Oil Stocks as a Squeeze Preventing Mechanism: Is Self-Regulation Possible?

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and
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Is Self-Regulation Possible?

by

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ABSTRACT: Squeezes are registered occasionally in the forward market for Brent crude oil. The squeezer accumulates forward contracts and creates artificial demand by refusing to close out, exploiting imperfections in the decentralized market clearing. The artificial demand in turn creates a price surge and the possibility of a squeeze thus introduces uncertainty about the market outcome. Squeezes therefore render the market institution less palatable to other market participants (traders and refineries) who may find other ways of accomplishing the economic functions of the forward market, so the producers have a long term interest in keeping market clearing smooth by supplying stocks to squeezed short. The extent to which such self-regulatory stocks should be held is analysed in the context of a repeated game. Unless the probability of a squeeze is very small, self-regulation should be possible.

1 The second author would like to thank Mrudula Patel for comments and discussions.
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Oil Stocks as a Squeeze Preventing Mechanism:  
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"Corners are, or were, consequences of an excessive freedom of enterprise which seems largely a thing of the past in American futures markets. The British grain trade has never permitted either corners or significant squeezes in its futures markets. Squeezes continue to occur in American futures markets, though they can and should be eliminated." (Working (1949))

0. Introduction

Recently the organised British forward market for crude oil has permitted significant squeezes. On the 11th of January, 1988, the Weekly Petroleum Argus (WPA) reported:

"Someone has got a lot of January Brent unsold. The name most frequently mentioned in this connection is Transworld Oil [TWO], a Netherlands based trader presided over by the buccaneering John Deuss who also owns a refinery in Philadelphia. Who if anyone is behind Mr. Deuss is a matter of speculation and Argus can report only that two traders out of three think there is an Arab Gulf state in the background.

The story begins at the end of November when TWO begins to accumulate claims to Brent for January lifting. That seemed odd at the time. The Opec ministers were about to meet and it would have been hard to find anyone in the oil industry optimistic about the outcome. To take a long position in crude to be lifted after the ministers had finished their deliberations appeared foolhardy. [...] There was a precedent. In April TWO had succeeded in cornering 15-Day Brent for lifting at the end of the month and had collected a premium of up to a dollar and a half over dated crude. That manoeuvre was clearly profitable in itself. [...] When the manoeuvre was repeated in December the major Brent producers, Esso and Shell, let it be known that they would do all in their power to frustrate it. Some cargoes were released from their corporate systems and other similar North Sea crudes, together with Nigerian, were made available. But they failed to prevent a serious distortion from developing in the spectrum of oil prices."

This paper is an attempt to understand the economics of self-regulating duopoly. The basic question is whether oligopolistic producers can carry what we shall call "regulatory stocks" in order to make sure that a squeezer cannot create artificial scarcity on the spot market. The last paragraph of the quotation from the WPA suggests that this type of self-regulation does not work.
Let us begin by explaining the jargon used in the quotation\(^1\). First of all, "Brent" designates Brent Blend, a mixture of the production from seventeen separate oil fields in the North Sea. Most of the Arabian oil imported in Europe is priced with reference to the price of Brent, which explains why Arab states may have an interest in pushing this price up. Esso and Shell, who control the production of Brent, have a long-term interest in keeping the Brent market liquid.

On any given day, there are two prices for Brent. The "dated" price is for a cargo that is lifted or to be lifted into a vessel on a particular date: the dated price is thus what is called the spot price in standard literature. A "15-Day" price is for a "paper" cargo and can be "first month" (for delivery on an unspecified date next month), or "second month" (for delivery in the second month to come), etcetera. A paper cargo is a claim to 500,000 barrels of Brent to be lifted at an unspecified date in a particular month to come. Dates of lifting must be determined 15 days in advance\(^2\). For April liftings, these dates can thus be determined up to the 13th of April\(^3\). Consequently, during the first two weeks of April there exists simultaneously a 15-Day April price (for still undated April cargoes) and a dated April price. A premium for 15-Day April Brent over dated crude is thus the (positive) difference between the price of a paper April cargo and a dated April cargo.

Section 1 gives the evidence on the price distortions that resulted from the April 1987 and January 1988 squeezes and three subsequent squeezes. The distortions occurred on the 15-Day forward market and implied huge premia of first month Brent over both second-month Brent and first-month West Texas Intermediate (WTI). The latter is an American crude that is a substitute for Brent and is traded on the New York Mercantile Exchange (NYMEX).

Section 2 sets up a model of a market with a potential squeeze. In Section 2.1 we examine the incentive to squeeze and for the producers to prevent the squeeze under the assumption that the producers coordinate their effort by setting up regulatory stocks. The opposing interests of the players are: Short-term profit to the squeezer against long-term profit

\(^1\) Further institutional details about the Brent market can be found in Mabro (1986) and Philips (1992).

\(^2\) To give the buyer time to charter a tanker and send it to Sullom Voe (Shetlands) where the loading takes place. Hence the name "15-Day market".

\(^3\) Since a loading date is in fact a three-day period. With 30 days in the month, the last loading period starts April 28 and 28-15=13.
to the producers. The question is whether and to what degree it pays to prevent a squeeze.
In Section 2.2 we then analyse whether the oligopolists' decision to keep regulatory stocks is any different from the monopolist's (it is not) and whether this affects the scope for cooperation in the repeated game (it does).

Section 3 concludes by pondering why regulatory stocks are not held even though this study indicates they should be. The realism of the assumptions of the model is discussed, as are variations of the model.

1. Squeezes on the 15-Day Brent Market

This section illustrates the price effects of the two squeezes mentioned in the introduction and gives an account of the major squeezes on the 15-Day market since then.

![Figure 1: The April 1987 Squeeze (10th of March to 15th of April 1987)](image)

Source: Platt's and Nymex
Figure 1 covers the period from 10 March to 15 April 1987 and represents the premium of first month Brent over second month Brent and the premium of first month Brent over first month WTI. From mid-March the premium of first over second month Brent started to climb up to $0.50. In the first week of April, it suddenly increased to $1.60 within 5 days. That this was due to a squeeze and not to market fundamentals is confirmed by the fact that the premium of first month Brent over first month WTI followed the same pattern, although WTI is a close substitute to Brent.

How does such a squeeze arise? To close out a position on the 15-Day market, there are two possibilities. The first is a "bookout" whereby a number of participants agree to cancel their contracts with a cash settlement. These participants have contracts that can be arranged in a chain starting and ending with the same participant. A squeezer has never sold forward and thus will not appear in a bookout. The contracts that are cleared by a bookout are not pertinent to the squeeze.

The second way of closing out a position is to pass on a 15-day notice to take delivery along a "daisy chain". The chain starts when a seller with entitlements to Brent serves a 15-day notice to take delivery to those who have buying contracts with him for April delivery. A buyer who receives such a notice can either accept it or pass it on to somebody who bought from him. The squeezer accepts the notice and thus builds up his stock of claims to Brent for April lifting.

What happens if a participant cannot close his position? If a participant without entitlement to Brent has sold a contract to the squeezer on the forward market and if the squeezer refuses to construct a bookout by selling the contract back to the participant, then the participant has a legal obligation to deliver Brent to the squeezer in April. The participant must buy a cargo on the spot market thereby creating an artificial demand for spot Brent. The squeezer has bought an significant amount of the entitlements to April Brent and thus more or less controls supply. With an inelastic supply and an artificially increased demand the spot price goes up. The other possibility is that the squeezer agrees to sell a paper cargo back to the short participant so that a bookout can be arranged. This time the squeezer sets the terms

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4 The sources are Platt's for Brent prices and Nymex for WTI prices. Note that for Brent, the months change as of the 10th of any particular month. Until 9 March, the first month is March. From 10 March to 9 April, the first month is April, and so on. For WTI, the first month is April until 20 March. As of 23 March, the first month is May.
of the paper trade. This leaves the spot price unchanged but raises the 15-Day price. A mixture of these two types of squeezing is of course also a possibility.

The squeeze results from the fact that the market participants (apart from the squeezer) on average are net short and the squeezer is very long in the market. In case of a corner, the squeezer has complete control over the dated April cargoes, so that the sellers have to buy from him at a premium. Implicit in all this is the idea that the integrated producers are not willing to supply cargoes from the stocks they hold for refinery (or other) purposes.

Figure 2: The January 1988 Squeeze (10th of December to 15th of January)

In late November 1987, TWO began buying January cargoes at a price at parity or

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5 Tax considerations may be involved: individual producers may have preferred to retain oil on their hands rather than selling it at arms-length at a higher price since they are taxed on the average of prices from the first day of the month preceding delivery (here 1st March) and ending on the middle day of the month of delivery, (WPA, 1st June 1987). This reasoning ignores the effect of producers' sales on the price, which is central in the analysis of section 2.
even at a discount to February cargoes. On 10 December 1987 (see Figure 2) a new price squeeze started, pushing the premium of first-month (January) over second-month (February) up to $1.37 within fifteen days. During the same time span the premium over first month WTI climbed to $1.66, while dated Brent remained at a discount of about 50 cents per barrel.

It is only in the second week of January that dated cargoes were sold at a small premium. So the squeeze happened on the 15-Day market, not on the spot market.

Figures 3-6 give detailed evidence on the daily deals of the January 1988 squeeze as published by the *Weekly Petroleum Argus*. TWO is reported to have had control of almost all dated January cargoes in the second half of December (Around the 10th of January it also owned the majority of the remaining undated January cargoes). Intuition suggests that TWO built up its long position at a time other longs were selling. Figure 4 shows that the majority of trading took place in the first three weeks of November. On one particular day, more than 50 deals were reported. This alone exceeds a month’s production. Since the second half of December, that is, once the dating of January cargoes started, almost no deals were reported. That is the period during which, supposedly, TWO squeezed by refusing to close the market’s open interest.

Figure 3 shows that the period of active trading in November was also a period of moderately decreasing prices. On a given day the differences in price for deals made were of moderate size reflecting a near concurrence on the expected spot price. Figures 5 and 6 show similar data for the February 1988 contract. This pattern of trade is typical for the 15-Day market. The important thing to note is that a squeezer can easily hide in this kind of data: The trading is decentralised and nobody keeps track of the traders’ positions, so the squeezer can quietly buy up contracts from different traders each of whom only has a moderately short position.

In the first week of January the premia were falling (See Figure 2). By the 11th of January the squeeze was over. The fall of the premia may have been related to some extent to Esso’s and Shell’s announcement that they would supply stocks from their corporate systems. At any rate, TWO had to take delivery of 41 out of the 42 cargoes produced in January and did not push the dated premium to high levels.

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6 Similar data do not exist for the April 1987 squeeze.
Figure 3: The January 1988 contract
High/Low Price

Source: Weekly Petroleum Argus

Figure 4: January 1988 contract
Number of deals

Source: Weekly Petroleum Argus
Figure 5: February 1988 contract
High/Low Price

Figure 6: February 1988 contract
Number of deals

Source: Weekly Petroleum Argus
It is worth noting that Esso in March 1988 proposed to its trading partners that sellers be given the option in the standard contract of substituting a number of other grades of crude oil from North Sea fields for Brent, in an attempt to deter a repetition of the squeeze. Substitution would incur a premium of 30 cents per barrel. Delivery options of this kind is the standard way of avoiding squeezes in futures markets (See Duffie (1989) pp. 323-4). After some discussion the proposal was not accepted. This alternative squeeze preventing proposal suggests that Esso had abandoned the idea that the main producers could deter squeezes by threatening to make stocks available.

Figure 7 shows how the differential between Platt’s quotations of the average price of paper barrels for delivery in the first and second month evolved from 1987 to 1991. The six peaks that can be identified (marked I-VI) are potential candidates for squeezes and in fact five of them (I-IV and VI) were. The huge peak between V and VI was caused by the Gulf Crisis that led to a maximum differential of 3.1 $/bl on 17 September 1992. The maxima of the six potential squeezes are given in Table 1.

<table>
<thead>
<tr>
<th>Peak</th>
<th>Date</th>
<th>Maximum Differential</th>
<th>Differential 2nd Month</th>
</tr>
</thead>
<tbody>
<tr>
<td>I</td>
<td>08/04/87</td>
<td>1.60 $/bl</td>
<td>8.9 %</td>
</tr>
<tr>
<td>II</td>
<td>31/12/87</td>
<td>1.37 $/bl</td>
<td>8.1 %</td>
</tr>
<tr>
<td>III</td>
<td>03/08/89</td>
<td>0.75 $/bl</td>
<td>4.6 %</td>
</tr>
<tr>
<td>IV</td>
<td>06/11/89</td>
<td>1.53 $/bl</td>
<td>8.1 %</td>
</tr>
<tr>
<td>V</td>
<td>10/01/90</td>
<td>1.17 $/bl</td>
<td>5.8 %</td>
</tr>
<tr>
<td>VI</td>
<td>12/04/91</td>
<td>0.90 $/bl</td>
<td>4.7 %</td>
</tr>
</tbody>
</table>

1) Due to unexpectedly strong demand for heating oil, cfr. text.

The two first peaks (I and II) are the two squeezes that were mentioned above and they both led to price surges that reached maxima of more than eight percent (measured as the differential over second month price7). The third and fourth squeezes were supposedly carried out either by a Wall Street refiner or by a trading house. Weekly Petroleum Argus

7 This measure may overestimate the size of the squeeze slightly if the market is in backwardation initially.
Figure 7: Five Squeezes in the 15-Day Market
wrote about IV:

"The premium for November 15-Day Brent over December widened from 40 c/bl between Friday morning and Thursday evening. This premium has talked wider all week [27/10 - 2/11/89] but in such secrecy that it appeared an attempt to force up the November quotations and price reporting services felt manipulated. However both numbers given above were confirmed and there is conjecture that one party sold short in expectation that the spread would close in. But it was eventually forced to cover at almost double the premium at which it bought [sold?]."  (Weekly Petroleum Argus, 6/11/89 p. 8)

And:

"... The price of North Sea crude is blurred by the pressure on November Brent which some feel is rubbing off on December prices. The possibility that an extra Brent cargo may be fitted into the November programme helped reduce November prices slightly but they remain out of line with the rest of the market. The monthly recurrence of so called squeezes since the summer, partly due to the reduction in Brent production, is angering traders who feel the future of the 15-Day market in jeopardy. ... (Weekly Petroleum Argus 13/11/89 p. 8).

The quotes show two important effects of a squeeze: first, a market participant that for speculative reasons was taking a short position had to cover at a large loss; and second, market participants are getting discontent with the market because of squeezes. Furthermore it shows that the possible availability of an additional cargo reduced the effect of the squeeze. These observations constitute a crucial part of our modelling of the market in Section 2.

V was as mentioned not a squeeze. This can be seen from the fact that spot prices raised even more than first month prices, and the cause of this was an increase in the demand for heating oil. Platt's Oilgram Price Report of 2/1/90 noted:

"... With Brent now turning wet into the second-half of Jan, and with the absence of any squeeze-related play in Jan Brent, paper and wet Brent are acting in unison."

Thus there seems to be general agreement that this was not a squeeze.

VI was a squeeze that was carried out by a large, Chinese trading house. The story was however not commented by the Weekly Petroleum Argus, nor by any other commentator that we know of and we are relying on confidential information in this case.

Summing up, we have identified five major squeezes in the 15-Day market for Brent crude oil in the period 1987 - 1991. The immediate effect of squeezes is that some market participants incur losses out of line with normal speculative gains and losses. The long-run effect is that market participants are worried to a degree where they feel that the future of the entire market is at stake. Finally, if extra cargoes (read: regulatory stocks) were available, the effects of squeezes could be dampened if not eliminated.
2. A Squeezer Round the Corner?

In this section we propose a model that captures the ideas that were presented above. We first model how the squeeze affects the market and what it costs to prevent a squeeze given that the producers can cooperate on holding regulatory stocks. We then move on to determining under what circumstances such regulatory stocks can be sustained in a non-cooperative repeated game.

Two points should be borne in mind throughout. The paper is about regulatory stocks, and that only. This means that the only reason to hold stocks is a potential squeeze. In particular, the producers face no other uncertainty than that arising from the squeeze so buffer stocks are ruled out; the price that the producers receive is constant before and after a squeeze so speculative stocks are ruled out; the marginal cost of production is constant (zero) so there is no incentive to hold transaction stocks; the time horizon is infinite so strategic stocks in the sense of Møllgaard and Phillips (1992) are not an issue either.

The second point is that in this framework production always equals sales except in the period where a squeeze occurs in which case sales equal production plus stocks.

2.1 The Squeezer’s Game

We ignore the presence of integrated oil companies and assume that the market participants belong to at most one of four groups:

1. refineries
2. traders
3. producers
4. squeezer.

The final demand for crude oil arises from the refineries, who buy the crude on the spot market. The producers hedge their entire production by selling it forward. The traders serve as intermediaries and buy forward from the producers to sell on the spot market. Thus, to simplify matters, we assume that the spot market consists of refineries, traders and possibly a squeezer, whereas the forward market consists of producers, traders and possibly a squeezer. Section 3 comments on the fact that integrated oil companies operate in the market.

Assume that the refineries’ demand for crude oil takes a form that allows us to write the (inverse) demand that traders face on the spot market as a random variable that is linear
in quantity, \( x_i \):

\[
\hat{P}_i = (\hat{\alpha}_i - RP) - x_i,
\]

(1)

where \( \hat{\alpha}_i \sim N(\alpha, \nu) \) with \( \alpha > 0 \) and \( RP \) is some positive constant to be determined later. We have \( E(\hat{P}) = P = \alpha_i - RP - x_i \), \( P_i \) can be thought of as the "market expectation" of the spot price. Traders are risk-averse profit maximisers. In particular their utility function may be negatively exponential in profits \( u(\pi) = -e^{\alpha \pi} \) implying, since \( \pi \) follows a normal distribution, that they maximise \( E(\pi) - AVar(\pi)/2 \). Their expected payoff is thus

\[
d(P_i - p) - d^2 AVar\hat{P}_i = d(P_i - p) - d^2 A^2 \nu,
\]

(2)

where \( A \) is the Arrow-Pratt coefficient of constant absolute risk aversion and \( d \) is the net long position of the trader (\( d \) for "deals"). A trader’s participation constraint is thus:

\[
P_i - p_i \geq d^2 A^2 \nu = RP.
\]

(3)

As long as the participation constraint holds with inequality, there are payoffs to be made from trading in forward contracts, so the market should attract more traders implying that the forward price \( p_i \) be competed up to equality of (3). But this argument holds for a fixed \( d \) and the right hand side of (3) is minimised for \( d = 1 \) (given that a deal is indivisible and that \( d > 0 \)), so it is in the interest of the producers to have at least as many traders in the market as the number of deals so that each of the traders buy one forward contract from the producers. Seen from the producers point of view, the risk premium they have to pay - the cost of hedging - is decreasing in the number of traders in the market.\(^8\) Note that (3)

\(^8\) A similar point is made in Working (1953). See especially the section entitled "The Cost of Hedging".

13
basically assumes a situation of normal backwardation in which the producers are so risk averse that they hedge completely. Given their (now certain) demand curve, they can go on and maximise profits in the usual way.

Equality of the constraint (3) with \( d = 1 \) implies that the producers face a certain demand function on the forward market of the simple form:

\[
p_t = P_t - RP = \alpha_t - x_t.
\]

In this subsection we assume that the producers can sustain a cooperative outcome in a non-cooperative repeated game (which we then study in Section 2.2). We also assume that the marginal cost of production is constant and normalise it to zero. The \( n \) producers therefore share monopoly profits in each period. The equilibrium is summarised in (5).

<table>
<thead>
<tr>
<th>Monopoly outcome</th>
</tr>
</thead>
<tbody>
<tr>
<td>with no stocks</td>
</tr>
<tr>
<td>Quantity:</td>
</tr>
<tr>
<td>( x_i = \frac{\alpha_t}{2} )</td>
</tr>
<tr>
<td>Price:</td>
</tr>
<tr>
<td>( p_t = \frac{\alpha_t}{2} )</td>
</tr>
<tr>
<td>Profit:</td>
</tr>
<tr>
<td>( \pi_t = \frac{\alpha_t^2}{4} )</td>
</tr>
</tbody>
</table>

The model above can explain a volume of trade on the forward market equal to the volume of production and it does not allow for squeezes since all traders are net long. As a matter of fact, the volume of trade on the 15-Day forward market is typically ten-fold the volume of production and squeezes do occur. It is argued elsewhere (see Møllgaard (1992)) that the volume on the forward market can be explained predominantly by speculation, implying that different traders enjoy different spot price expectations. This also explains why some traders short-sell contracts and thus how a squeezer can build up a large long position. We shall not pursue the issue of speculation here, but note that it may reduce the traders’ aversion to risk that the market is liquid (has a high volume), since they can then free themselves from whatever contract they have engaged in at any time incurring a moderate loss. Thus, the traders, as do the producers, favour a high number of trades.
The implicit assumption that we make and - more importantly - that the traders make, is that the market’s open interest at maturity is cleared by financial transactions using the spot price which is determined by the (known) production and the realisation of the refineries’ demand curve (1). The traders do not expect a squeeze.

A squeezer is indistinguishable from a normal trader. The squeezer uses imperfections in the clearing of the forward market strategically to squeeze a temporary profit out of the markets. As mentioned, the volume in the forward market is bigger than the volume of the spot market, but the difference is normally closed by financial transactions in a bookout or a daisy chain as maturity approaches. However, in the 15-Day market, no participant is legally obliged to enter in the clearing and the squeezer uses this opportunity to squeeze the market: By offering a price in the high end of the spectrum, the squeezer obtains legal rights to a substantial part if not all of the physical cargoes. Those who sold cargoes forward without having them (everybody but the producers) and who therefore are genuinely short of oil are put in a difficult situation: they have to buy either on the spot market thereby crowding out the usual buyers or take the squeezer’s terms on the forward market. This idea is modelled by assuming that a squeeze of size \( \alpha' \) equal to the open interest of the market at maturity affects the inverse demand in the following way:

\[
P_t = \alpha_t - \alpha' - x_t - RP^s = p_t - (\alpha' - RP^s) \quad ; \quad \alpha' > 0.
\]  

Here the assumption is that if the traders have to buy up paper cargoes on the forward market (from the squeezer) at a premium in order to satisfy their contractual obligations, they reduce their own risk premium (that may become negative):

\[
RP^s = RP - \alpha'
\]  

In this case we say that the squeeze was entirely on the 15-day market, since we will note a price surge only on the 15-day market as the traders close their positions.

The alternative is that the traders decide to deliver the oil, buying it on the spot
market. This will raise the spot price by \( \alpha' \) and will in the end have the same effect on traders' profits, but this time the spot price is affected and so we say that the squeeze is on the spot market. Indeed, it is this possibility of buying spot oil that puts a limit on the terms that the squeezer can set if the squeeze is settled in terms of paper cargoes. One could, of course, imagine a combination: the squeeze could be settled partly in paper barrels and partly in wet barrels, but this does not affect the analysis. Note that, since the producers sold their entire production forward, their immediate profit is not affected.

To summarise: Within each period \( t \), the producers first sell their production on the forward market to the traders. Some traders also sell forward contracts to other traders expecting financial settlement at maturity. The squeezer possibly buys a substantial number of forward contracts. Then the spot market opens. If the squeezer does not show up, those who bought forward from the producers clear the spot market with the refineries, and the resulting spot price is used in financial settlements of the remaining open interest on the forward market. If the squeezer does show up, short traders may appear on the spot market in desperate search for a possibility to fulfil their legal obligation. This would drive up the spot price. Alternatively, the short traders may seek a settlement with the squeezer on the forward market and this would drive up forward prices.

In the normal mode of functioning there is a friendly competitive environment on the forward market: friendly meaning that the imperfections of the clearing mechanism are not exploited in a squeezing game. When a squeeze occurs, an aggressive environment results: the imperfections of the clearing mechanism are exploited, traders and/or refiners are trapped and have to pay more for the crude oil than they expected and thus observe reduced expected profits or even losses.

A successful squeeze therefore results in a general dissatisfaction among the refiners or the intermediaries because of the malfunctioning and unpredictability of the market. They will tend to organise trade outside the market or in other markets or they get more averse to risk. After all, the squeeze proved that spot prices were more volatile than they thought initially and so a higher risk premium is required.

We assume (for simplicity of notation) that when a successful squeeze occurs, \( \alpha' \) squeezed refineries leave the market in the next period to never come back if the squeeze occurs on the spot market, or if the squeeze is on the forward market that the traders require a risk premium that is \( \alpha' \) higher. One could again imagine a hybrid case, but the important
thing to note is that in all periods following a squeeze, the producers are left with a demand curve with an intercept that is \( \alpha' \) lower. The inverse demand curve in all periods following a squeeze is:

\[
p_{t-1} = \alpha_{t-1} - x_{t-1} = (\alpha_i - \alpha') - x_{t-1} \tag{8}
\]

The producers thus have a long-run incentive to prevent a squeeze in order to keep the price of their hedged production from falling in the future; to keep the market liquid. They can achieve this by keeping stocks in order to match a squeeze if it occurs, i.e. by keeping stocks \( s = \alpha' \). The idea is that if the squeezer squeezes, then the producers throw their stocks on the market at the normal price to meet the artificially created demand. In a sense, the producers always keep a physical position of size \( \alpha' \) to match the squeezer’s long paper position. We assume that the cost of storage is the interest \( r \) per $ per barrel per period.

If they hold stocks to prevent a squeeze, i.e. \( s = \alpha' \), they have to subtract the interest on the value of the stocks \( r\alpha'p_i \) from revenue in each period and reoptimising we find that compared to (5) the producers raise output and lower price a bit to take the cost of storage into account:

\[
\begin{align*}
\text{Monopoly outcome} \\
\text{with stocks} \\
\text{Quantity:} & \quad x_i = \frac{\alpha_i - r\alpha'}{2} \\
\text{Price:} & \quad p_i = \frac{\alpha_i - r\alpha'}{2} \\
\text{Profit:} & \quad \pi_i = \frac{(\alpha_i - r\alpha')^2}{2}.
\end{align*}
\tag{9}
\]

If they fail to prevent a squeeze in period \( t \) (not holding stocks), profits will be \( (\alpha_i - \alpha')^2/4 \) in all future periods instead of \( (\alpha_i/2)^2 \). The discounted loss of not preventing a squeeze is therefore

\[
\text{Loss} = \frac{(2\alpha_i - \alpha')\alpha'}{4r} \tag{10}
\]

where we assume a discount rate of \( 1/(1+r) \) and that the effect of a squeeze in period \( t \) manifests from period \( t+1 \) onwards. The (reasonable) assumption underlying this modelling...
is that the producers’ output is inflexible in period $t$ so that production is decided upon before the squeezer reveals himself. Additional supply therefore has to come from the stocks if they exist. These stocks are supposedly made available immediately.

The price in case of a successful squeeze is the price of equation (6) taking the monopoly output as given:

$$ p_t^s = \frac{\alpha_t}{2} - \alpha^s = p_t - \alpha^s, \quad (11) $$

so a price surge of size $\alpha^s$ will occur on the forward market towards maturity if the squeeze is happening there or on the spot market if that is where the squeeze pops up.

The (gross) payoff of a squeeze to the squeezer is the size of his long position, $\alpha^s$, times the price difference between selling and buying, $p_t^s - p_t = \alpha^s$, i.e. $(\alpha^s)^2$. In the case of a squeeze, the squeezer has to dispose of the acquired oil elsewhere and he therefore suffers a loss, $\Theta$, of squeezing independently of whether the squeeze is successful or not.

The payoffs are summarised in Table 2. The producers’ payoff are reported as deviations from an eternal monopoly profit, with the value $((r-1)/r)(\alpha^2/4)$ today.

The producers’ entry in the lower, right corner of the matrix is due to the assumption that if the squeezer squeezes and if the producers sell their stocks, the squeezer cannot squeeze again since he has now been identified as such and so the producers do not need to

<table>
<thead>
<tr>
<th>Squeezer/Producers</th>
<th>No stocks</th>
<th>Stocks=$\alpha^s$</th>
</tr>
</thead>
<tbody>
<tr>
<td>No squeeze</td>
<td>$(0,0)$</td>
<td>$0, \frac{(1-r)\alpha^s(2\alpha^s-r\alpha^s)}{4}$</td>
</tr>
<tr>
<td>Squeeze</td>
<td>$(\alpha^s)^2-\Theta,\frac{(2\alpha^s-\alpha^s)\alpha^s}{4r}$</td>
<td>$(-\Theta,\alpha^s p)$</td>
</tr>
</tbody>
</table>

Table 2: Payoff Matrix for Squeezer vs. Producers
hold stocks any longer. On the other hand, the producers earn an additional profit \((\alpha'p_s)\) from satisfying the extra demand the squeezer has created.

There is no Nash equilibrium in pure strategies: if the producers do not carry stocks the squeezer will squeeze; if the squeezer squeezes, the producers will carry stocks; but then the squeezer will not squeeze and thus it does not pay for the producers to carry stocks.

Letting stocks be a continuous variable allows \(s\) to take a value above zero but below \(\alpha'\). What happens if \(s\) takes an internal value? In the case of a squeeze, a number of the squeezed traders can fulfil their obligation by buying the producers stocks at price \(p_s\) and delivering to the squeezer. This modifies the squeeze to one of size \((\alpha' - s)\) and thus affects prices by \((\alpha' - s)\) and so the squeezer’s profit, \(\Delta(s)\), becomes a quadratic function of the stocks:

\[
\Delta(s) = (\alpha' - s)^2 - \Theta \quad \text{(12)}
\]

Observe that the squeezer’s profit takes the value zero for \(s = \alpha'\) if the cost, \(\Theta\), of squeezing is strictly positive. The square root of \(\Theta\) translates the cost of squeezing into an equivalent number of cargoes. The presence of a cost of squeezing thus means that by keeping stocks equal to \(\alpha'\) the producers commit overkill, since stocks are costly and a squeeze will be unprofitable even if stocks are reduced by \(\sqrt{\Theta} \).

Now we exogenise the squeezer and assume that the producers perceive that there is an exogenous probability, \(\sigma\), of a squeeze of size \(\alpha'\) in each period. This means that the producers do not try to influence the squeezer’s behavior strategically, but that they rather observe the possibility of a squeeze and take it into account.

We choose a simple model of a squeeze probability: Assume that there is probability \(\sigma\) of a squeeze until the squeeze has occurred (if ever) whereafter the squeezer is identified and a squeeze cannot occur again.

Production can be changed from period to period and before a squeeze occurs it is chosen to maximise expected single period profit:

\[
E(\Pi(x;\alpha,p,r,s)) = (1 - \sigma)(\alpha - x)(x - rs) - \sigma(\alpha - x)(x - s) \quad \text{(13)}
\]

The first term says that if a squeeze does not occur (an event that happens with probability \(1-\sigma\)), stocks are held in vain and the producers have to pay the cost of storage, \(prs\). The second term says that if a squeeze does occur, the producers’ sales are increased by the size
of the stocks at the going price.

The equilibrium that obtains before a squeeze is therefore

\[
\begin{align*}
x &= \frac{1}{2}(\alpha_i - \gamma s) \\
p &= \frac{1}{2}(\alpha_i - \gamma s) \\
E(\Pi) &= \frac{1}{4}(\alpha_i - \gamma s)^2
\end{align*}
\]

where \( \gamma = \sigma - (1 - \sigma)r \).

If \( \gamma > 0 \), i.e. if \( \sigma > \frac{r}{1-r} \), then the quantity is lower and the price is higher than in the situation in which a squeeze cannot occur, see (5). Conversely if \( \gamma < 0 \).

In the period after a squeeze, i.e. once uncertainty has been resolved, production is chosen to maximise

\[
\Pi(x;\alpha_i, \alpha^i, s) = (\alpha_i - \alpha^i - s - x)x_i,
\]

since the new demand intercept is

\[
\alpha_{i,i} = \alpha_i - (\alpha^i - s) \quad \forall i > 0.
\]

The solution that obtains after a squeeze is therefore

\[
\begin{align*}
x &= \frac{1}{2}(\alpha_i - \alpha^i - s) \\
p &= \frac{1}{2}(\alpha_i - \alpha^i - s) \\
\Pi' &= \frac{1}{4}(\alpha_i - \alpha^i - s)^2
\end{align*}
\]

Seen from period \( t \), the producers’ expected, discounted profit takes the form of a complex, geometric progression:
\[ E(\Pi) = E(\Pi) \]
\[ - \frac{1}{1-r} [(1-\alpha)E(\Pi) - \sigma \Pi'] \]
\[ - \frac{1}{(1-r)^2} [(1-\alpha)^2 E(\Pi) - \sigma(1-(1-\alpha))\Pi'] \]
\[ - \frac{1}{(1-r)^3} [(1-\alpha)^3 E(\Pi) - \sigma(1-(1-\alpha)-(1-\alpha)^2)\Pi'] \]
\[ \ldots \]
\[ = \frac{1-r}{\sigma} E(\Pi) - \frac{\sigma}{r} \frac{1-r}{\sigma-r} \Pi' . \]

The first term in the square brackets multiplies the probability that a squeeze has not occurred up to a certain period with the expected stage game profit before a squeeze. The second term similarly multiplies the cumulated probability that a squeeze happened in a given period or before with the after-squeeze profits.

For later convenience, define

\[ K(s;\alpha,\alpha',\sigma,r) = 4/E(\Pi) - \frac{\sigma}{r} \Pi' / \]

\[ = (\alpha_i - \gamma s)^2 - \frac{\sigma}{r} (\alpha_i - \alpha_i^* - s)^2 \]

and observe that \( K \) is proportional to \( E(\Pi) \) and that these expressions therefore enjoy the same functional characteristics.

We employ the following assumptions in the sequel:
Assumption 1: The interest rate is positive, \( r > 0 \), and \( \sigma \) is a probability, i.e. \( \sigma \in [0;1] \).

Assumption 2: \( \alpha_i > \alpha' > 0 \).

Assumption 3: \( 0 \leq s \leq \alpha' \).

Assumption 1 is hardly controversial. Assumption 2 is a joint assumption: Firstly it means that a squeeze raises prices by two hundred percent at the most, which does not seem restrictive given the size of the squeezes reported in Table 1. Secondly it implies that the market does not vanish completely in the periods following a squeeze if the squeeze is completely unprevented \((s = 0)\) so that the problem is still well defined after a squeeze. Assumption 3 requires stocks to be non-negative and not to exceed \( \alpha' \). (If \( s > \alpha' \), \( \alpha_{ii} = \alpha_i - (\alpha' - s) > \alpha_i \) which is not a tenable assumption.)

Let \( s' \) denote optimal stocks. We immediately get the result that if a squeeze is totally unlikely to occur, it does not pay to hold stocks:

**Proposition 1 (Certainly No Squeeze):** \( \alpha = 0 \Rightarrow s' = 0 \).

**Proof:** \( K(s;\alpha,\alpha',0,r) = (\alpha_i - rs)^2 \) takes a global minimum = 0 for \( s = \alpha_i/r > 0 \). \( K(s) \) is real valued and continuous in \( s \). The restriction of \( K(s) \) to \( [0;\alpha'] \) therefore takes a maximum on this set. \( K \) is convex in \( s \) and the maximum thus is at \( s = 0 \) or \( s = \alpha' \). \( K(0;\alpha,\alpha',0,r) = \alpha_i^2 > K(\alpha';\alpha,\alpha',0,r) = (\alpha_i - \alpha'r)^2 \) if \( \alpha_i > \alpha'r/2 \). But if \( \alpha'r/2 > \alpha_i \), the corresponding price is negative, which does not make economic sense.

**Remark:** Given Assumption 2, non-negativity of prices is only an issue for interest rates above 200 per cent. More general comments about non-negativity of prices are found in the Appendix.
Proposition 2: A sufficient condition for \( s' = \alpha' \) is \( \sigma > \frac{r}{1-r} \).

Proof: \( \sigma > \frac{r}{1-r} \Rightarrow \gamma > 0 \Rightarrow \frac{dK(s)}{ds} > 0, \forall s \Rightarrow K(\alpha') > K(0) \)

Remark 1: An equivalent formulation of Proposition 2 has \( r < \frac{\sigma}{1-\sigma} \Rightarrow s' = \alpha' \).

Remark 2: \( \frac{r}{1-r} \) is the inverse of the present value of an eternal sequence of \( I \)'s. In other words, the condition in Proposition 2 compares the risk of a squeeze with time preferences.

Corollary (Certain Squeeze): \( \sigma = 1 \Rightarrow s' = \alpha' \).

Proof: Trivial: \( \frac{r}{1-r} < 1, \forall r \geq 0 \).

The necessary and sufficient conditions for \( s' = \alpha' \) and the complementary conditions for \( s = 0 \) are given in Proposition 3 of the Appendix. It is clear from the proof of Proposition 3 that the maximum must be obtained at \( s = 0 \) or at \( s = \alpha' \) since \( K \) is convex in \( s \) and since the domain is restricted to \( [0;\alpha'] \). Here we just indicate in Figure 8 the values of the parameters \( \sigma \) and \( r \) for which the necessary and sufficient conditions will always be fulfilled, where they can never be fulfilled and where the restrictions on \( \alpha' \) and \( \alpha \), compared to \( \sigma \) and \( r \) are effectively binding. We have here chosen what we consider a normal range for \( r \), namely \( r \in [0;0.5] \). We think of our basic unit of time as one month, and month-to-month interest rates very rarely exceed fifty per cent. The conclusion is that only for small squeeze-probabilities and (very) high interest rates is it the case that a monopoly will not hold stocks.

To give an idea of the content of the Appendix, let it suffice to be said that the parameter space is four dimensional and for each vector \( (\alpha,\alpha',r,\sigma) \in \mathbb{R}^4 \times [0;1] \) it is possible to check whether stocks will equal zero or \( \alpha' \). However, it turns out that the conditions can
be expressed in terms of an inequality relating \((\alpha'/\alpha_0)\) to some function of \(\sigma\) and \(r\), and so the problem is reduced to three dimensions: 
\[
(\frac{\alpha'}{\alpha_0}, \sigma, r) \in [0; 1] \times [0; 1] \times \mathbb{R},
\]
where we have used Assumption 2. The general necessary and sufficient condition for optimal stocks to be \(s = \alpha'\) is
where the functional forms of \( \mu \) and \( \nu \) are given in the Appendix. Zero stocks are preferred whenever the inequality is reversed.

This finalises the discussion of the profitability of stocks given that the producers cooperate. We now turn to a discussion of whether such cooperative outcomes is sustainable in a non-cooperative repeated game featuring squeezes and regulatory stocks.

### 2.2 The Producers’ Repeated Game

It is well known from the literature on repeated games with observed actions that cooperative outcomes can be sustained as subgame perfect equilibria of repeated games, this leading to payoffs over and above the equilibrium payoffs of the stage game, cf. for an early example Friedman (1971). These results are known as 'folk theorems' and roughly maintain that any outcome that all players prefer to a Nash equilibrium of the stage game can be a subgame perfect equilibrium of the repeated game if the interest rate is sufficiently low. The cooperative behaviour is sustained by the threat to revert to the Nash equilibrium of the stage game if a deviation occurs and play this equilibrium forever after. This punishment strategy is in itself (trivially) subgame perfect and thus a credible threat to lower the payoff of the deviator (and everybody else) in all periods following the deviation. If a potential deviator cares sufficiently about future payoffs, i.e. if she has a sufficiently high discount rate or a correspondingly low interest rate, then deviation is deterred by the threat. The deviator compares the immediate gain from deviating with the discounted loss from the eternal punishment starting the following period. There will be a threshold of the interest rate such that for all values above it, cooperation cannot be sustained. The questions that are treated in this section are whether a duopoly would hold stocks and how this and the possibility of a squeeze affect the scope for cooperation.

The analysis of the \( \pi \)-firm oligopoly case in which a squeeze may occur and stocks can be held follows similar considerations but gets somewhat more intricate because the interest rate and the probability of a squeeze enter the profit functions in a non-linear manner.
Before a squeeze, the firms’ profits are

\[
E(\Pi_i(s)) = (1 - \sigma)(\alpha_i - x)(x_i - rs) - \sigma(\alpha_i - x)(x_i - s)
\]

\[
= (\alpha_i - x)(x_i - rs), \quad i = 1, 2, \ldots, n
\]  \hspace{1cm} (21)

where \( x_i = \sum_{i=1}^{n} x_{it} \), \( s_i \) is firm \( i \)'s stock \( (s = \sum_{i=1}^{n} s_i) \), and equilibrium profits turn out to be

\[
E(\Pi_i) = \left( \frac{\alpha_i + \gamma s}{n-1} \right)^2, \quad \forall i.
\]  \hspace{1cm} (22)

After a squeeze, maximised profits are

\[
\Pi_i = \left( \frac{\alpha_i - (\alpha - s)}{n-1} \right)^2, \quad \forall i,
\]  \hspace{1cm} (23)

where we have used that the demand intercept equals \( \alpha_i - (\alpha' - s) \) after a squeeze of size \( \alpha' \) that was met with sales, \( s \), of stocks.

Importantly we get

**Proposition 7:** In equilibrium, the profits of the producers in \( n \)-firm oligoply do not depend on their share of overall stocks (i.e. on \( s_i \)), only on the level of overall stocks, \( s \).

**Proof:** Examination of (22) and (23) reveals that the expected profit before a squeeze and the profit after a squeeze depend on \( s \) but not on \( s_i \).

Proposition 7 means that if non-cooperative oligopolists agree that a certain level of stocks would be optimal to (partially) prevent a squeeze, then it does not matter whether they split the stocks equally or whether, say, one of them holds all stocks. This is so, because in equilibrium (before a squeeze) prices are set to balance the cost of holding stocks with the possible gain from holding these stocks should a squeeze occur.
Furthermore, the producers will agree on the optimal level of stocks and that level will coincide with that of the monopolist:

**Proposition 8**: In equilibrium, a certain level of stocks, $s$, is optimal to a non-cooperating oligopolist if and only if it is optimal for the monopolist.

**Proof**: Expected, discounted profits are

$$E(\Pi) = \frac{1-r}{\sigma - r} \frac{1}{(n-1)^2} \left[ (\alpha_i - \gamma s)^2 - \frac{\sigma}{r} (\alpha_i - \alpha' - s)^2 \right]$$

but $K(s)$ was exactly the function used to determine optimality of $s$ for the monopoly (see (19)).

**Remark**: For a given $s$, single firm oligopoly profit is the fraction $(2/(n+1))^2$ ($n \geq 1$) of the monopoly profit both in the short run (stage game) and in the long run (expected discounted profit).

Proposition 8 means that the oligopoly will hold either zero stocks or stocks equal to $\alpha'$, depending on the values of $r$ and $\sigma$ and possibly also of $\alpha'/\alpha$, exactly as would the monopoly, so Figure 8 and the analysis in the Appendix can be applied without modification.

The question that now comes to mind is whether the possibility of a squeeze and the ability to hold stocks change the firms’ incentive to deviate by increasing production. It does and quite significantly so. Observe that the profits mentioned in the proof of Proposition 8 are the result of the Nash equilibrium of the stage game. Thus if a deviator deviates from a situation of cooperation at time $t$, she will expect these profits from $t+1$. Since the discount rate is $1/(1+r)$, the discounted value at $t$ is $(\sigma+r)^1(n+1)^2K(s)$. This is the threat that may or may not prevent deviation, depending on $s$, $\alpha'$, $\alpha$, $\sigma$ and $r$.

To get a benchmark, firstly consider the standard model without the complications arising from squeezes and stocks. There is an incentive to deviate iff
\[ r > \frac{1 - \frac{4n}{(n-1)^2}}{\frac{4n}{(n-1)^2} - 1} = \frac{1 - m^{-1}}{m - 1} = C_0(n), \]  

where \( m = \frac{(n-1)^2}{4n} \). The critical value of \( r, C_0(n) \), decreases from .89 for \( n = 2 \) to .49 for \( n = 6 \) to .33 for \( n = 10 \), as also seen in Table 3.

<table>
<thead>
<tr>
<th>( n )</th>
<th>( C_0(n) )</th>
<th>( C_0(n) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>8/9</td>
<td>.89</td>
</tr>
<tr>
<td>3</td>
<td>3/4</td>
<td>.75</td>
</tr>
<tr>
<td>4</td>
<td>16/25</td>
<td>.64</td>
</tr>
<tr>
<td>5</td>
<td>5/9</td>
<td>.55</td>
</tr>
<tr>
<td>6</td>
<td>24/49</td>
<td>.49</td>
</tr>
<tr>
<td>7</td>
<td>7/16</td>
<td>.44</td>
</tr>
<tr>
<td>8</td>
<td>32/81</td>
<td>.40</td>
</tr>
<tr>
<td>9</td>
<td>9/25</td>
<td>.36</td>
</tr>
<tr>
<td>10</td>
<td>40/121</td>
<td>.33</td>
</tr>
</tbody>
</table>

In 1984 the concentration in the market for Brent has been calculated to correspond to 4.4 equal sized firms as measured by the inverse of the Herfindahl index (see Mabro (1986) pp. 40-45). The similar numbers for the British part of the North Sea and for the entire North Sea (i.e. including Norway’s part) were 5.66 and 8.23 respectively. Since 1984, the
British National Oil Corporation has been abolished and the Brent Blend has been redefined to include oil from other fields. Both of these events tended to decrease concentration (increase the equivalent number of equal sized firms) and for this reason, we have set \( n = 6 \) in the ensuing analysis. Note that a too high \( n \) tend to favour a non-cooperative outcome by lowering the critical value whereas a too low \( n \) has the opposite effect. The degree to which the critical value is over- or undervalued is indicated by Table 3. The critical value \( C_6(6) = 24/49 \approx .49 \) can be thought of as a benchmark in the following.

Now consider the incentive to deviate starting from a situation where a monopolist (and an oligopoly) would choose \( s = 0 \). In this case, we require \( s = 0 \) for all firms and assume that each firm produces \( 1/n \)'th of the monopoly output. There is an incentive to deviate if

\[
 r > \frac{-B \sqrt{B^2 - 4\sigma k^2/m}}{2(m-1)} = C(\sigma,k,n; s = 0),
\]

where \( B = m\sigma - \alpha \sigma - 1 + 1/m \) and where \( k = \frac{\alpha_i - \alpha'}{\alpha_i} \) is the percentage of the market that remains after a squeeze. Critical values for \( r, C(\sigma,k,n=6; s = 0) \) are found in Table 4 below. It is seen that the possibility of an unprevented squeeze reduces the scope for cooperation unless either the probability or the size of the squeeze is zero \((k = 1)\). It is not surprising that the first row of Table 4 falls rapidly to zero as \( \sigma \) increases, since a possible squeeze of size \( \alpha' = \alpha_i \) eats away the entire market and all future profits. In the last row, the size of the squeeze is zero and its probability does not matter, wherefore we get the no-squeeze critical value of \( 24/49 \). The same applies for the first column where the probability is zero and the size does not matter. In all other cases, the value is lower than the benchmark value of 0.49. For a given size of the squeeze \((a given k)\), the critical value of \( r \) falls dramatically as the probability of the squeeze increases - more so for big squeezes \((small k's)\) than for small ones. Similarly, for a given probability of a squeeze, the critical value falls off rapidly as the size of the squeeze gets bigger \((k \text{ goes from } 1 \text{ to } 0)\).

For \( \sigma = 0.4 \) and \( k = 0.5 \) we have the \( (six) \) oligopolists deviating if \( r > 0.143 \). But for
### TABLE 4: Critical Values for \( r, s = 0, n = 6 \).

<table>
<thead>
<tr>
<th>( k )</th>
<th>( \sigma )</th>
<th>0</th>
<th>0.2</th>
<th>0.4</th>
<th>0.6</th>
<th>0.8</th>
<th>1</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0.49</td>
<td>0.098</td>
<td></td>
<td>0</td>
<td></td>
<td></td>
<td>0</td>
</tr>
<tr>
<td>0.1</td>
<td>0.49</td>
<td>0.109</td>
<td>0.007</td>
<td>0.004</td>
<td>0.004</td>
<td>0.003</td>
<td></td>
</tr>
<tr>
<td>0.2</td>
<td>0.49</td>
<td>0.135</td>
<td>0.026</td>
<td>0.017</td>
<td>0.015</td>
<td>0.014</td>
<td></td>
</tr>
<tr>
<td>0.3</td>
<td>0.49</td>
<td>0.168</td>
<td>0.056</td>
<td>0.039</td>
<td>0.034</td>
<td>0.031</td>
<td></td>
</tr>
<tr>
<td>0.4</td>
<td>0.49</td>
<td>0.205</td>
<td>0.095</td>
<td>0.071</td>
<td>0.062</td>
<td>0.057</td>
<td></td>
</tr>
<tr>
<td>0.5</td>
<td>0.49</td>
<td>0.246</td>
<td>0.143</td>
<td>0.112</td>
<td>0.099</td>
<td>0.093</td>
<td></td>
</tr>
<tr>
<td>0.6</td>
<td>0.49</td>
<td>0.289</td>
<td>0.199</td>
<td>0.164</td>
<td>0.148</td>
<td>0.139</td>
<td></td>
</tr>
<tr>
<td>0.7</td>
<td>0.49</td>
<td>0.335</td>
<td>0.261</td>
<td>0.228</td>
<td>0.210</td>
<td>0.200</td>
<td></td>
</tr>
<tr>
<td>0.8</td>
<td>0.49</td>
<td>0.384</td>
<td>0.331</td>
<td>0.303</td>
<td>0.287</td>
<td>0.277</td>
<td></td>
</tr>
<tr>
<td>0.9</td>
<td>0.49</td>
<td>0.436</td>
<td>0.407</td>
<td>0.390</td>
<td>0.380</td>
<td>0.372</td>
<td></td>
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<tr>
<td>1</td>
<td>0.49</td>
<td>0.49</td>
<td>0.49</td>
<td>0.49</td>
<td>0.49</td>
<td>0.49</td>
<td></td>
</tr>
</tbody>
</table>

**LEGEND TO TABLES 4 & 5:** The tables show the critical value \( C(\sigma, r, n=6 ; s) \) for the interest rate: if \( r \) is greater than the value in a cell, there is an incentive to deviate and cooperation cannot be sustained. \( k \) is the percentage of the market that would be left after a squeeze if the squeeze were unprevented, \( k = \frac{\alpha_i - \alpha^2}{\alpha_i} \). \( \sigma \) is the probability of a squeeze.
### TABLE 5: Critical Values for $r, s = \alpha^2, n = 6$. 

<table>
<thead>
<tr>
<th>$k$</th>
<th>$\sigma$</th>
<th>0</th>
<th>0.2</th>
<th>0.4</th>
<th>0.6</th>
<th>0.8</th>
<th>1</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0.49</td>
<td>(1.95)</td>
<td>0.366</td>
<td>0.200</td>
<td>0.129</td>
<td>0.093</td>
<td></td>
</tr>
<tr>
<td>0.1</td>
<td>0.49</td>
<td>(2.12)</td>
<td>0.380</td>
<td>0.217</td>
<td>0.143</td>
<td>0.103</td>
<td></td>
</tr>
<tr>
<td>0.2</td>
<td>0.49</td>
<td>(2.32)</td>
<td>0.395</td>
<td>-0.236</td>
<td>0.159</td>
<td>0.117</td>
<td></td>
</tr>
<tr>
<td>0.3</td>
<td>0.49</td>
<td>0.682</td>
<td>0.409</td>
<td>0.258</td>
<td>0.179</td>
<td>0.133</td>
<td></td>
</tr>
<tr>
<td>0.4</td>
<td>0.49</td>
<td>0.614</td>
<td>0.422</td>
<td>0.282</td>
<td>0.202</td>
<td>0.153</td>
<td></td>
</tr>
<tr>
<td>0.5</td>
<td>0.49</td>
<td>0.577</td>
<td>0.435</td>
<td>0.309</td>
<td>0.229</td>
<td>0.178</td>
<td></td>
</tr>
<tr>
<td>0.6</td>
<td>0.49</td>
<td>0.551</td>
<td>0.448</td>
<td>0.339</td>
<td>0.262</td>
<td>0.210</td>
<td></td>
</tr>
<tr>
<td>0.7</td>
<td>0.49</td>
<td>0.531</td>
<td>0.459</td>
<td>0.373</td>
<td>0.303</td>
<td>0.251</td>
<td></td>
</tr>
<tr>
<td>0.8</td>
<td>0.49</td>
<td>0.515</td>
<td>0.470</td>
<td>-0.409</td>
<td>0.353</td>
<td>0.306</td>
<td></td>
</tr>
<tr>
<td>0.9</td>
<td>0.49</td>
<td>0.501</td>
<td>0.480</td>
<td>0.448</td>
<td>0.414</td>
<td>0.382</td>
<td></td>
</tr>
<tr>
<td>1</td>
<td>0.49</td>
<td>0.49</td>
<td>0.49</td>
<td>0.49</td>
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<td>0.49</td>
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</tr>
</tbody>
</table>

**LEGEND TO TABLE 5:** The incentive to deviate describes a fourth-degree polynomial in $r$, which typically (but not always) have one negative and three positive roots. The numbers in Table 5 are the lowest of the positive roots. Numbers in parentheses correspond to the third positive root in that for these values of $k$ and $\sigma$, there were two complex roots. See also text after (26).
this value of $\sigma$ and for all $r$ between 0 and 1.3973,\(^9\) that is for interest rates up to 139.73 per cent, the oligopolists should hold stocks, $s = \alpha'$, according to the necessary and sufficient conditions and in this case Table 5 is the one to look at. In Table 5 the critical value is found to be $C(0.5,0.5,n = 6; s = \alpha') = 0.435$, so that the duopolists should cooperate if the (month-to-month) interest rate does not exceed 43.5 per cent.

If $s = \alpha'$, the producers will deviate iff

$$\frac{1}{n-1} K(\alpha') - \frac{1}{\sigma - r} \frac{1}{(n-1)^2} K(\alpha') > \frac{1}{n} \frac{1}{\sigma - r} K(\alpha')$$  \hspace{1cm} (26)$$

The first term is the deviators expected profit in the period in which deviation takes place. The second term is the discounted value of the ensuing sequence of non-cooperative profits. The r.h.s. is $1/n$'th of expected discounted monopoly profits. (26) with equality describes a fourth degree polynomial in $r$ (remember that $\gamma$ is a function of $\sigma$ and $r$) where there generally is one negative root and possibly three positive roots. A general analytical solution to (26) was not found and numerical methods were applied to generate Table 5, which gives the first (lowest) positive root with $n = 6$. For interest rates above the first positive root and below the second, cooperation cannot be sustained, but if the interest rate is between the second and the third positive root cooperation is again sustainable. For interest rates above the third positive root deviation is to be expected again. Only the first positive root is tabled here since the second and the third root both are well above 1. As an example take $k = 0.5$, $\sigma = 0.4$ and $n = 6$. Then the first positive root is 0.435 as seen in Table 5, the second is 2.698 and the third is 4.948.

In Table 5, the critical value of $r$ is seen to vary much more with $\sigma$ than with $k$: the size of a squeeze attempt matters less now that it is prevented.

Comparing Tables 4 and 5, it can be concluded that for all probabilities and all sizes of the squeeze, the case for cooperation is stronger with than without stocks. This follows partly from the fact that $K(0) < K(\alpha')$ for most of the reported values of $\sigma$ unless $r$ is very high, so that future profits are worth more with than without stocks. In fact, if the interest rate is below the critical values in Table 5 (ignoring the $\sigma = 0$ column), then in all but five cases

\(^9\) That is for $r$ such that $\sigma = \sigma_4 = 0.5$, see the appendix.
the oligopolists should hold stocks. The five cases are the five first entries of the $\sigma = 0.2$ column. In these cases, if $r < 0.5196$, producers will hold stocks and will cooperate. The general conclusion from this analysis is that if the producers cooperate, they should also hold stocks. In a few cases this conclusion is reversed: if they hold stocks, they should also cooperate.

A caveat is appropriate here: Table 5 and inequality (26) are only valid in the completely symmetric case where the producers produce $1/n$'th of monopoly output and hold $1/n$'th of the stocks each. Only if the latter is also the case will each producer's expected profit be $1/n$'th of expected monopoly profit. At first sight this may seem at odds with Proposition 7. This, however is not the case: Proposition 7 describes a situation of Nash equilibrium in the stage game, whereas (26) describes the incentive to deviate from the cooperative scenario. Under the restriction that the $n$ firms' stocks sum to $\alpha'$, the asymmetric case adds the following term to the right hand side of (26):

$$\frac{1}{2} \frac{1-r}{\sigma - r} \left( \frac{n-1}{4n} \right) \left( \alpha_i - \gamma \alpha' \right) - \frac{1}{4} \gamma \left( s_D - \frac{\alpha'}{n} \right) \left( s_D - \frac{\alpha'}{n} \right).$$

$s_D$ is the potential deviator's stock. (27) is the effect on cooperation of asymmetric stocks. If the effect is negative, deviation is more likely to occur. The term in the square brackets can be shown to be positive for all values of $s_D$ between 0 and $\alpha'$, so the effect shares the sign with $(s_D - \alpha'/n)$: If a producer holds $s_D < \alpha'/n$, he is less likely to cooperate, whilst a producer that holds $s_D > \alpha'/n$ is more likely to do so. In particular, if one producer holds all of the stocks, he is less likely to deviate but all the other producers would be more inclined to do so. Whether they would in fact do so, depends on $k$, $\sigma$, $r$ and $n$.

Deviating by increasing production may not be an issue in the Brent market, where one producer (Shell UK) is in charge of organising liftings (see Philips (1992)). A production

---

10 $r$ such that $\sigma_4(r) = 0.2$.

11 We assume that the price is positive so $\alpha_i + \gamma \alpha' > 0$. Then note that $\gamma \leq 1$ and $2(1)/(4n) < 1/4$ for all $n \geq 1$. The second, negative term takes its minimum for $s_D = \alpha'$, but for this value it is a matter of manipulation to show that the entire expression is positive.

12 For a more general treatment of asymmetric oligopolies, see Waldmann (1991).
schedule is compiled well ahead of time and is negotiated and approved by the other producers. Production is thus "observed before it happens" and the response to a deviation could therefore be simultaneous rather than delayed a period, thus further discouraging deviation.

The overall conclusion of Section 2 is that unless the probability of a squeeze is very small or the (month-to-month) interest rate very high, self-regulation by means of squeeze-preventing stocks should be possible.

3. Conclusions and Extensions

The first four squeezes mentioned in Section 1 (see Table 1) occurred within a time range of three years (1987 - 1989), i.e. thirty six months. A crude estimate of $\sigma$ in this period is therefore $4/36 = 1/9 = 0.11$. For there to be any doubt that the producers should hold stocks to prevent the effects of a squeeze, the (month-to-month) interest rate should have been at least 33 pct. according to Proposition 4 of the Appendix. According to the sufficient condition (see remark to Proposition 2), $r < (1/9)/(8/9) = 0.125$ is a sufficient condition for $s' = \alpha'$. This condition was definitely met in the mentioned period. (A 12.5 pct. monthly interest rate corresponds to an interest rate of approximately 310 pct. on a yearly basis). So why did the producers not introduce regulatory stocks (or other squeeze-preventing mechanisms, for that matter)?

One partial answer is that they actually did release some cargoes in the December 1987 squeeze and a single cargo in the November 1989 squeeze was potentially made available, but these efforts were far from efficient - they "failed to prevent a serious distortion from developing in the spectrum of oil prices".

Another answer may be that the model may favour the incentive to keep stocks (or the model may be correct, but the producers do not realise this).

One feature of the model that may seem too strong is the assumption that $(\alpha' - s)$ market participants (namely the squeezed traders/refineries) leave the market immediately after a squeeze has occurred. In reality, they might leave (or reduce the amount traded) gradually over time because it takes time for traders to develop other markets to work in or for refineries to find other crudes to substitute Brent. It may also very well be that the market

---

13 If $r = 0.332215$, $\sigma_0(r) = 0.111111$. 

participants get uptight about the market immediately after a squeeze and contemplate to leave it for a while, but then relax and continue trading. Indeed, it seems boundedly rational that the same company can squeeze twice within eight months: the traders, knowing the squeezer from the first squeeze, should think that trading with him again may well mean trouble. At any rate, if the producers think that the market will not lose "customers", their incentive to assuage the effects of a squeeze is of course non-existent.

On the other hand, the increasing popularity of the International Petroleum Exchange (IPE) of London may be a response to the malfunctioning of the 15-Day market. The IPE trades futures in units of 1.000 barrels of Brent blend, i.e. 1/500'th of the size of the 15-Day contract. The IPE is a traditional futures market and thus features all the regulations you expect from such a market. The producers have traditionally favoured the 15-Day market - but maybe their customers more and more prefer to trade on the IPE?

Further to this, our model supposes that a squeeze can only happen once. It happens as mentioned frequently. Including this in the model would require a genuinely dynamic model (the demand intercept would be a non-increasing stochastic variable), but ceteris paribus our one-squeeze model would tend to underplay the role of stocks compared to a frequent-squeeze model.

Another feature of the model that may seem to favour stocks is the cost of storage. We have assumed that this cost only arises from the interest on the value of the stocks - what it costs to keep the crude off the market. In the real world, there may be substantial costs to keeping cargoes afloat or to renting tanks in Rotterdam. This would reduce the incentive to hold stocks. Stocks are furthermore used for a variety of other purposes, including strategic motives in games between the producers (on this, see Møllgaard and Philips (1992)), and producers seem to be secretive about the size of their stocks. (Our model actually requires that the size of the stocks be common knowledge).

Finally, an argument that weighs against self-regulation is that some major producers are part of integrated oil companies: Squeezes may hurt independent refineries without affecting integrated refineries. In the long run, this will reduce competition among refineries and benefit "survivors": the integrated companies.

The model presupposes that the size of a potential squeeze and the probability of a squeeze are known. In reality, there might be more uncertainty involved. Introducing probability distributions over squeezes of any size (non-negative numbers of cargoes, say),
would not alter the conclusions qualitatively.

The real world is of course much more complex and dynamic than the model of Section 2: demand and interest rates vary with time; the producers do not form a symmetric oligopoly; integrated oil companies complicate matters further. However, it is our belief that the model and the surrounding analysis shed light on the problem. The conclusion is clear: Self-regulation is possible.
References:


Appendix: Necessary and Sufficient Conditions, Parameter Space and Non-Negativity

s* designates optimal stocks.

Proposition 3: Necessary and sufficient conditions:

\[ s^* = \alpha^* \iff v \alpha^* - \mu \alpha_i \geq 0, \]

\[ s^* = 0 \iff v \alpha^* - \mu \alpha_i \leq 0, \]

where \( v = \gamma^2 r - \sigma \) and \( \mu = 2(\gamma r - \sigma) \).

Proof: \( K(s) \) is globally convex in \( s \):

\[ \frac{d^2 K(s)}{ds^2} = 2 \left( \frac{\gamma^2 - \sigma}{r} \right) > 0, \quad \forall \alpha, r > 0. \tag{A.2} \]

Thus it suffices to compare the values of \( K(s) \) at the lower and at the upper bounds for \( s \), i.e. \( K(0) \) and \( K(\alpha^*) \). It is easily checked that

\[ v \alpha^* + \mu \alpha_i > 0 \Rightarrow K(\alpha^*) > K(0) \Rightarrow s^* = \alpha^*; \text{and } s^* = \alpha^* \Rightarrow K(\alpha^*) \geq K(0) \Rightarrow v \alpha^* + \mu \alpha_i \geq 0. \]

\[ v \alpha^* + \mu \alpha_i < 0 \Rightarrow K(\alpha^*) < K(0) \Rightarrow s^* = 0; \text{and } s^* = 0 \Rightarrow K(\alpha^*) \leq K(0) \Rightarrow v \alpha^* + \mu \alpha_i \leq 0. \]

\[ v \alpha^* + \mu \alpha_i = 0 \Rightarrow K(\alpha^*) = K(0) \Rightarrow \{s^* = 0 \lor s^* = \alpha^*\}. \]

We state the following three lemmas without proof. (A parenthetical remark on notation: \( \sigma = \sigma(r) \) is taken to mean that \( r, \sigma \) = \( (r, \sigma(r)) \). Similarly, \( \sigma \in \{ \sigma(r) \}, \sigma(r) \} \) is taken to mean that \( (r, \sigma): \sigma(r) < \sigma < \sigma(r) \).

Lemma 1: \( \{\sigma = \sigma_1, \forall \sigma \neq \sigma_2\} \iff v = 0, \) where

\[ \sigma_1 = \frac{1 - 2r^2 - 2r^3 - \sqrt{1 - 4r^2 - 4r^3}}{2r(1 + r^2)}, \tag{A.3} \]

\[ \sigma_2 = \frac{1 - 2r^2 - 2r^3 - \sqrt{1 - 4r^2 - 4r^3}}{2r(1 + r^2)}. \]

Lemma 2: \( \sigma = \sigma_3 \iff \mu = 0, \) where \( \sigma_3 = \frac{r^2}{1 - r - r^2} \).
Lemma 3: \( \{ \sigma = \sigma_4 \lor \sigma = \sigma_5 \} \Leftrightarrow -\frac{\mu}{\nu} = 1 \), where

\[
\begin{align*}
\sigma_4 &= \frac{-\frac{1}{2} - r - r^3 - \sqrt{\frac{1}{4} - r - r^2 - r^3 - r^4}}{r(1 + r)^2} \\
\sigma_5 &= \frac{-\frac{1}{2} - r - r^3 - \sqrt{\frac{1}{4} - r - r^2 - r^3 - r^4}}{r(1 + r)^2}
\end{align*}
\] (A.4)

Lemma 4: \( \sigma_2 > \frac{r}{1 - r} > \sigma_4 > \sigma_3 > \sigma_i > \sigma_5 \), \( \forall r > 0 \).

(See Figure A.1)

Proof of Lemma 4:

1) \( \sigma_2 > r/(1 + r) \):

\[
r > 0 \Rightarrow 1 - \sqrt{1 - 4r^2 - 4r^3} > 0 \\
\Rightarrow 1 - 2r^2 - 2r^3 - \sqrt{1 + 4r^2 - 4r^3} > 2r^2(1 - r) \\
\Rightarrow \sigma_2 > \frac{r}{1 - r} \]

2) \( r/(1 + r) > \sigma_4 \):

\[
r > 0 \Rightarrow (1 - r - r^2)^2 = 1 - 2r - 3r^2 - 2r^3 + r^4 > \frac{1}{4} - r - r^2 - r^3 - r^4 = a \\
\Rightarrow 1 - r - r^2 > \sqrt{a} \\
\Rightarrow r^2(1 - r) > 1 - r - r^3 - \sqrt{a} \\
\Rightarrow \frac{r}{1 - r} > \sigma_4 \]
3) $\sigma_4 > \sigma_3$:

Let $b = \left(\frac{1}{2} - \frac{3}{2}r - \frac{3}{2}r^2 - r^3 + r^4\right)^2$
and $c = a(1 - r + r^3)^2$.

It is easily checked that

\[ r > 0 \Rightarrow c > b > 0 \Rightarrow \sqrt{c} > \sqrt{b} \Rightarrow \sigma_4 > \sigma_3 \]

4) $\sigma_4 > \sigma_4$: Follows from expanding the expressions and cancelling terms.

5) $\sigma_3 > \sigma_5$:

\[ r > 0 \Rightarrow 1 - r^2 - \sqrt{\frac{1}{4} - r^2 - r^3} > r - \sqrt{\frac{1}{4} - r - r^2 - r^3} \]
\[ \Rightarrow 1 - 2r^2 - 2r^3 - \sqrt{1 - 4r^2 - 4r^3} > -1 - 2r - 2r^3 - 2 \sqrt{\frac{1}{4} - r - r^2 - r^3} \]
\[ \Rightarrow \sigma_3 > \sigma_5 \]

Transitivity of $>$ completes the proof of Lemma 4.
Figure A1: The \((r, \sigma)\)-Plane

\[
\sigma = \frac{r}{1 + r}
\]

\[
\sigma_i, \sigma_j, \sigma_k
\]
Proposition 4: Division of the Parameter Space (See Figure A.2)

\( \forall \sigma \in [\sigma_1; 1], \ s^* = \alpha^i. \)  

(I)

\( \forall \sigma \in [\sigma_3; \sigma_4], \ 0 < -\frac{\mu}{\nu} \leq 1 \iff \left\{ s^* = \alpha^i \iff \frac{\alpha^i}{\alpha_i} \leq -\frac{\mu}{\nu}, \ s^* = 0 \iff \frac{\alpha^i}{\alpha_i} \geq -\frac{\mu}{\nu} \right\} \)  

(II)

\( \forall \sigma \geq 0 \ \sigma \in [\sigma_3; \sigma_4], \ s^* = 0. \)  

(III)

\( \forall \sigma \geq 0 \ \sigma \in [0; \sigma_1], \ 0 \leq -\frac{\mu}{\nu} \leq 1 \iff \left\{ s^* = \alpha^i \iff \frac{\alpha^i}{\alpha_i} \geq -\frac{\mu}{\nu}, \ s^* = 0 \iff \frac{\alpha^i}{\alpha_i} \leq -\frac{\mu}{\nu} \right\} \)  

(IV)

Proof: The proof follows the division (I, II, III and IV) of the Proposition.

(I) We divide this region of the parameter space into two sub-regions:

(I.a) \( \sigma \in [\sigma_3; r/(1+r)] \)

(I.b) \( \sigma \in [r/(1+r); 1] \).

In region (I.a), \( \gamma < 0 \), \( \nu < 0 \), \( \mu > 0 \) and \( -\mu/\nu \geq 1 \). The necessary and sufficient condition (A.1) for stocks (i.e. \( s^* = \alpha^i \)) becomes

\[ \frac{\alpha^i}{\alpha_i} \leq -\frac{\mu}{\nu}. \]

But the left hand side is by Assumption 2 smaller than or equal to one and the r.h.s. greater than or equal to one so the condition is always met.

Region (I.b) is exactly the sufficient condition of Proposition 2.

(II) In this region we have \( \gamma < 0 \), \( \nu < 0 \), \( \mu > 0 \) and \( 0 < -\mu/\nu \leq 1 \) and so the necessary and sufficient condition is binding.

(III) We divide this region into two subregions:

(III.a) \( \sigma \in [\sigma_1; \sigma_3] \)

(III.b) \( \sigma \in [\sigma_3; \sigma_1] \).
(III.a): Here we have $v \leq 0$, $\mu \leq 0$ and the necessary and sufficient condition for $s^* = \alpha'$ becomes

$$\frac{\alpha'}{\alpha_i} < -\frac{\mu}{\nu} < 0$$

for $v < 0$ (off $\sigma_1$) and $\mu \geq 0$ for $v = 0$ (on $\sigma_1$), both of which are impossible. So we have $s^* = 0$.

(III.b): In this region $v > 0$, $\mu < 0$ and $-\mu/\nu \geq 1$, so the necessary and sufficient condition for $s^* = \alpha'$ is $\alpha'/\alpha_i > -\mu/\nu$, which is impossible by Assumption 2.

(IV): $v > 0$, $\mu < 0$ and $0 \leq -\mu/\nu \leq 1$, so the necessary and sufficient condition for $s^* = \alpha'$ is $\alpha'/\alpha_i > -\mu/\nu$, which is binding. Note, however, that $s^* = \alpha'$ gives rise to a negative price, cf. below. ■

Remark: The reader may by now wonder what happened to $\sigma_2$. It is hidden in region (I.b):

$$\forall \sigma \leq 1: \sigma \in [\sigma_2, 1], v > 0, \mu > 0 \text{ and } v\alpha_i - \mu \alpha_i$$

will always hold true. For $\sigma \in [r/(1+r), \sigma_2]$ we have $v < 0$, $\mu > 0$ and $-\mu/\nu > 1$ so the condition for $s^* = \alpha'$ is always satisfied.

Remark: The function $-\mu/\nu = f(r, \sigma)$ has singularities along $\sigma_1$ and $\sigma_2$, where $v = 0$. Fix $r$ at, say, $r = 3$, and consider the function as $\sigma$ goes from 0 to 1 (See Figure A.3): As $\sigma$ goes from 0 to $\sigma_3$, $f(3, \sigma)$ goes from $2/3$ to 1. As $\sigma$ goes from $\sigma_3$ to $\sigma_n$, $f(3, \sigma)$ goes from 1 to $\infty$. As $\sigma$ goes from $\sigma_i$ to $\sigma_n$, $f(3, \sigma)$ goes from $-\infty$ to 0. As $\sigma$ goes from $\sigma_3$ to $\sigma_n$, $f(3, \sigma)$ goes from 0 to 1. As $\sigma$ goes from $\sigma_i$ to $\sigma_2$, $f$ goes from 1 to $\infty$. Finally, as $\sigma$ goes from $\sigma_2$ to 1, $f$ goes from $-\infty$ to $-4$. © The Author(s). © The Author(s). European University Institute. Digitised version produced by the EUI Library in 2020. Available Open Access on Cadmus, European University Institute Research Repository.
Figure A3: \( f(r = 3, \sigma) \)
Non-Negativity:

**Proposition 5:** Production is always positive.

**Proof:**

\[
x = \frac{1}{2} (\alpha_i - \gamma s) > 0 \iff \alpha_i > \gamma s \iff \alpha_i - \alpha s > -(1 - \sigma) rs,
\]

but the l.h.s. is always positive and the r.h.s. always non-positive.\[\]

**Lemma 5:** \(\forall (\sigma, r) \in [0, 1] \times \mathbb{R}, \gamma < -1: \frac{1}{\gamma} < -\frac{\mu}{\eta}\)

**Proof:** \[\{ \eta < -1 \iff \sigma < \frac{r}{2 - r - r^2} \} \Rightarrow \{ \eta > 0 \iff \mu < 0 \} \Rightarrow \left\{ \frac{1}{\gamma} < -\frac{\mu}{\eta} \iff \eta < \gamma \mu \right\}\]

\[
\sigma < \frac{r^2}{2 - r - r^2} \Rightarrow \gamma r - 2 \sigma < 0 \Rightarrow \gamma (\gamma r - 2 \sigma) > 0 \Rightarrow -\sigma \Rightarrow \gamma \mu > \eta.
\]

But the condition was that \(\sigma < \frac{r - 1}{r^2} < \frac{r^2}{2 - r - r^2}\), which then proves sufficient.\[\]

**Proposition 6:** In region (IV) (i.e. \(\{(\sigma, r) \in [0; 1] \times [2; \infty): \sigma \leq \sigma_5(r)\}\), \(s^* = \alpha^*\) implies a negative price.

**Proof:** By Proposition 5 we have that \(s^* > \alpha^* \iff \frac{\alpha^*}{\alpha_i} > -\frac{\mu}{\eta}\). Non-negativity of prices

\[
p = \frac{1}{2}(\alpha_i - \gamma \alpha^* ) > 0 \Rightarrow \frac{\alpha^*}{\alpha_i} < -\frac{1}{\gamma} (\gamma < 0), \text{ but by Lemma 5 we have } \frac{1}{\gamma} < -\frac{\mu}{\eta}, \text{ so}
\]

prices are negative if \(\frac{\alpha^*}{\alpha_i} > -\frac{\mu}{\eta}\)\[\]

**Remark:** The reason for this oddity is that the expected sales less the expected real cost of storage become largely negative due to very high interest rates \((r > 200 \%)\) and very large stocks. Multiplied by a negative price, profits become positive. The choice of \(s = 0\) always leads to positive prices, quantities and profits.
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