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by

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ABSTRACT

This paper discusses the properties of the univariate Dickey-Fuller test and the Johansen test for the cointegrating rank when there exist additive outlying observations in the time series. The model considered in the paper has some similarities with the classical measurement error model. We provide analytical as well as numerical evidence that additive outliers may produce spurious stationarity. Hence the Dickey-Fuller test will too frequently reject a unit root and the Johansen test will indicate too many cointegrating vectors. Through an empirical example we show how dummy variables can be used to remove this caveat.

Keywords: Unit Roots; Additive Outliers; Maximum likelihood cointegration.

JEL Codes: C22; C32.

AUTHORS' NOTES

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1. INTRODUCTION.

Testing for unit roots in univariate and multivariate time series has become a standard practice when modelling economic time series. This is because one of the main characteristics of such time series is that they show nonstationary patterns, although there may be occasions in which linear combinations of the series are stationary. In some recent applications as Brodin and Nymoen (1992) and Hendry and Mizon (1990), the modelling strategy is, first, to use the Dickey-Fuller (1979) test to check whether each time series is nonstationary, and, second, to apply the Johansen (1988) maximum likelihood approach to check whether linear combinations of the series are stationary. Given that the time series are nonstationary, the practitioner has to rely on critical values that have been simulated under the assumption of Gaussian error processes.

Besides nonstationarity, other properties of many economic time series are that the variance is time-varying, that there are structural breaks, and that there are outlying (or extreme) observations. Given the underlying assumptions of the generated critical values, it seems worthwhile to investigate whether these additional properties can effect proper practical inference. When the variance process is specified as some kind of an autoregressive conditional heteroscedastic type, it is shown in Kim and Schmidt (1991) and Haldrup (1992) that except from some borderline cases the empirical distributions of unit roots test statistics are generally not much affected. In Franses, Kofman and Moser (1992) it is argued that similar conclusions can be drawn for the multivariate case. The effects of structural breaks, of changing trends, and of innovational outliers are however more severe in some cases. These characteristics are likely to give the impression that a series shows nonstationary behaviour even when it does not contain a unit root, and hence the null hypothesis of a unit root is too easily not rejected. Consequently, it is readily imagined that the critical values for the tests should be modified in this case, see e.g., Perron (1989, 1990), Rappoport and Reichlin (1989), Hendry and Neale (1990), and the 1992-special issue of Journal of Business and Economic Statistics including Banerjee et al. (1992) and Perron and Vogelsang (1992).

In the papers by Perron (1990) and Perron and Vogelsang (1992) additive outliers are considered in the first difference of a time series, that is, under the null of a unit root. The alternative is assumed to be a stationary time series model with a changing mean. Hence, under the null hypothesis there will be a persistent jump in
the series at the times where outliers occur. In the present paper a different type of additive outliers is considered, namely in the levels of a time series, which is also the most common definition in the literature. Additive outliers of our type affect observations in isolation and may occur as a result of measurement errors or there may be special reasons for their existence. Examples include union-strikes, the hoarding behaviour of consumers when tax changes are announced and the effects of computer breakdowns on registrating, e.g. new car sales or unemployment. Our definition of an additive outlier implies that some shocks will only have a temporary effect and thus, provided they are sufficiently large or sufficiently frequent, may indicate that the series is really stationary. The latter is suggested by the results in Bustos and Yohai (1983) and Martin and Yohai (1986) where it is shown by simulation methods that an additive outlier biases the least squares estimator downward for the parameter in a stationary first order autoregressive process. Ledolter (1989) finds a crude first order approximation to the asymptotic bias in this case. A similar bias will be shown to appear in the nonstationary case. Hence, in some situations it can be expected that the additive outliers will establish the wrong impression that a time series is stationary when it is actually integrated.

The paper follows the following plan. In section 2, we derive an explicit asymptotic expression for the effects of frequency and size implied by additive outliers in case of the univariate Dickey-Fuller test statistic. The Johansen (1988) test method is a multivariate generalisation of this method, and hence we conjecture that similar results will apply for this case. It turns out that the new asymptotic distribution depends linearly on the frequency and on the square of the size of outliers relative to the innovation standard deviation. To verify whether the asymptotic results carry over to small samples, we report the outcomes of some Monte Carlo experiments in section 3. It emerges that even for small sample sizes, some of the asymptotic results closely match the small sample ones. This applies to the univariate as well as for the multivariate case although asymptotic distributions for the latter situation seem to be reached at a less rapid speed when the magnitude of outliers or the probability of outlier occurrences is large. In section 4 we show through an empirical example how practical inference can be affected by the presence of additive outliers. The final section concludes the paper with some remarks and with the recommendation that one strategy of avoiding the influence of additive outliers is to include dummy variables in the test equations, provided of course that the outliers have in fact been identified as additive outliers.
2. THE MODEL AND SOME ANALYTICAL RESULTS.

Consider the univariate process

\[ y_t = \phi y_{t-1} + \varepsilon_t \quad \text{for } t=1,2,\ldots,T \]  

(2.1)

where for simplicity \( \varepsilon_t \) is assumed to contain no autocorrelation and suppose that additive outliers of magnitude \( \pm \theta \) may occur with a given probability \( \pi \). Hence the time series we observe is

\[ z_t = y_t + 6 \delta_t, \]  

(2.2)

where \( \delta_t \) is a Bernoulli variable taking the value 1 or -1 with the specified probability and is zero otherwise. We can also write

\[ \delta_t = \delta_t^* + \delta_t^\prime, \]

such that the negative and positive entries of \( \delta_t \) can be separated and with a probability of a non-zero element in each of \( \delta_t^* \) and \( \delta_t^\prime \) being \( \pi/2 \). Of course, these will not be independent. Note that even in the case of a unit root in the \( y_t \)-process, \( \phi=1 \), an additive outlier will only have a temporary effect, since the contamination is added to \( y_t \). Hence the outlier will produce a once and for all "peak" in the series. This property is very different from innovation outliers, which are defined to be extreme realisations from the process generating the innovations \( \varepsilon_t \). For the latter situation any shock will have an everlasting influence in the case of a unit root, and will die out gradually when the process is stationary, \( |\phi|<1 \). An innovation outlier will thus give rise to a "bump" in the series. With respect to testing for unit roots we may expect, however, that additive outliers will be a cause for much greater concern, than innovation outliers, since the latter will affect subsequent observations whereas the former will not. This will rather yield isolated outlier points that are independently located. Note also the importance of defining the additive outlier in the levels of the series and not in the first difference as mentioned in the introduction.

As in the stationary case considered by Bustos and Yohai (1983), Martin and Yohai (1986) and Ledolter (1989), we may expect some bias of the least squares estimator when the time series is not stationary. In figures 1 and 2 the intuition behind this phenomenon is illustrated. In accordance with the above notation figure 1 displays
a scatter plot of $\Delta y_t$ on $y_{t-1}$ where $y_t$ is a generated random walk of 100 observations with $\varepsilon_t \sim N(0,1)$, and $\Delta y_t$ is defined by $\Delta y_t = y_t - y_{t-1}$. In figure 2 outliers have been added to the $y_t$ series in agreement with (2.2). The probability of an additive outlier is $\pi = 0.10$ and $\theta$ is set to 5. This corresponds to a magnitude of outliers which is 5 times the standard deviation of $\varepsilon_t$, which is a very extreme example, of course. The least squares regression line in figure 2 seems to suggest a negative bias of the regression coefficient, i.e. towards rejection of the unit root. When the parameters underlying a model have physical or economic interpretations, the presence of undetected influential observations, like an additive outlier, can therefore mislead the scientist about the properties of the model.

To give an analytical description of the above observation, assume for the case of no outliers that the regression

$$\Delta y_t = \rho y_{t-1} + \varepsilon_t$$  \hspace{1cm} (2.3)

is conducted where $\rho = \Phi - 1$. The least squares estimate of $\rho$ is

$$\hat{\rho} = (\sum_{t=1}^{T} y_{t-1} \Delta y_t)(\sum_{t=1}^{T} y_{t-1}^2).$$ \hspace{1cm} (2.4)

However, since the time series is observed with additive outliers it follows that

$$\hat{\rho}_{AO} = (\sum_{t=1}^{T} z_{t-1} \Delta z_t)(\sum_{t=1}^{T} z_{t-1}^2)$$ \hspace{1cm} (2.5)

and by writing out this expression we obtain

$$\hat{\rho}_{AO} = \frac{\sum_{t=1}^{T} y_{t-1} \Delta y_t + \theta \sum_{t=1}^{T} y_{t-1} \Delta \delta_t + \theta \sum_{t=1}^{T} \delta_{t-1} \Delta y_t + \theta^2 \sum_{t=1}^{T} \delta_{t-1} \Delta \delta_t}{\sum_{t=1}^{T} y_{t-1}^2 + \theta \sum_{t=1}^{T} \delta_{t-1}^2 + 2 \theta \sum_{t=1}^{T} \delta_{t-1} y_{t-1}}.$$ \hspace{1cm} (2.6)

The $t$-statistic for a zero coefficient null is defined as

$$t_{\rho_{AO}} = \frac{\hat{\rho}_{AO}}{\sqrt{\sum_{t=1}^{T} z_{t-1}^2}}.$$ \hspace{1cm} (2.7)
where
\[ \sigma^2 = T^{-1} \sum_{t=1}^{T} (\Delta z_t - \hat{\rho}_{AO} z_{t-1})^2 \] (2.8)

is the squared regression standard error. Further we define
\[ \sigma_t^2 = \lim_{T \to \infty} T^{-1} \sum_{t=1}^{T} E(\varepsilon_t^2). \] (2.9)

The limiting behaviour of \( \hat{\rho}_{AO} \) and its t-ratio can now be stated as follows.

**PROPOSITION.**

As \( T \to \infty \) we have that
\[
T \hat{\rho}_{AO} \Rightarrow \left( \int_0^1 W(r) dW(r) / \left( \int_0^1 W^2(r) dr \right) - (\theta / \sigma_e)^2 \pi \left( \int_0^1 W^2(r) dr \right)^{-1} \right)
\]
\[
t_{\hat{\rho}_{AO}} \Rightarrow \left( 1 + (\theta / \sigma_e)^2 \pi \right)^{-1/2} \left\{ \left( \int_0^1 W(r) dW(r) / \left( \int_0^1 W^2(r) dr \right)^{1/2} - (\theta / \sigma_e)^2 \pi \left( \int_0^1 W^2(r) dr \right)^{-1/2} \right) \right\}
\]

where "\( \Rightarrow \)" signifies weak convergence and where \( W(r) \) is a standard Brownian Motion defined on the unit interval \( r \in [0,1] \).

**Proof.** See Appendix.

Several interesting results are implied by this proposition. First, the asymptotics show that for either \( \pi = 0 \) or \( \theta = 0 \) the usual Dickey-Fuller distributions as tabulated by Fuller (1976) will apply. Note that for simplicity (but without loss of generality) we assume there is no weak dependence of the error process \( \varepsilon_t \) and hence no nuisance parameters entering the limiting distributions for the non-outlier case. If the probability of outlier occurrences is strictly positive and \( \theta = 0 \), \( \rho \) is still estimated (super-) consistently, but the limiting distributions contain nuisance parameters and will be shifted to the left and thus the Dickey-Fuller test will have an actual size in excess of the nominal size. Therefore we will reject the unit root hypothesis too often in favour of the stationary alternative. Note also that the distributions are invariant to decompositions of \( (\theta / \sigma_e)^2 \pi \) into the single components \( \theta \), \( \sigma_e \), and \( \pi \). Some similarities to the standard measurement error model that will generate MA(1) errors are apparent. Observe simply that if additive outliers are sufficiently large and sufficiently frequent, then the error term of \( \Delta z_t \), i.e. \( \varepsilon_t + \theta (/L) \delta_t \), will have MA(1)-like
characteristics which are known to produce nuisance parameters in unit root distributions, see e.g. Schwert (1989) and Pantula (1991).

With respect to the finite sample performance this will of course rely on the relative magnitude of the sample size $T$, $n$ and the magnitude of $\theta$ in relation to $\sigma_t$. That is, the appropriateness of asymptotic expressions will depend on whether the sample size is sufficiently large to ensure lower order terms in the expression (2.6) to become negligible.

The above results easily generalize to the multivariate case. Johansen (1988, 1991) and Johansen and Juselius (1990) show how for a $p$'th order vector autoregressive system the number of stationary (cointegrating) relations $r$, or alternatively, the number of common stochastic trends $p-r$, can be determined. It is shown that the testing procedure amounts to formulating a reduced rank regression problem which can be solved by a simple eigenvalue routine. If the number of cointegrating vectors is $r$, there are $r$ non zero eigenvalues. Testing for the number of cointegrating vectors therefore becomes a matter of testing for the number of zero eigenvalues. Johansen (1988, 1991) develops a LR-test for the null hypothesis that the $p-r$ smallest eigenvalues will equal zero, i.e. if the eigenvalues in descending order are denoted $\lambda_1, \lambda_2, \ldots, \lambda_r, \ldots, \lambda_p$, the so-called trace test for $H_0: \lambda_{r+1}=\lambda_{r+2}=\ldots=\lambda_p=0$ against the alternative of positive eigenvalues is defined

$$\Lambda_{\text{trace}} = -T \sum_{i=r+1}^{p} \ln(1-\lambda_i).$$ (2.10)

Instead of testing simultaneously the number of cointegrating vectors one could also test $r$ vectors against $r+1$, $r+1$ vectors against $r+2$ and so forth. Johansen denotes this procedure the $\lambda_{\text{max}}$ test procedure. The above tests have limiting distributions which are straightforward generalisations of the univariate Dickey-Fuller test to the multivariate case. In terms of asymptotics a univariate Brownian Motion process $W(r)$ is replaced by a vector Brownian Motion process, but otherwise the distributions are very similar. Empirical fractiles are tabulated in Johansen and Juselius (1990). It is therefore our conjecture that the results from the univariate model will carry over in a straightforward manner to the multivariate $\Lambda_{\text{trace}}$ and $\lambda_{\text{max}}$ tests.

3. NUMERICAL EVIDENCE.

In this section we provide some Monte Carlo evidence in order to illustrate the
practical implications of our analytical findings. The set-up of our experiments follows the notation used in the previous section. For the univariate case the data were generated according to (2.1) and (2.2) with \( \varepsilon \sim N(0, I) \), i.e. \( \sigma^2 = 1 \), and the experimental design included \( \theta = \{0, 3, 4, 5\} \), \( \pi = \{0.05, 0.10\} \) and \( T = \{25, 100, 400\} \). The regression model was

\[
\Delta z_t = \rho + \sigma \Delta z_{t-1} + \epsilon_t,
\]

from which the \( t \)-ratio was calculated as in (2.7). Inclusion of deterministic regressors in the auxiliary Dickey-Fuller regressions will affect the distributions, of course, but in order to focus on additive outliers only, the model with no deterministics provides a satisfactory simplification in this case. For the Dickey-Fuller regression results are reported in table 1.

With respect to the Johansen test we considered a first order bivariate VAR-system of non-cointegrated time series. Both series were generated as above and the same parameter space as in the univariate case was analyzed. In each experiment both series contained the same probability of outlier occurrences and the value of \( \theta \), the magnitude of outliers, were the same. However, the location of additive outliers were different by construction. Fractiles for the two lambda-max tests and the trace test statistics are reported in tables 2, 3 and 4, respectively. In all simulations 10000 replicates were used to construct the fractiles. See tables 2 and 3 for a detailed description of how the empirical distributions were generated.

The simulation experiments closely mimic our analytical findings. In fact, the change in distribution is quite dramatic in some cases when both the outlier probability and the magnitude of outliers become large. Of course this is of no surprise since the contribution to the total variation resulting from the outliers becomes relatively influential when both \( \pi \) and \( \theta \) increase, despite their temporary effect. Hence the signal from the random walk component will be annihilated by the noise from the outliers as these two parameters increase in magnitude. With respect to the fractiles of the univariate Dickey-Fuller test a probability of an outlier of \( \pi = 0.05 \), \( \theta = 3 \) standard deviations, and a sample size of \( T = 100 \) corresponds to an actual size of the test equal to .181 at a nominal 5% level.

Insert Table 1 about here

Insert Figure 3 about here
When $\pi$ increases to $\pi=.10$ the size is .269 and detoritates even more as the magnitude of outliers increases. Note that our asymptotic results suggest empirical fractiles to be similar when $(\theta/\sigma)^2\pi$ attains the same value $K$ for a given experiment. If for instance $(\pi=.05,\theta=4)$ and $(\pi=.10,\theta=3)$ then $K=.8$ and $K=.9$, respectively, and the fractiles appear to be very similar. We also constructed other experiments where $K$ attained the same value and, indeed, the simulated distributions appeared to be almost identical even in small samples. In figure 3 the empirical density functions for the Dickey-Fuller $t$-test are displayed for a sample size of $T=100$ for $K=0$, (i.e. the Dickey-Fuller distribution) and $K=.75$, and 1.5 respectively.

Concerning the Johansen tests many of the conclusions from the univariate model apply. All three tests entail huge shifts in the distributions as $\theta$ and $\pi$ increase, however, size distortions are generally worse in comparison with the Dickey-Fuller test. We will too frequently accept cointegration, i.e. the case of strictly positive eigenvalues. The results concerning a fixed value of $K$ also apply in the multivariate model. One respect in which the Johansen tests seem to differ from the Dickey-Fuller test is the rate by which empirical distributions will tend to their limiting distributions. If either $\theta$ and/or $\pi$ is of great magnitude there are huge differences between the empirical distributions for a small sample size and the empirical distributions for a large sample size, 400 say. However, we provided further evidence by increasing the number of observations and it appeared that there would only be a slight change in distributions in relation to those reported for $T=400$. In fact, the distributions for $T=1600$ were quite close to those reported for $T=400$. These results can be obtained from the authors upon request. The above observation seems to suggest, that for $\theta,\pi$ "large" there may still exist elements in the distributions that are not negligible in finite samples although these elements will be of a lower order and in the limit will vanish. In the above experiments the outliers are dated at different points in time for each series. We may expect, however, that if the dating of additive outliers is the same for each series then the influence on the empirical distributions will be very small,- probably there will no observable change in the distributions at all.

Insert Table 2 about here

Insert Table 3 about here

Insert Table 4 about here
4. AN EMPIRICAL EXAMPLE

In order to illustrate our analytical findings by an empirical example we considered the data set analyzed by Perron and Vogelsang (1992). The time series are the U.S./Finland real-exchange rate series based on both the CPI index and the GDP deflator. The data is annual and cover the periods 1900-1988 and 1900-1987, respectively. The series are displayed in figures 4 and 6. Perron and Vogelsang used the data in order to examine whether the time series were better described as stationary time series with a changing mean rather than integrated or unit root processes. By conducting an augmented Dickey-Fuller (ADF) test they found a $t$-ratio of -5.74 when the CPI measure was used with $p=1$ lag of the differenced series included in the auxiliary regression (5% critical value=-2.89). Hence, the unit root hypothesis was rejected for this series. On the contrary, when using the GDP deflator to define the real exchange rate the ADF-test with $p=2$ gave a test value of -1.62, thus accepting a unit root. Perron and Vogelsang challenged the view that the latter series was really a unit root process and showed by methods developed in the paper, that by allowing for a one-time change in the mean, the series was better described as being stationary.

Now, visual inspection of the charts 4 and 5 seems to indicate that both series, may, in fact, contain additive outlying observations. Spikes are observed in both series for 1918, 1921, 1932, 1948 and 1949. A minor spike is also observed in 1945. All the data irregularities have the characteristics of being additive outliers, since the series returns to the original level of the series after the extreme observation is observed. There also seem to be some intuitive reasons for the outliers, i.e. World War I, the crisis in the thirties, and the turbulence following World War II. Especially for the CPI-based measure a huge spike is observed in 1918, which may give a wrong impression of the scale of the $y$-axes. In figure 6, the years 1917-1919 have been removed from the series, and by so doing the impact of the remaining outliers become even more clear-cut.

In regression models additive outliers will normally show up in the residuals and give rise to a high kurtosis coefficient. The Jarque-Bera test for normality following from the regressions reported by Perron and Vogelsang give test values 835.67 and 26.83, respectively, which should be compared with fractiles from a $\chi^2(2)$-distribution. Of course the normality of the regression residuals is strongly rejected. Several methods can be used to cope with the presence of additive outliers. One
method is to consider robust estimation of the model by attaching less weight to extreme observations. How this will affect unit root inference is not yet clear. A different approach is to simply remove the outlier and treat the observation as a missing value. Maravall and Peña (1992) show how even for nonstationary time series the estimation of a missing value can be conducted by the Kalman filter smoother. For illustrative purpose we follow a much simpler pragmatic route by simply including dummy-variables in the auxiliary ADF-regression. It can be shown that the inclusion of impulse dummies will not affect the limiting distributions of the test statistics although the effective sample size when dealing with a fixed span of data may be reduced. Naturally this may cause minor changes in the finite sample distributions, if sufficiently many outliers are present.

The regression we suggest to conduct is of the form

$$\Delta z_t = (\alpha - 1)z_{t-1} + \sum_{i=0}^{p} \sum_{j=1}^{k} \omega_i D_{t-i} + \sum_{i=1}^{p} \psi_i \Delta z_{t-i} + e_t$$

where we then test whether $(\alpha - 1) = 0$ using the ADF-$t$-ratio. Of course a trend may be included in the regression as well. Note that for each impulse dummy variable, $D_t$, taking the value 1 at time $j$, $p$ lags should be included since the presence of lagged differences by construction will entail "lagged" additive outliers.

With respect to the CPI-based real exchange rate we included dummy variables corresponding to the peaks previously discussed and lags hereof. For $p = 2$ lags in the regression model and by excluding the most insignificant dummies (i.e. effectively including $D_{37}, D_{39}, D_{32}, D_{33}, D_{45}, D_{49}, D_{50}$) we obtained $ADF = -2.65$ whereby the null hypothesis is now not rejected. The estimate of the AR-root increased from .49 to .81 by including the dummy variables and the Jarque-Bera test value dropped to 12.63 indicating that the influence from outliers had been reduced considerably. We also conducted the same regression as above to the GDP based measure and obtained a Dickey-Fuller test value of -1.03, i.e. giving even stronger evidence for the presence of a unit root. The root estimate itself increased from .91 to .96.

As seen from our example the analytical findings are supported by the data. Whether the resulting time series are better described to be stationary with a changing mean is of course a different question. In fact, the nonstationarity of real exchange rate series has far reaching economic consequences, i.e. PPP not holding in the long run, so the alternative suggested by Perron and Vogelvang is probably right at the point. Notwithstanding our empirical application has illustrated another example of the
modesty by which the results from unit root tests should be interpreted.

5. CONCLUSIONS.

In the literature it is often argued that outlying observations and structural breaks may produce time series that are easily interchanged with unit root processes. This paper addresses a different kind of time series, namely processes with additive influential observations, and as shown the implications of this type of data irregularities will tend to produce spurious stationarity as opposed to the situations described above. The analytical and numerical results reported in the paper were based on the assumption that additive outliers of a certain magnitude will appear randomly with a given probability. Other descriptions of how additive outliers may occur can be considered but generally we may expect similar results. For instance, it is frequently observed that seasonal time series have a very strong (deterministic) pattern which, effectively, makes the seasonal variation of the data of a larger magnitude than the zero frequency variation of the data, at least for a finite stretch of data. The seasonal pattern in this case could therefore correspond very much to additive outliers of a deterministic kind, i.e. like when we model seasonality using dummy variables.

Although the use of dummy variables may at first sight appear a little arbitrary, it is extremely important to provide good economic interpretations for the particular dummy variables that appear to be required or at least one should find some reasons for the irregularities observed to avoid ad hoc-ness. If the removal of an additive outlier is done by including a dummy variable in the regression model it can be shown that this will remove the influence from the outlier and thus give rise to distributions under the null with a controllable size. However, the effective sample size will be reduced of course, which may affect the finite sample distributions. Alternatively we may replace outlying observations by those fitted using the procedures considered by Maravall and Peña (1992). As opposed to time series models with structural changes, changing means or regime shifts, say, there are no pretesting problems when additive outliers are considered, at least as long not too many additive outliers are present, see e.g. Perron (1991), Perron and Vogelsang (1992) and Zivot and Andrews for a discussion of pretesting problems.

In practice, innovation outliers and structural breaks are probably more frequently observed than additive outliers. However, the message of our paper is that outliers of any kind may invalidate inference when analyzing univariate and
multivariate time series models, and a careful detection and treatment of the outliers is therefore very important. Our findings also indicate the fragility of results from unit root tests. Testing for unit roots will only provide simple approximate formula for the basic properties of an observed time series. More fundamentally, the presence of outlying observations may indicate that a univariate model is inappropriate. This is also why a multivariate analysis including many explanatory variables may partially solve the problem, i.e. if the extreme observations occur jointly amongst the series and can be expected to cancel out when the simultaneous system is modelled.
APPENDIX. Proof of PROPOSITION.

The least squares estimator in case of additive outliers reads

$$\hat{\beta}_{AO} = \frac{\sum_{t} y_{t-1} \Delta y_{t} + \theta \sum_{t} y_{t-1} \Delta \delta_{t} + \theta \sum_{t} \delta_{t-1} \Delta y_{t} + \theta^{2} \sum_{t} \delta_{t-1} \Delta \delta_{t}}{\sum_{t} y_{t-1}^{2} + \theta^{2} \sum_{t} \delta_{t-1}^{2} + 2 \theta \sum_{t} \delta_{t-1} y_{t-1}}.$$  

With respect to the order of magnitude of the terms entering the expression we have the following Lemma:

**LEMMA.** As $T \to \infty$

1. $T^{-1} \sum_{t}^{T} y_{t-1} \Delta y_{t} \Rightarrow \sigma_{y}^{2} \int_{0}^{1} W(r) dW$
2. $T^{-2} \sum_{t}^{T} y_{t-1}^{2} \Rightarrow \sigma_{y}^{2} \int_{0}^{1} W^{2}(r) dr$
3. $T^{-1} \sum_{t}^{T} \delta_{t-1}^{2} \Rightarrow \pi$
4. $T^{-1} \sum_{t}^{T} \delta_{t-1} \Delta \delta_{t} - T^{-1} \sum_{t}^{T} \delta_{t-1}^{2} - T^{-1} \sum_{t}^{T} \delta_{t-1} \delta_{t} \Rightarrow -\pi$
5. $T^{-1/2} \sum_{t}^{T} \delta_{t-1} \Delta y_{t} - T^{-1/2} \sum_{t}^{T} \delta_{t-1} \Delta \delta_{t} \Rightarrow O_{p}(1)$ for $0 < \pi < 1$
6. $T^{-3/2} \sum_{t}^{T} \delta_{t-1} y_{t-1} \Rightarrow O_{p}(1)$ for $0 < \pi < 1$
7. $T^{-1/2} \sum_{t}^{T} y_{t-1} \Delta \delta_{t} - T^{-1/2} \sum_{t}^{T} \delta_{t-1} \Delta \delta_{t} \Rightarrow O_{p}(1)$ for $0 < \pi < 1$

**Proof.** For those familiar with the work by Phillips (1987) and his co-authors, (a) and (b) are obvious. Note however that no non-centrality parameters enter in (a) since for simplicity we have assumed innovations to be iid. (c) tells us that in the limit the number of non-zero entries in the dummy variable divided by the number of observations tends to the probability of an additive outlier occurrence. For (d) the second term on the RHS of the equality sign tends to zero as $T \to \infty$ which follows from a symmetry argument. Note simply that in accordance with the notation in the main text $E(\delta_{t}, \delta_{t-1}) \equiv lim_{T \to \infty} T^{-1} \sum_{t} \delta_{t}, \delta_{t-1} = E((\delta_{t} + \delta_{t-1})(\delta_{t} + \delta_{t-1})) = (\pi/2)^{2} - (\pi/2)^{2} - (\pi/2)^{2} + (\pi/2)^{2} = 0$. For (e), (f) and (g) it holds that in case $\pi = 1$ the orders of magnitude will be $O_{p}(1)$ which will set an upper limit for the expressions. In case $0 < \pi < 1$ elements of the stochastic series will be picked out and the orders will be of a lesser magnitude. $\blacksquare$
Now it follows that

\[
T\hat{\rho}_{AO} - \frac{T^{-1} \sum_t^T y_{t-1} \Delta y_t + \theta^2 T^{-1} \sum_t^T \delta_{t-1} \Delta \delta_t + o_p(1)}{T^{-2} \sum_t^T y_{t-1}^2 + o_p(1)} \Rightarrow \left( \int_0^1 W(r) dW \right) / \left( \int_0^1 W^2(r) dr \right) - \left( \theta / \sigma \right)^2 \pi \left( \int_0^1 W^2(r) dr \right)^{-1}
\]

Concerning the t-ratio as defined in (2.7) we have that

\[
\sigma^2 = T^{-1} \sum_t^T (\Delta z_t - \hat{\rho} z_{t-1})^2 - T^{-1} \sum_t^T (\Delta y_t + \theta \Delta \delta_t - \hat{\theta} y_{t-1} - \theta \delta_{t-1})^2 - T^{-1} \sum_t^T \epsilon_t^2 + \theta^2 \pi + o_p(1) \Rightarrow \sigma^2 + \theta^2 \pi.
\]

It is also easily verified that

\[
T^{-2} \sum_t^T z_{t-1}^2 = T^{-2} \sum_t^T (y_{t-1} + \theta \delta_{t-1})^2 - T^{-2} \sum_t^T y_{t-1}^2 + o_p(1) \Rightarrow \sigma^2 \int_0^1 W^2(r) dr
\]

such that by direct application of these results to (2.7) we get the desired results. \(\square\)
### TABLE 1. Empirical Fractiles for the univariate Dickey-Fuller t-test. Based on 10000 replications.

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**NOTE.** The sizes reported are actual sizes at a nominal 5%-level.

The fractiles were calculated as follows. Univariate time series were generated in accordance with (2.1) and (2.2) by letting $\sigma^2=1$. Next the Dickey-Fuller regression with no deterministics was generated from which the $t$-ratio, (2.7), was calculated. For each sample size and each value of the design parameters the test-statistics were calculated and replicated 10000 times.
### TABLE 2. Empirical Fractiles for the Johansen $\lambda_{max-1}$ test. Based on 10000 replications.

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**NOTE:** The sizes reported are actual sizes at a nominal 5%-level.

The fractiles were calculated as follows. The $\lambda_{max-1}$ test is constructed under the assumption of a first order system. Since the LR-test in this case is the squared $t$-ratio of the Dickey-Fuller test we generated data series in accordance with (2.1) and (2.2) such that $c_i$ and constructed the squared $t$-ratio as in (2.7). However, to facilitate comparison with the Johansen and Juselius (1990), Table A2 p. 208, a constant was included in the auxiliary regression. For each sample size and each value of the design parameters the test statistics were calculated and replicated 10000 times.
**TABLE 3. Empirical Fractiles for the Johansen $\lambda_{\max}^2$ test. Based on 10000 replications.**

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**NOTE:** The sizes reported are actual sizes at a nominal 5%-level. The fractiles were generated as follows. A bivariate time series of first order was generated in accordance with (2.1) and (2.2) for each series and with $\xi_i \sim N(0,1)$, $i=1,2$, and $\xi_2$ are uncorrelated temporarily as well as contemporaneously. For simplicity we ignored conditioning on short run dynamics in the estimation due to the assumption of a first order system, but the series were demeaned to facilitate comparison with the fractiles reported by Johansen and Juselius (1990), table A2 p. 208. Following the notation of Johansen, we let $X_t=(x_1, x_2)$ and define $R_0=AX^*_t$, $R_j=X^*_j$ where a '*' indicates that the series have been demeaned. Next the moment matrices $S_{ij}, S_{ij},$ for $i,j=0,1$ are calculated. If the VAR-model is written in ECM-form $\Delta X_t = \mu + \alpha \beta X_{t-1} + \epsilon$, the (reduced) rank of the matrix $\Pi = \alpha \beta'$ is found by solving the eigenvalue problem $\Pi \epsilon = 0$ which gives the eigenvalues $\lambda_1, \lambda_2$ in ascending order. The test-statistics are now easily constructed as

$$\lambda_{max} = -T \ln (1-\hat{\lambda}_2)$$

$$\lambda_{trace} = -T \ln (1-\hat{\lambda}_1) - T \ln (1-\hat{\lambda}_2)$$

For each sample size and each value of the design parameters the test-statistics were calculated and replicated 10000 times.
### TABLE 4. Empirical Fractiles for the Johansen $\lambda_{trace}$ test. Based on 10000 replications.

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<td>100</td>
<td>62.31</td>
<td>70.73</td>
<td>80.15</td>
<td>90.27</td>
<td>.930</td>
<td></td>
</tr>
</tbody>
</table>

**NOTE:** See table 3.
FIGURES

FIGURE 1. Scatter plot. No outliers.

FIGURE 2. Scatter plot. Additive outliers.
FIGURE 3. Density Functions for Dickey-Fuller t-test.
Sample size, $T=100$. Based on 50000 replications.

$K=0$

$K=0.75$

$K=1.5$

NOTE: The curves have been smoothed.
FIGURE 4. Logarithm of the U.S./Finland Real Exchange Rate Based on the Consumer Price Indexes (CPI); 1900-1988, Annual.

FIGURE 5. Logarithm of the U.S./Finland Real Exchange Rate Based on the GNP (GDP) Deflators as the Price Indexes; 1900-1987, Annual.
FIGURE 6. Logarithm of the U.S./Finland Real Exchange Rate Based on the Consumer Price Indexes (CPI); 1900-16, 1920-88, Annual.
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