii}(\phi) M_i \mu_i d\phi + \sum_{k \neq i} \int_{\phi^*_{kii}} r_{kii}(\phi) M_i \mu_i d\phi + \sum_{k \neq i} \int_{\phi^*_{iik}} r_{iik}(\phi) M_k \mu_k d\phi \\ &+ \sum_{k \neq j} \sum_{k \neq i} \int_{\phi^*_{kij}} r_{kij}(\phi) M_j \mu_j d\phi \ . \end{aligned} \tag{23}$$

We can re-write the aggregate price and GDP of country i in terms of weighted average productivities.[19] Let's define M_i^p as the mass of firms producing in country i and M_i^s as the mass of firms selling goods to country i. Then,

$$\begin{aligned} M_i^p &= M_i + \sum_{k \neq i} M_{iik} + \sum_{k \neq i} \sum_{i \neq j} M_{jik} \ , \\ M_i^s &= M_i + \sum_{k \neq i} M_{iik} + \sum_{k \neq j} \sum_{i \neq j} M_{ijk} \ . \end{aligned} \tag{24}$$

Having defined the mass of firms producing and selling in each country we can define

which I explain later.

[19]Following Melitz (2003).

the weighted average productivity of firms producing ($\tilde{\phi}_i^p$) and selling ($\tilde{\phi}_i^s$) as:

$$\begin{aligned}\tilde{\phi}_i^p &= \left\{\frac{1}{M_i^p}\left[M_{iii}\tilde{\phi}_{iii}^{\sigma-1} + \sum_{k\neq i} M_{kii}\frac{E_k^1}{E_i^1}\left(\frac{P_k}{\tau_{ki}P_i}\right)^{\sigma-1}\tilde{\phi}_{kii}^{\sigma-1} + \sum_{k\neq i} M_{iik}\left(\frac{1}{\gamma_{ik}}\right)^{\sigma-1}\tilde{\phi}_{iik}^{\sigma-1}\right.\right. \\ &+ \left.\left.\sum_{k\neq i}\sum_{i\neq j} M_{jik}\frac{E_j^1}{E_i^1}\left(\frac{P_j}{\tau_{ji}\gamma_{ik}}\right)^{\sigma-1}\tilde{\phi}_{jik}^{\sigma-1}\right]\right\}^{\frac{1}{\sigma-1}}, \qquad (25)\end{aligned}$$

$$\begin{aligned}\tilde{\phi}_i^s &= \left\{\frac{1}{M_i^s}\left[M_{iii}\tilde{\phi}_{iii}^{\sigma-1} + \sum_{k\neq i} M_{ikk}\left(\frac{w_k\tau_{ik}}{w_i}\right)^{1-\sigma}\tilde{\phi}_{ikk}^{\sigma-1} + \sum_{k\neq i} M_{iik}\gamma_{ik}^{1-\sigma}\tilde{\phi}_{iik}^{\sigma-1}\right.\right. \\ &+ \left.\left.\sum_{k\neq i}\sum_{i\neq j} M_{ijk}\left(\frac{\tau_{ij}\gamma_{jk}w_k}{w_i}\right)^{1-\sigma}\tilde{\phi}_{ijk}^{\sigma-1}\right]\right\}^{\frac{1}{\sigma-1}}. \qquad (26)\end{aligned}$$

Using these two equations we can define aggregate price and aggregate production in the differentiated good sector in country i as:[20]

$$P_i = (M_i^s)^{\frac{1}{1-\sigma}} p(\tilde{\phi}_i^s) = (M_i^s)^{\frac{1}{1-\sigma}} \frac{w_i}{\rho\tilde{\phi}_i^s}, \qquad (27)$$

$$GDP_i = M_i^p E_i \left(\frac{P_i\rho\tilde{\phi}_i^p}{w_i}\right)^{\sigma-1}. \qquad (28)$$

3.5 Trade and Multinational Production

The trade of a country will be given by the amount of exports and imports. Exports are composed by all the sales to foreign countries from firms (either domestic or foreign) producing in the domestic country. The expression for total exports in the differentiated good sector is given by:

$$\text{Exports}_i = X_i = \underbrace{\sum_{k\neq i} M_{kii} r_{kii}(\tilde{\phi}_{kii})}_{\text{Exports by Domestic Firms}} + \underbrace{\sum_{k\neq i}\sum_{k\neq j} M_{jik} r_{jik}(\tilde{\phi}_{jik})}_{\text{Exports by Foreign Firms}}.$$

In a similar way, imports in the differentiated good sector are all the goods consumed in the domestic country and produced in a foreign country. So total imports are:

$$\text{Imports}_i = IM_i = \sum_{k\neq i} M_{ikk} r_{ikk}(\tilde{\phi}_{ikk}) + \sum_{k\neq i}\sum_{k\neq j} M_{ijk} r_{ijk}(\tilde{\phi}_{ijk}).$$

The capital account is composed of the difference between the profits of domestic firms producing abroad and the profits of foreign firms producing in the domestic country.

$$\text{Capital Account}_i = \sum_k \sum_{j\neq i} M_{kji}\bar{\pi}_{kji} - \sum_k \sum_{j\neq i} Mkij\bar{\pi}_{kij}.$$

[20] Proof in the appendix.

The Current Account (CA) is the sum of Trade Balance (TB) $TB_i = (z_i - q_{i0}) + X_i - IM_i$ where $(z_i - q_{i0})$ is net exports of the homogeneous good, and the capital account balance. The current account balance equation can be written as:

$$\text{CA}_i = (z_i - q_{i0}) + X_i - IM_i + \sum_k \sum_{j \neq i} M_{kji} \bar{\pi}_{kji} - \sum_k \sum_{j \neq i} Mkij \bar{\pi}_{kij} \ . \qquad (29)$$

3.6 Equilibrium

Equation (21) defines the Zero Cut-off Profit Condition (ZCPC), which expresses the average profit of a firm from country i as a function of the domestic cut-offs, the mass of firms operating in each country, and wages. The net value of a firm from country i is then $v_i = \bar{\pi}_i$. As there is free entry, the expected profit of a firm before making a draw should be zero, otherwise more firms will enter until this condition is satisfied. Define the net value of an entering firm as v_i^e. In equilibrium v_i^e should be equal to zero. Then the free entry condition (FEC) can be expressed as:

$$v_i^e = \theta_{iii} \bar{\pi}_i - \kappa_i^e w_i = 0 \ , \qquad (30)$$

which says that the average value of a firm producing in country i times the probability of successful entry should be equal to the entry cost (the cost of making the productivity draw). θ_{iii} is a function of the scale (ϕ_i^m) and shape (α) parameters of the productivity distribution and of the domestic cut-off (ϕ_{iii}^*). Rearranging terms in equation (30) we get $\bar{\pi}_i = \frac{\kappa_i^e w_i}{\theta_{iii}}$.

In order to solve for the equilibrium we need to find $3 * N + 1$ variables: N cut-offs ($\phi_{iii} \ \forall i$); N numbers for the mass of firms for each country ($M_i \ \forall i$); N wages (w_i) and 1 price (p_0) . Normalizing the price of the homogeneous good to one we end up with $3 * N$ endogenous variables. The set of $3 * N equations$ are given by:

- Free entry condition equal to zero cut-off profit condition.
- Current account balance condition.
- Labor market clearing condition.

Definition: *Given z_{i0}, τ_{ij}, γ_{ij}, κ_i^e, κ_i^d, κ_i^x, κ_i^{MP}, $g_i(\phi)$, L_i and N $\forall i, j = 1, ..., N$, a* ***multinational production equilibrium*** *is a set of wages w_i, price indices, P_i, income, GNP_i, mass of firms M_i, mass of entrants, M_i^e, allocations for the representative consumer $q_{jki}(\phi)$ and prices, $p_{jki}(\phi)$, for firms such that:*

1. *In all countries, given prices and aggregate expenditure, consumers demand choices ($q_{jki}(\phi)$ and q_{io}) satisfy* (3) *and* (4)*.*
2. *In all countries, firms maximize profits from all activities (equation* (6) *solves* (5)*).*

3. *P_i satisfies equation (22)*

4. *Labor markets clear.*

5. *Free entry condition: $v_i^e = 0$ (see equation (30)).*

6. *Current Account balance condition is zero (see equation (29)).*

7. *The mass of firms producing is equal to the mass of firms taking the productivity draw times the probability that the draw is bigger than the domestic cut-off, $M_i = \theta_{iii} M_i^e$*

8. *World demand of the homogeneous good is equal to world supply: $\sum_i z_i = \sum_i q_{i0}$.*

4 Calibration

4.1 Data

I use data from four different sources to calibrate the model: The World Bank Enterprise Survey (WBES), the United Nations (UNCTAD), OECD Stan, and the database on bilateral trade flows from Waugh (2010).

World Bank Enterprise Survey: This database is a stratified sample of the universe of firms in developing countries. I use the standardized survey, which has data starting in 2006. This database is being updated continuously, and for many countries there is already a panel of two years. I use this database to obtain statistics related to firms' performance for South American countries : a) proportion of exporting firms; b) proportion of foreign firms. I consider only firms in the manufacturing sector.

UNCTAD: I use the Foreign Direct Investment profile for the Latin-American countries under study. I use data on the origin of the stock of FDI by country.

OECD Stan: I use data on the production by multinational firms and proportion of firms exporting for Europe.

Waugh(2010): This data-base contains information on trade for a large set of countries for the year 1996, including Latin-American and European countries. I use trade statistics (exports and imports) by origin and destiny in order to construct bilateral trade flows between countries and the absorption measure reported.

4.2 Calibration Strategy:

I calibrate the model separately for two regions: South America and Europe. I select these regions because they both present very different trade arrangements. While South America is characterized by high trade barriers, Europe is well-known as a low trade barrier area for the members of the European Union. Analyzing the differences in the gains from openness in these two regions for countries of different size provides information on how much countries in the closed region are losing compared to those in the open region, and how much could be the potential gains of becoming more open. To maintain symmetry I will include in both calibrations five countries, four belonging to the region and a fifth which stands for the rest of the world (RW). The countries included in each regions are (i) Argentina, Brazil, Chile, Uruguay in South America; and (ii)France, Italy, Netherlands and United Kingdom in Europe.

I will use data for 1996 whenever it is possible.[21] OECD Stan database has information on sales of multinationals only for the late 2000's. I will use data for 2007 which is the earliest year for which they have data for all countries. The parameters I need to calibrate are:

- **Size (L_i):** I use data from the UNCTAD on labor force. I normalize Uruguay's size to 1 ($L_{Uru} = 1$). Country sizes are then $L_{Arg} = 9.47$, $L_{Bra} = 48.94$, $L_{Chi} = 3.69$ and $L_{RW} = 1582.5$.[22] For Europe, country sizes are: $L_{Fra} = 16.8$, $L_{Ita} = 14.9$, $L_{UK} = 18.7$ and the $L_{RW} = 1567.3$.
- **Substitutability between varieties (σ):** I use a value of 6 which generates a mark-up of 20%, as is common in the literature (for example in Ghironi and Melitz (2005)).
- **Productivity distribution:** I assume that productivities are drawn from a Pareto distribution with scale parameter $\phi_m = 1$ for all countries. I will assume that all countries have the same shape parameter α. Given the Pareto assumption for productivities, sales are distributed Pareto with shape parameter $\alpha/(\sigma-1)$. There is a large discussion in the literature about the value of α and $\alpha/(\sigma-1)$. Chaney (2008) finds that $\alpha_i/(\sigma-1)$ is around 2 for the US, but he does not calculate the value of α and σ. Ramondo and Rappoport (2010) use $\alpha = 4$. Breinlich and Cuñat (2010) estimate $\alpha/(\sigma-1)$ and find values ranging from 1.13 to 4.88. Arkolakis et al. (2013) use $\alpha = 4.2$. Finally, Arkolakis and Muendler (2010) estimate $\alpha/(\sigma-1)$ from Brazilian data and find a value of 1.21. I use the estimate of Arkolakis and Muendler (2010) for two reasons. First, because they estimate the shape parameter of sales from Brazilian data, one of the countries I am studying. Second, because $\sigma = 6$ implies $\alpha = 6.05$ which is in the middle range of previous estimates.

[21] In 1995 the MERCOSUR members should have had the last reduction in tariffs for trade within the region, and a common tariff for the rest of the world. For a more detailed discussion on this see Bustos (2011).

[22] To calculate the RW I take out Russia and Germany from all the variables, two big countries not included in Waugh (2010).

- **Fixed entry cost (κ_i^e):** In order to make a productivity draw, firms in country i should pay a fixed cost κ_i^e. I calibrate this parameter to match the GDP per capita in each country relative to the RW for the year 1996.

- **Fixed operating cost (κ_i^d):** If a firm decides to operate, it has to pay a fixed cost (κ_i^d). I will set the value of this parameter such that the smallest firm producing in each country demands 10 workers. The amount of labor demanded by the smallest firm is:

$$\ell(\phi_{iii}^*) = \sigma\kappa_i^d \ . \tag{31}$$

  As equation (31) shows, labor demand of the firm with productivity level equal to the domestic cut-off productivity only depends on σ and κ_i^d.[23] As all countries have the same σ, all countries should have the same κ_i^d in order to obtain that the smallest firm demands ten workers in all countries. I thus set $\kappa_i^d = 10/6$ for all i.

- **Fixed cost of exporting (κ_i^x):** In order to export, a firm has to pay an additional fixed cost (κ_i^x). This cost directly affects the mass of firms deciding to export. I will use the proportion of firms exporting as a fraction of the total number of operating firms. For South America I use firm-level data from the World Bank Enterprise Survey to calculate this statistic in the data. For Europe, I use the OECD Stan dataset.

- **Fixed cost of doing MP (κ_i^{MP}):** To operate in a foreign country, a firm has to pay a fixed cost of (κ_i^{MP}) in the country where the firm will open the plant. I will calibrate this parameter to match the proportion of foreign firms in a given country. As this cost increases, the proportion of foreign firms decreases. I use data from the World Bank Enterprise Survey to construct this statistic in the data for South America and OECD Stan for Europe.

- **Iceberg cost of exporting (τ_{ji}):** In order to deliver one unit to country j, firm in country i has to deliver τ_{ji} units. These parameters are pinned down to target $Trade_{ji}$ over $Absorption_i$[24] across the countries in my study. I use data from Waugh (2010) on trade of manufactures to construct these targets.

- **Productivity shifter (γ_{ji}):** When a firm produces abroad the productivity of a firm is shift by γ_{ji}. The new productivity for a firm from country i producing in country j is $\hat{\phi} = \frac{\phi}{\gamma_{ji}}$. To calibrate this parameter I use the proportion of sales from foreign firms in the domestic country. I do not allow firms from the countries

[23] See proof in the appendix.

[24] $Trade_{ji}$ is imports of country i from country j plus exports from country i to country j. Absorption is calculated as $GDP_i + Imports_i - Exports_i$.

in the sample to perform FDI in the rest of the world. Using data from the WBES I compute the participation of foreign sales on total sales. Unfortunately, this database does not have the country of origin of foreign firms. So, I use the composition of FDI stock in manufactures to impute these values. The data on FDI stock in manufactures come from the UNCTAD Foreign Direct Investment profile for South America and OECD Stan for Europe.

- **Endowment of the homogeneous good (z_i):** I use the trade deficit in the manufacturing sector to calibrate this parameter.

4.3 Calibration Results

Tables 3 to 8 present the calibrated parameters. Panel A of each table presents the results for South America, while Panel B presents the results for Europe. The model performs well in matching the selected targets. The GDP per capita of the RW is normalized to 1. To match the much higher GDP per capita in Europe relative to the RW (see Table 8), I need to impose much lower entry costs in Europe than in South America (second column of Table 3). Table 4 shows that the model also matches the trade balance over absorption in the manufacturing sector, even though it slightly overestimates Italian trade surplus (9.5 in the model versus 8.9 in data). For the proportion of firms exporting (first column of Table 3) and the participation of foreign firms sales in total sales (Table 7), the model is able to match the data almost perfectly for all countries.

To match the trade statistics I use variable and fixed trade costs. Note that Argentina and Brazil, the two largest countries in South America, show lower ratios of Trade-to-Absorption, 35.8% and 22.8% respectively. On the other hand, Chile and Uruguay, the smallest countries, show much higher ratios: 59.4% and 58.3%. In order to match the large proportion of domestic firms exporting in Argentina, the model requires small fixed costs of exporting for this country (see column four of Table 3). This also allows smaller firms to enter into the export market, making it possible to match at the same time the large proportion of firms exporting and the relatively low trade-to-absorption ratio. The importance of the RW as a trade partner is also shown in the calibrated parameters. Participation of the RW in trade goes from 51% for Uruguay to 86% for Chile. As a result Chile, with high variable trade cost of exporting to the rest of South American countries, presents a low average variable trade cost (compared to the levels of Argentina and Brazil around 100%). Uruguay is the country in the region with the highest average variable trade costs (124%), something unexpected since it is the smallest country.[25]

[25] Small countries are those who benefit the most from openness according to traditional trade theory.

For Europe we can immediately observe that trade-to-absorption is much higher than in South America. Italy, the country with the lowest ratio has a value of 44.1%, while the Netherlands, the country with the highest ratio, exhibits a ratio of 118.1%. In order to match the higher ratio, the model requires much smaller trade costs. This is shown in Figure 1, which shows the average trade costs by country (both the simple average or a weighted by trade composition average). It can be easily seen that South American countries face much higher average trade costs than European countries. The weighted average trade cost in South America is 111% (so the average variable cost is $\tau = 2.11$), while in Europe it is 65% (the average variable cost in Europe is $\tau = 1.649$). Another interesting fact is that while in Europe the smallest country, the Netherlands, faces the lowest trade cost, in South America the smallest country, Uruguay, faces the highest average trade costs.

Similar observations apply to multinational production.[26] As in the case of trade costs, the efficiency parameter γ is much higher for South America than Europe (see Table 7). This implies that foreign firms are much less productive operating abroad in South America than in Europe. The average value of this parameter is 1.92 in South America, while it is 0.58 in Europe. The fact that in Europe the average γ is smaller than one is mainly driven by the productivity of firms from the RW operating abroad. Firms from the RW operating in Europe are three times more efficient than in their domestic countries. Then, as most of the MP comes from the RW, the average γ in Europe is smaller than one.

To sum up, the baseline model is consistent with cross country evidence on bilateral trade flows and multinational production for the set of selected countries. South America faces higher trade barriers than Europe, and these trade barriers vary with country size among regions. While the smallest country in Europe (the Netherlands) is the one with the smallest average trade costs, Uruguay, the smallest country in South America is the one with the highest average trade cost. Also, South American countries cannot attract as much MP as European countries because the productivity of multinationals operating in South America is much lower than the productivity of multinationals operating in Europe.

5 Experiments

I use the calibrated model to perform a set of counterfactual experiments. First, I investigate how much countries benefit from trade and MP by closing the economies (a

[26] I set a value of 100 to γ_{ij} when there are zeros in the data.

world in autarky), and study the role played by MP and BMP in countries of different size. Then, I reduce trade and MP costs in South America and study the potential gains in real GDP. Finally, I analyze the role of trade and MP in shaping the distribution of firm size in countries of different size in the two regions. In summary, I will quantitative study the following:

1. To assess the low gains from trade and MP attained by South America relative to Europe, I compute the losses (changes in real manufacturing GDP and GNP) of moving to autarky in South America relative to Europe.
2. To assess the role played by MP and BMP in explaining the previous results I perform three exercises:
    - To assess the role played by BMP, I compare the losses of moving to autarky in a world with and without BMP.
    - To assess the role of MP itself (without including BMP), I set up a world without BMP, and compare the losses of moving to autarky with and without MP.
    - To assess the role played by MP as a whole (including BMP), I compare the losses of moving to autarky in a world with and without MP.
3. To assess the potential gains from an improvement in the degree of openness in South America, I compute the changes in real manufacturing GDP and GNP of decreasing trade costs in South America to the average level in Europe with three different configurations:
    - Maintaining the same multinational production costs.
    - Increasing the efficiency of foreign firms producing in South America by 20%.
    - Increasing the efficiency of foreign firms producing only in Uruguay by 20%.
4. To assess the effects of MP and trade on firm size distribution, I compute the proportion of firms with more than 100 and 250 employees in the baseline economy, in an economy without MP and in an economy in autarky.

5.1 Gains from Openness

To study the gains from openness, I close the economies to trade and MP (a world in autarky). In autarky, $\gamma_{ij} = \tau_{ij} = \infty$. The first two columns of Table 9 present the changes in real manufacturing GDP and GNP using as benchmark the calibrated model economies. Panel A presents the results for South America and Panel B for Europe. Losses of moving to autarky in Europe are much larger than in South America (10.5%

versus 5.3% of real GDP) which indicates that Europe benefits much more from openness than South America. This is expected since trade costs are higher and a efficiency of foreign firms is lower in South America compared to Europe. Small countries lose more than large countries in both regions. The higher degree of openness in Europe results in larger differences between the country that loses the most and the country that loses the least compared to South America. In Europe, the Netherlands loses 20.3% of real GDP and Italy loses 5.6% (almost 15 percentage points difference), while in South America, the difference between the losses of Uruguay and Brazil is 8.5 percentage points.

The last two columns of Table 9 present the changes in real manufacturing GDP and GNP using as benchmark a modified version of the baseline economy, an economy where BMP is not allowed. Allowing for BMP introduces an extra possibility for foreign firms, the possibility of using a third country as an export platform. However, the extent to which they will be able to benefit from exports will be determined by trade barriers. Comparing the results of the third column, to the one of the first column, we can see that BMP is more important in small countries than in large countries, and that European countries benefit more from BMP. The losses for the Netherlands in a world without BMP are 4 p.p lower than in the benchmark economy, while for Uruguay are only 1.2 p.p. lower. Then, high trade barriers not only affect the exports of domestic firms but also the exports of foreign firms, and as a result the ability of small countries to attract multinational firms.

5.1.1 The role played by MP and BMP

To disentangle the role played by MP and BMP in the gains from openness, I perform three experiments. The results of these three exercises are presented in Table 10.

To assess the role played by BMP in explaining the gains from openness, I compute the ratio between the losses of going to autarky in the baseline economy and the losses of going to autarky in a world without BMP. If this ratio is 1 it means that BMP plays no role in explaining the gains from openness, while BMP becomes more important as this ratio increases. The first column of Table 10 presents the results. The ratio is larger for small countries than for large countries, and also tends to be higher in Europe than in South America. In Uruguay the losses of going to autarky in the baseline economy are 11.2% higher than in the case without BMP and in the Netherlands this number goes up to 19.9%, while for Brazil and Italy this ratio is 1.035 and 1.097 respectively. Then, small countries benefit more from BMP as expected.

To assess the role played by MP itself (this means MP without the possibility of BMP), I compute the ratio between the losses of going to autarky in a world without

BMP and the losses of going to autarky in a world without MP. This ratio shows the importance of MP itself in explaining the gains from MP as a whole. If the ratio is close to 1 it means that the gains from MP mostly come through BMP, while as this ratio increases it means that MP itself becomes more important (and as a result BMP becomes less important) in explaining the gains from MP. The second column of Table 10 presents the results. In Europe, Italy loses 84% more going to autarky in a world without MP compared to a world without BMP, but the Netherlands loses only 20% more. As expected MP itself is more important in explaining the gains from MP in the large country than in the small one. In South America both Brazil and Uruguay present the same ratio 1.51. This happens because the efficiency of multinational firms operating in Brazil is very low, and so MP is not a very cheap way of overcoming trade barriers. However, another large country like Argentina has a ratio much higher (2.033) than the one of Uruguay.

Finally, to asses the role played by MP through both channels, I compute the ratio between the losses of going to autarky in the baseline economy and the losses of going to autarky in a world without MP. If this ratio is 1 it means that MP plays no role in explaining the gains from openness, while as it increases it means that MP becomes more important in explaining the gains from openness. The third column of Table 10 presents the results. Large countries display the highest gains from MP. In South America the country that benefits the most is Argentina and in Europe Italy. In Europe, the country that benefit the least from MP is the Netherlands, the smallest country, while in South America it is Brazil, the largest country. The underlying message is the same as in the previous exercise: since in Brazil the efficiency of multinational firms is low, the role played by MP is lower. Also, as South America as a region is closed then the gains from trade are not very large, which increases the importance of MP in explaining the gains from openness.

To sum up, if countries face relatively low trade costs and high efficiency of foreign firms, large countries benefit more from MP as a whole, with small countries benefitting more from BMP. On the other hand, if trade costs are high and the efficiency of multinationals is low the large country may not benefit from MP more than the small country.

5.2 Reducing trade costs and improving efficiency

To study the potential gains in South America of an improvement in the degree of openness, I reduce the average trade costs for all countries in the calibration for South

America to the average level in Europe ($\tau = 1.64$).[27]

Panel A of Table 11 presents the result of reducing trade costs only in South America to the average level in Europe (i.e. imposing $\tau = 1.64$ to all South American countries). All countries gain by reducing trade costs, but the smallest country, Uruguay, gains significantly more. The gains in Uruguay are 29.9% of real manufacturing GDP, while in Brazil, the largest country, are just 4%. I find, as Eaton and Kortum (2002), that the gains from reducing trade costs are larger than the losses of going to autarky. Thus, just by reducing trade barriers South America can obtain large gains.

To assess the potential gains South American countries may obtain from the interaction of trade and MP, in addition to the reduction in trade costs I increase the productivity of multinational firms by 20%. Panel B of Table 11 presents the results of this experiment. There is a large gain in real manufacturing GDP in all countries, but specially in large countries. However, since multinational firms send their profits back, the increase is not reflected in a large increase in real manufacturing GNP, except for Uruguay. In Uruguay, real manufacturing GDP increases more than 9 percentage points relative to the previous experiment, while real manufacturing GNP increases almost 7 percentage points more relative to the previous experiment.

Panel C of Table 11 presents the result of increasing only the efficiency to multinationals operating in Uruguay by the same magnitude as in the previous exercise. Changes in real manufacturing GDP for the rest of countries are the same as in the case of only reducing trade costs, while in Uruguay it increases by 12 percentage points in addition. An interesting result from these experiments is that Uruguay would gain more if the efficiency improves only domestically compared to the case where it improves in all the countries of the region. This is because if the efficiency only improves in Uruguay there is a larger set of multinationals going to this country.

Discussion on Bridge Multinational Production

The previous experiments reflect the importance of BMP for a small country. In the absence of BMP, the gains in real manufacturing GDP of reducing trade barriers decrease for all countries, but they decrease significantly more for Uruguay. In Uruguay the gains are reduced by 6.2 p.p. while in Brazil they are only reduced in 0.3 p.p. (see Panel A of Table 11, column 3). When trade costs are reduced, small countries can attract more foreign firms who will locate there to export to the rest of countries, explaining the importance of BMP. This indicates that, for small countries like Uruguay to take

[27] Trade barriers can be reduced by reducing trade tariffs within the region and also with the RW, improving the available infrastructures, forcing countries to respect trade agreements, etc.

advantage of MP, it needs to be able to export to the rest of countries in the region.

Panel B of Table 11 shows in the third column the increase in manufacturing real GDP when in addition to the reduction in trade costs we increase the efficiency of foreign firms but we do not allow for BMP. Compared to the numbers in Panel A we see that Uruguay is the country with the smallest additional increase in manufacturing real GDP (0.7 p.p.) while the remaining countries show increases that go from 1.9 p.p. to 4 p.p. This result indicates that BMP is crucial for Uruguay to benefit from increases in the efficiency of multinationals since otherwise the gains would not be larger than the ones it would get by only reducing trade costs.

Finally, if we improved only the efficiency of foreign firms operating in Uruguay we again find that BMP is crucial to explain the gains. While in the baseline economy real manufacturing GDP increases 10.9 p.p. more than when we only reduce trade costs, if we shut down BMP the additional increase is only of 2.3 p.p. The result is explained because without the possibility of serving third countries Uruguay does not become an attractive location for multinational firms, even with the increase in productivity, because its domestic market is small.

Discussion on the Role of Assumption 1

Assumption 1 allows me to treat each activity as independent. With assumption 1 a firm located in Uruguay and exporting to Brazil is going to produce a different good than a firm that decided to locate in Brazil and sell in Brazil. Using the fact that activities are independent I can calculate profits for each activity separately which simplifies the solution of the problem. Without assumption 1 a firm would have to choose from which location to serve each market. With assumption 1 a firm can serve one market from all the locations. Then, assumption 1 reduces the degree of competition between countries to attract MP. The decrease in competition also reduces the importance of the efficiency of multinationals operating in my country. Without assumption 1, a firm will choose to locate in the country that is more efficient, and the remaining countries will not be able to attract this firm (as long as trade costs are low enough). Now, all countries may attract MP as long as the activity is profitable for a firm. Then the gains I obtained from reducing trade barriers and improving efficiency will be higher without assumption 1 for the most efficient country (the one that will be able able to attract larger amounts of MP).

The importance of assumption 1 is closely link to the role played by BMP. Without assumption 1, BMP is crucial for the most efficient market to be able to attract MP, specially if the most efficient country is small, since what the firm wants is to serve all countries from the cheapest location (and as opening plants in other countries involves paying a fixed cost, firms may want to minimize the number of locations). The small country, even though it is efficient is not going to attract MP since the domestic market is small. Then, BMP becomes a very important factor without assumption 1. Then, my results are a lower bound for the importance of BMP. The importance of BMP for a small country in the open region, like the Netherlands, might be underestimated if the country is used as an export platform to serve the remaining European countries. Also the differences between how much BMP contributes to the gains from openness between Uruguay and the Netherlands (the small countries in each region) will be enhanced without assumption 1.

To sum up, assumption 1 simplifies the solution of the problem by making each activity independent. Assumption 1 decreases the competition between countries to attract MP which reduces the importance of the efficiency of multinationals operating in the domestic country. Finally, even though with assumption 1 BMP is an important factor, without assumption 1 BMP will be crucial for attracting MP, specially for small countries.

5.3 Firm size distribution

There is a large literature studying the effects of different kinds of friction on the size distribution of firms. Previous studies have focused on the effects of size dependent policies (Guner et al. (2008), Restuccia and Rogerson (2008), García-Santana and Pijoan-Mas (2012)), capital market imperfections (Erosa (2001), Amaral and Quintin (2010), Buera et al. (2011), Greenwood et al. (2010)) and trade (Melitz (2003), Piguillem and Rubini (2012)) on firm size distribution. I contribute to this literature by assessing the effect of trade and MP on the distribution of firms' sizes and show that these effects vary across small and large countries within a region, and also among countries of similar size across regions with different degrees of openness.

Let us first study the total effect of trade and MP in the distribution of firm size. In autarky all countries will have the same distribution of firms,[28] while in the baseline economy this distribution differs significantly across countries. In the baseline economy, the small country in each region has a higher proportion of large firms than the large country. In South America, Uruguay has 1.1% of firms with more than 250 employees while Brazil has 0.8%, and in Europe this proportion is 4.2% for the Netherlands and 1.7% for Italy. It can also be observed that trade and MP has a larger impact on the size distribution of firms for countries in Europe (the open region) than for countries in South

[28] This comes from the fact that I am using a Pareto distribution with the same shape parameter for the productivity of firms.

America (the closed region). The proportion of firms with more than 250 employees is almost four times larger in the Netherlands than in Uruguay (4.4% vs 1.1%), and in Italy it doubles that of Brazil (1.7% vs 0.8%). As Europe is more open, they benefit more from trade and MP and these shape the distribution of firms increasing the proportion of large firms.

To disentangle the role played by trade and MP in shaping the size distribution of firms, I compute the distribution of firm sizes in a world without MP. Comparing the result of the column No MP to autarky we obtain the contribution of trade to the size distribution of firms, and comparing the result of the column named baseline to the one named No MP we obtain the contribution of MP.

For large countries, MP seems to be the most important factor. While the proportion of large firms is almost unchanged when allowing for trade compared to autarky, it increases significantly when we allow for MP. In Italy from the 1 percentage point increase explained by openness, 0.2 p.p. is explained by trade while 0.8 p.p. is explained by MP. For small countries this is not true. Both trade and MP have similar effects. In Uruguay allowing for trade increases the proportion of large firms 0.2 p.p. and allowing for MP increases the proportion 0.2 p.p. In the Netherlands, trade increases the proportion of large firms by 1.3 p.p. and MP by 2.2 p.p.

To sum up, trade and MP have important effects on the size distribution of firms, but this effect varies across countries and regions. Openness has a larger effect in the size distribution of firms on countries in the open region and in small countries compared to large countries.

6 Conclusions

In this paper I construct a heterogeneous firms model of trade with asymmetric countries, MP, and BMP to study the effects of trade barriers and country size in the location decision of multinational firms. I find that BMP is crucial for a small country to attract MP and to take full advantage of trade liberalization and efficiency improvements. BMP explains up to 20% of the gains from openness in the Netherlands while only 10% in Uruguay.

If trade costs are reduced in South America to the average level in Europe, Uruguay's real manufacturing GDP increases 30%. If I do not allow for BMP this increase is reduced by 6 percentage points. If in addition we improve the efficiency of multinationals operating in Uruguay by 20%, real manufacturing GDP increases 41.8%. However, almost all the additional increase in manufacturing real GDP is explained by BMP, since without BMP the increase is 26%, only 2.3 p.p. larger than without any improvement in the efficiency of multinationals.

Finally, MP and BMP shift the distribution of firms toward large firms reinforcing the effect of trade. While in autarky the Netherlands and Uruguay have the same distribution of firms, in the calibrated version of the model, the Netherlands has a proportion of firms with more than 100 employees which doubles that of Uruguay, and with more than 250 employees which is four times larger.

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Table 3: Calibrated Parameters

Panel A

	L_i	κ^e	κ^d	κ^x	κ^{MP}	z
Argentina	9.47	0.09	1.67	0.34	11.77	0.13
Brazil	48.94	1.95	1.67	1.15	2.07	0.35
Chile	3.69	0.13	1.67	2.05	19.07	0.04
Uruguay	1.00	0.07	1.67	0.82	9.17	0.01
Rest of the World	1582.5	3.00	1.67	1.00	2.67	12.66

Panel B

	L_i	κ^e	κ^d	κ^x	κ^{MP}	z
France	16.8	3.3e-6	1.67	0.89	5.42	0.78
United Kingdom	18.7	3.0e-6	1.67	1.50	10.07	0.90
Italy	14.9	1.0e-6	1.67	1.25	9.87	0.73
Netherlands	4.9	1.0e-6	1.67	3.32	10.97	0.25
Rest of the World	1567.3	1.00	1.67	1.00	2.67	15.67

Table 4: Calibration Results-Iceberg Export Costs

Panel A

	Exporting country				
Country	Argentina	Brazil	Chile	Uruguay	RW
Argentina	1	2.27	2.73	2.39	2.61
Brazil	1.48	1	2.36	1.76	2.03
Chile	1.66	2.07	1	2.27	1.93
Uruguay	1.75	2.19	2.57	1	2.68
Rest of the World	1.74	1.97	2.06	2.22	1

Panel B

	Exporting country				
Country	France	UK	Italy	Netherlands	RW
France	1	1.82	1.62	1.74	1.61
UK	1.59	1	1.70	1.50	1.36
Italy	1.80	1.94	1	1.74	1.81
Netherlands	1.52	1.49	1.55	1	1.32
Rest of the World	1.81	1.80	1.78	1.77	1

Table 5: Calibration Results-Efficiency of Multinational Firms

Panel A

	Country of origin				
Country	Argentina	Brazil	Chile	Uruguay	RW
Argentina	1	1.47	1.46	1.41	1.48
Brazil	3.75	1	3.08	2.45	2.49
Chile	2.49	2.35	1	2.15	1.81
Uruguay	100	100	100	1	2.02

Panel B

	Country of origin				
Country	France	UK	Italy	Netherlands	RW
France	1	1.62	2.15	1.83	0.33
UK	1.65	1	2.20	1.68	0.28
Italy	1.40	1.49	1	1.55	0.29
Netherlands	1.65	1.47	100	1	0.29

Table 6: Performance of the Model-Trade Composition

Trade (as % of Absorption)-Data vs Model

Panel A

	Arg		Bra		Chi		Uru		RW	
	Data	Model	D	M	D	M	D	M	D	M
Arg	-	-	2.9	3.3	3.5	3.8	9.5	9.8	0.2	0.2
Bra	9.6	9.1	-	-	4.2	4.4	17.2	16.8	0.5	0.5
Chi	1.5	1.1	0.5	0.4	-	-	1.8	1.5	0.2	0.1
Uru	0.8	0.9	0.5	0.5	0.4	0.5	-	-	0.0	0.0
RW	24.0	24.2	18.9	18.9	51.3	51.5	29.9	30.1	-	-
Total	35.8	35.1	22.8	23.2	59.4	60.1	58.3	58.2	0.9	0.9

Panel B

	Fra		UK		Ita		Neth		RW	
	Data	Model	D	M	D	M	D	M	D	M
Fra	-	-	8.5	6.3	8.6	7.5	13.8	11.2	2.3	1.7
UK	6.7	7.2	-	-	4.4	5.1	18.6	18.9	2.6	2.8
Ita	7.6	7.2	5.0	4.3	-	-	9.7	10.8	1.7	1.3
Neth	3.4	3.6	5.8	5.3	2.7	3.6	-	-	1.2	1.2
RW	34.2	34.7	49.5	49.7	28.4	28.0	76.2	77.2	-	-
Total	51.9	52.7	68.9	65.6	44.1	44.2	118.4	118.1	7.7	7.1

Table 7: Performance of the Model-Foreign Production Composition

Foreign Sales (as % of Total Sales)-Data vs Model

Panel A

	Argentina		Brazil		Chile		Uruguay	
	Data	Model	Data	Model	Data	Model	Data	Model
Argentina	-	-	0.1%	0.1%	0.7%	0.6%	0.0%	0%
Brazil	1.4%	1.5%	-	-	0.3%	0.3%	0.0%	0%
Chile	1.4%	1.4%	0.1%	0.1%	-	-	0.0%	0%
Uruguay	1.1%	1.0%	0.2%	0.2 %	0.3%	0.3%	-	-
RW	31.9%	31.7%	7.8%	8.0%	32.5%	32.3%	29.7%	30.7%

Panel B

	Fra		UK		Ita		Neth	
	Data	Model	Data	Model	Data	Model	Data	Model
France	-	-	2.5%	2.1%	2.5 %	2.6%	2.9%	2.6%
UK	3.0%	3.4%	-	-	1.6%	1.9%	4.9%	5.3%
Italy	1.6%	2.1%	1.2%	1.3%	-	-	0.0%	0.0%
Netherlands	1.4%	1.4%	2.1 %	1.6 %	1.3%	1.2	-	-
RW	20.1%	20.3%	38.9%	37.8%	13.2%	12.7%	35.0%	35.1%

Table 8: Calibration Results: Aggregate Targets

Panel A

	Data vs Model							
	% Exporting Firms		% Foreign Firms		GDP per Capita		Trade Balance	
	Data	Model	D	M	D	M	D	M
Arg	52.3	52.4	7.9	7.9	1.56	1.56	-6.0	-6.1
Bra	14.1	14.1	7.2	6.9	0.87	0.86	-1.5	-1.8
Chi	24.6	24.3	5.8	5.8	1.08	1.08	-8.9	-8.8
Uru	33.3	32.8	7.7	8.0	1.22	1.24	-10.1	-9.9

Panel B

	Data vs Model							
	% Exporting Firms		% Foreign Firms		GDP per Capita		Trade Balance	
	Data	Model	D	M	D	M	D	M
Fra	44.7	45.0	11.5	11.7	4.7	4.6	0.7	0.7
UK	37.0	36.7	12.6	12.8	4.6	4.6	-2.2	-2.4
Ita	28.4	28.9	3.6	3.9	5.5	5.5	8.9	9.5
Neth	42.2	42.5	12.9	12.5	5.0	5.0	-1.8	-1.6

Table 9: Experiment Results-Closing the Economies

Panel A

	Changes in %			
	Autarky with BMP		Autarky without BMP	
	Real GDP	Real GNP	Real GDP	Real GNP
South-America	-5.3	-3.4	-5.0	-3.5
Argentina	-9.5	-4.9	-9.0	-5.2
Brazil	-3.6	-2.5	-3.5	-2.5
Chile	-11.9	-8.7	-10.9	-8.9
Uruguay	-12.1	-10.8	-10.9	-10.4

Panel B

	Changes in %			
	Autarky with BMP		Autarky without BMP	
	Real GDP	Real GNP	Real GDP	Real GNP
Europe	-10.5	-7.3	-9.3	-7.3
France	-9.1	-6.4	-8.3	-6.3
UK	-13.4	-8.8	-11.9	-9.0
Italy	-5.6	-3.5	-5.1	-3.6
Netherlands	-20.3	-17.1	-17.0	-16.5

Table 10: Experiment Results-The Effects of MP and BMP

Panel A

	Relative losses in real GDP		
	$\frac{\text{Baseline}}{\text{World without BMP}}$	$\frac{\text{World without BMP}}{\text{World without MP}}$	$\frac{\text{Baseline}}{\text{World without MP}}$
Argentina	1.055	2.033	2.145
Brazil	1.035	1.510	1.563
Chile	1.099	1.478	1.623
Uruguay	1.112	1.510	1.680

Panel B

	Relative losses in real GDP		
	$\frac{\text{Baseline}}{\text{World without BMP}}$	$\frac{\text{World without BMP}}{\text{World without MP}}$	$\frac{\text{Baseline}}{\text{World without MP}}$
France	1.087	1.679	1.826
UK	1.128	1.671	1.885
Italy	1.097	1.841	2.019
Netherlands	1.199	1.203	1.443

Table 11: Experiment Results-Reducing Costs

Panel A

	Changes (in %)			
	Same MP costs		Same MP Costs-No BMP	
	Real GDP	Real GNP	Real GDP	Real GNP
South-America	6.2	6.1	5.3	6.0
Argentina	11.2	11.1	9.0	11.7
Brazil	4.0	4.0	3.7	4.0
Chile	13.1	12.4	9.4	12.4
Uruguay	29.9	29.1	23.7	27.1

Panel B

	Changes (in %)			
	Improve 20% efficiency		Improve 20% efficiency- No BMP	
	Real GDP	Real GNP	Real GDP	Real GNP
South-America	9.7	6.6	7.6	6.9
Argentina	17.7	11.7	13.0	13.0
Brazil	6.3	4.3	5.6	4.5
Chile	21.4	14.3	12.4	14.3
Uruguay	38.3	36.6	24.4	30.7

Panel C

	Changes (in %)			
	Improve 20% efficiency only in Uruguay		Improve 20% efficiency only in Uruguay- No BMP	
	Real GDP	Real GNP	Real GDP	Real GNP
South-America	6.3	6.1	5.3	6.2
Argentina	11.1	11.1	9.0	11.7
Brazil	4.0	4.0	3.7	4.0
Chile	13.1	12.4	9.4	12.4
Uruguay	41.8	29.3	26.0	28.1

For all the experiments I use the average trade costs in Europe ($\tau = 1.64$)

Table 12: Experiment Results-Firms Size Distribution

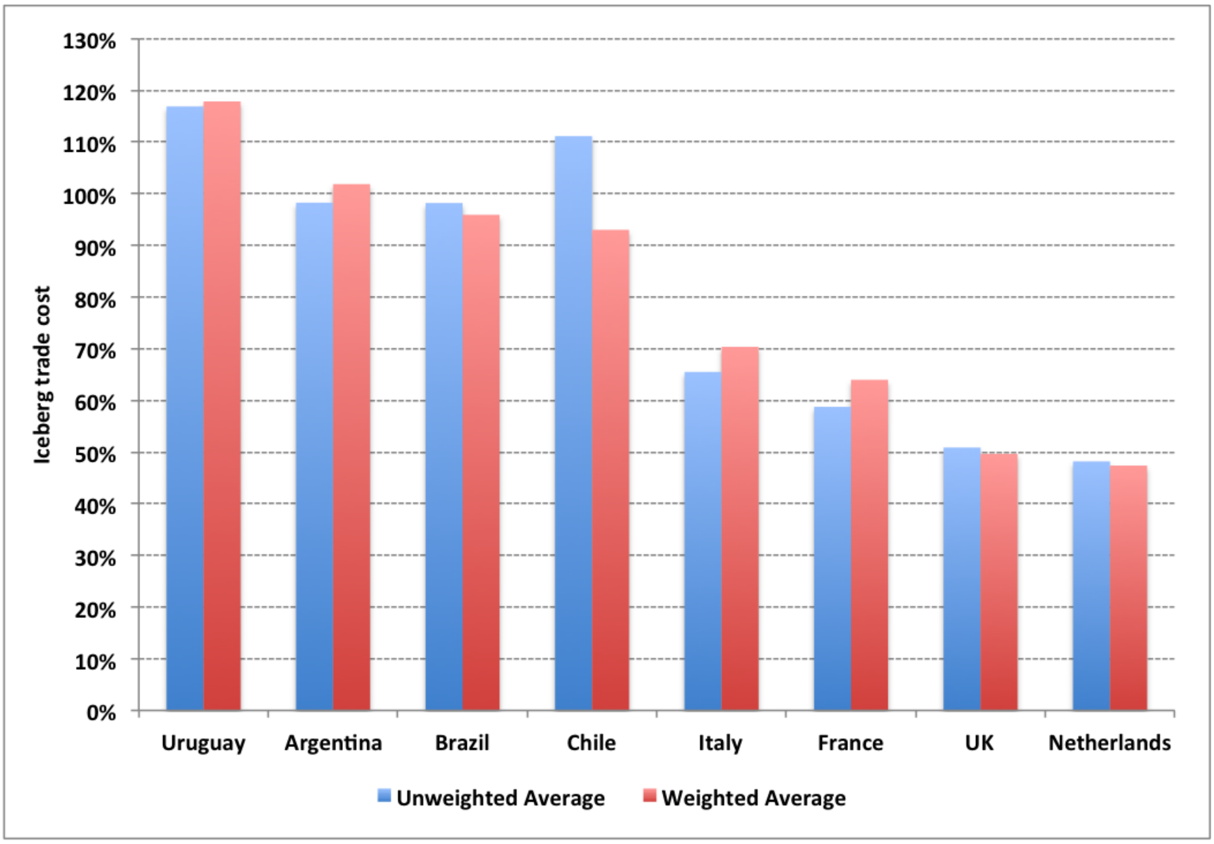
Panel A

	Proportion of firms with more than x employees							
	Benchmark		No BMP		No MP		Autarky	
	> 100	> 250	> 100	> 250	> 100	> 250	> 100	> 250
Uruguay	3.4	1.1	3.2	1.0	2.9	0.9	2.2	0.7
Netherlands	7.9	4.2	7.5	3.7	5.6	2.0	2.2	0.7

Panel B

	Proportion of firms with more than x employees							
	Benchmark		No BMP		No MP		Autarky	
	> 100	> 250	> 100	> 250	> 100	> 250	> 100	> 250
Brazil	2.4	0.8	2.3	0.8	2.4	0.8	2.2	0.7
Italy	3.5	1.7	3.5	1.6	2.8	0.9	2.2	0.7

Figure 1: Average iceberg trade costs



7 Appendix

7.1 Labor for the smaller firm operating

The amount of labor demanded by the smaller firm is:

$$\begin{aligned}
\ell(\phi^*_{iii}) &= \frac{q(\phi^*_{iii})}{\phi^*_{iii}} + \kappa^d_i \\
q(\phi^*_{iii}) &= \frac{r(\phi^*_{iii})}{p(\phi^*_{iii})} \\
\text{from equation (\textbf{??})} \rightarrow r(\phi^*_{iii}) &= \sigma w_i \kappa^d_i \\
\text{and from equation (6)} \rightarrow p(\phi^*_{iii}) &= \frac{\sigma}{\sigma - 1}\frac{w_i}{\phi^*_{iii}} \\
\text{then} \rightarrow q(\phi^*_{iii}) &= (\sigma - 1)\kappa^d_i \phi^*_{iii} \\
\ell(\phi^*_{iii}) &= \sigma \kappa^d_i
\end{aligned}$$

7.2 Aggregation

In this section I will show how to get the weighted average productivity of firms producing and selling in each country, as well as the aggregate price and production.

From equation 24 we obtain the total mass of firms producing and the total mass of firms selling in country i. Let us define the average productivity of firms performing each activity:

$$\begin{aligned}
\tilde{\phi}_{iii} &= \left[\int_{\phi^*_{iii}}^{\infty} \phi^{\sigma-1}\mu_i d\phi\right]^{\frac{1}{1-\sigma}} \\
\tilde{\phi}_{kii} &= \left[\int_{\phi^*_{kii}}^{\infty} \phi^{\sigma-1}\mu_i d\phi\right]^{\frac{1}{1-\sigma}} \\
\tilde{\phi}_{kki} &= \left[\int_{\phi^*_{kki}}^{\infty} \phi^{\sigma-1}\mu_i d\phi\right]^{\frac{1}{1-\sigma}} \\
\tilde{\phi}_{jki} &= \left[\int_{\phi^*_{jki}}^{\infty} \phi^{\sigma-1}\mu_i d\phi\right]^{\frac{1}{1-\sigma}}
\end{aligned}$$

Using the expressions from above we can define the weighted average productivity as:

$$\tilde{\phi}_i^p = \left\{\frac{1}{M_i^p}\left[M_{iii}\tilde{\phi}_{iii}^{\sigma-1} + \sum_{k\neq i} M_{kii}\frac{E_k^1}{E_i^1}\left(\frac{P_k}{\tau_{ki}P_i}\right)^{\sigma-1}\tilde{\phi}_{kii}^{\sigma-1} + \sum_{k\neq i} M_{iik}\left(\frac{1}{\gamma_{ik}}\right)^{\sigma-1}\tilde{\phi}_{iik}^{\sigma-1}\right.\right.$$
$$\left.\left. + \sum_{k\neq i}\sum_{i\neq j} M_{kij}\frac{E_k^1}{E_i^1}\left(\frac{P_k}{\tau_{ki}\gamma_{ij}}\right)^{\sigma-1}\tilde{\phi}_{kij}^{\sigma-1}\right]\right\}^{\frac{1}{\sigma-1}}, \qquad (32)$$

$$\tilde{\phi}_i^s = \left\{\frac{1}{M_i^s}\left[M_{iii}\tilde{\phi}_{iii}^{\sigma-1} + \sum_{k\neq i} M_{ikk}\left(\frac{w_k\tau_{ik}}{w_i}\right)^{1-\sigma}\tilde{\phi}_{ikk}^{\sigma-1} + \sum_{k\neq i} M_{iik}\gamma_{ik}^{1-\sigma}\tilde{\phi}_{iik}^{\sigma-1}\right.\right.$$
$$\left.\left. + \sum_{k\neq i}\sum_{i\neq j} M_{ijk}\left(\frac{\tau_{ij}\gamma_{jk}w_k}{w_i}\right)^{1-\sigma}\tilde{\phi}_{ijk}^{\sigma-1}\right]\right\}^{\frac{1}{\sigma-1}}. \qquad (33)$$

Let us write now the equation for aggregate price in country i (equation 22)

$$P_i = \left[\int_{\phi_{iii}^*}(p_{iii}(\phi))^{1-\sigma}M_i\mu_i(\phi)d\phi + \sum_{k\neq i}\int_{\phi_{ikk}^*}(p_{ikk}(\phi))^{1-\sigma}M_k\mu_k(\phi)d\phi\right. \qquad (34)$$
$$\left. + \sum_{k\neq i}\int_{\phi_{iik}^*}(p_{iik}(\phi))^{1-\sigma}M_k\mu_k(\phi)d\phi + \sum_{k\neq j}\sum_{k\neq i}\int_{\phi_{ikj}^*}(p_{ikj}(\phi))^{1-\sigma}M_j\mu_j(\phi)d\phi\right]^{\frac{1}{1-\sigma}}.$$

now, replace $p_{ikj}(\phi)$ $\forall i,j,k$ using equation 6 in the previous expression to obtain:

$$P_i = \left[\int_{\phi_{iii}^*}\left(\frac{w_i}{\rho\phi}\right)^{1-\sigma}M_i\mu_i(\phi)d\phi + \sum_{k\neq i}\int_{\phi_{ikk}^*}\left(\frac{w_k\tau_{ik}}{\rho\phi}\right)^{1-\sigma}M_k\mu_k(\phi)d\phi\right.$$
$$\left. + \sum_{k\neq i}\int_{\phi_{iik}^*}\left(\frac{w_i\gamma_{ik}}{\rho\phi}\right)^{1-\sigma}M_k\mu_k(\phi)d\phi + \sum_{k\neq j}\sum_{k\neq i}\int_{\phi_{ikj}^*}\left(\frac{w_k\gamma_{kj}\tau_{ik}}{\rho\phi}\right)^{1-\sigma}M_j\mu_j(\phi)d\phi\right]^{\frac{1}{1-\sigma}}$$

$$P_i = \left[\left(\frac{w_i}{\rho}\right)^{1-\sigma}M_i\int_{\phi_{iii}^*}(\phi)^{\sigma-1}\mu_i(\phi)d\phi + \sum_{k\neq i}M_{ikk}\left(\frac{w_k\tau_{ik}}{\rho}\right)^{1-\sigma}\int_{\phi_{ikk}^*}(\phi)^{\sigma-1}\mu_k(\phi)d\phi\right.$$
$$\left. + \sum_{k\neq i}M_{iik}\left(\frac{w_i\gamma_{ik}}{\rho}\right)^{1-\sigma}\int_{\phi_{iik}^*}(\phi)^{\sigma-1}\mu_k(\phi)d\phi + \sum_{k\neq j}\sum_{k\neq i}M_{ikj}\left(\frac{w_k\gamma_{kj}\tau_{ik}}{\rho}\right)^{1-\sigma}\int_{\phi_{ikj}^*}(\phi)^{\sigma-1}\mu_j(\phi)d\phi\right]^{\frac{1}{1-\sigma}}$$

We can replace the integral terms by each of the average productivities, and we get:

$$P_i = \frac{w_i}{\rho}\left[M_{iii}\tilde{\phi}_{iii}^{1-\sigma} + \sum_{k\neq i}M_{ikk}\left(\frac{w_k\tau_{ik}}{w_i}\right)^{1-\sigma}\tilde{\phi}_{ikk}^{1-\sigma}\right.$$
$$\left. + \sum_{k\neq i}M_{iik}\gamma_{ik}^{1-\sigma}\tilde{\phi}_{iik}^{1-\sigma} + \sum_{k\neq j}\sum_{k\neq i}M_{ikj}\left(\frac{w_k\gamma_{kj}\tau_{ik}}{w_i}\right)^{1-\sigma}\tilde{\phi}_{ikj}^{1-\sigma}\right]^{\frac{1}{1-\sigma}}$$

Note that the term inside brackets is $\frac{(M_i^s)^{\frac{1}{\sigma-1}}}{\tilde{\phi}_i^s}$, and that $p(\tilde{\phi}_i^s) = \frac{w_i}{\rho\tilde{\phi}_i^s}$. Then

$$P = (M_i^s)^{\frac{1}{1-\sigma}} p(\tilde{\phi}_i^s)$$

In a similar way we can derive the equation for aggregate GDP.

$$\begin{aligned} GDP_i &= \int_{\phi^*_{iii}} r_{iii}(\phi) M_i \mu_i d\phi + \sum_{k \neq i} \int_{\phi^*_{kii}} r_{kii}(\phi) M_i \mu_i d\phi \\ &+ \sum_{k \neq i} \int_{\phi^*_{iik}} r_{iik}(\phi) M_k \mu_k d\phi + \sum_{k \neq j} \sum_{k \neq i} \int_{\phi^*_{kij}} r_{kij}(\phi) M_j \mu_j d\phi \end{aligned}$$

Replacing $r(\phi)$ by the expressions found in equation 7 we get:

$$\begin{aligned} GDP_i &= \int_{\phi^*_{iii}} E_i^1 P_i^{\sigma-1} \left(\frac{\rho\phi}{w_i}\right)^{\sigma-1} M_i \mu_i d\phi + \sum_{k \neq i} \int_{\phi^*_{kii}} E_k^1 P_k^{\sigma-1} \left(\frac{\rho\phi}{w_i \tau_{ki}}\right)^{\sigma-1} M_i \mu_i d\phi \\ &+ \sum_{k \neq i} \int_{\phi^*_{iik}} E_i^1 P_i^{\sigma-1} \left(\frac{\rho\phi}{w_i \gamma_{ik}}\right)^{\sigma-1} M_k \mu_k d\phi \\ &+ \sum_{k \neq j} \sum_{k \neq i} \int_{\phi^*_{kij}} E_k^1 P_k^{\sigma-1} \left(\frac{\rho\phi}{w_i \gamma_{ij} \tau_{ki}}\right)^{\sigma-1} M_j \mu_j d\phi \\ GDP_i &= E_i^1 P_i^{\sigma-1} \left(\frac{\rho}{w_i}\right)^{\sigma-1} M_i \int_{\phi^*_{iii}} \phi^{\sigma-1} \mu_i d\phi + \sum_{k \neq i} E_k^1 P_k^{\sigma-1} \left(\frac{\rho}{w_i \tau_{ki}}\right)^{\sigma-1} M_i \int_{\phi^*_{kii}} \phi^{\sigma-1} \mu_i d\phi \\ &+ \sum_{k \neq i} E_i^1 P_i^{\sigma-1} \left(\frac{\rho}{w_i \gamma_{ik}}\right)^{\sigma-1} M_k \int_{\phi^*_{iik}} \phi^{\sigma-1} \mu_k d\phi \\ &+ \sum_{k \neq j} \sum_{k \neq i} E_k^1 P_k^{\sigma-1} \left(\frac{\rho}{w_i \gamma_{ij} \tau_{ki}}\right)^{\sigma-1} M_j \int_{\phi^*_{kij}} \phi^{\sigma-1} \mu_j d\phi \end{aligned}$$

We can replace again the integral terms by the average productivities for each occupation, and operating we get:

$$\begin{aligned} GDP_i &= E_i^1 P_i^{\sigma-1} \left(\frac{\rho}{w_i}\right)^{\sigma-1} \left[M_i \tilde{\phi}_{iii}^{1-\sigma} + \sum_{k \neq i} \frac{E_k^1}{E_i^1} \left(\frac{P_k}{P_i w_i \tau_{ki}}\right)^{\sigma-1} M_{kii} \tilde{\phi}_{kii}^{1-\sigma} \right. \\ &+ \left. \sum_{k \neq i} \left(\frac{1}{\gamma_{ik}}\right)^{\sigma-1} M_{iik} \tilde{\phi}_{iik}^{1-\sigma} \sum_{k \neq j} \sum_{k \neq i} \frac{E_k^1}{E_i^1} \left(\frac{P_k}{P_i \gamma_{ij} \tau_{ki}}\right)^{\sigma-1} M_j \tilde{\phi}_{kij}^{1-\sigma} \right] \end{aligned}$$

Note that the term in brackets is equal to $M_i^p * \left(\tilde{\phi}_i^p\right)^{\sigma-1}$, then

$$GDP_i = M_i^p E_i^1 P_i^{\sigma-1} \left(\frac{\rho}{w_i}\right)^{\sigma-1} \left(\tilde{\phi}_i^p\right)^{\sigma-1}$$

and as $r_{iii}(\tilde{\phi}_i^p) = E_i^1 P_i^{\sigma-1} \left(\frac{\rho \tilde{\phi}_i^p}{w_i}\right)^{\sigma-1}$ then

$$GDP_i = M_i^p r_{iii}(\tilde{\phi}_i^p)$$

7.3 Algorithm to solve for the equilibrium

In order to solve for the equilibrium we need to give $3*N$ guesses. We will give N guesses for the product of expenditure in differentiated goods and aggregate prices ($E_i^1 * P_i^{\sigma-1}$), N guesses for wages (w_i) and N guesses for the mass of firms in country i (M_i). With these guesses we can calculate the productivity cut-offs for each activity using equation 8, 11, and 14. Once we have all the cut-offs computed we need to follow the next steps for each country. Take country i:

1. Check if the exporting cut-offs (ϕ_{jii}^*), MP cut-offs (ϕ_{kki}^*) and the BMP cut-offs (ϕ_{jki}^*) are well computed.
   (a) If all the cut-offs for firms from country i producing in country k and selling to country j are bigger than the domestic cut-offs, then the domestic cut-offs are well computed and you have to go to step 2.
   (b) If at least one cut-off is smaller than the domestic cut-off:
   - If the smallest cut-off is an exporting or a MP cut-off, then:
     i. Re-calculated the domestic cut-off cut-off using equation 13.
     ii. Check that the new domestic cut-off is smaller than the rest of cut-offs (exporting, MP or BMP) or repeat the previous step incorporating the new smallest cut-off until there are no more cut-offs smaller than the domestic cut-off.
   - If the smallest cut-off is a BMP cut-off, then
     i. First re-calculated the new MP cut-off using equation 15.
     ii. If this new MP cut-off is above the domestic cut-off, then check if there are no more cut-offs smaller than the domestic one. If this is the case, go to step 2.
     iii. If this new MP cut-off is smaller than the domestic cut-off re-calculate the domestic cut-off using equation 35 and repeat the process until there are no more cut-offs smaller than the domestic one.

$$\pi_{iii}(\phi_{iii}^*)+\sum_{k\in K^x}\pi_{kii}(\phi_{iii}^*)+\sum_{k\in K_{ki}^{MP}}\pi_{kki}(\phi_{iii}^*)+\sum_{k\neq i}\sum_{j\in J_{ki}^{BMP}}\pi_{jki}(\phi_{iii}^*)=0\ , \quad (35)$$

2. Check that the MP cut-offs are well computed i.e. that all the BMP cut-offs are larger or equal than the MP cut-off in each case.
   (a) If all the BMP cut-off are above the MP cut-off, then the MP cut-off is well computed and we are done.
   (b) If at least one BMP cut-off is smaller than the MP cut-off, re-calculate the MP cut-off using equation 15.

(c) Repeat the process until there are no more BMP cut-offs smaller than the MP cut-off