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Measuring technical input efficiency for similar production units:

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Feb. 93

Abstract

This paper is a survey of the non parametric approach to efficiency measurement of similar production units. It gives a critical review on different aspects of the underlying theory. The main issues are static models (DEA, FDH) and the role played by the related efficiency indices, dynamic models (Malmquist approach, sequentiel efficiency etc.) and explanatory regression models used for implementation purposes.

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1. Introduction

This is the first of two papers on the measurement of technical input efficiency for similar production units where similarity refers to condition that all units must have the same production technology i.e. common inputs and outputs. The present paper will consider theoretical aspects of the issue whereas the second paper is concerned with an empirical analysis of the Danish hospital sector in order to illustrate the mechanism behind the theoretical framework put forward in this paper.

Performance evaluation of production units based on traditional production theory has received growing interest during the past decade. In particular the so called Data Envelopment Analysis (the DEA-analysis as proposed by Charnes, Cooper & Rhodes [1978]) which at present seems to be the most widespread method for productivity measurement among public as well as private organizations. The DEA-method relates to similar production units (or Decision Making Units as they are often called) where outputs and inputs are measurable whereas data on prices not necessarily are available. These characteristics are often found in production units which provide services i.e. libraries, hospitals etc. However, this paper will mainly focus on the so called Free Disposal Hull method (the FDH method) as proposed by Deprins, Simar & Tulkens [1984]. The reason will become clear as the underlying assumptions are investigated.

Looking in particular at indices of technical efficiency it is possible to distinguish between three main categories; output indices, input indices and a combination of the two called a graph indices (see Färe, Grosskopf & Lovell [1985]). We focus on input indices since the use of inputs as key-variables implicitly treat the production units as cost minimizers. Therefore input (or resource) control is assumed to be the overall decision parameter of the DMUs whereas output is considered as given. As an example consider the hospital sector where outputs in the form of patients are uncontrollable since nobody knows the exact number of patients arriving for treatment. The management therefore primarily has to focus on the consumption of resources taking the output level as given in order to optimize their performance.

Moreover, the framework chosen is non-parametric since this approach does not impose any a priori assumptions on the functional relationship between inputs and outputs in the production process. In particular the free disposal hull procedure represents the closest approach to a ranking procedure completely based on the information obtained from the production dataset. Free disposability does not include an assumption of convexity on the production possibility set contrary to the DEA-model. In fact the only technological assumption made is free disposability of inputs and outputs.

This paper is organized as follows. Section 2 contains some preliminary definitions from production theory. Given a set of observations on a group of similar activities the aim is to filter these activities such that good and bad performances are identified. One can attempt to apply such a filtering through the non-parametric approaches, FDH and DEA, as examined in section 3. Moreover, section 3 sketches a general procedure for data selection in order to secure the required similarity among the activities i.e. common inputs and outputs. In section 4, two indices of technical input efficiency are examined under various reference technologies since it is preferable to rank the activities according to an efficiency score. In Section 5 the purely technical information obtained from the efficiency scores are extended by the inclusion of institutional factors. Some part of the technical inefficiency may be explained by such factors based on a regression approach. Section 6 extends the previous static analysis to cover dynamic aspects. Section 7 summarizes in final remarks.

2. Preliminaries

Let $\mathbf{R}^m_+$ denote the non-negative Euclidean m-orthant. For every $x, x' \in \mathbf{R}^m_+$ we write $x \geq x'$ resp. $x > x'$, if for every $i = 1, ..., m, x_i \geq x'_i$ resp. $x_i > x'_i$. We shall consider the production of s outputs denoted by $y \in \mathbf{R}^s_+$ from a set of m inputs, where inputs are denoted by $x \in \mathbf{R}^m_+$.

Define the production possibility set as $Y = \{(x, y)|x \in \mathbf{R}^m_+, y \in \mathbf{R}^s_+, (x, y)$ is feasible$\}$. The production possibility set is said to satisfy free disposability of inputs if $(x, y) \in Y$ and $x' \geq x$ then $(x', y) \in Y$. Likewise it is said to satisfy free disposability of outputs if $(x, y) \in Y$ and $y' \leq y$ then $(x, y') \in Y$.

Let $L(y) = \{x|(x, y) \in Y\}$ and let $Q(x) = \{y|(x, y) \in Y\}$ then $x \in L(y*)$ is an efficient input vector for $y*$ if there is no $\mu \in [0, 1[$ such that $\mu x \in L(y*)$. Likewise $y \in Q(x*)$ is an efficient output vector for $x*$ if there is no $\delta \in]0, 1[$ such that $y/\delta \in Q(x*)$. The frontier S of Y is then defined as $S(Y) = \{(x, y)|x$ is efficient and y is efficient$\}$.

Let $Y_0 = \{(x_k, y_k)|k = 1, ..., n\}$ be a set of n production vectors e.g. observed production activities from n production units (DMUs). Moreover, let the production possibility set be defined by the free disposal hull technology of Y_0 - that is:

$$Y_{FDH} = \{(x, y)|(x, y) = (x_k, y_k) + \sum_j \mu_j[e^m_j, 0^s] - \sum_i \tau_i[0^m, e^s_i],$$

$$(x_k, y_k) \in Y_0 \cup \{0^m, 0^s\}, \mu_j \geq 0, \tau_i \geq 0, j = 1, ..., m, i = 1, ..., s\},$$

where e^s_i is the i'th column of the s-dimensional identity matrix and e^m_j is the j'th column of the m-dimensional identity matrix. Obviously this set satisfies the free disposability requirements defined above.

Define the dominated resp. the dominant set of a given production vector (x', y') as $DO(x', y', Y) = \{(x, y) \in Y | x \geq x' \text{ and } y \leq y'\}$ resp. $D(x', y', Y) = \{(x, y) \in Y | x \leq x' \text{ and } y \geq y'\}$. Notice that the free disposal hull technology can be expressed as $Y_{FDH} = \cup_{k=1}^{n} DO(x_k, y_k) \cup \{0^m, 0^s\}$.

Consider the following simple example concerning the production activities of four DMUs each producing one output (y) by two inputs (x_1, x_2):

DMU	(y, x_1, x_2)
A	(1,2,6)
B	(1,4,3)
C	(1,7,1)
D	(1,6,4)

Figure 2.1 illustrates the free disposal hull technology based on the four observations as it appears in the input space.

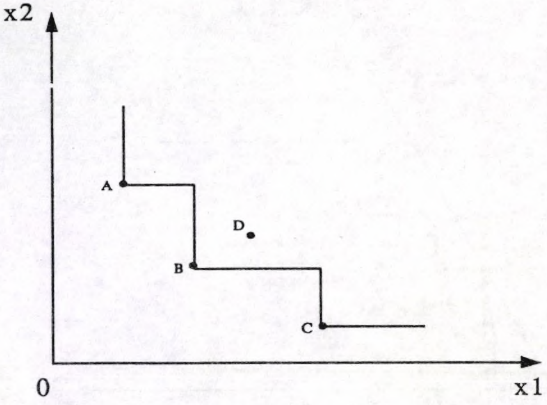

Figure 2.1: *The free disposal hull technology of A,B,C and D*

If we add the assumption that the production possibility set satisfies convexity it becomes necessary to distinguish between several types of technologies each identical to the free disposal convex hull (FDCH). If for example the technology is characterized by constant returns to scale (which was what Farrell [1957] originally assumed) we get the following possibility set:

$$Y_{CON} = \{(x, y) | (x, y) = \sum_k \Psi_k(x_k, y_k) + \sum_j \mu_j [e_j^m, 0^s] - \sum_i \tau_i [0^m, e_i^s],$$

$$\Psi_k \geq 0 \forall k, (x_k, y_k) \in Y_0 \cup \{0^m, 0^s\}, \mu_j \geq 0, \tau_i \geq 0, j = 1, ..., m, i = 1, ..., s\}.$$

Y_{CON} is different from Y_{FDH} because linear combinations of observations are allowed to be elements of the possibility set as well as the observations themselves. If we further add the assumption that $\sum_k \Psi_k \leq 1$, the technology will be characterized by decreasing returns to scale. Denote by $Y_{DEC} = \{Y_{CON}$ and $\sum_k \Psi_k \leq 1\}$ the decreasing returns to scale FDCH-technology. If the zero-vector $(\{0^m, 0^s\})$ is excluded and $\sum_k \Psi_k = 1$ is added, the technology is characterized by variable returns to scale. Denote by $Y_{VAR} = \{Y_{CON} \setminus \{0^m, 0^s\}$ and $\sum_k \Psi_k = 1\}$ the variable returns to scale FDCH-technology (note that due to convexity, increasing returns to scale is precluded as a general property). Notice that Y_{VAR} is contained within Y_{DEC} which again is contained within Y_{CON}. Thus in general:

$$Y_{FDH} \subseteq Y_{VAR} \subseteq Y_{DEC} \subseteq Y_{CON}$$

Figure 2.2 illustrates the convex hull of the different technologies in the simple one-input-one-output case.

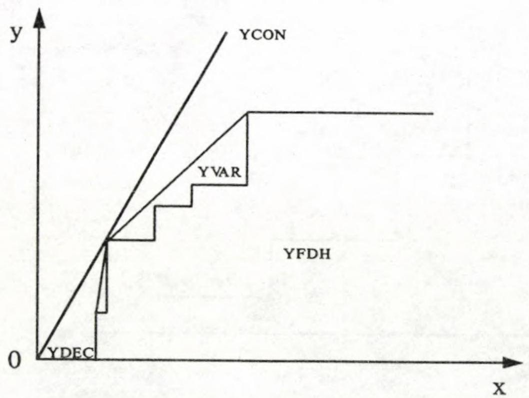

Figure 2.2: *Various types of production technologies*

3. Filtering the data set

Given the data set Y_0 we would like to divide the data into two subgroups according to their reflected performance level. Ideally this partitioning would result in one group containing the good performances, i.e. the efficient DMU's and another group containing the bad performances, i.e. the inefficient DMU's.

Among others such a partition is possible through the use of a free disposal hull technology filter, though one can discuss whether the procedure is fair to all DMU's.

3.1. The FDH-procedure

The free disposal hull technology provides a natural partitioning of a data set since the technology is based on a principle of dominance. Dominance appears to be a well suited concept for the filtering of the DMU's because if one DMU dominates another DMU then it uses less of at least one of the inputs and achieves the same output level of all and possible more of some of the outputs.

Formally, look at each element in Y_0 and construct the set $D(x_k, y_k, Y_0)$ where $(x_k, y_k) \in Y_0$.

Definition: The k'th DMU is *undominated* if and only if $D(x_k, y_k, Y_0)$ is a singleton (only contains the k'th element), otherwise the DMU is *dominated*.

Due to the definition of dominance there exists a group of DMU's which are undominated but not dominating any other DMU. This is obviously the case for highly specialized units where production is limited in either inputs or outputs but also for units which are dominated in some dimensions and dominating in others.

Consider the example from section 2. Clearly, D is dominated by B whereas A and C are undominated but not dominating any other DMU.

This highly simplified example illustrates a major problem concerning the use of the FDH-procedure; the relatively large number of undominated but non-dominating units. Empirical work by Tulkens [1990] seems to indicate that a large part, 50-90 %, of the data set will be declared undominated and of this part about 50-70 % are non-dominating units. In general the number of undominated but non-dominating units increases when the dimension of the product vector (x, y) increases and when the number of units (n) decrease. Thus there does not exist a clear cut relationship between dominance and efficiency. Though it would be convenient to declare all the undominated units efficient it raises serious questions to the role of the undominated but non-dominating units. The question is whether it is fair to assume that such units possess efficiency or whether they have obtained their position either due to the way we defined dominance or due to pure specialization. This is of particular importance when the strategic responses of the DMUs to FDH-evaluation (or control) are considered.

In general the FDH-procedure can be characterized as highly unrestrictive since the only assumption included is free disposability of inputs and outputs. This makes it difficult to characterize the efficient units as illustrated above. However, the dominated units are easier to characterize as inefficient since they are declared so under very weak restrictions.

3.2. The DEA-procedure

Convexity of the production possibility set is the underlying assumption of the DEA-procedure. Thus the choice between FDH and DEA methods fundamentally relates to whether one accepts the assumption of convexity or not. In DEA the data filtering is not directly built upon the concept of dominance, rather the procedure uses some kind of collective production function defined as the frontier of the chosen FDCH-technology defined in section 2. Hence, the observations placed on this frontier are declared efficient and the elements in the interior of the FDCH-technology are declared inefficient - that is:

Definition: The k'th DMU characterized by $(x_k, y_k) \in Y_{FDCH}$ is *efficient* if and only if $(x_k, y_k) \in S(Y_{FDCH})$ and otherwise *inefficient*.

Two notable properties occur in relation to the FDH-procedure. Firstly, an undominated observation in Y_{FDH} will not necessarily be an element of the FDCH-frontier (and hence efficient) on the other hand an element of the FDCH-frontier will always be undominated in Y_{FDH} as illustrated in figure 3.1. This follows directly from the fact that $Y_{FDH} \subseteq Y_{FDCH}$ as mentioned in section 2. Secondly, the dominated observations are not only dominated by observations in Y_0 but also by linear combinations of some of these observations. This fact may have consequenses which concerns the implementation of the filtering result. It is easier to explain to the manager of a dominated production unit that he is dominated by another actually existing unit than by a linear combination of e.g. the division in Copenhagen and the division in Stockholm – and perhaps more importantly, it is easier to suggest efficiency improvement strategies since it is straightforward to copy the strategy of the dominating (and actually observed) unit.

Consider the simplified example from section 2. Here the envelopment becomes the convex hull of the observations in the input space. D is still dominated, however not only by B but also by linear combinations of A and B as well as linear combinations of B and C (see figure 3.2).

Also important is the fact that the partitioning of the data set strongly depends on the assumed underlying type of FDCH-technology. In figure 2.2 it is easy to see that the same observation can be declared either efficient or inefficient depending upon the type of FDCH-technology (Y_{CON}, Y_{DEC} or Y_{VAR}) chosen. Since $Y_{VAR} \subseteq Y_{DEC} \subseteq Y_{CON}$, efficient observations in Y_{CON} will also be efficient under both Y_{DEC} and Y_{VAR}-technologies. Likewise efficient observations in Y_{DEC} will also be efficient under the Y_{VAR}-technology.

It is worth noticing that an assumption of convexity may seem inappropriate in the short run since activities constructed as linear combinations of existing activities are functioning as efficient references. Such pseudo observations

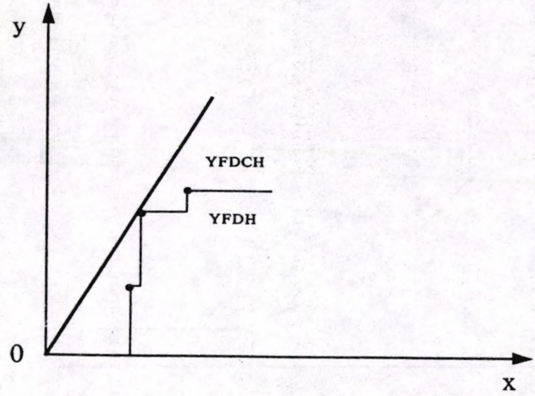

Figure 3.1: *Undominated observations are not necessarily efficient*

will rarely be attainable in the short run because of fixed technological constraints speaking in favor of free disposability as the sole assumption on the technology. The efficient references in the free disposal hull technology are existing (and dominating) activities and hence their performance level ought to be attainable even in the short run. However, in the long run convex combinations may of course represent attainable activities.

3.3. On the design of data categories

As mentioned above, specialization is partly to blame for the relatively large number of undominated but non-dominating observations within the FDH-technology which makes it hard to argue for a clear cut relationship between dominance and efficiency. Furthermore, many of the specialized production units will also appear as efficient observations under the FDCH-technology (in the DEA-method) making the over all number of efficient observations relatively large. This problem seems to appear in practice because the analyst often prefers a large number of observations (in order to be able to execute the analysis) to the comparability of the underlying production units as demanded by the theory. However, as we already have seen and also will discover in the following, the meaningfulness of the efficiency measurement methods are indeed very sensitive to the character of the data material. Hence, it is very important to design the data categories such that they insure the largest number of 'normal' observations as possible. The concept of normal roughly means that all the underlying units should use fair amounts of all categorized inputs to produce fair amounts of all categorized outputs as opposed to specialized units which only use some of the inputs to produce

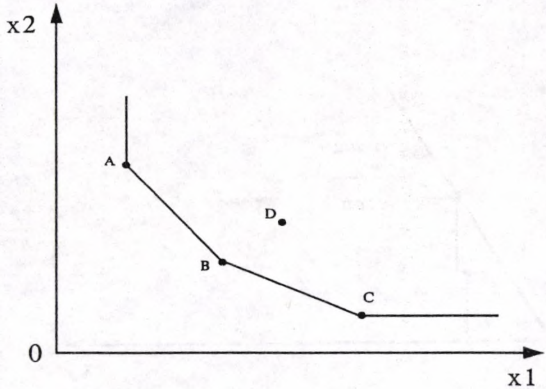

Figure 3.2: *Illustrating the example*

only some of the outputs.

In the following we will try to sketch a simple test procedure which tests whether the assumed data categories are acceptable in relation to a predefined concept of normality.

Definition: We say that an observation is *normal* if it belongs to the pseudo technology Y^N satisfying the following three conditions:

1. $0^m \in Y^N$. Y^N includes the origin.

2. $(Y^N + Y^N) \subseteq Y^N$. Additivity which means that the summation of any two elements in Y^N is again in Y^N.

3. Y^N is convex,

otherwise the observation is said to be *specialized*.

Notice that Y^N is a convex cone in the input space with vertex zero to be further defined by the evaluator as e.g. done in figure 3.3.

In order to examine the location of the data set we can determine the different clusters of Y_0 through density search techniques as e.g. reviewed by Silverman [1990] or SAS users guide on statistics [1985].

We say that the proposed data categories are acceptable if the cluster characterized by the highest data density is inside the above defined normal-cone – in other words the chosen categories are representable. The idea is pictured by figure 3.3.

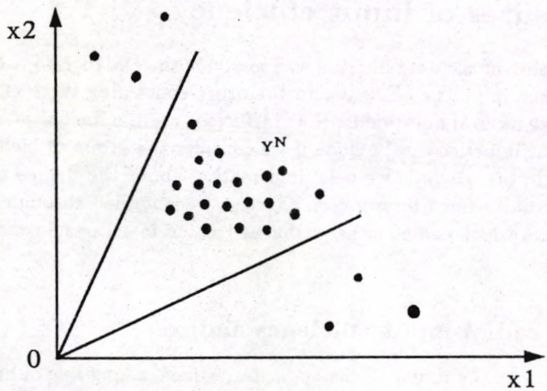

Figure 3.3: *An example of the normal-cone technology*

On the contrary, if the main cluster is not inside the normal-cone then the data categories (or units of measurement) must be redefined until they become so. If that is impossible without obvious violations of the general production profile of the chosen activity then we know already that the result of the analysis will be a large number of efficient units.

However, notice that even if the chosen data categories are acceptable in the above defined manner there may still be a problem with specialized units which will act as 'outliers'. An obvious solution to the problem connected with the presence of outliers is to exclude them from the data sample. However, if the filtering is carried out with both kind of samples it is possible to examine the impact of the outliers on the envelopment of the efficient frontier i.e. a sort of sensitivity analysis. The sensitivity analysis must be preferred to a mere exclusion of the outliers since useful information is gained with respect to the condition of similarity - the larger the impact of the exclusion on the envelopment, the less similar are the excluded units to the rest of the sample. It should be noted that with a free disposal hull reference technology (Y_{FDH}) the impact of outliers on the envelopment is limited relative to Y_{FDCH} since by exclusion of outliers in Y_{FDCH} the linear combinations are excluded as well. Hence, the DEA-methods are more sensitive to outliers in the data sample. Traditionally 'outliers' are detected using distance measures as e.g. Mahalanobis distance (see e.g. Weisberg [1980]).

4. Measures of input efficiency

The main purpose of the data filtering was to divide the DMUs into two subgroups; one containing the efficient and the other containing the inefficient units. The next natural question is; How inefficient are the inefficient units? Obviously great injustice can be done if we consider the group of inefficient units as one. In other words we need information about the degree of efficiency of each unit. Such information can e.g. be obtained through input efficiency indices which can be either radial as treated in 4.1 or non-radial as treated in 4.2.

4.1. The radial input efficiency index

In economic theory the radial efficiency measures have a long line of history. However, the concept of technical efficiency and hence the measurement of technical efficiency is considered to be introduced by Farrell [1957], who again was inspired by Debreu's coefficient of resource utilization (Debreu [1951]). Farrells input efficiency index is a radial index in the sense that it measures the efficiency of an observation along a ray from the origin in the input space to the frontier of the production technology.

Definition: Considering output as fixed we define Farrells index of input efficiency as:

$$E_F(x, L) = \min\{\theta | \theta x \in L\},$$

where $L = \{x | (x, y) \in Y\}$.

Notice that $0 < E_F \leq 1$ for $x \in L$. Thus E_F measures the maximal proportionate reduction in inputs given the feasibility constraint $x \in L$. The input efficiency index is illustrated by figure 4.1.

4.1.1. The FDH-procedure

Consider the n sets $D(x_k, y_k, Y_0), k = 1, ..., n$. These sets contain information about the observations which dominate the k'th observation (the evaluated unit). As mentioned in section 3.1, the k'th observation is undominated (efficient) if and only if $D(x_k, y_k, Y_0)$ is a singleton. Otherwise it is dominated (inefficient) by the other observations included in $D(x_k, y_k, Y_0)$.

Let the observations in $D(x_k, y_k, Y_0)$ be indexed by $h = 1, ..., H$. Consider a given observation $k0$ in Y_0. This observation is the observation under evaluation. Our aim is to measure the input efficiency of $k0$ based on the radial Farrell index defined above. One way to do this is to build a procedure directly upon the dominance set $D(x_k, y_k, Y_0)$. This was done in the one-input -multiple-output case by Deprins, Simar & Tulkens [1984] and in the multiple-

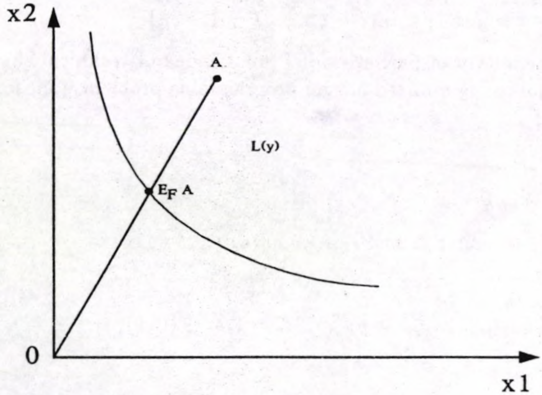

Figure 4.1: *Farrell's index of input efficiency*

input-one-output case by Thiry & Tulkens [1988]. The following stepwise procedure extends this approach to the multiple-input-multiple-output case:

(I)

Step 1. Determine $D(x_{k0}, y_{k0}, Y_0)$.

Step 2. For each of the H elements in $D(x_{k0}, y_{k0}, Y_0)$ calculate the input ratios $\theta_{hj} = x_{hj}/x_{k0j}, h = 1, ..., H, J = 1, ..., m$. θ_{hj} is the h'th units amount of input j in relation to the amount of input j used by the evaluated unit $k0$, hence $0 < \theta_{hj} \leq 1$. θ_{hj} indicates how much the evaluated unit $k0$ can reduce the amount of input j in order to be as good as unit h with respect to input j.

Step 3. For all h calculate $\theta_h = \max\{\theta_{hj}\}$. Notice that this implies that $k0$ is compared to each of the H units with respect to the most favorable input level. If this was not the case then $\theta_h x_{k0j}$ would not be included in Y_{FDH}.

Step 4. Calculate $\theta = \min \theta_h$ in order to move $(\theta x_{k0}, y_{k0})$ to the frontier of Y_{FDH}.

To demonstrate that the efficiency index of the above procedure in fact is a Farrell type radial index we show that the input efficiency measurement within the FDH-technology also can be solved by a mixed integer programming procedure. As mentioned in section 2, Y_{FDH} can be expressed as the union of the sets $DO(x_k, y_k, Y_0)$. More formally:

$$Y_{FDH} = \{(x, y) | \sum_k \delta_k x_{kj} \leq x_j, \sum_k \delta_k y_{ki} \geq y_i, \sum_k \delta_k = 1, \delta_k \in \{0,1\},$$

$$j = 1, ..., m, i = 1, ..., s, k = 1, ..., n\}.$$

Through the definition of Farrells radial input index of technical efficiency we obtain the following n mixed integer programming problems (one for each element in Y_0):

$\min \theta$

s.t. $(\theta x_k, y_k) \in Y_{FDH}$,

using the above formulation of Y_{FDH} we have:

(II) $\min \theta$

s.t. $\sum_k \delta_k x_{kj} \leq \theta x_{k0j}, \sum_k \delta_k y_{ki} \geq y_{k0i}, \sum_k \delta_k = 1, \delta_k \in \{0,1\}$,

$j = 1, ..., m, i = 1, ..., s, k = 1, ..., n$,

Proposition 1: The solution to (I) is identical to the solution to (II).

Proof: Consider the programming-formulation (II) and restrict the attention to the k's within $D(x_{k0}, y_{k0}, Y_0)$ since k's outside of $D(x_{k0}, y_{k0}, Y_0)$ are characterized either by $\theta_{kj} > 1$ or $\sum_k \delta_k y_{ki} < y_{k0i}$ which cannot be a solution to (II). Hence we are going to show that

$$\theta = \min\{\max\{x_{1j}/x_{k0j}\}, ..., \max\{x_{Hj}/x_{k0j}\}\}$$

is identical to the θ obtained from:

$\min \theta$

s.t. $\sum \delta_h x_{hj} \leq \theta x_{k0j}, \sum \delta_h y_{hi} \geq y_{k0i}, \sum \delta_h = 1, \delta_h \in \{0,1\}$,

$j = 1, ..., m, i = 1, ..., s, h = 1, ..., H$,

where $\sum \delta_h y_{hi} \geq y_{k0i}$ by definition.

Since $\sum \delta_h = 1$ and $\delta_h \in \{0,1\}$ only one of the H observations can obtain the weight 1 and the rest 0. Hence we can write the inequalities as H pairwise comparisons in m inputs:

$x_{1j} \leq \theta x_{k0j}$

.

.

.

$x_{Hj} \leq \theta x_{k0j}$

For each of these comparisons we have m subequalities all of which should be satisfied. Thus we must choose θ as the biggest of the m possible values. This is done for each of the H comparisons. Given the minimization of θ we obtain

$$\min\{\max\{x_{1j}/x_{k0j}\}, ..., \max\{x_{Hj}/x_{k0j}\}\} \quad \clubsuit.$$

At this stage two remarks seems necessary. Firstly, notice that $(\theta x_{k0}, y_{k0})$ does not have to be an element of $D(x_{k0}, y_{k0}, Y_0)$, though it belongs to $D(x_{k0}, y_{k0}, Y_{FDH})$. Therefore there may exist other units in $D(x_{k0}, y_{k0}, Y_0)$ which are using less than θx_{k0} of some of the inputs. This property is a consequence of the radial efficiency measure and the weak assumptions of the FDH-technology. We will return to this in section 4.1.3 and 4.2.

Secondly, notice that the focus on input efficiency can cause strongly counter intuitive efficiency results.
Consider the following three observations (x_1, x_2, y_1, y_2) :

A:	(2,4,8,10)
B:	(1,4,10,10)
C:	(2,4,100,100)

Obviously A is dominated by B and C since both these observations are characterized by input amounts smaller than or equal to A's and output amounts larger than or equal to A's. However, the input efficiency measure for A (θ^A) will be equal to 1 i.e. input efficient, since

$$\theta^A = \min(\max(1/2, 4/4), \max(2/2, 4/4), \max(2/2, 4/4)) = 1.$$

Thus we have an input efficient and yet dominated observation, since focusing on input efficiency implicitly assumes that the observations are output efficient which definitely does not have to be the case. Hence in the case of an input efficient but dominated observation it would be recommendable to check for output efficiency as well. One has to keep in mind that input efficiency measures only can be used to indicate whether the units are performing input efficiently or not. It is not a clear cut overall efficiency judgement and indeed no acceptable solution seems to be available.

4.1.2. The DEA-procedure

As mentioned in section 3.2, what separates the DEA-procedure from the FDH-procedure is the assumption of convexity. Therefore (as noticed by Banker, Charnes & Cooper [1984]) we can derive the DEA-model directly from Farrell's definition of technical input efficiency and the assumed technology just as it was done in (II) where $Y = Y_{FDH}$. Hence we have the following DEA-model:

$\min \theta$

s.t $(\theta x_k, y_k) \in Y_{FDCH}$,

where Y_{FDCH} can be either Y_{CON}, Y_{DEC} or Y_{VAR}. Thus, in the traditional Farrell case of $Y = Y_{CON}$ we have n LP-problems:

$\min \theta$

s.t. $\sum_k \delta_k x_{kj} \leq \theta x_{k0j}, \sum_k \delta_k y_{ki} \geq y_{k0i}, \delta_k \geq 0,$

$j = 1, ..., m, i = 1, ..., s, k = 1, ..., n.$

Seiford & Thrall [1990] review the different DEA-models both in terms of input and output efficiency. Notice that the degree of input efficiency strongly depends on the assumed underlying technology. As easily seen in figure 2.2 in section 2, $\theta_{CON} \leq \theta_{DEC} \leq \theta_{VAR} \leq \theta_{FDH}$.

4.1.3. Slacks

Consider the example from section 2 of four DMU's each producing one output by two inputs (y, x_1, x_2). Unit D was dominated by unit B. If we measure the input efficiency of D by Farrells radial measure of input efficiency we will have $\theta^D = 3/4$ and hence the efficiency reference of D is $\theta^D x^D = (4.5, 3)$ as illustrated by figure 4.2. Notice that D ought to be compared to B but in fact it is compared to the reference $\theta^D x^D$ which is as efficient as B in input 2 but still inefficient relative to B in input 1 where the slack is 0.5. In general, all slacks will equal zero if and only if θ satisfy $\theta x \in DI$ where $DI = \{x|(x, y) \in D(x, y, Y_0)\}$. Obviously there will be many of the n units where the radial efficiency reference has slacks in either one or more dimensions. At worst we will have an evaluated unit which is declared efficient ($\theta = 1$) by the radial efficiency index but has possitive slacks in $m - 1$ dimensions (as an example consider unit E in figure 4.2 below). Thus the radial efficiency measure can be very misleading if it is not corrected with respect to slacks. In other words, the efficiency concept has to be that of Koopmans [1951] where an observation is declared efficient if and only if all slacks equal zero.

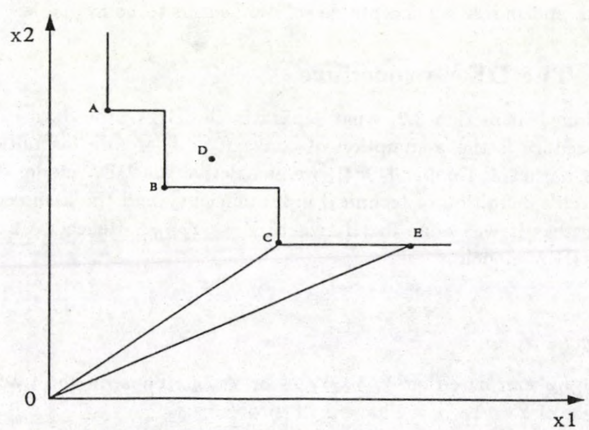

Figure 4.2: *Slacks*

Slacks can be included directly in the programming-models from section 4.1.1 and 4.1.2. Consider for example the following reformulation of (II) where $Y = Y_{FDH}$:

(III) $\min[\theta - \alpha(\sum_j s_j^+ + \sum_i s_i^-)]$

s.t. $\sum_k \delta_k x_{kj} + s_j^+ = \theta x_{k0j}, \sum_k \delta_k y_{ki} - s_i^- = y_{k0i}, \sum_k \delta_k = 1, \delta_k \in \{0,1\}$,

$s_j^+, s_i^- \geq 0, j = 1, ..., m, i = 1, ..., s, k = 1, ..., n$,

where s_j^+ resp. s_i^- is the slack in the j'th input resp. the i'th output and $\alpha > 0$ is a very small so called non-Archimedean number which is exogeneously chosen by the analyst. The non-Archimedian α was introduced by Charnes, Cooper, Lewin, Morey & Rousseau [1981]. Obviously, the choice of α has an impact on the reduction in the efficiency measure due to slacks. Notice that in order to be efficient in this model $\theta = 1$ and $s_j^+ = s_i^- = 0 \;\; \forall\, j, i$. For DEA- technologies inclusion of slacks is e.g. found in Banker, Charnes & Cooper [1984]. Originally Farrell [1957] solved the slack-problem within the Y_{CON}-technology by adding the pseudo observations $(\infty, 0, ...), (0, \infty, 0, ...), ..., (0, ..., \infty)$ to Y_0. However, this 'solution' is only possible when convexity of Y is assumed.

4.2. The non-radial input efficiency index

Instead of the slack correcting model (III) above we can build a new model where the input efficiency measure will be non-radial.

Definition: Consider output as fixed. In the case of strictly positive inputs define the non-radial Färe-Lovell input efficiency index as:

$E_{FL}(x, L) = \min\{\sum_j \theta_j/m | (\theta_1 x_1, ..., \theta_m x_m) \in L, \theta_j \in]0,1] \; \forall\, j, x \in R^m_{++}\}$,
where $L = \{x | (x, y) \in Y\}$.

If $\forall\, j,\ \theta_j = \theta$ then $E_{FL} = E_F$, making E_{FL} a generalization of E_F. E_{FL} is consistent with Koopmans concept of efficiency since $E_{FL} = 1$ if and only if $\theta_j = 1 \; \forall\, j$.

Consider $Y = Y_{FDH}$. As in (II), using the Färe-Lovell input efficiency index we obtain the following n mixed integer programming problems:

(IV) $\min \sum_j \theta_j/m$

s.t. $\sum_k \delta_k x_{kj} \leq \theta_j x_{k0j}, \sum_k \delta_k y_{ki} \geq y_{k0i}, 0 < \theta_j \leq 1, x_j > 0, \sum_k \delta_k = 1, \delta_k \in \{0,1\}$,

$j = 1, ..., m, i = 1, ..., s, k = 1, ..., n$.

Notice that the degree of efficiency obtained by (IV), $\sum_j \theta_j/m$, has not the same intuitive interpretation as Farrell's input efficiency index θ since

$((\sum_j \theta_j/m)x_k, y_k)$ is not necessarily an element of Y_{FDH}. However, the information contained by the partial θ_j's is in some sense more valuable (that is specific) than the aggregated information contained by Farrells θ.

Proposition 2: If $(\theta_1^*, ..., \theta_m^*)$ is a solution to IV with respect to unit $k0$ then,

$(\theta_1^* x_{k01}, ..., \theta_m^* x_{k0m}) \in DI$, where
$DI = \{x | (x, y) \in D(x_{k0}, y_{k0}, Y_0)\}$.

Proof: The proof is rather trivial since we know that a solution must be based on the units which are elements of $D(x_{k0}, y_{k0}, Y_0)$ and since no slack is allowed by definition of $E_{FL}, \theta_j^* x_{k0j} \in DI$ as defined above. ♣

Notice that the proposition does not hold if it is assumed that Y is convex as in the DEA-formulations.

(IV) can be formulated as a stepwise procedure similar to (I). Compared to the stepwise procedure (I) of section 4.1.1, step 1 and 2 are identical (and are therefore not repeated at this stage). However, step 3 and 4 must reformulated as follows:

Step 3. Calculate the fractions $Z = \sum \theta_{hj}/m \; \forall \; h$.

Step 4. Minimize Z.

Proposition 3: If the production technology is $Y_{FDH}, k0$ is compared to the same unit $h^* \in D(x_{k0}, y_{k0}, Y_0)$ in model II and IV if and only if:

$\max_j\{\theta_{jh}^*\} = \min_h\{\max_j\{\theta_{jh}\}\}$ and
$\sum_j \theta_{jh}^* = \min_h\{\sum_j \theta_{jh}\}$.

Proof: A straightforward consequence of proposition 1 and the definition of E_{FL}.♣

Proposition 3 implies that although $k0$ is compared to an existing unit both in model II and IV it does not have to be the same unit. Notice that in model II the actual efficiency reference does not have to be an existing unit though it is compared to one. In model IV the actual efficiency reference is an existing unit as stated by proposition 2.

Theorem 1: In the case of strictly positive inputs the Färe-Lovell input efficiency measure E_{FL} satisfies:

A) *Indication:*

a) $\forall \; x \in L(y), E_{FL}(x, L) \leq 1$

b) If $x \in Eff \; L(y)$, then $E_{FL}(x, L) = 1$, where $Eff \; L(y) = \{x \in L(y)|$ if $x^* \leq x, x^* \neq x$, then x^* is not in $L(y)\}$.

B) *Subhomogeneity:*

For any $x \in L(y)$ and any $\delta \leq 1, E_{FL}(\delta x, L) \leq (1/\delta) E_{FL}(x, L)$

C) *Commensurability:*

Let A denote a $(m \times m)$ diagonal matrix with strictly positive entries.

$$\forall\, x \in \mathbf{R}^m_{++}, E_{FL}(x, L) = E_{FL}(Ax, AL)$$

D) *Strict monotonicity:*

If $x^* \geq x, x^* \neq x$ then $E_{FL}(x^*, L) < E_{FL}(x, L)$.

Proof: It has been proved by e.g. Russell [1987] that E_{FL} in general satisfies axioms A)-C) along with weak monotonocity. Hence it remains to prove that E_{FL} satisfies strict monotonocity in the above case.
Rewrite E_{FL} as follows:

$$E_{FL}(x, L) = \inf\{\frac{1}{m}\sum_{j=1}^{m}\frac{\tilde{x}_j}{x_j}; \tilde{x} \in D(x, L)\},$$

where $D(x, L) = \{\tilde{x} \in L | \tilde{x} \leq x\}$. Since $D(x, L) \cap L$ is a compact set minimum will in fact be achieved for at least one solution $\tilde{x}^*$. Let $(\theta_1^*, ..., \theta_m^*)$ represent such a solution and let $\hat{x} \geq x, \hat{x} \neq x$ then we know that $(\theta_1^* x_1, ..., \theta_m^* x_m) \leq (\theta_1^* \hat{x}_1, ..., \theta_m^* \hat{x}_m)$. Hence there must be at least one $\theta_j < \theta_j^*$ for which

$$(\theta_1^* \hat{x}_1, ..., \theta_j \hat{x}_j, ..., \theta_m^* \hat{x}_m)$$

is closer to the efficient subset of L and hence $E_{FL}(\hat{x}) < E_{FL}(x)$. ♣

It is easy to see that the radial Farrell input index fails to satisfy indication as well as strict monotonicity which is two rather crucial features of a technical efficiency index.

Compared to the slack model in section 4.1.3 the model based on the Färe-Lovell index seems less ad hoc. Further, it has the advantage that the slack is a relative measure and hence is independent of the units of measurement as opposed to the approach of (III). In the above definition it does however require that inputs must be strictly positive which at first sight seems fairly harmless but in practice might involve some problems. Observations which include input amounts equal to zero will mainly originate from specialized units. Hence, either one has to design the data categories carefully e.g. by the approach mentioned in section 3.3 or one has to aggregate the input categories which seems less satisfactory. If we define the Färe-Lovell index such that $x \in \mathbf{R}^n_+$ we must settle for weak monotonicity in theorem 1.

4.3. Measuring the efficiency of the undominated units

Though we can measure the degree of input efficiency of each observation in the data set we are still left with the problem that a large number of the observations in fact will be input efficient (or undominated) as mentioned in section 3. Hence, it would be preferable if we could rank the efficient (or undominated) units by some measure which can be interpreted along the same lines as the original efficiency measure.

Andersen & Petersen [1989] propose a procedure to solve this problem within a DEA-framework. The basic idea is to construct a measure for each efficient observation which determines how much inputs can be increased proportionately, provided that the observation stays efficient relative to the data set. Consider the following reformulation of the DEA-model where $Y = Y_{CON}$:

(V) $\min \theta$

s.t. $\sum_k \delta_k x_{kj} \leq \theta x_{k0j}, \sum_k \delta_k y_{ki} \geq y_{k0i}, \delta_k \geq 0,$
$j = \{1, ..., m\}, i = \{1, ..., s\}, k = \{1, ..., n\} \setminus \{k0\}.$

The only difference between the original DEA-model and this reformulation is the exclusion of the $k0$'th observation from the envelopment of the frontier as illustrated by figure 4.3. Notice that the inefficient observations remain

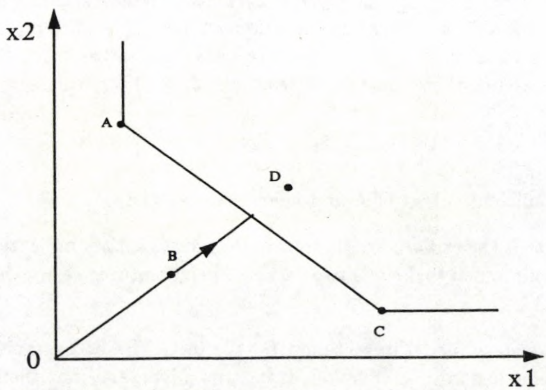

Figure 4.3: *Measuring B against the pseudo-frontier.*

inefficient and obtain the same degree of efficiency since their exclusion does not change the frontier. However, observations with $\theta = 1$ in the original DEA-model will receive a degree of efficiency larger than or equal to 1 in the respecified model. Those observations which maintain their efficiency degree equal 1 are those which in the original model have slacks in one or more input dimensions.

However, Andersen & Petersen notice that if the assumed technology is changed to Y_{DEC} or Y_{VAR} there may occur situations where (V) has no solution. Consider figure 4.4 where $Y = Y_{DEC}$. The observation C represents a situation where (V) has no solution since the removal of C from the frontier implies that the input level of C has no frontier reference.

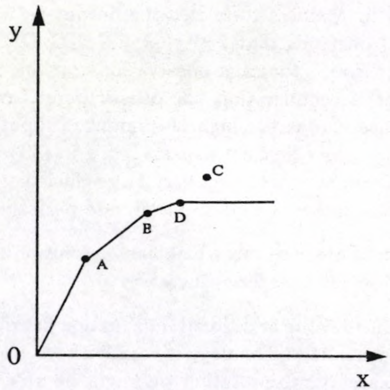

Figure 4.4: *No solution to model V*

Y_{FDH} is characterized by additional restrictions on the δ's compared to both Y_{DEC} and Y_{VAR}. Thus there is an even larger probability of finding observations within Y_{FDH} which has no solution to the programming-problem. For the free disposal hull technology Y_{FDH} we have the following n reformulated mixed integer programming-problems:

(VI) $\quad \min \theta$

s.t. $\sum_k \delta_k x_{kj} \leq \theta x_{k0j}, \sum_k \delta_k y_{ki} \geq y_{k0i}, \sum_k \delta_k = 1, \delta_k \in \{0,1\}$,
$j = \{1, ..., m\}, i = \{1, ..., s\}, k = \{1, ..., n\} \setminus \{k0\}$.

It is easy to see that (VI) has no solution when the evaluated observation is characterized by a maximal output amount in relation to all other units in Y_0 for at least one output category since it would imply that at least one of the i restrictions $\sum_k \delta_k y_{ki} \geq y_{k0i}$ are violated. Consider the following reformulation of the example introduced in section 2:

DMU	(y, x_1, x_2)
A	(2,2,6)
B	(1,4,3)
C	(1,7,1)
D	(1,6,4)

The only difference is that the output amount for A is doubled. The four observations are still characterized by the same input efficiency scores - that is $\theta^A = \theta^B = \theta^C = 1$ and $\theta^D = 0.75$ as found by (II). Observations A and C are undominated but non-dominating and B is dominating D. Solving the four programming-problems stated in (VI), we see that D recieves the same efficiency score. $\theta^B = 1.5$ (compared to D) and $\theta^C = 3$ (compared to B). However, there is no solution to A since A violates the output restriction. Output is equal to 2 but (VI) demands that $2 \leq y_k, k = \{B, C, D\}$, which obviously cannot be satisfied. Notice that the observations can be ranked as $C > B > D$. However, it is peculiar that the observation C receives the highest efficiency score since it uses the highest amount of input 1. In general, specialized units will receive efficiency scores $\theta >> 1$ in (VI) due to their incomparability. Thus we ought to interpret very large efficiency scores as an indication of specialization rather than extremely good performance.

Notice further that the number of observations which has no solution to (VI) increases when the number of output categories increases.

As a conclusion one must admit that the additional information provided by (VI) in relation to the free disposal hull technology is very limited. However, with careful data design and careful interpretation we might be able to use the obtained information in relation to an explanation of the efficiency scores through environmental factors.

5. Explaining inefficiency

Many attempts to decompose the radial technical efficiency measure into several sub-measures have been made in order to characterize the observed activities. Also measures of size efficiency have been proposed along with measures of most productive scale size (see e.g. Färe, Grosskopf & Lovell [1983] and Maindiratta [1990])

However, all these measures are build upon the same production data set as the 'original' technical efficiency measure and hence basically no new information is introduced by their calculation. Trying to "explain" the observed technical efficiency score by calculating e.g most productive scale size does not provide any explanation of the actual level of technical efficiency. Hence in order to be able to explain the outcome of DEA and FDH analysis, more information is obviously needed to be introduced. Information which relates to the characteristics not only of the purely technical side of the activity but also of the organizational environment.

One such way to provide explanations of the obtained efficiency scores is to interpret the calculated efficiency scores as a dependent variable which is determined by a set of environmental factors.

Let $\theta = (\theta_1, ..., \theta_n)$ denote the vector of efficiency scores for the n observations and Z be a $n \times L$ matrix of L environmental factors. Thus a general regression model can be formulated as:

$$\theta_k = f(z_k; \beta) + e_k, \quad k = 1, ..., n$$

where β are the parameters to be estimated, z_k are the vector of environmental factors for the k'th unit and e_k is a disturbance term for the k'th unit. In order to estimate the vector of parameters β, assumptions about the functional form of $f(z_k; \beta)$ have to be made. This specification could be non-linear and thus require non-linear estimation techniques. However since no apriori knowledge about the relationship between θ and z_k are available we follow the tradition of assuming a linear realtionship, i.e. the model:

$$\theta = Z\beta + e,$$

Notice that though the regression model above is linear it is still possible to consider non-linear transformations of the environmental factors provided the transformed variables are linear with respect to θ.

A classical problem connected with regression analysis – the selection of the independent variables – appears in this model through determination of the set Z of environmental factors. Obviously it is impossible to insure the inclusion of all relevant variables. Roughly the environmental factors can be divided into two categories. One consist of uncontrollable variables exogeneous to the DMU's. If we look at hospitals an example of such a variable could be the patient mix reflecting that the patients are not homogeneous with respect to their demand of resources. This cannot be covered by the standard model since it would require disaggregation of the outputs to an extent which is realisticly impossible and theoretically undesirable. The other group consists of variables which describes differences in the organizational structure of the DMU's. For hospitals an example could be whether a hospital has a research department or not. Assume that the research department affects the efficiency through the input vector since resources are used and hence measured as part of input, but if no outputs are registered the efficiency scores will be understated.

If the parameters in the linear model are estimated by OLS problems occur, because the vector θ of efficiency scores are restricted to take values between 0 and 1. This implies biased and inconsistent estimates of β. The estimates of β becomes biased (and thus inconsistent) since $0 < \theta \leq 1 \Rightarrow 0 < Z\beta + e \leq 1 \Leftrightarrow Z\beta + e > 0$ and $Z\beta + e \leq 1 \Leftrightarrow e > -Z\beta$ and $e \leq 1 - Z\beta$. e is thus a function of Z and therefore correlated with Z. But in order for the OLS to give unbiased estimates of β, Z and e must be uncorrelated. This can be derived from the formula for the OLS estimate b:

$b = (Z'Z)^{-1}Z'\theta \Rightarrow b = (Z'Z)^{-1}Z'(Z\beta + e) \Rightarrow$
$b = (Z'Z)^{-1}Z'Z\beta + (Z'Z)^{-1}Z'e.$

Taking expectations of the previous expression gives:

$$E[b] = E[(Z'Z)^{-1}Z'Z\beta] + E[(Z'Z)^{-1}Z'e] \Rightarrow E[b] = \beta + E[(Z'Z)^{-1}Z'e].$$

If Z and e were uncorrelated the last term would disappear but with $0 < \theta \leq 1$ this does not happen.

Some transformation of θ is needed to solve the problem concerning the restrictions on θ. If the procedure of ranking efficient observations is applied (see section 4.3), θ is only bounded below by 0. In this case it is sufficient to use a logaritmic transformation of θ to obtain an unrestricted dependent variable, i.e. the model $\ln\,\theta = Z\beta + e$. For $\theta \to 0 \Rightarrow \ln\,\theta \to -\infty$ and for $\theta \to +\infty \Rightarrow \ln\,\theta \to +\infty$. This approach is e.g. chosen by Lovell, Walters & Wood [1990].

However, another transformation of θ may prove more satisfactory. Consider the following reformulation of the general version:

$$\ln((1-\theta)/\theta) = Z\beta + e,$$

For $\theta \to 0 \Rightarrow (1-\theta)/\theta \to +\infty \Rightarrow \ln((1-\theta)/\theta) \to +\infty$ and for $\theta \to 1 \Rightarrow (1-\theta)/\theta \to 0 \Rightarrow \ln((1-\theta)/\theta) \to -\infty$, i.e. $\ln((1-\theta)/\theta) \in]-\infty, +\infty[$. Thus this transformation of θ alters the limited dependent variable to a dependent variable with unrestricted range and OLS can be applied. Moreover, using this kind of transformation we escape the problematic method of ranking efficient units proposed by Andersen & Petersen op. cit. Notice that the sign of the estimates of the β's relate to the transformation and not to θ itself, where the effect has the opposite sign.

If the variation in θ to a high degree can be explained by the Z variables this indicates that to a large extent a given efficiency scores can be explained by specific institutional conditions rather than mere excessive use of inputs.

6. Dynamic aspects

Up till now we have only considered single period problems – that is efficiency measurement of observations from one particular period relative to the technology of that period. But what if panel data are available? Not surprisingly, looking at efficiency in a dynamic context in order to examine long run efficiency trends proves to be more than just a smooth extension of the single period analysis. In general there are difficulties involved in the determination of the relevant time horizon of the analysis. The units may become uncomparable over time with respect to previously fixed standards such as production categories, sample size etc, and since uncomparability leads to meaningless results the long run efficiency results are easily distorted. Moreover, and just as important, there are purely methological problems involved as well.

As an introduction, consider the following extended version of the example in section 2 where the two periods are represented by their own technology:

DMU	(y_1, x_1^1, x_2^1)	(y_2, x_1^2, x_2^2)
A	(1,2,6)	(1,2,6)
B	(1,4,3)	(1,6,3)
C	(1,7,1)	(1,7,1)
D	(1,6,4)	(1,6,4)

The two periods are almost identical except for unit B where the amount

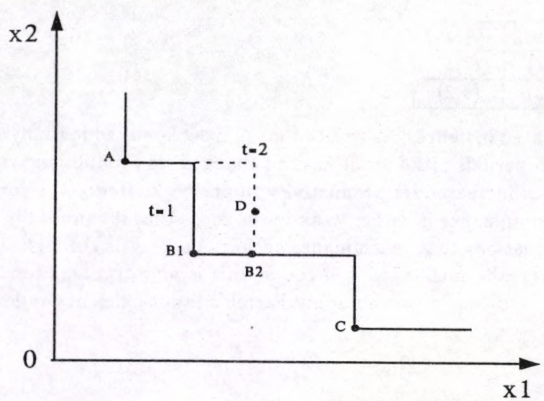

Figure 6.1: *Illustrating the example*

of input 1 is increased by two units in the second period. If input efficiency of the four units is measured in period 2 relative to the period 2 technology (the free disposal hull of Y_0^2), we would have; $\theta^{A2} = \theta^{B2} = \theta^{C2} = \theta^{D2} = 1$. Focusing on observation D we notice that moving from period 1 to period 2 it has become input efficient since $\theta^{D1} = 0.75$ and $\theta^{D2} = 1$. But the production vector of D is identical in the two periods. Hence we cannot interpret changes in the degree of efficiency for the same observation over time in a direct manner. We need additional information about the change in reference technology. The usual approach in such situations is the use of index numbers as when turning to the Malmquist index.

Let $x^t \in \mathbf{R}_+^m$ resp. $y^t \in \mathbf{R}_+^s$ denote input resp. output vectors at time t $(t = 1, ..., T)$ and let Y_t denote the production possibility set at time t based on the set of observations at time $t, Y_0^t = \{(x_{kt}, y_{kt}) | k = 1, ..., n, t = 1, ..., T\}$. Let the history of production data up till time t be given by the set:

$$YH^t = \{(x_k^\tau, y_k^\tau) \in Y_0^\tau | \tau \leq t\}.$$

In this way the production data can be viewed sequentially from $t = 1$ to $t = T$. Consider the dominant set $D^t(x_k^t, y_k^t, YH^t) = \{(x, y) \in YH^t; x \leq x_k^t, y \geq y_k^t\}$.

Definition: An observation $k0$ is ***sequentially undominated*** if and only if $D^t(x_{k0}^t, y_{k0}^t, YH^t)$ is a singleton. Otherwise the observation is declared ***sequentially dominated***, i.e. there exist previous observations which used the same amount or less of inputs and achieved the same or a higher amount of outputs.

Consider the following simple example of two DMU's in two periods:

DMU	(y_1, x_1)	(y_2, x_2)
A	(1,1)	(1,1)
B	(3,3)	(4,2)

YH^t is illustrated in figure 6.2. Notice that A is declared sequentially undominated in both periods although it has not changed its production vector. So is B, but B has increased its productivity in period 2. Hence the concept of sequential undominance is rather weak in the sense that it cannot distinguish between observations that reach higher performance levels through time and those which remain unchanged – of course this is an advantage for the specialized units. Obviously we can apply Farrell's input efficiency index to the

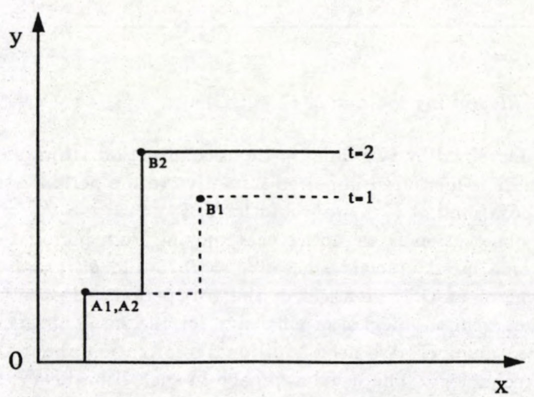

Figure 6.2: *Illustrating YH^t of the example*

sequential frontiers. This index will be well defined since $YH^t \subset YH^{t+1}$.

6.1. The Malmquist efficiency index

If we want to evaluate the change in activity for a given unit from one period to another relative to a given frontier at time d, we can use the Malmquist index approach.

Let E_F^{dt} denote the Farrell input efficiency index for an observation at time t relative to the technology (the frontier) at time d based on Y_0^d.

Definition: Following the approach of Berg et al. [1992] we define the Malmquist input efficiency index between two periods 1 and 2 as:

$$M_d(1,2) = \frac{E_F^{d2}}{E_F^{d1}} = \frac{E_F^{22}}{E_F^{11}} \frac{E_F^{d2}/E_F^{22}}{E_F^{d1}/E_F^{11}},$$

where $d = 1,2$ represent the reference technology. The first term is the ratio of input efficiency for the two periods i.e. a catching-up effect. The second term represents a frontier shift effect – that is it measures the distance between technology 2 and 1 based on the common reference technology. In figure 6.3, Q is inefficient in both periods. Hence both E_F^{22} and E_F^{11} are smaller than 1 and the cathing-up effect is equal to:

$$\frac{OA/OC}{OB/OD}.$$

The frontier shift effect relative to technology 1 can be written as:

$$\frac{(OE/OC)/(OA/OC)}{(OB/OD)/(OB/OD)} = \frac{OE}{OA}.$$

Thus, $M_1(1,2) = \frac{OE/OC}{OB/OD}$. Hence the Malmquist index captures two different aspects of an efficiency development; efficiency measured relative to the periods own technology, and a shift in the frontier due to a technological change. If $M_d(1,2) > 1$ there has been a positive efficiency development, if $M_d(1,2) = 1$ efficiency has been constant and finally if $M_d(1,2) < 1$ a negative efficiency development has occured.

Consider the above example once more. Focus on the observation D and consider M_1. We know that $E_F^{11} = 0.75$ and $E_F^{22} = 1$. Calculating E_F^{12} and E_F^{21} we find that $M_1(1,2) = 1$. Hence the Malmquist index reveals that the performance of D, relative to technology 1, is unchanged despite that the efficiency scores indicate a positive development. The frontier change and the change in efficiency work in opposite directions and cancels out through the Malmquist index.

As noticed by Berg et al. [1992] the Malmquist index satisfies the circular test i.e. $M_d(0,1)$ times $M_d(1,2)$ equals $M_d(0,2)$. However, it is worth noticing that we cannot be certain that all Farrell indices involved in the definition of the Malmquist index are well defined. If we consider the sequential frontier

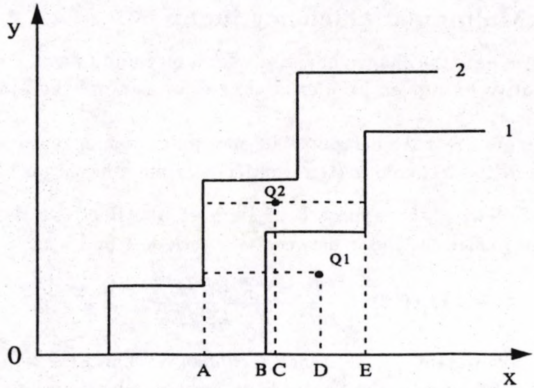

Figure 6.3: *The Malmquist input efficiency index*

unit A	YH^1	YH^2	YH^3	YH^4
period 1	E_F^{11}	E_F^{21}	E_F^{31}	E_F^{41}
period 2	-	E_F^{22}	E_F^{32}	E_F^{42}
period 3	-	-	E_F^{33}	E_F^{43}
period 4	-	-	-	E_F^{44}

Table 6.1: *Sequential performance indices*

in order to take the history of production data into account we can make tables like table 6.1. It is easily observed that the efficiency indices of the lower part of the table are not necessarily well defined due to possible observations from the period in question which dominate all previous technologies *d*. Hence the Malmquist index which involves any such Farrell indices is not generally defined. This may be seen as a major drawback of the Malmquist approach since although chainable the index cannot cover all stages of a given production development. Figure 6.4. illustrates a case where E_F^{12} is not defined.

7. Final remarks

Summarizing the analysis above, five main aspects immediately appear.

Firstly, as required by theory, it showed up to be extremely important that the analyzed units were in fact similar in the technological sense. The various models surveyed in the previous sections are all very sensitive to the structure of the activity data set, especially to "outliers" in the form of specialized units. Due to the underlying assumptions on the production technology, the

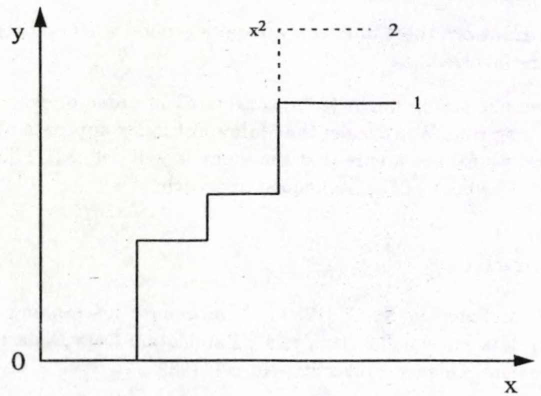

Figure 6.4: *A case where E_F^{12} has no solution*

FDH-model proved to be less sensitive than the various DEA-models.

Secondly, the choice between relevant production technologies is far from evident. At least two levels are involved when determining the technology. Whether choosing DEA or FDH depends on the attitude towards the assumption of convexity. Moreover, having accepted convexity which follows from the DEA-models, one has to determine the proper returns to scale. There seems to be some arguments in favour of the unrestictive formulation free disposability. However, it is recommendable to consider all various kinds of technologies in order to get a more complete picture of the activities as such.

Thirdly, when it comes to measuring technical efficiency the partial efficiency indices found by the non-radial Färe-Lovell input index appear to be more interesting than the traditional radial Farrell index. The whole point of FDH and DEA-models are to handle disaggregated activity data which makes it natural to operate with partial efficiency indices too. However, strictly speaking we have defined the Färe-Lovell index as the mean of the partial efficiency scores which makes it difficult to interpret as opposed to Farrell's radial index.

Fourthly, it is important to remember that technical efficiency indices are only technical of nature i.e. only related to the transformation of inputs into outputs. Hence, when it comes to explaining the obtained technical efficiency result it is necessary also to include institutional (environmental) factors. This can be done through a regression approach as illustrated but obviously there are problems related to the definition of relevant environmental variables. However, these problems are quite general whenever regression approaches are involved.

Finally, a dynamic set-up ought to be considered in order to examine efficiency in the long run. We consider the Malmquist index approach and show that in general we cannot assure that the index is well defined. This proves to be a major drawback of the Malmquist approach.

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