European University Institute

DEPARTMENT OF ECONOMICS

Essays in Financial Economics

Jeanine Baumert

Thesis submitted for assessment with a view to obtaining the degree of Doctor of Economics of the European University Institute

Florence, 26 April 2017

European University Institute
Department of Economics

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Acknowledgments

The completion of this doctoral dissertation was only possible with the support of several people. First of all I would like to thank both my advisors Ramon Marimon and David Levine for their continued support, guidance and encouragement throughout my PhD - their rigorous reading of my work and insightful comments with no doubt has tremendously increased the quality of this work. I am also thankful to Andrea Galeotti and Piero Gottardi for their useful comments, suggestions and challenging questions concerning my work. I have greatly benefited from the intensive training a the EUI and am extremely grateful to all faculty for the great effort they put into this.

I would like to thank Douglas Gale for giving me the opportunity to spend time at NYU and all other faculty who were available to meet me and provide me with their insightful comments on my research, in particular Boyan Jovanovic.

I would like to especially acknowledge the financial support from the German Academic Exchange Service and the EUI.

The experience at the EUI would not have been the same without the great team previously at Villa San Paolo and now at Villa La Fonte, namely Jessica Spataro, Lucia Vigna, Julia Valerio, Sarah Simonsen, Anne Banks, Thomas Bourke, Loredana Nunni and Sonia Sirigu.

This work as well as my understanding of economics in general has greatly profited from various discussions with colleagues at the Economics departments at EUI and NYU.

Without the support of my family and friends I could not have mastered the challenges of a PhD. In particular, I am thankful to my parents Ursula and Wolfgang for providing me with the foundations and love that enabled me to pursue my goals in life in general, and this PhD in particular. I am grateful to Niccolò for his infinite amount of love and patience, being there for me when things were tough and being an amazing father to our beautiful son Leonardo who brings so much joy into our lives.

This thesis is dedicated to my late father Wolfgang.

Florence, 19 April 2017

Introduction

This thesis addresses two phenomena, which are commonly observed in the financial markets but so far have received relatively little attention in the literature. By putting these issues into a theoretical framework I provide a deeper understanding of their mechanisms as well as their possible impact on the market as a whole.

The aim of the first part of the thesis is to provide a rationale why informed traders in the financial markets voluntarily share information with others. The success of online chatting platforms within the financial service industry shows that traders very much like to communicate with each other. Standard theory however suggests that by sharing information, traders reduce their informational advantage and thus decrease their profits. The aim of this work is to reconciliate theory with the facts by providing an explanation on why sharing information could be profitable for traders as well as to investigate the effects of information sharing on the market as a whole.

The second part of the thesis analyses the role of credit insurance on debt renegotiation. During the renegotiation of private debt in Greece it has become apparent that the existence of credit insurance significantly changes investor behavior when it comes to their willingness to accept a deal. This article explores the impact of credit insurance on the bargaining process as well as how it changes lending conditions for the issuer of debt.

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Chapter 1

Insider Trading and Communication among Peers

1.1 Introduction

In October 2014 Goldman Sachs together with 14 other financial institutions invested \$66M in launching the messaging platform "Symphony" that allows participants in the financial markets to communicate with each other. It must be their underlying belief that by enabling traders to share information with each-other, they provide a valuable service to the clients of Symphony for which the latter are willing to pay an annual subscription fee. In fact it can be frequently observed in the financial markets that some of the most successful investors like to share their insights with other investors. With the emergence of the powerful hedge fund industry, which is sometimes portrayed as an "old boy network", this raises a number of questions. Why do investors share information at all? Does communication among already better informed investors have detrimental effects on financial markets in general or on those that are less informed? Or on the contrary, does it lead to better informed decisions among investors, and thus to more financial stability?

In standard models of insider trading, following the seminal Kyle (1985) model, traders profit from having proprietary information that the rest of the market does not have. In these models, traders would not give away any information for free, as it would reduce their informational advantage and thus decrease their profits. The theory is thus at odds with what we can observe. The aim of this works is to reconcile the observation of communication among privately informed traders with the theory by providing a model where a strategic insider trader benefits from sharing (some) information with others. I then continue to analyze implications for the market as a whole.

In particular, I construct a three period version of the extended Kyle (1985) model of insider trading, where in contrast to the original model, I allow for long-term and short-term information about the liquidation value of the asset. The characteristic of short-term information is that it becomes publicly known in an intermediate period, so that the knowledge of it becomes worthless for trading after this period. Long-term information on the other hand is never perfectly revealed. Before the first round of trading, each informed trader is endowed with two signals, one which contains short-term information and one which contains long-term information. I show that if one trader is able to communicate its short-term information to other traders with some noise, he can profit from this communication. The intuition is the following: By sharing his information he introduces noise into the economy and only he himself knows its precise realization. All future trading decisions depend on the realization of this noise. In particular even when the short-term information becomes obsolete, the noise is not being revealed so that it continues to have impact on trading decisions. At this point the sender however still wants to infer the long-term information of other informed traders from the market price. Since he is better able to separate the noise from the real information contained in the price, he has an advantage in extracting information from the price compared to the market maker. By sending a noisy signal of short-term information he thus has endogenously created an informational advantage about the long-term information and consequently increased its profit.

Since traders who receive information also increase their profits, both types of informed traders, senders and receivers, can be better off communicating. This is to the cost of the uninformed noise traders, which has an important policy implication: If a regulator wanted to protect small, non-professional investors he might want to do anything possible to limit communication among informed traders. Regarding informational efficiency, I find that even though long-term price efficiency is lower (as this is exactly how the sender is profiting), in the short-term informational efficiency may actually improve. This is due to an equilibrium effect: Since in the long-term liquidity decreases and trading thus becomes more costly, the sender prefers to shift some trading on the long-term signal to the first period. By increasing his trading intensity he reveals more information about the long-term signal earlier on which increases informational efficiency in the short-term.

This paper falls into the broader literature of strategic manipulation in asset markets. This literature can be distinguished into three main categories depending on how manipulation is achieved (Allen and Gale (1992)): In the first category of models manipulation is obtained by actions which change the real or perceived value of assets, in the second category by communication of information that is relevant to the payoff of the asset and in the third category by manipulating the price through trading only. This paper falls into the second category, as the central trader increases his profit by sending messages. Other example in this strand of literature are e.g. Vila and Jean-Luc (1989), where a trader can make a profit by shorting a stock first, then spreading incorrect information and afterwards buying back the asset at a lower price. Benabou and Laroque (1992) on the other hand show in a reputational cheap-talk game that an informed trader can profit by communicating misleading information, as long as he is perceived as being honest. This paper differs to this line of research in that the insider trader profits not from communicating incorrect information, but from communicating noisy but true information. Adding noise to the economy thereby hinders endogenous learning of the market.

The observation that traders can profit from their information even after is has been revealed is thereby not new to the finance literature. Brunnermeier (2005) investigates the question of how an insider trader that receives noisy information before it is announced to the public can exploit on it even after its announcement. After the revelation of information the insider realizes with which noise he previously received the signal. Also in this model the trader thus benefits from knowing the realization of the noise that is incorporated into the price and thus giving him an advantage in interpreting the price. The insider is however passive in this model and his advantage comes form "mingling" with the entrepreneur and receiving the information before it becomes public. In this work on the other hand the sender creates this advantage actively by communicating his information to other traders with some noise, while ex-ante not having any superior information.

Another strand of literature related to this work investigates the effect of mandatory order disclosure on insider trading. Huddart et al. (2001) modify the standard Kyle model so that the (monopolistic) insider has to disclose his order before the next round of trading. The finding of that paper is that disclosure always decreases the informed trader's profits, leads to more liquidity and better market efficiency. Cao et al. (2013) extend this work to a multi-trader stetting where traders have heterogeneous signals. In this setting traders may increase their profits as trade disclosure lets them learn other traders signals at a faster pace than the market maker.

The effect of short-term information has first been investigated by Admati and Pfleiderer (1988). In contrast to the standard Kyle model the insiders' information remains private only for one period and then is publicly revealed. Additionally, they introduce a second type of noise trader that can decide on the timing of his trade but not on the actual order size. This leads to an equilibrium where there is a strategic complementarity to trade at the same time (as all traders prefer to trade when the market is liquid and the more traders trade the more liquid the market). This may explain the empirically documented U-shape in trading volume within a day.

This paper proceeds in the following way: Section 2 introduces the structure of the economy. Section 3 defines what is meant by equilibrium and characterizes an equilibrium in this economy. Section 4 presents the results.

1.2 Model

Consider a market for a single asset in fixed supply, which is traded in three trading rounds and liquidates thereafter. Following the extended setup of Kyle (1985) there are three types of market participants: n informed traders, noise traders and a competitive market maker. All market participants are risk neutral and none of them can observe the liquidation value of the asset v perfectly before it has matured.

The role of the market maker is to execute the orders of the informed and noise traders. When receiving orders, the market maker cannot distinguish between informed and noise trading, since he can only observe the aggregate order X_t. There is free entry into market making and he thus earns zero profits. This implies that the market clearing price equals to the market makers expectation of the liquidation value v of the asset given the information he can extract from the history of aggregate orders up until period t, $\{X_s\}_{s=1}^{t}$.

Informed traders receive signals about the liquidation value of the asset before the first round of trading and trade in order to exploit their informational advantage. Thereby each trader receives two types of signals: a short-term signal s_i, which becomes publicly revealed in an intermediate period as well as a long-term signal l_i, which remains private until the liquidation of the asset. As commonly assumed in the literature (e.g. Brunnermeier (2005), Cao et al. (2013) among others) I assume that information is dispersed among informed traders, in the sense that the sum of the signals of all traders equals the liquidation value of the asset

$$v = \sum_{i=1}^{n} s_i + \sum_{i=1}^{n} l_i$$

. All signals are thereby independently normally distributed with mean 0 and variance σ_s^2 for the short-term signals and σ_l^2 for the long-term signals respectively.

The new feature of this model is that after the first round of trading one trader, which will be referred to as the central trader in what follows, can send a noisy message

$$m^i = s_c + \delta^i$$

about his short-term information to all other $n-1$ traders which I will refer to as the peripheral traders. Peripheral traders are mute in the sense that they cannot exchange any information among them, nor do they return the favor to the central trader. This setup can thus be considered as a directed communication network with star shape. Each peripheral trader i receives the central trader's information disturbed by a different noise term δ^i, whereby all δ^i's are assumed to be independently normal distributed with mean 0 and variance σ_δ^2, which is exogenously given and not a strategic choice of the central trader. Messages are assumed to be sent truthfully in the sense that the central trader cannot lie about the signal he has received in order to mislead others.

Receiving a message allows a peripheral trader to improve his knowledge not only about the central players short-term signal s_c and thus the value of the asset, but also about the messages other traders have received and how this affects their orders and consequently the market price. Furthermore I assume that the sender of the message is able to observe the noise terms δ^i with which he is communicating his information. This allows him to exactly understand how his message will be used by the peripheral traders and gives him an advantage in interpreting the second period price as will be explained in more details below and will be an important driver of the results. The information structure is assumed to be common knowledge among all market participants. Based on his information, each informed trader i chooses his order size x_t^i when it is his turn to trade.

Noise traders on the other hand do not receive any private information and inelastically demand the asset. The noise traders' aggregate demand in period t, ϵ_t is assumed to be a random variable, normally distributed with mean zero and variance σ_ϵ^2. The presence of noise traders is a common assumption in models of heterogeneous information. Their role is to camouflage the informed traders' information. Without them, as noted by Grossman and Stiglitz (1980) the informed traders' (aggregate) information would be immediately revealed by the market price. Their presence also implies that the no-trade theorem does not hold in this economy, so that there will be trade after information has been communicated.

Timing

There are three periods of trading. Before the first period each informed trader i receives his private short-term signal s_i and private long-term signal l_i. In the first round of trading only the central trader is allowed to trade. Based on the signals he has received, he submits his order x_1^c. Noise traders submit a random order ϵ_1. The market maker observes the aggregate order $X_1 = x_1^c + \epsilon_1$ and sets the market price p_1, which is observed by everyone. The central trader sends a noisy message m^i about his short-term signal to each peripheral trader i. After observing the price p_1 and the message m^i he received from the central trader, each peripheral trader i updates his beliefs. In the second round of trading only peripheral traders are allowed to trade. Based on his updated beliefs each peripheral trader submits his order for the second period x_2^i. Noise traders demand again a random order ϵ_2. As in the first period the market maker observes the aggregate order $X_2 = \sum_{i \neq c} x_2^i + \epsilon_2$ and sets the market price p_2. After the second period the short-term information becomes public. While the central trader trades again in the third period in the same fashion as in the first period after having observed the period 2 market price and short-term information, peripheral traders are not allowed to trade again. After the third round of trading the asset liquidates and each trader receives an amount of v for each unit hold in the asset. These timing assumptions are chosen for the following reasons: The central trader needs to trade for a second time in order to enable him to profit from sending his signal as it is in the

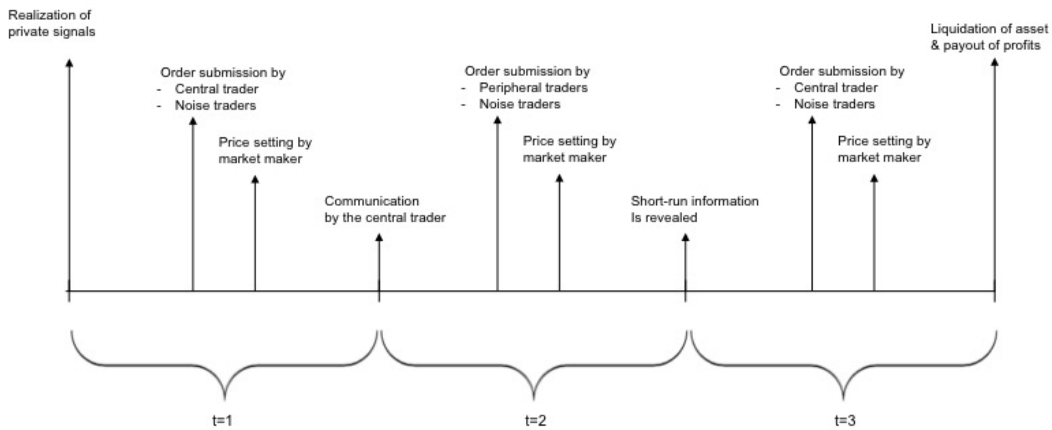

Figure 1.2.1: Timing

third round of trading where he gains from communication. This is not the case for the peripheral traders who gain only when receiving information in the second period of trading. A more general version of the model would allow both types of traders to trade in each period. This would increase competition among informed traders and thus decrease their profits. The claimed result of the central trader benefiting from sending information would still prevail, since he can still increase his forecast precision compared to the market maker in the third period. The analysis of the model would however be much less tractable for the following two reasons: The peripheral traders would need to solve a dynamic problem which increases complexity while not adding any insight. Secondly having two types of traders being active within one period makes the analysis less tractable as the market now needs to distinguish between two types of traders.

It is substantial to the results however to let the central trader trade on his short-term information before the peripheral traders can take advantage of his message. A two period model where the central trader can submit an order only at the same time or after the peripheral trader take advantage of his information might not lead to the same result as this would lead to a decline in profits in the first period (due to the short-term signal being less valuable as it is used by many traders) which might outweigh his future gain.

Market Maker's Pricing Rule

The market maker fulfills the orders of the informed and noise traders by acting as an intermediary and taking on any potentially arising net position. Since he is competitive he sets the market price equal to the expected liquidation value v of the asset. In order to form expectations he tries to infer information from the informed traders orders. He can however not observe individual orders but only the aggregate order of informed and noise traders X_t and thus a noisy signal of the informed traders demand. The prices of the asset in period 1 and 2 are then given as

$$p_1 = \mathbb{E}\left[v \mid X_1\right]$$

and

$$p_2 = \mathbb{E}\left[v \mid X_1, X_2\right]$$

After the second round of trading, the short-term information $\{s_i\}_{i=1,\dots,n}$ becomes publicly known, so that the market maker takes this into account when setting the third period price to

$$p_3 = \mathbb{E}\left[v \mid X_1, X_2, X_3, \{s_i\}_{i=1,\dots,n}\right]$$

Informed Traders Problem

According to their own valuation of the asset and conjecturing the market maker's as well as the other traders' strategies, each informed trader i decides on how many units x_t^i of the asset he wants to demand when it is his turn to trade, depending on the information he has available. When making his decision in period 1 the central trader takes into account how his current order impacts both current and future prices. This leads to the following optimization problem in period 1 for the central trader c

$$x_1^{c,*} \in argmax_{x_1^c} \mathbb{E}\left[x_1^c\left(v - p_1\left(x_1^c\right)\right) + x_3^{c,*}\left(x_1^c\right)\left(v - p_3\left(x_1^c\right)\right) \mid s_c, l_c, \left\{\delta^i\right\}_{i=2,\ldots,n}\right] \quad (1.2.1)$$

. After having observed the realized price of period 1 additionally to their own signals and the message they have received from the central trader, peripheral traders decide on their order in the second period . Their static optimization problem is given by

$$x_2^{i,*} \in argmax_{x_2^i} \mathbb{E}\left[x_2^i\left(v - p_2\left(x_2^i\right)\right) \mid s_i, l_i, m^i, p_1\right] \quad (1.2.2)$$

When the central trader gets to trade again in the third period he will have observed the period 1 and 2 prices additionally to the information he has had already in period 1. Also the short-term information will have been revealed at this point in time. Since after the third round of trading the asset liquidates, his optimization problem is of static nature and can be written as

$$x_3^{c,*} \in argmax_{x_3^c} \mathbb{E}\left[x_3^c\left(v - p_3\left(x_3^c\right)\right) \mid \left\{s_i\right\}_{i=1,\ldots,n}, l_c, \left\{\delta^i\right\}_{i=2,\ldots,n}, p_1, p_2\right] \quad (1.2.3)$$

1.3 Equilibrium

This section first establishes what is meant by an equilibrium and then proceeds to characterize a linear equilibrium of the trading game described above.

Definition 1. A sequentially rational Bayesian Nash equilibrium of the trading game is given by a strategy profile
$\left\{x_1^{c,*}, \left\{x_2^{i,*}\right\}_{i\in\{2,\ldots,n\}}, x_3^{c,*}, p_1^*, p_2^*, p_3^*\right\}$ such that

1. the central trader chooses his first period and third period order optimally as defined in (1.2.1) and (1.2.3)

2. peripheral traders choose their second period order optimally as defined in (1.2.2)

3. the market maker sets the price according to $p_1^* = \mathbb{E}\left[v \mid X_1^*\right]$, $p_2^* = \mathbb{E}\left[v \mid X_1^*, X_2^*\right]$ and $p_3 = \mathbb{E}\left[v \mid X_1^*, X_2^*, X_3^*, \{s_i\}_{i=1,\ldots,n}\right]$ and beliefs are consistent.

In what follows I will focus on a symmetric equilibrium in linear strategies, where each participant makes decisions based on a linear combination of the information available to him and all peripheral traders follow the same strategy. The following proposition demonstrates that such kind of strategies are indeed consistent with an equilibrium as defined above.

Proposition 2. *A sequentially rational Bayesian Nash equilibrium in which all pure* *trading strategies are of the linear form*

$$
\begin{aligned}
x_1^c &= \alpha^c s_c + \beta_1^c l_c \\
x_2^i &= \alpha^p s_i + \beta_2^p l_i + \gamma^p m^i + \theta^p p_1 \\
x_3^c &= \beta_3^c l_c + \varphi^c \sum_{i \neq c} \delta^i + \theta^c T + \zeta^c S
\end{aligned}
$$

and the market maker's pricing rule is of the linear form

$$p_1 = \lambda_1 X_1 \tag{1.3.1}$$

$$p_2 = \tilde{\xi}^1 p_1 + \lambda_2 X_2 \tag{1.3.2}$$

$$p_3 = \sum s_i + \tilde{\xi}^S S + \tilde{\xi}^T T + \lambda_3 X_3 \tag{1.3.3}$$

where the price signals S and T are given as $S = X_1 - \alpha^c s_c$ and

$T = X_2 - \left[\alpha^p \sum_{i \neq c} s_i + \gamma^p (n-1) s_c + \theta^p (n-1) p_1\right]$ *is characterized by the following equations*

Informed traders strategies

$$\begin{aligned}
\alpha_1^c &= \frac{1}{2\lambda_1} \\
\beta_1^c &= \frac{1-\tilde{\xi}^S\beta_3^c}{2\lambda_1+\tilde{\xi}^S\zeta^c}
\end{aligned} \tag{1.3.4}$$

$$\begin{aligned}
\alpha^p &= \frac{1}{2\lambda_2} \\
\beta_2^p &= \frac{1}{2\lambda_2} \\
\gamma^p &= \frac{1}{\lambda_2}\frac{\mu}{(2+(n-2)\,\mu)} \\
\theta^p &= \frac{\pi-1}{\lambda_2}-(n-2)\,\gamma^p\pi+(n-1)\left\{\gamma^p\frac{\varrho_c^{mm}}{\lambda_1}\right\}
\end{aligned} \tag{1.3.5}$$

$$\begin{aligned}
\beta_3^c &= \frac{1}{2\lambda_2} \\
\varphi^c &= -\frac{\xi_p^c\gamma^p}{2\lambda_3} \\
\theta^c &= \frac{\xi_p^c-\tilde{\xi}^T}{2\lambda_3} \\
\zeta^c &= -\frac{\tilde{\xi}^S}{2\lambda_3}
\end{aligned} \tag{1.3.6}$$

where

$$\begin{pmatrix} \mu \\ \pi \end{pmatrix} = \begin{pmatrix} \sigma_s^2 \\ \lambda_1\alpha^c\sigma_s^2 \end{pmatrix}\begin{pmatrix} \sigma_s^2+\sigma_\delta^2 & \lambda_1\alpha^c\sigma_s^2 \\ \lambda_1\alpha^c\sigma_s^2 & (\lambda_1)^2\left((\alpha^c)^2\sigma_s^2+(\beta_1^c)^2\sigma_l^2+\sigma_\epsilon^2\right) \end{pmatrix}^{-1}$$

$$\xi_p^c = \frac{(n-1)\,\beta_2^p\sigma_l^2}{(n-1)\left(\beta_2^p\right)^2\sigma_l^2+\sigma_\epsilon^2}$$

Market maker's pricing rule

$$\lambda_1 \quad = \quad \frac{\alpha^c\sigma_s^2+\beta_1^c\sigma_l^2}{\left(\alpha^c\right)^2\sigma_s^2+\left(\beta_1^c\right)^2\sigma_l^2+\sigma_\epsilon^2}$$

$$\tilde{\xi}^1 = 1-\lambda_2\,(n-1)\left(\frac{\gamma^p\varrho_c^{mm}}{\lambda_1}+\theta^p\right)$$

where

$$\varrho_c^{mm} = \frac{\alpha^c\sigma_s^2}{\left(\alpha^c\right)^2\sigma_s^2+\left(\beta_1^c\right)^2\sigma_l^2+\sigma_\epsilon^2}$$

$$\lambda_2 = \frac{(n-1)\,\alpha^p\sigma_s^2+(n-1)\,\beta_2^p\sigma_l^2+\gamma^p\,(n-1)\,var\left[s_c\mid X_1\right]+\gamma^p\,(n-1)\,cov\left[s_c,l_c\mid X_1\right]}{(n-1)\left(\alpha^p\right)^2\sigma_s^2+(n-1)\left(\beta_2^p\right)^2\sigma_l^2+\left(\gamma^p\right)^2(n-1)\,\sigma_\delta^2+\sigma_\epsilon^2+\left[\gamma^p\,(n-1)\right]^2 var\left[s_c\mid X_1\right]}$$

$$\tilde{\xi}^S = \left(1-\lambda_3\beta_3^c\right)\xi_c^{mm}-\lambda_3\zeta^c$$

$$\tilde{\xi}^T = \xi_p^{mm}-\lambda_3\left(\varphi^c\xi_\delta^{mm}+\theta^c\right)$$

where

$$\xi_c^{mm} = \frac{\beta_1^c\sigma_l^2}{\left(\beta_1^c\right)^2\sigma_l^2+\sigma_\epsilon^2}$$

$$\xi_p^{mm} = \frac{(n-1)\,\beta_2^p\sigma_l^2}{(n-1)\left(\beta_2^p\right)^2\sigma_l^2+(n-1)\left(\gamma^p\right)^2\sigma_\delta^2+\sigma_\epsilon^2}$$

$$\xi_\delta^{mm} = \frac{(n-1)\,\gamma^p\sigma_\delta^2}{(n-1)\left(\beta_2^p\right)^2\sigma_l^2+(n-1)\left(\gamma^p\right)^2\sigma_\delta^2+\sigma_\epsilon^2}$$

and

$$\lambda_3 = \frac{\beta_3^c var\left[l_c \mid S\right] + \varphi^c cov\left[\sum_{i\neq c} l_i, \sum_{i\neq c}\delta^i \mid T\right]}{\left(\beta_3^c\right)^2 var\left[l_c \mid S\right] + \left(\varphi^c\right)^2 var\left[\sum_{i\neq c}\delta^i \mid T\right] + \sigma_\epsilon^2}$$

Proof. The proof proceeds in two steps. First I will show that given the pricing rule of the market maker the informed traders' decisions are optimal indeed. Then I will confirm that given the informed traders' strategies the market maker's pricing rule is indeed of the claimed form.

Periperhal Traders

Conjecturing the linear strategies of other peripheral traders (1.3.5) and the linear price rule (1.3.2) the first order condition of the peripheral trader j's static maximization problem is given by

$$\begin{aligned} 2\lambda_2 x_2^j &= \mathbb{E}\left[\sum s_i + \sum l_i - \tilde{\xi}^1 p_1 - \lambda_2\left(\sum_{i\neq c,j}\left(\alpha^p s_i + \beta_2^p l_i + \gamma^p m^i + \theta^p p_1\right) + \epsilon_2\right) \mid s_i, l_i, m^i, p_1\right] \\ &= \left(1 - \lambda_2\left(n-2\right)\gamma^p\right)\mathbb{E}\left[s_c \mid m^j, p_1\right] + s_j + l_j - \lambda_2\left(n-2\right)\theta^p p_1 - \tilde{\xi}^1 p_1 \end{aligned}$$

where the last equality follows from collecting terms and realizing that the best estimate of trader j of other traders' messages is his estimate about s_c (since the noise terms with which messages are communicated are independently distributed with mean 0). When forming expectations about s_c the peripheral traders use both the message sent by the central trader as well as the period 1 price as it also contains information about s_c. Since all variables are distributed jointly normal we can invoke the multivariate version of the projection theorem in order to calculate expectations

$$\begin{aligned}
\mathbb{E}\left[s_c \mid m^j, p_1\right] &= \mathbb{E}\left[s_c\right] + \begin{pmatrix} cov\left[s_c, m^j\right] \\ cov\left[s_c, p_1\right] \end{pmatrix} \begin{pmatrix} var\left[m^j\right] & cov\left[m^j, p_1\right] \\ cov\left[m^j, p_1\right] & var\left[p_1\right] \end{pmatrix}^{-1} \begin{pmatrix} m^j \\ p_1 \end{pmatrix} \\
&= \begin{pmatrix} \sigma_s^2 \\ \lambda_1 \alpha^c \sigma_s^2 \end{pmatrix} \begin{pmatrix} \sigma_s^2 + \sigma_\delta^2 & \lambda_1 \alpha^c \sigma_s^2 \\ \lambda_1 \alpha^c \sigma_s^2 & \left(\lambda_1\right)^2 \left(\left(\alpha^c\right)^2 \sigma_s^2 + \left(\beta_1^c\right)^2 \sigma_l^2 + \sigma_\epsilon^2\right) \end{pmatrix}^{-1} \begin{pmatrix} m^j \\ p_1 \end{pmatrix}
\end{aligned}$$

. $\mathbb{E}\left[s_c \mid m^j, p_1\right]$ is thus a weighted sum of the message and the period 1 price. By defining μ as the weight on the message and π as the weight on the price we can thus write

$$\mathbb{E}\left[s_c \mid m^j, p_1\right] = \mu m^j + \pi p_1$$

Inserting this expression in the previously derived first-order condition and comparing coefficients results in the claimed coefficients (1.3.5). The second-order condition to this optimization problem is given by $\lambda_2 \geq 0$.

Central Trader

Conjecturing the linear price rule (1.3.3) the first order condition of the central trader's maximization problem of the third period can be written as

$$2\lambda_3 x_3^c = l_c + \mathbb{E}\left[\sum_{i \neq c} l_i \mid T\right] - \tilde{\xi}^S S - \tilde{\xi}^T T$$

When forming expectations about the other traders long-term information, the central trader can separate the noise term $\sum_{i \neq c} \delta^i$ from the price signal T and he can thus effectively observe the other traders' information disturbed only by the liquidity shock

ϵ_2

$$T^c \equiv T - \gamma^p \sum_{i \neq c} \delta^i = \beta_2^p \sum_{i \neq c} l_i + \epsilon_2$$

. His expectation is thus given by the projection theorem by

$$\begin{aligned} \mathbb{E}\left[\sum_{i \neq c} l_i \mid T^c\right] &= \xi_p^c T^c \\ &= \xi_p^c \left(T - \gamma^p \sum_{i \neq c} \delta^i\right) \end{aligned}$$

where

$$\begin{aligned} \xi_p^c &= \frac{cov\left[\sum_{i \neq c} l_i, T^c\right]}{var\left[T^c\right]} \\ &= \frac{(n-1)\,\beta_2^p \sigma_l^2}{(n-1)\left(\beta_2^p\right)^2 \sigma_l^2 + \sigma_\epsilon^2} \end{aligned}$$

Inserting the beliefs into the first order condition gives

$$2\lambda_3 x_3^c = l_c - \xi_p^c \gamma^p \sum_{i \neq c} \delta^i + \left[\xi_p^c - \tilde{\xi}^T\right] T - \tilde{\xi}^S S$$

and comparing coefficients implies the claimed result. The second-order condition to the period 3 optimization problem is given by $\lambda_3 \geq 0$.

The central traders' optimization problem in the first period is slightly more elaborate as the equilibrium concept I use requires that he does not find it profitable to deviate from his equilibrium strategy, taking into account also how this actions affects the future. I follow here the same proof strategy as outlined in Brunnermeier (2005).

First we need to consider how a deviation in period 1 strategy of the central trader would change the equilibrium outcome in period 2 and 3. Then I derive a condition that ensures that such a deviation would not be optimal. Because of the noisy structure of this economy, the other informed traders as well as the market maker would not be able to detect such a deviation but would instead attribute the change in the aggregate quantities to a different realisation of the noise. Thus they do not change their strategy as a consequence of the deviation in the central trader's strategy. Note also that since the short-term information of the central trader will already be public knowledge by the time he trades again, a deviation in his trading intensity α_1^c has no effect in the future.

So let us consider what happens after a deviation of the trading in intensity in the long-term signal from β_1^c to β_1^{dc} . Firstly the induced change in demand in period 1 will lead to a change in the first-period price to

$$
\begin{aligned}
p_1^{dc} &= \lambda_1 \left(\alpha^c s_c + \beta_1^{dc} l_c + \epsilon_1 \right) \\
&= p_1 + \lambda_1 \left(\beta_1^{dc} - \beta_1^c \right) l_c
\end{aligned}
$$

The change in the period 1 price has also an effect on the period 2 demand and price, which however does not have any impact on the price signal T since the period 2 strategy of the peripheral traders does not change as explained above. The pricing signal S changes however to

$$\begin{aligned} S^{dc} &= \beta_1^{dc} l_c + \epsilon_1 \\ &= S + \left(\beta_1^{dc} - \beta_1^c\right) l_c \end{aligned}$$

This changes the optimality condition in period 3 so that the optimal order size in period 3 after a deviation in period 1, $x_3^{dc,*}$ is characterized by

$$\begin{aligned} 2\lambda_3 x_3^{dc,*} &= l_c - \xi_p^c \gamma^p \sum_{i \neq c} \delta^i + \left[\xi_p^c - \tilde{\xi}^T\right] T - \tilde{\xi}^S \left[S + \left(\beta_1^{dc} - \beta_1^c\right) l_c\right] \\ &= 2\lambda_3 x_3^c - \tilde{\xi}^S \left(\beta_1^{dc} - \beta_1^c\right) l_c \end{aligned}$$

So that the continuation profit in period 3 after a deviation becomes

$$\begin{aligned} v_3\left(x_3^{dc,*}\right) &= x_3^{dc,*} \mathbb{E}\left[\left(v - p_3\left(x_3^{dc,*}\right)\right) \mid l_c, \sum_{i \neq c} \delta^i, S, T\right] \\ &= \lambda_3 \left(x_3^{dc,*}\right)^2 \\ &= \lambda_3 \left(x_3^c - \frac{\tilde{\xi}^S \left(\beta_1^{dc} - \beta_1^c\right) l_c}{2\lambda_3}\right)^2 \end{aligned}$$

where the second step follows by invoking the period 3 first order condition. The total profit from the view of period 1 after a deviation is then

$$\begin{aligned} v_1\left(\beta_1^{dc}\right) &= x_1^{dc}\mathbb{E}\left[v - p_1^{dc} \mid s_c, l_c\right] + \lambda_3 \mathbb{E}\left[\left(x_3^{dc,*}\right)^2 \mid s_c, l_c\right] \\ &= \left(\alpha^c s_c + \beta_1^{dc} l_c\right)\left[s_c + l_c - \lambda_1\left(\alpha^c s_c + \beta_1^c l_c\right) - \lambda_1\left(\beta_1^{dc} - \beta_1^c\right) l_c\right] \\ &\quad + \lambda_3 \mathbb{E}\left[\left(x_3^c - \frac{\tilde{\xi}^S\left(\beta_1^{dc} - \beta_1^c\right) l_c}{2\lambda_3}\right)^2 \mid s_c, l_c\right] \end{aligned}$$

The profit maximizing deviation in period 1 taking into account its effect in period 3 is characterized by the FOC with respect to β_1^{dc}

$$\begin{aligned} \lambda_1\left(\alpha^c s_c + \beta_1^{dc} l_c\right) &= \left[s_c + l_c - \lambda_1\left(\alpha^c s_c + \beta_1^c l_c\right) - \lambda_1\left(\beta_1^{dc} - \beta_1^c\right) l_c\right] \\ &\quad - \tilde{\xi}^S \mathbb{E}\left[\left(x_3^c - \frac{\tilde{\xi}^S\left(\beta_1^{dc} - \beta_1^c\right) l_c}{2\lambda_3}\right) \mid s_c, l_c\right] \end{aligned}$$

In equilibrium it must be that there is no profitable deviation so that $\beta_1^{dc} = \beta_1^c$ and hence

$$\begin{aligned} \lambda_1\left(\alpha^c s_c + \beta_1^c l_c\right) &= \left[s_c + l_c - \lambda_1\left(\alpha^c s_c + \beta_1^c l_c\right)\right] - \tilde{\xi}^S \mathbb{E}\left[x_3^c \mid s_c, l_c\right] \\ &= \left[s_c + l_c - \lambda_1\left(\alpha^c s_c + \beta_1^c l_c\right)\right] - \tilde{\xi}^S\left(\beta_3^c + \zeta^c \beta_1^c\right) l_c \end{aligned}$$

comparing the coefficient of l_c we have

$$\lambda_1 \beta_1^c = 1 - \lambda_1 \beta_1^c - \tilde{\xi}^S\left(\beta_3^c + \zeta^c \beta_1^c\right)$$

and rearranging for β_1^c gives the claimed result (1.3.6). The corresponding second-order condition is given by

$$2\lambda_1 + \tilde{\xi}^S \zeta^c \geq 0$$

A deviation in α^c has no implication on future profits for the central trader, since by the time the central trader trades again the short-term signal s_c has become public. We therefore have a static optimization problem and the optimality condition with respect to α^c (similarly to the one of the peripheral traders) implies

$$2\lambda_1 \alpha^c = s_c$$

with the correspond second-order condition $\lambda_1 \geq 0$

Market Maker

Given the linear-normal structure of the model all random variables are (jointly) normally distributed. This allows us to apply the projection theorem to derive the conditional expectations of the asset given the information available at period 1,2 and 3. In particular we have

$$\begin{aligned} p_1 &= \mathbb{E}\left[v \mid X_1\right] \\ &= \frac{cov\left[v, X_1\right]}{var\left[X_1\right]} X_1 \\ &= \frac{\alpha^c \sigma_s^2 + \beta_1^c \sigma_l^2}{\left(\alpha^c\right)^2 \sigma_s^2 + \left(\beta_1^c\right)^2 \sigma_l^2 + \sigma_\epsilon^2} X_1 \end{aligned}$$

so that the price rule in period 1 indeed has the claimed form with

$$\lambda_1 = \frac{\alpha^c \sigma_s^2 + \beta_1^c \sigma_l^2}{\left(\alpha^c\right)^2 \sigma_s^2 + \left(\beta_1^c\right)^2 \sigma_l^2 + \sigma_\epsilon^2}$$

$$\begin{aligned}
p_2 &= \mathbb{E}\left[v \mid X_1, X_2\right] \\
&= \mathbb{E}\left[v \mid X_1\right] + \frac{cov\left[v, X_2 \mid X_1\right]}{var\left[X_2 \mid X_1\right]}\left(X_2 - \mathbb{E}\left[X_2 \mid X_1\right]\right) \\
&= p_1 + \lambda_2\left[X_2 - (n-1)\left(\gamma^p \mathbb{E}\left[s_c \mid X_1\right] + \theta^p p_1\right)\right] \\
&= \left[1 - \lambda_2 (n-1)\left(\frac{\gamma^p \varrho_c^{mm}}{\lambda_1} + \theta^p\right)\right] p_1 + \lambda_2 X_2
\end{aligned}$$

So that also p_2 is following the proclaimed pricing rule with

$$\begin{aligned}
\lambda_2 &= \frac{cov\left[v, X_2 \mid X_1\right]}{var\left[X_2 \mid X_1\right]} \\
&= \frac{(n-1)\alpha^p \sigma_s^2 + (n-1)\beta_2^p \sigma_l^2 + \gamma^p (n-1) var\left[s_c \mid X_1\right] + \gamma^p (n-1) cov\left[s_c, l_c \mid X_1\right]}{(n-1)(\alpha^p)^2 \sigma_s^2 + (n-1)(\beta_2^p)^2 \sigma_l^2 + (\gamma^p)^2 (n-1)\sigma_\delta^2 + \sigma_\epsilon^2 + \left[\gamma^p (n-1)\right]^2 var\left[s_c \mid X_1\right]}
\end{aligned}$$

and

$$\tilde{\xi}^1 = 1 - \lambda_2 (n-1)\left(\frac{\gamma^p \varrho_c^{mm}}{\lambda_1} + \theta^p\right)$$

where

$$\begin{aligned}
\varrho_c^{mm} &= \frac{cov\left[s_c, X_1\right]}{var\left[X_1\right]} \\
&= \frac{\alpha^c \sigma_s^2}{(\alpha^c)^2 \sigma_s^2 + (\beta_1^c)^2 \sigma_l^2 + \sigma_\epsilon^2}
\end{aligned}$$

In period 3 the short-term information $\{s_i\}_{i=1}^n$ has been revealed. The market maker can thus eliminate the noise that came form the short-term signals from the aggregate order has observed

$$\begin{aligned}
T &= X_2 - \left[\alpha^p \sum_{i \neq c} s_i + \gamma^p (n-1) s_c + \theta^p (n-1) p_1\right] \\
&= \sum_{i \neq c} \left(\alpha^p s_i + \beta_2^p l_i + \gamma^p m^i + \theta^p p_1\right) + \epsilon_2 - \left[\alpha^p \sum_{i \neq c} s_i + \gamma^p (n-1) s_c + \theta^p (n-1) p_1\right] \\
&= \beta_2^p \sum_{i \neq c} l_i + \gamma^p \sum_{i \neq c} \delta^i + \epsilon_2
\end{aligned}$$

and

$$\begin{aligned}
S &= X_1 - \alpha^c s_c \\
&= \beta_1^c l_c + \epsilon_1
\end{aligned}$$

which improves his estimate about the central and peripheral trader's information. The price then becomes

$$\begin{aligned}
p_3 &= \mathbb{E}\left[v \mid X_1, X_2, X_3, \{s_i\}_{i=1}^n\right] \\
&= \mathbb{E}\left[v \mid X_1, X_2, \{s_i\}_{i=1}^n\right] + \frac{cov\left[v, X_3 \mid X_1, X_2, \{s_i\}_{i=1}^n\right]}{var\left[X_3 \mid X_1, X_2, \{s_i\}_{i=1}^n\right]} \left(X_3 - \mathbb{E}\left[X_3 \mid X_1, X_2, \{s_i\}_{i=1}^n\right]\right) \\
&= \sum s_i + \mathbb{E}\left[l_c \mid S\right] + \mathbb{E}\left[\sum_{i \neq c} l_i \mid T\right] + \frac{cov\left[v, X_3 \mid S, T\right]}{var\left[X_3 \mid S, T\right]} \left(X_3 - \mathbb{E}\left[X_3 \mid S, T\right]\right) \\
&= \sum s_i + \mathbb{E}\left[l_c \mid S\right] + \mathbb{E}\left[\sum_{i \neq c} l_i \mid T\right] + \lambda_3 \left(X_3 - \mathbb{E}\left[\beta_3^c l_c + \varphi^c \sum_{i \neq c} \delta^i + \theta^c T + \zeta^c S + \epsilon_3 \mid S, T\right]\right) \\
&= \sum s_i + \left[\left(1 - \lambda_3 \beta_3^c\right) \xi_c^{mm} - \lambda_3 \zeta^c\right] S + \left(\xi_p^{mm} - \lambda_3 \left(\varphi^c \xi_\delta^{mm} + \theta^c\right)\right) T + \lambda_3 X_3
\end{aligned}$$

with

$$\begin{aligned}\xi_c^{mm} &= \frac{cov\left[s_c, S\right]}{var\left[S\right]} \\ &= \frac{\beta_1^c \sigma_l^2}{\left(\beta_1^c\right)^2 \sigma_l^2 + \sigma_\epsilon^2}\end{aligned}$$

$$\begin{aligned}\xi_p^{mm} &= \frac{cov\left[\sum_{i \neq c} l_i, T\right]}{var\left[T\right]} \\ & \frac{\left(n-1\right)\beta_2^p \sigma_l^2}{\left(n-1\right)\left(\beta_2^p\right)^2 \sigma_l^2 + \left(n-1\right)\left(\gamma^p\right)^2 \sigma_\delta^2 + \sigma_\epsilon^2}\end{aligned}$$

$$\begin{aligned}\xi_\delta^{mm} &= \frac{cov\left[\sum_{i \neq c} \delta^i, T\right]}{var\left[T\right]} \\ & \frac{\left(n-1\right)\gamma^p \sigma_\delta^2}{\left(n-1\right)\left(\beta_2^p\right)^2 \sigma_l^2 + \left(n-1\right)\left(\gamma^p\right)^2 \sigma_\delta^2 + \sigma_\epsilon^2}\end{aligned}$$

and

$$\lambda_3 = \frac{\beta_3^c var\left[l_c \mid S\right] + \varphi^c cov\left[\sum_{i \neq c} l_i, \sum_{i \neq c} \delta^i \mid T\right]}{\left(\beta_3^c\right)^2 var\left[l_c \mid S\right] + \left(\varphi^c\right)^2 var\left[\sum_{i \neq c} \delta^i \mid T\right] + \sigma_\epsilon^2}$$

so that the pricing rule of period 3 does indeed have the conjectured form.

□

1.4 Results

This section presents the main findings of this paper. I illustrate theoretically and on hand of a numerical example that it can be profitable for an insider trader to share information and provide some intuition for the result. Since the central trader's profits and price efficiency in period 3 are inseparable from each other, they are discussed in the same section. Then some implications on the market as a whole are discussed. For the numerical results I choose a parametrization of n=5, $\sigma_s^2 = 1/5$, $\sigma_l^2 = 1/5$, $\sigma_\epsilon^2 = 1$ and solve for both the economy with communication as presented in the previous section as well as the economy without communication, where the central trader cannot send a message about his short-term signal.

Profits of the central trader and informational efficiency in period 3

Figure 1 panel A depicts the ex-ante profits of the central trader with and without communication depending on the precision with which the central trader shares information. It can be seen that, independently with how much noise the signal is being sent, the central trader is always increasing profits by sharing information. The intuition for this result is the following: by sharing his short-term information the central trader adds noise to the economy, and only he knows the exact realization of this noise. When inferring information about the other traders long-term information from the price signal T he can take advantage of this fact and can thus extract more information from it. In order to formalize this intuition let us look at the price signals that are observable to

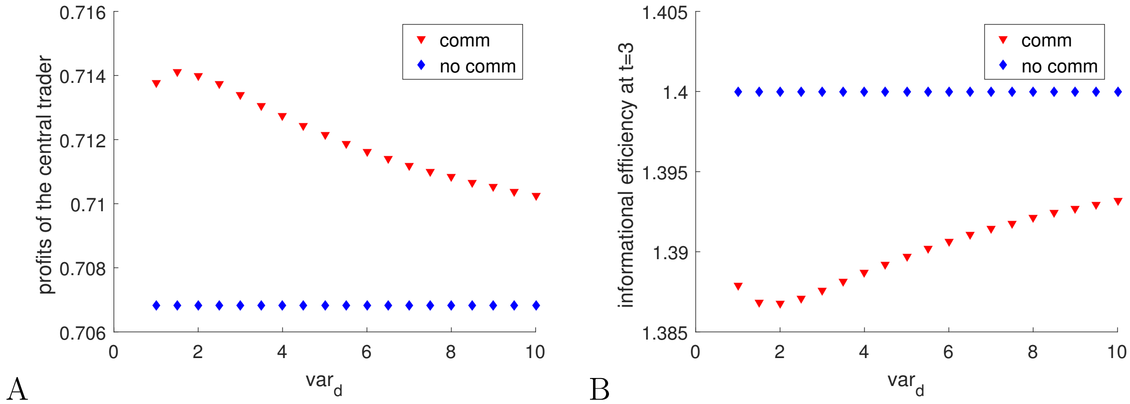

Figure 1.4.1: Ex-ante profits of the central trader and informational efficiency at t=3 measured as $1/\text{var}(v \mid S, T)$

the market maker and the central trader in turn. While the market maker can observe

$$T = \beta_2^p \sum_{i \neq c} l_i + \gamma^p \sum_{i \neq c} \delta^i + \epsilon_2$$

about the long-term signals of the peripheral traders, the central trader also knows the precise realization of the noise term $\sum_{i \neq c} \delta^i$, and can thus infer

$$T_c = \beta_2^p \sum_{i \neq c} l_i + \epsilon_2$$

. This gives him a more precise estimate of the information he is missing $\sum_{i \neq c} l_i$, which increases his informational advantage compared to the market maker. This intuition can be confirmed by looking at the graph. Figure 1.4.1 panel B shows the price informativeness at the beginning of t=3. While the actual profits depend on the informational advantage of the central trader over the market maker, i.e. the difference in posterior variances between them, we can see from the very similar shape of this graph, that the increase in profits of the central trader is mainly due to an decrease in price informativeness in period 3[1]. It can thus be summarized that by communicating his short-term information to other traders the central trader is deteriorating the long-term price informativeness and through this is increasing his profits.

The resulting increase in profits is thereby the highest at the point where the market maker's posterior variance (the inverse of the price informativeness) increases most compared to his own, in this example at a value around 2. If the central trader was able to choose the amount of noise with which he was communicating his information, this would be the level he would choose. This hump shape comes about from the overall impact of a change in the variance of the noise σ_δ^2 on the disturbance $\gamma^p \sum_{i \neq c} \delta^i$ of the price signal T in equilibrium. Everything else being equal, the higher σ_δ^2 , the less informative is the message to the peripheral traders and thus the less weight γ^p do they attach to the information they receive. The overall effect on the variance of noise the central trader adds to the economy $var\left(\gamma^p \sum_{i \neq c} \delta^i\right)$ is thus non-monotonic. For lower values the increase in the variance σ_δ^2 dominates, while for higher values the decrease of the weight γ^p is stronger so that it leads to total decrease in the variance of the added noise.

[1]in fact also the posterior variance of the central trader increases due to the lower trading intensity β_2^p of the peripheral traders in period 2 which is a result of the decreased liquidity as described in the next section. This effect is however much smaller compared to the increase in the posterior variance of the market maker, and hence the all-over effect is that the difference in posterior variance and hence the informational advantage increases

Profits of the peripheral traders

The peripheral traders are receiving additional information about a signal they cannot observe, which unambiguously increases their profits. Figure 1.4.2 panel A shows the profits of a peripheral trader in the economy with and without communication Clearly,

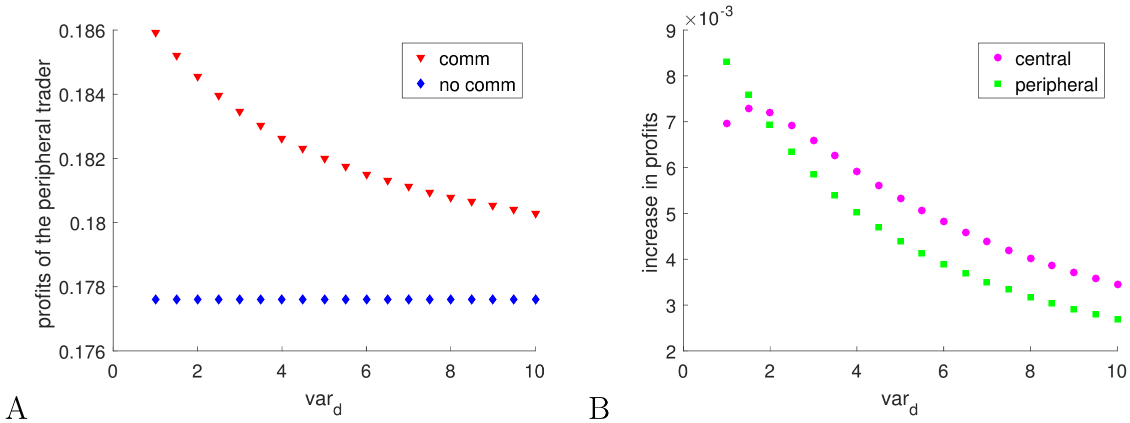

Figure 1.4.2: Ex-ante profits of a peripheral trader and comparison of profits

the more noise the central trader sends his signal with, the less valuable is this information to the peripheral trader and thus the lower is the resulting increase in profits.

When comparing the increase in profits from communication between the two types of traders, I find an ambiguous result. Even though only the peripheral trader receives any information about the fundamental value of the asset, it is not always him who profits more. Figure 1.4.2 panel B shows that this is only the case when the signal is sent very precisely, for larger values of the communication noise however, the central trader's increase in profit exceeds the peripheral traders increase. It is a surprising result that adding confusion to the market can be more valuable than receiving real information about the asset.

In this section we have seen that all informed traders profits increase after communication, those who sent and those who receive information. Since the market maker makes zero profit in expectation, this gain in profit is to the cost of the uninformed traders. This finding has an important policy implication: If a regulator's goal was to protect small, non-professional traders he should put his best efforts into limiting the amount of secret communication between informed traders.

Liquidity

The effects on market liquidity follow the intuition of the standard Kyle (1985) model: since the informational advantage of insider traders increases with communication, the market maker decreases the liquidity in order to compensate himself for bad trades due to the deterioration of the adverse selection problem he is facing. This is the case for both periods two and three as we can see in Figure 1.4.3. It is an interesting feature of the model though that communication about short-term information has still consequences for the liquidity of the asset even when this information has already become obsolete.

Informational efficiency in period 2

The decrease in liquidity in period 3 thereby also has some effect on the inter-temporal decision problem of the central trader. Since it becomes relatively more costly to trade in the third period compared to the first period, he anticipates some of the trading to the first period by increasing the weight he puts on the long-term signal in the first period while decreasing it in the third period. This increase can be seen in Figure

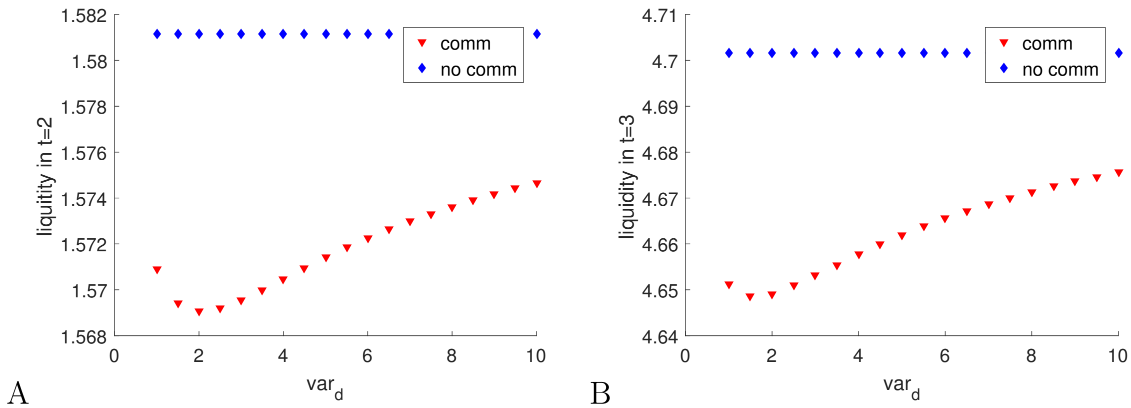

Figure 1.4.3: Liquidity

1.4.4 panel A. By increasing his trading intensity on the long-term signal, he releases more information to the market compared to the economy with no communication. This improves the informational efficiency of the market in period 2 as the posterior variance of the market maker declines. It needs to be highlighted that this is a purely endogenous effect as no additional information has been traded upon at this point in time.

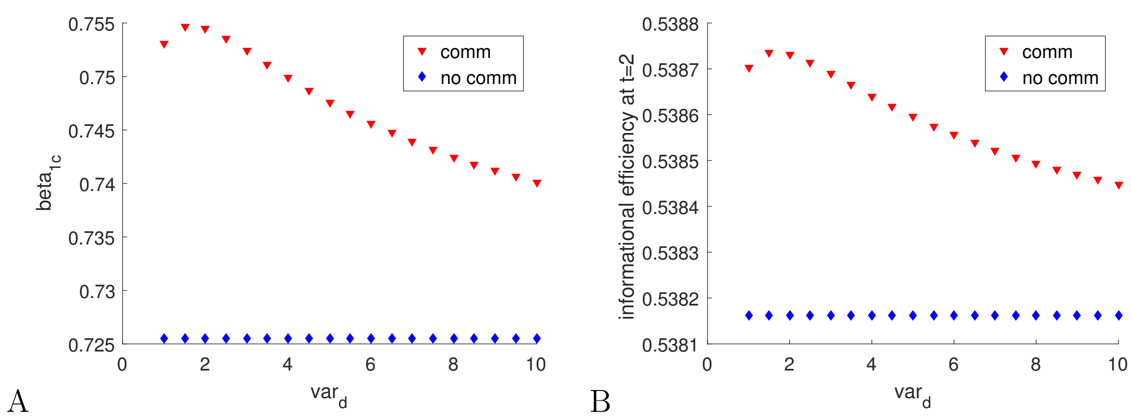

Figure 1.4.4: Trading intensity on the long-term signal in t=1 and informational efficiency in t=2 measured as $1/\text{var}(v \mid X_1)$

1.5 Summary

This work provides a theoretical explanation for the empirically observed phenomenon of communication among insider traders. I show that by sending a noisy message about his short-term signal, an informed trader can increase his profits as he adds noise to the economy and thus hinders learning of other market participants in the long-run, in particular of the market maker. Surprisingly though, communication leads to an increase in short-term informational efficiency due to an equilibrium effect. As liquidity becomes scarce in the final period, the central trader anticipates some of his trading to the first period which leads to a stronger dissemination of his information earlier on. Compared to an economy where communication is not possible, both the senders and receivers of information are better off. This comes to the cost of uninformed noise traders. If a regulator's aim was to protect this latter type of investors he should do anything possible to prevent secret communication.

1.6 References

Admati, Anat R. and Paul Pfleiderer (1988) "A Theory of Intraday Patterns: Volume and Price Variability," *Review of Financial Studies*, Vol. 1, No. 1, pp. 3–40, jan.

Allen, Franklin and Douglas Gale (1992) "Stock-Price Manipulation," *Review of Financial Studies*, Vol. 5, No. 3, pp. 503–29.

Benabou, R. and G. Laroque (1992) "Using Privileged Information to Manipulate Markets: Insiders, Gurus, and Credibility," *The Quarterly Journal of Economics*, Vol. 107, No. 3, pp. 921–958, aug.

Brunnermeier, Markus K. (2005) "Information Leakage and Market Efficiency," *Review of Financial Studies*, Vol. 18, No. 2, pp. 417–457.

Cao, Huining Henry, Yuan Ma, and Dongyan Ye (2013) "Disclosure, Learning, and Coordination."

Huddart, Steven, John S Hughes, and Carolyn B Levine (2001) "Public Disclosure and Dissimulation of Insider Trades," *Econometrica*, Vol. 69, No. 3, pp. 665–81.

Kyle, Albert S (1985) "Continuous Auctions and Insider Trading," *Econometrica*, Vol. 53, No. 6, pp. 1315–35.

Vila and Jean-Luc (1989) "Simple games of market manipulation," *Economics Letters*, Vol. 29, No. 1, pp. 21–26.

Chapter 2

Sovereign Debt and CDS - A Welfare Analysis

2.1 Introduction

Credit Default Swaps have been at the center of the discussion since the beginning of the recent financial crisis. Their impact on the market for sovereign debt has been highly debated. Most recently, during the Greek debt renegotiation, it was frequently argued in the press that a number of hedge funds who invested in CDS hindered the debt renegotiation process and thus made an efficient resolution of the Greek debt problem more difficult. While from an ex-post perspective hindering renegotiation is clearly welfare decreasing, it is not obvious whether this is also true from an ex-ante perspective. This work aims at analyzing the welfare implications of credit insurance on the renegotiation process for government debt in an analytical framework.

We analyze a model of government borrowing, where the lender can insure himself against government default by signing a contract with a third party. Under quite general specifications we characterize the sub-game perfect equilibrium and compare it to the second-best and an economy where no such insurance is available. As commonly assumed in the literature for government debt (e.g. Eaton and Gersovitz (1981), Arellano (2008), Aguiar and Gopinath (2007)) we allow for risk-averse preferences of the government, which enables us to study the impact of credit insurance on consumption smoothing. Previous work on this subject has focused on a risk-neutral borrower and was therefore not able to address this issue. A second difference to the existing literature in this field is that the level of borrowing is a decision variable, while in previous studies it has been taken as exogenous.

We find that by investing in credit insurance the lender can strengthen its outside option during renegotiation and consequently obtain a higher share of the bargaining surplus compared to when no such option is available. Even though credit insurance never pays in equilibrium, it enables the lender to enforce a higher debt repayment. Credit insurance thus works as a commitment device. In case of a risk neutral government this is ex-ante welfare improving: enforcing a higher amount of repayment leads to improved borrowing conditions and thus relaxes the borrowing constraint of the government. We also specify the assumption on the competitive structure of the economy that are needed for this mechanism to go through.

The government is however not the party who chooses the level of credit insurance. Under the assumption of risk-aversion the lender no longer always chooses the socially efficient level of credit insurance and credit insurance may even decrease welfare. The reason for this is that credit insurance has two opposing effects in this case. On one hand, by increasing the amount of borrowing available, credit insurance helps the government to transfer wealth between periods and therefore facilitates consumption smoothing across time. On the other hand renegotiation helps to add contingency to the otherwise non-contingent bond. By enforcing a higher repayment in the low state, credit insurance reduces this benefit and as a result makes consumption smoothing between states more difficult. Which effect prevails depends on the endowment in the low state. If the endowment in the low state is very low compared to the initial wealth and the endowment in the high state, the second effect is stronger so that credit insurance is welfare decreasing. This is because the marginal utility is decreasing in case of risk-averse preferences, and thus for a low endowment in the low state compared to the level of consumption in period 1, the government values the additional consumption in the first period less compared to the higher repayment in the low state. The previous literature in this field has not been able to address these opposing welfare effects, as the borrower is always assumed to be risk-neutral in these models. The contribution of this paper is thus to develop a model that is general enough to capture these opposing effects of credit insurance has on the welfare due to risk-aversion.

The work most closely related to the present is Sambalaibat (2011) who studies the impact of credit insurance on the moral hazard problem of the government. In her model investment is not observable and renegotiation is assumed to happen only in the low state. As a consequence the government does not fully take the losses in the low state and therefore has an incentive to invest less than the socially efficient amount. By enforcing a higher repayment in the low state, credit insurance ameliorates this moral hazard problem. We confirm that under risk-neutrality credit insurance has an welfare improving effect even in an environment of full information. Bolton and Oehmke (2011) study the impact of credit insurance in a corporate debt model with an exogenous amount of investment needed. They also find that credit insurance works as a commitment device and thus relaxes the borrowing constraint of the firm. This makes it possible to finance projects that were otherwise not possible to realize, which is welfare improving. In equilibrium however, they find that for high levels of borrowing, the lender may choose a higher level of credit insurance than what would be socially efficient as it leads to inefficient default in some states. We also find a similar result. In our model however the debt level is an endogenous choice of the government. This enables us to show that in case of risk-neutrality, the government never finds it optimal to choose such high debt levels that lead to inefficient default, so that there is no over-insurance in equilibrium.

This work is also related to Grossman and Van Huyck (1988) and Kehoe and Levine (2006) who show that default can add contingency to the otherwise non-contingent bond contract. In our work we show that credit insurance hinders this welfare improving aspect of default or renegotiation and thus might also be welfare decreasing.

2.2 Model

Suppose there is a government which can access the international credit markets either for consumption smoothing purposes and/or to invest into a productive technology. We make the model general enough to account for both, later we analyze the impact of credit insurance on investment and consumption smoothing separately. The economy lasts for two periods: In period 0 the government can borrow from one of the risk-neutral lenders a notional amount of debt b at a price $q(b)$. We make the standard assumption that while credit markets are competitive ex-ante, the government can only borrow from one of the lenders. The government is also endowed with some initial wealth w_0, which is assumed to be low enough so that the government wants to borrow. It can use its initial wealth and the proceeds from borrowing to either consume in period 0 or to invest into its productive technology. k units invested in period 0 produce $f(\theta, k)$ units of the consumption good in period 1, where f is increasing in both arguments and concave with respect to the second argument. The productivity factor θ is a random variable which can take two values: θ^H with probability π and θ^L with probability $1 - \pi$. The realization of θ is observable to both parties but not contractible, so that the amount of debt cannot be made contingent on the state. The government values consumption in period t according to a utility function $u(c_t)$. We first make some general observations based on u being continuous, increasing and twice differentiable, then we restrict the analysis further to the case of risk-neutrality and risk-aversion.

There is limited commitment on the side of the government so that it may decide not to repay its debt. In this case it suffers a loss of $\lambda \in [0, 1]$ to its output. This loss can be interpreted as corresponding to the cost of market exclusion in an infinite horizon economy. Default thus leads to an ex-post efficiency loss. This opens up the room for renegotiation: the government and lender can come together and renegotiate the amount of outstanding debt. Renegotiation is costly however, at a cost $\delta < \lambda$ to total output. The surplus from renegotiation is shared according to a Nash bargaining rule.

The lender has also the option to enter into an insurance contract with a third party, a risk-neutral insurer. This contract, in practice called credit default swap, pays a mutually agreed amount of $i \geq 0$ in case of a full default of the government for a premium $q^{CDS}(i)$. Similar to the bond market, the insurance market is perfectly competitive ex-ante, but the lender can only contract with one single insurer. As standard in the literature (e.g. Bolton and Oehmke (2011), Sambalaibat (2011)) renegotiation is considered voluntary, so that credit insurance does not pay after successful renegotiation. None of the agents discount the future.

Timing is as follows: At the beginning of period $t = 0$ the government simultaneously chooses the amount of investment k and the notional amount of debt b. In the middle of period $t = 0$, after having observed the level of investment, lenders give a quote at which price $q(b)$ they are willing to take on the full notional b. Then the government decides on which offer to accept or whether to reject all. If it accepts one offer it receives an amount $q(b)\,b$ of lending. At the end of period $t = 0$ insurers quote a price schedule to the lender at which they are willing to pay a notional amount i in case of default. The lender then decides on the level of credit insurance. At the beginning of period $t = 1$ the productivity shock θ is realized. After observing the shock the government and the lender can decide whether they want to enter into renegotiation. If renegotiation is rejected by one of the parties, the government decides on whether to fully repay or fully default. We make thus the assumption that renegotiation happens before the actual repayment decision of the government. This insures that the government can only renegotiate in states where it actually would default if there was no renegotiation in place.[1] The status quo of the bargaining game is thus state contingent and determined by the repayment decision of he government.

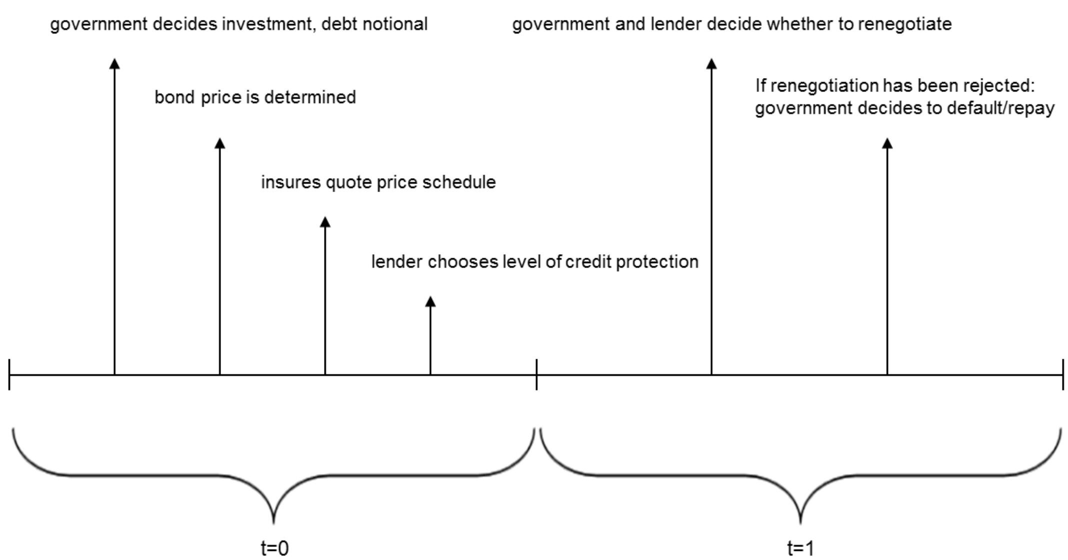

Figure 2.2.1: timing

[1]If we would assume that renegotiation happens after the default decision of the government, this would imply that the government claims opportunistically default in some states where it would fully repay if there was no renegotiation procedure in place as it knows to get a better deal during renegotiation.

2.3 Equilibrium

In this section we describe the equilibrium with and without CDS. The next section will characterize the equilibrium. We describe the decision problem of each agent at each decision node.

2.3.1 The government's problem in period 0

The government chooses the level of debt b and investment k at the beginning of period 0 correctly anticipating bond prices, in order to maximize lifetime consumption

$$u\left(c_0\right) + \pi u\left(c_H\right) + \left(1-\pi\right) u\left(c_L\right)$$

where c_0 denotes consumption in period 0 and is given by the initial wealth plus the proceeds from issuing debt minus the amount invested into the productive technology

$$c_0 = w_0 + q(b,k)b - k$$

. c_s denotes consumption in period 1 in state s and depends on whether both parties agree to renegotiation in period 1 and in case they do not, whether the government prefers to fully repay or default. In case of renegotiation the government's consumption in state s is given by

$$c_s^{ren} = \left(1-\delta\right) f\left(\theta^s, k\right) - d^{ren}\left(i, \theta^s, k\right)$$

as the government suffers a loss of δ to output and repays the renegotiated amount of debt $d^{ren}(i, \theta^s, k)$ determined by Nash bargaining in period 1. In case of full repayment the government consumes

$$c_s^{pay} = f(\theta^s, k) - b$$

as it suffers no loss to output but repays the full outstanding debt. Default leads to an output loss of λ while there is no repayment so that consumption in this case is given by

$$c_s^{def} = (1 - \lambda) f(\theta^s, k)$$

2.3.2 Bond price

In the middle of period 0, after having observed the governments investment k and the amount of debt b the government wants to borrow, lenders quote the price at which they are willing to take on the full amount of debt, correctly anticipating the choice of credit insurance and the repayment of debt. Its decision problem is thus given by

$$q(b,k) \in \arg\max_q -qb + \mathbb{E}_\theta \left[d^{eft}(i^*(k,b), \theta, k, b) \right] \tag{2.3.1}$$

where $d^{eft}(i, \theta, k, b)$ is the effective repayment of debt in period 1 and $i^*(k,b)$ is the optimal choice of credit insurance by lender as described next. We will see in section 2.4, that in equilibrium competition amongst lenders will lead to zero profits.

2.3.3 The lender's choice of CDS

At the end of period 0 the investor chooses the optimal level of credit insurance taking into account the effect its choice has on the bargaining outcome in the following period. In order to obtain credit insurance with a notional of i, the lender has to pay a premium $q^{CDS}(i,k,b)\,i$ to the insurer, where $q^{CDS}(i,k,b)$ is the price schedule quoted by the insurer. In period 1 the lender receives the effective repayment of debt $d^{eft}(i,\theta,k,b)$ depending on whether there is default, renegotiation or repayment. Only if the government defaults so that the effective repayment equals to zero, does the credit insurance pay out the promised amount i to the lenders. The lenders profit maximization problem is thus given by

$$i^*(k,b) \in \max_i \; -q^{CDS}(i,k,b)\,i + \mathbb{E}_\theta\left[d^{eft}(i,\theta,k,b) + I_{\left\{d^{eft}(i,\theta,k,b)=0\right\}}i\right] \tag{2.3.2}$$

where I denotes the indicator function[2].

2.3.4 Renegotiation

At the beginning of period 1 both parties must decide whether they are willing to renegotiate the bond contract. If renegotiation takes place the renegotiated amount of

[2]Note that since the amount of lending payed to the government has been already determined at the beginning of period $t = 0$ it is considered as sunk and therefore not taken into account in its decision problem at this point.

debt is determined according to Nash bargaining and thus solves the following problem

$$d^{ren}(i, \theta^s, k) = \arg\max_d \left[u\left((1-\delta) f(\theta^s, k) - d\right) - u\left(c^{quo}(\theta^s, k)\right)\right]\left[d - d^{quo}(\theta^s, k)\right] \tag{2.3.3}$$

Where $c^{quo}(\theta^s, k)$ is the amount of consumption the government would receive in the state-contingent status quo, which depends on the repayment decision of the government at the end of period 1 as described in the next section. If renegotiation is successful the government suffers a cost of $(1-\delta)$ to its output and needs to repay the renegotiated amount. The welfare improvement for the government from renegotiation is thus given by $u\left((1-\delta) f(\theta^s, k) - d\right) - u\left(c^{quo}(\theta^s, k)\right)$. The lender on the other hand receives and amount of d if renegotiation is successful, while $d^{quo}(\theta^s, k)$ is the amount of repayment he would receive either from the government or its insurance contract if renegotiation was not successful. In order to make both parties to agree into renegotiation, they must prefer the renegotiation outcome to the status quo.

2.3.5 Status Quo

In case renegotiation has been rejected by either the government or the lender, the government can choose only between full repayment or full default. The government will choose to repay, if the contractual amount of debt is smaller than the cost of default

$$f(\theta^s, k) - b \geq (1-\lambda) f(\theta^s, k) \tag{2.3.4}$$

otherwise it will prefer to default.

2.3.6 Equilibrium Definition and Second Best

The equilibrium concept we use is sub-game perfect equilibrium. The next section characterizes the equilibrium. We then compare the equilibrium to the second-best where a social planner can choose the amount of credit insurance before the economy starts and the economy evolves otherwise as previously described. Since lenders and insurers make zero profits in equilibrium this is equivalent to a situation where the government chooses the level of credit insurance. We also compare the equilibrium to a situation where the lender cannot insure itself against default so that the equilibrium level of credit insurance $i^* (k, b)$ is exogenously set to zero.

2.4 Characterization

We now characterize the sub-game perfect equilibrium with credit insurance by backwards induction starting from the final decision node. In case renegotiation is rejected the government needs to decide of whether to repay fully or default. Rewriting condition (2.3.4) we can see that the government repays if debt is small compared to output

$$b \leq \lambda f (\theta^s, k)$$

. The state-dependent status quo for the government in the renegotiation game can thus be written as

$$c^{quo}(\theta^s, k) = \begin{cases} f(\theta^s, k) - b & b \leq \lambda f(\theta^s, k) \\ (1-\lambda) f(\theta^s, k) & b > \lambda f(\theta^s, k) \end{cases}$$

The status quo for the lender on the other hand is given by

$$d^{quo}(\theta^s, k) = \begin{cases} b & b \leq \lambda f(\theta^s, k) \\ i & b > \lambda f(\theta^s, k) \end{cases}$$

as in case of a full default credit insurance pays out the contractual amount of i, while in case of a full repayment the lender receives the contractual amount of debt b. Renegotiation can only take place if both parties can be made better off than these levels. We can see immediately that in states such that there is full repayment there is no room for renegotiation. This is because the government would only agree into renegotiation if the renegotiated amount of debt was strictly below the contractual amount as it also suffers the cost of renegotiation. On the other hand the lender would not agree to such a deal, as he would receive a full repayment of the contractual amount of debt if he would reject renegotiation. Thus, there can be no efficiency improvement for states where there is full repayment and renegotiation therefore does not take place. If on the other hand debt levels are high enough $b > \lambda f(\theta^s, k)$ such that if renegotiation was rejected there would be default, renegotiation can lead to an ex-post efficiency improvement as it is less costly than full default. If however the level of credit insurance is higher than the full surplus from renegotiation $(\lambda - \delta) f(\theta^s, k)$ it is not possible to make the lender to agree and thus also in this case renegotiation is rejected and followed by full default. The next lemma summarized the effective repayment of debt.

Lemma 3. *In states where the debt-to-output ratio is low so that $b \leq \lambda f(\theta^s, k)$ there is full repayment of debt. When $b > \lambda f(\theta^s, k)$ and the level of credit insurance is low enough, so that $i \leq (\lambda - \delta) f(\theta^s, k)$ renegotiation takes place. If however $b > \lambda f(\theta^s, k)$ and $i > (\lambda - \delta) f(\theta^s, k)$ renegotiation is rejected and the government defaults.*

The effective amount of repayment can thus be written as

$$d^{eft}(i, \theta^s, k, b) = \begin{cases} b & b \leq \lambda f(\theta^s, k) \\ d^{ren}(i, \theta^s, k) & b > \lambda f(\theta^s, k), \; i \leq (\lambda - \delta) f(\theta^s, k) \\ 0 & b > \lambda f(\theta^s, k), \; i > (\lambda - \delta) f(\theta^s, k) \end{cases} \tag{2.4.1}$$

We can thus see that the level of credit insurance only matters for high enough levels of debt. Before proceeding to the choice of credit insurance of the lender we make an observation regarding the relation between the renegotiated amount of debt and credit insurance. We find the intuitive result that whenever there is renegotiation in period 2, the renegotiated amount of debt is increasing in the level of credit protection.

Lemma 4. *Whenever renegotiation is accepted the renegotiated amount of debt repayment $d^{ren}(i, \theta^s, k)$ is non-decreasing in the amount of credit insurance i.*

Proof. see appendix A.1 □

We now proceed to analyze the optimal choice of credit insurance by the lender. Competition among risk-neutral insurers implies that the price of credit insurance equals to the probability of default for each level of credit insurance chosen

$$\begin{aligned} q^{CDS}(i,k,b) &= P\left(\left\{d^{eft}(i,\theta,k,b)=0\right\}\right) \\ &= \mathbb{E}_\theta\left[I_{\left\{d^{eft}(i,\theta,k,b)=0\right\}}\right] \end{aligned}$$

so that the problem of the lender (2.3.2) simplifies to

$$i^*(k,b) \in \max_i \mathbb{E}_\theta\left[d^{eft}(i,\theta,k,b)\right] \tag{2.4.2}$$

From this expression we can see that the benefit of credit insurance to the lender comes purely from strengthening its bargaining power when there is renegotiation. This is because the lender internalizes that the price of credit insurance cancels out with its expected payment in case of default. Additionally, however, credit insurance also leads to a higher repayment to the lender in case of renegotiation. Credit insurance thus imposes a non-pecuniary externality on the bargaining process between the government and the lender. As we have seen in lemma 4 the renegotiated amount of debt is increasing with the level of credit insurance, so that the effective repayment of debt in each state is maximized by choosing an amount of credit insurance that equals the full bargaining surplus, $i=(\lambda-\delta)f(\theta^s,k)$. With this choice the lender is able to extract the full surplus from renegotiation[3]. Credit insurance thus effectively changes the

[3] Note that this observation is in fact not particular to Nash bargaining, as the lender is able to extract the full bargaining surplus under any bargaining protocol by choosing $i=(\lambda-\delta)f(\theta^s,k)$ as this choice pushes the government exactly to its participation constraint.

bargaining power between the government and the lender during renegotiation. Note however that the lender needs to choose the level of credit insurance before the uncertainty about θ has been resolved. Choosing $i = (\lambda - \delta) f\left(\theta^H, k\right)$ would lead to full surplus extraction in the high state but to default in the low state. It might be preferable to choose a level of credit insurance of $i \leq (\lambda - \delta) f\left(\theta^L, k\right)$ in order to allow for renegotiation in the low state, even though this implies that less than the full surplus is extracted in the high state. The following proposition characterizes the optimal choice of the lender:

Lemma 5. *For low levels of debt such that* $b \leq \lambda f\left(\theta^L, k\right)$ *the amount of credit insurance is irrelevant, so in particular setting* $i^*(k, b) = (\lambda - \delta) f\left(\theta^L, k\right)$ *is weakly optimal for the lender. For intermediate levels of the debt such that*

$b \in \left(\lambda f\left(\theta^L, k\right), \lambda f\left(\theta^H, k\right)\right]$ *the optimal choice of credit insurance is* $i^*(k, b) = (\lambda - \delta) f\left(\theta^L, k\right)$*. For high levels of debt* $b > \lambda f\left(\theta^H, k\right)$ *the optimal choice of credit insurance is either* $i^*(k, b) = (\lambda - \delta) f\left(\theta^L, k\right)$ *or* $i^*(k, b) = (\lambda - \delta) f\left(\theta^H, k\right)$*.*

Proof. From Lemma 3 it follows that there is full repayment of the contractual amount of debt in both states when $b \leq \lambda f\left(\theta^L, k\right)$, so that credit insurance has no impact on the debt repayment. The price of credit protection is also zero, as credit insurance does not pay out in neither of the states. Any level of credit insurance thus gives the same profit to the lender. For $b \in \left(\lambda f\left(\theta^L, k\right), \lambda f\left(\theta^H, k\right)\right]$ there is full repayment of debt in the high state, but either default or renegotiation in the low state, depending on the level of credit protection chosen by the lender. If he chooses $i \leq (\lambda - \delta) f\left(\theta^L, k\right)$ there is renegotiation in the low state so that the expected repayment to the lender is

$$\pi b + (1-\pi)\, d^{ren}\left(i, \theta^{L}, k\right)$$

while if he chooses $i > (\lambda - \delta)\, f\left(\theta^{L}, k\right)$ there is default in the low state so that his expected payoff is

$$\pi b$$

Clearly the first option is more profitable for the lender and since the amount of renegotiated debt $d^{ren}\left(i, \theta^{L}, k\right)$ is increasing with the level of credit insurance (Proposition 4) the optimal choice is $i^{*}(k, b) = (\lambda - \delta)\, f\left(\theta^{L}, k\right)$ in this case. For $b > \lambda f\left(\theta^{H}, k\right)$ there is either default or renegotiation in both states. If the lender chooses $i \leq (\lambda - \delta)\, f\left(\theta^{L}, k\right)$ there is renegotiation in both states and the expected payoff to the lender is

$$\pi d^{ren}\left(i, \theta^{H}, k\right) + (1-\pi)\, d^{ren}\left(i, \theta^{L}, k\right)$$

while if he chooses $i \in \left((\lambda - \delta)\, f\left(\theta^{L}, k\right), (\lambda - \delta)\, f\left(\theta^{H}, k\right)\right]$ there is default in the low state so that his expected payoff is

$$\pi d^{ren}\left(i, \theta^{H}, k\right)$$

. Note however that the lender is able to extract the full surplus in the high state if he chooses $i = (\lambda - \delta)\, f\left(\theta^{H}, k\right)$, which might be more profitable than choosing $i = (\lambda - \delta)\, f\left(\theta^{L}, k\right)$ in order to allow for renegotiation in the low state. Clearly, he would never choose any level in between, since it would still lead to full default in the low state, while extracting less than the full surplus in the high state. Which option is more profitable depends on the probability π as well as the relative difference of the shocks. Choosing an amount of credit insurance $i > (\lambda - \delta) f\left(\theta^H, k\right)$ on the other hand would lead to default in both states and thus an expected repayment of 0, which can never be optimal. $\square$

Note that by choosing $i^*(k, b) = (\lambda - \delta) f\left(\theta^H, k\right)$ the lender forces the government into default in the low state. This results in a potential an ex-ante efficiency loss as lenders and insurers make zero profit and the government suffers the higher cost of default compared to renegotiation. We will see in the next section that under risk-neutrality this efficiency loss never arises in equilibrium, as the government never chooses a debt level of $b > \lambda f\left(\theta^H, k\right)$ which would make this choice of credit insurance optimal for the lender.

We next derive the price of debt. Competition in the market for bonds implies that the lenders profit given by equation (2.3.1) must equal to zero. The price of debt $q(b, k)$ is thus given by the following equation

$$q(b,k)b = \mathbb{E}_\theta\left[d^{eft}\left(i^*(k,b), \theta, k, b\right)\right] \tag{2.4.3}$$

The value of debt must equal its expected repayment. The government when choosing the amount of debt b and investment k, correctly anticipates the bond prices quoted by the lenders and the effective repayment in period 1.We now analyze the governments problem for the case of risk-neutrality and risk-aversion separately and study the implications credit insurance on welfare for each case in turn. We will see that the effect of credit default swaps can work in different directions.

2.5 Risk-Neutrality

2.5.1 Equilibrium

The government anticipates that by borrowing less than $\lambda f\left(\theta^L, k\right)$ it will fully repay in both states and the bond prices therefore equals to 1. The maximal welfare it can achieve by borrowing such an amount is thus given by

$$\begin{aligned} \underline{W}^{CDS} &= \max_{b,k} \quad w_0 - k + b + \pi\left[f\left(\theta^H, k\right) - b\right] + (1-\pi)\left[f\left(\theta^L, k\right) - b\right] \\ s.t. &\quad b \leq \lambda f\left(\theta^L, k\right) \\ &\quad w_0 - k + b \geq 0 \end{aligned}$$

where the second constraint comes from the non-negativity of consumption requirement in period 0 (in period 1 it is automatically satisfied by the constraint on debt). Risk-neutrality implies that the proceeds from issuing debt in period 0 cancels out exactly with the expected repayment in period 1, the problem thus simplifies to

$$\underline{W}^{CDS} = \max_{b,k} \quad w_0 - k + \pi f\left(\theta^H, k\right) + (1-\pi) f\left(\theta^L, k\right)$$

$$\begin{aligned} s.t. \qquad & b \leq \lambda f\left(\theta^L, k\right) \\ & k \leq w_0 + b \end{aligned}$$

Borrowing $b \in \left(\lambda f\left(\theta^L, k\right), \lambda f\left(\theta^H, k\right)\right]$ on the other hand leads to renegotiation in the low state but full repayment in the high state. Already canceling out the proceeds from issuing debt and its expected repayment, the welfare the government can obtain with this choice is thus given by

$$\overline{W}^{CDS} = \max_{b,k} \quad w_0 - k + \pi f\left(\theta^H, k\right) + (1-\pi)(1-\delta) f\left(\theta^L, k\right)$$

$$\begin{aligned} s.t. \qquad & b \in \left(\lambda f\left(\theta^L, k\right), \lambda f\left(\theta^H, k\right)\right] \\ & k \leq w_0 + \pi b + (1-\pi) d^{ren}\left(i^*(k,b), \theta^L, k\right) \end{aligned} \tag{2.5.1}$$

as the government needs to pay the cost of renegotiation in the low state, but also repays only the renegotiated amount of debt which is anticipated by the lender and thus affects the borrowing constraint. Comparing the two problems we can see that if wealth is high enough such that the government can invest the efficient level of capital k^{**} by borrowing less than $\lambda f\left(\theta^L, k^{**}\right)$ it will always do so. If however the optimal investment is much higher than what it can achieve by borrowing $\lambda f\left(\theta^L, k\right)$ it might prefer to incur the cost of renegotiation in the low state in return for relaxing the borrowing constraint. The following lemma shows that the government never finds it optimal to borrow an amount $b > \lambda f\left(\theta^H, k\right)$ as this would lead to renegotiation in both states.

Lemma 6. *For any level of credit insurance, the government never finds it optimal to borrow strictly more than $\lambda f\left(\theta^H, k^*\right)$ where k^* denotes the optimal level of investment in equilibrium.*

Proof. Suppose not and it would be optimal to borrow amount b^* strictly higher than $\lambda f\left(\theta^H, k^*\right)$. This would result in renegotiation in the high and in the low state. Now consider the alternative choice $\tilde{b} = \lambda f\left(\theta^H, k^*\right)$. This choice implies that there is full repayment in the high state and renegotiation in the low state. Note that the level of debt only appears in the constraint on investment. Under the original choice b^* the constraint reads $\pi d^{ren}\left(i, \theta^H, k^*\right) + (1-\pi)\, d^{ren}\left(i, \theta^L, k^*\right)$ while under the alternative choice $\tilde{b}$ it is given by $\pi\lambda f\left(\theta^H, k^*\right) + (1-\pi)\, d^{ren}\left(i, \theta^L, k^*\right)$. Since the latter is larger for arbitrary i, the borrowing constraint is more slack when borrowing $\tilde{b}$ than under the original level of debt. Also the government does not suffer the cost of renegotiation δ in the high state under $\tilde{b}$. Welfare is thus strictly higher under the alternative choice of debt, the original choice can thus not have been optimal. □

2.5.2 Second-best

Let us now compare the equilibrium with the second-best. In the second-best the planner chooses the amount of credit insurance instead of the lender. Since for low levels equilibrium values of debt credit insurance does not have an impact on the repayment decision, the planner cannot improve on the equilibrium amount of welfare. If on the other hand the government chooses an amount of borrowing $b \in \left(\lambda f\left(\theta^L, k\right), \lambda f\left(\theta^H, k\right)\right]$ the planner could potentially improve on welfare by further relaxing the constraint (2.5.1) as he chooses the amount of credit insurance i directly which determines the bargaining outcome. This might lead to an efficiency improvement as the lender is less constraint on its investment decision. However, as derived in proposition (2.3.2) the lenders chooses a level of credit insurance of $i^*(k, b) = (\lambda - \delta) f\left(\theta^L, k\right)$ for intermediate levels of debt, so that he extracts the full surplus from renegotiation. The constraint thus cannot be further relaxed by a social planner.

The other inefficiency that might potentially arise in equilibrium is that the lender chooses such a high level of credit insurance that the government is forced into default in the low state and thus suffers the higher cost of default while renegotiation would have been possible for a lower level of credit insurance. In lemma 2.3.2 we found that this might occur for high levels of debt $b > \lambda f(\theta^s, k)$. Lemma 6 however shows that such a high amount of debt is never chosen in equilibrium, so that this kind of inefficiency never occurs in equilibrium. We summarize the finding in the following proposition

Proposition 7. *Under risk-neutrality the lender chooses the socially-efficient level of credit insurance.*

Since the planner could have chosen an amount of credit insurance which equals to zero but did instead choose the equilibrium level, it is an immediate consequence of the previous proposition that the equilibrium with credit insurance pareto-dominates the equilibrium without credit insurance.

Corollary 8. *The welfare in the economy with credit insurance is higher than in the* *economy without CDS.*

2.6 Risk-Aversion

We now proceed to analyze the welfare properties of the equilibrium when the government has risk-averse preferences

$$u(c)$$

with u strictly concave and the usual Inada condition

$$\lim_{c \to 0} u'(c) = \infty$$

applies. For the ease of exposition we will only consider pure endowment economies such that the production function becomes $f(\theta, k) = \theta$ from now on.[4] This setup is frequently used in the sovereign debt literature and therefore of particular interest. We will first show on hand of an example why the level of credit insurance chosen by the lender may no longer be constraint efficient. Then we derive the more general result that under risk-aversion the lender generally (weakly) over-insures. We then proceed to compare the economy with credit insurance to the economy without credit insurance. We first provide some intuition on the hand of two examples on what are the trade-offs between the two scenarios. We then give sufficient conditions under which the economy without credit insurance strictly pareto dominates and as a consequence the economy

[4]Clearly, under the assumption of risk-neutrality, if the output is independent of investment and only depends on the shock θ (as in a pure endowment economy) credit insurance does not have any effect. In the this section we will thus look at the more interesting case of a pure endowment economy with risk aversion.

with credit insurance is inefficient.

2.6.1 Equilibrium

As in the previous section, the governments problem depends on the level of debt. For low levels of debt $b \leq \lambda f\left(\theta^L, k\right)$ there is full repayment in both states and the government's problem becomes now

$$\begin{aligned} \underline{W}^{CDS} &= \max_b \quad u\left(w_0+b\right)+\pi u\left(\theta^H-b\right)+\left(1-\pi\right)u\left(\theta^L-b\right) \\ s.t. & \qquad b \leq \lambda\theta^L \end{aligned}$$

. The Inada condition implies that consumption in period 0 is positive, so we no longer need the extra constraint as in the case of risk-neutrality. By borrowing $b \in \left(\lambda f\left(\theta^L, k\right), \lambda f\left(\theta^H, k\right)\right]$ on the other hand we have renegotiation in the low state but full repayment in the high state. So that government's problem is

$$\begin{aligned} \overline{W}^{CDS} &= \max_b \quad u\left(w_0+\pi b+\left(1-\pi\right)\left(\lambda-\delta\right)\theta^L\right)+\pi u\left(\theta^H-b\right)+\left(1-\pi\right)u\left(\left(1-\lambda\right)\theta^L\right) \\ s.t. & \qquad b \in \left(\lambda\theta^L, \lambda\theta^H\right] \end{aligned} \tag{2.6.1}$$

Similarly as in the case of risk neutrality we can show that the government never finds it optimal to choose and amount of borrowing above $\lambda\theta^H$. However we need to make an extra assumption.

Assumption (A1)

$$\theta^H > \frac{2\lambda}{\lambda-\delta}\theta^L$$

This assumption ensures that the renegotiated amount of debt $d^{ren}\left(i,\theta^H\right)$ is always larger than $\lambda\theta^L$ (under any level of credit insurance), so that borrowing $d^{ren}\left(i,\theta^H\right)$ leads to renegotiation or default in the low state depending on the level of credit insurance.[5]

Lemma 9. *Under any level of credit protection, the government never finds it optimal to borrow strictly more than* $\lambda\theta^H$

Proof. Suppose not and it would be optimal to borrow amount b^* strictly higher than $\lambda\theta^H$. In what follows we will show that by choosing an alternative debt level of $\tilde{b} = d^{ren}\left(i,\theta^H\right) < \lambda\theta^H$ the government can do at least as good. We distinguish different cases depending on the level of credit insurance:

case 1 $i \leq \lambda\theta^L$: as we have seen in section 2.4 under $b^* > \lambda\theta^H$ this implies that there is renegotiation in both states

$$c_0 = w_0 + \pi d^{ren}\left(i,\theta^H\right) + (1-\pi)\,d^{ren}\left(i,\theta^L\right) \quad c_H = (1-\delta)\,\theta^H - d^{ren}\left(i,\theta^H\right)$$
$$c_L = (1-\delta)\,\theta^L - d^{ren}\left(i,\theta^L\right)$$

while under $\tilde{b} = d^{ren}\left(i,\theta^H\right)$ there is still renegotiation in the low state by assumption

[5] This is because under no credit insurance and risk-neutrality, the renegotiated amount of debt is $\frac{(\lambda-\delta)\theta^L}{2}$ as this ensures that the surplus from bargaining is shared to equal parts among the lender and the borrower. The assumption assures that $\frac{(\lambda-\delta)\theta^L}{2} > \lambda\theta^L$. Under risk-aversion or with some level of credit insurance the renegotiated amount of debt is even higher than $\frac{(\lambda-\delta)\theta^L}{2}$.

(A1) but full repayment in the high state

$$c_0 = w_0 + \pi d^{ren}\left(i,\theta^H\right) + (1-\pi)\, d^{ren}\left(i,\theta^L\right) \quad c_H = \theta^H - d^{ren}\left(i,\theta^H\right)$$
$$c_L = (1-\delta)\,\theta^L - d^{ren}\left(i,\theta^L\right)$$

we can see that the government has a higher consumption in the high state in period 1 as it does not suffer the cost of renegotiation, while other consumption levels are the same. Thus choosing $\tilde{b} = d^{ren}\left(i,\theta^H\right)$ gives a strictly higher welfare.

case 2 $i \in \left(\lambda\theta^L, \lambda\theta^H\right]$: under $b^* > \lambda\theta^H$ there is renegotiation in the high state while there is default in the low state

$$c_0 = w_0 + \pi d^{ren}\left(i,\theta^H\right) \quad c_H = (1-\delta)\,\theta^H - d^{ren}\left(i,\theta^H\right) \quad c_L = (1-\lambda)\,\theta^L$$

while under $\tilde{b} = d^{ren}\left(i,\theta^H\right)$ there is still default in the low state by assumption (A1) but full repayment in the high state

$$c_0 = w_0 + \pi d^{ren}\left(i,\theta^H\right) \quad c_H = \theta^H - d^{ren}\left(i,\theta^H\right) \quad c_L = (1-\lambda)\,\theta^L$$

also in this case the government does not suffer the cost of renegotiation in the high state by choosing $\tilde{b} = d^{ren}\left(i,\theta^H\right)$ and can thus increase welfare compared to $b^* > \lambda\theta^H$

case 3 $i > \lambda\theta^H$: under $b^* > \lambda\theta^H$ there is default in both states

$$c_0 = w_0 \quad c_H = (1-\lambda)\,\theta^H \quad c_L = (1-\lambda)\,\theta^L$$

while under $\tilde{b} = d^{ren}\left(i, \theta^H\right)$ there is still default in the low state by assumption (A1) but full repayment in the high state

$$c_0 = w_0 + \pi d^{ren}\left(i, \theta^H\right) \quad c_H = \theta^H - d^{ren}\left(i, \theta^H\right) + c_L = (1-\lambda)\,\theta^L$$

Since we have by the participation constraint of the that

$$d^{ren}\left(i, \theta^H\right) \leq (\lambda - \delta)\,\theta^H \leq \lambda\theta^H$$

consumption in the high state in period 1 is higher compared to choosing $b^* > \lambda\theta^H$ and also consumption in period 0 is higher since under this alternative choice the government repays in the high state and can thus borrow. Consumption in the low state of period 1 remains the same so that also in this last case welfare is higher under the alternative choice $\tilde{b} = d^{ren}\left(i, \theta^H\right)$. Thus $b^* > \lambda\theta^H$ cannot have been optimal. $\square$

2.6.2 Second best

We now proceed to show first on hand of an example and then in a general result that in equilibrium the lender chooses a (weakly) higher amount of credit insurance compared to the socially efficient level. In the following section we will compare the economy with credit insurance to the economy without credit insurance and this will also a deliver condition under which the lender strictly over-insures compared to the socially efficient level.

2.6.2.1 Example

Consider the economy as described above with the following parameters:

$$\theta^L = 2, \quad \theta^H = 4, \quad w_0 = 0, \quad \pi = 0.1, \quad \delta = 0, \quad \lambda = 0.8$$

In the efficient (first-best) allocation we have constant consumption across time and states

$$c^{**} = \frac{w_0 + \pi\theta^H + (1-\pi)\,\theta^L}{2} = 1.1$$

Now, if a social planner was to choose the level of credit insurance, he could implement the first best allocation by borrowing $b = 2.9 < 3.2 = \lambda\theta^H$ which would lead to full repayment in the high state and renegotiation in the low state. If he sets credit insurance to such a level i^{**} such that

$$d^{ren}\left(i^{**}, \theta^L\right) = 0.9$$

It is easy to check that this results in the efficient allocation. In equilibrium however the lender chooses the level of credit protection in order to extract the full renegotiation surplus. The renegotiated amount of debt in the low state is thus

$$d^{ren}\left(i^{*}, \theta^L\right) = (\lambda - \delta)\,\theta^L = 1.6$$

which results in the consumption level in the low state being smaller than the efficient allocation

$$c_L < c^{**}$$

The example shows that under risk-aversion it is no longer true that the lender chooses the socially efficient level of credit insurance. This is because in the incomplete contracts economy renegotiation works as implicitly adding contingency to the non-contingent bond contract. The lender however, does not internalize this effect on consumption smoothing when choosing the optimal level of credit insurance.

Proposition 10. *Under risk-aversion the lender over-insures with respect to the socially efficient choice of credit insurance, i.e.* $i^{**} \leq i^* = (\lambda - \delta)\,\theta^L$.

Proof. Suppose not and the socially efficient level was $i^{**} > (\lambda - \delta)\,\theta^L$.

If the government would consequently choose an amount of $b^{**} \in \left(\lambda\theta^L, \lambda\theta^H\right]$ we have seen in section 2.4 together with the level of credit insurance this implies default in the low state and repayment in the high state. The consumption allocation is thus

$$c_0 = w_0 + \pi b^{**} \quad c_H = \theta^H - b^{**} \quad c_L = (1-\lambda)\,\theta^L$$

If on the other hand the planner chooses an alternative level of credit insurance $\tilde{i} = (\lambda - \delta)\,\theta^L$ it is still feasible for the government to choose the same amount of debt $\tilde{b} = b^{**}$. The lower level of credit insurance implies that then there is renegotiation in the low state so that the consumption allocation is given by

$$c_0 = w_0 + \pi b^{**} + (1-\pi)\,(\lambda - \delta)\,\theta^L \quad c_H = \theta^H - b^{**} \quad c_L = (1-\lambda)\,\theta^L$$

Comparing the two expressions we can see that consumption in the initial period is higher under the alternative choice $\tilde{i} = (\lambda - \delta)\,\theta^L$ while consumption in the second period is the same. Hence $i^{**} > (\lambda - \delta)\,\theta^L$ cannot have been an optimal choice of the planner.

If on the other hand the government would choose a level of debt is low so that $b^{**} \leq \lambda\theta^L$, then there is full repayment in both states, so credit insurance does not have any effect. Thus choosing $i^{**} \leq (\lambda - \delta)\,\theta^L$ is weakly better. We have seen in the previous lemma that the government never chooses an amount of borrowing greater than $\lambda\theta^H$ which completes the proof.

□

2.6.3 CDS vs No-CDS

It is however not clear from the previous findings, what is the welfare effect of CDS chosen by the lender, compared to no CDS at all. Both are inefficient, so they are not trivial to compare. We will get some insight on this from the following examples.

2.6.3.1 Examples

Example 1

Consider the above economy with the following specifications. Utility is of CRRA form

$$u\left(c\right) = \frac{c^{1-\sigma}}{1-\sigma}$$

with the parameter of risk aversion taking a value of $\sigma = 5$. Other parameters are as follows: the cost of default is $\lambda = 0.2$, the cost of renegotiation is $\delta = 0$. The endowment in the high state is $\theta^H = 5$, in the low state $\theta^L = 1$. Both states are equally likely so that $\pi = 0.5$. It's a well-known result that under the assumption of risk-aversion the share of renegotiation surplus of the more risk-neutral party increases. In the case of CRRA utility it is easy to show that in the economy without credit insurance the renegotiated amount of debt is a constant proportion ρ of the bargaining surplus $d^{ren}(0, \theta^s) = \rho(\lambda - \delta)\theta^s$ where with the parameters above $\rho \approx 0.5657$ (compared to the case of risk-neutrality where $\rho = 0.5$)[6]. In the economy with credit protection on the other hand the lender buys credit insurance in order to extract the full surplus of renegotiation $\lambda\theta^L$ under any degree of risk-aversion as we have seen in section 2.3.3. We will first consider an economy with no initial wealth $w_0 = 0$.

Standard calculations give that in both economies the government chooses a debt level of

$$b^{*,NO} = b^{*,CDS} = \lambda\theta^H = 1$$

The government chooses thus to borrow the maximum amount possible in both economies. We thus have full repayment in the high state and renegotiation in the low state in both economies. The welfare of the government in an economy with credit insurance is

$$\begin{aligned} & u\left(\pi b^* + (1-\pi)\lambda\theta^L\right) + \pi u\left(\theta^H - b^*\right) + (1-\pi)u\left(\theta^L - \lambda\theta^L\right) \\ = \quad & u(0.6) + \pi u(4) + (1-\pi)u(0.8) \\ \approx \quad & -2.23 \end{aligned}$$

[6]it can be shown that ρ needs to satisfy the following non-linear equation $(1 - \delta - \rho(\lambda - \delta))^{1-\sigma} - (1-\lambda)^{1-\sigma} = (1-\sigma)(1 - \delta - \rho(\lambda - \delta))^{-\sigma}\rho(\lambda - \delta)$

while in the economy without credit insurance welfare is given by

$$\begin{aligned} & u\left(\pi b^* + (1-\pi)\,\rho\lambda\theta^L\right) + \pi u\left(\theta^H - b^*\right) + (1-\pi)\,u\left(\theta^L - \rho\lambda\theta^L\right) \\ \approx\ & u\,(0.5566) + \pi u\,(4) + (1-\pi)\,u\,(0.8869) \\ \approx\ & -2.81 \end{aligned}$$

We can see that welfare is higher in the economy with credit insurance. The intuition is similar to the production economy: credit insurance enables the lender to enforce a higher repayment in the low state. This leads to higher bond prices and thus higher proceeds from issuing debt. This results in a higher level of consumption in period 0 and facilitates inter-temporal consumption smoothing, which increases welfare because of the risk-aversion of the government.

Example 2

Let us now consider the similar example as in the previous section, with the only difference that now the initial wealth is given by

$$w_0 = 1$$

. Standard calculations show that the government still finds it optimal to choose

$$b^{*,NO} = b^{*,CDS} = \lambda\theta^H = 1$$

as constraint on borrowing is still binding. The welfare of the government in the economy with credit insurance is now

$$\begin{aligned} & u\left(w_0 + \pi b^* + (1-\pi)\lambda\theta^L\right) + \pi u\left(\theta^H - b^*\right) + (1-\pi)u\left(\theta^L - \lambda\theta^L\right) \\ = \; & u(1.6) + \pi u(4) + (1-\pi)u(0.8) \\ \approx \; & -0.34 \end{aligned}$$

while in the economy without credit insurance it is

$$\begin{aligned} & u\left(w_0 + \pi b^* + (1-\pi)\rho\lambda\theta^L\right) + \pi u\left(\theta^H - b^*\right) + (1-\pi)u\left(\theta^L - \rho\lambda\theta^L\right) \\ \approx \; & u(1.5566) + \pi u(4) + (1-\pi)u(0.8869) \\ \approx \; & -0.25 \end{aligned}$$

Observe, that as opposed to the previous example, the government would be better off in an economy without credit insurance. While increasing consumption in period 0, credit insurance also enforces a higher repayment in the low state and thus limits the amount of inter-state consumption smoothing. In this example the government values the additional proceeds from issuing debt in period 0 less, because even without credit insurance it has already a relatively high level of consumption compared to the low state. And since marginal utility is decreasing because of the concavity assumption it values additional consumption in period 0 less compared to the higher repayment in the low state so that the negative impact on inter-state consumption smoothing prevails. As we can see in the latter example the equilibrium choice of credit insurance by the lender no longer agrees with the socially efficient level. A social planner would choose the level of credit insurance taking both of the previously explained effects into account, while the lender chooses the level of credit insurance that maximizes the expected amount of repayment. The results that we found in the previous section, that credit insurance is welfare increasing and the lender always chooses the socially optimal level of credit insurance thus no longer hold under the assumption of risk-aversion, which is standard in the government debt literature.

2.6.3.2 General Result

We now proceed to provide sufficient conditions under which welfare in the economy without credit protection is strictly higher compared to welfare in the economy with credit protection. On one hand wealth needs to be low enough so that the government borrows an amount higher than $\lambda\theta^L$. We have seen in section 2.4 that otherwise credit insurance does not matter as the government repays fully in both states. The following lemma provides a condition on wealth that ensures that the optimal level of debt is higher than $\lambda\theta^L$ in the economy with credit insurance.

Lemma 11. *Suppose $\delta = 0$. If the initial wealth is low relative to the endowment in the low state so that $w_0 < (1 - 2\lambda)\,\theta^L$, in the economy with credit protection the government chooses to a debt level $b^{*,CDS} > \lambda\theta^L$ so that there is renegotiation in the low state.*

Proof. The proof proceeds by showing if the government would choose a low debt level $b^{*,CDS} \leq \lambda\theta^L$ such that there is full repayment in both states, it would find it optimal to borrow at the boundary $b^{*,CDS} = \lambda\theta^L$ and by borrowing more it can do strictly better.

We have a corner solution for low levels of debt $b^{*,CDS} \leq \lambda\theta^L$ if welfare has a strictly positive slope with respect to debt in the point $\lambda\theta^L$:

$$\begin{aligned} & u'\left(w_0 + \lambda\theta^L\right) - \pi u'\left(\theta^H - \lambda\theta^L\right) - (1-\pi)\, u'\left(\theta^L - \lambda\theta^L\right) \\ > \; & u'\left(w_0 + \lambda\theta^L\right) - \pi u'\left(\theta^L - \lambda\theta^L\right) - (1-\pi)\, u'\left(\theta^L - \lambda\theta^L\right) \\ = \; & u'\left(w_0 + \lambda\theta^L\right) - u'\left(\theta^L - \lambda\theta^L\right) > 0 \end{aligned}$$

where the last inequality follows from the assumption that $w_0 < (1-2\lambda)\,\theta^L$. We thus have to compare the welfare at higher levels of debt only with the welfare at the point $b^{*,CDS} = \lambda\theta^L$ in order to show that higher debt levels are optimal. Note that since $\delta = 0$ welfare is continuous in the point $\lambda\theta^L$ (see (2.6.1)). Since the derivative of welfare wrt to debt is still positive for slightly higher levels of debt $b^{*,CDS} = \lambda\theta^L + \epsilon$[7]

$$\begin{aligned} & \pi u'\left(w_0 + \pi\left(\lambda\theta^L + \epsilon\right) + (1-\pi)\,\lambda\theta^L\right) - \pi u'\left(\theta^H - \left(\lambda\theta^L + \epsilon\right)\right) \\ > \; & u'\left(w_0 + \lambda\theta^L + \pi\epsilon\right) - \pi u'\left(\theta^L - \lambda\theta^L + \epsilon\right) \\ > \; & 0 \end{aligned}$$

for ϵ small enough under the condition that $w_0 < (1-2\lambda)\,\theta^L$, we can increase welfare even further by choosing $b^{*,CDS} = \lambda\theta^L + \epsilon$ □

In order to for credit insurance to be welfare decreasing, it needs to be the case that the cost of credit insurance - having to repay a larger amount of debt after renegotiation in the low state - outweighs the benefit - to be able to transfer a higher amount from

[7] note that we do not take the derivative with respect to the low state, as there is renegotiation for $b^{*,CDS} > \lambda\theta^L$ so that the consumption in the low state is independent of the level of debt

the low state to period 0 (and through borrowing also indirectly to the high state in period 1). The following proposition shows that this is the case if the endowment in the low state is low enough relative to a weighted average between the initial wealth and the endowment in the high state.

Proposition 12. *Suppose $\delta = 0$. Then for $w_0 < (1-2\lambda)\,\theta^L$* and
$\theta^L < \min\left\{\frac{\pi\theta^H + w_0}{1-2\rho\lambda+\pi}, \frac{\pi\lambda\theta^H + w_0}{1-2\rho\lambda+\pi\rho\lambda}\right\}$ *the economy without credit insurance strictly pareto dominates the economy with credit insurance. The lender thus strictly over-insures relative to the efficient choice of credit insurance.*

Proof. The proof proceeds in three steps.

Step 1: If the government chooses a debt level higher than $\lambda\theta^L$ both in the economy with and without credit protection we have that consumption in period 0 and in the high state in period 1 is higher in the economy with credit protection compared to the economy without credit protection, while consumption in the low state is lower in the economy with credit protection.

Proof: In what follows we define by $\rho \equiv \frac{d^{ren}\left(0,\theta^L\right)}{\lambda\theta^L}$ the share of the bargaining surplus obtained by the lender in the economy without CDS. In the economy without credit protection we then have that the level of consumption is given by

$$c_L^{NO} = (1-\rho\lambda)\,\theta^L$$

. In an interior solution $b^{*,NO} \in \left(\lambda\theta^L, \lambda\theta^H\right)$ we have that the government perfectly smoothes consumption between period 0 and the high state of period 1 so that

$$c_0^{NO} = c_H^{NO} = \frac{w_0 + \pi\theta^H + (1-\pi)\,\lambda\rho\theta^L}{1+\pi}$$

as $\pi\theta^H + (1-\pi)\,\lambda\rho\theta^L + w_0$ is the total wealth to be shared among period 0 and the high state of period 1. At the upper bound for debt $b^{*,NO} = \lambda\theta^H$ we have that

$$c_0^{NO} = w_0 + \pi\lambda\theta^H + (1-\pi)\,\lambda\rho\theta^L$$

and

$$c_H^{NO} = \theta^H - \lambda\theta^H$$

. The optimal amount of borrowing can thus be written as

$$b^{*,NO} = \min\left\{\frac{\theta^H - (1-\pi)\,\lambda\rho\theta^L - w_0}{1+\pi}, \lambda\theta^H\right\}$$

consumption in period 0 as

$$c_0^{NO} = \min\left\{\frac{w_0 + \pi\theta^H + (1-\pi)\,\lambda\rho\theta^L}{1+\pi}, w_0 + \pi\lambda\theta^H + (1-\pi)\,\lambda\rho\theta^L\right\}$$

and consumption in the high state in period 1as

$$c_H^{NO} = \max\left\{\frac{w_0 + \pi\theta^H + (1-\pi)\,\lambda\rho\theta^L}{1+\pi}, \theta^H - \lambda\theta^H\right\}$$

. Similarly we have the in the economy with credit protection where $\rho = 1$ we have that

$$c_L^{CDS} = (1-\lambda)\,\theta^L$$

$$c_0^{CDS} = \min\left\{\frac{w_0 + \pi\theta^H + (1-\pi)\,\lambda\theta^L}{1+\pi}, w_0 + \pi\lambda\theta^H + (1-\pi)\,\lambda\theta^L\right\}$$

$$c_H^{CDS} = \max\left\{\frac{w_0 + \pi\theta^H + (1-\pi)\,\lambda\theta^L}{1+\pi}, \theta^H - \lambda\theta^H\right\}$$

and

$$b^{*,CDS} = \min\left\{\frac{\theta^H - (1-\pi)\,\lambda\theta^L - w_0}{1+\pi}, \lambda\theta^H\right\}$$

and. Comparing the expressions the result follows immediately.

Step 2: for $c_L^{NO} < c_0^{NO}$ the welfare in the economy without credit protection is strictly higher compared to the economy with credit insurance.

Proof: The intuition of this result is simple. If $c_L^{NO} < c_0^{NO}$ the government would like to shift consumption from period 0 to the low state in period 1 in the equilibrium of the economy without credit insurance. Credit insurance however works in the opposite direction as it further decreases consumption in the low state and increases consumption in period 0. Credit insurance is thus welfare decreasing. We now proceed to give the formal proof. Suppose the government borrows an amount of debt $b^{*,NO} > \lambda\theta^L$ as specified in step 1 in the economy without credit protection. The consumption allocations are then as given in step 1. The assumption $w_0 < (1-2\lambda)\,\theta^L$ together with lemma 11 ensures that the best the government can do in the economy with credit protection is to borrow $b^{*,CDS} > \lambda\theta^L$. Under these choices the results from step 1 follow immediately. By repeatedly applying a version of the intermediate value theorem[8] for

[8]the mean value theorem says that for a continuous function f and $x < y$, there exits a $\xi \in (x, y)$ s.t. $\frac{f(y)-f(x)}{y-x} = f'(\xi)$. Applying the fact that for concave functions the first derivative is decreasing

concave functions we get that

$$\begin{aligned}
& u\left(c_0^{NO}\right)+\pi u\left(c_H^{NO}\right)+(1-\pi)\,u\left(c_L^{NO}\right) \\
\geq\;& u\left(c_0^{NO}\right)+\pi u\left(c_H^{NO}\right)+(1-\pi)\,u\left(c_L^{CDS}\right)+(1-\pi)\,u'\left(c_L^{NO}\right)\left(c_L^{NO}-c_L^{CDS}\right) \\
>\;& u\left(c_0^{NO}\right)+\pi u\left(c_H^{NO}\right)+(1-\pi)\,u\left(c_L^{CDS}\right)+(1-\pi)\,u'\left(c_0^{NO}\right)\left(c_L^{NO}-c_L^{CDS}\right) \\
=\;& u\left(c_0^{NO}\right)+\pi u\left(c_H^{NO}\right)+(1-\pi)\,u\left(c_L^{CDS}\right)+u'\left(c_0^{NO}\right)\left(\pi c_H^{CDS}-\pi c_H^{NO}+c_0^{CDS}-c_0^{NO}\right) \\
=\;& u\left(c_0^{NO}\right)+u'\left(c_0^{NO}\right)\left(c_0^{CDS}-c_0^{NO}\right)+\pi u\left(c_H^{NO}\right)+\pi u'\left(c_0^{NO}\right)\left(c_H^{CDS}-c_H^{NO}\right)+(1-\pi)\,u\left(c_L^{CDS}\right) \\
\geq\;& u\left(c_0^{NO}\right)+u'\left(c_0^{NO}\right)\left(c_0^{CDS}-c_0^{NO}\right)+\pi u\left(c_H^{NO}\right)+\pi u'\left(c_H^{NO}\right)\left(c_H^{CDS}-c_H^{NO}\right)+(1-\pi)\,u\left(c_L^{CDS}\right) \\
\geq\;& u\left(c_0^{CDS}\right)+\pi u\left(c_H^{CDS}\right)+(1-\pi)\,u\left(c_L^{CDS}\right)
\end{aligned}$$

The first inequality follows from $f\left(y\right)\geq f\left(x\right)+f'\left(y\right)\left(y-x\right)$ for $y=c_L^{NO}$ and $x=c_L^{CDS}$. The strict equality in line 3 follows from the assumption that $c_L^{NO}<c_0^{NO}$ so that by concavity $u'\left(c_L^{NO}\right)\geq u'\left(c_0^{NO}\right)$ and $c_L^{NO}-c_L^{CDS}\geq 0$ as we have seen in step 1. The equality in line 3 follows from the fact that the total wealth in both economies is the same so that $c_0^{CDS}+\pi c_H^{CDS}+(1-\pi)\,c_L^{CDS}=c_0^{NO}+\pi c_H^{NO}+(1-\pi)\,c_L^{NO}$. The inequality in line 6 follows from the fact that in an interior solution we have that $c_0^{NO}=c_H^{NO}$ and if the government is constrained at $\lambda\theta^H$ $c_0^{NO}<c_H^{NO}$ so that $u'\left(c_0^{NO}\right)\geq u'\left(c_H^{NO}\right)$ and $c_H^{CDS}-c_H^{NO}\geq 0$ as we have shown step 1. The last inequality follows from applying $f\left(y\right)\leq f\left(x\right)+f'\left(x\right)\left(y-x\right)$ twice, once for $y=c_0^{CDS}$ and $x=c_0^{NO}$ and another time for $y=c_H^{CDS}$ and $x=c_H^{NO}$. We have thus shown that by choosing $b^{*,NO}$ the government can achieve a higher welfare compared to the equilibrium in the economy with credit insurance compared. Thus we have shown that the equilibrium welfare in the economy

we have that $f\left(y\right)\leq f\left(x\right)+f'\left(x\right)\left(y-x\right)$ and $f\left(y\right)\geq f\left(x\right)+f'\left(y\right)\left(y-x\right)$.

without credit insurance it higher under the conditions provided.

Step 3: if $\theta^L < \min\left\{\frac{\pi\theta^H+w_0}{1-2\rho\lambda+\pi}, \frac{\pi\lambda\theta^H+w_0}{1-2\rho\lambda+\pi\rho\lambda}\right\}$ then $c_L^{NO} < c_0^{NO}$.

Proof: Using the expressions derived in step 1 we have that

$$c_L^{NO} = (1-\rho\lambda)\,\theta^L < \min\left\{\frac{\pi\theta^H + (1-\pi)\,\lambda\rho\theta^L + w_0}{1+\pi}, w_0 + \pi\lambda\theta^H + (1-\pi)\,\lambda\rho\theta^L\right\} = c_0^{NO}$$

After rearranging terms gives that this is equivalent to

$$\theta^L < \min\left\{\frac{\pi\theta^H + w_0}{1-2\rho\lambda+\pi}, \frac{\pi\lambda\theta^H + w_0}{1-2\rho\lambda+\pi\rho\lambda}\right\}$$

The last statement follows from the fact that we already showed in proposition 10 that the lender weakly over-insures relative to the planner's choice of credit protection. Since the welfare in the economy without credit protection is strictly higher as we have seen, the welfare in the economy with credit insurance must be lower than in the second best. $\square$

2.7 Conclusion and Outlook

We have analyzed the welfare effect of credit insurance under different scenarios. We find that under risk-neutrality credit insurance is always welfare improving and the lender chooses the socially efficient level of credit insurance. This is no longer true under risk-aversion. Renegotiation implicitly adds a contingency to the non-contingent bond contract and thus enables the government to smooth consumption across states. By enforcing a higher repayment in the lower state credit insurance hinders this mechanism. In equilibrium, the lender might thus choose a level of credit protection that is higher compared to the socially efficient amount. Whether credit insurance is welfare increasing or decreasing in case of risk-aversion depends on how much the government values better terms of borrowing in period 0 compared to a lower consumption in the low state of period 1, which is implied by the endowment in the low state compared to the initial level of wealth and the level of endowment in the high state.

It would also be interesting to see what impact credit insurance has on equilibrium quantities such as bond prices and levels of debt and whether this impact also depends on the initial level of wealth. Furthermore it would be nice to have more concrete conditions under which credit insurance is welfare decreasing or increasing.

Appendix

A.1

Lemma. *For $i \leq (\lambda - \delta) f(\theta^s, k)$ the renegotiated amount of debt repayment $d^{ren}(i, \theta^s, k)$ is non-decreasing in the amount of credit insurance i*

Proof. An interior solution to the bargaining problem (2.3.3) must satisfy the following first order condition

$$-u'((1-\delta) f(\theta^s, k) - d^*)[d^* - i] + \quad u((1-\delta) f(\theta^s, k) - d^*) - u((1-\lambda) f(\theta^s, k)) = \quad 0$$

Applying the implicit function theorem to this equation gives

$$\begin{aligned}\frac{\partial d^*}{\partial i} &= -\frac{\overbrace{u'\left(\left(1-\delta\right)f\left(\theta^s,k\right)-d^*\right)}^{\geq 0}}{\underbrace{u''\left(\left(1-\delta\right)f\left(\theta^s,k\right)-d^*\right)\left[d^*-i\right]}_{\leq 0}\underbrace{-u'\left(\left(1-\delta\right)f\left(\theta^s,k\right)-d^*\right)}_{\leq 0}\underbrace{-u'\left(\left(1-\delta\right)f\left(\theta^s,k\right)-d^*\right)}_{\leq 0}} \\ &\geq 0\end{aligned}$$

so that any interior solution is increasing in i (by concavity and increasingness of the utility). A boundary solution at the upper bound is given by $d^* = (\lambda - \delta) f(\theta^s, k)$ and therefore constant with respect to i, while a boundary solution at the lower bound $d^* = i$ is trivially increasing in i. As a conclusion any solution to the bargaining problem is non-decreasing in i. $\square$

2.8 References

Aguiar, Mark and Gita Gopinath (2007) "Emerging Market Business Cycles: The Cycle Is the Trend," *Journal of Political Economy*, Vol. 115, pp. 69–102.

Arellano, Cristina (2008) "Default Risk and Income Fluctuations in Emerging Economies," *American Economic Review*, Vol. 98, No. 3, pp. 690–712, June.

Bolton, Patrick and Martin Oehmke (2011) "Credit Default Swaps and the Empty Creditor Problem," *Review of Financial Studies*, Vol. 24, No. 8, pp. 2617–2655.

Eaton, Jonathan and Mark Gersovitz (1981) "Debt with Potential Repudiation: Theoretical and Empirical Analysis," *Review of Economic Studies*, Vol. 48, No. 2, pp. 289–309, April.

Grossman, Herschel I and John B Van Huyck (1988) "Sovereign Debt as a Contingent Claim: Excusable Default, Repudiation, and Reputation," *American Economic Review*, Vol. 78, No. 5, pp. 1088–97, December.

Kehoe, Timothy J. and David K. Levine (2006) "Bankruptcy and collateral in debt constrained markets,"Technical report.

Sambalaibat, Batchimeg (2011) "Credit Default Swaps and Sovereign Debt with Moral Hazard and Debt Renegotiation," mimeo.