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Public Policy in a Two Sector Model of Endogenous Growth

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This paper studies a two sector endogenous growth model in which the government intervenes in the economy by financing research and education. We characterize both the balanced growth path and the transitional dynamics of the model showing the steady-state equilibrium to be a saddle point. We also show that while income taxation is distortive, a Pareto optimal outcome can be reached by means of a consumption tax in the decentralized setting.

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1. Introduction

This paper studies a two reproducible factors model of government investment spending and taxation. The model offers insights into the role of the public sector in generating the conditions for growth, in particular regarding the accumulation of knowledge through scientific research and of human capital through education.

In the model one of the factors (physical capital) is a stockpile of the consumption good and is privately produced while the other (which can be taken to represent disembodied or embodied knowledge) is publicly produced, its production being financed through tax revenue.

If we assume this second factor to be disembodied knowledge, we have, in fact, a public research model which extends the presentation by Helpman-Grossman (1991) of the classic contribution by Shell (1967). The essential idea in the model is that the creation of knowledge is not always the byproduct of other activities, as in "learning by doing" models of growth (see Arrow 1962, Sheshinski 1967, Romer 1986) but, in contrast, can be fostered by increasing the allocation of economic resources explicitly devoted to inventive activity. Since knowledge is non rivalrous so that its competitive price is zero, this diversion invites and often requires an active public role (particularly if we consider the problem of exclusion). To quote Shell: "Historically intervention on behalf of inventive activity has taken two basic forms: first, the establishment of a legal device, the patent, designed to bestow property rights on certain of the outputs of the inventive process. The second form of intervention is that of direct nonmarket

1An input in a production process can be defined to be non rivalrous if its use in one activity does not reduce the amount available for other activities. A good is non excludable if people cannot be prevented from using it. Both basic science and applied science are non rivalrous: Pythagoras' theorem as well as a metallurgical formula or a chip design, once discovered, can be used at zero marginal cost by anyone. In an economy in which technical ideas are commodities, the basic premises of classical welfare economics are violated and the optimality of the competitive mechanisms is not assured. This analysis of the public good characteristics of knowledge in the Shell paper has been developed recently by P. Romer, who investigates the link between non rivalry of inputs and nonconvexity of production sets (see Romer 1989a, 1990).
support of research and development. Universities have long played such a role in Western economies. In the United States, the Department of Agriculture has undertaken research activities since its inception. The Department of Commerce has initiated industrial research programs modelled after the agricultural research stations. The Department of Defence often uses the device of contracting research to private enterprises on a cost-plus-fixed-fee basis.  

The main differences between the treatment offered here and that of Helpman and Grossman are that we remove the restrictive assumption of identical production functions in the two sectors and assume a more general form for consumers preferences. Moreover we fully characterize the transitional dynamics of the model, while they limit themselves to offer an heuristic argument for the convergence of the system to the steady state which seems to be valid for the logarithmic specification of preferences they assume. Finally we also analyse the model from a normative point of view, showing that the command economy solution can be decentralized if a consumption tax rather than an income tax is imposed.

If we interpret the second factor as human capital we are in effect working in the Uzawa (1965)–Lucas (1988) tradition. The main development in this direction is that we model education as being a public not a private concern, which is generally consistent with the facts.  

2 From Shell (1967) p. 70. He also mentions as grounds for public intervention the extreme riskiness that characterizes the inventive process at the microeconomic level.

3 As regards the first kind of intervention mentioned by Shell, we recall that models of imperfect competition and R & D where firms enjoy monopoly power after having developed a prototype of a good are an important stream of the endogenous growth literature (see for example Romer (1987, 1989b), Grossman–Helpman (1991) chaps. 3 and 4). However a larger proportion of the generic knowledge relevant to a technology is written down, published in journals, discussed at national and international meetings (Nelson-Wright 1992, p.1958), i.e. is non excluded.

4 Following the literature but aware of committing an "abuse de langage", we define "steady state" as a situation in which variables grow at a constant rate.

5 From a prescriptive point of view, even if human capital as embodied knowledge is rival and excludable, the important externalities it generates can be invoked in favor of its public accumulation. Also equity
When seen as a contribution to the public finance literature, our model enriches the taxonomy put forward by Barro (1989) and Barro and Sala i Martin (1990), whose analysis is limited to one capital good models, while the difference from Rebelo (1990) and King and Rebelo (1990) is that it is assumed in these papers that government revenue is used to finance the provision of goods that do not affect the production possibilities of the private sector.

The model delivers persistent growth, whose engine is shown to be public activity, without which the return to physical capital would progressively decline, so discouraging private saving. Since we assume constant returns to the reproducible factors, their sustained accumulation could be anticipated on the basis of what we already know about growth models.

A distinctive feature of the model is that we are able to show that the steady-state equilibrium is a saddle point and to analyse the characteristics of the transition to the steady state. This is particularly interesting because the transitional dynamics of two sector models of endogenous growth have been fully described only in particular cases. 6

It is of course important to study the dynamics outside the steady-state, since if it is not possible to establish stability of the considerations for public education seem to be particularly important in the light of the results of endogenous growth models. In fact one implication of such models is that, coeteris paribus, the initial distribution in human and non human wealth tends to perpetuate itself, being transmitted down from parents to children, leading to persistent income inequality, even in models in which the utility of children enter the utility function of parents.

As is well known, other grounds for public education arise in a stochastic context: the incompleteness of capital markets makes it impossible to share the risk of human capital accumulation and the imperfection of credit markets makes it impossible to borrow to pay for it.

6 A numerical study of the dynamics outside the steady state is offered for a two sector model of endogenous growth with a Cobb-Douglas technology, whose steady state properties are studied, for instance, in Rebelo (1990) is due to Mulligan and Sala i Martin (1992). The transition to the steady state for this model is worked out analytically in chap 4 of Barro and Sala i Martin (1993), for the case in which the production function is the same in the two sectors and for the case in which the production function of one good is linear in that good, that is the hypothesis in Lucas (1988), if externalities to human capital are ruled out.
balanced growth path, and to rule out long adjustment times in the face of shocks such as a change in the parameters or an exogenous reduction in stocks, the characteristics of the steady state configuration become largely irrelevant for any applied or policy analysis and the emphasis on asymptotic rates of growth seems unwarranted.

The investigation of the transitional dynamics starts with the definition of new variables which are constant in steady state and which transforms the model into a stationary one where the dynamics are more amenable to analysis. We are also then able to analyse the behaviour of the system when linearized around the steady state showing that it has two real roots of opposite sign, yielding the saddle point structure.

Moving to normative aspects we compare the decentralized outcomes in our model with the Pareto optimal solution, which would be chosen by a "social planner" directly operating the technology of the economy. We show that with an income tax there is no way to attain the command optimum, which can however be reached by imposing a consumption tax.

2. A Two Factor Model of Growth with an Income Tax

To keep the analysis simple and highlight the questions of interest, several simplifying assumptions are used. The first is that the population and the supply of labour are both constant. This rules out an analysis of fertility, labour force participation, or variation in hours worked per worker. The other simplifying assumptions relate to factor intensities. We assume that physical capital can be accumulated as

This "elasticity optimism" regarding labour supply, as it is labeled in the public finance literature, is endemic among endogenous growth theorists. In fact leisure cannot grow in steady state so the income and substitution effect associated with sustained growth in labor productivity must not alter labour supply. This can be shown to happen for two classes of time separable preferences over leisure and consumption. In the first class instantaneous felicity takes the form:

\[ u(C, l) = C^{1-\sigma}v(l)^{1-\sigma}, \sigma > 0, \]  

or for \( \sigma = 1 \), \[ u(C, l) = \log(C) + v(l) \]

where \( v(.) \) must be increasing and concave if \( \sigma \leq 1 \) and increasing and convex if \( \sigma > 1 \). In the second class we have: \( u(C, lH) \), where \( l \) is leisure and \( H \) is a reproducible capital good (e.g. human capital) and the function is concave and homogeneous of degree \( b \). For the first class see King et al. (1988), for the second Rebelo (1990).
forgone output which means that capital goods are produced with the same
technology as the final output.

The production function of the tangible commodity is:

\[ Y = A_1 K^{1 - \gamma} L^{\gamma} E^\gamma \]  \hspace{1cm} (2.1)

where \( A_1 \) is a scale factor, \( K_k \) and \( L_k \) are capital and labour employed in
the sector and \( E \) can be read as representing the stock of scientific
ideas or the level of human capital. In both cases its increase leads to
Harrod neutral, i.e. labour augmenting technological progress.\(^8\)

Knowledge is assumed to accumulate according to:

\[ \dot{E} = A_2 K_E^{1 - \beta} L_E^\beta E^\beta \]  \hspace{1cm} (2.2)

where \( K_E \) and \( L_E \) are labour and capital employed in producing \( E \) (Research
and Development or Education), while \( A_2 \) is another scale factor.

The level of productivity increases with the cumulative output of
the research sector because the results of research are freely available.
When employing the interpretation of \( E \) as education we assume that all
workers learn at the same rate.\(^9\) In the representative agent framework,
in which the endogenous growth literature has been developed, this
assumption is not restrictive but it may be in more general and realistic
models. The government taxes all incomes at the exogenous rate \( t \) and uses
the revenues to finance expenses. We therefore have:

\(^8\) We recall that Harrod-neutral technical progress implies that if the
ratio between capital and labour measured in efficiency units is
constant, then the factor shares are constant. We can add that it
implies that when the ratio between output and capital is constant then
the marginal productivity of capital is constant. Unless technological
change is Harrod neutral, one sector models generally cannot approach an
equilibrium balanced growth path that is economically meaningful, i.e.
that is not in conflict with stylized facts (see, e.g. Burmeister and
Dobell 1970).

\(^9\) For ease of exposition, from now on, we will call \( E \) education and will
mention its alternative interpretation only when needed.
(wL + rK)t = wL_E + rK_E \tag{2.3}

where $w$ is the wage, $r$ the interest rate, $L = L_E + L_k$ and $K = K_E + K_k$. Notice that (2.3) constrains the government to run a balanced budget. That is the government can neither finance deficits by issuing debt nor run surpluses by accumulating assets. Profit maximization by firms imposes equality between marginal products and rental rates of factors so we have:

$$r = (1-\gamma)A_k K_k^{\gamma} (L_E)^{1-\gamma} \tag{2.4}$$

$$w = \gamma A_k^{1-\gamma} L_k^{1-\gamma} E_k^{\gamma} \tag{2.5}$$

We have assumed no depreciation occurs so the user cost of capital is equal to the interest rate. Notice that the representative producer assumes that changes in her quantity of output do not lead to any change in the amount of public services. Assuming public agencies apply the principle of cost minimization we also have:

$$r/w = (1-\beta) L_E / \beta K_E = (1-\gamma) L_k / \gamma K_k \tag{2.6}$$

Again the subscripts refer to the sector in which the factors are employed. We write, to simplify notation, $L_k = \chi L$ and $K_k = \phi K$. We also normalize labour force to one.

Using (2.3) and (2.6) we get to:

$$\phi = (t-1)/((\beta-\gamma)(1-\gamma)^{-1} t -1) \tag{2.7}$$

$$\chi = \gamma (t-1)/((\gamma-\beta) t - \gamma) \tag{2.8}$$

An immediate result is therefore that the proportion of each factor used in the public sector is constant through time, i.e. it does not
depend on the capital/labour ratio in the economy, and that it is increasing in the tax rate. We also have that the proportion of labour (capital) used in each sector relative to the total supply is increasing in the labour (capital) share in the sector.\footnote{In fact we have: \( \delta \phi / \delta t < 0, \delta \chi / \delta t < 0, \delta \phi / \delta \beta > 0, \delta \chi / \delta \beta < 0, \delta \phi / \delta \gamma < 0, \delta \chi / \delta \gamma > 0. \)}

We can rewrite the law of motion of accumulated knowledge as:

$$E/E = B_1 (K/E)^{1-\beta}$$

(2.9)

where

$$B_1 = \frac{\alpha (1+\gamma)/(\alpha+\gamma)}{\alpha+\gamma/\gamma}$$

The intertemporal budget constraint families face at time zero is:

$$W_0 = \int_0^s (C(s) - w(s)(1-t) - r(s)(1-t)W) \exp \int_r^v dv ds$$

(2.10)

where \( W \) is family wealth. Assuming CES preferences:

$$U = I^{\sigma} C^{1-\sigma} \frac{dU}{dt} < 0, \frac{dU}{d\chi} > 0, \frac{dU}{d\gamma} < 0, \frac{dU}{d\beta} > 0, \frac{dU}{d\theta} < 0, \frac{dU}{d\phi} > 0.$$

(2.11)

we obtain the following Euler condition:

$$C/C = -(\rho - r(1-t)) \sigma^{-1}$$

(2.12)

This simply says that the rate of consumption growth will be positive as long as the marginal rate of substitution of present for future consumption is lower than the marginal rate of transformation.

Using (2.4), (2.5) and the expressions found for \( \chi \) and \( \phi \), we write:

\footnote{If \( \sigma = 1 \) we have in fact that instantaneous felicity is logarithmic.}
\[ r = B_2 K^{-\gamma} E^\gamma \] 

\[ w = \gamma \left( \frac{B_2}{1-\gamma} \right)^{1-1/\gamma} A_1^{1/\gamma} K^{1-\gamma} E^\gamma \] 

where \( B = A_1 (1-\gamma)(\gamma(\beta-\gamma)t-\gamma(1-\gamma))^{\gamma(1-\gamma)(\gamma-\beta)t - \gamma(1-\gamma)^{-\gamma}} \)

Substituting these expression in the dynamic budget constraint of families and noting that in equilibrium aggregate wealth must be equal to aggregate physical capital we have the following equation of motion for capital:

\[ \frac{K}{K} = (1-t) B_2 (E/K)^{\gamma} - C/K \] 

where \( B_2 = B_2 /((\gamma-\beta)t +1-\gamma)) \)

From (2.12) and (2.13) we get:

\[ \frac{C}{C} = -\rho/\sigma + (1-t) B_2 (E/K)^{\gamma} \sigma^{-1} \] 

2.1 The Steady State Solution

One major fact of economic development is that national growth rates do not display long run trends so steady state models can be interpreted as a reasonable first approximation to reality.

An important step in the analysis is therefore to consider if a steady state solution exists and what are its characteristics. Differentiating (2.9) and (2.15) it is clear that in steady-state, the rate of increase of knowledge will be equal to the rate of increase of capital and of consumption. We indicate this common rate of growth by \( g \). To calculate it we first find from (2.16) an expression for the steady
state ratio of knowledge to capital in terms of the steady state rate of growth. We then substitute the ratio in (2.9), getting the following implicit expression for the steady state rate of growth:

\[ g = B_1 ((g \sigma + \rho)/(1-t)B_2) \]  

(2.1.1)

To interpret (2.1.1) we can use Fig. 1. The curve \( g \) represents the left hand side of (2.1.1), \( G \) the right hand side. The curves will always intersect for one positive value of \( g \), i.e. the rate of growth will be positive and unique. We have in fact:

\[ G'(g) = B_1(B_2(1-t)) \]

\[ \frac{(1-\beta)}{\gamma} - (\beta-1)\sigma^{-1}(g \sigma + \rho)^{(\beta-\gamma-1)/\gamma} < 0 \]

\[ G''(g) = B_1(B_2(1-t)) \]

\[ \frac{(1-\beta)}{\gamma} - (\beta-1)\sigma^{-2}(\beta-\gamma-1)(g \sigma + \rho)^{(\beta-2\gamma-1)/\gamma} > 0 \]

\[ \lim_{g \to 0} G(g) = +\infty \]

\[ \lim_{g \to +\infty} G(g) = 0 \]

The explanation for sustained growth in this economy is that research allows a technological progress which keeps the marginal product of physical capital from falling to the level of the subjective discount rate \( \rho \). As can be seen from (2.1.1) the long run rate of growth is affected by changes in both economic structure through \( \sigma, \rho, A_1, A_2, \beta, \gamma \) and public policy, through \( t \). Going back to the original parameters we have:

12 Of course, we could calculate the ratio from 2.9 and substitute it in 2.16, thus obtaining a different expression, which, however, would deliver the same comparative statics results.
For example an increase in the subjective discount rate $\rho$ or in the elasticity of the marginal utility of consumption $\sigma$ implies that the $G$ curve shifts down, so that the equilibrium growth rate decreases. It falls because consumers, being less patient or less willing to substitute consumption intertemporally save less. Lower capital per capita means lower income and a reduced tax base to finance research.

Now consider the long run implication of a variation in the tax rate. We have $G = 0$ for $t=0$ and $t=1$. In fact the function is first increasing then decreasing in $t$, as can be seen by calculating:

$$\frac{\partial}{\partial t} \left( \frac{(1-t)^{1-\beta}}{(1-\gamma)((\beta-\gamma)t+\gamma)} \right) = \frac{(1-t)^{(1-\beta-\gamma)}(1-t)^{(1-\gamma)}((\gamma-\beta)t^2-2\gamma t+\gamma)(\gamma+(\beta-\gamma)t)^{-2}}{((\gamma-\beta)t^2-2\gamma t+\gamma)^2}$$

The sign of this function is the same, for $0 < t < 1$, as that of:

$$(\gamma-\beta)t^2-2\gamma t+\gamma$$

This parabola has one positive root lower than unity and another root which can be discarded because if $\gamma < \beta$ it is bigger than unity and if $\gamma > \beta$ it is negative. The economically significant root, which is the value of the tax rate that maximizes the rate of growth is:

$$t_{\text{max}} = \frac{(\gamma-\sqrt{\gamma\beta})}{(\gamma-\beta)}$$

For values of $t$ smaller than this root the function takes positive values, for bigger values it takes negative values. If we are in the interval in which it is positive(negative), an increase in $t$ will shift the curve $G$ in Fig.1 to the right (left), so increasing (decreasing) the
rate of growth. The economic intuition is that when the tax rate is low, an increase in it will increase tax receipts so allowing the government to finance more research. This will prevent capital productivity from falling and so indirectly encourage saving and capital accumulation. However, when \( t \) is very high a further increase will reduce tax receipts, by discouraging saving: the lower capital per capita implies a lower income i.e. a smaller tax base. We have a form of the Laffer curve effect at work here and in fact a particular powerful one because it influences the growth rate and has therefore compound effects. It must be underlined however that the reason for the inverse relationship between tax rate and tax receipts is not the work discouragement effect on which the literature about the Laffer curve focuses.\(^{13}\)

Notice that if \( \gamma = \beta \), the "Helpman - Grossman hypothesis", which is in fact acceptable if we interpret \( E \) as being disembodied knowledge, \( t = \frac{1}{2} \), while if we assume \( \beta = 1 \), the "Lucas hypothesis", which is more reasonable if we interpret \( E \) as being education, then \( t = 0.456 \), assuming a labour share of 0.7 and a capital share of 0.3 in the physical good sector. We also recall that total government budgets as share of GDP are close to these levels in most OECD countries.

2.2 Transitional Dynamics

We now turn to the analysis of transitional dynamics in this model and to the issue of stability. This is important since in general there will be an initial imbalance between the two reproducible factors i.e. the initial ratio between the stock of physical capital and the stock of knowledge will presumably be different from the one required for a steady state. It is natural then to ask if and how the economy will ever get to the steady state proportions. A first question to ask is how such imbalance will affect the saving rate. The stock of the relatively scarce factor could be increased either by substituting away from investment in the other factor or substituting away from consumption. Assuming that the

\(^{13}\) First reference on this literature would be the cocktail napkin on which Arthur Laffer first sketched the relationship between tax revenue and marginal tax rate which carries his name. For a discussion see Fullerton (1982).
equilibrium is stable, investment could take place in both sectors and relatively more in the one producing the relatively scarce capital good or it could be positive only in the scarce factor so that the model behaves as a one capital good model until the steady state is reached. Alternatively the model could be unstable in the sense that investment is higher in the factor that is relatively more abundant.

The first step in our analysis is to define two new variables \( e \) and \( c \), which are constant in the steady-state, the first being the ratio between the stock of education capital \( E \) and the stock of physical capital \( K \) and the second being the ratio between consumption and \( K \).\(^{14}\)

We have, by subtracting, member by member, 2.15 from respectively 2.9 and 2.16:

\[
\frac{\dot{e}}{e} = B_1 e^{\beta-1} - (1-t)B_2 e^\gamma + c \tag{2.2.1}
\]

\[
\frac{\dot{c}}{c} = -\rho + (1-t)B_3 e^\gamma ((\gamma - \beta) t + 1 - \gamma - \sigma) \sigma^{-1} ((\gamma - \beta)t + 1 - \gamma)^{-1} + c \tag{2.2.2}
\]

We first analyse the \( e = 0 \) locus. Along it we have:

\[
\left. \frac{\dot{c}}{\dot{e}} \right|_{e=0} = (1-t)B_3 e^\gamma - B_1 e^{\beta-1}
\]

\[
\lim_{e \to +\infty} c|_{e=0} = -\infty
\]

\[
\lim_{e \to +\infty} c|_{e=0} = +\infty
\]

\[
\frac{dc}{de}|_{e=0} = (1-t)B_3 e^\gamma - B_2 (\beta-1)e^{\beta-2} > 0
\]

\(^{14}\) In Mulligan and Sala-i-Martin (1991, 1992) terminology \( c \) is a control-like variable and \( e \) a state-like variable.
\[
\lim_{e \to 0} \frac{dc}{de} \big|_{e=0} = 0
\]
\[
\lim_{e \to +\infty} \frac{dc}{de} \big|_{e=0} = +\infty
\]
\[
d^2c \big/ de^2 \big|_{e=0} = (1-t)B_3(y-1)e^{y-2} - B_2(\beta-1)(\beta-2)e^{\beta-3} < 0
\]
\[
\lim_{e \to +\infty} d^2c \big/ de^2 \big|_{e=0} = 0
\]

So the locus will be upward sloping and strictly concave. Moreover its intercept on the horizontal axis will be positive and it will have a vertical asymptote at the origin.

Now consider the \( c = 0 \) locus. We have:

\[
c \big|_{c=0} = \rho/\sigma - xe^y
\]

where \( x = (1-t)B_2((y-\beta)t+1-\gamma-\sigma)((y-\beta)t+1-\gamma)^{-1}\sigma^{-1} \)

\[
\lim_{e \to 0} c \big|_{c=0} = \rho\sigma^{-1}
\]

\[
\lim_{e \to +\infty} c \big|_{c=0} = \begin{cases} -\infty & \text{if } x > 0 \\ +\infty & \text{if } x < 0 \end{cases}
\]

\[
dc/de \big|_{c=0} = -\gamma xe^{y-1}
\]

\[
d^2c/de^2 \big|_{c=0} = -\gamma(y-1)xe^{y-2}
\]

If \( x \) is strictly positive (negative), the curve is downward(upward)-sloping and strictly convex (concave).\(^\text{15}\) The vertical

\(^{15}\) Notice that both cases are possible for reasonable values of the
intercept is positive and both the first and the second derivatives go to zero as a goes to infinity.

The phase diagram for the case in which the $c = 0$ isocline is upward sloping is represented in Fig 2. Everywhere above the $dc/dt = 0$ locus the consumption - capital ratio $c$ is increasing: consumption is above the level that would just maintain $c$ constant. Similarly $c$ is decreasing at points below the $dc/dt = 0$ locus. As regards the $de/dt = 0$ locus we have that at points to the left of the $de/dt = 0$ locus the $E/K$ ratio is increasing. Analogously, at every point to the right of the $de/dt = 0$ locus that same ratio is decreasing. As usual the arrows shown in the figure indicate the directions of motion and we can see that we have a saddlepoint equilibrium. This implies that for each initial level of the capital goods there is a unique initial level in consumption consistent with convergence to the steady state. Convergence of $c$ and $e$ to the steady-state values is monotonic.

Notice that even if the $de/dt = 0$ and $dc/dt = 0$ loci are both upward sloping they cannot intersect twice or be tangent.

The first point is implied by the previously established uniqueness of the steady state rate of growth $g$. From 2.9 we see that the steady state value for $e$ is also unique. Finally, substituting the steady state $e$ into (2.2.1) or (2.2.2) we see that there is only one steady state $c$.

A tangency solution would require that when the two loci meet they have the same slope:

$$\frac{dc}{de} \bigg|_{e=0} = \frac{dc}{de} \bigg|_{c=0}$$

In fact we should have, substituting the expressions already found for these derivatives:

parameters. In fact $B_2$ is positive and $(\gamma - \beta) + \delta$ is positive. For values of $\sigma$ near or below unity, suggested by empirical estimates, the sign of $x$ can be positive or negative. Coeteris paribus the higher is $\sigma$ the smaller will be $x$. But even in particular cases, for example for $\gamma = \beta$ or for $\beta = 1$ it is not possible to sign $x$, without knowing precisely the values of the other parameters.
(1-t)B_3 \gamma e^{\gamma-1} - B_1 (\beta-1) e^{\beta-2} = (1-t)B_2 ((\gamma-\beta)t+1-\gamma-\sigma)((\gamma-\beta)t+1-\gamma)^{-1} e^{-1} \gamma e^{\gamma-1}

or, after reordering and simplifying, and recalling the relationship between \( B_3 \) and \( B_2 \):

\[ B_1 (\beta-1) e^{\beta-2} = (1-t)B_2 \gamma e^{-1} e^{\gamma-1} \]

This equation has no solution since, for all values of \( e \), the expression on the left hand side is negative and that on the right hand side is positive.

In Fig. 3 the case is represented where the \( \frac{dc}{dt} = 0 \) locus is downward sloping. The diagram shows that we have a saddle point equilibrium in this case as well. The analysis of this case which can be conducted along lines analogous to those used previously is left to the reader.

2.3 The Saddle Point Equilibrium in the Linearized System

We now linearize the dynamic system in \( c \) and \( e \) around the steady state values \( e^* \) and \( c^* \):

\[
\begin{bmatrix}
\frac{dc}{dt} \\
\frac{de}{dt}
\end{bmatrix} =
\begin{bmatrix}
c^* (1-t) e^{-1} B_2 \gamma ((\gamma-\beta)t+1-\gamma-\sigma)((\gamma-\beta)t+1-\gamma)^{-1} e^{\gamma-1} & (c-c^*) \\
e^* B_1 \beta e^{\beta-1} ((1-t)(\gamma+1)B_3 e^{\gamma-1} + c^*) & (e-e^*)
\end{bmatrix}
\]

Recalling that along the \( \frac{da}{dt}= 0 \) locus we have:

\[ c^* = -B_1 e^{\beta-1} + (1-t)B_3 e^{\gamma} \]

we can write the determinant of the system as:
\[ \Delta = c(\beta-1)B e^{\beta-1} - c(1-t)B e^{\gamma((\gamma-\beta)t+1-\gamma)} < 0 \]

This confirms the conclusions reached by means of qualitative analysis that the equilibrium is a saddle point so that there is only one level of initial consumption that, given initial capital stocks is compatible with convergence to the steady state.

In the terminology of Blanchard-Kahn (1980) or Buiter (1990) the number of predetermined or backward-looking variables (i.e. \( e \)) equals the number of stable roots of the characteristic equation of the homogenous system, and the number of non-predetermined, jump variables (i.e. \( c \)) equals the number of unstable roots. We have in fact a linear two-point boundary value problem with linear boundary conditions, where one boundary condition takes the familiar form of an initial condition for the predetermined variable and the other is obtained from the terminal condition that the system should be convergent.


Having established the existence, uniqueness and stability of the steady-state solution, we now move on to assess the externalities implied by public expenditure and taxation, which lead to which the decentralized choices generating outcomes that are not Pareto optimal. The easiest way to do so is to compare the decentralized equilibrium with the solution to a planning problem in which the government dictates the accumulation rates of the capital goods and the consumption choices over time.

3.1 The Command Economy

We assume the social planner maximizes the representative household’s welfare, described by (2.11), subject to the constraints:

\[ K = A_1 (\phi K)^{1-\gamma} (\chi E)^\gamma - C \quad (3.1.1) \]
This is a well known two sector model with a linearly homogeneous Cobb-Douglas technology, so we can be brief. The FOC's are:

\[ C^{-\sigma} = \theta_1 \]  

\[ \theta_1 A_1 (1-\gamma) \phi^{-\gamma} K_1^{1-\gamma} (\chi E)^{\gamma} = \theta_2 A_2 (1-\phi)^{-\beta} (1-\beta)((1-\chi) E)^{\beta} \]  

where \( \theta_1 \) and \( \theta_2 \) are the costate variables relative to the states \( K \) and \( E \).

\[ \theta_1 A_1 (\phi K)^{1-\gamma} E^{\gamma} \chi^{-1} = \theta_2 A_2 (K(1-\phi))^{1-\beta} (1-\chi)^{\beta-1} E^{\beta} \]  

\[ \dot{\theta}_1 = \rho \theta_1 - A_1 \phi^{-\gamma} K_1^{1-\gamma} (1-\gamma) (\chi E)^{\gamma} - (1-\beta) \theta_2 A_2 (1-\phi)^{-\beta} ((1-\chi) E/K)^{\beta} \]  

\[ \dot{\theta}_2 = \rho \theta_2 - A_2 (\phi K)^{1-\gamma} E^{\gamma} \chi^{\gamma-1} A_1 - A_2 (K(1-\phi)/A)^{-\beta} (1-\chi)^{\beta} \]  

Dividing (3.1.4) by (3.1.5) member by member we obtain:

\[ \gamma \phi / (1-\gamma) \chi = \beta (1-\phi) / (1-\beta) (1-\chi) \]  

From (3.1.1) and (3.1.2), considering that \( \phi \) and \( \chi \) must be constant in steady state, we have that in steady state the rate of growth of knowledge and physical capital must be the same which we indicate by \( g^s \). Totally differentiating (3.1.4) we see that in steady-state:

\[ \theta_1 / \theta_1 = \theta_2 / \theta_2 \]  

16 For references on this model see note 6.
Substituting (3.1.4) in (3.1.6) and (3.1.7), we have:

\[
\frac{\theta_1}{\theta_1} = \rho - A_1 (1-\gamma)(E\chi/\phi K)\gamma
\]

(3.1.10)

\[
\frac{\theta_2}{\theta_2} = \rho - \beta A_2 (K(1-\phi)/(1-\chi)E)^{1-\beta}
\]

(3.1.11)

Since in steady state the left hand side of (3.1.10) and (3.1.11) are equal we deduce that:

\[
A_1 (1-\gamma)(E\chi/\phi K)\gamma = A_2 \beta ((1-\phi)K/(1-\chi)E)^{1-\beta}
\]

(3.1.12)

From (3.1.3) and (3.1.10) we get to:

\[
C/C = (-\rho + A_1 (1-\gamma)(E\chi/\phi K)\gamma)\sigma^{-1}
\]

(3.1.13)

Using (3.1.8) and (3.1.12) to calculate $E\chi/\phi K$, we then find:

\[
g^* = -\rho\sigma^{-1} + ((\beta A_2)^\gamma (\gamma (1-\beta))^{y(1-\gamma)} A_1 (1-\beta)/(1+\gamma-\beta))^{1/(1+\gamma-\beta)} \sigma^{-1}
\]

(3.1.14)

We therefore have that the steady state rate of growth is higher the more patient and the more willing consumers are to substitute consumption intertemporally.

3.2. Comparison between the Command and the Market Economy

It is easy to show that it is not possible to find an income tax rate path which gives a Pareto optimal equilibrium.

For ease of exposition we consider the case in which $\beta = \gamma$ and limit ourselves to the steady state configuration. In the command economy we have from (3.1.8) $\phi = \chi$ and therefore we can rewrite (3.1.1), (3.1.2),
(3.1.13) and (3.1.14) as:

\[ K = A_1 \phi^{1-\gamma}E^\gamma - C \quad (3.2.1) \]

\[ E = A_2 (1-\phi)K^{1-\gamma}E^\gamma \quad (3.2.2) \]

\[ \dot{C}/C = -\left( \rho - A_1 (1-\gamma)(E/K)^\gamma \right)\sigma^{-1} \quad (3.2.3) \]

\[ g^* = -\rho/\sigma + (A_1 (1-\gamma))^{1-\gamma}(A_2^{1-\gamma})\sigma^{-1} \quad (3.2.4) \]

We note that in steady state we have:

\[ 1-\phi = g^* (A_1 (1-\gamma)/A_2)^{1-\gamma}/A_2 \quad (3.2.5) \]

Considering the market economy, we have that in the case \( \beta = \gamma \) (2.15), (2.9) and (2.16) become:

\[ K = A_1 (1-t)K^{1-\gamma}E^\gamma - C \quad (3.2.6) \]

\[ E = A_2 tK^{1-\gamma}E^\gamma \quad (3.2.7) \]

\[ \dot{C}/C = -\left( \rho - A_1 (1-\gamma)(1-t)(E/K)^\gamma \right)\sigma^{-1} \quad (3.2.8) \]

If we compare the equations of motion describing the decentralized economy with those relative to the command economy we see that due to the presence of the tax (3.2.3) is different from (3.2.8), so that there is no way, given the tax system, to reproduce the optimal program in the market economy. The point is that the income tax is distorting since it drives a wedge between the social and the private return on investment. On the other hand if \( t=0 \) we have a no-growth steady state equilibrium, which is also inefficient.
What can be done is to impose a tax rate such that, in steady state, the command economy and the market economy grow at the same rate. Recalling that in steady state all variables grow at the same rate \( g \), we have from (3.2.7):

\[
\frac{K}{E} = \left(\frac{g}{A_t^2} t\right)^{1/1-\gamma}
\]  
(3.2.9)

Substituting (3.2.9) in (3.2.8), we have:

\[
g = (-\rho + A_1 (1-\gamma) (1-t)(g/A_t^2)^{\gamma/1-\gamma}) e^{-1}
\]  
(3.2.10)

It could then be possible to construct a taxation path in such a way that \( g \) is equal to the balanced growth path of the command economy, (provided, of course, this equation has a solution for \( 0 < t < 1 \)). However in general, this solution will be different from \( 1-\phi \), given by (3.2.5). Then from (3.2.2) and (3.2.7), the ratio between the stocks of the capital goods, and, from (3.2.1) and (3.2.6), the ratio between the stock of physical capital and consumption will be different in the two economies. This means that, for given \( K_0 \), i.e. physical capital at time 0, \( C_0 \), consumption at time 0, will be different from the optimal level \( C^*_0 \). But this implies from the definition of the optimum that even when the optimal growth rate is achieved with an income tax, the level of households' welfare will be lower in the decentralized equilibrium.

4. A Consumption Tax Model

In this section we show that the command optimum can be attained in a decentralized setting if the government revenue is obtained through a consumption tax rather than an income tax, a result analogous to those reached in different settings by, for instance, Barro (1990) and Rebelo (1990). The economic intuition for the equivalence between a consumption tax and a lump-sum tax is that, since leisure is exogenous in this model, a consumption tax does not alter any incentive the agents face, i.e. does not distort the only decision made by households in this economy, the
decision of consuming now versus later. To prove this formally we show below that the equations characterizing the steady state in the command economy will be equal to those characterizing the market economy, provided the tax rate is chosen optimally.

Our results apply to the steady-state; although it should be possible to analyze the transitional dynamics of this version of the model along the same lines used to study the income tax model above, this is not attempted here.

We begin by considering that, if all revenue comes from a consumption tax the government budget constraint will be:

\[ Ct = rK + wL \]  \hspace{1cm} (4.1)

where \( t \) is now the consumption tax rate. This can be rewritten, noticing that (2.4) and (2.5) still hold, since the conditions of profit maximization are not affected by the fiscal regime, as:

\[ Ct = A_t \gamma(\phi K)^{1-\gamma}(E_x)^\gamma(1-\chi)/\beta x \]  \hspace{1cm} (4.2)

The budget constraint of families is:

\[ w = rW + w - (1+t)C \]  \hspace{1cm} (4.3)

Recalling that in the aggregate \( K = W \), and again using (2.4) and (2.5) the equation of motion of physical capital becomes:

Notice that if leisure were endogenized by considering utility functions consistent with steady state growth (i.e. belonging to the kinds mentioned in note 7) it is very likely, even if it is not shown analytically here, that a consumption tax would still be equivalent to a lump sum tax. The same cancelation of income and substitution effects that makes leisure constant in spite of a growing wage rate would insure this.
while, keeping CES preferences, the Euler conditions imply:

\[
\frac{\dot{C}}{C} = -\left( \rho - A_1(1-\gamma)(E_x/K\phi)^{y} \right) \sigma^{-1}
\]  

(4.5)

Cost minimization by public agencies imposes:

\[
\gamma \phi / (1-\gamma) = \beta (1-\phi)/(1-\beta)(1-\gamma)
\]  

(4.6)

while the accumulation of the non material asset is described by:

\[
E = A_2(K(1-\phi))^{1-\beta}(1-\gamma)E^{\beta}
\]  

(4.7)

In the steady state the rates of growth of \(E, K, C\) are easily seen to be the same, so that the equations (4.2), (4.4), (4.5), (4.6), (4.7) can be solved for the five unknowns \(C/K, E/K, \phi, \chi, g\). Now we have that these equations, apart from the first, also characterise the command economy. This can be seen by comparing them to equations (3.1.1), (3.1.2), (3.1.8) and (3.1.10), that, together with (3.1.12), determine the optimal steady state \((C/K)^*, (E/K)^*, \phi^*, \chi^*, g^*\). To determine the optimal tax rate one can then simply substitute \(\phi^*, \chi^*, (C/K)^*, (E/K)^*\) in (4.2). We find:

\[
t^* = \gamma \beta (1-\chi^*/(1-\gamma)(\gamma(1-\beta) - (1-\gamma)\beta \chi^*/(1-\gamma)^{\beta}(1-\beta)) + \beta (1-\beta)\chi^*
\]

while:

\[
\chi^* = 1-g^* A_2^{-\gamma/(\beta-1)} A_1^{\beta/(\beta-1)} (1-\gamma) \gamma/(\gamma - \beta) \gamma(1-\beta)/(\beta-1) \gamma(1+\gamma-\beta)^{-1}
\]

The transversality conditions will then be used to calculate the initial level of \(C\).
5. Concluding Remarks.

We have analysed a two factor model in which public policy, by inducing knowledge accumulation, works as the engine of sustained growth. While endogenous growth models generally are solved only for the balanced growth path, we have been able to fully characterize the transitional dynamics of the model, which we show to be globally saddle path stable. 

Coming to policy implications from the analysis: we show that while a Pareto optimum cannot be achieved in the steady state with an income tax it can with a consumption tax.

This work is only a beginning and many further developments are possible. The first direction of further research would be to calibrate the model and simulate the impact of tax changes on the rate of growth. Another route would be to look at the empirical implications of the theory for relations between the size of investment expenditure by the government, in particular in research and education, and the rate of growth. Notice that since the analysis applies not only to steady state paths an empirical investigation could look at differences in performances across countries in the short run and not only simply at averages calculated over long periods of time. Other theoretical questions that could be asked starting from the model studied here are, for instance, whether the command optimum is attainable outside the steady state by resort to a consumption tax, and what are the effects of debt financing. Another experiment could also be conducted assuming that the government is not benevolent but, for instance, wants to maximize its own revenue.

As the perceptive reader will know by now, there is the Barro and Sala-i-Martin (1993) exception.
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