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The Spanish Case

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THE EFFECTS OF REPLACEMENT SCHEMES ON CAR SALES: THE SPANISH CASE

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This paper studies a model of car replacement designed to evaluate policies addressed to influence replacement decisions. An aggregate hazard function is computed from optimal replacement rules of heterogeneous consumers, which mimics the hump-shaped hazard function observed for the Spanish car market. The model is calibrated to evaluate quantitatively the Plan Prever, a replacement scheme introduced in Spain in 1997, finding that the positive effect of the subsidy is high in the short run but small in the long run for both sales and the average age of the stock.

Keywords: Car scrapping, replacement schemes, heterogeneous consumers.

(JEL D12, H31)

1. Introduction

Over the past recent years, Spanish governments have introduced some policy measures aimed at increasing road safety, reducing environmental pollution and stimulating car sales by the mean of subsidizing car replacement. We refer to these policies as *replacement schemes*. The aim of this paper is to study the main effects of such schemes on car sales and on the average age of the stock. To this end, we solve a model of car replacement with a continuum of ex-ante hetero-

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geneous consumers, where the individual decision to replace is endogenous and depends on car's age. The aggregate behavior of sales is computed through explicit aggregation of individual replacement rules. Among other things, we show that the presence of an age threshold — as is the case in several implemented replacement schemes, the Spanish included—, has the puzzling implication that some car owners optimally delay replacement, although a large fraction of them advance it, as aimed. Finally, the proposed model is used to simulate the effects of the replacement scheme, known as Plan Prever, introduced in Spain in 1997. We find that this policy increases notably new car sales in the short run, but in the long run the effect on sales and in the average age of the stock is small: with respect to the previous level, a transitory increase of around 16% in sales should follow the introduction of the subsidy, whereas in the long run a permanent increase of about 1.2% in car sales, and a permanent reduction of 8% in the average age of the stock of cars —from 8.7 to 8 years— should be observed.

Several reasons can be given to justify the finite lifetime of cars and their replacement. Some of them, which we call technical obsolescence, have to do with depreciation associated with usage or failures generated by some stochastic events. Others are related to economic factors, like technical progress, which induces the replacement of an old car by a new, more efficient one, even when the old car is still technically operative. This could be termed economic obsolescence. In this paper, we include both types of factors in a stylized fashion.

The efficacy of car replacement schemes has been already analyzed. Hahn (1995) and Baltas and Xepapadeas (1999), among others, focus on the environmental consequences of this type of policy. A different perspective is adopted by Adda and Cooper (2000), who analyze the French case focusing exclusively on the sales effect of the replacement subsidy. They embed a dynamic replacement model into a structural estimation procedure in the vein of Rust (1987).

This paper focuses on car sales and adopts a structural framework, but it differs in several aspects from Adda and Cooper (2000). Firstly, they assume that consumers face idiosyncratic shocks in preferences and income, uncorrelated both across time and consumers. In this paper, however, we assume persistent heterogeneity in preferences. In this sense, both approaches can be understood as two extreme cases of heterogeneity. Adda and Cooper also consider an age threshold to take advantage of the replacement subsidy, but contrary to our result,

it has no consequences on aggregate purchases. Secondly, we work in continuous time building a model in line with the real options literature and this, joint with our assumption about consumer preferences and heterogeneity, allows us to get an explicit expression for the replacement age as a function of different factors affecting replacement. Finally, given the low time interval covered by our database, we calibrate the model in contrast to Adda and Cooper's Generalized Method of Moments estimation procedure.

The remaining work is organized as follows. In Section 2, we present a description of the replacement schemes adopted in Spanish during the 1990's—in particular the Plan Prever—and some empirical evidence on car replacement for Spain. Section 3 describes a replacement model at the individual as well as at the aggregate level. It also studies the effects of introducing a replacement scheme on the replacement age. Section 4 is devoted to the calibration of the model on Spanish car market data. Section 5 quantifies the main effects of the Prever scheme both on car sales and on the average age of the stock, and reports some robustness checks. Finally, Section 6 summarizes and concludes.

2. Car Replacement and Replacement Schemes in Spain¹

Several measures have been introduced during recent years by Spanish governments to promote car replacement. The first was the introduction of compulsory periodic inspection in 1987, a mechanism that not only reinforces compliance with certain technical standards but also promotes car replacement by increasing the cost of maintaining aging cars. More recently, car replacement has been directly encouraged by the replacement schemes *Renove I* (1994), *Renove II* (1994–1995) and *Prever*—initiated in 1997 and still in force. Both programs have the purpose of lowering the average age of the stock of cars on the road, with subsequent positive effects on the road safety and the environment. To this end they give a subsidy to the acquisition of a new car provided that a car older than a given age is deregistered and scrapped by the same owner. Plan *Renove I* was in effect from April 12 to October 12, 1994. Plan *Renove II* applied from October 12, 1994 to June 30, 1995. Plan *Prever* started in April 11, 1997 and is of indefinite duration. Although it suffered recent modifications, during the first two years, the period to which we restrict our empirical analysis, Plan

¹The data and Gauss code used in this paper to calibrate and simulate the model can be downloaded from <http://oro1.usc.es/~aesamp/prever.zip>.

Prever reduced the new vehicle registration tax² by 480 euros if the scrapped car was aged 10 years or more. The subsidy has the vehicle registration tax as an upper bound. Table 1 summarizes the main elements characterizing these replacement schemes.

INSERT TABLE 1 ABOUT HERE

To analyze the effects of replacement schemes, we use annually recorded data by Dirección General de Tráfico (DGT). Data are given at December 31st and for one-year periods. Using this information, we compute aggregate empirical hazard rates for car deregistration, $h_\tau(I)$, as follows

$$h_\tau(I) = \frac{B_\tau(I)}{P_{\tau-1}(I-1)},$$

where $B_\tau(I)$ represents reported deregistration of cars aged I in year τ and $P_{\tau-1}(I-1)$ denotes the stock of cars aged $I-1$ at the end of year $\tau-1$. We compute the stock at the end of year τ starting from a reported initial stock at 1969, and the number of registered cars and deregistered cars for each age for successive years —see Appendix A1 for details. Figures 1 and 2 show these hazard rates for several years, as well as the average for the periods 1988–1993 and 1994–1996 which are used below for calibration purposes. It is worth noting that observed hazard rates are hump shaped.

INSERT FIGURE 1 ABOUT HERE

INSERT FIGURE 2 ABOUT HERE

The main difference between the two Renove schemes and the Plan Prever is that the later one is permanent whereas the former were temporary. As shown in Licandro and Sampayo (1997b), the temporary character precludes any long run effect of the scheme on car sales, as the positive initial effect is compensated with a subsequent negative effect once the subsidy disappears. As Figure 1 shows, the hazard moved up significantly in 1994, during the introduction of the Renove scheme and moved down in 1996 —the Renove scheme finished in the middle of 1995—, below the 1993 hazard. On the basis of the 1993–1996 observed aggregate hazard rates for deregistration

²New cars sales in Spain are taxed with two indirect ad-valorem taxes. The first is the value-added tax (Impuesto sobre el Valor Añadido, IVA). The second is known as the registration tax. At the time of the Prever scheme, the IVA was 16% and the registration tax 7% for small-medium car engine power and 12% for medium-high car engine power —with some exceptions for Canarias, Ceuta and Melilla.

of Spanish cars, Licandro and Sampayo (1997b) found that a rise in car sales by about 120,000 units prompted by Renove I during 1994 was followed by a subsequent fall in 1996—in 1995 Renove II helped to maintain sales roughly at 1993 levels. Unlike Renove I an II, the Prever scheme is of indefinite duration, implying that no depression in sales following the rise induced by its introduction should be expected.

INSERT TABLE 2 ABOUT HERE

INSERT FIGURE 3 ABOUT HERE

Table 2 shows some data on car stock and replacement for 1997 and 1998 as well as the averages for 1988–1993 and 1994–1996—see also Figure 3. The stock growth rates are very similar to the average observed for the period 1988–1993. The annual deregistration rates for 1997 and 1998 are also close to the average for 1988–1993. Although this might suggest that, contrary to expectations, the Prever scheme has had no significant effect on this variable—whereas Renove I had boosted the 1994 deregistration rate to 4.2%—, Figure 2 shows that the observed average deregistration hazard function for 1997–1998 lies above the same average for the periods 1988–1993 and 1994–1996.

Moral Rincón (1998) uses the same data set to analyze aggregate scrapping decisions in the Spanish car market. She estimates aggregate hazard rates adopting a reduced econometric framework, finding that car’s age is the main determinant of observed scrapping. She also finds a positive effect of Plan Renove on the hazards. In contrast, we use a theoretical model to quantify the effects of Plan Prever on sales and the average age of the stock, through the mean of its effect on the aggregate hazards. We show that monotonic increasing hazards at the individual level combined with heterogeneity among owners can generate non monotonic hazards at the aggregate level that mimic the observed hazards for cars in Spain.

3. The Model

Although we adopt a microeconomic perspective as a starting point for the analysis of replacement decisions, only aggregate data on car replacement are available. At the individual level, hazard functions are expected to be increasing for both technical and economic reasons. As it is shown below, the model in this paper delivers idiosyncratic stepwise hazard functions.

However, as can be observed in Figures 1 and 2, aggregate hazard functions for car replacement are hump-shaped. At an aggregate level, to highlight the dependence of car replacement on age, a hazard rate perspective is very useful as some previous work show —see for instance Caballero and Engel (1993) or Cooper, Haltiwanger and Power (1999). However, as these authors also point out, although at the individual level hazard rates are expected to be monotonic increasing functions, non monotonic hazard rates can result in the aggregate, provided there is enough heterogeneity. In this paper, and in order to replicate the Spanish aggregate hazards for cars, we introduce inter-individual differences in preferences that generate heterogeneity in replacement age. This allows us to generate a cross-sectional density of replacement age, which is the link between idiosyncratic stepwise hazard functions and hump-shaped aggregate empirical hazards. In this section, we first describe and solve the individual replacement problem and analyze the effects of a replacement scheme on individual replacement. Then, we study the aggregate consequences of individual behavior.

3.1. *The Microeconomic Replacement Problem*

Time is continuous. There is a continuum of heterogeneous consumers with preferences $\theta e^{\gamma t} c(t) + s(t - a(t))$ defined on nondurable consumption c and durable goods services s . For simplicity, consumers own one and only one car. Services of a car bought at time $t - a$ are defined as $s(t - a) = e^{\gamma(t-a)}$ where $e^{\gamma t}$ measures instantaneous services provided by a new car bought at time t , and $a \geq 0$ is car's age. The growth rate of new car quality is given by $\gamma > 0$. The utility of nondurable consumption is linear with marginal utility $\theta e^{\gamma t}$. We assume that $\theta \in [0, \theta_{\max}]$, so that consumers are different in their marginal utility of nondurables consumption.³ Note that we are also assuming that marginal utility of nondurables consumption and quality of new cars are growing at the same rate, which allows us to obtain a constant replacement age. Otherwise, the optimal replacement age would converge to zero as time goes to infinity. Finally, each consumer is

³Although here utility is linear and all consumers have the same income, allowing for different values of θ makes consumers with lower θ have a lower marginal utility of income. As is shown in Tirole (1988), pp. 96–97, in a similar context, this can be interpreted as if utility is concave in nondurables consumption and consumers have different income and therefore, different marginal rates of substitution between income and durables services.

endowed with a flow of exogenous income y measured in nondurable units.

Let us assume that all new cars have the same quality and can be purchased at a constant price p . The scrapping value of an old car is d_0 . Therefore, $p - d_0 > 0$ is the car replacement cost which is assumed to be exogenous. Further, a car may suffer an irreparable failure with probability $\delta > 0$, constant and exogenous, that forces the owner to replace the car by a new one. The existence of a second hand market is ignored.

In Appendix A2, the consumer's control problem is transformed into an equivalent stationary recursive problem. The optimal replacement age can be obtained as the solution to the following dynamic programming problem:

$$W(a) = \max \{V(a), V(0) - \theta(p - d_0)\}, \quad [1]$$

where $V(a)$ reflects the instantaneous value of owning a car of age a , and $V(0) - \theta(p - d_0)$ represents the value of replacing a car of age a by a new car. Notice that the replacement cost $p - d_0$ is weighted by the marginal utility of nondurables consumption, θ . The optimal consumer's strategy is to keep the car whenever a belongs to the "continuation region" $[0, T)$ and reinitialize the variable a to its initial value $a = 0$ —replace the car— at cost $\theta(p - d_0)$ whenever $a \geq T$. As the replacement age T is endogenous, this is a free boundary value problem.

Let $\rho > 0$ define the rate of time preference. As is shown in Appendix A2.1, the following assumptions guarantee that the previous replacement problem makes sense giving rise to a finite and nonnegative replacement age.

ASSUMPTION 1. $\gamma < \delta + \rho$.

ASSUMPTION 2. $0 \leq \theta < \frac{1}{(\delta + \rho)(p - d_0)}$.

Assumption 1 guarantees that utility is bounded and is also required for the optimal replacement age T be strictly positive for $\theta > 0$. Assumption 2 can be written as $\theta(p - d_0) < \frac{1}{\delta + \rho}$. This inequality says that the replacement cost times the marginal utility of nondurables, must be less than the discounted services of a car with an expected infinite lifetime. This assumption implies that the replacement age is

bounded above. Under these assumptions, the optimal replacement age is given by the solution to the following nonlinear equation

$$\theta = \frac{1}{p - d_0} \left(\frac{1 - e^{-(\delta+\rho)T}}{\delta + \rho} - \frac{e^{-\gamma T} - e^{-(\delta+\rho)T}}{\delta + \rho - \gamma} \right) \equiv \theta(T; \alpha_0), \quad [2]$$

where $\alpha_0 = \{\rho, p, d_0, \gamma, \delta\}$.

INSERT FIGURE 4 ABOUT HERE

Since the function $\theta(T; \alpha_0)$ defined in [2] is a monotonic function of T , for $T \geq 0$ it can be inverted to give T as a function of θ :

$$T = T(\theta; \alpha_0) \equiv \theta^{-1}(\theta; \alpha_0). \quad [3]$$

The thick line in Figure 4 represents the replacement age function. It must be noted that this function does not have an explicit expression and, as it is crucial to our model, this forces us to make computations numerically. The function $T(\theta; \alpha_0)$ allows us to define θ_{\max} as the type such that $T_{\max} = T(\theta_{\max}; \alpha_0)$, where T_{\max} is the highest age at which someone is observed to deregister a car. Therefore, we restrict the study of the replacement behavior to $\theta \in [0, \theta_{\max}]$ where $\theta_{\max} < \frac{1}{(\delta+\rho)(p-d_0)}$.

Concerning the comparative statics of the replacement age with respect to the parameters, the following proposition summarizes the results.

PROPOSITION 1. *For $\theta \in]0, \theta_{\max}]$, the replacement age is increasing with respect to $p - d_0$ and $\rho + \delta$, and decreasing with respect to γ .*

PROOF. First, the derivative of equation [2] with respect to $p - d_0$ is

$$\frac{dT}{d(p - d_0)} = \frac{(\delta + \rho - \gamma) \theta}{\gamma (e^{-\gamma T} - e^{-(\delta+\rho)T})}$$

and is always positive.

To check the sign of derivatives with respect to γ or $\rho + \delta$, it is useful to write [2] in integral form as follows

$$\theta(p - d_0) = \int_0^T (1 - e^{\gamma(z-T)}) e^{-(\delta+\rho)z} dz. \quad [4]$$

The derivative in [4] with respect to γ is

$$\frac{dT}{d\gamma} = \frac{(\delta + \rho - \gamma)}{\gamma (e^{-\gamma T} - e^{-(\delta+\rho)T})} \int_0^T (z - T) e^{\gamma(z-T)} e^{-(\delta+\rho)z} dz.$$

The integrand is the product of three functions which are continuous in the closed interval $[0, T]$. The first function $(z - T)$ is negative in the open interval $(0, T)$ and the other two are positive, implying that the derivative is negative.

Finally, taking the derivative in [4] with respect to $\rho + \delta$ results

$$\frac{dT}{d(\delta + \rho)} = \frac{(\delta + \rho - \gamma)}{\gamma (e^{-\gamma T} - e^{-(\delta + \rho)T})} \int_0^T (1 - e^{\gamma(z-T)}) e^{-(\delta + \rho)z} dz.$$

In this case, the integrand is the product of three positive functions in the open interval $(0, T)$ which are continuous in the closed interval $[0, T]$. Therefore the derivative is positive.

First, Proposition 1 states that replacement age is increasing with the replacement cost $(p - d_0)$. This cost can increase both because the price of new cars, p , increases and because the scrapping value, d_0 , decreases. In both cases the effect is the same and increases the replacement age. Second, concerning the effect of δ on the replacement age, equation [4] makes clear that the failure rate acts on the replacement age in the same way as the discount factor. This is usual in dynamic models where uncertainty is governed by a Poisson process as here. That is, an increase in the probability of a car failure reduces the expected present value of future gains from replacement, which are defined as the gain in services at each age times the probability of survival up to this age. Third, the effect of technical progress on the replacement age can be better understood by looking also at equation [4]: the replacement age is the value that equalizes the subjective replacement cost —on the left hand side— with the expected gain in durable services on the right hand side. This gain is computed as the discounted difference between the services provided by the newest and the oldest car in the economy, at each moment during the lifetime of the former. If technical progress increases, the distance between the services provided by both cars (the technological frontier) increases. As the replacement cost remains unaltered, reducing the replacement age restores equality in [4] by increasing the relative services of the oldest car in the economy and lowering the time over which this difference is computed. This is the mechanism through which embodied technical progress relates inversely to the replacement age and generates (economic) obsolescence of cars that are otherwise technically useful.

Finally, it is worth noting that the replacement behavior characterized above can be understood as a step hazard function: the conditional probability of replacement is constant and equal to the failure rate δ for age up to the optimal replacement value, and equal to one for higher values of age.

– *The Effects of a Replacement Scheme on the Replacement Age*

Let us assume that the replacement scheme, adopted at time t_0 , is a subsidy $s > 0$ for cars aged at least $\bar{T} > 0$. Consequently, the parameter vector α changes from its former value α_0 to $\alpha_1 = \{\rho, p, d_0 + s, \gamma, \delta\}$, conditional on the car being replaced at an age at least equal to \bar{T} . The replacement problem with subsidy is solved in Appendix A3 where Lemma A3.1 establishes the existence of a type $\underline{\theta}$ which is indifferent between taking advantage of the subsidy or not. As a by-product, this lemma also proves that: i) for consumers with $\theta < \underline{\theta}$ the replacement age is given by function $T(\theta; \alpha_0)$ as defined in equation [3], implying that consumers with $\theta < \underline{\theta}$ do not change their behavior; ii) $\underline{\theta} < \bar{\theta}$ where $\bar{\theta}$ is such that $T(\bar{\theta}; \alpha_0) = \bar{T}$. Note that consumers with $\theta \in [\underline{\theta}, \bar{\theta}]$ used to replace at age $T(\theta; \alpha_0) < \bar{T}$ but are induced by the scheme to delay replacement to take benefit of the subsidy. The following proposition completes the analysis of the replacement decision for the remaining types and summarize the results —the proof is also in Appendix A3.

PROPOSITION 2. *The optimal replacement rule under subsidy $s > 0$ and threshold $\bar{T} > 0$ is*

$$\hat{T}(\theta; \alpha_0, s, \bar{T}) = \begin{cases} T(\theta; \alpha_0) & \text{if } \theta < \underline{\theta} \\ \bar{T} & \text{if } \underline{\theta} \leq \theta \leq \bar{\theta} \\ T(\theta; \alpha_1) & \text{if } \theta \in [\bar{\theta}, \theta_{\max}] \end{cases},$$

where $T(\bar{\theta}; \alpha_1) = \bar{T}$.

Function $T(\theta; \cdot)$ in Proposition 2 has been previously defined in equation [3]. Figure 4 represents the optimal scrapping function $\hat{T}(\theta; \alpha_0, s, \bar{T})$. Firstly, all consumers with $\theta > \bar{\theta}$ would like to reduce the lifetime of cars to take advantage of the subsidy. Secondly, some among them, those with $\underline{\theta} < \theta < \bar{\theta}$, would be induced by the subsidy to reduce the scrapping age below \bar{T} , but that is not allowed by the replacement scheme. They replace then at age \bar{T} . Thirdly, consumers with $\theta < \underline{\theta}$ had a replacement age smaller than \bar{T} before the introduction

of the replacement scheme. However, some of them, those such that $\underline{\theta} \leq \theta \leq \bar{\theta}$, have incentives to delay their replacement to take advantage of the subsidy. Fourth, consumers with $\theta < \underline{\theta}$ do not have incentives to modify their behavior and replace at age $T(\theta; \alpha_0)$.

Although Proposition 1 establishes that a replacement subsidy reduces the replacement age, Proposition 2, the main theoretical result of the paper, stresses the fact that the existence of an age threshold induces a mass of owners with an otherwise heterogenous replacement age to concentrate replacement at the age threshold. On the one side, some car owners reduce their replacement age just to this limit. On the other side, the subsidy induces some car owners to delay replacement to take advantage of the subsidy. The quantitative importance of the delay effect depends on the distribution of the stock of cars around the age threshold. However, this result brings attention to the fact that, in implementing this type of policy, the intended reduction of the average age of the stock of cars can be partly offset.

3.2. Aggregation

The car-owning population $N(t)$ at time t is assumed to be

$$N(t) = \int_0^{\theta_{\max}} N(t, \theta) d\theta + N_{\infty}(t),$$

where $N(t, \theta)$ denotes the number of individuals of type $\theta \in [0, \theta_{\max}]$. As each owner owns a single car and he must replace it in order to buy a new one, $N(t)$ also measures the number of cars in the economy. Replacement decisions of individuals of type $\theta \in [0, \theta_{\max}]$ are governed by the rules described in the previous section. In addition, there is another group of car owners that never deregister their cars. They are denoted by $N_{\infty}(t)$ and referred as type-infinity. The members of this latter group, which largely represents individuals who in reality fail to deregister upon sending their cars to scrap, only buy a new car if forced to do so by an irreparable failure. The size of each group of consumers is assumed to be growing at the rate $n > 0$, which is taken to be exogenous and constant. Under these assumptions, population is distributed according to the stationary density function $\zeta(\theta)$, verifying

$$\int_0^{\theta_{\max}} \zeta(\theta) d\theta + \zeta_{\infty} = 1,$$

with ζ_{∞} representing the fraction of type-infinity car owners.

Assuming one car per individual implies that the deregistration of a car is automatically followed by the purchase of a new car, regardless of whether deregistration is forced by irreparable failure or is the result of a decision to replace a car that is aging and road worthless. Therefore, the following equations must be verified,

$$\int_{t-T(\theta;\alpha)}^t M(z, \theta; \alpha) e^{-\delta(t-z)} dz = \zeta(\theta) N(t), \quad \forall \theta \in [0, \theta_{\max}], \quad [5]$$

$$\int_{-\infty}^t M_{\infty}(z; \alpha) e^{-\delta(t-z)} dz = \zeta_{\infty} N(t), \quad [6]$$

where $M(t, \theta; \alpha)$ and $M_{\infty}(t; \alpha)$ denote the number of cars bought at time t by each consumer type, for a given parameters vector α .

From [5], cars bought by type θ car owners less than $T(\theta; \alpha)$ years ago have not yet been replaced except for car wrecks. The term $e^{-\delta(t-z)}$ represents the rate of consumers that having bought a car at moment z have not suffer a failure yet. In addition, those consumers that bought a car more than $T(\theta; \alpha)$ years ago have already replaced it and, therefore, we only consider car registrations from $t - T(\theta; \alpha)$ on. Taking time derivatives in [5] gives

$$M(t, \theta; \alpha) = M(t - T(\theta; \alpha), \theta; \alpha) e^{-\delta T(\theta; \alpha)} + (\delta + n) \zeta(\theta) N(t), \quad [7]$$

the first term on the right hand side representing unforced replacement of cars bought at time $t - T(\theta; \alpha)$ —economic obsolescence—, the second replacements forced by irreparable failure —technical obsolescence—, and the growth of the population of individuals of type θ .

Concerning type-infinity consumers, from [6]

$$M_{\infty}(t; \alpha) = (\delta + n) \zeta(\theta) N(t).$$

The total number of car registrations at time t , which we denote as $M(t; \alpha)$, depends on the parameters vector α and is given by

$$M(t; \alpha) = \int_0^{\theta_{\max}} M(t, \theta; \alpha) d\theta + M_{\infty}(t; \alpha). \quad [8]$$

4. Calibrating the Model

The model is calibrated in order to simulate the effects of Plan Prever on aggregate car sales and on the average age of the stock. The distribution of car buyers by type $\zeta(\theta)$ is calibrated in order to match the

average aggregate hazard rates for car replacement during the period 1988–1993. The period 1988–1996 defines a cycle on car sales, but we exclude the period 1994–1996 since in these years the Renove schemes were in practice, introducing severe distortions on hazard rates, as can be observed in Figure 1. This calibration is done conditional on the following numerical values for the remaining parameters:

- i) For the failure rate, we take $\delta = 0.0014$ from the observed average hazard rate for cars aged less than one year for the period 1988–1993.
- ii) The population growth rate is assumed to be $n = 0.04$, the average growth rate of the stock during the period 1988–1993.
- iii) Concerning technical progress, we rely on Izquierdo, Licandro and Maydeu (2001). They find that the increase in car’s quality, measured as the difference between the official car price index and a quality adjusted price of cars, from January 1997 to December 2000, was 3.1% per year. Consequently, we take $\gamma = 0.031$.
- iv) The price of new cars is normalized to one, since equation [2] does not change if divided by p .
- v) The scrapping value is taken to be $d_0 = 0.012$.⁴
- vi) As the discount rate, we take $\rho = 0.08$.

These numerical values, in particular the scrapping value and the discount rate, are arbitrary. The sensitivity of the analysis to some of these parameters is discussed in Section 5.

In this section, we firstly derive the theoretical relationship between the hazard function and the population distribution. Secondly, we use the observed aggregate hazard function to calibrate the population distribution.

4.1. Aggregate Hazard Rates and the Population Distribution

Under the assumption that the economy is in a steady state, car purchases must be growing at the population growth rate, for all types. This allows us to write equation [7] as follows:⁵

⁴The average price of new cars in the 1990 Encuesta de Presupuestos Familiares (EPF), the Spanish consumers survey, is 9,934 euros. Taking this value as reference, $d_0 = 0.012$ implies a scrapping price of around 120 euros.

⁵For $n = \delta = 0$, we have $M(t, \theta; \alpha_0) = \frac{N(t, \theta)}{T(\theta; \alpha_0)}$, implying that car registrations of type θ consumers are uniformly distributed in a time interval of length $T(\theta; \alpha_0)$. However,

$$M(t, \theta; \alpha_0) = \left(\frac{\delta + n}{1 - e^{-(\delta+n) T(\theta; \alpha_0)}} \right) \zeta(\theta) N(t). \quad [9]$$

Let us define the stationary density function for car sales as $\mathbf{m}(\theta; \alpha_0) = \frac{M(t, \theta; \alpha_0)}{M(t; \alpha_0)}$ for $\theta \in [0, \theta_{\max}]$, with total sales $M(t; \alpha_0)$, and type θ purchases $M(t, \theta; \alpha_0)$, defined in [8] and [9], respectively. There is an indirect map between $\zeta(\theta)$ and aggregate hazard rates which is derived into the three steps summarized below —see Appendix A4 for more details.

First, there is a direct relationship between $\mathbf{m}(\theta; \alpha_0)$ and $\zeta(\theta)$ given by

$$\zeta(\theta) = \frac{\mathbf{m}(\theta; \alpha_0) \left(1 - e^{-(\delta+n) T(\theta; \alpha_0)} \right)}{\Lambda(\alpha_0) + 1 - \mathbf{M}(\theta_{\max}; \alpha_0)}, \quad [10]$$

where $\Lambda(\alpha_0)$ is the integral of the numerator of equation [10] on the interval $[0, \theta_{\max}]$, and $\mathbf{M}(\theta_{\max}; \alpha_0)$ denotes the distribution function corresponding to the density $\mathbf{m}(\theta; \alpha_0)$ and evaluated at θ_{\max} .

Second, since from [2] $\theta = \theta(T; \alpha_0)$,

$$\mathbf{m}(\theta(T; \alpha_0); \alpha_0) = \frac{\mathbf{f}(T; \alpha_0)}{\theta'(T; \alpha_0)}, \quad [11]$$

where $\mathbf{f}(T; \alpha_0)$ denotes the unconditional density function for car scrapping age.

Third, the aggregate hazard rate for cars aged T is

$$h(T; \alpha_0) = \delta + \frac{\mathbf{f}(T; \alpha_0)}{1 - \mathbf{F}(T; \alpha_0)}. \quad [12]$$

where $\mathbf{F}(T; \alpha_0) = \mathbf{M}(\theta(T; \alpha_0); \alpha_0)$. Differentiating [12] with respect to age, we have

$$\mathbf{f}'(T; \alpha_0) = \left[\frac{h'(T; \alpha_0)}{(h(T; \alpha_0) - \delta)} - (h(T; \alpha_0) - \delta) \right] \mathbf{f}(T; \alpha_0). \quad [13]$$

Therefore, as $h(T; \alpha_0)$ can be computed using available information for hazard rates, solving equation [13] function $\mathbf{f}(T; \alpha_0)$ can also be computed and then, using expressions [10] and [11], $\zeta(\theta)$ can be obtained.

when population grows and cars crash, the relationship is more complex, as equation [9] shows.

4.2. Calibrating the Population Distribution

As we restrict to the period 1988–1993 for calibration, we are forced to consider only hazard rates from age 0 to 28, the range for which data are available in this period. The observed hazard function, for integer values of time τ and age $I = \{0, 1, \dots, 28\}$, $h_\tau(I)$, is defined and computed using official annual data as indicated in Section 2 and A1. It is worth noting that the recorded annual deregistration data constitute a smoothed version of $B_\tau(I)$, since cars recorded as of age I years when deregistered in year τ , may in fact have any age between $I - 1$ —if registered on December 31st, year $\tau - I$ and deregistered on January 1st, year τ —, and $I + 1$ years —if registered on January 1st year $\tau - I$ and deregistered on December 31st, year τ . This is important as we are modelling replacement decisions in continuous time. However, we are forced to ignore this smoothing as there are no data on deregistration of cars for shorter periods. This assumption amounts to assign an age I to all cars such that $T \in \left[I - \frac{1}{2}, I + \frac{1}{2} \right]$ which, applied in particular to the last interval, implies $T_{\max} = 28.5$.

We match the model hazard, $h(T; \alpha_0)$, to the average of annual hazards in the period 1988–1993, $h(I)$, defined as

$$h(I) = \frac{1}{6} \sum_{\tau=1988}^{1993} h_\tau(I), \text{ for } I = \{0, 1, \dots, 28\}.$$

A continuous time approximation of the average, scaled up to annual terms is given by

$$h(T; \alpha_0) = h(I) \left(\frac{n}{e^n - 1} \right),$$

for $T = I$, and interpolating cubic splines⁶ for a grid of intermediate values of T . In Figure 5 the line represents the calibrated distribution, $h(T; \alpha_0)$, and the observed hazard rates, $h(I)$, are drawn as dots.

INSERT FIGURE 5 ABOUT HERE

With the calibrated function $h(T; \alpha_0)$ in hand, we obtain $\mathbf{f}(T; \alpha_0)$ by solving equation [13] numerically with the initial condition $\mathbf{f}(0; \alpha_0) = h(0; \alpha_0) - \delta = 0$. The numerical integration of the function $\mathbf{f}(T; \alpha_0)$, shown in Figure 6, yields $\mathbf{F}(28.5) \approx 0.7$, i.e. a new car has about a 30% probability of not being deregistered in the following 28.5 years.

⁶For interpolation we take $h(T; \alpha_0) = h(28)$ for $T \in [28, 28.5[$.

Finally, to obtain $\zeta(\theta)$ —Figure 7—we use equations [10] and [11]. Integration of the function $\zeta(\theta)$ shows that about half of the population of car owners are type-infinity and never deregister their cars. Although data are about official car deregistration this evidence might indicate that not all scrapped cars are deregistered, pointing out a measurement problem.

INSERT FIGURE 6 ABOUT HERE

INSERT FIGURE 7 ABOUT HERE

5. Policy Simulations

We use the model to quantify the effects of the Prever scheme. To mimic it, we take $s = 0.048$, i.e. a subsidy of 4.8% (480 euros) of the new car price, and $\bar{T} = 10$. On a first step, we compute the new replacement function $\hat{T}(\theta; \alpha_0, s, \bar{T})$, see Figure 4. Figure 8 shows the difference between the new replacement age and the original one, as a function of the latter. The change in T ranges between about 1.77 and -1.2 years. This justifies the use of a continuous time framework, since it shows that serious errors might have arisen from using a discrete time model based on annual periods, the period for which official data are compiled.

INSERT FIGURE 8 ABOUT HERE

The replacement scheme brings about two qualitatively different effects: a transitory effect and a permanent effect. Let us first describe the transitory effect. It may be that at time t_0 , when the replacement scheme is introduced, some individuals of type $\theta > \bar{\theta}$ own cars aged more than $\hat{T}(\theta; \alpha_0, s, \bar{T})$. As can be seen in Figure 4, for any age $T > \bar{T}$, immediate replacement may be to the advantage of car owners of types θ between $\theta(T; \alpha_0)$ and $\theta(T; \alpha_1)$. The transitory effect, for $T \geq \bar{T}$, is given by

$$\begin{aligned} \text{TE}(T; \alpha_0, \alpha_1) &= \int_{\theta(T; \alpha_0)}^{\theta^*(T; \alpha_1)} M(t_0 - T, \theta; \alpha_0) e^{-\delta T} d\theta & [14] \\ &= \int_{\theta(T; \alpha_0)}^{\theta^*(T; \alpha_1)} \frac{(\delta + n) e^{-(\delta+n)T} N(t_0) \zeta(\theta)}{1 - e^{-(\delta+n)T} T(\theta, \alpha_0)} d\theta, \end{aligned}$$

where $\theta^*(T; \alpha_1) = \min \{\theta(T; \alpha_1), \theta_{\max}\}$.⁷ The second line has been derived under the assumption that the economy was in a stationary state before time t_0 and using therefore equation [9]. The total increase in replacements is given by integration on the interval $[10, T_{\max}]$. Although the adjustment is not formalized here, we should expect that this transitory effect does not occur instantaneously due to the time it takes search and buy a new car and possible temporary shortages, induced by the large increase in demand associated to the replacement scheme.

To compute the transitory effect we use equation [14]. Let us call this the *model simulation*. We take as the aggregate stock of cars, $N(t_0)$, the average for the period 1988–1993 for cars between age 0 and 28, evaluated at the beginning of the second quarter of 1997 — note that we are assuming that the stock grows at rate $n = 0.04$. This computation affords 163,541 car replacements, which reflects all the cars in the economy whose age is higher than 10, the threshold, and higher than the new optimal replacement age. This represents an increase of about 16% over total sales given by equation [8] in steady state.

It must be noted that there is a negative initial effect which is ignored in the previous computation, due to the behavior of types $\underline{\theta} \leq \theta \leq \bar{\theta}$ who optimally delay replacement. For each one of these types, the transitory effect consists in a temporary postponement in replacements from t_0 to $t_0 + 10 - T(\theta; \alpha_0)$. The omission does not affect our computation as it is only a temporary delay that almost disappears before the end of 1998, which is the horizon we are taking to evaluate the transitory effect. If we denote as \underline{T} the replacement age without subsidy of the indifferent owner $\underline{\theta}$, once the value of $\underline{\theta}$ is calculated according to Lemma A3.1 in Appendix A3, this age is computed as $\underline{T} = T(\underline{\theta}; \alpha_0)$ and is equal to 8.23 years. Therefore, cars that before the subsidy were replaced at an age $T \in [8.23, 10]$, delay their replacement during a period of time equal to $10 - T$. This means that almost all of them are replaced before the end of 1998 and only those aged from 8.23 to 8.25 delay their replacement to January, 1999. But note that, in the stationary state before the subsidy, our model predicts that only 33,899 cars are scrapped between age 8.23 and 10.

⁷For $T \geq \bar{T}$, the corresponding type θ is higher under α_1 than under α_0 . The min condition takes into account that types with $\theta > \theta_{\max}$ are not affected by the replacement scheme.

In fact, the Prever scheme does not affect the stationary stock, but the observed stock. Therefore, as we have data on deregistrations, we can use this evidence to check the accuracy of our predictions. We do this by trying to answer the following question: How many car deregistrations would be observed without the Prever scheme? Let us call *counterfactual simulation* the computation we make to answer this question. This exercise confronts two difficulties: we do not have a criterion to delimit the period over which the transitory effect extends, and we do not know how the hazard rate would have been without the Prever scheme. Concerning the former, as an approximation, we assume that the transitory effect spreads over 1997 and 1998. As for the latter, in order to extrapolate the trend of the pre-subsidy period, we project car deregistrations for 1997 applying the stationary hazard $h(T; \alpha_0)$ to the observed stock of cars at the end of 1996 for $T = I$ with $I = \{1, 2, \dots, 28\}$, and to 1997 car registrations for $I = 0$. We interpolate using cubic splines for values of T different from I . This allows us to estimate counterfactual deregistrations for 1997 and the stock at the end of this year. Then, we use this stock and the stationary hazard rate again to compute deregistrations during 1998, using also observed car sales in 1998 for $I = 0$ —the results for the stock are shown in Figure 9. The total number of deregistrations so calculated for 1997 and 1998, for cars aged 10 or more, were 803,969. We subtract this number from actual data on scrapping for cars aged 10 or more in both years—a total of 896,486 cars were actually deregistered along these two years—, giving a *counterfactual simulation* of the effect of Prever on car replacement of 92,517 cars for the period 1997–1998. If we compare this result with that of the *model simulation*, the latter — 163,541 cars— is higher concluding that, according to this comparison, our model may be overestimating the transitory effect of Plan Prever.

INSERT FIGURE 9 ABOUT HERE

The previous comparison must be taken carefully. Note that to compute the transitory effect in the *model simulation*, we are implicitly using the stationary stock as well as the stationary hazard. In contrast, in the *counterfactual simulation* results are computed as the difference between actual and stationary hazard rates applied to the observed stock. However, this analysis provides some interesting insights on the factors conditioning the efficacy of the policy.

The discrepancies between both simulations can be attributable to two factors: i) our model implies an excessive reduction in the optimal

replacement age compared to the observed one and, ii) the number of cars older than 10 is higher in the model stationary stock. To check item i), we can compare the hazard rate resulting from the transitory effect with the observed average for 1997–1998 as well as the average for 1988–1993 used in the *counterfactual simulation*. All these functions are shown in Figure 10. The hazard rate implied by the transitory effect is computed by assuming that this transitory effect splits evenly between 1997 and 1998 –see equation [A4.8] in Appendix A4. Figure 10 clearly shows that our computation overestimates the increase in hazard rates for age between 10 and 20 years. Concerning point ii) above, looking at Figure 9 we see that, although the stationary stock is slightly higher than the observed stock from age 10 to 15, it is considerably lower for almost all the remaining relevant values of age. Therefore, the overestimation does not seem to be caused by differences in the stock, but by an excessive reduction in the optimal replacement age implied by our model, which generates an excessive increase in the hazard rates.

INSERT FIGURE 10 ABOUT HERE

Two key assumptions may underlie the referred overestimation of the transitory effect of the Prever scheme: the absence of a second hand market and the assumption of a constant physical depreciation rate. Figure 10 shows that for age between 10 and 14 the observed hazard for 1997–1998 does not differ too much from the average 1988–1993. Moreover, for age between 20 and 28 the 1997–1998 average does not differ from the predicted hazard with transitory effect. It may be that the second hand market dominates the subsidy incentive up to age 14, operates partially from age 14 to 20, and is dominated by the replacement subsidy for higher values of age. It may also be that observed scrapping up to age 14 is basically physical depreciation, with an increasing depreciation rate. Consequently, the Prever scheme influences the hazard for age larger than 14 only.

Concerning the permanent effect, in the long run the number of cars replaced at any time t that would not have been replaced without subsidy —the permanent effect— is

$$\begin{aligned}
 \text{PE}(t) = & N(t) \int_{\underline{\theta}}^{\theta_{\max}} \left(\frac{n+\delta}{1-e^{-(n+\delta)T(\theta;\alpha_1)}} - \frac{n+\delta}{1-e^{-(n+\delta)T(\theta;\alpha_0)}} \right) \zeta(\theta) d\theta \\
 & + N(t) \int_{\underline{\theta}}^{\bar{\theta}} \left(\frac{n+\delta}{1-e^{-(n+\delta)T}} - \frac{n+\delta}{1-e^{-(n+\delta)T(\theta;\alpha_0)}} \right) \zeta(\theta) d\theta,
 \end{aligned}
 \tag{15}$$

where the integrand is, for each type, the difference between stationary car sales after and before the subsidy. The integral is defined over the range of types θ affected by the policy. This computation affords 14,509 additional car replacements of owners who are advancing their replacement, and a reduction in replacement of 2,155 cars caused by replacement delays. The net effect of 12,354 additional car replacements represents an increment of 1.2% in total sales.

Finally, we compute the influence of the Prever scheme on the stationary average age of the stock. The age distribution of cars older than 28 years is not available. Therefore, we are forced to compute this moment conditional on cars below 29 years of age—see Appendix A4 for details. We find that the subsidy reduces the average age of the stock from 8.7 to 8 years.

5.1. Robustness

To investigate the sensitivity of the model to a particular parameter, we recalibrate $\zeta(\theta)$ after changing it and simulate the policy. Concerning ρ , we have performed simulations with $\rho = 0.12$ and $\rho = 0.05$. Under the former value, the predicted transitory effect of Prever rises to 179,484, an increase of 9.7% with respect to the 163,541 predicted with $\rho = 0.08$, and the permanent effect rises also by 12.6% to 13,922. When $\rho = 0.05$, the predicted transitory effect is 7.7% lower than in the benchmark case and the permanent effect is also reduced by 10%.

We also find that an increase in the pre-subsidy scrapping value from $d_0 = 0.012$ to $d_0 = 0.024$, increases the transitory effect by 1.2% and the permanent effect by 1.3%. If instead, the subsidy is introduced with a pre-subsidy scrapping value $d_0 = 0.06$, the transitory effect increases by 5.1% and the permanent effect by 5.6%. If we calibrate $\zeta(\theta)$ using the mean deregistration hazard function for 1994–1996 instead of 1988–1993—other parameters at their original levels—, the transitory effect increases by 5.5% and the permanent effect by 7.8%. Concerning the failure rate, if we take $\delta = 0.01$ —around ten times the original value—, the transitory effect is reduced by 10% and the permanent effect increases by 4%.

Our calibration of hazard rates revealed that a large part of scrapped cars are never deregistered. However, it could be argued that the introduction of a replacement scheme induces a larger fraction of car owners to deregister. Although there is no evidence to test this hypo-

thesis, we can make a robustness analysis. Let us assume that 20% of type-infinity car owners change their behavior after the subsidy is introduced, so that they deregister their cars now. Let assume that they are distributed in $[0, \theta_{\max}]$ as the existing population. In this case, both the transitory and permanent effects increase by 17%.

6. Final Remarks

The model described in this paper allows for an evaluation of the effects of car replacement subsidies. On the theoretical side, it matches observed aggregate hazard rates starting from heterogeneous endogenous replacement decisions. It also highlights the fact that the presence of a threshold age in the replacement scheme induces a delay in replacement for some consumers. On the empirical side, we are able to make a quantitative evaluation of the Prever scheme introduced in Spain in 1997. Although our model seems to overestimate the short run effects of the Prever scheme, in the long run the increase in car sales predicted by our model is very small. Finally, we find that the subsidy reduces the average age of the stock of cars, as expected, although the reduction is small.

It is necessary, however, to make some remarks about the simplifications implied by the main assumptions. As it stands, the model embodies five major simplifications: there is no second-hand car market, the physical depreciation rate is age independent, the parameters affecting the optimal scrapping age are constant —except for the change in the scrapping value caused by the subsidy—, cars have no running or maintenance costs, and the transitory effect is instantaneous.

Ignoring the second-hand car market and assuming a time independent physical depreciation rate may be the main reasons for the model to be overestimating the effects of the Prever scheme, which only subsidizes the simultaneous purchase of a new car and deregistration of an old one. A more accurate estimate would be possible if information were available concerning the number of transactions and prices of cars of different age in the second-hand market. Also, consideration of the second-hand market would be desirable for evaluation of the consequences of making the subsidy available to all deregistered cars, whether or not deregistration is accompanied by purchase of a new car, because this measure would raise prices in the second-hand market and hence stimulate the replacement of newer as well as older cars.

The function $T(\theta; \alpha)$, which is constant in the present model, must in reality change over time, not only because of the time-dependence of some parameters in α —ignored in the present model—, but also because of such unmodelled factors as income shocks and employment stability.

Consideration of running and maintenance cost not only refines the model for analysis of schemes like Prever, but is essential for its application to the analysis of the effects on car sales and deregistration of changes in oil taxes or inspection standards.⁸

Finally, it would be desirable to take into account the various factors responsible for the transitory effect of Prever-like schemes not being instantaneous, as can be adjustment cost either on the demand or supply side of the market.

Appendixes

A1. Data

To compute the hazard rates, $h_\tau(I)$, as defined in Section 2, we use published yearly data by the Dirección General de Tráfico (DGT). Data are given at December 31st and for one-year periods. We have two options to compute $h_\tau(I)$.

On the one hand, for each year, statistical bulletins from DGT report data on the stock of cars classified by age between age 0 and 9, from 1969 to 1982. From 1984 onward, the age ranges from 0 to 21.⁹ This allows us to compute deregistrations of cars as,

$$B_\tau^r(I) = P_\tau^r(I) - P_{\tau-1}^r(I-1), \quad \forall \tau \geq 1984, \quad \forall I = 0, 1, 2, \dots, 21, \quad [\text{A1.1}]$$

where superscript “r” is for “reported” data, and use [A1.1] to calculate $h_\tau(I)$ as $\frac{B_\tau^r(I)}{P_{\tau-1}^r(I-1)}$.

On the other hand, from 1970 onward, the DGT publishes data on new car registrations, M_τ , as well as car deregistrations $B_\tau(I)$ classified by age

⁸The introduction of compulsory inspection in 1987 had a major effect on replacement—see Licandro and Sampayo (1997a) and Moral Rincón (1998).

⁹These data are not available for 1983.

between age 0 and a maximum age that varies from year to year but being always higher than 30. This allows us to compute the stock for each year as

$$\left. \begin{aligned} P_\tau(0) &= M_\tau - B_\tau(0), \quad \forall \tau \geq 1970, \\ P_{1970}(I) &= P_{1969}^r(I-1) - B_{1970}(I), \quad \forall I = 1, \dots, 10, \\ P_\tau(I) &= P_{\tau-1}(I-1) - B_\tau(I), \quad \forall \tau > 1970, \quad \forall I = 1, \dots, \tau - 1960. \end{aligned} \right\} \quad [\text{A1.2}]$$

$h_\tau(I)$ can be computed as $\frac{B_\tau(I)}{P_{\tau-1}(I-1)}$.

The second option is more useful as, for high values of τ , the stock is available for values of age higher than 21. For instance, in 1988 we can compute the stock classified by age between 0 and 28. Instead, if we use the first option, we have only the stock between age 0 and 21. It must be noted that, even for age less than 22, $P_\tau(I)$ computed using [A1.2] and $P_\tau^r(I)$ are not the same from 1983 onward. In general, it happens that, for $\tau > 1983$, $P_\tau(I) < P_\tau^r(I)$ when $I \leq 10$, and $P_\tau(I) > P_\tau^r(I)$ when $I \geq 10$. In the aggregate we have $P_\tau > P_\tau^r$. For example, $P_{1996} = 15, 223, 454$ and $P_{1996}^r = 14, 753, 809$.

Given the discrepancy mentioned above, we choose the second option as the first would only allow the computation of $h_\tau(I)$ for values of I lower than 22, leaving aside from the analysis an age range that is probably strongly affected by the policy under study.

A2. The Replacement Problem Without Subsidy

Under the assumptions of Section 3.1, consumers solve the following problem

$$W(A(0), a(0)) = \max_{c(t), \{t_k\}_{k=1}^\infty} E_0 \int_0^\infty (\theta c(t) + e^{-\gamma a(t)}) e^{-(\rho-\gamma)t} dt, \quad [\text{A2.1}]$$

subject to

$$\frac{dA(t)}{dt} = y + rA(t) - c(t) \quad [\text{A2.2}]$$

$$A(t_k) = \lim_{t \rightarrow t_k^-} A(t) - (p - d_0), \quad k = 1, 2, \dots \quad [\text{A2.3}]$$

$$a(t_k) = T \quad [\text{A2.4}]$$

$$da(t) = \begin{cases} -T & \text{if } t = t_k, \quad k = 1, 2, \dots \\ dt + dq & \text{otherwise} \end{cases} \quad [\text{A2.5}]$$

$$dq = \begin{cases} T - a(t) & \text{with probability } \delta dt, \\ 0 & \text{with probability } (1 - \delta) dt, \end{cases} \quad [\text{A2.6}]$$

$A(0), a(0)$ given,

where $A(t)$ denotes financial wealth, $a(t)$ durable's age, θ the idiosyncratic taste parameter and $c(t)$ nondurables consumption. The rate of time preference is denoted as ρ , and γ is the embodied rate of durable goods technical

progress. The exogenous instantaneous income and interest rate are y and r , respectively. We denote as p the price of new durable goods and as d_0 the scrapping value, both exogenous. We assume $d_0 < p$ reflecting partial irreversibility in purchases which, joint with the absence of second hand markets, generates infrequent replacement. Finally, k is an index for successive replacements.

Equations [A2.2] and [A2.3] represent the budget constraint where nondurables consumption is chosen continuously, and car replacement is chosen at random discrete times t_k motivated by partial irreversibility of car purchases. Equation [A2.4] defines the optimal replacement age T . Equations [A2.5] and [A2.6] reflect the evolution of age as a Poisson process, capturing the idea that at each time with a probability δ , the car age reaches the value T that renders the car wasteful and must be replaced by a new one. That is, the age evolution is composed of two effects: a deterministic one, where the age evolves with time if a failure does not happen, and a stochastic process represented by q with dq following a Poisson process reflecting the fact that, with probability δ , the car crashes and it is scrapped. T is endogenous and defines the optimal replacement age in the sense that if the car does not suffer a breakdown from age zero to T , it will be replaced optimally at this latter age. So we represent a failure as if it suddenly increases the age to the value where it is optimal to replace the car. In spite of the linearity of the problem, unappealing corner solutions on $c(t)$ are not considered, and next we look at conditions for an interior solution to apply.

Note that the problem only depends on time through the discount rate $\rho - \gamma$ so that it is stationary. This was made possible by both the inclusion of the term $e^{\gamma t}$ into the individual valuation of nondurable goods, and the consideration of a stationary stochastic process — a Poisson process — for the underlying uncertainty. Besides the simplification of the problem, this will allow us to get a constant replacement age for cars. The stationarity and recursivity of the problem makes possible to apply the Bellman principle of optimality to get:

$$W(A, a) = \max \{V(A, a), J(A, a)\}, \quad [\text{A2.7}]$$

where $J(A, a)$ is the value of scrapping the car and is given by¹⁰

$$\begin{aligned} (\rho - \gamma) J(A, a) = \max_c \{ & \theta c + 1 \\ & + J'_a(A, a) + J'_A(A, a)(y + rA - c) \}. \end{aligned} \quad [\text{A2.8}]$$

$V(A, a)$ is the value of keeping the car and is given by:

$$\begin{aligned} (\rho - \gamma) V(A, a) = \max_c \{ & \theta c + e^{-\gamma a} + \delta [J(A, a) - V(A, a)] \\ & + V'_a(A, a) + V'_A(A, a)(y + rA - c) \}. \end{aligned} \quad [\text{A2.9}]$$

¹⁰In deriving Bellman equations, we follow Dixit and Pindyck (1994). For what follows, note that when no failure takes place, $\frac{da}{dt} = 1$.

Both equations are subject to constraints [A2.4]–[A2.6].

If $c(A, a)$ is the interior solution for problems [A2.8] and [A2.9], the following first order conditions must be verified:

$$\theta = J'_A(A, a) \quad [\text{A2.10}]$$

$$\theta = V'_A(A, a). \quad [\text{A2.11}]$$

Applying the envelope theorem to [A2.8] and [A2.9] gives

$$\begin{aligned} (\rho - \gamma) J'_A(A, a) &= J'_A(A, a) r & [\text{A2.12}] \\ &+ J''_{AA}(A, a) (y + rA - c(A, a)) \\ &+ J''_{aA}(A, a) \end{aligned}$$

$$\begin{aligned} (\rho - \gamma) V'_A(A, a) &= \delta [J'_A(A, a) - V'_A(A, a)] & [\text{A2.13}] \\ &+ V'_A(A, a) r + V''_{AA}(A, a) (y + rA - c(A, a)) \\ &+ V''_{aA}(A, a). \end{aligned}$$

The implicit function theorem allows us to differentiate [A2.10] and [A2.11], resulting that all derivatives of functions J and V of order higher than one are equal to zero. Therefore, from equations [A2.12] and [A2.13] we obtain that $\rho - \gamma = r$. The other way around: if this equality applies, an interior solution for nondurables consumption can be obtained. Hereafter we will assume that this equality holds.

Now, we solve the problem. Integrating [A2.2] for each non replacement time interval and computing left limits,

$$\begin{aligned} \lim_{t \rightarrow t_k^-} A(t) e^{-r(z-t_{k-1})} \\ + \lim_{z \rightarrow t_k^-} \int_{t_{k-1}}^z c(t) e^{-r(t-t_{k-1})} dt &= A(t_{k-1}) \\ &+ \lim_{z \rightarrow t_k^-} \int_{t_{k-1}}^z y e^{-r(t-t_{k-1})} dt. \end{aligned}$$

Using [A2.3] gives

$$\begin{aligned} (A(t_k) + (p - d_0)) e^{-r(t_k-t_{k-1})} \\ + \int_{t_{k-1}}^{t_k} c(t) e^{-r(t-t_{k-1})} dt &= A(t_{k-1}) \\ &+ \int_{t_{k-1}}^{t_k} y e^{-r(t-t_{k-1})} dt. \end{aligned}$$

Substituting recursively for successive t_k ,

$$\begin{aligned} \lim_{k \rightarrow \infty} e^{-rt_k} A(t_k) + \int_0^\infty c(t) e^{-rt} dt \\ + (p - d_0) \sum_{k=1}^\infty e^{-rt_k} = A(0) \\ + \int_0^\infty y e^{-rt} dt. \end{aligned}$$

Ruling-out Ponzi schemes, $\lim_{k \rightarrow \infty} e^{-rt_k} A(t_k) = 0$, resulting

$$\int_0^\infty c(t) e^{-rt} dt + (p - d_0) \sum_{k=1}^\infty e^{-rt_k} = A(0) + \int_0^\infty y e^{-rt} dt.$$

This is the realized budget constraint. But, given that successive replacement times are random, at time 0 the budget constraint must be verified in expected value. Computing expectations in the previous equation gives:

$$\int_0^\infty c(t) e^{-rt} dt + (p - d_0) E_0 \sum_{k=1}^\infty e^{-rt_k} = A(0) + \int_0^\infty y e^{-rt} dt, \quad [\text{A2.14}]$$

as only scrapping times are random and the right hand side is deterministic. Given the linearity of the utility function and the assumption $r = \rho - \gamma$, the nondurables consumption path is determined as a residual after durable goods spending, and the financial wealth can be ignored as a state variable in the replacement problem. These assumptions joint with the absence of credit constraints make it possible to write the intertemporal budget constraint as we do and to ignore the consumer income in the objective function. Therefore, we substitute constraint [A2.14] into the objective function and the following problem results:

$$\begin{aligned} W(a) = \max_{\{t_k\}_{k=1}^\infty} E_0 \left\{ \int_0^\infty e^{-\gamma a(t)} e^{-(\rho-\gamma)t} dt \right. \\ \left. - \theta (p - d_0) \sum_{k=1}^\infty e^{-(\rho-\gamma)t_k} \right\}, \quad [\text{A2.15}] \end{aligned}$$

subject to [A2.4], [A2.5] and [A2.6], and with a given at $t = 0$.

To solve [A2.15], we again take advantage of stationarity and recursivity, and reformulate the problem as follows:

$$W(a) = \max \{V(a), J(a)\}, \quad [\text{A2.16}]$$

where, as before, $V(a)$ denotes the value function if no replacement takes place and $J(a)$ the value of replacing a car of age a . This formulation corresponds to problem [A2.7] after removing wealth as a state variable. Assuming differentiability of the value function, if the car is not replaced, equation [A2.15] can be written as

$$V(a) = \frac{e^{-\gamma a}}{\rho - \gamma} + \frac{\delta}{\rho - \gamma} [J(a) - V(a)] + \frac{1}{\rho - \gamma} V'_a(a). \quad [\text{A2.17}]$$

If the car becomes worthless for use, it must be scrapped and a new car bought at cost $\theta(p - d_0)$. From [A2.15], at some replacement time t_k the value of owning a car of age a , $J(a)$, must be equal to the value of a new car, $V(0)$, minus the replacement cost. Therefore, the value of replacing a failed car of age a , is given by $J(a) = V(0) - \theta(p - d_0)$. Taking this into account, the following differential equation results:

$$V(a) = \frac{e^{-\gamma a}}{\rho + \delta - \gamma} + \frac{1}{\rho + \delta - \gamma} V'_a(a) + \frac{\delta}{\rho + \delta - \gamma} [V(0) - \theta(p - d_0)], \quad [\text{A2.18}]$$

whose analytical solution is,

$$V(a) = \frac{e^{-\gamma a}}{\rho + \delta} + \frac{\delta}{\rho + \delta - \gamma} (V(0) - \theta(p - d_0)) + C e^{(\rho + \delta - \gamma)a}. \quad [\text{A2.19}]$$

Evaluating this expression at $a = 0$, gives

$$V(0) = \frac{(\rho + \delta - \gamma)}{(\rho + \delta)(\rho - \gamma)} + \frac{(\rho + \delta - \gamma)}{\rho - \gamma} C - \frac{\delta}{\rho - \gamma} \theta(p - d_0). \quad [\text{A2.20}]$$

Finally, taking [A2.20] into [A2.19]

$$V(a) = \frac{e^{-\gamma a}}{\rho + \delta} + \frac{\delta}{(\rho + \delta)(\rho - \gamma)} + \left(\frac{\delta}{\rho - \gamma} + e^{(\rho + \delta - \gamma)a} \right) C - \frac{\delta}{\rho - \gamma} \theta(p - d_0). \quad [\text{A2.21}]$$

To solve the value function $V(a)$, the replacement age T and the constant of integration C must be determined. To this end, we use the “boundary value” and “smooth pasting” conditions —see Dixit and Pindyck (1994). Note that problem [A2.16] can be written as

$$W(a) = \max \{ V(a), V(0) - \theta(p - d_0) \}. \quad [\text{A2.22}]$$

This gives the following boundary value condition

$$V(T) = V(0) - \theta(p - d_0). \quad [\text{A2.23}]$$

Replacing [A2.20] and [A2.21] into equation [A2.23] results,

$$\frac{e^{-\gamma T}}{\rho + \delta} - \frac{1}{\rho + \delta} + \left(e^{(\rho + \delta - \gamma)T} - 1 \right) C + \theta(p - d_0) = 0. \quad [\text{A2.24}]$$

On the other hand, the smooth pasting condition in this problem is the following¹¹

$$V'_a(T) = 0.$$

¹¹Dixit and Pindyck (1994) provide an heuristic derivation of the “smooth pasting” condition. This condition establishes that $V(T)$ and $V(0) - \theta(p - d_0)$ which are, respectively, the value of keeping and replacing a car aged T , must meet tangentially at the optimal replacement age. In our model, $\frac{\partial V(T)}{\partial a} = \frac{\partial [V(0) - \theta(p - d_0)]}{\partial a} = 0$.

To compute this equation, we compute the derivative in [A2.21] and evaluate the resulting expression at T :

$$-\frac{\gamma}{\delta + \rho}e^{-\gamma T} + (\rho + \delta - \gamma) Ce^{(\rho + \delta - \gamma)T} = 0. \quad [\text{A2.25}]$$

The solution to the equations system [A2.24] and [A2.25] gives C and $T(\theta; \alpha_0)$. The later defines the optimal replacement age as a function of parameters and is shown in equation [2] in Section 3.1.

A2.1. Existence of an Optimal Replacement Age

To assure that $T(\theta; \alpha_0)$ as defined in [3] is the solution to the replacement problem, we must prove that: i) the value function is decreasing as a function of age; ii) it is optimal to own a car during some time interval, and iii) exists a finite value of a for which it is optimal to replace the car.

The first assertion is true if and only if $V(a) \geq V(0) - \theta(p - d_0)$ for $a \in [0, T(\theta; \alpha_0)]$ and $V(a)$ decreases monotonically in the same interval. To verify the monotonicity of the value function, we differentiate $V(a)$ into equation [A2.21] resulting that the value function is decreasing if and only if

$$C \leq \frac{\gamma e^{-(\delta + \rho)a}}{(\delta + \rho)(\delta + \rho - \gamma)}, \quad [\text{A2.26}]$$

with equality for $a = T(\theta; \alpha_0)$. To check this inequality, we solve for C in [A2.24], resulting

$$C = \frac{1 - e^{-\gamma T(\theta; \alpha_0)}}{(\delta + \rho)(e^{(\delta + \rho - \gamma)T(\theta; \alpha_0)} - 1)} - \frac{\theta(p - d_0)}{e^{(\delta + \rho - \gamma)T(\theta; \alpha_0)} - 1}. \quad [\text{A2.27}]$$

Using [2] to substitute $\theta(p - d_0)$ we have

$$C = \frac{\gamma e^{-(\delta + \rho)T(\theta; \alpha_0)}}{(\delta + \rho)(\delta + \rho - \gamma)}. \quad [\text{A2.28}]$$

The inequality [A2.26] reduces to

$$\frac{e^{-(\delta + \rho)T(\theta; \alpha_0)}}{\delta + \rho - \gamma} \leq \frac{e^{-(\delta + \rho)a}}{\delta + \rho - \gamma},$$

which holds for $a \in [0, T(\theta; \alpha_0)]$ if and only if $\gamma < \rho + \delta$.

Moreover, given that the value function is decreasing during the tenure interval and that $p > d_0$, it is clear that $V(a) \geq V(0) - \theta(p - d_0)$ for all $a \in [0, T(\theta; \alpha_0)]$, resulting $T(\theta; \alpha_0) \geq 0$ and $T(\theta; \alpha_0) = 0$ if and only if $\theta = 0$. Therefore, for $a \in [0, T(\theta; \alpha_0))$ —known as the “continuation region”—, it is optimal to keep the car and $a = T(\theta; \alpha_0)$ is the replacement age.

Finally, we prove that $T(\theta; \alpha_0) < \infty$ if and only if $\theta < \frac{1}{(\delta+\rho)(p-d_0)}$. From [A2.20] and [A2.21],

$$V(a) - [V(0) - \theta(p - d_0)] = \frac{e^{-\gamma a} - 1}{\delta + \rho} + C \left(e^{(\delta+\rho-\gamma)a} - 1 \right) + \theta(p - d_0).$$

Using [A2.28] to replace C results

$$\begin{aligned} V(a) - [V(0) - \theta(p - d_0)] &= \frac{e^{-\gamma a} - 1}{\delta + \rho} + \theta(p - d_0) \\ &\quad + \frac{(e^{(\delta+\rho-\gamma)a} - 1) \gamma e^{-(\delta+\rho)T(\theta; \alpha_0)}}{(\delta + \rho)(\delta + \rho - \gamma)}. \end{aligned}$$

Taking limits when $T(\theta; \alpha_0) \rightarrow \infty$,

$$\lim_{T(\theta; \alpha_0) \rightarrow \infty} V(a) - [V(0) - \theta(p - d_0)] = \frac{e^{-\gamma a}}{\delta + \rho} - \left(\frac{1}{\delta + \rho} - \theta(p - d_0) \right).$$

This expression shows that if $T(\theta; \alpha_0) < \infty$ then $\theta(p - d_0) < \frac{1}{\delta + \rho}$. Otherwise, this limit can be positive no matter how high a is, contradicting the existence of a finite $T(\theta; \alpha_0)$. Conversely, if $\theta(p - d_0) < \frac{1}{\delta + \rho}$ there is some $\bar{a} < \infty$ such that for $a > \bar{a}$ this limit takes a negative value, which proves the existence of a finite value of $T(\theta; \alpha_0)$.

The conditions assuring that $0 \leq T(\theta; \alpha_0) < \infty$ are stated under Assumptions 1 and 2 in Section 3.1.

A3. The Replacement Problem with Subsidy

With the subsidy, the maximization problem of Appendix A2 remains unaltered except that equation [A2.3] is replaced by the following:

$$A(t_k) = \lim_{t \rightarrow t_k^-} A(t) - (p - d_0 - \mathbb{I}(a(t_k) \geq \bar{T})s); \quad k = 1, 2, \dots$$

where $\mathbb{I}(T(t_k) \geq \bar{T})$ is defined as:

$$\mathbb{I}(a(t_k) \geq \bar{T}) = \begin{cases} 1 & \text{if } a(t_k) \geq \bar{T} \\ 0 & \text{otherwise.} \end{cases}$$

Taking this into account, the reasoning of Appendix A2 can be reproduced here and the maximization problem now is the following:

$$\begin{aligned} W(a) &= \max_{\{t_k\}_{k=1}^{\infty}} \int_0^{\infty} e^{-\gamma a(t)} e^{-(\rho-\gamma)t} dt \\ &\quad - \theta(p - d_0 - \mathbb{I}(a(t_k) \geq \bar{T})s) \mathbb{E}_0 \sum_{k=1}^{\infty} e^{-(\rho-\gamma)t_k} \end{aligned}$$

subject to the same constraints as in problem [A2.15] in Appendix A2. The recursive formulation of this problem is given also by equation [A2.16] in Appendix A2 but now $J(a)$ is:

$$J(a) = \begin{cases} V(0) - \theta(p - d_0) & \text{if } a < \bar{T} \\ V(0) - \theta(p - d_0 - s) & \text{if } a \geq \bar{T}. \end{cases} \quad [\text{A3.1}]$$

The following Lemma establishes the existence of a type $\underline{\theta}$ which is indifferent between taking advantage of the subsidy or not.

LEMMA A3.1. *Let $\bar{\theta}$ define the type such that $T(\bar{\theta}; \alpha_0) = \bar{T}$. Without subsidy, types $\theta < \bar{\theta}$ where replacing at age $T(\theta; \alpha_0) < \bar{T}$. With subsidy these owners have two alternatives: keep their behavior given by $T(\theta; \alpha_0)$ without taking benefit of the subsidy or delay the replacement to age \bar{T} to get the subsidy. Let $V(a, \theta)$ and $V^*(a, \theta)$ define, respectively, the value of a car aged a if the replacement age continues to be $T(\theta; \alpha_0)$ or is just \bar{T} , for $\theta < \bar{\theta}$. Therefore, there exist a unique type $\underline{\theta} < \bar{\theta}$ such that $V(a, \underline{\theta}) = V^*(a, \underline{\theta})$. Moreover, for $\theta \in [0, \underline{\theta}[$, $V(a, \theta) > V^*(a, \theta)$ whereas $V(a, \theta) < V^*(a, \theta)$ for $\theta \in]\underline{\theta}, \bar{\theta}]$.*

PROOF. From Figure 4 the replacement age increases monotonically with respect to θ . Therefore, at the time of the subsidy implementation the replacement age is always lower than \bar{T} for $\theta < \bar{\theta}$. Types $\theta \in [0, \bar{\theta}[$ have to delay the replacement to age \bar{T} , in order to receive the subsidy. However, it is necessary to check if it is profitable to assume the cost implied by this decision.¹² To this end, we first compute the value function without subsidy which, from Appendix A1 is given by [A2.21] after replacing C by [A2.28], resulting

$$V(a, \theta) = \frac{e^{-\gamma a}}{\rho + \delta} + \frac{\delta}{(\rho + \delta)(\rho - \gamma)} - \frac{\delta}{\rho - \gamma} \theta(p - d_0) \quad [\text{A3.2}] \\ + \left(\frac{\delta}{\rho - \gamma} + e^{(\rho + \delta - \gamma)a} \right) \frac{\gamma e^{-(\delta + \rho)T(\theta; \alpha_0)}}{(\delta + \rho)(\delta + \rho - \gamma)},$$

where $T(\theta; \alpha_0)$ is given by [2] in Section 3.1. On the other hand, if the owner scraps the car when is \bar{T} years old, the value of holding a car of age $a \in [0, \bar{T}]$ is given by

$$V^*(a, \theta) = \frac{e^{-\gamma a}}{\rho + \delta} + \frac{\delta}{(\rho - \gamma)(\rho + \delta)} \quad [\text{A3.3}] \\ + \left(\frac{\delta}{\rho - \gamma} + e^{(\rho + \delta - \gamma)a} \right) \bar{C} - \frac{\delta}{\rho - \gamma} \theta(p - d_0),$$

¹²Note that, in contrast, for $\theta \geq \bar{\theta}$ the advance in replacement is not a prerequisite to receive the subsidy but it is a response. Taking advantage of the subsidy has no cost.

which is similar to [A2.21] in Appendix A1, except that now the boundary of the problem is exogenously given at \bar{T} . As the replacement age is given by \bar{T} , the constant \bar{C} must be computed as the solution to the equation $V^*(\bar{T}, \theta) = V^*(0, \theta) - \theta(p - d_0 - s)$, with $V^*(\bar{T}, \theta)$ and $V^*(0, \theta)$ given by [A3.3] when evaluated at $a = \bar{T}$ and 0 respectively. After doing this we have,

$$\bar{C} = \frac{1 - e^{-\gamma\bar{T}}}{(\rho + \delta)(e^{(\delta+\rho-\gamma)\bar{T}} - 1)} - \frac{\theta(p - d_0 - s)}{e^{(\delta+\rho-\gamma)\bar{T}} - 1},$$

and therefore

$$\begin{aligned} V^*(a, \theta) = & \frac{e^{-\gamma a}}{\rho + \delta} + \frac{\delta}{(\rho + \delta)(\rho - \gamma)} - \frac{\delta}{\rho - \gamma}\theta(p - d_0) \quad [A3.4] \\ & + \left(\frac{(1 - e^{-\gamma\bar{T}})\theta(\rho + \delta)(p - d_0 - s)}{(\rho + \delta)(e^{(\delta+\rho-\gamma)\bar{T}} - 1)} \right) \\ & \times \left(\frac{\delta}{\rho - \gamma} + e^{(\rho+\delta-\gamma)a} \right). \end{aligned}$$

Finally, to see which owners want to delay their replacement in order to take advantage of the subsidy, we compute the difference $V^*(a, \theta) - V(a, \theta)$, with $V^*(a, \theta)$ given by [A3.4] and $V(a, \theta)$ given by [A3.2]. Let $\Omega(a, \theta) = V^*(a, \theta) - V(a, \theta)$. Next, we prove the following:

i) $\Omega(a, \theta)$ is increasing and continuous as a function of θ in the interval $[0, \bar{\theta}]$.
Let

$$\Omega(a, \theta) = \left(\frac{1 - e^{-\gamma\bar{T}}}{(\rho + \delta)(e^{(\delta+\rho-\gamma)\bar{T}} - 1)} - \frac{\theta(p - d_0 - s)}{e^{(\delta+\rho-\gamma)\bar{T}} - 1} - \frac{\gamma e^{-(\delta+\rho)T(\theta; \alpha_0)}}{(\delta + \rho)(\delta + \rho - \gamma)} \right) \times \left(\frac{\delta}{\rho - \gamma} + e^{(\rho+\delta-\gamma)a} \right). \quad [A3.5]$$

The derivative with respect to θ gives

$$\begin{aligned} \frac{\partial \Omega(a, \theta)}{\partial \theta} = & \left(\frac{\gamma e^{-(\delta+\rho)T(\theta; \alpha_0)}}{\delta + \rho - \gamma} \frac{dT(\theta; \alpha_0)}{d\theta} - \frac{p - d_0 - s}{e^{(\delta+\rho-\gamma)\bar{T}} - 1} \right) \\ & \times \left(\frac{\delta}{\rho - \gamma} + e^{(\rho+\delta-\gamma)a} \right). \end{aligned}$$

If $\frac{dT(\theta; \alpha_0)}{d\theta}$ is computed from [2] in Section 3.1 and substituted into the previous equation, after some manipulations, we obtain

$$\begin{aligned} \frac{\partial \Omega(a, \theta)}{\partial \theta} = & \left(\frac{p - d_0}{e^{(\delta+\rho-\gamma)T(\theta; \alpha_0)} - 1} - \frac{p - d_0 - s}{e^{(\delta+\rho-\gamma)\bar{T}} - 1} \right) \quad [A3.6] \\ & \times \left(\frac{\delta}{\rho - \gamma} + e^{(\rho+\delta-\gamma)a} \right) > 0, \end{aligned}$$

given that $T(\theta; \alpha_0) < \bar{T}$ for $\theta \in [0, \bar{\theta}]$.

ii) $\Omega(a, \bar{\theta}) > 0$ for all $a \in [0, \bar{T}]$.

If, to compute [A3.2], equation [A2.27] is substituted into [A2.21] — instead of [A2.28] as we did in point **i)**—, results

$$V(a, \theta) = \left(\frac{1 - e^{-\gamma T(\theta; \alpha_0)}}{(\rho + \delta)(e^{(\delta + \rho - \gamma)T(\theta; \alpha_0)} - 1)} - \frac{\theta(p - d_0)}{e^{(\delta + \rho - \gamma)T(\theta; \alpha_0)} - 1} \right) \\ \times \left(e^{(\rho + \delta - \gamma)a} + \frac{\delta}{\rho - \gamma} e^{(\rho + \delta - \gamma)T(\theta; \alpha_0)} \right) \\ + \frac{e^{-\gamma a}}{\rho + \delta} + \frac{\delta e^{-\gamma T(\theta; \alpha_0)}}{(\rho + \delta)(\rho - \gamma)}.$$

If we use this equation to compute $\Omega(a, \theta)$ evaluated at $\bar{\theta}$, it is easy to see that

$$\Omega(a, \bar{\theta}) = \left(e^{(\rho + \delta - \gamma)a} + \frac{\delta}{\rho - \gamma} e^{(\rho + \delta - \gamma)\bar{T}} \right) \left(\frac{\bar{\theta}s}{e^{(\rho + \delta - \gamma)\bar{T}} - 1} \right) > 0.$$

Therefore, denoting as $\underline{\theta}$ the type that solves equation $\Omega(a, \theta) = 0$, if a solution exists, we have that $\underline{\theta} < \bar{\theta}$, and some owners optimally delay their replacement to take advantage of the subsidy.

iii) If $\bar{T} > 0$, then $\Omega(a, 0) < 0$ for all $a \in [0, \bar{T}]$.

Consider the following limit:

$$\lim_{\theta \rightarrow 0} \Omega(a, \theta) = \left(\frac{1 - e^{-\gamma \bar{T}}}{(\rho + \delta)(e^{(\delta + \rho - \gamma)\bar{T}} - 1)} - \frac{\gamma}{(\delta + \rho)(\delta + \rho - \gamma)} \right) \\ \times \left(\frac{\delta}{\rho - \gamma} + e^{(\rho + \delta - \gamma)a} \right).$$

For $\bar{T} > 0$ the previous limit is negative if and only if

$$\frac{e^{\gamma \bar{T}} - 1}{e^{(\delta + \rho)\bar{T}} - e^{\gamma \bar{T}}} < \frac{\gamma}{\delta + \rho - \gamma},$$

which is always the case provided that $\gamma < \rho + \delta$ is verified.

Therefore, from **i)**, **ii)** and **iii)** we can apply Bolzano theorem so that a unique value $\underline{\theta}$ exists such that $0 < \underline{\theta} < \bar{\theta}$, and can be found as the solution to the equation $\Omega(a, \theta) = 0$. This is a nonlinear equation whose solution in θ is independent of a —as can be seen in [A3.5]—, and must be solved numerically. Moreover, if $\bar{T} = 0$, the limit computed in **iii)** is equal to zero showing that $\theta = 0$ is the solution.

PROOF (OF PROPOSITION 2). Lemma 1 establishes the replacement age for $\theta \leq \bar{\theta}$. It remains to analyze the replacement for $\theta \geq \bar{\theta}$. As, from Proposition

1, the replacement age is inversely related to the scrapping subsidy, $\bar{\theta} < \bar{\theta}$. We first study the replacement behavior for $\theta > \bar{\theta}$ with $\bar{\theta}$ being the type such that $T(\bar{\theta}; \alpha_1) = \bar{T}$. In this case, the replacement decision is a free boundary problem equivalent to that solved in Appendix A2 but with different boundary conditions that now are:

$$\left. \begin{aligned} V(T) &= V(0) - \theta(p - d_0 - s) \\ V'_a(T) &= 0, \end{aligned} \right\} \quad [\text{A3.7}]$$

where V is given by [A2.21] with $p - d_0 - s$ replacing $p - d_0$. Solving this pair of equations as we did without subsidy, the resulting replacement age is given by equation [2] but with $p - d_0 - s$ instead of $p - d_0$.

Finally the interval $[\underline{\theta}, \bar{\theta}]$ is analyzed. Cars owned by these types have an age higher than \bar{T} at the time of the introduction of the subsidy. But, if their problem were solved using equations [A3.7] the replacement age with subsidy would be lower than \bar{T} , given the definition of $\underline{\theta}$. Therefore, if they want to take advantage of the subsidy, they must replace at age \bar{T} . To see if it pays to do that, the value of replacing at age \bar{T} that was previously computed and is given by [A3.4], must be compared with the value of not taking advantage of the subsidy. This latter value was computed in [A3.2] and the difference in [A3.5] as $\Omega(a, \theta)$ —although the proof of Lemma A3.1 was conducted for $\theta \in [0, \underline{\theta}]$, equations [A3.2]–[A3.6] remain valid for $\theta \in [\underline{\theta}, \bar{\theta}]$. Given that $T(\bar{\theta}; \alpha_1) = \bar{T}$, it must happen that $\Omega(a, \bar{\theta}) = 0$. Moreover, the derivative of equation [A3.6] with respect to θ is,

$$\frac{\partial^2 \Omega(a, \theta)}{\partial \theta^2} = - \frac{(p - d_0)(\delta + \rho - \gamma)}{(e^{(\delta + \rho - \gamma)T(\theta; \alpha_0)} - 1)^2} \frac{dT(\theta; \alpha_0)}{d\theta} < 0.$$

That is, $\Omega(a, \theta)$ is equal to zero for $\underline{\theta}$ and $\bar{\theta}$, positive for $\bar{\theta}$ and strictly concave for all $\theta \in [0, \bar{\theta}]$. Therefore $\Omega(a, \theta)$ is also positive for $\theta \in [\underline{\theta}, \bar{\theta}]$, concluding that it is also optimal to replace at age \bar{T} in this interval.

A4. Aggregate Hazard Rates and the Population Distribution

As noted in Section 4.1, the stationary density function for car sales is defined as $\mathbf{m}(\theta; \alpha_0) = \frac{M(t, \theta; \alpha_0)}{M(t; \alpha_0)}$, for $\theta \in [0, \theta_{\max}]$, with total sales $M(t; \alpha_0)$ and each type purchases $M(t, \theta; \alpha_0)$ defined in [8] and [9] respectively. Starting from equation [9], and after some manipulations,

$$\mathbf{m}(\theta; \alpha_0) = \frac{\zeta(\theta) (1 - e^{-(\delta+n)T(\theta; \alpha_0)})^{-1}}{\Omega(\alpha_0) + 1 - \zeta(\theta_{\max})}, \quad [\text{A4.1}]$$

where,

$$\Omega(\alpha_0) = \int_0^{\theta_{\max}} \zeta(\theta) (1 - e^{-(\delta+n)T(\theta; \alpha_0)})^{-1} d\theta.$$

Solving for $\zeta(\theta)$ affords

$$\zeta(\theta) = \frac{\mathbf{m}(\theta; \alpha_0) (1 - e^{-(\delta+n) T(\theta; \alpha_0)})}{\Lambda(\alpha_0) + 1 - \mathbf{M}(\theta_{\max}; \alpha_0)},$$

where

$$\Lambda(\alpha_0) = \int_0^{\theta_{\max}} \mathbf{m}(\theta; \alpha_0) (1 - e^{-(\delta+n) T(\theta; \alpha_0)}) d\theta, \quad [\text{A4.2}]$$

and $\mathbf{M}(\theta_{\max}; \alpha_0)$ is the distribution function corresponding to the density $\mathbf{m}(\theta; \alpha_0)$ evaluated at θ_{\max} .

Next, the replacement hazard function is defined as

$$h(t, T; \alpha_0) \equiv \frac{B(t, T; \alpha_0)}{P(t, T; \alpha_0)}, \quad [\text{A4.3}]$$

where T is the car age, $P(t, T; \alpha_0)$ is the number of cars of age T on the stock at time t , and $B(t, T; \alpha_0)$ is the number of cars of age T deregistered during some short period $[t, t + dt]$. $P(t, T; \alpha_0)$ and $B(t, T; \alpha_0)$ are related through

$$\begin{aligned} B(t, T; \alpha_0) &\equiv \frac{P(t, T(t); \alpha_0) - P(t + dt, T(t + dt); \alpha_0)}{dt} \quad [\text{A4.4}] \\ &= - \left(\frac{\partial P(t, T; \alpha_0)}{\partial t} + \frac{\partial P(t, T; \alpha_0)}{\partial T} \right). \end{aligned}$$

Note that by the definition of age T , $dT = dt$. Since individuals with $\theta < \theta(T; \alpha_0)$ have no car aged T , $P(t, T; \alpha_0)$ can be written as

$$\begin{aligned} P(t, T; \alpha_0) &= \int_{\theta(T; \alpha_0)}^{\theta_{\max}} M(t - T, \theta; \alpha_0) e^{-\delta T} d\theta \quad [\text{A4.5}] \\ &\quad + M(t - T, \infty; \alpha_0) e^{-\delta T}. \end{aligned}$$

Under stationarity, equation [A4.5] becomes

$$P(t, T; \alpha_0) = e^{-(n+\delta)T} M(t; \alpha_0) (1 - \mathbf{M}(\theta(T; \alpha_0); \alpha_0)). \quad [\text{A4.6}]$$

From equation [A4.4],

$$B(t, T; \alpha_0) = \delta P(t, T; \alpha_0) + e^{-(n+\delta)T} M(t; \alpha_0) \mathbf{f}(T; \alpha_0), \quad [\text{A4.7}]$$

where

$$\mathbf{f}(T; \alpha_0) = \mathbf{m}(\theta(T; \alpha_0); \alpha_0) \theta'(T; \alpha_0),$$

is the density \mathbf{m} after using the variable change $\theta(T; \alpha_0)$. Substituting [A4.6] and [A4.7] into equation [A4.3], results

$$h(T; \alpha_0) = \delta + \frac{\mathbf{f}(T; \alpha_0)}{1 - \mathbf{F}(T; \alpha_0)}.$$

To compare our simulation results with data in Section 5, we use the transitory hazard rate just after Plan Prever, under the assumption that the transitory effect splits evenly between 1997 and 1998. Let us denote this hazard as $h_{TE}(T)$. To compute this function, it must be taking into account that, with subsidy, car replacement in the age interval $[T(\underline{\theta}; \alpha_0), \bar{T}]$ is reduced to failures and that owners in the interval $[\underline{\theta}, \bar{\theta}]$ concentrate their replacement at age \bar{T} . Therefore, using definition [A4.3], if the subsidy is introduced at time t_0 , $h_{TE}(T)$ is given by

$$h_{TE}(T) = \begin{cases} h(T; \alpha_0) & \text{if } T < T(\underline{\theta}; \alpha_0) \\ \delta & \text{if } T(\underline{\theta}; \alpha_0) \leq T < \bar{T} \\ h(T; \alpha_0) + \frac{1}{2} \frac{\text{TE}(T; \alpha_0, \alpha_1)}{P(t_0, T; \alpha_0)} \\ + \frac{1}{2} \int_{\underline{\theta}}^{\bar{\theta}} \frac{M(t_0 - T(\theta; \alpha_0), \theta; \alpha_0)}{P(t_0, T(\theta; \alpha_0); \alpha_0)} d\theta & \text{if } T = \bar{T} \\ h(T; \alpha_0) + \frac{1}{2} \frac{\text{TE}(T; \alpha_0, \alpha_1)}{P(t_0, T; \alpha_0)} & \text{if } T > \bar{T}, \end{cases} \quad [\text{A4.8}]$$

where $\text{TE}(T; \alpha_0, \alpha_1)$ is given by [14] in Section 5.

Finally expression [A4.6] can be used to compute the stationary average age of the stock of cars with subsidy, reported in Section 5. Denoting as $p(T; \alpha_0) = \frac{P(t, T; \alpha_0)}{N(t)}$ the per capita stock of cars aged T , this function integrates less than one in the interval $[0, T(\theta_{\max}; \alpha_0)]$ because, as it is observed in the main text, $\mathbf{M}(\theta_{\max}; \alpha_0) < 1$. Since the distribution of age for higher values than $T(\theta_{\max}; \alpha_0)$ is unknown, we are forced to compute the average age of the stock conditional on the age being less than $T(\theta_{\max}; \alpha_0)$. If $p(\alpha_0)$ denotes the integral of the function $p(T; \alpha_0)$ over the interval $[0, T(\theta_{\max}; \alpha_0)]$, the conditional average age is

$$\frac{1}{p(\alpha_0)} \int_0^{T(\theta_{\max}; \alpha_0)} T p(T; \alpha_0) dT .$$

With subsidy, the stationary stock $p(T; \alpha_1)$ is given by

$$p(T; \alpha_1) = \begin{cases} e^{-(n+\delta)T} \frac{M(t; \alpha_1)}{N(t)} (1 - \mathbf{M}(\theta(T; \alpha_1); \alpha_1)) & \text{if } T < \underline{T} \\ e^{-(n+\delta)T} \frac{M(t; \alpha_1)}{N(t)} (1 - \mathbf{M}(\underline{\theta}; \alpha_1)) & \text{if } \underline{T} \leq T < \bar{T} \\ e^{-(n+\delta)\bar{T}} \frac{M(t; \alpha_1)}{N(t)} (1 - \mathbf{M}(\theta(T; \alpha_1); \alpha_1)) & \text{if } T \geq \bar{T}, \end{cases}$$

where $M(t; \alpha_1)$ is given by [8] using α_1 instead of α_0 and \underline{T} is defined as $\underline{T} = T(\underline{\theta}; \alpha_0)$. $\mathbf{M}(\theta(T; \alpha_1))$ is the distribution function whose corresponding density is given by [A4.1] with α_1 replacing α_0 . Note that the subsidy

makes the function $\mathbf{m}(\theta; \alpha_1)$ to be a mixed discrete–continuous function that remains constant in the interval $[\underline{\theta}, \bar{\theta}]$ and accumulates a mass at point $\bar{\theta}$. Consequently function $\mathbf{M}(\theta; \alpha_1)$ is discontinuous at this point and the stock is also discontinuous at the corresponding age, \bar{T} .

The average age of the stock with subsidy is:

$$\frac{1}{p(\alpha_1)} \int_0^{T(\theta_{\max}; \alpha_1)} T p(T; \alpha_1) dT,$$

where $T(\theta_{\max}; \alpha_1) < T(\theta_{\max}; \alpha_0)$ and $p(\alpha_1)$ is given by the integral of the function $p(T; \alpha_1)$ over the interval $[0, T(\theta_{\max}; \alpha_1)]$.

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TABLE 1
Replacement schemes for cars in Spain during the 1990's

	Plan Renove I April, 1994	Plan Renove II October, 1994	Plan Prever April, 1997
Starting date	April, 1994	October, 1994	April, 1997
Time in force	6 months	9 months	permanently
Requirements	To scrap a car aged 10 years or more	To scrap a car aged 7 years or more	<ul style="list-style-type: none"> • To scrap a car aged 10 years or more • Old car ownership ≥ 1 year at the replacement time • Less than 6 months between scrapping and purchase
Allowances in new car taxes (Euros)	<ul style="list-style-type: none"> • $\max\{508, TB\}$ if $\tau = 0.11$ • $\max\{600, TB\}$ if $\tau = 0.13$ • $\max\{4,600 \times \tau, TB\}$ otherwise 	<ul style="list-style-type: none"> • $\max\{480, TB\}$ if $\tau = \{0.07, 0.12\}$ • $\max\{3,700 \times \tau, TB\}$ otherwise 	<ul style="list-style-type: none"> • $\max\{480, TB\}$ and: • $\tau=0.07$ for small-medium engine power cars • $\tau=0.12$ for medium-high engine power cars

Definitions. τ = Vehicle registration tax rate; TB (New car registration tax bill) = $\tau \times$ price of new car. *Source:* The three decrees establishing the corresponding replacement schemes were gazetted under the name "REAL DECRETO-LEY" (RDL) in the "Boletín Oficial del Estado" (BOE), the Spanish State Official Gazette. Are the following: RDL 4/1994, BOE April 12, 1994; RDL 10/1994, BOE October 12, 1994; RDL 6/1997, BOE April 11, 1997. On the new car registration taxes, see Ley (Act) 38/1992, BOE December 29, 1992 and January 19, 1993, and successive modifications available at www.aeat.es.

TABLE 2
 Actual registrations, deregistrations and stock growth: average
 1988–1993, average 1994–1996, 1997 and 1998.

	1988–1993	1994–1996	1997	1998
	%stock	%stock	%stock	%stock
New car registrations	8.2	6.3	6.8	7.7
Stock growth	4.5	3.1	3.9	4.5
Cars scrapped (deregistered)	3.6	3.2	3.1	3.4

FIGURE 1
Observed aggregate hazard rates for car replacement in Spain
1993–1996

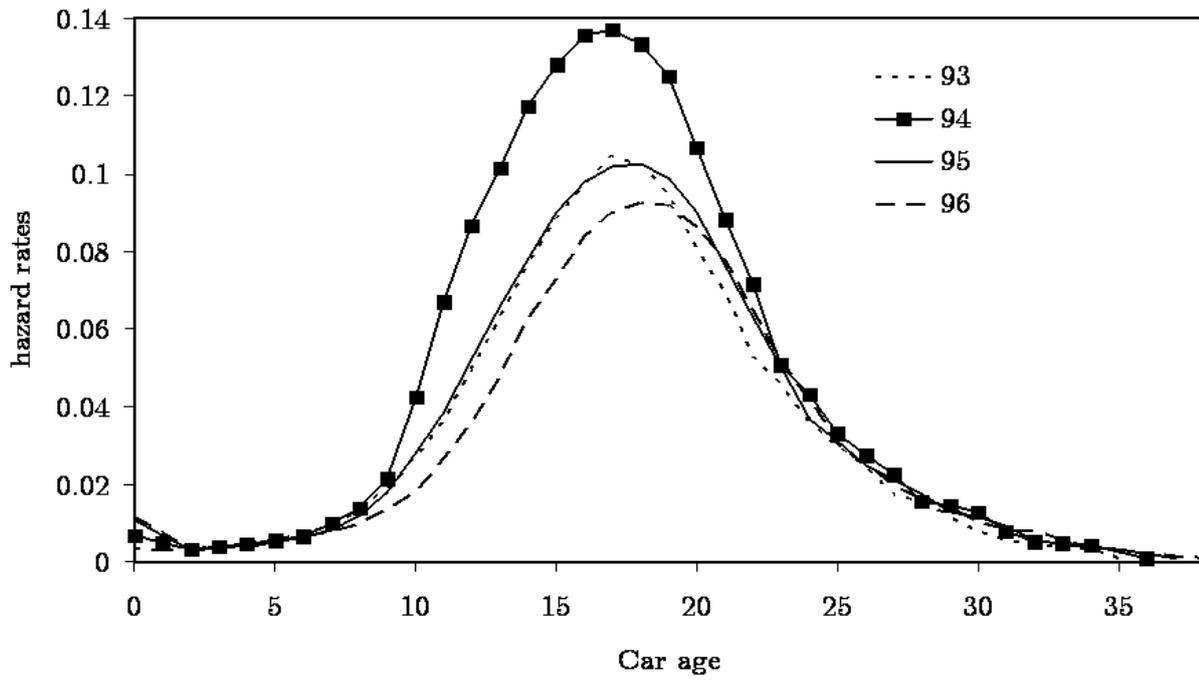


FIGURE 2
Observed hazard rates for several years in Spain

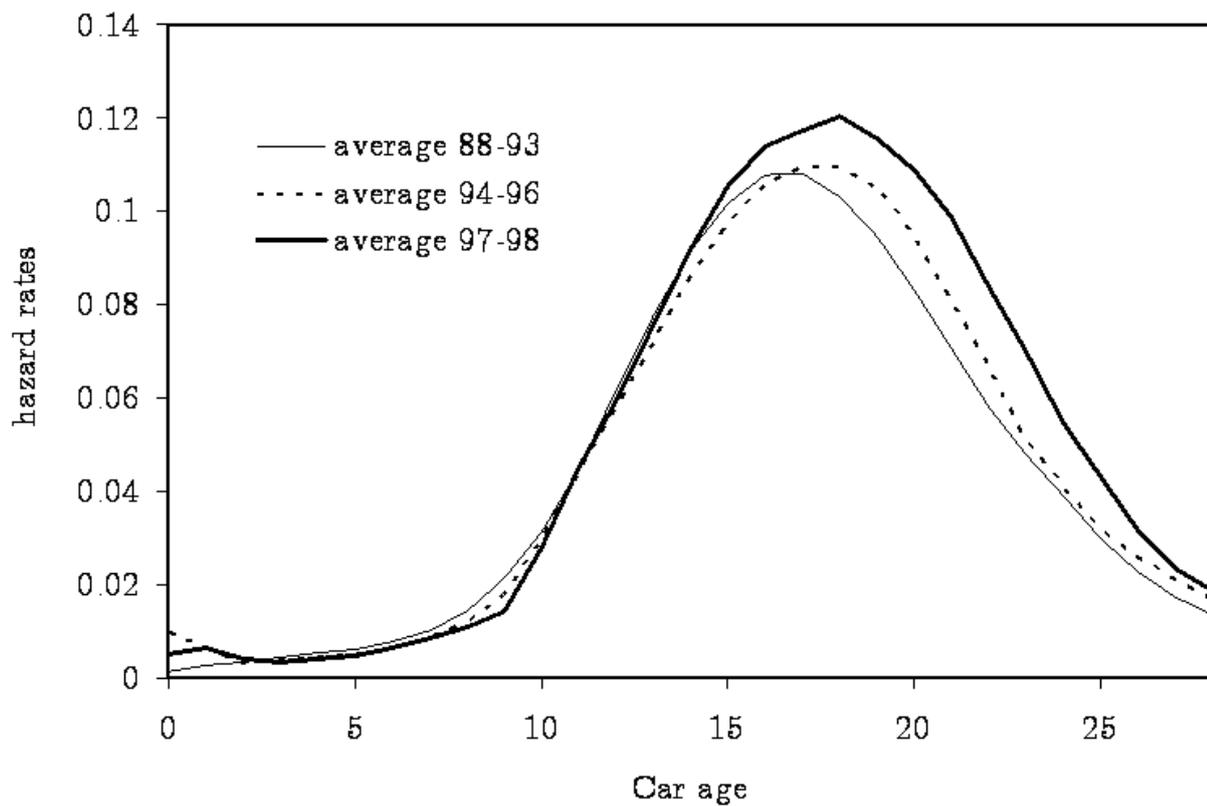


FIGURE 3
Growth rate of the stock of cars for 1988–1998 and its
composition

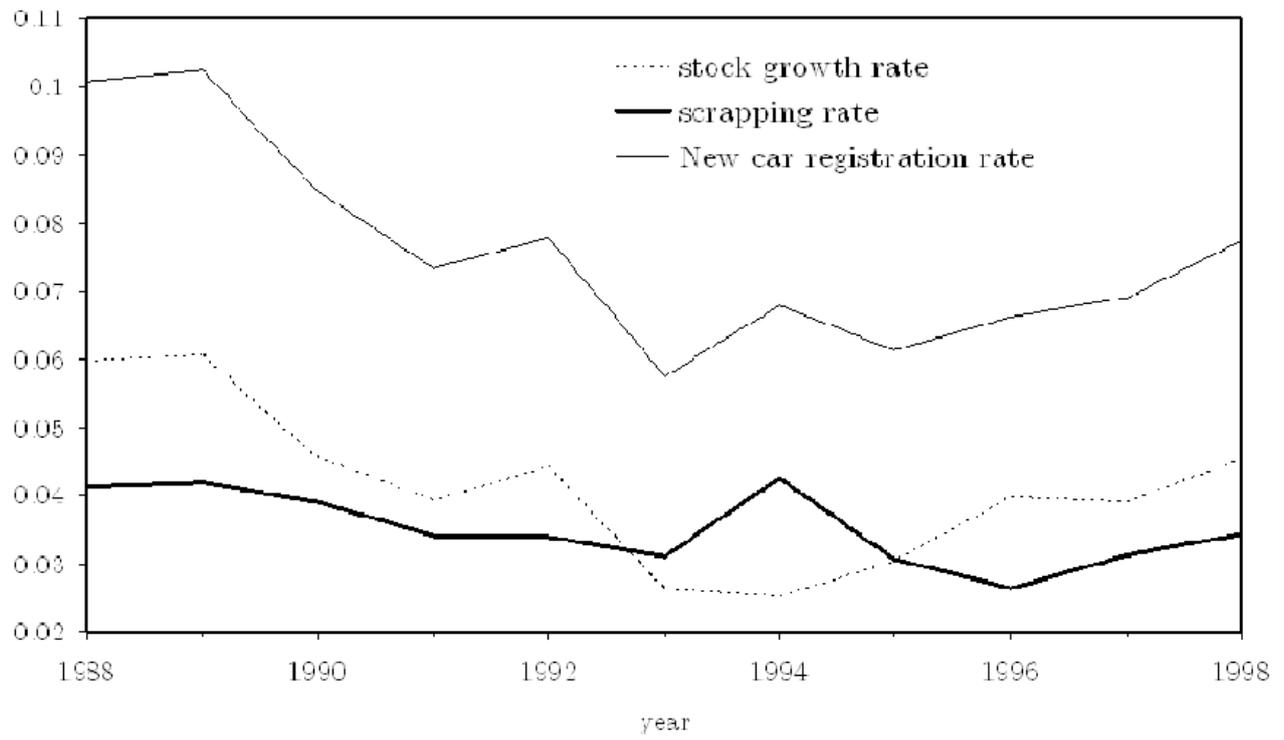


FIGURE 4

Optimal replacement age as a function of θ , before and after the subsidy. The parameter values for this Figure are: $p = 1$, $d_0 = 0.012$, $\rho = 0.08$, $\gamma = 3.1$, $\delta = 0.0014$. For the replacement age with subsidy, $s = 0.048$. Both functions are identical for $\theta < \underline{\theta}$

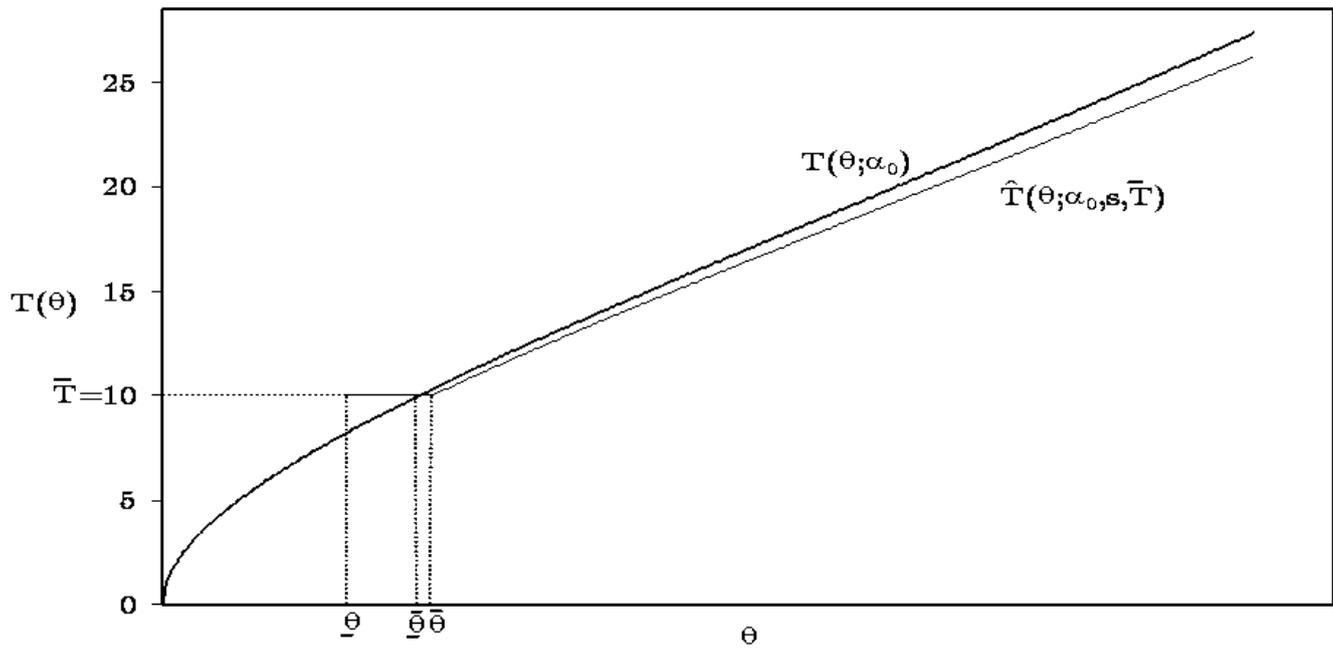


FIGURE 5
Calibrated deregistration hazard function $h(T; \alpha_0)$ (line) and
observed Spanish car deregistration hazard rates averaged over
the period 1988–1993 (dots)

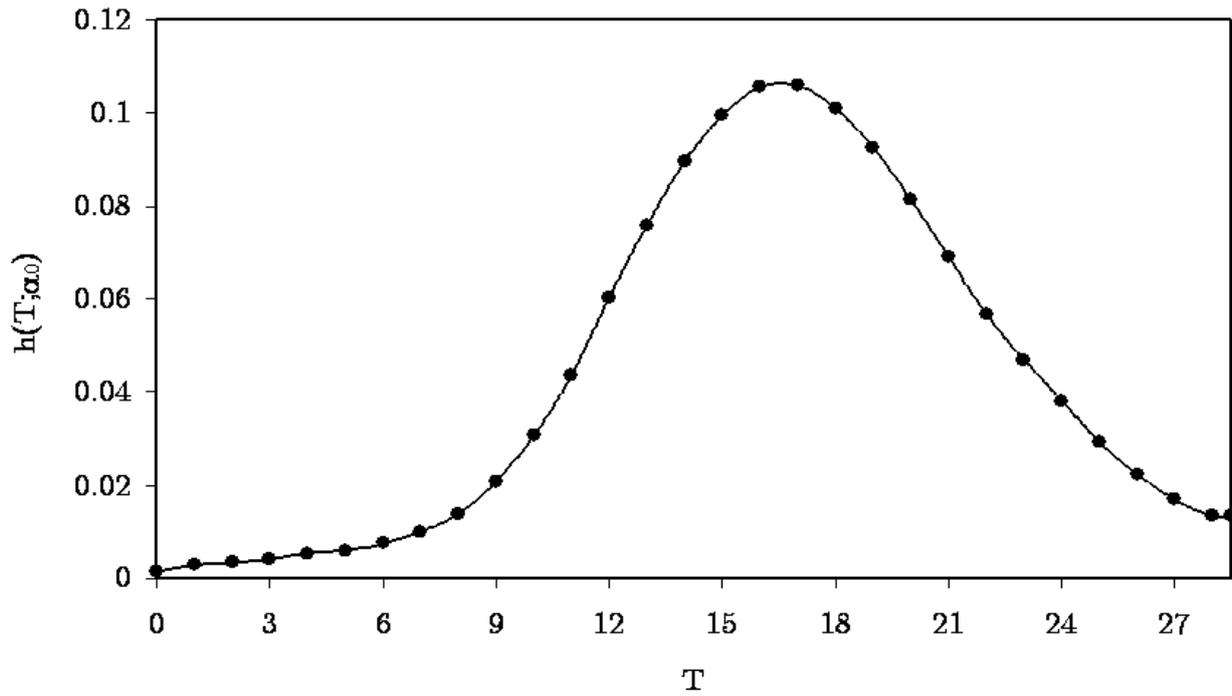


FIGURE 6
Calibrated density function $f(T; \alpha_0)$

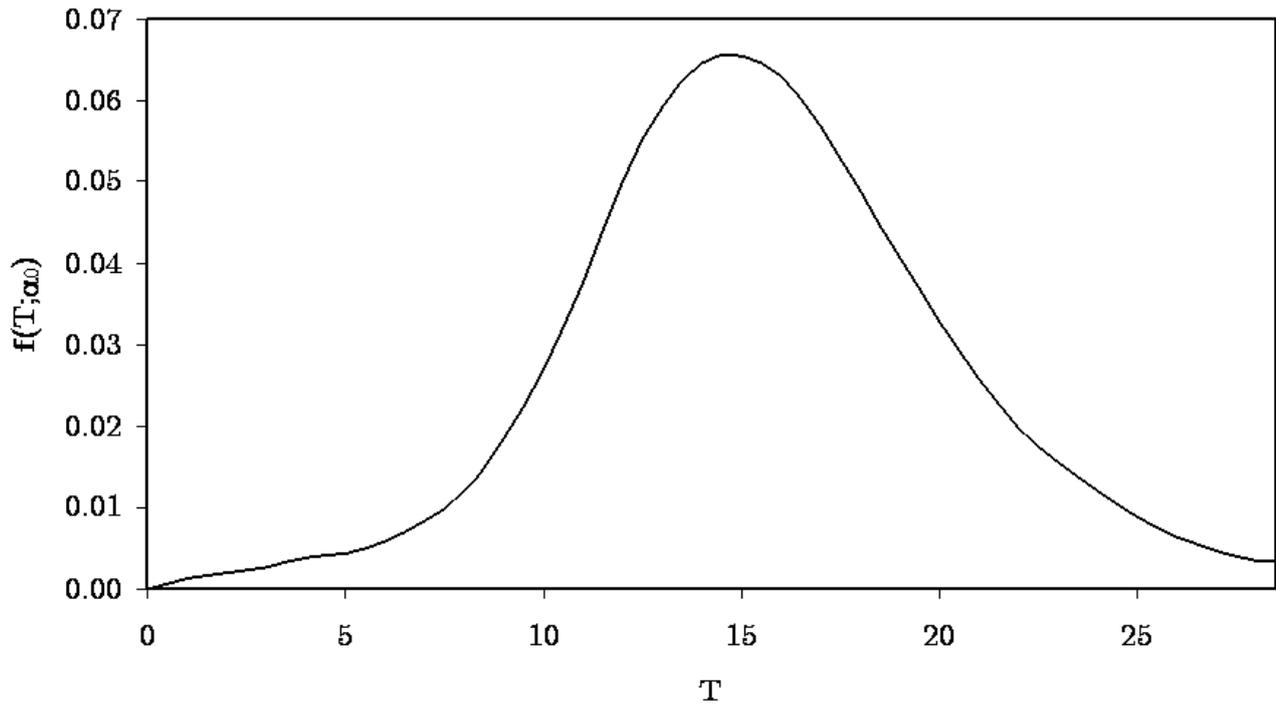


FIGURE 7
Calibrated distribution of the car-owning population by θ

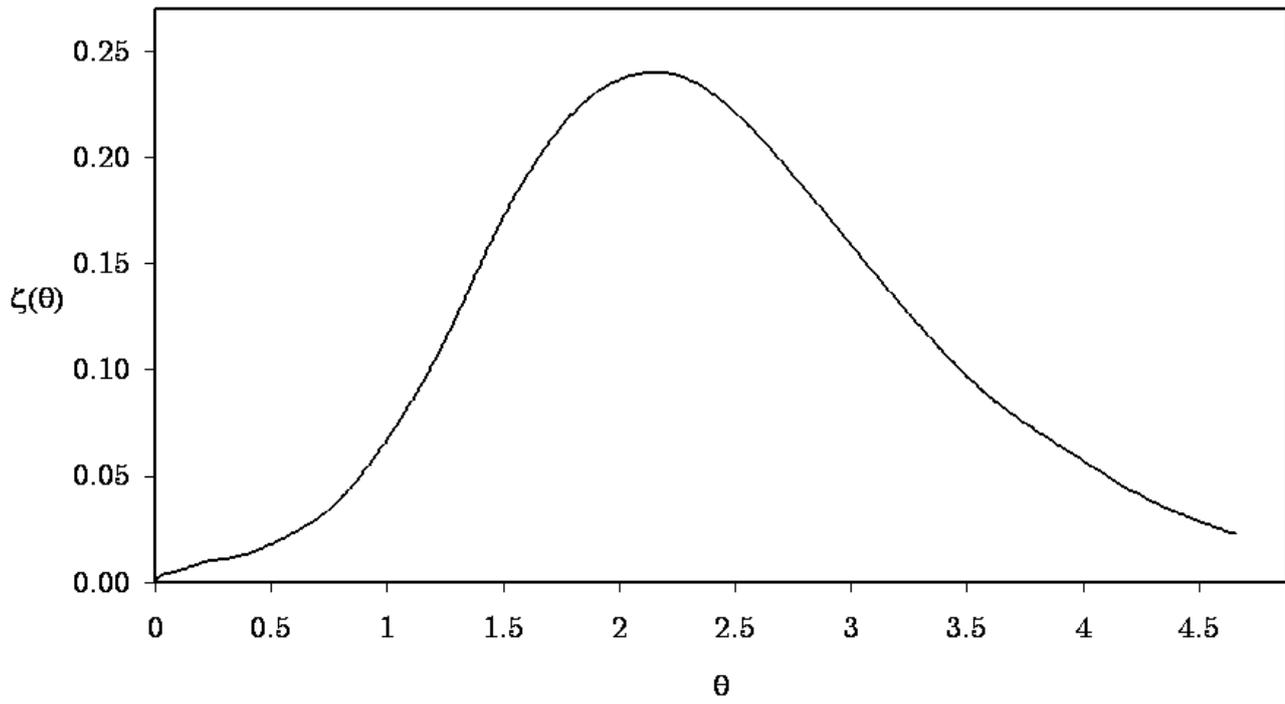


FIGURE 8
Change in optimal replacement age induced by the replacement
scheme, as a function of pre-Prever optimal replacement age

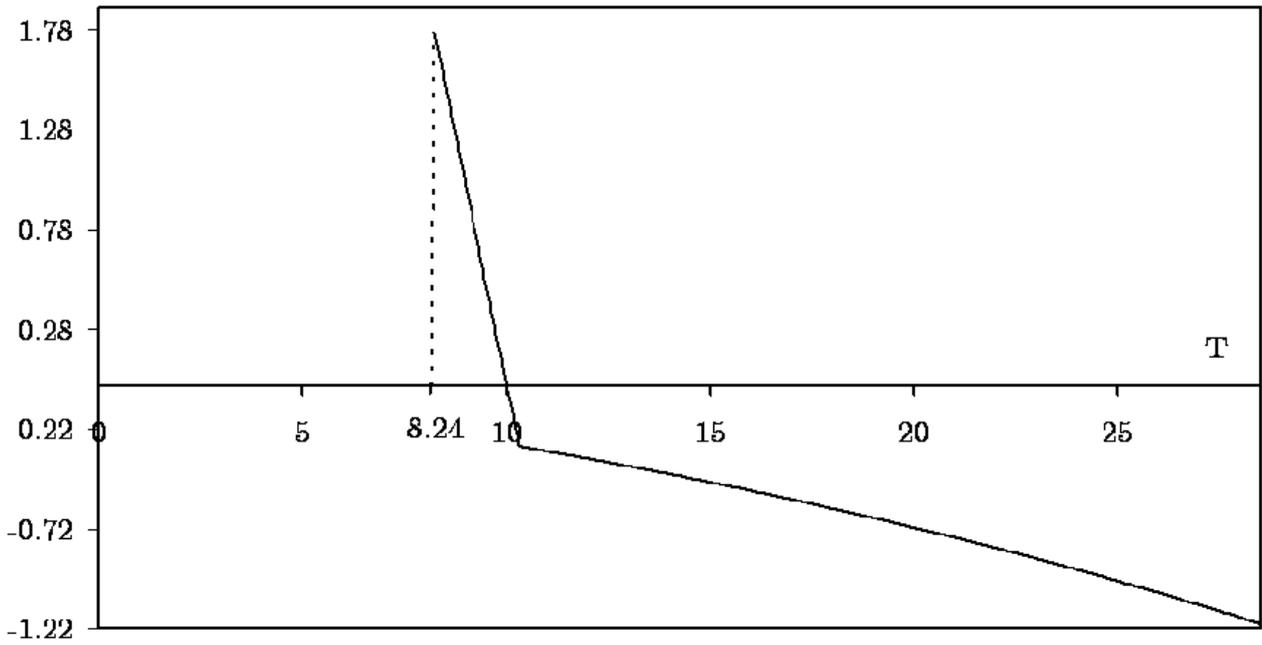


FIGURE 9
Observed age-wise distribution of car population on December 31st, 1996, simulation for December 31st, 1997 and stationary stock

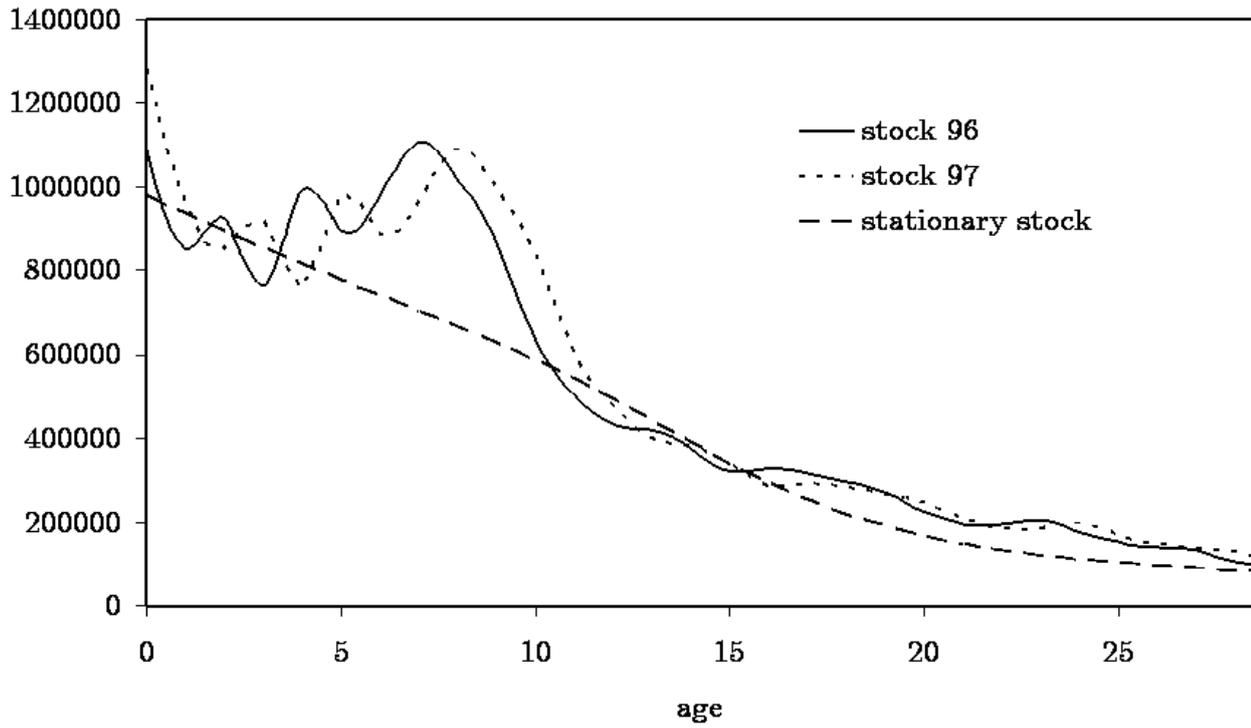


FIGURE 10
Average observed hazard rates for 1988–1993, average for
1997–1998 and simulated hazard with transitory effect

