Seigniorage, Optimal Taxation, and Time Consistency: A Review

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Abstract

This paper reviews the existing literature on seigniorage, optimal taxation, and time consistency. An extension of the simple optimal taxation model of Mankiw (1987) serves as the unifying framework. Particular emphasis is put on the derivation of the consolidated public sector flow budget constraint from the government budget constraint and the central bank profit and loss account and balance sheet. Within this framework, it is shown that a slightly modified version of the Persson, Persson and Svensson (1987) solution to the time consistency problem, that is, specific ways of debt structure management, is not only necessary but also sufficient for optimality. However, this will not necessarily be true in more general models [Calvo and Obstfeld (1990).] In addition, the different recommendations of the public finance literature on optimal inflation, notably the Friedman (1969) rule of constant deflation, positive inflation, or seigniorage maximizing inflation, are shown to be specific time consistent solutions to the optimal tax problem. The model is deliberately kept as simple as possible to make the analysis accessible for non-specialists.

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List of Symbols

If not mentioned explicitly, the following conventions are followed:
upper case letters denote nominal variables and lower case letters real
variables,
a time index $t$ denotes end of period $t$ values,
a superscript $e$ indicates values that were expected at the end of the
former period.
In addition, the following notation is applied:
\[ b_t : \text{stock of government bonds held by private agents at the end of period} \]
\[ b_t^b : \text{stock of government bonds held by the central bank at the end of} \]
\[ b_t^{tot} : \text{total stock of all government bonds outstanding at the end of period} \]
\[ c_t : \text{credit the central bank gives to the private sector (put differently: the} \]
\[ d_t : \text{real net indebtedness of the public sector, i.e. the difference between} \]
\[ g_t : \text{real government expenditure,} \]
\[ H_t : \text{Hamiltonian of the public sector optimization problem,} \]
\[ i_t : \text{nominal interest rate paid at the end of period} \]
\[ L_t : \text{all end of period} \]
$l_{t1}$: social losses from output taxation,

$l_{t2}$: social losses as a consequence of suboptimal holdings of real balances,

$l_{t3}$: all social losses from inflation surprises,

$m_t$: real balances private agents at the end of period $t - 1$ plan to hold at the end of period $t$,

$M_t$: nominal balances at the end of period $t$,

$\sigma_t$: operation costs of the central bank (will for convenience be set equal to zero later on),

$P_t$: nominal price level,

$r_t$: domestic real rate of interest between the end of period $t - 1$ and the end of period $t$,

$R_t = \prod_{i=1}^{t} 1/(1 + r_i)$: real interest or discount factor to calculate the present value of a variable at the end of period $t$,

$s_t$: all seigniorage revenues,

$s_t^e$: seigniorage from expected inflation,

$s_t^s$: seigniorage from surprise inflation,

$s_{t}^{*s}$: seigniorage from surprise inflation when the expected inflation rate is $\bar{\pi}$,

$x_t$: central bank profit,

$y_t$: real output,

$\Theta_t = \prod_{i=1}^{t} 1/(1 + \theta_i)$: time preference factor the public sector uses to discount the social losses of period $t$ to its period 0 present value,

$\lambda_t$: current value multiplier of the Hamiltonian $H_t$,

$\pi_t$: inflation rate between the end of period $t - 1$ and the end of period $t$,

$\pi_t^e$: inflation rate expected at the end of period $t - 1$ for period $t$,

$\pi_t^s$: unexpected inflation rate or inflation surprise between the end of period $t - 1$ and the end of period $t$,

$\bar{\pi}$: highest possible inflation rate,

$\tau_t$: average output tax,

$\Omega_t = \prod_{i=1}^{t} (1 + \omega_i)$: growth factor of real balances between the end of period 0 and the end of period $t$. 


Introduction

Seigniorage, or the public sector’s revenue from the creation of money, has attracted a great deal of attention in monetary theory following the initial discussion by Keynes (1923) and the pathbreaking formal analysis of Bailey (1956).¹ In part, recent interest in the issue has clearly been related to the role of the EMS and the establishment of a European central bank.² The budgetary problems of the Eastern European states after the structural break in 1989 have also drawn attention to the importance of seigniorage as a revenue instrument. Most of the emerging democracies are highly dependent on seigniorage because they usually have poorly developed excise and income taxation systems together with significant black market activities, rather underdeveloped domestic capital markets, and only limited access to international capital markets. Oblath and Valentinyi (1993) indicate how important seigniorage is quantitatively in Eastern Europe. For Hungary, which is among the more stable Eastern European countries, they value the average share of the revenues from money creation at 3.5 percent of GDP over the period of 1989–1992 with a peak of 5.1 percent in 1991. If one considers the recognized budgetary role that seigniorage already plays in Southern European countries it is seems that the scale of these figures is not unrealistic; Southern European countries after all have more fertile alternative revenue sources.³

Given the vast scale of the literature on seigniorage, it is surprising that no detailed survey of the public finance theory of seigniorage is available. Reasons for this may include the different modeling approaches

¹See also Friedman (1953).
³Fischer (1982) for instance estimates the Italian seigniorage between 1973 and 1978 as 3.9 percent of GNP. For the period from 1980 until 1990, Repullo (1991) finds in the case of Spain an average amount of seigniorage of 3.98 percent of GDP. The figure for Italy, however, seems to be a little too high, which could be explained with Fischer’s rather crude measure of seigniorage as being simply the change in base money, see Klein and Neumann (1990). Using a more sophisticated measure, Bruni, Penati and Porta (1989) calculate Italian seigniorage as 2.1 percent of GDP between 1976 and 1981 and as 1.7 percent of between 1982 and 1987. And finally, Fischer and Easterley (1990) on page 133 find it “safe to argue that rates of seigniorage of much more than 2.5 percent would not be sustainable ...”.
used in the literature as well as the range of results found. For instance, earlier authors often addressed the issue by asking which rate of inflation maximized public revenues whereas Phelps (1973) in his path-breaking integration of inflation within a modern public finance framework of optimal taxation stressed the optimal mix of all possible revenue sources. In this context, optimality (from a public finance point of view) means that a mix of the different revenue instruments raises the required revenues at the lowest possible welfare costs. However, as the literature stimulated by Phelps' idea has shown, optimal inflation rates may either be either positive or negative, depending on how efficient are alternative revenue instruments like output taxes. The only consensus in monetary theory seems to be that the optimal inflation rate in general does not imply maximizing seigniorage.

Another strand of the literature has focussed on the inherent time consistency problem, which has been well known since Calvo (1978) applied the classic ideas exposed in Kydland and Prescott (1977) to monetary economics. In this context, the question of time inconsistency arises because after individuals have chosen their holdings of nominal balances unexpected inflation allows the public sector to devalue the real value of nominal balances. Thereby it can reduce its liabilities vis a vis the private sector and collect revenues at social costs lower than the ones resulting from alternative revenue instruments. Irrespective of the importance of this problem, however, the time inconsistency problem is often assumed away in the public finance literature on optimal seigniorage by restricting attention to situations in which the policy maker is precommitted to the ex ante optimal policy.

In contrast, this paper does not ignore the time consistency problem, but offers a simple framework within which many of the results of the public finance literature on optimal inflation can be encompassed as time consistent solutions to the optimal tax problem. The plan of the paper is as follows: Section 2 contains an extension of the model suggested by Mankiw (1987), who incorporated inflation taxes into the optimal output

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4 As will become clear later, negative inflation rates result when it is optimal to drive the nominal interest rate to zero [Friedman (1969)].
taxation framework of Barro (1979). Particular emphasis is put on the derivation of the consolidated public sector flow budget constraint and of seigniorage from the government budget constraint, the central bank profit and loss account, and the central bank balance sheet [in the sub­sections 2.1 and 2.2, respectively]. The social losses resulting from the use of the different revenue instruments will be justified in 2.3. In section 3, the time inconsistency of the optimal policy is studied. To have a benchmark case, subsection 3.1 contains the solution of the optimal tax problem under precommitment. The time inconsistency of the optimal policy is shown in 3.2 while the time consistent solution under discretion is characterized in 3.3. Section 4 offers a brief review of the different solutions to the time consistency problem, notably reputational mechan­isms [in subsection 4.2] and specific management of public assets and debt as has been suggested by Persson et al. (1987) [in subsection 4.2]. The Persson, Persson and Svensson suggestion then is formally discussed as a solution to the time consistency problem in section 5. In particular, it will be shown that in our set–up a slightly modified version of their solution is not only necessary, but also sufficient for optimality, although this will not necessarily be true in more general models [Calvo and Obstfeld (1990) and Persson, Persson and Svensson (1989)]. Section 6 offers some interpretations of the optimal policy. In subsection 6.1, the Ramsey principle of optimal taxation is related to the solution found in this paper. The different recommendations of the public finance literature on optimal inflation [Friedman (1969) rule, positive or even seigniorage maximizing inflation rates] are shown to be specific time consistent solutions to the optimal tax problem [in subsection 6.2]. Subsection 6.3 briefly touched the question to which extent our results generalize to general equilibrium models of optimal taxation. Finally, section 7 concludes.

1 The Optimal Tax Problem

In what follows, we deal with the government and the central bank as if they were one institution, called the public sector. This is, for example,

\(^5\)For a discussion of Barro's model see Aschauer (1988).
appropriate when the central bank is totally dependent on the government, a paradigm that is true for many countries.\footnote{Concentrating on dependent central banks admittedly prevents the analysis of a number of important issues of “political economy.” Given the space constraint, however, we still decide to focus the present interest on this situation. For a recent discussion of central bank independence, see Walsh (1993).} The public sector in our closed economy has to finance an exogenous stream of expenditure through the revenues from taxing a homogeneous output good and through seigniorage.\footnote{See Fischer (1983a) or Rogers (1989) for generalizations to endogenous government expenditure.} The use of both revenue instruments causes welfare losses. The optimal tax problem is thus to find a sequence of taxes and seigniorage that minimize the present value of the social losses while satisfying the intertemporal budget constraint.\footnote{The term optimal is used in a normative sense.} In the following subsection, this constraint is derived.

1.1 The Consolidated Public Sector Budget Constraint

To concentrate on the issues related to seigniorage, we assume that the stream of the nominal homogeneous output good $Y_0, Y_1, Y_2, \ldots$ is exogenously given. For simplicity the analysis is moreover restricted to a deterministic environment; all realizations of exogenous variables other than the policy instruments are hence known with certainty. If $\tau_t$ denotes the average tax rate, the nominal tax revenues $T_t$ can be expressed as:

$$T_t = \tau_t Y_t.$$  \hspace{1cm} (1)

In addition to financing its expenditures $G_0, G_1, G_2, \ldots$ through output taxation and central bank profit $X_t$, the government also raises funds through increasing the stock $B_{t-1}^{tot}$ of nominal government bonds. The stylized government flow budget constraint in nominal terms, which requires the equality between nominal expenditure and revenues at the end of any period, can therefore be written as

$$G_t + i_t B_{t-1}^{tot} = T_t + X_t + \Delta B_{t}^{tot},$$  \hspace{1cm} (2)
where \( i_t \) denotes the nominal interest rate which is paid at the end of period \( t \) on assets issued either privately or publicly at the end of period \( t - 1 \).\(^9\) The budget constraint in real terms follows after division of (2) by the nominal price level \( P_t \). For notational convenience, let us denote real variables by lower case letters, which, for example, implies \( B_{t-1}^{\text{tot}} / P_t = b_{t-1}^{\text{tot}} / (1 + \pi_t) \). The government budget constraint in real terms then reads as:

\[
g_t + \frac{i_t}{1 + \pi_t} b_{t-1}^{\text{tot}} = \tau_t y_t + x_t + \Delta b_{t}^{\text{tot}}.
\]

(3)

In (3), \( \pi_t \) stands for the rate of inflation between the end of period \( t - 1 \) and the end of period \( t \):

\[
\pi_t = \frac{P_t - P_{t-1}}{P_{t-1}} = \frac{\Delta P_t}{P_{t-1}}.
\]

(4)

So far, money has not had a role in our endowment economy. We therefore assume that legal restrictions require individuals to pay any purchases of other individual’s endowment in cash.\(^{10}\) To determine the domestic price level and central bank profit we postulate the existence of a stable aggregate relation between output, nominal interest rates and the demand for real balances in the economy. Let \( m_t(y_t, i_t) \) denote the real balances private agents at the end of period \( t - 1 \) plan to hold at the end of period \( t \) when they expect an inflation rate of \( \pi_t^{\text{e}} \).\(^{11}\) The partial derivatives of real balances are assumed to reflect the standard transaction and store of value motive:\(^{12}\)

\[
\frac{\partial m_t(y_t, i_t)}{\partial y_t} > 0, \quad \frac{\partial m_t(y_t, i_t)}{\partial i_t} < 0.
\]

(5)

\(^9\)Note that for simplicity we assume the nominal interest rate on all bonds in the economy to be the same.

\(^{10}\)This is in some sense similar to the legal restriction usually assumed when cash in advance constraints are modeled.

\(^{11}\)The index \( t \) on \( m_t \) indicates that the demand for real balances may also depend on influences other than the nominal interest rate and real output, for example, on changes of the financial technology.

\(^{12}\)Note that it is possible to a certain extent to justify this form of the demand for real balances by intertemporal optimization behavior of private agents [See Aizenman (1989), in particular the appendix therein, and Calvo and Guidotti (1993)]. It should also be mentioned that the dependence of desired real balances on expected inflation is to be modeled particularly carefully in order to avoid the Lucas critique when working with a “reduced form economy” like the one used here.
In order to determine the demand for nominal balances at the end of period \( t - 1 \), we additionally assume that, given the expected price level for the next period \( P^e_t \), private agents achieve the desired level of expected real balances \( m_t \) at the end of \( t - 1 \); i.e.

\[
\frac{M_{t-1}^d}{P^e_t} = m_t(y_t, i_t).
\] (6)

For a given price expectation \( P^e_t \), the equality of the supply and demand of nominal balances then determines the price level at the end of period \( t - 1 \):

\[
M_{t-1} = m_t(y_t, i_t)P^e_t = m_t(y_t, i_t)P_{t-1}(1 + \pi_t^e)
\]

\[\implies P_{t-1} = \frac{M_{t-1}}{m_t(y_t, i_t)(1 + \pi_t^e)}.\] (7)

We may observe that the price level both directly and indirectly depends on the expected inflation rate, given that the expected inflation rate affects the nominal interest rate. To model this we make the simple assumption that the nominal interest rate is determined by the Fisher relation, i.e. that nominal interest payments fully compensate expected inflation:

\[
(1 + i_t) = (1 + r_t)(1 + \pi_t^e) \quad \text{or} \quad i_t = r_t + \pi_t^e + r_t \pi_t^e.\] (8)

If we finally assume that the domestic real interest rate is exogenously given too, we are left with the expected inflation rate as the only endogenous variable affecting the demand for real balances:13

\[
m_t(y_t, i_t) = m_t(y_t, r_t + (1 + r_t)\pi_t^e) = m_t(\pi_t^e).\] (9)

For any given expectations of the inflation rate there is therefore a one to one relation between the money supply and the price level and thus the actual inflation rate in our model. We may thus take the actual inflation rate and not the money supply as the policy instrument for analytical convenience.14

13In a closed economy, a constant real interest rate would obtain, for example, if consumers utility were linear in consumption [compare Obstfeld (1991)].

14Note that we implicitly assume too that the policy maker can control the money supply process.
We now explicitly derive an expression for central bank profit $x_t$, since this is the only item in the budget constraint of the public sector that has not yet been discussed. The stylized central bank profit and loss account may be written as: \(^{15}\)

\[
x_t = \frac{i_t B_{t-1}^{cb} + i_t C_{t-1} - O_t}{P_t},
\]

\(^{(10)}\)

where $B_{t-1}^{cb}$ and $C_{t-1}$ denote the nominal value of the stock of government bonds and of privately issued assets held by the central bank at the end of period $t - 1$, respectively. $O_t$ represents the operation costs of the central bank. Equation (10) expresses that real central bank profits equal the difference between interest payments on the end of last period’s stock of central bank assets and its operational costs. \(^{16}\)

In addition to the central bank profit and loss account, the monetary balance sheet of the central bank is crucial for the derivation of its profits. It shows that changes in the supply of (base) money $M_t$ result from open market operations:

\[
\frac{M_t - M_{t-1}}{P_t} = \frac{\Delta B_t^{cb} + \Delta C_t}{P_t}. \quad (11)
\]

Substituting expression (11) into (10), we find that central bank profit may be written as:

\[
x_t = \frac{\Delta M_t + i_t (B_{t-1}^{cb} + C_{t-1}) - \Delta (B_t^{cb} + C_t) - O_t}{P_t}. \quad (12)
\]

As shown in Appendix A, substituting (12) into the government budget constraint (3) yields a form of the consolidated budget constraint of the public sector, in which the role of the alternative revenue instruments becomes clear:

\[
g_t + (1 + r_t) b_{t-1} + o_t = \tau_t y_t + s_t + b_t, \quad (13)
\]

\(^{15}\)Similar approaches can be found in Klein and Neumann (1990) and in Repullo (1991).

\(^{16}\)Note that we have explicitly assumed that the nominal interest rate paid on government bonds is the same as the one paid on private bonds. This excludes the possibility of implicit subsidies of the private sector through below market interest rates on $C_{t-1}$ [Compare Klein and Neumann (1990)].
where $b_{t-1} = b_{t-1}^{\text{tot}} - b_{t-1}^c$ is the real value of the end of period $t-1$ stock of government bonds not held by the central bank and $s_t$ is seigniorage. In the next subsection we discuss the components of $s_t$; note that the potential revenues due to a devaluation of government debt after unexpected inflation are also included in $s_t$. This explains the term $(1 + r_t) b_{t-1}$ on the left hand side of the equality sign of (13).\footnote{See Calvo and Guidotti (1990) for a similar point of view.}

1.2 Seigniorage from Expected and from Surprise Inflation

In Appendix A, it is also shown that seigniorage in (13) comprises the following items:

$$s_t = \frac{\Delta M_t}{P_t} + i_t \frac{C_{t-1}}{P_t} + \frac{1}{P_t} - \frac{1}{P_t} (1 + i_t) B_{t-1} - \frac{\Delta C_t}{P_t} - \frac{O_t}{P_t}$$

$$= \left[ (1 + \pi_{t+1}^e) m_{t+1} (\pi_{t+1}^e - c_t) - \frac{1}{1 + \pi_t} m_t (\pi_t^e) - (1 + r_t) c_{t-1} \right]$$

$$+ \frac{\pi_t - \pi_t^e}{1 + \pi_t} (1 + r_t) b_{t-1} - o_t.$$ \hfill (14)

The real seigniorage revenues in (14) have five components, reflecting the revenues from expected \textit{and} from unexpected inflation:

1. The real value of the additional nominal balances the private sector has to acquire at the end of period $t$ to achieve expected real balances equal to desired real balances $m_{t+1}(\pi_{t+1}^e)$.\footnote{Note that government debt enters the expression for seigniorage differently from central bank assets. While all changes of the stock of central bank assets and the real value of interest payments affect seigniorage, only changes of the value of government debt resulting from surprise inflation are part of seigniorage.}

2. The real value of interest payments $i_t C_{t-1} / P_t$ on the end of last period's stock of private assets held by the central bank.

3. The end of period $t$ difference between the expected real value of government debt (issued at the end of period $t-1$) and its actual value.
4. The real value of additional credit $\Delta C_t/P_t$ the central bank gives to others than the government.

5. The operational costs of the central bank $o_t$, which for convenience we set equal to zero from now on.\textsuperscript{19}

Since we are interested in the time inconsistency issue of monetary policy within an optimal taxation framework, it is useful to distinguish between the seigniorage revenues due to both expected and unexpected inflation. In what follows, a superscript $e$ will indicate the component of a variable that was expected at the end of last period and a superscript $s$ will stand for the surprise or unexpected component. With this convention, the seigniorage revenues in the current period for instance are simply:

$$s_t = s^e_t + s^s_t. \quad (15)$$

By setting $\pi_t$ equal to $\pi^e_t$ so that there is no surprise inflation, the expected real seigniorage revenues follow directly from the expression (14) for the total real seigniorage revenues:

$$s^e_t(\pi^e_{t+1}) = \pi^e_{t+1}m_{t+1}(\pi^e_{t+1}) + [m_{t+1}(\pi^e_{t+1}) - m_t(\pi^e_t)] + r_tc_{t-1} - [c_t - c_{t-1}]
\quad = [(1 + \pi^e_{t+1})m_{t+1}(\pi^e_{t+1}) - c_t] - [m_t(\pi^e_t) - (1 + r_t)c_{t-1}]. \quad (16)$$

To understand the implications of this expression more deeply, the expected real seigniorage revenues for two extreme cases are considered now:\textsuperscript{20}

1. The central bank implements monetary policy solely through open market operations in government bonds:
   In this case, the credit extended to the private sector is equal to zero and the expected real seigniorage revenues are:
   $$s^e_t = \frac{\Delta M_t}{P_t} = \pi^e_{t+1}m_{t+1}(\pi^e_{t+1}) + [m_{t+1}(\pi^e_{t+1}) - m_t(\pi^e_t)]. \quad (17)$$

\textsuperscript{19}This not to say that these costs can be neglected. Klein and Neumann for instance estimate the operation costs of the Deutsche Bundesbank at around 10% of the total seigniorage in the 1980s. They are however assumed to be exogenous given the present interest.

\textsuperscript{20}Similar results in a continuous time framework can be found in Drazen (1985).
Expected seigniorage hence only results from changes in base money. For this reason, (17) has at times been called the cash-flow or monetary measure of seigniorage. In particular, the first term on the right hand side of (17) arises as a consequence of an erosion of the real value of nominal balances that private agents will want to hold constant in a stationary state (with constant real interest rates and real output). In contrast, the second term on the right hand side arises even if the inflation rate is constant because private agents want to adjust their real balances if the real interest rate, real output, or any other exogenous variable changes. For example, as Friedman (1971) has argued real output growth increases seigniorage for a given inflation rate by increasing desired real balances.

2. The central bank implements monetary policy solely through open market operations in bonds issued privately: The expected revenues from inflation then correspond to the interest payments on the end of the last period’s stock of central bank assets privately issued, which is equal to the end of the last period’s stock of money by construction:

\[ s_t^e = \frac{i_t C_{t-1}}{P_t^e} = \frac{i_t M_{t-1}}{P_t^e} = i_t m_t. \]  

(18)

The foregone interest payments for holding non interest bearing domestic money instead of interest bearing securities now measure the revenue from the creation of money. The right hand side of (18) is therefore usually called the opportunity cost measure of seigniorage.

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22 Some authors, for instance Anand and van Wijnbergen (1989), make a distinction between inflation tax and seigniorage. They coin the item \( \pi_{t+1} m_{t+1} \) in (17) the inflation tax (with \( \pi_{t+1} \) being the tax rate and \( m_{t+1} \) being the inflation base) and the item \( (m_{t+1} - m_t) \) seigniorage. The sum of both is then referred to as the revenues from money creation or monetization. We do not want to make this distinction here but rather call all revenues from money creation seigniorage (or equivalently inflation taxes), because what Anand and van Wijnbergen call seigniorage also has a tax aspect in that it implies a transfer of real resources from the private to the public sector.
Although these two measures look rather different and it has been dis­puted which is the right one to use for empirical work, they are intertemporally equivalent. More generally, the present value of expected seigniorage is independent of the implementation of monetary policy. To see this, define the market discount factor as:

\[ R_{-1} = 1 + r_0, \quad R_0 = 1, \quad R_t = \prod_{i=1}^{t} \frac{1}{1 + r_i}. \quad (19) \]

Using the Fisher relation (8) and the identity \( R_t(1 + r_t) = R_{t-1} \), we then find the present value of seigniorage is given by:

\[
\sum_{t=0}^{\infty} R_t \pi^e_t = \sum_{t=0}^{\infty} R_t \left\{ \left[ (1 + \pi_{t+1}^e) m_{t+1}(\pi_{t+1}^e) - c_t \right] \right\}
\]

\[
= \sum_{t=0}^{\infty} R_{t+1}(1 + i_{t+1}) m_{t+1}(\pi_{t+1}^e) - \sum_{t=0}^{\infty} R_t m_t(\pi_t^e)
\]

\[
+ \sum_{t=0}^{\infty} R_t c_t - \sum_{t=0}^{\infty} R_{t-1} c_{t-1}
\]

\[
= \sum_{t=1}^{\infty} R_t i_t m_t(\pi_t^e) + (1 + r_0)c_{-1} - m_0, \quad (20)
\]

where \( c_{-1} \) is the end of period -1 stock of privately issued bonds that are held by the central bank and \( m_0 \) is the real, expected end of period value of the end of period \( t - 1 \) stock of nominal balances.

We now turn to the revenues from surprise inflation. If we subtract the expected seigniorage (16) from total seigniorage (14), we get the revenues from surprise inflation:

\[
s^e_t(\pi^e_t) = \frac{\pi^e_t}{1 + \pi^e_t + \pi_t^e} \left[ m_t(\pi_t^e) + (1 + r_t)(b_{t-1} - c_{t-1}) \right]. \quad (21)
\]

As long as the public sector is a net debtor of the private sector, the revenues from unexpected inflation are positive and increase with increasing inflation surprises. These revenues come about for two reasons: On the one hand, government debt and private assets are not protected against inflation surprises as long as they are not indexed. If the public sector is

\[24\]This has been pointed out by, among others, Repullo (1991).
a net debtor vis à vis the private sector it can devalue its net liabilities through the use of unexpected inflation. On the other hand, we have assumed that the nominal balances carried over to the current period are determined at the end of the last period before the public sector chooses the actual rate of inflation. The public sector can thus always devalue these nominal balances more than expected by the private sector. Note that surprise inflation devalues all nominal public liabilities, whereas expected inflation only devalues the non-interest bearing ones. This holds true, because interest bearing public liabilities are protected against expected inflation when the Fisher relation is valid.

To simplify the notation, let us call the real net indebtedness of the public sector \( d_t \):

\[
d_t = b_t - c_t.
\]

(22)

After the substitution of (22) and the expressions for expected and unexpected seigniorage, (16) and (21), into the budget constraint (13), we arrive at a form of the consolidated public sector budget constraint that will be used in the following chapters:

\[
d_t = g_t + (1 + r_t)d_{t-1} - \tau_t y_t - s_t^e(\pi_{t+1}^t) - s_t^s(\pi_t^s).
\]

(23)

Equation (23) determines the real value of the net indebtedness \( d_t \) that the public sector has to issue at the end of period \( t \). It has to equal the difference between real expenditure plus the real indebtedness at the end of period \( t \), which was expected at the end of period \( t - 1 \) on the one hand and the sum of taxation revenues plus expected and surprise seigniorage on the other hand.

1.3 Social Losses from Taxation and Inflation

To formulate the optimal tax problem of the public sector, the welfare losses resulting from the use of the different policy instruments still need to be specified. We assume the social losses as perceived by the policy maker take the following additive form in each period:

\[
L_t = l_{t1}(\tau_t) + l_{t2}(i_{t+1}) + l_{t3}(\pi_t^s),
\]

(24)
with $l_{ti}(0) = l'_{ti}(0) = 0$ and $l''_{ti} > 0$ when the argument is not equal to zero ($i = 1, \ldots, 3$). The specification of equation (24) includes the following assumptions: The social losses at any point in time are supposed to depend positively and convexly on each of the different policy instruments. They are allowed to change over time, for instance with changes of real output or financial technology. Furthermore, social losses from the use of any policy instrument are assumed to be zero whenever that instrument is not used. Finally, our specification of the loss function is separable in the different arguments.\(^{25}\)

In particular, we may distinguish the following sources of social loss: \(^{26}\)

1. Social losses from the use of the endowment tax may arise from collection or enforcement costs. They can intuitively be motivated by assuming that individuals try to avoid taxes more rigorously, the more utility they have to sacrifice as a consequence of reduced consumption. If utility has the standard convexity properties, these activities are intensified more than proportionally with increasing average taxes. The collection costs hence increase convexly in the average tax rate too.\(^{27}\)

2. Anticipated inflation causes social losses because it leads to suboptimal holdings of real balances and thereby increased transaction costs and decreased social welfare. These additional costs do not arise when the satiation level of desired real balances is held, which is the case if the nominal interest rate is zero, implying zero opportunity costs of holding money.\(^{28}\)

\(^{25}\)This assumption is used for simplicity, although it excludes situations in which the use of a policy instrument influences the losses incurred from other instruments. For example, in our set-up, inflation cannot increase the real collection costs of the endowment tax if there are significant collection lags. [See Tanzi (1977) for the classic argument on collection costs due to inflation and Dixit (1991) for a reinterpretation.]

\(^{26}\)See Fischer and Modigliani (1979), Fischer (1983b), or Driffill, Mizon and Ulph (1990) for a more detailed discussion of the different losses from inflation.

\(^{27}\)Note that they do not appear in the public sector budget constraint, because we implicitly assume that private agents have to pay them in addition to their tax payments.

\(^{28}\)Leach (1983) appears to be the first to motivate these welfare costs in the context
3. In contrast to anticipated inflation, surprise inflation does not cause social losses from suboptimally low real balances because private agents have already determined their desired holdings of nominal balances when the authority decides whether to create inflation surprises or not. The costs of surprise inflation come instead from menu costs and from unwanted redistribution among private agents (if they use contracts based on nominal price expectations). Note that these costs do not arise from expected inflation because private agents will adjust their contracts.29

We have now extended Mankiw (1987)'s model to a framework, in which we can discuss the time consistency problem of the optimal policy.30

2 The Time Consistency Problem of the Optimal Inflation Policy

In this section, we show that the best achievable policy is not time consistent in general and discuss what the time consistent policy under discretion is.

of seigniorage and optimal taxation by a the use of a transactions technology. He assumed real balances to be an input good of this technology, which decrease the time devoted to unproductive 'shopping activities.'

29 In the literature, it has been debated whether the policy maker is only concerned with the social losses from expected inflation or also with the ones from unexpected inflation. [Compare for example Kydland and Prescott (1977), Barro (1983), Barro and Gordon (1983a), Barro and Gordon (1983b) and opposed to them Grossman (1990).] Since we are also discussing the time consistency problems of the optimal tax policy, we have to model the costs from surprise inflation. The omission of them would lead to the same mistake the original paper of Persson et al. (1987) was flawed with. [See Calvo and Obstfeld (1990) and Persson et al. (1989).] 30

30 More rudimentary versions of Mankiw's model have been applied by Roubini and Sachs (1989), Grilli (1989), Yashiv (1989), Poterba and Rotemberg (1990), Trehan and Walsh (1990), de Jong and van der Ploeg (1991), and Goff and Toma (1993).
2.1 The Solution to the Optimal Tax Problem Under Precommitment

As a benchmark case, we first want to study the optimal tax policy under precommitment. The term precommitment implies that the public sector can credibly bind itself to follow announced policies. In particular, unexpected inflation is not in this case available as a policy instrument. The possibility to precommit simplifies the public optimization problem considerably because the expected inflation rate must equal the actual rate. It is thus rational for the private sector to expect the inflation rate that is ex ante optimal from the authority’s point of view. Consequently, the public sector can act as a Stackelberg leader, who, taking account of the Stackelberg follower’s reaction, chooses the optimal expected inflation rate and precommits to stick to it in the future. On the other hand, the private sector is the Stackelberg follower, who reacts to the leader’s inflation announcement by adjusting its real balances.

If the public discount factor $\Theta_t$ is defined as:

$$\Theta_t = \prod_{i=1}^{t} \frac{1}{1 + \theta_i}, \quad 0 < \theta_i < 1,$$

the optimal tax problem under precommitment is to choose $\tau_t$ and $\pi_{t+1}^e$ so as to minimize $\sum_{t=0}^{\infty} \Theta_t \left[ l_{t1}(\tau_t) + l_{t2}(\pi_{t+1}) \right]$ subject to (23) with $s^a = 0$ and (8) and (16) substituted in. In order to characterize the solutions under precommitment, we define the discrete Hamiltonian with the co-state variable $\lambda_t$ (in current value notation) $t = 0, 1, 2, \ldots$:

$$\mathcal{H}_t(d_{t-1}, \tau_t, \pi_{t+1}^e, \lambda_t) = \left[ l_{t1}(\tau_t) + l_{t2}(\pi_{t+1}) \right] + \lambda_t \left[ g_t + (1 + r_t)d_{t-1} - \tau_t y_t - s^e_t(\pi_{t+1}) \right].$$

In addition to (23) with $s^a = 0$, the first order conditions are:\footnote{Note that output was exogenously given and thus does not depend on the average tax rate.}

$$\mathcal{H}_{\tau_t} = \frac{\partial l_{t1}(\tau_t)}{\partial \tau_t} - \lambda_t y_t = 0,$$  (27.a)
Moreover, the following transversality condition must be satisfied:

$$\lim_{t \to \infty} R_t \lambda t d_t = 0. \quad (27.d)$$

If all instruments cause social losses, the policy characterized by these first order conditions must result in the highest achievable social welfare, compared to all time consistent alternatives. We thus call it the ex ante optimal policy.32

### 2.2 The Time Inconsistency Problem Under Discretion

In this section, it is shown that the policy optimal under precommitment is in general time inconsistent when the authorities have discretion over the policy instruments. Under discretion, a policy plan is called time inconsistent when it is ex-ante but not ex-post optimal. Put differently, an optimal policy is time inconsistent when the announced or planned future policy determined as optimal at the initial date is no longer found to be optimal from the viewpoint of a later date, although no new information has arrived in the meantime.

One of the first to point out the potential importance of time inconsistency in monetary economics was Auernheimer (1974). Following the general analysis of Kydland and Prescott (1977), Calvo (1978) and Lucas and Stokey (1983) proved that the inflation pattern, which is optimal from a public finance point of view, is in general time inconsistent under discretion.33 The source of the time inconsistency in all of these models

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32In the literature, this policy is sometimes also called the “second best policy” or the “best second best policy”. Note that first best policies are not achievable since there is no loss free instrument available.

33More generally, Strotz (1955) appears to be the first to recognize the problem of time inconsistency in economic decision making. He shows that for an intertemporally
is exactly the same as in ours; the private sector has to determine its optimal holdings of real balances on the basis of its rational expectations of inflation before the public sector chooses the actual inflation rate.

To formally show the time inconsistency of the ex ante optimal policy under discretion, we first observe that it coincides with the optimal policy under precommitment. This holds true because inflation surprises are not an ex ante policy instrument. We thus have to prove that, given individuals believe in the authority’s inflation announcement, the optimal precommitment policy is no longer optimal under discretion. Without loss of generality, let us concentrate on the problems at the end of the periods zero and one. We want to argue indirectly now and thus assume that at the end of period zero individuals find the inflation rate credible that was optimal under precommitment. We then show that when the authority has discretion over the policy instruments at the end of period one, it will create surprise inflation. The optimal tax problem under discretion at the end of period one then is to choose \( r_t, \pi_t^e \), and \( \pi_t^s \) so as to minimize \( \sum_{t=1}^{\infty} \Theta_t [l_{t1}(\tau_t) + l_{t2}(i_{t+1}) + l_{t3}(\pi_t^s)] \) subject to (23) with (8), (16) and (21) substituted in. The discrete Hamiltonian for \( t = 1, 2 \ldots \) under discretion becomes:

\[
\mathcal{H}_t(d_{t-1}, \tau_t, \pi_{t+1}^e, \pi_t^s, \lambda_t) = [l_{t1}(\tau_t) + l_{t2}(i_{t+1}) + l_{t3}(\pi_t^s)] + \lambda_t [g_t + (1 + r_t) d_{t-1} - \tau_t y_t - s_t^e(\pi_{t+1}^e) - s_t^s(\pi_t^s)].
\]

In addition to (23) and the standard transversality condition, the first order conditions are:

\[
\mathcal{H}_{\tau_t} = \frac{\partial l_{t1}(\tau_t)}{\partial \tau_t} - \lambda_t y_t = 0, \tag{29.a}
\]

\[
\mathcal{H}_{\pi_{t+1}^e} = \frac{\partial l_{t2}(i_{t+1})}{\partial \pi_{t+1}^e} - \lambda_t \frac{\partial s_t^e}{\partial \pi_{t+1}^e} = 0, \tag{29.b}
\]

\[
\mathcal{H}_{\pi_t^s} = \frac{\partial l_{t3}(\pi_t^s)}{\partial \pi_t^s} - \lambda_t \frac{\partial s_t^s}{\partial \pi_t^s} = 0, \tag{29.c}
\]

\[
\lambda_{t-1} = \frac{1}{1 + \theta_t} \mathcal{H}_{d_{t-1}} = \lambda_t \left( \frac{1 + r_t}{1 + \theta_t} \right) \left( \frac{1 + \pi_t^e}{1 + \pi_t^s + \pi_t^s} \right), \quad t = 1, 2, \ldots \tag{29.d}
\]

optimizing individual, yesterday’s optimal choice will not necessarily be optimal tomorrow, unless the discount factor consistently translates today’s valuation of a future event into tomorrow’s valuation of the same event.
where:
\[ \frac{\partial s_t^s}{\partial \pi_t^s} = \frac{1 + \pi_t^s}{(1 + \pi_t^s + \pi_t^e)^2} \left[ m_t(\pi_t^e) + (1 + r_t)d_{t-1} \right]. \]

To see the time inconsistency, first note that by (29.a) and (29.b), the Lagrange multiplier is positive whenever the optimal tax rate or inflation rate is positive. Together with (29.c), this implies that the time consistent choice \( \pi_t^s = 0 \) is not optimal in general because at \( \pi_t^s = 0 \) the derivative of the Hamiltonian with respect to \( \pi_t^s \) is not equal to zero unless the net nominal liabilities of the public sector are zero:

\[
\mathcal{H}_{\pi_t^s} \bigg|_{\pi_t^s=0} = -\lambda_t \frac{1}{1 + \pi_t^e} \left[ m_t(\pi_t^e) + (1 + r_t)d_{t-1} \right] \neq 0
\]

\[ \iff m_t(\pi_t^e) + (1 + r_t)d_{t-1} \neq 0. \quad (30) \]

If, for example, the net nominal liabilities of the public sector are positive, the marginal increase of unexpected inflation away from zero generates additional revenues by devaluing the public net nominal liabilities while the marginal losses are zero. Surprise inflation will hence be used and the optimal policy is not time consistent.

### 2.3 The Time Consistent Solution Under Discretion

In this section, we briefly study the time consistent solution under discretion. Since rational private agents understand that the precommitment solution is time inconsistent under discretion, they will not base their decisions on the inflation expectations that would be optimal under precommitment. Agents rather expected an inflation rate that does not give the public sector an incentive to deviate and the authority loses its Stackelberg leadership. In our model, the highest technologically feasible inflation rate protects individuals from inflation surprises. Let us denote such an inflation rate by \( \bar{\pi} \), assuming that it is finite; for simplicity, we also assume that it is time invariant and exogenously given.\(^{34}\) The public optimization problem then changes substantially because the authority

\(^{34}\)See Grossman and Van Huyck (1986) for a discussion on the existence of such an upper boundary to inflation. Note that we would not have needed the existence
now has to take $\bar{\pi}$ as given and the two choice variables it is left with are $\tau_t$ and $\pi^*_t$. Since the marginal revenue from inflation surprises is positive,

$$\frac{\partial s_t^*}{\partial \pi^*_t} = \frac{1 + \bar{\pi}}{(1 + \bar{\pi} + \pi^*_t)^2} \left[ m_t(\bar{\pi}) + (1 + r_t)d_{t-1} \right] > 0. \tag{31}$$

and since $\bar{\pi}$ is already the highest feasible inflation rate, the public sector can only create surprise deflation. However, a marginal decrease of $\bar{\pi}$ would result in marginal revenue losses while the marginal social losses are zero; it thus cannot be optimal to create surprise inflation when individuals expect $\bar{\pi}$. In addition to (23) with $\pi^*_{t+1} = \bar{\pi}$ and $s^* = 0$, the first order conditions, which characterize the time consistent equilibrium under discretion, are:

$$H_{\tau_t} = \frac{\partial l_t(\tau_t)}{\partial \tau_t} - \lambda_t y_t = 0, \tag{32.a}$$

$$\lambda_{t-1} = \frac{1}{1 + \theta_t} H_{d_{t-1}} = \lambda_t \frac{1 + r_t}{1 + \theta_t}. \tag{32.b}$$

It should be stressed that this discretionary equilibrium is welfare inferior compared to the precommitment policy since the highest possible inflation rate is in general not the optimal one. Put differently, as a consequence of loosing its Stackelberg leadership, the public sector is left with the average tax rate as the only instrument compared to the two instruments under precommitment. This in general results in suboptimality.\(^{35}\)

### 3 A Brief Review of the Different Solutions to the Time Consistency Problem

Having realized the welfare inferiority of the outcome in a discretionary equilibrium, it is natural to look for possibilities to improve upon it. The

\(^{35}\)See Kydland and Prescott (1977), Fischer (1980), Barro and Gordon (1983a), and Barro and Gordon (1983b).
easiest way out of the time consistency problem of course is to assume that the public sector can credibly precommit not to use surprise inflation. Many authors, who wanted to concentrate on the public finance aspects of the optimal tax package, have made this assumption.\textsuperscript{36} Although it may be appropriate to concentrate on the precommitment solution when the central bank, like the Bundesbank, is institutionally independent of the government, it is generally accepted that many countries do not have access to such a precommitment technology.

A possible reaction to this problem is to advocate institutional reforms, for example, with the aim to give the central bank more independence from the government.\textsuperscript{37} Sometimes constitutional amendments have even been suggested as a solution to the time inconsistency problem in order to force the policy maker to follow a fixed rule, thus preventing the discretionary use of surprise inflation.\textsuperscript{38} Although these ideas are very important when there are political majorities in favor of a substantial reform of the institutional environment, it may not be easy to find democratic support for them in other situations, at least not in the short run. This is not to say that institutional reforms are an unrealistic or unworthy goal for the long run. It rather reflects our interest in solutions that can be implemented without the approval of the legislature, that is, reputational equilibria and specific ways of using asset and debt management to overcome the problem.


\textsuperscript{37}Rogoff (1985) suggestion of appointing a “conservative” central banker, who places a higher weight on price stability than society, goes in this direction. Moreover, in the vivid discussion on the set–up of a European central bank, this idea has frequently been taken up. See Eichengreen (1993).

\textsuperscript{38}Compare Kydland and Prescott (1977). It should be mentioned that it may not even be desirable to implement rules even because any gains from imposing a rule have to be traded off against potential disadvantages from lost monetary flexibility. After the adoption of a rule, it is for example not possible to respond to contingencies not foreseen or not describable in the rule. See Fischer (1990) on this.
3.1 Reputational Mechanisms

Taylor (1983) appears to be the first to argue that societies sometimes may already have found a way out of the discretionary equilibrium by using reputational mechanisms. To outline the main idea, we first observe that so far we have not allowed current behavior of the public sector to influence the future expectations of the private sector. The easiest way to model such an intertemporal link is to use so called trigger strategies. Following for example, Grossman and Van Huyck (1986), a trigger strategy may be modeled as follows in our set up: At the end of a period $t$, private agents observe last period’s actual inflation rate $\pi_{t-1}$. They can thus find out whether the public sector has followed its announcements or has created inflation surprises in the last period. At the same time, the public sector announces the inflation rate for the next period. Private agents are rational and hence understand that an announcement is time inconsistent if it is smaller than $\bar{\pi}$. However, as long as the public sector has never deviated from the announced policy until the end of period $t-1$, it is assumed to have built up a reputation for being able to resist the temptation to break the rule. In this case, private agents believe that the public sector will not deviate in period $t+1$ either. In contrast, if the public sector deviates once from its announcement, the trigger is pulled. It then loses its reputation forever and private agents expect it to always choose the discretionary policy, i.e. the highest technologically feasible inflation rate $\bar{\pi}$.

Confronted with a trigger strategy, the public sector faces the following trade-off: A one period deviation from the preannouncement results in additional revenues from surprise inflation, which allows it to make less use of the future revenue instruments. Ceteris paribus, this decreases the social losses. In all future periods, however, private agents will expect the highest possible inflation rate which leads to the discretionary equilibrium with higher social losses. The precommitment solution now is implementable as a so called reputational equilibrium if the present value of social losses without reneging is smaller than the one with reneging.

\[^{39}\text{That reputation is lost forever can be generalized by assuming that after some periods it is possible to build up the reputation again.}\]
and it is optimal for the public sector not to create inflation surprises.\footnote{See Stokey (1989) for a general discussion of trigger strategies.}

Although the idea of reputational equilibria is appealing at first sight, there are still many problems with this approach.\footnote{The most important one is that, in general, there exists more than one reputational equilibria. It is not clear how the private sector, which is composed of a large number of individuals, coordinates on the most favorable one.} Since there are excellent surveys available on the application of reputational equilibria in monetary policy games, we do not elaborate on this idea here but rather move on to the Persson et al. (1987) solution to the time consistency problem.\footnote{See Rogoff (1987), Alesina and Tabellini (1988), Driffill (1988), Persson (1988), Blackburn and Christensen (1989), Rogoff (1989) and the references therein.}

### 3.2 Asset and Debt Management

The solution to the time consistency problem through asset and debt management was first suggested by Lucas and Stokey (1983) for a closed barter economy without capital, in which the repudiation of public debt is excluded by assumption; capital is not modeled and a solution for Kydland and Prescott (1977)’s “classical” time consistency problem of optimal capital levies is thus not offered.\footnote{Note that we have implicitly also excluded taxes on capital income.} In Lucas and Stokey (1983)’s model the time consistency problem arises because the intertemporal consumption – leisure decision is sensitive to expected changes in the distortionary consumption tax but cannot be changed once the future period is entered. Lucas and Stokey show how a specific debt structure of real indexed bonds can solve the time inconsistency problem by shifting debt repayments into periods in which the authority would otherwise tolerate higher real interest rates than preannounced. Persson and Svensson (1984) have provided a more intuitive reinterpretation of this argument in terms of the public sector cash flow.

In an open economy, the Lucas and Stokey (1983) idea has also been investigated by Persson and Svensson (1986). They show that the argument can only be generalized to \textit{large} open economies. In \textit{small} open economies...

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\footnotetext[40]{See Stokey (1989) for a general discussion of trigger strategies.}
\footnotetext[41]{The most important one is that, in general, there exists more than one reputational equilibria. It is not clear how the private sector, which is composed of a large number of individuals, coordinates on the most favorable one.}
\footnotetext[43]{Note that we have implicitly also excluded taxes on capital income.
economies, the real rate of interest is determined by the world interest rate and the public sector loses the possibly to affect future optimal decisions through changing the real interest rate. It has thus been concluded that Lucas and Stokey’s argument is not valid in small open economies. However, in a recent paper, Huber (1992) has pointed out another way of restoring the time consistency of the optimal policy even in a small open economy. Consider for example a situation in which the public sector in a future period could increase social welfare by deviating from the taxation scheme that is optimal from today’s point of view. When it now is optimal from a future point of view to cut taxes (which would lead to higher consumption than optimal from today’s point of view) the potential welfare gains can be offset by the right choice of consumption indexed public bonds which pay higher interest payments in reaction to the higher consumption. According to Huber, clever debt management ensures that the net gain from a deviation from the preannounced policy can be made nil. The time consistency problem of a small open barter economy can thus be solved too.

Finally, Persson et al. (1987) have generalized the Lucas and Stokey approach to a closed monetary economy. They argue that the full indexation of all public debt to actual inflation in connection with the implementation of monetary policy via open market operations in non-public nominal bonds of a certain maturity structure can also ensure time consistency when money is present in the economy. If their suggestion is followed, the public sector has zero nominal liabilities vis a vis the private sector. It appears then to be quite intuitive that surprise inflation would not generate net gains or losses on nominal assets and liabilities of the public sector. Calvo and Obstfeld (1990), however, have pointed out that the Persson, Persson and Svensson device only satisfies the first-order condition for optimality, but not the second order conditions. They provide an example in which the interaction of today’s monetary policy with future interest rates can lead to future gains from surprise inflation although no current gains are generated. In their response to Calvo and Obstfeld (1990)’s comment, Persson et al. (1989) also take account of the social losses from surprise inflation and show that the second order conditions then are not necessarily violated anymore. However, they do
not succeed in proving that the second order conditions always hold, consequently, they assume that it is true and note that “this is obviously not very satisfactory, but it is the usual way of proceeding in optimum taxation problems that are too complex for detailed examination of the second-order conditions” [Persson et al. (1989) on page 5]. Because the Persson et al. (1987) suggestion has not been dealt with intensively in the literature, we will discuss it formally in our model now. Within the framework developed above, it will be shown below that a slightly modified version of the Persson et al. (1987) suggestion leads to a unique time consistent policy that satisfies the first and second order conditions for optimality.

4 The Sufficiency of the Persson, Persson and Svensson Solution

The model presented in the previous sections of this paper is more specific than the set-up used in the literature we have just briefly reviewed. In particular, the real rate of interest is given and the endowment tax is assumed to relate to an exogenously given tax base. The time consistency problem investigated by Lucas and Stokey is hence not present here by construction. This simplification allows us to focus on the fundamental problems arising from the existence of money in the economy. We recall that from (21) the real value of the net nominal liabilities of the public sector at the end of period $t$ is equal to

$$
\frac{1 + \pi_t}{1 + \pi_t} \left[ m_t(\pi_t^e) + (1 + r_t)(b_{t-1} - c_{t-1}) \right].
$$

(33)

As we have seen before, the public net nominal liabilities cause the time consistency problem. Consequently, Persson et al. (1987) suggest that they be zero to achieve time consistency. This can, for instance, be attained as follows:

1. All public debt is indexed to ex post inflation, which implies the full compensation for the actually experienced rate of inflation even if
it deviates from the expected inflation rate:

\[
\frac{1 + i_t}{1 + \pi_t} b_{t-1} = (1 + r_t)b_{t-1}.
\]  \( (34) \)

The real value of the public debt thus cannot be changed by surprise inflation anymore.

2. The nominal end of period \( t \) value of all private bonds held by the public sector at the end of period \( t - 1 \) has to be equal to the stock of nominal balances outstanding at the end of period \( t - 1 \):

\[
M_{t-1} = (1 + i_t)C_{t-1}.
\]  \( (35) \)

Because (35) is equivalent to

\[
m_t(\pi^e_t) = (1 + r_t)c_{t-1},
\]  \( (36) \)

the real value of the net liabilities at the end of period \( t \) of the public sector vis a vis the private sector is zero in this case.\(^{44}\)

A zero nominal position of the public sector at the end of each period can be achieved by implementing a share of \( 1/(1 + i_{t+1}) \times 100\% \) of all changes of the money supply through open market operations in non-indexed private bonds. Consequently, a share of \( i_{t+1}/(1 + i_{t+1}) \times 100\% \) of all base money changes must be realized through open market operations in domestic government bonds. For such a policy, the expected real seigniorage revenues at the end of period \( t \) become [compare equation (14)]:

\[
s_t^e(\pi^e_{i+1}) = \frac{i_{t+1}}{1 + r_{t+1}} m_{t+1}(\pi^e_{t+1}) = \frac{r_{t+1} + \pi^e_{t+1}(1 + r_{t+1})}{1 + r_{t+1}} m_{t+1}(\pi^e_{t+1}).
\]  \( (37) \)

As we have discussed earlier [compare (20)], the implied base money changes lead to the same intertemporal seigniorage revenues as the implementation of base money changes solely through open market operations.

\(^{44}\)Note that this interpretation of the Persson, Persson, and Svensson idea differs from the suggestion in their (1987) paper in which they claim that at the end of any period nominal balances and private assets held by the central bank ought to be equalized.
in government bonds. The public sector thus does not lose intertemporally when it implements monetary policy as described above. It is at this point natural to ask how a central bank that in the past has always transferred base money changes directly to the government and thus only holds government bonds in its portfolio can meet the requirements of the (modified) Persson, Persson, Svensson solution. In our simple model with Ricardian equivalence, in which the issue of government bonds has no real effects, this is straightforward: The central bank simply has to sell \( \frac{1}{1 + it} \times 100\% \) of its portfolio of government bonds to the private sector. In turn, it ought to use the proceeds to buy nominal private assets. We conclude that it is in principle possible for a central bank to change its portfolio composition so as to meet the requirements just outlined, although this may cause real effects if Ricardian equivalence does not hold.\(^{45}\)

We now formally prove the claims made above. If the net nominal liabilities of the public sector are always equal to zero, the revenues from surprise inflation are also zero independently of the choice of \( \pi_t^e \). The discrete Hamiltonian then simplifies to:

\[
\mathcal{H}_t(d_{t-1}, \tau_t, \pi_{t+1}, \pi_t^e, \lambda_t) = \left[ l_{t1}(\tau_t) + l_{t2}(\pi_{t+1}) + l_{t3}(\pi_t^e) \right] + \lambda_t \left[ g_t + (1 + \tau_t)d_{t-1} - \tau_t y_t - s_t^e(\pi_{t+1}) \right], \tag{38}
\]

where:

\[
s_t^e(\pi_{t+1}) = \frac{\pi_{t+1}}{1 + \tau_{t+1}} m_{t+1}(\pi_{t+1}).
\]

The first order conditions now are the law of motion for \( d_t \), (23), and:

\[
\mathcal{H}_{\tau_t} = \frac{\partial l_t(\tau_t)}{\partial \tau_t} - \lambda_t y_t = 0, \tag{39.a}
\]

\(^{45}\)Note that Klein and Neumann (1990) have implicitly shown that the policy of the German Bundesbank is broadly in line with the suggestion of Persson, Persson, and Svensson. In particular, between 1961 and 1974, the Bundesbank transferred only 13 percent of the revenues from money creation to the government while this value amounted to 73 percent between 1974 and 1987. Following Klein and Neumann, this phenomenon can be explained by the fact that the Bundesbank had built up a portfolio of net public sector wealth during the first period which then started to mature. At least part of the incentive to unexpectedly inflate was eliminated by these operations.
\[ \mathcal{H}_{\pi_{t+1}^e} = \frac{\partial l_2(i_{t+1})}{\partial \pi_{t+1}^e} - \lambda_t \frac{\partial s_t^e}{\partial \pi_{t+1}^e} = 0, \quad (39.\text{b}) \]
\[ \mathcal{H}_{\pi_t^s} = \frac{\partial l_3(\pi_t^s)}{\partial \pi_t^s} = 0, \quad (39.\text{c}) \]
\[ \lambda_{t-1} = \frac{1}{1 + \theta_t} H_{d_{t-1}} = \frac{1 + r_t}{1 + \theta_t} \lambda_t, \quad t = 1, 2, \ldots \quad (39.\text{d}) \]

with [by equation (37)]:
\[ \frac{\partial s_t^e}{\partial \pi_{t+1}^e} = m_{t+1}(\pi_{t+1}^e) + \frac{i_{t+1}}{1 + r_{t+1}} r_{t+1}'(\pi_{t+1}^e). \]

Finally, the standard transversality condition has to be met:
\[ \lim_{t \to \infty} R_t \lambda_t d_t = 0. \]

By recursive substitution of (39.d) into the transversality condition, we can equivalently express the transversality condition as:
\[ \lim_{t \to \infty} \Theta_t d_t = 0. \quad (39.\text{e}) \]

Condition (39.e) requires that the present value of the net public debt (calculated with the authority’s discount factor) converge to zero as time tends to infinity. If the public discount factor equals the real interest rate this is equivalent to the well known solvency or no Ponzi-game condition, namely that the growth rate of the net public debt be smaller than the real rate of interest.\(^{46}\) The first order condition (39.c) implies that in an optimum it is necessary to set \(\pi_t^s\) equal to zero. If a solution to the optimal tax problem exists, it therefore must be time consistent.\(^{47}\) In addition, from (39.a) and (39.b) it follows that in any period there is a unique pair \((\tau_t, \pi_{t+1}^e)\) that satisfies the first condition, implying that there is at most one solution to the optimal tax problem. In order to show that the first order conditions actually determine a minimum of the social losses, we need to check the second order conditions too. Unfortunately they do

\(^{46}\)For an empirical test that accepts this condition for the US compare Hamilton and Flavin (1986).

\(^{47}\)Note that we would not have reached this conclusion if we had neglected the social losses from surprise inflation. Compare the discussion on page (14) when we specify the social losses.
not hold in general even in our rather simple set-up. However, the second order conditions are met for any sequence of the revenue instruments that fulfills the first order conditions, if the expected seigniorage revenues exhibit a Laffer curve as depicted in figure 1.48

![Figure 1: Seigniorage Laffer Curve](image)

The proof of this claim is delegated to appendix B. Since the first order conditions are satisfied by exactly one sequence of tax rates and expected inflation rates, we have determined a unique solution to the optimal tax problem. It should be stressed that modeling the social losses from surprise inflation appropriately is crucial to ensure the validity of the second order conditions too.

48See for example figure 2 in Bailey (1956), Bruno and Fischer (1990), or Dornbusch and Fischer (1993) for a similar assumption. Note that we could be more general here. It would suffice to assume that the seigniorage revenues are concave whenever the first order conditions are fulfilled. Because the first order conditions imply that a minimum can only lie on the left side of the seigniorage Laffer curve, this could be guaranteed for example by the assumption that the seigniorage revenues have at most one maximum and are concave on the left side of it. Note that we are a little sloppy when we say that the optimum lies on the left side of the Laffer. As shall be seen later, the revenue maximizing inflation rate could also be optimal. This is thus always meant to be included when inflation rates are said to be on the left side of the Laffer curve.
5 An Interpretation of the Optimal Time Consistent Policy

In this section, we want to investigate the properties of the optimal solution [in the subsections 6.1 and 6.2]. In addition, we shall discuss which of our results may also be found in general equilibrium models of optimal taxation in subsection 6.3.

5.1 Optimal Inflation and the Ramsey Principle

Eliminating the co–state variable in (39.a) and (39.b), we find that the optimal tax scheme apart from zero surprise inflation is characterized by the following necessary conditions:

\[
\frac{\partial t_2}{\partial \pi_{t+1}^e} = \frac{\partial t_1}{y_t}, \tag{40.a}
\]
\[
\frac{\partial t_1}{\partial \pi_{t+1}^e} = \frac{1 + r_{t+1}}{1 + \theta_{t+1}} \frac{\partial t_1}{y_{t+1}}, \tag{40.b}
\]
\[
\frac{\partial t_2}{\partial \pi_{t+1}^e} = \frac{1 + r_{t+1}}{1 + \theta_{t+1}} \frac{\partial t_2}{\partial \pi_{t+2}^e}. \tag{40.c}
\]

Equation (40.a) is the static first order condition which requires the ratios of the marginal social losses to marginal taxation revenues to be equalized for both instruments in any period. The two following equations (40.b) and (40.c) are the intertemporal first–order conditions by which the discounted ratios of the marginal social losses and the marginal taxation revenues of either instrument must also be equalized between periods in an optimum, implying that taxes are smoothed over time. Optimality hence requires that the higher the marginal social loss of an instrument relative to its marginal revenue, the less revenue should be collected from that instrument. This result directly corresponds to Ramsey’s principle of optimal taxation, Ramsey (1927).\(^{49}\) Taking account of the public budget constraint, we finally observe that optimal output taxes and inflation

\(^{49}\)Note that if the economy is in a steady state, in which the marginal losses and revenues from either instrument grow with the same rate, and if the public discount
rates increase when the present value of the expenditure to be financed
increases.

These considerations imply that high inflation rates may be optimal for
essentially three reasons: First, tax enforcing and collecting authorities
work inefficiently or tax evasion is significant. This is captured in our
model by large marginal collection costs. Second, the interest elasticity
of real balances is small so that high expected inflation rates lead to small
reductions of real balances and hence small marginal social losses from
expected inflation.\textsuperscript{50} And third, desired real balances are large for any
given inflation rate, offering a large tax base for inflation. This would be
the case if the financial sector is relatively underdeveloped and the bulk
of transactions is thus facilitated in cash. Note that for these reasons,
an active black market or shadow economy tends to increase the optimal
inflation rate, because illegal transactions are mostly facilitated in cash
in order to avoid taxes.\textsuperscript{51}

As an additional comparative static exercise, we now want to briefly
discuss the effects financial innovations have on the optimal inflation rates
and on social welfare. This question for instance is important when the
welfare effects of the introduction of new financial technologies in Eastern
Europe are to be judged. For each given rate of expected inflation, a
financial innovation presumably reduces the desired holdings of real bal-
ances, leading to lower marginal social costs \textit{and} lower marginal revenues
from using expected inflation. The net effect on the optimal rate of in-
flation is thus ambiguous: If the marginal social costs decrease (increase)
by more than the marginal revenue, higher (lower) optimal inflation rates
rate is equal to the real interest rate, the first order conditions even call for constant
output taxes and constant inflation rates. This form of the tax smoothing result is
often celebrated in the literature. Temporary changes in expenditure are financed by
bond issues which do not have real effects in a Neo–Ricardian world. Opposed to this,
if the public discount rate is not equal to the real interest rate, tax rates will grow or
fall over time at a constant rate.
\textsuperscript{50}This corresponds to the intuition that more inelastic tax bases ought to be more
heavily taxed.
\textsuperscript{51}In the US, for example, currency is supplied at almost 1000$ per inhabitant (in-
cluding children!).
would result. Similarly, it is not at all clear how a financial innovation affects social welfare. Ceteris paribus, it of course increases welfare. However, it has to increase the tax rate and the rate of inflation when agents hold less real balances, because seigniorage revenues will also decrease. These higher average tax rates lower social welfare. If this effect is large enough, it may even overcompensate the initial welfare increase, thereby resulting in a negative net effect on welfare. This is the more likely to happen the more important seigniorage is as a source of public revenue. If individuals in this case reduce their desired holdings of real balances sharply, for example because credit cards become an accepted means of payment or an efficient check clearing system is installed, large increases in inflation may be inevitable to meet the revenue requirements. Authorities confronted with these sort of problems thus ought to be advised to improve the efficiency of their tax system in order to have alternative revenue sources when the revenues from inflation decrease.

5.2 The Optimal Inflation Rate in Specific Cases

The different recommendations of the public finance literature on optimal inflation rates are now shown to be special cases of our time consistent solution:

1. The ratio of the marginal social costs to marginal revenues of output taxation is large relative to that of inflation:

It is optimal to rely mainly on seigniorage to finance public expenditure. Rearranging (40.a) shows that for very large marginal losses from output taxes it is moreover optimal to choose the seigniorage maximizing inflation rate, for which the first derivative of expected seigniorage becomes zero:

\[
\frac{\partial s_t^e}{\partial \pi_{t+1}^e} = y_t \frac{\partial l_{t+1}}{\partial \pi_{t+1}^e} \approx 0 \quad \text{if} \quad \frac{\partial l_{t1}}{\partial \tau_{t1}} \approx \infty.
\]  

\[ (41) \]

\[ ^{52}\text{Compare de Gregorio (1991) for a more detailed discussion.} \]

\[ ^{53}\text{In a different context, Fischer and Summers (1989) have discussed a similar phenomenon.} \]

\[ ^{54}\text{Cagan (1956) introduced the notion of a revenue maximizing rate of inflation into monetary economics.} \]
Using (37), the seigniorage maximizing inflation rate can equivalently be characterized by a version of the well known unit elasticity rule:

\[ \frac{i_{t+1}}{m_t(y_{t+1}, i_{t+1})} \frac{\partial m_t(y_{t+1}, i_{t+1})}{\partial i_{t+1}} = 1. \quad (42) \]

It requires that a one percent increase of the nominal interest rate after a marginal increase of the expected inflation rate leads to a one percent decrease of real balances. This is the case when seigniorage is maximized.\(^{56}\)

2. The ratio of the marginal social costs to marginal revenues of output taxation is small relative to that of seigniorage:

In this case, the optimal policy implies that a major share of the revenues is collected through output taxes and only a small part through seigniorage. Moreover, if in the optimal policy the product of the marginal social losses from output taxes and the marginal revenue from seigniorage is negligible, then the marginal social losses from inflation are set equal to zero:

\[ \frac{\partial l_{t2}}{\pi_{t+1}} = \frac{(\partial l_{t1}/\partial \tau_t)(\partial s_t^e/\partial \pi_{t+1}^e)}{y_t} \approx 0 \quad \text{if} \quad \frac{\partial l_{t1}}{\partial \tau_t} \frac{\partial s_t^e}{\partial \pi_{t+1}^e} \approx 0. \quad (43) \]

In other words, it is optimal in this case requires to drive the opportunity costs of holding money, i.e. the nominal interest rate, to zero so as to guarantee that individuals hold the satiation level of real balances:

\[ i_{t+1} = r_{t+1} + (1 + r_{t+1})\pi_{t+1}^e = 0 \implies \pi_{t+1}^e = -\frac{r_{t+1}}{1 + r_{t+1}} \approx -r_{t+1}. \quad (44) \]

The optimal expected inflation rate thus ought to be (approximately) equal to the negative real interest rate. This is the Friedman (1969) rule of optimal deflation which requires a contraction of the money supply in

\(^{55}\)Not that this is not the standard form of the unit elasticity rule since the money demand is usually modeled to depend on the expected inflation rate rather than the on nominal interest rate.

any period and leads to negative seigniorage revenues, which are financed through output taxation.\textsuperscript{57} Intuitively, it comes about, because there is now a loss free revenue available through which all expenditure can be financed. Since money is produced without costs, it should thus be supplied until satiation is reached.\textsuperscript{58}

3. The ratios of the marginal collection costs to marginal revenues for both instruments are of the same order of magnitude:

Expenditures are then optimally financed from the revenues accruing from both instruments. As mentioned above, the optimal use of either taxation instrument is determined by the relative ratios of the marginal tax revenues to the marginal social losses. In particular, positive inflation rates are part of the optimal policy.

5.3 Some Generalizations to Equilibrium Models of Optimal Taxation

Having studied the optimal inflation tax above, the question remains as to whether the results given are a consequence of the simplicity of our model. Needless to say that the use of a general equilibrium framework would certainly be more satisfactory. In a more complete analysis, our tax on exogenous output ought to be substituted by a distortionary labor or consumption tax. However, one would expect that the introduction of additional social costs from output taxes would strengthen the case for a positive optimal inflation rate. This appears at least to be true when collecting the tax involves positive costs.\textsuperscript{59} The results of our cases 1. and 3. thus generalize to general equilibrium models.

Following this intuition would also lead to the conclusion that it no longer remains optimal to follow the Friedman (1969) rule in case 2. if

\textsuperscript{57}The Friedman rule has sometimes also been called the optimum quantity of money rule or the Chicago rule.

\textsuperscript{58}This parallels the usual argument that in a welfare optimum, whatever is demanded be supplied of goods that are costless to produce.

\textsuperscript{59}See Aizenman (1987) or Végh (1989a).
a consumption or labor income tax distorts the endogenous labor/leisure choice, thereby causing social losses irrespectively of the magnitude of collection costs. Phelps (1973) and essentially all authors who introduce money into general equilibrium by modeling it as an argument of the utility function have supported this view and reached the conclusion that positive inflation rates are even optimal in the absence of significant collection costs. On the other hand, the money-in-the-utility-function approach has often been criticized for it does not appear to be convincing that individuals derive utility just from holding money. As an alternative, Drazen (1979) and Leach (1983) have suggested modeling money as an intermediate input good of a transaction technology, yielding transactions services and thereby decreasing the time necessary for unproductive shopping activities. Following this idea, Guidotti and Végh (1993) have shown that the optimal inflation rate in general is still positive. On the other hand, as usual, there is an exception from the rule: if labor is taxed in a general equilibrium model and the transactions technology exhibits constant returns to scale, a zero inflation tax remains the optimal choice even under distortionary income taxation. This special case is the one, Kimbrough (1986a) and (1986b), exclusively dealt with. At times, many authors have thus concluded that the Friedman rule also holds in a general equilibrium framework when collection costs are negligible. As Guidotti and Végh (1993) have argued convincingly though, standard money demand theories do not yield to a transactions technology that satisfies the properties necessary to replicate Friedman’s result in a general equilibrium framework. In general, a positive optimal inflation rate may thus also be optimal if the social costs of output taxation result from its distortions rather than from positive collection costs as in case 3. above. However, it should be stressed that the results of Guidotti and Végh (1993) should not be viewed as the “final wisdom”. On p. 203, they themselves state: “On the other hand, given that the microfoundations of money demand still remain a controversial and, to a large

61 See, for example, Lucas (1986) or Phelps (1989).
63 Most prominently, Lucas (1986) has taken this view.
extent, unexplored territory, we do not wish to conclude that the inflation tax is likely to be always positive. Rather, we feel that until the microfoundations of the transactions technology are further explored and the profession reaches something of an consensus, no definite conclusions should be drawn."

6 Conclusion

In this paper, a simple version of the intertemporal optimal tax problem of the public sector was discussed, in which a given stream of expenditure needs to be financed through output taxation and seigniorage so as to minimize the social losses both instruments induce. It was shown how the time consistency problem of the optimal solution to this problem can be cured by the application of a modified and simplified version of the Persson et al. (1987) asset management scheme. Following this policy turned out to be not only necessary but also sufficient for the existence of a unique time consistent optimal policy, although this is not true in more general models.64

Apart from this, the different recommendations of the public finance literature on optimal inflation were found as specific cases of the general time consistent policy when public assets and debt are appropriately managed. The assumption of precommitment usually made was not necessary to arrive at this result. Furthermore, it was shown that when taxing output involves collection costs, the optimal taxation package contains both, output and inflation taxation. This replicates the classical result of Phelps (1973), who, however, was not concerned with time consistency problems. Only in extreme cases, in which the ratio of the marginal loss of one instrument to its marginal revenue clearly dominates this of the other, it does turn out to be optimal to rely almost exclusively on the revenue source with the lower marginal loss to revenue ratio.

What are the policy conclusions that can be drawn from these re-

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64Calvo and Obstfeld (1990) have shown that Persson, Persson, and Svensson's version in general leads to saddle points and not to welfare maxima.
sults? First, it is important to stress that positive inflation rates of "moderate" magnitude are not necessarily an undesirable outcome of an inflationary bias, which would arise in a discretionary equilibrium. As has been argued above, moderate positive inflation rates may well arise in the time consistent second best equilibrium as the optimal outcome when the output tax system is inefficient. A large underground economy, for instance, strengthens the case for positive inflation, because inflation may be the only possibility to indirectly tax the shadow economy. Programs with the goal to reduce inflation ought to take account of this fact in that they put emphasis on the improvement of the output and excise taxation system. This is likely to be an important issue in most of the states in Eastern Europe. 65

Second, Persson et al. (1987)'s suggestions offer two possibilities of how the authority may decrease its incentive to resort to inflation surprises. On the one hand, it may restrict direct and indirect borrowing of the government from the central bank as much as possible. This would allow the central bank to build up a portfolio of nominal private assets, which, at least, partly could help to reduce the net nominal liabilities of the public sector. 66 On the other hand, the authorities ought to be encouraged to issue indexed bonds. A casual reflection might lead to the conclusion that this instrument has hardly been used in the OECD countries. 67 However, many countries have actually indexed a quite substantial share of their public sector debt indirectly by denominating it

65Dornbusch and Fischer (1993) report evidence that successful stabilizations from high to moderate inflation rates have often been accompanied by drastic improvements of the efficiency of the tax system. On the other side, Cukierman, Edwards and Tabellini (1992) have analyzed why the authorities may not be eager to devote resources to a reform of the tax system. They argue that it can be optimal for the authority in a politically unstable and polarized economy not to reform the tax system. The reason for this is that the gains of such a reform are likely to be inherited by its successor public sector, which probably uses them for purposes not appreciated by the current public sector. The casual empirical observation that politically unstable countries often are also the ones with high inflation rates seems to suggest support of this view.

66Note that the German Bundesbank is not allowed to directly lend to the government and that open market purchases of domestic government bonds are only possible under very restrictive conditions.

67Israel and England are exceptions.
in foreign currencies. This is similar to an ex post indexation to inflation since the standard open economy macro-models would predict that foreign currency debt cannot be devalued by surprise inflation. Inflation surprises are rather likely to create a real exchange rate depreciation as a consequence of an overshooting reaction of the nominal exchange rate. This would increase the real value of foreign currency public debt as perceived from the domestic country's point of view and even reduce the incentive to resort to unexpected inflation. Further research in this direction seems promising.

At the end of this review the limited scope of the public finance approach to determine optimal inflation rates should be stressed. Of course, there are other important determinants of monetary policy and it may be already too much of an abstraction to take the inflation rate as the policy instrument that can be perfectly controlled. In addition, the framework used here is extremely monetaristic, that is, prices are perfectly flexible and output is not all affected by inflation, even if inflation is completely unexpected. In my opinion, rather than depriving the public finance approach to the determination of optimal inflation "as one of the most overstudied areas in monetary theory" [Summers (1991)], one should therefore interpret the results with some care. After all, economic models are not supposed to tell the truth but to organize thinking. As an example, even if the Friedman rule turns out to be the optimal choice in public finance models, objectives of monetary policy not modeled there, for instance stabilization goals or exchange rate targets, may well prevent its implementation in practice. On the other hand, public finance models of optimal inflation can well explain why it may not even be desirable to reduce inflation in countries which badly developed taxation systems before the institutions have not been reformed. As I have argued, this will have important implications for the design of an optimal policy.

\[68\text{See Dornbusch (1989) or Summers (1991).}\]
Appendix A

This appendix comprises the technical details of the derivation of (13) and (14). First, observe that

\[ P_t^e = (1 + \pi_t^e) P_{t-1}, \quad (A.1) \]

\[ P_t = (1 + \pi_t) P_{t-1} = \frac{1 + \pi_t}{1 + \pi_t^e} P_t^e. \quad (A.2) \]

In addition, remember that we have denoted nominal variables by upper case and real variables by lower case letters; for example \( c_t = C_t/P_t \).

Using these identities and (7), (8), equation (12) can be rewritten:

\[ x_t = \frac{\Delta M_t + i_t (B_{t-1}^c + C_{t-1}) - \Delta (B_{t}^c + C_t) - O_t}{P_t} \]

\[ = (1 + \pi_{t+1}^e) \frac{M_t}{P_{t+1}} - \frac{1 + \pi_t^e}{1 + \pi_t} \frac{M_{t-1}}{P_t} + \frac{1 + \pi_t^e}{1 + \pi_t} (1 + r_t) (b_{t-1}^c + c_{t-1}) \]

\[ - b_t^c - c_t - o_t \]

\[ = (1 + \pi_{t+1}^e) m_{t+1}(\pi_t^e) - \frac{1 + \pi_t^e}{1 + \pi_t} \left[ m_t(\pi_t^e) - (1 + r_t)(b_{t-1}^c + c_{t-1}) \right] \]

\[ - b_t^c - c_t - o_t. \quad (A.3) \]

Then, we put (A.3) into (3) and rearrange:

\[ g_t + \frac{i_t}{1 + \pi_t} b_{t-1}^{tot} = \tau_t y_t + (b_{t-1}^{tot} - b_t^c) + \left[ (1 + \pi_t^e) m_{t+1}(\pi_t^e) - c_t \right] \]

\[ - \frac{1 + \pi_t^e}{1 + \pi_t} \left[ m_t(\pi_t^e) - (1 + r_t)(b_{t-1}^c + c_{t-1}) \right] - o_t. \quad (A.4) \]

If we now bring \( b_{t-1}^c \) on the left side and use \( b_{t-1} = b_{t-1}^{tot} - b_{t-1}^c \), we get:

\[ g_t + \frac{1 + \pi_t^e}{1 + \pi_t} (1 + r_t) b_{t-1} = \tau_t y_t + b_t + \left[ (1 + \pi_t^e) m_{t+1}(\pi_t^e) - c_t \right] \]

\[ - \frac{1 + \pi_t^e}{1 + \pi_t} \left[ m_t(\pi_t^e) - (1 + r_t)c_{t-1} \right] - o_t. \quad (A.5) \]

Finally, adding \( (1 + r_t) b_{t-1} \) on both sides yields (13) and (14):
\[ g_t + (1 + r_t)b_{t-1} = \tau_t y_t + b_t + \left[ (1 + \pi_t^e)m_{t+1}(\pi_{t+1}^e) - c_t \right] - \frac{1 + \pi_t^e}{1 + \pi_t} \left[ m_t(\pi_t^e) - (1 + r_t)c_{t-1} \right] + \frac{\pi_t - \pi_t^e}{1 + \pi_t} (1 + r_t)b_{t-1} - \alpha_t \]
\[ = \tau_t y_t + b_t + s_t. \]  

\textbf{Appendix B}

We now prove that, under the assumption of a seigniorage Laffer curve, any set of the policy instruments that satisfies the first order conditions also fulfills the second order conditions for the existence of minimum of the social loss function.

Remembering that the social losses were assumed to be convex in the policy instruments and that \( \lambda_t \) must be positive in an optimum, we can sign most of the second derivatives of the Hamiltonian (38) without further assumptions:

\[ H_{\tau_t^2} = \frac{\partial^2 l_t}{\partial \tau_t^2} > 0, \quad H_{\tau_t \lambda_t} = -y_t < 0, \]
\[ H_{\pi_t^{e,2}} = \frac{\partial^2 l_t}{\partial \pi_t^{e,2}} + \lambda_t \frac{\partial^2 s_t^e}{\partial \pi_t^{e,2}} \leq 0, \quad H_{\pi_t^{e, \lambda_t}} = - \frac{\partial s_t^e}{\partial \pi_t^{e,1}} < 0, \]
\[ H_{\pi_t^{e,2}} = \frac{\partial^2 l_t}{\partial \pi_t^{e,2}} > 0, \quad H_{\pi_t^{e, \lambda_t}} = 0, \]
\[ H_{\tau_t \pi_t^{e,1}} = H_{\tau_t \pi_t^e} = H_{\pi_t^{e,1} \pi_t^e} = 0. \]  

That \( H_{\pi_t^{e,1} \lambda_t} \) is positive follows directly from (39.b), because for an optimum the expected inflation rate has to lie on the left side of the seigniorage Laffer curve. The sign of \( H_{\pi_t^{e,2}} \) causes the only problem:

\[ \frac{\partial^2 s_t^e}{\partial \pi_t^{e,2}} = m_t^{t+1}(\pi_t^{e,1}) + \frac{i_{t+1}}{1 + r_t} m_t''(\pi_{t+1}^e) \leq 0. \]  

However, when the expected seigniorage revenues exhibit a Laffer curve, this expression is negative, again because the expected inflation rate is
not larger than the seigniorage maximizing inflation rate. Under the Laffer curve assumption, we therefore find that, whenever the first order conditions are fulfilled, the bordered Hessian satisfies the second order conditions for the existence of a minimum:\footnote{For a mathematical discussion of these sufficient conditions compare for example Kamien and Schwartz (1991), p. 311.}

\[
\begin{bmatrix}
H_{\tau_t,\tau_t} & H_{\tau_t,\pi_{t+1}^{e}} & H_{\tau_t,\lambda_t} \\
H_{\pi_{t+1}^{e},\tau_t} & H_{\pi_{t+1}^{e},\pi_{t+1}^{e}} & H_{\pi_{t+1}^{e},\lambda_t} \\
H_{\lambda_t,\tau_t} & H_{\lambda_t,\pi_{t}^{e}} & 0
\end{bmatrix}
= \begin{bmatrix}
+ & 0 & - \\
0 & + & - \\
- & - & 0
\end{bmatrix} < 0, \quad (B.3)
\]

and

\[
\begin{bmatrix}
H_{\tau_t,\tau_t} & H_{\tau_t,\pi_{t+1}^{e}} & H_{\tau_t,\pi_{t}^{e}} & H_{\tau_t,\lambda_t} \\
H_{\pi_{t+1}^{e},\tau_t} & H_{\pi_{t+1}^{e},\pi_{t+1}^{e}} & H_{\pi_{t+1}^{e},\pi_{t}^{e}} & H_{\pi_{t+1}^{e},\lambda_t} \\
H_{\pi_{t}^{e},\tau_t} & H_{\pi_{t}^{e},\pi_{t+1}^{e}} & H_{\pi_{t}^{e},\pi_{t}^{e}} & H_{\pi_{t}^{e},\lambda_t} \\
H_{\lambda_t,\tau_t} & H_{\lambda_t,\pi_{t+1}^{e}} & H_{\lambda_t,\pi_{t}^{e}} & 0
\end{bmatrix}
= \begin{bmatrix}
+ & 0 & 0 & - \\
0 & + & 0 & - \\
0 & 0 & + & 0 \\
- & - & 0 & 0
\end{bmatrix} < 0. \quad (B.4)
\]

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