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**Endogenous Firm Efficiency
in a Cournot Principal-Agent Model**

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Abstract: In a market of Cournot firms managed by agents, the degree of firm efficiency is inversely related to the number of firms in the market.

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I. Introduction

In the neoclassical theory of the firm, it is simply taken as given that firms operate efficiently. While often argued that the degree of firm efficiency should be treated as endogenous (see in particular Leibenstein [1966, 1973, 1975, 1978, 1987] and Williamson [1967, 1975, 1985], there is as yet no neoclassical model of the determinants of firm efficiency.

In particular, much work remains to be done to clarify the links between market structure and firm efficiency. The theory of x-inefficiency argues that firm efficiency will be greater, the more competitive the markets in which the firm operates. This argument has been disputed, although perhaps more on semantic grounds than on the substantive point that there is a positive relationship between firm efficiency and market competitivity.[1]

Selten [1986] explores the consequences of firm inefficiency for market performance. In his model, however, the degree of firm efficiency is exogenous. Willig [1987] models the relationship between market structure and firm efficiency by examining the influence of changes in the price elasticity of demand on firm efficiency in a principal-agent model. But the links between market structure and the price elasticity of demand are not made explicit.

1. Stigler [1976] argues that the theory of X-inefficiency is ill-founded. In the real world, it is costly to enforce contracts. What this means is that an employee's performance should be monitored until the payoff to the firm of a marginal increase in efficiency equals the marginal cost of an increase in monitoring effort. If there are costs of monitoring, Stigler argues, it is illogical to the compare the level of efficiency that is attainable in the real world and the level of efficiency that would be attainable if monitoring costs were zero and call the difference "inefficiency." See also De Alessi [1983]).

I present here a Cournot principal-agent model of the determinants of firm efficiency. In this model, there is a random element to marginal cost, which is observed by the firm's manager. This random element is not observed by the firm's owner, who indirectly controls the manager's efforts by setting a fee schedule for the manager that depends on the realized value of marginal cost. This fee schedule determines the manager's efforts and therefore the firm's marginal cost. The firm's marginal cost is in turn a factor that determines the Cournot market equilibrium.

The main result of the model is that the degree of firm efficiency is inversely related to the number of firms in the market. The fee schedule which is optimal from the owner's point of view balances the marginal payoff from greater firm efficiency with the marginal cost of inducing greater managerial effort. The greater the number of firms in the market – the greater the degree of competition – the smaller the payoff associated with a marginal increase in firm efficiency and the less it is in the interest of the owner of the firm to set a fee schedule that will induce the manager to make a great effort to reduce marginal cost. The consequence is that the equilibrium level of marginal cost is greater, the larger the number of firms in the market.

II. Structure of the model

There are n firms. The product is homogeneous, and the inverse demand curve,

(1) $$p = a - bQ$$

(where $Q = q_1 + q_2 + \dots + q_n$ and q_i is the output of firm i) is linear. For each firm, there is an owner and a manager. Average and marginal

Mega.

cost for firm i is

(2) $$c_i(\epsilon_i) = \alpha + \epsilon_i e^{-L_i} ,$$

where $\alpha > 0$, ϵ_i is a nonnegative random variable and L_i is the labor of the manager of firm i. Without loss of generality, let $0 < \underline{\epsilon} \le \epsilon_i$

$\le \bar{\epsilon}$, and suppose that ϵ_i has a continuous density function $f(\epsilon_i)$. Suppose also that the manager's services are essential to the operation of the firm, and that the manager's income from his next best alternative employment is zero.

The manager of the firm observes ϵ_i and L_i; the owner of the firm observes neither. The owner of the firm indirectly controls the manager's actions by establishing a cost target $c(\hat{\epsilon}_i)$ and a fee schedule $\phi(\hat{\epsilon}_i)$ that depend on the value $\hat{\epsilon}_i$ of the random variable that the manager reports to the owner.

The manager must achieve the cost target if he is to receive any fee at all. Thus

(3) $$c(\hat{\epsilon}_i) = \alpha + \epsilon_i e^{-L_i} ,$$

and the manager's labor is

(4) $$L_i = \log \epsilon_i - \log [c(\hat{\epsilon}_i) - \alpha] .$$

If the true value of the random cost element is ϵ_i and the manager reports a value $\hat{\epsilon}_i$, the manager's utility is

(5) $$U(\hat{\epsilon}_i | \epsilon_i) = \phi(\hat{\epsilon}_i) - \lambda L_i ,$$

where λ is the manager's marginal disutility of labor. The manager selects L_i to maximize (5). The owner of the firm, unable to observe ϵ_i directly, maximizes his expected payoff, the expected profit of the firm after subtracting manager's fee $\phi(\hat{\epsilon}_i)$.

(8b) $\qquad U(\epsilon_i|\epsilon_i) - U(\hat{\epsilon}_i|\hat{\epsilon}_i) \geq \lambda(\log \hat{\epsilon}_i - \log \epsilon_i)$.

Running through the same arguments but reversing the roles of ϵ_i and $\hat{\epsilon}_i$ yields

(9) $\qquad \lambda(\log \hat{\epsilon}_i - \log \epsilon) \geq U(\epsilon_i|\epsilon_i) - U(\hat{\epsilon}_i|\hat{\epsilon}_i)$.

But (8b) and (9) can both be true only if both hold with equality; thus any feasible fee function and cost structure must produce a manager's utility that satisfies

(10) $\qquad U(\epsilon_i|\epsilon_i) - U(\hat{\epsilon}_i|\hat{\epsilon}_i) = \lambda(\log \hat{\epsilon}_i - \log \epsilon_i)$.

Since (10) is true for all ϵ_i and $\hat{\epsilon}_i$, it is true for $\hat{\epsilon}_i = \overline{\epsilon}$.

Substituting $\hat{\epsilon}_i = \overline{\epsilon}$ in (12) and rearranging terms gives

(11) $\qquad U(\epsilon_i|\epsilon_i) = U(\overline{\epsilon}|\overline{\epsilon}) + \lambda \log \dfrac{\overline{\epsilon}}{\epsilon_i}$.

Since $\overline{\epsilon} \geq \epsilon_i$, the last term on the right in (11) is positive. A feasible fee schedule and cost target will give the manager greater utility, the closer is ϵ_i to its lowest possible value.

No fee schedule that produced $U(\overline{\epsilon}|\overline{\epsilon}) > 0$ could be optimal for the principal, since the principal could always switch to a less costly feasible fee schedule that would make $U(\overline{\epsilon}|\overline{\epsilon}) = 0$ and still satisfy $U(\epsilon_i|\epsilon_i) \geq 0$ for all ϵ_i.

An optimal feasible fee schedule therefore satisfies

(12) $\qquad U(\epsilon_i|\epsilon_i) = \lambda \log \dfrac{\overline{\epsilon}}{\epsilon_i}$.

Thus any feasible fee schedule/cost target pair satisfies (12). Now suppose a fee schedule satisfies (12). Then $U(\epsilon_i|\epsilon_i) > 0$, which is one of the elements of feasibility.

Combining (4) and (5) gives

(13) $\qquad U(\hat{\epsilon}_i | \epsilon_i) = \phi(\hat{\epsilon}_i) - \lambda \log \epsilon_i + \lambda \log [c(\hat{\epsilon}_i) - \alpha].$

Using (13) and (13) evaluated for $\epsilon_i = \hat{\epsilon}_i$, one finds

(14) $\qquad U(\hat{\epsilon}_i | \epsilon_i) - U(\hat{\epsilon}_i | \hat{\epsilon}_i) = \lambda(\log \hat{\epsilon}_i - \log \epsilon_i)$

But (14) and (12) evaluated for $\epsilon_i = \hat{\epsilon}_i$ yield

(15) $\qquad U(\epsilon_i | \epsilon_i) = \lambda[\log \hat{\epsilon} - \log \epsilon_i] = U(\hat{\epsilon}_i | \epsilon_i) .$

This is the second condition for feasibility. This establishes

Proposition 1: A fee schedule/cost target pair $\{\phi(\epsilon), c_i(\hat{\epsilon}_i)\}$ is feasible if and only if it satisfies (12).

Equations (12) and (13) yield a relation between a feasible fee schedule, cost target pair:

(16) $\qquad \phi_i(\epsilon_i) = \lambda \log \dfrac{\overline{\epsilon}}{c_i(\epsilon_i) - \alpha} .$

This will be used to express the principal's optimization problem in terms of the cost target alone.

IV. Product market equilibrium

The product market is one of n-firm Cournot oligopoly with cost differences (although in equilibrium all firms have the same marginal cost). The realized value of firm 1's profit is

(17) $\qquad V_1 = b \left[S_1 - \dfrac{n}{n+1} \overline{S} \right]^2 - \phi(\epsilon_1),$

where $S_1 = [a - c(\epsilon_1)]/b$ and $\overline{S}$ is the average of all S_1's, or equivalently

(18)
$$V_1 = \frac{1}{b(n+1)^2}\{a - \alpha - n[c(\epsilon_1) - \alpha] + \sum_{j=2}^{n} [c(\epsilon_j) - \alpha]\}^2 - \lambda \log \overline{\epsilon}$$
$$+ \lambda \log [c(\epsilon_1) - \alpha] ,$$

if profit is expressed in terms of the cost target.

Imputing Cournot behavior to the principal of firm 1, we suppose that he picks a cost target $c_1(\epsilon_1)$ that maximizes his expected payoff, taking the cost targets of other firms as given. The principal of firm 1 thus seeks to maximize

$$E(V_1) = \frac{1}{b(n+1)^2} \int_{\epsilon_1} \cdots \int_{\epsilon_n} \{a - \alpha - n[c(\epsilon_1) - \alpha] + \sum_{j=2}^{n} [c(\epsilon_j) - \alpha]\}^2 f(\epsilon_n)\ldots f(\epsilon_1)d\epsilon_n\ldots d\epsilon_1$$

(19)

$$- \lambda \log \bar{\epsilon} + \lambda \int_{\epsilon_1} \log [c(\epsilon_1) - \alpha]f(\epsilon_1)d\epsilon_1 .$$

Application of the Euler condition of the calculus of variations shows that the first-order necessary condition for maximization of (19) is found by differentiating under the integral signs with respect to $c_1(\epsilon_1)$ and setting the result equal to zero. The first-order condition is

$$(20) \quad -\frac{2n}{b(n+1)^2}\{a - \alpha - n[c(\epsilon_1) - \alpha] + \sum_{j=2}^{n} [E(c_j) - \alpha]\} + \frac{\lambda}{c(\epsilon_1) - \alpha} = 0 ,$$

where E denotes an expected value.

For notational simplicity, write

$$(21) \qquad\qquad a^* = a - \alpha \qquad\qquad c_j^* = c_j - \alpha .$$

Then (20) can be rewritten as a quadratic equation in c_1^*:

$$(22) \qquad n(c_1^*)^2 - [a^* + \sum_{j=2}^{n} E(c_j^*)]c_1^* + \frac{(n+1)^2}{n}\frac{b\lambda}{2} = 0$$

Equation (22) defines the principal's payoff-maximizing c_1^* as a function of n, a^*, and the expected values of the fees offered other firms' managers. This is the equation of firm 1's cost target reaction surface. But this equation holds for all values of ϵ_1 in the interval $(\underline{\epsilon}, \bar{\epsilon})$. Thus for the technology (2), the optimal cost target is a constant, independent of the realized value of ϵ_1. By (16),

the fee schedule that is optimal for the principal is also constant.

Since the optimal cost target is a constant, $c_i^* = E(c_i^*)$. Substituting this in (22) yields the equation of a reaction surface that defines principal 1's payoff-maximizing fee as a function of n, a^*, and the fees offered other firms' managers.

Since firms are identical as regards the distribution of the random part of cost, managers' utility functions, and principals' utility functions, in equilibrium all principals will select the same cost target. Imposing symmetry in (22) gives an equation that is satisfied by the cost target that is optimal for the principals:

$$(23) \qquad (c^*)^2 - a^*c^* + \frac{(n + 1)^2}{n} \frac{b\lambda}{2} = 0.$$

The root of (23) that maximizes principals' payoffs is

$$(24) \qquad c^* = \frac{a^* - \sqrt{(a^*)^2 - 4\frac{(n + 1)^2}{n} \frac{b\lambda}{2}}}{2}$$

From (24), one obtains

$$(25) \qquad 2\frac{\partial c^*}{\partial n} = \frac{b\lambda \frac{n^2 - 1}{n^2}}{\sqrt{a^* - 2b\lambda \frac{(n + 1)^2}{n}}} > 0 .$$

This establishes

Proposition 2: In the principal-agent model of Cournot quantity-setting firms, equilibrium average cost rises with the number of firms.

Equilibrium values for a numerical example is shown in Table 1. In this Cournot principal-agent model, the cost target rises as the number of firms rises. The manager's fee and the principal's payoff fall as the number of firms rises.

These results are just opposite to that which would be predicted by X-inefficiency theory. But in the context of the model, they are not hard to understand. The principal sets a fee schedule, cost target pair that maximizes his expected return. The profit-maximizing

Table 1: Equilibrium Values, Cournot Principal-Agent Model

$(b = 1, a = 10, \alpha = 1, \bar{\epsilon} = 1)$

n	c^*	ϕ	V
2	0.2574	1.3573	7.1354
3	0.3068	1.1817	3.5416
4	0.3618	1.0168	1.9680
5	0.4196	0.8686	1.1766

pair will set the principal's marginal revenue equal to the marginal increase in the agent's fee, subject to constraints. But marginal revenue will be less, all else equal, the greater the number of firms in the market. The greater the number of firms, therefore, the smaller the incentive of the principal to set a high fee schedule and induce the agent to invest a great deal of labor in minimizing cost.

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