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Periodic Model Changes in Oligopoly

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ABSTRACT: In a single-period quantity-setting duopoly where the effect of periodic model changes is to maintain consumer uncertainty about the likelihood that the product will be satisfactory, both firms can earn greater profit by making model changes than by standardizing their products, but a prisoner's dilemma pattern of payoffs makes standardization by both firms the only noncooperative equilibrium. In a repeated version of the same game, persistent model changes may emerge as a noncooperative equilibrium.

JEL Classification Numbers: 022, 611

Responsibility for errors is my own.

Any customer can have a car painted any color that he wants, so long as it is black.

Henry Ford

I. Introduction

Economists' models of product differentiation fall into two broad classes. In nonspatial models, the degree of product differentiation is usually taken to be exogenous, and the model is used to explain the consequences of changes in the degree of product differentiation for various aspects of market performance.[1] In spatial models, the extent of product differentiation is endogenous, with the choice of product characteristics treated by analogy with the choice of location. Early spatial models, following Hotelling [1929], examined firms' equilibrium choices of location or product characteristics when firms' choices can be changed without cost. More recent spatial models examine equilibrium product differentiation when product characteristics, once chosen, cannot be changed.[2]

These literatures do not address a kind of product differentiation that is common for consumer durable experience goods. Producers of automobiles, big-ticket kitchen appliances, stereo equipment, video cassette recorders, and like goods routinely bring out new versions of their principal products every year.[3] The new model is usually described as embodying startling improvements, and

1. See Spence [1976a, 1976b] for seminal examples, or more recently Vives [1984] and Majerus [1988].

2. See Prescott and Visscher [1977], Lane [1980], and Neven [1987].

3. Sometimes the same sort of model changes appear for producer durable goods - construction equipment and tractors are examples. Revised editions of university textbooks may be another.

sometimes this may be the case. More often, however, differences from one year to the next appear, on any objective basis, to be relatively minor.

One effect of such model changes, particularly for durable experience goods, is to maintain consumer uncertainty about product quality. If the most recent automobile I purchased was a Model T Ford, if my uncle and father-in-law and several co-workers own Model T Fords, and Model T Fords are the same from one year to the next, then I have a very good idea whether or not a Model T Ford will satisfy my needs. If the most recent automobile I purchased was a year τ Model T Ford, while some relatives own newer or older versions, that are thought to differ in vaguely specified ways, and some co-workers own Ford automobiles that bear different names (but seem to be similar in some ways to the Model T), then I will have a much less precise idea whether or not next year's Model T Ford will satisfy my needs.

In the model outlined here, if product quality is uncertain, demand maximizes expected utility. Periodic model changes result in some sales to consumers who believe, incorrectly, that they will find the product satisfactory. If product quality is known, on the other hand, demand maximizes utility, and consumers who know they would find the product unsatisfactory drop out of the market. However, some consumers with high reservation prices remain in the market. Whether a firm prefers to engage in periodic model changes depends on which type of demand yields greater profit.

Given this type of demand, I investigate conditions under which standardization and/or model changes will emerge as equilibrium strategies in noncooperative duopoly. Preliminary to this is the analysis of monopoly.

II. Monopoly

Product quality is modeled very simply: either a consumer finds the product satisfactory, or he does not. Demand when product quality is known but the product is unsatisfactory to some consumers and demand when product quality is unknown but the product is expected to be unsatisfactory to some consumers are derived from the structure of demand when it is known that the product will be satisfactory to all consumers.[4]

The quality index χ is the probability that the product will be satisfactory. If the product is standardized, χ is known. A fraction $1 - \chi$ of consumers, who would purchase the product if it were satisfactory, do not buy because they know they will find it unsatisfactory.

Because the product is a durable good, individual consumers purchase relatively infrequently. If the producer engages in regular model changes, prospective purchasers are uncertain of the quality of the current model, even though they may have purchased an earlier model at some point in the past. In this case, demand maximizes expected utility.

If it were known that the product would be satisfactory to all consumers, social welfare would be

(1) $$U(q) = aq - \frac{1}{2}bq^2 + m,$$

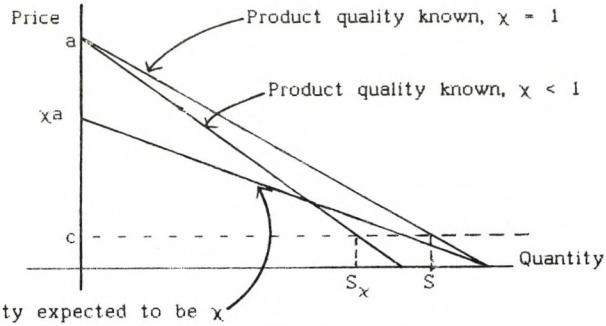
where m represents expenditure on all other goods. The implied inverse demand and demand curves are linear:

(2a) $$p = a - bq$$

(2b) $$q = \frac{a - p}{b}$$

4. For similar models, see Schmalensee [1983] or Milgrom and Roberts [1986].

Figure 1: Demand curves, Product Standardization vs. Model Changes



If the product is standardized and it is known that its quality $\chi < 1$, $1 - \chi$ of consumers, at any price, drop out of the market. Demand and inverse demand are then

(3a) $$q = \chi\frac{a - p}{b}$$

(3b) $$p = a - \frac{b}{\chi}q \ .$$

As shown in Figure 1, if product quality is known but $\chi < 1$, the demand curve rotates in a clockwise direction around its price-axis intercept, compared with the demand curve described by equations (2). Some consumers with high reservation prices stay in the market, but the quantity demanded at any price falls.

If the firm makes regular model changes, consumers do not know the quality of the particular model currently on the market. Expected quality is χ, expected utility is

(4) $$E(U) = \chi[aq - \tfrac{1}{2}bq^2] + m,$$

and the equations of the inverse demand and demand curves are

(5a) $$p = \chi(a - bq)$$

(5b) $$q = \frac{a - \frac{p}{\chi}}{b} \ .$$

When product quality is uncertain, the demand curve rotates in a counterclockwise direction around its quantity-axis intercept,

compared with the demand curve described by equations (2). Consumers with high reservation prices drop out of the market because they expect to find the product unsatisfactory. For some consumers with high reservation prices, this expectation is incorrect.

Assume that the cost function is

(6a) $$c(q_i) = cq_i,\ i = 1,2$$

if variety i is standardized and

(6b) $$C(q_i) = cq_i + F,\ i = 1,2$$

if the producer of variety i introduces a new model every year. F is the fixed and sunk cost of making periodic model changes.

Monopoly profit when the product is standardized is

(7) $$\pi^m_{ST} = \chi b\left(\frac{S}{2}\right)^2,$$

where $S = (a - c)/b$ is the quantity that would be demanded if the product were known to be satisfactory and price were equal to marginal cost.

With periodic model changes, monopoly profit is

(8) $$\pi^m_{MC} = \frac{b}{\chi}\left(\frac{S_\chi}{2}\right)^2 - F\ ,$$

where $S_\chi = (\chi a - c)/b$.

From equations (7) and (8), we have

(9) $$\pi^m_{ST} - \pi^m_{MC} = \frac{1 - \chi}{4\chi}c(\chi S + S_\chi) + F \geq 0\ ,$$

with equality holding only if $\chi = 1$ and $F = 0$. For the model considered here, therefore

Proposition 1: If it is costly to engage in model changes, a monopolist will maximize his payoff by standardizing his product.

III. Duopoly - The Structure of Demand

A. Both Varieties Known to be Satisfactory

Each of two firms produces one variety of a differentiated product. If it were known that both varieties would be satisfactory, the social welfare function would be

$$U(q_1,q_2,m) = a(q_1 + q_2) - \frac{b}{2}(q_1^2 + 2\theta q_1 q_2 + q_2^2) + m \tag{10}$$

The implied inverse demand curves are linear,

$$p_1 = \frac{\partial U}{\partial q_1} = a - b(q_1 + \theta q_2) \tag{11a}$$

$$p_2 = \frac{\partial U}{\partial q_2} = a - b(\theta q_1 + q_2) \quad . \tag{11b}$$

θ is a product differentiation parameter. If $\theta = 0$, products are completely differentiated. If $\theta = 1$, products are standardized.[5]

It is convenient to write the inverse demand curves in terms of deviations of prices from marginal cost. Using the cost functions (6a) and (6b), as appropriate, (11a) and (11b) become

$$p_1 = c + b[S - q_1 - \theta q_2] \tag{12a}$$

$$p_2 = c + b[S - \theta q_1 - q_2] \ . \tag{12b}$$

The demand curves implied by (12a) and (12b) are[6]

$$q_1 = \frac{S}{1 + \theta} - \frac{1}{b}\frac{1}{1 - \theta^2}[p_1 - c - \theta(p_2 - c)] \tag{13a}$$

$$q_2 = \frac{S}{1 + \theta} - \frac{1}{b}\frac{1}{1 - \theta^2}[p_2 - c - \theta(p_1 - c)] \ . \tag{13b}$$

5. This approach to the modeling of product differentiation is due to Spence [1976a]. The assumption that products are differentiated complicates the algebra of the model, but is necessary to permit the study of price-setting firms.

6. Prices and quantities must be nonnegative, which implies certain restrictions on the ranges over which the equations of the demand curves and inverse demand curves are valid. See Majerus [1988].

B. Both Varieties Standardized

If variety i is known to be of quality χ_i, i = 1,2, where $0 \le \chi_i \le 1$, then the fraction $1 - \chi_i$ of consumers who find the product unsatisfactory, at any price, drop out of the market. Demand curves are

$$q_1 = \chi_1\left\{\frac{S}{1+\theta} - \frac{1}{b}\frac{1}{1-\theta^2}[p_1 - c - \theta(p_2 - c)]\right\} \tag{14a}$$

and

$$q_2 = \chi_2\left\{\frac{S}{1+\theta} - \frac{1}{b}\frac{1}{1-\theta^2}[p_2 - c - \theta(p_1 - c)]\right\} \tag{14b}$$

respectively. By inversion, inverse demand curves if varieties are known to be of qualities χ_1 and χ_2, respectively, are

$$p_1 = c + b\left[S - \left(\frac{q_1}{\chi_1} + \theta\frac{q_2}{\chi_2}\right)\right] \tag{15a}$$

$$p_2 = c + b\left[S - \left(\theta\frac{q_1}{\chi_1} + \frac{q_2}{\chi_2}\right)\right] \tag{15b}$$

C. Neither Variety Standardized

If both firms engage in regular model changes, realized social welfare depends on whether or not the product is found to be satisfactory after purchase. If both varieties are found to be satisfactory, social welfare is given by equation (10). If after purchase variety i is found to be unsatisfactory, social welfare is given by equation (10) with q_i set equal to zero.

Expected social welfare is therefore

$$\begin{aligned} E(U) &= \chi_1\chi_2 U(q_1,q_2) + \chi_1(1-\chi_2)U(q_1,0) + (1-\chi_1)\chi_2 U(0,q_2) \\ &\quad + (1-\chi_1)(1-\chi_2)U(0,0) \\ &= \chi_1(aq_1 - \tfrac{b}{2}q_1^2) + \chi_2(aq_2 - \tfrac{b}{2}q_2^2) - b\chi_1\chi_2\theta q_1 q_2 + m\ . \end{aligned} \tag{16}$$

The implied inverse demand curves are

$$p_1 = \frac{\partial E(U)}{\partial q_1} = \chi_1[a - b(q_1 + \theta\chi_2 q_2)]$$

(17a)

$$= c + b[S_1 - \chi_1 q_1 - \theta\chi_1\chi_2 q_2]$$

$$p_2 = \frac{\partial E(U)}{\partial q_2} = \chi_2[a - b(\theta\chi_1 q_1 + q_2)]$$

(17b)

$$= c + b[S_2 - \theta\chi_1\chi_2 q_1 - \chi_2 q_2] ,$$

where $S_i = (\chi_i a - c)/b$, $i = 1,2$.

By inversion, demand curves if varieties are of expected qualities χ_1 and χ_2 are

(18a) $$q_1 = \frac{1}{1 - \theta^2\chi_1\chi_2}\left\{\frac{S_1}{\chi_1} - \theta S_2 - \frac{1}{b}\left[\frac{p_1 - c}{\chi_1} - \theta(p_2 - c)\right]\right\}$$

(18b) $$q_2 = \frac{1}{1 - \theta^2\chi_1\chi_2}\left\{\frac{S_2}{\chi_2} - \theta S_1 - \frac{1}{b}\left[\frac{p_2 - c}{\chi_2} - \theta(p_1 - c)\right]\right\} .$$

D. One Variety Standardized

Suppose now that variety 1 is standardized, while variety 2 brings out a new model every year, and is believed to be of quality χ_2. Expected utility is

$$E(U) = \chi_2 U(q_1,q_2) + (1 - \chi_2)U(q_1,0)$$

(19)

$$= aq_1 - \frac{b}{2}q_1^2 + \chi_2[aq_2 - \frac{b}{2}q_2^2 - \theta b q_1 q_2] + m .$$

The implied inverse demand curves and demand curves are

(20a) $$p_1 = \frac{\partial E(U)}{\partial q_1} = c + b(S - q_1 - \chi_2\theta q_2)$$

(20b) $$p_2 = \frac{\partial E(U)}{\partial q_2} = c + b(S_2 - \theta\chi_2 q_1 - \chi_2 q_2)$$

and

(21a) $$q_1 = \frac{1}{(1 - \chi_2\theta^2)}\left[S - \theta S_2 - \frac{p_1 - c - \theta(p_2 - c)}{b}\right]$$

(21b) $$q_2 = \frac{1}{\chi_2(1 - \chi_2\theta^2)}\left[S_2 - \chi_2\theta S - \frac{p_2 - c - \chi_2\theta(p_1 - c)}{b}\right]$$

However, (21a) holds only if it is known that variety 1 will be satisfactory. If $\chi_1 < 1$, demand for variety 1 is scaled back by a

factor χ_1, since consumers who would find the product unsatisfactory drop out of the market. If variety 1 is standardized and $\chi_1 < 1$, demand for variety 1 is

$$(21a)' \qquad q_1 = \frac{\chi_1}{(1 - \chi_2\theta^2)}\left[S - \theta S_2 - \frac{p_1 - c - \theta(p_2 - c)}{b} \right].$$

Inverting (21a)' and (21b), we obtain the inverse demand curves if variety 1 is standardized at quality χ_1 and variety 2 engages in model changes and is of expected quality χ_2:

$$(22a) \qquad p_1 = c + b\left[S - \left(\frac{q_1}{\chi_1} + \theta\chi_2 q_2 \right) \right]$$

$$(22b) \qquad p_2 = c + b\left[S_2 - \chi_2\left(\theta\frac{q_1}{\chi_1} + q_2 \right) \right].$$

Demand curves and inverse demand curves if variety 1 engages in model changes and variety 2 is standardized are obtained from equations (21) and (22) by permuting subscripts in an obvious way.

III. Quantity-setting Duopoly

Formally, this is a two-stage game. In the first stage, each firm picks a marketing strategy - to standardize its product (ST) or to engage in periodic model changes (MC). Choices are revealed, and in the second stage of the game each firm picks its profit-maximizing output, taking the output of the other firm as given. Each firm's strategy therefore specifies a marketing strategy and an output pair. Each firm's output is the Cournot quantity-setting equilibrium output, given choices of marketing strategy.

The inverse demand curves (15), (17), and (22) are used to express single-period profits as functions of outputs. General

Table 1: Single-period payoffs, quantity-setting duopoly

Firm 1 \ Firm 2	ST	MC
ST	$x_1 b\left(\frac{S}{2+\theta}\right)^2$ $x_2 b\left(\frac{S}{2+\theta}\right)^2$	$x_1 b\left(\frac{2S-\theta S_2}{4-x_2\theta^2}\right)^2$ $\frac{b}{x_2}\left(\frac{2S_2-\theta x_2 S}{4-x_2\theta^2}\right)^2 - F$
MC	$\frac{b}{x_1}\left(\frac{2S_1-\theta x_1 S}{4-x_1\theta^2}\right)^2 - F$ $x_2 b\left(\frac{2S-\theta S_1}{4-x_1\theta^2}\right)^2$	$\frac{b}{x_1}\left(\frac{2S_1-\theta x_1 S_2}{4-\theta^2 x_1 x_2}\right)^2 - F$ $\frac{b}{x_2}\left(\frac{2S_2-\theta x_2 S_1}{4-\theta^2 x_1 x_2}\right)^2 - F$

Note: upper term in each quadrant is firm 1's payoff, lower term in each quadrant is firm 2's payoff.

expressions for payoffs under alternative strategic combinations in the single-period quantity-setting game are shown in Table 1.[7]

Suppose first that the model describes a one-shot game. If the two products are sufficiently poor substitutes - if θ is sufficiently close to zero - each firm will be very nearly a monopolist in its own market. From the results of the previous section, it follows that if θ is sufficiently small, each firm will earn a greater equilibrium profit by standardizing its product than by making model changes. In noncooperative equilibrium, each firm will standardize its product.

7. For an outline of derivation of the payoffs, see the Appendix.

Table 2: Single-period payoffs, quantity-setting duopoly
$\theta = 1$, $a = 101$, $c = 1$, , $\chi_1 = \chi_2 = 0.75$

(a) F = 200

		Firm 2: ST	Firm 2: MC
Firm 1	ST	833.3 833.3	1,113.9 500.6
	MC	500.6 1,113.9	785.1 785.1

(b) F = 1

		Firm 2: ST	Firm 2: MC
Firm 1	ST	833.3 833.3	1,113.9 699.6
	MC	699.6 1,113.9	984.1 984.1

Note: upper term in each quadrant is firm 1's payoff, lower element in each quadrant is firm 2's payoff.

Even if θ is close to one, numerical evaluation shows that the only noncooperative equilibrium in a one-shot game is for both firms to standardize their product.[8] If the fixed cost of making model changes is sufficiently large, both firms will be strictly worse off if both make model changes, than if both standardize their products, as in Table 2(a). But if the fixed cost of making model changes is small, the pattern of payoffs is that of the prisoners' dilemma game. For the parameter values in Table 2(b), both firms would be strictly

8. Programs to evaluate payoffs for quantity- and price-setting games for arbitrary ranges of parameters are available from the author.

better off making regular model changes. In noncooperative equilibrium, however, they standardize their products.

Now suppose Table 1 is thought of as giving the single-period payoffs in an infinitely repeated game. Let firms maximize payoffs that are the present-discounted value of income streams built up out of the single-period payoffs given in Table 1. If the fixed cost of model changes is sufficiently large, as in Table 2(a), noncooperative equilibrium in the repeated game will have each firm standardize its product. But if the fixed cost of model changes is small, and the discount rate is sufficiently low, use of a trigger strategy that threatens reversion to the standardized product regime if a firm defects will allow the duopoly to sustain the use of model changes as a noncooperative equilibrium.[9,10]

The results of this section can be summarized as

Proposition 2: The noncooperative equilibrium strategy in a one-shot quantity-setting game calls for both firms to standardize their products. One noncooperative equilibrium in an infinitely repeated version of the same game repeats the single-period equilibrium, period after period. If the discount rate and the fixed cost of model changes are sufficiently small, another noncooperative equilibrium in the infinitely repeated game will be for both firms to make regular model changes.

9. See Friedman [1971] for discussion of a trigger strategy defined in terms of outputs.

10. If a single firm defected from (MC,MC) in an infinitely repeated game, it would presumably take some time for consumers to learn the quality of the newly standardized product. Without modeling this learning process in detail, the present discounted value of the income stream resulting from defection would be less than that implied by using the payoffs in Table 1, which assumes that consumers know the quality of the variety as soon as it is standardized. If a noncooperative pattern of model changes can be sustained for the payoffs in Table 1, therefore, it can also be sustained if it takes time for consumers to learn a products quality.

IV. Price-setting Duopoly

In the first stage of the price-setting game, each firm picks a marketing strategy. Choices are revealed, and in the second stage of the game each firm picks its profit-maximizing price, taking the price of the other firm as given. Each firm's strategy is therefore a marketing strategy-price pair. Each firm's price is the Bertrand price-setting equilibrium price, given choices of marketing strategy.

The demand curves (14), (18), and (21a)' and (21b) are used to express single-period profits as functions of prices. General expressions for single-period payoffs in the price-setting game are shown in Table 3.[11]

As in the quantity-setting game, it follows from consideration of the monopoly case that if θ is sufficiently close to zero, the only noncooperative equilibrium in the single-shot game will have both firms standardize their product.

Two types of noncooperative equilibria emerge in the single-shot price-setting game as θ approaches 1. For intermediate values of θ, each firm standardizes its product in noncooperative equilibrium. If the fixed cost of making model changes is sufficiently small, payoffs assume the prisoners' dilemma pattern that occurred in the quantity-setting game. An example is shown in Table 4(a).

11. If $\theta = 1$ and firms set prices and firms standardize products, noncooperative (Bertrand) equilibrium payoffs equal zero. If $\theta = 1$, firms can earn a positive profit, depending on the size of fixed cost, only if one or both firms make model changes.

Table 3: Single-period payoffs, price-setting duopoly

Firm 1	Firm 2: ST	Firm 2: MC
ST	$\chi_1 b \frac{1-\theta}{1+\theta}\left(\frac{S}{2-\theta}\right)^2$ $\chi_2 b \frac{1-\theta}{1+\theta}\left(\frac{S}{2-\theta}\right)^2$	$\frac{b}{1-\chi_2\theta^2}\left[\frac{(2-\chi_2\theta^2)S-\theta S_2}{4-\chi_2\theta^2}\right]^2$ $\frac{b}{\chi_2(1-\chi_2\theta^2)}\left[\frac{(2-\chi_2\theta)S_2-\chi_2\theta S}{4-\chi_2\theta^2}\right]^2 - F$
MC	$\frac{b}{\chi_1(1-\chi_1\theta^2)}\left[\frac{(2-\chi_1\theta)S_1-\chi_1\theta S}{4-\chi_1\theta^2}\right]^2 - F$ $\frac{b}{1-\chi_1\theta^2}\left[\frac{(2-\chi_1\theta^2)S-\theta S_1}{4-\chi_1\theta^2}\right]^2$	$\frac{b}{\chi_1(1-\theta^2\chi_1\chi_2)}\left[\frac{(2-\theta^2\chi_1\chi_1)S_1-\theta\chi_1 S_2}{4-\theta^2\chi_1\chi_2}\right]^2 - F$ $\frac{b}{\chi_2(1-\theta^2\chi_1\chi_2)}\left[\frac{(2-\theta^2\chi_1\chi_1)S_2-\theta\chi_2 S_1}{4-\theta^2\chi_1\chi_2}\right]^2 - F$

Note: upper term in each quadrant is firm 1's payoff, lower element in each quadrant is firm 2's payoff.

Table 4: Single-period payoffs, price-setting duopoly
$a = 101,\ c = 1,\ ,\ F = 1,\ \chi_1 = 0.75,\ \chi_2 = 0.5$

(a) $\theta = 0.5$

		Firm 2 ST	Firm 2 MC
Firm 1	ST	1,111.1 740.7	2,016.0 577.1
	MC	795.0 1,752.8	1,479.9 827.2

(b) $\chi = 0.9$

		Firm 2 ST	Firm 2 MC
Firm 1	ST	174.4 116.3	1,683.1 225.7
	MC	232.4 1,053.1	1,187.9 491.6

Note: upper term in each quadrant is firm 1's payoff, lower element in each quadrant is firm 2's payoff.

But for values of θ very near 1, noncooperative equilibria call for one firm to make model changes and the other to standardize its product and reveal its product's quality. For the example of Figure 4(b), (ST,MC) and (MC,ST) are both noncooperative equilibrium strategies. The firm which standardizes its product earns the greater return. Arriving first in a price-setting market for a durable consumer experience good is an advantage, if later varieties are close substitutes: the first mover can opt to standardize. In such markets, history will matter.

In an infinitely repeated version of the price-setting game, a trigger strategy could sustain periodic model changes as equilibrium marketing strategies, if stage game payoffs assume the prisoner's dilemma pattern and the discount rate is sufficiently low. We thus have

Proposition 3: The noncooperative equilibrium strategy in a one-shot price-setting game calls for both firms to standardize their products unless θ is near 1, and for one firm to standardize its product while the other makes model changes. One type of noncooperative equilibrium in an infinitely repeated version of the same game repeats the single-shot equilibrium, period after period. If the single shot equilibrium is for both firms to standardize their products and the discount rate and the fixed cost of model changes are sufficiently small, another noncooperative equilibrium in the infinitely repeated game will be for both firms to make regular model changes.

V. Final Remarks

The effect of periodic model changes is to maintain consumer uncertainty about product quality. In this model, it will never be profitable for a monopolist to engage in regular model changes. In one-shot quantity-setting games, and in one-shot price-setting duopoly games if products are poor substitutes, noncooperative equilibrium strategies call for both firms to standardize their product, although for sufficiently low values of fixed cost both firms would earn greater profit if both made periodic model changes. In dynamic versions of such games, periodic model changes can emerge as noncooperative equilibrium strategies.

Appendix: Derivation of payoffs under alternative marketing strategies

Quantity-setting firms

(ST,ST): firm 1's profit is

$$\pi_1 = b\left[S - \left(\frac{q_1}{\chi_1} + \theta\frac{q_2}{\chi_2} \right) \right] q_1 \tag{A.1}$$

Maximizing, one obtains firm 1's quantity reaction curve. The equations of the reaction curves are

$$\begin{pmatrix} 2/\chi_1 & \theta/\chi_2 \\ \theta/\chi_1 & 2/\chi_2 \end{pmatrix} \begin{pmatrix} q_1 \\ q_2 \end{pmatrix} = \begin{pmatrix} 1 \\ 1 \end{pmatrix} S \tag{A.2}$$

Equilibrium outputs are

$$\begin{pmatrix} q_1 \\ q_2 \end{pmatrix} = \frac{S}{2 + \theta} \begin{pmatrix} \chi_1 \\ \chi_2 \end{pmatrix} . \tag{A.3}$$

and substitution in the profit equations yields the payoffs for this case.

(MC,MC): firm 1's profit is

$$\pi_1 = b[S_1 - \chi_1(q_1 + \theta\chi_2 q_2)]q_1 - F . \tag{A.4}$$

The resulting system of reaction curves is

$$\begin{pmatrix} 2 & \theta\chi_2 \\ \theta\chi_1 & 2. \end{pmatrix} \begin{pmatrix} q_1 \\ q_2 \end{pmatrix} = \begin{pmatrix} S_1/\chi_1 \\ S_2/\chi_2 \end{pmatrix} . \tag{A.5}$$

Equilibrium outputs are

$$\begin{pmatrix} q_1 \\ q_2 \end{pmatrix} = \frac{1}{4 - \theta^2\chi_1\chi_2} \begin{pmatrix} 2 & -\theta\chi_2 \\ -\theta\chi_1 & 2 \end{pmatrix} \begin{pmatrix} S_1/\chi_1 \\ S_2/\chi_2 \end{pmatrix} \tag{A.6}$$

and substitution in the profit equations yields the payoffs for this case.

(ST,MC): Expressions for the firm's profits appear above. The reaction curves form the system of equations

$$\begin{pmatrix} 2/\chi_1 & \theta\chi_2 \\ \theta\chi_2/\chi_1 & 2\chi_2 \end{pmatrix} \begin{pmatrix} q_1 \\ q_2 \end{pmatrix} = \begin{pmatrix} S \\ S_2 \end{pmatrix} . \tag{A.7}$$

Equilibrium outputs are

$$\text{(A.8)} \qquad \begin{pmatrix} q_1 \\ q_2 \end{pmatrix} = \frac{x_1}{x_2} \frac{1}{4 - x_2\theta^2} \begin{pmatrix} 2x_2 & -\theta x_2 \\ -\theta x_2/x_1 & 2/x_1 \end{pmatrix} \begin{pmatrix} S \\ S_2 \end{pmatrix}$$

and substitution yields the payoffs for this case. Payoffs for the (MC,ST) case are symmetric.

Price-setting firms

(ST,ST): firm 1's profit is

$$\text{(A.9)} \qquad \pi_1 = \frac{x_1}{1 - \theta^2} \frac{p_1 - c}{b} [(1 - \theta)bS - (p_1 - c) + \theta(p_2 - c)]$$

Maximization yields the equation of firm 1's price reaction curve. The system of equations formed by the reaction curves is

$$\text{(A.10)} \qquad \begin{pmatrix} 2 & -\theta \\ -\theta & 2 \end{pmatrix} \begin{pmatrix} p_1 - c \\ p_2 - c \end{pmatrix} = (1 - \theta)bS \begin{pmatrix} 1 \\ 1 \end{pmatrix} .$$

Equilibrium prices are

$$\text{(A.11)} \qquad \begin{pmatrix} p_1 - c \\ p_2 - c \end{pmatrix} = \frac{1 - \theta}{2 - \theta} bS \begin{pmatrix} 1 \\ 1 \end{pmatrix} ,$$

and substitution yields equilibrium payoffs for this case.

(MC,MC): firm 1's profit is

$$\text{(A.12)} \pi_1 = \frac{p_1 - c}{1 - \theta^2 x_1 x_2} \left\{ \frac{S_1}{x_1} - \theta S_2 - \frac{1}{b} \left[\frac{p_1 - c}{x_1} - \theta(p_2 - c) \right] \right\} - F .$$

The system of price reaction curves that result from maximization of the firms' profits is

$$\text{(A.13)} \qquad \begin{pmatrix} 2/x_1 & -\theta \\ -\theta & 2/x_2 \end{pmatrix} \begin{pmatrix} p_1 - c \\ p_2 - c \end{pmatrix} = b \begin{pmatrix} 1/x_1 & -\theta \\ -\theta & 1/x_2 \end{pmatrix} \begin{pmatrix} S_1 \\ S_2 \end{pmatrix}$$

with resulting equilibrium prices

$$\text{(A.14)} \begin{pmatrix} p_1 - c \\ p_2 - c \end{pmatrix} = \frac{bx_1x_2}{4 - \theta^2 x_1 x_2} \begin{pmatrix} 2/x_2 & \theta \\ \theta & 2/x_1 \end{pmatrix} \begin{pmatrix} 1/x_1 & -\theta \\ -\theta & 1/x_2 \end{pmatrix} \begin{pmatrix} S_1 \\ S_2 \end{pmatrix} .$$

Substitution in the expressions for profit yields payoffs for this case.

(ST,MC): expressions for profits appear above. The system of equations of reaction curves is

$$\text{(A.15)} \quad \begin{pmatrix} 2 & -\theta \\ -\chi_2\theta & 2 \end{pmatrix} \begin{pmatrix} p_1 - c \\ p_2 - c \end{pmatrix} = b \begin{pmatrix} 1 & -\theta \\ -\chi_2\theta & 1 \end{pmatrix} \begin{pmatrix} S \\ S_2 \end{pmatrix},$$

with resulting equilibrium prices

$$\text{(A.16)} \quad \begin{pmatrix} p_1 - c \\ p_2 - c \end{pmatrix} = \frac{b}{4 - \chi_2\theta^2} \begin{pmatrix} 2 & \theta \\ \chi_2\theta & 2 \end{pmatrix} \begin{pmatrix} 1 & -\theta \\ -\chi_2\theta & 1 \end{pmatrix} \begin{pmatrix} S \\ S_2 \end{pmatrix}.$$

Substitution in the expressions for profit yields payoffs for this case. Payoffs for the case (MC,ST) are symmetric.

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