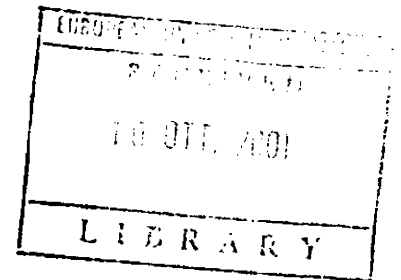**EUROPEAN UNIVERSITY INSTITUTE**
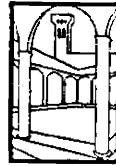**Department of Economics**

Essays on customer loyalty and on the competitive effects of frequent-flyer programmes

Pedro Fernandes

Thesis submitted for assessment with a view to obtaining
the degree of Doctor of the European University Institute

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EUROPEAN UNIVERSITY INSTITUTE
Department of Economics

Essays on customer loyalty and on the competitive effects of frequent-flyer programmes

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To my grandparents and their great-grandson

Contents

i

List of Figures

Acknowledgments

As Lady Bracknell might have said, to lose one supervisor may be regarded as misfortune but to lose two looks like carelessness. Luckily, my carelessness has ultimately gone unpunished.

Indeed, it has given me the opportunity to work with Massimo Motta and it is to him that this thesis owes the most. I am certain that the thesis would not have been completed were it not for Massimo, who set me back on my feet and offered much-needed guidance and encouragement just as my enthusiasm began to wane.

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viii

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Introduction

The three essays in this thesis share a common theme: customer loyalty.

Customer loyalty is a fundamental concept in strategic marketing, and it is accepted by its practitioners that the pattern of loyal behvaiour varies considerably across markets, and within a market, across consumers.[1] Chapter 1 explores this observation. In particular, it asks what drives some consumers to establish relations of loyalty with an individual seller, whilst others adopt a strategy of shopping around.

The chapter develops a model where buyers and sellers meet repeatedly and where, on each purchasing opportunity, boundedly rational buyers choose which seller to address on the basis of the records of his experience with the sellers. In turn, a buyer's record of a seller is reinforced by the experienced payoff and past outcomes are discounted as buyers are assumed to be forgetful. The model presented borrows heavily from that in Weisbuch et al. (1995) though it differs in one crucial respect. Weisbuch et al.'s (1995) setting is such that consumers are assured of having their demand met when meeting a seller and that the utility enjoyed by a buyer is the same irrepective of where the purchase is made. Given this, buyers appear to face no uncertainty and there is no motivation for consumers to learn about which seller to address. The model developed in Chapter 1, on the other hand, introduces this uncertainty by allowing for the possibility that upon visiting a seller, a buyer is faced with empty shelves. In such a setting, a buyer's expected utility is dependent on the number of other buyers that have selected to visit the same seller in the same period. The model allows to investigate how the number of buyers affects the shopping behaviour of the buyers themselves.

Whether buyers' shopping behaviour develops into one of exhibiting loyalty to a seller or into one of shopping around is dependent on the parameters that define the buyers' learning process. The influence of these parameters is

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[1] See, for example, Ehrenberg (1988) and Reichheld (1996).

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as intuition would suggest. The greater the payoff to a buyer from having his demand met and the less forgetful a buyer is of his past experience, the more likely it is that he will become loyal to a seller. Similarly, the greater the relevance attached by a buyer to his record of past experiences with sellers, the more likely it is that he will become loyal.

The results obtained echo those of Weisbuch et al. (1995). Indeed, the results are exactly reproduced when the number of consumers is allowed to approach infinity. Outside this special case, however, the results of the two models diverge. Under the model developed in the chapter, for given values of the parameters shaping the learning process of buyers and for a given number of buyers, buyers are more likely to shop around than they would be in the setting of Weisbuch et al. (1995). When the number of buyers in the market is relatively small the divergence in the predictions of the two models can be particularly poignant. Where Weisbuch et al (1995) would predict that buyers develop loyal relations with a seller, the model developed in Chapter 1 would predict that buyers shop around.

Chapter 1 also examines the effect on buyers' shopping behaviour as the number of buyers in the market changes. This comparative statics exercise is carried out under two scenarios. First, it is assumed that the aggregate supply of the good provided by the sellers is sufficient to meet aggregate demand. In such a context, buyers are more likely to develop loyal behaviour as the number of buyers in the market increases. However, for some values of the parameters defining the buyers' learning process, buyers will always adopt a searching behaviour, irrespective of market size. The second scenario allows for the possibility that there is a mismatch between aggregate supply and demand. In this context, the findings show that the greater the relative scarcity of the good, the more likely it is that consumers develop a behaviour of shopping around. Conversely, the greater the extent to which aggregate supply exceeds aggregate demand, the more likely it is that consumers become loyal to a seller.

The differing degree of loyalty exhibited by consumers can form a basis on which firms are able to price discriminate. As noted by Fudenberg and Tirole (1997), this type of discrimination falls outside the more familiar first, second or third degree forms of price discrimination.[2] Although such type of discrimination can be carried out at a fairly basic level - familiar customers may often enjoy a discount at the corner shop - the information required

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[2] See, for example, Wilson (1993).

for firms at large to practice this type of price discrimination is becoming increasingly available. Chapters 2 and 3 focus on one vehicle through which firms discriminate between consumers according to their loyalty. These two chapters examine the the role of loyalty-inducing schemes, in particular airlines' frequent-flyer programmes ($FFPs$).

Loyalty-inducing schemes, such as $FFPs$, reward customers that purchase repeatedly from the same firm. Typically, such loyal customers receive discounts, coupons for future purchases or they benefit from preferred treatment. Both Chapters 2 and 3 investigate the anti-competitive effects of loyalty schemes. The two chapters contrast markedly, however, in the assumption made over the ability of loyalty schemes in inducing customers to become loyal. The motivation for Chapter 2 stems from the observation that there exists scant evidence to show that loyalty-schemes are actually succesful in inducing customers to become loyal. Chapter 3, on the other hand, builds on the premise that the loyalty schemes of airlines are succesful in creating for the carriers a portfolio of loyal passengers. While the two premises cannot be reconciled, the findings of the two chapters should be interpreted as complementary. If it is accepted that $FFPs$ induce loyalty, Chapter 3 finds that the schemes enhance the market power of the larger airlines and may act as barriers to entry. If, on the other hand, it is assumed that $FFPs$ are not successful in inducing loyalty, Chapter 2 suggests that these schemes can still act as vehicles to raise prices above what they would be if such programs were not available. Either way, the findings point towards the anti-competitive effects of loyalty schemes.

Literature in industrial economics has generally noted that such schemes create switching-costs and consequently lock-in customers.[3] In turn, firms are able to exploit the set of locked-in customers by setting price above what they would otherwise be and thereby earn profits higher than those that would be earned if loyalty schemes did not exist. These results hinge on the assumption that such schemes do indeed induce switching costs. In fact, however, there is scant empirical evidence to back the claim that loyalty schemes are actually succesful in locking in customers, and thereby create switching costs.

Motivated by the lack of such evidence, Chapter 2 offers an alternative rationale for the ubiquity of these programmes. In particular, in a setting where two duopolists compete over price and over the value of the discount

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[3] See for example Klemperer (1995).

to hand to repeat-buyers, it is shown that although customers are not locked-in by the promise to future discounts, the firms will nevertheless find it in their interest to reward repeat buyers. Doing so enables firms to set higher prices and enjoy greater profits. This result stems from the fact that a firm's promise to offer a discount induces a rival firm to compete less aggressively since the latter knows that the former must face the burden of discounts. Diagrammatically, the reaasoning boils down to the observation that the launch of a loyalty-programme by one firm shifts the reaction curve of the rival outwards.

The model presented in Chapter 2 distinguishes between those consumer who participate in the market on every purchasing opportunity - the frequent consumers - and those that take part only once. It is shown that the equilibrium prices, the discount offered by the firms and the firms' profits are all increasing with the share of the frequent consumers. Lastly, the model presented suggests that while under some conditions, the set of frequent consumers is better off when firms adopt the loyalty-program, those that take part in the market only once are invariably worse off.

Chapter 3 addresses the concern that *FFPs* act as barriers to entry as they further insulate large incumbents from competition. The reasoning behind this view is based on the following logic. The convex structure of *FFPs* reward schedule is such that frequent flyers have the incentive to concentrate all flights on a few number of airlines. In order to be able to collect a better reward a traveller is likely to choose the airline which flies more often from the nearby airport, the one that flies most frequently on the more travelled route and the one whose menu of destinations to which a prize can be redeemed is more appealing. Other things equal, therefore, a *FFP* is more attractive when it is run by a larger airline.

Chapter 3 explores the extent to which *FFPs* tilts the playing field in favour of the larger airlines. It does so by exploiting a series of events that took place over the course of 1998 and involved six of the American major carriers. In the course of that year, three sets of pairwise agreements were established - US Airways partnered American Airlines, Continental joined forces with Northwest and Delta with United Airlines. With the exception of the link between Contintal and Northwest, the three sets of alliances were limited to marketing agreements and centred on making an airline treat its partner's *FFP* as if it were its own. This *FFP* reciprocity meant that a member of an airline's loyalty scheme could also clock up air miles when travelling on its partnering airline and could redeem prizes on either. Overnight, the

opportunities for a member of the FFP of one of the six airlines involved to earn air miles and redeem prizes doubled. If, as argued, $FFPs$ contribute to an airline's market power, then the boost received by an airline by allying itself with another should have a noticeable effect on fares.

The chapter attempts to establish whether such an effect did take place by estimating a price equation and testing for the size and signficance of explanatory variables reflecting the arrangements of the FFP alliances. Data were used on average fares and traffic covering the thousand largest domestic city-pair markets within the 48 contiguous states of America. An initial set of results is obtained by modelling the formation of alliances - be it of the observed airline or of a rival - through a pair of dummy variables. Under such a specification, the analysis of Chapter 3 suggests that an airline that has joined an alliance earns a premium of around 1 per cent. On the other hand, an airline serving a route where no competitor has formed an alliance enjoys a premium of around 1.9 per cent compared to those routes where a competitor has joined an alliance. Further insights are given when the size of the alliances is taken into account. Under this specification, the results suggest that the effect of forming an on fare levels is not due to changes in the relative attractiveness of $FFPs$ at the route level. Instead, the results suggest that airlines raise their average fare levels due to the greater diffusion of their FFP at the airport and in the national market at large. The results point towards the conclusion that the formation of the FFP alliances have enhanced the market power of those airlines involved. This finding is consistent with the hypothesis that $FFPs$ benefit the airlines which are able to offer a greater menu of flights, and lends support, therefore, to the concerns raised by the competition authorities over the anti-competitive effects of $FFPs$.

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Chapter 1

A study on the effects of market size on buyer-seller relationships

1.1 Introduction

In the second half of the nineteenth century, hundreds of men regularly sought work on a daily basis at unloading the ships in the docks of London and Liverpool. At each centre, in the early morning, the master called out the necessary number of names for the work at hand.[1] Those chosen were hired for the day. The supply of labour generally exceeded demand and the dockers who turned up at a centre faced the risk of not being called. For a given task, the larger the crowd at a centre the less likely that an individual worker would be called upon. In turn, not being called meant foregoing a day's work, which is to say, a day's pay. As described in Lovell (1969) and Taplin (1985), the behaviour of the men when seeking work differed markedly amongst them. On the one hand, the so called loafers would, on different days, look for work at different centres. On the other hand, the regulars - accounting for the greater proportion of the work force - invariably addressed the same centre.[2]

The above description begs a question: if a day's work was a day's work

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[1] See Mayhew (1861, pp.301 − 312) for a vivid description of the working world of London dock-labourers in the mid-nineteenth century.

[2] See the first chapter in Taplin (1985) for an account of these differences in behaviour and, in particular, of the high degree of immobility characterising the regulars.

and wages were the same across centres, why did some workers remain loyal to a centre whilst others searched around and chose to address different centres on different days?

Although broad, the strokes with which I have described the above work arrangements will have served, I hope, to highlight one feature of markets that is often overlooked in economic analysis. Namely that different buyers adopt different shopping habits - some will become loyal and address the same seller over time whilst others will visit a number of them, each with some frequency.

The varied behaviour amongst the dock labourers in the English ports shopping for a day's work is mirrored by buyers in other markets with similar characteristics, namely the high frequency with which individuals meet to trade. One that has been studied recently in some detail is the Marseille fish market where a stable population of sellers and buyers interact on a daily basis (see Weisbuch et al. 1995, Kirman and Vriend 1997, and Herreiner 1998). A finding in these studies is that while a large number of the buyers are loyal clients of a particular seller, others search and divide their patronage amongst two or three stores. Hence, the *strength of the relations* established between buyers and sellers is seen to vary across the population. Here the strength of a relation between a buyer-seller pair refers to the frequency with which the pair is matched.

To marketing practitioners the co-existence of these varied behaviours is not new.[3] Ehrenberg's (1988) study on repeat purchasing sets off from work in this field which established that "empirically, the finding is that most people *tend to develop habits* of buying one or some small number of brands, each fairly regularly" (Ehrenberg, 1988, p.5, my italics).

The quote from Ehrenberg, and the italicized phrase in particular, provides the cue to introduce the model presented in Weisbuch et al. (1995). In turn, this model serves as a platform for the model treated in this chapter.

At the heart of Weisbuch et al. (1995) is the view that the strength of a buyer-seller relation should be viewed as an evolving arrangement that is moulded by the experience of these individuals in the market. The authors treat the strength of buyer-seller relations as endogenous and examine the question of what drives some consumers to establish strong relations with a particular seller, repeatedly patronising him, whilst others adopt a strategy of

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[3]For a review of studies within the marketing literature which seek to account for varied shopping behaviour see McAllister and Pessemier (1982).

shopping around. Consumers are boundedly rational and learn about which seller to address on the basis of the outcomes of past visits. Whether buyers become loyal or not then depends on the parameters shaping their learning rule and on the level of utility obtained from each transaction.

In Weisbuch et al. (1995) stores are identical and they have unlimited stocks.[4] It follows that consumers are certain of being served by a store on every purchase occasion and that the benefit from being served does not differ amongst stores. In such a context there seems to be little reason for buyers to go through a learning process at all - why update the record one has of sellers and why pay attention to such records when all assure equal service? In the model analysed in the chapter, sellers stock a limited quantity of the perishable good. In the event that the demand at one store exceeds the available stock, some customers will be turned back unserved. It follows that the benefit enjoyed by a buyer when addressing a seller is dependent on the actions of the other buyers and that this benefit varies over time as buyers' choice of which seller to address also varies.

As in Weisbuch et al. (1995), buyers will be modelled as boundedly rational agents who must choose in each time period which of two sellers to address. The choice rule followed by a buyer is shaped by the records of his experience with the two sellers. In turn, a buyer's record of a seller is reinforced by the experienced payoff and past outcomes are discounted as buyers are assumed to be forgetful.

In this chapter I am interested in describing the steady-state behaviour of buyers though attention will be restricted to those steady-states where either all buyers are indifferent between visiting either seller or there are two groups of buyers where each group becomes loyal to one of the sellers. What leads consumers to exhibit one sort of behaviour or the other is the main question to be explored.

The results I obtain echo those established by Weisbuch et al. (1995). Indeed, their exact results are reproduced when the number of consumers in the market is allowed to approach infinity. Outside this special case, in particular when the number of market participants is small, the model explored in this chapter tells a different story from that of Weisbuch et al.

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[4]Weisbuch et al. (1995) relax the assumption that the utility from consuming a unit of the good at different stores is identical. However, in the models solved analytically the authors, they do not relax the assumption that the stock of good is unlimited. The latter assumption, though not explicit, is implicit in their assumption that consumers are always served.

12

(1995).

In comparison with the setting described in Weisbuch et al. (1995), buyers are more likely to randomise between sellers rather than become loyal to a particular one. By choosing appropriate parameter values, it is straightforward to construct an example where, in the setting of Weisbuch et al (1995), buyers become biased towards a seller and visit him on eight times out of every ten occasions. For these same parameter values, the model presented below suggests that buyers will remain indifferent between sellers unless there are more than 75 buyers in the market.[5] Only for a market size bigger than this will buyers also develop a preference for a particular seller, though the magnitude of the bias will always be smaller than in the Weisbuch et al. (1995) setting. It is only when the number of buyers is allowed to approach infinity that the results of the two models coincide.

In the model first described below, where the supply of the good in the market is such that the demand of all buyers could be matched, buyers are more likely to develop a loyalty towards a seller as the market size increases. Nevertheless, for some parameter values, buyers will always search.

By moving away from the assumption that aggregate supply equals aggregate demand, I am able to see how relative scarcity affects buyer behaviour in the steady-state. For a given number of buyers, which is to say, for a given aggregate demand, the larger the stock of the good supplied by sellers, the more likely it is that consumers become loyal to a seller. Given the discussion in the previous paragraph, this result implies that increasing the sellers' supply of the good, brings the result closer to that of Weisbuch et al. (1995). The intuition for this is immediate. As the stocks of each seller increase, it is less likely that consumers face empty shelves and so the closer one is to Weisbuch et al.'s (1995) assumption that buyers are certain of being served. On the other hand, the greater the number of buyers chasing a given supply, the more likely it is that consumers will search stores rather than become loyal to one. In fact, if the number of buyers is sufficiently large, then all buyers will be searchers.

Whether buyers adopt one behaviour or the other depends on the parameters that define the buyers' learning process. The influence on buyers' behaviour of the parameters defining their learning rule is as intuition would

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[5]This example corresponds to the setting where $\beta = 2$ and $\frac{\gamma}{U} = 0.96$ - the notation is explained in Section 1.3. For these parameter values, buyers in the setting of Weisbuch et al. (1995) will patronise their favourite seller with a probability of 0.8301.

suggest. The greater the benefit to a buyer of having his demand met and the less forgetful buyers are of their past experience, the more likely it is that consumers develop a loyalty towards a seller. Similarly, the greater the relevance attached by buyers to their past experience when selecting which seller to visit, the more likely it is that buyers become loyal to one of them.

The chapter is structured as follows. In Section 1.2, we present a review of the literature that has modelled exchange economies and paid explicit attention to the existence and development of the relations between buyers and sellers. This section introduces the work of Weisbuch et al. (1995) which, in turn, serves as a platform for the fully developed model of Section 1.3. Results and a discussion are presented in Section 1.4. Section 1.5 concludes.

1.2   Trade relations in exchange economies

A model intending to study how individual and market characteristics influence the development of relations between buyers and sellers should exhibit a number of features. First, the setting must be a dynamic one so that the same pair of agents - a buyer and a seller - are allowed to meet repeatedly. Second, buyers and/or sellers must be distinguishable amongst themselves in the eyes of the individuals of the opposite set. Third, agents must be able to learn so that they react to past experience and alter their behaviour accordingly.

Due to the imagery of a search equilibrium as one of active interactions (Kirman, 1994), search theory might seem, at first sight, a promising area in which to find such a framework. The models developed in this field aim at characterizing the equilibrium of an economy where a cost must be incurred by those wishing to have an opportunity to trade and where agents are ex-ante imperfectly informed of the returns from such a trade (see for example Stigler 1961, Lippman and McCall 1976, Cross 1983 and Diamond 1989). As a setting in which to discuss the development of trade relations, however, these models are not appropriate. Firstly, the equilibrium is static in that it is defined by the optimal stopping-rules of consumers and the optimal price to be set by the sellers. There are no repeated interactions. Secondly, even when the models are enriched by allowing both sets of agents - both buyers and sellers - to search and, in addition, these are allowed to choose how intensely to search, the matching mechanism which draws agents together is a "two sided random sampling procedure" (Stigler 1961, Benhabib and Bull

14

1983, Mortensen 1996; quote from Mortenson, p.6). In these models trade is amongst non-distinguishable agents in the sense that they are not identifiable by those on the opposite side of the trade.

A more recent strand in the literature, that using models of local interactions, on the other hand, sets off precisely by identifying very clearly who is allowed to interact with whom. There, the economy is described by a lattice where agents are distributed on the nodes and interactions are restricted to those between agents on neighbouring nodes. This setting has been used to examine the emergence of trade clusters or the diffusion of a technology or of a norm across an economy. Ioannides (1990), Albin and Foley (1992), Blume (1993), Ellison (1993) and Anderlini and Ianni (1996) are some examples that make use of such a set-up. Despite the well defined notion of which agents (are allowed to) interact with whom, these set-ups do not lend themselves to a discussion on pair-wise trade relations given that an agent's neighbours are implicitly assumed to be indistinguishable between themselves. In each time period, either an agent interacts with all his neighbours or he interacts with only one of them randomly. The question of how the strength of a relation between two neighbours develops is hijacked by this exogenous structure.

Within the context of the Iterated Prisoner's Dilemma (IPD), various studies have examined how the strength of the relations between players evolve over time (Smucker et al., 1994, Stanley et al., 1994 and Tesfastion, 1995).[6] In these models, players use their record of past interactions to choose whom to play with and from whom to refuse a proposal to play. Similarly to what is proposed here, the strength of the relations between the individuals playing the IPD evolve over time according to the outcome of past encounters.

The setting in Vriend (1996) is closer to the one studied in this model examined in this chapter. The central idea in Vriend (1996) is that consumers and shops, initially endowed with little information about their surroundings, learn to create and exploit trading opportunities. In particular, consumers learn what shopping strategy to follow in order to have a better chance of being served and shops learn the level of production and signalling activity that they should undertake each period in order to maximize profits.[7] Of

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[6]In these papers, players are modelled as automata associated, at each point in time, with a particular strategy such as Tit-for-Tat, Always-Cooperate or Tit-for-2-Tats. When playing the Prisoners' Dilemma with a partner, a player chooses to Cooperate or Defect according to their strategy and to his record of past interactions with that partner.

[7]The behaviour of both consumers and shops is modelled through a Classifier System and the learning process takes place using genetic algorithms. Such a setting is intuitively

particular interest is the result that, on average, consumers return to the store last visited on 30% of the occasions and, concealed by this average, the finding that some consumers are four times as likely as others to follow such a strategy. However, these results are statements on the likelihood that consumers shop randomly or follow a strategy of *'returning to the last seller '* rather than on the likelihood of addressing a particular seller or the other which is the goal of this chapter.

As highlighted in the Introduction, the issues examined in Weisbuch et al. (1995) coinicide with those tackled here. Given that the main model presented below relies heavily on Weisbuch et al. (1995), I shall move directly to describing the model and later, when appropriate, draw out the differences between the two settings.

1.3 Model

A market for an homogenous perishable good is made up of an even number, $2N$, of consumers indexed by i, $i = [1, 2, .., 2N]$, and by two sellers, shop A and shop B. The market operates in each period (day) t, with all agents actively participating in it. Consumers hold an inelastic demand for one unit of the good in each period and derive a constant net benefit, U, from consuming it.

The two identical sellers have a passive role in this model. Each sets an identical price in each period and they share the market potential equally, i.e. each produces N units of the goods in each period. The passive role of the two sellers - producing a constant quantity and setting a constant price in each period - implies that the evolution of relations between sellers and consumers is fuelled exclusively by the behaviour of the latter.

Consumer i holds a pair of parameters J_{ijt}, $j = \{A, B\}$ which are a summary statistic of how well i has fared in trading with shop j up to period t. This is the fidelity parameter described in Weisbuch et al. (1995). The value of J_{ijt} is updated as the experience of the consumer unfolds according to,

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appealing and lends itself to be implemented in simulations - as carried out by Vriend -, though it does not allow for analytic results to be derived..

$$J_{ij0} = 0 \qquad\qquad t = 0$$

$$J_{ijt} = (1-\gamma)\,J_{ijt-1} + U_{ijt-1} \quad t = 1, 2, \ldots \tag{1.1}$$

where γ, $0 < \gamma < 1$ and where,

$$U_{ijt-1} = \begin{cases} U & \text{if } i \text{ purchased from shop } j \text{ at time } t-1 \\ 0 & \text{otherwise} \end{cases} \tag{1.2}$$

The parameter γ reflects the relevance that i attaches to his past record of store j. If $\gamma = 0$, the consumer takes full consideration of the fidelity he held in the previous period. On the other hand, if $\gamma = 1$, the consumer's fidelity to a shop in period t will be equal to the utility received from that shop in period $t-1$ alone. An alternative and intuitive way to interpret γ, suggested in Weisbuch et al. (1995), is to view it as a proxy for the forgetfulness of consumers - the higher the value of γ the more forgetful consumers are.

In selecting which of the two shops to visit at time t, consumer i attaches a probability P_{ijt} of addressing seller j. This probability is given by the choice rule,

$$P_{ijt} = \frac{\exp\left(\beta J_{ijt}\right)}{\exp\left(\beta J_{iAt}\right) + \exp\left(\beta J_{iBt}\right)}\ for\ j = \{A, B\} \tag{1.3}$$

where $\beta \geq 0$ reflects the significance attached by consumers to their record of market experience. In particular, β measures the extent to which the choice probability is biased towards the store with whom the consumer holds a higher fidelity parameter. Note that,

$$\frac{\delta\left(\frac{P_{iAt}}{P_{iBt}}\right)}{\delta\beta} = (J_{iAt} - J_{iBt})e^{\beta(J_{iAt}-J_{iBt})} \tag{1.4}$$

The partial derivative is positive if and only if $J_{iAt} > J_{iBt}$. For given values of J_{iAt} and J_{iBt}, the higher the value of β the greater the probability of addressing the shop with whom the consumer holds a higher fidelity. It is interesting to consider the behaviour associated with the extreme values of β. If $\beta = 0$, it is clear from equation (1.3) that the probability of visiting either shop is $\frac{1}{2}$ whatever the consumers' past experience in dealing with either shop may have been, i.e. independent of the J_{ijt}. On the other hand, if $\beta = \infty$ then, with probability equal to 1, a consumer addresses the shop with the

highest J_{ijt}, irrespective of how small the difference in the fidelity actually is. Such a behaviour mirrors that of a consumer following a best-reply strategy.

The learning rule defined by (1.1) and (1.3) falls under the class of exponential fictitious play and support for this family of rules as "strong" learning rules is found in Fudenburg and Levine (1995) and Marimon (1995). In addition, Easley and Rustichini (1995) identify this class of rules as satisfying the axioms which they believe adaptive rules should possess.[8] Furthermore, Weisbuch et al. (1997) have provided support to the logit form as a choice function by showing that it is the outcome of an optimization exercise involving the trade-off between exploiting a known and well rewarded strategy and exploring new strategies with possibly better payoffs.

1.3.1 Solving the model

The model can be solved analytically by approximating the dynamics of the model by ordinary differential equations. This approach is in the spirit of the mean field theory found in statistical physics and it centres on replacing fluctuations around a mean by the expected value.[9] The approximation is valid provided the changes in the fidelity parameters, J_{ijt}, are small at each time step and that the population is homogeneously mixed, *i.e.* each buyer has equal access to each seller so that each individual behaves as if he dealt with the market average.

Recall that the law of motion of J_{ijt} described by equation (1.1) can be written as,[10]

$$J_{ijt+\tau} - J_{ijt} = -\gamma J_{ijt} + U_{ijt}$$

In line with the assumptions described above, the stochastic equation can be approximated by considering the expected value of the random variables U_{ijt},

$$J_{ijt+\tau} - J_{ijt} = -\gamma J_{ijt} + E\left(U_{ijt}\right) \tag{1.5}$$

where, using (1.2), $E\left(U_{ijt}\right)$ is given by,

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[8] These basic axioms are monotonicity, symmetry and independence of irrelevant alternatives.

[9] See Chapter 8 in Weisbuch (1991).

[10] This exposition relies partly on Herreiner (1998).

$$E\left(U_{ijt}\right) = P_{ijt}\, P\left(S_{ijt}\right)\, U \tag{1.6}$$

The expression for $E\left(U_{ijt}\right)$ is the product of i 's probability of addressing store j in period t, P_{ijt}, with the probability that store j has sufficient units of the good to satisfy i 's demand, $P\left(S_{ijt}\right)$, and the level of utility enjoyed if i 's demand is indeed met, U.

Taking the limit $\tau \to 0$ in (1.5) yields the deterministic differential equation,

$$\frac{dJ_{ijt}}{dt} = -\gamma J_{ijt} + E\left(U_{ijt}\right) \tag{1.7}$$

The steady-state is defined by the invariance of the fidelity parameters, $\frac{dJ_{ijt}}{dt} = 0$, which, from (1.7), gives the following characterisation of the steady-state,[11]

$$\gamma J_{ij} = E\left(U_{ij}\right) \tag{1.8}$$

1.3.2 Detour - Comparison with Weisbuch et al. (1995)

When visiting a seller, the expected utility to a consumer depends on the probability of being served. This is explicit in (1.6). In turn, the probability of being served is a function of the behaviour of all other consumers in the market.

In contrast, Weisbuch et al.(1995) make the assumption that consumers are certain of being served, $P\left(S_{ijt}\right) = 1$. Furthermore, as is assumed in the model presented here, that paper also takes prices as fixed over time and equal across shops. Consumers, therefore, face no uncertainty; they are sure of purchasing what they demand and they are aware of the (equal and constant) price charged. In such a context, however, what is the rationale for consumers to learn? What is there for consumers to learn about?

On the other hand, a rationale for consumers to go through a learning process, will exist if, as assumed above, consumers are uncertain of the outcome when visiting a shop. This uncertainty was introduced in the model by allowing for the possibility that a customer is greeted by empty shelves. Hence in any given period, whether a consumer is served or not depends on

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[11]Since the equation describes the steady-state the time sub-scripts are left out.

(a) the quantity of goods stocked by the visited shop and on (b) what the other consumers in the market are doing.[12]

1.3.3 Symmetric steady-state

The search is limited to the class of symmetric steady-states.[13] By *symmetric*, I refer to an equilibrium where half the consumers adopt a diametrically opposite behaviour from the other half.

In a symmetric steady-state there are an equal number - N - of two types of consumers. Let G_1 and G_2 be the two sets within which consumers of the same type are grouped. In the steady-state, the probability of a consumer $i \in G_1$ visiting shop A is equal to the probability of a consumer $j \in G_2$ consumer addressing shop B. And coversely, the probability with which $i \in G_1$ addresses B is equal to the probability with which $j \in G_2$ goes to A.

Given this, the steady-state behaviour of all consumers can be established by looking at the behaviour of one consumer in one of the two groups. This will be done with reference to a consumer belonging to G_1.

It follows from (1.8) that in the steady-state,

$$\gamma J_A = E(U_A) \qquad (1.9)$$
$$\gamma J_B = E(U_B)$$

where the sub-script i was dropped given consumers within G_1 are identical.

The task at hand now is to express $E(U_A)$ and $E(U_B)$ in terms of the variables J_A and J_B. The expression for $E(U_A)$ is tackled first.

Recall from (1.6) that

$$E(U_A) = P_A \, P(S_A) \, U \qquad (1.10)$$

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[12] This feature is also present in the model developed in Herreiner (1998) and largely based on Weisbuch et al. (1995). There, however, it is assumed that a buyer does not take into account the effect that his own actions will have on the probability of being served and it is also assumed that each buyer believes all other buyers search each shop with an equal probability.

[13] To investigate asymmetric steady-states would require solving a system of $2 * 2N$ equations with as many unknowns, as it would be necessary to establish the steady-state of the $2 * 2N$ fidelity parameters J_{ij}, $i = \{1, 2, ...2N\}$, $j = \{A, B\}$. Though such a system is, in principle, solvable the unwieldy functional forms -as will be clear below - do not allow for analytic solutions to be derived.

20

where P_A is the probability that i visits shop A and $P(S_A)$ is the probability that i is served when visiting it. The term $P(S_A)$ depends on the actions of the remaining $2N - 1$ consumers present in the market. Recalling that each shop brings N units to the market, it follows that

$$E(U_A) = \underbrace{U * P_A * [\Pr(n_A \leq N - 1)]}_{E(U_A^1)} + \underbrace{U * P_A * \sum_{k=N}^{2N-1} \left[\Pr(n_A = k) * \frac{N}{(k+1)}\right]}_{E(U_A^2)}$$

(1.11)

where n_A is the number of consumers, out of the remaining $2N - 1$, who address shop A.

The first component of (1.11), $E(U_A^1)$, reflects the event where strictly less than N other consumers visit shop A. In this case all consumers that choose to visit A are guaranteed the purchase of a unit of the good. The second component, $E(U_A^2)$, describes the contribution to $E(U_A)$ from the event that more than N consumers visit shop A. In this case, the N units of the good available must be rationed amongst the customers. Given that shops are unable to identify individual consumers it is assumed that the rationing takes the form of a random allocation of the N units among these customers. In other words, sellers are not allowed to give preferential treatment to some buyers.

Recall that P_A is the probability associated with the steady-state values of J_A and J_B for a consumer in G_1 addressing shop A and, by the symmetry assumption, it is equal to the steady-state probability of a consumer in G_2 addressing shop B. On the other hand, P_B denotes the probability with which $i \in G_1$ addresses shop B and it equals the probability of $j \in G_2$ of going to shop A. It follows that the two components in (1.11) can be written as,

$$E(U_A^1) = U P_A \sum_{k=0}^{N-1} \left[\sum_{l=0}^{k} C_l^{N-1} C_{k-l}^N P_A^l P_B^{k-l} P_B^{N-1-l} P_A^{N-(k-l)} \right]$$

$$E(U_A^2) = U P_A \sum_{k=N}^{2N-1} \left[\sum_{l=k-N}^{N-1} C_l^{N-1} C_{k-l}^N P_A^l P_B^{k-l} P_B^{N-1-l} P_A^{N-(k-l)} \frac{N}{k+1} \right]$$

where $C_k^N \equiv \frac{N!}{k!(N-k)!}$. After substituting out P_A and P_B using (1.3), the previous equations simplify to,

$$E\left(U_A^1\right) = \frac{U \sum_{k=0}^{N-1}\left[\sum_{l=0}^{k} C_l^{N-1} C_{k-l}^{N} \exp \beta\left[J_A(N-k+1+2l)+J_B(N+k-1-2l)\right]\right]}{\left(\exp \beta J_A + \exp \beta J_B\right)^{2N}}$$

$$E\left(U_A^2\right) = \frac{U \sum_{k=N}^{2N-1}\left[\sum_{l=k-N}^{N-1} C_l^{N-1} C_{k-l}^{N} \exp \beta\left[J_A(N-k+1+2l)+J_B(N+k-1-2l)\right]\right] \frac{N}{k+1}}{\left(\exp \beta J_A + \exp \beta J_B\right)^{2N}}$$

The last two expressions can be plugged into the top equation in (1.9) to obtain a first equation in terms of two variables J_A and J_B that characterises the steady-state.

Following similar steps, an expression $E\left(U_B\right)$ can also be written in terms of J_A and J_B. As before decompose $E\left(U_B\right)$ so that

$$E\left(U_B\right) = E\left(U_B^1\right) + E\left(U_B^2\right) \tag{1.12}$$

where the terms $E\left(U_B^1\right)$ and $E\left(U_B^2\right)$ are defined in an analogous way to $E\left(U_A^1\right)$ and $E\left(U_A^2\right)$ respecitvely. Following the same reasoning as before the expressions for $E\left(U_B^1\right)$ and $E\left(U_B^2\right)$ can be written as,

$$E\left(U_B^1\right) = \frac{U \sum_{k=0}^{N-1}\left[\sum_{l=0}^{k} C_l^{N-1} C_{k-l}^{N} \exp \beta\left[J_A(N+k-1-2l)+J_B(N-k+1+2l)\right]\right]}{\left(\exp \beta J_A + \exp \beta J_B\right)^{2N}}$$

$$E\left(U_B^2\right) = \frac{U \sum_{k=N}^{2N-1}\left[\sum_{l=k-N}^{N-1} C_l^{N-1} C_{k-l}^{N} \exp \beta\left[J_A(N+k-1-2l)+J_B(N-k+1+2l)\right]\right] \frac{N}{k+1}}{\left(\exp \beta J_A + \exp \beta J_B\right)^{2N}}$$

The above algebraic work allows me to write out the two equations in (1.9) in terms of the two unknowns that that must be solved for, J_A and J_B. However, the functional form of the expressions involved do not allow me to solve the system analytically. Instead, to best comment on the solutions to the system, the bottom equation is subtracted from the top one so that a single equation with a single unkown variable $\Delta \equiv J_A - J_B$ is derived. After much nettlesome algebra this step gives,

$$\gamma\Delta = \frac{U}{Z(\Delta)} \sum_{k=0}^{N-1}\left[\sum_{l=0}^{k} C_l^{N-1} C_{k-l}^{N} \frac{\left[\exp \beta\Delta(4l+2-2k)\right]-1}{\exp \beta\Delta(1+2l-k)}\right]$$
$$+ \frac{U}{Z(\Delta)} \sum_{k=N}^{2N-1}\left[\sum_{l=k-N}^{N-1} C_l^{N-1} C_{k-l}^{N} \frac{\left[\exp \beta\Delta(4l+2-2k)\right]-1}{\exp \beta\Delta(1+2l-k)} \frac{N}{k+1}\right] \tag{1.13}$$

where $Z\left(\Delta\right)$ is a function of Δ and is defined as $Z\left(\Delta\right) \equiv \sum_{j=0}^{2N} C_j^{2N} \exp\left[\beta\Delta(j-N)\right]$. Equation (1.13) can be further simplified by dividing both the numerator and the denominator of the fractions inside the square brackets by $\exp\left[\beta\Delta\left(1+2l-k\right)\right]$ and making use of the fact that $2\sinh\left(x\right) = \exp\left(x\right) - \exp\left(-x\right)$, where $\sinh\left(x\right)$ is the hyperbolic sine of x. The simplified equation is then given by,

$$\gamma\Delta \;=\; \frac{2U}{Z(\Delta)}\sum_{k=0}^{N-1}\left[\sum_{l=0}^{k}C_l^{N-1}C_{k-l}^N\sinh\left(\beta\Delta(2l+1-k)\right)\right]$$

$$+\;\frac{2U}{Z(\Delta)}\sum_{k=N}^{2N-1}\left[\sum_{l=k-N}^{N-1}C_l^{N-1}C_{k-l}^N\sinh\left(\beta\Delta(2l+1-k)\right)\frac{N}{k+1}\right]$$

$$\tag{1.14}$$

Let Δ^* be a (the) solution to (1.14). The steady-state shopping strategy of consumers can be expressed in terms of Δ^*. In particular, it follows from (1.3) that, in the steady-state, the probability for a consumer in G_1 of addressing shop A is $P_A^* = 1 - \frac{1}{1+\exp(\beta\Delta^*)}$ while its complement describes the probability of selecting shop B.

If $\Delta^* = 0$ is a solution to (1.14), the consumer will have an equally strong relation with both shops, $J_A^* = J_B^*$, and will be equally likely to address either shop, $P_A^* = P_B^* = \frac{1}{2}$. On the other hand, if the solution to (1.14) is large in absolute values then the steady-state will be characterised by consumers having established a bias towards addressing one of the two shops. If $\Delta^* > 0$ and large, then consumers in G_1 will have established a strong relation with shop A and a weaker one with shop B. The inverse will be true for consumers in G_2.

1.4 Results

Although I am not able to solve (1.14) and obtain an explicit expression for the solution Δ^*, I am able to characterize these solutions and hence describe the steady-state behaviour of consumers. The results of this exercise are presented below and a discussion of them will be presented in the subsequent section.

1.4.1 Results

To make the presentation clearer let $g(\Delta) \equiv LHS(\Delta) - RHS(\Delta)$, where $LHS(\Delta)$ is the term on the left-hand-side of equation (1.14) and $RHS(\Delta)$ the term on the right-hand-side of this same equation. It follows that solving for Δ in (1.14) is equivalent to finding the value(s) of Δ such that $g(\Delta) = 0$.

A series of observations on $g(\Delta)$ can be established- the proofs are left to Appendix 1.A.

Observation

(i) The function $g(\Delta)$ is continuous in Δ.

(ii) The function $g(\Delta)$ is odd, *i.e.*, $g(\Delta) = -g(-\Delta)$.

(iii) The function $g(\Delta)$ is convex for $\Delta \in \Re^+$.

(iv) There exists a $\bar{\Delta} \geq 0$ such that $g(\Delta) > 0$ and $g(-\Delta) < 0$ for $\Delta > \bar{\Delta}$.

Note that observations (i) and (ii) together imply that $g(0) = 0$ for all values of the paramaters U, β, γ and N. In addition, observations $(i), (ii)$ and (iii) together imply that $g(\Delta)$ is concave for $\Delta \in \Re^-$.

Taken together, the four observations point to the following: $g(\Delta)$ has either one or three solutions depending on whether its derivative at $\Delta = 0$ is positive or negative, respectively. In turn,[14]

$$\left.\frac{dg(\Delta)}{d\Delta}\right|_{\Delta=0} \gtreqless 0$$

$$\Longleftrightarrow \tag{1.15}$$

$$\frac{\gamma}{2\beta U} \gtreqless \underbrace{\frac{1}{2^{2N}}\left(\sum_{k=0}^{N-1}\sum_{l=0}^{k} C_l^{N-1}C_{k-l}^N (2l+1-k) + \sum_{k=N}^{2N-1}\sum_{l=k-N}^{N-1} C_l^{N-1}C_{k-l}^N (2l+1-k)\frac{N}{k+1}\right)}_{f(N)}$$

Let $f(N)$ denote the right-hand-side of (1.15). Note that $f(N)$ is a function of N alone. It is shown in Appendix 1.B that $f(N)$ has a range within the interval $(0, 0.25)$ and that, for $N > 2$, $f(N)$ is increasing in N.

The findings established so far are summarized by the following corollary:

Corollary 1.1 *If* $\frac{\gamma}{2\beta U} \geq f(N)$ *then there is a unique solution to* (1.14) *given by* $\Delta^* = 0$. *If, on the other hand* $\frac{\gamma}{2\beta U} < f(N)$ *then there will be three solutions of which* $\Delta^* = 0$ *it is one while the other two are opposite in sign but equal in absolute value.*

Proof. The proof follows from the four observations listed earlier. ∎

Figure 1.1 illustrates Corollary 1.1. The figure plots $g(\Delta)$ for different values of the ratio $\frac{\gamma}{2\beta U}$. For low values of β such that $\frac{\gamma}{2\beta U} > f(N)$, the function $g(\Delta)$ has a positive derivative at $\Delta = 0$ and it has a unique zero. On the other hand, for higher β, the slope of $g(\Delta)$ at $\Delta = 0$ is negative and the curve intersects the horizontal axis thrice.

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[14]See Appendix 1.B
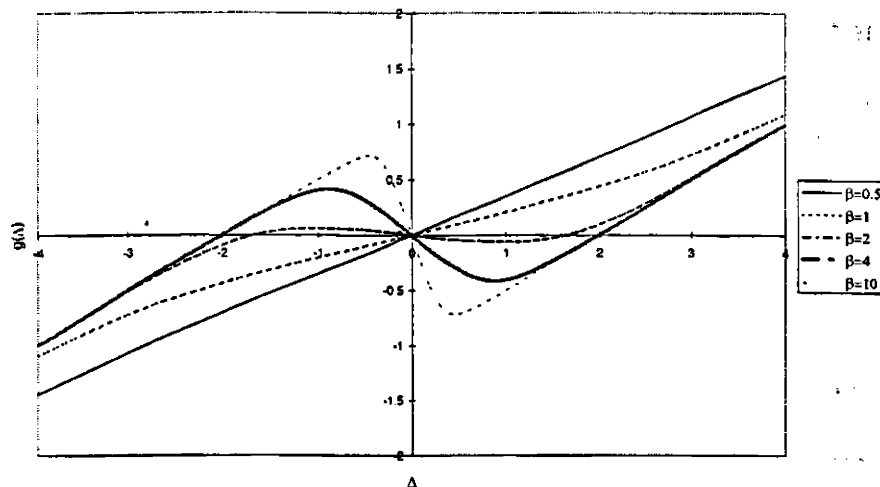
Figure 1.1: Plot of $g(\Delta)$ for different values of β. ($\gamma = 0.5, U = 1, N = 4$)

When the parameter values are such that $g(\Delta)$ has three solutions the central solution, $\Delta^* = 0$ describes an unstable stationary point. A small perturbation will set the consumers' mixed strategy to evolve to a new steady-state. The new steady-state will be associated with one of the other two exterior solutions which are stable. However, when $\Delta^* = 0$ is the unique solution it corresponds to a stable steady-state. Appendix 1.C lays out the proof of these claims.

When there are two stable steady-states, these are charactized by non-zero values of Δ^* that are equal in absolute terms and opposite in sign. In the steady-state associated with a positive Δ^*, consumers from G_1 have a bias towards addressing shop A, whilst consumers from G_2 will have an equally strong bias in favour of shop B. On the other hand, in the steady-state associated with the negative root Δ^*, the biases of these two groups towards the shops are reversed.

1.5   Discussion

In this section I carry out a comparative statics exercise to examine how the parameters β, γ, U and N affect the steady-state behaviour of the consumers. I first consider the parameters which can be interpreted as being characteristics of consumers - β, γ, U - and then examine the effect of market size, proxied by N.

1.5.1   Characteristics of consumers

As pointed out earlier, condition (1.15) implies that for low values of β the unique solution to (1.14) is $\Delta^* = 0$. This corresponds to the scenario where consumers address either store with an equal probability, $P_A^* = P_B^* = 0.5$. On the other hand, high values of β lead to one of the two possible stable steady-states where consumers have a bias towards visiting one of the two stores.

The parameter β influences not only the nature of the steady-state behaviour - whether consumers remain indifferent between the two stores or they develop a bias towards one of them - but it also affects the magnitude of the bias. Larger values of β are associated with larger absolute values of the solution Δ^*, which implies consumers exhibit a stronger bias towards one of the two stores. It is insightful to describe the intuition behind this here although the proof is left to Appendix 1.D.

By definition, a higher value of β means that consumers attach a greater relevance to the difference in the value of their fidelity parameters with the two shops. Consequently, when β is higher, the difference in the probability of visiting one shop over the other is wider in favour of the shop with whom the higher fidelity parameter is held.[15] In turn, this will increase the gap between a consumer's two fidelity parameters as the consumer expects to enjoy a greater expected utility from the store with whom he already has a bias towards. Two forces work in this direction. On the one hand, other things equal, a store's contribution to the utility a consumer expects to enjoy is greater the higher the probability of the consumer of visiting that store. On the other hand, conditional on being at a store, the difference in the utility a consumer expects to enjoy - which is to say, the difference in the probability

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[15] This reasoning is captured by the expression for the derivative presented in (1.4) and which can be re-written as $\frac{\partial \left(\frac{P_A}{P_B} \right)}{\partial \beta} = \Delta e^{\beta \Delta} > 0 \Leftrightarrow J_A > J_B$.
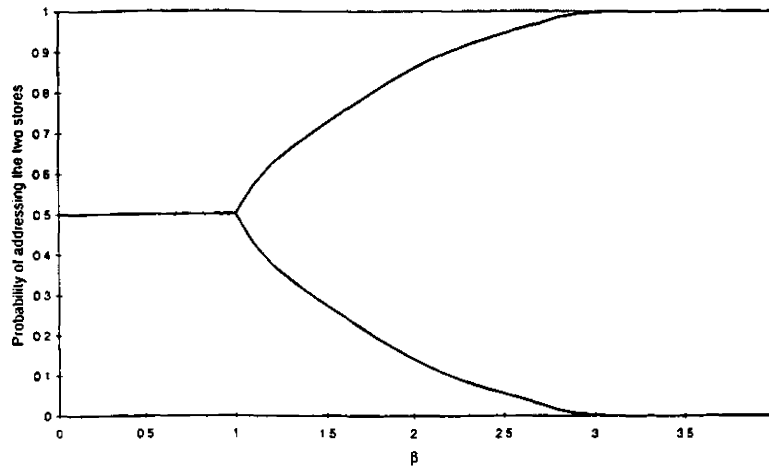
Figure 1.2: Plot of steady-state probabilities against β. ($\gamma = 0.5, U = 1, N = 30$)

of being served - at the store for which he has a preference compared to the other store is increasing with the size of the bias.[16]

I am not able to illustrate the previous intuition formally. Instead, resort is made to numerical simulations and the output from one such exercise is summarized in Figure 1.2. For a given set of values for N, γ and U, the figure draws the steady-state shopping strategies for values of β in the interval $[0, 4]$. For sufficiently low values of β, the symmetric solution is unique and the consumer will patronise both stores with a probability of $\frac{1}{2}$. Once β is large enough so that $\frac{\gamma}{2\beta U} < f(N)$ then consumers establish a preferential relation with one of the two stores. The strength of this relation, mirrored by the probability of addressing a store, is increasing in β and rapidly approaches the limit values of 0 and 1.

The effects of the parameters γ and U on the solutions Δ^* are as expected intuitively and match those established by Weisbuch et al. (1995). Condition (1.15) implies that high values of U and low values of γ are associated with

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[16]As consumers' bias approaches the limiting case of visiting one store with certainty and the other with zero probability, the difference in the conditional expected utility approaches $\frac{U}{N+1} > 0$.

consumers developing a preferential relation towards one of the stores. On the other hand, if U is low or γ large enough, consumers will be indifferent to puchasing from either store. In addition, if the values of U and γ are such that consumers' behaviour is characterized by a bias towards a given store, then for higher values of U and/or lower values of γ, the bias will be even stronger (see Appendix 1.D). The intuition for this is as follows. For a high U, a small sequence of succesful visits to the same store is sufficient to create a significant gap between the fidelity parameters a consumer holds with respect to the two stores. In turn, this gap will bias future choices towards the store just visited and future succesful visits to the same shop will widen this gap further. In addition, if γ is small, the gap will narrow only slightly in the event that a visit to the preferred shop does not reward the consumer with the good.

1.5.2   Market Size

In this section I will examine how the steady-state behaviour of consumers is affected by the number of consumers in the market.

Recall that the function $f(N)$ in (1.15) is increasing in N and that depending on whether the ratio $\frac{\gamma}{2\beta U}$ is smaller or greater than $f(N)$, consumers will be, respectively, indifferent between the stores or have a bias towards one.[17] This leads to the following corollary.

Corollary 1.2 *The parameter space for β, γ and U such that consumers develop a bias towards one of the shops is weakly greater as the number of consumers in the market increases.*

Proof. The proof follows from the previous discussion on the properties of $f(N)$. ∎

Corollary 1.2 states that consumers are more likely to become loyal to one of the shops as the market size increases. This does not imply, however, that if N is sufficiently high consumers will become loyal to a store or that if N is low enough, consumers will be indifferent between stores. Indeed, if the ratio $\frac{\gamma}{2\beta U}$ is sufficiently low consumers will establish a mixing behaviour between stores for any N. On the other hand, for sufficiently high values of

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[17]This statement should be qualified as, '$f(N)$ is increasing in N for $N > 2$ '. This is overlooked in the text in order to make the presentation more fluid.

28

$\frac{\gamma}{2\beta U}$ consumers will always develop a bias towards a store independently of market size.

However, if the parameter values are such that consumers exhibit a bias towards one store in the steady-state, then the *magnitude* of this bias is increasing with N.

The intuition for the role of N on the steady-state behaviour of consumers is the following.

In the steady-state of the model presented below, the shopping strategies across the population of consumers are such that all consumers are *expected* to be served - the expected number of customers addressing each shop is equal to the number of units of the good available for sale, N. However, the choice of store to visit by an individual consumer is the realization of a random variable - as made explicit by the choice rule (1.3). In terms of the market as a whole, the realized division of the patronage between the two shops is the result of $2N$ such realizations. It follows that the realized outcome is not necessarily equal to the expected outcome. In particular, it is likely that the realized division of consumers between the two stores on a given purchase opportunity is such that more than N will go to the same store. When this occurs, rationing will take place. By the law of large numbers, it follows that as N increases the deviation of the realized outcome from the expected outcome is likely to fall. In other words, as N increases, it will be less likely that rationing will have to take place and hence the higher the utility a consumer can expect to enjoy. In the limit, as N goes to infinity, the realized outcome will equal the expected outcome and consumers will be certain of being served by the store they address. This limiting point is equivalent to the setting described by Weisbuch et al. (1995) where consumers are always assured service.

It is timely to contrast the above results with those established by Weisbuch et al. (1995). In the latter paper, consumers exhibit a mixing behaviour in the steady-state if $\frac{\gamma}{2\beta U} \geq 0.25$ and they will have a bias towards one of the two shops otherwise.[18] Given that $f(N) \leq 0.25$ it follows that if $\frac{\gamma}{2\beta U} \geq 0.25$ then it will also be the case that $\frac{\gamma}{2\beta U} \geq f(N)$.

This discussion can be summarized by the following corollaries.

Corollary 1.3 *If β, γ and U are such that in the model of Weisbuch et*

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[18]See equation (12) in Weisbuch et al. (1995). The term Π in that paper is equivalent to the term U in the model presented here.

al. (1995) *consumers have mixing behaviour then it will be the case that this same behaviour will be displayed by consumers in the steady-state of the model presented above. The reverse, however, is not true.*

Corollary 1.4 *The lower the number of consumers in the market, the more likely it is that the results of Weisbuch et al. (1995) and those of the model of Section 1.3 diverge. Only when the number of consumers approaches infinity do the results of the two models coincide.*

Proof. The proof of Corollary 1.3 follows from the discussion set-out above. Corollary 1.4, on the other hand, follows directly from the fact that $f(N)$ is increasing in N and that $\lim_{N\to\infty} f(N) = 0.25$. ∎

The *divergence* between the two models mentioned in Corollary 1.4 refers to the differences in the nature of consumer behaviour - whether a bias in favour of a store is developed or not. Loyal behaviour by consumers is more likely to emerge in Weisbuch et al. (1995) than in the model presented above.

Relative scarcity

By construction, the number of units of the good stocked by the two shops changes one-to-one with the number of consumers: both are set equal to $2N$. Therefore, altering N does not change the relative scarcity of the good in the market. The goal of this section is to examine how the relative scarcity of the good in the market affects the consumers' steady-state behaviour. To carry this out it is necessary to separate the number of consumers from the number of units of the good in the market. This is done by assuming that each of the two stores stock M units in each period while there continue to exist $2N$ consumers in the market. M can be smaller, equal or greater than N.

The steps to tackle this modified model are the same as those taken to solve the original model of Section 1.4. While Appendix 1.E details how this modified model is solved, only a very brief account is presented here. The superscript ^{m} will be added to the notation used in Section 1.3 when referring to the *modified* model.

As before, a system of two equations with the two unknowns J_A and J_B is derived and from this a single equation -analogous to (1.14)- with the single unknown Δ^m is obtained. The observations (*i*) through (*iv*) made in Section 1.4.1 in relation to (1.14) also hold with respect to the analogous equation
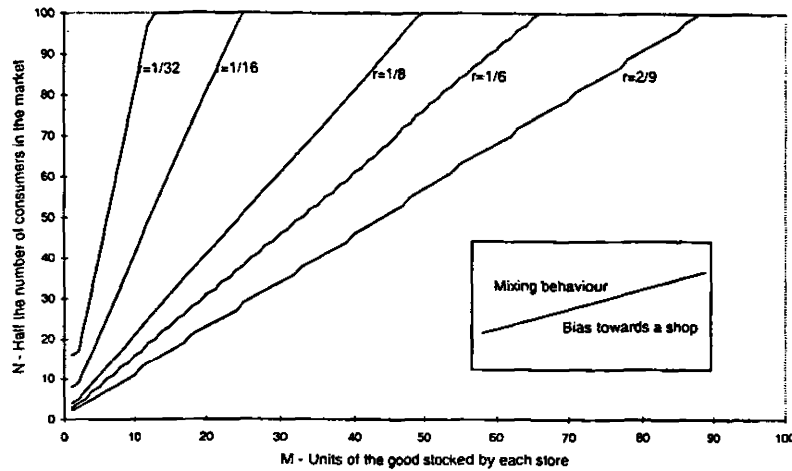
Figure 1.3: Boundaries in the $M \times N$ space between the twp types of steady-state beaviour for different values of the ratio $r = \frac{\gamma}{2\beta U}$.

derived from the modified model. It follows that, as before, it is possible to derive a condition which determines whether the modified model will have the unique stable solution $\Delta^{m*} = 0$ or three solutions of which $\Delta^{m*} = 0$ is one but is unstable and the other two are equal in absolute value. The condition determining whether there will be one or three solutions is derived in an analogous way to that used to obtain (1.15). There will be a unique solution if $\frac{\gamma}{2\beta U} \geq f^m(M, N)$ and three solutions when $\frac{\gamma}{2\beta U} < f^m(M, N)$. The function $f^m(M, N)$ is described in Appendix 1.F.

Note that if $M \geq 2N$ each store is able to satisfy the demand of all consumers single-handed which implies that customers are always certain of being served. Indeed, for $M \geq 2N$, $f^m(M, N) = 0.25$ and hence the condition determining the characterisation of the steady-state boils down to $\frac{\gamma}{2\beta U} \lessgtr 0.25$. This agrees with the condition established in Weisbuch et al. (1995) where, by assumption, consumers are always assured service.

The function $f^m(M, N)$ has a range in the interval $(0, 0.25)$ and it can be shown numerically that the function is increasing in M and decreasing

in N.[19] This implies that for a given set of parameters U, β and γ the condition $\frac{\gamma}{2\beta U} < f^m(M, N)$ is more likely to be satisfied for smaller M and larger N. Other things equal, therefore, consumers are more likely to develop a preference towards one of the shops for higher values of M and for lower values of N. Conversely, consumers are more likely to develop a mixing behaviour for low values of M and high values of N. This result is illustrated in Figure 1.3. For given values of the ratio $\frac{\gamma}{2\beta U}$, the figure draws the boundary in the $N \times M$ space separating the type of behaviour consumers exhibit in the steady-state. In the area above a line, the values of N and M are such that consumers have a mixing behaviour and address either store with an equal probability. On the other hand, the area below the line marks the region where consumers develop a bias towards patronising one of the shops. For a given value of $\frac{\gamma}{2\beta U}$ and fixing the number of consumers - fixing N - increasing the number of units of the good stocked by each store, one moves from the region where consumers are indiffernt between stores to that where consumers have a preference towards one. The opposite is true if for a given M, the value of N is increased.

As Figure 1.3 illustrates, the smaller the ratio $\frac{\gamma}{2\beta U}$, the larger the area in the $M \times N$ space where consumers will have a bias towards shopping at one of the stores. It is not the case, however, that there will always be two distinct regions. In particular, the region where consumers exhibit a preference towards a store might be empty. This follows directly from the fact that the function $f^m(M, N)$ is contained within the interval $(0, 0.25)$ and that if the parameter values are such that $\frac{\gamma}{2\beta U} > 0.25$, then the condition $\frac{\gamma}{2\beta U} > f^m(M, N)$ is satisfied for all N and M. In turn, this means that in the entire $N \times M$ space the steady-state behaviour will be characterized by consumers being indifferent towards either store.

When the relative values of M and N are such that consumers develop a bias towards a given store, the magnitude of the bias itself is also influenced by M and N. Although I am not able to show analytically how this bias varies with N and M, numerical simulations can be carried out. Doing so, points towards the following result. Consumers' bias towards a shop is increasing in M and decreasing in N. For a given value of M, Figure 1.4 shows how the steady-state probabilities of consumers addressing both shops change with N. For low values of N, consumers have a strong preference towards one of the shops, patronising it on more than 90% of the occasions. As N increases,

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[19]See Appendix 1.F.
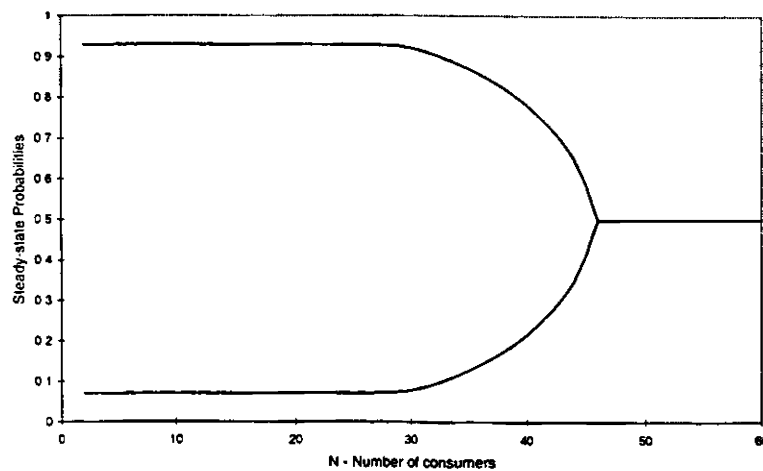
Figure 1.4: Steady-state behaviour as a function of the number of consumers in the market. ($M = 30, U = 1, \gamma = 1, \beta = 1$)

the bias towards the shop falls, first gradually and then rapidly, until it does not exist any longer.

At first, the relation established above between the relative scarcity of the good in the market and the existence of preferential relations between a seller and a shop might appear to be counter-intuitive. It could be expected that the greater the scarcity, the more likely it would be that a consumer returns to a familiar shop in order to ensure that he gets served. While this reasoning could be valid in a more general context, it cannot hold in the model discussed here where sellers treat all consumers equally. Consequently, being loyal to a shop is no guarantee of being served. Instead, the relation between relative scarcity and loyalty is better, and more simply, explained by the fact that the greater the scarcity - the lower the ratio $\frac{M}{N}$ - the lower is the utility that a consumer expects to gain from each visit. This in turn, lowers the tendency to create strong relations as it has an effect analogous to that of lowering U. The opposite is true when $\frac{M}{N}$ increases so that there are less consumers for each unit of the good placed in the market.

1.6 Conclusion

The starting point of the analysis in this chapter is the view that the relations between sellers and buyers are the result of an on-going process where agents learn from their experience whom to trade with in the future. The model presented here simplifies this view by assuming that buyers alone have the ability to learn and that sellers play a passive role. In doing so, it follows closely the work of Weisbuch et al. (1995). In the latter paper, however, consumers are assured of having their demand met whichever seller they choose to address and so there is no rationale for why they should learn in the first place. The above analysis addresses this by allowing for the possibility that a consumer is faced with empty shelves when visiting a store.

In comparison with the setting where such uncertainty does not exist, buyers are more likely to adopt a mixing behaviour rather than become loyal to a particular store. The size of the market - given by the number of buyers - influences the behaviour of buyers in the steady-state. As the number of buyers in the market grows, the more likely it is that buyers become loyal to a particular store.

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36

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Appendices

1.A Proof of observations $(i) - (iv)$

i) The function $g(\Delta)$ can be decomposed as the sum, difference, product and quotient of contunuos functions. It follows that $g(\Delta)$ itself will be continuous.

ii) Express the function $g(\Delta)$ as $g(\Delta) = \gamma\Delta - RHS(\Delta)$ where $RHS(\Delta)$ is the right-hand-side expression in (1.14). It follows that $g(-\Delta) = -\gamma\Delta - RHS(-\Delta)$. Taking care to explicitly write Z as a function of Δ, a typical term, RHS_{lk}, within the summation signs of $RHS(-\Delta)$ is given by,

$$RHS_{lk}(-\Delta) = C_l^{N-1} C_{k-l}^N \frac{U}{Z(-\Delta)} \sinh\left(-\beta\Delta\left(1 + 2l - k\right)\right)$$

Note that $\sinh(-x) = -\sinh(x)$ and that $Z(-\Delta) = Z(\Delta)$. Hence, the term $RHS_{lk}(-\Delta)$ can be expressed as

$$
\begin{aligned}
RHS_{lk}(-\Delta) &= -C_l^{N-1} C_{k-l}^N \frac{U}{Z(\Delta)} \sinh\left(\beta\Delta\left(1 + 2l - k\right)\right) \\
&= -RHS_{lk}(\Delta)
\end{aligned}
$$

The above relation holds for each individual summand of $RHS(\Delta)$ and therefore $g(-\Delta) = -\gamma\Delta + RHS(\Delta) = -g(\Delta)$. This proves that the function $g(\Delta)$ is odd.

iii) It is proved that $g(\Delta)$ is convex for $\Delta \in \Re^+$, by showing that $\frac{d^2 g(\Delta)}{d\Delta^2} > 0$ for $\Delta > 0$. Note that $g(\Delta)$ can be expressed as,

$$
\begin{aligned}
g(\Delta) &= \gamma J_A - E(U_A) - (\gamma J_B - E(U_B)) \\
&= \gamma\Delta - (E(U_A) - E(U_B))
\end{aligned}
$$

which is the equation in the background of (1.14).

First, consider the term $E(U_A)$. Recall from (1.11) that $E(U_A) = E(U_A^1) + E(U_A^2)$ where,

$$E(U_A^1) = U P_A \sum_{k=0}^{N-1} \left[\sum_{l=0}^{k} C_l^{N-1} C_{k-l}^N P_A^l P_B^{k-l} P_B^{N-1-l} P_A^{N-(k-l)} \right] \quad (1.16)$$

$$E(U_A^2) = U P_A \sum_{k=N}^{2N-1} \left[\sum_{l=k-N}^{N-1} C_l^{N-1} C_{k-l}^N P_A^l P_B^{k-l} P_B^{N-1-l} P_A^{N-(k-l)} \frac{N}{k+1} \right]$$

Therefore, the first derivative $\frac{dg(\Delta)}{d\Delta}$ is given by

$$
\begin{aligned}
\frac{dg(\Delta)}{d\Delta} &= \gamma - \frac{dE(U_A)}{d\Delta} + \frac{dE(U_B)}{d\Delta} \\
&= \gamma - \left(\frac{dE(U_A^1)}{d\Delta} + \frac{dE(U_A^2)}{d\Delta} \right) + \left(\frac{dE(U_B^1)}{d\Delta} + \frac{dE(U_B^2)}{d\Delta} \right)
\end{aligned}
$$

The expressions for $\frac{dE(U_j^c)}{d\Delta}$, $c = \{1,2\}$, $j = \{A, B\}$ can be obtained using the chain rule $\frac{dE(U_j^c)}{d\Delta} = \frac{dE(U_j^c)}{dP_A} \frac{dP_A}{d\Delta} + \frac{dE(U_j^c)}{dP_B} \frac{dP_B}{d\Delta}$. In turn $\frac{dP_A}{d\Delta} = \beta \frac{\exp(\beta\Delta)}{(1+\exp(\beta\Delta))^2} = \beta P_A P_B = -\frac{dP_B}{d\Delta}$ whilst the terms for $\frac{dE(U_j^c)}{dP_A}$ and $\frac{dE(U_j^c)}{dP_B}$ can be derived straight-forwardly from (1.16).

For the sake of presentation I do not present either and only note that both $\frac{dE(U_A)}{d\Delta}$ and $\frac{dE(U_B)}{d\Delta}$ are functions of P_A and P_B.

The second derivative $\frac{d^2g(\Delta)}{d\Delta^2}$ is given by,

$$
\frac{d^2 g(\Delta)}{d\Delta^2} = -\frac{d^2 E(U_A)}{d\Delta^2} + \frac{d^2 E(U_B)}{d\Delta^2}
$$

which simplifies to

$$
\frac{d^2 g(\Delta)}{d\Delta^2} = \beta^2 U \left[\sum_{k=0}^{N-1} \sum_{l=0}^{k} C_l^{N-1} C_{k-l}^{N} H(P_A, P_B) + \sum_{k=N}^{2N-1} \sum_{l=k-N}^{N-1} C_l^{N-1} C_{k-l}^{N} H(P_A, P_B) \frac{N}{k+1} \right]
$$

where,

$$
\begin{aligned}
H(P_A, P_B) &= (T_1 P_A - T_2 P_B)^2 \left(P_A^{T_2} P_B^{T_1} \right) - (T_2 P_A - T_1 P_B)^2 \left(P_A^{T_1} P_B^{T_2} \right) \\
&\quad + 2N \left(P_A^{T_3} P_B^{T_4} - P_A^{T_4} P_B^{T_3} \right) \\
T_1 &= N + 2l - k + 1 \\
T_2 &= N - 2l + k - 1 \\
T_3 &= N + 2l - k + 2 \\
T_4 &= N - 2l + k
\end{aligned}
$$

The challenge now is to show that this expression is non-negative for all $\Delta \geq 0$. Note that when $\Delta \geq 0$ then $P_A \in (0.5, 1)$. In addition, given that both β and U are positive by assumption, the sign of the expression for $\frac{d^2g(\Delta)}{d\Delta^2}$ depends on P_A and N alone. It follows that the convexity of $g(\Delta)$ in the
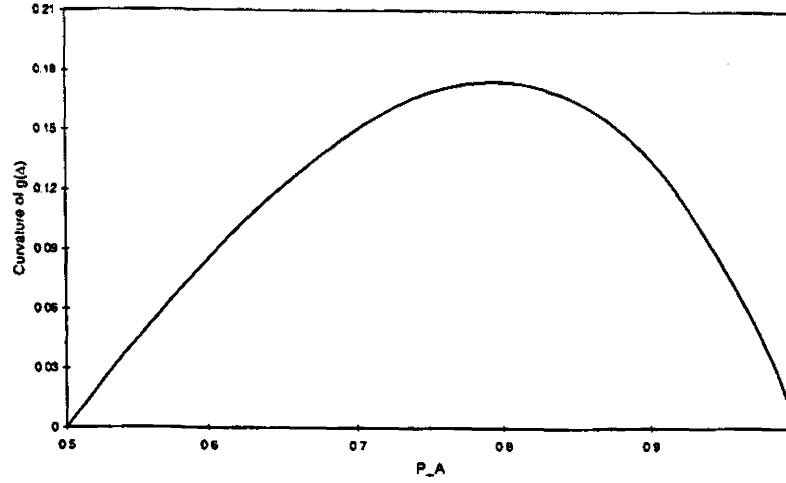
Figure 1.5: Plot of curvature of $g(\Delta)$.

positive quadrant can be proved by showing that $\frac{d^2 g(\Delta)}{d\Delta^2} \geq 0$ for $P_A \in (0.5, 1)$ for all N. To show this I resorted to numerical analysis and plotted $\frac{d^2 g(\Delta)}{d\Delta^2}$ in the range $P_A \in (0.5, 1)$ and for all values of N up to 100. Figure 1.5 illustrates the typical plot obtained from this exercise. For all values of N used, the curve was above the horizontal axis which implies that the curvature of $g(\Delta)$ is positive and hence that the function is convex for $\Delta \geq 0$. This completes the proof.

iv) The function $g(\Delta)$ can be re-written as,

$$g(\Delta) = \gamma\Delta - U\left[P_A * P(S_A) - P_B * P(S_B)\right] \tag{1.17}$$

where P_A is the probablity of consumer $i \in G_1$ addressing shop A and $P(S_A)$ is the probability of being served at A. P_B and $P(S_B)$ are similarly defined in relation to shop B.

It follows from the choice rule (1.3) that $\lim_{\Delta\to\infty} P_A = 1$ and that $\lim_{\Delta\to\infty} P(B) = 0$. In turn, from (1.6), this implies that $\lim_{\Delta\to\infty} P(S_A) = 1$ and $\lim_{\Delta\to\infty} P(S_B) = \frac{N}{N+1}$. Putting these results together, it follows from (1.17) that $\lim_{\Delta\to\infty} g(\Delta) = \lim_{\Delta\to\infty} \gamma\Delta - U > 0$. In turn, this result coupled with the fact that $g(\Delta)$ is

continuous, odd and convex for $\Delta \in \Re^{+}$ implies that there exists a $\bar{\Delta} \geq 0$ such that $g(\Delta) > 0$, for $\Delta > \bar{\Delta}$. Lastly, given $g(\Delta)$ is an odd function, it follows that $g(-\Delta) < 0$ for $\Delta > \bar{\Delta}$.

1.B Deriving condition (1.15) and the properties of $f(N)$

Note that $\frac{d\sinh(\Delta)}{d\Delta} = \cosh(\Delta)$ and that $\cosh(0) = 1$, $\sinh(0) = 0$ and $Z|_{\Delta=0} = \sum_{j=0}^{2N} C_j^{2N} = 2^{2N}$. It is then straight-forward to derive that,

$$\frac{dg}{d\Delta}\Big|_{\Delta=0} = \gamma - \frac{23U}{2^{2N}}\left(\sum_{k=0}^{N-1}\sum_{l=0}^{k} C_l^{N-1}C_{k-l}^{N}(2l+1-k) + \sum_{k=N}^{2N-1}\sum_{l=k-N}^{N-1} C_l^{N-1}C_{k-l}^{N}(2l+1-k)\frac{N}{k+1}\right)$$

Condition (1.15) in the text follows directly.

The proof that $f(N) \in (0, 0.25)$ is post-poned to Appendix 1.E where it will be shown that the function $f^m(M, N)$ - of which $f(N)$ is a special case - is contained within the interval $(0, 0.25)$.

I have not been able to show analytically that $f(N)$ is increasing in N. However, the plot of this function shows that for $N > 2$, $f(N)$ is increasing.

1.C Stability of steady-states

It follows from (1.7) that,

$$\begin{aligned}
\frac{d\Delta}{dt} &= \frac{dJ_A}{dt} - \frac{dJ_B}{dt} = E(U_A) - E(U_B) - \gamma\Delta \qquad (1.18) \\
&= -g(\Delta)
\end{aligned}$$

The stability of the stationary points can be shown by using the properties of $g(\Delta)$ described in Section 1.5.1

It is straight-forward to show that if $\Delta^* = 0$ is the unique solution then it is a stable stationary point. As established in Section 1.5.1, if $\Delta^* = 0$ is the unique solution when the slope of $g(\Delta)$ is positive at $\Delta = 0$. When this is the case, it follows that $g(\varepsilon) > 0$, and that $g(-\varepsilon) < 0$, where $\varepsilon > 0$ and small. Hence, the direction of change in the neighbourhood of $\Delta^* = 0$, given by $\frac{d\Delta}{dt} = -g(\Delta)$, is towards the point $\Delta^* = 0$. The steady-state is therefore stable.

Now consider the case where there are 3 solutions.

It was shown in the text that $g(\Delta) = 0$ has 3 solutions when $\frac{dg}{d\Delta}\big|_{\Delta=0} < 0$. When this is the case, $g(\varepsilon) < 0$ and $g(-\varepsilon) > 0$ for $\varepsilon > 0$ and small. Therefore, it follows that $\Delta^* = 0$ is not a stable stationary point.

It remains to examine the steady-states associated with the two non-zero solutions. Consider first the steady-state associated with the positive solution, denoted by Δ^+. Given that for such a solution to exist it must be the case that $\frac{dg}{d\Delta}\big|_{\Delta=0} < 0$ and given $g(\Delta)$ is convex in $\Re^+$ and that that for a sufficiently high value of Δ the function $g(\Delta)$ is positive, it follows that at Δ^+ the function $g(\Delta)$ intersects the horizontal axis from below. Furthermore, given the convexity of $g(\Delta)$ it follows that $g(\Delta^+ + \varepsilon) > 0$ and $g(\Delta^+ - \varepsilon) < 0$. Together with (1.18), this ensures that around Δ^+ the movement is towards Δ^+ and hence the steady-state associated with Δ^+ is stable. An analogous reasoning can be made to show that the steady-state associated with the the symmetric solution $-\Delta^+$ is also stable.

1.D Effect of β, γ and U on solution Δ^*

1.D.1 Parameter β

Let $\Delta^* \neq 0$ be the absolute value of the non-zero solutions to (1.14) for $\beta = \bar{\beta}$ and $\Delta^{*\prime}$ be the absolute value of the non-zero solutions for $\beta = \bar{\beta}'$. It is shown that if $\bar{\beta}' > \bar{\beta}$ then $\Delta^{*\prime} > \Delta^*$.

To simplify the presentation I restrict the attention in the rest of the proof to the positive quadrant where $\Delta > 0$. Analogous steps could be repeated to tackle the solution lying in the negative quadrant.

Recall that (1.14) is obtained from expanding the equation, $\gamma\Delta = E(U_A) - E(U_B)$. Note also that $E(U_A) - E(U_B) = 0$ for $\Delta = 0$ and, from observation (iii) in Section 1.5.1, that $E(U_A) - E(U_B)$ is concave in Δ for $\Delta > 0$. It follows that the curve which plots $E(U_A) - E(U_B)$ will cut the curve for $\gamma\Delta$ from above and that the intersection point corresponds to the solution Δ^*. Therefore, to show that if $\bar{\beta}' > \bar{\beta}$ then $\Delta^{*\prime} > \Delta^*$ one has to show that the curve for $E(U_A) - E(U_B)$ shifts upwards as β increases.

$$\frac{d\left(E\left(U_A\right)-E\left(U_B\right)\right)}{d\beta}=\frac{d\left(E\left(U_A\right)-E\left(U_B\right)\right)}{dP_A}\frac{dP_A}{d\beta}+\frac{d\left(E\left(U_A\right)-E\left(U_B\right)\right)}{dP_B}\frac{dP_B}{d\beta}$$

This derivative can be signed by recalling that $\frac{d(E(U_A)-E(U_B))}{dP_A}>0, \frac{dP_A}{d\beta}>0$ and that $\frac{d(E(U_A)-E(U_B))}{dP_B}<0$ and $\frac{dP_B}{d\beta}<0$. Therefore, $\frac{d(E(U_A)-E(U_B))}{d\beta}>0$. This completes the proof.

1.D.1 Parameters γ and U

It is clear from (1.14) that γ and U only affect the solution to that equation if their ratio $\frac{\gamma}{U}$ changes.

Define the function $h\left(\Delta\right)\equiv\frac{\gamma}{U}\Delta-h'\left(\Delta\right)$ where $h'\left(\Delta\right)\equiv\frac{RHS(\Delta)}{U}$ and $RHS\left(\Delta\right)$ is the term on the right-hand-side of (1.14). Note that the term U cancels out so that $h'\left(\Delta\right)$ is a function of N and Δ alone. In addition, note that, like the function $g\left(\Delta\right)$ defined in 1.4.1, $h\left(\Delta\right)$ is convex in the positive quadrant and concave in the negative quadrant.

The steady-state solutions Δ^* are given by solving $h\left(\Delta\right)=0$, or graphically, by the intersection of the curve $h\left(\Delta\right)$ with the horizontal axis. It follows from the definition of $h\left(\Delta\right)$ that higher values of $\frac{\gamma}{U}$ would shift $h\left(\Delta\right)$ upwards in the region $\Delta>0$ and downwards in the region $\Delta<0$. Given the convexity of $h\left(\Delta\right)$ in the region where $\Delta>0$ and its concavity when $\Delta<0$, it follows that the intersection of the curve $h\left(\Delta\right)$ with the horizontal axis will occur at lower absolute values of Δ for higher values of $\frac{\gamma}{U}$. This completes the proof.

1.E Deriving and solving the modified model of Section 1.5.2.1

The steps taken to set-up and solve the modified model follow very closely those taken in Section 1.4. Accordingly, I shall not describe all the steps involved to solve this modified model and instead will only highlight the key stepping-stones.

As in Section 1.4.1.1 the first step is to derive an expression for the expected utility enjoyed at each store. For store A,

$$E\left(U_A\right) = \underbrace{U * P_A * \left[\Pr\left(n_A \le N - 1\right)\right]}_{E\left(U_A^1\right)} + \underbrace{U * P_A * \sum_{k=N}^{2N-1} \left[\Pr\left(n_A = k\right) * \frac{N}{(k+1)}\right]}_{E\left(U_A^2\right)}$$

$$(1.19)$$

where, as before, the first component $E\left(U_A^1\right)$ reflects the case where less than M consumers visit store A and hence are assured to have their demand met. On the other hand, $E\left(U_A^2\right)$ is the contribution to the expected utility from the event that at least M other consumers visit store A so that rationing must take place. The two components can be expressed in terms of P_A and P_B as,

For $\quad M \le N$

$$E\left(U_A^1\right) \quad = \quad U P_A \sum_{k=0}^{M-1} \left(\sum_{l=0}^{k} C_l^{N-1} C_{k-l}^N P_A^l P_B^{k-l} P_B^{N-1-l} P_A^{N-(k-l)} \right)$$

$$E\left(U_A^2\right) \quad = \quad U P_A \left[\sum_{k=M}^{N-1} \left(\sum_{l=0}^{k} C_l^{N-1} C_{k-l}^N P_A^l P_B^{k-l} P_B^{N-1-l} P_A^{N-(k-l)} \frac{M}{k+1} \right) \right.$$

$$+ \quad \left. \sum_{k=N}^{2N-1} \left(\sum_{l=k-N}^{N-1} C_l^{N-1} C_{k-l}^N P_A^l P_B^{k-l} P_B^{N-1-l} P_A^{N-(k-l)} \frac{M}{k+1} \right) \right]$$

For $\quad M > N$

$$E\left(U_A^1\right) \quad = \quad U P_A \left[\sum_{k=0}^{N-1} \left(\sum_{l=0}^{k} C_l^{N-1} C_{k-l}^N P_A^l P_B^{k-l} P_B^{N-1-l} P_A^{N-(k-l)} \right) \right.$$

$$+ \quad \left. \sum_{k=N}^{M-1} \left(\sum_{l=k-N}^{k} C_l^{N-1} C_{k-l}^N P_A^l P_B^{k-l} P_B^{N-1-l} P_A^{N-(k-l)} \right) \right]$$

$$E\left(U_A^2\right) \quad = \quad U P_A \sum_{k=M}^{2N-1} \left(\sum_{l=k-N}^{N-1} C_l^{N-1} C_{k-l}^N P_A^l P_B^{k-l} P_B^{N-1-l} P_A^{N-(k-l)} \frac{M}{k+1} \right)$$

The terms P_A and P_B can be substituted out so that the two components $E\left(U_A^1\right)$ and $E\left(U_A^2\right)$ are expressed as functions of J_A and J_B. This allows us to write out an equation analogous to (1.8) where the unkown variables are J_A and J_B.

Repeating the same reasoning as above with respect to the utility expected to be enjoyed at store B one derives an expression for $E(U_B)$ and from this obtain a second equation in terms of the two unknowns J_A and J_B.

Subtracting the second equation from the first, yields after much algebra a single equation where the unknown variable is $\Delta \equiv J_A - J^B$,

For $\underline{M} \leq \underline{N}$

$$
\gamma\Delta = \frac{2U}{Z(\Delta)} \left[\sum_{k=0}^{M-1} \left(\sum_{l=0}^{k} C_l^{N-1} C_{k-l}^N \sinh\left(\beta\Delta(2l+1-k)\right) \right) \right.
$$

$$
+ \sum_{k=M}^{N-1} \left(\sum_{l=0}^{k} C_l^{N-1} C_{k-l}^N \sinh\left(\beta\Delta(2l+1-k)\right) \frac{M}{k+1} \right)
$$

$$
\left. + \sum_{k=N}^{2N-1} \left(\sum_{l=k-N}^{N-1} C_l^{N-1} C_{k-l}^N \sinh\left(\beta\Delta(2l+1-k)\right) \frac{M}{k+1} \right) \right]
$$

For $\underline{M} > \underline{N}$

$$
\gamma\Delta = \frac{2U}{Z(\Delta)} \left[\sum_{k=0}^{N-1} \left(\sum_{l=0}^{k} C_l^{N-1} C_{k-l}^N \sinh\left(\beta\Delta(2l+1-k)\right) \right) \right.
$$

$$
+ \sum_{k=N}^{M-1} \left(\sum_{l=k-N}^{k} C_l^{N-1} C_{k-l}^N \sinh\left(\beta\Delta(2l+1-k)\right) \right)
$$

$$
\left. + \sum_{k=M}^{2N-1} \left(\sum_{l=k-N}^{N-1} C_l^{N-1} C_{k-l}^N \sinh\left(\beta\Delta(2l+1-k)\right) \frac{M}{k+1} \right) \right]
$$

$$(1.20)$$

As before, $Z(\Delta) \equiv \sum_{j=0}^{2N} C_j^{2N} \exp\left[\beta\Delta(j-N)\right]$.

Whether $M \leq N$ or $M > N$, the relevant equation is of the same form as the one obtained for the original model in Section 1.4. The four observations made in Section 1.4.1 with respect to equation (1.14) of the original model are also applicable to (1.20) and their proof follows the same steps as those shown in Appendix 1.A and will not be repeated here.

Similarly, the steps taken to derive the function $f^m(M, N)$ are identical to those taken to obtain the function $f(N)$ in the original model. The function

$f^m(M,N)$ is given by,

<u>$If \quad M \leq N$</u>

$$
\begin{aligned}
f(M,N) & \quad \frac{1}{2^{2N}} \left[\sum_{k=0}^{M-1} \left(\sum_{l=0}^{k} C_l^{N-1} C_{k-l}^N (2l+1-k) \right) \right. \\
f(M,N) &= + \sum_{k=M}^{N-1} \left(\sum_{l=0}^{k} C_l^{N-1} C_{k-l}^N (2l+1-k) \frac{M}{k+1} \right) \\
&+ \sum_{k=N}^{2N-1} \left(\sum_{l=k-N}^{N-1} C_l^{N-1} C_{k-l}^N (2l+1-k) \frac{M}{k+1} \right) \left. \right]
\end{aligned}
$$

$$(1.21)$$

<u>$If \quad M > N$</u>

$$
\begin{aligned}
f(M,N) & \quad \frac{1}{2^{2N}} \left[\sum_{k=0}^{N-1} \left(\sum_{l=0}^{k} C_l^{N-1} C_{k-l}^N (2l+1-k) \right) \right. \\
f(M,N) &= + \; + \sum_{k=N}^{M-1} \left(\sum_{l=k-N}^{N-1} C_l^{N-1} C_{k-l}^N (2l+1-k) \right) \\
&+ \sum_{k=M}^{2N-1} \left(\sum_{l=k-N}^{N-1} C_l^{N-1} C_{k-l}^N (2l+1-k) \frac{M}{k+1} \right) \left. \right]
\end{aligned}
$$

1.F Properties of $f(M,N)$

It remains to show that $f^m(M,N)$ lies in the interval $(0, 0.25)$ and that it is increasing in M and decreasing in N.

1.F.1 Proof that $f^m(M,N) \leq 0.25$

Take the case where $M \leq N$. Then,

$$
\begin{aligned}
f^m(M,N) &\equiv \frac{1}{2^{2N}}\left[\sum_{k=0}^{M-1}\sum_{l=0}^{k} C_l^{N-1}C_{k-l}^N(2l+1-k)\right. \\
&\quad + \sum_{k=M}^{N-1}\sum_{l=0}^{k} C_l^{N-1}C_{k-l}^N(2l+1-k)\frac{M}{k+1} \\
&\quad \left. + \sum_{k=N}^{2N-1}\sum_{l=k-N}^{N-1} C_l^{N-1}C_{k-l}^N(2l+1-k)\frac{M}{k+1}\right] \\
&\leq \frac{1}{2^{2N}}\left[\sum_{k=0}^{N-1}\sum_{l=0}^{k} C_l^{N-1}C_{k-l}^N(2l+1-k)\right. \\
&\quad \left. + \sum_{k=N}^{2N-1}\sum_{l=k-N}^{N-1} C_l^{N-1}C_{k-l}^N(2l+1-k)\right] \\
&= \frac{1}{2^{2N}}\left(\sum_{k=0}^{N-1}\sum_{l=0}^{k} C_l^{N-1}C_{k-l}^N\right) \\
&= \frac{2^{2N-2}}{2^{2N}} \\
&= 0.25
\end{aligned}
$$

where the first step is obtained by noting that the term $\frac{M}{k+1} < 1$. A similar reasoning also carries through for the case where $M > N$. Hence, $f^m(M,N) \leq 0.25$, $\forall\ N,\ M > 0$.

1.F.2 Proof that $f^m(M,N) > 0$.

Equation (1.21) shows that the function $f^m(M,N)$ can be decomposed as the sum of three elements, each one involving a double summation: first summing over the index l and then over the index k. The typical summand in each of these elements is given by $C_l^{N-1}C_{k-l}^N(2l+1-k)$ where $l, k \geq 0$.

I will show that for a given k, the sum over the index l is positive. It will then follow that when summing over k, the result will also be positive.

If $k \leq 1$, then the term $(2l+1-k) \geq 0$ and hence the sum over l must also be greater than or equal to 0.

Consider now the case where $k = \bar{k} \geq 2$. Consider the contribution to the summation when $l = \tilde{l}$, where $\tilde{l}$ is such that $2\tilde{l}+1-\bar{k} < 0$. This condition

48

is satisfied when $\bar{l} < \frac{k-1}{2}$. For every $\tilde{l}$ there exists a $\tilde{l}' = \bar{k} - 1 - \tilde{l}$ such that $2\tilde{l} + 1 - \bar{k} = - \left(2\tilde{l}' + 1 - \bar{k}\right)$. In other words, when, for a given $\bar{k}$, one sums over the index l, for each value of the index l such that $\left(2l + 1 - \bar{k}\right)$ is negative, there is a corresponding value of l for which $\left(2l + 1 - \bar{k}\right)$ is positive and equal in absolute value. It follows that, for a given $\bar{k}$, the sum over the index l will be positive if,

$$
\begin{array}{rccc}
& -C_{\tilde{l}}^{N-1}C_{\bar{k}-\tilde{l}}^{N}\left(2\tilde{l}+1-\bar{k}\right) & < & C_{\tilde{l}'}^{N-1}C_{\bar{k}-\tilde{l}'}^{N}\left(2\tilde{l}'+1-\bar{k}\right) \\
\Leftrightarrow & C_{\tilde{l}}^{N-1}C_{\bar{k}-\tilde{l}}^{N} & < & C_{\tilde{l}'}^{N-1}C_{\bar{k}-\tilde{l}'}^{N} \\
\Leftrightarrow & C_{\tilde{l}}^{N-1}C_{\bar{k}-\tilde{l}}^{N} & < & C_{\bar{k}-1-\tilde{l}}^{N-1}C_{1+\tilde{l}}^{N} \\
\Leftrightarrow & \frac{(N-1)(N-2)\cdots(N-\tilde{l})}{\tilde{l}(\tilde{l}-1)\cdots 1} \ast \frac{N(N-1)(N-2)\cdots(N-\bar{k}+\tilde{l}+1)}{(\bar{k}-\tilde{l})(\bar{k}-\tilde{l}-1)\cdots 1} & < & \frac{(N-1)(N-2)\cdots(N-\bar{k}+1+\tilde{l})}{(\bar{k}-\tilde{l}-1)(\bar{k}-\tilde{l}-2)\cdots 1} \ast \frac{N(N-1)(N-2)\cdots(N-\tilde{l})}{(1+\tilde{l})\tilde{l}\cdots 1} \\
\Leftrightarrow & \frac{1}{\bar{k}-\tilde{l}} & < & \frac{1}{1+\tilde{l}} \\
\Leftrightarrow & \tilde{l} & < & \frac{\bar{k}-1}{2}
\end{array}
$$

By definition of $\tilde{l}$ the last condition is true.

Therefore, for a given $\bar{k}$, the result of summing over the index l is positive. It follows that the subsequent summing over all k will also be positive. Hence, the function $f^m(M, N) > 0$.

Note finally that since the function $f(N)$ defined in Section 1.5.1 is a special case of $f^m(M, N)$ - obtained when $M = N$ - it follows that the range of $f(N)$ must also be contained in the interval $(0, 0.25)$.

1.F.3 Proof that $f^m(M, N)$ is increasing in M and decreasing in N for $M \leq 2N$.

It remains to show that $f^m(M, N)$ is increasing in M and decreasing in N. It did not prove possible to show analytically and resort was made to numerical methods. The value of $f^m(M, N)$ was found for each pair of M and N in a $100 \ast 100$ grid and it was checked that given a value of M, $f^m(M, N)$ fell as N increased and conversely, that for a given value of N the function increased as M increased. For $M \geq 2N$, however, the function $f^m(M, N) = 0.25$. Figure 6 summarizes the results of this numerical exercise.
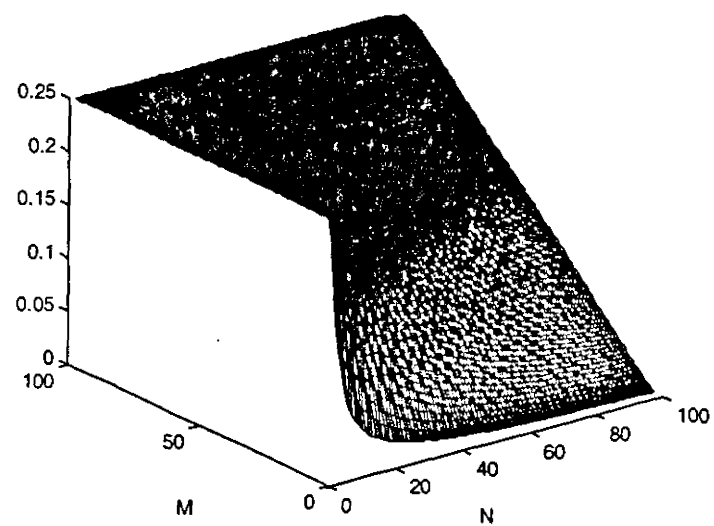
Figure 1.6: Plot of $f^m(M, N)$ in $[2, 100] \times [2, 100]$ grid.

Chapter 2

A study into loyalty-inducing programmes which do not induce loyalty

2.1 Introduction

Keeping up with its reputation as a pioneer in the airline industry, in May 1981 American Airlines launched AAdvantage, the first frequent-flyer programme (FFP). Within a week, United Airlines countered by introducing a similar programme and in the next few months all of the main American carriers followed.[1] The response outside the United States was slower though today all of the more important players in the industry have their own FFP.

The concept behind these programmes is that of rewarding passenger loyalty to a carrier. Rewards come in the shape of free flights, gifts or upgrades and loyalty is measured in air miles, which are calculated as a combination of money spent and distance travelled, so that business and first-class tickets generate more air miles than economy tickets on the same route. Membership in a FFP is, almost always, free.

The popularity of $FFPs$ is considerable. Today, AAdvantage can boast over 30 million members whilst its main American competitors, United Airlines and Delta, have in their loyalty programmes around 23 million members

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[1]Chapter 1 in Nako (1992) provides a detailed description of the early developments of $FFPs$ in the United States.

each.[2] In Europe, British Airways has enrolled in its Air Miles scheme over four million people and in 1996 half a million of these had made use of their rewards and flown for free.[3]

Throughout this chapter I will refer to airlines, passengers and to frequent-flyers. It should be understood, however, that the analysis aims to be pertinent to the class of loyalty-inducing programmes as a whole and hence to go beyond the air transport industry. As with airlines, the proliferation of these schemes across supermarkets, fuel retailers, hotel chains, car rental companies and other retailers has been phenomenal. In 1995, according to Andersen Consulting a quarter of American consumers had access to frequent-shopping programmes at their local supermarket. In Britain, by the end of 1997, the three largest supermarkets, Tesco, Sainsbury and Safeway, had 25 million card holders between them, which accounted for more than two thirds of their customers.[4]

How can the proliferation of *FFPs* be accounted for? Why have.they become an industry standard and why are they so popular with travellers? At first sight the answers to these questions appear straight-forward.

The airlines' rationale to implement a loyalty-scheme, as announced by the carriers themselves, is to increase the repeat purchase rate of its customers. The mechanism by which *FFPs* achieve this is the following. Due to the equity customers build in the programmes via the collection of air miles, a customer faces the opportunity cost of foregone miles when he decides to patronise a second airline. In order to avoid this cost, travellers stick to one airline. On the other hand, the prospect of a free trip to Paris or an upgrade to Executive Class provides a clear motivation for travellers to join an airline's programme.

As it stands, the above explanation is backed by strong intuition. Upon scrutiny, however, the intuition wobbles. Firstly, the causal relation between loyalty inducing schemes and locked-in consumers does not perform well when confronted with empirical evidence. Secondly, it is not clear that the possibility of receiving an award in the future is the best way through which to raise the value of an airline in the eyes of travellers. I now turn to these two

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[2]Frequent Flier Newsletter available on the Internet address, http://www.frequentflier.com.

[3]Skypala, P. (1997). *How taking off is taking off: Air Miles.* Financial Times, 18th October, p. 3, London Edition.

[4]Brown-Humes, C. (1997), *Points have a blunt edge*, Financial Times (p.24), 25th October, London Edition.

points.

While advertising, improved distribution or sales promotion aim, through different routes, to raise a company's market penetration, loyalty programmes have the precise goal of increasing the purchase frequency of customers. As such an evaluation of a FFP should hinge on its success on this front.

To the best of my knowledge, the only such appraisal of a loyalty-programme carried out within academic circles is that conducted by Sharpe and Sharpe (1997). These authors investigate the success of the Australian Fly Buy programme in raising the loyalty of customers of the participating brands to levels that are in excess of what would be expected.[5] Their findings leave little room for enthusiasm: the results show that although there is a weak level of excess loyalty the expected deviation is not consistently observed for all the participating brands.[6] The findings of Sharpe and Sharpe (1997) second a 1997 report from the Mintel research group which finds that consumers do not become more loyal to a retailer despite being a member of its loyalty-scheme.[7]

An informal confirmation of the lukewarm performance of FFP can be read from what is not said by the industry practitioners in comments and interviews to the press. Indeed, while they are keen to herald the launch of loyalty schemes as a means to obtain a loyal clientele, they are suspiciously silent on the actual outcome of such programmes. It is also notable that in Reichheld's (1996) extensive examination of companies that have implemented (successful) policies to induce loyalty no mention is made of $FFPs$.

The few empirical studies and the silence of airline executives are far from being watertight evidence of the shortcomings of $FFPs$ in fulfilling the proclaimed objective of raising repeat purchase rates. However, they do cast doubts. These doubts are reinforced by noting that the strength of the intuitive link between $FFPs$ and high repeat purchase rate is eroded in light of the following observations.

1. All of the main airlines have their own FFP.

2. An individual is able to join more than one programme. Furthermore,

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[5] The Fly Buy programme is a multi-collection scheme whereby points can be collected for the same reward scheme from any of the participating suppliers.

[6] The *excess* is in relation to the degree of loyalty predicted by the Dirichlet and by the Negative Binomial Distribution models of repeat buying as set out in Ehrenberg (1988) and applied widely in the marketing literature.

[7] Loyalty cards fail to impress, *Financial Times*, 7th January, p.7, London edition.

membership is generally free and the time cost involved in filling in forms is negligible. Not surprisingly, individuals join the programmes of more than one airline. A survey carried out by Toh and Hu (1988) reported that the average number of multiple membership among frequent flyers was 2.3. A 1999 survey amongst business travelers found that on average travelers belonged to three separate loyalty-programs and suggested that this figure was rising.[8]

3. Holding multiple FFP membership cards implies that a traveller may vary his choice of airline without necessarily losing out on air miles. The opportunity costs which FFP seek to induce onto consumers are, in this way, mitigated, although the non-linearity of most schemes and the existence of a mileage expiration date in most programmes ensure that they are not totally lost.

The above exposition casts doubts on the effectiveness of loyalty-programmes in fulfilling the objective announced by their implementers - raising the repeat purchase rate of customers. There remains the suggestion that $FFPs$ are tools through which to raise the general value of the airline in the eyes of travellers and so contribute to an airline's market penetration. After all, travellers are attracted by discounts and gifts and their demand can be competed for via the generosity of $FFPs$. However, it is questionable whether loyalty programmes are the most effective tools with which to lever customer value. Are there no better policies with which to motivate consumers to select a given airline?

Surveys carried out amongst travellers consistently report that price levels, punctuality and on-board service are the three criteria to which passengers pay more attention to in their choice of airline.[9]

In addition, loyalty-schemes are costly. A recent estimate placed the costs of running a FFP between 3 and 6% of an airline's revenue.[10] Furthermore, airlines should cost the lost revenue that comes about from the award of free flights. On the one hand, some passengers use the collected air miles to go on a flight they would otherwise have been willing to pay for. On the

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[8] OAG Business Travel Lifestyle Survey, quoted in *Insideflyer* January 1999.

[9] Survey carried out by the International Air Transport Association (IATA) referred to in Roger Bray, *When Work Gets the Perks*, in The Financial Times, 9th February 1998, p.14, London Edition. The survey of Toh and Hu (1988) arrives at the same conclusions.

[10] "Extra Lift for Airlines", *Asian Business*, August 1993, pp.44-46, quoted in Dowling and Uncles (1997).

other hand, passengers flying on their awards might displace regularly paying travellers. Admittedly, this problem is limited by the general excess capacity in the airline industry as well as by the restrictions imposed by airlines on the flights against which an award may be claimed.[11]

The discussion presented so far has aimed at dismembering the frequently heralded view of $FFPs$ as win-win arrangements between airlines and travellers. As was sketched above, it is not clear that these schemes succeed in providing airlines with a portfolio of loyal customers nor are they the most direct means of raising the value proposition to customers and, through this, an airline's market share.

Banerjee and Summers (1987) and Caminal and Matutes (1990) offer an alternative and insightful explanation for the ubiquity of loyalty schemes. Their central idea is that $FFPs$ are a tool for airlines to transfer some of the consumer surplus to themselves - they are win-lose arrangements where airlines are on the winning side. In both papers, the ability of airlines to reap the surplus arises from the switching costs that such programmes induce on consumers. In turn, the switching costs emerge due to the following reason. The models analysed in both of these papers consider a two-period time horizon so that travellers must patronise, by construction, the same airline in the second period as they did in the first period in order to collect the discount offered to repeat buyers. A traveller who switches airline, on the other hand, foregoes the entitlement to the discount. It follows that in the second period travellers are induced to stick to their first period choice. In the second period, therefore, airlines compete less aggressively as it becomes harder to attract passengers who chose the rival in the past. In addition, there is an incentive to be less aggressive in the first period as well. It is in an airline's interest to ensure that the competitor has a sufficient share of 'old customers' to induce it not to behave aggressively in the second period. To ensure this, an airline will resist lowering prices in the first period and take over the entire market.[12] Hence, the benefit of $FFPs$ to airlines accrues from the higher prices which the segmentation of the market allows carriers to set, rather than from the rewards of holding a portfolio of loyal customers *per se*.

While some travellers will choose their airline according to the balance

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[11] A general rule offered by the Frequent Flier Newsletter is that 5% of an airline's seats are allocated for use by FFP members making use of a reward.

[12] This intuition for the less aggressive first period behaviour is valid in the setting of Banerjee and Summers (1987) although it does not apply in Caminal and Matutes (1990).

on their air miles account, the arguments presented earlier suggest that the behaviour of others runs against the behaviour hypothesized by the two papers. As mentioned earlier, travellers are generally enrolled in more than one loyalty scheme and tend to distribute their purchases over several airlines. This conduct suggests that even if $FFPs$ give rise to switching costs, travellers do not seem to be greatly limited by them. Given this, the mechanism identified by Banerjee and Summers (1987) and Caminal and Matutes (1990) which allows airlines to charge higher prices is no longer present. An appropriate question which follows is whether airlines still find $FFPs$ appealing if these schemes are not successful in imposing switching costs on travellers. Are airlines still able to extract consumer welfare through such programmes?

In this chapter I examine the role of loyalty-programmes in a setting where the schemes do not induce a switching-cost on consumers. This will be carried out by analysing a model which extends the two-period horizon of the papers mentioned above to a three-period setting. The relevance to the analysis, however, is not in the number of periods *per se*, but rather in the notion that to benefit from the discount offered by the FFP a traveller does not have to choose the same airline in all periods.[13]

Running parallel with the above inquiry, the analysis in this chapter also attempts to shed some light on the welfare of travellers who participate in the market rarely *vis-à-vis* those who fly frequently. How do these two groups fare when $FFPs$ are launched? The interest in this question is grounded on the idea that $FFPs$ are targeted at rewarding frequent customers. It follows that one would expect this group to benefit from these programmes. Here, I will show that these two groups of consumers do benefit differently from the implementation of a loyalty-scheme. The results obtained point out that the group of occasional travellers, those that fly rarely, invariably lose. On the other hand, whether frequent-flyers benefit or not from the introduction of a FFP will be shown to depend on the weight that this group has in the population of consumers as a whole.

The following questions, reflecting the above discussion, summarize the points which I will seek to address in the chapter.

1. Can a FFP which does not lock in customers be the outcome of competitive practice?

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[13] Banerjee and Summers (1987) on p. 16 extend their model to T periods. They maintain, however, the restriction that consumers must have patronised the airline in all $T - 1$ periods before the coupons can be used.

2. How do prices compare between the scenario in which firms launch a *FFP* and one where firms are unable to discriminate between customers (have no record of their past behaviour) and are therefore unable to offer special treatment to loyal customers?

3. Do travellers benefit or lose from the implementation of a *FFP* ? Do occasional and frequent traveller benefit or lose differently?

4. How does the composition of the consumer population - the ratio of frequent to occasional travellers- influence the prices set and the coupon values set by airlines?

The role of discount coupons as a means to discriminate between consumers involved in repeat buying has been studied in contexts outside loyalty programmes.[14] Fudenburg and Tirole (1997) and Chen (1997) turn loyalty programmes on their head and study the behaviour of firms poaching customers of competitors by offering a discount to these if they switch. As expected, the ability to poach affects the degree to which customers switch between firms. Whether poaching leads to too much or too little switching in comparison to the socially efficient level is shown to depend on the nature of consumers' relative preferences for the two brands - whether these are constant or independent over time - and on whether the firm is able to commit in the first period to its behaviour in second period.

The rest of this chapter is structured as follows. In Section 2.2, I describe the model with which I intend to tackle the questions laid out above. Section 2.3 solves this model under the special case where airlines are unable to discriminate passengers by their past choices. The results derived there will serve as an appropriate benchmark for the findings obtained in Section 2.4 where, in contrast, airlines are allowed to discriminate in favour of repeat travellers. Section 2.5 ties the results obtained with those of the relevant literature and Section 2.6 concludes.

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[14]There is also an interesting literature examining those coupons generally distributed through newspaper or mail and unrelated to repeat buying. Such coupons have been regarded as a means to price discriminate, both in settings where the coupons are targetted at a particular group (Bester and Petrakis, 1994) and where they are untargetted (Narasimhan, 1984 and Caminal , 1996).

2.2   Model

The model borrows its basic structure from von Weizsäcker (1984) and Klemperer (1987) who, to my knowledge, first grafted inter-temporally changing tastes onto a Hotelling-like model of product differentiation.

There are two competing airlines to be denoted by A and B who offer services at a constant marginal cost c. The services offered by airline A differ from those of airline B along dimensions such as the menu of travel schedules offered and the available connecting flights.[15] Following Hotelling (1929), these differences are captured by picturing the two airlines as if located at the end-points of the interval $I \equiv [0,1]$. Let airline A be located at point 0 and airline B at point 1. Consumers are distributed along I. The location of a consumer at time t is given by $i_t \in I$. The location i_t reflects a consumer's ideal point, and the distance from it to the end-points measures the disutility from purchasing a less preferred ticket. A consumer who purchases a ticket from airline A in Period t enjoys a utility $R - p_t^A - i_t$, where R is the reservation price, p_t^A is the effective price charged by A and i_t is the distance that separates the consumer from A. Similarly, if he buys the service from B, the benefit will be given by $R - p_t^B - 1 + i_t$. In line with the terminology of models employing the Hotelling (1929) setting, though at the risk of causing some confusion, I shall refer to the disutility associated with buying a less preferred ticket as the transportation cost.

The model considers a 3-period setting. If present in the market in period t, a consumer will demand one unit of the service offered by A or one unit of the service offered by B.

There are two types of consumers. The *frequent travellers* take part in the market in each of the three periods. It is assumed that these consumers are uniformly distributed along I in each period and that their location in one period is independent of that in the previous period. As in Caminal and Matutes (1990) the change in the location of these travellers can be interpreted as "a change in travel plans: connecting flights and time schedules are more or less appropriate in one airline or the other depending on the origin and destination of the plane" (Caminal and Matutes 1990, p. 356). Hence, *ceteris paribus*, which of the two airlines is more attractive may vary from one period to the other.

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[15]What I wish to exclude are differences in services which give rise to vertical product differentiation.

The *occasional travellers* are the second type of consumers, and these take part in the market for one period only. It is assumed that this group of consumers is also uniformly distributed along I. At the end of each period, however, they exit the market and are replaced by a new mass of occasional travellers whose locations are independent of the consumers they replace. To ease computation, I assume that the market serves a constant unit mass of consumers over time which requires that the density of consumers joining the market equals the density of consumers leaving the market at the end of a period. The share of regular and occasional travellers in the market is, therefore fixed. Let μ be the proportion of frequent travellers so that $1 - \mu$ is the proportion of occasional ones. The value of the parameter μ is common knowledge. Lastly, I assume that a consumer knows which type of traveller he is: whether he will leave after one period or whether he will be in the market for all 3 periods.

The two categories of travellers described above are not an exhaustive description of the types of travellers one might wish to consider. The absence of travellers who are present in the market over the three periods and have a fixed location throughout seems particularly critical. Such a set of consumers corresponds to those who need to take the same trip - at the same hour, to and from the same airports- and hence are likely to hold a constant relative preference between the two airlines. I have not made room for them in order to keep the analysis tractable.[16]

In addition, there are dimensions other than that of frequency of consumption and location on the interval I along which passengers can be distinguished. An obvious one is that between business class and economy class passengers. In the framework of the model, making this distinction would call for the modelling of consumers with different reservation prices and heterogeneous unit transportation costs.[17] A second, closely related, distinction is that between travellers whose tickets are paid for by their employer and those who have to cover the cost themselves. Characterizing travellers along either or both of these lines appears natural in the context of a study on

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[16]On the other hand, the model purposefully rules out travellers who are present in the market for two out of the three periods. Their presence would have attributed to the *FFPs* the ability to create switching costs which would run against the premise of the model.

[17]Different reservation prces could be easily incoorporated into the above model. Provided these prices were such that they guranteed that consumers alwyas purchased a unit of the good, the analysis carries through unaltered.

60

FFPs.[18] Doing so, however, would burden the analysis and put at risk the ability to yield any clear answers to the questions laid out at the end of Section 1.

Having added the above parenthesis, I now return to the description of the model. The two airlines recognise past customers and are, accordingly, able to discriminate between travellers on the basis of the revenue that they have generated to the airline in the past. In other words, airlines are able to launch *FFPs*. Here, the structure of the *FFPs* which airlines are allowed to implement is restricted to the following class: customers receive a discount - a coupon - on patronising an airline for the second time. In restricting the class of admissible *FFPs* to this I have aimed at finding a compromise between parsimony and the need to portray the most salient features of a *FFP*. Hence, and in line with the discussion in Section 1, the class of *FFPs* that I consider does not impose switching costs as it allows consumers to collect a discount from an airline even if he has addressed the rival in the past. Furthermore, the *FFPs* considered are such that they ensure that frequent travellers collect a discount over the three periods.

The timing of the decision taken by the players in the model is as follows. Prior to Period 1, airline A selects its price p_1^A and the (absolute) value of the coupon α it offers to repeat buyers. Simultaneously, airline B selects its price p_1^B and its coupon β. Whilst the value of the coupons remain unaltered throughout the 3 periods, the price levels are reviewed at the start of both Period 2 and Period 3 before the redistribution of consumers along I takes place. Let p_2^A be the price selected by A in the second period and p_3^A that chosen by this same airline in Period 3. The analogous prices chosen by airline B will be denoted by p_2^B and p_3^B respectively. Consumers, on the other hand, must decide at the beginning of each period which airline to patronise. Their choices are made once they have been distributed along I and, hence, once their location i_t is known. It follows that consumers base their choice on their relative distance to the end-points as well as on the relative prices and coupons offered by the two airlines.

In selecting its price and coupon airlines aim to maximise expected profits over the three periods. Travellers, on the other hand, intend to maximise the sum of the expected utility gained during their stay in the market. Future income and utility are not discounted.

---

[18]See Cairns and Galbraith (1990) for a study of *FFPs* which pivots on the existence of travellers whose fare is partially covered by a third party.

2.3 Benchmark: no discrimination

An appropriate benchmark for the analysis that follows is to consider the above model in a setting where airlines are unable monitor consumers' past decisions. In such a scenario, airlines cannot identify repeat customers and, hence, are unable to offer them coupons. The absence of a reward for repeat-buyers breaks the inter-temporal link in the (frequent)travellers' decision rules thereby making consumers' decision in one period independent of that in the others. The independence of consumers' behaviour over time implies that the policy which maximises airlines' profits over the 3 periods coincides with that which maximises profits over a single period.

Consider then the behaviour of consumers and airlines in Period 1, say. A consumer present in the market in this period collects a utility level of $R - p_1^A - i_1$ if he patronises airline A and a level $R - p_1^B - 1 + i_1$ if he chooses airline B. Comparing the two expressions, it follows that the optimal behaviour is to address A if $i_1 \in (0, \Psi)$, where $\Psi \equiv \frac{1 + p_1^B - p_1^A}{2}$ and address B otherwise. Given this behaviour, it is simple to show that price competition between the two airlines gives rise to the unique equilibrium prices $p_1^{A*} = p_1^{B*} = 1 + c$. The excess of price over marginal cost arises from the local monopoly power that airlines possess due to the spatial setting of the model.[19] In turn, the symmetric solution implies that $\Psi = \frac{1}{2}$ in equilibrium. Travellers located in the first half of I address A and the remaining address B. Given this, airlines expect to collect a revenue of $\frac{1+c}{2}$ in each period and thereby expect to make profits over the three periods equal to $\frac{3}{2}$.

2.4 Solving the general model

I now turn to the case where airlines are able to discriminate between consumers on the basis of their past purchases.

The model is solved by working backwards from Period 3. For each of the periods, the analysis establishes the consumers' set of optimal decision rules. For a frequent traveller, an example of a typical rule within this set takes the form: "In Period 2, given that prices and the coupon levels offered by airlines are p_2^A, p_2^B, α and β, and given that in Period 1, I addressed airline A and that

---

[19]More generally, equilibirum prices are given by $t + c$, where t is the unit cost of transport. The higher the cost of transport faced by travellers the greater is the ability of the two airlines to extract consumer surplus.

in the following period I expect to be located at i_3 and expect to face prices p_3^A and p_3^B then I will return to A if $i_2 \leq i^*$ and go to B otherwise." The set of rules are optimal in the sense that they ensure consumers maximise their expected future utility at each point in time. It is assumed that travellers have completely rational expectations. Given that consumers abide by these optimal rules, I then calculate the prices and coupon levels that airlines set in order to maximise the sum of their expected profits over the three periods.

To make the presentation as fluid as possible I leave to Appendix 2.A many of the algebraic stepping-stones involved.

2.4.1 Period 3

Consider the behaviour of an occasional traveller who takes part in the market in Period 3. Given prices p_3^A and p_3^B, an occasional traveller located at i_3 will address airline A if $i_3 \leq \frac{1+p_3^B-p_3^A}{2}$. Otherwise he will purchase from airline B.

The behaviour of a frequent traveller, on the other hand, depends on the history of his past purchases. By Period 3, a frequent traveller will have either bought a ticket once from each airline or he will have bought twice from the same. If the latter, he will have received the discount offered to repeat buyers already and, by construction, will not be able to benefit from a further coupon. Accordingly, the decision of such a consumer will depend only on his location i_3 and on the relative prices of tickets. It follows that he will address airline A if $i_3 \leq \frac{1+p_3^B-p_3^A}{2}$ and airline B otherwise. On the other hand, if the consumer has addressed different airlines in the past, then his choice in Period 3 determines the airline from which the discount is to be received. Consequently, his decision takes into account the relative generosity of the two coupons. Patronising airline A yields him a utility level of $R - p_3^A + \alpha - i_3$ while the utility from choosing B is $R - p_3^B + \beta - 1 + i_3$. Comparing the two utility levels, it is straight-forward to see that he will address A if $i_3 \leq \frac{1+p_3^B-\beta-p_3^A+\alpha}{2}$ and address B otherwise. In sum, the optimal decision rule for a frequent traveller in Period 3 can be written as,

<u>Period 3</u>

$$
\text{If}
\begin{cases}
\text{have received discount in Period 2 and }
\begin{cases}
i_3 \leq \frac{1+p_3^B - p_3^A}{2} \equiv \Omega_1 & \text{then go to } A. \\
i_3 > \Omega_1 & \text{then go to } B.
\end{cases} \\[2em]
\text{have not yet received discount and }
\begin{cases}
i_3 \leq \frac{1+p_3^B - \beta - p_3^A + \alpha}{2} \equiv \Omega_2 & \text{then go to } A. \\
i_3 > \Omega_2 & \text{then go to } B.
\end{cases}
\end{cases}
\tag{2.1}
$$

Using the above decision rules it is possible to construct the expressions for the airlines' profits in Period 3. For airline A this will be given by,

$$
\Pi_3^A = \left(p_3^A - c\right)\left((1-\mu) + \mu(1-s)\right)\Omega_1 + \left(p_3^A - \alpha - c\right)\mu s \Omega_2 \tag{2.2}
$$

where s is the share of frequent travellers who have switched between the two airlines in the first two periods. The first component of (2.2) reflects the contribution to A's profits from those travellers who will not collect the discount α in the third period: that is to say, the occasional travellers and the share of frequent travellers who have already received the discount in the past. The second component, on the other hand, picks up the portion of A's third period profits generated from those who are yet to receive the discount.

Airline A chooses p_3^A to maximise Π_3^A. Solving this optimization problem it is possible to derive A's reaction function as,

$$
p_3^A = \frac{1}{2}\left(1 + c + p_3^B + \mu s\left(2\alpha - \beta\right)\right). \tag{2.3}
$$

An analogous expression can be derived for airline B. Solving the two reaction functions simultaneously it is then possible to derive the equilibrium prices. These are described in Proposition 2.1.

Proposition 2.1 *The unique equilibrium in Period 3 is for airline A to set a price $p_3^{A*} = 1 + c + \alpha\mu s$ and airline B a price $p_3^{B*} = 1 + c + \beta\mu s$.*
Proof. The result is obtained by solving the two reaction functions simultaneously. The necessary second-order conditions are also met. ∎

Proposition 2.1 establishes that the third-period equilibrium prices lie above the benchmark level. The intuition behind this result can be explained with reference to the airlines' reaction functions.
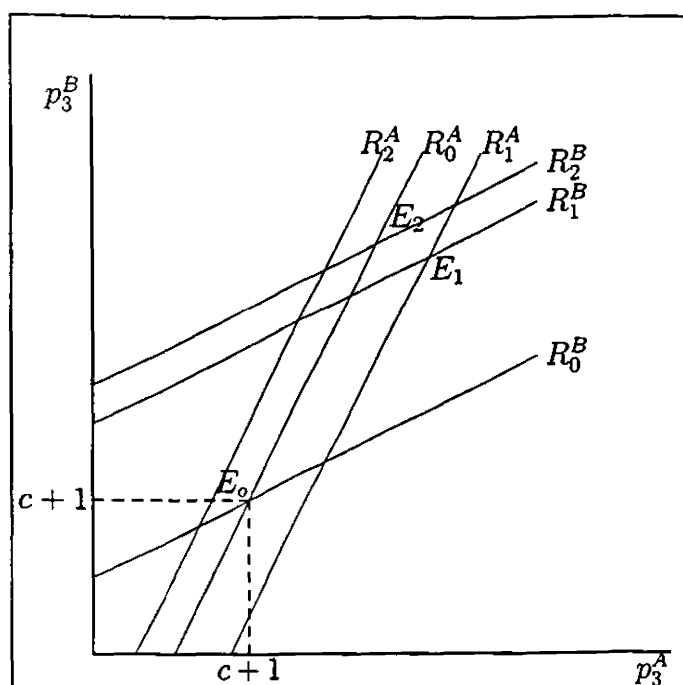
Figure 2.1: Shift of airlines' reaction functions due to $FFPs$

First, note from A 's best-reply (2.3) that airlines' prices are strategic complements - if airline B raises its price, airline A 's optimal policy requires that it raises p_3^A as well. Second, offering a discount affects an airline's behaviour in Period 3 through two distinct routes. On the one hand, the coupon acts as a second tool through which to compete for the demand of frequent travellers. Consequently, the higher the value of the discount offered, the less aggressive will an airline be in its price competition. On the other hand, the commitment to pay out a coupon raises the costs that airlines face compared to the benchmark case. Graphically, both of these effects - the less aggressive behaviour and the need to cover committed costs - lead to an outward shift of the airlines' reaction functions. For airline A, say, each of the two effects is responsible for a shift of size $\frac{\alpha\mu s}{2}$ (for airline B, the expression would be $\frac{\beta\mu s}{2}$). Third, given the value of airline A's discount, a higher value β leads to a more aggressive competition over base price by

A. Graphically, this is represented by an inward shift of the best-reply curve by $\frac{\beta \mu s}{2}$ for airline A and by $\frac{\alpha \mu s}{2}$ for airline B.

The net effect of introducing FFP on the reaction functions depends on the relative values of α and β as seen in (2.3) . If an airline's discount is greater than half of that of the rival's then its reaction curve will shift outwards. Otherwise, the shift will be inwards. Figure 2.1 illustrates the former case by the shift of the best-reply curves from the benchmark case $\left(R_0^A, R_0^B\right)$ to $\left(R_1^A, R_1^B\right)$. The pair of curves $\left(R_2^A, R_2^B\right)$, on the other hand, depicts the case where $\alpha < \frac{\beta}{2}$.

Note that an airline's discount shifts that airline's best-reply curve outwards by twice the amount that it shifts the rival's inwards. Hence, and independent of whether the pair $\left(R_1^A, R_1^B\right)$ or $\left(R_2^A, R_2^B\right)$ is the appropriate one, the resulting equilibrium prices will be greater than those of the benchmark case.

The higher equilibrium prices do not imply that the profits earned by the two airlines are necessarily higher than those derived in the benchmark case since discounts must now be handed out. To see this, note that at the equilibrium prices, the third period profit of the two airlines are given by,

$$\Pi_3^{A*} = \frac{1}{2}\left(1 + \alpha \mu s \left(1 - \mu s\right)\left(\beta - \alpha\right)\right) \tag{2.4}$$

$$\Pi_3^{B*} = \frac{1}{2}\left(1 + \beta \mu s \left(1 - \mu s\right)\left(\alpha - \beta\right)\right)$$

It follows, that an airline's expected profit increases with the value of the discount offered by its rival. Furthermore, they will be higher, equal to or lower than the benchmark level - $\frac{1}{2}$ - depending on whether the coupon it offers is lower, equal to or higher than that of its rival. The intuition behind this is the following. Say $\beta > \alpha$. For travellers who will benefit from a discount in Period 3, the effective price will be lower at airline B since $p_3^{B*} - \beta = 1 + c - \beta\left(1 - \mu s\right) < 1 + c - \alpha\left(1 - \mu s\right) = p_3^{A*} - \alpha$. Given this, a greater proportion of these travellers will opt to address airline B. This airline must increase its base price, p_3^B, to cover the costs of handing out discounts and, at the same time, to curtail the demand by this set of travellers. However, raising base price also induces a greater share of those consumers who will not benefit from the discount in Period 3 to choose airline A. While the former set of travellers pay an effective price lower than $1 + c$, the effective price paid by the latter is above the competitive level. To sum

up, offering a more generous coupon attracts those who pay a lower effective price and repels those who pay the higher price.

Lastly, it should be pointed out that at the start of Period 3, the airline burdened with the more geneorus discount would have the incentive to review the value of the discount it had chosen in Period 1 and set it equal to 0. In the context of this model, such an action is assumed to be not possible. In turn, the assumption can be supported on the grounds of reputation effects and on the fact that it would damage the airline's ability to set up a new FFP in the future.[20]

2.4.2 Period 2

Using the equilibrium prices of Period 3, I now work backwards to determine the optimal choice rules of consumers in Period 2 which, in turn, will allow me to solve the optimization problem of the two airlines at the start of Period 2.

Like his Period 3 counterpart, an occasional traveller in Period 2 will be unaffected by the generosity of the discounts offered. Therefore, he will patronise airline A if $i_2 \leq \frac{1+p_2^B-p_2^A}{2}$ and airline B otherwise.

Now consider the behaviour of a frequent traveller in Period 2. It is necessary to distinguish these travellers by their choice in Period 1.

Consider first a frequent traveller who purchased from A in Period 1. If he returns to airline A in the second period he will receive the discount offered by A to repeat buyers. The expected utility of a consumer returning to A in Period 2 is therefore given by $R - p_2^A + \alpha - i_2 + E\left(U^3|xx\right)$, where $E\left(U^3|xx\right)$ is the expected utility enjoyed in Period 3 by a consumer who has in the previous two periods visited the same airline. Similarly, the expected utility of a consumer addressing airline B in Period 2, conditional on having bought a ticket from A in Period 1, is equal to $R - p_2^B - (1 - i_2) + E\left(U^3|xy\right)$. The term $E\left(U^3|xy\right)$ reflects the utility a consumer can expect in Period 3 given that he has addressed different airlines in the first two periods. Therefore, a consumer who has addressed A in Period 1 will return to it in Period 2 if $i_2 \leq \frac{1+p_2^B-p_2^A+\alpha+E\left(U^3|xx\right)-E\left(U^3|xy\right)}{2}$ and will address B otherwise.

---

[20]Interestingly, airlines are typically within their rights to review the discounts offered in their schemes. In the conditions laid out by $FFPs$ that I have come across, airlines reserve the right to change the awards, the rules for earning mileage credit and, with a few month's notice, to end the programme (see for example AAdvantage 2000 and Qualiflyer 2000).

Following an analogous reasoning, the decision rule for a consumer who addressed B in Period 1 will be given by: patronise A if $i_2 \leq \frac{1+p_2^B - \beta - p_2^A + E(U^3|xy) - E(U^3|xx)}{2}$ and B otherwise.

The expressions $E(U^3|xx)$ and $E(U^3|xy)$ are given by,[21]

$$
\begin{aligned}
E(U^3|xx) &= \int_0^{\Omega_1} \left(R - p_3^A - i\right) di + \int_{\Omega_1}^1 \left(R - p_3^B - 1 + i\right) di \\
&= R - p_3^B - \frac{1}{2} + \Omega_1^2
\end{aligned}
$$

$$
\begin{aligned}
E(U^3|xy) &= \int_0^{\Omega_2} \left(R - p_3^A + \alpha - i\right) di + \int_{\Omega_2}^1 \left(R - p_3^B + \beta - 1 + i\right) di \\
&= R - p_3^B + \beta - \frac{1}{2} + \Omega_2^2
\end{aligned}
$$

where, recall, $\Omega_1 = \frac{1 + p_3^B - p_3^A}{2}$ and $\Omega_2 = \frac{1 + p_3^B - \beta - p_3^A + \alpha}{2}$. Note that a consumer who addresses the same airline in the first two periods, benefits from the discount offered by the FFP in the second period and hence cannot expect to benefit further in Period 3. Consequently, the expression for $E(U^3|xx)$ does not include either of the terms α and β. On the other hand, a consumer who has chosen different airlines in the first two periods can still expect to receive the coupon offered by the $FFPs$. It follows that $E(U^3|xy)$, is a function of the coupon values.

Making use of the expressions derived for $E(U^3|xx)$ and $E(U^3|xy)$, the second period optimal decision rules can be written as,

Period 2

$$
\text{If} \begin{cases} \text{went to } A \text{ in } t = 1 \text{ and} \begin{cases} i_2 \leq \frac{1 + p_2^B - \beta - p_2^A + \alpha + \Omega_1^2 - \Omega_2^2}{2} \equiv \Omega_3 & \text{then go to } A. \\ i_2 > \Omega_3 & \text{then go to } B. \end{cases} \\ \\ \text{went to } B \text{ in } t = 1 \text{ and} \begin{cases} i_2 \leq \frac{1 + p_2^B - p_2^A - \Omega_2^2 + \Omega_1^2}{2} \equiv \Omega_4 & \text{then go to } A. \\ i_2 > \Omega_4 & \text{then go to } B. \end{cases} \end{cases}
$$

$$(2.5)$$

[21] It is assumed that the values of Ω_1 and of Ω_2 lie in the unit interval. In equilibrium this condition is assured.

Both Ω_3 and Ω_4 are functions of the prices in the third period and hence a function of s, the share of frequent travellers who address different airlines in the first two periods. By construction,

$$s = \sigma\left(1 - \Omega_3\right) + \left(1 - \sigma\right)\Omega_4 \tag{2.6}$$

where σ is the share of frequent travellers who addressed airline A in Period 1. Using (2.5), equation (2.6) can be written as,

$$s = \frac{3 + 4\left(1 - 2\sigma\right)\left(p_2^B - p_2^A\right) + \left(1 + \alpha - \beta\right)^2 + 4\sigma\left(\beta - \alpha\right)}{2\left(4 + \mu\left(\alpha - \beta\right)^2\right)} \tag{2.7}$$

The problem facing the two airlines at the start of Period 2 -to maximize profits over Periods 2 and 3 - can now be written out. For airline A this problem is given by,

$$\begin{aligned}
\underset{\{p_2^A\}}{Max}\Pi_2^A &= \left(p_2^A - c\right)\left(1 - \mu\right)\left(\frac{1 + p_2^B - p_2^A}{2}\right) + \left(p_2^A - c\right)\mu\left(1 - \sigma\right)\Omega_4 \\
&\quad + \left(p_2^A - \alpha - c\right)\mu\sigma\Omega_3 + \Pi_3^A
\end{aligned}$$

The first three terms on the right hand side of (2.8) pick up the contribution to A's profits respectively from: the occasional travellers, the frequent travellers who addressed B in Period 1 and the frequent travellers who addressed A in Period 1. An analogous expression can be written down for airline B and the equilibrium second period prices are given by solving the two maximization problems simultaneously.

Proposition 2.2. *There is a unique pair of p_2^{A*} and p_2^{B*} which forms the equilibrium to the second period pricing game.*
Proof. See Appendix 2.A. ∎

The expressions defining the equilibrium prices assured by Proposition 2.2 are, however, unwieldy and offer little insight. Instead, I will draw attention to two special cases.

Corollary 2.1. a) *If the demand generated by the frequent travellers in Period 1 is divided equally between the two airlines, $\sigma = \frac{1}{2}$, then the equilibrium second period prices are given by $p_2^{A*} = 1 + c + \frac{\mu\alpha}{2}$ and $p_2^{B*} = 1 + c + \frac{\mu\beta}{2}$.*
b) *If the value of the discounts set by the two airlines is equal, say λ, then the equilibrium second period prices are given by $p_2^{A*} = 1 + c + \mu\lambda\frac{1+\sigma}{3}$ and $p_2^{B*} = 1 + c + \mu\lambda\frac{2-\sigma}{3}$.*

Proof. See Appendix 2.A. ∎

As is clear in the two special cases considered in Corollary 2.1, equilibrium prices are set above the competitive level. The intuition for this result is identical to the one presented in Section 2.4.1 for the third period equilibrium prices. Here, as was the case in Period 3, the best-reply curves of the two airlines are also shifted out due to the $FFPs$. The forces behind these shifts are the same as those identified earlier. Recall that an airline's own discount lessens its competitive aggressiveness in prices and raises its costs. Both effects lead to an outward shift of its best-reply curve. On the other hand, the discount offered by the rival leads to an inward movement of an airline's best-reply curve, representing the incentive to price more aggressively. A graphical representation of the effect of $FFPs$ on the second period best-reply curves of the two airlines is similar, therefore, to those drawn in Figure 2.1.

The second special case offers an added insight to the model as it illustrates that the price set by an airline in Period 2 increases with σ, its first period market share of frequent travellers. Althought, this feature is shared with the model of Caminal and Matutes (1990), the intuition behind it is markedly different. In the latter paper, a higher market share in the first period increases the incentive of an airline to exploit their repeat-buyers and lowers that of attracting first time buyers so that price competition is less aggressive. In the model considered here, on the other hand, the relation between second period prices and first period market share comes about because the mass of repeat buyers, and therefore the mass of travellers entitled to a discount in Period 2 is increasing in an airline's first period share of frequent travellers. The higher this share is, the greater the total value of discounts to be handed out in Period 2. Faced with this burden, the optimal response of an airline is to raise its price.

2.4.3 Period 1

I now examine the choices of travellers and airlines in Period 1.

An occasional traveller in Period 1 will base his choice on the relative prices of the two airlines. In particular, he will address airline A in Period 1 if his location $i_1 \leq \frac{1+p_1^B-p_1^A}{2}$ and will address airline B otherwise.

A frequent traveller, on the other hand, will take into account the effect of his decision on expected future utility. Let $E\left(U^2|A\right)$ be the expected surplus a traveller expects to collect in Periods 2 and 3 given that he addresses airline A in Period 1 and let $E\left(U^2|B\right)$ be the analogous term for a passenger who addresses airline B in Period 1. It follows that a frequent traveller will patronise airline A in Period 1 if and only if,

$$R - p_1^A - i_1 + E\left(U^2|A\right) \geq R - p_1^B - (1 - i_1) + E\left(U^2|B\right) \qquad (2.8)$$

The terms $E\left(U^2|A\right)$ and $E\left(U^2|B\right)$ can be explicitly worked out as,[22]

$$
\begin{aligned}
E\left(U^2|A\right) &= \int_0^{\Omega_3}\left(R-p_2^A+\alpha-i+E\left(U^3|xx\right)\right)di+\int_{\Omega_3}^1\left(R-p_3^B-(1-i)+E\left(U^3|xy\right)\right)di \\
&= R+\Omega_3^2-p_2^B-\tfrac{1}{2}+E\left(U^3|xy\right)
\end{aligned}
$$

and

$$
\begin{aligned}
E\left(U^2|B\right) &= \int_0^{\Omega_4}\left(R-p_2^A-i+E\left(U^3|xy\right)\right)di+\int_{\Omega_4}^1\left(R-p_2^B+\beta-(1-i)+E\left(U^3|xx\right)\right)di \\
&= R+\Omega_4^2-p_2^B+\beta-\tfrac{1}{2}+E\left(U^3|xx\right)
\end{aligned}
$$

Substituting these expressions into (2.8) yields after some simplification the following first period optimal decision rule for frequent travellers,

<u>Period 1</u> $\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad$ (2.9)

$$
\text{If } \begin{cases} i_1 \leq \frac{1+p_1^B-p_1^A+\Omega_3^2-\Omega_4^2+\Omega_2^2-\Omega_1^2}{2} \equiv \Omega_5 & \text{then go to } A. \\ i_1 > \Omega_5 & \text{then go to } B. \end{cases}
$$

Recall that the terms $\Omega_1, \Omega_2, \Omega_3$ and Ω_4 are a function of $E\left(\sigma\right)$. However, by construction, Ω_5 is equal to σ. It follows that an expression for σ can be computed through the implicit function J,

---

[22] Again it is implictly assumed that both Ω_3 and Ω_4 lie in the unit interval.

$$J \equiv \frac{1 + p_1^B - p_1^A + \Omega_3^2 - \Omega_4^2 + \Omega_2^2 - \Omega_1^2}{2} - \sigma \qquad (2.10)$$

and then substituted into the definition of Ω_5.

The rule defined in (2.9) completes the description of the optimal behaviour of frequent travellers at each point in time. Using the full set of optimal rules, it is straight-forward to derive the expected demand facing the two airlines over the 3 periods and hence to formulate the optimization problem of the two airlines. For airline A, this problem is given by,

$$\underset{\{p_1^A, \alpha\}}{Max} \Pi_1^A = \left(p_1^A - c\right) \left((1 - \mu) \left(\frac{1 + p_1^B - p_1^A}{2} \right) + \mu \Omega_5 \right) + \Pi_2^A \qquad (2.11)$$

while an analogous problem can be constructed for airline B.

Given that I restrict the search for an equilibrium to the class of symmetric equilibria, it will be sufficient to limit the work to the optimization problem of one of the airlines. Appendix 2.A presents a full description of the work involved. Here, only the result is presented.

Proposition 2.3. *In the unique symmetric equilibrium airlines set first period prices $p_1^{A*} = p_1^{B*} = 1 + c + \frac{4\mu}{13 - 10\mu}$ and issue coupons with a value of $\alpha^* = \beta^* = \frac{6}{13 - 10\mu}$. It then follows from Propositions 2.1 and 2.2 that in the second and third period $p_2^{A*} = p_2^{B*} = p_3^{A*} = p_3^{A*} = 1 + c + \frac{3\mu}{13 - 10\mu}$.*
Proof. The first step is to derive the candidate symmetric equilibrium by solving the first order conditions of the maximization problem once the symmetry conditions are imposed. To then show that the pair of prices and discounts thus obtained constitute an equilibrium it is necessary to show that neither airline has an incentive to deviate from them. To carry out this second step, I resort to numerical simulations and show that the equilibrium is indeed robust to small deviations as well as to the deviation of an airline opting to offer no discount. Appendix 2.A presents the details of the work involved. ∎

It follows from Proposition 2.3 that at equilibrium, airlines will earn the benchmark profit of $\frac{1}{2}$ in Period 2 and in Period 3. Although prices are above $1 + c$ in both of these periods, profits are kept down to the competitive
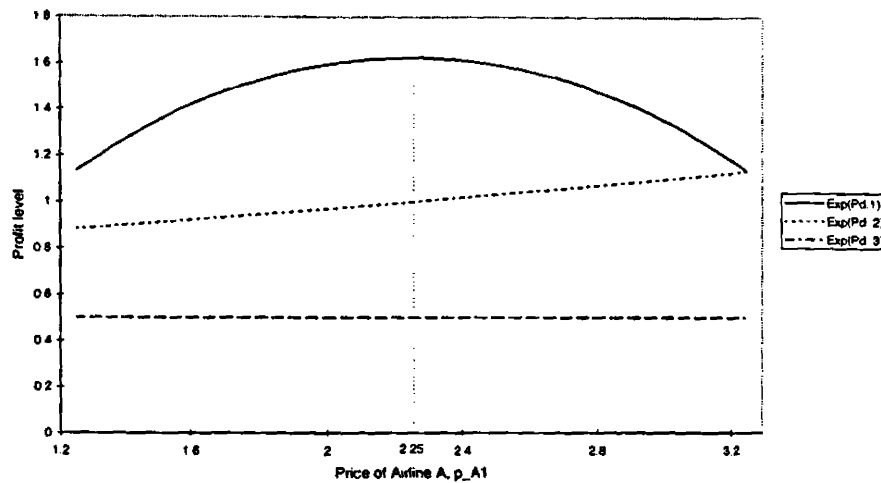
Figure 2.2: Expected profit of airline A at the start of each of the 3 periods. $\left(c = 1, \mu = 0.5, \alpha = \beta = 0.75, p_1^B = 2.25\right)$.

level due to the discounts that are handed out. In Period 1, however, this reasoning does not apply. There, prices are above the competitive level and, by construction, no discounts are given. It follows that airlines yield non-competitive levels of profits in this period. To see how this can be sustained consider the effects of an airline deviating from the equilibrium.

Other things equal, a reduction from the equilibrium level of, say, airline A's price would increase the share of travellers - both occasional and frequent - that this airline would attract in Period 1. However, this would imply that in Period 2, airline B would be facing a smaller mass of frequent travellers that would qualify for its discount. Accordingly, B would be willing to compete more aggressively in price, which would have negative effects on A's Period 2 profits. First, airline A would be unable to compete as aggressively as airline B, as it faces a greater mass of travellers qualifying for discounts and would therefore attract a lower share of travellers. Second, of the mass of frequent travellers that do address A in Period 2 a greater proportion of these will qualify for A 's discount thereby reducing A 's profits from its non-deviation level.

Figure 2.2 illustrates the previous discussion. The figure draws. for $\mu =$

0.5, the profit that airline A can expect at the start of each period as a function of the price it sets at the start of Period 1, given that its rival sets the equilibrium strategy. As shown in the graph for $\mu = 0.5$ and $c = 1$, the equilibrium discount is 0.75 and the first period price is 2.25. A deviation to a price below this level, but above 2, the benchmark price, allows airline A to collect a higher profit in the first period - the vertical distance between the two upper curves - though a smaller one in Period 2 - given by the distance between the two lower curves. The profit over the 3 periods, given by the upper curve, falls.

In sum, prices are sustained above the competitive benchmark level in Period 1 as it is in the airlines' interest to ensure that the rival attracts a sufficiently large market share in that period, so that the incentive to compete aggressively in the subsequent period is reduced.

Note that the ability to charge prices above $1 + c$ in the first period hinges on the coupon having a non-zero value. If this was not the case, *i.e.* $\alpha = \beta = 0$, a firm which deviates by undercutting its rival in the first period would go unpunished in Period 2 since it would not suffer from serving a higher mass of repeat buyers. It is in the interest of both airlines, therefore, to set non-zero coupons.

2.5   Discussion

In this section I will discuss the results obtained above. Particular attention is paid to the effect of population mix ie the proportion of travellers in a period which are frequent travellers, on the equlibrium prices, discounts and airline profits. Lastly, the impact of the FFP on the welfare of each type of traveller and on social welfare as a whole is considered.

2.5.1   Effect of population mix on equilibrium prices and discounts

Equilibrium prices are the same in both Period 2 and Period 3 and they are above the benchmark prices. This result can be understood by noting that in equilibrium, as far as an individual airline is concerned, the proportion of different types of travellers is the same in both periods. In particular, there are $1 - \mu$ occasional travellers, $\frac{\mu}{2}$ frequent travellers that qualify for that airline's discount and $\frac{\mu}{2}$ that do not qualify.
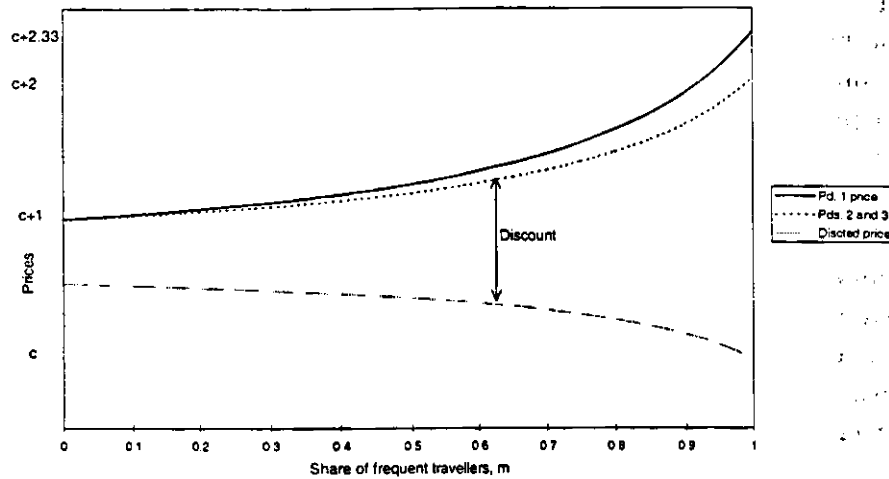
Figure 2.3: Equilibrium prices and discounts as a function of the share of frequent travellers.

It is immediate from Proposition 2.3 that the equilibrium price levels and the value of the discount are positively related to the share of frequent travellers in the market. This is graphed in Figure 2.3. Note that the curve describing the discounted price is not defined at $\mu = 0$ as the notion of a discount does not exist in such a setting. For this value of μ equilibrium prices are at their lowest and are equal to the benchmark level, $c + 1$. On the other hand, when the entire mass of consumers are frequent travellers, $\mu = 1$, the equilibrium prices are at their highest. In Period 1, the price equals to $c + 2\frac{1}{3}$ while in the last two period it is equal to $c + 2$. The equilibrium discount level also reaches its maximum value, 2, when $\mu = 1$. Note that the effective price paid by a repeat buyer, base price minus discount is given by $c + \frac{7 - 7\mu}{13 - 10\mu}$ and is decreasing in μ.

As a percentage of the price mark-up, the discount is equal to $\frac{6}{13 - 7\mu}$ which is increasing in μ. For very low values of μ this percentage is below 50% but it rises with μ until it reaches 100% when $\mu = 1$. Hence, when the market is made up exclusively of frequent travellers, the effective price paid by a consumer benefiting from the discount is equal to an airline's marginal cost.

The intuition for the positive relation between μ with both equilibrium

prices and coupon levels is the following. Low values of μ imply that, over the 3 periods, a greater proportion of purchases is carried out by occasional rather than by frequent travellers. Accordingly, airlines place a greater weight in competing for the former set of travellers. Given that the behaviour of an occasional traveller is not influenced by the discount offered, airlines can only compete for their business by offering low base prices. At the same time, airlines do not wish to offer higher discounts since these would raise base prices, as noted in Proposition 2.1, and hence drive away a share of the more numerous, by assumption, occasional travellers. On the other hand, when the share of frequent travellers is high, airlines are able to compete for their patronage through the generosity of their coupons and are less concerned with the adverse effect that higher prices have in the demand generated by the (small) group of occasional travellers.

2.5.2   Airline profits and travellers' welfare

The previous section noted that both equilibrium prices and discounts are positively related to the share of frequent travellers in the market. Other things equal, higher prices across the two airlines raise their profits and higher discounts lower them. The reverse is true with respect to travellers' welfare. What then is the net effect of μ on airline profits and travellers' welfare?

Travellers' welfare

When airlines are allowed to introduce $FFPs$, their equilibrium profits are above the benchmark level. It follows that the set of travellers, taken as a whole, are worse off when these loyalty schmes are introduced.[23] However, this does not imply that each individual fares badly from the introduction of the programmes. To see this consider the welfare of the two sets of travellers separately.

Occasional travellers are in the market for one period only. Given that they cannot receive a coupon and that the price paid by them is always above the benchmark level it follows that all occasional travellers are worse off if

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[23]Provided the prices and discounts set by the two airlines are equal - so that the equilibrium is symmetric -the total surplus to be divided between travellers and airlines is equal to $3\left(R - c - \frac{1}{2}\right)$ and is independent of the prices and discounts offered. The constant level of total surplus follows from the assumption that travellers hold an inelastic demand for one unit of the good in each period.

the $FFPs$ are launched. Furthermore, given that prices increase with μ, the higher the share of frequent travellers the worse off will the occasional travellers be. This set of travellers will always prefer the benchmark scenario in which airlines are unable to launch $FFPs$.

To see the effect of the $FFPs$ on the welfare of a frequent traveller it is necessary to calculate his expected life-time utility. This expression is given by $3\left(R - c - \frac{1}{2}\right) - \frac{33 - 20\mu}{13 - 10\mu}$ which is a decreasing function of μ. It follows that a frequent traveller does better when he is one of the few frequent travellers. Compared to the expected lifetime surplus in the benchmark model, given by $3\left(R - c - \frac{1}{2}\right) - 3$, it is quickly established that frequent travellers are better off in a market which offers $FFPs$ if and only if $\mu < \frac{6}{10}$. When the share of frequent travellers is above this level, the higher coupon value received from being a repeat buyer does not make up for the higher prices faced.

Corollary 2.2 summarizes the previous discussion.

Corollary 2.2. *Provided the set of frequent travellers is not empty, airlines are able to collect higher profits if they are allowed to launch FFPs. Introducing these programmes makes an occasional consumer necessarily worse off while frequent travellers benefit if and only if they account for less than $\frac{6}{10}$ of the passengers.*

Airline profits

In equilibrium, the airlines' profits over the 3 periods can be derived as,

$$\Pi_1^j = \frac{39 - 26\mu}{2\left(13 - 10\mu\right)}, \ j = A, B. \qquad (2.12)$$

and it follows directly from Proposition 2.3. Note that the profit earned over the three periods is increasing in μ. When $\mu = 0$ profits are equal to the benchmark level of $\frac{3}{2}$ as one would expect. On the other hand, profits are at their highest, $\Pi_1^j = 2\frac{1}{6}$, when the entire market is composed of frequent travellers.

The positive relation between μ and profits suggests that airlines would find it appealing to divide the market into two segments as described below in Corollary 2.3.

Corollary 2.3. *Consider the following policy.*

i) Split the market into two such that occasional travellers patronise one segment whilst frequent travellers patronise the other segment;

ii) Set equilibrium prices and discounts in each segment according to Proposition 2.3:

Segment	p_1^{J*}	p_2^{J*}	p_3^{J*}	$\alpha^* = \beta^*$
Occasional	$1 + c$	$1 + c$	$1 + c$	–
Frequent	$2\frac{1}{3} + c$	$2 + c$	$2 + c$	2

In a symmetric equilibrium where both carriers adopt this policy, airline profits are given by $\frac{6+5\mu}{4}$. Airlines will find it attractive to split the market in the way described by the above policy.

Proof. By construction $\mu = 0$ n the segment of the market patronised by occasional travellers and $\mu = 1$ in the segment patronised entirely by frequent travellers. Given this, the equilibrium prices and discount set out in the Corollary follow immediately from Proposition 2.3. Using (2.12), the profits to an airline when it segments the market as described in the Corollary is given by, $\mu \frac{39-26(1)}{2(13-10(1))} + (1 - \mu)\frac{3}{2} = \frac{9+4\mu}{6}$. To see that this profit level is higher than that which would be achieved if the market was not divided note that $\frac{9+4\mu}{6} = \frac{(9+4\mu)(13-10\mu)}{6(13-10\mu)} = \frac{117-38\mu-40\mu^2}{6(13-10\mu)} \leq \frac{117-78\mu}{6(13-10\mu)} = \frac{39-26\mu}{2(13-10\mu)}$ which is the profit level of the airlines when the market is not split. ∎

Corollary 2.3 assumes that airlines are able to segment the market between frequent and occasional travellers. The menu of price and discounts laid out in the Corollary will not, by themselves, achieve such a segmentation - frequent travellers would prefer to pay the price charged occasional travellers and forefeit the chance of benefitting from a discount.[24] However, in the air transport industry, as indeed with passenger transportation in general, an imperfect segmentation is obtained through the offer of First/Business Class and Economy Class seats. The former tend to be occupied exclusively by frequent travellers who place greater value on comfor and flexibility whilst the latter are typically taken up by occasional passengers. Clearly, factors other than those related to FFP lead airlines to offer Business Class and

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[24] In other words, the menu of prices and discounts are not a separating equilibrium.

78

Economy Class seats and to charge a higher price for the former. Nevertheless, Corollary 2.3 provides an addition reason why, in conjunction with launching a FFP, an airline find this segmentation profitable.

Social welfare

Given that consumers hold an inelastic demand for one unit of the good in each period, maximizing social welfare is tantamount to minimising total consumer transportation costs. In turn, the latter are minimised if, in each period, consumers address the airline closest to them: those located in the first half of I patronise A whilst those in the second half of the unit interval address airline B.

In the absence of $FFPs$ total transportation costs are minimized since consumers' optimal policy is to address the closest airline in each period. Introducing the $FFPs$ described by Proposition 2.3 does not alter this result. It can be checked that when the prices and coupons offered by the two airlines are equal, as they are in the symmetric equilibrium described above, then consumers will also patronise the closest airline in each period. Social welfare is, therefore, maximized.[25]

This result contrasts with that of Caminal and Matutes (1990). These authors report that the launch of $FFPs$ lead consumers to incur higher transportation costs than they would otherwise. Their result is driven by the fact that some consumers will be willing to travel further in order to address the same airline that they had done in the past and so be eligible for the coupon offered to repeat buyers. In the model presented here, on the other hand, consumers can always address the closest airline, minimising travelling costs and be sure that at some point in time - if not in the second period then in the third- they will be entitled to a discount. This result hinges on the assumption that airlines are symmetric and that travellers do not discount future gains.

To close this section, I should note - as do Caminal and Matutes (1990, $p.361$) - that in a more general model where consumers are endowed with an elastic demand function, the increase in the prices that results from the introduction of $FFPs$ will have negative welfare effects. In this light, the benchmark case which does not allows for discrimination between first-time and repeat buyers would be superior in terms of total social welfare.

---

[25] If the prices and discount offered by airlines are equal then it can be checked that $\Omega_j = \frac{1}{2}$ for $j = 1, 2, 3, 4, 5$.

To lock-in or not?[26]

The previous sections have argued that a *FFP* need not create switching costs to allows an airline to collect supra-competitive profits. Given this, one might expect that an airline can do even better if it implements a programme which *does* induce switching costs on travellers as this would add one other force driving airlines towards an outcome away from the competitive benchmark setting. In this section I ask whether such reasoning is valid.

To approach this question it is necessary to first recast the model of Section 2.2 so that it features *FFPs* which induce switching-costs. The modified model is then solved and the equilibrium prices and profits of airlines compared with those that were derived in the original model.

There are two simple alternative ways of altering the model so that the *FFPs* considered induce switching costs on travellers. Firstly, the generosity of the schemes may be tightened so that a discount is handed out to a traveller when he patronises the same airline for the third time. A second possibility is to reduce the time horizon of the model to two periods. Either modification would give rise to a setting where frequent travellers must patronise the same airline at every purchasing opportunity in order to benefit from the discount offered.

However, it should be noted that neither of these alternatives is ideal. In both cases, the *locking-in feature* is introduced at the cost of altering other aspects of the model. If the first route is followed, then the number of purchases required for a traveller to earn the discount increases from two to three. On the other hand, if the second suggestion is taken, the time horizon over which airlines compete is shortened to two periods. In either case, the structure of the model is altered. Therefore, these structural differences must be kept in mind when the results of the modified model are compared with those of the original one, as it would be wrong to attribute the differences in the outcome of the models entirely to the presence or absence of a *locking-in feature*.

The above problem cannot, however, be overcome. Introducing *FFPs* which induce switching costs will necessarily alter other features of the model.[27]

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[26]To be rigorous, the term *lock-in* should be replaced by *induce switching costs* since travellers' choices are never forcefully tied to their past actions. With this in mind, I will use in this section the term *lock-in* as it makes the exposition easier.

[27]A third alternative is to alter the original model in the following way. Let the population of frequent travellers be composed of three groups, -label them F, G, H - which take

80

Of the two possibilities described above, the second one is chosen - shortening the time horizon of the model to two periods - as it corresponds to the setting described in Banerjee and Summers (1987) and Caminal and Matutes (1990).

2.5.3 Modified model

The modified model differs from the one described in Section 2.2 due to the shortening of the time horizon from 3 to 2 periods. Hence, airlines set prices at the start of both periods while the coupon value is decided at the start of Period 1. The occasional travellers are in the market for only one of the periods while the frequent travellers participate in both. Lastly, note that - as before - a frequent traveller who patronises an airline for the second time receives that airline's discount.[28]

The steps involved in solving the modified model are analogous to those taken in solving the 3-period model which were presented in Section 2.3. To avoid repeating the presentation of similar reasoning most of the work involved in solving the model is left to the Appendix 2.B.

As before, the model is solved by working backwards. By solving the optimization problems of the two airlines in Period 2, their reaction functions can be derived. For airline A, this is given by,

$$p_2^{A'} = \frac{1}{2}\left(1 + c + p_2^{B'} + 2\mu\alpha'\sigma' + \mu\beta'\left(1 - \sigma'\right)\right)$$

where the notation is equivalent to the one used in Sections 2.2 and 2.3 and the suffix $'$ is used to denote the modified model. An analogous expression can be derived as B's reaction function. Solving the two equations simultaneously yields the second period equilibrium prices,

$$
\begin{aligned}
p_2^{A'} &= 1 + c + \mu\alpha'\sigma' \\
p_2^{B'} &= 1 + c + \mu\beta'\left(1 - \sigma'\right)
\end{aligned}
\tag{2.13}
$$

turns in participating in the market: in Period 1, groups F and G take part, in Period 2 groups G and H and in the last period groups F and H. This alternative has the merit of not altering the time horizon nor the generosity of the $FFPs$. This alternative was explored but it proved hard to derive any results from it.

[28]When all travellers are frequent travellers, $\mu = 1$, this model is identical to one of the models examined by Caminal and Matutes (1990).

This result points out that an airline's second period price is above the benchmark level and increases both with the size of the discounts set in the first period and with the mass of travellers eligible to receive the discount in Period 2 - given by the product of μ and the airline's share of the demand generated by frequent travellers in Period 1.

Using the above equilibrium prices, it is instructive to construct the expressions which describes the airlines' profits in the second period. For airline A, this will be given by,

$$\Pi_2^{A'} = \frac{1}{2}\left(1 + \alpha'\mu\sigma'\left(\mu\sigma' - 1\right)\left(1 + \beta\mu\right)\right) \tag{2.14}$$

and a similar expression holds for airline B. Given that $\mu\sigma' \leq 1$, it follows that $\Pi_2^{A'} \leq \frac{1}{2}$, which is competitive benchmark level..

Now consider the airlines' problems in Period 1. Here, the airlines' objective is to maximise the sum of its first period profits and those expected in the second period - given by (2.14)for airline A. The equilibrium prices and disounts will be given by solving the reaction function of each airline simultaneously. It is shown in Appendix 2.A, that equilibrium prices and discounts ds exist though it is not possible to derive an explicit expression for them. Instead, for a given value of the parameter μ their values can be worked out numerically. Figure 2.4 summarizes the results by plotting the base prices as well as the discounted price paid by repeat buyers in Period 2. Note that the curve drawing the discounted price is not defined at $\mu = 0$.

It is clear from Figure 2.4 that equilibrium prices increase with the share of frequent travellers and that they are above the competitive level, $c + 1$, for all $\mu > 0$. The discount offered to repeat buyers also increases with μ although only just slightly. For $\mu = \varepsilon$ - a share just above 0 - the discount is equal to 0.65 while for $\mu = 1$, its value is 0.67. Other readings of the results summarized in Figure 2.4, and an interpretation of them, are left to the next section where a comparison with the results derived in Section 2.3 for the three-period model is carried out.

2.5.4 Comparing the two models

In order to contrast the outcome of the two models more easily and given that the symmetric equilibrium of the modified model can only be characterized numerically, it is appropriate to compare the prices, discounts and profits resulting from the two models for a given value of μ.
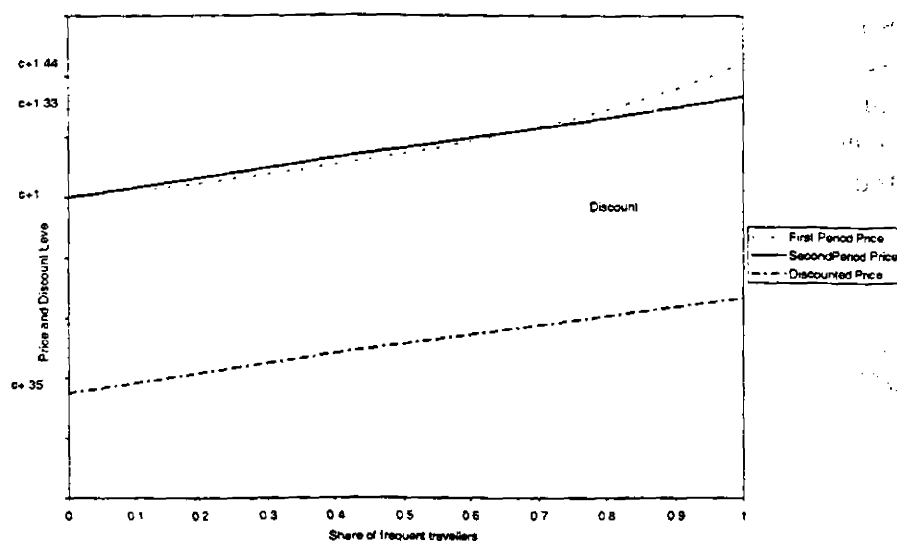
Figure 2.4: Two-period model equilibrium prices as a function of the share of frequent travellers.

Table 2.1 below characterizes the symmetric equilibrium of the games described in the two models for $\mu = 1$. The figures presented in the table would be different if other values of μ had been chosen, although the qualitative comparison which follows would still be valid. The first three columns report the equilibrium mark-up of the airlines in each of the periods. The absolute value of the discount as well as the percentage of the mark-up which it represents are given in the subsequent two columns. The remaining four columns show the equilibrium profits earned by the airlines over the time horizon of the model and the average profit per period. To make the following discussion simpler, the original three-period model is referred to as Model I while the modified, two-period model will be labelled as Model II.

Table 2.1: Comparison of the three models for $\mu = 1$.

Model	Price mark-up			Discount		Profit per Period			
	Pd. 1	Pd. 2	Pd. 3	Abs.	%	Pd. 1	Pd. 2	Pd. 3	Avg.
Benchmark	1	1	1	—	—	0.50	0.50	0.50	0.50
Model I	2.33	2	2	2	100	1.17	0.50	0.50	0.72
Model II	1.44	1.33	—	0.67	50	0.72	0.39	—	0.56

As had been noted earlier, the second period price in Model II is above the benchmark level. The intuition for this was described in Section 1 and it stems from the airlines' ability to exploit the switching costs which their $FFPs$ induce on travellers.

The second period price in Model II is considerably lower than the price level in Period 2 and 3 in Model I. Thisdifference reflects the more generous discounts offered in Model I. Give equal discounts, both settings would give rise to equal second period prices.

To see how airlines are able to sustain higher equilibrium prices in the first period in Model I than in Model II, recall equation (2.14) describing the second period profits of airline A in Model II,

$$\Pi_2^{A'} = \frac{1}{2} \left(1 + \alpha' \mu \sigma' \left(\mu \sigma' - 1 \right) \left(1 + \beta \mu \right) \right) \tag{2.15}$$

In the symmetric equilibrium $\sigma' = \frac{1}{2}$. Any deviation in the first period prices away from the symmetric equilibrium will shift σ' away from $\frac{1}{2}$ and, given (2.14), will increase airline A's second period profits. If the symmetric equilibrium is to be sustained it is necessary that a downward deviation price from the equilibrium level harms an airline's first-period profits. In other words, the symmetric equilibrium prices must be such that the increase in the first-period market share of such a deviant does not make up for the lower price charged. In turn, this implies that prices cannot be sustained at high values.

On the other hand, as was discussed in Section 2.3.4, in Model I,the temptation of an airline to cut its first-period price from the equilibrium value is checked by the negative effects that this has on its second-period profits. This allows for higher prices to be sustained in Period 1 than those of Model II.

The figures provided in Table 2.1 show that average profits per period are higher in Model I than in Model II for $\mu = 1$. This is in fact a general result as be read from Figure 2.5 which contrasts the average profit per period earned by an airline in the benchmark case, under Model I and under Model II. The relative ranking of the models in terms of average profit per period is largely accounted by the fact that airlines are able to sustain higher prices in Model I than in Model II. In addition in Model II, a large proportion, $\frac{5}{6}$, of the frequent travellers addressing an airline in Period 2 are repeat buyers
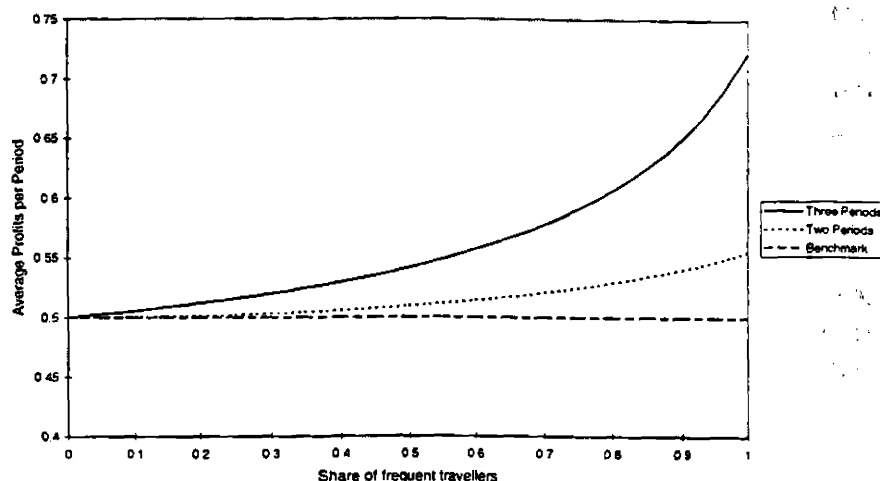
Figure 2.5: Average profit per period in three settings: the two-, the three-period and the benchmark model.

who qualify for a discount and accordingly, pay a low effective price.[29] On the other hand, in Model I, in each of the Periods 2 and 3, only half of the frequent travellers qualify for the discount and the revenue foregone due to these discounts is made up by those paying the high full price. The net result is that, both in Period 2 and in Period 3, airlines' profits do not fall below the benchmark level as occurs in the Period 2 of Model II.

Before closing this section, it is appropriate to consider a setting where airlines compete over three periods, as in Model I, and frequent travellers qualify for a discount only if they patronise the same airline throughout. This was an alternative modification discussed at the start of this section and it reflects the setting discussed briefly in Banerjee and Summers (1997). While the formal treatement of this model proved unworkable, it is possible, through intuition, to characterise the outcome in such a setting. It is in the interest of an airline to ensure that the rival attracts a mass of frequent travellers that will qualify for a discount in the third period. If this were not the case, the airline with no loyal travelllers would have the incentive to

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[29]The share of repeat buyers in Period 2 is given by the sum of the terms $\Lambda_1 + (1 - \Lambda_2)$ which are defined in Appendix 2.B For $\mu = 1$, this sum is equal to $\frac{5}{6}$.

undercut and prices would be driven to the benchmark level. Compared to the two period setting of Model II, price competition would be less aggressive and lower discounts would be necessary to sustain the cooperation between the airlines. This intuition points to the result that the airlines fare better when the number of periods during which customers must be loyal increases. It does not, however, establish a comparison with the profit level earned in the setting described in the original model described in Section 2.4 where the FFP induced no switching costs.

So how can the heading of Section 2.5 be answered? Are airlines better off with a FFP which imposes switching-costs on its travellers or not? The above discussion does not allow this question to be answered. However, one - trivial - point is apparent: the structure of the $FFPs$ plays a significant impact on the profitability of the scheme.

2.6   Conclusion

This chapter aimed at exploring some issues surrounding customer loyalty schemes. Its main concern was to examine whether loyalty-schemes need to induce switching costs on travellers in order to have a *raison d'être*. The answer is in the negative. It was shown that even when $FFPs$ do not induce such costs, they are a tool which facilitate tacit-collusion amongst airlines.

The composition of the population was seen to influence equilibrium prices and the level of the coupons awarded to repeat buyers. Both increased with the share of frequent travellers in the population.

The analysis also pointed out that, typically, travellers have little to be enthusiast about $FFPs$. Those who participate in the market rarely, and hence cannot hope to benefit from any coupon, lose out due to the high prices. On the other hand, for frequent travellers to benefit from the launch of a FFP it is necessary that the share of travellers which they account for is not very high so that the coupon received offsets the higher prices practised.

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Appendices

2.A Solving the Three Period Model

The proof is divided into two parts. Part 1 sets out the candidate symmetric equilibrium. Part 2 then shows that this candidate equilibrium is robust to deviations and so does indeed form an equilibrium.

In the process of going through the solution to the model Propositions 2.2 and 2.3 and Corollary 2.1 will be proved.

2.A.1 Part 1 - Finding the candidate symmetric equilibrium

The optimal decision rules of travellers and airlines are traced backwards in time, as presented in the main body of the text.

Period 3   Recall that the equilibrium prices in Period 3 were shown in Section 2.4.1 to be,

$$
\begin{aligned}
p_3^{A*} &= 1 + c + \alpha\mu s \\
p_3^{B*} &= 1 + c + \beta\mu s
\end{aligned}
$$

This was proven in the main text and is not repeated here.

Given the third period equilibrium prices, it will be useful to re-write the terms Ω_1 and Ω_2 as,

$$
\begin{aligned}
\Omega_1 &= \frac{1 + p_3^{B*} - p_3^{A*}}{2} = \frac{1 + \mu s\,(\beta - \alpha)}{2} \\
\Omega_2 &= \frac{1 + p_3^{B*} - \beta - p_3^{A*} + \alpha}{2} = \frac{1 + (\mu s - 1)\,(\beta - \alpha)}{2}
\end{aligned}
\tag{2.16}
$$

Period 2   Section 2.4.2 defined the terms Ω_3 and Ω_4 as,

$$
\begin{aligned}
\Omega_3 &= \frac{1 + p_2^B - p_2^A + \alpha + \Omega_1^2 - \Omega_2^2 - \beta}{2} \\
\Omega_4 &= \frac{1 + p_2^B - p_2^A + \Omega_1^2 - \Omega_2^2}{2}
\end{aligned}
\tag{2.17}
$$

These terms will be useful for later.

Following the presentation in Section 2.4.2, the maximization problems of the two airlines at the start of Period 2 are given by,

$$
\begin{aligned}
\underset{\{p_2^A\}}{Max\Pi_2^A} \;=\;& \left(p_2^A - c\right)\left(1 - \mu\right)\left(\frac{1 + p_2^B - p_2^A}{2}\right) + \left(p_2^A - c\right)\mu\left(1 - \sigma\right)\Omega_4 + \\
& \left(p_2^A - \alpha - c\right)\mu\sigma\Omega_3 + \Pi_3^A
\end{aligned}
$$

$$
\begin{aligned}
\underset{\{p_2^B\}}{Max\Pi_2^B} \;=\;& \left(p_2^B - c\right)\left(1 - \mu\right)\left(\frac{1 - p_2^B + p_2^A}{2}\right) + \left(p_2^B - c\right)\mu\sigma\left(1 - \Omega_4\right) + \\
& \left(p_2^B - \beta - c\right)\mu\left(1 - \sigma\right)\left(1 - \Omega_3\right) + \Pi_3^B
\end{aligned}
$$

The second period equilibrium prices are found by solving the two problems simultaneously. By solving the two first order conditions, it can be shown that the reaction functions of the two airlines are given by the following algebraically cumbersome expressions:

$$
\begin{aligned}
p_2^A &= \frac{1}{D}\left(A + Bc + Cp_2^B\right) \\
p_2^B &= \frac{1}{G}\left(E + Bc + Fp_2^A\right)
\end{aligned}
\tag{2.18}
$$

where,

$$A = 2\sigma \quad (\sigma - 1)\mu^3 \left((\alpha - \beta)^5 + \alpha(\alpha - \beta)^4\right) + 8\sigma^2\mu^2 \left((\alpha - \beta)^3 + 3\alpha(\alpha - \beta)^2\right) +$$

$$2\sigma \left(\begin{array}{c} (\mu^2 - \mu^3)(\alpha - \beta)^4 - 4\mu^2(\beta - \alpha)(\beta - 3\alpha) + \\ 2\mu^2 \left(2(\beta - \alpha)^3 - 7\alpha(\alpha - \beta)^2\right) + 4\mu\left((\beta - \alpha)(\beta - 3\alpha) - 2\alpha\right) \end{array} \right) +$$

$$(\alpha - \beta)^4(\mu^3 - 2\mu^2) + 2\mu^2\left((\beta - \alpha)(\beta^2 - \alpha^2) + 2(\beta - \alpha)(\beta - 3\alpha)\right) +$$

$$4\mu\left(2(\beta - \alpha) - (\alpha - \beta)(5\alpha - 3\beta)\right) - 16$$

$$B = 4\sigma \quad (\sigma - 1)\mu^2\left(\mu(\alpha - \beta)^4 + 4(\alpha - \beta)^2\right) + (\alpha - \beta)^4(\mu^3 - \mu^2) +$$

$$4\mu(\mu - 2)(\alpha - \beta)^2 - 16$$

$$C = 4\sigma \quad (\sigma - 1)\mu^2\left(\mu(\alpha - \beta)^4 + 4(\beta - 3\alpha)(\beta - \alpha)\right) + (\alpha - \beta)^4(\mu^3 - \mu^2) +$$

$$4\mu(\mu - 2)(\alpha - \beta)^2 - 16$$

$$D = 8\sigma \quad (\sigma - 1)\mu^2\left(\mu(\alpha - \beta)^4 + 4(2\alpha - \beta)(\alpha - \beta)\right) + 2(\alpha - \beta)^4(\mu^3 - \mu^2) +$$

$$8\mu^2(\beta - 2\alpha)(\alpha - \beta) - 16\mu(\alpha - \beta)^2 - 32$$

$$E = 2\sigma \quad (\sigma - 1)\mu^3\left((\beta - \alpha)^5 + \beta(\alpha - \beta)^4\right) + 8\sigma^2\mu^2\left((\beta - \alpha)^3 + 3\beta(\alpha - \beta)^2\right) +$$

$$2\sigma(\mu^3 - \mu^2)(\alpha - \beta)^4 + 2\sigma\mu^2\left(4(\alpha - 3\beta)(\alpha - \beta) + 4(\alpha - \beta)^3\right) -$$

$$2\sigma\mu^2\left(10(\alpha - \beta)^2 - 4\mu((\alpha - 3\beta)(\alpha - \beta) - 2\beta)\right) -$$

$$\mu^3(\alpha - \beta)^4 + 2\mu^2\left((\alpha - \beta)^3 - 2(3\beta - \alpha)(\beta - \alpha)\right) +$$

$$4\mu(2\alpha - \alpha^2 + \beta^2 - 6\beta) - 16$$

$$F = 4\sigma \quad (\sigma - 1)\mu^2\left(\mu(\alpha - \beta)^4 + 4(\alpha - 3\beta)(\alpha - \beta)\right) + (\alpha - \beta)^4(\mu^3 - \mu^2) +$$

$$4\mu^2(\alpha - 3\beta)(\alpha - \beta) - 8\mu(\alpha - \beta)^2 - 16$$

$$G = 8\sigma \quad (\sigma - 1)\mu^2\left(\mu(\alpha - \beta)^4 + 4(2\beta - \alpha)(\beta - \alpha)\right) + 2(\alpha - \beta)^4(\mu^3 - \mu^2) +$$

$$8\mu^2(\alpha - 2\beta)(\alpha - \beta) - 16\mu(\alpha - \beta)^2 - 32$$

The equilibrium prices are found by solving the system of equations given by the two reaction functions, (2.18). It follows that the second period equiliobrium prices are given by,

$$p_2^{A*} = \frac{AG + CE + (C + G)Bc}{DG - FC}, \qquad p_2^{B*} = \frac{DE + FA + (F + D)Bc}{DG - FC}$$

$$(2.19)$$

It was checked that at this equilibrium point the necessary second-order conditions are met.

The expressions for p_2^{A*} and p_2^{B*} in terms of the model's parameters are too unwieldy to provide any insight. However, two special cases can be considered.

First, when $\alpha = \beta = \lambda$ it can be shown that $A = -16\sigma\mu\lambda - 16$, $B = C = F = -16$, $D = G = -32$ and $E = 16\sigma\mu\lambda - 16\mu\lambda - 16$. Substituting these terms into (2.19), the equilibrium second prices simplify to $p_2^{A*} = 1 + c + \mu\lambda\frac{(1+\sigma)}{3}$ and $p_2^{B*} = 1 + c + \mu\lambda\frac{(2-\sigma)}{3}$.

Second, when $\sigma = 0.5$ it can be shown that $A =$, $B =$, $C =$, $D =$, $E =$, $F =$ and $G =$. Substituting these terms into (2.19), the equilibrium second prices simplify to $p_2^{A*} = 1 + c + \frac{\mu\alpha}{2}$ and $p_2^{B*} = 1 + c + \frac{\mu\beta}{2}$.

This proves Corollary 2.1.

Period 1 The maximization problem facing airline A at the start of Period 1 is given by,

$$
\begin{aligned}
\underset{\{p_1^A, \alpha\}}{Max}\, \Pi_1^A &= \left(p_1^A - c\right)\left(\mu\sigma + (1-\mu)\left(\frac{1 + p_1^B - p_1^A}{2}\right)\right) + \Pi_2^A \\
&= \left(p_1^A - c\right)\left(\mu\sigma + (1-\mu)\left(\frac{1 + p_1^B - p_1^A}{2}\right)\right) + \\
&\quad \left(p_2^A - c\right)\left(\mu(1-\sigma)\Omega_4 + (1-\mu)\left(\frac{1 + p_2^B - p_2^A}{2}\right)\right) + \\
&\quad \left(p_2^A - \alpha - c\right)\mu\sigma\Omega_3 + \\
&\quad \left(p_3^A - c\right)\left(\mu(1-s)\Omega_1 + (1-\mu)\left(\frac{1 + p_3^B - p_3^A}{2}\right)\right) + \\
&\quad \left(p_3^A - \alpha - c\right)\mu s\Omega_2
\end{aligned}
$$

An analogous expression can be written for airline's B maximization problem. However, given that the I will be searching for a symmetric equilibrium it will be sufficient to work with (2.20).

The two first order conditions of A's optimisation problem are given by,

$$\frac{d\Pi_1^A}{dp_1^A} = \mu\sigma + (1-\mu)\left(\frac{1+p_1^B-p_1^A}{2}\right) + (p_1^A-c)\left(\mu\frac{d\sigma}{dp_1^A} - \frac{(1-\mu)}{2}\right) +$$

$$\left(\mu(1-\sigma)\Omega_4 + (1-\mu)\left(\frac{1+p_2^B-p_2^A}{2}\right)\right)\frac{dp_2^A}{dp_1^A} + \mu\sigma\Omega_3\frac{dp_2^A}{dp_1^A}$$

$$(p_2^A-c)\left(-\mu\Omega_4\frac{d\sigma}{dp_1^A} + \mu(1-\sigma)\frac{d\Omega_4}{dp_1^A}\right) + \mu(p_2^A-\alpha-c)\left(\sigma\frac{d\Omega_3}{dp_1^A} + \Omega_3\frac{d\sigma}{dp_1^A}\right) +$$

$$(p_2^A-c)\left(\frac{1-\mu}{2}\right)\left(\frac{dp_2^B}{dp_1^A} - \frac{dp_2^A}{dp_1^A}\right) + (p_3^A-c)\left(-\mu\Omega_1\frac{ds}{dp_1^A} + \mu s\frac{d\Omega_1}{dp_1^A}\right) +$$

$$\left(\mu(1-s)\Omega_1 + (1-\mu)\left(\frac{1+p_3^B-p_3^A}{2}\right)\right)\frac{dp_3^A}{dp_1^A} + \mu s\Omega_2\frac{dp_3^A}{dp_1^A} +$$

$$(p_3^A-c)\left(\frac{1-\mu}{2}\right)\left(\frac{dp_3^B}{dp_1^A} - \frac{dp_3^A}{dp_1^A}\right) + (p_3^A-\alpha-c)\left(\mu\Omega_2\frac{ds}{dp_1^A} + \mu s\frac{d\Omega_2}{dp_1^A}\right)$$

$$\frac{d\Pi_1^A}{d\alpha} = (p_1^A-c)\mu\frac{d\sigma}{d\alpha} + \left((1-\sigma)\Omega_4\mu + (1-\mu)\left(\frac{1+p_2^B-p_2^A}{2}\right)\right)\frac{dp_2^A}{d\alpha} +$$

$$(p_2^A-c)\left(-\Omega_4\mu\frac{d\sigma}{d\alpha} + (1-\sigma)\mu\frac{d\Omega_4}{d\alpha} + \frac{(1-\mu)}{2}\left(\frac{dp_2^B}{d\alpha} - \frac{dp_2^A}{d\alpha}\right)\right) +$$

$$\sigma\Omega_3\mu\left(\frac{dp_2^A}{d\alpha}-1\right) + (p_2^A-\alpha-c)\mu\left(\Omega_3\frac{d\sigma}{d\alpha} + \sigma\frac{d\Omega_3}{d\alpha}\right) +$$

$$\left((1-s)\Omega_1\mu + (1-\mu)\left(\frac{1+p_3^B-p_3^A}{2}\right)\right)\frac{dp_3^A}{d\alpha} +$$

$$(p_3^A-c)\mu\left(-\Omega_1\frac{ds}{d\alpha} + (1-s)\frac{d\Omega_1}{d\alpha}\right) + \left(\frac{dp_3^A}{d\alpha}-1\right)s\Omega_2\mu +$$

$$(p_3^A-c)\left(\frac{1-\mu}{2}\right)\left(\frac{dp_3^B}{d\alpha} - \frac{dp_3^A}{d\alpha}\right) + (p_3^A-\alpha-c)\mu\left(\Omega_2\frac{ds}{d\alpha} + s\frac{d\Omega_2}{d\alpha}\right)$$

I now set out to evaluate the derivatives of various terms with respect to the two choice variable which build up the two first order conditions

To derive $\frac{d\sigma}{dp_1^A}$ and $\frac{d\sigma}{d\alpha}$ recall the implicit equation (2.10) described in Section 2.4.3 ,

$$J \equiv \frac{1 + p_1^B - p_1^A + \Omega_3^2 - \Omega_4^2 + \Omega_2^2 - \Omega_1^2}{2} - \sigma \qquad (2.20)$$

where the terms $\Omega_1, \Omega_2, \Omega_3$ and Ω_4 are functions of σ, p_1^A and α. Based on (2.20), it is possible to use the implicit function theorem to calculate $\frac{d\sigma}{dp_1^A}$ and $\frac{d\sigma}{d\alpha}$. The partial derivatives of J - after imposing the symmetry conditions, i.e. $\sigma = 0.5$, $\alpha = \beta$, $p_1^A = p_1^B$ - are given by $\frac{\partial J}{\partial p_1^A} = -\frac{1}{2}, \frac{\partial J}{\partial \alpha} = \frac{1}{4}$ and $\frac{\partial J}{\partial \sigma} = -1$. Therefore, given the symmetry,

$$\frac{d\sigma}{dp_1^A} = -\frac{\partial J}{\partial p_1^A} \div \frac{\partial J}{\partial \sigma} = -\frac{1}{2}$$

$$\frac{d\sigma}{d\alpha} = -\frac{\partial J}{\partial \alpha} \div \frac{\partial J}{\partial \sigma} = \frac{1}{4}$$

To derive the expressions for $\frac{dp_2^A}{dp_1^A}, \frac{dp_2^B}{dp_1^A}$ note that $\frac{dp_2^A}{dp_1^A} = \frac{\partial p_2^A}{\partial p_1^A} + \frac{d\sigma}{dp_1^A} \frac{\partial p_2^A}{\partial \sigma}$ and $\frac{dp_2^B}{dp_1^A} = \frac{\partial p_2^B}{\partial p_1^A} + \frac{d\sigma}{dp_1^A} \frac{\partial p_2^B}{\partial \sigma}$. From (2.19) it follows that $\frac{\partial p_2^A}{\partial p_1^A} = \frac{\partial p_2^B}{\partial p_1^A} = 0$.

Obtaining the expressions for $\frac{\partial p_2^A}{\partial \sigma}$ and $\frac{\partial p_2^B}{\partial \sigma}$, requires a bit more work as it requires to look at the terms A, B, C, D, E, F and G which define p_2^A and p_2^B. Under symmetry $\frac{\partial A}{\partial p_1^A} = -16\mu\alpha$, $\frac{\partial E}{\partial p_1^A} = 16\mu\alpha$ and $\frac{\partial B}{\partial p_1^A} = \frac{\partial C}{\partial p_1^A} = \frac{\partial D}{\partial p_1^A} = \frac{\partial F}{\partial p_1^A} = \frac{\partial G}{\partial p_1^A} = 0$. In addition it is straight-forward to calculate that under symmetry, $A = E = -8\mu\alpha - 16$, $B = C = F = -16$ and $D = G = -32$. It then follows from (2.19) that $\frac{\partial p_2^A}{\partial p_1^A} = \frac{\left(G\frac{\partial A}{\partial p_1^A} + C\frac{\partial E}{\partial p_1^A}\right)}{(DG - FC)} = \frac{\mu\alpha}{3}$ and that $\frac{\partial p_2^B}{\partial p_1^A} = \frac{\left(D\frac{\partial E}{\partial p_1^A} + F\frac{\partial A}{\partial p_1^A}\right)}{(DG - FC)} = -\frac{\mu\alpha}{3}$.

Putting these previous results together, it follows that,

$$\frac{dp_2^A}{dp_1^A} = -\frac{\mu\alpha}{6}$$

$$\frac{dp_2^B}{dp_1^A} = \frac{\mu\alpha}{6}$$

An analogous procedre can be followed to obtain the following expressions

for $\frac{dp_2^A}{d\alpha}$ and $\frac{dp_2^B}{d\alpha}$,

$$
\begin{aligned}
\frac{dp_2^A}{d\alpha} &= \frac{\partial p_2^A}{\partial \alpha} + \frac{d\sigma}{d\alpha}\frac{\partial p_2^A}{\partial \alpha} = \frac{\mu}{2} + \frac{1}{4}\frac{\mu\alpha}{3} \\
&= \frac{\mu}{2} + \frac{\mu\alpha}{12} \\
\frac{dp_2^B}{d\alpha} &= \frac{\partial p_2^B}{\partial \alpha} + \frac{d\sigma}{d\alpha}\frac{\partial p_2^B}{\partial \alpha} = 0 + \frac{1}{4}\frac{-\mu\alpha}{3} \\
&= \frac{-\mu\alpha}{12}
\end{aligned}
$$

After imposing the symmetry conditions it can be shown that $\frac{d\Omega_1}{dp_1^A} = \frac{d\Omega_2}{dp_1^A} = 0$, $\frac{d\Omega_3}{dp_1^A} = \frac{d\Omega_4}{dp_1^A} = \frac{\mu\alpha}{6}$ and $\frac{dp_3^A}{dp_1^A} = \frac{dp_3^B}{dp_1^A} = \frac{ds}{dp_1^A} = 0$. Similarly, it can be shown that $\frac{d\Omega_1}{d\alpha} = -\frac{\mu}{4}$, $\frac{d\Omega_2}{d\alpha} = \frac{2-\mu}{4}$, $\frac{d\Omega_3}{d\alpha} = \frac{d\Omega_4}{d\alpha} = \frac{1}{4} - \frac{\mu}{4} - \frac{\mu\alpha}{12}$, $\frac{dp_3^A}{d\alpha} = \frac{\mu}{2}$, $\frac{dp_3^B}{d\alpha} = 0$ and $\frac{ds}{d\alpha} = 0$. Lastly, it is easy to see that under symmetry $\Omega_i = \frac{1}{2}$ $i = 1,2,3,4$ and $s = \frac{1}{2}$

Substituting the various expressions obtained into the two first-order conditions yields after some simplifcation the following system of equations,

$$
\begin{aligned}
\frac{d\Pi_1^A}{dp_1^A} &= \frac{1}{2} - \frac{p_1^A}{2} + \frac{c}{2} + \frac{\mu\alpha}{3} \qquad\qquad (2.21) \\
\frac{d\Pi_1^A}{d\alpha} &= \frac{\left(p_1^A - c\right)\mu}{4} - \frac{13\mu\alpha}{24} + \frac{\mu^2\alpha}{4}
\end{aligned}
$$

The equilibrium price and coupon level is obtained by equating the two conditions to 0 and solving the system. Carrying this out yields, for $\mu \neq 0$,

$$
\begin{aligned}
p_1^{A*} &= 1 + c + \frac{4\mu}{13 - 10\mu} \qquad\qquad (2.22) \\
\alpha^* &= \frac{6}{13 - 10\mu}
\end{aligned}
$$

Given the symmetry, it follows $p_1^{B*} = 1 + c + \frac{4\mu}{13-10\mu}$ and $\beta^* = \frac{6}{13-10\mu}$. Substituting these values into the previously derived expressions for the equilibrium prices in Period 2 and in Period 3, gives the equilibrium prices described in Proposition 2.3.

The second-order have been seen to be satisfied locally. It is necessary to check that the candidate symmetric equilibrium described in (2.22) is robust to deviations. This is set out below, in Part 2 of the proof.

2.A.2 Part 2 - Robustness of candidate equilibrium

With no loss of generality, it is assumed that the deviating airline in Period 1 is A whilst airline B selects the candidate equilibrium's price and discount, $\beta^* = \frac{6}{13-10\mu}$ and $p_1^{B*} = 1 + c + \frac{4\mu}{13-10\mu}$.

In Part 1 of this proof, it was implicitly assumed that the terms Ω_j, $j = 1, ..., 5$ lied within the unit interval. At the proposed equilibrium point, such conditions are indeed met. However, when testing for the robustness of the candidate equilibrium, it is necessary to consider the possibility that following A's initial deviation, the optimal behaviour of airlines in subsequent periods will be such that, Ω_j lies outside the unit interval. This alters the functional form of the airlines' profit function and therefore requires that the reaction function be re-examined.

Following on from this consideration, the rest of this section is structured as follows. First, it is shown numerically that there is no profitable first-period deviation by airline A provided airlines' choices of prices and discount are such that $0 \leq \Omega_j \leq 1$, $j = 1, ..., 5$ .This restriction is then set aside at the cost, however, of confining the space of A's potential deviations to those where $\alpha = 0$. As before, it will be shown that under the new equilibrium prices following A's deviation, airline A earns a lower profit level than if it followed the strategies described by the (candidate) symmetric equilibrium.

2.A.2.1 Restrict Ω_j to the unit interval, $j = 1, 2..., 5$. When the choice of A's first period deviation and the choice of ensuing equilibrium prices in Periods 2 and 3 are limited such that $0 \leq \Omega_j \leq 1$, then the airlines' profit functions described above are correct. It follows, that it is then possible to use the reaction functions derived above to establish the equilibrium prices in the last two periods for a given deviation by A. Due to the cumbersome expressions involved I carried out this task numerically using the following algorithm:

1. Let $\mu = 0$.

2. In 0.1 fine grid, consider all pairs of α and p_1^A, and for each pair use the expressions derived in Part 1 of the proof to work out the equilibrium prices in Periods 2 and 3.

3. If the equilibrium prices derived are such that any of the terms Ω_j, $j = 1, 2, ..., 5$ lie outside the unit interval, then discard the relevant

pair $\{\alpha, p_1^A\}$ as a possible deviation.

4. For each admissible deviation, use the respective equilibrium prices to construct the expected profit of airline A over the 3 period and check whether this is higher or lower than the expected profits earned at the candidate symmetric equilibrium.

5. Let $\mu = \mu + 0.05$.

6. Repeat steps 2 - 5, until $\mu = 1$.

The results from this programme show that there is no admissible deviation, where admissible has the peculiar definition described above, from the candidate symmetric equilibrium which is profitable to airline A.

2.A.2.2 Allowing $\Omega_j, j = 1, 2, ..., 5$ to lie outside the unit interval

As mentioned previously the space of A 's deviations will be restricted to those where it chooses to offer no discount, *ie* $\alpha = 0$..

To derive the new equilibrium strategies following A 's deviation in the first period, it is necessary to work backwards from Period 3.

Period 3 Given that $\alpha = 0$, the objective functions of the two airlines in Period 3 can be written as,

$$\Pi_3^A = \left(p_3^A - c\right)(1 - \mu s)\,\widehat{\Omega}_1 + \mu s\widehat{\Omega}_2 \tag{2.23}$$

$$\Pi_3^B = \left(p_3^B - c\right)(1 - \mu s)\left(1 - \widehat{\Omega}_1\right) + \left(p_3^B - \beta^* - c\right)\mu s\widehat{\Omega}_2$$

where,

$$\widehat{\Omega}_i = \begin{cases} 0 & \text{if } \Upsilon_i < 0 \\ \Upsilon_i & \text{if } \Upsilon_i \in [0, 1] \\ 1 & \text{if } \Upsilon_i > 1 \end{cases}, i = 1, 2 \text{ where } \Upsilon_1 = \frac{1 + p_3^B - p_3^A}{2}, \Upsilon_2 = \frac{1 + p_3^B - p_3^A - \beta^*}{2}.$$

$$\tag{2.24}$$

The objective functions of the two airlines are piece-wise functions in third period prices. This complicates matters as it becomes necessary to consider $3 * 3 = 9$ cases and solve the maximization problem of each airline. Note,

however, that $\widehat{\Omega}_2 \leq \widehat{\Omega}_1$ so that the cases to be considered are reduced to 6 as tabled below,

	Range	
	$\widehat{\Omega}_1$	$\widehat{\Omega}_2$
Case A_3	0	0
Case B_3	$\in I$	0
Case C_3	1	0
Case D_3	$\in I$	$\in I$
Case E_3	1	$\in I$
Case F_3	1	1

$$(2.25)$$

The Period 3 equilibrium prices are found by carrying out the following steps.

First, calculate for each of the six cases the *constrained* reaction functions of the two airlines. The constrained reaction function gives an airline's best response to its rival's price conditional on the values of $\widehat{\Omega}_1$ and $\widehat{\Omega}_2$ remaining within the range defining the relevant case.

Second, use the six constrained reaction functions to construct the overall best reply function for each airline. This is done by comparing the profit level obtained by following the constrained reaction function across each of the six cases and selecting the price response which yields the highest profit.

Third, the equilibrium to the pricing game is given by the intersection of the two overall best reply functions

The work involved in constructing the six constrained reaction functions will not be presented here. However, to act as an illustration case D_3 is considered in detail.

Case D_3

It follows from (2.24) and from (2.25) that the conditions which define Case D_3 are given by $0 \leq \frac{1+p_3^B-p_3^A}{2} \leq 1$ and $0 \leq \frac{1+p_3^B-p_3^A-\beta^*}{2} \leq 1$. These conditions can be re-written as,

$$p_3^A + \beta^* - 1 \leq p_3^B \leq 1 + p_3^A \tag{2.26}$$

Solving the first-order conditions of the airlines' optimization problem (2.23) gives,

$$\frac{d\Pi_3^A}{dp_3^A}\bigg|_{case.D_3} = \frac{1}{2}\left(1 + c + p_3^B - 2p_3^A - \beta^*\mu s\right) = 0$$

$$\Rightarrow \quad p_3^A = \frac{1}{2}\left(1 + c + p_3^B - \beta^*\mu s\right)$$

$$\frac{d\Pi_3^B}{dp_3^B}\bigg|_{case.D_3} = \frac{1}{2}\left(1 + c + p_3^A - 2p_3^B + 2\beta^*\mu s\right) = 0$$

$$\Rightarrow \quad p_3^B = \frac{1}{2}\left(1 + c + p_3^A + 2\beta^*\mu s\right) \qquad (2.27)$$

The system of equations (2.27) together with the conditions (2.26) and the fact that the third period profit functions of the airlines are concave in own prices allow me to describe the optimal behaviour of the two airlines conditional on $\widehat{\Omega}_1$ and $\widehat{\Omega}_2$ being in the unit interval, ie remain within case D_3. For a given p_3^B, airline A will set its price p_3^A according to its reaction function given in (2.27). However, if this choice of p_3^A is below the smallest value of p_3^A necessary to satisfy (2.26) then A will set the lowest price consistent with (2.26). On the other hand, if the price is above the maximum price consistent with(2.26), then it will set the highest admissible price. Airline B follows a similar behaviour. Figure 2.5 below illustrates this discussion. The kinked solid curves graph the reaction functions of the two airlines and they are constructed from the 4 dotted lines which describe the two conditions (2.26) and the two equations in (2.27).

An analogous procedure can be followed for the other five cases. The following tables summarize the results for all 6 cases by reporting the best-reply of each airline to a rival's price. The tables should be read as follows. If the price of the rival is below the critical value $crit.$ 1, then the best-reply of an airline is to the set its price as given in BR_{low}. On the other hand, if the rival sets a price above the critical value $crit.$ 2, then the airline should reply according to BR_{high}. Lastly, if the price of the competitor lies between the two critical values, the best response id given by BR_{mid}. Note that while in cases A_3 and F_3 only one of the airlines is active, in case C_3, the best-reply of the airlines is invariant to the rival's strategy.

Case A_3	Airline A	Airline B
BR	-	$p_3^A - 1$
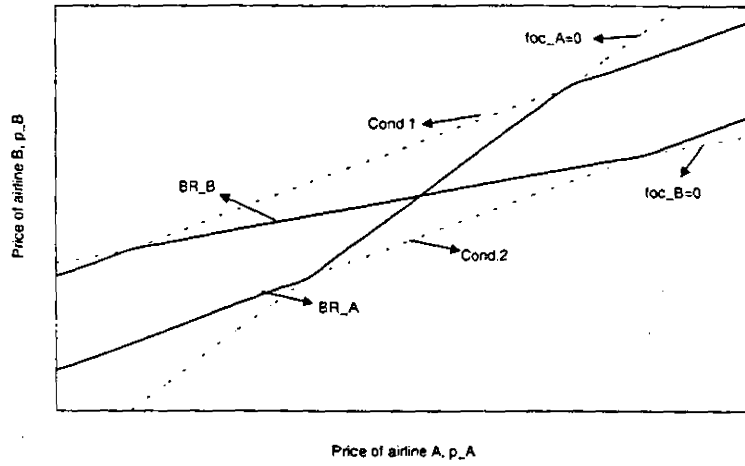
Figure 2.6: Airlines' reaction functions under case D.

Case B_3	Airline A	AirlineB
$crit.1$	$c - 1$	$c + 3 - 2\beta^* + \frac{2\mu s}{1-\mu s}$
$crit.2$	$c + 2\beta^* - 1$	$c + 3 + \frac{2\mu s}{1-\mu s}$
BR_{low}	$p_3^B + 1$	$p_3^A + \beta^* - 1$
BR_{mid}	$\frac{1}{2}\left(p_3^B + c + 1\right)$	$\frac{1}{2}\left(p_3^A + c + 1 + \frac{2\mu s}{1-\mu s}\right)$
BR_{high}	$p_3^B + 1 - \beta^*$	$p_3^A - 1$

Case C_3	Airline A	Airline B
BR	$p_3^B - 1$	$p_3^A + 1 + \beta^*$

Case D_3	Airline A	Airline B
$crit.\ 1$	$c - 1 + \beta^*\left(2 - \mu s\right)$	$c - 1 + 2\beta^*\mu s$
$crit.2$	$c + 3 - \beta^*\mu s$	$c + 3 - 2\beta^*\left(1 - \mu s\right)$
BR_{low}	$p_3^B + 1 - \beta^*$	$p_3^A + 1$
BR_{mid}	$\frac{1}{2}\left(p_3^B + c + 1 - \beta^*\mu s\right)$	$\frac{1}{2}\left(p_3^A + c + 1 + 2\beta^*\mu s\right)$
BR_{high}	$p_3^B - 1$	$p_3^A - 1 + \beta^*$

Case E_3	Airline A	Airline B
crit. 1	$c+1+\frac{2}{\mu s}-\beta^*$	$c-1$
crit . 2	$c+1+\frac{2}{\mu s}+\beta^*$	$c-1+2\beta^*$
BR_{low}	p_3^B-1	$p_3^A+1+\beta^*$
BR_{mid}	$\frac{1}{2}\left(p_3^B+c-1-\beta^*+\frac{2}{\mu s}\right)$	$\frac{1}{2}\left(p_3^A+c+1+2\beta^*\right)$
BR_{high}	$p_3^B-1-\beta^*$	p_3^A+1

Case F_3	Airline A	Airline B
BR	$p_3^B-1-\beta^*$	–

Equipped with the 6 constrained reaction functions it is possible to construct the overall reaction curve for each airline and therfore solve for the equilibrium prices. This is a particularly cumbersome task in terms of the algebra involved and, for the sake of exposition, the working is not shown here. Nevertheless, it can be shown that for $\beta^* \leq 1 \Leftrightarrow \mu \leq \frac{7}{10}$ an equilibrium exists and it is given by $p_3^{A*} = 1+c$ and $p_3^B = 1+c+\beta^*\mu s$. At these prices, the relevant case is Case D_3. For $\mu > \frac{7}{10}$, no equilibrium exists.

Period 2 Henceforth, I assume that the solutions to the Period 3 subgame are given by $p_3^{A*} = 1+c$ and $p_3^{B*} = 1+c+\beta^*\mu s$. In other words, I will restrict my attention to the case where $\mu < \frac{7}{10}$ as this allows me to proceed analytically with the proof.

Recall, that A's objective function at the start of Period 2 is,

$$\Pi_2^A = \left(p_2^A - c\right)\left((1-\mu)\,\Psi_1 + \mu\sigma\widehat{\Omega}_3 + \mu(1-\sigma)\,\widehat{\Omega}_4\right) + \Pi_3^A$$

where,,

$$\widehat{\Omega}_j = \begin{cases} 0 & \text{if} \quad \Upsilon_j < 0 \\ \Upsilon_j & \text{if} \quad \Upsilon_j \in [0,1] \\ 1 & \text{if} \quad \Upsilon_j > 1 \end{cases} , \quad j = 3,4 \qquad (2.28)$$

$$\text{where} \quad \Upsilon_3 = \frac{1 + p_2^B - p_2^A + E\left(U^3|xx\right) - E\left(U^3|xy\right)}{2},$$

$$\Upsilon_4 = \frac{1 + p_2^B - p_2^A - \beta^* - E\left(U^3|xx\right) + E\left(U^3|xy\right)}{2}.$$

$$\text{and} \quad \Psi_1 = \begin{cases} 0 & \text{if} \quad \frac{1+p_2^B-p_2^A}{2} < 0 \\ \frac{1+p_2^B-p_2^A}{2} & \text{if} \quad \frac{1+p_2^B-p_2^A}{2} \in [0,1] \\ 1 & \text{if} \quad \frac{1+p_2^B-p_2^A}{2} > 1 \end{cases}$$

Using the Period 3 equilibrium prices established above, it follows that $\widehat{\Omega}_1 = \frac{1+\beta^*\mu s}{2}$, $\widehat{\Omega}_2 = \frac{1-\beta^*(1-\mu s)}{2}$ and consequently that $E\left(U^3|xx\right) - E\left(U^3|xy\right) = \frac{\beta^*}{2}\left(\beta^*\mu s - \frac{\beta^*}{2} - 1\right)$. In turn, it is straightforward to establish that $\Upsilon_3 \le \Upsilon_4$ if and only if $\mu s \le \frac{1}{2}$. Below I will assume that the latter condition is satisfied. In equilibrium this will indeed be the case. Lastly, note that Ψ_1 is always greater than both $\widehat{\Omega}_3$ and $\widehat{\Omega}_4$.

The procedure followed to solve the pricing game in this period is similar to that adopted for Period 3.

Firstly, it is necessary to distinguish the cases where the airlines' objective functions change due to the discontinuities that arise from (2.28).There are five possible cases that must be analysed as tabled below.

	$\widehat{\Omega}_3$	$\widehat{\Omega}_4$	Ψ_1	Constraint 1	Constraint 2
Case A_2	0	0	$\in I$	$\Upsilon_3 < 0$	$\frac{1+p_2^B-p_2^A}{2} > 0$
Case B_2	0	$\in I$	$\in I$	$\Upsilon_3 < 0$	$\Upsilon_4 > 0$
Case C_2	$\in I$	$\in I$	$\in I$	$\frac{1+p_2^B-p_2^A}{2} < 1$	$\Upsilon_3 > 0$
Case D_2	$\in I$	$\in I$	1	$\Upsilon_4 < 1$	$\frac{1+p_2^B-p_2^A}{2} > 1$
Case E_2	$\in I$	1	1	$\Upsilon_3 < 1$	$\Upsilon_4 > 1$

The constraints 1 and 2 described in the table are those constraints which define the relevant case.

Secondly, for each of the 5 cases, it is necessary to work out the *constrained* reaction functions. To do so, the following procedure must be carried out for

each of the 5 cases. Using the equality $s = \sigma \left(1 - \widehat{\Omega}_3\right) + (1 - \sigma)\widehat{\Omega}_4$, and the relevant expressions for $\widehat{\Omega}_3$ and $\widehat{\Omega}_4$, derive an expression for s in terms of p_3^A, p_3^B, μ and σ . This expression is then substituted in for s in the definitions of $\widehat{\Omega}_3$, $\widehat{\Omega}_4$ and Ψ_1. In a similar way to what was done above, the constrained reaction functions of each airline are obtained by maximizing the relevant Period 2 profit function with respect to their second-period price subject to the two constraints defining the case at hand. The expressions describing these functions are too cumbersome and will not be present here. Graphically, however, the constrained reaction functions of the two airlines are similar to those presented in Figure 2.5.

Lastly, to obtain the global reaction function, it is necessary to i) calculate the best reply to a rival's price under the five different cases and ii) select the reply which yields the highest profit. The solution to the game is then given by the intersection of the two overall reaction functions.

As it was not feasible to carry out these steps algebraically, I did so by resorting to numerical simulations. The result obtained is the following. For $\mu \leq \frac{7}{10}$, the region being considered, an equilibrium to the second period sub-game is given by the set of prices described in Proposition 2.2 and in (2.19) once the substitutions $\alpha = 0$ and $\beta = \beta^*$ are made.

Period 1 By assumption, $p_1^B = p_1^{B*}$, $\beta = \beta^*$ and $\alpha = 0$. It is sufficient to the consider the optimisation problem facing airline A. The objective function of this airline is,

$$\underset{\{p_1^A\}}{Max} \quad \Pi_1^A = \left(p_1^A - c\right)\left(\mu\sigma + (1 - \mu)\,\Psi_2\right) + \Pi_2^A,$$

$$\text{where } \Psi_2 = \begin{cases} 0 & \text{if } \frac{1+p_1^{B*}-p_1^A}{2} < 0 \\ 1 & \text{if } \frac{1+p_1^{B*}-p_1^A}{2} > 1 \\ \frac{1+p_1^{B*}-p_1^A}{2} & \text{otherwise} \end{cases}$$

$$\text{and } \sigma = \begin{cases} 0 & \text{if } \frac{1+p_1^{B*}-p_1^A+E\left(U^2|A\right)-E\left(U^2|B\right)}{2} < 0 \\ 1 & \text{if } \frac{1+p_1^{B*}-p_1^A+E\left(U^2|A\right)-E\left(U^2|B\right)}{2} > 1 \\ \frac{1+p_1^{B*}-p_1^A+E\left(U^2|A\right)-E\left(U^2|B\right)}{2} & \text{otherwise} \end{cases}$$

As before, the piece-wise nature of the objective function, forces me to resort to numerical calculations. The price p_1^A was allowed to take values
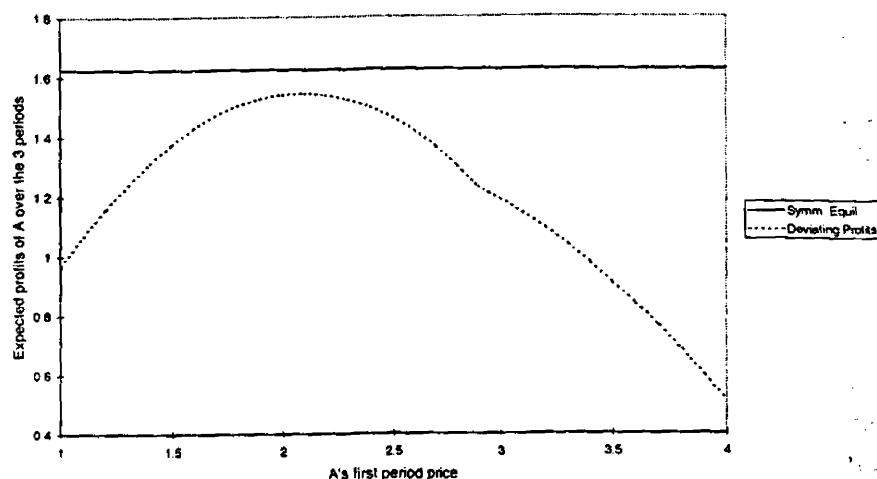
Figure 2.7: Expected profits of airline A as a function of its first period price.($c = 1, \mu = 0.5$)

between $c + 1$ and $c + 4$ at intervals of 0.05. For each level of p_1^A, I derived the respective value of σ and consequently the profits of A over the three periods. The results derived show that A 's profits are everywhere below what it would receive if it did not deviate from the candidate symmetric equilibrium. This is illustrated by Figures 2.6 and 2.7. For $\mu = 0.5$, Figure 2.6 graphs A 's profit level as a function of its first period price, p_1^A, when this airline offers no discount and follows in subsequent periods the equilibrium pricing strategies that were derived above. There is an optimal price to be charged by A, though it is clear that its profits are below those that it would receive had it not deviated - this profit level is given by the horizontal line. Figure 2.7 summarizes the numerical simulations carried out by showing how the equilibrium profits of airline A varies with μ. For the sake of comparison, the figure also draws out the profits that this airline would earn had it not deviated. It is clear that A is better off if it does not deviate from the candidate symmetric equilibrium. This completes the proof.
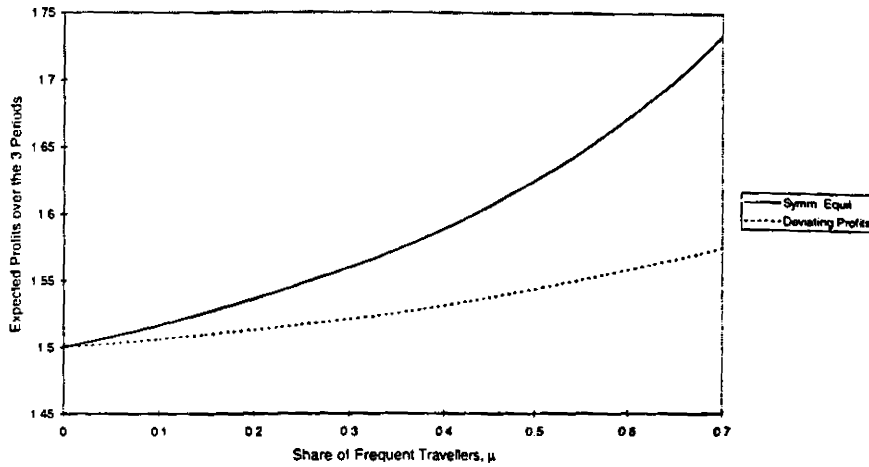
Figure 2.8: Profits of airline A in the symmetric equilibrium and under its optimal deviating behaviour given $\alpha = 0$, as a function of μ.

2.B Solving the Two Period Model

The steps involved in solving the two-period model follow closely the presentation in Caminal and Matutes $(1990, p.370)$. The same notation is used as in the three-period model, though the suffix $'$ (prime) is added to distinguish the two. The model is solved by working backwards starting in Period 2.

2.B.1 Period 2

An occasional traveller will patronise airline A if and only if $i_2 \leq \frac{1+p_2^{B'}-p_2^{A'}}{2} \equiv \Lambda_1$. Otherwise he will address B. A frequent traveller conditional on having address A in Period 1 will return to it in Period 2 if and only if $i_2 \leq \frac{1+p_2^{B'}-p_2^{A'}+\alpha'}{2} \equiv \Lambda_2$. On the other hand, if he addressed B in the first period, he will purchase from A in Period 2 if and only if $i_2 \leq \frac{1+p_2^{B'}-\beta-p_2^{A'}}{2} \equiv \Lambda_3$. Let σ' be the share of frequent travellers who selected A in Period 1. The profit of airline A in the second period is given by,

$$\Pi_2^{A'} = \left(p_2^{A'}-c\right)\left((1-\mu)\Lambda_1 + \mu(1-\sigma')\Lambda_3\right) + \left(p_2^{A'}-\gamma-c\right)\mu\left(\sigma' * \Lambda_2\right)$$

After substituting out the terms Λ_1, Λ_2 and Λ_3 in the above and upon simplification, one can write the profit function of A in the second period as,

$$\Pi_2^{A'} = \frac{1}{2}\left(p_2^{A'} - c\right)\left(1 + p_2^{B'} - p_2^{A'} + \mu\left(\beta'\left(\sigma' - 1\right) + \alpha'\right)\right) - \frac{\mu\sigma'\alpha'}{2}\left(1 + \alpha' + p_2^B - p_2^A\right)$$

An analogous expression can be derived for airline B. Differentiating the two profit functions with respect to the choice variables, p_2^A and p_2^B yields the following first order conditions,

$$\frac{d\Pi_2^{A'}}{dp_2^{A'}} = 1 - 2p_2^{A'} + p_2^{B'} + c - \mu\beta' + \mu\sigma'\beta + 2\mu\sigma'\alpha'$$

$$\frac{d\Pi_2^{B'}}{dp_2^{B'}} = 1 + p_2^{A'} - 2p_2^{B'} + c + 2\mu\beta' - 2\mu\sigma'\beta' - \mu\sigma'\alpha'$$

Setting the two first order conditions to 0 and solving the ensuing system gives the equilibrium second period prices,

$$p_2^{A'*} = 1 + c + \mu\sigma'\alpha$$
$$p_2^{B'*} = 1 + c + \mu\left(1 - \sigma'\right)\beta'$$

It is straight-forward to see that the second-order conditions are met since $\frac{d^2\Pi_2^{A'}}{dp_2^{A'}} = \frac{d^2\Pi_2^{B'}}{dp_2^{B'}} = -2 < 0$.

2.B.2 Period 1

An occasional traveller patronises A in Period 1 if and only if $i_1 \leq \frac{1 + p_1^{B'} - p_1^{A'}}{2}$.

On the other hand a frequent traveller will address A if $i_1 \leq \frac{1 + p_1^{B'} - p_1^{A'} + E\left(U^2|A\right) - E\left(U^2|B\right)}{2} \equiv \Lambda_4$ and will address B otherwise. The term $E\left(U^2|J\right)$, $J = \{A, B\}$, is the expected utility gained by a frequent traveller in Period 2 given that he addressed J in Period 1. The value of these expected utilities can be calculated as,

$$E\left(U^2|A\right) = \int_0^{\Lambda_2}\left(R - p_2^{A'} + \alpha' - i\right)di + \int_{\Lambda_2}^1\left(R - p_2^{B'} - 1 + i\right)di$$

$$= R - p_2^{B'} - \frac{1}{2} + \Lambda_2^2$$

$$E\left(U^2|B\right) = \int_0^{\Lambda_3}\left(R - p_2^{A'} - i\right)di + \int_{\Lambda_3}^1\left(R - p_2^{B'} + \beta' - 1 + i\right)di$$

$$= R - p_2^B + \beta' - \frac{1}{2} + \Lambda_3^2$$

Using these results it is possible to express Λ_4 as

$$\Lambda_4 = \frac{1 + p_1^{B'} - p_1^{A'} - \beta' + \Lambda_2^2 - \Lambda_3^2}{2} \tag{2.29}$$

The profit earned by airline A over the two periods is given by,

$$\Pi_1^{A'} = \left(p_1^{A'} - c\right)\left(\mu\sigma' + (1 - \mu)\left(\frac{1 + p_1^{B'} - p_1^{A'}}{2}\right)\right) + \Pi_A^2$$

where, by construction, $\sigma' = \Lambda_4$.

Maximizing $\Pi_1^{A'}$ with respect to $p_1^{A'}$ and α' and then imposing the symmetry conditions $p_1^{B'} = p_1^{A'}$, $\beta' = \alpha'$ and $\sigma' = \frac{1}{2}$ yields,

$$\frac{d\Pi_1^{A'}}{dp_1^{A'}} = \frac{1}{2}\left(1 - \left(p_1^{A'} - c\right)(1 - \mu)\right) + \left(p_1^{A'} - c\right)\mu\frac{d\sigma'}{dp_1^{A'}} - \frac{\mu\alpha'^2}{2}(1 - \mu)\frac{d\sigma'}{dp_1^{A'}}$$

$$\frac{d\Pi_1^{A'}}{d\alpha'} = \left(p_1^{A'} - c\right)\mu\frac{d\sigma'}{d\alpha'} + \frac{\mu^2\alpha'}{8} - \frac{\mu\alpha'}{2} - \frac{\mu\alpha'^2}{2}(1 - \mu)\frac{d\sigma'}{d\alpha'} \tag{2.30}$$

Using (2.29) and recalling that $\Lambda_4 = \sigma'$, it is straight-forward to compute $\frac{d\sigma'}{dp_1^{A'}}$ and $\frac{d\sigma'}{d\alpha}$ at the symmetric point as,

$$\frac{d\sigma'}{dp_1^{A'}} = -\frac{1}{2\left(1 + \mu\alpha'^2\right)}$$

$$\frac{d\sigma'}{d\alpha'} = \frac{1 + \alpha'(1 - \mu)}{4\left(1 + \mu\alpha'^2\right)}$$

Substituting these expressions into (2.30) and solving the first order conditions gives the following system of equations,

$$0 = p_1^{A'} - c - \frac{2 + \mu\alpha'^2(3 - \mu)}{2 + 2\mu\alpha'^2(1 - \mu)} \tag{2.31}$$

$$0 = \mu\frac{2\left(p_1^{A'} - c\right) + \alpha(1 - \mu)\left(2p_1^{A'} - 2c - \alpha'\right) + \mu\alpha - 4\alpha - \alpha'^3(2\mu + 1)}{8\left(1 + \mu\alpha'^2\right)}$$

The candidate symmetric equilibrium prices and coupon levels are given by the solution to this system of equations. This systemn can not be solved analytically and it is necessary to resort to numerical computations to derive the equilibrium price and discount levels.

The equilibrium values are those plotted in Figure 2.4 in Section 2.5.1.

The equilibrium prices and discount are a function of μ. It was checked numerically that for each μ in a 0.02 grid in the $[0, 1]$ interval, the second-order conditions were met at the symmetric equilibrium point.

As with the three-period model, it is necessary to check that the solutions to (2.31) actually form an equilibrium. In other words, it is necessary to check that neither airline has the incentive to deviate. The steps involved to carry this out were similar to those presented in Part 2 of Appendix 2.A. For the sake of exposition, that work is not presented here. It is noted, however, that the analysis concluded that the symmetric equilibrium is robust to deviations to corner solutions, ie. to deviations by a firm not to offer no discount. This completes the proof.

Chapter 3

An empirical study of the effects of FFP alliances

3.1 Introduction

One hundred and twenty nine scheduled flights take off every week from Salt Lake City, Utah to Boston, Massachussets. Of these, 68 are operated by United Airlines and both Delta and Southwest Airlines both fly 21. United Airlines flies to 29 other destinations from Salt Lake City, whilst Delta and Southwest Airlines serve 67 and 22 other cities respectively. [1] Delta's network of flights across America as a whole is considerably greater than that of the other two airlines. [2]

All three airlines offer their passengers the opportunity to enrol in their own frequent-flyer programmes ($FFPs$).

$FFPs$ influence a traveller's choice of airline by creating a link between all of an airline's flights. Independent of when the journey is made or where to or from, air miles are earned by those participating in the scheme. [3]

The convex structure of FFP's reward schedule gives the incentive for a

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[1] These figuers are based on the airlines' schedule in the week between the 8 and the 14 May 1999, as reported in the OAG Pocket Flight Guide to North America. Only flights to US destinations have been included, though these cover both direct as well as connecting flights.

[2] See Table 3.3 further below

[3] Cairns and Galbraith (1990) refer to this link created by the $FFPs$ as an *artificial compatiblity* between the different routes serviced by the same airline.

traveller to concentrate her flights on a few number of airlines, one or two.[4]
Other things equal, a traveller is likely to choose the airline with which she
expects to fly the most in the future - the one with the most service on
the route she flies more often and the one that flies the most routes from
her home airport. In Salt Lake City, travellers keen on $FFPs$ and flying
most frequently to Boston will lean towards United Airlines whilst those
whose destination varies will be attracted to Delta. In addition, the value
a traveller attaches to a free flight in the future increases with the size of
the menu of destinations amongst which she can choose to fly to. Given
its wider domestic, and indeed international network, Delta would come out
better than its rivals on this count too.[5]

The above reasoning suggests that the effectiveness of an airline's FFP
in influencing a traveller's choice is closely related to the size of that airline
- both its share of activity on the route and at the nearby airport as well as
the size of its overall network of routes.[6]

For those concerned with the competitiveness of the airline industry this
is worrying as it implies that the larger players are (further) insulated from
competition and allowed to price less aggressively than they would otherwise.
In addition, the entry of a new airline into the air transport industry, or that
of an airline into an airport dominated by an incumbent is made that much
harder. Either entry takes place on a large enough scale so that the entrant
places itself on the same level as the incumbent, or the entrant faces the
prospect of competing at a disadvantage.

The sub-committees of the American Senate and Congress supervising
aviation, competition, anti-trust and business rights have had the above is-
sue repeatedly brought to their attention, most recently in the GAO-99-37
report by the US General Accounting Office.[7] An underlying theme in these

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[4]In 1999, members of Delta's Sky Miles program for example, required 40,000 air
miles to redeem one free First Class round ticket within or between the continental US
(including Alaska) and Canada whilst 20,000 miles would not be sufficient to earn one
Coach/Economy Class ticket for the same trip (25,000 miles would have been necessary).

[5]The attractiveness of a FFP also depends on the generosity with which air miles
are handed out. There is considerable homogeneity in the rate at which airlines reward
travellers though periodic special offers - when twice or thrice as many air miles are
distributed for the same length of travel - make it hard to establish a clear comparison
between programmes.

[6]Borenstein (1996) notes that hotels with facilities in many cities have used loyalty-
programmes more extensively than those with a more limited number of outlets.

[7]See also GAO-90-147, GAO-97-120, GAO-98-112 and GAO-98-176

reports is that certain marketing schemes, of which $FFPs$ are the most prominent, have added to the existent operating barriers to entry - namely the control over airport slots and gate leases by the major airlines and the existence of perimeter rules at some busy airports - and thereby made effective competition by new entrants harder.[8]

The concerns over the anti-competitive effects of $FFPs$ found in the GAO reports echo those laid out in the reviews in Levine (1987), Bailey and Williams (1988), Clifford and Whinston (1989) and Borenstein (1992) of the effects of deregulation on the American airline industry.

This chapter addresses this concern. It explores the extent to which $FFPs$ contribute further to the market power of larger airlines. It will do so by exploiting a natural experiment occurring over 1998 involving six of the American major carriers. In the course of that year, three sets of pairwise agreements were established: between Continental with Northwest, between American with US Airways and between Delta and United. With the exception of the alliance between Continental and Northwest, the three sets of agreements were exclusively of a marketing nature and centred on each airline treating its partner's FFP as if it were its own. This FFP reciprocity implied that a member of one airline's loyalty scheme could also clock up air miles when travelling with the partnering airline and could redeem prizes on either airline.

Overnight, the opportunities to earn air miles and redeem prizes for a member of one of the six airlines' FFP doubled.

Consider once again the traveller based at Salt Lake City and assume that she is a member of Delta's Sky Miles program. Following Delta's alliance with United she can now collect air miles on her Sky Miles account whenever she flies with United including its 68 weekly flights to Dallas. In addition, once enough air miles are accumulated, she has the option of redeeming a free flight on either United's or Delta's routes. The alliance between the two airlines has in effect increased the benefit of being a Sky Miles member. More generally, members of the loyalty-programs of any of the other five airlines involved in these agreements will benefit similarly.

If, as argued, $FFPs$ contribute to an airline's market power, benefitting those airlines with the greater network of routes, then the boost received by

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[8]Perimeter rules prohibit flights from an airport that exceed certain distances. The existence of such rules at the airports La Guardia, New York and National, Washington DC, limits the ability of airlines based in the west to compete at these airports (GAO/RCED-98-176).

an airline's loyalty-program by allying itself with that of another - particularly when both carriers involved are major carriers - should have noticeable effects on fares. The analysis of this chapter attempts to establish whether such an effect did take place, and, in so doing, explores the routes through which $FFPs$ contribute to the market power of an airline. In carrying this out, it will also be addressing the question left open in the 1999 report by the US General Accounting Office asking for a value to be put on the anti-competitive effects of the three alliances due to the expanded $FFPs$.[9]

The chapter is structured as follows. In the following section, I review other studies that have quantified the market power enjoyed by carriers due to their $FFPs$. In Section 3.3, I present a brief description of the three pairwise alliances among the six US major carriers. Section 3.4 lays out the background to the econometric analysis, the results and interpretation of which are presented in Section 3.5. Section 3.6 closes the chapter.

3.2 Estimates of the value of frequent-flyer programmes

Passenger surveys reveal that following concerns over security, price, punctuality and service, $FFPs$ are a factor travellers pay attention to when choosing with which airline to travel (Toh and Hu, 1988 and GAO/RCED-90-147, 1990).

Nako (1992) and Morrison and Whinston (1995) tackle the problem of placing a money value on the importance travellers attach to an airline's FFP. In both papers, travellers' choice of airline is formulated as a logit choice model where price, frequency of flights and membership in $FFPs$ are included in the set of explanatory variables. Morrison and Whinston (1995) estimate that the marginal value to a traveller of clocking an extra air mile is 8.2 cents. The paper qualifies this finding in two respects. First, it shows that the value placed on an air mile depends on the air miles a traveller has already accumulated. Second, it is also shown that the value is greater for those travellers who are likely to travel often enough to clock up enough air miles to redeem a prize but not enough that they are certain to do so. Nako (1992) uses data on business travellers and shows that the value a traveller attributes to a FFP varies from airline to airline though the reasons for these

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[9]GAO/RCED-99-37, p.3

differences are not wholly accounted for. However, Nako (1992) does obtain the interesting result that the value of an airline's FFP is raised by \$4.16 for every 10% increase in its airport market share.

In this chapter, I propose to look at the opposite side of the coin studied by Nako (1992) and Morrison and Whinston (1995). I will attempt to quantify the premium an airline enjoyes due to its FFP. Clearly, the two issues are related - a premium can only come about if travellers attach a value to these loyalty programmes and realign their preferences accordingly.

I have not come across a study that has attempted to put a figure on the impact of $FFPs$ on the level of fares. However, pinning together the results of two separate strands of research, there is evidence that offering a FFP does allow an airline to set a premium on its fares.

Levine (1987) and Borenstein (1989) suggest that an airline with a dominant presence at an airport has a significant advantage in attracting travellers setting off from that airport, regardless of the specific route flown.· This is best understood through an example. Consider an airline with a significantly higher presence in airport A than in airport B. Levine and Borenstein note that on the route between A and B, such a carrier is likely to transport a greater share of the passengers setting off from A and travelling on the round-trip $A - B - A$ than the share of passengers setting off from B and travelling on the $B - A - B$ round-trip. Borenstein (1991) sets out to explain the mechanisms through which airlines with a high presence at an airport enjoy this advantage. After controlling for factors such as cost and quality levels that are likely to favour the larger airlines, Borenstein (1991) suggests that the advantage of the dominating airline is likely to be due to marketing devices such as $FFPs$ and Travel Agency Commission Override (TACOs), rather than due to factors such as reputation or information spillovers.[10]

There is a second strand in the literature concerned with explaining the level of fares across airlines and across routes using a series of explanatory variables to reflect the costs of the airlines and the structure of the market. Borenstein (1989, 1990), Evans and Kessides (1993) and Abramowitz and Brown (1993) find that a carrier's share of traffic at the airports on the end-points of a route and the share of passengers transported by the airline on the observed route are two of the variables with a significant positive effect

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[10]$TACOs$ are the travel-agency equivalent of $FFPs$. Through such programmes, travel agencies are rewarded by the airline company according to the number of tickets the agency sells on that airline's flights.

on the level of fares.[11] Borenstein (1989) estimates that an airline with 50% of the originations at both ends of a route is estimated to charge for tickets likely to be purchased by business travellers 6% more than those of an airline with a smaller presence at both ends. Evans and Kessides (1993) suggests that the effect is in fact twice as high.

The empirical studies cited above suggest that *FFPs* are one channel through which airlines with a significant presence at the airport level have an advantage in attracting travellers on any route from that airport and that in turn, an airline with a significant share of the activity at an airport is able to charge a premium on flights to/from that airport. Put together, the two strands of the literature suggest that *FFPs* are a vehicle through which the larger airlines exercise their market power and are able to set higher fares.

There is a small body of literature[12] that has explored the competitive effects of airline alliances. The alliances explored by these studies, however, are more far-reaching than *FFP* reciprocity as they typically cover code-sharing agreements.[13] Within such studies it is then difficult to attribute an observed price change to the expansion of an airline's *FFP* rather than due to the advantages of having a code-sharing partner. On the other hand, by looking at alliances that have centred exclusively on marketing arrangement, the analysis presented below is able to circumvent this difficulty.

3.3   Review of the alliances

In this section, the alliances formed amongst the six major American airlines during the course of 1998 are briefly reviewed. In doing so, care is taken to note the chronology of the alliances' milestones and to highlight what the agreements cover and what they do not cover. This section will also paint a brief picture of how the alliances expand the network of routes on which air miles can be earned and prizes redeemed for each of the loyalty programmes.

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[11]Evans and Kessides (1993) do not, however, find the route market share to be statistically significant.

[12]See for example Brueckner and Whalen (2000) and Bamberger et al (2001)

[13]Two airlines are said to code-share when each of the airlines uses its own designator code to market flights operated by the second airline as its own. For example, a flight on a Continental operated plane will be listed in the computer reservation systems used by travel agencies (and on the flight information screens at airports) with both a Continental and a Northwest code.

3.3.1 Timing and nature of agreements

Over the course of 1998, six of the major American airlines formed pair-wise agreements. The more significant steps in setting up these agreements are reported in Table 3.1 and a more detailed description is given in Appendix 3.A.[14]

In essence, the alliances between United and Delta and between American and US Airways were exclusively of a marketing nature. Passengers flying with one of the airlines were allowed to use the aiport lounges of the partner and, more importantly, members of an airline's loyalty-scheme were allowed to clock up air miles when flying with the partner and allowed to redeem prices with the partner.

The alliance between Continental and Northwest, on the other hand, went beyond a marketing agreement. Northwest proposed and was successful in purchasing a controlling interest in Continental. The Department of Justice (DOJ) and the Department of Transport (DOT) challenged this plan on the grounds that Continental-Northwest would jointly have a dominant presence in a number of airports and on certain routes. Northwest did go ahead with its purchase of equity stock in Continetal, though it gave assurances that it would not exercise its voting rights in Continental for six years and that the two airlines would be run separately. Contrary to the other two alliances, the agreement between Continental and Northwest extended beyond the reciprocity of their FFP and also included the decision for the two airlines to code-share on some domestic and international flights.

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[14]The information used in this section was collected from the press releases of the airlines as well as from various issues of the *Frequent Flier* and the 1998 company accounts of the six airlines. These sources are detailed in Appendix 3.A.

Table 3.1: Milestones in setting-up the three alliances

United - Delta	
30-4-1998	Announce a global alliance under which airlines plan to engage in code- sharing agreements, reciprocal FFP and co-operation in other marketing areas. Agreement will be put to play first in US market. Airlines foresee FFP reciprocity within the year and code-sharing by early 1999.
31-8-1998	Announce that, as of 1st September, members of either FFP can earn air miles on partner's flights within the US, Puerto Rico and US Virgin Islands. Starting from the 15th October, air miles can be redeemed for flights with partner. Due to opposition from Delta's pilots, plans to code-share are discontinued.
American- US Airways	
23-4-1998	Announce agreement on broad marketing relation, including FFP reciprocity, to be implemented by late summer.
27-7-1998	Announce that as of 1st August FFP members can redeem air miles on each other's flights and benefit from each other's lounge facilities.
24-8-1998	Announce that members of both programmes can combine miles to redeem an award on either, provided a minimum of 1000 miles are used from each account.
Continental-Northwest	
26-1-1998	Announce strategic global alliance including code-sharing arrangements and FFP reciprocity. Subject to DOJ and DOT review Northwest is to buy a controlling interest in Continental.
29-10-1998	DOJ launches suit challenging Northewest's proposed purchase. Notwithstanding, the two airlines announce that they will proceed with the stock transaction and the marketing alliance.
20-11-1998	Announce the completion of Northwest's equity stake in Continental and the agreement for Northwest to restrict its ability to vote its stock.
6-12-1998	Reciprocity in mileage earning takes effect and announce that, subject to regulatory review, members of one FFP may apply for reward from the other programme starting 1-2-1999 for travel beginning 1-3-1999.
28-12-1998	Announce that, starting on the 29th December, the two partners will code-share on flights between US and Japan and beyond.
6-12-1998	Announce that, starting on the 7th January, the two partners will begin code-sharing on domestic and international flights to 95 destinations.

Table 3.2: Size of airlines and alliances, 1997

Airline or alliance	Enplaned Passengers	
	(m)	Share (%)
Delta	97.3	17.6
United	72.9	13.2
Delta-United	**170.2**	**30.8**
American	66.1	12.0
US-Airways	57.4	10.4
American-US Airways	**123.5**	**22.3**
Northwest	47.1	8.5
Continental	34.2	6.2
Northwest-Continental	**81.3**	**14.7**
Alliance subtotal	**375.0**	**67.8**

Note: Figures refer to US domestic travel.

Source: GAO/RCED 99-37, Appendix 1.

3.3.2 Domestic relevance of the alliances

The six carriers involved in the marketing agreements are among the largest seven American carriers.[15] In 1997, they transported 375 million passengers within the United States accounting for 67.8% of all domestic travelling. As reported in Table 3.2, the Delta-United partnership is the largest - accounting for 30.8% of the market - and is more than twice as large as the Northwest and Continental alliance who account for 14.9% of the market between the two.

The size of an airline's partner provides a rough idea of the added attractiveness that the airline's loyalty programme is likely to enjoy from the alliance. However, a finer picture can be drawn by looking at how the alliances increase the share of flights from a given airport on which a member of an airline's FFP can earn air miles and by examining how the network of routes and airports of an airline's partner augments that airline's own network.

These issues turn on a discussion of the size and of the complementarity

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[15]In terms of enplaned passengers on domestic flights, Southwest airlines is the fifth largest with a market share in 1997 of 10.1%

Table 3.3: Complementarity of partners' domestic network

Airline	Routes	Cities	Region of Influence
Delta	314	18	Central, Atlantic
Overlap	72	74	
United	229	10	Pacific, Midwest, Mideast
American	240	32	Pacific, Midwest, Eastern
Overlap	27	50	
US-Airways	210	11	Northeast, Middle Atlantic, South-east
Northwest	134	6	Mideast, Northern
Overlap	10	53	
Continental	171	26	Eastern, Southern

Source: Calculations based on Domestic Airline Fares Consumer Report for the four quarters of 1998. The areas of influence were read off the maps included in the airlines' 1998 annual reports.

of the networks of each pair of partners. Bob Crandall, chairman and CEO of American Airlines noted that his airline's match with US-Airways was a good one "because the two airlines' network complement each other very well".[16] Other airlines voiced similar views with respect to their choice of partner. Emphasizing the complementarity of networks has two purposes. On the one hand, it aims to mitigate concerns the compeition authority might have that a marekting alliance might feed into a slack in competition on those routes where the two partnering airlines operate. A significant overlap between the airlines' activities would heighten this concern.[17] On the other hand, by emphasizing the complementarity of networks, the airlines publicize the expanded network from which its FFP members stand to gain.

Table 3.3 and Figures 3.1-3.3 illustrate the degree to which the activities of partnering airlines complement each other and the extent to which they overlap.

Table 3.3 illustrates that there is considerable overlap between an airline

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[16] AMR Press Release, 23rd April 1998.
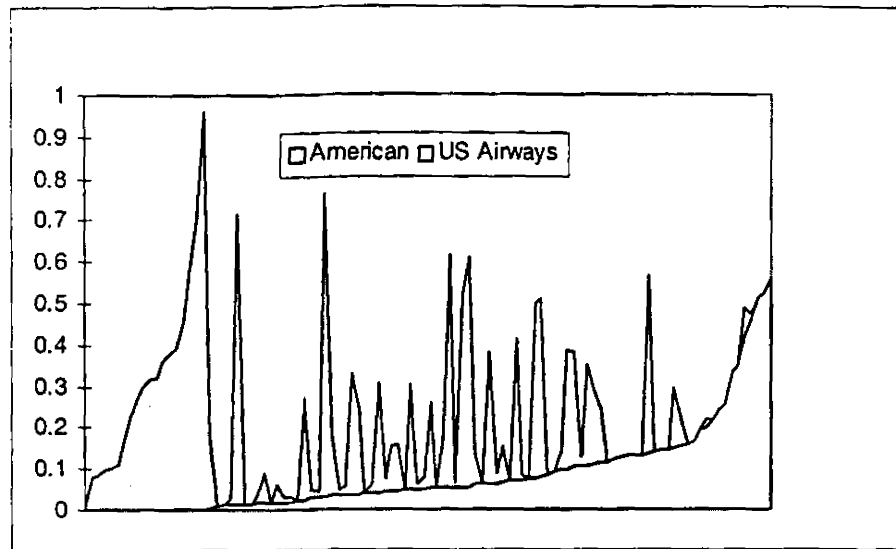[17] This concern is highlighted throughout GAO/RCED-98-215.

Figure 3.1: US-Airways and American Airline's share of activity across US cities.

and its partner in terms of the number of cities covered by their operations. This is not surprising as the data on routes, and thereby cities, are drawn from the *Domestic Airline Fares Consumer Report* and cover the top 1,000 city-pair routes within the US and one would expect the main airlines to have some activity in the more important cities. Figures 3.1-3.3 provides a better idea how much overlap between partnering airlines there is at the city level. For each alliance, the share of passengers transported by each partner to/from each of the cities in which at least one of them competes. In each of the three graphs, the cities have been ordered in terms of increasing share of one of the partners.

Together, Table 3.3 and Figure 3.1 suggest that members of US-Airways' *FFP* will benefit substantially from the alliance with American Airlines' loyalty-scheme. These members will be able to earn air miles on a further 240 domestic routes, up from US Airways' existent 237, and will cover 32 additional cities. Of the three alliances, that between Continental and Northwest appears as the one with the lowest degree of overlap. Continental operates in less than 8% of those routes where Northwest flies, while the converse percentage is even smaller. In contrast, there appears to be a significant overalp
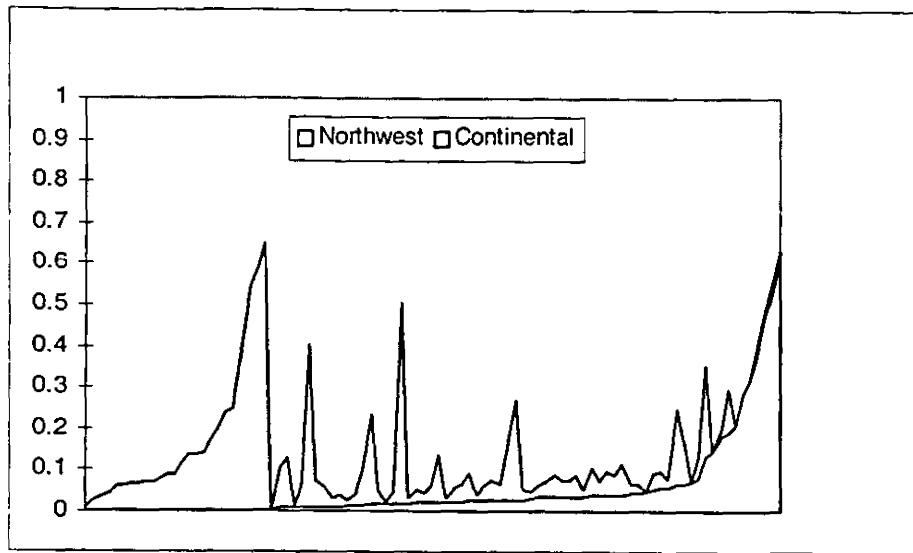
Figure 3.2: Northwest and Continental's share of activity across US cities.
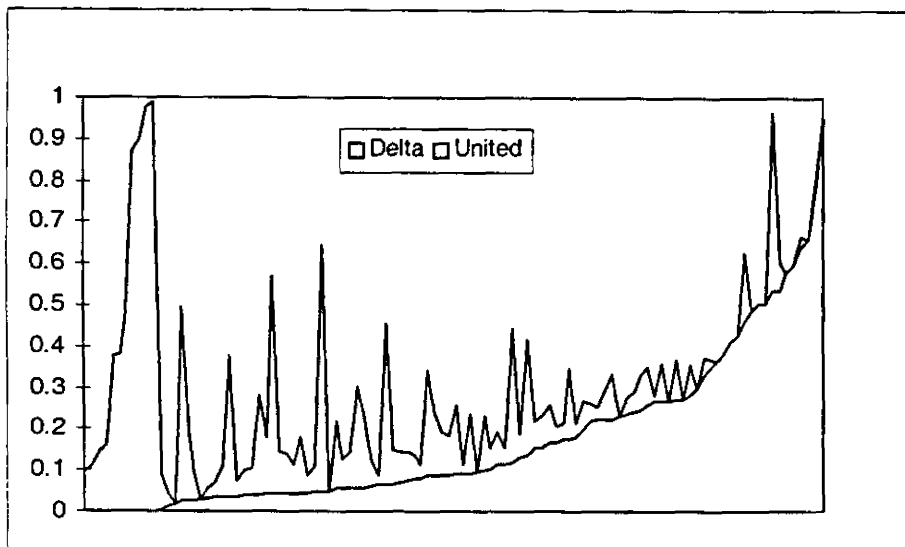
Figure 3.3: Delta and United's share of activity across US cities.

in the activities of United and Delta. Prior to forming the alliance, United already flew to just under a fifth of the routes flown by Delta while Delta flew in just under a fourth of those flown by United. Figure 3.3 illustrates that the overlap in activity of these two airlines in the cities they operate in is also considerable.

The brief discussion over the operations of the six carriers had the aim of underlining the sizeable impact that the pair-wise alliances have on expanding the network of flights and routes across which a member of an airline's FFP will be able to earn and redeem air miles.

3.4   Estimating the impact of the marketing alliances

In light of the discussion presented earlier, I specify a price equation which will allow to test whether:

- An airline that has formed a FFP alliance with a second airline is able to exploit this through higher fares. The premium will be greater the greater the size of the partnering airline at the route, airport and at the national level; and

- An airline facing competition on a route from a rival that has formed a FFP alliance is at a competitive disadvantage. The disadvantage is greater the more significant is the presence of the rival and its partner at the route, airport and national level.

3.4.1   Price equation

The effect of the alliances on carriers' prices is estimated by regressing the route fare level on a vector of airline and route characteristics and on variables reflecting the existence and size of these alliances.

The strategy is to estimate a price equation of the form,

$$Fare_{ijt} = \beta X_{ijt} + \gamma A_{ijt} + \tau T_t + v_j + \varepsilon_{ijt}, \text{ for } \begin{array}{l} i = 1, 2, ..., f_j \\ j = 1, 2, ..., n \\ t = 1, 2, ..., T \end{array} \qquad (3.1)$$

Table 3.4: Variables reflecting market structure

Variable	Description	Expected Sign
$Passengers_{jt}$	Logarithm of the total number of passengers flying on route j.	$(-)$
$Route\ Share_{ijt}$	Share of passengers travelling on route j that fly with airline i.	$(+)$
$City\ Share_{ijt}$	Weighted average of airline i's share of passengers on routes that start from or end at either-point of route j. The weights are given by the proportion of traffic at the two end-points.	$(+)$
$National\ Share_{it}$	Airline i's share of US domestic market.	$(+)$
Hub_{ij}	Dummy variable is equal to 1 if at least one of the end-points of route i is a hub of airline j and is equal to 0 otherwise.	$(+)$
$Low\ Cost\ Airline_i$	Dummy variable equal to 1 if airline i is a low cost carrier and is equal to 0 otherwise.	$(-)$

where $Fare_{ijt}$ is the natural logarithm of the average fare of airline i on route j at time t, X_{ijt} is a vector of market structure characteristics that vary with the firm's identity within the route and over time, A_{ijt} is a vector of characteristics to reflect the alliance arrangements, T_t is a vector of time dummy variables and v_j is a route fixed effect. The random error ε_{ijt} is assumed to be i.i.d. with zero mean and variance σ_ε^2.

The structure of (3.1) follows closely that estimated by Evans and Kessides (1993) and Morrison and Whinston (1995) though the set of explanatory variables in these two papers does not include variables reflecting the existence of airline alliances.

Market structure variables

Table 3.4 summarizes the variables included in the vector X_{ijt} reflecting the market structure and suggests their expected effect on the fare level.

The variables described in Table 3.4 are common to various studies that estimate a fare equation and a fuller discussion of them is left to Appendix 3.B.[18]

Tourism interaction variables

$FFPs$ tend to benefit frequent flyers rather than occasional passengers flying for leisure or tourism. Consequently, the impact of $FFPs$ is likely to be lower in popular tourist routes than on routes with a higher proportion of business travellers.

To explore this the variables reflecting an airline's presence at the route, airport and national level, $Route\ Share_{ijt}$, $City\ Share_{ijt}$ and $National\ Share_{ijt}$, are interacted with an index measuring the extent to which the observed route is flown by tourist travellers to yield the variables $Tourism * Route\ Share_{ijt}$, $Tourism * City\ Share_{ijt}$ and $Tourism * National\ Share_{ijt}$ respectively.

The tourism index was constructed based on the income per capita generated from hotels and other accomodation in the metropolitan areas where the endpoint cities of a route are located.[19] If the advantages to an airline of having a significant presence at the route, airport or national level are driven by FFP effects, then it is expected that the sign of the interaction variables is negative. On the other hand, the intuition that airlines with a greater share of the market at the route, airport or national level benefit particularly from $FFPs$ would be questioned if these interaction variables are not significant.[20]

FFP alliance variables

Vector A_{ijt} reflects the arrangements of the FFP alliances and it is appropriate to consider in some detail the variables to be included in this vector.

A first strategy is to construct variables to signal whether an airline has entered into an alliance and whether it faces competition from an airline that has. Accordingly, let $Own\ Alliance_{it}$ be equal to 1 if in quarter t the observed carrier belongs to one of the FFP alliances and 0 otherwise. Similarly, let

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[18] In addition to the studies cited in Section 2 see also Morrison and Whinston (1995).

[19] Appendix 3.B describes in detail how the tourism index is constructed.

[20] Borenstein (1991) discusses togreater length the competing explanations (such as information spillovers and economies of scale in advertising) for the advantage enjoyed by the airline with the strongest presence at an airport and looks at the role of the tourism interaction variables in descriminating between these explanations.

Rival Alliance$_{ijt}$ be equal to 1 if in quarter t carrier i faces competition on route j from at least one carrier that has formed a *FFP* alliance with a third one. Following on from the reasoning laid out earlier, the variable *Own Alliance$_{it}$* is expected to have a positive effect on the fare level of airline i and *Rival Alliance$_{ijt}$* to have a negative effect.

These two dummy variables reflect the presence or absence of airline alliances. On the other hand, it was put forward in Section 3.1 and in Section 3.2 that the impact of the alliances is likely to be influenced by the size of the airlines involved - partnering an airline with a particularly significant presence on a route is likely to have a greater impact than if the alliance was with a small player. Furthermore, when assessing the size of an alliance, it is necessary to consider the three distinct spheres in which the alliance can be measured: at the route, at the airport and at the national level.

Given this reasoning, a second strategy to capture the effect of the *FFP* alliances is to include in the vector A_{ijt} the variables *Partner's Route Share$_{ijt}$*, *Partner's City Share$_{ijt}$* and *Partner's National Share$_{it}$* which measure the market share of airline i's partner at the route, airport and national level respectively. Where an airline does not have a partner, these variables take the value of 0. Under the hypothesis that an airline's *FFP* is enriched by the partnership with a second loyalty programme, it is expected that these three variables have a positive effect on the fare level.

Similarly, the increase in the competitive pressure faced by an airline if two of its rivals agree to form a *FFP* alliance is likely to be dependent on the market share of these rivals.

To capture this, the change in the competitive pressure brought about by one or more rivals forming an alliance will be measured by the increase in concentration which comes about by treating two partnering airlines as one entity. A formal description requires some notation. Let n_{jt} be the number of airlines operating on route j at time t, let N_{jt} be the number of *FFP* alliances at time t in which the two partnering airlines are both serving route j, let m index the airlines from 1 to n_{jt} such that the last N_{jt} refer to those with a partnering airlines among the first $n_{jt} - N_{jt}$ airlines. Finally, let s_{mjt} be the market share of airline m on route j and $s_{m'jt}$ be the market share of m's partner if airline m has a partner and it also competes in route j. If m does not have a partner or its partner does not compete on route j then $s_{m'jt} = 0$. Using this notation, the variable constructed to reflect the change in the competitive pressure due to the formation of alliances by competing airlines is given by,

Table 3.5: Calculating *Concentration Increment Route,* **example**

Airline	Market Share	*Concentration Increment Route*
A	30%	0.05
B	35%	0.05
C	10%	0.22
D	15%	0.22
E	10%	0.24

$$Concentration\ Increment\ Route_{ijt} = \frac{\sum\limits_{m,m\neq i}^{n_{jt}-N_{jt}} \left(s_{mjt} + s_{m'jt}\right)^2 - \sum\limits_{m,m\neq i,m\neq i'}^{n_{jt}} s_{mjt}^2}{1 - \left(s_{ijt} + s_{i'jt}\right)^2}.$$

The increase in concentration is normalized by the contribution to the Herfindahl index of the market share held by the observed airline and that of its partner (where one exists). Where no rival airline has formed an alliance then *Concentration Increment Route*$_{ijt}$ is equal to 0.

The value of *Concentration Increment Route*$_{ijt}$ is increasing in the market share of the rival airlines that have formed an alliance. As an example, Table 3.5 reports the value of this variable in a scenario where five airlines serve a given route and two pairs of alliances are established - between airlines A and B and another between airlines C and D.

The added competitive pressure faced by the two smaller partnering airlines due to the alliance formed by the larger A and B, is considerably greater than the change in concentration facing A and B due to the alliance of C and D. Clearly, the greatest increase in pressure is felt by airline E - rather than facing four distinct competitors, this airline will now face two pairs of allied airlines. Note, that by definition the value of *Concentration Increment Route*$_{ijt}$ is the same for two partnering airlines.

In an analogous way, the variable *Concentration Increment City*$_{ijt}$ is constructed to capture the increase in concentration at the airport level due to the formation of FFP alliances by competing airlines. Similarly, the variable *Concentration Increment National*$_{it}$ is defined at a nationwide level. It is expected that the increase in loyalty scheme concentration amongst

126

competitors resulting from the alliances will have a downward impact on the observed airline's fares. It follows that the coefficients of these three variables when estimating the price equation (3.1) is expected to be negative.

3.4.2 Econometric Issues

In estimating Equation (3.1), two issues stand out: the use of route fixed effects and the endogeneity of variables.

Fixed Effects

Equation (3.1) includes the term v_j which is intended to capture demand and cost differences that do not vary across the airlines serving route j and are constant over time. The more clear example of these are the distance of the route, the degree to which a route is tourist oriented, the level of congestion at each end-point airport and the presence of slot controlled airports or flight limitations at either end of the route. The fixed effects capture these differences without having to explicitly measure them and it is attractive inasmuch as most of the variation in price within the sample is due to differences at the route level, as will be clear by examining the regression results.

The use of fixed effects, however, is not without criticism. First, the use of fixed effects curtails the ability to make out of sample predictions. This concern is legitimate if the results of the analysis were intended to be transposed to, say, the European context or if the sample of routes accounted for a small fraction of total American market. The former exercise will not be done - not least because the airline industry in the two continets is structurally different and operates in a different regulatory framework - and the latter concern is unwarranted as the sampled routes account for over 70 per cent of all traffic. A second drawback in the use of fixed effects is that no use is made of the information in the variability between routes and, related to this, efficiency is lost due to fall in the degree of freedom (See Greene 1993, Chapter 16 and Ichino, 2000). This criticism stands.

A way out would be to change the framework and model Equation (3.1) with random effects whereby the estimation of the parameters fully exploits the information in the variability between and within routes. However, the random effect estimator is only consistent if the explanatory variables are not correlated to the route specific effects. After running each of the specifications detailed in Section 5 below, this condition was tested using the Hausmann

test and consistently rejected. Faced with this, the remedies are either to find instruments which are correlated with the explanatory variables but not with the route specific effects or to abandon the random effects framework and fall back on fixed effects. Given the difficulties in finding appropriate instruments I resorted to the latter strategy.

Endogeneity of explanatory variables

It can be expected that $Route\ Share_{ijt}$ is a function of the price charged and therefore correlated with the error term ε_{ijt}. A carrier whose actual price is above the predicted value - a positive ε_{ijt} - is expected to lose market share to its competitiors. This potential correlation would lead to overestimate the true effect of market share on prices.

This problem is tackled by using an instrument for market share. As in Evans and Kessides (1993) I use a carrier's intra-route rank as an instrument for its route market share. The variable $Rank_{ijt}$ is constructed by equating it with the rank in terms of market share of each airline within a given route. The rank is defined in descending order, so that the largest carrier takes the rank of one.

The variable $Rank_{ijt}$ is a valid instrument if it is correlated with market share and orthogonal to the error term. The first requirement is met. By construction, $Rank_{ijt}$ is negatively correlated with market share. The second requirement will hold if, for example, a change in a carrier's price leads to a change in its market share but not by enough to alter its rank within the route. This is more likely to occur where the difference in the market share of the ith and the $i + 1^{th}$ ranked firm is large. On the other hand, this requirement is less likely to hold within the set of airlines with smaller market shares. It can be expected that a price change by one of the smaller airline might be sufficient to reverse a market share gap with a similarly small airline and thereby alter its own rank. Table 3.6 illustrates the extent to which this problem might come about. The table shows the average difference in the market share between successively ranked airlines.

One way round the problem, adopted by Evans and Kessides (1993), is to set the rank of all airlines ranking third or smallest to be equal to 3. Hence, all but the two largest carriers are grouped together into one category and the implicit assumption is made that changes in prices for these small carriers

Table 3.6: Average difference in market share

Rank	Difference in Market Share
First-Second	30.3%
Second-Third	8.3%
Third-Fourth	3.51%
Fourth-Fifth	1.89%

Source: Calculations based on Consumer Report, 1998 1st Quarter

are not large enough to move them out of this group. The figures reported in Table 3.6 suggest that this is a feasible assumption.

3.4.3 Data

The data on fares and traffic used are from the *Domestic Airline Fares Consumer Report (Consumer Report)* published by the U.S. Department of Transport. The report provides information on passenger data and average prices paid by travellers in the 1,000 largest domestic city-pair markets within the 48 contiguous states of America.[21] The report applies a screen so that for each route, data are only supplied for those airlines judged to be competitors - these are defined as those transporting at least 10% of the passengers on that route. The markets covered by the *Consumer Report* account for 75% of all 48-state passengers and 70% of total domestic passengers. The *Consumer Report* is elaborated quarterly and I make use of the data in the five quarterly reports from the first quarter 1998 up to and including the first quarter of 1999.

The data are corrected to take into consideration that two cities are sufficiently close to each other so that their airports compete with each other for travellers. This is the case of Oakland, CA which is close to San Francisco, CA and of Baltimore, MD which is close to Washington DC.

The average fares were deflated using quarterly data on air carrier jet fuel prices from the *Aviation Industry Overview* for the fiscal years of 1998 and 1999 issued by the Office of Aviation Policy and Plans.

The indices on the tourism orientation of the cities at the end-points

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[21] In turn, the information presented in the *Consumer Report* is constructed from the Department of Transportation's Origin and Destination Survey of Airline Passenger Traffic which is a continuous survey of 10% of all the passengers travelling on US certified carriers.

of the routes were constructed using data from 1997 *Economic Census - Accomodation and Foodservices* and from the *State and Metropolitan Areas Data Book, 1997-98.*

Appendix 3.B provides further details on the data used.

3.5   Results and discussion

The results of estimating Equation (3.1) are reported in Table 3.7. For ease of presentation, estimates of the route fixed effects and the quarterly dummy variables are not shown.

The bottom two rows of Table 3.7 show the very low correlation between the fixed effect variables and the remaining explanatory variables, and the result of the Hausmann specification test. Both results suggest that the use of fixed rather than random effects is appropriate.

The first two regressions, labelled Regression *A* and *B*, estimate Equation (3.1) using the two dummy variables *Own Alliance* and *Rival Alliance* to reflect, respectively, whether the observed airline has formed an alliance and whether it is competing on the observed route with an airline that has. Regression *C*, on the other hand, uses the set of variables described earlier to reflect the size of an airline's own alliance and that of competing alliances. Lastly, note that Regressions *B* and *C* include the set of three variables that interact an airline's share of activity at the route, airport and national level with the tourism index on the observed route.

Table 3.7: Regression results

Dependent variable	Regression		
Fare	**A**	**B**	**C**
Passengers	−0.151 (0.007)	−0.173 (0.008)	−0.172 (0.008)
Rank	−0.014 (0.002)	−0.018 (0.003)	−0.016 (0.003)
City Share	0.147 (0.015)	0.122 (0.020)	0.125 (0.020)
National Share	0.720 (0.036)	0.849 (0.050)	0.702 (0.052)
Hub	0.051 (0.006)	0.052 (0.006)	0.056 (0.006)
Low Cost Airline	−0.177 (0.006)	−0.187 (0.006)	−0.168 (0.006)
*Tourism * Route Share*		0.006 (0.003)	0.006 (0.003)
*Tourism * City Share*		0.041 (0.016)	0.039 (0.016)
*Tourism * National Share*		−0.104 0.038	−0.087 (0.038)
Rival Alliance	−0.020 (0.004)	−0.018 (0.004)	
Own Alliance	0.011 (0.004)	0.010 (0.004)	
Concentration Increment Route			−0.107 (0.054)
Concentration Increment City			−1.169 (0.120)
Concentration Increment National			0.583 (0.114)
Partner's Route Share			0.000 (0.000)
Partner's City Share			0.038 (0.023)
Partner's National Share			0.302 (0.045)
Observations	13,203	12,388	12,388
Prob ≥F	0.0	0.0	0.0
Correl.(v_j, Xb)	0.092	0.018	0.001
Hausmann: Prob≥ χ^2	0.0	0.0	0.0
R^2	0.370	0.378	0.387

Note: Standard deviations are given in brackets. The Hausmann specification tests
the equivalence of fixed and random effects estimates. The test statistic is distributed
as chi-squared with 19 degrees of freedom. The degrees of freedom are given by the
number of estimable parameters in the fixed effects model which includes the four time
dummy variables. All variables are significant at the 5% level except for
Partner's City Share and *Partner's Route Share* in Regression
C which are significant at the 10% level.

3.5.1 Discussion

Influence of market structure

An interpretation of the negative sign on the variable *Passengers* is that it reflects the more fierce competition one expects to exist on the more busy routes.

All three regressions found the variables for an airline's share of the market at the route, airport and at the national level to be significant.

Recall that *Rank* is negatively related to an airline's route market share and note the significant negative coefficient of coefficients for *Rank* in Table 3.7. It follows that route market share has consisently a positive effect on average fare levels. An airline's share of activity at the airport level also has a positive effect on an airline's fares. Based on the results from regression *A* and evaluated at the sample mean, the elasticity of price with respect to *City Share* is 0.040. [22] A one standard deviation increase in *City Share* over the sample mean will increase the fare level by 2.7 per cent. The results also reveal that an airline's share in the national market will earn it a premium: a one standard deviation increase in this share over the sample mean will add 7.4 per cent to an average return fare. It is also worth noting that airlines flying to or from their hub are in a further advantaged position and, on average, are able to set prices 5.2 per cent higher.

First evidence on the role of *FFPs*

Borenstein (1991) interpreted the negative sign on *Tourism * City Share* as an indication that the impact of having a large share of traffic at an airport was due to the *FFPs*. While the reported coefficients for regressions *B* and *C* in Table 3.7 do not point in this direction as far as airport dominance is concerned, they do so at the route and national level.

The tourism orientation index used in the three regressions on Table 3.7 has a range from 0.16 to 2.5 and a mean of 0.77. Consider regression *B*. Given the estimated coefficient on *Tourism * National Share* of −0.104, the advantage of having a significant share of the nationwide market falls by 31 per cent on the most touristic routes compared to what it would be on the least touristic.[23] This fall in the premium is also mirrored at the route level,

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[22]Focusing on regression *A* allows to put aside the effect of the tourism interaction variables.

[23]The tourism index of the most touristic route, 2.5, multiplied by the coefficient of

132

where the advantage of holding a given rank in the most touristic route is roughly one fifth of the advantage of holding that rank in the least touristic route.[24]

An interpretation of the reduced impact of route and national dominance on touristic route is to attribute it to FFP effects. As discussed in the Introduction, $FFPs$ are one mechanism through which airline can earn a premium from their significant market share. Such schemes, however, are likely to be less effective on touristic routes where the propotion of frequent flying business travellers is comparatively low and where the remaining passengers, those flying for tourism or leisure, attach less importance to $FFPs$.

The above findings do not carry over to the airport level. The regression results suggest that the impact of dominance at this level is greater the higher the tourist index on a route. This contrasts with Borenstein (1991) where the reverse effect was found.

The robustness of these results to the choice of the tourism index was tested on two fronts. In computing the tourism index, I followed Borenstein (1991) and truncated its value at the upper end in order to correct for some accounting inconsistencies in Nevada cities. The data reported in Nevada on hotels' income includes income from gambling and thereby distorts the overall data. To correct for this, the index was truncated at the top.[25] The regression results were found to be robust to the choice of the cut-off level. In addition, an alternative tourism index was computed based on the share of income generated in the metroplitan area of the routes' endpoints from hotels and other accomodation (Abramowitz and Brown 1993). The results of the latter exercise are reported in Appendix 3.C and they are consistent with the results presented in Table 3.7.

Interpreting the alliance variables

The interpretation of the tourism interaction variables is a first suggestion that FFP effects matter. A clearer suggestion of this, however, is had from examining the estimated coefficients of those variables reflecting the formation of the FFP alliances. Consider first Regressions A and B which include

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$Tourism * National Share$, -0.104, implies that the impact of having an average market share at the national level in the most touristic routes falls by 31 per cent (calculated by $\frac{2.5*0.104}{0.849-0.16*0.104} = 0.31$).

[24]Calculated by $\frac{-0.018+2.5*0.0058}{-0.018+0.16*0.0058} = 0.21$

[25]See Appendix 3.B.

Table 3.8: Impact on Fares of FFP Alliances

Level	Change in Concentration + 0.0051	Partner's Share +10%
Route	0.0 %	0.0 %
Airport	-0.6 %	0.4%
National	0.3 %	3.1

the two dummy variables *Rival Alliance* and *Own Alliance* in the set of explanatory variables.

The coefficient on *Rival Alliance* suggests that those airlines serving routes where no competitor has formed an alliance enjoy a premium of around 1.9 per cent over those routes where such an alliance has been made. On the other hand, the premium an airline earns from having joined an alliance herself can be read from the coefficient of *Own Alliance* and is around 1 per cent.

Both of these results are significant and point towards the finding that more is lost by facing competitors that have formed an alliance than is gained by taking part in one. Further insight is given by Regression C where the size of the alliances are considered.

The estimated coefficients in Regression C show that the increase in FFP concentration at the route and airport level due to a FFP alliance involving one or more of an airline's rivals has a downward effect on prices. In contrast, at the national level the effect is positive though the magintude is considerably small. Table 3.8 below illustrates the size of these effects. The first column of figures in Table 3.8 reports the estimated effect on the fares of an airline due to the formation of a FFP alliance involving rival airlines. The example assumes that the rival partnering airlines each have a 5% at the route, city and national level and that the observed carrier's share at each of these levels is 10% so that the increment in concentration (as measured by the Herfindahl index) is 0.0051.

The second column of figures in Table 3.8 illustrate the premium earned by an airline taking part in a FFP alliance arising from the share of acitivity of its partnering airline. The figures in this column are based on the estimates of Regression C and on the assumption that the partner's share at the route, city and national level is 10%.

The results reported in Table 3.8 suggest that the effect of the alliances on fare levels do not come about due to the change in concentration or market shares at the route level. Instead, the impact of the alliances come about due to the way in which they change the market structure at the airport and at the national level.

3.6 Conclusion

In the wake of the alliances involving six of the seven largest US airlines, the report *Aviation Competition- Effects on Consumers from Domestic Airline Alliances Vary* (GAO/RCED-99-37) was presented before the chairmen of the relevant committees of the American Senate. The report sees the reciprocity of $FFPs$ between partnering airlines as beneficial to consumers. However, it also raises the concern over the potential role that such marketing agreements have on conferring additional market power to those airlines with a significant presence in the market. The report notes that it is unable to quantify this aspect. In this chapter, I have attempted to take on this challenge.

The empirical analysis carried out suggest that the formation of FFP alliances have enhanced the market power of partnering airlines. The ability to charge higher prices are positively related to the partner's share of activity at the airport and at the national level. On the other hand, the partner's share of activity at the route level does not have a sizeable impact on an airline's average fares. Conversely, the formation of a marketing alliance by competing carriers was shown to have a downward effect on an airline's fares. The results are consistent with the hypothesis that $FFPs$ enhance the market power of the larger airlines which are able to offer a greater menu of flights and it therefore lends support to the concerns raised by competition authorities over the anti-competitive effects of $FFPs$.

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135

136

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Appendices

3.A Timing and nature of agreements

3.A.1 Continental-Northwest[26]

The announcement of the formation of a strategic global alliance between Continental and Northwestern was made at the end of January 1998. The agreement foresaw code-sharing arrangements, the reciprocity of the *FFPs* and the co-operation between Continental and Northwest's important partner KLM Royal Dutch Airlines. Furthermore, Northwest was to purchase 51% of Continental's voting stock and Continental's Board of Directors was to expand to allow the appointment of one director designated by Northwest. Contrary to the intentions of the two airlines, however, the agreement took long to materialise.

Due to the cross-ownership clause, the agreement required the approval of the Department of Justice (DOJ) to go ahead. However, the necessary thumbs-up from the DOJ, did not arrive as this Department began to investigate the alliance on anti-trust grounds. Despite Northwest's pledge to maintain the two operations separate and to not exercise its voting rights for 10 years, the DOJ initiated a law suit challenging Northwest's proposed purchase of Continental. Nevertheless, the two airlines went ahead with their plans and in late November 1998, they announced that Northwest had purchased the stake in Continental. In addition, it was announced that Northwest would not use its voting rights in Continental for the next six years at least.

The marketing side of the alliance was phased in over the ensuing months. Subject to regulatory review, reciprocity in mileage earnings was announced on the 3rd of December and taken into effect three days later. In addition it was announced that beginning on the 1st February 1999 members of one program may request awards from the other program for travel beginning the 1st of March. Code-shared flights between the two airlines started on the 29th December 1998 and covered 28 weekly flights between US and Japan and 21 weekly flights beyond Japan.

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[26]Information was collected from Continental Press Releases dated 26 January 1998, 28 December 1998, on *Frequent Flier* issues dated 22 and 29 October 1998, 5 and 19 November 1998, 3 December 1998 and on the October 1998 issue of *Inside Flyer*.

3.A.2 American-US Airways[27]

The agreement between American Airlines and US Airways was announced towards the end of April 1998. The alliance was to centre almost exclusively on the pooling and reciprocity of the loyalty programmes offered by the two airlines and the proposed plans were introduced over the following 4 months.

Effective from the start of August 1998, members of American's AAdvantage program could earn air miles when flying on US Airways' flights. Similarly, members of US Airways' Dividend Miles program were able to earn air miles on this program when flying on an American Airlines flight. In addition, air miles from either program could be redeemed on either airline. At the end of August 1998, the two loyalty programs were brought further together. From then on, members of both airlines' FFP would be able to combine the miles from both accounts to claim an award for travel in either airline. It should be noted, however, that the pooling of miles could not be used to apply for awards on other airlines that are also partners in either American Airlines' AAdvantage or US Airways' Dividend Miles program.

The two airlines also agreed to allow reciprocal access to the domestic and international club facilities operated by the two airlines.

Finally, it is interesting to point out that in their initial announcements the two airlines hinted at the desire to set-up code-shared flights. The two airlines acknowledged, however, that such plans could not take place without consulting the pilots and they were not pursued during the period analysed in this chapter.

3.A.3 Delta-United Airlines[28]

On the 29th of April 1998, within a week of American and US Airways' announcement, Delta and United Airlines let their intention to form an alliancebe known. According to the initial plans, the alliance was to revolve around the agreement to run code-shared flights and on the reciprocity of the airlines' loyalty programs.

The implementation of the code-sharing agreement, however, was subject to the approval of both carriers' pilot unions. Following lengthy negotiations,

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[27] Information was collected from : AMR Press Release, April 23 1998, US Airways Press Release, July 27th and August 24th 1998 and on the *Frequent Flier* issues of 1st August, 8th October.

[28] Information based on: Delta Air Lines Annual Report 1998, United Airlines Press Release, 31st August 1998, *Frequent Flier* issues from 4th July, 5th September.

Delta was not willing to pay the price asked for by its pilots - a voting seat on its board of directors -and consideration of code-shared flights was discontinued in August.

The steps to bring the *FFPs* of the two airlines closer, however, went ahead. Starting from the 1st September 1998 members of both United's Mileage Plus and Delta's Sky Miles programmes could earn miles and class-of-service bonus when travelling on either carrier within the US, Puerto Rico and the US Virgin Islands.

3.B Definition of variables and sources of data

$Fare_{ijt}$ is the logarithm of the average one-way fare on the given route. *Source: Domestic Airline Fares Consumer Report*, DOT. Data from the first quarter 1998 through to and including the first quarter 1999 was used.

$Passengers_{ijt}$ is the logarithm of the total number of passengers flying on observed route in the given quarter. *Source:* Market size was extracted from Table 5 of *Domestic Airline Fares Consumer Report*, DOT.

$Route\,Share_{ijt}$ is the share of passengers transported by carrier i on route j in quarter t. *Source: Domestic Airline Fares Consumer Report*, DOT.

$City\,Share_{ijt}$ is the weighted average of airline i's share of passengers travelling on routes that start from or end at either end-point of route j at time t. This variable was calculated as,

$$City\,Share_{ijt} = \frac{Pass1_{jt} * City\,Share1_{ijt} + Pass2_{jt} * City\,Share2_{ijt}}{Pass1_{jt} + Pass2_{jt}}$$

where $Pass1_{jt}$ are the number of passengers travelling on routes that land or take off from one end-point of route j and $Pass2_{jt}$ the number of number of passengers on routes that land or take off from the other end-point of route j. The respective share of passengers of carrier i at each of the two end-points are denoted by $City\,Share1_{ijt}$ and $City\,Share2_{ijt}$. *Source:Domestic Airline Fares Consumer Report*, DOT.

$National\,Share_{it}$ is the number of passengers travelling on carrier i at time t divided by the total number of travellers at time t across all routes. *Source: Domestic Airline Fares Consumer Report*, DOT.

Hub_{ij} is equal to 1 if route j starts or finishes at a hub of carrier i and is 0 otherwise. The following cities were considered as hubs.

Table 3.B.1: Hubs

Airline	Hubs
Continental	New York, Houston, Cleveland
Northwest	Minneapolis/St. Paul, Detroit, Memphis
American	Dallas, Chicago, Miami
US Airways	Pittsburgh, Charlotte, Philadelphia
Delta Airlines	Atlanta, New York, Dallas, Cincinnati, Salt Lake City
United	San Francisco, Denver, Chicago, Los Angeles
TWA	St. Louis, New York
America West	Columbus, Phoenix
Reno Air	Reno, Las Vegas
American Trans Air	Chicago
AirTran Airways	Atlanta
Frontier Airlines	Denver

Source: Aiports were identified as hubs on the basis of information collected from the 1998 annual reports of the various companies and from the companies' web-sites.

Low Cost$_i$ is equal to one if, on average, airline i's price on a route relative to the industry's average price on routes of similar distances was 30 per cent or more lower. *Source:* Calculations based on *Domestic Airline Fares Consumer Report*, DOT.

Tourism$_j$ is defined as the maximum of the tourist index of the two end-point cities of route j. In turn, the tourism index of a city is calculated as the income per capita generated from hotels and other accomodation in the metropolitan areas where the city is located. This index was truncated at 2.5 at the upper end.

An alternative tourism index for a city is used for the regressions reported in Appendix 3.C. This alternative index is defined as the share of income of the metropolitan area where the end-point city is located that is generated from hotels and other accomodation. Here, the index was truncated at 0.6.

Table 3.B.2: Quarters in which the alliances operated

	1998_1	1998_2	1998_3	1998_4	1999_1
American-US Airways			√	√	√
United-Delta			√	√	√
Continental-Northwest					√

Source: Company press releases and various issues of Frequent Flier

Source: Calculations based on *Economic Census - Accomodation and Food-services, 1997* and *State and Metropolitan Areas Data Book, 1997-98.*

$Tourism * Route\,Share_{ijt}$, $Tourism * City\,Share_{ijt}$ and $Tourism * National\,Share_{ijt}$ are defined as the product of $Tourism_j$ with, respectively $Route\,Share_{ijt}$, $City\,Share_{ijt}$ and $National\,Share_{ijt}$.

The remaining variables, those capturing the existence and size of the FFP alliances, were defined in detail in the main text. Their construction is based on data from *Domestic Airline Fares Consumer Report*, DOT and on the interpretation of company press releases and of various issues of *Frequent Flier* as to when the alliances operated. Table 3.B.2 shows the latter.

3.C Robustness of results to choice of tourism index

Table 3.C.1 below presents the results of running Regressions B and C under an alternative tourism index. As explained above in Appendix 3.B, this alternative index is based on the share on income in the metropolitan area where the end-point cities are located that is generated from hotels and other accomodation.

Table 3.C.1: Regression results

Dependent variable	Regression	
Fare	**B'**	**C'**
Passengers	−0.173 (0.008)	−0.172 (0.008)
Rank	−0.018 (0.003)	−0.016 (0.003)
City Share	0.134 (0.020)	0.137 (0.019)
National Share	0.824 (0.047)	0.675 (0.050)
Hub	0.052 (0.006)	0.056 (0.006)
Low Cost	−0.187 (0.006)	−0.168 (0.006)
*Tourism * Route Share*	0.083 (0.034)	0.083 (0.034)
*Tourism * City Share*	0.394 (0.204)	0.373 (0.203)
*Tourism * National Share*	−1.109 0.482	−0.824 (0.480)
Rival Alliance	−0.018 (0.004)	
Own Alliance	0.010 (0.004)	
Concentration Increment Route		−0.109 (0.054)
Concentration Increment City		−1.168 (0.120)
Concentration Increment National		0.583 (0.114)
Partner's Route Share		0.000 (0.000)
Partner's City Share		0.036 (0.023)
Partner's National Share		0.305 (0.045)
Observations	12,388	12,388
Prob $\geq$ F	0.0	0.0
Correl.(v_j, Xb)	0.017	0.008
Hausmann: Prob$\geq \chi^2$	0.0	0.0
R^2	0.378	0.386

Note: The Hausmann specification tests the equivalence of fixed and random
effects estimates. The test statistic is distributed as chi-squared with 19
degrees of freedom. The degrees of freedom are given by the number of estimable
parameters in the fixed-effects model, which includes the four time dummy variables.
All variables are significant at the 5% level except for *Tourism * City Share*,
*Tourism * National Share*, *Partner's Route Share* and *Partner's City Share*.
in Regression C'. The first three are significant at the 10% level.