Cooperatives With Privately Optimal Price Indexed Debt Increase Membership When Demand Increases

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Abstract: In a simple model of a cooperative, an increase in the price offered for the cooperative’s product causes a reduction in membership and production. This perverse response called the Ward effect is often proposed as an explanation of why cooperatives are rare. When the model is augmented by allowing the cooperative to issue debt instruments which pay an arbitrary continuous function of the price of their product the Ward effect is reversed, that is, the cooperative has an upward sloping supply curve. In the model presented below, this results from the privately optimal decisions of a core group which is not concerned about efficiency or employment.

This note is the continuation of joint research between Stephen Smith and myself. I would like to thank him for stimulating conversations and for teaching me about the economics of labour managed firms. The usual caveat applies. Our collaboration was supported by funds of the Research Council of the European University Institute.
I. Introduction

If cooperatives are financed with conventional debt paying a fixed interest rate and are controlled by workers whose continued membership is not in doubt, the cooperative will reduce membership and output when the price they are offered for their product increases (Ward 1958, Domar 1966, Ireland and Law 1982, Bonin and Fukuda 1986). This perverse optimal response, known as the Ward effect, is widely considered to be one of the principal reasons that cooperatives are rare. It is driven by the fact that price suprises create quasi rents and that current members of the cooperative benefit if they share speculative losses with new members and do not share speculative profits. Speculative losses and profits are privately undesirable, since risk averse members would benefit if the risk were born by investors with diversified portfolios. The interests of members and investors are better served by bonds indexed to the price of output, which enable investors to sell insurance to the cooperative. This paper notes that the use of such assets can reverse the Ward effect and cause an upward sloping supply response.

Below it is shown in some generality that cooperatives which have access to debt instruments which pay an arbitrary continuous function of the price of their product, will choose a financial strategy which makes upward sloping supply privately optimal. An earlier paper shows that this reversal of the Ward effect occurs if the class of instruments is arbitrarily restricted and if the members are sufficiently risk averse (Waldmann and Smith 1992). Other proposed solutions to the risk diversification problem have not been shown to reverse the Ward effect when voluntarily implemented by cooperatives controlled by members who care only about the members share (see e.g. McCain 1977, Thomson 1982, Guesnerie and Laffont 1984, Suvakovic-Olgin 1992). The result presented below is at least somewhat novel, since complete freedom and selfishness of members of the cooperative is assumed and a robust reversal of the Ward effect is obtained.

This paper has four sections the first of which is this introduction. The second is a fairly simple example which illustrates the claim above. The third is a more general proof. The fourth draws conclusions.
II. A Model

There is a cooperative with some small number of members. It is required to pay present and future members equal shares of value added minus interest payments. Current members decide on financial and productive decisions in order to maximise their own expected utility. The current members are certain that they will continue to be members of the cooperative. Admitting new members can create a positive externality, benefiting new members if the share they receive is greater than the going wage. This presumably makes membership decisions inefficient. In particular if the cooperative is self financed or financed with ordinary debt, an increase in the price of its product will cause it to reduce output. This is the well known Ward effect.

Assume the cooperative can issue debt instruments which pay any continuous function of the price of their product. Assume that this price is set on competitive markets and that price risk is diversifiable. The privately optimal financing and output choices give an upward sloping supply curve. The existence of such assets would reverse the Ward effect if the members of cooperatives were rational utility maximisers. This theoretical result holds for fairly general production and utility functions and price distributions. The proof is by contradiction. Given any cooperative which has chosen a financial strategy that implies downward sloping supply a strictly preferable strategy with upward sloping supply can be described. Therefore the optimal strategy must involve upward sloping supply.

A simple model illustrating the above claims follows. There are two time periods. In period zero the cooperative issues liabilities which require them to pay investors a function of the price of the product in period 1 equal to \( h(p) \). The cooperative invests the proceeds in productive capital \( K \). In period 1 the price of output \( p \) is revealed and the cooperative chooses membership \( L \) and produces and sells output \( F(L,K) \) pays \( h(p) \) and divides the rest among its members giving each share.
\[ s = \frac{F(L, K) - h(p)}{L} \]  

(1)

The cooperative chooses \( h \) and, after \( p \) is revealed, \( L \) in order to maximise the expected utility of its members \( U(s) \). \( F \) is twice differentiable increasing in \( L \) and \( K \) and concave. \( F(0, K) = 0 \). \( U \) is monotonically increasing in \( s \) and as noted is a function only of \( s \). To shorten long equations define \( L(p) \) to be the optimal membership given \( K \), price \( p \) and optimal payments to investors \( h(p) \). Define \( s(p) \) to be the resulting share.

If \( h(p) \) is less than or equal to zero, \( s \) is maximised at \( L = 0 \) making nonsense of the assumption that decisions are made by members of the cooperative whose continued membership is not in doubt. Therefore it is assumed that \( h(p) \) is positive for every \( p \). Similarly it is assumed as is standard in the literature that the cooperative can always attract new members no matter how low \( s \) is. If the members of the cooperative are not very risk averse, it is possible that, for optimal plans, \( s \) will sometimes be zero. Clearly a minimum membership and minimum share should be defined. Fortunately, the proofs below show that any plan with downward sloping supply is inferior to another plan with the same distribution of membership and shares. This means that no such plan can be optimal with (or without) constraints on membership or shares.

To close the model it is necessary to decide how much investors are willing to pay in period zero for income \( h(p) \) in period one. This must be equal to \( K \). The analysis below is valid if investors are risk neutral and care only about the expected value of \( h(p) \). Similarly it is valid if investors are risk averse but have consumption in period one independent of \( p \). If so the risk in \( h(p) \) is diversifiable and investors care only about the expected value of \( h(p) \). Investors are assumed to care only about the expected value of \( h(p) \).

I will first present a fairly simple model in which \( p \) takes a finite number \( n \) of values each with equal probability. This simplifies the proof considerably, but is absolutely not necessary for the result.
The remainder of this section is a proof by contradiction that the cooperatives optimal financial and membership decisions give an upward sloping supply curve. Assume not, that for the optimal financial scheme \( h \) that there are prices \( p_1 \) and \( p_2 \) with \( p_1 < p_2 \) such that membership at price \( p_1 \) is greater than or equal to membership at price \( p_2 \). In symbols

\[
L_1 = L(h(p_1), K, p_1) \geq L_2 = L(h(p_2), K, p_2).
\]

(2)

define \( s_1 \) as the share of each member when price is \( p_1 \) and \( s_2 \) as the share when price is \( p_2 \). Now consider a change in the cooperatives behavior so they have \( L_1 \) members each of whom gets share \( s_1 \) when the price is \( p_2 \) and have \( L_2 \) members each of whom gets share \( s_2 \) when the price is \( p_1 \). This clearly increases investors expected income by \((p_2-p_1)[F(L_1,K)-F(L_2,K)] > 0\). It clearly has no effect on members expected utility which depends only on the univariate distribution of \( s \). It is not yet clear how this improvement can be achieved if employment decisions are made unilaterally by the cooperative.

Now consider a new financing scheme in which the cooperative pays investors \( h \sim(p) \) where

\[
h \sim(p_1) = p_1 F(L_2,K) - L_2 s_2 - (p_2-p_1)[F(L_1,K)-F(L_2,K)]
\]

(3)

\[
h \sim(p_2) = p_2 F(L_1,K) - L_1 s_1
\]

\[
h \sim(p) = h(p) \text{ otherwise}
\]

\[E(h(P)) = E(h \sim(p))\) so investors will supply \( K \) in either case. Assume the cooperative has \( L_1 \) members at price \( p_2 \) and \( L_2 \) members at price \( p_1 \) and the same number of members as before for other prices. If so the distribution of \( s \) is the same as with the original scheme except for a \( 1/n \)
probability that it will be increased by $(p_2 - p_1)[F(L_1, K) - F(L_2, K)]/L_2$. If $L_1$ is strictly greater than $L_2$, $h^-$ is clearly a better financing scheme than $h$. For optimally chosen membership expected utility is at least as great as with the arbitrary membership assumed above and is therefore greater than that under the original scheme. For $L_1$ equals $L_2$, $h^-$ is an improvement unless the arbitrary membership happens to be optimal. It is not optimal as is shown below, so $h^-$ gives higher expected utility than $h$.

Consider $s$ as a function of $L$ given $K$ and $h$ and $p$

$$s = [pF(L, K) - h(p)]/L$$

(4)

this gives the derivative

$$ds/dL = [pLF(L, K) - pF(L, K) + h(p)]/L^2 = [pLF(L, K) - s]/L^2$$

(5)

since $L_1$ was optimally chosen given $p_1$ and $h(p_1)$ the first order condition equation 6 holds.

$$p_1L_1F(L_1, K) - s_1 = 0$$

(6)

This means that for $h^-$, at price $p_2$ and employment $L_1$, $s = s_1$ so the derivative of $s$ with respect to $L$ is given by equation 7.

$$ds/dL = [p_2L_1F(L_1, K) - s_1]/L_1^2 = (p_2 - p_1)[L_1F(L_1, K)/L_1^2 > 0.$$ 

(7)

5
This means that $L_1$ is not the optimal employment level at price $p_2$ with liabilities equal $h\sim(p_2)$. Therefore the expected utility of members of the cooperative is greater if they use $h\sim(p)$ instead of $h(p)$, so $h(p)$ is not optimal.

This proves by contradiction that employment increases in the price of output if the cooperative uses optimal financial instruments which give liabilities as a function of $p$ and if investors are risk neutral or can diversify the resulting risk away.\(^1\)

III. A More General Proof

The assumption that $p$ takes only a finite number of values each with equal probability simplifies the proof, but is not necessary at all. A proof for a more general distribution of $p$ can be constructed using the same logic. This section presents a proof based on weaker smoothness and continuity assumptions.

First assume that $p$ is distributed according to the cumulative distribution function $G$ which is continuous and strictly monotonic at every possible value of $p$. That is assume that $p$ has an atomless distribution with connected support. Assume that $F$ is twice differentiable and quasi-concave with $F_L$ and $F_K$ positive and $F_{LL}$ and $F_{KK}$ strictly negative. Assume that the cooperative attempts to maximise the expected value of $U(s)$ for $U$ strictly concave and that investors are risk neutral. Finally assume that cooperatives can issue any asset which pays $h(p)$ continuous in $p$. The final assumption is particularly troublesome as it confines the allowed strategies of the cooperative not tastes or technology.

All these assumptions make it possible to reproduce the simple proof of section two for neighbourhoods of single points. Again I will

\(^1\)Note that with the strong assumptions about the distribution of $p$, the only assumption about $F$ used in the proof are that it is monotonically increasing in $L$ and once differentiable with respect to $L$. The only assumption about $U(s)$ is that it is increasing in $s$. 

6
derive a contradiction by assuming that with optimal liabilities $H(p)$ there is a $p_1$ and $p_2$ with $p_1 < p_2$ and $L_1 \geq L_2$. Again the basic idea will be to switch high and low membership levels and shares. First assume $L_1 > L_2$ — membership is greater at the lower price. The total derivative of the first order condition for membership implies that optimal membership is a continuous function of $p$ and $h$. The assumption that $h$ is continuous implies that membership is a continuous function of $p$ given $h(p)$. This means that there is an $e$ so small that employment is higher for all $p$ in $[p_1, p_1 + e]$ than in any point in $[p_2 - e, p_2]$. Let $d$ be the lesser of $G(p_1 + e) - G(p_1)$ and $G(p_2) - G(p_2 - e)$. By the assumption that the support of $G$ is connected, $d$ must be positive. Define $e_1$ as $G^{-1}(G(p_1) + d) - p_1$ and $e_2$ as $p_2 - G^{-1}(G(p_2) - d)$. Since $G$ is monotonic and continuous $G^{-1}$ exists and is monotonic and continuous. For $p$ between $p_1$ and $p_1 + e_1$ define

$$y(p) = G^{-1}(G(p_2) + G(p_1) - G(p))$$

(8)

$y$ maps points near $p_1$ to points near $p_2$ and maps a range of prices which occurs with probability $d$ to a region which appears with probability $d$. Since $G$ and $G^{-1}$ are continuous, $y$ is continuous. It makes it possible to define and improved financing scheme $h^-$. 

if $p_1 < p < p_1 + e_1 \quad h^-(p) = pF(L(y(p), K) - L(y(p))s(y(p))) - q$  

(9)

if $p_2 - e_2 < p < p_2 \quad h^-(p) = pF(L(y^{-1}(p), K) - L(y^{-1}(p))s(y^{-1}(p))) - q$

otherwise \quad $h^-(p) = h(p) - q$
for \( q = \int_{p_1}^{p_1+\epsilon_1} (y(p) - p)[F(L(p), K) - F(L(y(p)), K)]g(p)dp \)

and \( g(s) = G'(s) \).

As in the previous example the modified scheme gives the same expected return to investors as the original scheme and gives higher expected utility to members of the cooperative. The resulting function \( h^- \) is continuous in \( p \) since \( L(p), s(p), F(L, K), y \) and \( y^{-1} \) are continuous in \( p \).

The change is very similar to the change described in section II. The only difference from the earlier example is that the benefits are distributed to the cooperative equally for all prices. This is done to make \( h^- \) continuous. Again the improvement proves by contradiction that the optimal financing scheme makes employment \( L \) and supply non-decreasing in price.

Again it is easy to prove that supply is strictly increasing in price. Assume for optimal \( h \) there are \( p_1 \) and \( p_2 \) with \( p_1 < p_2 \) and \( L(p_1) = L(p_2) \). Since \( L(p) \) is non-decreasing membership must be constant for all \( p \) between \( p_1 \) and \( p_2 \). Define \( h^- \) as above for \( \delta = (G(p_2) - G(p_1))/2 \). In words switch the workers shares for low and high \( p \) between \( p_1 \) and \( p_2 \). As in section 2 this has no effect on investors return or on the expected utility of members if employment is chosen as it would be for liabilities \( h(p) \). Again, these employment levels are not optimal so expected utility is higher for \( h^- \) and optimal employment. This proves by contradiction that with optimal liabilities that are a continuous function of price, employment and supply are strictly increasing in the price of output and the Ward effect is reversed.
IV. Conclusions and Directions for Further Research

In the model presented above a cooperative is allowed to issue debt which pays an arbitrary continuous function of the price of their product. A core of members (founders) of the cooperative is assumed to be guaranteed continued membership and to make all financing decisions in order to maximise some concave function of the share per member. Then, when the price is revealed they are assumed to choose membership in order to maximise the share given the restriction that the share must be equal for all members. This separation of control and reward might be expected to cause inefficient behavior and indeed, in simple models where only debt paying a fixed return is allowed, the cooperative reduces membership and production when the price of its product increases. This perverse response is reversed when debt which pays an arbitrary continuous function of the price is allowed. The result that a new financial instrument eliminates an inefficiency which seemed to follow directly from an unrelated institutional restriction is very striking. The conclusion that with a little financial innovation cooperatives might be able to survive in market economies is very appealing.

Some final caveats are required however. First and most importantly I assumed that the price of the cooperatives product is observable and exogenous. If the cooperative is part of an imperfectly competitive industry, it could influence the price and therefore its price indexed debt obligations. Even if the price can't be manipulated, price indices by industry are generally difficult to calculate and are calculated with delay. This limits the potential application of the proposed instrument to cooperatives in competitive industries which produce a homogenous product.

I assumed that the risk in the price paid for the cooperatives product is fully diversifiable. If investors require higher expected returns for price indexed bonds than for regular bonds, the optimal amount of hedging will be reduced. I assumed that the debt instruments must pay a continuous function of the price of output. If the cooperative can
influence the price at all this is clearly necessary to prevent manipulation. Finally I considered a two period model. In practice a sluggish response of membership to demand may be caused by the firing costs implied by membership rights. Would current members of cooperatives use price indexed bonds to hedge such costs and eliminate their effect on membership decisions? The evaluation of the effect of new financial instruments on dynamic labour demand of imperfectly competitive cooperatives is a direction for future research.

The simple model of this paper was chosen to illustrate the most extreme form of perverse behavior of cooperatives proposed in the literature. The fact that privately optimal financial innovation can eliminate such behavior is very striking.

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