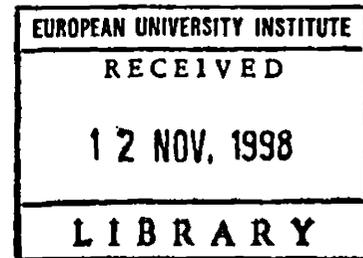


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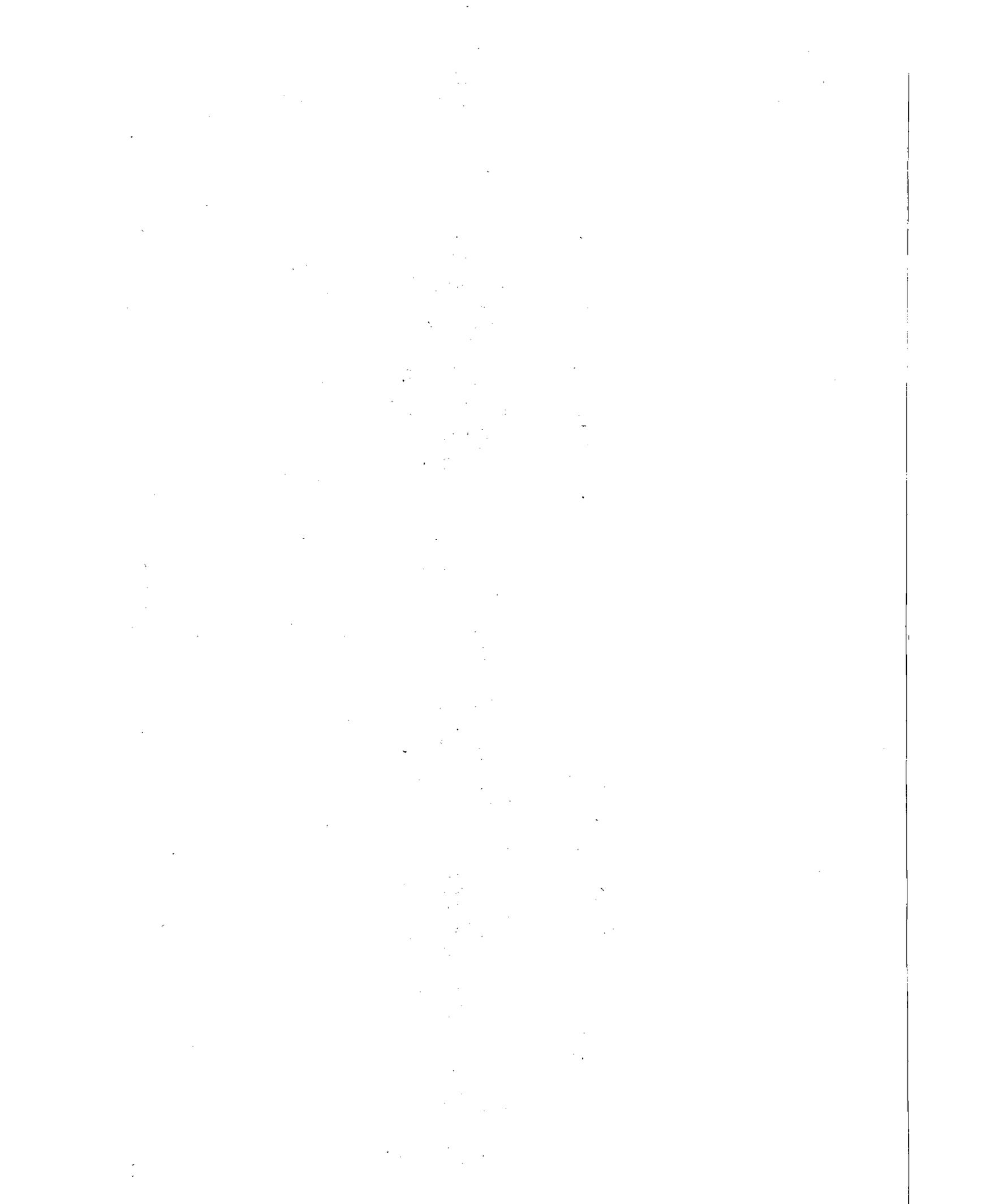
**Essays on Distributive Policies
and Economic Growth**

by
Güther Rehme

*Thesis submitted for assessment with a view to obtaining
the degree of Doctor of the European University Institute*

Florence, November 1998







Rosso

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Essays on Distributive Policies and Economic Growth

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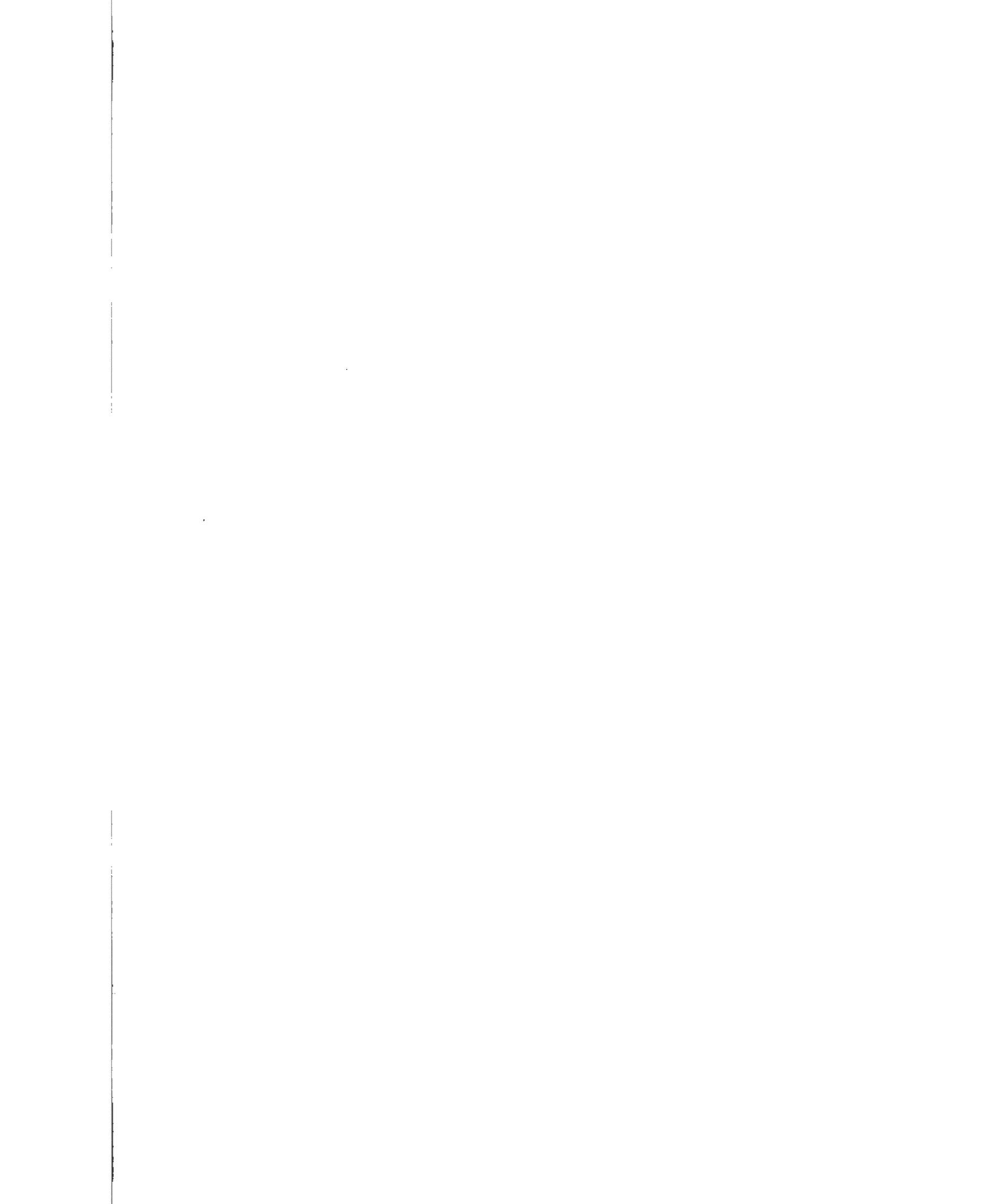
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To my beloved parents



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Introduction

The interest of economists in the determinants of economic growth is almost as old as the subject itself. But the systematic study of modelling growth may be traced back to two classic approaches which also formed the basis for a distinction that has remained valid until today. Optimal growth models as formulated in Ramsey (1928) address normative issues in long-term development. Positive growth models as in Harrod (1939) or Domar (1946) attempt to explain observable facts in the development process.

The Harrod-Domar model suffers from the well known 'knife edge' problem, according to which it is unlikely that the 'warranted rate of growth' and the natural rate of growth coincide. Solow (1956) showed that allowing for substitution between factors of production removes that problem and leads to the existence of a stable, steady state growth rate. In all of these theoretical explanations of long-term growth the driving force of economic development such as technical progress has been viewed as outside the realm of economic explanations. That is one reason why it has become common to refer to models of this kind as *exogenous* growth models. For a good introduction into that literature see, for instance, Burmeister and Dobell (1970), Intriligator (1971) or Takayama (1985).

However, for almost a decade now there has been interest in explaining economic growth *endogenously*, that is, by factors that are governed by economic

'laws'. The seminal contributions in the field that spurred this interest were made by Arrow (1962), Uzawa (1965), Romer (1986), Lucas (1988), and Romer (1990). One of the crucial differences compared to a typical Solow model is that one usually assumes the existence of some form of positive externality in the production function. For instance, Romer (1986) employs the concept of broadly defined capital as a factor producing positive spillovers. The important implication of such models is that one gets an inefficiency, because private agents are not able to take account of the spillover effects when maximizing their objective functions. Often the social optimum implies a constant return on capital in those models, leading to positive per capita growth. Other forms of endogenous growth models such as Romer (1987) or Romer (1990) identify knowledge as an externality creating factor. It is beyond the scope of the dissertation to review the endogenous growth literature. The interested reader may wish to consult Grossman and Helpman (1991), Barro and Sala-i-Martin (1995) or Aghion and Howitt (1998) who provide excellent introductions into and presentations of endogenous growth models.

At the same time this literature opened new avenues to address an old topic, namely, the question of how distribution and growth interact over time. That question has been empirically investigated in the seminal article by Kuznets (1955) who found an inverted U-shaped relationship between income inequality and income growth in the course of development. However, the relationship has not been found to hold in general and there seems to be some evidence that the opposite relationship may hold. (See, for instance, Cline (1975), Fields (1987), Fields and Jakubson (1992), or Bourguignon and Morrisson (1992).)

Theoretically, there has been a certain tradition to argue that growth determines distribution. In a standard neo-classical, positive or optimal exogenous

growth model à-la Solow (1956), or Ramsey (1928) growth determines the distribution of factor rewards in a steady state. That line of reasoning is most clearly presented in Kaldor (1956) and Kaldor (1957).¹ In contrast, most endogenous growth models take a different, almost opposite route and argue that distribution determines growth as, for example, shown in Alesina and Rodrik (1994), Bertola (1993) or Persson and Tabellini (1994) to name only a few. It is then an interesting question how growth reacts to changes in distribution. For instance, the researchers mentioned show that taxation leading to an income distribution that is more favourable to the non-accumulated factor of production such as labour, causes lower growth.

If one takes the view that distribution determines growth, it appears plausible to introduce welfare judgments to the analysis. That adds a normative element to the problem in that one may view the link between the income or wealth distribution and economic growth as an intertemporal trade-off. In the dissertation the welfare judgments are represented by governments, that is, the governments represent agents with different preferences. That raises the problem how one can succinctly represent the preferences of many, possibly very heterogeneous agents. Throughout the thesis I rely heavily on the assumption of representative consumers and that markets are perfectly competitive for the private sector. The notion of a representative consumer may be misleading as has been shown by Kirman (1992). In defence of employing that notion in macroeconomic models it

¹It is possible to make distribution matter for growth in a neo-classical framework as has been shown in, for instance, Stiglitz (1969) or Blinder (1975). Suppose markets work perfectly and agents have different savings rates, e.g. due to non-linearities in the consumption function. A different income or wealth distribution may then affect the propensity to save and so investment, affecting growth. However, the rates of return of the production factors would still be determined by growth in a steady state, balanced growth equilibrium with perfect markets. In most endogenous growth models a positive externality is often introduced that fixes the rates of return, which in turn determine the growth rate. In that sense endogenous growth models take an opposite route, since distribution might matter even though all agents' consumption functions are linear in income or wealth.

has become standard to argue as follows: Firstly, under well-defined, but rather stringent conditions individual behaviour can be aggregated exactly, as shown by Deaton and Muellbauer (1980), chpt. 6. Secondly, in international contexts, the assumption of differences between (typical) residents across countries is a useful and simple device to downplay differences within countries. (See, for instance, Obstfeld and Rogoff (1996), chpt. 1.) Thirdly, in an intertemporal framework the macroeconomic relevance of preference heterogeneity would fade over time as relatively rich individuals become increasingly rich and 'representative'. (See Bertola (1997), fn. 7.) Fourthly, if the private sector acts price-takingly, preference heterogeneity does not affect the determination of rates of return and so - in many models - growth.

Heterogeneity is, however, present in the dissertation by analyzing the welfare of different groups in the economy. The governments are taken to represent the groups, sometimes in different proportions. The governments' optimal policies are shown to determine the income or wealth distribution and growth. That describes the common thread which runs through the dissertation and explains the title.

The thesis comprises three parts that contain theoretical models on the optimal choices of governments that determine distribution and growth by some instruments in different economic environments. Although the three parts are linked by the common theme mentioned, the dissertation is really made up of four articles which are in principle separate entities. As they revolve around the same topic, the problem of unnecessary repetitions arises. I have tried to keep them at a minimum, but I have also endeavoured to enable the reader to consult each chapter separately. That has the disadvantage of allowing for some repetitions on the one hand, but it has the advantage that one may read a chapter of

interest without too much recourse to another one. For the reasons mentioned I sometimes use the terms 'chapter' and 'paper' interchangeably.

Each part is self-contained so that no general conclusion or summary is necessary. In order to aid the reader each part contains appendices relevant for that part only. That avoids skimming through a lot of pages. I have attempted to minimize repetitions so that the reader may find proofs close to where they belong. I have also tried to keep appendices to a minimum. The only thing that applies 'globally' is the bibliography, which the reader will find at the end of the dissertation. The remainder of the introduction provides a brief overview of the contents of the chapters in each part.

Part I

In this part I investigate public policies and economic growth in a closed economy. I focus on two results that have been shown in the literature. The first one states that distortionary taxation that distributes too much income or wealth towards the non-accumulated factor of production (labour) slows down growth (Alesina and Rodrik (1994)) and the second one says that income taxation is not optimal for investment subsidy financing and so not growth promoting (Bertola (1993)). In chapter 1 I concentrate on the first point.

Chapter 1: Economic Growth, (Re-)Distributive Policies: A Comparative Dynamic Analysis

Chapter 1 consists of two parts that analyze the link between economic growth and (re-)distributive policies in closed economies. It also provides a model framework used and referred to in the following two chapters.

In the first part I build on Alesina and Rodrik (1991) and present a theoretical

model which hypothesizes that redistribution is bad for growth. The structure of the model is such that a 'right-wing' (entirely pro-capital) policy is growth maximizing. I conduct a comparative dynamic analysis, which allows me to use as much information as is provided by such a model. The analysis works with optimizing governments. All optimal policies represent examples of the fact that to some extent public policy is economically endogenous in that policy variables depend on fundamental economic variables that are given to an optimizer. I analyze various public policies and find - among other things - that other than 'right-wing' objectives may also lead to a growth maximizing policy. I show that an increase in technological efficiency generally raises growth, but it also raises the optimal steady state tax rates or lowers any optimal wealth redistribution. I show that this has interesting implications for the post-tax factor income distribution.

In the second part, I use the model's theoretical results to interpret some findings in the cross-country growth econometrics literature. Many authors take averages of their data over time and run simple cross-country OLS regressions over these averaged data. A number of others use pooled time-series cross-country data to pay explicit attention to the time series dimension. The chapter's analysis focuses on the large number of contributions which use the former approach. In particular, I use the model's theoretical results to sign correlations between policy variables employed in simple OLS cross-country growth regressions and some important, possibly unobservable fundamental economic variables. Signing the correlations allows me to provide explanations for some puzzling findings in that empirical literature. For example, I show that in many cases the point estimates of the effect of redistributive transfer variables on growth may be biased downwards, suggesting that the hypothesis 'redistribution is bad for growth' may be inherently untestable by simple OLS cross-country growth regressions. A

negative bias is, however, perfectly compatible with the signs found for that estimate in the literature and it is also compatible with the opposite hypothesis that redistribution is not bad for growth.

The model employs a capital tax scheme, which raises the question whether such a distortionary tax scheme really captures a necessary relationship between distributive taxation and economic growth in a world with optimizing governments. That problem is addressed in more depth in the next chapter.

Chapter 2: Economic Growth, Distributive Policies, and an Income cum Investment Subsidy Tax Scheme

In chapter 2 I employ a capital income cum investment subsidy tax to investigate if distribution towards the non-accumulated factor of production (labour) retards growth and if income taxes are bad instruments to finance subsidies. I identify conditions under which the tax scheme is better for growth than other distorting tax schemes. I show that a 'left-wing' (pro-labour) government acts growth maximizing and that distributing income towards labour may raise growth. A 'right-wing' (pro-capital) government's preferred policy is not growth maximizing under the tax scheme, but may generate higher growth than its optimal, growth maximizing policy under another tax scheme. For growth maximizing policies the tax scheme's post-tax factor income distribution is generally biased towards labour compared to other tax schemes.

Part II

I return to a wealth tax scheme in this part, but in contrast to chapters 1 and 2 I focus on the interaction of economic growth and public policies in open economies. The trade-off between distribution and growth is placed in a non-cooperative

setting which adds an interesting and empirically important dimension.

Chapter 3: Growth, Redistribution, Capital Mobility and Tax Competition in Open Economies

Chapter 3 investigates the effect of redistributive policies on growth in open economies. In closed economies redistribution is taken to reduce growth. I show that in open economies tax competition leads optimizing, redistributing ('left-wing') governments to mimic 'right-wing' policies if capital mobility is very high. In the model 'right-wing' governments are strategically passive and just maximize capital income and growth. For domestic left-wing governments it is shown that 'left-right' competition leads to more redistribution and lower GDP growth than 'left-left' competition. Efficiency differences allow for higher GDP growth *and* redistribution than one's opponent. Irrespective of efficiency differences 'left-wing' governments are shown to have higher GDP growth when competing with other 'left-wing' governments. Furthermore, I discuss some empirical implications of the model and compare them to those of chapter 1.

Part III

Part III presents a theoretical model that concentrates on the role, human capital plays in economic growth. The recent growth performance of some East Asian countries suggests that there is a positive link from the provision of public education to high economic growth and a rather equal income distribution. Part III complements the other chapters in that the focus is on labour rather than on capital. In the literature and in some OECD countries' policy debates human capital is considered an important element for improving or maintaining international competitiveness. The chapter contributes to these debates by arguing

that education policy plays an important role when coping with the effects of 'globalization'.

Chapter 4: Public Policies and Education, Economic Growth and Income Distribution

The chapter provides a theoretical analysis relating economic growth, human capital composition, income distribution and public education policies. In the model human capital is 'lumpy' and only high skilled people carry it. The government chooses capital taxes to finance education, which directly affects growth, the number of high skilled people and wages. Growth and equality in the present value of lifetime wage earnings depend positively on the productivity of the education sector. The preferred policy of the *unskilled* is shown to be growth maximizing, whereas that of *skilled* labour leads to less education, lower growth and more income inequality. A *utilitarian* government chooses more than the growth maximizing education, and less inequality than what the unskilled choose. The chapter's public policy analysis offers an explanation for the recent observation of high growth and relatively low income inequality in some, highly competitive economies.

Conventions and Frequently Used Symbols

Variables depending on time are indexed by subscript t . Thus, x_t means that variable x depends on time. Changes of x_t over time are written as \dot{x} so that $\dot{x} \equiv \frac{\partial x_t}{\partial t}$. For all variables other than t the partial derivative of a function y with respect to variable x is written as y_x or $\frac{\partial y}{\partial x}$, which are used interchangeably.

Mathematical Symbols

\Leftrightarrow	'is equivalent to'
\Rightarrow	'implies'
\wedge	logical 'and'
\vee	logical 'or'
$x \in [a, b]$	$a \leq x \leq b$
$x \in (a, b]$	$a < x \leq b$
$x \in [a, b)$	$a \leq x < b$
$x \in (a, b)$	$a < x < b$
$a \rightarrow b$	a approaches b
$a \rightarrow b_+$	a approaches b from the right
$E(\cdot)$	expectation operator
$cov(i, j)$	covariance between variables i and j

Other Symbols

* index for variables in foreign country

Upper Case Latin Letters

A	index of development
C^W	consumption of workers
C^*	consumption of capital owners
F	measure of total factor income inequality
G	public inputs to production
H	index of effective labour
K	productive capital stock
L	total labour supply
L_1	number of high skilled workers
L_0	number of low skilled workers
N	size of population
V	discounted utility stream
$W(\cdot)$	welfare function
Y	output

Lower Case Latin Letters

av_1	ratio of tax revenues to tax base
av_2	ratio of tax revenues to GDP
c_0	consumption of low skilled workers
c_1	consumption of high skilled workers
g	public inputs to production
h	human capital
k	capital stock
l	number of workers
m	marginal tax rate
n	number of capital owners
r	(pre-tax) return on capital
t_c	consumption tax rate
w	wage rate
w_1	wage rate for high skilled labour
w_0	wage rate for low skilled labour
x	percentage of high skilled people in population
y	output
z	measure of degree of capital mobility

Greek Letters

α	share of capital (income in total income), chpts. 1-3; share of effective labour, chpt. 4
β	social weight attached to the welfare of the non-accumulated factor of production, chpts. 1-3
γ	rate of growth
γ_j	rate of growth of variable j
ϵ	productivity of the education sector, chpt. 4; any small number, chpts. 1-3
ζ	social weight attached to the welfare of a representative high skilled worker
η	capital adjusted wage rate, $\frac{w}{k}$
θ	tax rate on capital income
λ	degree of resource redistribution
ω	share of domestically owned capital that is employed in domestic production
ϕ	share of domestically owned capital that is productive in foreign production
ρ	rate of time preference
τ	tax rate on wealth

Gothic Letters

b	ratio of redistributive transfers to GDP
p	ratio of government expenditure to GDP
q	ratio of government expenditure to total (private and public) investment

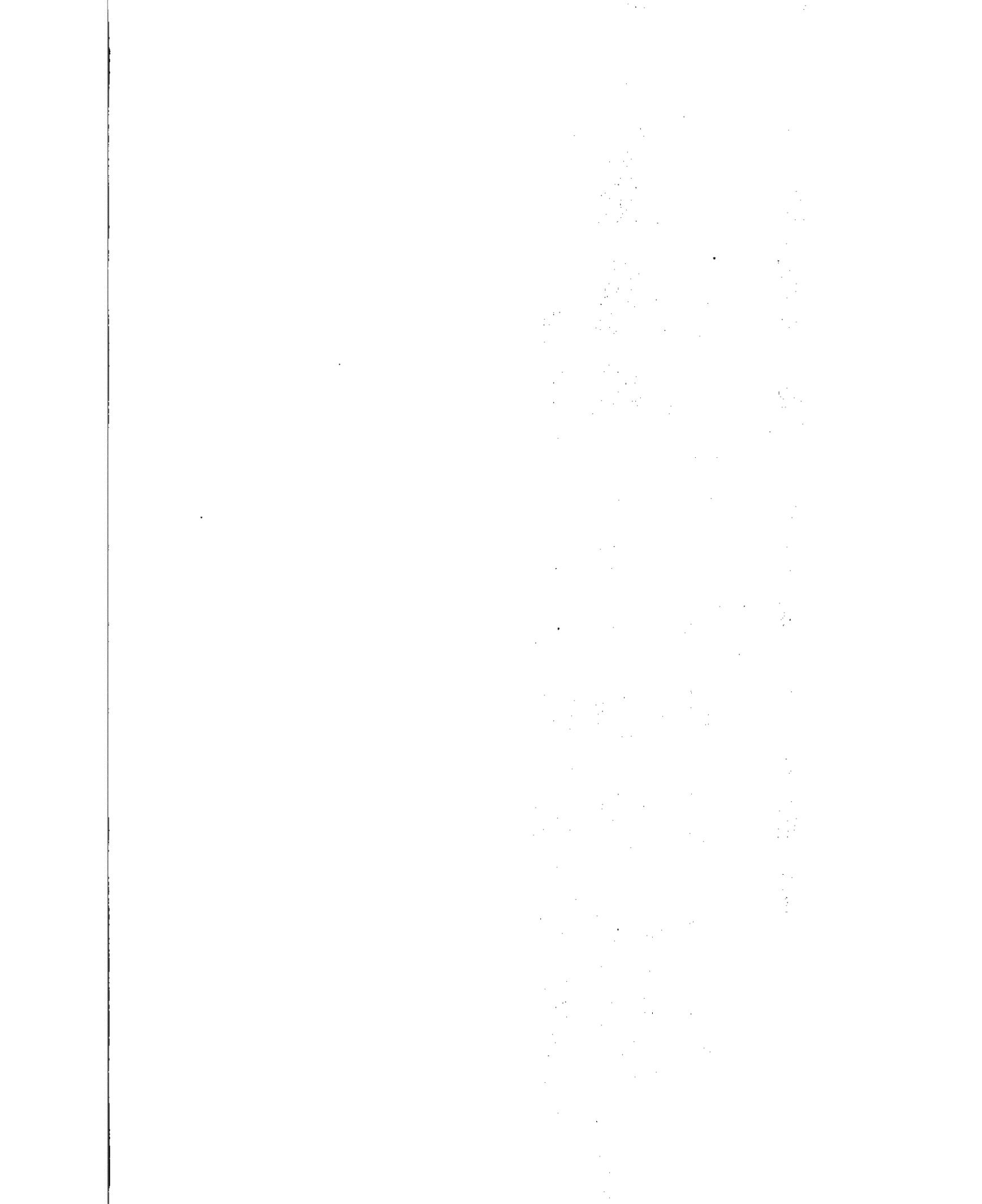
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Part I

**Economic Growth and
Distributive Policies in Closed
Economies**



Chapter 1

Economic Growth and (Re-)Distributive Policies: A Comparative Dynamic Analysis

1.1 Introduction

This chapter consists of two parts that investigate the link between economic growth and (re-)distributive policies in closed economies. In the first part (sections 1.2 to 1.5) I employ a common theoretical formulation on the interplay of public policy and economic growth, using as much information as is provided by a model. More specifically, I conduct a comparative dynamic analysis to study the effect of once-and-for-all changes in underlying fundamental economic variables on optimal steady state policies and growth. The focus in the first part is as much on theoretical as on normative predictions of the effects of distributive policies and policy changes on growth. The second part (section 1.6) uses the results of the comparative dynamic study to offer explanations of some, sometimes

puzzling empirical findings in the literature.

Part 1

Recent contributions on the interplay between economic growth and distributive policies in a closed economy such as Alesina and Rodrik (1994) and Bertola (1993) suggest that increasing taxes for redistributive purposes slows down growth. The reason why that conclusion is reached is given by the following line of argument: If a government taxes the income or wealth of the accumulated factor of production at a level that is higher than that which is optimal for growth, the government may redistribute resources towards the non-accumulated factor of production, but will exhibit lower steady state growth. The result crucially depends on the tax arrangement as is shown in the next chapter.

In this chapter, however, I use the structure of models in which the optimal policy of the accumulated factor maximizes growth. The model below follows Barro (1990) and Barro and Sala-i-Martin (1990) who study the impact of government spending on the private return on capital in growing economies using an endogenous growth set-up. Their results suggest that the government has room to influence the private return on capital and through that the growth rate.

Alesina and Rodrik have analyzed how a benevolent government in a closed economy with *wealth taxes* solves the problem posed by a trade-off between growth and redistribution. The chapter mainly builds on Alesina and Rodrik (1991). Although wealth taxes are only one particular policy instrument, I follow their argument that the wealth tax scheme is meant to capture a broad set of redistributive policies. In order to fix ideas and in line with most of the literature on capital taxation I abstract from taxation of the non-accumulated factor of production (labour). That facilitates the analysis and allows me to focus on

the problems associated with taxing capital. I also assume that expropriation of capital is ruled out for the governments. Although a command optimum in the model would involve expropriation of capital even for a government maximizing the welfare of the capital owners, I rule it out since it is not very common in the real world. Modelling why and when expropriation may come about is outside the scope of the chapter.

The economy is assumed to consist of two classes, namely 'capitalists' who own the accumulated factor of production (capital) and 'workers' who represent the non-accumulated factor of production (raw labour). To formulate the model in terms of classes serves to keep matters simple and allows one (1) to concentrate on the problem of growth and redistribution and (2) to relate to the literature on majority voting on tax rates.¹

In their 1994 article Alesina and Rodrik define redistribution as any policy that distributes income to the non-accumulated factor of production while reducing the incentive to invest. Thus, they assess income distributions relative to growth maximizing policies. Such policies seem somewhat unattractive as benchmark policies for analyzing income distributions. Therefore, I follow their 1991 paper which analyzes the trade-off between growth and wealth redistribution. Thus, I define redistribution as any policy that takes real resources from the accumulated factor of production by giving them to the non-accumulated factor of production. That allows me to investigate income distributional issues separately, using a different and more natural benchmark policy.

Given the optimal tax policies of a government I analyze what post-tax factor income distribution a policy induces. For instance, a government may be '*right-wing*' and only care about the owners of capital, or it may be '*left-wing*'

¹See e.g. Romer (1975), Roberts (1977), Meltzer and Scott (1981), and Mayer (1984).

caring only about the owners of raw labour. As a benchmark policy I take an '*income egalitarian*' policy. The reason for introducing the rather special egalitarian objective of granting all agents an equal income is the following:

Firstly, many people tend to associate left-wing with egalitarian, especially income egalitarian objectives. The two objectives are clearly distinct, because egalitarian objectives are mainly concerned about the relative well-being of agents, whereas the model's left-wing objective cares about the absolute welfare of the workers. However, this difference in objectives has interesting implications for tax policy, income distribution and growth comparisons.

Secondly, a policy that would grant equal income to all agents induces a post-tax factor income distribution that most people would agree on as a benchmark distribution, that is, it seems natural to compare any policy's post-tax factor income distributions relative to a policy that gives all agents equal incomes.

Thirdly, among all possible egalitarian objectives income egalitarianism involves the observable variable income which is more readily obtainable than information on such things as 'utilities'.² Where appropriate I discuss possible implications for other egalitarian objectives.

In the model the 'right-wing' government chooses the growth maximizing tax rate in the optimum. The 'left-wing' government sets higher taxes in the optimum in order to redistribute capital or secure a high steady state wage to capital ratio. Thus, even if the left-wing government does not redistribute wealth it will generally set its tax rate higher than the growth maximizing one. That means that one may distinguish between a redistributing, left-wing and a non-redistributing, left-wing government.

Given the optimal choices I analyze steady states varying some fixed param-

²For a discussion of egalitarian objectives see Sen (1982) or Atkinson and Stiglitz (1980).

eters that represent fundamental economic variables. The procedure may be justified by appeal to the 'Correspondence Principle', which roughly speaking states that there exists a mutually supportive relationship between economic dynamics and comparative statics. See Samuelson (1941) or Samueslon (1942). Thus, I implicitly assume that there exist convergent processes leading from one steady state equilibrium to another.

I show that all policies in the model depend on three fundamental economic variables that do not follow an explicit time path: the rate of time preference, an index of the technological efficiency of the production process and the (pre-tax) share of capital (income in total income).

In the model aggregate production is of the Barro (1990), Cobb-Douglas type so that the (pre-tax) share of capital equals the elasticity of output with respect to (privately owned) capital. The index of technological efficiency is meant to capture such diverse things as purely technological, or cultural, or institutional factors bearing influence on the way production is undertaken. Sometimes I interpret an element of the index as representing the state of the technology, permitting me to speak of technological change if that element changes. Within this framework the main results of the first part of the chapter are the following:

1. Growth maximizing and left-wing policies may be observationally indistinguishable, if the workers as a group are sufficiently patient. Thus, one may not be able to differentiate right-wing and left-wing policies by observation only.
2. Under a left-wing policy the conditions for wealth redistribution and positive growth are restrictive. In the model wealth transfers take place only if the share of capital is large, that is, if the pre-tax factor income distribution

is biased towards capital.

3. Under the optimal redistributing, left-wing policy an increase in technological efficiency leads to less resources being transferred to the non-accumulated factor of production. That is a testable implication of the model which is discussed in part 2. Normatively, it implies that if one compares two economies that are lead by redistributing governments, the one with a more efficient economy redistributes relatively less wealth per units of taxes collected in steady state, but it has higher growth. Thus, there is an interesting trade-off between growth, redistribution and technological efficiency in the model. If one interprets an increase in technological efficiency as technological progress, the model predicts that redistributing governments of more advanced economies grant less wealth transfers per units of taxes collected and place more weight on growth.
4. For all policies considered, higher technological efficiency leads to higher growth and either higher tax rates or no change in taxes, but lower redistribution. The reason for the result lies in the externality productive government expenditure exerts on the private return on capital. An increase in efficiency for given taxes raises growth. For given efficiency an increase in taxes lowers growth. But in the model public policy is endogenous in that all optimal steady state tax rates depend on fundamental economic variables. The combined effect of an increase in efficiency is to raise the growth rate and the tax rates. Thus, higher tax rates per se do not indicate that growth is lower. Possible implications of the result for empirical research are discussed in part 2.
5. A change in efficiency does not change the post-tax factor income distribu-

tion under right-wing and income egalitarian policies and shifts relatively more post-tax factor income to the accumulated factor of production (capital) under redistributing and non-redistributing left-wing policies. The result looks a bit odd and is explained by the fact that 'left-wing' governments are only concerned about the welfare level of the workers. An increase in efficiency raises the workers' wages and their welfare. A left-wing government does not care about relative incomes. That explains why it may be optimal for a left-wing policy to choose a policy that raises the workers' welfare and makes the capital owners get relatively more after-tax income.

6. There exists a particular (low) share of capital where an income egalitarian policy maximizes growth. Thus, income egalitarianism is not necessarily bad for growth. The result is interesting, because it shows - contrary to conventional wisdom - that other than 'right-wing' objectives may lead to growth maximizing policies.
7. There exist instances where left-wing and income egalitarian policies coincide. However, in general left-wing and income egalitarian policies are different in the model and induce different combinations of growth and post-tax factor income distributions. Furthermore, it is generally ambiguous whether a left-wing or an income egalitarian policy induces higher or lower growth in comparison to the growth maximizing policy.
8. An increase in the share of capital unambiguously raises the growth rate under all, but the income egalitarian policies. Taxes increase under an income egalitarian policy, do not change under a redistributing, left-wing policy and respond in an ambiguous way under all other policies considered.

The theoretical results of the chapter are based on the assumption that public policy is economically endogenous. That means that optimizing governments take fundamental economic variables into account when making their decisions. The implications of that assumption and of the theoretical results for empirical research are investigated in the next part of the chapter.

Part 2

In the cross-country growth empirics literature some authors use pooled time-series cross-country data to pay explicit attention to the time series dimension. However, a large number of authors takes averages of their data over time and runs simple cross-country OLS regressions over these averaged data.³ For instance, Koester and Kormendi (1989), Barro (1991), Levine and Renelt (1992) or Easterly and Rebelo (1993a) have used simple cross-country OLS regressions to study the effect of public policy on growth. All these papers assume that public policy is exogenous. In contrast, in this chapter it is assumed that public policy contains an element of economic endogeneity in that public policy takes account of country-specific, fundamental economic variables that may or may not be included in the regressions. The assumption of endogenous policy implies that the OLS estimates of the effect of public policy on growth are generally biased, implying that correct statistical inferences are not possible. Instead of focussing on the exact source of a bias, I use the theoretical results, that is, the theoretically derived correlations between public policy variables and fundamental economic variables of part 1 to sign the biases. That allows me to explain some empirical findings that seem puzzling from a theoretical viewpoint.

³In the chapter 'simple cross-country OLS regression' is meant to reflect that procedure of handling the time series dimension of the data. Of course, 'simple' does not mean simplistic, since the availability of data may not allow for another or better method of analysis.

I show that the estimated coefficients of the effects on growth of average or marginal tax rates, measured by variables closely related to the tax base most relevant for growth, are generally overestimated. In particular, the average tax rates, defined as the 'ratio of tax revenues to tax base', or marginal tax rates, defined as 'the ratio of the change in tax revenues to the change in tax base', are generically biased upwards. The result suggests that any reported negative effect of taxes on growth is understated, if measured by these variables.

However, the determination of the tax base that is of primary importance for growth is very difficult and in the literature taxes are usually measured by variables related to GDP or aggregate income. It is then shown that estimates of the effect on growth of average tax rates, measured by 'the ratio of tax revenues to GDP', or marginal tax rates, measured by 'the ratio of the change in tax revenues to the change in GDP' and obtained by regressing tax revenues to GDP, are biased downwards unless all governments in the sample pursue growth maximizing or income egalitarian policies. In that case any reported negative (positive) effect of taxes on growth would be overstated (understated), if measured by those variables. The systematic underestimation of the effect of these variables implies that the theoretical prediction that taxes negatively affect growth may be inherently untestable. Hence, it matters what variables one uses for taxes when running simple cross-country growth regressions. For instance, suppose a researcher starts with initial income included in a growth regression and then adds the variable 'tax revenues to GDP'. Most people find that the point estimate of the effect of that variable on growth is close to zero, insignificant and negative. A downward bias in that estimate means that the 'true' value of the reported coefficient on that variable may be positive and significant, when controlling for initial income. Usually the insignificant value of the estimated coefficient is attributed to the

high correlation between taxes and initial income. In that sense the chapter provides a different explanation why one may find such a point estimate for the effect of the variable 'tax revenues to GDP' in a growth regression that controls for initial income.

If public policy is assumed to be exogenous, estimated coefficients of the effect on growth of the ratio of productive public expenditure to total investment or GDP are biased downwards under all policies considered. If public policy is assumed to be endogenous, the point estimates of the effect on growth of the ratio of public investment to total investment are biased downwards.

That has interesting implications for any hypothesis that claims that the breakdown of total investment between public and private investment does not materially affect growth. It also casts doubt on the hypothesis that a typical country comes close to the quantity of public investment that maximizes growth. (See Barro (1990), p. S124.)

Under the assumption of endogenous policy, the estimated coefficients of the effect on growth of the ratio of redistributive transfers to GDP are generically biased downwards. That renders the hypothesis that redistribution is bad for growth untestable for the following reason: The theoretical prediction of part 1 is that redistributive transfers are bad for growth. However, in the model an increase in efficiency makes an optimizing, redistributing government grant less transfers to the non-accumulated factor of production. This last effect is ignored in cross-country growth regressions when one assumes that public policy is exogenous. For theoretical reasons many researchers expect a negative coefficient, when measuring the effect of redistributive transfers on growth. However, many people find positive coefficients (for example, for the effect of social security contributions on growth, see Sala-i-Martin (1996)). As any downward bias of the

estimated coefficient on redistributive transfer variables may be as large as minus infinity, a reported negative coefficient cannot corroborate the hypothesis that redistribution is bad for growth. On the other hand, any downward bias, found in these kinds of studies, is perfectly consistent with many empirical findings and the alternative hypothesis that redistribution is not bad for growth.

The chapter is organized as follows: In the first part, section 1.2 presents the model set-up, derives the market equilibrium and some of its properties. Section 1.4 presents the optimal policy choices of the governments. Section 1.5 provides a comparative dynamic analysis of the optimal policies. It contains the chapter's major theoretical results which are stated in propositions. The second part (section 1.6) uses theoretical correlations derived from the model to analyze the effect of tax, public investment and redistributive transfer variables on growth. Section 1.7 provides concluding remarks for the whole chapter.

1.2 The Model

There are two types of many identical individuals in the economy who are all equally patient and have the same rate of time preference ρ . One type of individuals owns capital equally and does not work. The other type owns (raw) labour equally, but no capital. Call the latter group 'workers' (W) and the former group 'capitalists' (k). Population is stationary and consists of l workers and n capitalists of whom there are less, that is, $l > n$. Capital is broadly defined and human capital is taken to be strictly complementary to physical capital. So in the model capitalists who, for instance, own computers know how to operate them as well. That eliminates a separate treatment of how human capital is accumulated and entails that the return on human capital services equals that of

physical capital services in a perfectly competitive economy. For a justification of such an approach in a different context see Mankiw, Romer and Weil (1992). The simplification allows one to concentrate on the distributional conflict between the accumulated and the non-accumulated factor of production.

The group of workers and the group of capitalists derive logarithmic utility from the consumption of a homogeneous, malleable good. That means that the individual preferences of the members of each group can be aggregated and represented by a single utility function. I ignore problems associated with aggregation within classes that are clearly important, but beyond the scope of the paper.

There are many price-taking, profit maximizing firms which are owned by the capital owners.⁴ Aggregate output is produced according to the following Barro (1990) production technology

$$y_t = A k_t^\alpha g_t^{1-\alpha} L_t^{1-\alpha} \quad , \quad 0 < \alpha < 1 \quad (1.1)$$

where y_t is output, k_t the installed real capital stock, L_t is labour supplied, and g_t are public inputs to production. Like Barro one may assume that the government owns no capital and that it buys a flow of output from the private sector and makes it available to the *individual* firm in the form of productive services. Then public inputs to production would be rival. Alternatively, *total* government expenditure may be taken to affect private production in a non-rival way. By assumption this empirically relevant distinction does not matter analytically in the model. It may be interesting to note that in the absence of a government, for instance, due to civil war or other forms of unrest, the economy breaks down and the workers and the capitalists 'starve'.

⁴The reason for proceeding in this fashion is to formulate the problem in a decentralized economy framework with perfect competition as in Cass (1965) and Koopmans (1965).

The variable A is an efficiency index, which depends on cultural, institutional and technological development. I eliminate all exogenous factors that may play a role in the growth process by assuming that A is constant over time.

Furthermore, I assume that each worker inelastically supplies $\frac{1}{l}$ units of labour at each point in time. Thus, the total labour endowment is equal to unity, that is, $L_t = 1$ because there are l workers in the economy. I abstract from problems arising from the depreciation of the capital stock so that output and factor returns are really defined in net terms. That has no consequences for the price-taking, market clearing logic of the model. (See, for instance, Bertola (1991).)

The Cobb-Douglas technology implies that the elasticity of output with respect to (broad) capital ($\frac{dy_t}{dk_t} \frac{k_t}{y_t}$) is constant over time. Later on I couple this with the assumption of perfect competition and profit maximization. Then α also denotes the (constant) *share of capital (income)* in total income $\frac{r_t k_t}{y_t}$. Thus, α allows for two interpretations in the model, one referring to technology and one referring to factor shares, that is, distribution.

1.2.1 The Public Sector

The government taxes wealth at the constant rate τ and redistributes a constant share λ of its tax revenues to the workers.⁵ Thus, τ is levied on k_t , that is, the capital owners' wealth at time t . The tax on capital is to be viewed as a tax on all resources that are accumulated, including human capital. The unskilled labour force, which is constant in the model, is not subject to taxation. I use the assumption to allow the government to discriminate between the two types

⁵Given the structure of the model the optimal τ and λ would be constant over time as is e.g. shown in Alesina and Rodrik (1991). I impose the restriction here to facilitate the analysis. A possible justification is that one does not usually observe changes in fundamental tax arrangements and policies for long periods of time.

of factors of production and to undertake (re-)distributive policies.

Alesina and Rodrik (1994) do not explicitly analyze wealth redistribution, that is, their model assumes $\lambda = 0$. Instead, they call 'redistribution' any policy that distributes *income* to the non-accumulated factor of production while reducing the incentive to accumulate. Hence, they assess income redistribution relative to a growth maximizing policy. In terms of income distribution it is far from clear why a growth maximizing policy should serve as a benchmark. For example, it may well be the case that moving from a growth maximizing policy to some other, e.g., left-wing policy increases income inequality and decreases growth. Most people assess such an income redistributing policy shift with reference to a policy that grants equal incomes. Thus, I define *redistribution* as taking real resources (wealth) from the accumulated factor of production by giving them to the non-accumulated factor of production. That is captured by the variable λ .

Given the Barro-type production function the government faces the following budget constraint, which is taken to be balanced at each point in time

$$\tau k_t = g_t + \lambda \tau k_t.$$

The LHS depicts the tax revenues and the RHS public expenditures. The workers receive $\lambda \tau k_t$ as transfers and g_t is spent on public inputs to production. As human and physical capital are strict complements by assumption, a strong form of redistribution is contemplated. It implies that if a capital good is given to the workers the corresponding services necessary to operate that good are also given to them. As a one good economy is contemplated, giving the capital good to the workers for consumption does not cause a problem, however. Rearranging I,

therefore, contemplate

$$g_t = (1 - \lambda)\tau k_t \quad (1.2)$$

as the government's budget constraint.

1.2.2 Property Structure and Firms

There are many identical, profit maximizing firms which operate in a perfectly competitive environment. The firms are owned by the capital owners, who rent capital to and demand shares of the firms which are collateralized one-to-one by capital. The markets for assets and capital clear at each point in time. The representative firm faces a given path of the market clearing rental rate, $\{r_t\}$, of capital, k_t , and takes the amount of productive government services, g_t , as given.⁶

The firms rent capital and hire labour in spot markets in each period. The output price of y_t is employed as numéraire and set equal to one. Given constant returns to capital and labour, factor payments exhaust output. Profit maximization entails that firms pay each factor of production its marginal product,

$$\begin{aligned} r_t &= \frac{\partial y_t}{\partial k_t} = \alpha A \left(\frac{g_t}{k_t} \right)^{1-\alpha} \\ w_t &= \frac{\partial y_t}{\partial L_t} = (1 - \alpha) A \left(\frac{g_t}{k_t} \right)^{1-\alpha} k_t, \quad L_t = 1, \forall t. \end{aligned}$$

Thus, from equation (1.2) one obtains

$$r = \alpha A [(1 - \lambda)\tau]^{1-\alpha} \quad (1.3)$$

$$w_t \equiv \eta(\tau, \lambda) k_t = (1 - \alpha) A [(1 - \lambda)\tau]^{1-\alpha} k_t. \quad (1.4)$$

⁶The assumptions that assets are collateralized one-to-one and the rental rate of capital is - later on - uniform and for simplicity constant can be relaxed without altering the results. For a justification of these assumptions cf. Barro and Sala-i-Martin (1995), Chpt. 2.

Hence, the tax rate has a bearing on the marginal product of capital and the wage rate. Notice that in contrast to the return on capital, wages are not constant over time and increase with the capital stock.⁷

For this and subsequent chapters let $E \equiv (1 - \alpha)A \left(\frac{q_t}{k_t}\right)^{-\alpha}$. Analyzing how the rate of return on capital and the wages are affected by changes in government policy, I use (1.2), (1.3) and $E = (1 - \alpha)A[(1 - \lambda)\tau]^{-\alpha}$. Then

$$\frac{\partial r}{\partial \tau} = \alpha E(1 - \lambda) > 0 \quad , \quad \frac{\partial r}{\partial \lambda} = \alpha E(-\tau) < 0. \quad (1.5)$$

Thus, redistribution has a negative effect on the rate of return and increases in the tax rate raise the rate of return. For the wage rate, (ηk_t) , and a given capital stock I obtain

$$\frac{\partial \eta}{\partial \tau} = (1 - \alpha)E(1 - \lambda) > 0 \quad , \quad \frac{\partial \eta}{\partial \lambda} = (1 - \alpha)E(-\tau) < 0. \quad (1.6)$$

Hence, for a given capital stock an increase in τ leads to a positive change in the rate of return on capital and in wages. Redistribution lowers each of them.

1.2.3 Capital Owners

There are many capitalists who choose how much to consume or invest. They have perfect foresight about the prices and tax rates. The capitalists as a group maximize their intertemporal utility according to the following programme taking

⁷Throughout the dissertation and for simplicity 'return to capital' is meant to be the pre-tax return to capital. Whenever it is necessary to distinguish between pre-tax and after-tax returns I make it explicit.

prices and tax rates as given

$$\max_{C_t^k} \int_0^{\infty} \ln C_t^k e^{-\rho t} dt \quad (1.7)$$

$$s.t. \quad \dot{k}_t = (r - \tau)k_t - C_t^k \quad (1.8)$$

$$k(0) = \bar{k}_0, \quad k(\infty) = \text{free}. \quad (1.9)$$

Equation (1.8) is the dynamic budget constraint of the capitalists. Note that the capitalists earn capital income $r k_t$ and pay taxes τk_t . The necessary first order conditions for this problem are given by (1.8), (1.9) and the equations

$$\frac{1}{C_t^k} - \mu_t = 0 \quad (1.10a)$$

$$\dot{\mu}_t = \mu_t \rho - \mu_t (r - \tau) \quad (1.10b)$$

$$\lim_{t \rightarrow \infty} k_t \mu_t e^{-\rho t} = 0, \quad (1.10c)$$

where μ_t is a positive co-state variable which can be interpreted as the instantaneous shadow price of one more unit of investment at date t . Equation (1.10a) equates the marginal utility of consumption to the shadow price of more investment, (1.10b) is the standard *Euler* equation which relates the costs of foregone investment (LHS) to the discounted gain in marginal utility (RHS), noting $\frac{1}{C_t^k} = \mu_t$, and (1.10c) is the transversality condition which ensures that the present value of the capital stock approaches zero asymptotically.

Depending on the after-tax return, the growth rate of consumption can be calculated in a standard way from (1.10a) and (1.10b) and it is given by

$$\gamma \equiv \frac{\dot{C}_t^k}{C_t^k} = (r - \tau) - \rho. \quad (1.11)$$

Growth of consumption is increasing in the after-tax return on capital and constant over time. Furthermore, equations (1.8), (1.10c) and (1.11) imply that in the optimum the instantaneous consumption of the capitalists is given by $C_t^k = \rho k_t$. Hence, $\gamma_k = \gamma$ so that the capitalists' wealth and consumption grow at the same rate in the optimum.⁸

1.2.4 Workers

The workers derive a utility stream from consuming their entire wages and government transfers. They do not invest and are not taxed by assumption. Their intertemporal utility is given by

$$\int_0^{\infty} \ln C_t^W e^{-\rho t} dt \quad \text{where} \quad C_t^W = \eta(\tau, \lambda)k_t + \lambda\tau k_t. \quad (1.12)$$

This assumption is reminiscent of growth models such as Kaldor (1956), where different proportions of profits and wages are saved. In the extreme case capitalists save and workers do not, which is the "*Classical Savings Rule*". In Kaldorian models the capitalists' investment decision is determined by the exogenously given growth rate. Recently, Bertola (1993) has derived the "*Classical Savings Rule*" result from utility maximization, which endogenizes the investment decisions and therefore the growth rate. In that sense the model's set-up reflects that result. However, Bertola does not use a two-class model and there are important differences to post-Keynesian models of growth, the most important of which is that the causality in both approaches is running in opposite directions. Whereas in Kaldorian models growth determines the factor share incomes, in endogenous growth models the direction is rather from factor shares to growth.

⁸The optimal choices are derived in Appendix A.1.

1.3 Market Equilibrium

The constancy of τ implies constancy of r and hence γ . The overall resource constraint in the economy is $I_t = y_t - g_t - C_t$ and given by

$$I_t = \dot{k}_t = (r - \tau)k_t + (\eta + \lambda\tau)k_t - C_t^k - C_t^W. \quad (1.13)$$

As the workers' consumption is $C_t^W = (\eta + \lambda\tau)k_t$, this constraint is binding so that $\gamma_{C^W} = \gamma_k$. The capitalists' consumption and wealth optimally grow at the constant rate $\gamma = \gamma_k$. In *steady state* all variables grow at the same constant rate. To verify that consider (1.1). Use the definition of g_t , that is, $g_t = (1 - \lambda)\tau k_t$, and substitute in (1.1). Recalling that $L_t = 1$ and taking logarithms and time derivatives yields $\gamma_Y = \gamma_k$. Hence, the *steady state* is characterized by *balanced growth* with $\gamma_Y = \gamma_k = \gamma = \gamma_{C^W}$. That describes the dynamic market equilibrium of the economy.

1.3.1 Properties of the Market Equilibrium

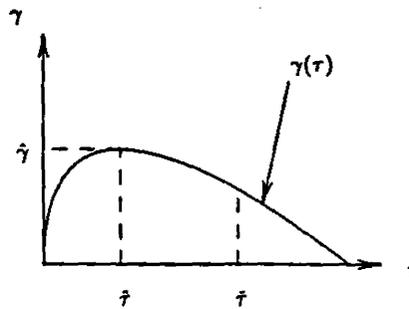
The steady state growth rate is given by $\gamma = (r - \tau) - \rho$ where $r = \alpha A[(1 - \lambda)\tau]^{1 - \alpha}$. The (τ, λ) combination that maximizes growth must satisfy $r_\lambda \leq 0$ and $r_\tau = 1$. From (1.5) $r_\lambda < 0$ so that $\lambda = 0$ and from $r_\tau - 1 = 0$ one obtains $\hat{\tau} = [\alpha(1 - \alpha)A]^{\frac{1}{\alpha}}$ as the growth maximizing (τ, λ) combination.

Lemma 1.1 *The (τ, λ) combination that maximizes growth is given by $\lambda = 0$ and $\hat{\tau} = [\alpha(1 - \alpha)A]^{\frac{1}{\alpha}}$ where $\hat{\tau}$ solves $r_\tau = 1$.*

The relationship between wealth taxes and growth is visualized in Figure 1.1.

At $\hat{\tau}$ the growth rate is maximal. If higher taxes - for example for wealth redistribution - are levied, the growth rate is lower so that growth is traded off against

Figure 1.1: The relationship between γ and τ



redistribution at a point such as $\hat{\tau}$ with $\lambda > 0$.

Notice that $r - \tau = \alpha A [(1 - \lambda)\tau]^{1-\alpha} - \tau = \tau (\alpha A [(1 - \lambda)\tau]^{-\alpha} - 1)$. Substitution of the growth maximizing (τ, λ) combination establishes that $\hat{r} - \hat{\tau} = \hat{\tau} \left(\frac{\alpha}{1-\alpha} \right)$. The effect of a marginal increase in efficiency on maximum growth is given by $\frac{d(\hat{r} - \hat{\tau})}{dA} = \left(\frac{\alpha}{1-\alpha} \right) \frac{d\hat{\tau}}{dA}$ where $\frac{d\hat{\tau}}{dA} = \hat{\tau} [\alpha A]^{-1} > 0$. Thus,

Lemma 1.2 *The maximum after-tax return is given by $\hat{r} - \hat{\tau} = \hat{\tau} \left(\frac{\alpha}{1-\alpha} \right)$. An increase in efficiency raises the maximum growth rate, the maximum after-tax return and the growth maximizing tax rate, that is, $\frac{d\hat{j}}{dA} > 0$, $j = \hat{\gamma}, (\hat{r} - \hat{\tau}), \hat{\tau}$.*

The result that a more efficient economy has higher maximum growth corresponds to common economic intuition and is hardly surprising. However, notice that under the model's growth maximizing policy higher efficiency implies higher taxes and a higher after-tax return on capital. In terms of Figure 1.1 one may think of an increase in A as an upward shift of the concave relationship between taxes and growth.

1.4 The Government

As in Alesina and Rodrik (1991) consider a government that cares about the two groups in the economy. Respecting the right of private property, it chooses τ and λ in order to maximize the welfare function

$$\max_{\tau, \lambda} (1 - \beta) V^r + \beta V^l \quad s.t. \quad \lambda \geq 0$$

where V^r and V^l denote the intertemporal utility of the capital owners and the workers, respectively, and are given by

$$V^r = \frac{\ln(\rho k_0)}{\rho} + \frac{\gamma}{\rho^2} \quad \text{and} \quad V^l = \frac{\ln[(\eta(\tau, \lambda) + \lambda\tau)k_0]}{\rho} + \frac{\gamma}{\rho^2}. \quad (1.14)$$

These expressions are obtained by integrating the utility functions in (1.7) and (1.12). For a derivation see appendix A.2. The condition $\lambda \geq 0$ restricts the governments in such a way that even a right-wing government does not tax workers. In that sense even a right-wing government is 'nice' to the workers. A negative λ would effectively amount to a tax on wages. The parameter $\beta \in [0, 1]$ represents the welfare weight attached to the two groups in the economy. If $\beta = 1, (0)$ the government cares about the workers (capitalists) only. The constancy of β may be justified by interpreting β as reflecting the socio-economic institutions in an economy. Then the fact that governments alternate in office becomes less of an issue since institutional features are usually constant for long periods of time.

The solution to the government's problem is derived in appendix A.3 and it is given by

$$\text{If } \beta\rho \geq [(1 - \alpha)A]^{\frac{1}{\alpha}} \text{ then:}$$

$$\tau = \beta\rho, \quad \lambda = 1 - \frac{[(1 - \alpha)A]^{\frac{1}{\alpha}}}{\beta\rho}.$$

If $\beta\rho < [(1 - \alpha)A]^{\frac{1}{\alpha}}$ then:

$$\tau[1 - \alpha(1 - \alpha)A\tau^{-\alpha}] = \beta\rho(1 - \alpha), \quad \lambda = 0.$$

From the last equation it follows that a right-wing government, $\beta = 0$, is only concerned about the after-tax return on capital and therefore growth in the model. It chooses $\lambda = 0$ and $\hat{\tau} = [\alpha(1 - \alpha)A]^{\frac{1}{\alpha}}$, that is, the growth maximizing (τ, λ) combination. (See Lemma 1.1.)

For $\beta > 0$ it follows that $\tau > \hat{\tau}$ when $\lambda \geq 0$ so that growth is not maximized. That can be visualized using Figure 1.1 on page 34. At $\hat{\tau}$ the growth rate is maximal. If higher taxes are levied for wealth redistribution ($\lambda > 0$) then the growth rate decreases. Thus, such a government trades off growth against wealth redistribution. It is interesting to note that β is inversely related to the growth rate. (For a proof see appendix A.3 or Alesina and Rodrik (1991).)

Proposition 1.1 (Alesina and Rodrik) *The growth rate $\gamma(\tau)$ is inversely related to β , the social weight attached to the welfare of the non-accumulated factor of production.*

Hence, under the model's wealth tax scheme placing more social weight on the welfare of non-accumulated factor of production causes optimizing governments to choose high tax rates and thereby relatively low growth.

Another implication is that there is a wide range of values where no wealth

redistribution takes place. Note that if ρ is a lot lower than β and the agents are patient, then the government does not redistribute.⁹

In order to concentrate on the distributional conflict I restrict the subsequent analysis to entirely pro-labour, 'left-wing' ($\beta = 1$) and entirely pro-capital, 'right-wing' ($\beta = 0$) governments. Thus, I assume that a government once in power is either right-wing or left-wing, so either $\beta = 0$ or $\beta = 1$. For that case denote the (τ, λ) combination, solving the equations above, by $\lambda \equiv \bar{\lambda}$ and $\bar{\tau}$.

The *left-wing* government chooses

If $\rho \geq [(1 - \alpha)A]^{\frac{1}{\alpha}}$ then:

$$\bar{\tau} = \rho, \quad \bar{\lambda} = 1 - \frac{[(1 - \alpha)A]^{\frac{1}{\alpha}}}{\rho}. \quad (1.15)$$

If $\rho < [(1 - \alpha)A]^{\frac{1}{\alpha}}$ then:

$$\bar{\tau}[1 - \alpha(1 - \alpha)A\bar{\tau}^{-\alpha}] = \rho(1 - \alpha), \quad \bar{\lambda} = 0. \quad (1.16)$$

The *right-wing* government chooses

$$\hat{\tau} = [\alpha(1 - \alpha)A]^{\frac{1}{\alpha}}, \quad (1.17)$$

does not redistribute and acts growth maximizing in the model.

⁹It is also worth noting that the optimal tax rates are non-zero. That is due to the assumption that λ is non-negative and labour supply is inelastic. As has been shown by Jones, Manuelli and Rossi (1993a) and Jones, Manuelli and Rossi (1993b) and in contrast to, for instance, Chamley (1986) this leads to non-zero tax rates on capital income.

1.5 A Comparative Dynamic Analysis

If the time preference rate is very low, $\rho \rightarrow 0$ in (1.16), a non-redistributing, left-wing government will mimic a growth maximizing policy. Thus,

Proposition 1.2 *A left-wing government will mimic a growth maximizing, right-wing government's policy only if it is very patient.*

This is a special, but interesting case. It shows that political preferences per se do not rule out the possibility of choosing a growth maximizing policy. In particular, a government placing maximal weight on the non-accumulated factor of production ($\beta = 1$), but at the same time putting almost equal weight to the welfare of future generations (low ρ) may act like a growth maximizer.¹⁰ Furthermore, in terms of *observed* tax and growth rates a sufficiently low ρ may make the measured tax and growth rates under an optimal left-wing or a growth maximizing policy indistinguishable.

Notice that $\tilde{\tau}$ in (1.15) is not affected by changes in efficiency. I now verify that the tax rate in (1.15), call it $\tilde{\tau}_1$, is higher than the one in (1.16), call it $\tilde{\tau}_2$. Equation (1.16) entails

$$x = \tau[1 - \alpha(1 - \alpha)A\tau^{-\alpha}] - \rho(1 - \alpha) = 0,$$

$$x_\tau = 1 - \alpha(1 - \alpha)^2 A\tau^{-\alpha} > 0, \text{ for } \tau > \hat{\tau}; \quad x_\rho = -(1 - \alpha)$$

so that $x_\tau > 0$ and $x_\rho < 0$ where $\rho < [(1 - \alpha)A]^{\frac{1}{\alpha}}$ in (1.16). By the *implicit function theorem* this implies $\frac{d\tilde{\tau}_1}{d\rho} > 0$. Substitute in (1.16) a particular (optimal)

¹⁰Of course, theoretically the two policies coincide only if $\rho \rightarrow 0$ which causes problems for the convergence of the utility indices. For the observability argument it suffices that ρ is very low while the utility indices still converge.

$\tilde{\tau}$. If one finds that

$$\tau[1 - \alpha(1 - \alpha)A\tau^{-\alpha}] < \rho(1 - \alpha)$$

then $x_\tau > 0$ implies that any optimal $\tilde{\tau}$ would have to be higher to satisfy (1.16).

The converse holds if the inequality sign is reversed. Setting the minimum tax rate in (1.15), that is, $\tilde{\tau}_1 = [(1 - \alpha)A]^{\frac{1}{\alpha}}$ equal to τ_2 in (1.16) I obtain

$$[(1 - \alpha)A]^{\frac{1}{\alpha}}(1 - \alpha) > \rho(1 - \alpha)$$

so that τ_2 would have to be lowered. Hence, $\tilde{\tau}_1 > \tilde{\tau}_2$.

Lemma 1.3 *The tax rate chosen by a non-redistributing, left-wing government satisfies $\tilde{\tau} < [(1 - \alpha)A]^{\frac{1}{\alpha}}$ and increases with the rate of time preference, $\frac{d\tilde{\tau}}{d\rho} > 0$.*

The result is not surprising, but the lemma is useful below.

In this model a right-wing government is only concerned about guaranteeing the maximum after-tax return on capital and therefore maximum growth. The growth maximizing tax rate $\hat{\tau} = [\alpha(1 - \alpha)A]^{\frac{1}{\alpha}}$ is increasing in A .

Suppose the left-wing government redistributes wealth in the optimum. Then equation (1.15) applies. I will now check under what conditions $\gamma > 0$ with $\lambda > 0$. If (1.15) holds then $(1 - \tilde{\lambda})\tilde{\tau} = [(1 - \alpha)A]^{\frac{1}{\alpha}}$. If $\mathcal{T} \equiv (1 - \alpha)A$, then

$$r = \alpha A[(1 - \lambda)\tilde{\tau}]^{1-\alpha} = \alpha A \mathcal{T}^{\frac{1-\alpha}{\alpha}} = \left(\frac{\alpha}{1 - \alpha}\right) \mathcal{T}^{\frac{1}{\alpha}}.$$

Equation (1.15) requires $\tilde{\tau} = \rho \geq \mathcal{T}_\alpha^{\frac{1}{2}}$, and $\gamma > 0$ implies $r - \tilde{\tau} - \rho > 0$. So $\tilde{\tau} = \rho$ has to satisfy

$$\tilde{\tau} > \mathcal{T}_\alpha^{\frac{1}{2}} \wedge \left(\frac{\alpha}{1-\alpha}\right) \mathcal{T}_\alpha^{\frac{1}{2}} > 2\tilde{\tau} \Leftrightarrow \tilde{\tau} \left(\frac{\alpha}{1-\alpha}\right) \mathcal{T}_\alpha^{\frac{1}{2}} > \mathcal{T}_\alpha^{\frac{1}{2}} 2\tilde{\tau} \Leftrightarrow \alpha > \frac{2}{3}.$$

Thus, the share of capital has to be sufficiently more important than that of public inputs or labour. Furthermore, for an increase in A I find $\frac{d\lambda}{dA} < 0$ so that λ would be lower in a new optimum.

Proposition 1.3 *For a redistributing ($\lambda > 0$), left-wing government $\gamma > 0$ only if $\alpha > \frac{2}{3}$. An increase in efficiency makes the left-wing government redistribute less wealth given its optimal policy, that is, $\frac{d\lambda}{dA} < 0$.*

This is an interesting implication of the model in Alesina and Rodrik (1991), which they do not discuss. The proposition is empirically relevant and testable. It entails that an increase in efficiency causes a left-wing government to redistribute *less* resources and place more weight on growth. Empirical implications of the result are discussed in the chapter's second part. Theoretically, it suggests that there is an interesting trade-off between growth, redistribution and technological efficiency.

Next, I turn to a non-redistributing, left-wing government, i.e., $\lambda = 0, \tilde{\tau} > \hat{\tau}$. The effect of an increase in efficiency on the tax rate in (1.16) is

$$\begin{aligned} (1 - \alpha(1 - \alpha)^2 A \tau^{-\alpha}) d\tau - (\alpha(1 - \alpha)\tau^{1-\alpha}) dA &= 0 \\ \frac{d\tau}{dA} &= \alpha(1 - \alpha)\tau (\tau^\alpha - \alpha(1 - \alpha)^2 A)^{-1}. \end{aligned} \quad (1.18)$$

From (1.15), (1.16) the left-wing government chooses a policy $\tilde{\tau} > \hat{\tau}$. Thus, $\frac{d\tilde{\tau}}{dA}$ is positive.¹¹ Hence, an increase in efficiency makes a non-redistributing, left-wing government increase its optimal tax rate. Recall that

$$r = \alpha A[(1 - \lambda)\tau]^{1-\alpha} \quad , \quad \eta = (1 - \alpha)A[(1 - \lambda)\tau]^{1-\alpha} = \frac{1 - \alpha}{\alpha} r.$$

For the effect of higher A on growth I check if $\frac{d\gamma}{dA} = r_A + (r_\tau - 1)\frac{d\tau}{dA} > 0$, that is, whether

$$\begin{aligned} \alpha\tau^{1-\alpha} &> (1 - \alpha(1 - \alpha)A\tau^{-\alpha}) \left[\alpha(1 - \alpha)\tau (\tau^\alpha - \alpha(1 - \alpha)^2A)^{-1} \right] \\ 1 &> (\tau^\alpha - \alpha(1 - \alpha)A) \left[(1 - \alpha) (\tau^\alpha - \alpha(1 - \alpha)^2A)^{-1} \right] \\ \tau^\alpha - \alpha^2(1 - \alpha)^2A &> (1 - \alpha)\tau^\alpha - \alpha^2(1 - \alpha)^2A \\ 1 &> 1 - \alpha, \end{aligned}$$

which is true since $\alpha < 1$. Thus, $\frac{d\tilde{\gamma}}{dA} > 0$ if $\lambda = 0$ in (1.16).

Suppose the left-wing government redistributes. Proposition 1.3 and equation (1.15) imply that $\frac{d\tilde{\tau}}{dA} = 0$ and $\frac{d\tilde{\lambda}}{dA} < 0$. Then $\frac{d\tilde{\gamma}}{dA} = r_A + r_\lambda \frac{d\tilde{\lambda}}{dA} > 0$ since $r_\lambda < 0$. Also, Lemma 1.2 implies $\frac{d\hat{\gamma}}{dA} > 0$ for a right-wing government. That establishes

Proposition 1.4 *The optimal policies of a right-wing or left-wing government imply that higher efficiency leads them to choose either higher taxes when there is no redistribution $\lambda = 0$, or the same taxes and reduce redistribution. An increase in efficiency leads to higher growth under a right-wing or a left-wing government's optimal policy.*

¹¹To see this notice that $\frac{d\tilde{\tau}}{dA} > 0$ requires $\tau^\alpha > \alpha(1 - \alpha)^2A$ which is equivalent to $\tau > \hat{\tau}(1 - \alpha)^{\frac{1}{\alpha}}$ and always satisfied since $\tilde{\tau} > \hat{\tau}$ and $(1 - \alpha)^{\frac{1}{\alpha}} < 1$.

1.5.1 The Factor Income Distribution

The optimal right-wing and left-wing policies lead to different steady state distributions of income. I define the following ratios as measures of total factor income inequality:

$$F^g = \frac{\text{total pre-tax capital income}}{\text{total pre-tax wage income}} = \frac{\tau k_t}{\eta k_t} = \frac{\alpha}{1 - \alpha}.$$

Thus, steady state *pre-tax* factor income inequality in the economy is the same under either government, and it is constant over time and independent of the capital stock. Notice that F^g is increasing in the share of capital α . Similarly, I define the *post-tax total factor income inequality* as

$$F(\tau, \lambda) = \frac{\text{total post-tax capital income}}{\text{total post-tax wage income}} = \frac{(r - \tau)k_t}{(\eta + \lambda\tau)k_t} = \frac{(r - \tau)}{(\eta + \lambda\tau)}. \quad (1.19)$$

This measure is also independent of the capital stock and constant over time, but depends on the government's optimal policy. By assumption there are more workers than capital owners in the economy, and the capitalists own the capital stock equally, implying no inequality in intra-group capital income. Also the workers supply labour in equal amounts so that there is no intra-group inequality in wage income.

Obviously, these 'inequality measures' are extremely crude. They ignore intra-group inequality, the population composition and other things. The justification for employing them lies in the following argument: In the model any policy change affects the personal and the factor income distribution. Thus, changes in tax policy have a direct impact on total factor income inequality between people which is not always the case when analyzing total personal income distributions.

Suppose person i gets income 10 and person j gets income 20. If the government gives 10 to i and takes 10 from j , person i and j would swap places in the total personal income distribution. That is sometimes not recorded as a change in total personal income inequality, especially if i and j have the same utility functions. In the model, however, such a transfer would affect factor income inequality since j may be a capital owner and i may be a worker. The income transfer would make one worker better off and increase total wage income, and it would make one capitalist worse off and reduce total capital income. Of course, if one used a personal income inequality measure that is decomposable so that one can group capital income and wage income recipients, where the groups are weighted, an income transfer from a capitalist to a worker would be recorded as a change in inequality, because intra-group and inter-group inequality would change. On the complexity of moving from a factor share to a personal income distribution analysis see, for example, Atkinson (1983).

Thus, by working with the factor income distribution I concentrate on situations where policy changes have a direct impact on measured income inequality via changes in post-tax factor shares. Furthermore, I am only interested in how different policies compare to each other in terms of growth and post-tax factor shares. The simplifying framework, therefore, serves to focus on the implications of different policies on the income distribution and growth.

In what follows I analyze how the optimal left-wing or right-wing policies compare to a (strictly) income egalitarian policy. Such a policy is strictly committed to granting equal after-tax incomes to workers and capitalists.¹² The reason for introducing this policy is twofold.

Firstly, it allows one to compare the left-wing and right-wing policies' induced

¹²Below I will loosely refer to such a policy as income egalitarian with the implicit understanding that it is strict.

after-tax factor income distribution to one where all agents get the same income. In that sense, the strictly income egalitarian policy provides a benchmark from which one may assess how much income inequality a left-wing or right-wing policy entails. However, for consistency with the paper's definition of 'redistribution', which is restricted to wealth, I make it explicit which benchmark I use for comparisons of alternative distributive policies.

Secondly, many people tend to associate left-wing with income egalitarian policies. The two clearly involve distinct objectives. A left-wing policy represents the interests of one particular group in the economy. In this model it tries to maximize the welfare of unskilled labour and it is therefore concerned about the welfare *level* of labour. In contrast, the income egalitarian objective is *relative* in nature in that it compares a worker's and a capitalist's income. Thus, levels do not feature as an objective for an income egalitarian. The model brings out that the two objectives may lead to very different tax, income distribution and growth combinations. However, one has to be careful with the paper's egalitarian objective. Many other egalitarian policies are possible and interesting to analyze. For example, a *utilitarian* attempts to equalize marginal utilities of the agents. A strictly *utility egalitarian* government tries to make everybody equally happy in terms of total individual utility. Furthermore, a *Rawlsian* objective involves comparing utilities of the least well-off. Also, the complicated issue arises whether the objectives require equality at each point in time or equality of intertemporal welfare. These issues and other egalitarian objectives are discussed in more depth by, for instance, Sen (1982) or Atkinson and Stiglitz (1980), chpt. 11.¹³ The

¹³Furthermore, these authors show that under some conditions the utility Rawlsian and the utility egalitarian solutions coincide. However, if the social welfare function takes individual utilities as its arguments but is no longer monotonically increasing in them, that is, if it is individualistic, but non-Paretian, the Rawlsian objective will no longer necessarily satisfy the egalitarian principle of equalizing utilities.

reason for not considering such policies lies in the aim to analyze the factor income distribution. As data on income are more readily available, everybody getting equal factor incomes is a natural reference point for policy assessment. In comparison, a factor income distribution that would make everybody equally happy, requiring knowledge about the exact form of the welfare function, appears far more difficult to determine - even in this simple model. That may justify restricting the analysis to strictly income egalitarian policies.¹⁴

To facilitate the analysis I assume that a government which pursues a strictly income egalitarian objective is unable to redistribute wealth to the workers. That does not affect the qualitative results below. For all governments that do not redistribute ($\lambda = 0$), the post-tax factor income ratio F is given by

$$F(\tau) = \frac{r - \tau}{\eta} = \frac{\alpha}{1 - \alpha} - \frac{\tau^\alpha}{A(1 - \alpha)}$$

and decreases in τ . In the model an increase in taxes raises the total wage income, and shifts relatively more income towards labour, reducing F .

As there are many more workers (l) sharing the total wage income equally than capitalists (n) sharing total capital income equally, the *strictly income egalitarian* government chooses a policy that grants each individual an equal after-tax income. That is achieved if

$$\frac{\eta k_t}{l} = \frac{(\tau - \tau)k_t}{n}.$$

Thus, it does not matter in the model whether the income egalitarian objective

¹⁴The clarification is important since the results presented below apply *only* to the income egalitarian policy. Other egalitarian objectives may lead to different results. Furthermore, notice that in the model a strictly income egalitarian policy coincides with that of an income leximin policy, which may not be the case for total utility egalitarian and utility leximin policies.

requires equality of income at each point in time or over the entire planning horizon. Notice that the objective is directly related to F and fixes it at

$$F^* = \frac{n}{l} \quad \text{where } n < l.$$

The income egalitarian government's objective is satisfied when setting taxes such that $F(\tau)$ equals its target F^* .¹⁵ The tax rate τ_e that satisfies this is given by

$$\tau_e = [A(\alpha - (1 - \alpha)F^*)]^{-\frac{1}{\alpha}}. \quad (1.20)$$

Note that F^* depends on the number of agents in each group. For consistency I require that $F^* < \min\{1, F^g\}$ which is easily met for $l \ll n$ and reasonable values of α . If there are only a few capital owners, then the income egalitarian government chooses a very low F^* , and a high τ_e . At the other extreme, assume that there are as many capitalists as workers. That would correspond to a representative agent economy where each household would derive equal income and under intra-group income equality would get equal wage or capital income. A (strictly) income egalitarian government would charge relatively lower taxes in that case. The income egalitarian policy implies

$$\frac{d\tau_e}{dA} = \frac{1}{\alpha} [A(\alpha - (1 - \alpha)F^*)]^{-\frac{1}{\alpha}-1} (\alpha - (1 - \alpha)F^*) = \tau_e [\alpha A]^{-1} > 0$$

so that an increase in A leads to a higher choice of τ_e . For the growth rate I find

$$\gamma_e = \tau_e [\alpha A \tau_e^{-\alpha} - 1]$$

¹⁵The reason for working with F^* rather than with l/n directly is to avoid confusion with the distribution of groups in the economy. Of course, they coincide in the model.

$$= \tau_e \left[\frac{\alpha A}{A(\alpha - (1 - \alpha)F^*)} - 1 \right]$$

and $\frac{d\tau_e}{dA} > 0$. Thus, as under a right-wing or a left-wing policy an increase in A also increases the growth rate under the income egalitarian policy.

Proposition 1.5 *Under a strictly income egalitarian policy $\frac{d\tau_e}{dA} > 0$ and $\frac{d\gamma_e}{dA} > 0$.*

Consider the optimal right-wing policy with $\hat{\tau} = [\alpha(1 - \alpha)A]^{\frac{1}{\alpha}}$. Denote F under a right-wing policy by \hat{F} . Then

$$\hat{F} = \frac{\alpha^2}{1 - \alpha}. \quad (1.21)$$

which depends positively on the share of capital (elasticity of output with respect to capital). Thus, as the share of capital increases, the optimal policy of a right-wing government shifts more total income towards capital. Note that \hat{F} is independent of the level of development A .

Depending on the rate of time preference the left-wing government chooses to redistribute or not in the optimum. Suppose $\lambda > 0$. From (1.15) $\tilde{\tau}(1 - \lambda) = [(1 - \alpha)A]^{\frac{1}{\alpha}}$ and $\tilde{\tau} = \alpha A [(1 - \alpha)A]^{\frac{1 - \alpha}{\alpha}}$ so that $\tilde{\tau}$ and $\tilde{\eta}$ are independent of λ . Thus,

$$\eta + \lambda \tilde{\tau} = (1 - \alpha)A [(1 - \alpha)A]^{\frac{1 - \alpha}{\alpha}} + \rho - [(1 - \alpha)A]^{\frac{1}{\alpha}} = \rho$$

so that the inequality measure (1.19) in the optimum becomes

$$\tilde{F}(\lambda > 0) = \frac{\tilde{\tau} - \rho}{\rho} = \frac{\alpha A [(1 - \alpha)A]^{\frac{1 - \alpha}{\alpha}} - \rho}{\rho} = \frac{\alpha [(1 - \alpha)A]^{\frac{1}{\alpha}}}{(1 - \alpha) \rho} - 1 \quad (1.22)$$

where $\rho > [(1 - \alpha)A]^{\frac{1}{\alpha}}$ and $\alpha > \frac{2}{3}$ if $\tilde{\gamma}(\lambda > 0) > 0$. Clearly, $\frac{d\tilde{F}(\lambda > 0)}{dA} > 0$.

Next, suppose $\rho < [(1 - \alpha)A]^{\frac{1}{\alpha}}$ so that $\lambda = 0$. Then $\tilde{\tau}$ in (1.16) solves

$$\tau[1 - \alpha(1 - \alpha)A\tau^{-\alpha}] = \rho(1 - \alpha)$$

and is only implicitly defined. However, we know that $\tilde{\tau} > \hat{\tau}$ if $\rho > 0$. Income inequality is therefore given by

$$\check{F}(\lambda = 0) = \frac{\alpha}{1 - \alpha} - \frac{\tilde{\tau}^\alpha}{A(1 - \alpha)}. \quad (1.23)$$

and such that $\check{F}(\lambda = 0) < \hat{F}$ for $\tilde{\tau} > \hat{\tau}$. For the effect of efficiency (higher A) on $\check{F}(\lambda = 0)$ I calculate

$$\frac{d\check{F}(\lambda = 0)}{dA} = -\frac{\alpha\tilde{\tau}^{\alpha-1}(1 - \alpha)A\frac{d\tilde{\tau}}{dA} - (1 - \alpha)\tilde{\tau}^\alpha}{(A(1 - \alpha))^2}.$$

Simplifying and substituting for $\frac{d\tilde{\tau}}{dA}$ from (1.18) yields

$$\begin{aligned} \frac{d\check{F}(\lambda = 0)}{dA} &= \frac{(1 - \alpha)\tilde{\tau}^\alpha}{((1 - \alpha)A)^2} - \frac{\alpha\tilde{\tau}^{\alpha-1}}{(1 - \alpha)A} \frac{d\tilde{\tau}}{dA} \\ &= \frac{(1 - \alpha)\tilde{\tau}^\alpha}{((1 - \alpha)A)^2} - \frac{\alpha\tilde{\tau}^{\alpha-1}}{(1 - \alpha)A} \alpha(1 - \alpha)\tilde{\tau} (\tilde{\tau}^\alpha - \alpha(1 - \alpha)^2A)^{-1} \\ &= \frac{(1 - \alpha)\tilde{\tau}^\alpha}{((1 - \alpha)A)^2} - \frac{\alpha^2\tilde{\tau}^\alpha}{A} (\tilde{\tau}^\alpha - \alpha(1 - \alpha)^2A)^{-1}. \end{aligned}$$

The expression is positive if

$$\begin{aligned} \frac{(1 - \alpha)\tilde{\tau}^\alpha}{((1 - \alpha)A)^2} &> \frac{\alpha^2\tilde{\tau}^\alpha}{A} (\tilde{\tau}^\alpha - \alpha(1 - \alpha)^2A)^{-1} \\ \frac{1}{((1 - \alpha)A)} &> \alpha^2 (\tilde{\tau}^\alpha - \alpha(1 - \alpha)^2A)^{-1} \\ \tilde{\tau}^\alpha &> \alpha(1 - \alpha)^2A + \alpha^2(1 - \alpha)A \\ \tilde{\tau}^\alpha &> \alpha(1 - \alpha)A[(1 - \alpha) + \alpha] \end{aligned}$$

$$\bar{\tau} > \hat{\tau}.$$

As a left-wing government chooses $\bar{\tau} > \hat{\tau}$, $\frac{d\bar{F}(\lambda=0)}{dA}$ is positive. I summarize in

Lemma 1.4 *Under the optimal right-wing or the income egalitarian policy technological progress does not change the steady state total post-tax factor income distribution, $\frac{d\bar{F}}{dA} = 0$ and $\frac{dF^*}{dA} = 0$.*

Under the optimal left-wing policy technological progress shifts the steady state total post-tax factor income distribution towards capital, $\frac{d\bar{F}(\lambda=0)}{dA} > 0$, $\frac{d\bar{F}(\lambda>0)}{dA} > 0$.

It is noteworthy that an advance in development (higher A) causes the optimal left-wing government's policy to shift income towards the accumulated factor of production. As $\frac{d\bar{F}(\lambda=0)}{dA} > 0$ and $\frac{d\bar{F}(\lambda>0)}{dA} > 0$ that holds no matter whether the left-wing government redistributes or not.

1.5.2 Comparative Dynamics under Different Policies

In this section I hypothetically compare the effects of the different policies on an otherwise identical economy. It is then an interesting question how the income egalitarian policy compares to that of a growth maximizing government. For $\gamma_e = \hat{\gamma}$ one needs $\tau_e = \hat{\tau}$ which is satisfied if

$$\begin{aligned} A[\alpha - (1 - \alpha)F^*] &= \alpha(1 - \alpha)A \\ a^* &= \frac{\sqrt{4F^* + F^{*2}} - F^*}{2}. \end{aligned}$$

Proposition 1.6 *If $a^* = \frac{\sqrt{4F^* + F^{*2}} - F^*}{2}$, then $\tau_e = \hat{\tau}$, $\gamma_e = \hat{\gamma}$ and $F^* = \hat{F}$.*

Thus, there exists a particular value of the share of capital where $\gamma_e = \hat{\gamma}$ so that the income egalitarian policy is equivalent to a growth maximizing policy. Under that condition income egalitarianism is not bad for growth. The result crucially depends on the income egalitarian government's target F^* .¹⁶ Of course, the result allows for another interpretation in the model. If the share of capital is equal to α^* , then a growth maximizing policy leads to minimal post-tax factor income inequality. Seen from that angle, the model provides an example that there may be instances where efficiency and equity orientated policies lead to the same outcome.¹⁷ For all other α 's it follows that

$$\alpha \geq \alpha^* \Leftrightarrow \tau_e \geq \hat{\tau} \Leftrightarrow \gamma_e < \hat{\gamma}.$$

so that in general the income egalitarian government does not maximize growth.

Suppose $\alpha < \alpha^*$. Then $\tau_e < \hat{\tau} < \check{\tau}$, because a left-wing government chooses $\bar{\tau} > \hat{\tau}$. As $F(\tau)$ is decreasing in τ , it follows that $\bar{F} < \hat{F} < F^*$, implying that the average capital owner would have lower income than the average worker. It would also entail that the right-wing government chooses a policy that would grant more income to a worker than to a capitalist. That is consistent with the model's right-wing government's objective, which is not concerned with relative income, but seems very implausible and unrealistic. I will therefore not investigate that case any further.

¹⁶It may be interesting to note that if there are as many capital owners as workers, then strict income egalitarianism calls for $F^* = 1$ in which case $\alpha^* = \frac{\sqrt{5}-1}{2}$. This means that α^* would correspond to the *golden ratio*.

¹⁷In general, the two interpretations are not equivalent, however. There may be instances where an income egalitarian policy leads to maximum growth. This may be so if that policy targets a particular personal income distribution, which implies a particular factor income distribution which *may* lead to maximum growth, as the proposition shows. The growth maximizer targets growth, implying a particular factor income distribution, implying many, possibly different personal income distributions. Thus, to get equivalence more assumptions are required. In the model the equivalence is due to the assumption of no intra-group inequality.

Suppose $\alpha > \alpha^*$. Then $\tau_e > \hat{\tau}$ and one gets

$$\begin{aligned} \hat{\tau} < \tau_e < \check{\tau} &\Leftrightarrow \hat{F} > F^* > \check{F} &\Leftrightarrow \hat{\gamma} > \gamma_e > \check{\gamma} \\ \hat{\tau} < \check{\tau} < \tau_e &\Leftrightarrow \hat{F} > \check{F} > F^* &\Leftrightarrow \hat{\gamma} > \check{\gamma} > \gamma_e. \end{aligned}$$

Thus, in this case the income egalitarian policy may be closer to the optimal policy of a left-wing government. The two policies coincide if there exists a ρ , call it ρ_e , such that $\check{\tau} = \tau_e$. In general, however, the exact relationship between the growth rates under the income egalitarian or the optimal left-wing policies are ambiguous. Also, it is ambiguous whether the left-wing or the right-wing policy is closer to the income egalitarian government's policy in terms of post-tax income inequality. In either case the right-wing policy shifts more income towards capital in comparison to the income egalitarian government. That is what one would expect. In contrast, the left-wing government may shift relatively more income to capital or labour depending on how patient the workers are. If they are very impatient, $\tau_e < \check{\tau}$ and they will shift relatively more income to the workers, leading to lower growth than under the income egalitarian policy. If they are patient, the left-wing government will choose to shift relatively more income to capital and there will be higher growth than under the income egalitarian policy. With $\frac{d\hat{F}(\lambda=0)}{dA} > 0$ and $\frac{d\check{F}(\lambda>0)}{dA} > 0$ the effect of technological progress under a left-wing policy is also ambiguous. It is inequality reducing if $\check{F} < F^*$ and inequality enhancing if $\check{F} > F^*$.

Proposition 1.7 *If $a > a^*$ the right-wing government always grants more income to the capital owners in comparison to the income egalitarian government.*

The optimal policies imply

$$1. \quad \rho > \rho_e: \quad \hat{\tau} < \tau_e < \bar{\tau} \Leftrightarrow \hat{F} > F^* > \check{F} \Leftrightarrow \hat{\gamma} > \gamma_e > \check{\gamma}$$

or

$$2. \quad \rho < \rho_e: \quad \hat{\tau} < \bar{\tau} < \tau_e \Leftrightarrow \hat{F} > \check{F} > F^* \Leftrightarrow \hat{\gamma} > \check{\gamma} > \gamma_e.$$

Thus, the left-wing government grants relatively more or less income to the workers, and it has lower or higher growth than the income egalitarian government. Technological progress may reduce or increase income inequality under the left-wing policy compared to the income egalitarian policy. All these results depend on how patient the agents are. Interestingly, for patient workers the optimal left-wing policy leads to a post-tax factor income distribution that is more favourable to capital in comparison to the income egalitarian policy. The workers are, however, compensated for that by higher income growth.

That brings out clearly that the objective of an income egalitarian government is strictly concerned about relative incomes, whereas the left-wing government cares about the welfare of the workers. Thus, $\check{F} > F^*$, implying that the capitalists get relatively more income than the workers under a left-wing government, is consistent with that government's objective, because it gives the workers the highest welfare. According to Proposition 1.2 the left-wing government may mimic a right-wing, growth maximizing policy if the workers are very patient. Therefore, the workers do not prefer an income egalitarian policy, although it would give them relatively more income at each date t . That is so, because growth is higher under the left-wing policy, granting them higher consumption in the future which they prefer if they are patient. That shows how misleading

it may be to identify income egalitarian with left-wing policies.

It is worth reiterating that the income egalitarian is a very special egalitarian objective. Suppose a utility egalitarian chooses a tax policy somewhere between those optimal for the income egalitarian or a Rawlsian. Suppose further that one compares policies relative to the strictly income egalitarian policy F^* . If an outcome with some income inequality is better for the worse-off group than a strictly income egalitarian outcome then it is better according to a Rawlsian (utility leximin) policy. Thus, if $\tilde{F} > F^*$ and the workers are worse off than the capitalists under a left-wing policy, then a Rawlsian would choose lower taxes and move closer to the workers' preferred, left-wing policy. That is so, because such a move would be Paretian as all agents would prefer an F such that $F > F^*$. Thus, a (total) utility egalitarian objective satisfying the Pareto-principle would also imply a policy $F > F^*$. Thus, for patient agents a large class of egalitarian objectives satisfying the Pareto-principle would shift relatively more income to capital than to labour. The argument is different when $\hat{F} < F^* < \tilde{F}$, and the income egalitarian policy leads to higher growth than the left-wing policy. If evaluated relative to F^* , it is not so clear what a utility leximin policy entails, that is, whether it would be closer to \hat{F} or \tilde{F} . A precise analysis would require whether that policy is concerned about the worst-off at any point in time or the worst-off, intertemporal welfare of the agents. In either case any policy away from F^* would violate the Pareto-principle, because it would make one group better off and another group worse off. In that case it is also unclear what a total utility egalitarian would choose relative to a Rawlsian.

For the purpose of the paper's analysis, however, it suffices to conclude that strictly income egalitarian and left-wing policies are generally not the same and lead to quite different post-tax factor income distributions and growth rates.

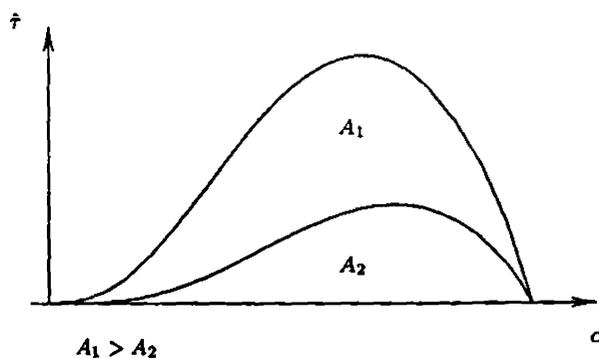
From the discussion so far it is clear that α plays a major role in the model's analysis. The importance of the share of capital for the relationship between taxes and growth has, for instance, been emphasized by Stokey and Rebelo (1995). In appendix A.4 I find the following reactions of γ , τ and F due to changes in α .

Proposition 1.8 *An increase in α leads to*

1. $\frac{d\hat{\tau}}{d\alpha} \geq 0$, $\frac{d\hat{\gamma}}{d\alpha} > 0$ and $\frac{d\hat{F}}{d\alpha} > 0$ under the right-wing policy,
2. $\frac{d\tau_e}{d\alpha} > 0$, $\frac{d\gamma_e}{d\alpha} \geq 0$ and $\frac{dF^*}{d\alpha} = 0$ under the income egalitarian policy,
3. $\frac{d\check{\tau}}{d\alpha} \geq 0$, $\frac{d\check{\gamma}}{d\alpha} > 0$ and $\frac{d\check{F}}{d\alpha} > 0$ under a left-wing, $\lambda = 0$ policy,
4. $\frac{d\check{\tau}}{d\alpha} = 0$, $\frac{d\lambda}{d\alpha} \geq 0$, $\frac{d\check{\gamma}}{d\alpha} > 0$ and $\frac{d\check{F}}{d\alpha} \geq 0$ under a left-wing, $\lambda \geq 0$ policy.

According to the proposition unambiguous responses of policy variables to changes in the share of capital do not exist in general. For instance, a plot of the growth maximizing tax rate $\hat{\tau}$ for different α and A reveals a pattern as in Figure 1.2. Thus, it is possible for given A that two values of α lead to the same $\hat{\tau}$, but

Figure 1.2: The growth maximizing τ for different α and A



different growth rates $\hat{\gamma}$, because $\frac{d\hat{\gamma}}{d\alpha} > 0$. Similar reasoning applies for the non-

redistributing, left-wing government. That suggests that information about tax rates alone may not explain growth rates in a cross-country OLS analysis.

Up to this point the comparative dynamic analysis has emphasized normative issues about growth and the income distribution. In terms of positive economics the model endogenizes public policy in that optimal policy variables are functions of underlying economic variables. Except for F^* of the income egalitarian government all policy variables are functions of α and A in the chapter. Endogenous fiscal policy has important implications for estimating the effect of fiscal policy on growth by cross-country OLS regressions. I turn to the implications for these empirical models below. To that end I assume that policy has a certain element of endogeneity and I use this model with complete endogeneity to point out some difficulties that have been encountered in the literature and offer some explanations why these difficulties arise when assuming that policy is exogenous.

1.6 Implications for Empirical Research

The presence of endogeneity elements in fiscal policy raises various problems for estimation in any cross-country growth regression as has been mentioned in Easterly and Rebelo (1993a), section 4. Instead of addressing the fundamental problem associated with endogenous regressors I will focus on some empirical findings in the simple cross-country OLS regression literature that appear to be at odds with theoretical results. Throughout this section 'simple (cross-country) OLS regression' refers to the very common procedure of taking averages of data over time and running cross-country OLS regressions over these averaged data.

Many authors have analyzed the effects of taxes or other fiscal variables on growth. For instance, Koester and Kormendi (1989), Barro (1991), Levine and

Renelt (1992), Easterly and Rebelo (1993a), or Sala-i-Martin (1997) have run simple cross-country OLS regressions to investigate the issue. Most of them find that tax rates or some other, tax financed fiscal variables have a negative, but - when controlling for initial income - insignificant effect on growth. That casts some doubt on theoretical findings such as Barro (1990) and Barro and Sala-i-Martin (1990) who show that some of the tax financed, fiscal policy variables should have a significant and sometimes positive effect on growth. The reason for the discrepancy may be due to the fact that policy is endogenous to some extent and that treating tax variables as exogenous causes simple cross-country OLS regressions to give a misleading picture of the relationship between fiscal policy and economic growth.

In what follows I set empirical findings in the literature against implications of the model above when running simple OLS regressions. According to the chapter's theoretical model the empirical steady state relationship between growth and public policy for a country i is of the form

$$\gamma_i = f(\tau_i(\alpha_i, A_i), \lambda_i(\alpha_i, A_i), \alpha_i, A_i, \rho_i) = f(\alpha_i, A_i, \rho_i) \quad (1.24)$$

where $f(\cdot)$ is a highly non-linear function of the fundamental variables α_i , A_i and ρ_i . Most authors assume that policy is exogenous which would mean that

$$\gamma_i = g(\tau_i, \lambda_i, \alpha_i, A_i, \rho_i)$$

where τ_i and λ_i are independent of the other variables included in $g(\cdot)$. The important point to notice is that $f(\cdot)$ and $g(\cdot)$ may be observationally equivalent when particular combinations of α_i , A_i , and ρ_i lead to the same growth rate under either assumption. To capture this feature I assume that empirical and theoretical

researchers agree that the Data Generating Mechanism (DGP) can be expressed theoretically by the joint probability distribution $D(\gamma, \tau, \lambda, \alpha, A, \rho)$. Notice that the probability distribution is expressed in terms of steady state variables and thus ignores any time dependence. That reflects the common procedure by empirical researchers to take averages over time of variables they consider of interest. Furthermore, most researchers would agree that ρ is a *soft* variable which is almost impossible to observe. Therefore, I ignore any impact the time preference rate ρ has on growth. Concerning the fundamental variables α_i and A_i I assume that they are country-specific, that is, independent and thus uncorrelated across countries.

In the rest of the chapter I discuss implications for running simple OLS regressions under the assumption that policy is exogenous when in fact it is endogenous. The following table presents a summary of what has been shown in the previous sections, that is, it summarizes theoretical correlations between the growth or tax rates and the share of capital or the state of technology under different (endogenous) policy regimes.

Table 1: Growth and Policy Effects

Right-Wing			
	$\hat{\tau}$	$\hat{\gamma}$	\hat{F}
A	+	+	0
α	?	+	+

Income Egalitarian			
	τ_e	γ_e	F^*
A	+	+	0
α	+	?	0

Left-Wing, $\lambda = 0$			
$\lambda = 0$	$\hat{\tau}$	$\hat{\gamma}$	\hat{F}
A	+	+	+
α	?	+	+

Left-Wing, $\lambda \geq 0$				
	$\hat{\tau}$	λ	$\hat{\gamma}$	\hat{F}
A	0	-	+	+
α	0	?	+	?

sign: (+) - positive, (-) - negative, (?) - ambiguous

If policy is endogenous and the true model is as in (1.24), then a linearized

version of it leads to the OLS regression

$$\gamma_i = \beta'_0 + \beta'_1 \alpha_i + \beta'_2 \tau_i(\alpha_i, A_i) + \beta'_3 \lambda_i(\alpha_i, A_i) + \beta'_4 A_i + \epsilon_i$$

where ϵ_i is a country-specific disturbance term which is assumed to be uncorrelated with each of the regressors and $E(\epsilon_i) = 0$. If that model were estimated by OLS, problems of identification and multicollinearity would arise. But there are other problems. In the model A_i captures the level of development. One could in principle construct indices that would reflect the level of development using some economically meaningful criteria. On the other hand, it makes sense to assume that A_i changes over time. The model has analyzed the effect of exogenous, once-and-for-all changes in A_i . An analysis of that kind is similar in spirit to models with exogenous technological change, which is commonly thought to be unobservable. The discussion about the *Solow-Residual* reflects these difficulties. (See, for instance, Barro and Sala-i-Martin (1995), chpt. 10.4.) Therefore, I assume either that countries differ widely in the level of development, which is hardly disputable, and that data on A_i are not available, or that development changes over time so that correct information about the level of A_i over time would be unobservable. Both assumptions imply that information on A_i would also be contained in the disturbance term and that A_i would not feature in the OLS regressions. If that is the case, the estimated coefficients in the regression above are generally biased.¹⁸ Instead of analyzing the 'true' model further, I will

¹⁸To be more precise, the estimates of $\beta'_j, j = 2, 3$ will surely be biased unless $\beta'_4 = 0$, that is, unless A_i is *weakly exogenous*, in which case it need not have been in the model to begin with. See Engle, Hendry and Richard (1983). By assumption A_i plays a role so that $\beta'_4 \neq 0$. Under what conditions the assumption of endogenous policy produces biased estimates in simple OLS cross-country growth regressions has been analyzed in a more general framework in 'Why Run a Million Regressions? On Some Difficulties of Doing OLS in Cross-Country Growth Analyses', mimeo, TU Darmstadt, Nov. 1997.

investigate the implications for simple OLS regressions that have been presented in the literature and interpret the results in the light of this model.

Given the measurement problem of A_i , I simplify the analysis by analyzing the second-best, but operationally viable model

$$\gamma_i = \beta_0 + \beta_1 \alpha_i + \beta_2 \tau_i(\alpha_i, A_i) + \beta_3 \lambda_i(\alpha_i, A_i) + v_i \quad (1.25)$$

where $v_i = v_i(A_i, \epsilon_i)$ is a country-specific disturbance term which depends on A_i . If that model were estimated by OLS, multicollinearity and the omission of a relevant variable would be a problem. However, I will focus on the problem caused by assuming that policy is exogenous. A standard justification for treating policy as exogenous is given by a randomization argument. For example, Barro (1989a) argues that in a large sample the optimal policies of governments may be treated as randomly generated. That comes close to saying, the optimal policies are exogenous. But in light of this paper's model the argument would not hold. Even if all countries had different governments with different welfare functions so that e.g. the tax rates looked randomly chosen, the model predicts that *all* policies would be influenced by fundamental economic variables included or not included in the sample. Below I focus on exactly that problem.

1.6.1 Taxes

Most studies investigating the relation between taxes and growth use the variables

$$av_1 = \frac{TAR}{TAB}, \quad m_1 = \frac{dTAR}{dTAB}, \quad av_2 = \frac{TAR}{GDP}, \quad m_2 = \frac{dTAR}{dGDP}$$

where TAR denotes the total tax revenues in a country, and TAB denotes the tax base. Usually, av_1 and av_2 are taken to be indicators of the *average tax rate*, whereas m_1 and m_2 are often used as proxies for the *marginal tax rate*. All these variables are problematic representations for aggregate relationships between fiscal policy and growth. For instance, the 'marginal tax rate' m_2 is commonly obtained by regressing tax revenues on GDP, where the resulting regression coefficient is then interpreted as the 'marginal tax rate'. (See, for instance, Koester and Kormendi (1989) or Easterly and Rebelo (1993b).) That raises the question whether GDP really is the relevant tax base.

In terms of the theoretical model the constant capital tax rate τ is equal to both av_1 and m_1 , because the latter are directly proportional to the level of and the change in the *tax base*. That appears problematic as most researchers would in general expect the average and the marginal tax rates to differ, in particular one would often expect that marginal tax rates (m) are not lower and often higher than average tax rates (av). One should bear in mind, however, that for economies with flat-rate tax systems, which are often analyzed and sometimes advocated to be adopted in the theoretical literature, $av = m$ would certainly hold. Thus, a discussion of the effect of such tax variables on growth in simple OLS cross-country growth regressions may be worth for that and for theoretical reasons. Furthermore, the theoretical model has the advantage that the expected correlations between av_1 or m_1 and A or α are of the same sign. These signs are of primary interest in this section and are succinctly captured by the properties of τ . For these reasons I will concentrate on regressor variables related to the tax base, that is, on av_1 , m_1 and τ first.

Suppose the simple cross-country OLS regression

$$\gamma_i = \delta_0^1 + \delta_1^1 \tau_i + u_i^1 \quad (1.26)$$

is run, treating τ_i as exogenous, when in fact it is endogenous as in (1.25). The superscript 1 indicates that the regression is run on tax variables related to the tax base. The OLS estimator of δ_1^1 , call it d_1^1 , is in general biased. Note that the OLS estimator is given by

$$d_1^1 = \frac{\sum_i (\tau_i - \bar{\tau}) \gamma_i}{\sum_i (\tau_i - \bar{\tau})^2}$$

Substituting in the 'true' model (1.25) yields

$$\begin{aligned} d_1^1 &= \frac{\sum_i (\tau_i - \bar{\tau}) (\beta_0 + \beta_1 \alpha_i + \beta_2 \tau_i (\alpha_i, A_i) + \beta_3 \lambda_i (\alpha_i, A_i) + v_i)}{\sum_i (\tau_i - \bar{\tau})^2} \\ &= \beta_2 + \beta_1 \sum_i c_i \alpha_i + \beta_3 \sum_i c_i \lambda_i + \sum_i c_i v_i \end{aligned}$$

where $c_i = \frac{(\tau_i - \bar{\tau})}{\sum_i (\tau_i - \bar{\tau})^2}$ and $v_i = v_i(A_i, \epsilon_i)$. Taking expectations under the assumption that policy is in fact endogenous I obtain¹⁹

$$E(d_1^1) = \beta_2 + \beta_1 E \left[\sum_i c_i \alpha_i \right] + \beta_3 E \left[\sum_i c_i \lambda_i \right] + E \left[\sum_i c_i v_i \right].$$

Hence, the OLS estimator d_1^1 is generally biased because the sum of the expectations of the expressions on the RHS is non-zero. The bias is due to three factors. First, the expression $\sum_i c_i v_i$ is positive for all non-redistributing governments since $cov(\tau_i, A_i) > 0$ from Table 1. Second, $cov(\tau_i, \lambda_i) \geq 0$, which is

¹⁹Notice that under exogenous policy the expectation would be conditional on $(\tau_i, \lambda_i, A_i, \alpha_i)$, since all researchers agree on the 'true' DGP by assumption. Under endogenous policy the expectation is only conditional on (A_i, α_i) , of course.

zero if all governments do not redistribute wealth, or positive if they do. Third, for economies with A_i, α_i such that $cov(\tau_i, \alpha_i) \geq 0$, the bias is clearly positive. Alesina and Rodrik (1994), fn. 7, cite theoretical support for the prediction that an increase in α raises redistributive pressure leading to higher taxes. However, if $cov(\tau_i, \alpha_i) < 0$ the sign of the bias may be ambiguous. I conclude that for a large class of policies the reported OLS estimate of the effect of tax rates on growth is biased upwards, when the tax rates are measured by variables like τ , av_1 or m_1 , which are exactly related to the tax base. Thus, in many cases any reported bad effect of taxes on growth may be understated. It is, however, also possible that the coefficients are biased downwards so that any reported bad effect would be overstated. That is the case when $cov(\tau_i, \alpha_i)$ is negative and larger in absolute value than $cov(\tau_i, \lambda_i)$ and $cov(\tau_i, v_i)$.

As the error term correlates with the regressor τ_i , or any transformation of τ_i depending on α or A , the conditions of *weak exogeneity* in the sense of Engle et al. (1983) are violated. (See also Spanos (1986), chpts. 19.3.) Hence, statistical inferences on d_1^1 are not really possible. So reported t -statistics on d_1^1 will not report the true significance of the estimated coefficient if one wants to know whether d_1^1 is significantly different from zero. It is also known that for omitted variable or misspecification problems the estimator may have lower or higher variance than another estimator, if one knows the true variance of v_i . (For that see, for instance, Greene (1991) chpt. 9, or Johnston (1984), chpt. 6.) But v_i is estimated with d_1^1 when doing OLS so that the estimated sum of squared residuals, used for hypothesis testing, may be higher or lower than the true variance of the residuals. That provides another reason why the $var(d_1^1)$ may be affected by the bias.

Proposition 1.9 *Let the average tax rate av_1 (the ratio of tax revenues to tax base) or the marginal tax rate m_1 (the ratio of the change in tax revenues to the change in tax base) have the same properties as τ . Simple cross-country OLS regressions of the growth rate on τ , av_1 or m_1 assuming that policy is exogenous, when in fact it is endogenous, yields generically biased estimates for the effect of taxes on growth. In many cases and definitely if $cov(\tau, \alpha) \geq 0$, the bias is positive. In the model correct statistical inferences of the effect of taxes on growth are not possible when using OLS estimators.*

A positive bias in regressors like τ , av_1 or m_1 implies that coefficients measuring the effect of these variables on growth are *overestimated*. Thus, either any reported negative effect of these tax variables on growth is understated or any reported positive effect is overstated.²⁰ In the theoretical literature taxes usually negatively affect growth. Hence, if one found a negative effect in empirical studies (simple OLS cross-country growth regressions) using regressors like τ , av_1 or m_1 , with the properties derived in this chapter's theoretical model, one could be 'sure' that the effect is indeed negative. Even a reported positive effect would not invalidate the theoretical prediction that the 'true' effect may really be negative. In that sense Proposition 1.9 provides a negative result for hypothesis testing, but it offers an important 'positive' link to empirical research in that it provides an argument that the theoretically postulated relationship between taxes and growth is 'true', if the regressors are τ , av_1 or m_1 and have the theoretical model's properties.

However, Proposition 1.9 rests on some strong assumptions. The determination of a country's most relevant tax base for growth is extremely difficult.

²⁰Notice that from the statistical literature a positive (negative) bias in $\hat{\beta}$ is equivalent to a systematic overestimation (underestimation) of the 'true' value $\beta \in \mathbb{R}$.

Usually governments raise all sorts of taxes on different tax bases. Furthermore, countries do not necessarily use the same tax bases and for some tax bases data are difficult to obtain. Thus, for researchers working with aggregate data it has become common and it is convenient to relate tax revenues to GDP or aggregate income, on which data are most commonly available. In fact, the vast majority of cross-country growth regression analyses use av_2 or m_2 as an indicator of taxes or tax policy. (For instance, all authors quoted so far use at least one of these variables.) However, the choice of regressor variables may affect the underlying structure of the estimated and the statistical model. Going from τ or av_2 to m_2 often entails a non-linear reparametrization of an original estimated model or a different model altogether.²¹ For instance, the choice of av_2 as a regressor variable means that

$$\gamma = f^2\left(\frac{\tau k_t}{y_t}, \alpha_i, A_i, \rho_i\right) = f^2(\alpha_i, A_i, \rho_i)$$

which is usually not the same as $f(\cdot)$ in (1.24). Instead of trying to find a reparametrization of the original model I assume all policy variables react to the fundamental variables as in Table 1 and $f^2(av_2, \cdot)$ can be linearized as

$$\gamma_i = \beta_0^2 + \beta_1^2 \alpha_i + \beta_2^2 av_{2i} + v_i^2(A_i, \epsilon_i).$$

The same assumptions and arguments apply for m_2 and $f^2(m_2, \cdot)$. From the

²¹It should be borne in mind that the theoretical model has been assumed to capture a *broad class* of tax structures. In that sense the results derived from the simple theoretical model are supposed to capture essential properties of the relation between taxes and growth in general. Thus, it is perfectly valid and common scientific procedure to reduce a problem to a simple model and use some, but not necessarily the closest estimable and statistical models to test the theoretical results. On this point see, for instance, Spanos (1986), chpts. 1.2 and 26.

theoretical model it is not difficult to verify that²²

$$av_2 \equiv \frac{\tau k_t}{y_t} = \frac{\tau^\alpha}{A(1-\lambda)^{1-\alpha}} \quad \text{and} \quad m_2 \equiv \frac{d(\tau k_t)}{dy_t} = \frac{av_2}{1-\alpha}. \quad (1.27)$$

Thus, $m_2 > av_2$ which is a property most researcher would expect. For instance, Koester and Kormendi (1989), p. 370/1, find that marginal tax rates m_2 are significantly different from average tax rates av_2 and that 'across countries marginal tax rates average about one-and-one-half to two times average tax rates'.

For the growth maximizing policy $av_2(\hat{\tau}) = \alpha(1-\alpha)$, $m_2(\hat{\tau}) = \alpha$ and so $\frac{dav_2(\hat{\tau})}{dA} = \frac{dm_2(\hat{\tau})}{dA} = 0$. For all non-redistributing polices I find

$$\frac{dav_2}{dA} = \frac{\tau^\alpha}{A^2} \left[\frac{\alpha A}{\tau} \frac{d\tau}{dA} - 1 \right] \quad \text{and} \quad \frac{dm_2}{dA} = \left(\frac{1}{1-\alpha} \right) \frac{dav_2}{dA}.$$

Consider the non-redistributing, left-wing government with $\frac{d\hat{\tau}}{dA}$ given by (1.18).

The expression $\frac{\alpha A}{\tau} \frac{d\tau}{dA} - 1$ is negative if

$$\begin{aligned} \alpha^2(1-\alpha)A(\tau^\alpha - \alpha(1-\alpha)^2A)^{-1} &< 1 \\ \alpha(1-\alpha)^2A + \alpha^2(1-\alpha)A &< \tau^\alpha \\ \alpha(1-\alpha)A &< \tau^\alpha \\ \hat{\tau} &< \tau \end{aligned} \quad (1.28)$$

which is true for a non-redistributing, left-wing government because $\hat{\tau} > \tau$. Hence, $\frac{dav_2(\hat{\tau})}{dA}$ and $\frac{dm_2(\hat{\tau})}{dA}$ are negative. Under the income egalitarian policy av_2 is given by $av_2 = \alpha - (1-\alpha)F^*$ and so independent of A . Hence, under that policy

²²Recall $y_t = Ak_t^\alpha g_t^{1-\alpha}$ and $g_t = (1-\lambda)\tau k_t$. Then $av_2 = \frac{\tau k_t}{y_t} = \frac{\tau k_t}{Ak_t^\alpha ((1-\lambda)\tau k_t)^{1-\alpha}}$ which reduces to the expression in (1.27). Furthermore, $\frac{dy_t}{d(\tau k_t)} = (1-\alpha)Ak_t^\alpha (1-\lambda)^{1-\alpha}(\tau k_t)^{-\alpha}$. Taking the inverse yields $\frac{d\tau k_t}{dy_t} = \frac{\tau^\alpha}{(1-\alpha)A(1-\lambda)^{1-\alpha}} = \frac{av_2}{1-\alpha}$.

m_2 does not depend on A as well. For a redistributing government

$$av_2 = \frac{\rho}{A[(1-\alpha)A]^{\frac{1-\alpha}{\alpha}}}$$

which decreases in A . By a similar argument one verifies that m_2 decreases in A under that policy. Hence, $cov(av_{2i}, A_i) \leq 0$ and $cov(m_{2i}, A_i) \leq 0$. To keep matters simple I assume that the covariance of av_{2i} (m_{2i}) and α_i is non-positive. For instance, for *given* tax policies of all non-redistributing governments an increase in α reduces av_2 . Thus, the results below are conditional on that or that $cov(av_{2i}, A_i)$ ($cov(m_{2i}, A_i)$) is more important than $cov(av_{2i}, \alpha_i)$ ($cov(m_{2i}, \alpha_i)$). The condition is testable and any results derived from it are therefore falsifiable. By these arguments av_2 co-varies with A or α in the same way as m_2 does. To simplify I present the following arguments in terms of av_2 with the understanding that they also apply for m_2 .

Suppose one assumes exogenous policy and runs the regression

$$\gamma_i = \delta_0^2 + \delta_1^2 av_{2i} + u_i^2.$$

where the superscript 2 indicates that model $f^2(\cdot)$ is being analyzed. Proceeding as above reveals that the estimate d_1^2 is biased. If $cov(av_{2i}, \alpha_i) \leq 0$, then d_1^2 is generically biased downwards, that is, it is biased down unless all governments are income egalitarian or growth maximizing and $cov(av_{2i}, \alpha_i) = 0$. Hence, for many data sets any measured negative effect of av_2 on γ may be overstated. Similar arguments hold for m_2 .

Proposition 1.10 *Assume policy variables react to fundamental economic variables as in Table 1, the DGPs for $f(\cdot)$ and $f^2(\cdot)$ are the same, and $\text{cov}(av_{2i}, \alpha_i)$ and $\text{cov}(m_{2i}, \alpha_i)$ are non-positive where av_2 denotes the ratio of tax revenues to GDP and m_2 denotes the ratio of the change in tax revenues to the change in GDP. A simple cross-country OLS regression of γ on av_2 or m_2 yields generically biased estimates. Unless all governments in the sample pursue growth maximizing or income egalitarian policies, the estimates are biased downwards.*

The theoretical model offers the following explanation for a positive bias: Tax rates higher than the growth maximizing ones lead to relatively lower growth. That explains why many authors find a negative point estimate of δ_1^2 . For instance, Koester and Kormendi (1989) report $d_1^1 = -0.074$ (-2.18) for a simple regression of growth on average tax rates.²³ (All t-statistics are shown in parentheses.) However, in the model the bad direct effect of high taxes is compensated by the good effect of taxes channelled into production via public services. That good effect is larger when A is higher and it is ignored when assuming exogenous policy in simple OLS cross-country regressions.

Many authors include initial income y_0 in their regressions (see, for instance, Levine and Renelt (1992), Engen and Skinner (1992), or Sala-i-Martin (1997)) and find that controlling for initial income renders their estimated effects of taxes on growth insignificant. Notice, however, that having initial income in the regression first and then adding a tax variable often renders the estimated coefficient of the effect of initial income on growth insignificant. For instance, Easterly and Rebelo (1993a) find in their growth regressions that seven(!) out of thirteen tax variables render the estimated coefficients for the effect of initial income on growth insignificant. This possibility is often ignored, although it provides a rationale

²³The reported coefficient on m_2 was -0.025 (-1.87).

for not including *either* tax measures *or* initial income in growth regressions. Thus, from a statistical viewpoint alone there is no reason why some authors (e.g. Levine and Renelt (1992) or Sala-i-Martin (1997)) conclude that fiscal variables are mostly non-robust regressors when fiscal variables are highly correlated with initial income. Of course, insignificance is only a statistical indicator that the point estimate of a variable is close to zero.

A downward bias of d_1^2 plays an important role in explaining why one is likely to obtain insignificant coefficients when including initial income in a regression such as

$$\gamma_i = \xi_0^2 + \xi_1^2 av_{2i} + \xi_2^2 y_{0i} + w_i^2. \quad (1.29)$$

Elementary econometrics (see, for example, Johnston (1984), chpt. 3.4) tells one that the OLS estimator of ξ_1^2 is given by

$$\hat{\xi}_1^2 = \frac{d_1^2 - \hat{\xi}_{13}^2 \hat{\xi}_{32}^2}{1 - \hat{\xi}_{23}^2 \hat{\xi}_{32}^2}$$

where d_1^2 reflects the simple regression of γ_i on av_2 , and $\hat{\xi}_{13}^2$ the regression of γ_i on y_{0i} , $\hat{\xi}_{23}^2$ the regression of av_2 on y_{0i} and $\hat{\xi}_{32}^2$ the regression of y_{0i} on av_2 . For instance, Koester and Kormendi report the estimates

$$\begin{aligned} d_1^2 &= -0.074 (-2.18) \quad , \quad \hat{\xi}_{13}^2 = -0.053 (-3.52), \\ \hat{\xi}_{23}^2 &= +0.293 (+6.32) \quad , \quad \hat{\xi}_{32}^2 = -0.005 (-0.11), \end{aligned}$$

from which the imputed value of $\hat{\xi}_{32}^2$ is 1.3389. The signs and magnitudes of the estimates are shared by many other papers. It is instructive to note that including initial income reduces the point estimate for the effect of taxes on

growth (d_1^2 vs. $\hat{\xi}_1^2$) dramatically (by a factor of 14.8) and makes it statistically insignificant, that is, makes it assume a value close to zero.

Firstly, notice that taxes and initial income are positively and comparatively strongly correlated ($\hat{\xi}_{23}^2$), indicating that *Wagner's Law* holds which asserts a positive relation between the size of the government and per capita income. Secondly, initial income is negatively related to growth, which is interpreted by some authors as indicating convergence in the growth process, that is, countries with lower initial income tend to have higher growth rates. The theoretical model implies that some of those simple regression coefficients are biased. For the argument I wish to make that is not a problem, though, because what matters are the signs of the simple estimates.

So suppose that $\hat{\xi}_{13}^2$, $\hat{\xi}_{23}^2$ and $\hat{\xi}_{32}^2$ were unbiased estimates. From Proposition 1.10 it follows that any negative value of d_1^2 is overstated so that the true δ_1^2 is less negative than the value reported. If one fixes the other estimates one gets

$$\hat{\xi}_1^2 = \frac{d_1^2 - (-0.053) (+1.3389)}{1 - (+0.293) (+1.3389)} = (d_1^2 + 0.0710) \times 1.6455.$$

Underestimation means that a negative estimate d_1^2 would overstate a true (negative) value δ_1^2 , which raises the problem of how big the bias is. Proposition 1.10 implies that the 'true' value may well be positive. For the moment suppose $\delta_1^2 = d_1^2 \times x$ where $x \in (-\infty, 1)$ and that the unknown 'true' value is non-positive, $\delta_1^2 \leq 0$. The following table reports imputed 'true' estimates of $\hat{\xi}_1^2$ for different magnitudes of the bias for the reported value of $d_1^2 = -0.074$.

Table 2: Bias Effects on $\hat{\xi}_1^2$ when $d_1^2 = -0.074$

x	0.99	0.95	0.9	0.85	0.80	0.5	0.10
$\hat{\xi}_1^2$	-0.004	+0.001	+0.007	+0.013	+0.019	+0.056	+0.104

From the table a five-percent bias raises the 'true' point estimate of $\hat{\xi}_1^2$ from -0.005 to $+0.001$. Thus, a slight bias in d_1^2 pushes the 'true' point estimate of $\hat{\xi}_1^2$ towards zero. That is interesting for the following reason: Many researchers start with growth regressions including initial income first and then add policy variables. Most of them find that variables such as av_2 take on values close to zero. Given the bias in d_1^2 the 'true' point estimate of $\hat{\xi}_1^2$ is likely to be positive and maybe statistically significant. Most authors argue that the reason they find insignificant values for variables such as av_2 may be due to the high correlation between initial income and av_2 , that is, high $\hat{\xi}_{23}^2$ or $\hat{\xi}_{32}^2$. In that sense the above calculations provide a different explanation why one may get insignificant statistics. Estimates close to zero of $\hat{\xi}_1^2$ may mean that the 'true' value is positive and that the bias in d_1^2 pushes that value down towards zero. Hence, one cannot exclude the possibility that a statistic such as the ratio of tax revenues to GDP may have a positive effect on growth.

The same holds for m_2 , of course, that is, one cannot exclude the possibility that marginal tax rates positively affect growth. For example, Perotti (1996) is puzzled to find positive and significant coefficients for the effect of marginal tax rates on growth in cross-country growth regressions using 2SLS.

A negative bias in the coefficients for av_2 or m_2 , obtained in simple OLS cross-country growth regressions, implies that the theoretical prediction that taxes negatively affect growth is inherently untestable. Any reported value of the effect of tax variables such as av_2 or m_2 on growth is always *underestimated*, that is, a reported negative effect would be overstated and a reported positive effect would be understated. As a reported negative effect may well be positive in that case, the theoretical prediction cannot be corroborated when using variables as such av_2 or m_2 and the chapter's theoretical model captures general properties of the

effect of taxes and policy on growth.

Recall that the argument runs in an opposite direction when employing the tax rate variables m_1 or av_1 . Proposition 1.9 tells us that the estimated effect of these tax rate variables on growth is biased upwards so that any measured negative effect of tax rates would be understated.

Hence, employing the chapter's theoretical correlations implies that in often used, simple cross-country OLS growth regressions the estimated coefficients of the effect of tax rate variables like av_1 or m_1 on growth are generally biased upwards and that the estimated coefficients of the effect of tax rate variables like av_2 or m_2 on growth are generally biased downwards. Thus, it matters a great deal which tax variables one includes in simple cross-country OLS growth regressions when one assumes that policy is endogenous and the most simple regressions produce biased estimates.

1.6.2 Public Investment

Sometimes growth is regressed on the ratio of government investment to GDP or total (private and public) investment to test how public investment such as changes in infrastructure affect growth. Theoretically, a positive relationship is expected. (See, for instance, Aschauer (1989), Barro (1990), Barro and Sala-i-Martin (1990), or Barro and Sala-i-Martin (1995), chpt. 4.4.) Surprisingly, in many empirical studies such as Barro (1989b), (1991), or Barro and Sala-i-Martin (1995) chpt. 12.3 the effect of these variables on growth turns out to be statistically insignificant. I analyze this somewhat puzzling phenomenon from the viewpoint of the chapter's theoretical model in which steady state total in-

vestment is given by $i_t = \gamma k_t$ and

$$\begin{aligned} p &\equiv \frac{g_t}{y_t} = \frac{1}{A} \left(\frac{g_t}{k_t} \right)^\alpha = \frac{((1-\lambda)\tau)^\alpha}{A}, \\ q &\equiv \frac{g_t}{\gamma k_t} = \frac{(1-\lambda)\tau}{\gamma}. \end{aligned} \tag{1.30}$$

Note that g_t are public services that are provided as a public input in production. Barro (1991) uses public expenditure data to test his theoretical predictions which are based on g_t arguing that it is a proxy for the change in the stock of public capital. That is a problematic assumption, possible implications of which he discusses. For consistency with empirical results, which are often based on that, I assume that g_t is indeed a good proxy for public investment, that is, a proxy for the change in the stock of public capital for which data are difficult to obtain. Furthermore, in line with the previous discussion on taxes and for simplicity I assume that the correlation between these ratios and α_i is less important than that between the ratios and A_i . Thus, all results below are conditional on that assumption. It is interesting to note, however, that for *given* tax policies (exogenous policy), that is, for given τ and λ both p and q are decreasing in A or α , since $\gamma_A, \gamma_\alpha > 0$ for given tax policies, so that the OLS coefficients measuring the effect of these variables on growth would be biased downwards even under the assumption of exogenous policy.

Lemma 1.5 *For given tax policies (exogenous policy), simple cross-country OLS regressions of the growth rate on p or q produce coefficients that are biased downwards.*

For endogenous policy I will discuss the ratio of public investment to total

investment (q) first. To this end let $b \equiv (1 - \lambda)\tau$. Then

$$\frac{db}{dA} = -\frac{d\lambda}{dA} \tau + \frac{d\tau}{dA} (1 - \lambda)$$

which is positive for all policies considered. I want to show that $\frac{dq}{dA} < 0$. Then

$$\text{sgn} \left(\frac{dq}{dA} \right) = \frac{db}{dA} \gamma - \frac{d\gamma}{dA} b = \frac{db}{dA} (r - \tau - \rho) - \left(\frac{dr}{dA} - \frac{d\tau}{dA} \right) b = q_1 + q_2$$

where $q_1 \equiv \left(\frac{db}{dA} r - \frac{d\tau}{dA} b \right)$ and $q_2 \equiv \frac{db}{dA} (-\tau - \rho) + \frac{d\tau}{dA} b$. Clearly, $q_2 < 0$, since

$$\begin{aligned} q_2 &= \left(-\frac{d\lambda}{dA} \tau + \frac{d\tau}{dA} (1 - \lambda) \right) (-\tau - \rho) + \frac{d\tau}{dA} (1 - \lambda)\tau \\ &= \frac{d\lambda}{dA} \tau(\tau + \rho) - \frac{d\tau}{dA} (1 - \lambda)\rho < 0 \end{aligned}$$

with $\frac{d\lambda}{dA} \leq 0$ and $\frac{d\tau}{dA} \geq 0$ and at least one of the derivatives being non-zero for all policies considered. For q_1 I obtain

$$\begin{aligned} q_1 &= \frac{db}{dA} r - \left(r_A + r_b \frac{db}{dA} \right) b \\ &= (\tau - r_b b) \frac{db}{dA} - r_A b \\ &= \alpha^2 A b^{1-\alpha} \frac{db}{dA} - r_A b \\ &= \alpha b^{1-\alpha} \left[\alpha A \frac{db}{dA} - b \right] \end{aligned}$$

which is non-positive for all non-redistributing policies since for $\lambda = 0$ one gets $\alpha A \frac{d\tau}{dA} \leq \tau$ as has been shown in (1.28) for non-redistributing, left or right-wing policies. For the income egalitarian policy $q_1 = 0$ since $b = [A(\alpha - (1 - \alpha)F^*)]^{1/\alpha}$ and $\frac{db}{dA} = \frac{b}{\alpha A}$. Also, for a redistributing, left-wing government $b = [(1 - \alpha)A]^{1/\alpha}$ and $\frac{db}{dA} = \frac{b}{\alpha A}$ so that $q_1 = 0$. Hence, $q_1 + q_2 < 0$ and so $\frac{dq}{dA} < 0$.

The ratio of government investment to GDP is $p = \frac{b^\alpha}{A}$ so that

$$\text{sgn} \left(\frac{dp}{dA} \right) = \alpha A b^{\alpha-1} \frac{db}{dA} - b^\alpha = b^\alpha \left[\frac{db}{dA} \frac{\alpha A}{b} - 1 \right]$$

which is zero for growth maximizing, income egalitarian and redistributing, left-wing policies from what has been shown above. For non-redistributing, left-wing policies it is negative.

Proposition 1.11 *Assume policy variables react to fundamental economic variables as in Table 1, the DGP is the same as that for $f(\cdot)$ and the growth rate is regressed on the ratio of public investment to total investment (q) or the ratio of public investment to GDP (p). The theoretical model predicts that if $\text{cov}(q_i, \alpha_i)$ and $\text{cov}(p_i, \alpha_i)$ are non-positive, then in simple cross-country OLS regressions*

1. *the estimated coefficients measuring the effect of q on growth are biased downwards under all policies considered.*
2. *the estimated coefficients measuring the effect of p on growth are biased downwards if all governments pursue non-redistributing, left-wing policies. The estimated coefficients may be unbiased if all governments pursue growth maximizing, income egalitarian or redistributing, left-wing policies and p_i is uncorrelated with α_i , i.e. $\text{cov}(p_i, \alpha_i) = 0$.*

The proposition allows one to interpret some estimates of the effect of those variables on growth as presented in the literature. For instance, Barro (1991), Table IV, reports that the estimated coefficient for q was 0.014 (0.636) and that for p was 0.13 (1.3) for regressions that included many variables. When also including the significantly positive variable i/y the estimated coefficient for p became negative and was -0.015 (-0.126).

Barro (1990), p. S124, interprets the estimated coefficient for q as corroboration of his hypothesis that '*the typical country comes close to the quantity of public investment that maximizes the growth rate.*' As violations of the standard assumptions for hypothesis testing make such an argument invalid, Proposition 1.11 provides one reason why such a hypothesis may rest on shaky ground. As any measured positive point estimate of the effect of q on growth is *underestimated* according to the proposition, it is likely that the ratio of public to total investment positively affects growth, *even if* all governments pursue growth maximizing policies. For a sample with heterogeneous policies and a 'true' estimate that is likely to be significantly positive, a hypothesis compatible with the proposition should be that a typical country that increases public investment, given everything else, is likely to increase the growth rate, no matter what policy is pursued.²⁴ Given the downward biases for the measured effects of q on growth one is lead to conclude that an increase in q raises the growth rate.

Barro interprets the statistically insignificant, but negative coefficient on p , when the variable i/y is controlled for, as an indication that '*there is no separate effect on growth from the breakdown of total investment between private and public components*'. That suggests the hypothesis that an optimizing public sector invests according to the same criteria as the private sector does. For instance, the public sector may wish to maximize the social return on its investment, just as private sector investors wish to maximize the private return on their investments. Barro implicitly assumes that maximization of the social return on public investment also maximizes the growth rate.

First of all, it is worth noting that by Lemma 1.5, that is, under the as-

²⁴Notice that a significantly positive coefficient does not entail that it should be growth maximizing to push q up to 1 when tax base or collection constraints curtail such choices. Those and other provisos are meant by 'given everything else'.

sumption of exogenous policy the estimated coefficients for the effect of p on growth are biased downwards, which suggests that the 'true' effect of public investment may be significantly positive. Under the assumption of endogenous policy the estimated coefficients for the effect of p would be biased downwards if the sample contained non-redistributing, left-wing governments. Secondly, under the assumption of endogenous policy the negative point estimate suggests that the breakdown matters. A reported value of -0.015 for the effect of p on growth means that holding total investment constant and raising public investment with a compensating cut in private investment reduces growth. Thus, a negative point estimate in that regression implies that a typical country's policy that crowds out private (steady state) investment is bad for growth. That implication may be a valid result (unbiased estimates) for samples where, according to Proposition 1.11, all countries are led by growth maximizing, income egalitarian or redistributing, left-wing policies and $cov(p, \alpha_i) = 0$. But given unbiasedness and statistical insignificance, it follows that other than growth maximizing policies would also lead to no crowding out of public and private investment. That, in turn, implies that a statistically insignificant value does not mean that governments have maximized the social return on public investment. Thus, other welfare objectives are compatible with an insignificant coefficient.

Barro views his empirical results as 'ongoing research' and is aware so some of the econometric problems mentioned above. However, he does not investigate the implications of endogenous policy in depth. From the chapter's theoretical model it follows that under endogenous policy the estimated coefficients for the effect on growth of the ratio of public investment to total investment are generally biased downwards. The estimated coefficients for the effect on growth of the ratio of public investment to GDP may be unbiased if that ratio is uncorrelated with the

share of capital and other than non-redistributing, left-wing policies are pursued. Otherwise, there is a downward bias in the estimated coefficients. Interestingly, if one assumes exogenous policy, the coefficients measuring the effect of that variable on growth are definitely biased downwards. Hence, reported insignificant negative point estimates for that coefficient in simple cross-country OLS regressions may be either biased downwards and so the 'true' coefficient may really be significantly positive or they are unbiased, but provide no corroboration of the hypothesis that all countries are lead by growth maximizing policies.

1.6.3 Redistributive Transfers

Some researchers test the effect of redistributive transfers such as social benefits, social security contributions etc. on growth. Transfers of that kind are political instruments to correct for socially unwanted pre-tax income inequality. In that context the growth rate is often regressed on the ratio of redistributive transfers to GDP, which - in this model - is given by²⁵

$$\frac{\lambda \tau k_t}{y_t} = \frac{\lambda \tau k_t}{A k_t^\alpha g_t^{1-\alpha}} = \frac{\tau k_t \left(\frac{g_t}{k_t}\right)^\alpha}{A g_t} = \frac{\lambda ((1-\lambda)\tau)^\alpha}{A(1-\lambda)}$$

where $\tau = \rho$ from (1.15) and $g_t = (1-\lambda)\tau k_t$. I assume that some governments in the sample are redistributing, left-wing governments so that data on redistribution (λ) are actually available. Then a regression of the growth rate on

$$b \equiv \lambda(1-\lambda)^{\alpha-1} \tau^\alpha A^{-1}$$

²⁵Notice that the arguments below and Table 1 imply that a regression of the growth rate on the ratio of transfers to tax revenues, that is, λ would also yield coefficients that are biased downwards.

would produce biased estimates for the effect of b on growth by the following reasoning: Firstly, for *given* policy and $cov(b_i, \alpha_i) \leq 0$, the estimated effect of ratio of redistributive transfers to GDP (b) on growth are biased downwards. Secondly, under the assumption of endogenous policy Table 1 implies that $\frac{db}{d\alpha}$ and so $cov(b_i, \alpha_i)$ are ambiguous in sign. Then

$$\frac{db}{dA} = b_\lambda \frac{d\lambda}{dA} + \lambda_A < 0$$

since $\lambda_A < 0$, $b_\lambda > 0$, $\frac{d\lambda}{dA} < 0$ (Proposition 1.3) and $\tau = \rho$ which is independent of A by equation (1.15), suggesting that simple OLS estimates for the effect of b_i on γ_i are biased downwards, that is, if $cov(b_i, \alpha_i) \leq 0$ in the sample or the covariance is less important than that of $cov(b_i, A_i)$ the estimated effect of transfers on growth are biased downwards.

Proposition 1.12 *If $cov(b_i, \alpha_i) \leq 0$ and some countries in the sample are lead by redistributing governments the model predicts that in simple cross-country OLS regressions the estimates measuring the effect on growth of the ratio of redistributive transfers to GDP are biased downwards.*

According to the proposition any reported bad effect of redistributive transfers is overstated and any reported good effect is understated. Notice that the downward bias would even be present if one assumed that policy was exogenous. The model's prediction, which is shared by many other models in the theoretical literature, is that transfers are bad for growth. The reason for overstating any measured negative effect of redistributive transfers on growth in this model is due to the prediction that countries with more efficient economies and redistributing governments choose to redistribute *less* resources per units of taxes collected.

As Proposition 1.12 does not say anything on the magnitude of the bias, the latter may be as large as minus infinity. That means, the hypothesis that redistributive transfers are bad for growth cannot be validated by any reported negative coefficient on transfers in simple OLS growth regressions. Thus, for simple OLS growth regressions Proposition 1.12 renders the hypothesis generically untestable.

In fact, these arguments suggest the opposite hypothesis, namely that redistributive transfers affect growth in a positive way. For instance, Perotti (1994) is surprised to find a significantly positive coefficient on redistributive transfers in his regressions of investment on policy variables. He quotes other studies that have found a positive relation between transfers and growth. (See fn. 8 of his paper.) Sala-i-Martin (1996) states that it is surprising that among the three components of public spending - public investment, public consumption and public transfers - the only one that seems to be positively related to growth is the redistributive transfer variable. For instance, social security contributions are often found to affect growth in a positive way.

In that sense Proposition 1.12 explains why one may get positive coefficients on redistributive transfer variables in simple OLS cross-country growth regressions under the hypothesis that redistribution is bad for growth. On the other hand, the proposition is perfectly compatible with the hypothesis that redistributive transfers positively affect growth.

1.7 Conclusion

In this chapter a comparative dynamic analysis is conducted to investigate the link between (re-)distributive policies and growth. Within a common theoretical

framework it is shown that optimizing governments take account of fundamental economic variables when making their decisions so that public policy is economically endogenous. In the model the optimal policies of growth maximizing, 'right-wing', non-redistributing and redistributing 'left-wing' and 'income egalitarian' governments are analyzed. I show that changes of fundamental economic variables have interesting effects on optimal steady state policies and through the latter on growth and the post-tax factor income distribution. Two findings of the chapter are noteworthy.

First, under certain conditions the policies optimal for the accumulated factor of production (growth maximization) may also be pursued if a government has other welfare objectives. An increase in technological efficiency generally raises optimal taxes and growth under the policies considered. That result suggests that the estimated coefficients on tax variables are generically biased in simple cross-country OLS growth regressions. The direction of the biases are deduced from the theoretical model, providing some explanations for certain, sometimes puzzling point estimates found in the empirical literature.

Second, in many models redistribution towards the non-accumulated factor of production slows down growth. In this paper redistribution is only optimal under quite restrictive conditions. In the optima considered an increase in technological efficiency reduces the incentive to redistribute so that a testable implication of the model is whether a more advanced country relies more or less on wealth transfers as a means to pursue some welfare objective.

Under the assumption that policy is economically endogenous, I show for simple cross-country OLS growth regressions that the point estimates of the effect of redistributive transfer variables on growth are generally biased downwards. That suggests, the hypothesis that redistribution is bad for growth may not be

testable by simple cross-country OLS growth regressions. The downward bias is, however, perfectly consistent with empirical findings in the literature which find a positive association between redistributive transfers and growth. It may also represent corroboration of the hypothesis that redistribution is not bad for growth.

Several caveats apply. I have only considered wealth taxes as a tax base. Other tax bases may entail different optimal policies. It is an interesting question whether the results carry over to other forms of taxation. Also, in reality workers own capital and capital owners supply labour so that it would be desirable to use a less aggregated set-up for the investigation of the trade-off between economic growth and personal income inequality. These questions leave room for further research on the topic.

Chapter 2

Economic Growth, Distributive Policies, and an Income cum Investment Subsidy Tax Scheme

2.1 Introduction

In the theoretical literature models are often presented in which increasing taxes for redistributive purposes slows down growth. For instance, Alesina and Rodrik (1994), or Bertola (1993) argue that a government that redistributes resources towards the non-accumulated factor of production levies higher taxes and induces lower steady state growth. Their results crucially depend on the tax arrangement. As has been pointed out by Bertola, any policy that subsidizes investment is good for growth. (Alesina and Rodrik make a similar point in their paper.) That raises the question how investment subsidies are financed and what their distributional consequences are. Bertola (1993), p. 1192, rules out *capital income taxation* as a means of subsidy financing as it would *defeat the purpose of enhancing growth*.

However, he analyzes the effect of consumption taxes on growth enhancement and their distributional consequences.

In this chapter I investigate whether capital income taxation really does defeat the purpose of enhancing growth. In particular, I provide a comparison of a particular capital income tax with a *wealth tax* scheme. As in Alesina and Rodrik (1994) the wealth tax scheme is supposed to represent a broad class of tax arrangements that distort the investors' incentive to accumulate capital. For simplicity and in line with the models mentioned above I assume that the non-accumulated factor of production is labour and the accumulated factor is capital. The workers never save and consume their entire income. The capital owners do not work and accumulate capital.¹

In the chapter the tax scheme is designed as a *capital income cum investment subsidy* tax. The tax rate on capital income and for investment subsidies is assumed to be *equal*, and therefore tantamount to a tax on the capital owners' consumption.² But notice that in terms of implementability there are important differences. As a consumption tax scheme the government would effectively tax the capital owners' consumption so that a government representing their interests may not want to use it. On the other hand a pro-labour government may wish to use it. For both governments it would be difficult to determine whether a homogeneous consumption good was bought by a capital owner or a worker. Thus, viewing the tax arrangement as a consumption tax raises various difficulties. But as an income cum investment subsidy scheme these difficulties do not arise. Suppose the government provides public inputs to production as in Barro (1990) and uses its tax revenues to finance them. Raising an income tax may then

¹Thus, the model is Kaldorian in spirit. For a justification why the workers may choose not to invest in an optimizing framework see Bertola (1993).

²For simplicity and in order to elucidate the effect of that tax on growth I abstract from wage taxes throughout.

be good in terms of the pre-tax return on capital if the public inputs positively affect production and it may be bad in terms of the post-tax return to capital. However, coupled with an investment subsidy, a 'right-wing' (capital owners') government may want to use the tax scheme. Analogous reasoning holds for a 'left-wing' (workers') government.

Setting the tax rate equal seems a very special arrangement at first sight, but it may be justified by the following observation. A right-wing government acts in the interests of capital owners, consequently it would wish to set the income tax rate equal to zero. As the presence of a Barro-type production function is assumed where public services feed back into production, it would like to subsidize investment as much as possible. Hence, setting the tax rate on income and investment equal seems to be a reasonable choice.

Similar reasoning applies for a 'left-wing' government. It would wish to set a very high tax rate on capital income for redistributive reasons. But that hinders investment, when one recalls that in a simple endogenous growth model à la Barro (1990) or Romer (1986) workers enjoy ever increasing wages along a balanced growth path. Thus, a left-wing government would have to strike a balance between financing investment and redistribution. Hence, setting the tax rate equal again appears to be a reasonable choice.

With these justifications the *uniform* tax rate on capital income and for investment subsidies is simply assumed. As in the model of chapter 1 the governments may want to expropriate the capital stock and run the economy more efficiently themselves. As that is rather unrealistic, I assume that each government respects the right of private property.

Given these assumptions I show that the economy's steady state growth rate depends only on the *pre-tax* return to capital and the time preference rate. That

is interesting for the following reason. The capital owners pay income taxes and receive an investment subsidy. The tax arrangement is such that for optimizing capital owners the distorting effect of income taxes is removed by the investment subsidy and the positive effect of public inputs to production. The income tax only has a negative effect on the capital owners' instantaneous consumption level.

The relationship between growth and taxes in the model is neither inverted U-shaped as in Barro (1990) or Alesina and Rodrik (1994) nor U-shaped as in Persson and Tabellini (1994), but strictly positive. Thus, in the model it is growth maximizing, if the capitalists are taxed maximally. Thus, capital income taxes *raise* the growth rate in the model, but reduce the capitalists' consumption level. It is shown that the *capital income cum investment subsidy tax* scheme allows higher growth than a *wealth tax* scheme if a government targets the same ratio of public inputs in production to the capital stock under either tax scheme. Furthermore, if the capital owners are sufficiently impatient setting the wealth and the income tax rate equal reveals that the income cum investment tax scheme allows for higher growth. Consequently, the model's tax scheme is conducive to high growth.

In contrast to most optimal growth models such as, for instance, Cass (1965) or Koopmans (1965) impatience is *not* necessarily bad for growth in the model. It is shown that there exists a concave relationship between growth and the time preference rate and there exists a time preference rate which maximizes growth for given taxes. The reason for that lies in the removal of the distorting effect of income taxation by the investment subsidy, which is taken into account by optimizing agents. As the tax scheme operates like a consumption tax, more impatience raises the capital owners' instantaneous level of consumption and with it the tax revenues the government channels into production. That raises

the return on capital and so the steady state growth rate. Furthermore, I show that an inverse relationship holds for maintaining a given growth rate, that is, the more impatient the capital owners are, the lower the income tax rate has to be for a given growth rate.

Next, the chapter provides a public policy analysis and asks what tax rates a government that represents a mixture of the workers' or capital owners' welfare would choose. I show that a 'right-wing' government never redistributes resources towards the workers and that its optimal tax rate is less than the growth maximizing one. That is hardly surprising since the 'right-wing' government represents the interests of the capital owners only. As capital income taxes reduce the investors' instantaneous consumption level, the 'right-wing' government chooses a tax rate that represents the optimal trade-off between generating high income through raising enough tax revenues in order to raise the return on capital and reducing consumption.

It is then an interesting question whether the capital owners are better off under the model's income tax scheme or a wealth tax scheme. I show that the capital owners prefer a wealth tax scheme when the wealth taxes are chosen so that their welfare is maximized. As growth may be higher under this model's tax scheme, the preferred choice of the capital owners implies that they value the direct tax effect on their consumption level higher than the intertemporal effect of having higher income and consumption in the future. I identify conditions under which the capitalists' preferred tax policy generates higher growth under the chapter's scheme than under the wealth tax scheme. From that I conclude that the accumulated factor of production does not always choose the growth maximizing tax base. Furthermore, it does not necessarily act growth maximizing, that is, it chooses higher growth under the model's tax scheme than under a

tax scheme where it actually maximizes growth.

Next, I show that if the social planner uses an income cum investment subsidy tax arrangement, placing *more weight* on the welfare of the *non-accumulated factor of production* (workers) raises the optimal tax rate on the income of the *accumulated factor of production* (capital) and through that *the growth rate*. Hence, under the tax arrangement it is not optimal for growth maximization to shift political power to the accumulated factor of production. The result is in direct contrast to what is shown in Alesina and Rodrik (1994). A result similar to the chapter's is obtained in Bertola (1993), but notice that in comparison to a wealth tax scheme, taxation of capital income *does not* defeat the purpose of enhancing growth in this model.

The 'right-wing' government acts like a growth maximizer under the wealth tax scheme. With the chapter's tax scheme a 'left-wing' government that does not redistribute towards labour acts like a growth maximizer. Thus, a switch from a wealth to an income cum investment subsidy tax scheme induces an important switch in optimal policies. I show that a 'left-wing' government only redistributes if the economy is sufficiently inefficient or the workers as a group are rather impatient. I conclude that one may observe an economy with a government that represents only the interests of the non-accumulated factor of production (labour) to have redistribution *and* higher growth than an economy represented by a government solely concerned about the accumulated factor of production (capital).

Next, I analyze the distributional consequences of the two tax schemes. I restrict the analysis to a comparison of growth maximizing policies for simplicity and in order to eliminate politically given distributional preferences. I focus on the distribution of total factor income of the capital owners and the workers.

The procedure is motivated by the observation that income shares of factors of production are still of considerable relevance for the study of income distribution and that usually property income is of great importance at the top of the income scale in personal income distribution analyses. (See, for instance, Atkinson (1983), chpt. 9.) I show that the pre-tax total factor income distribution is equal under either tax scheme for given growth maximizing policies. The pre-tax and post-tax income distributions depend on the share of capital. If the share of capital is very low and equals the minimum tax rate of the income tax scheme, the post-tax factor income distributions coincide. That is a special, but interesting case, as it shows that two economies might have the same factor income distributions, but exhibit different growth performances.

Furthermore, it is shown that the chapter's post-tax factor income distribution is in general more favourable to labour for growth maximizing policies and may generate higher growth than under a wealth tax scheme.

The chapter is organized as follows: Section 2.2 presents the model set-up, introduces the capital income tax cum investment subsidy tax scheme and the government's behaviour, and derives the market equilibrium. Section 2.4 provides a public policy analysis and compares the optimal tax choices of different governments with those obtained under a wealth tax scheme. The distributional consequences for growth maximizing policies are analyzed. In propositions I state the main results of the chapter. Section 2.6 draws some conclusions.

2.2 The Model

The model set-up is very similar to the one in chapter 1.³ I assume that the economy is populated by two types of many identical individuals who all have the same rate of time preference ρ . The capitalists (k) own capital equally and no labour and the workers (W) own labour equally, but no capital. Both groups derive logarithmic utility from the consumption of a homogeneous, malleable good.

There are many firms in the economy. Those who own capital, own shares of the firms. Aggregate output is produced according to a Barro (1990) technology

$$y_t = A k_t^\alpha g_t^{1-\alpha} L_t^{1-\alpha} \quad , \quad 0 < \alpha < 1 \quad (2.1)$$

where y_t is total output, k_t is the economy-wide real capital stock, g_t are total public inputs to production and A is an efficiency index, which depends on cultural, institutional and technological development. I ignore all exogenous factors that play a role in the growth process by assuming that A is constant over time.

Furthermore, I set $L_t = 1$ so that at each point in time (raw) labour is inelastically supplied and the total labour endowment is equal to unity. For simplicity, I assume that the technology uses raw labour and capital. I abstract from problems arising from the introduction of human capital. Alternatively, one may assume that k_t is broad capital and that human and physical capital are strict complements. For a justification of the latter approach see, for instance, Mankiw et al. (1992), p. 416.⁴ Throughout the chapter I abstract from problems

³Some of the assumptions have already been explained and justified in chapter 1. For the sake of brevity I will not repeat them here.

⁴Both assumptions would allow me to concentrate on the distributional conflict between the accumulated and the non-accumulated factor of production.

arising from the depreciation of the capital stock.

2.2.1 The Public Sector

The government taxes the capital income of and grants an investment tax subsidy to the capital owning households. Let θ be the tax rate on real capital income which is held by the investors. Thus, the constant tax rate θ is levied on $r_t k_t$. The government also grants an investment subsidy of $\theta \dot{k}_t$, i.e. it subsidizes the individuals' total investment at the rate θ . So I consider the special case where the tax rates on capital income and the investment subsidy are equal. The tax arrangement amounts to a tax on the capitalists' *consumption*, but in terms of implementability I refer to it as an *income cum investment subsidy tax*.⁵ The government respects the right of private property and it is impossible to tax all income. For simplicity let $\theta \in [0, 1 - \epsilon]$ where ϵ is small.⁶ Then *total tax revenues* (net of investment subsidies) at date t are given by

$$\Theta_t \equiv \theta[r_t k_t - \dot{k}_t]. \quad (2.2)$$

⁵In fact, I contemplate a *Ramsey Tax Problem*. See, for instance, Diamond and Mirrlees (1971) or Atkinson and Stiglitz (1980), chpt. 12. In order to see the equivalence let $q = 1 + t_c$ where q is the price consumption goods command in terms of producer prices normalized to be one and fixed. The government taxes consumption at rate t_c . Let Y^k denote the capital owners' pre-tax income minus pre-tax investment. Then a consumption tax is equivalent to an income cum investment tax if the capital owners' budget constraints satisfy

$$(1 + t_c)C^k = Y^k \Leftrightarrow C^k = (1 - \theta)Y^k,$$

which is true if $t_c = \frac{\theta}{1-\theta}$.

⁶A small ϵ captures that the upper bound on tax rates consistent with no expropriation may still be large, that is, it may be close to, but it is less than one. For ease of calculations it is often assumed that $\epsilon \rightarrow 0$ when the effects of maximal taxation are analyzed. Then the reader should bear in mind that maximal taxation in this market economy model with private property is not meant to be the same as outright expropriation.

The government faces the following budget constraint, which is assumed to be balanced at each point in time,

$$\Theta_t = g_t + \lambda\Theta_t.$$

The LHS depicts the total tax revenues and the RHS total public expenditure at time t . The workers receive the fraction $\lambda\Theta_t$ of tax revenues as transfers and g_t is spent on public inputs to production. The parameter λ measures the degree of redistribution in the economy and is constant over time. Rearranging I consequently contemplate the budget constraint,

$$g_t = (1 - \lambda)\Theta_t. \quad (2.3)$$

By assumption the government sets g_t/k_t constant for all t , which might appear to be in conflict with a balanced budget, because (2.2) and (2.3) imply

$$\frac{g_t}{k_t} = (1 - \lambda)\theta(\tau_t - \gamma_t) = \text{constant}, \quad (2.4)$$

which is only satisfied if $\dot{r}_t = \dot{\gamma}_t$ where γ_t denotes the growth rate of the aggregate capital stock. I show below that the assumption of a constant ratio of public inputs to production to the capital stock leads to $\dot{r}_t = \dot{\gamma}_t$ in equilibrium.⁷

2.2.2 The Private Sector

The *firms* operate in a perfectly competitive environment, maximize profits, and take g_t as given. They are owned by the capital owners who rent capital to

⁷Alternatively, I may impose the steady state condition when deriving the equilibrium later on, which would also call for constant g_t/k_t .

and demand shares of the firms. The capitalists' assets are their shares of the firms. The shares are collateralized one-to-one by capital. The markets for assets, capital and labour clear at each point in time and a representative firm faces a path of a uniform, market clearing rental rate, $\{r_t\}$, of the capital stock, k_t and wage rate, $\{w_t\}$, for labour.

The firms rent capital and hire labour in spot markets in each period. I set the price of y_t equal to 1 at each t , which implies that the price of k_t in terms of overall consumption stays at unity. (For a justification see, for instance, Barro and Sala-i-Martin (1995), chpt. 2.2) Given constant returns to capital and labour, factor payments exhaust output. Profit maximization entails that firms pay each factor of production its marginal product

$$\begin{aligned} r_t &= \frac{\partial y_t}{\partial k_t} = \alpha A \left(\frac{g_t}{k_t} \right)^{1-\alpha}, \\ w_t &= \frac{\partial y_t}{\partial L_t} = (1-\alpha) A \left(\frac{g_t}{k_t} \right)^{1-\alpha} k_t, \quad L_t = 1, \forall t. \end{aligned} \quad (2.5)$$

Notice that (2.5) implies constancy of the marginal product of capital over time. That follows from the assumption that the government sets g_t/k_t constant over time. Hence, $r_t = r$. The wages vary over time and the ratio of public inputs to production to the capital stock (g_t/k_t) has a positive bearing on the marginal product of capital and the wage rate.

The *capital owners* form a homogeneous group. As a class they choose how much income to consume or invest, and they take the paths of (r, θ, λ) as given. Their intertemporal problem is given by

$$\max_{C_t^k} \int_0^{\infty} \ln C_t^k e^{-\rho t} dt \quad (2.6a)$$

$$\text{s.t.} \quad C_t^k = (1-\theta)[rk_t - \dot{k}_t] \quad (2.6b)$$

$$k(0) = k_0, \quad k(\infty) = \text{free.} \quad (2.6c)$$

Equation (2.6b) is the capital owners' dynamic budget constraint. We see that higher income raises consumption and more investment lowers it. For given income and investment higher taxes have a negative effect on consumption. For finding the solution to this problem I construct the Lagrangean

$$H(C_t^k, k_t, \dot{k}_t) = \ln C_t^k e^{-\rho t} + \mu_t e^{-\rho t} [(1 - \theta)[rk_t - \dot{k}_t] - C_t^k]$$

where μ_t is a co-state variable associated with the budget constraint (2.6b) and represents the current-value marginal utility of wealth.⁸ Note that the objective function $H(\cdot)$ has two state variables, (C_t^k, k_t) , as its arguments and that the corresponding *Euler-Lagrange Equations* are given by

$$\text{Euler-Lagrange Equations: } H_x - \frac{d}{dt}H_{\dot{x}}, \quad x = C_t^k, k_t.$$

Then the FOC involves the static optimality conditions for C_t^k

$$\frac{1}{C_t^k} e^{-\rho t} - \mu_t e^{-\rho t} = 0$$

so that in the optimum the capital owners equate the marginal utility of consumption to the marginal utility of wealth. The dynamic condition for k_t is given by

$$\begin{aligned} \mu_t e^{-\rho t} (1 - \theta)r - \frac{d}{dt} [-(1 - \theta)\mu_t e^{-\rho t}] &= 0 \\ \mu_t e^{-\rho t} (1 - \theta)r + \dot{\mu}_t e^{-\rho t} (1 - \theta) - \rho \mu_t e^{-\rho t} (1 - \theta) &= 0. \end{aligned}$$

⁸For a similar procedure see Turnovsky (1995), chpt. 9. For the solution of variational problems see, for instance, Chiang (1992).

By cancellation in and rearrangement of this equation one obtains

$$\begin{aligned}\dot{\mu}_t &= \mu_t \rho - \mu_t r, \\ \lim_{t \rightarrow \infty} k_t \mu_t e^{-\rho t} &= 0,\end{aligned}$$

where the last equation ensures that the present value of the capital stock approaches zero asymptotically. The static optimality condition yields $\frac{\dot{\mu}}{\mu} = -\frac{\dot{C}^k}{C^k}$ and so consumption grows at

$$\gamma \equiv \frac{\dot{C}^k}{C^k} = r - \rho \quad (2.7)$$

which is constant for all t . Thus, if g_t/k_t is constant, then $\dot{\gamma} = \dot{r} = 0$, as asserted above. The growth rate depends on the *pre-tax* return on capital, because the capital owners have perfect foresight and know that they receive an investment subsidy. In the optimum the distorting effect of capital income taxation is exactly offset by the accumulation inducing effect of the investment subsidy which is, of course, due to the assumption of a uniform tax rate.

Equations (2.6b), (2.7) and the transversality condition imply that in the optimum the capitalists' instantaneous consumption is given by⁹

$$C_t^k = (1 - \theta)\rho k_t. \quad (2.8)$$

Hence, the distorting effect of income taxes is present in the capital owners' instantaneous level of consumption. Furthermore, it follows that the capitalists' wealth and consumption optimally grow at the same rate, that is, $\gamma = \gamma_k$. Thus, the model's tax scheme is highly growth promoting.

⁹For a derivation see Appendix A.1.

The *workers* derive a utility stream from consuming their entire income. They do not invest and they are not taxed by assumption. Their intertemporal utility is given by

$$\int_0^{\infty} \ln C_t^W e^{-\rho t} dt \quad \text{where } C_t^W = W_t + TR_t .$$

where W_t, TR_t denote the workers' total wage income, resp. transfers received. Thus, as in chapter 1.2.4 I assume the classical savings rule for the workers and the capital owners.

2.3 Market Equilibrium

The overall resource constraint in the economy is

$$I_t = \dot{k}_t = rk_t + W_t + TR_t - \theta rk_t + \theta \dot{k}_t - C_t^k - C_t^W. \quad (2.9)$$

As the workers' consumption is $C_t^W = W_t + TR_t$ with $TR_t = \lambda\theta(rk_t - \dot{k}_t)$, this constraint is binding, simplifying (2.9) to

$$\dot{k}_t = rk_t - \frac{C_t^k}{1 - \theta}$$

which corresponds to the capital owners' budget constraint which holds as an equality given the optimal behaviour of the capital owners, that is, $\gamma = \gamma_k$ and $C_t^k = (1 - \theta)\rho k_t$. Note that more impatience causes the capital owners to value current consumption more than future consumption. That makes them consume more per units of capital at each t . Furthermore, an increase in the tax rate θ reduces instantaneous consumption of the capital owners per units of capital.

Next consider production as given in (2.1). As $\frac{g_t}{k_t}$ is constant, $\gamma_y = \gamma_g = \gamma_k$. As $\gamma = \gamma_k$ one gets $r_t - \gamma_t = \rho$ in (2.4) so that

$$\frac{g_t}{k_t} = (1 - \lambda)\theta\rho$$

in steady state equilibrium. Substituting this into (2.5) one obtains

$$r = \alpha A[(1 - \lambda)\theta\rho]^{1-\alpha} \quad (2.10)$$

$$w_t \equiv \eta(\theta, \lambda)k_t = (1 - \alpha)A[(1 - \lambda)\theta\rho]^{1-\alpha}k_t, \quad L_t = 1, \forall t, \quad (2.11)$$

which looks a bit surprising because the marginal products depend on preference parameters. It is, of course, due to the fact that in a model with productive government inputs a financing tax scheme that operates like a consumption tax should depend on preference parameters. It is also noteworthy that the return on capital is higher the more *impatient* the investors are. The reason is that higher ρ makes the capital owners consume more per units of capital at each date, which increases the tax revenues that are channelled into production by the government to raise the return on capital and the growth rate.¹⁰

From equation (2.3) instantaneous consumption of the workers is given by

$$C_t^W = \eta(\theta, \lambda)k_t + \lambda\theta\rho k_t. \quad (2.12)$$

¹⁰In appendix A.5 I show that for the more general case of iso-elastic utility with preference for consumption smoothing the return on capital is also increasing in ρ , that is, if the intertemporal elasticity of substitution of consumption between different dates varies between zero and one, more impatience raises the steady state rate of return on capital. Interestingly, the comparative dynamic effect of consumption smoothing on the rate of return in steady state is shown to be ambiguous. The effects of less patience and more consumption smoothing on steady state consumption of the capital owners per units of capital is positive. The model is thus compatible with the finding that in endogenous growth models more consumption smoothing and more impatience imply a lesser willingness to save. See, for instance, Barro and Sala-i-Martin (1995), p. 144.

so that $\gamma_{C^w} = \gamma$. Hence, the steady state market equilibrium is characterized by balanced growth with $\gamma_y = \gamma_k = \gamma = \gamma_{C^w}$.

2.3.1 Properties of the Market Equilibrium

The steady state growth rate depends positively on the return on capital. As public inputs to production affect the return on capital, the following derivatives are useful. Let $E \equiv (1 - \alpha)A((1 - \lambda)\theta\rho)^{-\alpha}$. Then

$$\begin{aligned} r_\theta &= \alpha E(1 - \lambda)\rho & , & \quad r_\lambda = -\alpha E\theta\rho, \\ \eta_\theta &= (1 - \alpha)E(1 - \lambda)\rho & , & \quad \eta_\lambda = -(1 - \alpha)E\theta\rho \end{aligned} \tag{2.13}$$

so that an increase in the tax rate raises wages (given k_t) and the return on capital, and redistribution lowers them. Recall that the growth rate is given by

$$\gamma = r - \rho = \alpha A[(1 - \lambda)\theta\rho]^{1-\alpha} - \rho. \tag{2.14}$$

From this and the expressions above one readily verifies

Lemma 2.1 *The economy's growth maximizing tax rate, θ , is $1 - \epsilon$.*

The result establishes what a *growth maximizing* government would choose. It is clear that a growth maximizing policy is bad for the capital owners as it implies a very low level of their instantaneous consumption. The surprising implication is that the *income cum investment subsidy tax* scheme calls for maximal taxation of the reproducible factor of production, *if* the objective is to maximize growth.

2.3.2 Comparison to a Wealth Tax Market Equilibrium

Following Alesina and Rodrik I have presented a model in chapter 1 that is almost identical to then one developed so far. Assume that technology, preferences etc. are as in this chapter and that the only difference is that the government taxes the capital owners' *wealth*. The government runs a balanced budget and uses its tax revenues to finance public inputs to production. The capitalists' dynamic budget constraint is then given by $C^k = (r - \tau)k_t - \dot{k}_t$ where τ is the tax rate, levied on the capital owners' wealth. Solving a problem analogous to the one presented above shows that the economy wide growth rate (cf. also chapter 1.3) is given by $\gamma(\tau) = r(\tau) - \tau - \rho$, where $r(\tau)$ is given as in (2.5), that is, $r = \alpha A \left(\frac{g_t}{k_t}\right)^{1-\alpha}$ and $g_t = (1 - \lambda)\tau k_t$. I will now compare the two tax schemes and their implications for growth. To this end denote $\gamma(\theta)$ ($\gamma(\tau)$) as the capital income cum investment subsidy tax (wealth tax) induced growth rate.

First, suppose the two governments maintain the same ratio of g_t to k_t , that is, they set $g_t(\theta)/k_t = g_t(\tau)/k_t > 0$ for all t , then $r(\theta) = r(\tau)$ from (2.5) and so $\gamma(\theta) > \gamma(\tau)$. Thus,

Proposition 2.1 *If $\frac{g_t(\theta)}{k_t} = \frac{g_t(\tau)}{k_t} > 0$ for all t , then $\gamma(\theta) > \gamma(\tau)$.*

Thus, if a government sets a particular target ratio of productive government expenditure to the capital stock, it may fare better in terms of growth with this model's income cum investment subsidy tax scheme.

But notice that the return on capital depends on the rate of time preference in this model (*Ramsey result*) and so one needs to know exactly under what conditions the ratio result holds. For this I will investigate under what conditions $\gamma(\theta) > \gamma(\tau)$ with $\theta = \tau$ and $\lambda = 0$. From (2.10) and $r(\tau) = \alpha A ((1 - \lambda)\tau)^{1-\alpha}$ I

check whether $\gamma(\theta) > \gamma(\tau)$, that is, whether

$$\alpha A (\theta \rho)^{1-\alpha} - \rho > \alpha A (\tau)^{1-\alpha} - \tau - \rho$$

$$\alpha A (\theta \rho)^{1-\alpha} > \alpha A (\tau)^{1-\alpha} - \tau.$$

With $\theta = \tau$ and $\lambda = 0$ the inequality is equivalent to

$$\rho^{1-\alpha} > 1 - \frac{\tau^\alpha}{\alpha A}$$

$$\rho > \left[1 - \frac{\tau^\alpha}{\alpha A} \right]^{\frac{1}{1-\alpha}}.$$

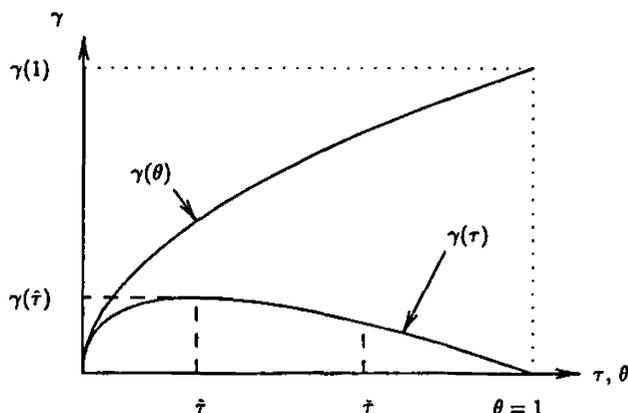
The growth maximizing wealth tax rate is given by $\hat{\tau} = [\alpha(1-\alpha)A]^{\frac{1}{\alpha}}$. (Cf. Lemma 1.1 of chapter 1.3.1.) Suppose $\theta = \hat{\tau}$, then the condition amounts to $\rho > \alpha^{\frac{1}{1-\alpha}}$.

Proposition 2.2 *If $\rho > \alpha^{\frac{1}{1-\alpha}}$, $\lambda = 0$, and $\theta = \hat{\tau}$ where $\hat{\tau}$ maximizes $\gamma(\tau)$, then*

$$\gamma(\theta)_{\theta=\hat{\tau}} > \gamma(\tau)_{\tau=\hat{\tau}}.$$

Figure 2.1 below visualizes the result when the proposition holds and $\epsilon \rightarrow 0$.

Figure 2.1: γ as a function of θ or τ



Given sufficient impatience the income cum investment subsidy tax scheme

generates higher growth than a wealth tax scheme. Notice that the tax scheme neither generates an inverted U-shaped (Barro (1990) or Alesina and Rodrik (1994)) nor a U-shaped relationship (Persson and Tabellini (1994)) between growth and taxes. Instead, a strictly positive relation between growth and taxes holds under the model's tax arrangement. The reason is that in terms of growth the positive effect of granting investment subsidies outweighs the negative effect of levying income taxes on capital. Another noteworthy implication of the model is that in contrast to most optimal growth models such as Cass (1965) or Koopmans (1965) impatience (higher ρ) is in general *not bad* for growth.¹¹ In order to see that let $\lambda = 0$ in (2.14) and calculate

$$\gamma_{\rho} = \alpha(1 - \alpha)A[\theta\rho]^{-\alpha} - 1$$

to establish that γ is a concave function of the rate of time preference ρ and that $\gamma_{\rho} \geq 0$ for given taxes. The time preference rate that maximizes the growth rate for given taxes is given by

$$[\alpha(1 - \alpha)A]^{\frac{1}{\alpha}} = \theta\rho \Leftrightarrow \rho = \hat{\tau}/\theta$$

so that there exists an interesting relationship between the growth maximizing wealth tax rate $\hat{\tau}$ and the growth maximizing time preference rate ρ under the model's tax scheme. The lower the ratio of the growth maximizing wealth tax rate to the income tax rate under this model's scheme, the more patient the capital owners are required to be for growth maximization. That comparative dynamic result is stated as

¹¹Notice that from Lemma 1.2 in chapter 1.5 it follows that the lower ρ is, the higher the growth rate under a wealth tax scheme will be.

Lemma 2.2 *Under a wealth tax scheme growth is maximized if ρ is very small. Under an income cum investment subsidy tax scheme $\gamma(\theta)$ is a concave function of ρ and maximized if $\rho = \hat{\tau}/\theta$.*

The reason for the different implications of the tax schemes for growth is that the tax arrangement is equivalent to a tax on consumption. The capital owners' instantaneous consumption depends positively on their time preference rate. If the capital owners are more impatient, they will choose higher consumption (per units of capital) and that will raise the tax revenues available to the government. The revenues may in turn be used to provide productive services, thereby raising the return on capital and so the growth rate.

Next I ask which θ yields $\gamma(\theta) = \gamma(\hat{\tau})$ given $\lambda = 0$. For this note that

$$\gamma(\hat{\tau}) = \frac{\alpha}{1-\alpha} \hat{\tau} - \rho \quad \text{where} \quad \hat{\tau} = [\alpha(1-\alpha)A]^{\frac{1}{\alpha}}.$$

Solving $\gamma(\theta) = \gamma(\hat{\tau})$ for θ involves

$$\begin{aligned} \alpha A [\theta \rho]^{1-\alpha} &= \frac{\alpha \hat{\tau}}{1-\alpha} \\ \theta &= \left[\frac{\hat{\tau}}{(1-\alpha)A} \right]^{\frac{1}{1-\alpha}} \frac{1}{\rho}. \end{aligned}$$

Thus, a lower ρ calls for higher θ if $\gamma(\theta)$ is to be set equal to $\gamma(\hat{\tau})$.

Proposition 2.3 *If $\theta = \left[\frac{\hat{\tau}}{(1-\alpha)A} \right]^{\frac{1}{1-\alpha}} \frac{1}{\rho}$ and $\lambda = 0$, then $\gamma(\theta) = \gamma(\hat{\tau})$.*

Thus, there is an interesting trade-off between tax rates and the time preference rate for given growth. The more impatient the capital owners are, the lower the taxes have to be for maintaining a given growth rate. Again that is due to the fact that more impatient capital owners consume more, but also generate higher

tax revenues for the government, which may be used for productive services.

2.4 A Public Policy Analysis

Consider a government that cares about the workers and the capital owners. Respecting the right of private property, it chooses θ and λ in order to maximize

$$W(\theta, \lambda, \beta) = (1 - \beta) V^r + \beta V^l$$

where V^r, V^l are the intertemporal utility indices of the capitalists, resp. workers. These are derived in appendix A.2 and are given by

$$V^r = \int_0^\infty \ln C_t^k e^{-\rho t} dt = \frac{\ln[(1 - \theta)\rho k_0]}{\rho} + \frac{\gamma}{\rho^2}, \quad (2.15)$$

$$V^l = \int_0^\infty \ln C_t^w e^{-\rho t} dt = \frac{\ln[(\eta + \lambda\theta\rho)k_0]}{\rho} + \frac{\gamma}{\rho^2}. \quad (2.16)$$

As in chapter 1 the parameter $\beta \in [0, 1]$ represents the welfare weight attached to the two groups in the economy. If $\beta = 1, (0)$, the government cares about the workers (capitalists) only. I refer to the government's choice of β as being a

$\beta = 1, (0)$ - left-wing (right-wing) government.

The constancy of β is justified by interpreting β as reflecting the socio-economic institutions in an economy. Then the fact that governments alternate in office becomes less of an issue since institutional features are usually constant for long periods of time.

Before proceeding the following result for a wealth tax scheme is useful. Let technology, preferences, welfare function etc. be as in this paper with the notable exception that the government raises wealth taxes. Denote $\gamma(\tau)$ as the growth

rate generated by the optimal policy under a wealth tax scheme. Then this chapter's model reduces to the one analyzed in chapter 1. Proposition 1.1 of chapter 1 states that the growth rate under a wealth tax scheme is inversely related to the social weight β attached to the welfare of the non-accumulated factor of production. For the purpose of this chapter I restate the proposition

Proposition 1.1 (Alesina and Rodrik) *The growth rate $\gamma(\tau)$ is inversely related to β , the social weight attached to the welfare of the non-accumulated factor of production.*

Thus, under a wealth tax scheme a government placing more weight on the welfare of the non-accumulated factor of production chooses a higher than the growth maximizing ('right-wing') tax rate. Below I will compare their result with this paper's tax scheme.

From (2.15) and (2.16) the welfare function of the government is given by

$$W(\theta, \lambda, \beta) = (1 - \beta) \frac{\ln[(1 - \theta)\rho k_0]}{\rho} + \beta \frac{\ln[(\eta + \lambda\theta\rho)k_0]}{\rho} + \frac{\gamma}{\rho^2}. \quad (2.17)$$

The government maximizes this function under the constraint $\lambda \geq 0$ which restricts the governments in that even a right-wing government does not tax workers. Let $v_i, i = \theta, \lambda$ denote the partial derivatives of $W(\cdot)$ with respect to θ, λ . Then maximization involves the following expressions of marginal welfare

$$v_\theta = -\frac{1 - \beta}{(1 - \theta)\rho} + \frac{\beta(\eta_\theta + \lambda\rho)}{(\eta + \lambda\theta\rho)\rho} + \frac{\gamma_\theta}{\rho^2} \quad \text{and} \quad v_\lambda = \frac{\beta(\eta_\lambda + \theta\rho)}{(\eta + \lambda\theta\rho)\rho} + \frac{\gamma_\lambda}{\rho^2}. \quad (2.18)$$

I am interested in the conditions under which one obtains an interior solution such that $\lambda > 0$. I will show that these conditions are restrictive and qualitatively similar to the ones presented in Alesina and Rodrik (1991). For an interior

solution the government solves

$$v_\theta = 0 \quad \text{and} \quad \lambda(v_\lambda) = 0$$

where the $\lambda(v_\lambda)$ expression enters because of complementary slackness for the constraint $\lambda \geq 0$. First rearrange the condition $v_\theta = 0$ using (2.18) to obtain

$$\begin{aligned} \frac{\beta(\eta_\theta + \lambda\rho)}{(\eta + \lambda\theta\rho)\rho} &= \frac{1 - \beta}{(1 - \theta)\rho} - \frac{\gamma_\theta}{\rho^2} \\ \frac{\beta}{(\eta + \lambda\theta\rho)} &= \frac{1 - \beta}{(1 - \theta)(\eta_\theta + \lambda\rho)} - \frac{\gamma_\theta}{\rho(\eta_\theta + \lambda\rho)}. \end{aligned} \quad (2.19)$$

Similarly, for an interior solution rearrange the condition $v_\lambda = 0$ to get

$$\begin{aligned} \frac{\beta(\eta_\lambda + \theta\rho)}{(\eta + \lambda\theta\rho)\rho} &= -\frac{\gamma_\lambda}{\rho^2} \\ \frac{\beta}{(\eta + \lambda\theta\rho)} &= \frac{-\gamma_\lambda}{\rho(\eta_\lambda + \theta\rho)}. \end{aligned} \quad (2.20)$$

Setting these equations equal yields

$$\begin{aligned} \frac{1 - \beta}{(1 - \theta)(\eta_\theta + \lambda\rho)} - \frac{\gamma_\theta}{\rho(\eta_\theta + \lambda\rho)} &= \frac{-\gamma_\lambda}{\rho(\eta_\lambda + \theta\rho)} \\ \frac{1 - \beta}{(1 - \theta)(\eta_\theta + \lambda\rho)} &= \frac{\gamma_\theta}{\rho(\eta_\theta + \lambda\rho)} - \frac{\gamma_\lambda}{\rho(\eta_\lambda + \theta\rho)} \\ \frac{\rho(1 - \beta)}{(1 - \theta)(\eta_\theta + \lambda\rho)} &= \frac{\gamma_\theta(\eta_\lambda + \theta\rho) - \gamma_\lambda(\eta_\theta + \lambda\rho)}{(\eta_\theta + \lambda\rho)(\eta_\lambda + \theta\rho)} \\ \frac{\rho(1 - \beta)(\eta_\lambda + \theta\rho)}{(1 - \theta)} &= \gamma_\theta(\eta_\lambda + \theta\rho) - \gamma_\lambda(\eta_\theta + \lambda\rho). \end{aligned} \quad (2.21)$$

Thus, for an interior solution equation (2.21) has to be satisfied. In appendix A.6 I show that the following relationship between θ and λ must then hold

$$\lambda = 1 - \frac{B^{\frac{1}{\alpha}}}{\theta\rho} \quad \text{where} \quad B \equiv \frac{(1-\theta)\alpha(1-\alpha)A}{1-\beta} + (1-\alpha)^2A. \quad (2.22)$$

Whenever the optimal (θ, λ) combination is such that $B^{\frac{1}{\alpha}} \geq \theta\rho$ there is no redistribution. Clearly, the optimal combination is such that θ, λ are functions of α, A, ρ, β . Suppose the optimal θ were increasing in A . Then a very high A would lead to a high θ . The maximum value for θ is $1 - \epsilon$. If A is such that $((1-\alpha)^2A)^{\frac{1}{\alpha}} > \rho$, then $\lambda = 0$. Thus, a high A rules out $\lambda > 0$. If θ were decreasing in A , the argument would even be simpler. An analogous argument holds for a low ρ . Hence, for very efficient economies or very patient agents there is no interior solution with $\lambda > 0$ and so redistribution does take place under those conditions. Thus, this chapter's model reaches the same qualitative conclusions as Alesina and Rodrik, namely that redistribution is bad for growth and only optimal if the agents are sufficiently impatient or the economy is not very efficient.

It is not difficult to see that a right-wing ($\beta = 0$) government does not redistribute, since if $\beta = 0$ in (2.18) one is left with $\frac{\gamma\lambda}{\rho}$ which is negative and so $v_\lambda < 0$ implies that there is no redistribution under a right-wing government.

Next, suppose $\beta \in [0, 1)$ and $\lambda = 0$. The government wants to find θ that solves $v_\theta = 0$. The optimal θ is denoted by $\bar{\theta}$. Setting v_θ equal to zero with $\lambda = 0$ implies

$$\frac{1-\beta}{1-\theta} = \frac{\beta\eta_\theta}{\eta} + \frac{\gamma_\theta}{\rho}$$

where I have already multiplied through by ρ . Let $E \equiv (1 - \alpha)A((1 - \lambda)\theta\rho)^{-\alpha}$. From (2.10), (2.11) and (2.13) one obtains $\frac{\eta_\theta}{\eta} = \frac{(1-\alpha)E\rho}{E\theta\rho} = \frac{1-\alpha}{\theta}$ and $\gamma_\theta = \alpha E\rho$ so that

$$\begin{aligned} \frac{1-\beta}{1-\theta} &= \frac{\beta(1-\alpha)}{\theta} + \alpha E \\ \frac{(\theta-\beta) + \alpha\beta(1-\theta)}{\theta(1-\theta)} &= \alpha(1-\alpha)A[\theta\rho]^{-\alpha} \end{aligned} \quad (2.23)$$

which must be solved by $\tilde{\theta}$ and is only implicitly defined and such that $\tilde{\theta} = f(\alpha, A, \rho, \beta)$. The optimal tax rate $\tilde{\theta}$ is *unique*. To show that I calculate $v_{\theta\theta}$ which is given by¹²

$$v_{\theta\theta} = -\frac{1-\beta}{\rho(1-\theta)^2} - \frac{(1-\alpha)\beta}{\rho\theta^2} + \frac{\gamma_{\theta\theta}}{\rho^2} < 0, \quad \forall \theta \in [0, 1] \quad (2.24)$$

as $\gamma_{\theta\theta} < 0$. Thus, v_θ is *strictly decreasing* in θ so that the optimal θ must be *unique*.

The tax rate chosen by a right-wing $\beta = 0$ government satisfies

$$\theta = \frac{\hat{\tau}(1-\theta)^{\frac{1}{\alpha}}}{\rho} \quad (2.25)$$

where $\hat{\tau} = [\alpha(1-\alpha)A]^{\frac{1}{\alpha}}$ which is the growth maximizing tax rate under a wealth tax scheme. Let $\tilde{\theta}^r$ denote the solution to the equation above. It is obvious that $\theta = 1$ ($\epsilon \rightarrow 0$) does not solve the equation and that $\tilde{\theta}^r$ is decreasing in the time preference rate. I summarize for a right-wing government in

Proposition 2.4 *A right-wing ($\beta = 0$) government does not redistribute, $\lambda = 0$, and its optimal tax rate is determined by $\tilde{\theta}^r = \frac{\hat{\tau}(1-\tilde{\theta}^r)^{\frac{1}{\alpha}}}{\rho} < 1$.*

¹²Note $v_\theta = -\frac{1-\beta}{(1-\theta)\rho} + \frac{(1-\alpha)\beta}{\rho\theta} + \frac{\gamma_\theta}{\rho^2}$ if $\lambda = 0$.

The intuition for $\tilde{\theta}^r < 1$ is not difficult to understand. On the one hand the right-wing government wants to set a high tax rate, since that is good for the capital owners' income and the growth rate, which positively affects the capitalists' utility. On the other hand higher taxes reduce consumption, which negatively affects their utility. $\tilde{\theta}^r$ represents the optimal trade-off for this problem.

It is an interesting question whether the capital owners are better off under the income tax scheme or under a wealth tax scheme. The capital owners' welfare under the wealth tax scheme (see appendix A.2) is given by

$$V^r(\tau) = \frac{\ln[\rho k_0]}{\rho} + \frac{\gamma(\tau)}{\rho^2}.$$

It is now shown that the capital owners prefer the wealth tax scheme to this model's capital income tax scheme. Under the income cum subsidy scheme the highest welfare to be obtained by the capital owners is given by $V^r(\tilde{\theta}^r)$ and under the wealth tax scheme it is $V^r(\hat{\tau})$. Clearly, $V^r(\tau) < V^r(\hat{\tau})$ for any $\tau \neq \hat{\tau}$. Without loss of generality assume that $\tau = \theta\rho$ so that $V^r(\tau = \theta\rho) < V^r(\hat{\tau})$. I will show that for positive taxes $V^r(\theta) - V^r(\tau = \theta\rho) < 0$. The difference is given by

$$\begin{aligned} & \frac{\ln[(1-\theta)\rho k_0]}{\rho} + \frac{\gamma(\theta)}{\rho^2} - \frac{\ln[\rho k_0]}{\rho} - \frac{\gamma(\tau = \theta\rho)}{\rho^2} \\ = & \frac{\ln(1-\theta)}{\rho} + \frac{\gamma(\theta) - \gamma(\tau = \theta\rho)}{\rho^2} \\ = & \frac{\ln(1-\theta)}{\rho} + \frac{\alpha A(\theta\rho)^{1-\alpha} - \rho - \alpha A(\theta\rho)^{1-\alpha} + \theta\rho + \rho}{\rho^2} \\ = & \frac{\ln(1-\theta) + \theta}{\rho}. \end{aligned}$$

The term $\ln(1 - \theta)$ is negative, but θ is positive. By the *mean value theorem* one gets $\frac{1}{p} = \frac{\ln 1 - \ln(1-\theta)}{1 - (1-\theta)}$, that is, $\ln(1 - \theta) = \frac{-\theta}{p}$ where $p \in (1 - \theta, 1)$ so that

$$\ln(1 - \theta) + \theta = \frac{-\theta}{p} + \theta = \frac{(p-1)\theta}{p} \quad (2.26)$$

which is negative since $p \in (1 - \theta, 1)$. Thus, $V^r(\theta) < V^r(\tau = \theta\rho)$ implying that $V^r(\tilde{\theta}^r) < V^r(\tau = \tilde{\theta}^r\rho) < V^r(\hat{\tau})$. Hence, the capital owners' optimal policies are such that they prefer a wealth tax to this model's income cum subsidy tax scheme.

Proposition 2.5 $V^r(\tilde{\theta}^r) < V^r(\tau = \tilde{\theta}^r\rho) < V^r(\hat{\tau})$ so that the capital owners' optimal policies under either tax scheme imply that they prefer a wealth tax scheme to a capital income cum investment subsidy tax scheme.

The result may not look very surprising because the capital income tax cum subsidy arrangement works like a tax on the capital owners' consumption, reducing their utility. However, the growth rate may be higher under the model's income tax scheme. Thus, the result establishes that the capital owners value the direct effect on their consumption level higher than the intertemporal effect of having higher income and so higher consumption in the future.

The right-wing government represents the accumulated factor of production and acts growth maximizing under a wealth tax scheme, but does not do so under the capital income cum subsidy scheme. I will show that for a wide range of parameter values the optimal policy for the capital owners under this model's tax scheme generates higher growth. Thus, even though the optimal right-wing policy is not growth maximizing under the capital income cum subsidy scheme, it may generate higher growth than the optimal right-wing policy under a wealth tax scheme. From Proposition 2.3 it follows that $\gamma(\theta) > \gamma(\hat{\tau})$ is equivalent to

$\theta > \left[\frac{\hat{\tau}}{(1-\alpha)A} \right]^{\frac{1}{1-\alpha}} \frac{1}{\rho}$. The optimal right-wing policy is $\tilde{\theta}^r = \frac{\hat{\tau}(1-\tilde{\theta}^r)^{\frac{1}{\alpha}}}{\rho}$. Substituting in $\gamma(\theta) > \gamma(\hat{\tau})$ yields

$$\begin{aligned} \frac{\hat{\tau}(1-\tilde{\theta}^r)^{\frac{1}{\alpha}}}{\rho} &> \left[\frac{\hat{\tau}}{(1-\alpha)A} \right]^{\frac{1}{1-\alpha}} \frac{1}{\rho} \\ (1-\tilde{\theta}^r)^{\frac{1}{\alpha}} &> \hat{\tau}^{\frac{\alpha}{1-\alpha}} \left[\frac{1}{(1-\alpha)A} \right]^{\frac{1}{1-\alpha}} \\ (1-\tilde{\theta}^r) &> \alpha^{\frac{\alpha}{1-\alpha}}. \end{aligned}$$

Whether this inequality holds is not easily analyzed analytically, but the following table presents a numerical simulation showing that there exist parameter values for which $\gamma(\hat{\tau}) < \gamma(\tilde{\theta}^r)$.¹³

Numerical Simulation for $A = 1$							
	ρ	α	$\hat{\tau}$	$\tilde{\theta}^r$	$\gamma(\hat{\tau})$	$\gamma(\tilde{\theta}^r)$	Δ
1.	0.01	0.25	0.001	0.086	-0.010	-0.009	+
2.	0.01	0.50	0.063	0.672	0.053	0.032	-
3.	0.01	0.75	0.107	0.851	0.312	0.218	-
4.	0.05	0.25	0.001	0.023	-0.050	-0.049	+
5.	0.05	0.50	0.063	0.420	0.013	0.023	+
6.	0.05	0.75	0.107	0.611	0.272	0.263	-
7.	0.10	0.25	0.001	0.012	-0.100	-0.099	+
8.	0.10	0.50	0.063	0.303	-0.038	-0.013	+
9.	0.10	0.75	0.107	0.466	0.222	0.248	+

where $\Delta = \text{sgn}(\gamma(\tilde{\theta}^r) - \gamma(\hat{\tau}))$. From the table it follows that for given α an increase in ρ causes the optimal right-wing policy to generate higher growth

¹³How the simulation was carried out is explained in Appendix A.8.

under the income cum subsidy than under the wealth tax scheme. A similar conclusion can be reached for given ρ and increases in α . That establishes

Proposition 2.6 $\exists \alpha, A$ and ρ such that $\gamma(\hat{\tau}) < \gamma(\bar{\theta}^r)$, that is, for a wide range of parameter values the preferred policy of the accumulated factor of production generates higher growth under the capital income cum subsidy tax than under the wealth tax scheme.

The proposition is interesting for the following reason: The right-wing government represents the owners of the accumulated factor of production and acts growth maximizing under a wealth tax scheme. Therefore, the proposition casts doubt on models such as Alesina and Rodrik's that identify growth maximizing and optimal policies of the owners of the accumulated factor of production. In this model the owners of the accumulated factor of production prefer a wealth tax scheme (Proposition 2.5) and a government representing their interests acts growth maximizing given that scheme. But that choice is not growth maximizing in comparison to a tax scheme that the owners of the accumulated factor of production would *not* choose and under which their optimal policy is not growth maximizing, but may still generate higher growth than under the accumulated factor owners' preferred (wealth) tax scheme (Proposition 2.6). Hence, the model provides an example that the owners of the accumulated factor of production do not always choose a growth maximizing tax base.

Next, I show that an increase in β , that is, an increase in the weight attached to the welfare of the non-accumulated factor of production (workers) increases the optimal θ for all non-redistributing governments ($\lambda = 0$). If $\beta > 0$ then $\bar{\theta}$

solves (2.23) so that

$$v(\tilde{\theta}(\beta), \beta)_\theta = 0$$

must hold in the optimum. One may view this as an implicit function and totally differentiate with respect to β to obtain¹⁴

$$v_{\theta\theta} \frac{\partial \tilde{\theta}}{\partial \beta} + v_{\theta\beta} = 0.$$

Equation (2.24) implies $v_{\theta\theta} < 0$, and notice that from (2.18)

$$v_{\theta\beta} = \frac{1}{(1-\theta)\rho} + \frac{1-\alpha}{\theta\rho} > 0, \quad \forall \theta \in [0, 1]$$

which entails that

$$\frac{\partial \tilde{\theta}}{\partial \beta} = -\frac{v(\tilde{\theta})_{\theta\beta}}{v(\tilde{\theta})_{\theta\theta}} > 0 \quad (2.27)$$

so that any optimal $\tilde{\theta}$ is increasing in β . But then an increase in β also raises the growth rate since $\gamma_\theta > 0$. Thus,

Proposition 2.7 *If there is no redistribution ($\lambda = 0$), placing more social weight on the welfare of the non-accumulated factor of production (higher β) raises the optimal tax rate ($\tilde{\theta}$) and growth (higher $\gamma(\tilde{\theta})$).*

This is one of the major results of the paper and in direct contrast to Proposition 1.1. It shows that if the social planner uses an income cum investment subsidy tax arrangement, and shifts political power to the *non-accumulated factor of production* (workers), the optimal tax rate on the income of the *accumulated*

¹⁴For a similar proof in a different context see Mirrlees (1986).

factor of production (capital) is raised, implying *higher growth*. Hence, under the tax arrangement it is not optimal for high growth to shift all political power to the accumulated factor of production. A similar result is obtained in Bertola (1993), but notice that taxation of capital income *does not* defeat the purpose of enhancing growth in this model.

Consider now a left-wing ($\beta = 1$) government. From (2.18) one readily verifies that $v_\theta(\beta = 1) > 0$ so that a left-wing government would choose the maximum tax rate $\theta^l = 1 - \epsilon$. I will ask under what conditions a left-wing government would want to redistribute.¹⁵ As $\theta^l = 1 - \epsilon$ I assume that ϵ is so small that $\theta \approx 1$. That facilitates the analysis without altering the qualitative results. A left-wing government wants to redistribute if $v_{\lambda|\lambda=0} > 0$, that is, if redistribution increases the workers' welfare. So from (2.18) with $\theta \approx 1$ and $\lambda = 0$ one has to check under what conditions

$$v_{\lambda|\lambda=0} = \frac{\eta_\lambda + \rho}{\eta \rho} + \frac{r_\lambda}{\rho^2} > 0$$

where $\gamma_\lambda = r_\lambda$. Recall $E \equiv (1 - \alpha)A((1 - \lambda)\theta\rho)^{-\alpha}$ and that

$$r_\lambda = -\alpha E\theta\rho, \quad \eta = E(1 - \lambda)\theta\rho, \quad \eta_\lambda = -(1 - \alpha)E\theta\rho.$$

Making the appropriate substitutions for θ, λ I get that

$$v_{\lambda|\lambda=0} = -\frac{1 - \alpha}{\rho} + \frac{1}{(1 - \alpha)A\rho^{1-\alpha}} - \frac{\alpha(1 - \alpha)A}{\rho^{1+\alpha}} > 0 \quad (2.28)$$

¹⁵In appendix A.7 I show that the optimal $\lambda \geq 0$ depends in a complicated, non-linear way on the parameters of the model. The exact solution is not of interest in this paper.

must hold if the government wishes to redistribute.¹⁶ The condition does *not* hold if $\rho \rightarrow 0$ or A is very large. Thus, a left-wing government does not want to redistribute if the economy is very efficient or the agents are very patient.

Proposition 2.8 *A left-wing government always sets the growth maximizing tax rate, $\theta^l = 1 - \epsilon$. If A is large or ρ is low, a left-wing government does not redistribute, $\lambda = 0$.*

Under a wealth tax scheme a right-wing government acts like a growth maximizer in the optimum. In contrast, this paper's tax arrangement establishes that a left-wing government that does not redistribute acts like a growth maximizer. Thus, a switch from a wealth to an income cum investment subsidy tax scheme induces an important switch in optimal policies. In particular, it makes a right and left-wing government switch roles in terms of who maximizes growth.

2.5 Distributional Implications

The distributional consequences of the optimal policies considered in the previous section are ambiguous if one wishes to compare different policies of the various governments and the two tax schemes. For simplicity, I concentrate on the policies chosen by growth maximizing governments employing the wealth tax or this model's income tax scheme. As an indicator of the income distribution I use the ratio of the *total* factor income of the capital owners to that of the workers. That is obviously only a crude measure, but as has been pointed out by, for instance, Atkinson (1983), the relationship between the income shares of factors of production and the distribution of income among persons is compli-

¹⁶With $\lambda = 0$ and $\theta \approx 1$ one gets $E = (1 - \alpha)A\rho^{-\alpha}$ and so $r_\lambda = -\alpha(1 - \alpha)A\rho^{1-\alpha}$, $\eta = (1 - \alpha)A\rho^{1-\alpha}$ and $\eta_\lambda = -(1 - \alpha)^2 A\rho^{1-\alpha}$. Simplifying yields the expression above.

cated, but factor shares remain of considerable relevance. Also, as regards the personal distribution of income it is still broadly true that income from property is of greater importance at the top of the income scale. Let F^g denote the pre-tax factor income ratio of the workers' and capitalists' factor incomes if the government pursues a growth maximizing ($\lambda = 0$) policy, then

$$F^g = \frac{r(\theta)k_t}{\eta(\theta)k_t} = \frac{r(\tau)k_t}{\eta(\tau)k_t} = \frac{\alpha}{1 - \alpha}$$

which follows from (2.10) and (2.11) and the corresponding expressions under a wealth tax scheme. So the pre-tax factor income distribution is equal under growth maximizing policies and independent of the capital stock.¹⁷ Depending on the share of capital α , pre-tax factor income inequality F^g is greater or less than one. Turning to post-tax factor income inequality I obtain these ratios

$$F(\hat{\tau}) = \frac{r(\hat{\tau}) - \hat{\tau}}{\eta(\hat{\tau})} \quad \text{and} \quad F(\hat{\theta}) = \frac{(1 - \theta) r(\theta)}{\eta(\theta)}$$

which reduce to

$$F(\hat{\tau}) = \frac{\alpha^2}{1 - \alpha} \quad \text{and} \quad F(\hat{\theta}) = \frac{\epsilon \tau}{\eta} = \frac{\epsilon \alpha}{1 - \alpha}, \quad \hat{\theta} = 1 - \epsilon. \quad (2.29)$$

under growth maximizing policies. From that I immediately get

Proposition 2.9 *If $\epsilon = \alpha$, then $F(\hat{\tau}) = F(\hat{\theta})$. If $\epsilon < \alpha$, then $F(\hat{\tau}) > F(\hat{\theta})$.*

¹⁷A word of caution is in order here. F^g should not be viewed as *the* pre-tax factor income ratio. In reality the ratio of total capital income to total wage income is less than one. For instance, in most OECD countries the factor share of physical capital is about 40 percent. Recall, however, that α is the share of broad capital, including human capital, in the model. Therefore, one may view F^g and F as inequality measures of the stylized economy under study. Rescaling the measures would not affect the qualitative results in any significant way.

Throughout it has been assumed that ϵ is small. If the share of capital is very low and equals the minimum tax rate ϵ , the post-tax factor income distributions under either tax scheme with a growth maximizing policy coincide. That is a special, but interesting case, since I have already shown that under the model's income tax scheme growth is in general higher. Thus, one may observe two economies with the same post-tax factor income distribution, but very different growth performances. In the more general case, $\epsilon < \alpha$, and the post-tax factor income distribution is more favourable to labour under the capital income cum investment subsidy tax scheme.

Hence, the only thing that one may conclude from this section's factor income distribution analysis is that the pre-tax factor income distribution is the same under both tax schemes with growth maximizing governments. The growth maximizing policy under this paper's income tax scheme is in general biased towards the post-tax income of the non-accumulated factor of production and may generate higher growth than the growth maximizing policy under the wealth tax scheme.

2.6 Conclusion

In many theoretical models high taxation of the accumulated factor of production (capital) for (re-)distributive purposes is unfavourable for high growth. The rationale for these results is not difficult to convey. If the government uses taxes that distort the private investors' incentive to accumulate capital, then any tax rate that is higher than the one that maximizes growth and that is good for the non-accumulated factor of production must slow down growth. Any policy that subsidizes investment is clearly good for growth. But that raises the question

how the subsidies are financed.

The chapter addresses two points that have been made in this context. The first point is that increasing taxes for redistributive purposes slows down growth and the second is that capital income taxation defeats the purpose of enhancing growth when used as a means to finance investment subsidies.

A *capital income cum investment subsidy* tax scheme is analyzed. The tax rate on capital income and for investment subsidies is uniform. The tax scheme is tantamount to a tax on the capital owners' consumption. I justify the implementability of the income tax scheme and the uniformity of the tax rate for a right-wing and a left-wing government. One reason why even a right-wing government may implement the tax scheme is that the government uses tax revenues to provide public inputs in production, which raise the return on capital. However, in the model the capital owners prefer another, namely a wealth tax scheme.

For the capital income cum investment subsidy tax scheme I show that investment subsidies remove the distorting effect of capital income taxation for optimizing agents. In equilibrium growth depends strictly positively on the pre-tax return to capital, the income tax rate and the time preference rate. There exists a growth maximizing time preference rate and it is growth maximizing to tax capital income maximally. The reason for these results is that the tax scheme operates like a consumption tax scheme. More impatience causes the capital owners to consume more, raising the government's tax revenues that are channelled into production as public inputs, thereby raising the return to capital and growth.

The chapter then provides a public policy analysis and compares optimal policies with those generated under a wealth tax scheme. It is shown that a right-wing government never redistributes and does not choose the growth maximizing

tax rate. Notice in Alesina/Rodrik a wealth tax scheme causes the right-wing government to act like a growth maximizer. Interestingly, I find that the capital owners' preferred policy is not growth maximizing under the capital income cum investment subsidy tax scheme. But their preferred policy under that tax scheme may still generate higher growth than their optimal, growth maximizing policy under the wealth tax scheme. Thus, the preferred policy of the accumulated factor of production is not always good for growth.

It is shown that placing *more weight* on the welfare of the *non-accumulated factor of production* (workers) leads the social planner to *raise* the optimal *tax rate* on the income of the accumulated factor of production (capital) and through that the *growth rate*. Hence, under the chapter's tax arrangement it is not optimal for growth maximization to shift political power to the accumulated factor of production. The result is in direct contrast to what is shown in Alesina and Rodrik (1994). A similar result is obtained by Bertola (1993), but notice that in comparison to a wealth tax scheme taxation of capital income *does not* defeat the purpose of enhancing growth in this model.

With the model's tax scheme a 'left-wing' government that does not redistribute towards labour acts like a growth maximizer. Thus, a switch from a wealth to an income cum investment subsidy tax scheme induces an important switch in optimal policies. In the model a 'left-wing' government only redistributes if the economy is sufficiently inefficient or the workers as a group are rather impatient. Thus, one may observe an economy with a government that represents only the interests of the non-accumulated factor of production (labour) to have redistribution *and* higher growth than an economy represented by a government solely concerned about the accumulated factor of production (capital).

Finally, I analyze the total factor income distribution consequences under

growth maximizing policies and the two different tax arrangements. I find that the chapters's tax scheme is in general biased towards generating relatively more post-tax factor income for the non-accumulated factor of production, while often inducing higher growth than under the wealth tax scheme.

However, one should be cautious about the results derived in the chapter. The set-up of the model is highly aggregated. In reality workers own capital and capital owners supply labour. It would be desirable to know more about how exactly the government achieves targeting personal investment. The model has worked with the distribution of total factor income, which is only a crude measure of the distribution of personal incomes that is usually considered to be of interest and more importance. Even though these problems provide ample room for further research, do I believe that the model captures some important aspects of taxation, economic growth and factor income distribution.

Appendix A

A.1 Proof that $\gamma = \gamma_k$

Let the capital owners' problem be

$$\max_{C_t} \int_0^{\infty} \ln C_t e^{\rho t} dt \quad (\text{A.1.1})$$

$$s.t. \quad \dot{k} = R k_t - C_t \quad (\text{A.1.2})$$

$$k(0) = \text{given}, \quad k(\infty) = \text{free}. \quad (\text{A.1.3})$$

where R denotes a constant after-tax return.¹⁸ Setting up the present value Hamiltonian for this problem yields

$$H = \ln C_t + \mu_t (R k_t - C_t)$$

where μ_t is the current value shadow price of one more unit of investment at date t . The necessary FOC for the maximization of $H(\cdot)$ are given by equations (A.1.2), (A.1.3) and the equations

$$\frac{1}{C_t} = \mu_t \quad (\text{A.1.4})$$

¹⁸The arguments below would not change if R followed a time path that the private sector agents had to take as given. See, for instance, Barro and Sala-i-Martin (1995), chpts. 2.1.2 and 4.1.4.

$$\dot{\mu}_t = \mu_t \rho - R \mu_t \quad (\text{A.1.5})$$

$$\lim_{t \rightarrow \infty} k_t \mu_t e^{-\rho t} = 0 \quad (\text{A.1.6})$$

where the last equation is a transversality condition which ensures that the present value of the capital stock approaches zero asymptotically. The shadow price evolves according to $\mu_t = \mu_0 e^{(\rho-R)t}$ where μ_0 is a positive constant which equals $\frac{1}{C_0}$. Consequently the transversality condition boils down to

$$\mu_0 \lim_{t \rightarrow \infty} k_t e^{-Rt} = 0.$$

Equations (A.1.4) and (A.1.5) imply that in the optimum consumption grows at the constant rate

$$\gamma \equiv \frac{\dot{C}_t}{C_t} = R - \rho. \quad (\text{A.1.7})$$

Thus, consumption at any date is described by

$$C_t = C_0 e^{(R-\rho)t}$$

where the initial level of consumption, C_0 , remains to be determined. Substituting for C_t in (A.1.2) implies

$$\dot{k}_t = R k_t - C_0 e^{(R-\rho)t}$$

which is a first order, linear differential equation in k_t . It is solved as follows

$$\dot{k}_t - R k_t = -C_0 e^{\gamma t}$$

$$e^{-Rt} (\dot{k}_t - R k_t) = -e^{-Rt} C_0 e^{\gamma t}$$

$$\int e^{-Rt} (\dot{k}_t - R k_t) dt = - \int C_0 e^{-\rho t} dt.$$

The last equation is an exact differential equation with integrating factor e^{-Rt} . The LHS is solved by $k_t e^{-Rt} + b_0$ and the RHS is solved by $\frac{C_0}{\rho} e^{-\rho t} + b_1$, where b_0, b_1 are arbitrary constants. Thus,

$$k_t = \frac{C_0}{\rho} e^{(R-\rho)t} + b e^{Rt} \quad (\text{A.1.8})$$

where $b = b_1 - b_0$. Substituting this into the transversality condition implies

$$\frac{1}{C_0} \lim_{t \rightarrow \infty} \left(\frac{C_0}{\rho} e^{(R-\rho)t} + b e^{Rt} \right) e^{-Rt} = \lim_{t \rightarrow \infty} \left(\frac{1}{\rho} e^{-\rho t} + \frac{b}{C_0} \right) = 0$$

which holds if the arbitrary constant b is set equal to zero. Then equation (A.1.8) becomes

$$k_t = \frac{C_0}{\rho} e^{\gamma t} \Rightarrow \gamma_k = \gamma = R - \rho$$

so that consumption and wealth grow at the same constant rate in the optimum. Furthermore, the optimal level of consumption at each date is given by $C_t = \rho k_t$.

Relation to different chapters

- Chapter 1: $R = (r - \tau)$ and $C_t = C_t^k$ and so $C_t^k = \rho k_t$ in the optimum.
- Chapter 2: $R = r$ and $C_t = \frac{C_t^k}{1-\theta}$ so that $C_t^k = (1 - \theta)\rho k_t$ in the optimum.
- Chapter 3: In the optimum $\omega_t = \omega$. Thus, $R = (r - \tau)\omega_t + (r^* - \tau^*)\phi(\omega_t)$ is constant. Hence, $C_t^k = \rho k_t$ in the optimum.

A.2 Derivation of the welfare measures

It is convenient to recall the *integration by parts* formula

$$\int_a^b v_2 dv_1 = [v_1 v_2]_a^b - \int_a^b v_1 dv_2.$$

The capitalists' and workers' intertemporal welfare is equivalent to

$$V^r = \int_0^t \ln C_t^k e^{-\rho t} \quad \text{and} \quad V^l = \int_0^t \ln C_t^W e^{-\rho t}$$

when letting $t \rightarrow \infty$. For the integration by parts define $v_2 = \ln C_t^j$, $dv_1 = e^{-\rho t} dt$ where $j = k, W$. In chapters 1 and 2 the optimal choices of the private sector agents imply that $dv_2 = \frac{\dot{C}_t^j}{C_t^j} = \gamma$ and constant. Then $v_1 = -\frac{1}{\rho} e^{-\rho t}$ so that

$$\begin{aligned} \int_0^\infty \ln C_t^j e^{-\rho t} dt &= \frac{1}{\rho} \left[-\ln C_t^j e^{-\rho t} \right]_0^\infty + \frac{1}{\rho} \int_0^\infty \gamma e^{-\rho t} dt \\ &= \frac{\ln C_0^j}{\rho} - \frac{1}{\rho^2} \left[\gamma e^{-\rho t} \right]_0^\infty. \end{aligned}$$

In chapter 1 $C_0^k = \rho k_0$ and $C_0^W = (\eta + \lambda \tau) k_0$ so that evaluation at the particular limits yields the expression for V^r and V^l as functions of τ and λ in (1.14).

In chapter 2 $C_0^k = (1 - \theta) k_0$ and $C_0^W = (\eta + \lambda \theta \rho) k_0$ leading to the expressions of V^r and V^l as functions of θ and λ in (2.15) and (2.16).

A.3 Derivation of the optimal policies

Under the wealth tax scheme the government solves

$$\max_{\tau, \lambda} (1 - \beta) V^r + \beta V^l \quad \text{s.t.} \quad \lambda \geq 0$$

The first order conditions for τ and λ are given by

$$\beta \frac{\eta_\tau + \lambda}{(\eta + \lambda\tau)\rho} + \frac{\gamma_\tau}{\rho^2} = 0 \quad , \quad \lambda \left(\beta \frac{\eta_\lambda + \tau}{(\eta + \lambda\tau)\rho} + \frac{\gamma_\lambda}{\rho^2} \right) = 0$$

Concentrating on an interior solution for λ , simplifying and rearranging yields

$$\beta \frac{\eta_\tau + \lambda}{(\eta + \lambda\tau)} = -\frac{\gamma_\tau}{\rho} \quad , \quad \beta \frac{\eta_\lambda + \tau}{(\eta + \lambda\tau)} = -\frac{\gamma_\lambda}{\rho}. \quad (\text{A.3.9})$$

Notice that γ_τ must be negative for the first equation to hold, so in the optimum $\tau > \hat{\tau}$ by Lemma 1.1. Division of these two equations by one another yields

$$\frac{\eta_\tau + \lambda}{\eta_\lambda + \tau} = \frac{\gamma_\tau}{\gamma_\lambda} \quad (\text{A.3.10})$$

and must hold in an optimum with $\lambda > 0$. Then $\gamma_\lambda = r_\lambda$ and $\gamma_\tau = r_\tau - 1$ imply $(\eta_\tau + \lambda)r_\lambda = (\eta_\lambda + \tau)(r_\tau - 1)$ which upon multiplying out becomes

$$\eta_\tau r_\lambda + \lambda r_\lambda = r_\tau \eta_\lambda + r_\tau \tau - \eta_\lambda - \tau. \quad (\text{A.3.11})$$

Notice that $r_\lambda \eta_\tau = r_\tau \eta_\lambda$ and that $\eta = \frac{1-\alpha}{\alpha} r$. Then $\lambda r_\lambda = r_\tau \tau - \frac{1-\alpha}{\alpha} r_\lambda - \tau$ and so

$$\left(\lambda + \frac{1-\alpha}{\alpha} \right) r_\lambda = \tau r_\tau - \tau \Leftrightarrow \left(\lambda + \frac{1-\alpha}{\alpha} \right) = \frac{\tau r_\tau}{r_\lambda} - \frac{\tau}{r_\lambda}.$$

Recall $r_\tau = \alpha E(1-\lambda)$, $r_\lambda = \alpha E(-\tau)$ where $E = (1-\alpha)A[(1-\lambda)\tau]^{-\alpha}$ as in (1.5).

Thus, $\frac{\tau r_\tau}{r_\lambda} = -\frac{\tau \alpha E(1-\lambda)}{\alpha E \tau} = -(1-\lambda)$. So for above

$$\lambda + (1-\lambda) + \frac{1-\alpha}{\alpha} = -\frac{\tau}{r_\lambda} \Leftrightarrow \frac{r_\lambda}{\alpha} = -\tau$$

which means that $E = 1$ and so

$$\tau = \frac{[(1-\alpha)A]^{\frac{1}{\alpha}}}{1-\lambda}. \quad (\text{A.3.12})$$

Notice that for this τ we have $E = 1$. For the first order condition for τ we note that $\eta = (1-\alpha)A[(1-\lambda)\tau]^{1-\alpha} = E[(1-\lambda)\tau] = [(1-\alpha)A]^{\frac{1}{\alpha}}$. Furthermore, $\eta_\tau = (1-\alpha)(1-\lambda)$, $\tau_\tau = \alpha(1-\lambda)$. Then (A.3.12) implies $\lambda = 1 - \frac{[(1-\alpha)A]^{\frac{1}{\alpha}}}{\tau}$ so that

$$\eta + \lambda\tau = [(1-\alpha)A]^{\frac{1}{\alpha}} + \tau \left(1 - \frac{[(1-\alpha)A]^{\frac{1}{\alpha}}}{\tau} \right) = \tau.$$

Then the first order condition for τ becomes

$$\beta \frac{\eta_\tau + \lambda}{(\eta + \lambda\tau)} = -\frac{\gamma_\tau}{\rho} \Leftrightarrow \frac{\eta_\tau + \lambda}{\tau} = -\frac{\gamma_\tau}{\beta\rho} \Leftrightarrow \frac{\eta_\tau + \lambda}{\gamma_\tau} = -\frac{\tau}{\beta\rho}.$$

For an interior solution τ must solve this equation, but the solution must also satisfy (A.3.10). Let $D \equiv \frac{\tau}{\beta\rho}$ and note that $\frac{\eta_\tau + \lambda}{\gamma_\tau} = -D = \frac{\eta_\tau + \tau}{\gamma_\tau}$ and so $\frac{\gamma_\tau}{\eta_\tau + \lambda} = -\frac{1}{D} = \frac{\gamma_\tau}{\eta_\tau + \tau}$ and hence $\frac{\eta_\tau + \lambda}{\gamma_\tau} = -D = \frac{\gamma_\tau}{\eta_\tau + \lambda} = -\frac{1}{D}$. Thus, $-\frac{1}{D} = -D$, $D = 1$ so that the solution satisfies $\tau = \beta\rho$. Thus,

$$\tilde{\tau} = \beta\rho \quad \text{and} \quad \tilde{\lambda} = 1 - \frac{[(1-\alpha)A]^{\frac{1}{\alpha}}}{\beta\rho}. \quad (\text{A.3.13})$$

which is equation (1.15) when $\beta = 1$. Recall that these equations hold for $\lambda \geq 0$, thus for $\beta\rho \geq [(1-\alpha)A]^{\frac{1}{\alpha}}$.

Suppose $\lambda = 0$, then the first order condition becomes

$$\frac{\eta_\tau}{\eta} = -\frac{\tau_\tau - 1}{\beta\rho} \Leftrightarrow \frac{(1-\alpha)E}{\tau E} = -\frac{\alpha E - 1}{\beta\rho} \Leftrightarrow (1-\alpha)\beta\rho = \tau - \alpha\tau E$$

so that the solution with $\lambda = 0$ is given by

$$(1 - \alpha)\beta\rho = \tau [1 - \alpha(1 - \alpha)A\tau^{-\alpha}] \quad (\text{A.3.14})$$

which holds only if $\beta\rho < [(1 - \alpha)A]^{\frac{1}{\alpha}}$. For $\beta = 1$ this is equation (1.16) in the text.

For the right-wing government ($\beta = 0$) the first order conditions are given by

$$\frac{\gamma_\tau}{\rho^2} = 0 \quad , \quad \lambda \left(\frac{\gamma_\lambda}{\rho^2} \right) = 0. \quad (\text{A.3.15})$$

Since $\gamma_\lambda = r_\lambda < 0$ it follows that $\lambda = 0$ and $\gamma_\tau = r_\tau - 1 = 0$ implies $\tau = \hat{\tau}$ so that by Lemma 1.1 the right-wing government acts like a growth maximizer.

Lemma $\gamma(\tau)$ is inversely related to β .

Proof: $\gamma_\tau < 0$ for $\tilde{\tau} > \hat{\tau}$ and $\rho > 0$ as given in (1.15) and (1.16). Also

$$\gamma(\tau) = \alpha A ((1 - \lambda)\tau)^{1-\alpha} - \tau - \rho.$$

Clearly, if $\lambda > 0$, then $\frac{d\tilde{\tau}}{d\beta} > 0$ in (A.3.13), and $(1 - \lambda)\tau = [(1 - \alpha)A]^{\frac{1}{\alpha}}$. Thus, $\frac{d\gamma}{d\beta} < 0$. Suppose $\beta \geq 0$ and $\lambda = 0$. Then $\tilde{\tau}$ is given as in (A.3.14) so that by the implicit function theorem $\frac{d\tilde{\tau}}{d\beta} > 0$. Thus, $\frac{d\gamma}{d\beta} = \gamma_\tau \frac{d\tilde{\tau}}{d\beta} < 0$ which proves the lemma.

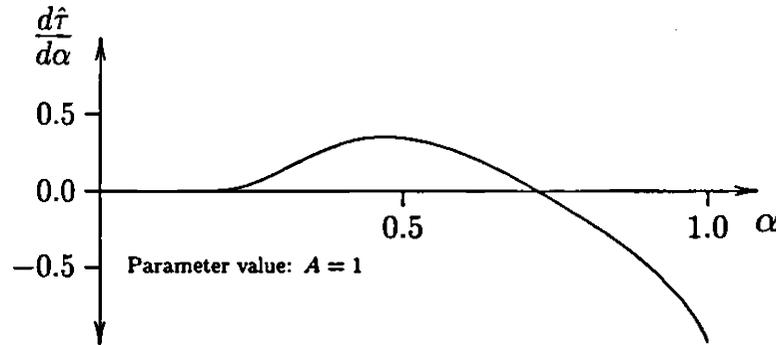
A.4 Effects of α on τ , γ and F under different policies

A.4.1 Growth Maximizing Policies

Under a *growth maximizing* policy $\hat{F} = \frac{\alpha^2}{1-\alpha}$, $\hat{\gamma} = \frac{\alpha \hat{\tau}}{1-\alpha} - \rho$, and $\hat{\tau} = [\alpha(1-\alpha)A]^{\frac{1}{\alpha}}$. Then $\frac{d\hat{F}}{d\alpha} = \frac{2\alpha(1-\alpha)+\alpha^2}{(1-\alpha)^2} > 0$ which is positive for all $\alpha \in (0, 1)$. For the growth maximizing tax rate I find

$$\begin{aligned} \frac{d\hat{\tau}}{d\alpha} &= \frac{[\alpha(1-\alpha)A]^{\frac{1}{\alpha}-1} (1-2\alpha)A}{\alpha} - \frac{[\alpha(1-\alpha)A]^{\frac{1}{\alpha}} \ln[\alpha(1-\alpha)A]}{\alpha^2} \\ &= \frac{\hat{\tau} (1-2\alpha)}{\alpha^2(1-\alpha)} - \frac{\hat{\tau} \ln \hat{\tau}}{\alpha} \end{aligned} \quad (\text{A.4.16})$$

which is not easy to evaluate. Clearly, $\ln \hat{\tau} < 0$ so that $-\frac{\hat{\tau} \ln \hat{\tau}}{\alpha} > 0$. But for $\alpha > \frac{1}{2}$ the first expression is negative so that the sign of $\frac{d\hat{\tau}}{d\alpha}$ seems to depend on α . The following plot establishes that $\frac{d\hat{\tau}}{d\alpha} \geq 0$ for a particular level of A .



Thus, there exist levels of A such that $\hat{\tau}$ first increases and then decreases in α . That means that for two different values of the share of capital, $\alpha_1 > \alpha_2$ such that $\hat{\tau}(\alpha_1) = \hat{\tau}(\alpha_2)$, the same growth maximizing tax rate $\hat{\tau}$ is induced. As $\frac{d\hat{\tau}}{d\alpha} \geq 0$ it is not clear what sign $\frac{d\hat{\gamma}}{d\alpha}$ takes. For the calculation of $\frac{d\hat{\gamma}}{d\alpha}$ rearrange to

get $(\hat{\gamma} + \rho) = \frac{\alpha \hat{\tau}}{1-\alpha}$. Then

$$\begin{aligned} \ln(\hat{\gamma} + \rho) &= \ln \alpha - \ln(1 - \alpha) - \ln \hat{\tau} \\ &= \ln \alpha - \ln(1 - \alpha) - \frac{\ln(\alpha(1 - \alpha)A)}{\alpha} \\ &= \left(\frac{\alpha + 1}{\alpha}\right) \ln \alpha + \left(\frac{1 - \alpha}{\alpha}\right) \ln(1 - \alpha) + \left(\frac{1}{\alpha}\right) \ln A. \end{aligned}$$

For the effect of a change in α on this expression I get

$$\begin{aligned} \frac{d \ln(\hat{\gamma} + \rho)}{d\alpha} &= -\left(\frac{1}{\alpha^2}\right) \ln \alpha + \left(\frac{\alpha + 1}{\alpha^2}\right) - \left(\frac{1}{\alpha^2}\right) \ln(1 - \alpha) - \frac{1}{\alpha} - \left(\frac{1}{\alpha^2}\right) \ln A \\ &= \frac{1}{\alpha} \left[\frac{1}{\alpha} - \left(\frac{1}{\alpha}\right) \ln(\alpha(1 - \alpha)A) \right] \\ &= \frac{1}{\alpha} \left[\frac{1}{\alpha} - \ln \hat{\tau} \right]. \end{aligned}$$

As $\hat{\tau} < 1$ the expression is positive. Then $\frac{d \ln(\hat{\gamma} + \rho)}{d\alpha} > 0$ which implies $\frac{d\hat{\gamma}}{d\alpha} > 0$. Thus, an increase in the share of capital raises the maximum growth rate.

A.4.2 Income Egalitarian Policies

Under the *egalitarian* policy $\frac{dF^*}{d\alpha} = 0$. Rearranging I obtain

$$\gamma_e + \rho = \tau_e(\alpha A \tau_e^{-\alpha} - 1) \quad \text{where} \quad \tau_e = [A(\alpha - (1 - \alpha)F^*)]^{\frac{1}{\alpha}}.$$

Then

$$\frac{d\tau_e}{d\alpha} = -\frac{1}{\alpha^2} \tau_e \ln \tau_e + \frac{1}{\alpha} [A(\alpha - (1 - \alpha)F^*)]^{\frac{1}{\alpha}-1} A(1 + F^*) > 0, \quad (\text{A.4.17})$$

that is, $\frac{d\tau_e}{d\alpha}$ is positive. Substitution and simplification imply

$$\ln(\gamma_e + \rho) = \ln \tau_e + \ln(1 - \alpha) + \ln F^* - \ln(\alpha - (1 - \alpha)F^*)$$

$$= \frac{1}{\alpha} \ln A + \frac{1-\alpha}{\alpha} \ln(\alpha - (1-\alpha)F^*) + \ln(1-\alpha) + \ln F^*.$$

Taking the derivative with respect to α yields

$$\begin{aligned} \frac{d(\gamma_e + \rho)}{d\alpha} &= -\frac{1}{\alpha^2} \ln A + -\frac{1}{\alpha^2} \ln(\alpha - (1-\alpha)F^*) \\ &\quad + \frac{1-\alpha}{\alpha} \left[\frac{1+F^*}{\alpha - (1-\alpha)F^*} \right] - \frac{1}{1-\alpha} \\ &= -\frac{1}{\alpha} \ln \tau_e + \frac{1}{\alpha(1-\alpha)} \left[\frac{(1-\alpha)^2(1+F^*)}{\alpha - (1-\alpha)F^*} - \alpha \right]. \end{aligned}$$

The expression is positive if

$$\begin{aligned} (1-\alpha)^2(1+F^*) &> \alpha^2 - \alpha(1-\alpha)F^* \\ F^* &> \frac{2\alpha-1}{1-\alpha} \end{aligned}$$

and $\alpha \leq \frac{1}{2}$. For $F^* < \frac{2\alpha-1}{1-\alpha}$ it may be negative, if α is sufficiently large. So if $\alpha \leq \frac{1}{2}$, then definitely $\frac{d\gamma_e}{d\alpha} > 0$. Thus, $\frac{d(\gamma_e + \rho)}{d\alpha}$ is positive for $\alpha \leq \frac{1}{2}$ and may be ambiguous if $\alpha > \frac{1}{2}$.

A.4.3 Redistributing, Left-Wing Policies

A redistributing, *left-wing* government chooses $\lambda > 0$. From equation (1.15) it follows that $\frac{d\tilde{\tau}}{d\alpha} = 0$ since $\tilde{\tau} = \rho$. For $\lambda = 1 - \frac{[(1-\alpha)A]^{\frac{1}{\alpha}}}{\rho}$ let $c \equiv (1-\alpha)A$, then

$$\begin{aligned} \frac{d\lambda}{d\alpha} &= -\frac{1}{\rho} \left[-\frac{c^{\frac{1-\alpha}{\alpha}}}{\alpha} - \frac{c^{\frac{1}{\alpha}} \ln c}{\alpha^2} \right] \\ &= \frac{c^{\frac{1}{\alpha}}}{\alpha\rho} \left[\frac{1}{1-\alpha} + \frac{\ln(1-\alpha) + \ln A}{\alpha} \right]. \end{aligned}$$

Suppose A is low (e.g. $A < e^{-\frac{\alpha}{1-\alpha}}$), then $\frac{d\lambda}{d\alpha} < 0$. Next, suppose $c \approx 1$, then $\frac{d\lambda}{d\alpha} > 0$. Thus, the sign of $\frac{d\lambda}{d\alpha}$ is ambiguous in general and depends on A . Since $\tilde{F} = \frac{\alpha}{1-\alpha} - \frac{\rho^\alpha}{(1-\alpha)A}$ I get

$$\frac{d\tilde{F}}{d\alpha} = \frac{1}{(1-\alpha)^2} - \frac{A(1-\alpha)\rho^\alpha \ln \rho + A\rho^\alpha}{A^2(1-\alpha)^2}.$$

For $A \rightarrow 0$ the expression becomes negative. For $\rho < e^{-\frac{1+\alpha}{1-\alpha}}$ with some small, positive x , the expression becomes positive. Hence, the sign of $\frac{d\tilde{F}}{d\alpha}$ is ambiguous and depends on A and ρ .

Under the $\lambda > 0$ policy (see (1.15))

$$r = \alpha A [(1-\lambda)\tau]^{1-\alpha} = \frac{\alpha}{1-\alpha} [(1-\alpha)A]^{\frac{1}{\alpha}}.$$

The growth rate can be rearranged as follows

$$\ln(\gamma + 2\rho) = \ln \alpha + \frac{1-\alpha}{\alpha} \ln(1-\alpha) + \frac{1}{\alpha} \ln A.$$

Taking the derivative yields

$$\frac{d \ln(\gamma + 2\rho)}{d\alpha} = \frac{1}{\alpha} - \frac{1}{\alpha^2} \ln[(1-\alpha)A] - \frac{(1-\alpha)}{\alpha(1-\alpha)}$$

which is positive since $(1-\alpha)A = (1-\lambda)\tau < 1$ in (1.15). Hence, under a redistributing policy $\frac{d\gamma}{d\alpha} > 0$.

A.4.4 Non-Redistributing, Left-Wing Policies

For $\lambda = 0$ equation (1.16) applies. The optimal tax rate $\hat{\tau}$ solves

$$z = \frac{\tau}{1-\alpha} - \alpha A \tau^{1-\alpha} - \rho = 0$$

The partial derivatives of z are given by

$$z_\tau = \frac{1}{1-\alpha} - (1-\alpha)\alpha A \tau^{-\alpha}$$

which is positive for all $\tau > \hat{\tau}$, and

$$z_\alpha = \frac{\tau}{(1-\alpha)^2} - A \tau^{1-\alpha} + \alpha A \tau^{1-\alpha} \ln \tau$$

so that

$$\begin{aligned} \frac{d\tau}{d\alpha} &= -\frac{z_\alpha}{z_\tau} = -\frac{\frac{\tau}{(1-\alpha)} - (1-\alpha)A\tau^{1-\alpha} + \alpha(1-\alpha)A\tau^{1-\alpha} \ln \tau}{1 - (1-\alpha)^2\alpha A\tau^{-\alpha}} \\ &= \frac{\tau \left[-\frac{1}{(1-\alpha)} + (1-\alpha)A\tau^{-\alpha} - \alpha(1-\alpha)A\tau^{-\alpha} \ln \tau \right]}{1 - (1-\alpha)^2\alpha A\tau^{-\alpha}} \end{aligned}$$

which is ambiguous in sign since the first term in z_α is positive, but the sum of the other two terms is negative. However, $\hat{\tau} \in \left((\alpha(1-\alpha)A)^{\frac{1}{\alpha}}, ((1-\alpha)A)^{\frac{1}{\alpha}} \right)$. Suppose $\hat{\tau} \rightarrow (\alpha(1-\alpha)A)^{\frac{1}{\alpha}}$ and $A = 1$. Then the $\frac{d\tau}{d\alpha}$ reduces to

$$\frac{\hat{\tau}(1-2\alpha)}{\alpha^2(1-\alpha)} - \frac{\hat{\tau} \ln \hat{\tau}}{\alpha}$$

which is the same expression as that for $\frac{d\hat{\tau}}{d\alpha}$ in (A.4.16). For $\alpha = \frac{1}{2}$ the expression is positive. For $\alpha \rightarrow 1$ a plot of the expression is similar to the one under a growth maximizing policy and reveals that the expression becomes negative.

Hence, there exist A, ρ, α such that $\frac{d\tilde{\tau}}{d\alpha} \geq 0$.

The change in the growth rate is given by

$$\begin{aligned}\frac{d\tilde{\gamma}}{d\alpha} &= A\tau^{1-\alpha} - \frac{d\tau}{d\alpha} - \alpha A [\tau^{1-\alpha} \ln \tau] + \alpha(1-\alpha)A\tau^{-\alpha} \frac{d\tau}{d\alpha} \\ &= A\tau^{1-\alpha} - \alpha A [\tau^{1-\alpha} \ln \tau] - [1 - \alpha(1-\alpha)A\tau^{-\alpha}] \frac{d\tau}{d\alpha}.\end{aligned}$$

I want to show that $\frac{d\tilde{\gamma}}{d\alpha} > 0$ for any $\tilde{\tau} \in ((\alpha(1-\alpha)A)^{\frac{1}{\alpha}}, ((1-\alpha)A)^{\frac{1}{\alpha}})$. For that it suffices to show that $\frac{d\tau}{d\alpha} < A\tau^{1-\alpha}$, since $-\alpha A [\tau^{1-\alpha} \ln \tau]$ is non-negative. For the rest of the proof it is convenient to represent the solution space $\tilde{\tau}$ in the form

$$\tilde{\tau} = x((1-\alpha)A)^{\frac{1}{\alpha}} \quad \text{where } x \in (\alpha^{\frac{1}{\alpha}}, 1) \Leftrightarrow \tilde{\tau} \in ((\alpha(1-\alpha)A)^{\frac{1}{\alpha}}, ((1-\alpha)A)^{\frac{1}{\alpha}}).$$

A higher x means that the optimal $\tilde{\tau}$ is higher. I want to show that $\frac{d\tau}{d\alpha} < A\tau^{1-\alpha}$, that is,

$$\begin{aligned}\frac{\frac{\tau}{(1-\alpha)} - (1-\alpha)A\tau^{1-\alpha} + \alpha(1-\alpha)A\tau^{1-\alpha} \ln \tau}{1 - (1-\alpha)^2\alpha A\tau^{-\alpha}} &< A\tau^{1-\alpha} \\ \frac{\tau^\alpha}{A(1-\alpha)^2} - 1 + \alpha \ln \tau &< \frac{1}{1-\alpha} - \alpha(1-\alpha)A\tau^{-\alpha}.\end{aligned}$$

Substituting $\tilde{\tau}$ for τ yields

$$\begin{aligned}\frac{x^\alpha}{1-\alpha} - 1 + \alpha \ln \tilde{\tau} &< \frac{1}{1-\alpha} - \frac{\alpha}{x^\alpha} \\ \frac{\alpha}{x^\alpha} - 1 + \alpha \ln \tilde{\tau} &< \frac{1-x^\alpha}{1-\alpha}\end{aligned}$$

and holds since $\alpha \ln \tilde{\tau}$ is unambiguously negative, $\frac{\alpha}{x^\alpha} < 1$ and $x^\alpha > 1$ for all $x \in (\alpha^{\frac{1}{\alpha}}, 1)$. Hence, $\frac{d\tilde{\gamma}}{d\alpha} > 0$.

For \tilde{F} with $\lambda = 0$ I obtain

$$\begin{aligned}\frac{dF}{d\alpha} &= \frac{1}{(1-\alpha)^2} - \frac{\tau^\alpha}{(1-\alpha)^2 A} - \frac{\tau^\alpha \ln \tau + \alpha \tau^{\alpha-1} \frac{d\tau}{d\alpha}}{(1-\alpha)A} \\ &= \frac{A - \tau^\alpha - (1-\alpha)\tau^\alpha \left[\ln \tau + \alpha \tau^{-1} \frac{d\tau}{d\alpha} \right]}{(1-\alpha)^2 A}.\end{aligned}$$

I want to show that this expression is positive. Its denominator is positive. Thus, for checking the sign of $\frac{dF}{d\alpha}$ it suffices to check the sign of the numerator. For simplicity

$$\begin{aligned}\frac{d\tau}{d\alpha} &= \frac{\tau H}{1 - \alpha(1-\alpha)^2 A \tau^{-\alpha}} \quad \text{where} \\ H &= \left[-\frac{1}{(1-\alpha)} + (1-\alpha)A\tau^{-\alpha} - \alpha(1-\alpha)A\tau^{-\alpha} \ln \tau \right].\end{aligned}$$

Then the numerator becomes

$$A - \tau^\alpha - \frac{(1-\alpha)\tau^\alpha}{1 - \alpha(1-\alpha)^2 A \tau^{-\alpha}} \left[(1 - \alpha(1-\alpha)^2 A \tau^{-\alpha}) \ln \tau + \alpha H \right] \quad (\text{A.4.18})$$

The expression in the square brackets is given by

$$(1 - \alpha(1-\alpha)^2 A \tau^{-\alpha}) \ln \tau + \alpha \left[-\frac{1}{(1-\alpha)} + (1-\alpha)A\tau^{-\alpha} - \alpha(1-\alpha)A\tau^{-\alpha} \ln \tau \right]$$

and simplifies to

$$(1 - \alpha(1-\alpha)A\tau^{-\alpha}) \ln \tau - \frac{\alpha}{(1-\alpha)} + \alpha(1-\alpha)A\tau^{-\alpha}$$

which upon substituting back into (A.4.18) and simplification yields

$$A - \tau^\alpha - \frac{[(1-\alpha)\tau^\alpha - \alpha(1-\alpha)^2 A] \ln \tau - \alpha\tau^\alpha + \alpha(1-\alpha)^2 A}{1 - \alpha(1-\alpha)^2 A \tau^{-\alpha}}.$$

Expressing this as a fraction of $(1 - \alpha(1 - \alpha)^2 A \tau^{-\alpha})$ amounts to

$$\begin{aligned} (1 - \alpha(1 - \alpha)^2 A \tau^{-\alpha})(A - \tau^\alpha) &- [(1 - \alpha)\tau^\alpha - \alpha(1 - \alpha)^2 A] \ln \tau \\ &+ \alpha\tau^\alpha - \alpha(1 - \alpha)^2 A \end{aligned} \quad (\text{A.4.19})$$

as the corresponding numerator. Now evaluate at $\tilde{\tau}$ and use $\tilde{\tau} = x((1 - \alpha)A)^{\frac{1}{\alpha}}$. Clearly, $1 > \alpha(1 - \alpha)^2 A \tilde{\tau}^{-\alpha} = \alpha(1 - \alpha)x^{-\alpha}$ since the lowest value x could assume is $\alpha^{\frac{1}{\alpha}}$. Thus, the denominator of the fraction is positive. Turning to the numerator in (A.4.19) I find

$$\begin{aligned} (1 - \alpha(1 - \alpha)x^{-\alpha})A(1 - (1 - \alpha)x^\alpha) &- [(1 - \alpha)^2(x^\alpha - \alpha)] A \ln \tilde{\tau} \\ &+ \alpha(1 - \alpha)A(x^\alpha - (1 - \alpha)) \end{aligned}$$

The term $-[(1 - \alpha)^2(x^\alpha - \alpha)] A \ln \tilde{\tau}$ is non-negative since $x^\alpha > \alpha$. I wish to show that the sum of the other two terms is positive. Multiplying out and collecting terms I get

$$\begin{aligned} A - \alpha(1 - \alpha)x^{-\alpha} - (1 - \alpha)Ax^\alpha + \alpha(1 - \alpha)^2 A + \alpha(1 - \alpha)Ax^\alpha - \alpha(1 - \alpha)^2 A, \\ A(1 - \alpha(1 - \alpha)x^{-\alpha} - (1 - \alpha)^2 x^\alpha) \equiv M(x). \end{aligned}$$

It is not difficult to verify that if $x \rightarrow \alpha^{\frac{1}{\alpha}}$, then $M \rightarrow A\alpha(1 - (1 - \alpha)^2) > 0$ and if $x \rightarrow 1$, then $M \rightarrow \alpha A > 0$. Thus, at the boundaries of x the numerator is positive. For showing that it is positive for all values in x , I look for extrema of $M(x)$. The function M is differentiable in x and so continuous. I look for either maxima or minima of M . If one finds a unique x in $(\alpha^{\frac{1}{\alpha}}, 1)$ that maximizes M , then by the sign found for the endpoints in that interval, it follows that M is positive. I will now show that all extrema in the relevant range maximize M so

that it cannot be negative. Taking the derivative yields

$$\frac{dM}{dx} = A(\alpha^2(1-\alpha)x^{-\alpha-1} - \alpha(1-\alpha)^2x^{\alpha-1})$$

and setting it equal to zero establishes

$$x^* = \left(\frac{\alpha}{1-\alpha}\right)^{\frac{1}{2\alpha}}$$

as the value of x that yields a unique extremum of M for given α . Suppose the extremum were a minimum and $\alpha \geq \frac{1}{2}$. Then $x^* \geq 1$ and by the boundary argument M would be positive. Thus, I concentrate on $\alpha < \frac{1}{2}$ for which it is possible that $\alpha^{\frac{1}{2}} < x^* < 1$. For showing that x^* maximizes M I calculate

$$\begin{aligned} \frac{d^2M}{dx^2} &= A(-\alpha^2(1-\alpha)(1+\alpha)x^{-\alpha-2} + \alpha(1-\alpha)^2x^{\alpha-2}) \\ &= Ax^{-2}(\alpha(1-\alpha)^3x^\alpha - \alpha^2(1+\alpha)(1-\alpha)x^{-\alpha}). \end{aligned}$$

Substituting in x^* one obtains

$$\begin{aligned} \frac{d^2M}{dx^2} &= Ax^{-2} \left(\alpha(1-\alpha)^3 \left(\frac{\alpha}{1-\alpha}\right)^{\frac{1}{2}} - \alpha^2(1+\alpha)(1-\alpha) \left(\frac{1-\alpha}{\alpha}\right)^{\frac{1}{2}} \right) \\ &= Ax^{-2} \alpha^{\frac{3}{2}}(1-\alpha)^{\frac{3}{2}} ((1-\alpha) - (1+\alpha)) < 0. \end{aligned}$$

Hence, $x^* \in (\alpha^{\frac{1}{2}}, 1)$ maximizes M . Thus, the infimum of M is at $\alpha^{\frac{1}{2}}$ which establishes that M is positive. As all other terms of $\frac{d\bar{F}}{d\alpha}$ are positive it follows that $\frac{d\bar{F}}{d\alpha} > 0$.

A.5 Iso-elastic utility functions and comparative dynamics

Suppose the capital owners have the instantaneous utility function

$$U(C_t^k) = \frac{C_t^{1-\nu} - 1}{1-\nu}, \quad \nu > 1 \quad (\text{A.5.20})$$

where the constant $\sigma = \frac{1}{\nu}$ represents the elasticity of intertemporal substitution. If $\nu \rightarrow 1$, $U(\cdot)$ reduces to logarithmic utility. A high ν implies a low elasticity intertemporal substitution, low σ . This means that the capital owners like to smooth consumption. In contrast, a high elasticity of substitution implies that the investors are indifferent to the timing of consumption. In that case the agents may defer consumption for a long time while investing in order to consume a large amount at a future date. By restricting $\nu > 1$ I rule out such behaviour implying $\sigma \in (0, 1)$.¹⁹ Notice that a high time preference rate ρ implies that the investors *value* future consumption less than current consumption.

The capital owners solve a problem similar to (2.6a) under the dynamic budget constraint

$$C_t^k = (1 - \theta) [rk_t - \dot{k}_t].$$

Proceeding as in section 2.2.2 it is not difficult to verify (see also e.g. Barro and Sala-i-Martin (1995), chpt. 2.1.2) that the steady state, balanced growth rate in

¹⁹Notice that in principle one could have assumed $0 < \nu < 1$ as well which may, however, cause problems below. For the argument I wish to make it may suffice to show that the model generalizes to all functions where $\nu > 1$.

a market equilibrium with arbitrary taxes is given by

$$\gamma = \frac{r - \rho}{\nu} = \sigma(r - \rho). \quad (\text{A.5.21})$$

Then the optimal level of consumption is determined by

$$C_t^k = (1 - \theta)[r - \gamma]k_t = (1 - \theta)[(1 - \sigma)r + \sigma\rho]k_t$$

where $k_t = k_0 e^{\gamma t}$. Thus, σ and ρ have an effect on both the level and growth of the capital owners' consumption. For *given* θ and r an increase in ρ (more impatience) or a decrease in σ (more consumption smoothing) lower the growth rate and raise the constant fraction $\frac{C_t^k}{k_t}$, that is, the capital owners' steady state consumption per units of capital. Thus, more impatience or more consumption smoothing make the capitalists *less willing to save* for *given* tax rates and *given* $\frac{g_t}{k_t}$ and r .

By assumption the government sets $\frac{g_t}{k_t} = \text{constant}$. For convenience assume $\lambda = 0$. Then with the balanced budget condition (2.4) one gets

$$b \equiv \frac{g_t}{k_t} = \theta(r - \gamma) = \theta(r - \sigma(r - \rho)).$$

In equilibrium the marginal product equals the return on capital, that is, $r = \alpha A(b)^{1-\alpha}$. Then b is implicitly defined as

$$\begin{aligned} b &= \theta \left[(1 - \sigma)\alpha A(b)^{1-\alpha} + \sigma\rho \right] \\ b^\alpha &= \theta \left[(1 - \sigma)\alpha A + \sigma\rho b^{\alpha-1} \right]. \end{aligned}$$

It is constant by the following argument. Define

$$x \equiv b^\alpha - \theta \left[(1 - \sigma)\alpha A + \sigma \rho b^{\alpha-1} \right]$$

then $x_b = \alpha b^{\alpha-1} - \theta \sigma \rho (\alpha - 1) b^{\alpha-2} > 0$ for all $b \in (0, 1)$. Then the solution to $x = 0$ must be independent of time since $\dot{x} = 0$. Thus, $\frac{db}{dt} = \dot{b} = 0$ by the implicit function theorem so that

$$g_t = c(\theta, \sigma, \rho) k_t \Leftrightarrow r = f(\theta, \sigma, \rho) \quad (\text{A.5.22})$$

which follows from (2.5). With $x_b > 0$ one verifies that $x_\rho < 0$ so that $\frac{db}{d\rho} > 0$ and hence $\frac{dr}{d\rho} > 0$. Also, $x_\sigma = \theta(\alpha A - \rho b^{\alpha-1}) \geq 0$ depending on ρ, α and A for given θ . Hence, the effect of an increase in σ on b and r is ambiguous.

A.6 Conditions for an interior solution

From (2.21) we must have that

$$\frac{\rho(1 - \beta)(\eta_\lambda + \theta\rho)}{(1 - \theta)} = \gamma_\theta(\eta_\lambda + \theta\rho) - \gamma_\lambda(\eta_\theta + \lambda\rho).$$

Recall that

$$r = \alpha A ((1 - \lambda)\theta\rho)^{1-\alpha}, \quad \eta = (1 - \alpha)A ((1 - \lambda)\theta\rho)^{1-\alpha}$$

Let $E \equiv (1 - \alpha)A ((1 - \lambda)\theta\rho)^{-\alpha}$. The partial derivatives of r, η are given by

$$r_\theta = \alpha E (1 - \lambda)\rho, \quad r_\lambda = -\alpha E \theta \rho, \quad \eta_\theta = (1 - \alpha)E (1 - \lambda)\rho, \quad \eta_\lambda = -(1 - \alpha)E \theta \rho$$

Multiplying out the LHS of (2.21) noting that $\gamma_i = r_i, i = \theta, \lambda$ one gets

$$r_\theta \eta_\lambda + r_\theta \theta \rho - r_\lambda \eta_\theta - r_\lambda \lambda \rho$$

The derivatives imply that $r_\theta \eta_\lambda - r_\lambda \eta_\theta = 0$. So the LHS reduces to

$$\begin{aligned} r_\theta \theta \rho - r_\lambda \lambda \rho &= \alpha E(1 - \lambda)\theta \rho^2 + \alpha E\theta \lambda \rho^2 \\ &= \alpha E[\rho^2 \theta - \lambda \theta \rho^2 + \lambda \theta \rho^2] \\ &= \alpha E \rho^2 \theta \end{aligned}$$

Then equation (2.21) is really given by

$$\begin{aligned} \frac{\rho(1 - \beta)(\eta_\lambda + \theta \rho)}{(1 - \theta)} &= \alpha E \rho^2 \theta \\ \frac{\rho(1 - \beta)(\theta \rho - (1 - \alpha)E\theta \rho)}{(1 - \theta)} &= \alpha E \rho^2 \theta \\ \frac{(1 - \beta)(1 - (1 - \alpha)E)}{(1 - \theta)} &= \alpha E \\ ((1 - \lambda)\theta \rho)^\alpha - (1 - \alpha)^2 A &= \frac{(1 - \theta)\alpha(1 - \alpha)A}{1 - \beta} \\ ((1 - \lambda)\theta \rho)^\alpha &= B \end{aligned}$$

where $B \equiv \frac{(1 - \theta)\alpha(1 - \alpha)A}{1 - \beta} + (1 - \alpha)^2 A$ and B is increasing in A and β for given θ .

From this one verifies that the optimal relationship between θ and λ must satisfy

$$\lambda = 1 - \frac{B^{\frac{1}{\alpha}}}{\theta \rho}$$

which is equation (2.22) in the text.

A.7 The optimal λ for a left-wing government

The left-wing government solves $v_\lambda = 0$ if $\lambda \geq 0$ in the optimum. From (2.18) this entails

$$(\eta_\lambda + \theta\rho)\rho = -\gamma_\lambda (\eta + \lambda\theta\rho)$$

As $\theta^l = 1 - \epsilon$, substitute for θ above and let $\theta \approx 1$. Recall $\gamma_\lambda = r_\lambda$, then

$$(\eta_\lambda + \rho)\rho = -r_\lambda (\eta + \lambda\rho).$$

Since $E \equiv (1 - \alpha)A((1 - \lambda)\theta\rho)^{-\alpha}$ so that

$$r_\lambda = -\alpha E\theta\rho, \quad \eta = E(1 - \lambda)\theta\rho, \quad \eta_\lambda = -(1 - \alpha)E\theta\rho$$

Making the appropriate substitutions with $\theta \approx 1$ I get

$$\begin{aligned}(\rho - (1 - \alpha)E\rho) &= (1 - \alpha)E\rho(E(1 - \lambda)\rho + \lambda\rho) \\ 1 - (1 - \alpha)E &= (1 - \alpha)E(E(1 - \lambda) + \lambda).\end{aligned}$$

Thus, λ depends in a complicated and non-linear way on the parameters of the model.

A.8 Numerical Simulation Procedure

I have defined the following variables in Mathematica

```
t := (a*(1 - a)*A)^a^(-1)
gt := -rho - t + a*A*t^(1 - a)
```

```

gth := -rho + a*A*(rho*th)^(1 - a)
c   := FindRoot[ths*rho - t*(1 - ths)^(1/a) == 0, {ths, 0}]
tst :=(1 - th) - a^(a/(1 - a))

```

where $th = \theta$, $t = \hat{r}$, $gt = \gamma(\hat{r})$ and $gth = \gamma(\theta)$. Setting $A = 1$ and for given values of α and ρ I have calculated $ths = \bar{\theta}^r$, set $ths = th$ and calculated gt and gth , recording the values in the table. I have checked, but not recorded, the results with calculating tst .

Part II

**Economic Growth and
Distributive Policies in Open
Economies**

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Chapter 3

Economic Growth, Redistribution, Capital Mobility and Tax Competition in Open Economies

3.1 Introduction

In this chapter I investigate the effects of redistributive taxation on economic growth in economies linked by factor mobility. In many policy discussions that address the issue of growth vs. redistribution, setting higher taxes for redistributive purposes is deemed to slow growth. Yet most developed and some developing countries redistribute a significant share of their GDP. Does that always lead to lower GDP growth? In the chapter's model the experience of higher or lower GDP growth, when governments opt for redistribution, depends on who their opponents are when setting taxes in a non-cooperative environment. Further-

more, it is shown that the growth/redistribution trade-off crucially hinges on technological efficiency.

As in the previous chapters I assume that the government provides public services that feed back into production. The model follows Alesina and Rodrik (1991), (1994) in that a government that cares about the non-accumulated factor of production in a closed economy chooses lower growth if it redistributes resources to that factor.

The present chapter extends the growth redistribution trade-off problem to a two-country world.¹ I identify the accumulated factor of production with capital and the non-accumulated factor of production with labour. The workers never save, and supply labour inelastically.² The capital owners do not work, accumulate capital and decide where to install their capital. That model specification allows one to concentrate on the problem of growth and distributive taxation.

Capital is internationally mobile in the chapter, and capital mobility has a direct bearing on the productivity of capital employed in production. The underlying forces governing the varying degrees of capital mobility are left unmodelled.

As Alesina and Rodrik I assume that the governments tax the capital owners' wealth, but not the non-accumulated factor of production. The wealth tax scheme is meant to represent a broad class of tax arrangements and captures problems associated with taxation of the accumulated factor of production in the growth process.³ As in the previous chapters expropriation of capital is ruled out for the

¹The chapter is a revised version of 'Economic Growth, (Re-)Distributive Policies, Capital Mobility and Tax Competition in Open Economies', EUI Working Paper, ECO 97/24, 1997.

²Bertola (1993) derives this behaviour for utility maximizing agents.

³In light of chapter 2 the choice of tax base is not at all innocuous. For instance, Bertola (1993) or Alesina and Rodrik (1994) point out that indirect taxation may lead to different results as regards the growth redistribution trade-off. In the previous chapter it has been shown that a capital income taxation cum investment subsidy tax scheme designed to equal a tax on consumption of the accumulated factor of production may guarantee higher growth in a closed economy for left-wing governments than right-wing ones. An extension of this closed

governments by assumption. Two principles for capital income taxation in open economies are common.⁴

Under the '*residence principle*' residents are taxed uniformly on their worldwide income regardless of the source of income (domestic or foreign), while non-residents are not taxed on income originating in a country.

Under the '*source principle*' all types of income originating in a country are taxed uniformly, regardless of the place of residence of the income recipients.

In the chapter it is simply assumed that the *source principle* for wealth taxation is adopted as a tax rule. If a country loses capital, it suffers in terms of welfare of the capital owners or the workers. Given the danger of losing capital and absent any problems arising from transfer pricing, the source principle appears more suited as a tax principle, because in a non-cooperative environment governments cannot perfectly monitor their residents' income or wealth. Another, empirical justification for the assumption is that capital income paid by subsidiaries at home to the center of multinational enterprises abroad (where it is often tax exempt) is typically taxed at source and at the same rate as the domestic firms' capital income.

In the optimum the capital owners allocate their capital depending on the economy result to open economies has been done in 'Redistribution, Income cum Investment Subsidy Tax Competition and Capital Flight in Growing Economies', EUI Working Paper, ECO 95/16, 1995. The results there suggest that any policy that guarantees high after-tax returns may maximize growth and attract capital. Even though right-wing and left-wing governments may switch roles in who guarantees high after-tax returns, the basic return-growth relationship for attraction of capital exists in this chapter and the open economy version of chapter 2. I have chosen the wealth tax base to relate to the literature that treats optimal policies of the accumulated factor of production as similar to growth maximizing policies.

⁴Razin and Yuen (1992) use an endogenous growth set-up to show that the *residence principle* is Ramsey efficient. Their result seems to suffer from a time inconsistency problem since distortionary capital or wage taxation may produce time inconsistent solutions. [Cf. Fischer (1980), Chamley (1985).] Capital taxation in economies with high capital mobility has received quite some attention recently in e.g. Chamley (1992), Canzonieri (1989), Roubini and Sala-i-Martin (1992), Gosh (1991) and Devereux and Shi (1991).

after-tax returns on capital in the economies.⁵ For given public policies in the two economies, I show that in the market equilibrium domestic GDP growth depends crucially on the capital allocation decision of the investors.

For the public policy analysis I assume that the governments in each country are either 'right-wing' and only care about the capital owners, or they are 'left-wing' caring about the workers only. The governments' objectives are such that a right-wing government wants to maximize the national investors' worldwide income, whereas a left-wing government is concerned about redistribution, GDP and GDP growth. In a closed economy the right-wing government acts growth maximizing.

The governments of open economies are taken to engage in *tax competition*.⁶ The objectives imply that the welfare maximizing governments implicitly compete for capital.⁷ The governments (left-wing or right-wing) in each country (domestic and foreign) move simultaneously, but before the private sector.⁸

For technologically similar economies it is shown that in the Nash Equilibria there is no room for redistribution for two left-wing governments. The result holds for sufficiently high capital mobility. For very low capital mobility the governments redistribute, but less than in a closed economy. The intuition for the result is the following: The left-wing governments face the trade-off between growth and redistribution. For the latter they need capital which is internationally mobile.

⁵For two-sector growth models with shifting of resources for investment see, for instance, Inada (1963), Rahman (1963), Intriligator (1964), Ryder (1969), Hamada (1969), Pitchford (1977) or Bagchi, Olsder and Strijbos (1981).

⁶Tax competition between fiscal authorities has been studied in numerous papers such as, for instance, Gordon (1983), Zodrow and Mieszkowski (1986), Wilson (1986), Wildasin (1988), Wildasin (1989), Gordon (1992), Bond and Samuelson (1989), Coates (1993), Kanbur and Keen (1993), or Lockwood (1993).

⁷Competition for capital has, for example, been analyzed in Sinn (1993).

⁸For a model that studies the related problem of solving the trade-off between the provision of government consumption goods and growth in a Barro (1990) world see Devereux and Mansoorian (1992).

They can only get more capital if they set a tax rate that approaches the one guaranteeing the maximum after-tax return. For redistribution they want to set higher taxes. Since it is capital that is redistributed, tax competition causes a left-wing government to concentrate on securing high enough wages. By that the effects of the concern for wealth inequality are reduced. The result is driven by capital mobility and strategic interaction between two governments which have the same preferences.

If a left-wing and a right-wing government compete in taxes, the strategic interaction is shown to be *less*. The reason is that the right-wing government is not concerned about redistribution. It just wishes to maximize the capital owners' utility by securing them a maximum after-tax return on capital. As the after-tax return on capital determines growth, it maximizes growth and by that it also attracts foreign capital. The lack of redistributive concern results in an extremely simple reaction function which the right-wing government possesses regardless of who its opponent is. Given the fixed right-wing reaction function, the left-wing government knows it cannot attract foreign capital. As a consequence it chooses to redistribute, albeit less than in the closed economy, and experience relatively low GDP growth.

As capital mobility increases, the left-wing governments are shown to begin mimicking right-wing policies. For high capital mobility it follows that two competing left-wing governments optimally set tax rates closer to the growth maximizing one than under left-right competition.

Next, the chapter analyzes the effect of technological efficiency differences. I show that as long as an efficiency gap can be maintained the efficient economy's government gets more capital. That is especially true for a left-wing government with an efficient economy. If the gap is large enough, it may redistribute *and* have

higher GDP growth than a right-wing opponent. Thus, the growth redistribution trade-off may not be a question of being right-wing or left-wing (preferences), but rather a problem of being efficient or not (technology). Interestingly and contrary to some policy debates, very high factor mobility ('globalization') may *not* constrain a nationally preferred redistribution policy, if the economy is sufficiently efficient.

Finally, for some degrees of capital mobility the model has the surprising implication that a left-wing government may be better off in terms of GDP growth if it faces competition from another left-wing government. That goes with the cost of a reduction in or no redistribution. Competing against a right-wing government in turn allows for some redistribution in the optimum, at the cost of reduced GDP growth.

Thus, one may conclude that high GDP growth and redistribution are possible if the economy is sufficiently efficient. Government preferences alone may not adequately explain the pattern of growth and redistribution in open economies with wealth tax competition, differences in strategic behaviour and varying degrees of factor mobility.

At the end of the chapter I discuss some empirical implications of the model and compare them to the results of chapter 1.

The chapter is organized as follows: Section 3.2 presents the model set-up, derives the dynamic market equilibrium and discusses the optimal policies in a closed economy. Section 3.6 analyzes tax competition among governments with different objectives. The main results of the chapter are stated in propositions. Section 3.7 discusses some empirical implications and section 3.8 concludes.

3.2 The Model

Consider a two-country world with a "domestic" and a "foreign" country. Denote variables in the foreign country by a (*). There are many identical individuals in each country, who all have the same rate of time preference ρ . The capitalists (k) own capital and no labour and the workers (W), own labour, but no capital. All agents derive logarithmic utility from the consumption of a homogeneous, malleable good that is produced in the two countries, and the domestic consumers prefer to consume the domestic good. By assumption all goods as well as the capital stocks can be transferred costlessly between the economies. Following Barro (1990) aggregate domestic production takes place according to

$$Y_t = A K_t^\alpha G_t^{1-\alpha} L_t^{1-\alpha}, \quad \text{where } 0 < \alpha < 1 \quad (3.1)$$

where Y_t is output produced in the home country, G_t are public inputs to production and A is an efficiency index. The foreign country has the same technology and technological differences between the countries are due to differences in efficiency. The economies are called *similar* if $A = A^*$ because the countries may well be different in terms of institutional or cultural development. I call economies *different* if $A \geq A^*$, which may capture the situation when one compares a developed Northern with a less-developed Southern economy. I abstract from problems arising from depreciation of the capital stock. Furthermore, $L_t^* = L_t = 1$ so that the labour endowment is inelastic and equal in both economies. K_t is an *index* of the domestically productive capital stock in the home economy. It takes the form

$$K_t = f(\omega_t k_t, (1 - \omega_t^*) k_t^*) = \omega_t k_t + \phi_t^* k_t^* \quad (3.2)$$

where k_t (k_t^*) is the real capital stock owned by domestic (foreign) capitalists. The variable $\omega_t \in [0, 1]$ denotes the fraction of real capital at date t owned by domestic capitalists that is retained at home for domestic production. The foreign owned capital stock k_t^* that becomes domestically productive depends on $\phi_t^*(\omega_t^*; z)$ which satisfies

$$\begin{aligned} 0 \leq \phi_t^*(\omega_t^*; z) \leq 1 - \omega_t^* \quad , \quad \phi_{\omega_t^*}^* ; \phi_{\omega_t^*, \omega_t^*}^* \leq 0 \\ \lim_{z \rightarrow \infty} \phi_t^* = 1 - \omega_t^* \quad , \quad \lim_{z \rightarrow 0} \phi_t^* = 0. \end{aligned} \tag{3.3}$$

I assume symmetry for both economies so that ϕ_t^* and ϕ_t are symmetric functions. The parameter z measures capital mobility imperfection in the sense to be explained below. The function ϕ_t captures the following: The amount of domestically owned capital, $\omega_t k_t$, that enters domestic production is fully productive at home. In contrast, the domestic capital owners may send $(1 - \omega_t)k_t$ abroad, but their capital stock is not as productive abroad as at home. In the model it is generally less productive abroad and enters foreign production via, K_t^* , that is, through $\phi_t k_t$ which is less than or equal to $(1 - \omega_t)k_t$. Analogous reasoning holds for domestic production Y_t and foreign owned capital in (3.1).

Thus, domestically and foreign owned capital stocks are imperfect substitutes in production. The degree of substitutability is due to productivity differences. The assumption that ϕ_t^* is a function of ω_t^* means that the productivity of foreign capital depends on how much the foreign capital owners have sent to the domestic economy. If more foreign capital (lower ω_t^*) enters the domestic economy's production, foreign capital becomes more productive (higher ϕ_t^*) in the domestic economy. One may think of foreign capital as a basket of foreign owned capital goods.⁹ As more of these goods enter domestic production, the possibility of

⁹The following is worth noting: I have assumed imperfect capital mobility of the same good.

finding ways to use these goods in a way that produces as efficiently as domestic capital goods increases. That justifies $\phi_{\omega_i}^* < 0$, since a reduction in ω_i^* increases ϕ^* . The parameter z measures the ease with which that is possible. A higher z means that it is easier to find ways so that foreign owned capital is as productive as domestically owned capital. The ease of achieving domestic capital productivity (increasing capital substitutability) is supposed to reflect the degree of capital mobility imperfection between the economies. I motivate that as follows:

1. From the trade literature it is well known that highly integrated economies like France and Germany or the states within the U.S. have very large intra-industrial trade, including trade in capital goods. This suggests that a machine produced in Germany (capital good) is as productive in France as in Germany when used as a production input and vice versa. In this model I extend that by assuming that a machine (stock) owned by a German is as productive in French as in German production.
2. Institutions play a powerful role in restricting the usefulness of foreign (real) capital at home. For example, if a firm in the U.S. would buy a car manufactured in Europe according to European safety standards, the U.S. firm would not be able to use that car on U.S. roads, because European cars are not required to have blinkers on the car's sides and have different bumpers. Another example is provided by domestically imposing certain norms and laws that may make foreign capital domestically unproductive.

For these reasons I call z a measure of capital mobility. If $z \rightarrow \infty$ capital

Call it *ectoplasm*, cf. Barro and Sala-i-Martin (1995). Ectoplasm and its fruits are assumed to be edible (consumption good), the fruits can also be used as investment for growing more ectoplasm. I assume that capital is in general less productive abroad and that shipping is costless. Then domestic capital will be sent abroad if the return is higher there. Wherever the owner's ectoplasm trees are located, investment in new ectoplasm trees in the particular country is assumed to be costless.

mobility becomes perfect, the different capital stocks become perfect substitutes in production and become equally productive. If $z \rightarrow 0$, the economies become autarkic, the capital stocks become unsubstitutable and foreign capital is getting completely unproductive in the domestic economy's production. The exact influence of the (institutional or technological) factors determining z are left unmodelled.

The *firms* in each country operate in a perfectly competitive environment, act as profit maximizers and cannot influence the public inputs to production. The firms are generally owned by domestic and foreign capital owners who rent capital to and demand shares of the domestic firms. The same holds for the foreign firms. The domestic capitalists' assets are their shares of the firms. The shares of the domestic and foreign firms are collateralized one-to-one by capital. The markets for assets and capital clear at each point in time.¹⁰

If the domestic capitalists send their capital abroad, they incur a loss $\phi_t < 1 - \omega_t$ per unit of domestic capital, as domestically owned capital is less productive abroad. However, domestically owned capital may be used for the production of foreign type or domestic type good in the foreign economy. If the domestic capitalists send their capital abroad in order to produce there, they choose to pay the foreign workers (and the government) in foreign type good, and they choose to pay themselves in domestic type good. That is so, because all domestic agents prefer to consume domestic goods. I assume that the foreign (domestic) firms using domestically (foreign) owned capital can produce both types of good and that once the capitalists have chosen which type of good is to be produced, one cannot change a domestic type good into a foreign type good. Thus, if

¹⁰The assumptions that assets are collateralized one-to-one and the rental rate of domestic and foreign capital is - later on - constant can easily be relaxed without altering the results. For a justification of these assumptions cf. Barro and Sala-i-Martin (1995), Chpt. 2.

there is any domestically owned capital abroad (or any foreign owned capital in the domestic economy), the prices of the two types of goods must be the same, because, otherwise, a profit maximizing firm would produce only that type of good which commands the higher price. Hence, a firm that uses foreign owned capital in domestic production will produce both types of goods only if the prices of the goods are equal. That price serves as numéraire and is set equal to 1.

Given perfect competition the firms in the domestic economy generally rent foreign and domestic capital and hire labour in spot markets in each period in their country. For constant returns to capital and labour, factor payments exhaust output. Profit maximization for given ω_t, ω_t^* entails that firms pay each factor of production its marginal product

$$\begin{aligned} \frac{\partial Y_t}{\partial(\omega_t k_t)} = \alpha A \mathfrak{F} = r_t & \quad , \quad \frac{\partial Y_t}{\partial((1 - \omega_t^*) k_t^*)} = \left(\frac{\phi_t^*}{1 - \omega_t^*} \right) r_t \\ \frac{\partial Y_t}{\partial L_t} = (1 - \alpha) A \mathfrak{F} K_t = w_t & \quad , \quad \text{where } L_t = 1, \forall t \text{ and } \mathfrak{F} \equiv \left(\frac{G_t}{K_t} \right)^{1-\alpha} . \end{aligned} \quad (3.4)$$

Due to the productivity differences the marginal product of foreign owned capital $(1 - \omega_t^*) k_t^*$ in domestic production is lower than that of domestic capital $\omega_t k_t$.¹¹ It is clear that the marginal products depend on government policy through the amount of public services supplied.

3.2.1 The Public Sector

The governments in both countries redistribute and tax the market value of capital at constant rates at each point in time. Let τ be the tax rate on the market value of capital (wealth) which is held domestically by domestic investors. Thus,

¹¹If foreign owned capital is domestically productive, then $K_t = \omega_t k_t + \left(\frac{\phi_t^*}{1 - \omega_t^*} \right) (1 - \omega_t^*) k_t^*$ so that a marginal increase of foreign owned capital (more $(1 - \omega_t^*) k_t^*$) in the domestic economy raises K_t by $\left(\frac{\phi_t^*}{1 - \omega_t^*} \right)$.

τ is levied on $\omega_t k_t$. The government also taxes the market value of real foreign capital located in the home country, $\phi_t^* k_t^*$, that is, the government demands $\tau \phi_t^* k_t^*$ of the domestic type good owned by the foreign capitalists or, equivalently, it takes the fraction τ of foreign owned, foreign type good which is less productive in producing domestic public services G_t . The reason that the government taxes less than $(1 - \omega_t^*) k_t^*$ lies in the fact that if the government raised $\tau(1 - \omega_t^*) k_t^*$ in order to buy capital goods in the domestic market to provide them as public inputs in production, the buyers of this type of capital good would only be willing to pay ϕ_t^* per unit of k_t^* for it, since foreign capital is less productive at home.¹² Analogous reasoning applies for the foreign government. This way of taxing wealth means that the countries adopt the *source principle* as a tax rule which requires that all types of wealth present in a country be taxed uniformly, regardless of the place of residence of the owners of wealth.

The government faces the following budget constraint, which is assumed to be balanced at each point in time

$$\tau K_t = G_t + \lambda \tau K_t.$$

The LHS depicts the tax revenues and the RHS public expenditures. The workers receive the fraction λ of tax revenues, that is, $\lambda \tau K_t$, as transfers and G_t is spent on public inputs to production. The variable λ represents the degree of redistribution in the economy. Rearranging and taking into account that the domestic

¹²Recall that foreign capital yields income $\frac{\phi_t^* r_t}{1 - \omega_t^*}$ at home. Therefore, the price per unit of foreign capital at home equals $\frac{\phi_t^*}{1 - \omega_t^*}$ which is less than that of domestic capital. Then the total market value of foreign capital is given by $\phi_t^* k_t^*$. That way of taxing the market value of capital is compatible with the source principle of capital *income* taxation.

government may have two sources of tax revenues the budget constraint satisfies

$$G_t = (1 - \lambda)\tau K_t. \quad (3.5)$$

Notice that τ is set by the government independently of other factors in the economy which corresponds to the uniform taxation of wealth as required by the strict form of the source principle. Differential taxation of foreigners and residents in the presence of perfect capital mobility has been investigated in a working paper version of the chapter.¹³ In contrast, in this paper the strict form is assumed to hold. As shown in e.g. Razin and Sadka (1994) or Bovenberg (1994) the source principle entails a *uniform* taxation of residents' and foreigners' capital income. The model can in principle allow for discriminatory taxation at the expense of considerable technical complications. Of course, the question whether tax discrimination plays a major role in the equilibria below is of interest, but then all the equilibria found in this paper can be interpreted as and shown to be results about average tax rates in a model with discriminatory taxation.

3.2.2 The Private Sector

The private sector is made up of many identical firms, workers and capital owners. Equation (3.4) implies that for given ω_t ω_t^* and given public policy the returns to domestically owned capital and labour are given by

$$r = \alpha A[(1 - \lambda)\tau]^{1-\alpha}, \quad (3.6)$$

$$w_t \equiv \eta(\tau, \lambda)K_t = (1 - \alpha)A[(1 - \lambda)\tau]^{1-\alpha}K_t, \quad L_t = 1, \forall t. \quad (3.7)$$

¹³See 'Redistribution, Wealth Tax Competition and Capital Flight in Growing Economies', EUI Working Paper ECO 95/9, 1995.

so that the return r on domestically owned capital is constant and higher than the constant return $\frac{\phi_t^* r}{(1-\omega_t^*)}$ on foreign owned capital. The wages are not constant, but grow with the index K_t of domestically productive capital. It can be seen that taxes and redistribution have a bearing on the marginal product of capital. Use the definitions given in (3.5), and (3.6), assume $0 < \omega_t \leq 1$, $0 \leq \phi^*(\omega_t^*) < 1$ and *fixed* for the home country and let $E \equiv (1 - \alpha)A[(1 - \lambda)\tau]^{-\alpha}$. Then

$$\frac{\partial r}{\partial \tau} = \alpha E(1 - \lambda) > 0 \quad , \quad \frac{\partial r}{\partial \lambda} = \alpha E(-\tau) < 0 . \quad (3.8)$$

So redistribution has a negative effect on the return on capital and increases in the tax rate raise the rate of return. For the wages, (ηK_t) , I obtain

$$\frac{\partial \eta}{\partial \tau} = (1 - \alpha)E(1 - \lambda) > 0 \quad , \quad \frac{\partial \eta}{\partial \lambda} = (1 - \alpha)E(-\tau) < 0 . \quad (3.9)$$

Thus, for *given* ω_t, ω_t^* and given K_t an increase in τ leads to a positive change in the rate of return and in wages. Redistribution lowers each of them.

The *workers* derive the following utility stream from consuming their entire wages and government transfers

$$\int_0^{\infty} \ln C_t^W e^{-\rho t} dt \quad \text{where} \quad C_t^W = \eta(\tau, \lambda)K_t + \lambda \tau K_t. \quad (3.10)$$

They do not invest and are not taxed by assumption.

The *capitalists* in each country cannot move, choose how much to consume or invest, and take the paths of (r, r^*, τ, τ^*) as given. As the capital owners have the opportunity to invest in either country, they determine where their capital is to be located, ω_t . They have perfect foresight and maximize their intertemporal

utility according to

$$\max_{C_t^k, \omega_t} \int_0^{\infty} \ln C_t^k e^{-\rho t} dt \quad (3.11a)$$

$$s.t. \quad \dot{k}_t = (r - \tau)\omega_t k_t + (r^* - \tau^*)\phi(\omega_t)k_t - C_t^k \quad (3.11b)$$

$$0 \leq \omega_t \leq 1 \quad (3.11c)$$

$$k(0) = \bar{k}_0, \quad k(\infty) = \text{free}. \quad (3.11d)$$

Equation (3.11b) is the capitalists' dynamic budget constraint and it captures the following: The capital owners allocate their capital stock to the home or foreign country depending on the return, they receive in a particular country. If they allocate ωk_t to the home country they receive the rate of return r . If they allocate $(1 - \omega)k_t$ to the foreign country, only $\phi(\omega)k_t$ will become productive and they will only receive the rate of return on the amount of capital that has become productive abroad. Thus, they receive capital income $r^* \left(\frac{\phi(\omega)}{1 - \omega} \right) (1 - \omega)k_t = r^* \phi(\omega)k_t$ in the foreign country. By assumption consumption and investment goods as well as capital stocks can travel freely and re-investment of profits earned in a country is costless in that particular country. The worldwide investment of the domestic capital owners i_t^n is the sum of what they invest at home, i_{1t}^n , and of what they invest abroad, i_{2t}^n . As goods and stocks can travel freely, $i_{1t}^n = \omega_t \dot{k}_t$ and $i_{2t}^n = (1 - \omega_t) \dot{k}_t$ which explains the \dot{k}_t term on the LHS of the budget constraint.

The necessary first order conditions for the problem above are given by (3.11b), (3.11c), (3.11d) and

$$\frac{1}{C_t^k} - \mu_t = 0 \quad (3.12a)$$

$$\mu_t(r - \tau)k_t + \mu_t(r^* - \tau^*)\phi'(\omega_t)k_t = 0 \quad (3.12b)$$

$$\dot{\mu}_t = \mu_t \rho - \mu_t [(r - \tau)\omega_t + (r^* - \tau^*)\phi(\omega_t)] \quad (3.12c)$$

$$\lim_{t \rightarrow \infty} k_t \mu_t e^{-\rho t} = 0. \quad (3.12d)$$

where μ_t is a positive co-state variable which is interpreted as the instantaneous shadow price of one more unit of investment at each date. Equation (3.12a) equates the marginal utility of consumption to the shadow price of more investment, (3.12c) is the standard *Euler* equation which relates the costs of foregone investment (LHS) to the discounted gain in marginal utility (RHS), noting $\frac{1}{C_t^k} = \mu_t$, and (3.12d) is the transversality condition for the capital stock which ensures that the present value of the capital stock approaches zero asymptotically. Equation (3.12b) describes the investors' capital allocation decision. It depends on the after-tax returns of the domestically owned capital stocks in either economy, since

$$\phi'(\omega_t) = -\frac{r - \tau}{r^* - \tau^*}.$$

If one totally differentiates the expression one sees that the allocation decision ω_t is increasing in the ratio of the domestic to the foreign after-tax returns,

$$\frac{d\omega_t}{d\left(\frac{r-\tau}{r^*-\tau^*}\right)} = -\frac{1}{\phi''(\omega_t)} \geq 0, \quad \text{where } \phi''(\omega_t) < 0.$$

Thus, if the domestic after-tax return increases, the capitalists leave more capital at home. To fix ideas and keep matters simple consider the explicit ϕ function

$$\phi(\omega_t) = \frac{z}{z+1} - \frac{z}{z+1} \omega_t^{1+\frac{1}{z}}. \quad (3.13)$$

which obeys all the restrictions I have put on ϕ earlier on.¹⁴ The parameter z

¹⁴It is immediate that $\phi', \phi'' \leq 0$. In order to check if $\phi(\omega) \leq (1 - \omega)$ let $d \equiv (1 - \omega) - \phi(\omega)$. Then $d_\omega = -1 + \omega^{\frac{1}{z}}$ and $d_{\omega,\omega} = \frac{1}{z}\omega^{\frac{1}{z}-1}$ establishes that $\omega = 1$ minimizes d for any $z \in [0, \infty)$ so

measures the degree of capital mobility imperfection. As required by (3.3) the limiting case $z \rightarrow \infty$ (0) captures perfect capital mobility (autarky). Then the optimal decision rule is given by

$$\omega = \min \left\{ \left(\frac{r - \tau}{r^* - \tau^*} \right)^z, 1 \right\} \quad (3.14)$$

which is increasing in the domestic after tax return for $\frac{r - \tau}{r^* - \tau^*} < 1$ and constant over time. If the after-tax return ratio is larger or equal to 1, the investors leave their capital in their home country, $\omega = 1$, $\phi(\omega) = 0$. If the foreign after-tax return goes up at $\frac{r - \tau}{r^* - \tau^*} \geq 1$, the investors may shift their capital abroad. Depending on the after-tax returns in the two countries the growth rate of consumption follows in a standard way from (3.12c) and (3.14):

$$\gamma \equiv \frac{\dot{C}_t^k}{C_t^k} = (r - \tau)\omega + (r^* - \tau^*)\phi(\omega) - \rho. \quad (3.15)$$

Consumption grows at a constant rate and increases in the after-tax returns. Suppose the capitalists' capital stock and their consumption grow at the same rate. That means that the capital stock of the capitalists grows at a rate which depends on the after-tax returns in the two countries. If the after-tax returns are such that $\omega = 1$, all the growth of domestically owned capital takes place in the home country. If the foreign after-tax return is sufficiently high, it becomes attractive for the domestic investor to shift capital abroad. The domestically owned capital stock grows then at a rate that is a mixture of contributions of home and foreign investment.

that $d \geq 0$. Now let $m \equiv \frac{z}{z+1}$ and note that $m_z = \frac{1}{(z+1)^2} \geq 0$. Then $m \rightarrow 0$ (1) as $z \rightarrow 0$ (∞). Suppose $\omega < 1$. Then for $z \rightarrow 0$ we have $\omega^{1+\frac{1}{z}} \rightarrow 0$ and $m \rightarrow 0$ so that $d \rightarrow 1 - \omega > 0$. For $z \rightarrow \infty$ we have $\omega^{1+\frac{1}{z}} \rightarrow \omega$ and $m \rightarrow 1$ so that $d \rightarrow 0$. From this I conclude that $(1 - \omega) \geq \phi(\omega)$ for all $z \in [0, \infty)$ and $\omega \in [0, 1]$.

The equations (3.11b), (3.12d), (3.14) and (3.15) imply that in the optimum the capitalists' consumption is given by $C_t^k = \rho k_t$. Thus, $\gamma = \gamma_k$ so that the capitalists' consumption and wealth grow at the same rate.¹⁵

3.3 Market Equilibrium

The constancy of τ, τ^* implies constancy of r, r^* and ω, ω^* and hence γ, γ^* .

3.3.1 Closed Economy

For a closed economy $\omega = 1$ and $K_t = k_t$. It is not difficult to see that the model reduces to the one analyzed in chapter 1. Hence, the market equilibrium is characterized by steady state, balanced growth with $\gamma_Y = \gamma_k = \gamma = \gamma_{CW}$ where $\gamma = (r - \tau) - \rho$ and $r = \alpha A[(1 - \lambda)\tau]^{1-\alpha}$. Lemma 1.1 of chapter 1 implies that growth is maximized by $\lambda = 0$ and $\hat{\tau} = [\alpha(1 - \alpha)A]^{\frac{1}{\alpha}}$ where $\hat{\tau}$ solves $r_{\tau} = 1$.

Figure 3.1: The relationship between γ and τ in a closed economy

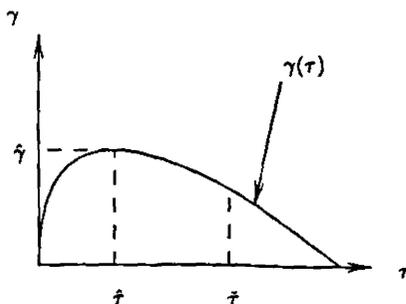


Figure 3.1 shows that growth is traded off against redistribution at a point such as $\tilde{\tau}$ with $\lambda > 0$. The after-tax return is given by

$$r - \tau = \alpha A[(1 - \lambda)\tau]^{1-\alpha} - \tau = \tau (\alpha A[(1 - \lambda)\tau]^{-\alpha} - 1).$$

¹⁵For a derivation see Appendix A.1 on p. 119.

and from Lemma 1.2 of chapter 1 the maximum after-tax return is $\hat{r} - \hat{\tau} = \hat{\tau} \left(\frac{\alpha}{1-\alpha} \right)$. As $\frac{d\hat{\tau}}{dA} > 0$, $j = \hat{\gamma}$, $(\hat{r} - \hat{\tau})$, $\hat{\tau}$, an increase in efficiency raises the maximum growth rate, the maximum after-tax return and the growth maximizing tax rate. That describes the dynamic market equilibrium of the closed economy.

3.3.2 Two-Country World

In this section the dynamic market equilibrium of the domestic economy is derived for given, arbitrary tax rates and so for given ω and ω^* as the latter are functions of the tax rates. Define $\nu_t \equiv \frac{\omega k_t}{K_t}$ and $\nu_t^* \equiv \frac{\phi^*(\omega^*)k_t^*}{K_t^*}$ as the shares of domestic and foreign capital in productive capital.

The capital owners' optimum implies $\gamma = \gamma_k$ where γ is given by (3.15) and is constant. Hence, in an open economy equilibrium the domestic capitalists' consumption grows at the same, constant rate as their wealth. The same holds for the foreign capitalists. The total wealth of the domestic capitalists at any point in time is k_t and the budget constraint satisfies

$$\dot{k}_t = r\omega k_t - \tau\omega k_t + r^*\phi k_t - \tau^*\phi k_t - C^k$$

In the two-country world with given ω, ω^* the world resource constraint is given by

$$\dot{k}_t + \dot{k}_t^* = rK_t + \eta K_t + r^*K_t^* + \eta^*K_t^* - G_t - G_t^* - C_t^k - C_t^{k^*} - C_t^W - C_t^{W^*}$$

where $K_t = \omega k_t + \phi^*k_t^*$, $K_t^* = \omega^*k_t^* + \phi k_t$, $G_t = (1-\lambda)\tau K_t$ and $G_t^* = (1-\lambda^*)\tau^*K_t^*$ since the governments run balanced budgets. The production functions imply $Y_t = rK_t + \eta K_t$ and $Y_t^* = r^*K_t^* + \eta^*K_t^*$. From the private sector optimality and

the steady state conditions the world resource constraint satisfies

$$\gamma k_t + \gamma^* k_t^* = (r - \tau)K_t + (r^* - \tau^*)K_t^* - \rho k_t - \rho k_t^*.$$

In equilibrium $GDP_t = Y_t$ so that GDP grows at the same rate as output. From the production function it follows that output Y_t must grow at same rate as K_t , because G_t grows at the same rate as K_t . Then the evolution of the domestic economy is determined by the growth rate of the aggregate, domestically productive capital stock which is given by

$$\Gamma_t \equiv \frac{\dot{K}_t}{K_t} = \frac{\gamma \omega e^{\gamma t} k_0 + \gamma^* \phi^* e^{\gamma^* t} k_0^*}{\omega e^{\gamma t} k_0 + \phi^* e^{\gamma^* t} k_0^*} = \nu_t \gamma + \nu_t^* \gamma^*. \quad (3.16)$$

Notice that this growth rate is a weighted average of the growth rates of the domestic and the foreign capitalists' capital (wealth) and it is in general not constant over time. To see that I calculate

$$\frac{d\Gamma_t}{dt} = \frac{(\gamma^2 \omega k_0 e^{\gamma t} + \gamma^{*2} \phi^* k_0^* e^{\gamma^* t}) K_t - (\dot{K}_t)^2}{K_t^2} = (\gamma - \gamma^*)^2 \Delta$$

where $\Delta = \frac{\omega k_0 e^{\gamma t} \phi^* k_0^* e^{\gamma^* t}}{K_t^2}$. Thus, Γ_t is increasing over time, unless $\gamma = \gamma^*$. Next, I wish to find $\lim_{t \rightarrow \infty} \Gamma_t$ if $\gamma > \gamma^*$ which is given by

$$\lim_{t \rightarrow \infty} \Gamma_t |_{\gamma > \gamma^*} = \frac{\gamma \omega k_0 + \lim_{t \rightarrow \infty} \gamma^* \phi^* e^{(\gamma^* - \gamma)t} k_0^*}{\omega k_0 + \lim_{t \rightarrow \infty} \phi^* e^{(\gamma^* - \gamma)t} k_0^*} = \gamma.$$

Lemma 3.1 *The GDP growth rate Γ_t is increasing over time, $\frac{d\Gamma_t}{dt} > 0$, for any $\gamma \neq \gamma^*$. If $\gamma > \gamma^*$, then $\lim_{t \rightarrow \infty} \Gamma_t |_{\gamma > \gamma^*} = \gamma$.*

Thus, the GDP growth rate approaches the domestic, closed economy growth

rate, if $\gamma > \gamma^*$.

Recall that $\gamma = \frac{\dot{c}_t^k}{c_t^k}$ so that there is a difference between GDP or GNP consumption growth of the capital owners. As all goods can travel costlessly I assume that GDP consumption adjusts at each point in time so as to maintain equilibrium. Thus, the domestic economy is characterized by balanced, but not necessarily steady state growth.

It is clear that ω, ω^* play a crucial role in determining the open economy equilibrium. But as the tax rates are given arbitrarily at this stage, one cannot say anything about the exact form of the equilibrium. Arbitrary levels and combinations of tax rates sustain multiple dynamic market equilibria. Economically, one cannot say very interesting things about the economies unless more structure is put on the way taxes are set. That is the objective of the tax competition game I contemplate below.

3.4 The Government

The domestic government is assumed to maximize the welfare of its domestic clientele. The governments take the intertemporal utility of their clientele as their welfare measure. From the theory of optimal taxation it is known that a government's objective can be stated in terms of the indirect utility function. However, a government's welfare function need not necessarily coincide with the individual agents' utilities as noted in e.g. Diamond and Mirrlees (1971) or Atkinson and Stiglitz (1980), chpt. 12. The working paper version of this chapter implicitly assumed that the domestic right-wing government represented the interests of the (domestic and foreign) capital owners as a class.¹⁶ Then a right-wing government

¹⁶See 'Redistribution, Wealth Tax Competition and Capital Flight in Growing Economies', EUI Working Paper ECO 95/9, 1995.

would be interested in GDP growth and would have an objective function very similar to a left-wing government's one. Both, right-wing and left-wing governments would then compete in taxes. But as governments are voted for by their national constituencies, that kind of international class objective is inconsistent with a truly representative, national democracy. It is an interesting question, however, whether pro-capital governments are truly and *only* representing their national voters (capital owners) in reality. The answer to that question is outside the model. Instead, in the chapter I assume that the governments really *do* represent the national capital owners' interests only.

In appendix B.1 I show that a domestic right-wing (strictly pro-capital) government (superscript r) has the objective function

$$V^r = \int_0^{\infty} \ln C_t^k e^{-\rho t} dt = \frac{\ln C_0^k}{\rho} + \frac{\gamma}{\rho^2} \quad (3.17)$$

$$\text{where } C_0^k = \rho k_0, \quad \gamma = (r - \tau)\omega + (r^* - \tau^*)\phi(\omega) - \rho.$$

The growth rate here is that of k_t and not K_t . That follows from the fact that the right-wing government serves domestic capitalists only and is therefore concerned about the capital owners' worldwide income and not GDP.

Similarly, the welfare measure of a domestic left-wing (strictly pro-labour) government (superscript l) integrates to

$$V^l = \int_0^{\infty} \ln C_t^W e^{-\rho t} dt = \frac{\ln C_0^W}{\rho} + \frac{1}{\rho} \int_0^{\infty} \Gamma_t e^{-\rho t} dt \quad (3.18)$$

$$\text{where } C_0^W = (\eta(\tau, \lambda) + \lambda\tau) K_0, \quad \Gamma_t = \nu_t \gamma + \nu_t^* \gamma^*,$$

$$\nu_t \equiv \frac{\omega k_0 e^{\gamma t}}{K_t}, \quad \nu_t^* \equiv \frac{\phi^*(\omega^*) k_0^* e^{\gamma^* t}}{K_t}, \quad K_t = \omega k_0 e^{\gamma t} + \phi^*(\omega^*) k_0^* e^{\gamma^* t}.$$

As the wages depend on the productive capital stock, the left-wing government is concerned about the level and the growth rate of GDP.

The two objective functions are increasing in (k_0, γ) and continuous in tax rates for given ω, ω^* . Thus, given everything else getting more domestic capital is in the interest of right-wing and left-wing governments in the model. Furthermore, the workers' welfare is also increasing in k_0^* and γ^* . Thus, each government's objective is implicitly compatible with another objective, namely that of increasing the growth rate of domestically owned capital. Hence, any policy that generates higher domestic capitalists' capital growth is in the interest of both types of government as it raises each group's welfare.

3.5 The Government in a Closed Economy

Respecting the right of private property, a right-wing or a left-wing government chooses τ and λ to maximize its clientele's welfare. Exactly the same problem has been analyzed in chapter 1. I present the optimal policies again for convenience.

The *left-wing* government chooses

If $\rho \geq [(1 - \alpha)A]^{\frac{1}{\alpha}}$ then:

$$\tau = \rho, \quad \lambda = 1 - \frac{[(1 - \alpha)A]^{\frac{1}{\alpha}}}{\rho}. \quad (3.19)$$

If $\rho < [(1 - \alpha)A]^{\frac{1}{\alpha}}$ then:

$$\tau[1 - \alpha(1 - \alpha)A\tau^{-\alpha}] = \rho(1 - \alpha), \quad \lambda = 0. \quad (3.20)$$

Again, I denote the (τ, λ) combination that solves (3.19) or (3.20) by $\tilde{\tau}$ and $\tilde{\lambda}$.

The *right-wing* government chooses the the growth maximizing (τ, λ) combination (see Lemma 1.1 of chapter 1)

$$\hat{\tau} = [\alpha(1 - \alpha)A]^{\frac{1}{\alpha}} \quad \text{and} \quad \lambda = 0. \quad (3.21)$$

Thus, a redistributing, left-wing government ($\lambda > 0$) trades off growth against redistribution with a policy $\tilde{\tau}, \lambda > 0$. According to Proposition 1.3 of chapter 1 the conditions for redistribution are restrictive. As the proposition is important in this chapter I restate it for convenience.

Proposition 1.3 *For a redistributing ($\lambda > 0$), left-wing government $\gamma > 0$ only if $\alpha > \frac{2}{3}$. An increase in efficiency makes the left-wing government redistribute less wealth given its optimal policy, that is, $\frac{d\lambda}{dA} < 0$.*

Thus, an increase in efficiency causes the left-wing government of a closed economy to redistribute less wealth in the optimum. The normative and empirical implications of the result for a closed economy have been discussed in chapter 1.

3.6 Tax Competition in a Two-Country

World with Capital Mobility

What happens to the optimal, closed economy policies if governments have to decide in a two-country world with capital mobility and the governments cannot coordinate their policies? That is a relevant question for countries among which full tax harmonization is not feasible. As a consequence governments may engage in tax competition. (For a similar point see, for instance, Sinn (1990) or Bovenberg (1994).) I model that problem as tax competition, that is, as a non-cooperative game between two governments. The strategies of the

two governments are the choices of τ, λ and τ^*, λ^* and only pure strategies are considered. I assume that perfect knowledge prevails about all the parameters, objective functions, the strategies and the sequence of moves. Everybody acts non-cooperatively. The governments move simultaneously and the private sector agents move simultaneously. But both governments move before the private sector. At each point in time the agents are confronted with the same problem. The agents remember at date t only what they have done at date 0. The capitalists in either economy have the same initial capital stock, $k_0 = k_0^*$, and the economies are equally efficient, $A = A^*$, unless stated otherwise. All agents are taken to be equally patient across countries. If the capital owners can invest in a global environment, it is reasonable to assume that they have the same rate of time preference.

These assumptions reduce the game to a simple two-stage game.¹⁷ The governments act as Stackelberg leaders and the private sectors act as Stackelberg followers. Given the optimal capital allocation decision of the capitalists (ω, ω^*) the governments decide on the tax rates and redistribution. Given the tax rates and λ, λ^* the private sector decides on where to invest. Solving backwards in this way requires a domestic government to maximize (3.17) or (3.18) with respect to its instruments, taking its opponent's choices of (τ^*, λ^*) as given. I will consider each possible match between a right-wing and a left-wing government under the assumption that the economies are *similar* ($A = A^*$) or significantly *different*

¹⁷In terms of dynamic games I therefore contemplate a *repeated game* with an open loop information structure. I justify the information structure by the requirement that democratic governments of either political leaning must constantly be reminded of their pre-election promises so that the outcome of the game in the first stage provides a benchmark for their decisions at time t . If the governments could remember the whole history of the game, time inconsistency issues might emerge. Modelling problems of time inconsistency and possibilities to cope with them is beyond the scope of the chapter. Thus, I assume that governments commit themselves to their decisions. How the commitment is enforced is outside of the model. References for dynamic games are e.g. Petit (1990) and Basar and Olsder (1995).

$(A > A^*)$.¹⁸

3.6.1 The Right-wing Government's Problem

A right-wing government does not redistribute in the model and solves

$$\max_{\tau} V^r \quad \text{s.t.} \quad \tau^*, \lambda^* \text{ given; } \lambda = 0.$$

I suppress the time subscript 0 in what follows. From (3.17) the FOC involves

$$\frac{C_{\tau}^k}{C^k \rho} + \frac{\gamma_{\tau}}{\rho^2} = 0. \quad (3.22)$$

As $C^k = \rho k$, $C_{\tau}^k = 0$. For the growth rate I find

$$\gamma_{\tau} = (r - \tau)\omega_{\tau} + (r_{\tau} - 1)\omega + (r^* - \tau^*)\phi_{\tau}.$$

Define $\Sigma_1 \equiv \frac{r-\tau}{r^*-\tau^*}$, $\Omega_{\tau}^1 = \frac{r_{\tau}-1}{r-1}$ and $\Omega_{\tau}^2 = \frac{r_{\tau}-1}{r^*-\tau^*}$ so that from (3.14)

$$\omega_{\tau} = z\Sigma_1^z \Omega_{\tau}^1, \quad \phi_{\tau} = -z\Sigma_1^z \Omega_{\tau}^2.$$

The domestic capitalists' FOC directly implies $\phi_{\tau}(r^* - \tau^*) + \omega_{\tau}(r - \tau) = 0$ and so $\gamma_{\tau} = (r_{\tau} - 1)\omega$. Thus, equation (3.22) is solved by $\hat{\tau} = [\alpha(1 - \alpha)A]^{\frac{1}{\alpha}}$ and is identical to the optimal right-wing policy in a closed economy. Note that the optimal tax rate is independent of the degree of capital mobility (z), the efficiency in the foreign country A^* and the opponent government's tax choice.

¹⁸To formulate the distributional conflict between capitalists and workers in a closed economy as a dynamic game has, for example, been done in Lancaster (1973), Pohjola (1983), Basar, Haurie and Ricci (1985), Mehrling (1986), Haurie and Pohjola (1987), Shimonura (1991), de Zeeuw (1992), or Seierstad (1993).

Proposition 3.1 *For any A, A^*, z and τ^* a domestic right-wing government chooses the growth maximizing tax rate $\hat{\tau} = [\alpha(1 - \alpha)A]^{\frac{1}{\alpha}}$, irrespective of who its opponent is.*

Hence, in any Nash Equilibrium a right-wing government pursues a policy that maximizes the domestic investors' worldwide capital income. As $\hat{\tau}$ is independent of foreign policy instruments, the right-wing government has a completely fixed reaction function.

3.6.2 The Left-wing Government's Problem

Given Nash behaviour a domestic left-wing government has to take the foreign policy choices as given. Hence, the left-wing government's problem is given by

$$\begin{aligned} & \max_{\tau, \lambda} \int_0^{\infty} \ln C_t^W e^{-\rho t} dt \\ \text{s.t. } & C_t^W = (\eta(\tau, \lambda) + \lambda\tau) K_t; K_t = \omega k_0 e^{\gamma t} + \phi^* k_0^* e^{\gamma^* t}; \\ & \lambda \geq 0; \tau^*, \lambda^* \text{ given.} \end{aligned} \quad (3.23)$$

That problem is not easily solved in general unless restrictions are put on how the opponent behaves or one rules out certain consumption paths. I will state these restrictions as I go along.

3.6.3 Left-wing vs. Right-wing

Suppose the domestic left-wing government competes in taxes with a foreign right-wing government, when both economies are similar. By Proposition 3.1 the right-wing government always sets $\tau = \hat{\tau}$. Under common knowledge the

left-wing government's problem becomes

$$\max_{\tau, \lambda} V^l \quad s.t. \quad \tau^* = \hat{\tau}^*, \lambda \geq 0$$

where V^l is given by (3.18). As the foreign right-wing government always sets $\tau^* = \hat{\tau}^*$, and given $A = A^*$, the domestic left-wing government cannot guarantee a higher after-tax return than the foreign right-wing government. But then it must be that $\omega^* = 1$, $\phi^*(\omega^*) = 0$ and $\Gamma_t = \gamma$. Thus, the GDP and GNP growth rate are the same for the domestic left-wing government. As a consequence it is not able to attract any foreign capital so that the FOC is given by¹⁹

$$V_\tau^l = \frac{C_\tau^W}{\rho C_0^W} + \frac{\gamma_\tau}{\rho^2} = 0, \quad (3.24)$$

$$V_\lambda^l = \lambda \left(\frac{C_\lambda^W}{\rho C_0^W} + \frac{\gamma_\lambda}{\rho^2} \right) = 0. \quad (3.25)$$

As in the closed economy the left-wing government wants to set $\tau \geq \hat{\tau}$ for either maximization of wages or redistribution. But any tax rate $\tau > \hat{\tau}$ makes γ_τ in (3.24) negative, since $r_\tau < 1$ for $\tau > \hat{\tau}$. Recall $C_0^W = (\eta + \lambda\tau)K_0$ and notice that

$$\Gamma_{|\phi^*=0} = \gamma = (r - \tau)\omega + (r^* - \tau^*)\phi(\omega) - \rho$$

since $\omega \leq 1$ as $r^* - \tau^* \geq r - \tau$. From that I get

$$\gamma_\tau = (r_\tau - 1)\omega + (r - \tau)\omega_\tau + (r^* - \tau^*)\phi_\tau. \quad (3.26)$$

¹⁹The $\lambda(\cdot)$ expression enters because of complementary slackness for $\lambda \geq 0$.

Let $\Sigma_1 \equiv \frac{r-\tau}{r^*-\tau^*}$, $\Omega_\tau^1 = \frac{r_\tau-1}{r-\tau}$, $\Omega_\tau^2 = \frac{r_\tau-1}{r^*-\tau^*}$, $\Omega_\lambda^1 = \frac{r_\lambda}{r-\tau}$, $\Omega_\lambda^2 = \frac{r_\lambda}{r^*-\tau^*}$. Then

$$\omega_\tau = z\Sigma_1^z \Omega_\tau^1, \quad \phi_\tau = -z\Sigma_1^z \Omega_\tau^2, \quad \omega_\lambda = z\Sigma_1^z \Omega_\lambda^1, \quad \phi_\lambda = -z\Sigma_1^z \Omega_\lambda^2. \quad (3.27)$$

Some algebra reveals that $\phi_\tau(r^* - \tau^*) + (r - \tau)\omega_\tau = 0$. Hence $\gamma_\tau = (r_\tau - 1)\omega$ and analogously $\gamma_\lambda = r_\lambda\omega$. Substituting the expressions above in (3.24) and (3.25) yields for an interior solution

$$\frac{\eta_\tau + \lambda}{\eta + \lambda\tau} + \frac{\omega_\tau}{\omega} = -\frac{r_\tau - 1}{\rho} \omega, \quad (3.28)$$

$$\frac{\eta_\lambda + \tau}{\eta + \lambda\tau} + \frac{\omega_\lambda}{\omega} = -\frac{r_\lambda}{\rho} \omega. \quad (3.29)$$

Multiplying these equations by the inverses of $r_\tau - 1$ and r_λ resp., setting the resulting equations equal, rearranging and noting that $r_\tau\eta_\lambda = r_\lambda\eta_\tau$ establishes²⁰

$$\tau = \frac{[(1-\alpha)A]^\frac{1}{\alpha}}{1-\lambda}. \quad (3.30)$$

The same relation holds in the closed economy. So again $\tau > \hat{\tau}$. Rearrange (3.28) to obtain

$$\frac{\eta_\tau + \lambda}{\eta + \lambda\tau} \left[\frac{z}{r-\tau} + \frac{\omega}{\rho} \right]^{-1} = -(r_\tau - 1). \quad (3.31)$$

²⁰To see that this is true let $a_1 \equiv \frac{\eta_\tau + \lambda}{\eta + \lambda\tau}$ and $a_2 \equiv \frac{\eta_\lambda + \tau}{\eta + \lambda\tau}$. Then

$$(3.28): \quad \frac{a_1}{r_\tau - 1} + \frac{\omega_\tau}{\omega(r_\tau - 1)} = -\frac{1}{\rho} \omega, \quad (3.29): \quad \frac{a_2}{r_\lambda} + \frac{\omega_\lambda}{\omega r_\lambda} = -\frac{1}{\rho} \omega.$$

Thus, $\frac{a_1}{r_\tau - 1} + \frac{\omega_\tau}{\omega(r_\tau - 1)} = \frac{a_2}{r_\lambda} + \frac{\omega_\lambda}{\omega r_\lambda}$. The definitions of $\Omega_\tau^1, \Omega_\lambda^1$ imply $\frac{\omega_\tau}{\omega(r_\tau - 1)} = \frac{z}{r-\tau} = \frac{\omega}{\omega r_\lambda}$. So one is left with $\frac{a_1}{r_\tau - 1} = \frac{a_2}{r_\lambda}$. Multiplication of both sides by $(\eta + \lambda\tau)$ yields $(\eta_\tau + \lambda)r_\lambda = (\eta_\lambda + \tau)(r_\tau - 1)$. Equations (3.8) and (3.9) imply $r_\tau\eta_\lambda = r_\lambda\eta_\tau$ so that $\eta_\lambda + \lambda r_\lambda = \tau r_\tau - \tau$ which is identical to equation (A.3.11) in appendix A.3. Solving yields (3.30).

Letting $\tau \rightarrow \hat{\tau}$, $r_\tau < 1$ and so the RHS is positive. From that one immediately gets that $\tau \rightarrow \hat{\tau}$ if $z \rightarrow \infty$. Turning to redistribution I rearrange (3.29), use (3.30) and simplify in order to check whether $V_\lambda^l > 0$ evaluated at $\lambda = 0$. The condition for that is²¹

$$[(1 - \alpha)A]^{-\frac{1}{\alpha}} > \left[\frac{z}{r - \tau} + \frac{\omega}{\rho} \right]. \quad (3.32)$$

The LHS is positive. Suppose the workers are impatient²² (ρ large) and $z \rightarrow 0$ (near autarky), then it is possible that $V_\lambda^l > 0$. However, since $z \in [0, \infty)$ there must be a z^0 , where $V_\lambda^l \leq 0$. Thus, there exists a $z > z^0$ where $\lambda = 0$. Refer to the tax rate chosen for $z \in [0, z^0]$ as τ^0 and the one chosen for $z > z^0$ as τ^1 . Recall that $\Gamma_t = \gamma$ for $\phi^* = 0$. Identify the GDP growth rate associated with τ^0 as Γ^0 and the corresponding ω as ω^0 , then it must be that $\Gamma^0 < \Gamma_t^*$ and $\omega^0 < 1$ with $\lambda > 0$. Also, for every $z > z^0$, call it z^1 , there is an optimal tax rate τ^1 . Call Γ^1, ω^1 the GDP growth rate and capital allocation decision for that τ^1 . Then $\omega^1 < 1$, $\lambda = 0$, $\gamma^* > \gamma$ and $\Gamma^1 < \Gamma_t^*$. To see this note that for the foreign right-wing government $\Gamma_0^* = \gamma^* + \nu^* \gamma$ so that $\Gamma < \Gamma_0^*$ is equivalent to $\gamma^* > \gamma(1 - \nu^*)$ in the foreign economy. By Lemma 3.1 Γ_t^* is increasing over time and in the limit it is equal to γ^* . But then $\Gamma < \Gamma_t^*$ for τ^1 and τ^0 and all t . From the argument about $z \rightarrow \infty$ it follows that $\tau \rightarrow \hat{\tau}$. But any optimal τ is a function of z . With $\tau \rightarrow \hat{\tau}$ as $z \rightarrow \infty$ it may happen that in the limit $\omega = 0$ and so $V^l \rightarrow -\infty$ which cannot be optimal. Thus, I conclude that if $z \rightarrow \infty$ a domestic left-wing government

²¹Again let $a_2 \equiv \frac{\eta_\lambda + \tau}{\eta + \lambda \tau}$. Using (3.27) it is true for (3.29) that $a_2 \geq -\left(\frac{\omega_\lambda}{\omega} + \frac{r_\lambda \omega}{\rho}\right) = -r_\lambda \left(\frac{z}{r - \tau} + \frac{\omega}{\rho}\right)$. Then $\frac{a_2}{-r_\lambda} = \frac{\eta_\lambda + \tau}{-r_\lambda(\eta + \lambda \tau)}$. Evaluating at $\lambda = 0$ implies $\frac{a_2}{-r_\lambda} = \frac{\eta_\lambda + \tau}{-r_\lambda \eta}$. Recall $E \equiv (1 - \alpha)A[(1 - \lambda)\tau]^{-\alpha}$ and so $\eta = E\tau, r_\lambda = -\tau\alpha E, \eta_\lambda = -\tau(1 - \alpha)E$ from (3.7), (3.8), and (3.9). At $\lambda = 0$ this implies $\frac{a_2}{-r_\lambda} = \frac{\tau - (1 - \alpha)\tau}{E\tau\alpha\tau E}$. Now substitute for τ from (3.30). $\lambda = 0$ implies $E = 1$ and so $\frac{a_2}{-r_\lambda} = [(1 - \alpha)A]^{-\frac{1}{\alpha}}$. Thus, for $V_\lambda^l > 0$ equation (3.32) must hold.

²²Extreme patience $\rho \rightarrow 0$, too, causes the left-wing government to mimic a right-wing policy.

will definitely set $\tau = \hat{\tau}$ in the optimum. For this case define ω^2 and Γ^2 . Then $\omega^2 = 1$ and $\Gamma^2 = \Gamma^*$. Consequently $\Gamma^0 < \Gamma^1 < \Gamma^2$ where $\Gamma^2 = \Gamma^*$. From these arguments I obtain²³

Proposition 3.2 *If a domestic, left-wing government competes in taxes with a foreign, right-wing government and the economies are similar, $A = A^*$, then*

1. *For low degrees of capital mobility, $z \in [0, z^0]$ the left-wing government sets $\tau > \hat{\tau}$ and $\lambda > 0$. Then $\omega^0 < 1$ and $\Gamma^0 < \Gamma_t^*$.*
2. *If $z > z^0$, there is no redistribution, $\lambda = 0$. The left-wing government just maximizes wages. Then $\omega^0 < \omega^1 < 1$ and $\Gamma^1 < \Gamma_t^*$.*
3. *If capital mobility is nearly perfect, $z \rightarrow \infty$, the left-wing government begins to mimic the right-wing government and will choose $\tau = \hat{\tau}$. Then $\omega^2 = 1$ and $\Gamma^2 = \gamma = \Gamma_t^*$ and constant.*
4. *$\Gamma^0 < \Gamma^1 < \Gamma^2$ where $\Gamma^2 = \Gamma_t^*$.*

Comparing the solutions with $\lambda = 0$ for the left-wing government in the closed economy, (3.19), and the open economy case, (3.31), I find that wages are lower when opening the economies up. Define $\tilde{\tau}_{\lambda=0, \omega \equiv 1}$ as the tax rate that the left-wing government sets in the closed economy and $\tau_{\lambda=0, \omega \leq 1}$ as the one in the two-country world. Then, if $z > 0$, one will observe $\tilde{\tau}_{\lambda=0, \omega \equiv 1} > \tau_{\lambda=0, \omega \leq 1}$ so that wage maximization by a domestic left-wing government is adversely affected by a high degree of capital mobility in the optimum.

²³Notice that the proposition establishes that there is steady state growth in the domestic economy, which may not be the case for the foreign right-wing government's economy. Γ_t^* and so GDP increase over time. I interpret this as being really good for the foreign workers. However, the main focus of this section is on the optimal behaviour of the *domestic* government given the optimal behaviour of the foreign government.

Corollary 3.1 *For a domestic left-wing government facing a right-wing foreign government it follows that $\bar{\tau}_{\lambda=0, \omega \equiv 1} \geq \bar{\tau}_{\lambda=0, \omega \leq 1}$. Thus, wages are lower in an open than in a closed economy.*

Two important features of Proposition 3.2 merit attention. First, left-wing governments do not redistribute in equilibrium if capital mobility is high. The reason is that the effects of the concern for inequality are competed away by fear of losing capital. Capital is good for redistributive reasons and for wages. Facing tax competition when capital mobility is high, the left-wing government is better off if - instead of redistribution - it puts more emphasis on securing high wages. With perfect capital mobility the after-tax returns are equal across countries. In that case capital is indifferent where to go because when both economies are equally efficient *and* perfect capital mobility prevails both governments optimally act as a right-wing government would by setting the tax rates that maximize the domestic capitalists' worldwide income.²⁴

Second, as the right-wing government chooses a rather inflexible tax profile, the left-wing government can take that into account and chooses to redistribute, if capital mobility is not high. That is due to a lack of strategic interaction. The left-wing government knows that it cannot attract foreign capital. Its best response is to take the optimal choice of the right-wing government as given and then solve its tax, redistribution problem.²⁵

²⁴If $z \rightarrow \infty$ the solution to the capitalists' problem in (3.12b) takes a 'bang-bang' form. That case is analyzed in 'Redistribution, Wealth Tax Competition and Capital Flight in Growing Economies', EUI Working Paper, ECO 95/9, 1995.

²⁵Note that this is still an outcome of a game. The right-wing government uses the ω reaction function of the second stage of the game. So even though the game collapses in the first stage (fixed reaction), the optimal tax choice of the right-wing government is still the result of a sequential (two-stage) game.

3.6.4 Left-wing vs. Left-wing

Now the domestic left-wing government's problem is to choose taxes when facing a foreign left-wing government. Again, assume $A = A^*$. As capital is good for left-wing governments, the best the domestic left-wing government can do is to find optimal policies for *given* tax choices of its left-wing opponent. Then the domestic left-wing government's problem in (3.23) is given by

$$\max_{\tau, \lambda} V^l \quad \text{s.t.} \quad \tau^*, \lambda^* \text{ given; } \lambda \geq 0.$$

Maximization involves setting the derivatives $\frac{\partial V^l}{\partial \tau}$ and $\frac{\partial V^l}{\partial \lambda}$ equal to zero for finding the optimum. I restrict the analysis to *steady state* paths with $\gamma = \gamma^*$. Thus, τ, λ must solve $\left(\frac{\partial V^l}{\partial j}\right)_{\gamma=\gamma^*} = 0$ where $j = \tau, \lambda$. In appendix B.2 it is shown that under the *steady state* restriction, $\gamma = \gamma^*$, the following FOC must be satisfied

$$\tau : \quad \frac{\eta_\tau + \lambda}{\eta + \lambda\tau} + \frac{\omega_\tau k_0 + \phi_\tau^* k_0^*}{K_0} + \frac{\gamma_\tau \nu + \gamma_\tau^* \nu^*}{\rho} = 0 \quad (3.33)$$

$$\lambda : \quad \lambda \left(\frac{\eta_\lambda + \tau}{\eta + \lambda\tau} + \frac{\omega_\lambda k_0 + \phi_\lambda^* k_0^*}{K_0} + \frac{\gamma_\lambda \nu + \gamma_\lambda^* \nu^*}{\rho} \right) = 0 \quad (3.34)$$

where $\nu = \frac{\omega k_0}{K_0}$, $\nu^* = \frac{\phi^* k_0^*}{K_0}$ and the $\lambda(\cdot)$ expression enters because of complementary slackness for $\lambda \geq 0$. The first two expressions on the LHS in (3.33) and (3.34) represent the effects of changes in the wage rate. The third expression shows how the growth rate, weighted by its contribution to the overall capital stock, reacts to changes in policy. The steady state assumption implies that $\Gamma = \gamma = \gamma^*$ so that in an equilibrium GDP and GNP grow at the same rate.

Equation (3.27) implies $\gamma_\tau = (\tau_\tau - 1)\omega$. For evaluation of γ_τ^* let $\Sigma_1 \equiv \frac{\tau_\tau - \tau}{\tau^* - \tau^*}$,

$\Sigma_2 \equiv \frac{r-r^*}{r-\tau}$, $\Omega_\tau^1 \equiv \frac{r_\tau-1}{r-\tau}$ and $\Omega_\lambda^1 \equiv \frac{r_\lambda}{r-\tau}$. Then

$$\omega_\tau^* = -z\Sigma_2^z \Omega_\tau^1, \quad \phi_\tau^* = z\Sigma_2^{z+1} \Omega_\tau^1, \quad \phi_\lambda^* = z\Sigma_2^{z+1} \Omega_\lambda^1. \quad (3.35)$$

and $\omega_\tau^*(r^* - \tau^*) + \phi_\tau^*(r - \tau) = 0$ so that

$$\nu\gamma_\tau = \nu(r_\tau - 1)\omega \quad \text{and} \quad \nu^*\gamma_\tau^* = \nu^*(r_\tau - 1)\phi^*. \quad (3.36)$$

Let $a_1 \equiv \frac{\eta_\tau + \lambda}{\eta + \lambda\tau}$, $a_2 \equiv \frac{\eta_\lambda + \tau}{\eta + \lambda\tau}$, $b_1 \equiv \frac{\omega_\tau k_0 + \phi_\tau^* k_0^*}{K_0}$ and $b_2 \equiv \frac{\omega_\lambda k_0 + \phi_\lambda^* k_0^*}{K_0}$. Assume an interior solution for λ exists. From (3.33) and (3.34) it is true that

$$\tau: \frac{a_1 + b_1}{r_\tau - 1} = -\frac{\nu\omega + \nu^*\phi^*}{\rho} \quad \text{and} \quad \lambda: \frac{a_2 + b_2}{r_\lambda} = -\frac{\nu\omega + \nu^*\phi^*}{\rho}.$$

Setting these equations equal yields

$$(a_1 + b_1) r_\lambda = (a_2 + b_2) (r_\tau - 1).$$

It is not difficult, but cumbersome to verify that $b_1 r_\lambda = b_2 (r_\tau - 1)$.²⁶ Thus, one is left with expressions that yield the same result as in the closed economy case, i.e. $\tau = \frac{[(1-\alpha)A]^\frac{1}{\alpha}}{1-\lambda}$. (See footnote 20.) I need the result later on and make it

Lemma 3.2 *Under left-right or left-left competition, for a (τ, λ) solution with $\lambda \geq 0$ the condition $\tau = \frac{[(1-\alpha)A]^\frac{1}{\alpha}}{1-\lambda}$ has to be satisfied.*

Given everything else, two left-wing governments would like to set $\tilde{\tau} > \hat{\tau}$. But

²⁶The equality may be verified using (3.27) and (3.35)

$$\begin{aligned} b_1 r_\lambda &= 1/K \left\{ z\omega \Omega_\tau^1 k + z\omega^* \Omega_\tau^1 \Sigma_2 k^* \right\} r_\lambda \\ b_2 (r_\tau - 1) &= 1/K \left\{ z\omega \Omega_\lambda^1 k + z\omega^* \Omega_\lambda^1 \Sigma_2 k^* \right\} (r_\tau - 1) \end{aligned}$$

where $r_\lambda \Omega_\tau^1 = (r_\tau - 1)\Omega_\lambda^1$.

higher taxes mean that b_1 is definitely negative since $r_\tau < 1$ for $\tau > \hat{\tau}$. Hence, taxes, and so wages are smaller in the open economy than in the closed one. (Cf. Corollary 3.1 in the previous section.) Notice that

$$\begin{aligned} b_1 &= \frac{\omega_\tau k_0 + \phi_\tau^* k_0^*}{K_0} \\ &= z \left(\frac{r_\tau - 1}{r - \tau} \right) \left(\frac{r - \tau}{r^* - \tau^*} \right)^z \frac{k_0}{K_0} + z \left(\frac{r_\tau - 1}{r - \tau} \right) \left(\frac{r^* - \tau^*}{r - \tau} \right)^{z+1} \frac{k_0^*}{K_0} \end{aligned}$$

so that one can rearrange equation (3.33) as follows

$$\begin{aligned} a_1 \left[\frac{z}{r - \tau} \left(\left(\frac{r - \tau}{r^* - \tau^*} \right)^z \frac{k_0}{K_0} + \left(\frac{r^* - \tau^*}{r - \tau} \right)^{z+1} \frac{k_0^*}{K_0} \right) + \frac{\nu\omega + \nu^*\phi^*}{\rho} \right]^{-1} \\ = -(r_\tau - 1). \end{aligned} \tag{3.37}$$

Let τ^0 solve the equation. If $k_0 = k_0^*$, the problem is completely *symmetric* in terms of strategy spaces (action sets), agents etc. As the strategies are continuous variables and *symmetric*, I contemplate a *symmetric game*. One can then apply the following theorem (see Lemma 6 in Dasgupta and Maskin (1986) or Theorem 5.10 in Rasmusen (1989), p.127, presented here)

Theorem 1 (Symmetric Equilibrium Theorem) *Every symmetric game that has an equilibrium has a symmetric equilibrium; that is, every game in which players' actions sets are identical at each point in time has an equilibrium in which mixing probabilities (perhaps equal to one) are identical.*

The theorem establishes that if the game has any Nash Equilibria at least one of them must be symmetric. For what follows I only consider symmetric equilibria. (In appendix B.3 I show that under the assumption $\gamma = \gamma^*$ all Nash Equilibria of the game are indeed unique and symmetric.)²⁷ Symmetry entails $\tau^0 = \tau^{*0}$

²⁷Under the assumption of $\gamma = \gamma^*$, that is, balanced growth, I show that the after-tax returns must be equal and that this together with the FOC in (3.33) and (3.34) implies uniqueness

with $\tau > \hat{\tau}$. Then it must be that $r - \tau = r^* - \tau^* \leq \hat{r} - \hat{\tau}$ so that $\omega = 1$, $\phi^* = 0$, ($\omega^* = 1$) and $K_0 = k_0$. That modifies b_1 and leaves a_1 as it is. Now τ^0 would have to solve

$$\begin{aligned} V'_{\tau|\tau^0=\tau^*0} &= a_1 + b'_1 + c'_1 \\ &= \frac{\eta_\tau + \lambda}{\eta + \lambda\tau} + z \left(\frac{r_\tau - 1}{\tau - \tau} \right) \left(\frac{k + k^*}{k} \right) + \frac{(r_\tau - 1)}{\rho} = 0. \end{aligned}$$

Rearrange the equation to obtain

$$\frac{\eta_\tau + \lambda}{\eta + \lambda\tau} \left[\frac{z}{\tau - \tau} \left(\frac{k + k^*}{k} \right) + \frac{1}{\rho} \right]^{-1} = -(r_\tau - 1). \quad (3.38)$$

If $z \rightarrow \infty$ then $\tau^0 \rightarrow \hat{\tau}$, and if $z \rightarrow 0$ then $\tau^0 \rightarrow \bar{\tau}$, where $\bar{\tau}$ is the left-wing government's preferred tax choice in the closed economy. The last result already tells one that for low enough capital mobility $\lambda > 0$. Proceeding as before I impose symmetry on (3.34), rearrange it, use (3.30), simplify and check whether $V'_\lambda > 0$ evaluated at $\lambda = 0$. Then the condition for $V'_\lambda > 0$ becomes

$$[(1 - \alpha)A]^{-\frac{1}{\alpha}} > \left[\left(\frac{z}{\tau - \tau} \right) \left(\frac{k + k^*}{k} \right) + \frac{1}{\rho} \right]. \quad (3.39)$$

The derivation is identical to the one presented in footnote 21 for the condition $\lambda > 0$ under left-right competition. It is clear that as z becomes infinite, λ is definitely set equal to zero and with $z \rightarrow 0$ one may get a positive λ . Furthermore, substituting (3.30) in (3.32) and solving for z establishes that there must be a $z > 0$, call it z^3 , where $V'_\lambda < 0$ and so $\lambda = 0$. Given the symmetry of the

and symmetry. An alternative procedure would have been to say at the beginning of this section that with $k_0 = k_0^*$ the game is symmetric and that I only consider symmetric equilibria. The justification for proceeding as above is that requiring $\gamma = \gamma^*$ restricts the analysis to balanced growth paths and requires equal after-tax returns, which does not automatically require symmetry or uniqueness in tax rates. So the adopted route is slightly more general.

problem, it follows that both left-wing governments set the same tax rate so that $\omega = \omega^* = 1$ and $\Gamma = \Gamma^*$. Hence,

Proposition 3.3 *If two left-wing governments compete in taxes, the economies are similar, $A = A^*$, and the agents have the same initial wealth $k_0 = k_0^*$, then*

1. *For $z > z^3 > 0$ there is no redistribution $\lambda = \lambda^* = 0$.*
2. *The governments set the same tax rates so that $\omega = \omega^*$ and $\Gamma = \Gamma^*$.*
3. *If $z \rightarrow \infty$, then $\tau \rightarrow \hat{\tau}$ and the left-wing governments act like right-wing ones.*

3.6.5 Comparison of Left-Right with Left-Left Tax Competition

Consider a domestic left-wing government that faces either another foreign left-wing government or a foreign right-wing government. Comparing the conditions for redistribution, i.e. equation (3.39) of section 3.6.3 and equation (3.32) in 3.6.4, I find that the condition for redistribution is weaker if the domestic government faces a foreign right-wing government. To see this consider the RHS of (3.39) and the RHS of (3.32). Notice that the LHS's of (3.39) and (3.32) are equal. Then for any $z > 0$ one obtains by comparison of the RHS's that

$$\left[\left(\frac{z}{r - \tau} \right) \left(\frac{k + k^*}{k} \right) + \frac{1}{\rho} \right] > \left[\frac{z}{r - \tau} + \frac{\omega}{\rho} \right].$$

Hence, the marginal utility of redistribution is reduced if a left-wing government competes in taxes with another left-wing government.

One may check that exactly the same condition applies for a comparison of the optimal tax choices, namely (3.31) and (3.38). I conclude that a domestic left-wing government sets lower taxes when competing with a foreign left-wing government than when confronting a right-wing government. The domestic left-wing government only chooses the same tax rates if $z = 0$ or $z \rightarrow \infty$. Call the domestic left-wing government's tax choice τ^4 when it faces a foreign right-wing opponent and τ^5 when confronted with a foreign left-wing government. Similarly, define Γ^4 , Γ^5 , λ^4 and λ^5 .

Proposition 3.4 *If the economies are similar, $A = A^*$, and the domestic left-wing government faces either a foreign right-wing or a foreign left-wing government, then $\tau^5 \geq \tau^4$, $\lambda^4 \leq \lambda^5$ and $\Gamma^5 \leq \Gamma^4$ for any $z \in [0, \infty)$.*

Thus, the proposition establishes that in two equally efficient economies a domestic left-wing government competing in taxes redistributes *at least* as much when facing a right-wing as when facing a left-wing government. Also, it taxes wealth *at least* as much when facing a right-wing as when facing a left-wing government. Thus, for a left-wing government it matters a lot who it competes with in taxes. The comparison between the two possible opponents suggests that one may observe more GDP growth at the expense of less redistribution if the domestic left-wing government faces a foreign left-wing government. Alternatively, one may observe more redistribution and less GDP growth if the domestic left-wing government competes with a foreign right-wing government.

The reason for the result lies in the fact that in the model a foreign right-wing government guarantees that $\omega^* = 1$ so that the left-wing government has no chance to attract foreign capital. The strategic interaction between two left-wing governments is more intense because each government may get some capital off

its left-wing opponent. That is in conflict with more redistribution (higher λ), but may be worthwhile in terms of wages.

If $z \rightarrow 0$ the strategic element in finding the optimal tax rate becomes less important. Then it makes sense that a left-wing government sets the same taxes as in the closed economy. On the other hand, if capital mobility is nearly perfect, $z \rightarrow \infty$, the left-wing government optimally mimics a right-wing, growth maximizing policy for fear of losing capital.

3.6.6 Different Technological Efficiency

The results so far only apply under the assumption of equal efficiency. In contrast, suppose now that the domestic government has a more efficient economy so that $A > A^*$. Lemma 1.2 of chapter 1 implies $\hat{r} > \hat{r}^*$ and $\hat{r} - \hat{\tau} > \hat{r}^* - \hat{\tau}^*$ if $A > A^*$. From Proposition 3.1 two *right-wing* governments always choose policies that maximize their domestic capitalists' income. Then it is clear that the right-wing government with a more efficient economy gets more capital and experiences higher GDP growth.

The other constellations of the game, i.e. left-right and left-left competition, are not easily analyzed. I will only make a few observations on the nature of possible equilibria in this section. Take the left-wing government. Before imposing symmetry I have shown in section 3.6.4 for a domestic left-wing government that the optimal choice of τ is obtained by solving equation (3.37), that is,

$$a_1 \left[\frac{z}{r - \tau} \left(\left(\frac{r - \tau}{r^* - \tau^*} \right)^z \frac{k_0}{K_0} + \left(\frac{r^* - \tau^*}{r - \tau} \right)^{z+1} \frac{k_0^*}{K_0} \right) + \frac{\nu\omega + \nu^*\phi^*}{\rho} \right]^{-1} \\ = -(\tau_\tau - 1)$$

where $a_1 = \frac{\eta_\tau + \lambda}{\eta + \lambda\tau}$. For *any* foreign vs. domestic after-tax return combinations and

$A - A^* = \epsilon$ with ϵ very small and $z \rightarrow \infty$ one gets $\tau \rightarrow \hat{\tau}$ and $\lambda = 0$. Hence, for slight efficiency differences and very high degrees of capital mobility, the domestic, more efficient economy with a left-wing government has higher GDP growth than the foreign economy with a right-wing government.

Proposition 3.5 *If capital mobility is very high, $z \rightarrow \infty$, and there are small efficiency differences between the economies, i.e. $A - A^* = \epsilon$, $\epsilon > 0$ and small, a domestic left-wing government will have higher GDP growth than a foreign right-wing government, $\Gamma > \Gamma^* = 0$.*

It is interesting to note that $A > A^*$ and $z \rightarrow \infty$ is really bad for the workers in the inefficient, foreign country, because as $z \rightarrow \infty$ one gets $\omega^* = 0, \omega = 1$ and $(\eta^* + \lambda^* \tau^*) K^* = 0$ and so $V^{l^*} \rightarrow -\infty$. Thus, it is crucial for the workers under a left-wing government to have an efficient economy if capital mobility is very high. As the left-wing government only represents domestic workers in this model (a form of left-wing nationalism), the policy of a domestic left-wing government with an efficient economy may cause the foreign workers to 'starve' in the inefficient economy. Interestingly, the proposition also captures the more general point that a national redistribution policy may *not* be constrained by very high capital mobility ('globalization').

Next, for sufficiently large differences in (A, A^*) it may well be possible that a domestic left-wing government redistributes, grows more than and gets at least as much capital as its right-wing counterpart, even if capital mobility is very low. Suppose $z \approx 0$, then one gets approximately the same solution as for the closed economy case. From Lemma 3.2 the solution satisfies $\tau = \frac{[(1-\alpha)A]^{\frac{1}{\alpha}}}{1-\lambda}$. Proposition 1.3 tells us that for positive growth and wealth redistribution capital must be more important in production than public inputs, that is, $\alpha > \frac{2}{3}$, which I assume

to hold now. I wish to check whether it is possible for a domestic left-wing government to have $\gamma \geq \gamma^*$ with $\lambda > 0$, if the foreign government is right-wing and has an inefficient economy. Now $\gamma \geq \gamma^*$ and $A > A^*$ involves

$$r - \tau - \rho \geq \hat{r}^* - \hat{\tau}^* - \rho \Leftrightarrow r - \tau \geq \frac{\alpha}{1 - \alpha} \hat{\tau}^* \quad (\text{see Lemma 1.2}).$$

Lemma 3.2 implies $\tau = \frac{\alpha}{1 - \alpha} [(1 - \alpha)A]^{\frac{1}{\alpha}}$ so that $\gamma \geq \gamma^*$ requires

$$\frac{\alpha}{1 - \alpha} \left([(1 - \alpha)A]^{\frac{1}{\alpha}} - [\alpha(1 - \alpha)A^*]^{\frac{1}{\alpha}} \right) \geq \tau.$$

For redistribution ($\lambda > 0, \tau = \rho$) one needs $\tau > [(1 - \alpha)A]^{\frac{1}{\alpha}}$. Now if $P \geq x$ and $x > Q$ then $Px > Qx$ so that

$$\begin{aligned} & \frac{\alpha}{1 - \alpha} \left([(1 - \alpha)A]^{\frac{1}{\alpha}} - [\alpha(1 - \alpha)A^*]^{\frac{1}{\alpha}} \right) \tau > \tau [(1 - \alpha)A]^{\frac{1}{\alpha}} \\ \Leftrightarrow & \frac{\alpha}{1 - \alpha} \left(1 - \alpha^{\frac{1}{\alpha}} \left(\frac{A^*}{A} \right)^{\frac{1}{\alpha}} \right) > 1 \\ \Leftrightarrow & \left(\frac{2\alpha - 1}{\alpha} \right)^{\alpha} \frac{1}{\alpha} > \frac{A^*}{A}. \end{aligned}$$

Assume Proposition 1.3 holds and $\alpha > \frac{2}{3}$, then the LHS is smaller than 0.946. Letting $A = xA^*$, $x > 1$, the inequality holds if $x > 1.056$. Thus, in the model an efficiency advantage of, say, 6 percent in a world of very low capital mobility ($z \approx 0$) is enough for a left-wing government to be able to redistribute and grow at least at the same rate as its right-wing opponent's economy.

Proposition 3.6 *If capital mobility is very low, $z \approx 0$, and the share of capital sufficiently high, $\alpha > \frac{2}{3}$, then for some efficiency differences, $A > A^*$, a domestic left-wing government may redistribute and grow at least at the same rate as its right-wing opponent's economy, that is, $\gamma \geq \gamma^*$ and $\lambda > 0$ and $\Gamma \geq \Gamma^*$.*

For all other degrees of capital mobility exact solutions are difficult to obtain. One may conjecture that there should always be (z, A) combinations that guarantee that a domestic government redistributes wealth and has higher GDP growth so that the solutions should be somewhere between those found in Propositions 3.5 and 3.6. Hence, it is important for the workers, as well as the capital owners, to live in an efficient economy, because that is good in terms of welfare and GDP growth and may mitigate negative effects of tax competition in a non-cooperative environment.

3.7 Some Implications for Empirical Research

The chapter's model extends some of the results of chapter 1 to open economies. There I have discussed the signs of possible biases for coefficients measuring the effect on growth of policy variables that are frequently used in 'simple cross-country OLS regressions', a procedure that has been defined in chapter 1. (See fn. 3 on p. 22.) As this chapter's optimal policies are highly non-linear, I will not investigate the signs of possible biases rigorously. Instead, I will outline possible implications of the model for simple cross-country OLS growth regressions. In order to focus on distributional conflicts I assume that the samples contain countries in which left-wing and right-wing policies are pursued. Furthermore, I restrict the discussion to two variables that are often used in the literature, namely the ratio of tax revenues to GDP (av_2 of section 1.6) and redistributive transfers.

One important assumption made in chapter 1 has been that economic policy is endogenous. That was meant to capture the fact that optimizing governments take fundamental economic variables into account when making decisions. With

tax competition and ignoring the rate of time preference, the fundamental economic variables that could in principle determine a country i 's growth in a large cross-section sample are such that

$$\gamma_i = f(\alpha_i, \alpha_{-i}^*, A_i, A_{-i}^*, z_i, z_{-i})$$

where the subscript $-i$ indexes all the countries other than country i . In principle, public policy of country i might depend on fundamental economic variables of each other country. For instance, one could assume that a country competes in taxes pairwise and perhaps alternately with each other country in the sample. That would raise tremendous problems in determining the exact optimal, public policies. Theoretically one cannot rule out that each country competes in taxes with all other countries, but it seems quite unrealistic. For that reason I rule out such a possibility.

An alternative that is more in line with the model is to assume that the countries of a sample can be (statistically) clustered into two groups of countries. One is taken to be governed by right-wing policies and the other one is led by left-wing policies. I assume that the clustering preserves all the behavioural patterns of the two policies. That allows one to concentrate on all cases of left-wing, right-wing competition. By running simple cross-country OLS growth regressions for each group separately one would find the behaviour of a typical member in that group. For the whole sample growth in a typical country would depend on

$$\gamma = f(\alpha, \alpha^*, A, A^*, z)$$

where all arguments of $f(\cdot)$ represent averages of each group's fundamental economic variables. (The '*' indexes the other group.) It is important to realize that

z is taken to be independent of which country is being considered. It reflects the symmetry assumption of the model and may be justified when one only considers typical countries.

Suppose one assumes that policy is exogenous when in fact it is endogenous and uses the whole sample to run a simple cross-country OLS regression of the growth rate on a policy variable. The resulting point estimate for the effect of the policy variable on growth is generally biased as has been shown in chapter 1, section 1.6. There the bias is due to country-specific effects, fundamental economic variables have on policy variables. For instance, if the efficiency index is difficult to observe and, therefore, not included in the regression, it will affect the point estimate through the non-random component of a country-specific error term. When the economies are linked by factor mobility or trade, the assumption of country-specific error terms may be justified, if one attributes greater weight to these factors in determining unobservable, random components. The assumption appears plausible and convenient in many situations and it is used by most authors in the literature.

The correlation of error terms across countries affects the sign of a bias. As a consequence it may lead to qualifications of some results of chapter 1. For country-specific error terms Proposition 1.10 has established that the point estimate of the effect of the ratio of tax revenues to GDP (av_2) on growth is biased downwards, unless all governments pursue right-wing or income egalitarian policies. For open economies and correlated error terms Proposition 3.2 has shown that for samples with technologically similar countries, linked by very high capital mobility, left-wing governments mimic right-wing policies. That suggests, the estimated coefficient on av_2 may be unbiased in such a case. However, it seems unlikely that nearly perfect capital mobility prevails between countries in a large

sample. Also, if the economies are different, Proposition 3.5 tells one that due to efficiency differences left-wing policies may lead to relatively high GDP growth, in which case it is almost certain that the point estimates for the effect of av_2 on growth are biased.

In section 1.6.3 I have shown that under the assumption of country-specific error terms the estimated coefficients on the ratio of transfers to taxes collected (λ) or of transfers to GDP ($\frac{\lambda rk}{y}$) are generally biased downwards in simple cross-country OLS growth regressions. (See Proposition 1.12.) In this chapter Propositions 3.2, 3.5 and 3.6 state that redistribution takes place if capital mobility is low and the economies are similar, or the economies are technologically different. For instance, suppose the conditions for Proposition 3.6 hold. In that case the typical economy under a left-wing policy has higher GDP growth with redistribution than the typical economy with a right-wing government. The result is due to sufficiently high efficiency differences. That suggests that in some cases a positive λ may be empirically correlated with a relatively high growth rate, that is, with a higher growth rate than for a right-wing government. That is so even though a higher λ causes lower growth in the theoretical model. Depending on efficiency differentials it is then possible that the point estimate for the effect of λ on γ may be biased upwards, which is the opposite direction of that derived for the closed economy. The upward bias would depend on what value z assumes. Thus, the possibility of an upward bias in the estimated coefficient on the effect of redistributive transfers on growth is conditional on particular realizations of the degree of capital mobility. An upward bias would support the hypothesis that redistribution is bad for growth. It is worth stressing, however, that the upward bias result holds for specific values of z only. As has been pointed out in chapter 1, most authors find significantly positive point estimates for the effect

of redistributive transfer variables on economic growth in simple cross-country OLS regressions. (See, for instance, Perotti (1994) or Sala-i-Martin (1996).) Furthermore, if the conditions for Proposition 3.2 are satisfied, a downward bias in the coefficients would follow.

Summarizing, I conclude that it matters for empirical analyses of the effect of public policy on economic growth what one assumes about the links that exist between countries (country-specific vs. cross-country linked error terms or the degree of capital mobility), whether policy is exogenous or endogenous and how governments interact with each other internationally.

3.8 Conclusion

Employing the framework of a simple growth model with distributional conflicts seems to imply that if one taxes wealth, the growth rate is reduced by redistribution. (See chapter 1.) That is the argument presented, for example, in Alesina and Rodrik (1994), Bertola (1993) and others, and it would suggest that redistribution always implies lower growth.

In this chapter I have extended the growth redistribution trade-off problem to a two-country world with varying degrees of capital mobility. Introducing non-cooperative behaviour (tax competition between governments) it is shown that the possibility of losing capital features saliently in the optimal decisions of a government that wishes to redistribute.

I show that when the opponent's economy is equally efficient and capital mobility is sufficiently high, no redistribution takes place in the optimum if *two left-wing governments* compete in taxes. That holds even though both care about redistribution. The intuitive reason for it is that capital is good for left-wing

governments. Losing capital reduces wages and the utility loss of a government incurred by a drop in wages outweighs the utility gain derived from redistribution. However, the workers are compensated for this by higher wages.

If a *right-wing* and a *left-wing* government compete in taxes, the right-wing government optimally pursues its domestically preferred policy and is not influenced by its opponent's tax choice. The right-wing government's reaction to any opponent's tax choice is very unresponsive in the model. As the right-wing government guarantees the maximum after-tax return for its capital owners when both economies are equally efficient, a competing left-wing government is unable to attract foreign capital. It is then optimal for the left-wing government to redistribute at least as much as it would when competing with another left-wing government. It is shown that as capital mobility increases, tax competition intensifies and the left-wing governments begin to mimic right-wing, growth maximizing policies.

If the economies are technologically different, i.e. one economy is more efficient than another one, then more capital will locate in the efficient economy. If the efficient economy wishes to redistribute, it can afford to do so at the expense of losing some capital. Hence, the amount of redistribution depends on who the opponent is and on the efficiency gap that distinguishes it from its opponents. Hence, policies that are geared to make an economy more efficient are in the interest of both workers and capital owners. That holds especially true for workers with a left-wing government.

Finally, in the model one would observe left-wing governments to behave differently in the optimum when facing different opponents. If the opponent is left-wing (same preferences) it will choose higher GDP growth and higher wages at the cost of reduced redistribution. If it confronts a right-wing government it

redistributes at least as much at the expense of lower GDP growth. The result again hinges on the intensity of strategic interaction and the degree of capital mobility.

In this paper it is argued that high GDP growth *and* redistribution may be possible with a large enough efficiency gap or low enough capital mobility. For instance, very high capital mobility ('globalization') may *not* constrain a national redistribution policy, if an economy is efficient. Government preferences alone may not adequately explain the pattern of redistribution and GDP growth in open economies with tax competition, differences in strategic behaviour and varying degrees of factor mobility.

Several caveats apply. I have abstracted from questions of time inconsistency. If governments could condition on the whole history of a more complicated dynamic game the outcome might well be different. I have not analyzed the effects of tariffs on capital flows. It is quite likely that a government whose economy experiences capital outflows will set up tariffs. It would also be desirable to use a less aggregated set-up when investigating the trade-off problem. In reality workers own capital and some of the well capital endowed work. These and other problems provide room for more research on the trade-off between growth and redistribution in a non-cooperative environment.

Appendix B

B.1 The governments' welfare measures

By assumption τ, τ^* and so ω, ω^* and γ, γ^* are constant. For the *right-wing* government the welfare integral is given by $V^r = \int_0^t \ln C_t^k e^{-\rho t}$. Let $t \rightarrow \infty$ and use integration by parts. For this define $v_2 = \ln C_t^k$, and $dv_1 = e^{-\rho t} dt$. Recall that $C_t^k = \rho k_t$. Then $dv_2 = \dot{C}^k / C^k = \gamma = \text{constant}$, and $v_1 = -\frac{1}{\rho} e^{-\rho t}$. Then

$$\begin{aligned} \int_0^\infty \ln C_t^k e^{-\rho t} dt &= -\frac{1}{\rho} [\ln C_t^k e^{-\rho t}]_0^\infty + \frac{1}{\rho} \int_0^\infty \gamma e^{-\rho t} dt \\ &= \frac{\ln C_0^k}{\rho} - \frac{1}{\rho^2} [\gamma e^{-\rho t}]_0^\infty \end{aligned} \quad (\text{B.1.1})$$

where $C_0 = \rho k_0$. Evaluation at the particular limits yields the expression of V^r in (3.17).

The *left-wing* government's welfare integral is given by $V^l = \int_0^t \ln C_t^W e^{-\rho t}$. Now let $t \rightarrow \infty$ and define $v_2 = \ln C_t^W$, and $dv_1 = e^{-\rho t} dt$. Recall $C_t^W = (\eta + \lambda\tau)K_t$. Then $dv_2 = \dot{C}_t^W / C_t^W = \Gamma_t$, and $v_1 = -\frac{1}{\rho} e^{-\rho t}$. Thus,

$$\int_0^\infty \ln C_t^W e^{-\rho t} dt = -\frac{1}{\rho} [\ln C_t^W e^{-\rho t}]_0^\infty + \frac{1}{\rho} \int_0^\infty \Gamma_t e^{-\rho t} dt \quad (\text{B.1.2})$$

which is equivalent to the expression for V^l in (3.18). From (3.16)

$$\Gamma_t = \frac{\gamma \omega e^{\gamma t} k_0 + \gamma^* \phi^* e^{\gamma^* t} k_0^*}{\omega e^{\gamma t} k_0 + \phi^* e^{\gamma^* t} k_0^*}$$

which depends on time so that one would have to evaluate $\int_0^\infty \Gamma_t e^{-\rho t} dt$ if one wanted to find a definite solution.

Next turn to the properties of V^l and V^r with respect to k_t, k_t^* . First note that $\frac{d \ln C_t^i}{d j} \geq 0$, for $i = W, k$, $j = k_t, k_t^*$ and $\forall t$. For two paths $k_{1t} > k_{2t}$ for all t , one has $\ln C_{1t}^i > \ln C_{2t}^i$ for $i = W, k$. But then welfare must also be higher, that is, $\int_0^\infty \ln C_{1t}^i e^{-\rho t} dt > \int_0^\infty \ln C_{2t}^i e^{-\rho t} dt$. Similarly, for $k_{1t}^* > k_{2t}^*$ one obtains $\int_0^\infty \ln C_{1t}^W e^{-\rho t} dt > \int_0^\infty \ln C_{2t}^W e^{-\rho t} dt$ and $\int_0^\infty \ln C_{1t}^k e^{-\rho t} dt = \int_0^\infty \ln C_{2t}^k e^{-\rho t} dt$ since $\frac{d \ln C_t^k}{d k_t^*} = 0$. So increases in k_t raise the welfare of capitalists and workers and increases in k_t^* raise the workers' welfare only. Since $k_t = k_0 e^{\gamma t}$ and $k_t^* = k_0^* e^{\gamma^* t}$ increases in k_0, k_0^* and in γ, γ^* increase k_t, k_t^* and hence the workers' welfare. Furthermore, increases in k_0, γ increase the capital owners' welfare.

B.2 The left-wing government's problem

Recall the left-wing government's problem in (3.23), that is,

$$\begin{aligned} & \max_{\tau, \lambda} \int_0^\infty \ln C_t^W e^{-\rho t} dt \\ \text{s.t. } & C_t^W = (\eta(\tau, \lambda) + \lambda \tau) K_t, \quad K_t = \omega k_0 e^{\gamma t} + \phi^* k_0^* e^{\gamma^* t}, \quad \lambda \geq 0. \end{aligned}$$

I look for constant policies in the optimum. To this end employ the *Leibniz Rule* and differentiate through the integral

$$\int_0^\infty \left(\frac{\partial C_t^W}{\partial \tau} \frac{1}{C_t^W} \right) e^{-\rho t} dt = 0, \quad \lambda \left(\int_0^\infty \left(\frac{\partial C_t^W}{\partial \lambda} \frac{1}{C_t^W} \right) e^{-\rho t} dt \right) = 0 \quad (\text{B.2.3})$$

where the expression for λ enters because of complementary slackness. The derivatives in the brackets are given by

$$\begin{aligned}\frac{\partial C_t^W}{\partial \tau} \frac{1}{C_t^W} &= \frac{(\eta_\tau + \lambda)K_t + (\eta + \lambda\tau)\frac{\partial K_t}{\partial \tau}}{(\eta + \lambda\tau)K_t} = \frac{\eta_\tau + \lambda}{\eta + \lambda\tau} + \frac{\partial K_t}{\partial \tau} \frac{1}{K_t} \\ \frac{\partial C_t^W}{\partial \lambda} \frac{1}{C_t^W} &= \frac{(\eta_\lambda + \tau)K_t + (\eta + \lambda\tau)\frac{\partial K_t}{\partial \lambda}}{(\eta + \lambda\tau)K_t} = \frac{\eta_\lambda + \tau}{\eta + \lambda\tau} + \frac{\partial K_t}{\partial \lambda} \frac{1}{K_t}.\end{aligned}$$

Now let $\Delta_1 \equiv \frac{\eta_\tau + \lambda}{\eta + \lambda\tau}$ and $\Theta_1 \equiv \frac{\eta_\lambda + \tau}{\eta + \lambda\tau}$ and notice that Δ_1, Θ_1 are constant. For the change in K_t I obtain

$$\begin{aligned}\frac{\partial K_t / \partial \tau}{K_t} &= \Delta_2 + \Delta_3 = \frac{(\omega_\tau + \gamma_\tau \omega t) e^{\gamma t} k_0}{\omega k_0 e^{\gamma t} + \phi^* e^{\gamma^* t} k_0^*} + \frac{(\phi_\tau^* + \gamma_\tau^* \phi^* t) e^{\gamma^* t} k_0^*}{\omega k_0 e^{\gamma t} + \phi^* e^{\gamma^* t} k_0^*} \\ \frac{\partial K_t / \partial \lambda}{K_t} &= \Theta_2 + \Theta_3 = \frac{(\omega_\lambda + \gamma_\lambda \omega t) e^{\gamma t} k_0}{\omega k_0 e^{\gamma t} + \phi^* e^{\gamma^* t} k_0^*} + \frac{(\phi_\lambda^* + \gamma_\lambda^* \phi^* t) e^{\gamma^* t} k_0^*}{\omega k_0 e^{\gamma t} + \phi^* e^{\gamma^* t} k_0^*}\end{aligned}$$

where I have defined $\Delta_2 \equiv \frac{(\omega_\tau + \gamma_\tau \omega t) e^{\gamma t} k_0}{\omega k_0 e^{\gamma t} + \phi^* e^{\gamma^* t} k_0^*}$ and $\Delta_3, \Theta_2, \Theta_3$ analogously. Notice that these expressions are not constant, but depend on time in a complex way. With the definitions I reformulate the optimality conditions in (B.2.3) as

$$\tau : \int_0^\infty (\Delta_1 + \Delta_2 + \Delta_3) e^{-\rho t} dt = 0 \quad (\text{B.2.4})$$

$$\lambda : \lambda \left(\int_0^\infty (\Theta_1 + \Theta_2 + \Theta_3) e^{-\rho t} dt \right) = 0. \quad (\text{B.2.5})$$

By Lemma 3.1 Γ_t is increasing over time and approaches $\max(\gamma, \gamma^*)$. In what is to follow I will concentrate on *steady state* paths. For *steady state* growth $\gamma = \gamma^*$. Let us focus on the optimality condition for τ . Imposing $\gamma = \gamma^*$ entails $K_t = \omega k_0 e^{\gamma t} + \phi^* k_0^* e^{\gamma t}$ and so $\Delta_2 = \frac{(\omega_\tau + \gamma_\tau \omega t) e^{\gamma t} k_0}{K_t} = \frac{(\omega_\tau + \gamma_\tau \omega t) k_0}{K_0}$

and $\Delta_3 = \frac{(\phi_\tau^* + \gamma_\tau^* \phi^* t) e^{\gamma_\tau^* k_0^*}}{K_t} = \frac{(\phi_\tau^* + \gamma_\tau^* \phi^* t) k_0^*}{K_0}$. Then (B.2.4) becomes

$$\int_0^\infty \Delta_1 e^{-\rho t} dt + \int_0^\infty \left(\frac{(\omega_\tau + \gamma_\tau \omega t) k_0}{K_0} \right) e^{-\rho t} dt + \int_0^\infty \left(\frac{(\phi_\tau^* + \gamma_\tau^* \phi^* t) k_0^*}{K_0} \right) e^{-\rho t} dt = 0.$$

For the evaluation of each integral notice that

$$\int_0^\infty e^{-\rho t} dt = \left[-\frac{1}{\rho e^{\rho t}} \right]_0^\infty = \frac{1}{\rho}, \quad \int_0^\infty t e^{-\rho t} dt = \left[-\frac{1}{\rho^2 e^{\rho t}} - \frac{t}{\rho e^{\rho t}} \right]_0^\infty = \frac{1}{\rho^2}.$$

Then it is not difficult to verify that

$$\begin{aligned} \int_0^\infty \Delta_1 e^{-\rho t} dt &= \frac{\Delta_1}{\rho} \\ \int_0^\infty \left(\frac{(\omega_\tau + \gamma_\tau \omega t) k_0}{K_0} \right) e^{-\rho t} dt &= \frac{\omega_\tau k_0}{\rho K_0} + \frac{\gamma_\tau \omega k_0}{\rho^2 K_0} \\ \int_0^\infty \left(\frac{(\phi_\tau^* + \gamma_\tau^* \phi^* t) k_0^*}{K_0} \right) e^{-\rho t} dt &= \frac{\phi_\tau^* k_0^*}{\rho K_0} + \frac{\gamma_\tau^* \phi^* k_0^*}{\rho^2 K_0}. \end{aligned}$$

From (B.2.3) and (B.2.4) I may now express the FOC for τ under the *steady state* condition $\gamma = \gamma^*$ as

$$\begin{aligned} \int_0^\infty \left(\frac{\partial C_t^W}{\partial \tau} \frac{1}{C_t^W} \right)_{|\gamma=\gamma^*} e^{-\rho t} dt &= 0 \Leftrightarrow \\ \frac{\Delta_1}{\rho} + \frac{\omega_\tau k_0 + \phi_\tau^* k_0^*}{\rho K_0} + \frac{\gamma_\tau \omega k_0 + \gamma_\tau^* \phi^* k_0^*}{\rho^2 K_0} &= 0 \end{aligned}$$

which is equivalent to the expression in (3.33). Analogous reasoning establishes that for an interior solution for λ

$$\int_0^{\infty} \left(\frac{\partial C_t^W}{\partial \lambda} \frac{1}{C_t^W} \right)_{|\gamma=\gamma^*} e^{-\rho t} dt = 0 \Leftrightarrow$$

$$\frac{\Theta_1}{\rho} + \frac{\omega_\lambda k_0 + \phi_\lambda^* k_0^*}{\rho K_0} + \frac{\gamma_\lambda \omega k_0 + \gamma_\lambda^* \phi^* k_0^*}{\rho^2 K_0} = 0$$

which corresponds to equation (3.34) in the text.

B.3 Symmetry and Uniqueness of Nash Equilibria in the Left-Left Tax Competition Game when $\gamma = \gamma^*$

If $\gamma = \gamma^*$, then $(r - \tau)\omega + (r^* - \tau^*)\phi(\omega) - \rho = (r^* - \tau^*)\omega^* + (r - \tau)\phi^*(\omega^*) - \rho$ must hold. Rearranging and cancelling ρ implies

$$\left(\frac{r - \tau}{r^* - \tau^*} \right) [\omega - \phi^*] = \omega^* - \phi. \quad (\text{B.3.6})$$

Now let $\Sigma_1 \equiv \left(\frac{r - \tau}{r^* - \tau^*} \right)$ and recall

$$\omega = \Sigma_1^z, \quad \phi = \frac{z}{z+1} (1 - \Sigma_1^{z+1}),$$

$$\omega^* = \Sigma_1^{-z}, \quad \phi^* = \frac{z}{z+1} (1 - \Sigma_1^{-z-1}).$$

Any Nash Equilibrium in $(\tau, \lambda, \tau^*, \lambda^*)$ must be such that either $\Sigma_1 < 1$ or $\Sigma_1 > 1$ or $\Sigma_1 = 1$. Suppose $\Sigma_1 < 1$. Then $\omega^* = 1$ by the optimal behaviour of the capital

owners, equation (3.14), and so $\phi^* = 0$. Then (B.3.6) reduces to

$$\begin{aligned}\Sigma_1 \omega &= 1 - \phi \\ \Sigma_1^{z+1} &= 1 - \frac{z}{z+1} + \frac{z}{z+1} \Sigma_1^{z+1} \\ (z+1) \Sigma_1^{z+1} &= (z+1) - z + z \Sigma_1^{z+1} \\ \Sigma_1^{z+1} &= 1\end{aligned}$$

which can only be satisfied if $\Sigma_1 = 1$, hence, contradicting our assumption that $\Sigma_1 < 1$. Next, suppose $\Sigma_1 > 1$. Then $\omega = 1$ and $\phi = 0$ by equation (3.14). From (B.3.6) this implies that

$$\begin{aligned}\Sigma_1 (1 - \phi^*) &= \Sigma_1^{-z} \\ \Sigma_1 - \frac{\Sigma_1 z}{z+1} + \frac{\Sigma_1 z}{z+1} \Sigma_1^{-(z+1)} &= \Sigma_1^{-z} \\ (z+1) \Sigma_1 - \Sigma_1 z + z \Sigma_1^{-z} &= (z+1) \Sigma_1^{-z} \\ \Sigma_1^{z+1} &= 1\end{aligned}$$

which again can only be satisfied if $\Sigma_1 = 1$, contradicting the assumption that $\Sigma_1 > 1$. Therefore, $\Sigma_1 = 1$ in equilibrium with $\gamma = \gamma^*$, that is, the after-tax returns must be equal. It is not difficult to see that $\Sigma_1 = 1$ indeed satisfies the condition $\gamma = \gamma^*$. So I conclude that any Nash Equilibrium in $(\tau, \lambda, \tau^*, \lambda^*)$ that satisfies $\gamma = \gamma^*$ must satisfy $r - \tau = r^* - \tau^*$.

Given symmetry with respect to technology, preferences, action sets etc. the after-tax functions $(r - \tau, r^* - \tau^*)$ are symmetric. Given that $r - \tau$ first (strictly) increases and then (strictly) decreases in τ , two tax rates τ sustain the same after-tax return as can be seen from Figure 3.1 on page 159. Thus, in order to get uniqueness I must check whether only one of the tax rates is chosen in

equilibrium. To this end I distinguish two cases.

First, suppose an interior solution with $\lambda \geq 0, \lambda^* \geq 0$ would be Nash Equilibrium. Then the FOC in (3.33), (3.34) and Lemma 3.2 tell us that

$$\tau = \frac{[(1-\alpha)A]^{\frac{1}{\alpha}}}{1-\lambda} \quad \text{and} \quad \tau^* = \frac{[(1-\alpha)A]^{\frac{1}{\alpha}}}{1-\lambda^*}$$

must hold in any possible Nash Equilibrium with $\lambda \geq 0, \lambda^* \geq 0$. Since $r = \alpha A[(1-\lambda)\tau]^{1-\alpha}$ and $\tau(1-\lambda) = [(1-\alpha)A]^{\frac{1}{\alpha}}$ it is not difficult to see that

$$r = \frac{\alpha}{1-\alpha} [(1-\alpha)A]^{\frac{1}{\alpha}} = r^*.$$

So Lemma 3.2 implies $r = r^*$ and under the condition $\gamma = \gamma^*$, that is, $\Sigma_1 = 1$, this implies that $\tau = \tau^*$ and so $\lambda = \lambda^*$. From that I conclude that any Nash Equilibrium with $\lambda \geq 0, \lambda^* \geq 0$ satisfying $\gamma = \gamma^*$ must be symmetric. Furthermore, from the optimality condition of Lemma 3.2 and for $\lambda \geq 0$,

$$0 \leq \lambda = 1 - \frac{[(1-\alpha)A]^{\frac{1}{\alpha}}}{\tau} \Leftrightarrow \tau \geq [(1-\alpha)A]^{\frac{1}{\alpha}} > [\alpha(1-\alpha)A]^{\frac{1}{\alpha}} = \hat{\tau}$$

so that $\tau > \hat{\tau}$ and so by symmetry $\tau^* > \hat{\tau}$. But since then either after-tax return function is (strictly) decreasing in τ or τ^* the tax rates chosen in equilibrium would have to be unique. Hence, in any Nash Equilibrium with $(\lambda, \lambda^* \geq 0)$ and $\gamma = \gamma^*$, the equilibrium must be *symmetric* and *unique*.

Next, I have to check whether any equilibrium with $\lambda = 0, \lambda^* = 0$ will be symmetric and unique under the assumption $\gamma = \gamma^*$. In this case equation (3.33) with $\lambda = \lambda^* = 0$ must be analyzed. Now $\gamma = \gamma^*$ requires

$$r - \tau = r^* - \tau^*$$

which allows for two symmetric and two asymmetric equilibria. (See Figure 3.1.) I will now show that the condition of equal after-tax returns allows for a unique equilibrium only. In equilibrium the FOC in (3.33) must be satisfied for an interior solution in τ . Under the condition $\gamma = \gamma^*$ and so equal after-tax returns

$$V_{\tau}(\lambda = \lambda^* = 0) = \frac{\eta_{\tau}}{\eta} + z \left(\frac{r_{\tau} - 1}{r - \tau} \right) \left(\frac{k + k^*}{k} \right) + \frac{(r_{\tau} - 1)}{\rho} = 0$$

where $\frac{\eta_{\tau}}{\eta} > 0$ for all $\tau \in [0, 1]$. Suppose $\tau < \hat{\tau}$ would hold in equilibrium. Since $r_{\tau} - 1 > 0$ for $\tau < \hat{\tau}$, marginal utility V_{τ} would be positive which cannot be the case in equilibrium. Thus, any equilibrium combination (τ, τ^*) must be such that $\tau, \tau^* \geq \hat{\tau}$. But if $\gamma = \gamma^*$ in equilibrium, the equilibrium (τ, τ^*) combination must be unique and symmetric. That follows since the after-tax functions are symmetric and so only $\tau = \tau^*$ satisfies $r - \tau = r^* - \tau^*$. Also, for $\tau = \tau^* > \hat{\tau}$ the after-tax functions are (strictly) decreasing so that any equilibrium combination must be unique.

Hence, any Nash Equilibrium in $(\tau, \lambda, \tau^*, \lambda^*)$ that satisfies $\gamma = \gamma^*$ must be *unique and symmetric*.

Part III

Human Capital and Economic Growth

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Chapter 4

Public Policies and Education, Economic Growth and Income Distribution

4.1 Introduction

As markets become more integrated ('globalization'), human capital assumes a pivotal role in policy debates (especially in OECD countries) on the maintenance of international competitiveness. The experience of some East Asian, high growth countries suggests that empirically there is a positive link from the provision of education to income equality and growth. (See, for instance, Bertola (1997), Fig. 6.) In this chapter I present a model that offers an explanation of the stylized fact and contributes to the policy debates, by recourse to three issues that have been put on the agenda by growth theorists.

One of the issues is that *human capital* formation may explain long term patterns of economic growth very well. (See, for instance, Lucas (1988), Tamura

(1991), Glomm and Ravikumar (1992) or Caballé and Santos (1993).)

A second issue concerns *population*. For instance, Kremer (1993), Deardorff (1994) or Romer (1996), chpt. 3.7, show that the size of the population and its growth may explain patterns of economic growth, if one looks at very long time horizons (Kremer's is more than one million years!). In those models the larger the population, the more likely technological progress and so higher growth is. Furthermore, Becker, Murphy and Tamura (1990) or Rosenzweig (1990) show that economically driven fertility choices and human capital investments may lead to growth.

Thirdly, the chapter considers the theory of *distribution* and *growth*, which has been analyzed in a vast number of contributions. Just to name a recent few suffice it to mention Bertola (1993), Alesina and Rodrik (1994) or Garcia-Peñalosa (1995b) who derive interesting conclusions on the relationship between (re-)distribution and growth, mainly showing that the relationship is negative if too many resources are (re-)distributed to the non-accumulated factor of production.

The chapter takes as its starting point the stylized fact that high growth economies have large or efficient *public education* systems and show low degrees of income inequality. In defence of considering public education one should bear in mind that even in countries such as the US a very large fraction of education is carried out publicly. Furthermore, governments have fiscal and institutional instruments other than direct provision of education at their disposal that have a significant bearing on the working of any private education systems.¹

The population in the model presented below is made up of two types of agents. They are either *high skilled* or *low skilled*. So I argue that the *composition*

¹For a discussion of public vs. private education see Glomm and Ravikumar (1992).

of the population matters in the growth process. I assume that human capital can be identified with 'degrees'. Thus, I take education to be '*lumpy*' in the sense that high skilled people are hired in the labour market only if they have obtained a degree.

In the model the agents own the initial capital stock *equally*. The assumption is motivated by the literature on human capital investment. There it is usually shown that given some distribution of innate abilities and costly education, optimizing agents sort themselves into high and low skilled workers depending on the path of the wages and the distribution of wealth. (See, for instance, Mincer (1958) or Findlay and Kierzkowski (1983).) If markets work perfectly, the sorting will see to it that the lifetime utility of a high skilled and a low skilled worker is equalized, and that there is no inequality in the present value of lifetime earnings. However, should the capital market not function perfectly so that, for example, agents wishing to fund education cannot borrow against future earnings, the sorting will be distorted and one would observe inequality in the present value of lifetime earnings. The assumption of equal capital ownership eliminates that effect on the people's choice of education.

Instead, the source of income inequality is taken to lie in the production process itself. In the model high skilled people carry human capital in the form of degrees that enable them to perform all the tasks a low skilled person can do and *more*. In particular, I assume *effective labour* in production to depend on *basic skills* and *high skills* and that *basic skills* and *high skills* are *imperfect substitutes* in production, but that *low and high skilled people* are *perfect substitutes in basic skills*. Thus, high skilled people may always perform the tasks of low skilled people, but low skilled people can never execute tasks that require a degree.² In

²For instance, Lindbeck and Snower (1996) show that firms may organize production so

a perfectly competitive labour market this entails that the high skilled workers get a wage *premium* over and above what their low skilled colleagues receive. Thus, *ex ante* every agent would like to and may get a degree which means that innate ability differences are not important in the set-up.³ I thus assume that all people have the same innate ability. Even if people have the same innate abilities and have the same initial endowments and although the capital market functions perfectly, there will in general be inequality in the present value of lifetime earnings in the model.⁴ That is so, because I assume that education is provided as a public good.

Throughout the paper income inequality refers to inequality in the present value of lifetime wage earnings, and I abstract from problems arising from the time spent receiving education. Of course, students forgo wage earnings for some time and do so with the expectation that they are compensated by higher wages in the future. But if one follows the traditional approach in the human capital literature (see e.g. Mincer (1958)) and views education as an effort demanding process, causing students to experience disutility while learning, the model may easily account for that by endowing the low skilled by some fixed positive and

that people perform one particular task (Tayloristic organization) or various tasks (holistic organization). In this model I argue that only high skilled people are capable of performing several tasks and that firms use a mixture of Tayloristic and holistic organization.

³I only contemplate agents that are endowed by some *basic* ability, which may be low, and that receive basic education, which can be provided and produced costlessly. In the paper education is always meant to be higher education. One may argue that *ex ante* everybody having a basic ability level is a candidate for receiving (higher) education and once chosen to be in the education process will complete the degree. So the education process is taken to be sufficiently productive in converting no (low) skills into high skills using the basic ability of agents.

⁴Recently, Chiu (1998) has presented a model that studies the positive (and causal) link from income equality to human capital accumulation and high growth. He attributes the source of inequality to innate ability differences and liquidity constraints. This paper provides a different, technology based explanation with a positive (causal) link from human capital to income equality and high growth for given policy. Notice also that in the previous chapters the source of factor income inequality was due to an unequal wealth distribution. In that sense this chapter complements the previous results.

endowing the high skilled by some fixed negative amount of 'happiness', without altering the qualitative results of the chapter. Along these lines I consider only adults who are at least as old as the ones with a degree. Then the low skilled adults joining the labour pool as adults can be taken to have received utility by being idle during adolescence. In contrast, the high skilled would have studied and suffered disutility up to adulthood.⁵

For a given capital stock the wage premium for high skilled labour is shown to depend negatively on the percentage of high skilled people in the total population. Thus, if the percentage of high skilled people in population increases, the wage premium falls. That captures an important and realistic aspect in the explanation of wage inequality. (For empirical studies on this issue see, for instance, Freeman (1977), Katz and Revenga (1989), Bound and Johnson (1989), Tilak (1989) or Londoño (1990).)

The government's task is to provide and finance education as a public good.⁶ Respecting the right of private property, the government finances education by raising a tax on the wealth of all individuals.⁷ Thus, even those who have not received (higher) education contribute to financing it. That is realistic in most public education systems and - as shown below - is in the low skilled people's interest. The model allows for other forms of taxation, but it is realistic to say that the government takes real resources (capital) from the private sector and uses them to finance education.

The model postulates a simple relationship between government revenue and

⁵Alternatively, one may simply assume that all people spent the same time in school, but attend different courses leading to different degrees.

⁶In the model the government implicitly chooses the percentage of high skilled people in the population and provides the education resources freely. Thus, education as a public good is *excludable*, *non-rival* and *optional* in the model.

⁷The command optimum would involve expropriation of the capital stock. As that is not common in the real world, I rule it out. For a similar justification cf. Alesina and Rodrik (1994).

education output, that is, high skilled people. In particular, I assume that the percentage of high skilled in population is directly related to the tax rate chosen by the government. An example is provided to demonstrate that the relationship is compatible with more general set-ups.

The agents solve a simple investment decision problem and in the market equilibrium, characterized by *steady state, balanced growth*, the growth rate is positively related to the percentage of high skilled people in population up to a certain point. The reason is that the government takes resources away from the private sector in order to finance education, which reduces growth. On the other hand, it generates more high skilled people which exert a positive effect on production, growth and equality in the present value of lifetime wage income. The model allows for the possibility that no high skilled labour is present in the economy and that there is a particular number of high skilled agents that generates the same rate of growth as that when there are no high skills. For maximum growth, taxes and so the number of high skilled people must not be too high. Furthermore, growth and income equality depend positively on the productivity of the education sector.

Next, I conduct a public policy analysis. If the government's welfare function attaches fixed weights to the representative high or low skilled individual's utility, a government that only represents the *low skilled worker* acts like a *Rawlsian* government. Both choose the growth maximizing number of high skilled people in the model. The intuition for this is that in the model the low skilled worker's wage does not depend on x , the percentage of high skilled people in population. But their capital income does depend on x , which explains why the low skilled worker chooses the growth maximizing x and the highest after-tax return on capital. A striking implication of the model is that growth maximization and

Rawlsian preferences yield identical policies.

The opposite holds for a government that only represents the average *high skilled* worker. It acts like an *Anti-Rawlsian* government. Both choose x lower than the growth maximizing one, because the wage premium depends negatively on x . Although a higher x raises their capital income, high skilled workers do not like too many of their own kind, because it reduces their wage premium. In the optimum they choose a x lower than the growth maximizing one.

A *strictly egalitarian* government that is rigidly committed to guaranteeing the same utility level for the average low and high skilled worker chooses *either* only high skilled *or* only low skilled people. A comparison of the agents' utility reveals that they may be better off if the government does not produce education. But a priori the strictly egalitarian objective does not preclude either possibility. The results are theoretically interesting, but in the rest of the paper and to add realism I concentrate on cases where there is heterogeneity in skills.

A *strictly utilitarian* government faces a *non-trivial* problem in the model. It maximizes the individual utility indices *and* the weight, it attaches to them. It is shown that it optimally sets a x higher than the growth maximizing one, implying that it attaches more weight to having high skilled people in the economy than making the average high or low skilled individual 'happy'.

A comparison of the different policies indicates the following pattern for growth and inequality in the present value of lifetime wages. Strict egalitarianism is generally bad for growth in this chapter. A government serving the average low skilled worker pursues a growth maximizing policy and grants the highest after-tax return on capital. It is ambiguous whether in comparison to each other the strictly utilitarian or the average high skilled labour serving government has higher growth, but both choose less than maximal growth.

For the distribution of the present value of lifetime wage income the model shows this pattern: The strictly egalitarian policy generates zero inequality, at the expense of relatively low growth. The preferred low skilled worker's policy implies a more equitable income distribution than the high skilled workers' one. The strictly utilitarian government chooses a more equal wage rate ratio than the low skilled workers' government. Interestingly, the optimal, *strictly utilitarian* policy is *more egalitarian* in the model than the optimal, *Rawlsian* policy.

High and low skilled people form the clientele of representative governments. The public policy analysis suggests that the recent high growth, relatively low income inequality experience of some, highly competitive countries is due to a combination of public education policies and improvements in the productivity of the education sector which is difficult to measure. If the productivity is equal across some countries, a policy favouring low skilled labour maximizes growth, may attract capital by granting high after-tax returns and reduces inequality compared to the optimal policy of high skilled labour.

The chapter is organized as follows: Section 4.2 describes the economy. Section 4.2.1 derives the optimal behaviour of the private sector and the market equilibrium. Section 4.3 investigates optimal policies for governments with different welfare functions. Section 4.4 compares the optimal policies and section 4.5 provides some concluding remarks.

4.2 The Model

Consider an economy that is populated by N (large) members of two representative dynasties of infinitely lived individuals. The two dynasties are *high skilled* workers, L_1 , and *low skilled* workers, L_0 , where L_1, L_0 denote the total numbers of

the respective agents in each dynasty. I assume that the difference between high and low skilled labour takes on a "lumpy" character, that is, *either* an individual has received education in the form of a degree and is then considered high skilled *or* it has no degree and remains in the low skilled labour pool. Thus, even if an agent has received some education, but has not obtained a degree, she or he will not be hired as a high skilled worker. By assumption the population is stationary so that the number of high and low skilled workers is given by

$$L_1 \equiv xN \quad \text{and} \quad L_0 \equiv (1 - x)N \quad (4.1)$$

where x denotes the percentage of high skilled people in the population. The members of the dynasties supply one unit of either high or low skilled labour inelastically over time. So the total high and low skilled labour supplied equals L_1 and L_0 , respectively. Each high or low skilled worker initially owns an equal share of the total capital stock in the economy, which is held in the form of shares of many identical firms operating in a world of perfect competition. Thus, all agents receive wage and capital income and make investment decisions.

I assume that aggregate production is given by

$$Y_t = A_t K_t^{1-\alpha} H^\alpha, \quad H^\alpha = [(L_1 + L_0)^\alpha + L_1^\alpha], \quad 0 < \alpha < 1, \quad (4.2)$$

where K_t denotes the aggregate capital stock including disembodied technological knowledge⁸, H measures effective labour in production, and A_t is a productivity index at time t . The production function is a reduced form of the following relationship between high and low skilled labour: I assume that *effective labour*

⁸Thus, technological knowledge is taken to be a sort of capital good which is used to produce final output in combination with other factors of production. For an up-to-date discussion of these kinds of endogenous growth models see, for instance, Aghion and Howitt (1998), chpt. 1.

depends on *basic skills* and *high skills* and that *basic skills and high skills are imperfect substitutes* in production. On the other hand it is assumed that *low and high skilled people are perfect substitutes in basic skills*. This argues that high skilled people may always perform the tasks of low skilled people, but that low skilled people can never execute tasks that require a degree.⁹ Notice that even if either no high skilled or no low skilled labour is present in the economy, production will take place. Thus, each type of labour alone is *not* an essential input in production.

I will analyze the effects of different public policies on economic growth and the income distribution. Postulating an equal initial wealth (capital) distribution is clearly a simplifying assumption, but it serves to focus on the effects of different policies on the income distribution. An alternative set-up may be that there is another type of agent in the economy that owns *all* the initial capital stock. Then the reader may verify that at least all the results for governments representing high or low skilled workers hold. Also, it is shown below that the workers' utility depends on the balanced growth rate so that analyzing high and low skilled workers embodies the problem, a capital owning class would have.

The government runs a balanced budget at each point in time, uses its tax

⁹To see this more clearly I assume $Y_t = A_t H_t^\alpha K_t^{1-\alpha}$, where the index of effective labour H depends on labour requiring *basic skills* (B) and labour requiring *high skills* (S). Labour requiring basic skills is performed by high and low skilled *persons*, $B = B(L_0, L_1)$, whereas high skilled labour is only performed by high skilled persons, $S = S(L_1)$. High and low skilled people are perfect substitutes to each other when performing basic skill (routine) tasks, i.e. $B(L_0, L_1) = L_0 + L_1$. Thus, high skilled people also perform those routine tasks a low skilled person may do. On the other hand, only high skilled people can perform high skilled tasks (labour) and for simplicity I let $S(L_1) = L_1$. To capture the relationship between labour inputs I assume

$$H = [B^\rho + S^\rho]^{\frac{1}{\rho}} = [(L_1 + L_0)^\rho + L_1^\rho]^{\frac{1}{\rho}}.$$

For $\rho < 1$ labour requiring basic skills (B) and labour requiring high skills (S) are imperfect (less than perfect) substitutes. For ease of calculations let $\rho = \alpha < 1$ which yields equation (4.2). For a similar set-up in a different context see Garcia-Peñalosa (1995a).

revenues to finance public education, and chooses its expenditure G_t so as to maintain a constant ratio of G_t to its tax base over time. With those assumptions the economic effects of various tax bases such as wage or capital income taxes would be possible to investigate in the model. However, in order to simplify let the government raise a tax on the wealth holdings of the agents. The government taxes the agents' wealth holdings at a constant rate τ on the capital stock of the representative agent $k_t = \frac{K_t}{N}$. So $G_t = \tau k_t N = \tau K_t$ and $\frac{G_t}{K_t} = \tau$ for all t . Thus, real resources are taken away from the private sector and used to finance public education, which generates high skilled workers. In general, public education is 'produced' using government resources and other factors such as, for instance, high skilled labour itself. That is captured by the following *reduced form* representation of the education technology sector

$$x = \tau^\epsilon \quad \text{where } 0 < \epsilon \leq 1, \quad (4.3)$$

$x_\tau = \epsilon \tau^{\epsilon-1} > 0$ and $x_{\tau\tau} = \epsilon(\epsilon-1)\tau^{\epsilon-2} \leq 0$. Thus, if the government channels more resources into the education process, it will generate more high skilled people, $x_\tau > 0$. However, doing this generally becomes more difficult on the margin ($x_{\tau\tau} < 0$). This is supposed to reflect that, if $x_{\tau\tau} < 0$, more public resources provided to the education sector lead to a decreasing marginal product of those resources due to e.g. congestion or other effects. The parameter ϵ measures the productivity of the education sector.¹⁰ If $\epsilon < 1$, the education sector is

¹⁰The reduced form education technology directly relates the percentage of high skilled people (x) to the percentage of resources (wealth) going into the education sector (τ). Then ϵ may be viewed as the elasticity of education output to education financing (input) and may be interpreted as a productivity measure. As $x_\tau = \tau^\epsilon \ln(\tau) < 0$ because $\tau < 1$, a higher ϵ reduces output for given τ . Thus, for given policy a decrease in ϵ reflects a more productive education technology. Also and more conventionally, let $pr = \frac{x}{\tau}$ denote productivity. Then $pr = \tau^{\epsilon-1}$, which is decreasing in ϵ as well. Hence, for given policy productivity would be decreasing in ϵ according to both productivity concepts.

productive and a marginal increase in the wealth tax rate increases education output substantially. Underlying that is the description of an education sector with spillovers from, for instance, high skilled to new high skilled people or where the capital equipment such as computers makes the education technology very productive. The case $\epsilon = 1$ looks as though the education technology were quite productive as well, since then the number of high skilled people rises one-to-one with an increase in the tax rate. However, $x_\epsilon < 0$ for given policy so that a higher ϵ leads to less high skilled people (education output). Combining this property with the assumption of a non-increasing marginal product ($x_{\tau\tau} \leq 0$) due to e.g. educational congestion effects may justify calling $\epsilon = 1$ a relatively unproductive education technology.¹¹

Equation (4.3) is compatible with many models that also use high skilled labour as an input generating education. The following argument justifies the use of the set-up. Let h_t denote the *total* stock of human capital in the economy in a discrete time model. Following, for instance, Azariadis and Drazen (1990) assume that human capital evolves according to

$$h_{t+1} = f(G_t, K_t, h_t) h_t$$

where new human capital h_{t+1} is produced by non-increasing returns. Here human capital formation would depend on the level of the stock of knowledge h_t , government resources provided for education G_t and the tax base K_t . The function $f(\cdot)$ governs the evolution of human capital. Assume that it is separable in

¹¹In an earlier version the model was extended to cases where $\epsilon > 1$ at the expense of adding more constants and functional forms without adding any significant qualitative insight.

the form $f(g(G_t, K_t), h_t)$. Let $g = c\left(\frac{G_t}{K_t}\right) = c(\tau)$ and for simplicity

$$h_{t+1} = c(\tau) h_t^\beta, \text{ where } c \geq 0, c' > 0, c'' \leq 0, 0 < \beta < 1.$$

where β measures the productivity of the education sector and $c(\tau)$ captures the efficiency or quality of education, depending on the government resources channeled into education. (For a similar expression in an optimizing agent framework, see Nerlove, Razin, Sadka and von Weizsäcker (1993) eqn. (7).) That is a widely used specification. See, for example, eqns. (1), (2) in Glomm and Ravikumar (1992), eqn. (1) in Eckstein and Zilcha (1994), or eqn. (2) in Razin and Yuen (1996). What distinguishes this model from those contributions is that in this paper human capital is carried discretely by the agents and so $h_t = x_t N$. Now normalize population, that is, set $N = 1$. Then total human capital at date t is given by x_t . In a steady state $\bar{x} = x_t = x_{t+1}$ and so

$$\bar{x} = c(\tau)^{\frac{1}{1-\beta}}.$$

Next suppose that the efficiency of the education sector is described by $c(\tau) = \tau^\mu$ where $0 < \mu < 1$. For non-increasing returns to scale it is necessary that $\mu + \beta \leq 1$. Let $\frac{\mu}{1-\beta} \equiv \epsilon$ then the more explicit set-up would be equivalent to (4.3) in steady state. As $\bar{x}_\epsilon < 0$, any increase ϵ would mean that less human capital is generated in steady state. From the assumption of non-increasing returns to scale it follows that $\mu \leq 1 - \beta$ so that $\epsilon \leq 1$. Hence, $\epsilon = 1$ would represent a relatively unproductive human capital formation process.

Finally, notice that from equation (4.3) choosing τ is equivalent to choosing x . For the rest of the chapter it is convenient to use the inverse relationship $\tau = x^{\frac{1}{\epsilon}}$ whenever the government chooses taxes.

4.2.1 The Private Sector

There are as many identical firms as individuals and the firms face perfect competition. I assume that there is a capital spillover, which takes the form $A_t = \left(\frac{K_t}{N}\right)^\eta = k_t^\eta$, where $\eta \geq \alpha$, so that the *average* capital stock is the source of a positive externality.¹² In order to simplify I set $\eta = \alpha$ which allows me to concentrate on steady state behaviour. For a justification see Romer (1986). As the firms cannot influence the externality, it does not enter their decision directly so that the marginal products are given by

$$\begin{aligned} r &= (1 - \alpha)k_t^\alpha K_t^{-\alpha} H^\alpha, \\ w_1 &= \alpha k_t^\alpha K_t^{1-\alpha} [(L_1 + L_0)^{\alpha-1} + L_1^{\alpha-1}], \\ w_0 &= \alpha k_t^\alpha K_t^{1-\alpha} (L_1 + L_0)^{\alpha-1}. \end{aligned} \quad (4.4)$$

The workers own all the assets which are collateralized one-to-one by capital. A representative worker takes the paths of r, w_1, w_0, τ as given. Assuming the workers have logarithmic utility, the representative worker solves the problem

$$\max_{c_i} \int_0^\infty \ln c_i e^{-\rho t} dt \quad (4.5)$$

$$s.t. \quad \dot{k} = w_i + (r - \tau)k - c_i \quad i = 0, 1 \quad (4.6)$$

$$k_0 = \text{constant}, \quad k_\infty = \text{free}.$$

Equation (4.6) is the worker's dynamic budget constraint. The problem is a standard one (see, e.g. Chiang (1992), chpt. 9.) and its solution involves the following growth rate of the average high or low skilled worker's consumption

$$\gamma = \gamma_{c_0} = \gamma_{c_1} = (r - \tau) - \rho. \quad (4.7)$$

¹²The model also works if the externality depends on the *entire* capital stock instead.

Thus, consumption of all workers grows at the same rate in the optimum and depends on the after-tax return on capital. As the workers own the initial capital stock equally and have identical utility functions, their investment decisions are the same. Thus, the wealth distribution does not change over time and all agents continue to own equal shares of the total capital stock over time. The only difference in utility stems from different wage incomes which affect the instantaneous levels of steady state consumption.

4.2.2 Market Equilibrium

For the rest of the paper normalize by setting $N = 1$ so that the factor rewards in (4.4) are given by

$$r = (1 - \alpha)(1 + x^\alpha), \quad (4.8)$$

$$w_1 = \alpha k_t (1 + x^{\alpha-1}), \quad (4.9)$$

$$w_0 = \alpha k_t. \quad (4.10)$$

The return on capital is constant over time and the wages grow with capital. As $w_1 = w_0 (1 + x^{\alpha-1})$, high skilled labour receives a premium over what their low skilled counterpart gets. That reflects the fact that the high skilled may always perfectly substitute for low skilled labour so that both types of labour receive the same wage w_0 for routine tasks and that performing high skilled tasks is remunerated by the additional amount $w_0 x^{\alpha-1}$. The premium depends on the percentage of high skilled labour in the population, grows over time at the rate γ and is decreasing in x for a given capital stock.

From the production function one immediately gets $\gamma_y = \gamma_k$ so that per capita output and the capital-labour ratio grow at the same rate. Since N does not grow,

$\gamma_Y = \gamma_K$ so that total output grows at the same rate as the aggregate capital stock. From (4.7) the consumption of the representative worker grows at γ . Each worker owns $k_0 = \frac{K_0}{N}$ units of the initial capital stock. Equation (4.6) implies $\dot{k} = w_i + (r - \tau)k - c_i$ so that

$$\gamma_k = \frac{w_i - c_i}{k} - (r - \tau) \quad \text{for } i = 0, 1.$$

where $(r - \tau)$ is constant. In steady state, γ_k is constant by definition. But $\frac{w_i}{k}$ is constant as well, because from (4.9) and (4.10)

$$\frac{w_1}{k_t} = \frac{\alpha k_t (1 + x^{\alpha-1})}{k_t} = \alpha(1 + x^{\alpha-1}) \quad \text{and} \quad \frac{w_0}{k_t} = \alpha,$$

which implies $\gamma_k = \gamma$. Thus, the economy is characterized by *balanced growth* in *steady state* with $\gamma_Y = \gamma_K = \gamma_y = \gamma_k = \gamma_{c_1} = \gamma_{c_0}$.

The levels of instantaneous consumption in steady state are determined as follows: From (4.6) and using $\gamma_k k = \dot{k}$ and $\gamma_k = \gamma_{c_1} = \gamma_{c_0}$ in steady state one obtains $(r - \tau - \rho)k_t = w_i + (r - \tau)k_t - c_i$. Thus,

$$c_1 = w_1 + \rho k_t \quad \text{and} \quad c_0 = w_0 + \rho k_t \quad (4.11)$$

are the instantaneous consumption levels of a representative high or low skilled worker in steady state.

From (4.8) and $\tau = x^{\frac{1}{\epsilon}}$ one obtains $\gamma = (1 - \alpha)(1 + x^\alpha) - x^{\frac{1}{\epsilon}} - \rho$ so that for given τ an increase in x raises growth. The necessary first order condition for growth maximization involves $\alpha(1 - \alpha)x^{\alpha-1} = \frac{x^{\frac{1}{\epsilon}-1}}{\epsilon}$ which upon solving for x

establishes that

$$\hat{x} = [\epsilon\alpha(1 - \alpha)]^{\frac{\epsilon}{1-\epsilon\alpha}} \quad (4.12)$$

is the growth maximizing percentage of high skilled workers in the population and $\hat{\tau} = [\epsilon\alpha(1 - \alpha)]^{\frac{1}{1-\epsilon\alpha}}$ is the growth maximizing tax rate.

Proposition 4.1 *A growth maximizing government chooses $\hat{x} = [\epsilon\alpha(1 - \alpha)]^{\frac{\epsilon}{1-\epsilon\alpha}}$ and $\hat{\tau} = [\epsilon\alpha(1 - \alpha)]^{\frac{1}{1-\epsilon\alpha}}$.*

Growth is a concave function of x since for $\epsilon \leq 1$ and any x

$$\frac{d^2\gamma}{(dx)^2} = -\alpha(1 - \alpha)^2 x^{\alpha-2} - \frac{1}{\epsilon} \left(\frac{1}{\epsilon} - 1 \right) x^{\frac{1-2\epsilon}{\epsilon}} < 0$$

and the marginal growth rate for $x \rightarrow 0$ is infinity,

$$\lim_{x \rightarrow 0} \frac{d\gamma}{dx} = x^{\alpha-1} \left[\alpha(1 - \alpha) - \frac{x^{\frac{1}{\epsilon}-\alpha}}{\epsilon} \right] = +\infty$$

since $\frac{1}{\epsilon} \geq 1$ by assumption. By the concavity of γ and given the above properties there is a $\gamma(x)$, generating the same growth as $\gamma(0)$. In that case the government chooses a rather high tax rate, implying a high x . Thus, in the model it is possible that an economy has high skilled workers, but does not do better than another economy with no high skilled people. That x is given by

$$\begin{aligned} \gamma(0) - \gamma(x) &= (1 - \alpha) - \rho - (1 - \alpha)[1 + x^\alpha] + x^{\frac{1}{\epsilon}} + \rho = 0 \\ \bar{x} &= (1 - \alpha)^{\frac{\epsilon}{1-\epsilon\alpha}} \end{aligned} \quad (4.13)$$

and clearly $\bar{x} > \hat{x}$. The effect of a change in the productivity of the education

sector for a given $x \in (0, 1)$ is given by

$$\frac{d\gamma}{d\epsilon} = \frac{\ln(x) x^{\frac{1}{\epsilon}}}{\epsilon^2} < 0.$$

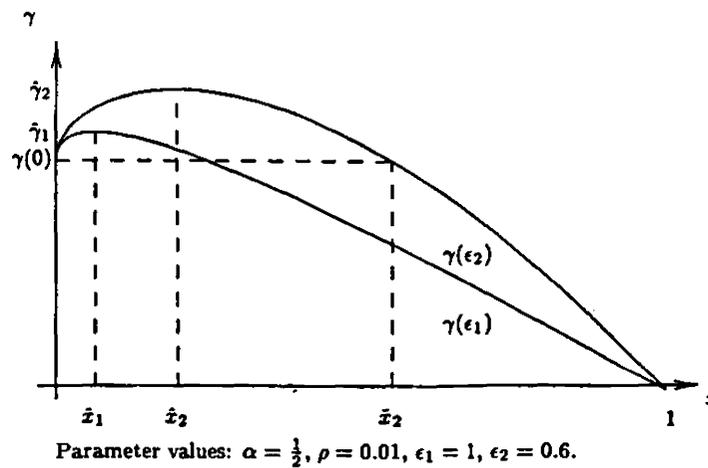
An increase in ϵ makes the production of education *more difficult*. So a reduction in ϵ , that is, making the education technology more productive, raises the growth rate. Hence, the growth maximizing x must also be higher.

Lemma 4.1 *The growth rate γ has the following properties:*

1. γ is concave in x .
2. $\lim_{x \rightarrow 0} \frac{d\gamma}{dx} = +\infty$.
3. $\frac{d\gamma}{d\epsilon} < 0$ for $x \in (0, 1)$.
4. If $\bar{x} = (1 - \alpha)^{\frac{\epsilon}{1-\alpha}}$, then $\gamma(0) = \gamma(\bar{x})$.

The properties can be read off from Figure 4.1. It can be seen that the growth

Figure 4.1: γ as a function of x for different ϵ



maximizing x increases with an increase in the productivity (lower ϵ) of the education technology and that there exists a \bar{x} where $\gamma(0) = \gamma(x)$.

4.2.3 Income Inequality

In the model all income differences between individuals are due to differences in wage income. Therefore, I will concentrate on the wage income distribution in this section. If one wants to relate growth to income inequality it makes sense to look at an average of personal wage incomes over time. If the agents can sell their income stream in a perfect capital market, they will discount their stream of wages by the market rate of return on assets, $r - \tau$. Consequently, the present value of their lifetime sum of discounted wages is

$$\int_0^{\infty} w_{it} e^{-(r-\tau)t} dt = \int_0^{\infty} w_{i0} e^{\gamma t} e^{-(r-\tau)t} = \frac{w_{i0}}{\rho} \equiv w_i^d \quad \text{where } i = 0, 1.$$

Thus, the variable w_i^d denotes the sum of an individual's wage income discounted by the market rate of return on assets. That is the income concept used when analyzing the wage income distribution in this section.¹³ Notice

$$w_0^d = \frac{w_0}{\rho} = \frac{\alpha k_0}{\rho} \quad \text{and} \quad w_1^d = \frac{w_1}{\rho} = \frac{\alpha k_0(1 + x^{\alpha-1})}{\rho} \quad (4.14)$$

and that the mean of the discounted sum of wage incomes is

$$\mu^d = (1 - x)w_0^d + xw_1^d = \frac{(1 + x^{\alpha})\alpha k_0}{\rho}. \quad (4.15)$$

¹³Other income variables one may want to use are current wage income w_{it} , detrended initial wages w_{i0} , or capital adjusted wages $\frac{w_{it}}{k_i}$. All of these concepts suffer from the problem that do not fully reflect the path the wages follow.

implying $\frac{dw_0^d}{dx} = 0$, $\frac{dw_1^d}{dx} < 0$ and $\frac{d\mu^d}{dx} > 0$, that is, the mean of the PV of lifetime wage income is increasing in x . In order to compare any two cumulative distribution functions of discounted lifetime wage income assume $x_1 > x$. Then the different values of x will give rise to two cumulative distribution functions, $F(w_i^d(x_1))$ and $G(w_i^d(x))$, which have unequal means.

If F dominates G in the sense of *Second Order Stochastic Dominance* (SOSD), then F will be preferred to G by any increasing, concave social welfare function according to Atkinson (1970). Geometrically, a distribution $F(w)$ dominates another distribution $G(w)$ in the sense of SOSD if over every interval $[0, c]$, the area under $F(w)$ is never greater (and sometimes smaller) than the corresponding area under $G(w)$.¹⁴

Second Order Stochastic Dominance is equivalent to *Generalized Lorenz Curve* (GLC) dominance. (For a proof see, for example, Lambert (1993), pp. 62-66.) Generalized Lorenz Curves are frequently employed for the comparison of income distributions with unequal means. A GLC is obtained by multiplying the values of the y-axis of an ordinary Lorenz Curve, which relates the share of the population (x-axis) to the share in total income (y-axis) that that population share receives, by mean income, i.e. (share of total income) \times (mean income). The GLC for the distribution of the PV of lifetime wages is presented below.

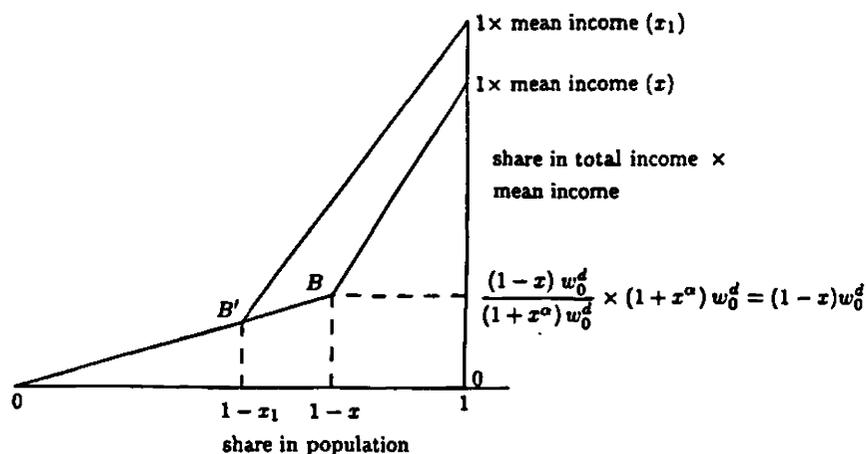
A GLC *dominates* another one if the two curves do not cross and one is completely above the other one. In the figure the income distribution associated with $x_1 > x$ GLC-dominates the income distribution for x . The reason is that

¹⁴The concept derives from evaluating risky returns under conditions of uncertainty. Formally and for non-negative incomes, *Second Order Stochastic Dominance* requires

$$\int_0^c F(w)dw \leq \int_0^c G(w)dw.$$

The introduction of the concept here follows Hirshleifer and Riley (1992), chpt. 3.4.

Figure 4.2: Generalized Lorenz Curve



an increase in x raises μ^d and shifts the kink at B to a point B' which is to the left and on the old $GLC(x)$.

One can then invoke the theorem by Shorrocks (1983) according to which GLC dominance carries with it welfare approval according to every increasing, strictly concave utility-of-income function. Put another way: Every individualistic additively separable symmetric and inequality-averse social welfare function would prefer the GLC dominating income distribution. That means that according to the GLC dominance criterion there exists a *unanimous* preference for the (PV of lifetime wage) income distribution with the higher GLC. Thus, all people with an increasing, concave social welfare function (SWF) would agree to prefer the GLC dominating income distribution. Even the high skilled would prefer the distribution with a higher x under e.g. a veil of ignorance. In that sense the model's society as a whole would prefer the distribution of the PV of lifetime wage income generated by the higher percentage of high skilled people.¹⁵

¹⁵It is straightforward to see that exactly the same holds for the distribution of detrended (initial) wage incomes w_{i0} and capital adjusted wages $\frac{w_{it}}{k_t}$. It also holds if one works with current wage rates w_{it} and $x \leq \hat{x}$. In that case an increase in x causes the new GLC to be everywhere above the old GLC for $t > 0$, because the capital stock would be higher at each

Let $I(x)$ be any inequality measure reflecting that a higher x leads to a GLC dominating distribution.¹⁶ Then $I(0) = I(1) = 0 < I(x)$ and $\frac{dI}{dx} < 0$ for $x \in (0, 1)$. Thus, according to $I(x)$ and for the PV of lifetime wage incomes there is no measured inequality if all agents get the same wage and they are all either equally high or low skilled. If there is any skill heterogeneity, producing more skills reduces inequality in the PV of lifetime wage incomes if measured by $I(x)$. Furthermore, as $x = \tau^\epsilon$ and so $I(\tau)$, a decrease in ϵ for a given policy τ , would lower $I(x)$.

Proposition 4.2 *If there is heterogeneity in skills, $x \in (0, 1)$, an increase in the percentage of high skilled people or an increase in the productivity of the education technology (lower ϵ) for given policy reduce inequality in the present value of lifetime wage incomes in the sense of Generalized Lorenz Curve Dominance.*

Thus, according to the proposition and in terms of the PV of lifetime wage income an increase in the number of high skilled people represents an equalizing income transfer from a rich, high skilled person to a relatively poor, low skilled person.

4.3 The Government

The government takes the optimal decision of the workers as given and chooses taxes to generate education output. That is equivalent to choosing x in the date and mean income would rise. However, if $x > \hat{x}$ it does not necessarily hold.

¹⁶A simple measure satisfying the properties of $I(x)$ is $I = \frac{w_1^2}{w_0^2} - 1$ which Fields (1987), axiom A5, calls *relative gap inequality*, defined in terms of mean wage incomes of the two groups. Notice that the measure as such does not take explicit account of the population composition. However, in the model it implicitly does, because the wage rates depend on the relative number of high skilled people. In appendix C.1 I check the properties of $I(x)$ against those of some other commonly used inequality measures. It is shown that, for instance, the variance and the coefficient of variation have the properties of $I(x)$ if $\alpha \leq \frac{1}{2}$.

model. I assume that the government has different objectives. For instance, it may represent only high skilled workers, only low skilled workers or a mixture of the two. Integrating the utility of the representative low and high skilled worker as given by (4.5) one obtains

$$V^h = \frac{\ln c_1}{\rho} + \frac{\gamma}{\rho^2} = \frac{\ln((\alpha(1+x^{\alpha-1}) + \rho)k_0)}{\rho} + \frac{\gamma}{\rho^2} \quad \text{and} \quad (4.16)$$

$$V^l = \frac{\ln c_0}{\rho} + \frac{\gamma}{\rho^2} = \frac{\ln((\alpha + \rho)k_0)}{\rho} + \frac{\gamma}{\rho^2}. \quad (4.17)$$

These expressions are derived in appendix C.2. Superscript h (l) stands for high (low) skilled.

4.3.1 A Strictly Egalitarian Government

A strictly (utility) egalitarian government wants to make all agents equally well off. Committed to perfect equality it sets $V^h = V^l$ which amounts to

$$\frac{\ln c_1}{\rho} + \frac{\gamma}{\rho^2} = \frac{\ln c_0}{\rho} + \frac{\gamma}{\rho^2} \Leftrightarrow c_1 = c_0.$$

But $c_1 = c_0$ is not possible unless *either* $x = 1$ and all individuals have high skills *or* $x = 0$ and all agents are low skilled. That is what the strictly egalitarian government chooses and it makes intuitive sense for such a government in the economy under consideration. In the model strict total utility egalitarianism is equivalent to strict income egalitarianism since $c_1 = c_0$ requires $w_1 = w_0$ with the agents having equal capital income by assumption.¹⁷ It is important to realize that a Rawlsian, who maximizes the utility of the least well-off, may also choose such a policy in $x \in [0, 1]$. The strictly egalitarian policy implies $I = 0$, that is,

¹⁷For a discussion of egalitarian objectives see Sen (1982) or Atkinson and Stiglitz (1980), chpt. 11 and the discussion in section 1.5.1 of chapter 1.

no inequality in the PV of lifetime wages, but the growth rates and consumption levels are different. A comparison of possible policies reveals that

$$\begin{aligned} V(x = 1) &\geq V(x = 0) \\ \frac{\ln((2\alpha + \rho)k_0)}{\rho} + \frac{\gamma(1)}{\rho^2} &\geq \frac{\ln((\alpha + \rho)k_0)}{\rho} + \frac{\gamma(0)}{\rho^2}. \end{aligned}$$

As $\gamma(0) - \gamma(1) = (1 - \alpha) - \rho - 2(1 - \alpha) + 1 + \rho = \alpha$, growth is higher if the strictly egalitarian government chooses $x = 0$. But the utility difference depends on

$$[(2\alpha + \rho) - (\alpha + \rho)] k_0 \geq e^{\frac{\alpha}{\rho}}$$

so that the agents under such a government would be better off with $x = 0$ if they are very patient (low ρ) and may want $x = 1$ if they are sufficiently impatient and their initial capital stock is large enough. Notice, however, that for $x = 1$ and positive growth, ρ must not be too large so that $x = 1$ may only be better, if the initial capital stock is very large. Thus, a clear welfare ranking is not possible, but it seems more likely that the agents would have higher utility with a $x = 0$ policy that generates higher growth. However, a priori one cannot rule out either policy, as the strictly egalitarian government is not concerned about welfare levels.

Proposition 4.3 *A strictly egalitarian government chooses either $x = 0$ or $x = 1$ implying $\gamma(0) > \gamma(1)$ and $I = 0$. If the agents are very patient, they will prefer $x = 0$.*

The proposition is theoretically interesting and points out an important indeterminacy in egalitarian policies. However, one rarely observes people having the same skills or all having the same degree. For the sake of realism I assume for the

rest of the paper that $x \in (0, 1)$. That rules out the class of strictly egalitarian policies above. Notice it also rules out the Rawlsian policies $x = 0$ or $x = 1$ which belong to this class. The restriction implies that an increase in x decreases inequality in the PV of lifetime wages and is equalizing individual utilities. I use the concept of equalizing utilities as the egalitarian principle. Thus, a policy that grants higher x will be called more egalitarian. For a similar point see Atkinson and Stiglitz (1980), chpt. 11.

4.3.2 A Class of Governments

Suppose the government has the social welfare function

$$W^b(V^h, V^l) = \zeta V^h + (1 - \zeta) V^l, \quad \zeta \in [0, 1] \quad (4.18)$$

where V^h, V^l are given by (4.16) and (4.17). The welfare function attaches fixed weights on the individual high and low skilled agent's utility. If $\zeta = 1$ the government is only concerned about the welfare of the representative high skilled worker, and if $\zeta = 0$ it cares about the average low skilled worker only. For all other values of ζ it represents a mixture of the representative agents' utility. The government chooses x in order to maximize W^b . The FOC is given by

$$\zeta \frac{\partial V^h}{\partial x} + (1 - \zeta) \frac{\partial V^l}{\partial x} = \frac{\zeta}{\rho} \left(\frac{\partial c_1}{\partial x} \frac{1}{c_1} + \frac{\partial \gamma}{\partial x} \frac{1}{\rho} \right) + \frac{(1 - \zeta)}{\rho} \left(\frac{\partial c_0}{\partial x} \frac{1}{c_0} + \frac{\partial \gamma}{\partial x} \frac{1}{\rho} \right) = 0.$$

Notice that $\frac{\partial c_0}{\partial x} = 0$ because low skilled labour's wages and consumption do not depend on x . Simplification yields

$$- \left(\frac{\zeta}{\rho} \right) \left(\frac{\alpha(1 - \alpha)k_0 x^{\alpha-2}}{\alpha k_0(1 + x^{\alpha-1}) + \rho k_0} \right) + \frac{\gamma_x}{\rho^2} = 0 \quad (4.19)$$

where $\gamma_x = \frac{\partial \gamma}{\partial x}$. From that one immediately obtains an important result. As the first expression on the LHS is negative for $\zeta > 0$, it follows that γ_x must be positive. Given the concavity of γ the government chooses x so that $x < \hat{x}$. Thus, if the government attaches positive weight to a representative high skilled worker, it does not choose the growth maximizing x , but rather a smaller percentage of people with high skills.

The case $\zeta = 0$ is of special interest because it is equivalent to the choice of a *Rawlsian* government. A Rawlsian government has a welfare function $W = \min(V^h, V^l)$. As the wages of the high skilled are always higher than those of the low skilled, $c_1 > c_0$ and so $V^h > V^l$ for all $x \in (0, 1)$. But then $\zeta = 0$ captures the preferences of a Rawlsian government with *leximin* preferences over the individuals' utilities.¹⁸ Thus, a Rawlsian government, maximizing the utility of the *least well-off*, and a government representing the average low skilled worker, set the growth maximizing tax rate ($x_t = \hat{x}$) and grant the maximum after-tax return on capital.

If $\zeta = 1$, the government acts in the interest of the average high skilled worker. That government's choice is equivalent to an *Anti-Rawlsian* government with *leximax* preferences such that $W = \max(V^h, V^l)$. Recall $\gamma_x = \alpha(1-\alpha)x^{\alpha-1} - \frac{x^{\frac{1}{\epsilon}-1}}{\epsilon}$ and use (4.19) to get the FOC for $\zeta = 1$

$$\begin{aligned} \frac{\epsilon\rho\alpha(1-\alpha)k_0x^{\alpha-2}}{\alpha k_0(1+x^{\alpha-1}) + \rho k_0} &= \epsilon\alpha(1-\alpha)x^{\alpha-1} - x^{\frac{1}{\epsilon}-1} \\ \frac{\epsilon\rho\alpha(1-\alpha)}{\alpha(1+x^{\alpha-1}) + \rho} &= \epsilon\alpha(1-\alpha)x - x^{\frac{1}{\epsilon}-1+2-\alpha} \\ (\epsilon\alpha(1-\alpha) - x^{\frac{1}{\epsilon}-\alpha})(\alpha(x+x^\alpha) + \rho x) &= \rho\epsilon\alpha(1-\alpha). \end{aligned}$$

¹⁸Notice that the Rawlsian objective is also satisfied if $V^h = V^l$, which is an objective that has been analyzed above. Recall that from now on the analysis is restricted to choices $x \in (0, 1)$ which makes setting $V^h = V^l$ impossible.

Notice $\hat{x}^{1-\alpha} = \epsilon\alpha(1 - \alpha)$ so that

$$(\hat{x}^{1-\alpha} - x^{1-\alpha})(\alpha(x + x^\alpha) + \rho x) = \rho\hat{x}^{1-\alpha}. \quad (4.20)$$

Hence, an increase in ζ makes a government choose a x lower than \hat{x} . The lowest x is chosen by a government representing the representative high skilled worker.

Call the x chosen by a $\zeta = 1$ government x_h . Then $x_h < x < \hat{x}$ for $0 < \zeta < 1$.

Proposition 4.4 *If $\zeta = 0$ and $x \in (0, 1)$, the government represents the average low skilled worker only and acts like a Rawlsian government. Both choose \hat{x} and maximize the after-tax return on capital and growth.*

If $\zeta = 1$, the government represents the average high skilled worker only and acts like an Anti-Rawlsian government. Both set $x_h < \hat{x}$ and have lower growth than the Rawlsian government.

Any other government with a welfare function $W^b = \zeta V^h + (1 - \zeta)V^l$ and $\zeta > 0$ sets $x < \hat{x}$. The optimal choices imply $\hat{\gamma}$ if $\zeta = 0$ and $\gamma(x_h)$ if $\zeta = 1$ where $\hat{\gamma} > \gamma(x_h)$. Furthermore, for $\zeta \in (0, 1)$, $\gamma(x_h, 1) < \gamma(x, \zeta) < \gamma(\hat{x}, 0)$ and $I(x_{\zeta=1}) > I(x_\zeta) > I(x_{\zeta=0})$.

Thus, inequality is lower and growth higher under a Rawlsian than under any other government with a welfare function as considered above. Notice that the proposition does not claim that the Rawlsian government chooses to eliminate all income inequality.

4.3.3 A Strictly Utilitarian Government

The strictly utilitarian government has the welfare function

$$W^u(V^h, V^l) = x V^h + (1 - x) V^l \quad (4.21)$$

where V^h, V^l again are the individual utility indices. This government is *strictly utilitarian* because it wants to maximize the sum of the individual utility indices. The utilitarian government's problem is *non-trivial* in the model, because maximization of (4.21) does not only involve maximizing the individual utility indices, but also choosing the weights attached to the indices. Using (4.17) one may express $W^u(\cdot)$ as

$$\begin{aligned} W^u(x) &= \frac{x \ln c_1}{\rho} + \frac{x \gamma}{\rho^2} + \frac{(1-x) \ln c_0}{\rho} + \frac{(1-x) \gamma}{\rho^2} \\ &= \frac{x \ln c_1}{\rho} + \frac{(1-x) \ln c_0}{\rho} + \frac{\gamma}{\rho^2}. \end{aligned}$$

The utilitarian government maximizes $W^u(\cdot)$ with respect to x . The derivative of W^u with respect to x is given by

$$v(x) \equiv \frac{\partial W^u}{\partial x} = \frac{1}{\rho} \left(\ln c_1 + \frac{\partial c_1}{\partial x} \frac{x}{c_1} - \ln c_0 + \frac{\partial c_0}{\partial x} \frac{1-x}{c_0} \right) + \frac{\gamma_x}{\rho^2}$$

where $v(x)$ denotes marginal welfare and $\gamma_x = \frac{\partial \gamma}{\partial x}$. As the consumption of the low skilled does not depend on x in steady state ($\frac{\partial c_0}{\partial x} = 0$), I simplify to obtain

$$v(x) = \frac{1}{\rho} \left(\ln \left(\frac{c_1}{c_0} \right) + \frac{\partial c_1}{\partial x} \frac{x}{c_1} \right) + \frac{\gamma_x}{\rho^2}. \quad (4.22)$$

For an optimum $v(x) = 0$ is required. Recall $c_1 = \alpha k_0(1 + x^{\alpha-1}) + \rho k_0$ so that

$$\Delta_1(x) \equiv \frac{\partial c_1}{\partial x} \frac{x}{c_1} = -\frac{\alpha(1-\alpha)x^{\alpha-1}}{\alpha(1+x^{\alpha-1}) + \rho} < 0.$$

Also for $\ln \left(\frac{c_1}{c_0} \right)$ one verifies that

$$\Delta_2(x) \equiv \ln \left(\frac{c_1}{c_0} \right) = \ln \left(\frac{\alpha(1+x^{\alpha-1}) + \rho}{\alpha + \rho} \right) > 0$$

since $\frac{\alpha(1+x^{\alpha-1})+\rho}{\alpha+\rho} > 1$. Thus, unless $\Delta_1(x) + \Delta_2(x) = 0$ one gets $x \neq \hat{x}$ in (4.22) which follows from the concavity of γ . I will show that $\Delta_2(x) + \Delta_1(x) > 0$ for all $x \in [0, 1]$. To this end let $\Delta_3(x) \equiv \alpha(1+x^{\alpha-1}) + \rho$ and $S(x) \equiv \Delta_2 + \Delta_1$ so that S is the sum of two functions. For showing that $S(x)$ is *strictly decreasing* in x for any $x \in [0, 1]$, I take the derivative of $S(x)$ with respect to x which is given by

$$\begin{aligned} \frac{\partial S}{\partial x} &= \frac{\partial \Delta_2}{\partial x} + \frac{\partial \Delta_1}{\partial x} = -\frac{\alpha(1-\alpha)x^{\alpha-2}}{\Delta_3} + \frac{\alpha(1-\alpha)^2x^{\alpha-2}}{\Delta_3} - \frac{\alpha^2(1-\alpha)^2x^{2\alpha-3}}{\Delta_3^2} \\ &= -\frac{\alpha^2(1-\alpha)^2x^{\alpha-2}}{\Delta_3} - \frac{\alpha^2(1-\alpha)^2x^{2\alpha-3}}{\Delta_3^2} \end{aligned}$$

and is negative for any non-negative x . Thus, $S(x)$ is strictly decreasing in x . Next, I wish to show that $\lim_{x \rightarrow \infty} S(x) = 0$ implying $\inf S(x) = 0, \forall x \in (0, \infty)$. So I need to show that $\lim_{x \rightarrow \infty} \Delta_2 + \lim_{x \rightarrow \infty} \Delta_1 = 0$. Notice that

$$\Delta_1 = -\frac{\alpha(1-\alpha)x^{\alpha-1}}{\alpha(1+x^{\alpha-1})+\rho} = -\frac{(1-\alpha)x^{\alpha-1}}{1+x^{\alpha-1}+\frac{\rho}{\alpha}} = -\frac{(1-\alpha)}{x^{1-\alpha}(1+\frac{\rho}{\alpha})+1}.$$

Then the claim is true since

$$\begin{aligned} \lim_{x \rightarrow \infty} \Delta_1 &= \lim_{x \rightarrow \infty} \left(-\frac{(1-\alpha)}{x^{1-\alpha}(1+\frac{\rho}{\alpha})+1} \right) = 0 \text{ and} \\ \lim_{x \rightarrow \infty} \Delta_2 &= \lim_{x \rightarrow \infty} \ln \left(\frac{\alpha(1+x^{\alpha-1})+\rho}{\alpha+\rho} \right) = 0. \end{aligned}$$

As $x \in [0, 1]$ one has $x < \infty$ and so $\inf S(x) > 0$ for all $x \in [0, 1]$ and so $S(x) > 0$. But then $v(x) > 0$ at $x = \hat{x}$ and by the concavity of γ one must have $\gamma_x < 0$ and so $x_u > \hat{x}$ in an optimum. (Subscript u denotes the optimal choice of the utilitarian government.)

Thus, marginal welfare $v(\hat{x}) > 0$ and so the optimal x chosen by a strictly utilitarian government must satisfy $x > \hat{x}$. Next, I want to show that the level

of welfare $W(x)$ is lower at $x \rightarrow 1$ than at \hat{x} . Let $\bar{x} = 1$. Then the difference in welfare levels is given by

$$W^u(\hat{x}) - W^u(1) = \frac{\hat{x} \ln \hat{c}_1 - \ln \check{c}_1}{\rho} + \frac{(1 - \hat{x}) \ln c_0 - (1 - \bar{x}) \ln c_0}{\rho} + \frac{\hat{\gamma} - \bar{\gamma}}{\rho^2}.$$

where c_0 is independent of x . Furthermore, using (4.12),

$$\begin{aligned} \hat{\gamma} - \bar{\gamma} &= (1 - \alpha) [1 + \hat{x}^\alpha] - \hat{x}^{\frac{1}{\alpha}} - \rho - [(1 - \alpha)2 - 1 - \rho] = \alpha + (1 - \alpha)\hat{x}^\alpha - \hat{x}^{\frac{1}{\alpha}} \\ &= \alpha + \hat{x}^\alpha [(1 - \alpha)(1 - \epsilon\alpha)] \equiv B > 0. \end{aligned}$$

Then the condition for the difference in welfare levels to be positive is

$$\frac{\hat{x} \ln \hat{c}_1 - \bar{x} \ln \check{c}_1}{\rho} + \frac{(\bar{x} - \hat{x}) \ln c_0}{\rho} + \frac{B}{\rho^2} > 0 \Leftrightarrow \left(\frac{\hat{c}_1}{c_0}\right)^d e^{\frac{B}{\rho^2}} > \frac{\check{c}_1}{c_0}$$

where $d = \frac{1}{\hat{x}}$ and $\bar{x} = 1$. Note that $\left(\frac{\hat{c}_1}{c_0}\right)^d > 1$ and $e^y = \left(1 + \frac{y}{1!} + \frac{y^2}{2!} + \frac{y^3}{3!} + \dots\right)$.

Then a sufficient condition for the inequality with $\bar{x} = 1$ to hold is

$$\begin{aligned} \left(1 + \frac{B}{\bar{x}\rho} + \dots\right) &> 1 + \frac{\alpha\bar{x}^{\alpha-1}}{\alpha + \rho} \\ C + B &> \frac{\rho\alpha}{\alpha + \rho} \\ C + \hat{x}^\alpha [(1 - \alpha)(1 - \epsilon\alpha)] &> \frac{\rho\alpha}{\alpha + \rho} - \alpha \end{aligned} \quad (4.23)$$

where C is a positive constant. Thus, the inequality holds because the RHS is negative. Hence, $W^u(\hat{x}) > W^u(1)$ implying that $v(1) < 0$, that is, marginal welfare is negative if x is close to one.¹⁹ But then the optimal solution must be such that $\hat{x} < x_u < 1$, because $S(x)$ is strictly decreasing for any $x > \hat{x}$ and γ is

¹⁹It is not difficult to verify that a similar result may be obtained for $\bar{x} > \hat{x}$. If $\gamma - \bar{\gamma} > \rho$ then $W^u(\hat{x}) > W^u(\bar{x})$ and $v(\bar{x}) < 0$ by reasoning as above.

concave in x . As $W^u(\hat{x}) > W^u(1)$ and all the derivatives exist, there must be one $x \in (\hat{x}, 1]$ where $v(x) = 0$. Then, the choice $\hat{x} < x_u < 1$ implies $I(1) < I_u < I(\hat{x})$ and $\gamma(1) < \gamma_u < \hat{\gamma}$.

Proposition 4.5 *A strictly utilitarian government chooses $\hat{x} < x_u < 1$ implying $I(1) < I_u < I(\hat{x})$ and $\gamma(1) < \gamma_u < \hat{\gamma}$.*

That is an interesting result and the intuition for it is not as straightforward as it seems. The strictly utilitarian government maximizes the individual utility indices and the weights, the groups contribute to overall welfare. More precisely, it trades off a higher individual high skilled worker's utility requiring a low x with its desire to maximize the number of high skilled people. On the other hand, it has to trade off higher welfare of each low skilled person which would imply choosing \hat{x} with its desire to minimize the number of low skilled people. In the optimum, it attaches more welfare weight on having high skilled people in the economy than choosing the growth maximizing number of high skilled persons. Thus, it chooses lower than maximum growth, but by this pushes down inequality in the PV of lifetime wages compared to a growth maximizing government.

4.4 Comparison of the Different Policies

In the previous section I have analyzed governments with different objectives and their optimal choices. In this section I compare these different choices under the realistic assumption that there is heterogeneity in skills. Propositions 4.1 to 4.5 imply that $x_h < x_l < x_u$ where x_u denotes the strictly utilitarian, x_h the high skilled labour and x_l the low skilled labour governments' optimal choice. The following proposition summarizes the results.

Proposition 4.6 *The optimal choices of the governments are such that*

1. $0 < x_h < x_l < x_u < 1$.

2. $\gamma_u, \gamma_h, \gamma_l > \gamma(1)$.

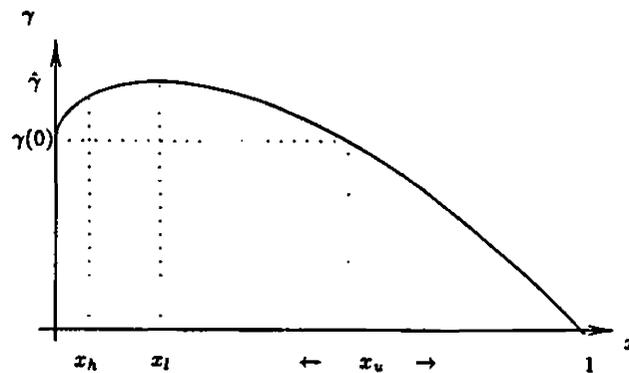
3. $\gamma_l = \hat{\gamma} > \gamma_u, \gamma_h$.

4. $\gamma_u \geq \gamma_h$.

5. $I_h > I_l > I_u$.

Figure 4.3 below visualizes the proposition's results.

Figure 4.3: The governments' optimal policies



The strictly egalitarian policy with *either* $x = 0$ *or* $x = 1$, is generally bad for growth compared to most other governments' choices. A government representing the average low skilled worker acts like a growth maximizing government and generates less inequality in the PV of lifetime wages than a government representing the average high skilled worker. The rationale for that is not difficult to see. For the low skilled workers an increase in x increases the growth rate and that is good for them. Due to the externality, high skilled people exert on production

in the model, the low skilled workers' wages do not depend on x in equilibrium. Thus, they choose maximum growth and the highest after-tax return on capital.

For the average high skilled labour representing government things are quite different. The positive externality of more high skills in production (higher x) has a negative impact on high skilled people's wages and that makes them choose a low x . On the other hand, a high x increases their capital income. The resulting trade-off is solved by not choosing the growth maximizing x so that the wage income component of their utility in steady state dominates.²⁰ Thus, it is socially desirable to have sufficient high skilled labour, but for the representative high skilled person it is privately bad, if too many of its kind are present.

The strictly utilitarian government maximizes the sum of the individual utility indices. It chooses *more* than the growth maximizing number of high skilled workers. The problem for this government is that it faces the non-trivial problem of solving simultaneously for the number of individuals of each type and maximizing each type's individual welfare. In the model it weighs the number of individuals more than the average utility of each type and that leads to a policy inducing a more equal distribution of the PV of lifetime wages. From the analysis it is not clear whether a utilitarian government has higher or lower growth than a government representing high skilled labour, but it definitely has less inequality in the PV of lifetime wage income.²¹

Interestingly, in the model the strictly *utilitarian* government chooses a policy that is *more egalitarian* in terms of the PV of lifetime wages as well as utility than a government representing low skilled labour. Thus, in the model the utilitarian

²⁰For a representative high skilled worker and absent any costs to education, the preferred x would actually be zero in this model.

²¹That follows since the implicit solutions with $x > 0$ for the Anti-Rawlsian and the utilitarian governments in (4.20), resp. $v(x) = 0$ in (4.22) are not easily solved and depend in a non-linear way on the parameters of the model. As an exact solution does not add significantly to the qualitative results, I leave this an open question.

policy is more egalitarian than a *Rawlsian* one. This provides an example that a Rawlsian welfare function does not always imply more egalitarianism than a utilitarian welfare function.²²

4.5 Conclusion

The experience of high growth economies suggests that there is a positive link from providing education to income equality and growth. The chapter presents a model that attempts to explain that stylized fact.

In the model the *composition* of human capital matters in the growth process. Assuming that human capital is 'lumpy' so that only those people who have received a degree can take high skilled jobs, it is shown that the public choice of human capital directly affects income inequality and economic growth.

In this chapter high skilled labour contributes more to effective labour in the production process than its unskilled counterpart. That is meant to reflect the fact that often the unskilled are not hired for high skilled tasks due to market imperfections or institutional restrictions. Hence, the number of people carrying high skills plays a crucial role in the model.

The government levies wealth taxes on all individuals and provides public education which produces human capital in the form of high skilled people. It is shown that the productivity of the education sector has a positive influence on growth and equality in the present value of lifetime wage earnings. In the market equilibrium an increase in the number of high skilled people lowers inequality in the present value of lifetime wages, raises growth up to a certain point and there

²²The textbook comparison of utilitarian and leximin welfare functions usually argues that the choice of a utilitarian leads to *more* and not less inequality. See, for instance, Mas-Colell, Whinston and Green (1995), p. 828.

exists a growth maximizing number of high skilled agents.

Introducing governments with different objectives reveals that strictly egalitarian policies are generally bad for growth.

A government representing the average *unskilled* worker acts like a *Rawlsian* government and chooses the *growth maximizing* number of high skilled people. Under both policies the after-tax return on capital is maximal.

The average *high skilled* worker's government acts *Anti-Rawlsian* and both choose less skilled people than a Rawlsian government. This policy makes the distribution of the present value of lifetime wages *more unequal* and growth *lower* than the policy chosen by the *Rawlsian*.

A strictly *utilitarian* government chooses more high skilled people than the *Rawlsian* government, because it simultaneously maximizes the utility of the average individual in each group and the weights the groups contribute to overall welfare. In the optimum the utilitarian government values the weight effect more. It is ambiguous how the strictly utilitarian policy compares to the policy preferred by the average *high skilled* worker in terms of growth. But in the model the utilitarian policy implies a more equitable income distribution than the *Rawlsian* one. That is theoretically interesting, because it is usually argued that *Rawlsian* choices lead to more egalitarian outcomes than *utilitarian* ones.

The main insight of the chapter, however, lies in the result that a government representing the average *low skilled* worker chooses maximum growth, the highest after-tax return on capital and an income distribution that is more equitable than the one chosen by a government representing the average *high skilled* worker. That stresses the importance of education in the growth process, its distributional implications and - in light on the results of chapter 3 - suggests consequences for maintaining international competitiveness. The model may provide a theoretical

explanation why some, highly competitive East Asian countries have empirically been found to exhibit low income inequality and high growth rates.

The results have to be interpreted with some caution, however. It would be desirable to know more about the exact link between government revenues channelled into education and the education output. The level of human capital that individuals carry may be important. Human capital acquisition may entail more than one degree for different levels of human capital. These and other questions are left for further research.

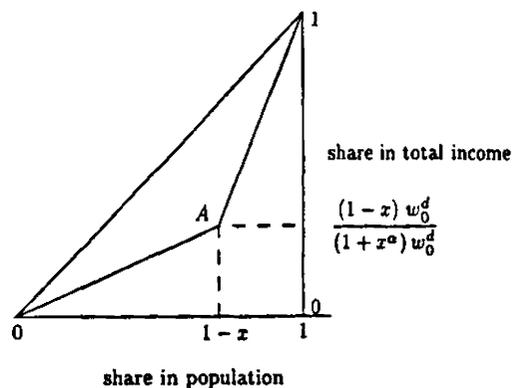
Appendix C

C.1 The relationship between $I(x)$ and some measures of income inequality

In this appendix and for convenience, the PV of lifetime wage income will simply be called (wage) income.

Lorenz Curve A Lorenz Curve (LC) relates the population to the income shares. Total wage income is $\mu^d N$. Furthermore, $L_0 = xN$, $L_1 = xN$ and mean income μ^d is increasing in x . The share of total income going to the low skilled is $\frac{w_0^d L_0}{\mu^d N} = \frac{(1-x)w_0^d}{\mu^d}$ so that the Lorenz curve looks like Figure 4.4 below.

Figure 4.4: Ordinary Lorenz Curve



The Lorenz Curve (LC) has a kink at the point A at which $(1 - x)$ percent of the population receive $\frac{1-x}{1+x^\alpha}$ percent of total wage income. On the margin an increase in x shifts A to the left by 1 unit *for a given income share*. On the other hand, a marginal increase in x *reduces* the income share by

$$\frac{(1 + x^\alpha) + \alpha x^{\alpha-1}(1 - x)}{(1 + x^\alpha)^2}$$

for a given population share. If such a change would move A up to any new position above the old Lorenz Curve (LC dominance), then inequality would unambiguously have been reduced. Thus, one must analyze whether the movement of A to the left is greater or less than the movement up or down. In the model A moves down. Thus, the condition amounts to

$$(LHS \downarrow) : \frac{(1 + x^\alpha) + \alpha x^{\alpha-1}(1 - x)}{(1 + x^\alpha)^2} < 1 \quad : (RHS \leftarrow)$$

If x is rather low ($x \rightarrow 0$), the inequality does not hold. Hence, in general the LCs cross no unambiguous ranking of the wage income distributions is possible according to the LC dominance criterion.

Gini Coefficient From the LC one may calculate the *Gini* coefficient as

$$G = 1 - 2 \left[\frac{(1-x)^2}{2(1+x^\alpha)} + \frac{x(1-x)}{1+x^\alpha} + \frac{x^2(1+x^{\alpha-1})}{2(1+x^\alpha)} \right] = \frac{x^\alpha(1-x)}{1+x^\alpha}$$

where the expression in square brackets represents the area under the LC. Then

$$\begin{aligned} sgn(G_x) &= [\alpha x^{\alpha-1}(1-x) - x^\alpha] (1+x^\alpha) - \alpha x^{\alpha-1} x^\alpha (1-x) \\ &= x^{\alpha-1} ([\alpha(1-x) - x] (1+x^\alpha) - \alpha x^\alpha (1-x)) \end{aligned}$$

For low x , $x \rightarrow 0$, an increase in x raises the Gini index, whereas for higher values of x a higher x reduces it. Hence, the Gini coefficient does not produce unambiguous rankings of the wage income distribution.

The Variance and the Coefficient of Variation The variance of personal wage income $V^d(x)$ is

$$\begin{aligned}
 V^d &= (w_1^d - \mu^d)^2 x + (w_0^d - \mu^d)^2 (1 - x) \\
 &= [w_0^d(1 + x^{\alpha-1}) - w_0^d(1 + x^\alpha)]^2 x + [w_0^d - w_0^d(1 + x^\alpha)]^2 (1 - x) \\
 &= w_0^{d2} \left[x^{2\alpha} \left(\frac{1-x}{x} \right)^2 x + (-x^\alpha)^2 (1-x) \right] \\
 &= w_0^{d2} x^{2\alpha-1} (1-x)
 \end{aligned}$$

which is decreasing in x if $\alpha < \frac{1}{2}$. The coefficient of variation is defined by

$C^d \equiv \frac{\sqrt{V^d}}{\mu^d}$ and amounts to

$$C^d(x) = \frac{w_0^d x^\alpha \left(\frac{1}{x} - 1 \right)^{\frac{1}{2}}}{w_0^d (1 + x^\alpha)}$$

The sign of $\frac{dC}{dx}$ depends on

$$\begin{aligned}
 &\left[\alpha x^{\alpha-1} \left(\frac{1-x}{x} \right)^{\frac{1}{2}} - \frac{x^\alpha}{2} \left(\frac{1-x}{x} \right)^{-\frac{1}{2}} x^{-2} \right] (1 + x^\alpha) - \alpha x^{\alpha-1} x^\alpha \left(\frac{1-x}{x} \right)^{\frac{1}{2}} \\
 &= \alpha x^{\alpha-1} \left(\frac{1-x}{x} \right)^{\frac{1}{2}} \left[\left(1 - \left(\frac{1-x}{x} \right)^{-1} \left(\frac{1}{2\alpha x} \right) \right) (1 + x^\alpha) - x^\alpha \right]
 \end{aligned}$$

which is definitely negative for all $x \in [0, 1]$ if $\alpha \leq \frac{1}{2}$.

C.2 Welfare Measures

The workers' welfare integral is given by $U_t^j = \int_0^t \ln c_{j,t} e^{-\rho t} dt$ where $j = 0, 1$.

Let $t \rightarrow \infty$ and use integration by parts. Define $v_2 = \ln c_{j,t}$, $dv_1 = e^{-\rho t}$. Then $dv_2 = \dot{c}_j/c_j = \gamma = \text{constant}$, and $v_1 = -\frac{1}{\rho} e^{-\rho t}$. That implies

$$\begin{aligned} \int_0^\infty \ln c_{j,t} e^{-\rho t} dt &= -\frac{1}{\rho} [\ln c_{j,t} e^{-\rho t}]_0^\infty + \frac{1}{\rho} \int_0^\infty \gamma e^{-\rho t} dt \\ &= \frac{\ln c_j(0)}{\rho} - \frac{1}{\rho^2} \Big|_0^\infty \gamma e^{-\rho t}, \end{aligned} \quad (\text{C.2.1})$$

where $j = 0, 1$. Evaluation of the expression at the particular limits establishes V^h in (4.16) and V^l in (4.17).

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