



# EUI WORKING PAPERS IN ECONOMICS

EUI Working Paper ECO No. 93/8

**Informed Speculation:  
Small Markets Against Large Markets**

FREDERIC PALOMINO

WEP

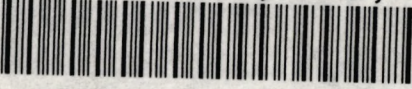
330

EUR

European University Institute, Florence



European University Library



3 0001 0014 9676 1



*Please note*

As from January 1990 the EUI Working Paper Series is divided into six sub-series, each sub-series is numbered individually (e.g. EUI Working Paper LAW No. 90/1).

© The Author(s), European University Institute.

Digitised version produced by the EUI Library in 2020. Available Open Access on Cadmus, European University Institute Research Repository.

**EUROPEAN UNIVERSITY INSTITUTE, FLORENCE**

**ECONOMICS DEPARTMENT**

EUI Working Paper ECO No. 93/8

**Informed Speculation:  
Small Markets Against Large Markets**

FREDERIC PALOMINO

**BADIA FIESOLANA, SAN DOMENICO (FI)**

All rights reserved.  
No part of this paper may be reproduced in any form  
without permission of the author.

© Frederic Palomino  
Printed in Italy in February 1993  
European University Institute  
Badia Fiesolana  
I – 50016 San Domenico (FI)  
Italy



# INFORMED SPECULATION: SMALL MARKETS AGAINST LARGE MARKETS

by

Frederic PALOMINO  
European University Institute  
Florence (Italy)

## Abstract

Informed speculation has become a popular area of research in Economics. However, the main part of the existing literature focuses on the amount of information revealed by price. In this paper, we will try to derive some properties of informed traders' welfare in imperfectly competitive markets. Two cases will be studied: a market with both heterogeneously informed speculators and noise traders and a purely speculative market. A discussion on the possible existence of several informationally isolated small markets will follow.

I would like to thank Alan Kirman and Robert Waldmann for helpful comments, Stuart Dixon, Melanie Lund and John Stanton-Ife for help in editing. All remaining errors are mine.



## I INTRODUCTION

In 50 BC, all of Gaul is occupied by Romans... All? No! A village populated by unshakeable Gauls still offers resistance to the invader.

Among the residents of this village is Obelix, chairman and unique employee of Obelix and Co., a firm specialized in menhir production. It is rumoured that he has discovered a revolutionary production process for menhirs. However, if some of the inhabitants of the village believe it, there are others who are rather skeptical.

As Obelix does not want to answer any of the insistent questions from the board of administration of Obelix and Co., the shareholders decide, individually, that during their daily boar hunt, they will spy on him in his quarry. However, as all of them did so at different hours of the day, they all gathered different pieces of information about the new production process.

While a spontaneous meeting of Obelix and Co. shareholders was taking place in the market place, Getafix, the druid, was conceiving one of his great plans. Getafix, whose fame was built upon his discovery of the famous magic potion, used to defeat the Romans, and his precious gift of tea to the Britons<sup>1</sup>, happened to be a pacifist, and thus, in order to avoid the general brawl of the last annual general meeting, he decided to organize a unique large stock market. He observed that not only risk-averse heterogeneously informed shareholders trying to maximize their utility were trading but also a group of people buying a totally random amount of stock just for fun. Getafix wondered if he had chosen the best market structure and whether he would have increased the welfare of the shareholders by organizing several small markets.

More than two thousand years later, informed speculation has become a popular area of research in Economics. However, the main part of the existing literature focuses on the amount of information revealed by prices. Grossman (1976, 1978) established that, under specific assumptions about the distribution of the private information and traders' utility, competitive markets with heterogeneously informed traders are informationally efficient. It follows that if the acquisition of information is endogenous and costly, then an equilibrium does not exist<sup>2</sup>.

Tirole (1982, proposition 1) explained this result by stating that in a Rational Expectation Equilibrium of a purely speculative market with traders of homogeneous prior beliefs, risk-averse traders will not trade, risk neutral traders may trade but they will not expect any gain from trade. If any of these assumptions is dropped then static speculation occurs. In most of the models, this has led to adding some irrational demand or supply. A reason for introducing irrational traders is given by Shleifer and Summers (1990):

"Some investors are not fully rational and their demand for risky asset is affected by

---

<sup>1</sup> See Gosciny and Uderzo: "Asterix chez les bretons"

<sup>2</sup> See Grossman and Stiglitz (1980).



their beliefs or sentiments that are not fully justified by fundamental news."

For example, in France, we can observe that the results of the national football team affect the behaviour of some traders the day after a game.

Irrational demand is most of the time described as noise trading. In the literature two definitions are usually found. Kyle (1984, 1985, 1989) defines noise trading as a random, exogenous, inelastic demand of assets. De Long, Shleiffer, Summers, Waldmann (1989, 1990a) assume that noise traders are "the ones who falsely believe that they have special information about the future price of risky assets". The main difference between the two definitions is that in the latter, noise trading is also the result of a maximizing behaviour and thus is not infinitely inelastic with respect to prices. As Shleiffer and Summers explain, there is an implicit assumption behind these two definitions:

"These demand shifts will only matter if they are correlated across noise traders. If all investors trade randomly, their trades cancel out and there are no aggregate shifts in demand. Undoubtedly, some trading in the market brings together noise traders with different models who cancel each other out. However, many trading strategies based on pseudo-signals, noise, and popular models are correlated, leading to aggregate demand shifts. The reason for this is that judgement biases afflicting investors in processing information tends to be the same. Subjects in psychological experiments tend to make the same mistake; they do not make random mistakes."

In dynamic models, irrational demand may also be modeled as feedback trading: traders' demand at time  $t$  is a function of the price variation between  $t-2$  and  $t-1$ <sup>3</sup>.

As Grossman and Tirole's results led to fully revealing equilibria, several papers focused on the aggregation of information in noisy rational expectation economies. Hellwig (1980), Diamond and Verecchia (1981) concluded that as long as the number of informed traders is finite, private information has an impact on equilibrium prices. Therefore rational traders should take this effect into account when computing their asset demands and then act as imperfect competitors. The only situation where competitive behaviour is rational is where it is assumed that there is a continuum of informed speculators. The main contribution under the alternative assumption of imperfect competition is due to Kyle (1989) but once more it is mainly informational properties that are highlighted.

As far as we know, only few papers have focused on traders' welfare. Under perfect competition assumptions, Laffont (1985) has demonstrated that partially revealing rational expectation equilibria are not Pareto optimal but that fully revealing REE are. Under the assumption that traders act strategically, Stein (1987) wondered if "more informed speculation was better than less". In other words, we would like to know if informed speculators prefer to trade in large markets rather than in small ones, given that they do not have perfect information, and so they would learn more

---

<sup>3</sup> See for example De Long, Shleifer, Summers and Waldmann (1990b) and Sentana and Wadhvani (1992).

information from prices in a large market.

The goal of this paper is to try to answer partially this question, when considering a market with heterogeneously informed speculators. We will proceed as follows. In section 2, Kyle's model and main properties will be presented. Within this framework, in section 3 some welfare results in a market with both noise traders and heterogeneously informed traders will be given. In section 4, speculators' welfare will be analyzed when they trade in a purely speculative market and are endowed with both private information and a random amount of the risky asset traded.

## II KYLE'S MODEL

### a) Presentation of the model:

One risky asset is traded at a clearing price  $p$ . After trade occurs the liquidation value  $v$  is realized.  $v$  is normally distributed with mean zero and variance  $\tau_v^{-1}$ .

Three kinds of traders participate in the market: noise traders, informed speculators and uninformed speculators.

Noise traders trade in aggregate an exogeneous random quantity  $z$ , normally distributed with mean zero and variance  $\sigma_z^2$ .

There are  $N$  informed speculators, indexed  $n=1, \dots, N$ . Each trader is endowed with a signal  $y_n = v + e_n$ , where  $e_1, \dots, e_N$  are normally and independently distributed with mean zero and variance  $\tau_e^{-1}$  and are independent of  $v$  and  $z$ . Each trader has an exponential utility function and choose a demand schedule  $K_n(\cdot, y_n)$ . Given  $p$ , the quantity traded is  $k_n = K_n(p, y_n)$ .

Then the utility can be written  $U_n = -\exp(-b_i \Pi_{in})$  where  $b_i$  is the constant absolute aversion and  $\Pi_{in} = (v-p)k_n$ .

There are  $M$  uninformed speculators, indexed  $m=1, \dots, M$ . They have an exponential utility function and choose a demand schedule  $X_m(\cdot)$ . Given  $p$ , the quantity traded is  $x_m = X_m(p)$ .

Then the utility can be written  $V_m = -\exp(-b_{im} \Pi_{im})$  where  $b_{im}$  is the constant absolute risk aversion and  $\Pi_{im} = (v-p)x_m$ .

All speculators have non stochastic initial endowment which are normalized to zero.

We define a Rational Expectation Equilibrium of Imperfect Competition as vectors of strategies  $K = (K_1, \dots, K_N)$ ,  $X = (X_1, \dots, X_M)$  and a random variable  $p$  such that the following three conditions hold:

1: For all  $n=1, \dots, N$ , and for any alternative vector of strategies  $X'$  differing from  $X$  only in the  $n$ th component  $X_n$ , the strategy  $X$  yields no less utility than  $X'$ :

$$E[U_n((v-p(X,K))x_n(X,K))] \geq E[U_n((v-p(X',K))x_n(X',K))]$$

2: For all  $m=1, \dots, M$ , and for any alternative vector of strategies  $K'$  differing from  $X$  only in the  $m$ th component  $K_m$ , the strategy  $K$  yields no less utility than  $K'$ :

$$E\{U_m((v-p(X,K))k_m(X,K))\} \geq E\{U_m((v-p(X,K'))k_m(X,K'))\}$$

3: Markets clear with probability one

$$\Sigma K_n + \Sigma X_m = z$$

However, we will consider a particular category of equilibria.

**Definition:** A symmetric linear equilibrium is an equilibrium in which the strategies  $K_n$  ( $n=1, \dots, N$ ) are identical linear functions and strategies  $X_m$  ( $m=1, \dots, M$ ) are identical linear functions. Thus, there exist constants  $\beta$ ,  $\gamma_i$ ,  $\gamma_U$ ,  $\theta_i$ ,  $\theta_U$  such that (for all  $n=1, \dots, N$  and  $m=1, \dots, M$ ) the strategies  $K_n$  and  $X_m$  can be written:

$$(1) \quad K_n(p, \bar{y}_n) = \theta_i - \beta \bar{y}_n - \gamma_i p \quad X_m(p) = \theta_U - \gamma_U p$$

b) Existence and Uniqueness of a symmetric linear REE:

Given the market equilibrium condition:

$$(2) \quad \sum_{n=1}^N K_n(p, \bar{y}_n) + \sum_{m=1}^M X_m(p) + \bar{z} = 0$$

we can compute  $Var(v|p)$  and  $Var(v|p, y_n)$ .

$$(3) \quad \tau_U = Var^{-1}(\bar{v} | \bar{p}) \quad \tau_i = var^{-1}(\bar{v} | \bar{p}, \bar{y}_n)$$

then

$$(4) \quad \tau_U = \tau_v + \phi_U N \tau_e \quad \tau_i = \tau_v + \tau_e + \phi_i (N-1) \tau_e$$

where

$$(5) \quad \phi_U = \frac{N\beta^2}{N\beta^2 + \sigma_e^2 \tau_e} \quad \phi_i = \frac{(N-1)\beta^2}{(N-1)\beta^2 + \sigma_e^2 \tau_e}$$

proof: See Kyle (1989), appendix A.

$\phi_U$  and  $\phi_i$  are parameters of measuring the informational efficiency. Prices become fully revealing when the paramaters are equal to one.

From these results, we can compute  $E(v|p)$  and  $E(v|p, y_n)$ :

$$(6) \quad E(\bar{v} | \bar{p}) = \frac{\phi_U \tau_e}{\beta \tau_U} [(N\gamma_i + M\gamma_U)\bar{p} - N\theta_i - M\theta_U]$$



$$(7) \quad E(\tilde{v} \mid \bar{p}, \bar{y}_n) = \frac{(1-\phi_1)\tau_c}{\tau_I} \bar{y}_n + \frac{\phi_1\tau_c}{\beta\tau_I} [(N\gamma_I + M\gamma_U)\bar{p} - N\theta_I - M\theta_U]$$

Proof: see Kyle (1989), appendix A.

We now compute the asset demand of the trader. From equations (1) and (2), we can write:

$$(8) \quad \bar{p} = \bar{p}_{in} + \lambda k_n$$

Then Kyle demonstrates the following lemma:

**Lemma 1:** Assume  $p_{in}$ ,  $v$  and  $y_n$  are jointly normally distributed. Let  $a_1$ ,  $a_2$ ,  $a_3$ , and  $\tau^*$  be constant such that:

$$(10) \quad E(\tilde{v} \mid \bar{p}_{in}, \bar{y}_n) = a_1 \bar{p}_{in} + a_2 \bar{y}_n + a_3 \quad (9), \quad \text{Var}^{-1}(\tilde{v} \mid \bar{p}_{in}, \bar{y}_n) = \tau^*$$

Let  $k_n^*$  denote the maximizing quantity and  $p^*$  be the maximizing price and assume that the second order condition  $2\lambda_1 + b_1 \text{Var}(v \mid p_{in}, y_n) > 0$  holds:

- If  $\lambda_1(1+a_1) + b_1/\tau^* \neq 0$ , then  $p_{in}$  can be expressed as a function of  $y_n$  and  $p$ , and  $k_n^*$  as a function of  $y_n$  and  $p^*$  as follows:

$$(11) \quad \bar{k}_n^* = \frac{E(\tilde{v} \mid \bar{p}^*, \bar{y}_n) - \bar{p}^*}{\lambda_1 + b_1/\tau^*}$$

- If  $\lambda_1(1+a_1) + b_1/\tau^* = 0$ , then  $p_{in}$  cannot be expressed as a function of  $y_n$ . The demand schedule gives  $p^*$  as a function of  $y_n$ , but not  $p_{in}$ :

$$\bar{p}^* = \frac{\lambda_1(a_2 \bar{y}_n + a_3)}{2\lambda_1 + b_1/\tau^*} \quad (12)$$

Proof: see Kyle (1989), pp 326-327.

**Theorem:** Assume  $\sigma_v^2 > 0$  and  $\tau_c > 0$ . If  $N \geq 2$  and  $M \geq 1$ , or if  $N \geq 3$  and  $M = 0$ , or if  $M \geq 3$  and  $N = 0$ , then there exists a unique symmetric linear equilibrium. If  $N = 1$  and  $M \geq 2$ , a symmetric linear equilibrium exists if  $M$  is sufficiently large (holding other parameters constant) and does not exist if  $\rho_U$  is sufficiently large (holding other parameters constant). If  $N + M \leq 2$ , a symmetric linear equilibrium does not exist.

Proof: See Kyle (1989), pp 329-330.

Kyle discussed extensively the properties of the equilibrium in the case  $N \geq 2$  and  $M \geq 1$ . An interesting result is that strictly less than a half of the private information is incorporated into the price. Thus, when the number of informed traders is large the

equilibrium price never becomes fully revealing. This means that the equilibrium does not converge to the equilibrium of perfect competition when the number of insiders become large.

However, Kyle did not focus on the welfare aspect of his results.

### III SOME WELFARE IMPLICATIONS

In this section, using Kyle's results, we will try to find out if traders are better off when the number of informed traders increases. This point is of importance since the case where only one trader has some private information is quite unrealistic. As already explained in the introduction, the problem is to know if traders with private information will trade on markets where many other traders also have private information or rather trade on markets where the number of informed traders is small. To answer this question, we compute the expected utility of a trader just after the observation of his private signal as a function of the number of traders in the market. Two cases will be studied. In the first one, the size of noise trading measured by its variance is fixed. In the second case, the size of the noise trading is proportionnal to the number of informed traders.

For simplicity, we will consider the case where all traders have some private information ( $M=0$ ), so we will drop the index  $I$ .

#### a) Characterization of the Equilibrium:

$\phi$ ,  $\tau$  are still given by equations (5) and (4) and  $E(v | p, y_n)$  is simplified:

$$(13) \quad E(\bar{v} | p, y_n) = \frac{(1-\phi)\tau_\epsilon}{\tau} y_n + \frac{N\phi\gamma\tau_\epsilon}{\beta\tau} p - \frac{N\phi\theta\tau_\epsilon}{\beta\tau}$$

Then,

$$(14) \quad K_n = -\frac{N\theta\phi\tau_\epsilon}{\beta\tau^*} + \frac{(1-\phi)\tau_\epsilon}{\tau} y_n - \frac{\beta\tau_\epsilon - N\gamma\phi\tau_\epsilon}{\beta\tau^*} p$$

$$\frac{\beta\tau^*}{(N-1)\gamma} + b\beta \quad \frac{\tau_\epsilon}{(N-1)\gamma} + b \quad \frac{\beta\tau^*}{(N-1)\gamma} + b$$

From the definition of a symmetric linear equilibrium, we have to solve the following system of equations:

$$(15) \quad \beta = \frac{(1-\phi)\tau_\epsilon}{\tau} \frac{1}{(N-1)\gamma} + b \quad \theta = -\frac{N\phi\theta\tau_\epsilon}{\tau} \frac{1}{(N-1)\gamma} + b \quad \gamma = \frac{\beta\tau - \phi N\gamma\tau_\epsilon}{\beta\tau} \frac{1}{(N-1)\gamma} + b$$

under the conditions:  $\frac{1}{(N-1)\gamma} [1 + \frac{N\gamma\phi\tau_\epsilon}{\beta\tau^*}] + b/\tau^* \neq 0$  and  $\frac{2}{(N-1)\gamma} + b/\tau^* > 0$

The solution is:

$$(16) \quad \theta=0 \quad \gamma = \frac{\beta[\tau_v + \tau_e + (N-1)\varphi\tau_e]}{\tau_e[1+(N-1)\varphi]} \quad \varphi = \frac{(N-2)}{2(N-1)} - \frac{b}{2\tau_e}\beta$$

with 
$$\varphi = \frac{(N-1)\beta^2}{(N-1)\beta^2 + \sigma_e^2\tau_e}$$

and  $\beta$  is the unique positive solution of the third equation above.

*Proof:* Rearranging (15.c) yields (16.b). From (16.b),  $\gamma$  is positive, then the only solution for (14.b) is  $\theta=0$ . Substituting (16.b) in (15.a) and rearranging yields (16.c).

We know from the previous theorem that the equilibrium is unique. (16.c) admits 3 solutions and one of them ( $\beta > 0$ ) satisfies the two inequalities. QED

**Proposition 1:** *There exists  $N^0$  such that for all  $N > N^0$   $\beta(N)$  is decreasing. Furthermore,  $\lim_{N \rightarrow \infty} \beta(N) = 0$ .*

*Proof:* see Appendix 1.

Before interpreting the proposition, it is useful to compute the equilibrium price.

Let  $\bar{y} = \frac{1}{N} \sum_{n=1}^N y_n$  then  $\sum_{n=1}^N K_n + \bar{z} = 0$  implies

$$(17) \quad p = \frac{\beta \bar{y}}{\gamma} + \frac{1}{N\gamma} \bar{z}$$

From (5) and (16),  $\tau = \tau_e + [N\tau_e - (N-1)\beta b]/2$ . Then,

$$(18) \quad \gamma = \frac{\beta[2\tau_e + N\tau_e - (N-1)\beta b]}{N\tau_e - (N-1)\beta b}$$

So,  $p$  can be rewritten as follows:

$$(19) \quad p = \left[ \frac{N\tau_e - (N-1)\beta b}{\beta[2\tau_e + N\tau_e - (N-1)\beta b]} \right] \left( \bar{y} + \frac{1}{N\beta} \bar{z} \right)$$

Looking at the expressions of  $p$  and  $\varphi$ , we can find a possible explanation for proposition 1. When  $N$  is small, the price reveals few information but the amount of information conveyed by the signal of a supplementary trader is large ( $\varphi$  is an increasing and concave function of  $N$ ). Then, by increasing  $\beta$ , traders increase the amount of information conveyed by the signal of a supplementary trader.

Looking at equation (19), we see that when  $\beta$  increases, the relative weight of the noise trading decreases.

However, when  $N$  becomes large,  $\bar{y}$  converges to  $v$ , then, by decreasing  $\beta$ , insiders keeps the price from being too informative. We can also verify this last point by looking at the efficiency parameter  $\varphi$ . If  $\beta$  is constant,  $\varphi$  converges to 1, when  $N$



increases ( prices become fully revealing), then, by decreasing  $\beta$ , insiders decrease the information conveyed by prices.

b) Expected utility:

We now compute the expected utility of an informed trader, just after having observed  $y_n$ , as a function of  $N$  and  $y_n$ . From the assumptions of normality and exponential utility the expected utility is equivalent to:

$$(20) \quad [E(\tilde{v} \mid p, \tilde{y}_n) - p]K_n - \frac{b}{2}K_n^2\tau^{-1}$$

From the existence theorem of the previous subsection, if  $N \geq 3$  then  $K_n$  is still given by equation (11). So  $U_n$  can be rewritten:

$$(21) \quad U_n = \frac{[E(\tilde{v} \mid p, y) - p]^2}{\left[ \frac{1}{(N-1)\gamma} + \frac{b}{\tau} \right]} \left[ \frac{1}{(N-1)\gamma} + \frac{b}{\tau} \right]$$

Substituting  $p$  and  $\tau$  in the previous equation yields

$$(22) \quad U_n = \frac{N\beta\tau_e}{(N-1)[2\tau_v + N\tau_e - (N-1)b\beta]} [(y_n - \bar{y}) - \frac{z}{N\beta}]^2$$

We can now compute the expected utility of a trader just after he has observed his signal. Using

$$E(\bar{y} \mid y_n) = \frac{y_n}{N(\tau_v + \tau_e)} (\tau_v + N\tau_e)$$

$$E(\bar{y}^2 \mid y_n) = \frac{y_n^2}{N^2(\tau_v + \tau_e)^2} (\tau_v + N\tau_e)^2 + \frac{(N-1)(2\tau_e + \tau_v)}{N^2\tau_v\tau_e(\tau_v + \tau_e)}$$

It follows that

$$(23) \quad E(U_n \mid y_n) = \frac{\beta\tau_e}{2\tau_v + N\tau_e - (N-1)b\beta} \left[ \frac{N(N-1)}{(N-1)^2} \frac{\tau_v^2}{(\tau_v + \tau_e)^2} y_n^2 + (N-1) \frac{2\tau_e + \tau_v}{\tau_v\tau_e(\tau_v + \tau_e)} + \frac{\sigma_z^2}{\beta^2} \right]$$

**Proposition 2:** *There exists  $N_1$  such that for all  $N > N_1$   $E(U_n \mid y_n)$  is decreasing. Furthermore  $\lim_{N \rightarrow \infty} E(U_n \mid y_n) = 0$ .*

**Proof:** See appendix 1.

**Interpretation:** When  $N$  is small, the expected utility of profits is low because the risk that noise traders "drive" the price far from the fundamental value is high. When  $N$

increases, the noise trading risk decreases and leads to an increase of the expected utility of profits. However, when  $N$  becomes large ( $>N_1$ ), the probability that an informed trader trades against another informed trader, instead of trading against a noise trader, is high and then the probability to make profits based on the private information decreases. It follows that the expected utility of profits decreases.

Then it would be interesting to find out what happens when the amount of noise trading increases proportionally to the number of informed traders. So, now we will assume the variance of  $z$  is  $N\sigma_z^2$ .

Equations (14), (15) and (16) remain unchanged as functions of  $\phi$ , but we have now,

$$(25) \quad \phi = \frac{(N-1)\beta^2}{(N-1)\beta^2 + N\sigma_z^2\tau_v}$$

**Proposition 3:** For all  $N > 1 + 1/(2\sigma_z^2\tau_v)$ ,  $\beta$  is an increasing function of  $N$  and  $\lim_{N \rightarrow \infty} \beta \leq (\sigma_z^2\tau_v)^{1/2}$ .

Proof: see appendix 2.

Interpretation: When the number of insiders increases, the price is affected in two ways. First, the risk of divergence from the fundamental value increases (noise trading effect). Second,  $\bar{y}$  becomes a better estimator of  $v$  (information effect).

(i) If  $\sigma_z^2 > 1/(4\tau_v)$ , then for all  $N > 3$ ,  $\beta$  is increasing. The noise trading effect always dominates the information effect. Increasing  $\beta$ , traders reduce the relative weight of the noise trading in the price and increase the relative weight of the information.

(ii) If  $\sigma_z^2 < 1/(4\tau_v)$ , then when  $N$  is small  $\beta$  is decreasing. The information effect dominates the noise trading effect. As soon as  $N > 1 + 1/2\sigma_z^2\tau_v$ , the noise trading effect dominates the information effect and  $\beta$  increases.

**Corollary:**  $\phi < 1/3$

Proof:  $\phi$  is an increasing function of  $\beta$ .  $\beta \leq (\sigma_z^2\tau_v)^{1/2}$  then  $\phi \leq (N-1)/3N < 1/3$ . QED.

The expected utility of an informed trader after the observation of his private signal is now:

$$(25) \quad E[U_n | y_n] = \frac{\beta\tau_v}{N(N-1)} \left[ (N-1)^2 \frac{\tau_v^2}{(\tau_v + \tau_e)^2} y_n^2 + \frac{(N-1)\tau_v + 2\tau_e}{\tau_v\tau_e} y_n + \frac{N\sigma_z^2}{\beta^2} \right]$$

**Proposition 4:** There exists  $N'$  such that for all  $N > N'$   $E(U_n | y_n)$  is decreasing. Furthermore  $\lim_{N \rightarrow \infty} E(U_n | y_n) = 0$ .

Proof: see appendix 2.

As in the previous case, there exists an optimal number of informed trader independent the private signals.

However, a large range of simulation have been performed and results show that, except for a set of signals of small measure,  $E[U_n | y_n]$  is decreasing for all  $N \geq 3$ .

To explain this result, let us write  $E[U_n | y_n]$  as follows:  $E[U_n | y_n] = A(N)y_n^2 + B(N)$   
The following observations have been made:

- For all vectors of parameters  $(b, \tau_v, \tau_v, \sigma_z)$  and for all  $N \geq 3$ ,  $B(N)$  is decreasing.
- There exists vectors  $(b, \tau_v, \tau_v, \sigma_z)$ , such that for all  $N \geq 3$ ,  $A(N)$  is decreasing.
- For all vectors  $(b, \tau_v, \tau_v, \sigma_z)$  such that  $A(N)$  admits a maximum  $A(N^*)$  with  $N^* > 3$ ,  $E[U_n | y_n]$  is always decreasing except for large values of  $y_n^2$ :  $y_n^2$  larger than 5 standard errors. The probability for such signals is less than  $10^{-5}$ , so this set of signals can be neglected.

A possible explanation is that noise traders dominate the market and that the domination increases with  $N$ . Then, despite their informational advantage, insiders, when their number increases, cannot keep the price from diverging far from the fundamental value. As a consequence, informed traders' expected utility of profit is decreasing.

c) Comparison with other results:

Pagano (1989) considered a model similar to Kyle's: traders choose linear strategies and act as imperfect competitors. However, only speculators participate in the market. Traders, instead of being endowed with private information, are endowed with a random number of shares of the risky asset ( $z_n$ ) where, for all  $n=1, \dots, N$ ,  $z_n$  is distributed with mean zero and variance  $\sigma_z^2$ .

So, as in the modified version of Kyle's model, the noise in this model increases proportionally to the number of traders.

The expected utility of a trader just after  $z_n$  has been observed is:

$$E[U_n | z_n] = \frac{b}{2\tau_v} \left[ \frac{(N-2)\sigma_z^2}{N(N-1)} - \frac{2z_n^2}{N} \right]$$

Pagano establishes the conditions under which equilibrium implies the concentration of all traders on the same market.

Suppose that each agent can choose between market A and market B. The choice between the two markets is open to each agent only ex ante: having selected one of them, he can trade only on that market.

Let  $N_A$  and  $N_B$  denote the number of agents expected to trade on market A and B and by  $S_A$  and  $S_B$  the corresponding sets of agents.

A trader  $n$  will choose market A if:



$$\frac{(N_A-2)\sigma_z^2}{(N_A-1)N_A} - \frac{2z_n^2}{N_A} > \frac{(N_B-2)\sigma_z^2}{(N_B-1)N_B} - \frac{2z_n^2}{N_B}$$

which is equivalent to

$$\Delta_{AB,n} = \sigma_z^2 \left[ \frac{N_A-2}{N_A(N_A-1)} - \frac{N_B-2}{N_B(N_B-1)} \right] - 2z_n^2 \left( \frac{1}{N_A} - \frac{1}{N_B} \right) > 0$$

Pagano defines a Two Market Conjectural Equilibrium (TMCE) as one in which the conjecture of agent about the number of agents trading on the two markets ( $N_A, N_B$ ) are fulfilled in equilibrium.

**Proposition (Pagano, p 262):** *In the absence of differential transaction costs, no TMCE exists, unless the two markets are identical ( $N_A=N_B$ ).*

Proof: see Pagano p 269.

Pagano comments this proposition as follows:

"Thus, if trade is costless, all traders tend to concentrate on a single market. In equilibrium two markets can only coexist in the knife-edge case where they are identical. This equilibrium however is not robust since a slight perturbation in conjectures is sufficient to revert the economy to the one-market equilibrium. For instance, if people conjectured that on market A they would find at least one more trader than in market B, they would all converge on market A and the other would disappear."

In the modified version of Kyle's model there may exist a TMCE with  $N_A \neq N_B$  if the two markets are informationally isolated.

**Proposition 5:** *In the absence of differential transaction costs, for any vector of parameters ( $b, \tau_v, \tau_w, \sigma_z$ ) such that  $N' \leq 3$ , there exists a two markets informationally isolated conjectural equilibrium with  $N_A = N_B + 1$ .*

Proof: As  $N' \leq 3$ , for all  $n=1, \dots, N$ ,  $E[U_n | y_n]$  is decreasing. Then no trader in  $S_B$  has an incentive to trade on market A. Traders in  $S_A$  are indifferent between the two markets and they trade on market A.

Expectations are fulfilled. As a consequence, a TMCE exists.

Q.E.D.

So, in this model, the two market equilibrium is robust in the sense that traders will never concentrate on a single market.

#### IV A PURELY SPECULATIVE MARKET

One can wonder which result, Pagano's proposition or proposition 5, will be verified if we consider a purely speculative market, i.e. a market in which only heterogeneously informed speculators trade.

In the original version of Kyle's model, it is assumed that traders' initial wealth is zero. We now assume that each trader  $n$  ( $n=1, \dots, N$ ) is endowed with a random amount of risky asset  $z_n$  (where  $z_n$  is distributed with mean zero and variance  $\sigma_z^2$ ) and is endowed with a fixed amount of risk free bond with return normalized to 1.

First, we need to redefine a symmetric linear equilibrium.

**Definition:** A symmetric linear equilibrium is an equilibrium in which the strategies  $K_n$  ( $n=1, \dots, N$ ) are identical linear functions. Thus there exist constants,  $\beta$ ,  $\delta$ ,  $\gamma$ , and  $\theta$  such that:

$$(26) \quad K_n = \theta + \beta y_n + \delta z_n - \gamma p$$

a) Existence and uniqueness of an equilibrium:

**Proposition 6:** Under the assumption that traders are endowed with a random amount of risky asset

$$\tau = \text{Var}^{-1}(\tilde{v} \mid p, y_n, z_n) = \tau_v + \tau_e + (N-1)\varphi\tau_e$$

with

$$(27) \quad \varphi = \frac{\beta^2}{\beta^2 + (1-\delta)^2 \sigma_z^2 \tau_e}$$

$$(28) \quad E(\tilde{v} \mid p, y_n, z_n) = \frac{(1-\varphi)\tau_e}{\tau} y_n + \frac{\varphi\tau_e}{\beta\tau} [N\gamma p - N\theta + (1-\delta)z_n]$$

Proof: See Appendix 3.

Since  $\delta$  is endogeneous, we can see that traders can act strategically on the amount of noise in the model. Traders can influence the amount of information revealed by price via two variables: the sensitivity of demand to private information ( $\beta$ ) and the sensitivity of demand to endowment ( $\delta$ ).

We also need to adapt lemma 1 to the new assumption.

**Lemma 2:** Assume that  $p = p_n + \lambda K_n$  and that  $p_n, v, y_n$  are jointly normally distributed for all  $n=1, \dots, N$ . Let  $a_1, a_2, a_3, a_4$  and  $\tau^*$  be constant such that

$$E(\tilde{v} \mid \tilde{p}_n, \tilde{y}_n, \tilde{z}_n) = a_1 \tilde{p}_n + a_2 \tilde{y}_n + a_3 \tilde{z}_n + a_4 \quad \text{Var}^{-1}(\tilde{v} \mid \tilde{p}_n, \tilde{y}_n, \tilde{z}_n) = \tau^*$$

Let  $K_n^*$  denote the maximizing quantity and  $p^*$  be the maximizing price and assume that the second order condition  $2\lambda + b\text{Var}(\tilde{v} \mid \tilde{p}_n, \tilde{y}_n, \tilde{z}_n) > 0$  holds:

Case 1: If  $\lambda(1+a_1) + b/\tau^* \neq 0$  then

$$(31) \quad K_n^* = \frac{(a_1-1)p^* + a_2y_n + (a_3+\lambda)z_n + a_4}{\lambda(1+a_1)+b/\tau^*} = \frac{E(\bar{v} \mid p^*, y_n, z_n) - p^* + \lambda z_n}{\lambda + b/\tau}$$

Case 2: If  $\lambda(1+a_1)+b/\tau^*=0$  then

$$(32) \quad p^* = \frac{\lambda[a_2y_n + (a_3+\lambda)z_n + a_4]}{2\lambda + b/\tau^*}$$

Proof: We proceed as Kyle did to prove lemma 1.

Trader n's final wealth is

$$(33) \quad W_n = (v-p)K_n + pz_n + B_0$$

Let  $I_n = (p_n, y_n, z_n)$ , then to maximize  $U_n$  is equivalent to maximize

$$(34) \quad [E(\bar{v} \mid I_n) - p]K_n + pz_n + B_0 - \frac{b}{2}K_n^2 \text{Var}(\bar{v} \mid I_n) = E(\bar{v} \mid I_n) - p_n - \lambda K_n K_n + (p_n + \lambda K_n)z_n + B_0 - \frac{b}{2}K_n^2/\tau^*$$

The first order condition for utility maximization yields

$$(35) \quad E(\bar{v} \mid I_n) - p_n + \lambda z_n - (2\lambda + b/\tau^*)K_n = 0$$

Assuming that the second order condition holds, we can write

$$(36) \quad K_n^* = \frac{E(\bar{v} \mid I_n) - p_n + \lambda z_n}{2\lambda + b/\tau^*}$$

$$p^* = p_n + \lambda K_n^*$$

$$(37) \quad K_n^* = \frac{(a_1-1)p_n + a_2y_n + (a_3+\lambda)z_n + a_4}{2\lambda + b/\tau^*}$$

The rest of the proof is similar to Kyle's demonstration with the term  $(a_3+\lambda)z_n + a_4$  substituting  $a_3$  in the numerator of  $K_n^*$ . QED

**Proposition 7:** Assume  $\tau_e^{-1} > 0$  and  $\sigma_z^2 > 0$ . If  $N > 2$  and  $\tau_e < \frac{(N-2)b^2\sigma_z^2}{N}$ , there exists a unique symmetric linear equilibrium. This equilibrium is such that

$$(38) \quad \theta = 0 \quad \beta = \left( \frac{N-2}{N-1} - \frac{2\tau_e}{\tau_e + b^2\sigma_z^2} \right) \frac{\tau_e}{b} \quad \delta = \frac{1}{N-1} + \frac{2\tau_e}{\tau_e + b^2\sigma_z^2} \quad \gamma = \frac{(N-2)\beta\tau}{(N-1)(\beta b + N\phi\tau_e)}$$

with



$$(39) \quad \varphi = \frac{\tau_e}{\tau_e + b^2 \sigma_z^2}$$

Proof: From the market equilibrium condition  $\sum_{n=1}^N K_n = \sum_{n=1}^N z_n$ , we can write

$$(40) \quad (N-1)\gamma p = K_n + (N-1)\theta + \beta \sum_{j \neq n} y_j + \delta \sum_{j \neq n} z_j - \sum_{i=1}^N z_i$$

$$\text{Let } \lambda = \frac{1}{(N-1)\gamma} \text{ and } p_n = \frac{1}{(N-1)\gamma} [(N-1)\theta + \beta \sum_{j \neq n} y_j + \delta \sum_{j \neq n} z_j - \sum_{i=1}^N z_i]$$

It follows that

$$(41) \quad p = p_n + \lambda K_n$$

Step 1: let us assume that case 1 of lemma 2 holds. It follows that

$$(42) \quad K_n = \frac{E(\bar{v} | p, y_n, z_n) - p + \frac{z_n}{(N-1)\gamma}}{\frac{1}{(N-1)\gamma} + b/\tau}$$

Putting (28) into (42) yields

$$(43) \quad K_n = -\frac{N\varphi\tau_e\theta}{\beta\tau + \beta b} + \frac{(1-\varphi)\tau_e}{\tau + b} y_n + \frac{(N-1)\gamma\varphi\tau_e(1-\delta) + \beta\tau}{\beta\tau + (N-1)\gamma\beta b} z_n - \frac{\beta\tau - N\gamma\varphi\tau_e p}{\beta\tau + \beta b}$$

As by definition  $K_n = \theta + \beta y_n + \delta z_n - \gamma p$ , we have to solve the following system of equations

$$(44.a) \quad \theta = -\frac{N\varphi\tau_e\theta}{\beta\tau + \beta b} \quad (44.b)$$

$$\beta = \frac{(1-\varphi)\tau_e}{\tau + \beta b}$$

$$(44.c) \quad \gamma = \frac{\beta\tau - N\gamma\varphi\tau_e}{\beta\tau + \beta b} \quad (44.d)$$

$$\delta = \frac{(N-1)\gamma\varphi\tau_e(1-\delta) + \beta\tau}{\beta\tau + (N-1)\gamma\beta b}$$

(44.c) implies

$$(45) \quad (N-1)\gamma = \frac{(N-2)\beta\tau}{\beta b + N\varphi\tau_e}$$

(44.b) and (45) imply

$$(46) \quad 2\varphi\tau_e = \frac{N-2}{N-1}\tau_e - \beta b$$

(44.d) and (45) imply

$$(47) \quad 2\varphi\tau_e = \frac{\beta b[(N-1)\delta - 1]}{(N-1)(1-\delta)}$$

So, from (46) and (47)

$$(48) \quad 1 - \delta = \beta b/\tau_e$$

Putting (48) into (27) yields

$$(49) \quad \varphi = \frac{\tau_e}{\tau_e + b^2\sigma_z^2}$$

Then (49) and (45) imply

$$(50) \quad \beta = \left[ \frac{N-2}{N-1} - \frac{2\tau_e}{\tau_e + b^2\sigma_z^2} \right]$$

and (49) and (50) imply

$$(51) \quad \delta = \frac{1}{N-1} + \frac{2\tau_e}{\tau_e + b^2\sigma_z^2}$$

If  $\beta$  is positive then the only solution to (44.a) is  $\theta=0$ . If  $\beta \leq 0$  then  $\theta$  is indeterminate.

We now look for the conditions under which the two inequalities are satisfied.

From (45), if  $\beta$  is strictly positive then  $\gamma$  is strictly positive, so  $[(N-1)\gamma]^{-1} + b/\tau > 0$ .

From (50), if  $N < 2$  or if  $N > 2$  and  $\tau_e > \frac{(N-2)b^2\sigma_z^2}{N}$ ,  $\beta < 0$ .

It follows that if  $N > 2$  and  $\tau_e < \frac{(N-2)b^2\sigma_z^2}{N}$ ,  $\beta > 0$ .

From the expression of  $E(v | p, y_n, z_n)$  and  $p = p_n + \lambda K_n$ , if  $\gamma$  is strictly positive, the coefficient of  $p_n$  in  $E(v | p, y_n, z_n)$  is strictly positive. Then, As  $b/\tau > 0$ , it follows that

$$(52) \quad \frac{(1+a_1)}{(N-1)\gamma} + b/\tau \neq 0$$

---

<sup>4</sup> Coefficient  $a_1$  in lemma 2.

Then if  $N > 2$  and  $\tau_e < \frac{(N-2)b^2\sigma_z^2}{N}$ , the two constraints are satisfied.

Step 2: Assume that case 2 of lemma 2 holds.

It means that we have  $2\lambda + b\text{Var}(\bar{v} | \bar{p}_n, \bar{y}_n, \bar{z}_n) > 0$  and  $\lambda(1+a_1) + b/\tau_e = 0$

It follows that

$$(53) \quad p = \frac{\lambda}{2\lambda + b/\tau_e} [a_2 y_n + (a_3 + \lambda) z_n + a_4] \quad \text{for any } y_n \text{ and } z_n.$$

The only solution is  $a_2 = 0$ ,  $a_3 = -\lambda$  and  $a_4 = 0$ .

$a_2 = 0$  means that  $y_n$  gives no information given  $p_n$  and  $z_n$ . Given the expression of  $p_n$ , the only possibility is  $p_n = v$ . This solution is impossible if  $\tau_e^{-1} > 0$  and  $\sigma_z^2 > 0$ .

Step 3: Assume that the second order condition of profit maximization does not hold. This implies that

$$(54) \quad E(v | y_n, z_n, p_n) - p_n + \lambda z_n = 0$$

which is equivalent to

$$(55) \quad (a_1 - 1)p_n + a_2 y_n + (a_3 + \lambda)z_n + a_4 = 0 \quad \text{for any } y_n \text{ and } z_n.$$

The only solution is  $a_1 = 1$ ,  $a_2 = 0$ ,  $a_3 = -\lambda$  and  $a_4 = 0$ . This solution is impossible.

So, there exists a symmetric linear equilibrium. Q.E.D

**Corollary 2:**  $\varphi$  is independent of  $N$  and  $\varphi < 1/2$ .

Proof: Equation (49) states that  $\varphi$  is independent of  $N$  and that  $\varphi$  is an increasing function of  $\tau_e$ . Then, from the condition of existence of an equilibrium,

$$\varphi < \frac{N-2}{2(N-1)} < \frac{1}{2} \quad \text{Q.E.D.}$$

The proposition states that in a purely speculative market, for any given number of traders and variance of endowment, there exists a critical level for the precision of the information beyond which an equilibrium does not exist any more. We can understand this by looking at equation (49). If  $\varphi$  is too large then price become too informative.

b) Traders' welfare:

From (52) and the market equilibrium condition, it follows that



$$(53) \quad p = \frac{\beta \bar{y}}{\gamma} + \frac{(\delta - 1) \bar{z}}{\gamma}$$

plunging (48) into (53) yields

$$(54) \quad p = \frac{\beta}{\gamma} (\bar{y} - \frac{b}{\tau_e} \bar{z})$$

then

$$(55) \quad K_n = \beta (y_n - \bar{y}) + (1 - \frac{\beta b}{\tau_e})(z_n - \bar{z}) + \bar{z}$$

$$(56) \quad U_n = [E(v | y_n, p, z_n) - p] K_n + p z_n - \frac{b}{2\tau} K_n^2 = [E(v | y_n, p, z_n) - p - \frac{b}{2\tau} K_n] K_n + p z_n$$

Substituting parameters  $\beta, \delta$  and  $\gamma$  for their value yields

$$(57) \quad U_n = \left[ \frac{N\tau_e}{2(N-1)\tau} (y_n - \bar{y}) - \frac{b}{2(N-1)\tau} (z_n - \bar{z}) + \frac{b}{2\tau} \bar{z} \right] \left[ \beta (y_n - \bar{y}) + (1 - \frac{\beta b}{\tau_e})(z_n - \bar{z}) + \bar{z} \right] + \frac{\beta}{\gamma} (\bar{y} - \frac{b}{\tau_e} \bar{z}) z_n$$

We now compute  $E(U_n | y_n, z_n)$

$$(58) \quad E(U_n | y_n, z_n) = \frac{y_n^2}{2b\tau c} \frac{\tau_e^2 \tau_v^2}{(\tau_e + \tau_v)^2} \left[ \frac{N-2}{N} b^2 \sigma_z^2 - \tau_e \right] - \frac{z_n^2}{\tau c} \frac{b}{(N-1)\tau_e} \left[ \tau_e + \frac{b^2 \sigma_z^2}{N} \right] + \frac{y_n z_n}{2\tau c} \left[ \frac{2b^2 \sigma_z^2}{N} (\tau_v + \frac{\tau_e}{N-1} (\tau_v + N\tau_e)) + \tau_e \left( \frac{N+1}{N} \tau_v + \frac{2}{N-1} (\tau_v + N\tau_e) \right) \right] + \frac{1}{(N-1)\tau} \left[ \frac{N-2}{N} b^2 \sigma_z^2 - \tau_e \right] \left[ \frac{1}{2bc} \frac{(2\tau_v + \tau_e)\tau_e}{(\tau_v + \tau_e)^2 \tau_v} + b^2 \sigma_z^2 \right]$$

with  $\tau = \tau_v + \tau_e + (N-1)\varphi\tau_e$  and  $c = \tau_e + b^2\sigma_z^2$

**Proposition 8:** *When traders are heterogeneously informed and heterogeneously endowed with shares of stock, no TMCE exists unless the two markets are identical.*

Proof: Assume that a TMCE exists. It implies that for any trader  $n$ , the choice between market A and market B is independent of the realizations  $y_n$  and  $z_n$  since on both markets endowments and informations are realizations of normally distributed variables with variances  $\sigma_z^2$  and  $\sigma_y^2$  given.

Then it is sufficient to demonstrate that for some sets of realizations of  $y_n$  and  $z_n$ ,

traders have an expected utility that is an increasing function of  $N$ . For those traders, the optimal strategy is to concentrate in one single market.

Let

$$(59) \quad A(x) = \frac{1}{2b\tau c} \frac{\tau_e^2 \tau_v^2}{(\tau_e + \tau_v)^2} \left[ \frac{x-2}{x} b^2 \sigma_z^2 - \tau_e \right]$$

$$(60) \quad B(x) = \frac{1}{2\tau c} \left[ \frac{2b^2 \sigma_z^2}{N} \left\{ \tau_v + \frac{\tau_e}{x-1} (\tau_v + x\tau_e) \right\} + \tau_e \left\{ \frac{x+1}{x} \tau_v + \frac{2}{x-1} (\tau_v + x\tau_e) \right\} \right]$$

$$(61) \quad C(x) = \frac{1}{\tau c} \frac{b}{(x-1)\tau_e} \left[ \tau_e + \frac{b^2 \sigma_z^2}{x} \right]$$

$$(62) \quad D(x) = \frac{1}{(x-1)\tau} \left[ \frac{x-2}{x} b^2 \sigma_z^2 - \tau_e \right] \left[ \frac{1}{2bc} \frac{(2\tau_v + \tau_e)\tau_e}{(\tau_v + \tau_e)^2 \tau_v} + b^2 \sigma_z^2 \right]$$

and,

$$(63) \quad G_n(x) = A(x)y_n^2 + B(x)y_n z_n + C(x)z_n^2 + D(x)$$

where the function  $A(\cdot)$ ,  $B(\cdot)$ ,  $C(\cdot)$  and  $D(\cdot)$  are defined on  $[3, \infty[$ . Then

$$(64) \quad E[U_n | y_n, z_n] = G_n(N)$$

and

$$(65) \quad G'_n(x) = A'(x)y_n^2 + B'(x)y_n z_n + C'(x)z_n^2 + D'(x)$$

where  $(\cdot)'$  denotes the derivative. It is immediate that  $B'(x)$  is strictly negative. Then  $G'_n(x) > 0$  is equivalent to

$$(65) \quad \frac{A'(x)}{B'(x)} y_n^2 + y_n z_n + \frac{C'(x)}{B'(x)} z_n^2 < -\frac{D'(x)}{B'(x)}$$

Computing the functions  $A'/B', C'/B', D'/B'$  one can show that they are all continuously differentiable on  $[3, \infty[$  and that they have a finite limit. Then they are all bounded. Furthermore it is immediate that  $C'(x)$  is strictly positive.

Then, for any  $x$ , and for any interval  $I_n \subset \mathbb{R}^+$  such that  $y_n \in I_n$ , there exists  $z'(I_n)$  such that

for all  $z_n \in [z'(I_n), \infty[$ ,  $\frac{dE[U_n | y_n, z_n]}{dN} > 0$  ; and for any interval  $I_n \subset \mathbb{R}^+$  such that  $y_n \in I_n$ ,

there exists  $z_-(I_n)$  such that for all  $z_n \in ]-\infty, z_-(I_n)[$ ,  $\frac{dE[U_n | y_n, z_n]}{dN} > 0$  .

Then there exists some sets  $S_y$  and  $S_z$  of strictly positive measure such that for any  $y_n \in S_y$  and  $z_n \in S_z$  the expected utility of traders is increasing with  $N$ . Q.E.D.

## V CONCLUSION

This paper highlights some of the differences between purely speculative markets in which noise is added to price by endowment shocks and markets with both informed speculators and noise traders:

- On the existence of an equilibrium: In a purely speculative market, for any given number of speculators in the market, there exists a critical level for the precision of the information beyond which a symmetric linear equilibrium does not exist any more. This results from the fact that speculators can act strategically on the amount of noise added to price.

In a market with both insiders and noise traders, a symmetric linear equilibrium always exists when there are more than two insiders.

- On traders' welfare: Propositions 2 and 4 establish that in a market with noise traders, for any signal, there exists an optimal number of traders in the market.

In a purely speculative market, we can see from the proof of proposition 8 that, for some endowments  $z_n$  and a signal  $y_n$ , traders have an increasing expected utility and some traders with the same endowment but a different signal have a decreasing expected utility.

As a consequence and as proposition 5 and proposition 8 show, in the absence of differential transaction costs, if ex-ante informed can choose between one large on two smaller informationally isolated markets, traders may choose to trade on small markets only if noise traders also participate in the market. In a purely speculative economy, informed speculators will only trade in large markets.

## APPENDIX 1

### Proof of proposition 1:

Let 
$$f(\beta, x) = \frac{(x-1)\beta^2}{(x-1)\beta^2 + \sigma_z^2 \tau_c} - \frac{(x-2)}{2(x-1)} + \frac{\beta b}{2\tau_c} \quad \text{where } x \in [3, \infty[.$$

We consider the implicit function  $f(\beta, x) = 0$ .

$$\frac{d\beta}{dx} = - \frac{\partial f / \partial x}{\partial f / \partial \beta}, \quad \frac{\partial f}{\partial \beta} = \frac{2(x-1)\beta\sigma_z^2\tau_c}{[(x-1)\beta^2 + \sigma_z^2\tau_c]^2} + \frac{b}{2\tau_c}, \quad \frac{\partial f}{\partial x} = \frac{\beta^2\sigma_z^2\tau_c}{[(x-1)\beta^2 + \sigma_z^2\tau_c]^2} - \frac{1}{2(x-1)^2}$$

We can see that for all  $x$ ,  $\partial f / \partial \beta > 0$ . So  $d\beta / dx < 0$  when  $\partial f / \partial x > 0$ .  $\partial f / \partial x > 0$  is equivalent to

$$\beta^2 [2\sigma_z^2\tau_c - \beta^2] (x-1)^2 - 2(x-1)\beta^2\sigma_z^2\tau_c - \sigma_z^4\tau_c^2 > 0$$

If, for all  $x$ ,  $2\sigma_z^2\tau_c - \beta^2 > 0$ , then there exists  $x^0$  such that, for all  $x > x^0$ ,  $\partial f / \partial x > 0$ .



Let  $h_0$  be the solution of  $\frac{(x-1)h^2}{(x-1)h^2 + \sigma_z^2 \tau_e} = \frac{x-2}{2(x-1)}$  then  $h_0^2 = \frac{(x-2)\sigma_z^2 \tau_e}{x(x-1)}$

$h_0$  is such that  $\beta^2 < h_0^2 < \sigma_z^2 \tau_e$  then  $2\sigma_z^2 \tau_e - \beta^2 > 0$ .

So, there exists  $x^0$  such that for all  $x > x^0$ ,  $d\beta/dx < 0$ .

Let  $N^0 = \text{int}(x^0) + 1$  and we have the desired result.

we now compute  $\lim_{N \rightarrow \infty} \beta(N)$

$$0 < \beta^2 < \frac{N-2}{N-1} \frac{\sigma_z^2 \tau_e}{N} \text{ then, } \lim_{N \rightarrow \infty} \beta(N) = 0$$

QED.

**Proof of proposition 2:** The proof is divided in three steps.

Step 1: For all  $N > N^0$ ,  $(N\tau_e - (N-1)\beta b)$  is an increasing function of  $N$  and  $\lim_{N \rightarrow \infty} (N\tau_e - (N-1)\beta b) = \infty$

Let  $f(x) = x\tau_e - (x-1)b\beta(x)$ .

$$\frac{df}{dx} = \tau_e - b\beta - (x-1)b \frac{d\beta}{dx}$$

From the proof proposition 1, for all  $x > x^0$ ,  $-(x-1)b(d\beta/dx) > 0$ .

From equation (16.c),  $(N-2)\tau_e - (N-1)b\beta > 0$  then,  $\tau_e - b\beta > 0$ .

So, for all  $x > x^0$ ,  $d[x\tau_e - (x-1)\beta b]/dx > 0$ .

It follows that for all  $N > N^0$ ,  $(N\tau_e - (N-1)\beta b)$  is an increasing function of  $N$ .

$$\lim_{N \rightarrow \infty} \left( \frac{N}{N-1} \tau_e - \beta b \right) = \tau_e, \text{ so } \lim_{N \rightarrow \infty} (N\tau_e - (N-1)\beta b) = \infty$$

$$\text{Step 2: Let } g(N) = \frac{\beta \tau_e}{N(N-1)} \left[ (N-1)^2 \frac{\tau_v^2}{(\tau_v + \tau_e)^2} y_n^2 + \frac{(N-1) \tau_v + 2\tau_e}{\tau_v \tau_e (\tau_v + \tau_e)} + \frac{\sigma_z^2}{\beta^2} \right]$$

There exists  $N_1$  such that, for all  $N > N_1$ ,  $g(N)$  is decreasing.

Let  $g_1(x) = (x-1)\beta/x$ ,  $g_2(x) = \beta/x$  and  $g_3(x) = [x(x-1)\beta]^{-1}$  where  $x \in [3, \infty[$ .

(i) there exists  $x_2$  such that, for all  $x > x_2$ ,  $g_1(x)$  is decreasing.

$$\frac{dg_1(x)}{dx} = \frac{1}{x^2} \left[ \beta + x(x-1) \frac{d\beta}{dx} \right] \text{ so, if } \beta + x(x-1) \frac{d\beta}{dx} < 0 \text{ then } (dg_1/dx) < 0$$

For all  $x > x^0$ ,  $(d\beta/dx) < 0$  then, For all  $x > x^0$ ,  $\beta + x(x-1)(d\beta/dx) < \beta + (x-1)^2(d\beta/dx)$

$$\beta + (x-1)^2 \frac{d\beta}{dx} = \frac{(x-1)^2 \beta^2 [\beta^2 \tau_e + \beta b - 2\sigma_z^2 \tau_e^2] + 2\beta^2 \sigma_z^2 \tau_e (3+2b\beta)(x-1) + \sigma_z^4 \tau_e (b\beta - \tau_e)}{4\beta \sigma_z^2 \tau_e (x-1) + b[(x-1)\beta^2 + \sigma_z^2 \tau_e]^2}$$

As for all  $x > x^0$ ,  $\beta(x)$  is decreasing and converges to 0 when  $x$  goes to infinity, there exists  $x_3 > x^0$  such that, for all  $x > x_3$ ,  $\beta^2 \tau_e + \beta b - 2\sigma_z^2 \tau_e^2 < 0$ .

Then, there exists  $x_4 > x_3$  such that, for all  $x > x_4$ ,  $\beta + (x-1)^2(d\beta/dx) < 0$

Let  $x_2 = x_4$ , we have the desired result.

(ii) The proof of proposition 1 implies that for all  $x > x_0$ ,  $g_2(x)$  is decreasing.

$$(iii) \frac{d[x(x-1)\beta]}{dx} = (2x-1)\beta + x(x-1) \frac{d\beta}{dx} > (x-1)[\beta + x \frac{d\beta}{dx}]$$

If there exists  $x_5$  such that, for all  $x > x_5$ ,  $[\beta + x(d\beta/dx)] > 0$  then for all  $x > x_5$ ,  $g_3(x)$  will be decreasing.

The denominator of  $d\beta/dx$  is a polynomial of degree 4 and the numerator of  $x(d\beta/dx)$  is a polynomial of degree 3 that is always positive. Then the numerator of  $[\beta + x(d\beta/dx)]$  is a polynomial of degree 4 and the coefficient of  $x^4$  is  $\beta^5$ . As  $\beta > 0$ , there exists  $x_5$  such that, for all  $x > x_5$ ,  $[\beta + x(d\beta/dx)] > 0$ .

Let  $N_1 = \text{int}\{\text{Max}(x_2, x_5)\} + 1$ , we have the desired result.

We now compute  $\text{Lim}_{N \rightarrow \infty} E(U_n | y_n)$ .

$\text{Lim}_{x \rightarrow \infty} [x\tau_e - (x-1)b\beta] = \infty$ ,  $\text{Lim}_{x \rightarrow \infty} g_1(x) = 0$ ,  $\text{Lim}_{x \rightarrow \infty} g_2(x) = 0$ , and on  $[x_5, \infty[$ ,  $g_3(x)$  is upper bounded. Then  $\text{Lim}_{N \rightarrow \infty} E(U_n | y_n) = 0$ . QED.

## APPENDIX 2:

### Proof of proposition 3:

(i)  $\beta^2 < \sigma_z^2 \tau_e / 2$

Let  $X_0$  be the solution of  $\frac{(N-1)X^2}{N(X^2 + \sigma_z^2 \tau_e) - X^2} = \frac{N-2}{2(N-1)}$  where  $X \in [3, \infty[$ ,

$$\text{then } X_0^2 = \frac{N-2}{2(N-1)} \sigma_z^2 \tau_e$$

$\beta^2 < X_0^2$  then  $\beta^2 < \sigma_z^2 \tau_e / 2$ .

(ii) Let  $X_0^* = 1 + 1/2\sigma_z^2 \tau_e$ . For all  $X > X_0^*$ ,  $\beta$  is an increasing function of  $X$ .

Let  $f(\beta, X) = \frac{(X-1)\beta^2}{(X-1)\beta^2 + X\sigma_z^2\tau_e} - \frac{X-2}{2(X-1)} + \frac{b\beta}{2\tau_e}$

We consider the implicit function  $f(\beta, X) = 0$ , then  $\frac{d\beta}{dX} = -\frac{\partial f/\partial X}{\partial f/\partial \beta}$

$$\frac{\partial f}{\partial \beta} = \frac{2\beta\sigma_z^2\tau_e(X-1)}{[X(\beta^2 + \sigma_z^2\tau_e) - \beta^2]^2} + \frac{b}{2\tau_e} > 0$$

$$\frac{\partial f}{\partial X} = \frac{-X^2(\beta^4 + \sigma_z^4\tau_e^2) + X\beta^2(\beta^2 - 2\sigma_z^2\tau_e) + \beta^2(2\sigma_z^2\tau_e + 1)}{2(X-1)^2[X(\beta^2 + \sigma_z^2\tau_e) - \beta^2]^2}$$

Let  $H(X)$  be the numerator of  $\partial f/\partial X$ .  $H(X)$  admits a maximum for  $X = \frac{\beta^2 - 2\sigma_z^2\tau_e}{2(\beta^4 + \sigma_z^4\tau_e^2)} < 0$

and is always decreasing for all  $X$  larger than this value.

We can rewrite  $H(X)$  as follows:  $H(X) = -\beta^4 X(X-1) - X^2\sigma_z^4\tau_e^2 + \beta^2[1 - 2(X-1)\sigma_z^2\tau_e]$

So, if  $2(X-1)\sigma_z^2\tau_e > 1$  then  $H(X) < 0$ .  $2(X-1)\sigma_z^2\tau_e > 1$  is equivalent to  $X > 1 + 1/2\sigma_z^2\tau_e$ .

So, for all  $X > 1 + 1/2\sigma_z^2\tau_e$ ,  $d\beta/dX > 0$ . It follows that for all  $N > 1 + 1/2\sigma_z^2\tau_e$ ,  $\beta$  is decreasing with  $N$ .

We now establish that  $\lim_{N \rightarrow \infty} \beta \leq (\sigma_z^2\tau_e)^{1/2}$ .

Let  $h^0$  be the solution of  $\frac{(x-1)h^2}{(x-1)h^2 + x\sigma_z^2\tau_e} = \frac{x-2}{2(x-1)}$  then  $h^2 = \frac{(x-2)\sigma_z^2\tau_e}{(x-1)}$

$\beta^2 < h_0^2$  then,  $\lim_{N \rightarrow \infty} \beta \leq (\sigma_z^2\tau_e)^{1/2}$ .

Q.E.D.

**Proof of proposition 4:** The proof is divided in four steps.

Step1:  $N\tau_e - (N-1)b\beta$  is increasing when  $\beta$  is increasing.

From equation (24),  $N\tau_e - (N-1)b\beta = 2\tau_e[1 + (N-1)\frac{(N-1)\beta^2}{N(\beta^2 + \sigma_z^2\tau_e) - \beta^2}]$

Let us consider the function  $f_1(x) = \frac{(x-1)\beta^2}{x(\beta^2 + \sigma_z^2\tau_e) - \beta^2}$  defined on  $[3, \infty[$

When  $d\beta/dx > 0$ ,  $\frac{df_1}{dx} = \frac{\sigma_z^2 \tau_e \beta [2x(x-1) \frac{d\beta}{dx} + \beta]}{[x(\beta^2 + \sigma_z^2 \tau_e) - \beta^2]^2} > 0$

Step2: Let  $f_2(x) = \frac{(x-1)\beta\tau_e}{2\tau_v + x\tau_e - (x-1)b\beta}$ , there exists  $x_1^*$  such that, for all  $x > x_1^*$ ,  $h(x)$  is decreasing.

$$\frac{df_2(x)}{dx} < 0 \Leftrightarrow \left[ \frac{\beta}{x} + (x-1) \frac{d\beta}{dx} \right] \left[ \frac{2\tau_v}{x} + \tau_e \right] - \frac{(x-1)b\beta^2}{x^2} - \frac{(x-1)}{x} \beta(\tau_e - b\beta) < 0$$

On  $]X_0^*, \infty[$ ,  $d\beta/dx > 0$ , then  $\frac{\beta}{x} + (x-1) \frac{d\beta}{dx} > 0$  and  $\frac{x-1}{x} \beta(x) > \frac{x_0^* - 1}{x_0^*} \beta(x_0^*)$ .

Let  $A(x) = \frac{x_0^* - 1}{x_0^*} [\tau_e - b\beta(x)]$ . From the implicit function  $f(\beta, x) = 0$ , it follows that

$$\frac{N-2}{2(N-1)} - \frac{\beta b}{2\tau_e} > 0 \text{ then } \tau_e - b\beta > 0.$$

$$\Leftrightarrow \left[ \frac{\beta}{x} + (x-1) \frac{d\beta}{dx} \right] \left[ \frac{2\tau_v}{x} + \tau_e \right] - \frac{(x-1)b\beta^2}{x^2} - \frac{(x-1)}{x} \beta(\tau_e - b\beta) < \left[ \frac{\beta}{x} + (x-1) \frac{d\beta}{dx} \right] \left( \frac{2\tau_v}{x} + \tau_e \right) - A(x)$$

for all  $x > X_0^*$ .

From the fact that  $\beta$  is bounded and the expression of  $(d\beta/dx)$ , it is immediate that

$$\lim_{x \rightarrow \infty} \left[ \frac{\beta}{x} + (x-1) \frac{d\beta}{dx} \right] = 0$$

Since  $A(x) > 0$ , there exists  $x_1^*$  such that, for all  $x > x_1^*$ ,  $dh/dx < 0$ .

Step3:

- When  $\beta$  is increasing,  $\frac{\sigma_z^2}{(x-1)\beta}$  is decreasing.

- Proceeding as in the proof of proposition 2 (step 2, (ii)), one can show that there exists  $x_2^*$  such that, for all  $x > x_2^*$ ,  $\beta/x$  is decreasing.

Let  $N^* = \text{int}(\text{Max}(x_1^*, x_2^*)) + 1$ , then for all  $N > N^*$ ,  $E(U_n | y_n)$  is decreasing when  $N$  is increasing.



Step 4:  $\lim_{N \rightarrow \infty} E(U_n | y_n) = 0$ .

$$\lim_{N \rightarrow \infty} \frac{\beta \tau_e}{N(N-1)} \left[ (N-1)^2 \frac{\tau_v^2}{(\tau_v + \tau_e)^2} y_n^2 + \frac{N-1}{\tau_v \tau_e} \frac{\tau_v + 2\tau_e}{\tau_v + \tau_e} + \frac{N\sigma_z^2}{\beta^2} \right] = \frac{\tau_v^2 \tau_e \beta_L}{(\tau_v + \tau_e)^2} y_n^2$$

where  $\beta_L = \lim_{N \rightarrow \infty} \beta(N)$ .

$\lim_{N \rightarrow \infty} [N\tau_e - (N-1)b\beta] = \infty$ , so  $\lim_{N \rightarrow \infty} E(U_n | y_n) = 0$ .

QED.

### APPENDIX 3:

#### Proof of proposition 6:

The market equilibrium condition is equivalent to

$$v + \frac{1}{N-1} \sum_{j \neq i} e_j - \frac{1-\delta}{(N-1)\beta} \sum_{j \neq i} z_j = \frac{1}{(N-1)\beta} [N\gamma p - N\theta - \beta y_i + (1-\delta)z_i]$$

Let 
$$h_i = \frac{1}{(N-1)\beta} [N\gamma p - N\theta - \beta y_i + (1-\delta)z_i]$$

then 
$$\text{Var}(v | y_i, z_i, h_i) = \text{Var}(v | y_i, z_i, p)$$

and 
$$\tau = \text{Var}^{-1}(v | y_i, z_i, p) = \tau_v + \tau_e + \left[ \frac{1}{N-1} \tau_e^{-1} + \frac{(1-\delta)^2}{(N-1)\beta^2} \sigma_z^2 \right]$$

which is equivalent to 
$$\tau = \tau_v + \tau_e + (N-1) \frac{\beta^2}{\beta^2 + (\delta-1)^2 \sigma_z^2 \tau_e} \tau_e$$

So 
$$\varphi = \frac{\beta^2}{\beta^2 + (\delta-1)^2 \sigma_z^2 \tau_e}$$

Using lemma 4.1 from Kyle (1989), it follows that

$$E(\bar{v} | p, y_n, z_n) = \frac{(1-\varphi)\tau_e}{\tau} y_n + \frac{\varphi\tau_e}{\beta\tau} [N\gamma p - N\theta + (1-\delta)z_n] \quad \text{Q.E.D.}$$

## REFERENCES:

- Black F. (1986): "Noise", *Journal of Finance*, 41: 529-543.
- De Long B., Shleiffer A., Summers L. and Waldmann R. (1989): "The size and incidence of losses from noise trading", *Journal of Finance*, 44: 681-696.
- De Long B., Shleiffer A., Summers L. and Waldmann R. (1990a): "Noise trader risk in financial markets", *Journal of Political Economy*, 98, 703-738.
- De Long B., Shleiffer A., Summers L. and Waldmann R. (1990b): "Positive feedback investment strategies and destabilizing rational speculation", *Journal of Finance*, 45: 379-395.
- Diamond, D. and Verrechia, R. (1981): "Information aggregation in a noisy rational expectations economy", *Journal of Financial Economics*, 9: 221-235.
- Gosciny, Uderzo: "Asterix chez les bretons".
- Grossman, S. (1976): "On the efficiency of competitive stock markets where traders have diverse information", *Journal of Finance*, 31: 573-585.
- Grossman, S. (1978): "Further results on the informational efficiency of competitive stock markets", *Journal of Economic Theory*, 18: 81-101.
- Grossman, S. and Stiglitz J. (1980): "On the impossibility of informationally efficient markets", *American Economic Review*, 70: 393-408.
- Hellwig, M. (1980): "On the aggregation of information in competitive markets", *Journal of Economic Theory*, 22: 447-498.
- Kyle, A. (1984): "A theory of futures market manipulations", in "The Industrial Organization of Futures Markets", R.W. Anderson Ed.: 141-191
- Kyle, A. (1985): "Continuous auctions and insider trading", *Econometrica*, 53: 1315-1335.
- Kyle, A. (1989): "Informed speculation with imperfect competition", *Review of Economic Studies*, 56: 317-356.
- Laffont, J.J. (1985): "On the welfare analysis of rational expectation equilibria with asymmetric information", *Econometrica*, 53: 1-29.
- Pagano, M. (1989): "Trading volume and asset liquidity", *Quarterly Journal of Economics*, 255-274.
- Sentana, E. and Wadhvani, S. (1992): "Feedback traders and stock return autocorrelation: evidence from a century of daily data", *The Economic Journal*, 102:

415-425.

Shleiffer, A. and Summers, L. (1990): "The noise trader approach to finance", *Journal of Economic Perspectives*, 4: 19-33.

Stein, J. (1987): "Informational externalities and welfare reducing speculation", *Journal of Political Economy*, 95: 1123-1145.

Tirole, J. (1982): "On the possibility of speculation under rational expectations", *Econometrica*, 50: 1163-1182.



# EUI WORKING PAPERS

EUI Working Papers are published and distributed by the  
European University Institute, Florence

Copies can be obtained free of charge  
– depending on the availability of stocks – from:

The Publications Officer  
European University Institute  
Badia Fiesolana  
I-50016 San Domenico di Fiesole (FI)  
Italy

**Please use order form overleaf**



# Publications of the European University Institute

To The Publications Officer  
European University Institute  
Badia Fiesolana  
I-50016 San Domenico di Fiesole (FI)  
Italy

From Name .....

Address .....

.....

.....

.....

.....

- Please send me a complete list of EUI Working Papers
- Please send me a complete list of EUI book publications
- Please send me the EUI brochure Academic Year 1993/94
- Please send me the EUI Research Report

Please send me the following EUI Working Paper(s):

No, Author .....

Title: .....

No, Author .....

Title: .....

No, Author .....

Title: .....

No, Author .....

Title: .....

Date .....

Signature .....



**Working Papers of the Department of Economics  
Published since 1990**

**ECO No. 90/1**

Tamer BASAR and Mark SALMON  
Credibility and the Value of Information  
Transmission in a Model of Monetary  
Policy and Inflation

**ECO No. 90/2**

Horst UNGERER  
The EMS – The First Ten Years  
Policies – Developments – Evolution

**ECO No. 90/3**

Peter J. HAMMOND  
Interpersonal Comparisons of Utility:  
Why and how they are and should be  
made

**ECO No. 90/4**

Peter J. HAMMOND  
A Revelation Principle for (Boundedly)  
Bayesian Rationalizable Strategies

**ECO No. 90/5**

Peter J. HAMMOND  
Independence of Irrelevant Interpersonal  
Comparisons

**ECO No. 90/6**

Hal R. VARIAN  
A Solution to the Problem of  
Externalities and Public Goods when  
Agents are Well-Informed

**ECO No. 90/7**

Hal R. VARIAN  
Sequential Provision of Public Goods

**ECO No. 90/8**

T. BRIANZA, L. PHLIPS and J.F.  
RICHARD  
Futures Markets, Speculation and  
Monopoly Pricing

**ECO No. 90/9**

Anthony B. ATKINSON/ John  
MICKLEWRIGHT  
Unemployment Compensation and  
Labour Market Transition: A Critical  
Review

**ECO No. 90/10**

Peter J. HAMMOND  
The Role of Information in Economics

**ECO No. 90/11**

Nicos M. CHRISTODOULAKIS  
Debt Dynamics in a Small Open  
Economy

**ECO No. 90/12**

Stephen C. SMITH  
On the Economic Rationale for  
Codetermination Law

**ECO No. 90/13**

Elettra AGLIARDI  
Learning by Doing and Market Structures

**ECO No. 90/14**

Peter J. HAMMOND  
Intertemporal Objectives

**ECO No. 90/15**

Andrew EVANS/Stephen MARTIN  
Socially Acceptable Distortion of  
Competition: EC Policy on State Aid

**ECO No. 90/16**

Stephen MARTIN  
Fringe Size and Cartel Stability

**ECO No. 90/17**

John MICKLEWRIGHT  
Why Do Less Than a Quarter of the  
Unemployed in Britain Receive  
Unemployment Insurance?

**ECO No. 90/18**

Mrudula A. PATEL  
Optimal Life Cycle Saving With  
Borrowing Constraints:  
A Graphical Solution

**ECO No. 90/19**

Peter J. HAMMOND  
Money Metric Measures of Individual  
and Social Welfare Allowing for  
Environmental Externalities

**ECO No. 90/20**

Louis PHLIPS/  
Ronald M. HARSTAD  
Oligopolistic Manipulation of Spot  
Markets and the Timing of Futures  
Market Speculation

**ECO No. 90/21**

Christian DUSTMANN  
Earnings Adjustment of Temporary  
Migrants

**ECO No. 90/22**

John MICKLEWRIGHT  
The Reform of Unemployment  
Compensation:  
Choices for East and West

**ECO No. 90/23**

Joerg MAYER  
U. S. Dollar and Deutschmark as  
Reserve Assets

**ECO No. 90/24**

Sheila MARNIE  
Labour Market Reform in the USSR:  
Fact or Fiction?

**ECO No. 90/25**

Peter JENSEN/  
Niels WESTERGÅRD-NIELSEN  
Temporary Layoffs and the Duration of  
Unemployment: An Empirical Analysis

**ECO No. 90/26**

Stephan L. KALB  
Market-Led Approaches to European  
Monetary Union in the Light of a Legal  
Restrictions Theory of Money

**ECO No. 90/27**

Robert J. WALDMANN  
Implausible Results or Implausible Data?  
Anomalies in the Construction of Value  
Added Data and Implications for Esti-  
mates of Price-Cost Markups

**ECO No. 90/28**

Stephen MARTIN  
Periodic Model Changes in Oligopoly

**ECO No. 90/29**

Nicos CHRISTODOULAKIS/  
Martin WEALE  
Imperfect Competition in an Open  
Economy

\*\*\*

**ECO No. 91/30**

Steve ALPERN/Dennis J. SNOWER  
Unemployment Through 'Learning From  
Experience'

**ECO No. 91/31**

David M. PRESCOTT/Thanasis  
STENGOS  
Testing for Forecastable Nonlinear  
Dependence in Weekly Gold Rates of  
Return

**ECO No. 91/32**

Peter J. HAMMOND  
Harsanyi's Utilitarian Theorem:  
A Simpler Proof and Some Ethical  
Connotations

**ECO No. 91/33**

Anthony B. ATKINSON/  
John MICKLEWRIGHT  
Economic Transformation in Eastern  
Europe and the Distribution of Income\*

**ECO No. 91/34**

Svend ALBAEK  
On Nash and Stackelberg Equilibria  
when Costs are Private Information

**ECO No. 91/35**

Stephen MARTIN  
Private and Social Incentives  
to Form R & D Joint Ventures

**ECO No. 91/36**

Louis PHLIPS  
Manipulation of Crude Oil Futures

**ECO No. 91/37**

Xavier CALSAMIGLIA/Alan KIRMAN  
A Unique Informationally Efficient and  
Decentralized Mechanism With Fair  
Outcomes

**ECO No. 91/38**

George S. ALOGOSKOUFIS/  
Thanasis STENGOS  
Testing for Nonlinear Dynamics in  
Historical Unemployment Series

**ECO No. 91/39**

Peter J. HAMMOND  
The Moral Status of Profits and Other  
Rewards:  
A Perspective From Modern Welfare  
Economics



**ECO No. 91/40**

Vincent BROUSSEAU/Alan KIRMAN  
The Dynamics of Learning in Mis-  
Specified Models

**ECO No. 91/41**

Robert James WALDMANN  
Assessing the Relative Sizes of Industry-  
and Nation Specific Shocks to Output

**ECO No. 91/42**

Thorsten HENS/Alan KIRMAN/Louis  
PHLIPS  
Exchange Rates and Oligopoly

**ECO No. 91/43**

Peter J. HAMMOND  
Consequentialist Decision Theory and  
Utilitarian Ethics

**ECO No. 91/44**

Stephen MARTIN  
Endogenous Firm Efficiency in a Cournot  
Principal-Agent Model

**ECO No. 91/45**

Svend ALBAEK  
Upstream or Downstream Information  
Sharing?

**ECO No. 91/46**

Thomas H. McCURDY/  
Thanasis STENGOS  
A Comparison of Risk-Premium  
Forecasts Implied by Parametric Versus  
Nonparametric Conditional Mean  
Estimators

**ECO No. 91/47**

Christian DUSTMANN  
Temporary Migration and the Investment  
into Human Capital

**ECO No. 91/48**

Jean-Daniel GUIGOU  
Should Bankruptcy Proceedings be  
Initiated by a Mixed  
Creditor/Shareholder?

**ECO No. 91/49**

Nick VRIEND  
Market-Making and Decentralized Trade

**ECO No. 91/50**

Jeffrey L. COLES/Peter J. HAMMOND  
Walrasian Equilibrium without Survival:  
Existence, Efficiency, and Remedial  
Policy

**ECO No. 91/51**

Frank CRITCHLEY/Paul MARRIOTT/  
Mark SALMON  
Preferred Point Geometry and Statistical  
Manifolds

**ECO No. 91/52**

Costanza TORRICELLI  
The Influence of Futures on Spot Price  
Volatility in a Model for a Storable  
Commodity

**ECO No. 91/53**

Frank CRITCHLEY/Paul MARRIOTT/  
Mark SALMON  
Preferred Point Geometry and the Local  
Differential Geometry of the Kullback-  
Leibler Divergence

**ECO No. 91/54**

Peter MØLLGAARD/  
Louis PHLIPS  
Oil Futures and Strategic  
Stocks at Sea

**ECO No. 91/55**

Christian DUSTMANN/  
John MICKLEWRIGHT  
Benefits, Incentives and Uncertainty

**ECO No. 91/56**

John MICKLEWRIGHT/  
Gianna GIANNELLI  
Why do Women Married to Unemployed  
Men have Low Participation Rates?

**ECO No. 91/57**

John MICKLEWRIGHT  
Income Support for the Unemployed in  
Hungary

**ECO No. 91/58**

Fabio CANOVA  
Detrending and Business Cycle Facts

**ECO No. 91/59**

Fabio CANOVA/  
Jane MARRINAN  
Reconciling the Term Structure of  
Interest Rates with the Consumption  
Based ICAP Model

**ECO No. 91/60**

John FINGLETON  
Inventory Holdings by a Monopolist  
Middleman



\*\*\*

**ECO No. 92/61**

Sara CONNOLLY/John  
MICKLEWRIGHT/Stephen NICKELL  
The Occupational Success of Young Men  
Who Left School at Sixteen

**ECO No. 92/62**

Pier Luigi SACCO  
Noise Traders Permanence in Stock  
Markets: A Tâtonnement Approach.  
I: Informational Dynamics for the Two-  
Dimensional Case

**ECO No. 92/63**

Robert J. WALDMANN  
Asymmetric Oligopolies

**ECO No. 92/64**

Robert J. WALDMANN /Stephen  
C. SMITH  
A Partial Solution to the Financial Risk  
and Perverse Response Problems of  
Labour-Managed Firms: Industry-  
Average Performance Bonds

**ECO No. 92/65**

Agustín MARAVALL/Víctor GÓMEZ  
Signal Extraction in ARIMA Time Series  
Program SEATS

**ECO No. 92/66**

Luigi BRIGHI  
A Note on the Demand Theory of the  
Weak Axioms

**ECO No. 92/67**

Nikolaos GEORGANTZIS  
The Effect of Mergers on Potential  
Competition under Economies or  
Diseconomies of Joint Production

**ECO No. 92/68**

Robert J. WALDMANN/  
J. Bradford DE LONG  
Interpreting Procyclical Productivity:  
Evidence from a Cross-Nation Cross-  
Industry Panel

**ECO No. 92/69**

Christian DUSTMANN/John  
MICKLEWRIGHT  
Means-Tested Unemployment Benefit  
and Family Labour Supply: A Dynamic  
Analysis

**ECO No. 92/70**

Fabio CANOVA/Bruce E. HANSEN  
Are Seasonal Patterns Constant Over  
Time? A Test for Seasonal Stability

**ECO No. 92/71**

Alessandra PELLONI  
Long-Run Consequences of Finite  
Exchange Rate Bubbles

**ECO No. 92/72**

Jane MARRINAN  
The Effects of Government Spending on  
Saving and Investment in an Open  
Economy

**ECO No. 92/73**

Fabio CANOVA and Jane MARRINAN  
Profits, Risk and Uncertainty in Foreign  
Exchange Markets

**ECO No. 92/74**

Louis PHLIPS  
Basing Point Pricing, Competition and  
Market Integration

**ECO No. 92/75**

Stephen MARTIN  
Economic Efficiency and Concentration:  
Are Mergers a Fitting Response?

**ECO No. 92/76**

Luisa ZANCHI  
The Inter-Industry Wage Structure:  
Empirical Evidence for Germany and a  
Comparison With the U.S. and Sweden

**ECO NO. 92/77**

Agustín MARAVALL  
Stochastic Linear Trends: Models and  
Estimators

**ECO No. 92/78**

Fabio CANOVA  
Three Tests for the Existence of Cycles  
in Time Series

**ECO No. 92/79**

Peter J. HAMMOND/Jaime SEMPERE  
Limits to the Potential Gains from Market  
Integration and Other Supply-Side  
Policies

**ECO No. 92/80**

Víctor GÓMEZ and Agustín MARAVALL  
 Estimation, Prediction and Interpolation for Nonstationary Series with the Kalman Filter

**ECO No. 92/81**

Víctor GÓMEZ and Agustín MARAVALL  
 Time Series Regression with ARIMA Noise and Missing Observations  
 Program TRAM

**ECO No. 92/82**

J. Bradford DE LONG/ Marco BECHT  
 "Excess Volatility" and the German Stock Market, 1876-1990

**ECO No. 92/83**

Alan KIRMAN/Louis PHLIPS  
 Exchange Rate Pass-Through and Market Structure

**ECO No. 92/84**

Christian DUSTMANN  
 Migration, Savings and Uncertainty

**ECO No. 92/85**

J. Bradford DE LONG  
 Productivity Growth and Machinery Investment: A Long-Run Look, 1870-1980

**ECO NO. 92/86**

Robert B. BARSKY and J. Bradford DE LONG  
 Why Does the Stock Market Fluctuate?

**ECO No. 92/87**

Anthony B. ATKINSON/John MICKLEWRIGHT  
 The Distribution of Income in Eastern Europe

**ECO No.92/88**

Agustín MARAVALL/Alexandre MATHIS  
 Encompassing Univariate Models in Multivariate Time Series: A Case Study

**ECO No. 92/89**

Peter J. HAMMOND  
 Aspects of Rationalizable Behaviour

**ECO 92/90**

Alan P. KIRMAN/Robert J. WALDMANN  
 I Quit

**ECO No. 92/91**

Tilman EHRBECK  
 Rejecting Rational Expectations in Panel Data: Some New Evidence

**ECO No. 92/92**

Djordje Suvakovic OLGIN  
 Simulating Codetermination in a Cooperative Economy

**ECO No. 92/93**

Djordje Suvakovic OLGIN  
 On Rational Wage Maximisers

**ECO No. 92/94**

Christian DUSTMANN  
 Do We Stay or Not? Return Intentions of Temporary Migrants

**ECO No. 92/95**

Djordje Suvakovic OLGIN  
 A Case for a Well-Defined Negative Marxian Exploitation

**ECO No. 92/96**

Sarah J. JARVIS/John MICKLEWRIGHT  
 The Targeting of Family Allowance in Hungary

**ECO No. 92/97**

Agustín MARAVALL/Daniel PEÑA  
 Missing Observations and Additive Outliers in Time Series Models

**ECO No. 92/98**

Marco BECHT  
 Theory and Estimation of Individual and Social Welfare Measures: A Critical Survey

**ECO No. 92/99**

Louis PHLIPS and Ireneo M'guel MORAS  
 The AKZO Decision: A Case of Predatory Pricing?

**ECO No. 92/100**

Stephen MARTIN  
 Oligopoly Limit Pricing With Firm-Specific Cost Uncertainty

**ECO No. 92/101**

Fabio CANOVA/Eric GHYSELS  
Changes in Seasonal Patterns: Are They  
Cyclical?

**ECO No. 92/102**

Fabio CANOVA  
Price Smoothing Policies: A Welfare  
Analysis

\*\*\*

**ECO No. 93/1**

Carlo GRILLENZONI  
Forecasting Unstable and Non-Stationary  
Time Series

**ECO No. 93/2**

Carlo GRILLENZONI  
Multilinear Models for Nonlinear Time  
Series

**ECO No. 93/3**

Ronald M. HARSTAD/Louis PHLIPS  
Futures Market Contracting When You  
Don't Know Who the Optimists Are

**ECO No. 93/4**

Alan KIRMAN/Louis PHLIPS  
Empirical Studies of Product Markets

**ECO No. 93/5**

Grayham E. MIZON  
Empirical Analysis of Time Series:  
Illustrations with Simulated Data

**ECO No. 93/6**

Tilman EHRBECK  
Optimally Combining Individual  
Forecasts From Panel Data

**ECO NO. 93/7**

Víctor GÓMEZ/Agustín MARAVALL  
Initializing the Kalman Filter with  
Incompletely Specified Initial Conditions

**ECO No. 93/8**

Frederic PALOMINO  
Informed Speculation: Small Markets  
Against Large Markets











